Astrophysical factor for the radiative capture reaction $\alpha+d\rightarrow{}^6\mathrm{Li}+\gamma$

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We consider the radiative capture process $\alpha+d\rightarrow{}^6\mathrm{Li}+\gamma$ at energies, $E_{\mathrm{c.m.}}\leqslant 300$ keV, that are relevant for astrophysical processes. Due to the peripheral character of the reaction, the overall normalization of the astrophysical factor S_{24} is entirely governed by one quantity, the asymptotic normalization coefficient C_{01} for ${}^6\mathrm{Li}\rightarrow\alpha+d$. Using the recently well established value for this constant $C_{01}=2.3\pm0.12$ fm$^{-1/2}$, we calculated S_{24} taking into account both $E1$ and $E2$ contributions. Our recommended value for S_{24} is 2.57 MeV nb at the most effective energy for the capture reaction in astrophysical processes, $E_{\mathrm{c.m.}}=70$ keV, which gives a reaction rate 0.036 cm^3 mole^{-1} s^{-1} at the temperature 0.8×10^9 K. We found a significant energy dependence of S_{24} at astrophysical energies. At energies of less than 110 keV, the $E1$ component dominates over the $E2$ component. At $E_{\mathrm{c.m.}}=70$ keV, the $E1$ contribution to the total transition is about 58%.

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The radiative capture reaction

$$\alpha+d\rightarrow{}^6\mathrm{Li}+\gamma \tag{1}$$

is the only process that produces ^{6}Li in the big bang model. The astrophysically important α-d relative kinetic energies for the process are $E_{\mathrm{c.m.}}\leqslant 300$ keV [1]. The available experimental data for this reaction cover the region 100 keV $\leqslant E_{\mathrm{c.m.}}\leqslant 4$ MeV [2,3]. The low energy experimental data have been obtained indirectly from measurements using the Coulomb breakup process ${}^6\mathrm{Li}+{}^{208}\mathrm{Pb}\rightarrow\alpha+d+{}^{208}\mathrm{Pb}$ [3]. The most recent theoretical calculations of the low energy cross section for reaction (1) have been performed within the framework of the microscopic resonating group method (RGM) [4], in the quasimicroscopic potential model [5,6], and in the framework of the multicluster dynamic model [7]. The importance of the $E1$ capture at astrophysically relevant energies was noted in Refs. [2,4–7]. Despite the fact that the $E1$ transition is forbidden in the long wavelength approximation, the $E1$ multipole can dominate over the $E2$ [5,7] because of the significant difference between the mass of the free α particle and twice the mass of the free deuteron at low energies.

Because of the small binding energy of ^{6}Li in the two-body channel $\alpha+d$, the capture reaction becomes almost totally peripheral at low energies and the α particle and d in ^{6}Li can be treated as free particles, including their mass defect. Peripheral radiative capture processes possess an important feature: The reaction amplitude is completely determined by the tail of the bound state wave function of the final nucleus in the two-body channel corresponding to the colliding nuclei [8–10]. However, the work in Refs. [4–6] did not take this fact into account. The bound state wave function of ^{6}Li used in Refs. [5,6] was taken from Ref. [11] where it had been derived in the three-body $(\alpha+n+p)$ model. The overlap integral derived with the wave function taken from [11] did not possess the proper asymptotic behavior (correct r dependence and normalization). The same shortcoming occurs for the α-d relative wave function in the RGM [4], which is approximated by the sum of Gaussian functions when solving the Schrödinger equation. The most crucial problem when calculating peripheral astrophysical radiative capture processes is to properly normalize the tail of the wave function since the overall normalization of the S factor for such reactions is governed only by the amplitude of the tail or, equivalently, the asymptotic normalization coefficient (ANC) of the corresponding overlap function [8–10].

