High energy octupole resonance in $^{116}$Sn

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The region of excitation energy from $70E_x<38$ MeV in $^{116}$Sn was studied with inelastic scattering of 240 MeV $\alpha$ particles. Parameters obtained for the isoscalar giant monopole resonance and the isoscalar giant quadrupole resonance are in agreement with accepted values. A peak exhausting $67\pm10\%$ of the E3 energy-weighted sum rule at $E_x=21.8\pm0.5$ MeV with width $\Gamma=7.1\pm0.5$ MeV was identified as the isoscalar high energy octupole resonance (HEOR). The energy and width are consistent with previous measurements made with proton, $^3$He, and $\alpha$ particle projectiles. The observed strength of the HEOR is in agreement with measurements made with $^3$He inelastic scattering, but is roughly three to four times larger than measurements made with $\alpha$ particle and proton inelastic scattering. [S0556-2813(96)01407-0]

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I. INTRODUCTION

The high energy octupole resonance (HEOR) has been studied in a wide range of nuclei ranging from $40\leq A\leq238$ with a variety of projectiles including electrons [1–5], protons [6,7], $^3$He [8], $\alpha$ particles [9,10], and heavy ions [11,12]. The tin isotopes $^{116,118,120}$Sn have been investigated extensively and the energy-weighted sum rule (EWSR) strengths, widths ($\Gamma$), and excitation energies ($E_x$) obtained are shown in Figs. 1(a), 1(b), and 1(c), respectively. Figures 1(b) and 1(c) show that in the tin isotopes the excitation energy and width are consistently around 23 and 7 MeV, respectively. Figure 1(a) shows that the measured E3 strengths vary tremendously, and those obtained for $^{116}$Sn from inelastic $\alpha$ scattering ($\leq19\%$ of the E3 EWSR) are well below the expected values. Measurements made with 129 MeV inelastic $\alpha$ scattering on $^{116}$Sn identified $(31\pm6)\%$ of the E3 EWSR in a low lying $J^\pi=3^-$ state and the low energy octupole resonance (LEOR) [13]. The remainder of the E3 strength ($\sim70\%$) should reside in the HEOR.

Inelastic scattering of $\alpha$ particles at energies between 90 and 172 MeV has been very effective for investigating isoscalar giant resonances ($T=0$), in part because isovector giant resonances ($T=1$) are weakly excited. However, this beam energy range is not suitable for investigating the HEOR in $^{116}$Sn. At these energies, $\alpha$ particles from the pickup-breakup reactions ($\alpha$, $^3$He)→($\alpha$,a,n) and ($\alpha$, $^3$Li)→($\alpha$,a,p) show up as broad bumps in the continuum over the excitation region of the HEOR. At $E_\alpha=96$ MeV, $\alpha$ particles from these processes have energies between 59 and 79 MeV [14], and thus appear the same in $\alpha$ particle spectra as inelastic scattering peaks from states with excitation energies between 17 and 37 MeV, which clearly overlap the HEOR. As the bombarding energy is increased, the apparent excitation energy of the pickup-breakup peaks increases (this is the method used to distinguish them from inelastic excitations). However, even at 172 MeV, the pickup-breakup peaks begin at $E_x=26$ MeV, which overlaps the tail of the HEOR and may complicate the assessment of the continuum. $\alpha$ particle studies of the HEOR in $^{116}$Sn have been carried out at $E_\alpha=340$ and 480 MeV [10], where the pickup-breakup peaks appear above an equivalent excitation energy of 50 MeV and are not a factor. However, in these studies, very low EWSR strengths were obtained for even the well-known isoscalar giant quadrupole (GQR) and isoscalar giant monopole (GMR) resonances in $^{116}$Sn. Furthermore, isovector strength is more strongly excited at these energies [15], and the isovector quadrupole resonance at $E_x\sim27$ MeV [16] may make observation of the HEOR more difficult. We have therefore investigated the isoscalar HEOR in $^{116}$Sn with 240 MeV $\alpha$ particles. At this energy, the products from $^3$He-$^3$Li pickup-breakup reactions are well past the region of interest ($E_x\geq35$ MeV) and the contribution from isovector resonances should be negligible.

II. EXPERIMENTAL TECHNIQUE

Beams of 240 MeV $\alpha$ particles from the Texas A&M K500 superconducting cyclotron bombarded a self-supporting foil enriched to 95.74% in $^{116}$Sn in the target chamber of the multipole-dipole-multipole (MDM) spectrometer [17]. The target thickness, determined by weighing, was $5.74\pm0.57$ mg/cm$^2$. Scattered particles which passed the spectrometer acceptance were analyzed by the MDM spectrometer and were detected by the modified Oxford detector. Details of the detector and spectrometer were reported in Refs. [18–21] and [22], respectively.

