Flux motion in anisotropic type-II superconductors near H_{c2}

Zhidong Hao and Chia-Ren Hu*
Texas Center for Superconductivity, University of Houston, Houston, Texas 77204
(Received 28 July 1993)

Flux motion in anisotropic type-II superconductors is studied in the framework of the time-dependent Ginzburg-Landau theory. Expressions for the flux-flow resistivity tensor (including all the longitudinal and Hall elements) are obtained for the case that the applied magnetic field **H** is parallel to one of the principal axes of the sample and H is near the upper critical field H_{c2}. A simple method is proposed for obtaining the anisotropy ratios from the **H** dependences of the longitudinal resistivities.

It is generally believed that the energy dissipation in the flux-flow state of a type-II superconductor is due to (i) Joule heating of normal excitations,[1] and (ii) relaxation of the order parameter.[2] The time-dependent Ginzburg-Landau (TDGL) theory[3–7] accounts for both of these two dissipation mechanisms. Many works have been carried out using the TDGL theory to study the transport properties and the dynamic structures of vortices.[3–7] With a slight generalization by allowing the order parameter relaxation constant to be complex, the TDGL theory can also be used to study the Hall effect in the flux-flow state.[8–13] The present effort is an extension of the previous works[3–13] to the case of anisotropic superconductors. In this paper we restrict ourself to the simplest case that the applied magnetic field **H** is parallel to one of the principal axes of the sample and H is near the upper critical field H_{c2}.

In a simple version of the TDGL theory for an anisotropic superconductor, the equation of motion for the order parameter ψ is

$$-\gamma(\partial_t - i\phi)\psi = -\psi(1-|\psi|^2) + \mu_{ij}\Pi_i\Pi_j\psi, \tag{1}$$

where $\mathbf{\Pi} = (i\kappa)^{-1}\nabla + \mathbf{a}$; the electric current density is

$$\mathbf{J} = \mathbf{J}^{(n)} + \mathbf{J}^{(s)}, \tag{2}$$

where the normal current density $\mathbf{J}^{(n)}$ and supercurrent density $\mathbf{J}^{(s)}$ obey

$$J_i^{(n)} = \sigma_{ij}^{(n)} E_j = \sigma_{ij}^{(n)}\left(-\frac{1}{\kappa}\partial_j\phi - \partial_t a_j\right) \tag{3}$$

and

$$J_i^{(s)} = -\mu_{ij}\mathrm{Re}\,(\psi^*\Pi_j\psi)\,. \tag{4}$$

Here $\gamma = \gamma_1 + i\gamma_2$ ($\gamma_1 > 0$) is the complex order parameter relaxation constant. $\partial_t = \partial/\partial t$ and $\partial_i = \partial/\partial x_i$ ($i = 1,2,3$). ϕ and $\mathbf{a}$ are the scalar[14] and vector potentials, respectively. μ_{ij} is the inverse of the normalized effective mass tensor $m_{ij} = M_{ij}/\bar{M}$ with M_{ij} the effective mass tensor[15–18] and $\bar{M} \equiv \det(M_{ij})^{1/3}$. [Note that $\mu_{ij}m_{jk} = \delta_{ik}$ and $\det(\mu_{ij}) = \det(m_{ij}) = \mu_1\mu_2\mu_3 = m_1m_2m_3 = 1$, where $\mu_i = 1/m_i$ and m_i ($i = 1,2,3$) are the principal values of μ_{ij} and m_{ij}, respectively.] $\sigma_{ij}^{(n)}$ is the normal state conductivity tensor. **E** is the electric field. The convention of summing over repeated indices is employed. We use the usual dimensionless units[19] (with a slight generalization[11]), which corresponds to measuring the magnitude of the order parameter in units of $(|\alpha|/\beta)^{1/2}$ (where α and β are the Ginzburg-Landau coefficients[19]), length in units of the mean penetration depth $\lambda = (\bar{M}c^2\beta/16\pi e^2|\alpha|)^{1/2}$, time in units of $\hbar/|\alpha|$, magnetic field in units of $\sqrt{2}H_c$ (H_c is the thermodynamic critical field), electric field in units of $|\alpha|\kappa/2e\lambda$, vector potential in units of $\sqrt{2}H_c\lambda$, scalar potential in units of $|\alpha|/2e$, electric current density in units of $\sqrt{2}H_c c/4\pi\lambda$, and conductivity in units of $\bar{M}c^2/h\kappa^2$. The mean coherence length ξ and the Ginzburg-Landau parameter κ are defined in terms of the mean mass $\bar{M}$ by the usual relations[19] $\xi = \phi_0/2\pi\sqrt{2}H_c\lambda$ ($\phi_0 = hc/2e$ is the flux quantum) and $\kappa = \lambda/\xi$.