Recently, significant progress has been achieved in the derivation of the ANC C_{01} (the quantum numbers 0 and 1 are defined below) for the virtual decay ${}^6\mathrm{Li}\rightarrow\alpha+d$ [12] . This ANC has been found to be $C_{01}=2.3\pm0.12$ fm$^{-1/2}$, using the analytic continuation of the elastic α-d $^3S_1$ partial scattering amplitude to the pole in the $E_{\mathrm{c.m.}}$ plane corresponding to the ^{6}Li ground state. The result of the analytic extrapolation has been checked in Ref. [12] by three other techniques: first by directly solving the inverse problem for α-d scattering and then by two different methods of finding a solution for the three-body $(\alpha+n+p)$ Faddeev equations. There was an independent derivation of the ANC for ${}^6\mathrm{Li}\rightarrow\alpha+d$ [13] using the three-body variational bound state wave function of ^{6}Li [11] and the method developed in Ref. [14]. All these calculations confirmed the result from the analytic continuation. Thus the value of C_{01} seems to be very reliable and can be used in practical calculations.

An immediate use of this result in the calculation of the astrophysical factor of the capture reaction (1), $S_{24}(E_{\mathrm{c.m.}})$, at very low $E_{\mathrm{c.m.}}$ has been indicated in a paper by one of us [10]. This idea has been used in [7], where the cross section for $\alpha+d$ radiative capture has been calculated within the framework of the multicluster dynamic model with the correct tail of the ^{6}Li bound state wave function in the $\alpha+d$ channel. In this paper, we have used the idea in Ref. [10] to calculate $S_{24}(E_{\mathrm{c.m.}})$ and its energy dependence for astrophysically relevant energies using the well-defined value of the ANC C_{01}. Simultaneously, we investigate the relative contribution of $E1$ and $E2$ transitions to $S_{24}(E_{\rm c.m.})$ and compare theoretical predictions with the low energy experimental data [3]. In contrast to Ref. [7], we present the detailed investigation of the $S_{24}(E_{\rm c.m.})$ behavior at astrophysical energies, give the recommended value for S_{24} (70 keV), and discuss the problems in the low energy experimental data derived from the Coulomb breakup [3].

In the long wave approximation, the cross section $\sigma(E_{\rm c.m.})$ of the direct radiative capture reaction is given by the sum of the cross sections $\sigma(E\lambda;E_{\rm c.m.})$ with $\lambda=1,2$ corresponding to the $E1$ and the $E2$ transitions [4]:

$$\sigma(E_{\rm c.m.})=\sigma(E1;E_{\rm c.m.})+\sigma(E2;E_{\rm c.m.}). \qquad (2)$$

The astrophysical factor is related to the corresponding cross section by $S(E\lambda;E_{\rm c.m.})=E_{\rm c.m.}\exp(2\pi\eta_i)\sigma(E\lambda;E_{\rm c.m.})$, where $\eta_i=Z_\alpha Z_d e^2\sqrt{\mu/2E_{\rm c.m.}}$ is the Coulomb parameter for α and d in the initial channel, $Z_\alpha e$ and $Z_d e$ are their charges, and μ is their reduced mass, with $\hbar=c=1$. Possible corrections to the long wave approximation for the $E1$ transition due to the spin-dependent part of the $E1$ operator and retardation effects are very small [5]. The influence of the internal quadrupole moments of d and ^{6}Li is found to be insignificant [15]. The $M1$ multipole is also practically negligible in this reaction [5]. Consequently, we take into account only the expression for the cross section in the classical long wave approximation. The cross section $\sigma(E\lambda;E_{\rm c.m.})$ is proportional to $B(E\lambda;E_{\rm c.m.})$ [4], which is expressed in terms of the matrix element

$$M^{\lambda}_{l_f l_i J_i}=A_\lambda\int_{R_c}^{\infty}dr I^{^6\rm Li}_{\alpha d\,01}(r)r^{2+\lambda}\psi_{l_i J_i}(r), \qquad (3)$$

where

$$A_\lambda=\mu^\lambda\left(\frac{1}{m_d^\lambda}+\frac{2(-1)^\lambda}{m_\alpha^\lambda}\right). \qquad (4)$$

