Reconstruction of an entangled state in a cavity via Autler-Townes spectroscopy

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We present a scheme to reconstruct a two-mode entangled state in a high-Q cavity from the spontaneous emission spectrum in a driven four-level atomic system. The two modes of the cavity field drive the upper three levels of the atom and thus the spontaneous emission spectrum contains the information about the photon statistics of the cavity field. Wigner function of the driving field is recovered from the photon statistics in a straightforward manner.

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Entanglement and nonlocality are some of the most emblematic concepts embodied in quantum mechanics [1]. The nonlocal character of an entangled system is usually manifested in quantum correlations between subsystems that have interacted in the past but are no longer interacting. The existence of nonlocal correlations, or entanglement between parts of a composite system is at the heart of quantum information theory [2–5]. In this Brief Report we present a scheme for the reconstruction of a two-mode entangled state in a high-Q cavity as

$$|\psi(AB)\rangle=\sum_{n_1,n_2=0}^{N-1} C_{n_1,n_2}|n_1\rangle_A|n_2\rangle_B, \tag{1}$$

where C_{n_1,n_2} is the probability amplitude of having n_1 photons in mode A and n_2 photons in mode B.

The quantum state of a radiation field whether single-mode or multimode, is completely described by the state vector $|\psi\rangle$ for a pure state and by the density operator ρ for a mixed state. There are also some other representation in terms of quasiprobability distributions such as P representation, Q representation, and Wigner function. One of the most fundamental problems of state measurement is the reconstruction of full information of the quantum state of a given field. In general, we cannot measure precisely the quantum state in a single experiment. Instead, we perform different experiments on identically prepared objects. Then we may infer the quantum state from the recorded statistical distributions of measured quantities. This idea was experimentally realized [6,7] in a quantum optical system proposed by Vogel and Risken [8]. In their scheme quadrature distributions of equally prepared light pulses were measured by homodyne detection and from the set of quadrature distribution, Wigner function was reconstructed. Some other methods have also been proposed for the measurement of the field distribution function [9].

Most of these schemes are for the quantum state reconstruction of a single-mode field in high-Q cavities. There are a few schemes also for the reconstruction of a multimode field inside a cavity [10–12]. Recently a scheme has been presented for the reconstruction of an entangled state in a cavity [12]. They used the idea that probability of atomic inversion after a two-level atom interacts with a cavity field is directly related to the Wigner characteristic function [13]. But under their approximations, the reconstructed Wigner function does not contain all the information of the field. We propose another scheme to reconstruct the Wigner function of an entangled state, which contains full information of the field. For this purpose we used the idea that quantum state of radiation field can also be measured by measuring the absorption and emission spectrum [14] in a driven system. In the presence of driving fields, the atomic levels display Stark splitting proportional to the Rabi frequency of the driving field. The emission or absorption spectra then have peaks, which are displaced from the resonance by the Rabi frequency. For a quantized driving field, the associated Rabi frequencies are distributed according to the photon distribution function from which we can recover the photon statistics. Wigner function can then be reconstructed from the knowledge of photon statistics in a straightforward manner [15].

For the determination of photon statistics of an entangled state (1) we propose to use Autler-Townes spectroscopy [16]. Consider an entangled field state (1) inside a high-Q cavity containing n_1 photons in cavity mode A and n_2 photons in cavity mode B, as shown in Fig. 1. Photon distribution function of this field can be obtained by sending a four-level atom initially prepared in level $|a\rangle$, which interacts with two modes of the cavity field. The atomic transition $|c_1\rangle-|a\rangle$ is resonant with cavity mode A while transition $|c_2\rangle-|a\rangle$ is resonant with cavity mode B. The interaction time t of the atom with the field is kept large as compared to the inverse of decay rate γ of the atom so that during the passage of the atom through the cavity, the atom radiates spontaneously from level $|a\rangle$ to $|b\rangle$ at a rate γ. The frequency of the spontaneously emitted photon is measured. A complete spectrum is obtained by repeating the experiment a number of times. The spectrum contains the informations about photon statistics of the entangled cavity field, which can be recovered by the procedure mentioned below.