Data on both elastic and inelastic scattering were taken at two separate times which utilized slightly different experimental setups. In the first run (set 1), data were acquired over the angle range $4^\circ\leq\theta\leq8^\circ$ using a solid-angle-defining collimator with $\Delta\theta=4^\circ$ and $\Delta\phi=2^\circ$. In the second run (set 2), data were acquired over the angle range $2^\circ\leq\theta\leq6^\circ$ using a solid-angle-defining collimator with $\Delta\theta=4^\circ$ and $\Delta\phi=4^\circ$. The newly installed beam analysis system (BAS) [23] was also utilized for data acquired in set 2, which helped to reduce beam halo and improve beam momentum resolution. For the giant resonance data, elastically scattered $\alpha$ particles were stopped at the detector entrance.

Ray tracing was used to determine the horizontal component of the scattering angle $\theta$, however, $\phi$ was not measured, so the effects of the different vertical openings were accounted for by angle averaging and will be discussed later. An angular resolution of $\Delta\theta=0.15^\circ$ was obtained by placing
a software window on angle deviation as described in Ref. [20]. The spectrometer angle was verified from the kinematic crossover of elastic scattering from hydrogen with $^{12}$C excited states. For a given spectrometer field setting, the focal plane position was calibrated in particle momentum from the locations of peaks from $^{12}$C excited states 7.65 to 24.0 MeV.

### III. DATA ANALYSIS

Each data set was divided into ten subsets of data (20 spectra in all), each corresponding to $\Delta \theta = 0.4^\circ$ using the angle obtained from ray tracing. A giant resonance spectrum taken at the average center-of-mass angle $\bar{\theta}_{\text{c.m.}} = 5.22^\circ$ is shown in Fig. 2(a). The average angle ($\bar{\theta}$) was obtained by integrating over the angle bin width and height (determined by the vertical dimension of the collimator). Absolute cross sections were determined by normalizing the elastic scattering data to optical-model calculations. Cross sections for the giant resonance data were obtained by normalizing the giant resonance spectra to the elastic spectra in the region of excitation energy between 10 and 20 MeV.

Optical-model calculations were performed with the code PTOLEMY [21,25] using $^{116}$Sn parameters [21] (listed in Table I) we obtained in a previous study. Angle-weighted-average cross sections were obtained for the different vertical dimensions of the collimators. Figure 3 shows the elastic scattering data and the optical-model calculations plotted versus average angle. The error bars include statistical error and angle uncertainty (summed in quadrature). The angular distributions obtained with different vertical dimensions of the collimators differ significantly only for $\bar{\theta}_{\text{c.m.}} < 2.5^\circ$.

The giant resonance data were analyzed by first subtracting a background/continuum and then performing a multiple peak fit. For each spectrum, several assumptions for the background/continuum were made to test the effects of different backgrounds. They were generally drawn as indicated in Fig. 2(a)] from high to low excitation energy as a smooth curve (fourth-order polynomial) that was extrapolated from the slope of the continuum at high excitation energy (beyond $\sim 30$ MeV) and then curved up to the minimum in the spectrum on the low excitation side. For the HEOR region ($E_x \geq 20$ MeV), the curves obtained with the different assumptions were nearly the same. They differed only in the

### Table I. Optical-model parameters [21] for $^{116}$Sn at $E_a = 240$ MeV.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V$ (MeV)</td>
<td>88.0</td>
</tr>
<tr>
<td>$W$ (MeV)</td>
<td>21.4</td>
</tr>
<tr>
<td>$R_v$ (fm)</td>
<td>5.89</td>
</tr>
<tr>
<td>$R_w$ (fm)</td>
<td>7.05</td>
</tr>
<tr>
<td>$a_v$ (fm)</td>
<td>0.83</td>
</tr>
<tr>
<td>$a_w$ (fm)</td>
<td>0.80</td>
</tr>
<tr>
<td>$R_c$ (fm)</td>
<td>6.34</td>
</tr>
</tbody>
</table>
region $E_x \leq 20$ MeV. The effects of the different background/continuum show mainly in the EWSR strengths and will be discussed later.

