ACCOUNTING FOR THE EFFECTS OF REHABILITATION ACTIONS ON THE RELIABILITY OF FLEXIBLE PAVEMENTS: PERFORMANCE MODELING AND OPTIMIZATION

A Thesis

by

VIGHNESH PRAKASH DESHPANDE

Submitted to the Office of Graduate Studies of
Texas A&M University
in partial fulfillment of the requirements for the degree of

MASTER OF SCIENCE

August 2008

Major Subject: Civil Engineering

ACCOUNTING FOR THE EFFECTS OF REHABILITATION ACTIONS ON THE RELIABILITY OF FLEXIBLE PAVEMENTS: PERFORMANCE MODELING AND OPTIMIZATION

A Thesis

by

VIGHNESH PRAKASH DESHPANDE

Submitted to the Office of Graduate Studies of
Texas A&M University
in partial fulfillment of the requirements for the degree of

MASTER OF SCIENCE

Approved by:

Co-Chairs of Committee,	Ivan Damnjanovic
	Paolo Gardoni
Committee Member,	Thomas Wehrly
Head of Department,	David V. Rosowsky

August 2008

Major Subject: Civil Engineering

ABSTRACT

Accounting for the Effects of Rehabilitation Actions on the Reliability of Flexible Pavements: Performance Modeling and Optimization. (August 2008)

Vighnesh Prakash Deshpande, B.E., Sardar Patel College of Engineering, Mumbai

Co-Chairs of Advisory Committee: Dr. Ivan Damnjanovic
Dr. Paolo Gardoni

A performance model and a reliability-based optimization model for flexible pavements that accounts for the effects of rehabilitation actions are developed. The developed performance model can be effectively implemented in all the applications that require the reliability (performance) of pavements, before and after the rehabilitation actions. The response surface methodology in conjunction with Monte Carlo simulation is used to evaluate pavement fragilities. To provide more flexibility, the parametric regression model that expresses fragilities in terms of decision variables is developed. Developed fragilities are used as performance measures in a reliability-based optimization model. Three decision policies for rehabilitation actions are formulated and evaluated using a genetic algorithm. The multi-objective genetic algorithm is used for obtaining optimal trade-off between performance and cost.

To illustrate the developed model, a numerical study is presented. The developed performance model describes well the behavior of flexible pavement before as well as after rehabilitation actions. The sensitivity measures suggest that the reliability of flexible pavements before and after rehabilitation actions can effectively be improved by providing an asphalt layer as thick as possible in the initial design and improving the subgrade stiffness. The importance measures suggest that the asphalt layer modulus at the time of rehabilitation actions represent the principal uncertainty for the performance after rehabilitation actions. Statistical validation of the developed response model shows that the response surface methodology can be efficiently used to describe pavement responses. The results for parametric regression model indicate that the developed regression models are able to express the fragilities in terms of decision variables. Numerical illustration for optimization shows that the cost minimization and reliability maximization formulations can be efficiently used in determining optimal rehabilitation policies. Pareto optimal solutions obtained from multi-objective genetic algorithm can be used to obtain trade-off between cost and performance and avoid possible conflict between two decision policies.

DEDICATION

To my Mother and Grandfather

ACKNOWLEDGEMENTS

I would like to thank my committee chairs, Dr. Damnjanovic and Dr. Gardoni, and my committee member, Dr. Wehrly, for their guidance and support throughout the course of this research. I would like to thank the Texas Transportation Institute and Texas A&M University for supporting me with funds during my research and coursework. Also I am grateful to Dr. Reinschmidt for providing guidance throughout my masters program. Thanks also go to my friends and colleagues and the department faculty and staff for making my time at Texas A&M University a great experience. Finally, thanks to my grandmother and father for their encouragement and to my wife, Maithilee for her patience and love.

TABLE OF CONTENTS

Page

Page

CHAPTER

Page

LIST OF FIGURES

Page

Page

LIST OF TABLES

CHAPTER I

INTRODUCTION

Pavements represent a major part of the nation's transportation infrastructure. With utilization and aging, the condition of pavements deteriorates requiring periodic repairs and maintenance to sustain their functionality. It is estimated that flexible pavements comprise approximately 60 percent of the total paved public roads in the U.S., or approximately 500,000 miles [1]. Just to maintain the current condition of these roads, the American Association of State Highway and Transportation Officials (AASHTO) estimates that transportation agencies across the U.S. will need to increase spending by approximately 42 percent [2].

Managing such large network of flexible pavements requires timely preventive maintenance and planned rehabilitation actions. In pavement engineering applications, preventive maintenance represents a planned strategy of treatments such as fog seals, microsurfacing, crack seals and other treatments designed to slow down the deterioration process without increasing the pavement structural capacity. On the other hand, rehabilitation actions represent activities that increase pavement structural capacity, such as pavement overlays. Both preventive maintenance and rehabilitation actions should be planned in an optimal manner as they require substantial financial, manpower, and equipment recourses. Typically, preventive maintenance and rehabilitation actions are planned as a part of a design strategy that minimizes pavement's life-cycle costs.

Life-cycle cost analysis takes into account all the costs incurred during pavement

---

This thesis follows the style of Journal of Reliability Engineering and System Safety.

maintenance costs, rehabilitation costs, as well as users' costs (e.g., time related, vehicle operating, safety, and environmental costs) [3]. Hence, for the assessment of life-cycle costs, it is crucial to evaluate pavement performance throughout its service life – before, as well as after the application of preventive maintenance and rehabilitation actions. Therefore, performance prediction models are essential to the economical design of pavements.