We start with the interaction picture Hamiltonian in the dipole and rotating-wave approximation as

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$$H=\hbar\kappa_1(|c_1\rangle\langle a|a_1+a_1^\dagger|a\rangle\langle c_1|)+\hbar\kappa_2(|c_2\rangle\langle a|a_2+a_2^\dagger|a\rangle \times\langle c_2|)+\hbar\sum_k (g_k e^{i\delta_k t}|a\rangle\langle b|b_k+g_k^* e^{-i\delta_k t}b_k^\dagger|b\rangle\langle a|). \quad (2)$$

This Hamiltonian contains three parts. First, two parts are for the interaction of the upper three levels of the atom with two modes of cavity field having vacuum Rabi frequencies κ_1 and κ_2 for each mode, whereas $a_1(a_1\dagger)$ and $a_2(a_2^\dagger)$ are the annihilation (creation) operators of the two modes A and B, respectively. The third part is for the interaction of level $|a\rangle-|b\rangle$ with reservoir mode k having coupling constant g_k and annihilation (creation) operator $b_k(b_k^\dagger)$. The frequency of the spontaneously emitted photon is ν_k, which is detuned with the atomic transition frequency ω_{ab} as given by the relation $\delta_k=\omega_{ab}-\nu_k$. The atom-field state vector $|\Psi^{(t)}(AB)\rangle$ can be written as

$$|\Psi^{(t)}(AB)\rangle=\sum_{n_1,n_2}\Big[C_{a,n_1,n_2,0_k}(t)|a,n_1,n_2,0_k\rangle +C_{c_1,n_1,n_2,0_k}(t)|c_1,n_1,n_2,0_k\rangle +C_{c_2,n_1,n_2,0_k}(t)|c_2,n_1,n_2,0_k\rangle +\sum_k C_{b,n_1,n_2,1_k}(t)|b,n_1,n_2,1_k\rangle\Big], \quad (3)$$

where $C_{a,n_1,n_2,0_k}(t)$, $C_{c_1,n_1,n_2,0_k}(t)$, and $C_{c_2,n_1,n_2,0_k}(t)$ represent the probability amplitudes for the atom to be in the states $|a\rangle$, $|c_1\rangle$, and $|c_2\rangle$, respectively, with n_1 photons in cavity mode A, n_2 photons in cavity mode B, and no photon in any reservoir mode. Whereas $C_{b,n_1,n_2,1_k}(t)$ is the probability amplitude for the atom in level $|b\rangle$ with n_1 and n_2 photons in cavity modes A and B, respectively, and with one photon in the reservoir mode k. The interaction time t of the atom with the field is kept large as compared to decay time γ^{-1} of the atom so that $\gamma t\gg 1$. Under this approximation the atom decays spontaneously during the interaction with the field. A detector placed outside the cavity measures the frequency of the emitted photon. The spontaneous emission spectrum is obtained by repeating the experiment a number of times. The spectrum is proportional to the steady-state expression of $C_{b,n_1,n_2,1_k}(t)$ as

$$S(\delta_k)=\sum_{n_1,n_2=0}^{N-1}|C_{b,n_1,n_2,1_k}(\infty)|^2 =\sum_{n_1,n_2=0}^{N-1}p(n_1,n_2)S_{n_1,n_2}(\delta_k), \quad (4)$$

where $p(n_1,n_2)=|C_{n_1,n_2}|^2$ is the photon statistics of the two-mode driving field, and

$$S_{n_1,n_2}(\delta_k)=|g_{k_0}|^2\delta_k^2/[(\kappa_1^2 n_1+\kappa_2^2 n_2-\delta_k^2)^2+\delta_k^2\gamma^2/4]. \quad (5)$$

Here we have replaced g_k by its value at $k_0=\omega_{ab}/c$, which is a reasonable approximation in the region of interest. The

