Exact near-onset analysis of the spin-density-wave instability in ferromagnetic superconductors: The linearly polarized state

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(Received 29 August 1983)

Using an approach similar to Abvikosov's theory of the vortex state near H_{c2}, we have performed an exact, near-onset analysis of a spin-density-wave instability leading to the "linearly polarized state" of Greenside *et al.* in ferromagnetic superconductors. The approach is based on a generalized Ginzburg-Landau theory for such materials, as formulated by Blount and Varma. Two models have been considered. In the (α,β) model, where the bulk magnetic energy is taken to be $\frac{1}{2}\alpha_m M^2+\frac{1}{4}\beta_m M^4$, we find the transition to be second order, and obtain explicit formulas for various physical quantities to leading order in the deviation from onset. We have also rigorously analyzed the most favored spatial structure just below onset, among all possibilities allowed by the instability, and have concluded that a plane-wave-like structure is favored in a physical limit considered. In the (α,γ) model, where the bulk magnetic energy is taken to be $\frac{1}{2}\alpha_m M^2+\frac{1}{6}\gamma_m M^6$, as is supported by recent experiments for $ErRh_4B_4$, we find the transition to be first order. This approach is then confined to an unphysical branch, which does not permit us to calculate various physical quantities on the physical branch.

I. INTRODUCTION

Based on a Ginzburg-Landau approach which emphasized the magnetic field interaction, Blount and Varma[1] predicted a spiral-magnetic (SM) phase for superconductivity to coexist with ferromagnetism, in, for example, such ternary compounds as $ErRh_4B_4$ and $HoMo_6S_8$. These compounds (and several others) have been called reentrant, or ferromagnetic, superconductors, because they first become superconducting as the temperature T is lowered through an upper transition temperature T_{c1}, and then they reenter into a normal ferromagnetic state as T is further lowered through a lower (or reentrant) transition temperature T_{c2}. Neutron scattering on polycrystalline[2,3] $ErRh_4B_4$ and $HoMo_6S_8$ did confirm the existence of a characteristic wave number $\vec{q}_0$ associated with the magnetic order, in a narrow temperature range just above T_{c2}, which appeared consistent with the prediction of Ref. 1. However, subsequently, neutron scattering was performed on a single-crystal $ErRh_4B_4$, which revealed that the magnetization is linearly polarized.[4] An independent magnetic study of the same single crystal also showed that at least this material is strongly anisotropic in its magnetic properties, with the easy axis being either of the two a axes in a tetragonal symmetry.[5] Greenside *et al.*[6] then analyzed the effect of anisotropy on the Blount-Varma theory of the spiral-magnetic phase. They found that as the strength of anisotropy was increased, the spiral-magnetic phase first changed into an elliptically polarized (EP) phase, and then to a linearly polarized (LP) phase, with the magnetization confined to a single easy axis. This last change occurred when a *finite* critical strength of anisotropy is exceeded. While the solution for the SM state may be expressed solely in terms of analytical expressions, the solutions for the EP and LP states, as obtained by Greenside *et al.*, relied heavily on numerical methods, in order to solve a coupled set of nonlinear differential equations with two-point boundary conditions, which described the states. Such numerical approaches may be quite accurate with the help of modern computers, but many physical quantities are still not easily computed. In this paper, we show that an exact, *near-onset* analysis may be obtained for the LP state, which is the end product of a spin-density-wave instability. This allows us to obtain explicit analytic expressions for many physical quantities including the onset temperature T_s, the "order parameter" $\langle M^2\rangle(T/T_s)$, which is the spatial average of the magnetization squared, and many thermodynamic quantities, such as the free-energy density $\mathscr{F}$, the entropy density S, the specific-heat jump at T_s, ΔC, the thermodynamic critical field H_c, etc. The order of the transition is also explicitly determined in this process. In addition, two more goals have been achieved by this analysis.

(1) In the numerical analysis of Greenside *et al.*, they assumed that the LP state was a plane-wave-like state, i.e., varying in one direction only, without any attempt to first rule out other spatial structures (such as one which varies periodically in two directions) by a free-energy-minimization principle. In the present exact analysis valid just below T_s, we can start with the most general spatial structure allowed by the instability, and use a free-energy-minimization principle to select the optimum one. This structure does turn out to be plane-wave-like for the "standard model" of Blount and Varma in a physical limit considered, but for a slight variation of this model [see point (2) below], this conclusion appears to be no longer true. Unfortunately, we can only analyze the preferred spatial structure for an *unstable* branch in this second model, because the transition turns out to be *first* order!

(2) In the "standard model" of Blount and Varma, the magnetic free energy is assumed to be composed of

$\frac{1}{2}\alpha M^2+\frac{1}{4}\beta M^4$, as in the standard Ginzburg-Landau theory of a second-order transition. However, evidence is now available[7] that for $ErRh_4B_4$, the β coefficient appears to be accidentally (or for reasons yet unknown) zero, so that the next-order term $\frac{1}{6}\gamma M^6$ must be included in order to form a consistent theory. We shall call these two models the (α,β) and (α,γ) models, respectively. In this work, we have analyzed the effect of changing from the (α,β) model to the (α,γ) model in order to compare their predictions. As already mentioned in point (1) above, we have found important qualitative differences in these two models studied, which will be elaborated in the later sections.

The organization of this paper is as follows. In Sec. II we introduce the general Ginzburg-Landau free-energy functional as proposed by Blount and Varma and then introduce a normalized version of it for the (α,β) model. In Sec. III we performed a right-at-onset analysis within the (α,β) model, in order to determine the characteristic wave number for the spin-density-wave instability, and its onset temperature. In Sec. IV we proceed to study the just-below-onset analysis, in a way similar to Abrikosov's theory[8] of the vortex state near H_{c2}, in order to determine the growth rate of the appropriate order parameter, and obtain explicit formulas for various thermodynamic quantities to leading order in the deviation from onset. In Sec. V we present a rigorous analysis of the preferred spatial structure, using free-energy minimization as the criterion, and taking into account all possibilities allowed by the instability. We find that a plane-wave-like structure is favored for a physical limit considered. In Sec. VI we turn our attention to the (α,γ) model, and perform similar linear and nonlinear analyses of the spin-density-wave instability. We find that the onset transition must be *first* order in this case, but cannot predict various thermodynamic quantities, since the present approach is valid on the unstable branch only, once the transition is first order. The preferred spatial structure in this unstable branch appears to be not even periodic in space, but it is not clear what it implies for the physical branch. Finally, Sec. VII contains the conclusion.

