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Probing the softest region of the nuclear equation of state

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An attractive, energy-dependent mean-field potential for baryons is introduced in order to generate a soft region in the nuclear equation of state, as suggested by recent lattice QCD calculations of baryon-free matter at finite temperature. Based on a hadronic transport model, we find that although this equation of state has negligible effects on the inclusive hadronic spectra, it leads to a minimum in the energy dependence of the transverse collective flow and a delayed expansion of the compressed matter. In particular, the transverse flow changes its direction as the colliding system passes through the softest region in the equation of state. [S0556-2813(98)50409-8]

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The primary goal of experiments on relativistic heavy-ion collisions is to create and study the quark gluon plasma (QGP) [1]. Lattice QCD calculations of baryon-free matter at finite temperature [2] have shown that although both the energy and entropy densities increase appreciably at about 150 MeV where the phase transition from the hadronic matter to the QGP occurs, the change in pressure is considerably weaker, leading thus to a small sound velocity which is given by the pressure gradient with respect to energy density, i.e., $V_s^2 \equiv dP/de$. This soft region in the nuclear equation of state is expected to have significant influence on the collective dynamics of the hot and dense matter formed in relativistic heavy ion collisions. In particular, a small sound velocity delays the expansion of the compressed matter and leads also to a reduced transverse collective flow. Indeed, recent studies based on the hydrodynamical model have demonstrated that as one varies the incident energy of a heavy ion collision, the system may reach the softest region in the equation of state and forms a long-lived fireball, which then results in the appearance of a minimum in the energy dependence of the transverse collective flow [3–6]. Although the hydrodynamical model provides a convenient framework to study the effects of nuclear equation of state on the collective behavior of colliding nuclei [5,7], it requires the introduction of an *ad hoc* freeze-out procedure to compare the calculated results with the experimental measurements.

In this Rapid Communication, we take a different approach to explore the effect of a softened equation of state on experimental observables by using the hadronic transport model ART [11], which treats consistently the final freeze-out. The possibility of describing the main features of lattice QCD phase transition in a hadronic model has recently been demonstrated by allowing hadron masses to decrease in medium, which has been argued to be related to the restoration of chiral symmetry [8]. Although neither QGP phase transition nor chiral symmetry restoration actually happens in our approach as in the hydrodynamical models, results from our studies are useful for understanding the signals for a real phase transition. Indeed, we find that the softened equation of state leads to both a minimum in the incident energy dependence of the transverse collective flow and a delayed expansion of the compressed matter as in the hydrodynamical model. Our results further show that such an equation of state has almost no effect on the inclusive hadron spectra.

To introduce a soft region in the equation of state, we consider for the baryon sector of the reaction system the following relation between the pressure P and energy density e in the mean-field approach [9,10]:

$$P=\frac{2}{3}E_k\rho+\rho\frac{dW}{d\rho}-W, \tag{1}$$

$$e=(m+E_k)\rho+W, \tag{2}$$

where E_k, W, ρ, and m are the average kinetic energy per baryon, potential energy density, baryon density, and nucleon mass, respectively. It was shown in Ref. [10] that W generally depends on both ρ and E_k. We require that at low energy densities, it coincides with the normal soft equation-of-state obtained from the well-known Skyrme interaction, i.e.,

$$P=\frac{2}{5}E_f\rho-179\rho^2/\rho_0+164\left(\frac{\rho}{\rho_0}\right)^{7/6}\rho \quad (\mathrm{MeV/fm}^3), \tag{3}$$

where E_f and ρ_0 are, respectively, the Fermi energy and normal nuclear matter density. Above the soft region, the pressure is taken to be

$$P=200\left(\frac{\rho}{\rho_0}\right)^{1/3}\rho-153 \quad (\mathrm{MeV/fm}^3), \tag{4}$$

which is similar to that of a noninteracting quark gas confined in a MIT bag.

To satisfy the condition at high energy densities, the following potential energy density is introduced:

$$W=\rho\left[600\left(\frac{\rho}{\rho_0}\right)^{1/3}-\frac{3}{5}E_f\left(\frac{\rho}{\rho_0}\right)^{2/3}+153/\rho+C_1\right] \quad (\mathrm{MeV/fm}^3), \tag{5}$$

where C_1 is a constant depending on the energy density at which Eq. (4) is reached, i.e., the upper boundary of the soft region.