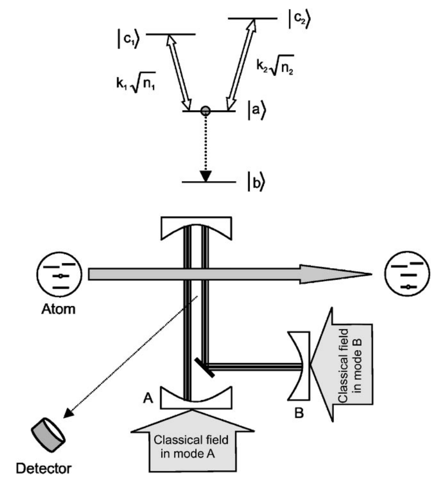

FIG. 1. An entangled field state inside a high-Q cavity having two modes of radiation A and B. Two classical field generators are used to displace the two modes of the field to reconstruct the Wigner function. A detector is used to measure the frequency of the spontaneously emitted photon. A four-level atom interacts with cavity modes A and B via atomic transitions $|c_1\rangle-|a\rangle$ and $|c_2\rangle-|a\rangle$, respectively.

function $S_{n_1,n_2}(\delta_k)$ has double peaks (the well-known Autler-Townes doublet) located at $\delta_k=\pm\sqrt{\kappa_1^2 n_1+\kappa_2^2 n_2}$. The height of each peak is the same and is given by $4|g_{k_0}|^2/\gamma^2$. So at this position of δ_k the peaks have a contribution only from photon probability function $p(n_1,n_2)$. The plot of $S(\delta_k)$ versus δ_k^2 gives a complete spectrum showing peaks located at $\kappa_1^2 n_1+\kappa_2^2 n_2$ for each set of n_1 and n_2. The spontaneous emission spectrum thus depends on the photon numbers in the two modes A and B driving the atomic transitions $|c_1\rangle-|a\rangle$ and $|c_2\rangle-|a\rangle$, respectively. Therefore the photon statistics can be recovered from the knowledge of the spectrum. The spectrum has N^2 peaks at the positions $\kappa_1^2 n_1+\kappa_2^2 n_2$. Each peak in the spectrum $S(\delta_k)$ will be clearly resolved if the spacing between two consecutive peaks is greater than the full width at half maximum (FWHM) of each peak. The FWHM of these peaks are determined as

$$\Delta_{n_1,n_2}=\tfrac{1}{4}\sqrt{\gamma^4+16\gamma^2(k_1^2 n_1+k_2^2 n_2)}. \quad (6)$$

Now comes the problem of determining the spacing between two consecutive peaks, which can be determined if the order of the peak positions are known. The question of determination of the order of peaks is nontrivial. It depends upon k_1, k_2, and γ. For k_1 and k_2 close to each other with a ratio of about 0.9, we get the peaks at $n|k_1^2-k_2^2|$, where n ranges from 0 to N^2-1. For $n=0$ it represent a peak due to joint photon probability $p(0,0)$. For $n=1$, there are two peaks due to photon probabilities $p(0,1)$ and $p(1,0)$, and so on until the last peak represents the photon probability $p(n,n)$. The spacing between two consecutive peaks then comes out as $|k_1^2-k_2^2|$. The condition for the resolution of peaks then becomes