II. GINZBURG-LANDAU FREE-ENERGY FUNCTIONAL AND (α,β) MODEL

We begin with the Ginzburg-Landau free-energy functional for a ferromagnetic superconductor as is proposed by Blount and Varma:[1]

$$\mathcal{F}=\int d^3x\left[\left[\tfrac{1}{2}\alpha_s|\psi|^2+\tfrac{1}{4}\beta_s|\psi|^4+\tfrac{1}{2}p_0|(\vec{\nabla}-ir_0\vec{A})\psi|^2\right]+\left(\tfrac{1}{2}\alpha_m|\vec{M}|^2+\tfrac{1}{4}\beta_m|\vec{M}|^4+\tfrac{1}{6}\gamma_m|\vec{M}|^6+\tfrac{1}{2}\Gamma^2|\vec{\nabla}\vec{M}|^2\right)+\left[\frac{1}{8\pi}(\vec{B}-4\pi\vec{M})^2+\tfrac{1}{2}\eta_1|\psi|^2|\vec{M}|^2+\tfrac{1}{2}\eta_2|\psi|^2|\vec{\nabla}\vec{M}|^2\right]\right], \tag{1}$$

where $r_0\equiv 2e/\hbar c$, with e the electronic charge, $\hbar$ the reduced Planck constant, and c the speed of light. We have also added the M^6 term in this equation in order to include the possibility that β_m might be zero. In this expression, ψ is the superconducting pair—wave-function order parameter, $\vec{A}$ is the vector potential, $\vec{B}$ the magnetic induction, $\vec{M}$ the magnetization due to the rare-earth local moments, and all coefficients are constants independent of temperature, except that $\alpha_s=\alpha_{s0}(T/T_c-1)$ and $\alpha_m=\alpha_{m0}(T/T_{m0}-1)$, where T_c and T_{m0} are the bare mean-field transition temperatures for superconductivity and ferromagnetism, respectively. (Note that T_{m0} is not the T_m' of Ref. 1, but is the T_m^0 there, and α_m is already the combination $\alpha-4\pi$ of Ref. 1.) In the (α,β) model, we assume $\beta_m\neq 0$, and neglect the γ_m term. We can then introduce the following normalized quantities:

$$fe^{i\chi}\equiv\psi/\psi_\infty,\quad \vec{\mathcal{M}}\equiv\vec{M}/M_\infty,\quad \vec{\mathcal{B}}\equiv\vec{B}/4\pi M_\infty\ ,$$
$$\overline{\mathcal{F}}\equiv\mathcal{F}/F_{s0}\lambda^3\ ,\quad \vec{Q}\equiv(\vec{A}-r_0^{-1}\vec{\nabla}\chi)/4\pi M_\infty\lambda\ , \tag{2}$$

and the following dimensionless parameters:

$$\kappa\equiv\lambda/\xi_s,\quad \zeta\equiv\lambda/\xi_m\ ,$$
$$\mu^2/\nu^2=F_{m0}/F_{s0},\quad \nu^2\equiv 4\pi/|\alpha_m|\ . \tag{3}$$

In these definitions,

$$\psi_\infty\equiv(-\alpha_s/\beta_s)^{1/2},\quad F_{s0}\equiv\alpha_s^2/4\beta_s\equiv\tfrac{1}{4}|\alpha_s|\psi_\infty^2\ , \tag{4a}$$

$$M_\infty\equiv(-\alpha_m/\beta_m)^{1/2},\quad F_{m0}\equiv\alpha_m^2/4\beta_m=\tfrac{1}{4}|\alpha_m|M_\infty^2, \tag{4b}$$

and

$$\xi_s\equiv(p_0/|\alpha_s|)^{1/2},\quad \xi_m\equiv(\Gamma^2/|\alpha_m|)^{1/2}\ ,$$
$$\lambda\equiv(4\pi p_0)^{-1/2}(r_0\psi_\infty)^{-1}\ . \tag{4c}$$

Clearly ξ_s and λ are the usual superconducting coherence length and penetration depth, respectively, κ is the usual Ginzburg-Landau parameter, and ξ_m is an analogous magnetic coherence length, which diverges as $|T/T_{m0}-1|^{-1/2}$ as $T\to T_{m0}$. We shall be principally working at temperatures $T\leq T_{m0}$, where M_∞ is well defined and positive. The η_1 and η_2 terms in Eq. (1) are introduced by Blount and Varma to represent the effects of spin-flip scattering and conduction-electron spin polarization. For them we introduce the dimensionless combinations

$$\overline{\eta}_1\equiv(4\pi)^{-1}\eta_1\psi_\infty^2,\quad \overline{\eta}_2=(4\pi)^{-1}\eta_2\psi_\infty^2/\lambda^2\ . \tag{5}$$

Then we have the following reduced free-energy functional for the (α,β) model:

$$\Delta\overline{\mathcal{F}}\equiv\overline{\mathcal{F}}-\overline{\mathcal{F}}_{s0}=\int d^3x\{[(1-f^2)^2+2\kappa^{-2}|\nabla f|^2]+2\mu^2[(\vec{\mathcal{B}}-\vec{\mathcal{M}})^2+\vec{Q}^2f^2+\overline{\eta}_1f^2\vec{\mathcal{M}}^2+\overline{\eta}_2f^2|\vec{\nabla}\vec{\mathcal{M}}|^2]+(\mu/\nu)^2(-2\vec{\mathcal{M}}^2+\vec{\mathcal{M}}^4+2\zeta^{-2}|\vec{\nabla}\vec{\mathcal{M}}|^2)\}\ , \tag{6}$$

where $\overline{\mathscr{F}}_{s0}$ is the value of $\overline{\mathscr{F}}$ for $f=1$, $\mathscr{M}=\mathscr{B}=0$. In this equation, length is already measured in units of λ, and note that in this length scale, $\vec{\nabla}\times\vec{Q}=\vec{\mathscr{B}}$ is true. So far, Eq. (6) appears to be for an isotropic system, with no anisotropy energy included. However, when the anisotropy energy becomes sufficiently large that $\vec{\mathscr{M}}$ becomes confined to an easy axis (which we denote as $\hat{z}$), then the leading anisotropy energy terms will reduce to the form $\alpha_a M^2+\beta_a M^4$, and therefore may be included in Eq. (1) by introducing effective values for α_m and β_m for this particular linear polarization. Then Eq. (6) is *unchanged* as long as it is understood that in Eqs. (4b) and (4c) α_m and β_m now mean these effective values.