Spectra which did not contain the peak due to elastic scattering off hydrogen were fitted simultaneously with a multipeak-multispectra fitting routine with the requirement that all peaks have the same excitation energy and width in each spectrum [26]. Excellent fits to the spectra could be obtained with three peaks in addition to the well-known GMR and GQR. Figures 2(a) and 2(b) illustrate a spectrum taken at $\theta_{c.m.} = 5.22^\circ$ with and without background/continuum subtracted, and the results of the five-peak multispectra fit. The positions and widths from Ref. [27] were used as initial parameters for the GMR and GQR. The best multipeak-multispectra fit ($\chi^2 - 2$) was obtained for positions and widths of the GQR and GMR fixed at $E_x = 13.2$ MeV and $\Gamma = 3.3$ MeV for the GQR and $E_x = 15.6$ MeV and $\Gamma = 4.1$ MeV for the GMR [27]. The heights of the GQR and GMR as well as all parameters of the other three peaks were allowed to vary. The positions and widths obtained for the other peaks are given in Table II. For the spectra where elastic scattering off hydrogen in the target obscured the GMR and GQR peaks ($\theta_{c.m.} \geq 6.45^\circ$), three-peak single-spectra fits were performed to the excitation region $10 \leq E_x \leq 30$ MeV. In these spectra, the positions and widths of the $E_x = 21.8$ and 26.1 MeV peaks were fixed at the values obtained from the five-peak multispectra fit. The heights of these peaks and the parameters of the peak due to elastic scattering off hydrogen were free to vary. Spectra taken at $\theta_{c.m.} = 3.08^\circ$, 3.47$^\circ$, and 3.86$^\circ$ were excluded from the analysis because, at the spectrometer field setting used, elastically scattered $\alpha$ particles were deflected off the spectrometer exit and into the detector, which created a broad bump in the region of interest. This bump was well below the region of interest for $\theta_{c.m.} \geq 2.70^\circ$ spectra and well above the region of interest for spectra with $\theta_{c.m.} \geq 4.25^\circ$.

A four-peak multispectrum fit was also attempted. The GMR and GQR positions and widths were fixed at the values stated above. The rest of the parameters were allowed to vary, and the best fit was obtained with a narrow peak at 11.4 MeV and a broad peak ($\Gamma = 11.2$ MeV) at $E_x = 23.0$ MeV. The quality of the fit ($\chi^2 - 10$) was much worse than the five-peak fit because the broad peak had different shapes in the different spectra.

The angular distributions obtained for the peaks are shown in Fig. 4. The angular distribution of the narrow peak at $E_x = 11.4$ MeV was inconsistent and is not shown. Distorted-wave Born approximation (DWBA) calculations are also shown, and with the exception of the peak at $E_x = 26.1$ MeV, agree very well with the data. The error bars on the data points represent the combined uncertainty (summed in quadrature) due to statistics and the quality of the multipeak fit. The cross sections for the background/continuum in the region $15 \leq E_x \leq 30$ MeV from both data sets are also shown in Fig. 4, and the error bars are due statistical uncertainties only. The reduction in the cross section for the background/continuum in set 2 (closed circles) is due to the use of the BAS (which reduced beam halo) and the larger vertical dimension of the collimator (allowing $\Delta \phi = 4^\circ$, which reduced elastic slit scattering). The vertical dimension of the collimator was smaller (allowing $\Delta \phi = 2^\circ$) implies the values were fixed in the fits.

### Table II. Five-peak multispectra fit parameters and EWSR values from this work and results from other work on $^{116}$Sn.