$$(|k_1^2-k_2^2|)/(\gamma\sqrt{k_1^2 n_1+k_2^2 n_2})>1. \quad (7)$$

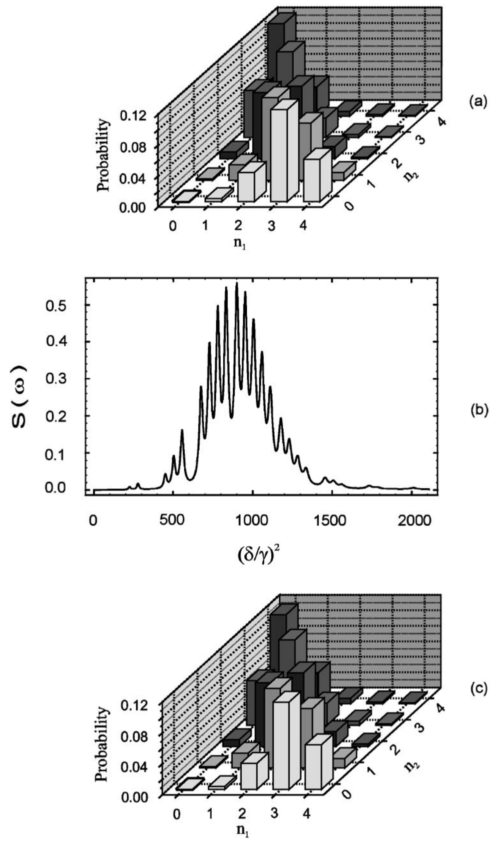

FIG. 2. (a) Original photon statistics of entangled field state $|\psi(AB)\rangle = \Sigma_{n_1,n_2=0}^{4} C_{n_1,n_2}|n_1\rangle_A|n_2\rangle_B$. (b) Spontaneous emission spectrum when the above three levels of a four-level atom are driven by the above-mentioned entangled state with $(k_1^2 - k_2^2)/(2\gamma\sqrt{k_1^2+k_2^2}) = 1.16$. (c) Reconstructed photon statistics.

The peaks of the spectrum merge with each other with an increase in the difference between k_1 and k_2. The peaks again becomes distinguishable when k_1 and k_2 are far apart with the consecutive peaks difference k_1^2 if $k_1^2 < k_2^2$ and k_2^2 if $k_1^2 > k_2^2$. Then the condition for the resolution of the peaks comes out to be

$$(k_i^2)/(\gamma\sqrt{k_1^2 n_1 + k_2^2 n_2}) > 1, \tag{8}$$

where $k_i = k_1$ or k_2 whichever is less. Under these conditions, the function $S_{n_1,n_2}(\delta_k)$ behaves like a delta function centered at $k_1^2 n_1 + k_2^2 n_2$. The complete spontaneous emission spectrum consists of contributions from all the photons excitations in the photon distribution function $p(n_1,n_2)$. So by knowing the spectrum values only at the points where the peaks occur and then after proper normalization we can get the photon distribution function $p(n_1,n_2)$. The two-mode photon statistics and spontaneous emission spectrum for $N = 5$ are shown in Fig. 2. This spectrum is taken for the case when the values of k_1 and k_2 are close together and for a minimum value of the parameter $(k_1^2 - k_2^2)/(2\gamma\sqrt{k_1^2+k_2^2}) = 1.16$. Photon statistics is then reconstructed from this spectrum by the procedure mentioned earlier. The result shows that the original and reconstructed photon distributions of the entangled state (1) are in good agreement.

Now we shall discuss how the Wigner function can be reconstructed from the spontaneous emission spectrum. The photon statistics of the cavity field allows us to calculate the Wigner function at the origin of the phase space. It has been pointed out by Royer [17] that the complete Wigner function can be obtained by shifting the system or equivalently the frame of reference in phase space. It is shown by Banaszek and Wodkiewicz that the shifting of the system can be achieved by attaching a coherent state with the signal field [18]. The scanned quasidistribution then gives the Wigner function of the signal field. In our scheme to get the Wigner function of the entangled state (1) we propose to displace each mode by injecting coherent states $|-\alpha_1\rangle$ and $|-\alpha_2\rangle$ into the cavity. Where α_i $(i=1,2)$ are the complex numbers characterizing the amplitude and phase of the shifts. Experimentally this operation is carried out by coupling two resonant classical oscillators to the cavity modes A and B, respectively (see Fig. 1). The displaced entangled field is then written as

$$|\psi_D(AB)\rangle = D^\dagger(\alpha_1)D^\dagger(\alpha_2)|\psi_0(AB)\rangle, \tag{9}$$

where $|\psi_0(AB)\rangle$ is the initially prepared entangled state in the cavity, while $D(\alpha_i)(i=1,2)$ are the displacement operators given as

$$D(\alpha_i) = \exp(\alpha_i a_i^\dagger - \alpha_i^* a_i). \tag{10}$$

In this scheme we show that spectrum with no injected field $(\alpha_1 = \alpha_2 = 0)$ yields the photon distribution function. In addition the spectrum for each value of α_1 and α_2 gives the two-mode Wigner function $W(\alpha_1,\alpha_2,\alpha_1^*,\alpha_2^*)$ in a straightforward way as [15]

$$W_s(\alpha_1,\alpha_2,\alpha_1^*,\alpha_2^*) = \frac{4}{\pi^2}\sum_{n_1,n_2=0}^{N-1}(-1)^{n_1+n_2} \times p_s(n_1,n_2,\alpha_1,\alpha_2), \tag{11}$$