We conclude this section by pointing out that due to the temperature dependence of α_s and α_m, three of the four dimensionless parameters in Eq. (3) are temperature dependent:

$$\mu^2=\mu_0^2(1-T/T_{m0})/(1-T/T_c)^2 , \tag{7a}$$

$$\nu^2=\nu_0^2/(1-T/T_{m0}) , \tag{7b}$$

$$\zeta^2=\zeta_0^2(1-T/T_{m0})/(1-T/T_c) , \tag{7c}$$

so that μ, ν^{-1}, and ζ all approach zero like $(1-T/T_{m0})^{1/2}$ as $T\to T_{m0}$, while κ, $\nu\zeta$, and $\mu\nu$ are finite at $T=T_{m0}$. On the other hand, the two parameters in Eq. (5) are only weakly T dependent near T_{m0},

$$\overline{\eta}_1=\overline{\eta}_{10}(1-T/T_c),\quad \overline{\eta}_2=\overline{\eta}_{20}(1-T/T_c)^2 . \tag{8}$$

These temperature dependences are important for the proper near-onset analysis of the LP state.

III. RIGHT-AT-ONSET ANALYSIS FOR THE LP STATE IN THE (α,β) MODEL

As we have already explained in the preceding section, Eq. (6) may be regarded as to have already included a strong anisotropy energy, if we assume that $\vec{\mathscr{M}}$ and $\vec{\mathscr{B}}$ are both confined to a z component only:

$$\vec{\mathscr{M}}=\mathscr{M}\hat{z},\quad \vec{\mathscr{B}}=\mathscr{B}\hat{z} . \tag{9}$$

Varying $\Delta\overline{\mathscr{F}}$ with respect to f, $\mathscr{B}$, and $\mathscr{M}$ then gives the equations

$$f-f^3+\kappa^{-2}\nabla^2 f-\mu^2(Q^2+\overline{\eta}_1\mathscr{M}^2+\overline{\eta}_2\,|\,\vec{\nabla}\mathscr{M}\,|^2)f=0 , \tag{10a}$$

$$\vec{\nabla}\times[(\mathscr{B}-\mathscr{M})\hat{z}]+\vec{Q}f^2=0 , \tag{10b}$$

$$\nu^{-2}(\mathscr{M}-\mathscr{M}^3)+(\nu\zeta)^{-2}\nabla^2\mathscr{M} +(\mathscr{B}-\mathscr{M})-\overline{\eta}_1 f^2\mathscr{M}+\overline{\eta}_2\vec{\nabla}\cdot(f^2\vec{\nabla}\mathscr{M})=0 , \tag{10c}$$

$$\vec{\nabla}\times\vec{Q}=\mathscr{B}\hat{z} . \tag{10d}$$

At $T=T_{m0}$, these equations reduce to

$$f-f^3+\kappa^{-2}\nabla^2 f=0 , \tag{11a}$$

$$\vec{\nabla}\times[(\mathscr{B}-\mathscr{M})\hat{z}]+\vec{Q}=0,\quad \vec{\nabla}\times\vec{Q}=\mathscr{B}\hat{z} , \tag{11b}$$

$$(\nu\zeta)^{-2}\nabla^2\mathscr{M}+(\mathscr{B}-\mathscr{M})-\overline{\eta}_1\mathscr{M}+\overline{\eta}_2\nabla^2\mathscr{M}=0 . \tag{11c}$$

Since f is decoupled from $\mathscr{B}$ and $\mathscr{M}$, Eq. (11a) implies $f=1$, which has been used in Eqs. (11b) and (11c).

These equations are linear equations and may be solved with the ansatz

$$\begin{Bmatrix}\mathscr{M}\\ \mathscr{B}\end{Bmatrix}=\begin{Bmatrix}\overline{\mathscr{M}}\\ \overline{\mathscr{B}}\end{Bmatrix}\exp(i\vec{q}\cdot\vec{r})\ \ (\text{with } \vec{q}\perp\hat{z}) . \tag{12}$$

Then,

$$(q^2+1)\overline{\mathscr{B}}-q^2\overline{\mathscr{M}}=0 , \tag{13a}$$

$$\{[(\nu\zeta)^{-2}+\overline{\eta}_2]q^2+1+\overline{\eta}_1\}\overline{\mathscr{M}}-\overline{\mathscr{B}}=0 , \tag{13b}$$

which implies

$$\{[(\nu\zeta)^{-2}+\overline{\eta}_2]q^2+(q^2+1)^{-1}+\overline{\eta}_1\}\overline{\mathscr{M}}=0 . \tag{14}$$

This equation has no solution for real positive q (assuming $\overline{\eta}_1+\overline{\eta}_2 q^2>0$, which is obviously physical), indicating that the spin-density-wave (SDW) instability can only occur at a temperature T_s below T_{m0}. Assuming that the amplitude of the SDW is infinitesimal at T_s, we can still linearize Eqs. (10a)–(10d). This again gives Eqs. (11a) and (11b), but Eq. (11c) picks up one more term $\nu^{-2}\mathscr{M}$ on its left-hand side. Again solving with the ansatz, Eq. (12), we find that Eq. (13b) picks up a term $-\nu^{-2}$ in its curly brackets. Equation (14) is then changed to

$$\nu^{-2}\equiv\nu_0^{-2}(1-T/T_{m0}) =\overline{\eta}_1+[(\nu\zeta)^{-2}+\overline{\eta}_2]q^2+(q^2+1)^{-1} . \tag{15}$$