The flux-flow conductivity tensor σ_{ij} is defined by the relation

$$J_i^T = \sigma_{ij}\langle E_j\rangle, \tag{5}$$

where $\mathbf{J}^T = \langle\mathbf{J}\rangle$ is the transport current density and the angular brackets indicate spatial average.

The component of σ_{ij} due to normal current is easily obtained as follows. Assuming a uniform translation with the velocity **v**, we have $\partial_t\mathbf{a} = -\mathbf{v}\cdot\nabla\mathbf{a}$, where **a** is the static solution. The electric field can then be expressed as

$$\mathbf{E} = -\mathbf{v}\times\mathbf{b} + \nabla\left(-\frac{1}{\kappa}\phi + \mathbf{v}\cdot\mathbf{a}\right), \tag{6}$$

where $\mathbf{b} = \nabla\times\mathbf{a}$ is the local magnetic flux density. The second term on the right-hand side of Eq. (6) contributes to the local electric field,[5,13] but it does not contribute to the spatial average, since the integration can be converted to a vanishing surface term. Therefore, we have

$$\langle\mathbf{E}\rangle = -\mathbf{v}\times\mathbf{B}, \tag{7}$$

where $\mathbf{B} \equiv \langle\mathbf{b}\rangle$, and

$$\langle J_i^{(n)}\rangle = \sigma_{ij}^{(n)}\langle E_j\rangle. \tag{8}$$

To compute $\mathbf{J}^{(s)}$, it is necessary to solve Eq. (1) for the moving order parameter. In the system of coordinates whose axes coincide with the principal axes, the tensor

μ_{ij} is diagonalized, i.e., $\mu_{ij} = \mu_i\delta_{ij} = (1/m_i)\delta_{ij}$. To the lowest order in $|\psi|^2$, Eq. (1) becomes

$$\gamma(\mathbf{v}\cdot\boldsymbol{\nabla} + i\kappa\mathbf{v}\cdot\mathbf{a})\psi = -\psi + \mu_i\Pi_i^2\psi, \tag{9}$$

where a uniform translation ($\partial_t = -\mathbf{v}\cdot\boldsymbol{\nabla}$) is assumed and the potentials $\phi = \kappa\mathbf{v}\cdot\mathbf{a}$ and $\mathbf{a}$ correspond to uniform fields $\langle\mathbf{E}\rangle$ and $\mathbf{B}$ [since the amplitudes of the spatial variations of $\mathbf{E}$ and $\mathbf{b}$ are of $O(|\psi|^2)$].