where $p_s(n_1,n_2,\alpha_1,\alpha_2)$ is the photon statistics of a displaced entangled state. Thus, the two-mode Wigner function of an entangled field state can be found directly if the photon statistics of the displaced state (9) is known for all values of α_1 and α_2. The Autler-Townes spectrum $S(\delta_k)$ depends on the complex amplitudes α_1 and α_2 and it has peak values at each set of n_1 and n_2. If we want to reconstruct the photon distribution function $p_s(n_1,n_2,\alpha_1,\alpha_2)$ from the spectrum, the only meaningful values of δ_k^2 are $\kappa_1^2 n_1 + \kappa_2^2 n_2$ for different sets of n_1 and n_2. The reconstructed photon distribution of the displaced state is therefore given by those values in the spectrum where $\delta_k^2 = \kappa_1^2 n_1 + \kappa_2^2 n_2$. We may write it as

$$p_s(n_1,n_2,\alpha_1,\alpha_2) = \frac{1}{N_u}\sum_{m_1,m_2=0}^{N-1} \times \frac{p(m_1,m_2,\alpha_1,\alpha_2)|g_{k_0}|^2(\kappa_1^2 n_1 + \kappa_2^2 n_2)}{(\kappa_1^2(m_1-n_1)+\kappa_2^2(m_2-n_2))^2 + (\kappa_1^2 n_1 + \kappa_2^2 n_2)\gamma^2/4}, \tag{12}$$

where N_u is the normalization constant. It may be noted that the spontaneous emission spectrum for given values of injected fields $|-\alpha_1\rangle$ and $|-\alpha_2\rangle$ gives the Wigner function at points α_1 and α_2 in the complex plane and we have to obtain the spontaneous emission spectra for different values of α_1 and α_2 to reconstruct the complete Wigner function.

Since we are considering an entangled state (1), the photon statistics of the entangled state after the injection of coherent states $|-\alpha_1\rangle$ and $|-\alpha_2\rangle$ is given by

$$p(n_1,n_2,\alpha_1,\alpha_2)=|\langle n_1,n_2|D^\dagger(\alpha_1)D^\dagger(\alpha_2)|\psi_0(AB)\rangle|^2, \tag{13}$$

where $|\psi_0(AB)\rangle$ is the initial entangled state (1). On substituting it in Eq. (13) and using

$$\langle m|D^\dagger(\alpha_i)|n\rangle=\sqrt{\frac{m!}{n!}}\exp\left[-\frac{|\alpha_i|^2}{2}\right](\alpha_i^*)^{n-m}L_m^{n-m}(|\alpha_i|^2), \tag{14}$$

where $L_m^{n-m}(|\alpha_i|^2)$ are associated Laguerre polynomials, we get

$$p(n_1,n_2,\alpha_1,\alpha_2) = e^{-(|\alpha_1|^2+|\alpha_2|^2)}\left|\sum_{r_1=n_1}^{N}\sum_{r_2=n_2}^{N}C_{r_1,r_2}\sqrt{\frac{n_1!\times n_2!}{r_1!\times r_2!}} \times(\alpha_1^*)^{r_1-n_1}(\alpha_2^*)^{r_2-n_2}L_{n_1}^{r_1-n_1}(|\alpha_1|^2)L_{n_2}^{r_2-n_2}(|\alpha_2|^2)\right|^2. \tag{15}$$

This expression for photon distribution of the displaced entangled state is then substituted in Eq. (12) to obtain the reconstructed photon statistics and finally the Wigner function is obtained using expression (11).