This equation defines an onset temperature for *each* wave number q of the SDW. The physically realized one T_s is obtained by optimization with respect to q. This gives

$$q_c^2=[(\nu\zeta)^{-2}+\overline{\eta}_2]^{-1/2}-1 , \tag{16}$$

and

$$1-T_s/T_{m0}=\nu_0^2\{\overline{\eta}_1+2[(\nu\zeta)^{-2}+\overline{\eta}_2]^{1/2} -[(\nu\zeta)^{-2}+\overline{\eta}_2]\} , \tag{17}$$

where $(\nu\zeta)^{-2}$, $\overline{\eta}_1$, and $\overline{\eta}_2$ are to be evaluated at $T=T_s$. Since $(\nu\zeta)^{-2}=\Gamma^2/4\pi\lambda^2$ is expected to be very small (10^{-4}–10^{-6}), these expressions may very possibly be dominated by the $\overline{\eta}_1,\overline{\eta}_2$ terms. We note that for $\overline{\eta}_2=0$, $q_c^2=(\nu\zeta)_s-1$, and $(\nu\zeta)_s=(4\pi)^{1/2}(\lambda/\Gamma)(1-T_s/T_c)^{-1/2}$. For $\lambda/\Gamma\approx 50$–500, $T_s\approx 1$ K, $T_c\approx 9$ K, we find $q_c\simeq 13.7$–43.3. This gives the characteristic wavelength $\Lambda_c\equiv 2\pi/q_c$ in units of λ to be 0.46–0.15. If $\overline{\eta}_2$ is not small in comparison with $(\nu\zeta)^{-2}$, q_c will be smaller than the above estimate. For example, if $\overline{\eta}_2=10^{-2}$–$10^{-3}$, then $q_c\approx 3$–5.5, and $\Lambda_c\approx(2.1$–$1.1)\lambda$. Thus, Λ_c may well be of the order of λ, rather than much smaller than λ!

IV. JUST-BELOW-ONSET ANALYSIS FOR THE LP STATE IN THE (α,β) MODEL

We now consider T slightly away from T_s, with $|T-T_s|\ll T_s$. If the transition is second order, we shall find a growing SDW as T falls *below* T_s. On the other hand, if the transition is first order, then the growth of the SDW will occur as T rises *above* T_s, making T_s a supercooling transition temperature. (See Sec. VI for an example.)

We first notice that Eq. (16) determines the magnitude of $\vec{q}_c$, but not its direction. We thus have a *degeneracy* of fluctuation modes which can compete for growth away from T_s. Only one appropriate combination of these modes will actually grow, which minimizes the total free energy of the system. To find this combination, and the rate of growth of this order parameter, we assume

$$f=1-\delta f\ , \tag{18a}$$

$$\mathcal{M}=\sum_i \overline{\mathcal{M}}_i e^{i\vec{q}_i\cdot\vec{r}}+\delta\mathcal{M}\equiv\mathcal{M}_0+\delta\mathcal{M}\ , \tag{18b}$$

$$\mathcal{B}=[q_c^2/(q_c^2+1)]\mathcal{M}_0+\delta\mathcal{B}\equiv\mathcal{B}_0+\delta\mathcal{B}\ , \tag{18c}$$

$$\vec{Q}=\sum_i \frac{i\vec{q}_i\times\hat{z}}{q_c^2+1}\overline{\mathcal{M}}_i e^{i\vec{q}_i\cdot\vec{r}}+\delta\vec{Q}\equiv\vec{Q}_0+\delta\vec{Q}\ , \tag{18d}$$

where $\vec{q}_i\perp\hat{z}$, and $|\vec{q}_i|=q_c$, for all i. Substituting these equations into Eqs. (10a)–(10d), linearizing them with respect to $\delta f,\delta\mathcal{M},\delta\mathcal{B},\delta\vec{Q}$, and using

$$\vec{\nabla}\times[(\mathcal{B}_0-\mathcal{M}_0)\hat{z}]+\vec{Q}_0=0,\ \vec{\nabla}\times\vec{Q}_0=\mathcal{B}_0\hat{z}\ , \tag{19a}$$

$$\nu_s^{-2}\mathcal{M}_0+(\nu\xi)^{-2}\nabla^2\mathcal{M}_0+(\mathcal{B}_0-\mathcal{M}_0)-\overline{\eta}_1\mathcal{M}_0+\overline{\eta}_2\nabla^2\mathcal{M}_0=0\ , \tag{19b}$$

where ν_s^{-1} is the value of ν^{-2} at $T=T_s$, we obtain the following equations:

$$2\delta f-\kappa^{-2}\nabla^2\delta f=\mu_s^2(\vec{Q}_0^{\,2}+\overline{\eta}_1\mathcal{M}_0^2+\overline{\eta}_2|\vec{\nabla}\mathcal{M}_0|^2)\ , \tag{20a}$$

$$\vec{\nabla}\times[(\delta\mathcal{B}-\delta\mathcal{M})\hat{z}]+\delta\vec{Q}=2\vec{Q}_0\delta f,\ \vec{\nabla}\times\delta\vec{Q}=\delta\mathcal{B}\hat{z}\ , \tag{20b}$$

$$\nu_s^{-2}\delta\mathcal{M}+(\nu\xi)^{-2}\nabla^2\delta\mathcal{M}+(\delta\mathcal{B}-\delta\mathcal{M})-\overline{\eta}_1\delta\mathcal{M}+\overline{\eta}_2\nabla^2\delta\mathcal{M}=(\nu_s^{-2}-\nu^{-2})\mathcal{M}_0+\nu_s^{-2}\mathcal{M}_0^3-2\overline{\eta}_1\mathcal{M}_0\delta f+2\overline{\eta}_2\vec{\nabla}\cdot(\delta f\vec{\nabla}\mathcal{M}_0)\ . \tag{20c}$$

Multiplying Eq. (20c) by $\mathcal{M}_0$, averaging over space, and assuming that the structure is periodic in space, so that one can integrate by parts without worrying about surface terms, we obtain

$$\begin{aligned}(\nu_s^{-2}-\nu^{-2})\langle\mathcal{M}_0^2\rangle+\nu_s^{-2}\langle\mathcal{M}_0^4\rangle-2\overline{\eta}_1\langle\delta f\mathcal{M}_0^2\rangle-2\overline{\eta}_2\langle\delta f|\vec{\nabla}\mathcal{M}_0|^2\rangle\\ =\langle\delta\mathcal{M}[\nu_s^{-2}\mathcal{M}_0+(\nu\xi)^{-2}\nabla^2\mathcal{M}_0-\mathcal{M}_0-\overline{\eta}_1\mathcal{M}_0+\overline{\eta}_2\nabla^2\mathcal{M}_0]+\delta\mathcal{B}\mathcal{M}_0\\ =\langle(\delta\mathcal{B}-\delta\mathcal{M})\mathcal{B}_0\rangle-\langle\delta\mathcal{B}(\mathcal{B}_0-\mathcal{M}_0)\rangle\\ =\langle\vec{\nabla}\times[(\delta\mathcal{B}-\delta\mathcal{M})\hat{z}]\cdot\vec{Q}_0\rangle-\langle\delta\vec{Q}\cdot\vec{\nabla}\times[(\mathcal{B}_0-\mathcal{M}_0)\hat{z}]\rangle\\ =2\langle Q_0^2\delta f\rangle\ ,\end{aligned}$$

where we have used Eqs. (19b), (19a), and (20b). Noting that $\nu_s^{-2}-\nu^{-2}=\nu_0^2(T-T_S)/T_{m0}$, we find

$$\nu_0^{-2}[(T_s-T)/T_{m0}]\langle\mathcal{M}_0^2\rangle=\nu_s^{-2}\langle\mathcal{M}_0^4\rangle-2\langle\delta f(\vec{Q}_0^{\,2}+\overline{\eta}_1\mathcal{M}_0^2+\overline{\eta}_2|\vec{\nabla}\mathcal{M}_0|^2)\rangle\ . \tag{21}$$