It is easy to show that Eq. (9) can be converted to an isotropic form by a simple transformation of variables. For $\mathbf{B}\|\mathbf{e}_3$ [here $\mathbf{e}_i$ $(i = 1,2,3)$ are the unit vectors along the principal axes], the transformation reads

$$x_i = \sqrt{\mu_3\mu_i}\,\tilde{x}_i, \tag{10}$$

$$\partial_i = \tilde{\partial}_i/\sqrt{\mu_3\mu_i}\,, \tag{11}$$

$$a_i = \tilde{a}_i/\sqrt{\mu_i}\,, \tag{12}$$

$$B_i = \tilde{B}_i \quad (B_i = B\delta_{i3}), \tag{13}$$

$$v_i = \sqrt{\mu_3\mu_i}\,\tilde{v}_i, \tag{14}$$

$$\kappa = \tilde{\kappa}/\sqrt{\mu_3}\,. \tag{15}$$

This transformation [except the addition of Eq. (14)] is the same as that of Ref. 16 (when $\mathbf{B}\|\mathbf{e}_3$), which was proposed for the study of time-independent problems. Note that the above transformation preserves the relations $\boldsymbol{\nabla}\cdot\mathbf{B} = 0$ and $\mathbf{B} = \boldsymbol{\nabla}\times\mathbf{a}$.

The transformed form of Eq. (9) is exactly the same as its isotropic counterpart:

$$\gamma(\tilde{\mathbf{v}}\cdot\tilde{\boldsymbol{\nabla}} + i\tilde{\kappa}\tilde{\mathbf{v}}\cdot\tilde{\mathbf{a}})\psi = -\psi + \tilde{\Pi}^2\psi, \tag{16}$$

where $\tilde{\boldsymbol{\Pi}} = (i\tilde{\kappa})^{-1}\tilde{\boldsymbol{\nabla}} + \tilde{\mathbf{a}}$, and $\tilde{\mathbf{a}} = \mathbf{e}_2B\tilde{x}_1$. The solution of Eq. (16) is obtained immediately by slightly generalizing that of Ref. 5 to allow an imaginary part of γ:

$$\psi = \sum_q C_q \exp\left\{-\frac{\tilde{\kappa}B}{2}\left[\tilde{x}_1(t) + \frac{q}{\tilde{\kappa}B} + \frac{i\tilde{\kappa}(\gamma_2\tilde{v}_1 - \gamma_1\tilde{v}_2)}{2B}\right]^2 + i\left[q - \frac{\tilde{\kappa}^2(\gamma_1\tilde{v}_1 + \gamma_2\tilde{v}_2)}{2}\right]\tilde{x}_2(t)\right\}, \tag{17}$$

where $\tilde{\mathbf{x}}(t) = \tilde{\mathbf{x}} - \tilde{\mathbf{v}}t$. From this result we first obtain the "isotropic supercurrent density" $\tilde{\mathbf{J}}^{(s)} = -\mathrm{Re}(\psi^*\tilde{\boldsymbol{\Pi}}\psi)$:

$$\tilde{\mathbf{J}}^{(s)} = \tilde{\mathbf{J}}_0^{(s)} + \frac{\tilde{\kappa}}{2}[-\gamma_1\tilde{\mathbf{v}}\times\mathbf{e}_3 + \gamma_2\tilde{\mathbf{v}}]\,|\psi|^2, \tag{18}$$

where $\tilde{\mathbf{J}}_0^{(s)}$ is the "isotropic equilibrium vortex current density," of which the spatial average is zero. The supercurrent density $\mathbf{J}^{(s)}$ is related to the quantity $\tilde{\mathbf{J}}^{(s)}$ by

$$J_i^{(s)} = \sqrt{\mu_i}\,\tilde{J}_i^{(s)}. \tag{19}$$

A straightforward calculation then gives

$$\begin{pmatrix}\langle J_1^{(s)}\rangle \\ \langle J_2^{(s)}\rangle\end{pmatrix} = \frac{\tilde{\kappa}\langle|\psi|^2\rangle}{2B}\begin{pmatrix}\gamma_1/m_1 & \gamma_2\sqrt{m_3} \\ -\gamma_2\sqrt{m_3} & \gamma_1/m_2\end{pmatrix}\begin{pmatrix}\langle E_1\rangle \\ \langle E_2\rangle\end{pmatrix}, \tag{20}$$