<table>
<thead>
<tr>
<th>Work</th>
<th>Projectile, $E$ (MeV)</th>
<th>$J^\pi$</th>
<th>$E_x$ (MeV)</th>
<th>$\Gamma$ (MeV)</th>
<th>%EWSR</th>
</tr>
</thead>
<tbody>
<tr>
<td>Present</td>
<td>$\alpha$, 240</td>
<td>$3^-$</td>
<td>21.8$\pm$0.5</td>
<td>7.1$\pm$0.5</td>
<td>67$\pm$10</td>
</tr>
<tr>
<td>Ref. [10]</td>
<td>$\alpha$, 340</td>
<td></td>
<td>23.5$\pm$0.3</td>
<td>5.0$\pm$0.4</td>
<td>14$\pm$5</td>
</tr>
<tr>
<td>Ref. [10]</td>
<td>$\alpha$, 480</td>
<td></td>
<td>23.2$\pm$0.3</td>
<td>4.5$\pm$0.3</td>
<td>11.5$\pm$4</td>
</tr>
<tr>
<td>Ref. [8]</td>
<td>$^3$He, 110–140</td>
<td></td>
<td>24.4$\pm$1.5</td>
<td>7.0$\pm$1.5</td>
<td>74$\pm$20</td>
</tr>
<tr>
<td>Ref. [6]</td>
<td>$^{3}$H, 800</td>
<td></td>
<td>22.9$\pm$1.0</td>
<td>6.5$\pm$1.0</td>
<td>22$\pm$6</td>
</tr>
<tr>
<td>Present</td>
<td>$\alpha$, 240</td>
<td>$0^+$</td>
<td>15.6$^*$</td>
<td>4.1$^*$</td>
<td>100$\pm$15</td>
</tr>
<tr>
<td>Ref. [27]</td>
<td>$\alpha$, 129</td>
<td></td>
<td>15.6$\pm$0.3</td>
<td>4.1$\pm$0.3</td>
<td>180$\pm$60</td>
</tr>
<tr>
<td>Present</td>
<td>$\alpha$, 240</td>
<td>$2^+$</td>
<td>13.2$^*</td>
<td>3.3$^*$</td>
<td>95$\pm$20</td>
</tr>
<tr>
<td>Ref. [27]</td>
<td>$\alpha$, 129</td>
<td></td>
<td>13.2$\pm$0.2</td>
<td>3.3$\pm$0.2</td>
<td>84$\pm$24</td>
</tr>
<tr>
<td>Present</td>
<td>$\alpha$, 240</td>
<td></td>
<td>26.1$\pm$0.6</td>
<td>7.3$\pm$0.4</td>
<td></td>
</tr>
<tr>
<td>Present</td>
<td>$\alpha$, 240</td>
<td></td>
<td>11.4$\pm$0.4</td>
<td>0.4$\pm$0.1</td>
<td></td>
</tr>
</tbody>
</table>
Formed with the code PTOLEMY with internal collective form factors \([U(r)]\) of the usual form,

\[
U(r) = R \frac{dU_0(r)}{dr},
\]

where \(R\) is the optical-model nuclear radius and \(U_0 = V + iW\) is the optical-model potential with the real \((V)\) and imaginary \((W)\) potentials of the Woods-Saxon form. For such states located at excitation energy \(E_x\) that exhaust 100% of the EWSR strength, the theoretical deformation length is

\[
\beta_l^2 R^2 = L(2L + 1) \frac{\hbar^2}{2mE_x} \frac{4\pi}{3A},
\]

where \(\beta_l\) is the deformation parameter and \(L\) is the angular momentum of the state. Recently, Beene, Horen, and Satchler [28] showed that the deformed optical potential model does not correctly predict \(E3\) strength for low-lying states excited by \(^{17}\)O inelastic scattering; however, the value we obtained [21] using Eqs. (3.1) and (3.2) for the \(E3\) deformation length of the 2.266 MeV state in \(^{116}\)Sn measured with 240 MeV inelastic \(\alpha\) scattering agrees with values obtained from inelastic electron scattering [29,30].

The isovector giant dipole resonance (IVGDR) calculation was also performed with PTOLEMY by reading in an external form factor. The form factor and EWSR for the IVGDR are described in Ref. [31]. For convenience, the GMR and isoscalar giant dipole resonance (ISGDR) calculations were performed with the code DWUCK4 [32] using internal form factors. A comparison of calculations with DWUCK4 and PTOLEMY for 240 MeV \(\alpha\) particles is described in Ref. [21]. The GMR and ISGDR form factors are expressed as

\[
U(r) = -3U_0(r) - r \frac{dU_0(r)}{dr} \quad \text{for } L = 0,
\]

\[
U(r) = \frac{1}{R \sqrt{3}} \left(-3r^2 \frac{dU_0(r)}{dr} - 10rU_0(r)
\right. \\
\left. + \frac{5}{3} \langle r^2 \rangle_{\text{WS}} \frac{dU_0(r)}{dr}\right) \quad \text{for } L = 1,
\]

from Refs. [33] and [34], respectively, where \(\langle r^2 \rangle_{\text{WS}}\) is the mean squared radius of the Woods-Saxon optical-model potential. For the GMR and ISGDR which exhaust the EWSR limit, the theoretical deformation lengths are expressed as

\[
\beta_l^2 R^2 = \frac{\hbar^2}{2mE_x} \frac{20\pi}{3A} \quad \text{for } L = 0,
\]

\[
\beta_l^2 R_{1/2}^2 = \frac{6\pi\hbar^2 R_{1/2}^4}{mAE_x} (11\langle r^4 \rangle_{\text{FM}} - \frac{24}{7} \langle r^2 \rangle_{\text{FM}}^2)^{-1} 
\]

for \(L = 1\),

from Refs. [33] and [34], respectively, where \(R_{1/2}\) and \(\langle r^n \rangle_{\text{FM}}\) are the half-density radius and mean radius to the \(n\)th power of the Fermi mass distribution. For both the Woods-Saxon