To illustrate the effects of the soft region of nuclear equation of state, we consider the extreme case of a vanishing sound velocity. This can be achieved by keeping a constant

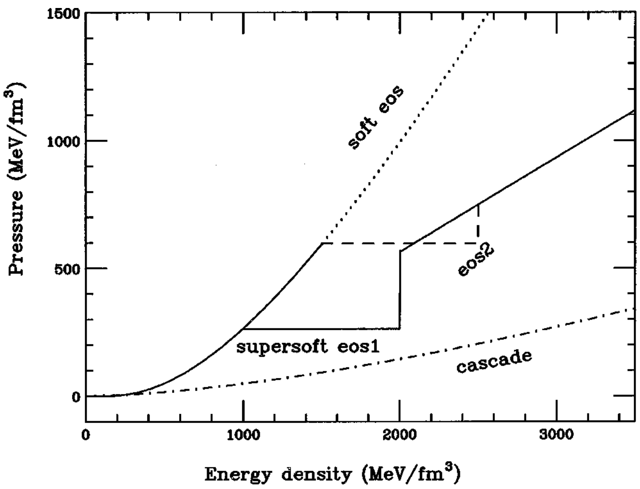

FIG. 1. Nuclear equation of state at zero temperature.

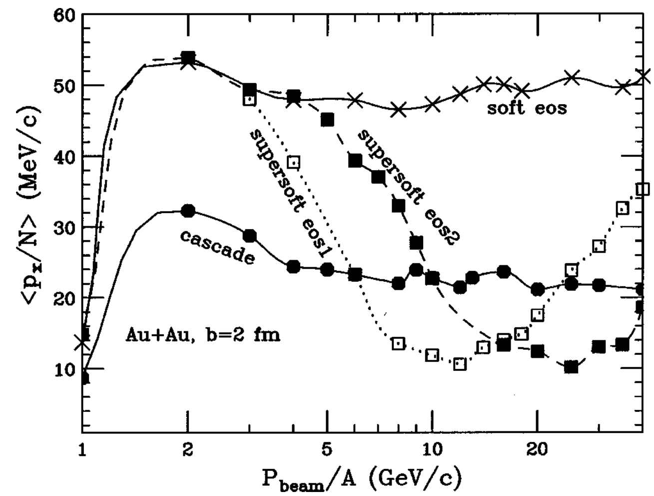

FIG. 2. Excitation function of total in-plane transverse momentum for the reaction of Au+Au at an impact parameter of 2 fm.

pressure at P_c while changing the energy density via the following potential energy density in Eq. (1):

$$W=\rho\left(C_2-\frac{P_c}{\rho}-E_k\right) \quad (\mathrm{MeV/fm^3}), \tag{6}$$

where C_2 is again a constant determined from the energy density of the lower boundary of transition region. In deriving the above expression we have assumed that E_k scales with $\rho^{2/3}$ as in an adiabatic process.

The resulting equation of state is shown in Fig. 1 as super-soft eos1 and eos2, with the softest region in the energy density range of $1<e<2$ GeV/fm^3 and $1.5<e<2.5$ GeV/fm^3, respectively. For comparison, we have also included the kinetic pressure, corresponding to a pure cascade model, as a function of energy density. The mean-field potential to be used in the transport model can be obtained from the relation $U\equiv\partial W/\partial\rho$, which is repulsive outside but attractive inside the softest region. In particular, the mean-field potential in the softest region is energy dependent in order to balance the increasing kinetic pressure as the energy density increases. We note that both parameters C_1 and C_2 have no effect on the dynamics as only the second derivative of the potential energy density appears in the equation of motion.

Effects of the softened equation of state can be studied using the relativistic transport model ART. We refer the reader to Ref. [11] for the detail of the model and its applications in studying various aspects of relativistic heavy-ion collisions at AGS energies. It has been found in these studies that the transverse collective flow, defined by

$$\langle P_x/N\rangle_t=\frac{1}{N}\int\langle P_x/N\rangle(y)\cdot\frac{dN}{dy}\cdot\mathrm{sgn}(y)\cdot dy, \tag{7}$$

is sensitive to the nuclear equation of state. We expect that the super-soft equation of state we introduced here will also affect the transverse collective flow if the energy density achieved in the collisions reaches the softest region, as in the recent study based on the hydrodynamical model [5,7]. In the latter study, a minimum in the excitation function of $\langle P_x/N\rangle_t$ at a beam energy of about 6 GeV/nucleon has been found when a softened equation of state due to the QCD phase transition is used.

In Fig. 2, we show the excitation function for the average transverse momentum predicted by the ART model using the four equations of state shown in Fig. 1. For the case of either the cascade or the normal soft equation of state, details can be found in our previous publications [11]. It is interesting to see that the super-soft equation of state with the softest region leads to a minimum in the excitation function of the transverse flow, as in hydrodynamical calculations. Furthermore, the incident energy at which the minimum occurs depends on the location of the softest region in the equation of state. For both super-soft eos1 and eos2, the average transverse momenta near the minima are even smaller than those in the cascade calculations as attractive mean-field potentials have been introduced in these equations of state to counterbalance the positive gradient of the thermal pressure. This is more clearly seen in Fig. 3, where we compare the average transverse momentum distribution using the cascade model and the super-soft eos1 for the Au+Au reaction at $P_{\mathrm{beam}}/A=12$ GeV/c. It is seen that the flow parameter, defined as