To obtain δf, we solve Eq. (20a),

$$2\delta f-\kappa^{-2}\nabla^2\delta f=\mu_s^2\sum_{i,j}\overline{\mathcal{M}}_i\overline{\mathcal{M}}_j\left[\frac{-\vec{q}_i\cdot\vec{q}_j}{(q_c^2+1)^2}+\overline{\eta}_1+\overline{\eta}_2(-\vec{q}_i\cdot\vec{q}_j)\right]e^{i(\vec{q}_i+\vec{q}_j)\cdot\vec{r}}\ ,$$

$$\delta f=\mu_s^2\sum_{i,j}\overline{\mathcal{M}}_i\overline{\mathcal{M}}_j\frac{(-\vec{q}_i\cdot\vec{q}_j)[(q_c^2+1)^{-2}+\overline{\eta}_2]+\overline{\eta}_1}{2+\kappa^{-2}(\vec{q}_i+\vec{q}_j)^2}e^{i(\vec{q}_i+\vec{q}_j)\cdot\vec{r}}\ . \tag{22}$$

If we now introduce the structural constants β_1 and β_2,

$$\langle\mathcal{M}_0^4\rangle=\beta_1\langle\mathcal{M}_0^2\rangle^2\ , \tag{23a}$$

$$\langle 2\delta f(\vec{Q}_0^{\,2}+\overline{\eta}_1\mathcal{M}_0^2+\overline{\eta}_2|\vec{\nabla}\mathcal{M}_0|^2)\rangle=\mu_s^2\beta_2\langle\mathcal{M}_0^2\rangle^2\ , \tag{23b}$$

we can rewrite Eq. (21) as

$$\langle\mathcal{M}_0^2\rangle=\left[\frac{T_s-T}{T_{m0}-T_s}\right]\Big/(\beta_1-\mu^2\nu^2\beta_2)\ , \tag{24}$$

which implies a second-order transition at T_s. In this whole derivation, we have neglected $(T_s-T)/T_c$ with respect to $(T_s-T)/T_{m0}$, which allows us to treat $(\nu\xi)$, $\overline{\eta}_1$, $\overline{\eta}_2$, and $\mu\nu$ as constants independent of temperature. The total free energy may now be evaluated by using Eq. (6):

$$\begin{aligned}\frac{\mathcal{F}-\mathcal{F}_{s0}}{|\mathcal{F}_{s0}|}&=\langle[2\delta f(2\delta f-\kappa^{-2}\nabla^2\delta f)-(\mu/\nu)_s^2\mathcal{M}_0^4]\rangle\\ &=-(\mu/\nu)_s^2[\beta_1-(\mu\nu)^2\beta_2]\langle\mathcal{M}_0^2\rangle^2\ ,\end{aligned}$$

or

$$\frac{\mathcal{F}-\mathcal{F}_{s0}}{|\mathcal{F}_{s0}|}=-\left[\frac{\mu}{\nu}\right]_s^2\frac{[(T_s-T)/(T_{m0}-T_s)]^2}{\beta_1-(\mu\nu)^2\beta_2}\ . \tag{25}$$

Rewriting this equation as average free-energy density in natural units,

$$F \simeq -\frac{\alpha_s^2}{4\beta_s}\Bigg|_s - \frac{\alpha_m^2}{4\beta_m}\Bigg|_s \frac{[(T_s-T)/(T_{m0}-T_s)]^2}{\beta_1-(\mu\nu)^2\beta_2} , \quad (26)$$

we may evaluate the average entropy density

$$S = S_{sc} - \frac{\alpha_{m0}^2}{2\beta_m}\frac{(T_s-T)/T_{m0}^2}{\beta_1-(\mu\nu)^2\beta_2} , \quad (27)$$

and the specific-heat jump at T_s,

$$\Delta C\,|_{T_s} = (\alpha_{m0}^2/2\beta_m)/T_{m0}[\beta_1-(\mu\nu)^2\beta_2] . \quad (28)$$

If we compare this with the specific-heat jump at T_{c1} due to the superconducting transition,

$$\Delta C\,|_{T_{c1}} = \alpha_{s0}^2/2\beta_s T_{c1} , \quad (29)$$

we find the ratio

$$\Delta C\,|_{T_s}/\Delta C\,|_{T_{c1}} \approx \frac{\mu_0^2}{\nu_0^2}\frac{T_{c1}}{T_{m0}} \gtrsim 100 , \quad (30)$$

where we have used the estimate of μ_0^2/ν_0^2 in Ref. 9. We have taken $\beta_1-(\mu\nu)^2\beta_2$ as a constant of the order of unity which will be shown in the next section. The specific-heat jump Eq. (28) is reduced from the corresponding ferromagnetic specific-heat jump, which would have occurred at T_{m0} if superconductivity did not interact with magnetism, only by the factor $\beta_1-(\mu\nu)^2\beta_2$, which should be larger than 1. (See Sec. V.) Next we set Eq. (26) equal to $H_c^2/8\pi$, and find that the thermodynamic critical field H_c has a continuous value and first derivative at T_s, but a discontinuous second derivative,

$$\frac{H_c(T_s)}{4\pi}\left[\frac{d^2H_c}{dT^2}\Bigg|_{T_s+\epsilon} - \frac{d^2H_c}{dT^2}\Bigg|_{T_s-\epsilon}\right] = \frac{\alpha_{m0}^2}{2\beta_m T_{m0}^2[\beta_1-(\mu\nu)^2\beta_2]} = \frac{\Delta C\,|_{T_s}}{T_{m0}} . \quad (31)$$

This formula assumes that the system is type I in the vicinity of T_s, as is observed.[10] In the next section, we analyze the preferred spatial structure, which determines the constants β_1 and β_2.