Here m_j is the mass of the particle j; R_c is the cutoff radius introduced to check the peripheral character of the reaction; $I^{^6\rm Li}_{\alpha d\,01}(r)=\langle\phi_\alpha\phi_d|\phi_{^6\rm Li}\rangle$ is the radial overlap function of the bound state wave functions of ^{6}Li, α, and d; and r is the distance between the centers of mass of the α particle and the d. The only bound state of ^{6}Li has relative α-d angular orbital momentum $l_f=0$ and spin parity $J_f^\pi=1^+$. The wave function $\psi_{l_i J_i}(r)$ describes the relative α-d motion in the continuum with the relative angular orbital momentum l_i and the total angular momentum J_i. When calculating the cross section, we took into account all the possible initial angular momenta including $J_i^\pi=3^+, 2^+, 1^+$ for the $E2$ transition arising from $l_i=2$ and $J_i^\pi=2^-, 1^-, 0^-$ for the $E1$ transition arising from $l_i=1$ [4]. When deriving the matrix elements for the $E1$ and $E2$ transitions, only the parts of the $E1$ and $E2$ electromagnetic transition operators which depend on the relative α-d coordinate r have been taken into account due to the peripheral character of the capture reaction at astrophysically relevant energies. To calculate the $E1$ transition, the exact nuclear masses of the α and d must be used in the factor A_1 [4,5].

The most crucial part of the calculation is the overlap function $I^{^6\rm Li}_{\alpha d\,01}(r)$. Because of the peripheral character of the reaction under consideration, only the asymptotic part of the overlap function contributes to the radial integral in Eq. (3). This will be explicitly demonstrated below in our calculations. The asymptotic behavior of the overlap function is well known and given by

$$I^{^6\rm Li}_{\alpha d\,01}(r)\approx C_{01}\frac{W_{-\eta,1/2}(2\kappa r)}{r}, \qquad (5)$$

where $W_{-\eta,l_f+1/2}(2\kappa r)$ is the Whittaker function, $\eta=Z_\alpha Z_d e^2\mu/\kappa$, $\kappa=\sqrt{2\mu\varepsilon}$, and $\varepsilon=1.475$ MeV is the binding energy of ^{6}Li in the channel $\alpha+d$. In our calculations $I^{^6\rm Li}_{\alpha d\,01}(r)$ has been approximated by

$$I^{^6\rm Li}_{\alpha d\,01}(r)=J_{01}^{1/2}\phi_0(r). \qquad (6)$$

Here J_{01} is the spectroscopic factor of the configuration $\alpha+d$ in ^{6}Li, $\phi_0(r)$ is the single-particle wave function of the bound state α-d calculated in a Woods-Saxon potential with the standard parameters, $r_0=1.20$ fm, and $a=0.70$ fm. No spin-dependent term appears in the case under consideration. The depth of the potential was found using the well-depth procedure. It is very important to stress that the calculated S factor does not depend on the form of the adopted α-d potential and, hence, on the model approximation of Eq. (6). It is almost entirely defined by the tail of the overlap function $I^{^6\rm Li}_{\alpha d\,01}(r)$ or by the tail of its approximation $J_{01}^{1/2}\phi_0(r)$. To provide the correct normalization of that tail, the spectroscopic factor J_{01} used satisfied the asymptotic relation

$$J_{01}^{1/2}\phi_0(r)\approx C_{01}\frac{W_{-\eta,1/2}(2\kappa r)}{r}, \quad r\geqslant R_N, \qquad (7)$$

where R_N is the α-d nuclear interaction radius. It follows from Eqs. (5)–(7) that

$$C_{01}=J_{01}^{1/2}b_0, \qquad (8)$$

where b_0 is the ANC of the single-particle wave function $\phi_0(r)$. For the parameters chosen for the Woods-Saxon potential, the spectroscopic factor is $J_{01}=0.733$. It is clear from Eqs. (3), (6), and (7) that for a peripheral reaction the normalization of the matrix element $M^{\lambda}_{l_f l_i J_i}$ and, hence the cross section, for both the $E1$ and $E2$ transitions is entirely defined by the ANC C_{01} and not by the individual factors J_{01} and b_0. Because of the peripheral character of the capture reaction, the S factor is not sensitive to the details of the adopted α-d optical potential in the initial channel. We used the conventional Woods-Saxon form [6] with the depth of the central part $V_0=74.23$ MeV for $l_i=1$ and $V_0=79.23$ MeV for $l_i=2$, the spin-orbit part $V_{\rm s.o.}=3.305$ MeV, $r_0=r_C=1.85$ fm, and $a=0.71$ fm.