where we have used Eqs. (7) and (14) to express $\tilde{\mathbf{v}}$ in terms of $\mu_i = 1/m_i$ and $\langle\mathbf{E}\rangle$, and the quantity $\tilde{\kappa}\langle|\psi|^2\rangle/2B$ is the same as its isotropic counterpart[19] except κ is replaced by $\tilde{\kappa}$:

$$\frac{\tilde{\kappa}\langle|\psi|^2\rangle}{2B} = \frac{\tilde{\kappa}^2}{(2\tilde{\kappa}^2 - 1)\beta_A + 1}\left(1 - \frac{B}{H_{c2\|3}}\right) \tag{21}$$

$$\simeq \frac{1}{2\beta_A}\left(1 - \frac{B}{H_{c2\|3}}\right) \quad (\text{ for } \tilde{\kappa} \gg 1), \tag{22}$$

where the Abrikosov constant $\beta_A = 1.16$ and $H_{c2\|i}$ is the upper critical field along the x_i axis.

Combining Eqs. (8) and (20), we obtain the flux-flow conductivity tensor

$$\sigma_{ij} = \begin{pmatrix}\sigma_{11}^{(n)} + \frac{\gamma_1\tilde{\kappa}\langle|\psi|^2\rangle}{2Bm_1} & \sigma_{12}^{(n)} + \frac{\gamma_2\tilde{\kappa}\langle|\psi|^2\rangle\sqrt{m_3}}{2B} \\ \sigma_{21}^{(n)} - \frac{\gamma_2\tilde{\kappa}\langle|\psi|^2\rangle\sqrt{m_3}}{2B} & \sigma_{22}^{(n)} + \frac{\gamma_1\tilde{\kappa}\langle|\psi|^2\rangle}{2Bm_2}\end{pmatrix}. \tag{23}$$

Note that the Onsager relation alone gives $\sigma_{21}(\mathbf{B}) = \sigma_{12}(-\mathbf{B})$; but most anisotropic superconductors have additional symmetries when m_{ij} is diagonal in the chosen coordinate system and $\mathbf{B}$ is along a principal direction (i.e., twofold rotation or inversion about the x_1 or x_2 axis). We then have $\sigma_{12}(-\mathbf{B}) = -\sigma_{12}(\mathbf{B})$, and therefore $\sigma_{21}(\mathbf{B}) = -\sigma_{12}(\mathbf{B})$, which we will assume in the following analysis. Since $\gamma_1 > 0$, the two terms in each of the longitudinal conductivities (σ_{11} and σ_{22}) add constructively. However, depending on the properties of the material, γ_2 may be positive or negative.[8–10] Therefore, the two terms in the Hall conductivity σ_{12} (or $\sigma_{21} = -\sigma_{12}$) add constructively if $\sigma_{12}^{(n)}$ and γ_2 have the same sign;[20] the opposite is the case otherwise.

The inverse of σ_{ij} is the flux-flow resistivity tensor

$$\rho_{ij} = \frac{1}{\det(\sigma_{ij})}\begin{pmatrix}\sigma_{22} & -\sigma_{12} \\ \sigma_{12} & \sigma_{11}\end{pmatrix} \tag{24}$$

or, to the lowest order in $\langle|\psi|^2\rangle$,

$$\frac{\rho_{11}}{\rho_{11}^{(n)}} = 1 - \frac{\tilde{\kappa}\langle|\psi|^2\rangle}{2B}\frac{\xi_1^2}{\zeta_1^2\left(1 + (\sigma_{12}^{(n)})^2/\sigma_{11}^{(n)}\sigma_{22}^{(n)}\right)}\left[1 + \left(\frac{\gamma_2}{\gamma_1}\right)^2 - \left(\frac{\gamma_2}{\gamma_1} - \sqrt{\frac{m_1}{m_2}}\frac{\sigma_{12}^{(n)}}{\sigma_{22}^{(n)}}\right)^2\right], \tag{25}$$