V. THE $\vec{q}$-STAR ANALYSIS AND THE PREFERRED SPATIAL STRUCTURE IN THE (α,β) MODEL

So far we have not specified the set of $\vec{q}_i$, which is in the summation of Eqs. (18b)–(18d), nor the expansion coefficients $\{\overline{\mathcal{M}}_i\}$ in these equations. They should be determined by minimizing the total free energy. In view of Eq. (26), this should mean minimizing $\beta_1-(\mu\nu)^2\beta_2$. Since this combination depends on the parameters $(\mu\nu)^2$, $\bar{\eta}_1$, $\bar{\eta}_2$, q_c^2, and κ^2 [see Eqs. (22) and (23)], a general minimization is difficult. We shall therefore look at a limiting case of this expression only. It is probably reasonable to set $q_c^2 \approx 10^2$–10^3, and $\kappa \lesssim 5$. Then Eq. (22) may be approximated by keeping just the $\vec{q}_i+\vec{q}_j=0$ terms. This gives $\beta_2 \simeq \{q_c^2[(q_c^2+1)^{-2}+\bar{\eta}_2]+\bar{\eta}_1\}^2$, independent of the choice of $\{q_i\}$ and $\{\overline{\mathcal{M}}_i\}$. We must then choose $\{\vec{q}_i\}$ and $\{\overline{\mathcal{M}}_i\}$ to minimize β_1. The set $\{\vec{q}_i\}$ must satisfy the conditions that (1) all $\vec{q}_i \perp \hat{z}$, (2) all $|\vec{q}_i| = q_c$, (3) if $\vec{q} \in \{\vec{q}_i\}$, then $-\vec{q} \in \{\vec{q}_i\}$. It is customary to call such a set a "star." The coefficients $\{\overline{\mathcal{M}}_i\}$ also must satisfy a condition, $\overline{\mathcal{M}}_{\vec{q}} = \overline{\mathcal{M}}^*_{-\vec{q}}$, because $\mathcal{M}(\vec{r})$ must be a real function. If the $\vec{q}$ star is made of $(\vec{q},-\vec{q})$ only, with associated coefficients $(\overline{\mathcal{M}},\overline{\mathcal{M}}^*)$, then $\langle \mathcal{M}_0^2\rangle = 2|\overline{\mathcal{M}}|^2$, and $\langle\mathcal{M}_0^4\rangle = 6|\overline{\mathcal{M}}|^4$, which give $\beta_1=\frac{3}{2}$. This corresponds to a plane-wave-like structure. If the $\vec{q}$ star is made of $(\vec{q}_1,\vec{q}_2,-\vec{q}_1,-\vec{q}_2)$, with the associated coefficients $(\overline{\mathcal{M}}_1, \overline{\mathcal{M}}_2, \overline{\mathcal{M}}_1^*, \overline{\mathcal{M}}_2^*)$, then we must require $\cos\theta_{12} \equiv \vec{q}_1\cdot\vec{q}_2/q_c^2$ to be a rational number n/m, in order for the structure to be periodic in both x and y. We then find

$$\langle\mathcal{M}_0^2\rangle = 2(|\overline{\mathcal{M}}_1|^2+|\overline{\mathcal{M}}_2|^2) ,$$

$$\langle\mathcal{M}_0^4\rangle = 6(|\mathcal{M}_1|^2|\mathcal{M}_2|^2)^2+12|\mathcal{M}_1|^2|\mathcal{M}_2|^2 ,$$

so that

$$\beta_1 = \tfrac{3}{2}[1+2\gamma(1-\gamma)],$$

where

$$\gamma \equiv |\overline{\mathcal{M}}_1|^2/(|\overline{\mathcal{M}}_1|^2+|\overline{\mathcal{M}}_2|^2) .$$

This expression is minimized at $\gamma=0$ or 1, with $\beta_1=\frac{3}{2}$, still corresponding to a plane-wave structure. Proceeding to

$$\{q_i\} = (q_1,q_2,q_3,-q_1,-q_2-q_3)$$

and

$$\{\overline{\mathcal{M}}_i\} = (\overline{\mathcal{M}}_1,\overline{\mathcal{M}}_2,\overline{\mathcal{M}}_3,\overline{\mathcal{M}}_1^*,\overline{\mathcal{M}}_2^*,\overline{\mathcal{M}}_3^*) ,$$

we must require

$$\cos\theta_1 \equiv q_1\cdot q_2/q_c^2 = n/m ,$$

$$\cos\theta_2 \equiv q_1\cdot q_3/q_c^2 = -n/m ,$$

or $\cos\theta_1 = n_1/m_1$, $\cos\theta_2 = n_2/m_2$, such that (n_1,l_1,m_1) and (n_2,l_2,m_2) are two independent sets of Pythagorean numbers. We then find for either case

$$\beta_1 = \tfrac{3}{2}\left[1+2\sum_{i<j}|\overline{\mathcal{M}}_i|^2|\overline{\mathcal{M}}_j|^2\Big/\left[\sum_i|\overline{\mathcal{M}}_i|^2\right]^2\right] ,$$

which is still minimized by letting one $\overline{\mathcal{M}}_i \neq 0$ only. The $\vec{q}$ star may be further generalized to include more $\vec{q}$ vectors, since there are infinite sets of Pythagorean numbers. However, it is straightforward to show that β_1 is still minimized by letting one $\overline{\mathcal{M}}_i \neq 0$ only. We thus have shown that at least for the limit considered, the preferred structure is the plane-wave structure assumed by Greenside *et al.*

VI. THE (α,γ) MODEL AND THE CORRESPONDING NEAR-ONSET ANALYSIS

We next consider the possibility that $\beta_m=0$, $\gamma_m\neq 0$, which is supported by recent experiments. We shall use the same normalization scheme, Eqs. (2)–(5), except that Eq. (4b) must be replaced by

$$M_\infty = (-\alpha_m/\gamma_m)^{1/4}, \quad F_{m0} \equiv \tfrac{1}{3} |\alpha_m|^{3/2}/\gamma_m \ . \tag{4b'}$$