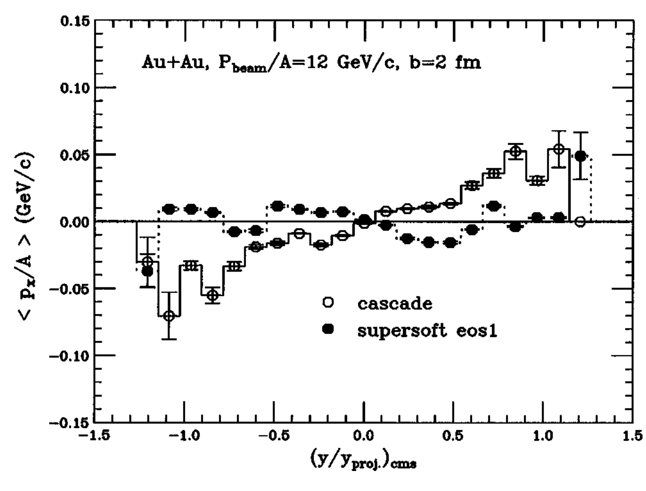

FIG. 3. In-plane transverse momentum distribution for the reaction of Au+Au at $P_{\mathrm{beam}}/A=12$ GeV/c and an impact parameter of 2 fm.

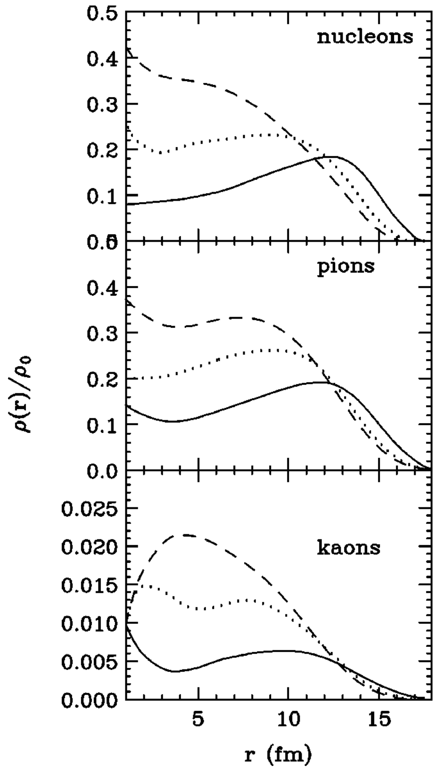

FIG. 4. Radial density distribution of nucleons, pions, and kaons in the reaction Au+Au at $P_{\mathrm{beam}}/A=12$ GeV/c and an impact parameter of 2 fm. The solid, dotted, and dashed lines are calculated using the soft eos, the super-soft eos1, and the cascade model, respectively.

the slope of the transverse momentum distribution at midrapidity, changes from positive to negative sign as a result of the attractive mean-field potential in the softest region. Our results also show that the flow parameter changes gradually from positive to negative then to positive as the beam energy increases. A change in the sign of flow parameter as the beam energy increases is another clear indication of the softening of the nuclear equation of state. To observe this effect experimentally, one needs to measure the absolute direction of the transverse flow besides its strength. This can be done by studying the shadowing effect on pion flow [12–15] and the circular polarization of the γ ray emitted from the reaction [16], as in heavy ion collisions at low energies.

A softened equation of state is expected not only to slow down the expansion of the system but also to affect the amount of compression in the collision. Since the pressure decreases from the normal soft, super-soft to the cascade at the same energy density, one expects to reach the highest compression in the cascade model. Shown in Fig. 4 are the radial density distributions of nucleons, pions, and kaons at 20 fm/c from the Au+Au reaction at an impact parameter of 2 fm and a beam momentum of 12 GeV/c. At this already rather late time of the reaction, the normal soft equation of state gives the most extended particle distributions, indicating thus the fastest compression and expansion. The particle distributions from calculations using the super-soft eos1 are more compact, implying thus a slower compression and expansion.

It is worth mentioning that we have also studied other inclusive hadronic observables using the four equations of state and have found that it is almost impossible to distinguish the predictions using the softened equation of state from others.

In summary, we have studied effects of the softened nuclear equation of state on heavy ion collisions at relativistic energies within a hadronic transport model. To keep a zero pressure gradient in the softest region, an attractive mean field for baryons has been introduced. The softened equation of state is found to lead to a minimum in the excitation function of transverse flow and a delayed expansion but has almost no effect on the inclusive hadronic observables. Although it is questionable to use the hadronic degrees of freedom at very high energy densities, our results do indicate that a super-soft equation of state can produce the effects expected from the phase transition. Experiments on the measurement of the transverse collective flow at different incident energies will be extremely useful in our search for the signatures of chiral and deconfinement transitions.

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