$$\frac{\rho_{22}}{\rho_{22}^{(n)}} = 1 - \frac{\tilde{\kappa}\langle|\psi|^2\rangle}{2B}\frac{\xi_2^2}{\zeta_2^2\left(1 + (\sigma_{12}^{(n)})^2/\sigma_{11}^{(n)}\sigma_{22}^{(n)}\right)}\left[1 + \left(\frac{\gamma_2}{\gamma_1}\right)^2 - \left(\frac{\gamma_2}{\gamma_1} - \sqrt{\frac{m_2}{m_1}}\frac{\sigma_{12}^{(n)}}{\sigma_{11}^{(n)}}\right)^2\right], \tag{26}$$

$$\frac{\rho_{12}}{\rho_{12}^{(n)}}=1-\frac{\tilde{\kappa}\langle|\psi|^2\rangle}{2B}\frac{1}{1+(\sigma_{12}^{(n)})^2/\sigma_{11}^{(n)}\sigma_{22}^{(n)}}\left[\frac{\xi_1^2}{\zeta_1^2}+\frac{\xi_2^2}{\zeta_2^2}-\frac{\gamma_2}{\gamma_1}\frac{\xi_1\xi_2}{\zeta_1\zeta_2}\frac{1-(\sigma_{12}^{(n)})^2/\sigma_{11}^{(n)}\sigma_{22}^{(n)}}{\sigma_{12}^{(n)}/\sqrt{\sigma_{11}^{(n)}\sigma_{22}^{(n)}}}\right], \tag{27}$$

and $\rho_{21}=-\rho_{12}$, where $\xi_i=\xi/\sqrt{m_i}$ and $\zeta_i=\sqrt{\sigma_{ii}^{(n)}/\gamma_1\kappa^2}$ ($\zeta_i=\sqrt{\lambda^2\sigma_{ii}^{(n)}h/\gamma_1\bar{M}c^2}$ in conventional units) are, respectively, the coherence length and the electric field screening length[21] along the x_i axis. Note that, in the above expressions [Eqs. (25)–(27)], the terms of $O(\gamma_2^2,\gamma_2\sigma_{12}^{(n)},(\sigma_{12}^{(n)})^2)$ are usually small and can be neglected; then, these expressions can be further simplified [for example, see Eq. (37) below].

The Hall angle is readily obtained from σ_{ij} or ρ_{ij}. If $\mathbf{J}^T\|\mathbf{e}_1$, we have

$$\tan\theta_2=\frac{\langle E_2\rangle}{\langle E_1\rangle}=\frac{\sigma_{12}}{\sigma_{22}}=\frac{\rho_{21}}{\rho_{11}} \tag{28}$$

$$=\tan\theta_2^{(n)}\left\{1-\frac{\tilde{\kappa}\langle|\psi|^2\rangle\xi_2^2}{2B\zeta_2^2}\left[1-\frac{\gamma_2}{\gamma_1}\sqrt{\frac{m_2}{m_1}}\left(\tan\theta_2^{(n)}\right)^{-1}\right]\right\}, \tag{29}$$

where the subscript i of the angles θ_i and $\theta_i^{(n)}$ indicates that the Hall component of $\langle\mathbf{E}\rangle$ is parallel to the x_i axis. (The Hall angle is measured counter clockwise relative to the orientation of $\mathbf{J}^T$.)

If $\mathbf{J}^T\|\mathbf{e}_2$, we have

$$\tan\theta_1=-\frac{\langle E_1\rangle}{\langle E_2\rangle}=\frac{\sigma_{12}}{\sigma_{11}}=\frac{\rho_{21}}{\rho_{22}} \tag{30}$$

$$=\tan\theta_1^{(n)}\left\{1-\frac{\tilde{\kappa}\langle|\psi|^2\rangle\xi_1^2}{2B\zeta_1^2}\left[1-\frac{\gamma_2}{\gamma_1}\sqrt{\frac{m_1}{m_2}}\left(\tan\theta_1^{(n)}\right)^{-1}\right]\right\}. \tag{31}$$