The results for S_{24}, for different energies of astrophysical interest, are given in Table I and Fig. 1. These show a strong energy dependence of $S_{24}(E_{\rm c.m.})$ at astrophysical energies, confirming the results of [4,5]. It is clear from Fig. 1 that $S_{24}(E2;E_{\rm c.m.})$ has a stronger energy dependence than

TABLE I. Calculated astrophysical factors S_{24} as a function of $E_{\rm c.m.}$ and cutoff radius R_c; $S_{24}(E\lambda;E_{\rm c.m.})$ is the astrophysical factor for the $E\lambda$ transition; $S_{24}(E_{\rm c.m.})$ is the total astrophysical factor for $E1+E2$ transitions; $S_{24}^{\rm as}(E_{\rm c.m.})$ is the astrophysical factor for $E1+E2$ transition calculated with use of the tail of the radial overlap function $C_{0\,1}W_{-\eta,1/2}(2\kappa r)/r$.

E (keV)	R_c (fm)	Astrophysical factors (MeV nb)			
		$S_{24}(E1;E_{\rm c.m.})$	$S_{24}(E2;E_{\rm c.m.})$	$S_{24}(E_{\rm c.m.})$	$S_{24}^{\rm as}(E_{\rm c.m.})$
10	0	1.12	0.33	1.45	
	3.0	1.12	0.33	1.45	
	4.0	1.09	0.32	1.41	
	4.5	1.06	0.32	1.38	1.38
70	0	1.50	1.07	2.57	
	3.0	1.50	1.06	2.56	
	4.0	1.45	1.04	2.49	
	4.5	1.41	1.03	2.44	2.44
100	0	1.73	1.60	3.33	
	3.0	1.73	1.59	3.32	
	4.0	1.68	1.56	3.24	
	4.5	1.62	1.55	3.17	3.17
210	0	2.42	4.19	6.60	
	3.0	2.42	4.14	6.56	
	4.0	2.32	4.05	6.38	
	4.5	2.24	4.00	6.24	6.25
300	0	2.94	7.21	10.15	
	3.0	2.93	7.096	10.03	
	4.0	2.80	6.90	9.70	
	4.5	2.69	6.79	9.48	9.5

$S_{24}(E1;E_{\rm c.m.})$. The $E1$ transition dominates over the $E2$ at the energies of less than 110 keV as derived in [7]. At the most effective astrophysical energy $E_{\rm c.m.}\approx 70$ keV [2], the $E1$ transition contributes about 58% to the total transition. At an energy of $E_{\rm c.m.}=300$ keV, which is also astrophysically relevant, the $E1$ capture gives about 30% of the total capture cross section. In that aspect, our results for the $E1$ contribution confirm the conclusion that the contribution of the isoscalar $E1$ transition is very important at energies of astrophysical interest despite the fact that the $E1$ transition is forbidden in the long wavelength approximation [4,5,7]. However, the relative contribution of the $E1$ transition in our calculations and the absolute magnitude of our cross section are significantly higher than the results from [4] (Fig. 1). The reason lies in the peripheral character of the reaction under consideration. Since in [4–6] the α-d bound state wave function was taken as the sum of the Gaussians which die off very quickly, the contribution of the peripheral part of the integral to the capture rate was suppressed compared to our calculations where the tail of the wave function has been taken to be the correct Whittaker form. Since the tail of the α-d bound state wave function in [6] is closer to the realistic tail than that used in [4], the S_{24} calculated in [6] is closer to ours than that found in [4].

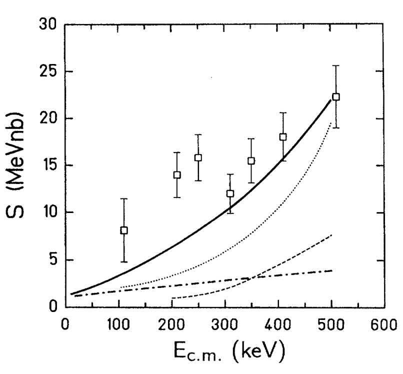

FIG. 1. The S_{24} as a function of the α-d relative kinetic energy $E_{\rm c.m.}$. The squares are the experimental data [3,16]. The solid line indicates our calculation for $E1+E2$ transitions. The dash-dotted line is our calculation for $E1$ transition only. The dashed line displays the calculations for $E1+E2$ transitions from [4]. The dotted line shows the results for $E1+E2$ transitions from [6].