Equation (6) is then changed to

$$\Delta\mathscr{F} = \int d^3x \{[(1-f^2)^2 + 2\kappa^{-2}|\vec{\nabla} f|^2] + \tfrac{3}{2}\mu^2[(\vec{\mathscr{B}} - \vec{\mathscr{M}})^2 + \vec{Q}^2 f^2 + \overline{\eta}_1 f^2 \vec{\mathscr{M}}^2 + \overline{\eta}_2 f^2 |\vec{\nabla}\vec{\mathscr{M}}|^2] + \tfrac{1}{2}(\mu/\nu)^2(-3|\vec{\mathscr{M}}|^2 + |\vec{\mathscr{M}}|^6 + 3\zeta^{-2}|\vec{\nabla}\vec{\mathscr{M}}|^2)\} \ , \tag{6'}$$

while Eq. (7a) must be changed to

$$\mu^2 = \mu_0^2(1 - T/T_{m0})^{1/2}/(1 - T/T_c)^2 \ . \tag{7a'}$$

Thus $\mu\nu$ is no longer finite at T_{m0}, but diverges as $(1-T/T_{m0})^{-1/4}$. The linear analysis is completely unaltered, with Eqs. (16) and (17) remaining valid. The nonlinear analysis, however, must be redone, with much similarity with the (α,β) model. Equations (10a)—(10d) remain practically unchanged, except that μ^2 is replaced by $\frac{3}{4}\mu^2$, and $\mathscr{M}^3$ is replaced by $\mathscr{M}^5$. The structural constants must now be defined as

$$\langle \mathscr{M}_0^6 \rangle = \beta_3 \langle \mathscr{M}_0^2 \rangle^3 \ , \tag{23a'}$$

$$\langle 2\delta f(Q_0^2 + \overline{\eta}_1 \mathscr{M}_0^2 + \overline{\eta}_2 |\nabla\mathscr{M}_0|^2) \rangle = \tfrac{3}{4}\mu^2 \beta_2 \langle \mathscr{M}_0^2 \rangle^2 \ , \tag{23b'}$$

while Eq. (24) must now be replaced by

$$\frac{T_s - T}{T_{m0} - T_s} = \beta_3 \langle \mathscr{M}_0^2 \rangle^2 - \tfrac{3}{4}(\mu\nu)_s^2 \beta_2 \langle \mathscr{M}_0^2 \rangle \ . \tag{24'}$$

This is a *quadratic* equation for $\langle \mathscr{M}_0^2 \rangle$, whose solution is schematically shown in Fig. 1. It contains a physical (i.e., stable) branch *AB*, and an unphysical (i.e., unstable) branch *BC*. This is a typical solution for a first-order transition, with T_s identified as the supercooling critical temperature. The superheating critical temperature T_s^{sh} is where the two branches meet. A Gibbs construction is needed to obtain the thermodynamic critical temperature T_s^{th}. We shall not attempt to evaluate T_s^{th}, nor any thermodynamic quantities, since the present approach is strictly valid near T_s only, where the unphysical branch lies. Near this point, Eq. (24') may be solved as

$$\langle \mathscr{M}_0^2 \rangle = \frac{4}{3} \frac{(1-T_s/T_c)^2}{\mu_0^2 \nu_0^2 \beta_2} \frac{T - T_s}{[T_{m0}(T_{m0} - T_s)]^{1/2}} \ , \tag{32}$$

which is valid for $T \gtrsim T_s$. The free-energy density is found to be

$$\frac{\mathscr{F} - \mathscr{F}_{s0}}{|\mathscr{F}_{s0}|} = (\tfrac{3}{4}\mu^2)^2 \beta_2 \langle \mathscr{M}_0^2 \rangle^2 - (\mu/\nu)_s^2 \beta_3 \langle \mathscr{M}_0^2 \rangle^3 \simeq \frac{1}{\beta_2 \nu_0^4} \left[\frac{T - T_s}{T_{m0}} \right]^2 \quad \text{for } T \gtrsim T_s \ . \tag{33}$$

Comment on the preferred spatial structure remains to be made, but we can again look at the unphysical branch only. In view of Eq. (33), we must now *maximize* β_2. For $q_c^2 \gg \kappa^2$ we still find the same β_2 given in Sec. V, which is independent of the spatial structure. Thus we must go beyond this approximation. In the opposite limit $q_c^2 \ll \kappa^2$, we find

$$\beta_2 \cong \left\langle \sum_{ijkl} \overline{\mathscr{M}}_i \overline{\mathscr{M}}_j \overline{\mathscr{M}}_k \overline{\mathscr{M}}_l \left[\frac{-\vec{q}_i \cdot \vec{q}_j}{(q_c^2+1)^2} + \overline{\eta}_1 + \overline{\eta}_2(-\vec{q}_i \cdot \vec{q}_j) \right] \left[\frac{-\vec{q}_k \cdot \vec{q}_l}{(q_c^2+1)^2} + \overline{\eta}_1 + \overline{\eta}_2(-\vec{q}_k \cdot \vec{q}_l) \right] \times \exp[i(\vec{q}_i + \vec{q}_j + \vec{q}_k + \vec{q}_l) \cdot \vec{r}] \right\rangle \bigg/ \left[\sum_i |\overline{\mathscr{M}}_i|^2 \right]^2 \ . \tag{34}$$

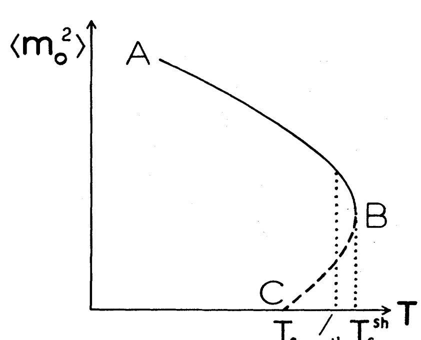

FIG. 1. Schematic plot of the solution of Eq. (24') as $\langle \mathscr{M}_0^2 \rangle$ vs T, indicating a first-order transition. T_s, T_s^{th}, and T_s^{sh} denote the supercooling, thermodynamic, and superheating critical temperatures, respectively. The solid line *AB* stands for the physical (i.e., stable) branch while the dashed line *BC* stands for the unphysical (i.e., unstable) branch.

If $\overline{\eta}_1$ dominates, we would find $\beta_2 = \overline{\eta}_1^2 \beta_1$. Thus for this case we must *maximize* β_1. Examining the $4q$ and $6q$ stars, we find the maxima to occur at a square and a hexagonal lattice, respectively, with $\beta_1 = \frac{9}{4}$ and $\frac{5}{2}$ for the two cases. Expanding the members of the star, we find that the preferred structure to occur at larger and larger number of $\vec{q}$'s, with the cell size also expanding to infinity. Thus the preferred structure seems to be not even periodic in space. We might call this state a "macroscopic spin-glass," which differs from the usual spin-glass in that here the nearest-neighbor spins are still aligned.