The viscosity of the flux motion in anisotropic superconductors is a tensor quantity: η_{ij}. The relation between η_{ij} and σ_{ij} is found as follows. The viscous force (per unit length) $\mathbf{f}^V$ acting on a single vortex is given by

$$f_i^V=-\eta_{ij}v_j. \tag{32}$$

Since we are using the coordinates whose axes coincide with the principal axes, the tensor η_{ij} is diagonalized: $\eta_{ij}=\eta_i\delta_{ij}$, where $i=1,2$ for $\mathbf{B}\|\mathbf{e}_3$. The density of the dissipation rate is (in conventional units)

$$W=-n\mathbf{f}^V\cdot\mathbf{v}=\frac{B}{\phi_0}\left(\eta_1v_1^2+\eta_2v_2^2\right), \tag{33}$$

where $n=B/\phi_0$ is the density of vortices. This should be equivalent to

$$W=\mathbf{J}^T\cdot\langle\mathbf{E}\rangle=\sigma_{11}\langle E_1\rangle^2+\sigma_{22}\langle E_2\rangle^2. \tag{34}$$

From Eqs. (33) and (34), and with the help of Eq. (7), we obtain

$$\eta_1=(\phi_0B/c^2)\sigma_{22}, \tag{35}$$

$$\eta_2=(\phi_0B/c^2)\sigma_{11}. \tag{36}$$

So far, $\mathbf{B}\|\mathbf{e}_3$ is assumed. If $\mathbf{B}$ is aligned parallel to $\mathbf{e}_1$ or $\mathbf{e}_2$, the corresponding results can be obtained by cyclic permutation: $1\to2\to3\to1$. Further consideration for the case that $\mathbf{B}$ is oriented arbitrarily with respect to the principal axes will be given elsewhere.[22]

We now point out a way for obtaining the anisotropy ratios, $H_{c2\|1}:H_{c2\|2}:H_{c2\|3}=m_1^{-1/2}:m_2^{-1/2}:m_3^{-1/2}$ ($H_{c2\|i}=\tilde{\kappa}=\kappa/\sqrt{m_i}$), from the $\mathbf{B}$ dependences[23] of the longitudinal flux-flow resistivities. We consider a high-temperature superconductor [of which $\kappa\sim O(10^2)\gg1$]. The anisotropy ratio, for example, between the a and c axis, $\Gamma_{ac}=H_{c2\|a}/H_{c2\|c}=\sqrt{m_c/m_a}$, can be obtained from the $\mathbf{B}$ dependence of the longitudinal resistivity along the b axis (we allow the possibility that m_a and m_b may be different). Note that [in Eqs. (25) and (26)] the ratios $\sigma_{ab}^{(n)}/\sigma_{bb}^{(n)}$, $\sigma_{bc}^{(n)}/\sigma_{cc}^{(n)}$, etc., are usually small [$\sim O(10^{-3})$],[24–26] and so is the ratio γ_2/γ_1 [$\sim O(k_BT_c/\varepsilon_F)$, where T_c is the critical temperature and ε_F is the Fermi energy].[8–10] Therefore, we can neglect the $\sigma_{ij}^{(n)}$ ($i\neq j$) and γ_2 terms (although they play important roles for the Hall effect), and obtain the longitudinal resistivity for $\mathbf{J}^T\|\hat{b}$,

$$\frac{\rho_{bb}}{\rho_{bb}^{(n)}}=1-\frac{\xi_b^2(1-h)}{2\beta_A\zeta_b^2}, \tag{37}$$

where the reduced field

$$h=B/H_{c2}(\theta); \tag{38}$$

θ is the angle between the c axis and $\mathbf{B}$ in the ac plane, i.e., $H_{c2}(0)=H_{c2\|c}$ and $H_{c2}(\pi/2)=H_{c2\|a}$. Note that Eq. (37) [with h given by Eq. (38)] is derived in the present paper only for $\mathbf{B}$ parallel to a principal axis (i.e., $\theta=0$ or $\pi/2$); a further analysis[22] shows that it is also valid for $0<\theta<\pi/2$.