![Figure 4](Image)

**FIG. 4.** Angular distributions of the differential cross sections of the peaks and the underlying background/continuum plotted versus the average center-of-mass angle. The curves represent DWBA calculations for the indicated resonances where the solid lines and dashed lines have been angle averaged for \(\Delta \phi = 2^\circ\) and \(4^\circ\), respectively. The solid triangles and circles represent the background/continuum from set 1 and set 2, respectively.
IV. DISCUSSION

We obtained consistent fits to the spectra over the excitation energy range 11 ≤ Eₓ ≤ 30 MeV with five peaks, of which three could be identified as the GMR, GQR, and HEOR. The strengths obtained for the GMR and GQR were (100±15)% of the E0 EWSR and (95±20)% of the E2 EWSR, respectively, and are consistent with the results for ¹¹⁶Sn from Refs. [10] and [27]. The strength obtained for the HEOR was (67±10)% of the E3 EWSR. In other ¹¹⁶Sn work, (31±6)% of the E3 EWSR has been identified in the sum of the low lying Jπ=3− state and the LEOR [13]. Therefore the strength of these states plus our strength for the HEOR then exhaust roughly 100% of the E3 EWSR. The width (Γ=7.1 MeV) of the HEOR is also in good agreement with previous measurements on the tin isotopes performed with protons [6,7], ³⁷He [8], and α particles [9].

Bonin et al. [10] report much lower strength for the HEOR [(14±5)% and (11.5±4)%] in ¹¹⁶Sn from 340 and 480 MeV inelastic α scattering, but they also only see 43% and 49% of the E2 EWSR in the GQR and 28% and 22% of the E0 EWSR in the GMR, where ~100% of each are observed in our measurement and are reported in numerous other experiments. Carey et al. [6] report 22% of the E3 EWSR in the HEOR in ¹¹⁶Sn using 800 MeV inelastic proton scattering, but do not report the strengths they obtained for the other resonances they observed. This is particularly important, since the GMR overlaps the HEOR, and the two resonances can only be separated by a multiple peak fitting routine. However, Bertrand et al. [7], also using inelastic proton scattering, report similar low values of the E3 EWSR for the HEOR in ¹¹⁸Sn but also see only ~35% of the E2 EWSR in the GQR.

The position we obtained for the HEOR (Eₓ=21.8 MeV) is slightly lower than previous measurements (Eₓ~23.5 MeV); however, the peak at Eₓ=26.1 MeV has not been previously reported in ¹¹⁶Sn. If the excitation region containing these two peaks is fitted by one peak, the position and width are Eₓ=23.0 MeV and Γ=11.2 MeV, respectively. This position is consistent with the previous average for the HEOR, but the width is much too large. Furthermore, as the angular distributions for the 21.7 and 26.1 MeV states differ, the χ² for the fit of the broad peak becomes considerably worse. As shown in Fig. 4, the ISGDR DWBA calculation fits the angular distribution for the 26.1 MeV peak above θc.m.=4°, but the θc.m.=2.7° data point is much too low. From the present data we cannot clearly identify the 26.1 MeV peak as the ISGDR due to this discrepancy, and also since the cross section for this peak at large angles has large errors, as it is near the detector cutoff. A future measurement in ¹¹⁶Sn and other nuclei, in this excitation region and over the angles 1°≤θ_p≤8°, is presently being planned. If the peak is indeed the ISGDR it should be much stronger than the HEOR at small angles θc.m.<3°.

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[12] R. L. Neto, P. Roussel-Chomaz, L. Rochais, N. Alamanos, F. Auger, B. Fernandez, J. Gastebois, A. Gillibert, R. Lacey, A. Miczaika, D. Pierre-out, J. Barrett, S. K. Mark, R. Turcotte, Y. Blumenfeld, N. Frascaria, J. P. Roynette, J. A. S particular importance, since the GMR overlaps the HEOR, and the two resonances can only be separated by a multiple peak fitting routine. However, Bertrand et al. [7], also using inelastic proton scattering, report similar low values of the E3 EWSR for the HEOR in ¹¹⁸Sn but also see only ~35% of the E2 EWSR in the GQR.

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