Discovery of such a state would be extremely interesting, if this study was not confined to an unphysical branch. Unfortunately, whether this is a peculiar property of this branch or it is also shared by the physical branch is a question which cannot be anwered within the present approach. We speculate that this peculiar property might be intimately tied to the order of the transition, since if the transition is first order, or even if the transition is just *near* a first-order transition, then the quantity to be minimized will not be as simple as β_1, and the preferred spatial structure will likely be non-plane-wave-like. Since q_c^2 is probably larger than κ^2 in the known ferromagnetic superconductors, the different spatial structures will be very nearly degenerate, if not exactly degenerate. Thus other energy contributions not considered here (such as the interaction with the lattice) might be important in selecting the spatial structure(s) to be realized.

One might wonder whether it is appropriate to discuss the preferred spatial structure of an unphysical branch, knowing that it is not stable. We believe that the answer is yes, since the branch being unstable does not mean that it is a free-energy local maximum. Rather, it means that it is a free-energy saddle point in a huge, if not infinite, dimensional space. In other words, the state is actually a local free-energy minimum with respect to variations in all except usually a few directions in this space. That structural variations as are discussed in this section do not include variations in any one of these "unsafe" directions can be seen by the existence of, for example, an *upper* bound 3 to β_1, which is attained by an *Nq* star in the limit of letting $N \rightarrow \infty$.

Physically, one can see that the unsafe directions include letting the *amplitude* of the fluctuation either go to 0 (hence, reaching the uniform superconducting state), or grow (hence, reaching the *stable* branch of the LP state), but these are clearly not included in the structural variations.

VIII. CONCLUSIONS

In this paper we have presented an exact, near-onset analysis within the Ginzburg-Landau formulation of the *linearly polarized* state for the coexistence of ferromagnetism and superconductivity in reentrant superconductors. This theory is valid for $|T_s - T| \ll T_s$ only, where T_s is the onset temperature. We have considered the (α,β) model, where the magnetic bulk energy is taken to be the *standard* form $\frac{1}{2}\alpha_m M^2 + \frac{1}{4}\beta_m M^4$, and the (α,γ) model, where the magnetic bulk energy is taken to be of the form $\frac{1}{2}\alpha_m M^2 + \frac{1}{6}\gamma_m M^6$, which is supported by some recent experiments on polycrystalline and single crystal $ErRh_4B_4$.

For the (α,β) model, we find the transition to be second order, and obtain explicit, closed-form formulas for the characteristic wave number (16), the onset temperature (17), the initial growth rate of the appropriate SDW order parameter (24), the free-energy density (26), the entropy density (27), the specific-heat jump at T_s (28), and the change of curvature of the thermodynamic critical field H_c at T_s (31), where the numbers enclosed in parentheses are the relevant equation numbers. We have also rigorously analyzed the preferred spatial structure, and have concluded that it is plane-wave-like in the physical limit considered.

For the (α,γ) model, our most important conclusion is that the transition into the linearly polarized state must be first order, even without a negative β coefficient. We could not calculate the thermodyanmic quantities for this case because our approach is then valid for an unphysical branch only. Another curious fact discovered for this case is that at least for this unphysical branch the preferred spatial structure can be not even periodic in space (or, equivalently, it can have an infinitely large cell size) for which we coined the term "macroscopic spin-glass." (See Sec. VI for details.) Whether this has any relevance to reality (i.e., the physical branch) is unknown at the present.

In a sequel to this paper, we shall include anisotropy energy explicitly, and investigate the elliptically and circularly polarized states, by a similar near-onset analysis.

Finally, a comment is in order about the magnetic stiffness energy term $\frac{1}{2}\Gamma^2 |\vec{\nabla}\vec{M}|^2$ used in Eq. (1), which is invariant with respect to separate rotations in the coordinate and spin spaces. This symmetry is violated by the magnetic dipole-dipole interaction, so to include this interaction, one must allow Γ^2 to have different values for longitudinal and transverse variations of $\vec{M}$. That is, the magnetic stiffness energy must be generalized to $\frac{1}{2}\Gamma_l^2(\vec{\nabla}\cdot\vec{M})^2 + \frac{1}{2}\Gamma_t^2|\vec{\nabla}\times\vec{M}|^2$. Correspondingly, one must introduce ζ_l and ζ_t, as may be seen from Eqs. (3) and (4c) for the definitions of ζ and ξ_m. However, this generalization leads to a trivial extension of the present theory, with ζ replaced by ζ_t in every result and conclusion. This may be seen by first considering the linear equations describing the SDW fluctuations at their respective onset temperatures. Since the equations are linear in $\mathscr{M}$ and $\mathscr{B}$, there is a clear separation of longitudinal and transverse fluctuations, with $\vec{\mathscr{M}}$ and $\vec{\mathscr{B}}$ parallel to and perpendicular to $\vec{q}$, respectively, and there can be no coupling between the two types of fluctuations. For the transverse fluctuations, $\vec{\nabla}\cdot\vec{M}=0$, the discussion is exactly the same as we have presented, except for the replacement of ζ by ζt. For the longitudinal fluctuations, Eq. (11b) with $\vec{\mathscr{M}}$ and $\vec{\mathscr{B}}$ left in arbitrary directions, which is valid at the onset temperature, implies $\vec{Q}=\vec{\mathscr{B}}=0$. The onset temperature as a function of $\vec{q}$ for such fluctuations is then given by Eq. (15) with ζ replaced by ζ_l, *and with the term* $(q^2+1)^{-1}$ *replaced by 1.* This is optimized at $\vec{q}=\vec{0}$, and the resulting optimum onset temperature is much lower than that for the transverse SDW's. Thus just below the highest onset temperature T_s, when the discussion may be confined to the degenerate subspace of the optimum transverse SDW fluctuation modes, the longitudinal fluctuation modes may be totally neglected. This is equivalent to setting $\vec{\nabla}\cdot\vec{M}=0$ and $|\vec{\nabla}\times\vec{M}|^2 = |\vec{\nabla}\vec{M}|^2$ for the whole near-onset anlaysis presented here. We thus have shown the sufficiency of the magnetic stiffness energy used in Eq. (1), in spite of the possible importance of the magnetic dipole inteaction in real magnetic superconductors.

ACKNOWLEDGMENTS

This work was supported by National Science Foundation Grant No. DMR82-05697. It was partially worked out while the author was visiting the Argonne National Laboratory (ANL). The dislocation support from ANL and the warm hospitality of Dr. G. W. Crabtree are gratefully acknowledged.

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