The weak field dependence of $\rho_{bb}^{(n)}$ is usually negligible; then, Eq. (37) implies the relation

$$\rho_{bb}(B;\mathbf{B}\|\hat{c})=\rho_{bb}(\Gamma_{ac}B;\mathbf{B}\|\hat{a}), \tag{39}$$

which means, for example, that ρ_{bb} for $B=1$ T and $\mathbf{B}\|\hat{c}$ is the same as that for $B=\Gamma_{ac}$ T and $\mathbf{B}\|\hat{a}$. This scaling relation can be used to obtain Γ_{ac}. In particular, the ratio between the initial ($B\to H_{c2}$) slopes measures directly the anisotropy ratio, i.e.,

$$\frac{[\partial\rho_{bb}(\mathbf{B}\|\hat{c})/\partial B]_{H_{c2\|c}}}{[\partial\rho_{bb}(\mathbf{B}\|\hat{a})/\partial B]_{H_{c2\|a}}}=\frac{H_{c2\|a}}{H_{c2\|c}}=\Gamma_{ac}. \tag{40}$$

By the same way, one finds similar scaling relations for

$\rho_{aa}(\mathbf{B})$ and $\rho_{cc}(\mathbf{B})$, which can be used to obtain Γ_{bc} and Γ_{ab}, respectively. The same scaling relation as Eq. (39) for the longitudinal flux-flow resistivities has been realized in Ref. 27, based mainly on the analysis of various experimental data; the present work provides a theoretical justification for the empirical result of Ref. 27 within the mean-field Ginzburg-Landau regime.

In summary, we have considered the flux motion in anisotropic type-II superconductors near H_{c2} by using the time-dependent Ginzburg-Landau theory. We have obtained expressions for the flux-flow resistivity tensor (including all the longitudinal and Hall elements) for the case that the vortices are aligned parallel to one of the principal axes. We have proposed a simple method for obtaining the anisotropy ratios from the field dependences of the longitudinal flux-flow resistivities.

This work was supported by the Texas Center for Superconductivity at the University of Houston under the prime grant from DARPA and the State of Texas and a grant from the Robert A. Welch Foundation. We thank C. S. Ting for useful discussions.

---

*Permanent address: Department of Physics, Texas A&M University, College Station, Texas 77843.

[1] J. Bardeen and M. J. Stephen, Phys. Rev. **140A**, 197 (1965).

[2] M. Tinkham, Phys. Rev. Lett. **13**, 803 (1964).

[3] A. Schmid, Phys. Kondens. Mater. **5**, 302 (1966).

[4] C. Caroli and K. Maki, Phys. Rev. **164**, 591 (1967).

[5] R. S. Thompson and C.-R. Hu, Phys. Rev. Lett. **27**, 1352 (1971).

[6] C.-R. Hu and R. S. Thompson, Phys. Rev. B **6**, 110 (1972).

[7] L. P. Gor'kov and N. B. Kopnin, Usp. Fiz. Nauk **116**, 413 (1975) [Sov. Phys. Usp. **18**, 496 (1975)].

[8] K. Maki, Phys. Rev. Lett. **23**, 1223 (1969); Prog. Theor. Phys. **41**, 902 (1969).

[9] H. Fukuyama, H. Ebisawa, and T. Tsuzuki, Prog. Theor. Phys. **46**, 1028 (1971); H. Ebisawa and H. Fukuyama, *ibid.* **46**, 1042 (1971).

[10] H. Ebisawa, J. Low Temp. Phys. **9**, 11 (1972).

[11] A. T. Dorsey, Phys. Rev. B **46**, 8376 (1992).