To confirm our approach, we calculated the dependence of the $S_{24}(E\lambda;E_{\rm c.m.})$ and $S_{24}(E_{\rm c.m.})$ on the cutoff radius at different energies. The results, shown in Table I, explicitly demonstrate that at astrophysical energies the contribution of the nuclear interior to the capture reaction is negligibly small. For example, at $E_{\rm c.m.}=70$ keV, the contribution of the region $r\leqslant 4.5$ fm to the S factor is less than 5%. At $r\geqslant 4.5$ fm the difference between the model radial overlap function $J_{01}^{1/2}\phi_0(r)$ and its tail $C_{0\,1}W_{-\eta,1/2}(2\kappa r)/r$ is only about 4%. That is why the S factor, $S_{24}^{\rm as}(E_{\rm c.m.})$, calculated with $R_c=4.5$ fm and with the overlap integral approximated by Eq. (5) (the last column of Table I), practically coincides with the exact $S_{24}(E_{\rm c.m.})$ (the fifth column of Table I) calculated with the same cutoff radius. Both of these S factors are very close to the exact result obtained without any cutoff. We note that the dominant contribution to the matrix element at $E_{\rm c.m.}=70$ keV comes from the region $r\geqslant 4.5$ fm.

In Fig. 1, we compare the calculated S factor with data at energies relevant to astrophysical processes. The only available data at low energies are those found in [3] from analysis of the Coulomb breakup $^6\text{Li}+{}^{208}\text{Pb}\rightarrow\alpha+d+{}^{208}\text{Pb}$. These data have about 30% uncertainty in the absolute normalization and about 15% statistical uncertainty. In addition, the experimental data are different for different branches: $v_\alpha \leqslant v_d$ and $v_d \leqslant v_\alpha$. In Fig. 1 we show only the results of an updated analysis of Coulomb breakup data for the branch $v_d \leqslant v_\alpha$ (v_d and v_α are the velocities of the outgoing deuteron and α particle after Coulomb breakup of the projectile ^{6}Li). These data were kindly provided by Kiener [16].

Our calculations show an explicit energy dependence in low energy region and are in fair agreement with data points [3] for $E_{\text{c.m.}} \geqslant 300$ keV. Moreover, our calculations also reproduce the higher energy data including the resonance region [2]. However, here we only give the results of the calculations in the low energy region relevant for nuclear astrophysics where the capture reaction is almost entirely peripheral. In our calculations both $E1$ and $E2$ transitions are important. We note that in Coulomb breakup the $E1$ relative contribution is suppressed compared to $E2$ [3]. Hence, at energies where $E1$ and $E2$ contributions are comparable, Coulomb dissociation will not provide an accurate determination of S_{24}. However, we can conclude from our calculations (Fig. 1) that for energies $E_{24} \geqslant 300$ keV, where $E2$ capture dominates over $E1$, Coulomb breakup can provide reliable information about S_{24}. We note that in the analysis of the Coulomb breakup data in work [3], it was assumed that only the $E2$ transition occurs [3]. Hence we would expect that at low energies $E_{24} < 300$ keV the experimental data should be lower than our calculations.