We presented a scheme based on Autler-Townes spectroscopy to reconstruct the Wigner function for the two-mode entangled state in a high-Q cavity. There are a few conditions that should be satisfied for the reconstruction of Wigner function. The most important one is the selection of k_1/γ and k_2/γ. These two parameters should be selected in such a way that the peaks in the spectrum associated with different values of n_1 and n_2 are clearly resolved. We proposed our scheme for a general kind of entangled state in which there is not any relation between the number of photons in two modes. However, the same procedure can be applied to other cases such as the case where the number of photons in two modes are the same or the case where the number of photons in two modes are fixed [19]. For the resolution of the peaks in the Autler-Townes spectrum we arrived at the conditions as mentioned in Eqs. (7) and (8). Both these conditions require large ratios of vacuum Rabi frequencies (κ_1 and κ_2) for transitions $|c_1\rangle-|a\rangle$ and $|c_2\rangle-|a\rangle$ to the atomic decay rate γ from level $|a\rangle-|b\rangle$. In Fig. 2 we have plotted the spontaneous emission spectrum and reconstructed photon statistics for the case when $(k_1^2-k_2^2)/(2\gamma\sqrt{k_1^2+k_2^2})=1.16$. The values of k_1/γ and k_2/γ for this case come out as 16.67 and 15, respectively. The recent experiments [20–22] show that these ratios are quite accessible.

Another important limitation is that the atomic decay rates from levels $|c_1\rangle-|a\rangle$ and $|c_2\rangle-|a\rangle$ should be much smaller than the decay rate γ from level $|a\rangle-|b\rangle$. This condition is necessary because these decay may change the photon statistics of the field inside the cavity.

[1] A. Einstein *et al.*, Phys. Rev. **47**, 777 (1935).
[2] P. W. Shor, SIAM J. Comput. **26**, 1484 (1997); L. K. Grover, Phys. Rev. Lett. **79**, 325 (1997); **79**, 4709 (1997).
[3] A. K. Ekert, Phys. Rev. Lett. **67**, 661 (1991); C. H. Bennett *et al.*, *ibid.* **68**, 557 (1991).
[4] C. H. Bennett and S. J. Wiesner, Phys. Rev. Lett. **69**, 2881 (1992).
[5] C. H. Bennett *et al.*, Phys. Rev. Lett. **70**, 1895 (1993).
[6] D. T. Smithey *et al.*, Phys. Rev. Lett. **70**, 1244 (1993).
[7] S. Schiller *et al.*, Phys. Rev. Lett. **77**, 2933 (1996).
[8] K. Vogel and H. Risken, Phys. Rev. A **40**, 2847 (1989).
[9] D.-G. Welsch *et al.*, Prog. Opt. **34**, 63 (1999).
[10] M. G. Raymer *et al.*, Phys. Rev. A **54**, 2397 (1996).
[11] H. Kuhn *et al.*, Phys. Rev. A **51**, 4240 (1995).
[12] M. S. Kim and G. S. Agarwal, Phys. Rev. A **59**, 3044 (1999).
[13] M. S. Kim *et al.*, Phys. Rev. A **58**, R65 (1998); M. Wilkens and P. Meystre, *ibid.* **43**, 3832 (1991); S. M. Dutra *et al.*, *ibid.* **48**, 3168 (1993).
[14] M. S. Zubairy, Phys. Lett. A **222**, 91 (1996); Phys. Rev. A **57**, 2066 (1998); T. Azim and M. S. Zubairy, Phys. Lett. A **250**, 344 (1996); M. Mahmoudi *et al.*, J. Opt. B: Quantum Semiclassical Opt. **2**, 315 (2000).
[15] K. E. Cahill and R. J. Glauber, Phys. Rev. **177**, 1857 (1969); **177**, 1882 (1969).
[16] S. H. Autler and C. H. Townes, Phys. Rev. **100**, 703 (1955). See also P. L. Knight and P. W. Milonni, Phys. Rep. **66**, 23 (1980).
[17] A. Royer, Phys. Rev. Lett. **55**, 2745 (1985); Phys. Rev. A **15**, 449 (1977).
[18] K. Banaszek and K. Wodkiewicz, Phys. Rev. Lett. **76**, 4344 (1996).
[19] M. Ikram *et al.*, Opt. Commun. **184**, 417 (2000).
[20] G. Raithe *et al.*, in *Advances in Atomic, Molecular, and Optical Physics*, edited by P. Berman (Academic, New York, 1994).
[21] Q. A. Turchette *et al.*, Appl. Phys. B: Photophys. Laser Chem. **60**, S1 (1995).
[22] C. J. Hood *et al.*, Phys. Rev. Lett. **80**, 4157 (1998).