[12] N. B. Kopnin, B. I. Ivlev, and V. A. Kalatsky, J. Low Temp. Phys. **90**, 1 (1993).

[13] R. J. Troy and A. T. Dorsey, Phys. Rev. B **47**, 2715 (1993).

[14] In Eqs. (1) and (3), in the places of ϕ should be the quantity $\tilde{\phi} = -\tilde{\mu}/e$, where $\tilde{\mu}$ is the electrochemical potential. The relation between $\tilde{\phi}$ and the scalar potential ϕ is $\rho = (\tilde{\phi} - \phi)/4\pi\lambda_{\mathrm{TF}}^2$, where ρ is charge density and λ_{TF} is the Thomas-Fermi screening length. Since the difference between $\tilde{\phi}$ and ϕ is generally small (Refs. 3–7), we ignore this difference in this paper.

[15] D. R. Tilley, Proc. Phys. Soc. London **85**, 1177 (1965).

[16] R. A. Klemm and J. R. Clem, Phys. Rev. B **21**, 1868 (1980).

[17] V. G. Kogan and J. R. Clem, Phys. Rev. B **24**, 2497 (1981).

[18] Z. Hao and J. R. Clem, Phys. Rev. B **43**, 7622 (1991).

[19] A. L. Fetter and P. C. Hohenberg, in *Superconductivity*, edited by R. D. Parks (Marcel Dekker, New York, 1969).

[20] Our result for σ_{ij} reduces to that of Ref. 13 if $m_1 = m_2 = m_3 = 1$, except a sign difference in the expressions for the Hall conductivities. The origin of this difference can be seen by comparing our Eq. (1) with Eq. (2.1) of Ref. 13: In our Eq. (1) the electric charge $q = -2e < 0$, while in Eq. (2.1) of Ref. 13 $q = 2e > 0$.

[21] This is a generalization of the definition for the characteristic screening length introduced in Refs. 5 and 6 to the anisotropic case. The imaginary part of Eq. (1) multiplied by ψ^* and the equation of continuity $\nabla\cdot\mathbf{J} = 0$ [the term $\partial_t\rho$, which is of $O(v^2)$, is discarded] give $-\mathrm{Im}[\gamma\psi^*(\partial_t - i\phi)\psi] = -\kappa^{-1}\nabla\cdot\mathbf{J}^{(s)} = \kappa^{-1}\nabla\cdot\mathbf{J}^{(n)}$. Substituting $\mathbf{J}^{(n)}$ by using Eq. (3), one obtains an equation for the scalar potential ϕ; this equation introduces the lengths ζ_i ($i = 1, 2$ for $\mathbf{B}\|\mathbf{e}_3$) characterizing the spatial variations of ϕ along these principal axes. For reference, $\zeta^2 = \xi^2/12$ for isotropic superconductors with high concentrations of paramagnetic impurities; L. P. Gor'kov and G. M. Eliashberg, Zh. Eksp. Teor. Fiz. **54**, 612 (1968) [Sov. Phys. JETP **27**, 328 (1968)].

[22] Z. Hao and C.-R. Hu (unpublished).

[23] For high-κ materials, the magnitude of the magnetization ($-4\pi\mathbf{M} = \mathbf{H} - \mathbf{B}$) is small compared with both B and H when $H \gg H_{c1}$ (the lower critical field); therefore, we can ignore the difference between $\mathbf{H}$ and $\mathbf{B}$.

[24] Y. Iye, S. Nakamura, and T. Tamegai, Physica C **159**, 616 (1989).

[25] T. R. Chien, T. W. Jing, N. P. Ong, and Z. Z. Wang, Phys. Rev. Lett. **66**, 3075 (1991); J. M. Harris, N. P. Ong, and Y. F. Yan, *ibid.* **71**, 1455 (1993).

[26] S. J. Hagen *et al.*, Phys. Rev. B **47**, 1064 (1993).

[27] Z. Hao and J. R. Clem, Phys. Rev. B **46**, 5853 (1992).