In summary, the $\alpha+d$ capture reaction is almost entirely peripheral at astrophysical energies. Hence, the overall normalization of the cross section of the direct radiative capture process is totally defined by only one quantity, the ANC for $^6\text{Li}\rightarrow\alpha+d$. At energies important for astrophysical processes, $E_{\text{c.m.}} \leqslant 300$ keV, due to the peripheral character of reaction (1), the overlapping of the clusters α and d is negligibly small. Hence, to calculate the S_{24} factor with an uncertainty of a few percent, it is sufficient to use either the model overlap integral in the form of Eq. (6) or simply to use its tail given by Eq. (7). Since the ANC we used has been derived from the α-d elastic scattering data, the results given here for $S_{24}(E_{\text{c.m.}})$ can be considered as an "indirect measurement" of the astrophysical factor for reaction (1). Taking into account the accuracy of the extracted ANC C_{01} in Ref. [12], the approximations made in the model overlap integral, and in the electromagnetic operators, neglecting contributions from the quadrupole moments of d and ^{6}Li, we conclude that the overall uncertainty of the calculated astrophysical factor S_{24} at the energy 70 keV, is about 12%. We find a strong energy dependence of the astrophysical factor $S_{24}(E_{\text{c.m.}})$ at astrophysically relevant energies and importance of the $E1$ transition. Our recommended values for $S_{24}(E_{\text{c.m.}})$ are given in the fifth column of Table I (solid line in Fig. 1). Our calculated reaction rate at the temperature $T=0.8\times10^9$ K is given by $Y=0.036$ cm^3 mole^{-1} s^{-1}. Our results confirm the conclusion of [2] that the production rate of ^{6}Li via the reaction (1) in the big bang is a small contribution to the abundance of the universal ^{6}Li, which is presumably produced mainly by the galactic cosmic rays, for example, as a result of bombardment of high energy protons on different heavier elements, so-called high energy spallation reactions [17].

Finally, we would like to draw attention to the fact that the use of the ANC for normalizing the cross sections is an extremely powerful tool for nuclear astrophysics. In particular, one ANC for $^6\text{Li}\rightarrow\alpha+d$ defines the overall normalization of both $E1$ and $E2$ transitions. Thus the use of the ANC to calculate the S_{24} factor has some advantage compared, for example, to determining it from Coulomb breakup, since the relative contribution from $E1$ capture is suppressed in Coulomb breakup compared to that in the direct radiative capture. In principle, the Coulomb post-acceleration effects can also cloud the analysis of some Coulomb breakup data. Of course, for ^{6}Li breakup post-acceleration effects should be negligible because the charge/mass ratio for the fragments α and d is practically the same. Contributions from nuclear processes also complicate the interpretation of the breakup data. It clearly would be useful to repeat the ^{6}Li breakup experiment to extract more accurate information about astrophysical factor S_{24} at energies $E_{\text{c.m.}} \geqslant 300$ keV, where the $E2$ transition dominates over $E1$.

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[1] D. N. Schramm and R. V. Wagoner, Annu. Rev. Nucl. Sci. **27**, 37 (1977).
[2] R. G. Robertson *et al.*, Phys. Rev. Lett. **47**, 1867 (1981).
[3] J. Kiener *et al.*, Phys. Rev. C **44**, 2195 (1991).
[4] S. Typel *et al.*, Z. Phys. A **339**, 335 (1991).
[5] N. A. Burkova *et al.*, Phys. Lett. B **248**, 15 (1990).
[6] M. A. Jusupov *et al.*, Izv. Akad. Nauk Kaz. SSR (Ser. Fiz. Mat.) **2**, 30 (1991).
[7] G. G. Ryzhikh, R. A. Eramzhyan, and S. Shlomo, Phys. Rev. C **51**, 3240 (1995).
[8] A. M. Mukhamedzhanov and N. K. Timofeyuk, JETP Lett. **51**, 282 (1990).
[9] H. M. Xu *et al.*, Phys. Rev. Lett. **73**, 2027 (1994).
[10] L. D. Blokhintsev and A. M. Mukhamedzhanov, in *Proceedings of the 14th International Union of Pure and Applied Physics Conference on Few-body Problems in Physics*, Williamsburg, 1994, edited by F. Gross (AIP Press, New York, 1995), p. 801.
[11] V. I. Kukulin *et al.*, Nucl. Phys. **A417**, 128 (1984); **A453**, 365 (1986).

[12] L. D. Blokhintsev *et al.*, Phys. Rev. C **48**, 2390 (1993).
[13] A. M. Mukhamedzhanov *et al.*, Phys. At. Nucl. **56**, 937 (1993).
[14] A. M. Mukhamedzhanov and N. K. Timofeyuk, Sov. J. Nucl. Phys. **51**, 431 (1990).
[15] K. Langanke and C. Rolfs, Z. Phys. A **325**, 193 (1994).
[16] J. Kiener (private communication).
[17] W. A. Fowler, J. L. Greenstein, and F. Hoyle, Geophys. J. R. Astron. Soc. **6**, 148 (1962).