In general, performance prediction can be either deterministic, using sometimes conservative (biased) estimates that ignore the inherent uncertainties in the pavement performance and deterioration, or probabilistic. The probabilistic nature of pavement deterioration arises from two different sources of uncertainty: uncertainty in pavement utilization (random input) and uncertainty in pavement response (random output). Since pavement structures are type of infrastructure facilities associated with large response and utilization uncertainties, it is important to explicitly account for them in developing pavement performance models.

Over the years, a number of researchers have developed probabilistic pavement performance models for both project-and network-level applications. Typically network-level performance models [4-6] take into account the effects of rehabilitation, but generally do not consider pavement characteristics and fatigue failure mechanics.

Reliability models are probabilistic models that can take into account pavement characteristics and utilization patterns in the specification of propensity functions. Reliability models predict the probability that pavement will perform its intended function under a given set of conditions over a specified period of time. If the failure event is well defined, reliability models can be effectively used to predict the performance of flexible pavement [7]. The concept of reliability has been implemented in modeling pavement performance [7-11]. Zhang and Damnjanovic [7] developed a model based on the Method of Moments technique (MOM) that has ability to express the reliability function as a closed-form function of basic random variables. The advantage of a closed-form function is its suitability for implementation in optimization models. Alsherri and George [8] developed structural reliability model based on Monte Carlo simulation (MCS). Zhou and Nowak [9] developed system and individual component reliability models based on special sampling technique. Chua *et al*. [10] and Darter *et al*. [11] developed models based on a mechanistic approach for predicting pavement distresses in terms of material behavior and structural responses. However, these models do not explicitly consider the effect of rehabilitation actions on pavement reliability, which is an important shortcoming for their effective implementation in life-cycle cost analysis.

The objective of the research is to develop a model that is able to take into account the effects of planned rehabilitation actions on the reliability of flexible pavements. The developed model considers multiple failure criteria (fatigue cracking and rutting). The model is based on the solution from a multilayer linear-elastic analysis to obtain pavement mechanistic responses (tensile and compressive strains) before and after the application of rehabilitation actions. In the linear elastic theory, directional stresses and strains are obtained by assuming a stress function that satisfies the differential equation for specified boundary conditions. Since the differential equation for the layered system cannot be solved analytically, it is solved numerically for specified boundary conditions. Hence the relation between pavement responses and input decision variables that controls responses are implicit and pavement response model can be termed as black-box model.

Conventionally, the reliability is evaluated using Monte Carlo simulation (MCS) technique. However, the MCS technique typically requires a relatively large number of simulations in order to obtain sufficiently accurate estimates of failure probabilities and it becomes impractical to simulate the black-box model thousands of times. In the research, an alternative approach of response surface methodology (RSM) is explored for evaluating the reliability. The objective of RSM in reliability analysis is to approximate the implicit responses into a closed-form function. The developed response model is computationally simple and can be easily simulated to obtain reliability estimates.

Typically in reliability analysis, the performance is modeled in terms of fragilities. The fragility in the simple words can be defined as the conditional probability of failure given the level of demand. However, the fragilities are the functions of decision variables (layer thickness, layer modulus of elasticity) in the sense that stronger the pavement lesser is the failure probability and vice versa. The fragilities that are expressed in terms of decision variables can be efficiently used in optimization formulations. In the research, a parametric regression model is developed to express pavement fragilities as the function of decision variables.

The primary objective while determining the optimal rehabilitation action is safety in performance and economy in design. In addition to balance between safety and economy, since the decision variables that control the performance of flexible pavements are uncertain, it is necessary to account for the uncertainty in performance. Therefore probabilistic optimization technique that accounts for uncertainties is necessary while optimizing the rehabilitation actions for flexible pavements. One of the probabilistic optimization techniques is reliability-based optimization (RBO). The RBO can be efficiently used in balancing the needs between safety in performance and economy in design. Though the use of RBO seems attractive and has advantages, the RBO problems are complex and require a robust optimization technique that can provide a global optimal solution. Traditional optimization techniques which include gradient projection algorithms are robust in finding a single local optimal solution. However, complex domain like in RBO can have more than one optimal solutions and therefore more robust technique is required that can find a near-global solution. In the research, Genetic Algorithm (GA) is used because of its efficiency in finding a near-global solution. The GA performs a global and probabilistic search thus increasing the likelihood of obtaining a near-global solution.

The remainder of the report is organized as follows. In the next section, models for pavement reliability are introduced followed by a review on general repair models that account for the effects of rehabilitation actions. In the Chapter II, formulation of pavement responses using multilayer linear-elastic analysis is presented. Next the model formulation and solution approach is described followed by a numerical example. In

Chapter III, response surface methodology and parametric regression modeling of fragilities is described followed by numerical example. In Chapter IV, reliability-based optimization and problem formulations is discussed followed by discussion on Genetic Algorithm. A numerical example is presented to illustrate the developed methodology for optimization. Finally a conclusion is presented that summarizes the entire research along with important observations.

1.1 Pavement Reliability

Reliability models are probabilistic models that predict the probability that a component or system will perform its intended function under a given set of conditions at a particular instant or over a specified period of time. Limit state functions can be defined in a number of different ways to describe whether a specified level of performance is met or not. Examples of performance level include safety against collapse, and loss of serviceability. Based on design equations and practice, failure events for flexible pavements can be mathematically defined using transfer and traffic utilization functions. Hence structural limit state functions can be mathematically defined and used to develop pavement performance models.

Pavement reliability generally considers the remaining life expressed as a difference between the number of load applications, N_C (capacity), a pavement can withstand before failing to meet a specified performance measure, such as roughness or rutting, and the number of load applied, N_D (demand) [8]. The failure of a pavement section occurs when $N_D \geq N_C$. The corresponding limit state function $g(\mathbf{x},t)$, where $\mathbf{x}$

denotes a vector of n basic variables and t is the time, can be defined as $g(\mathbf{x}) = \left[N_C(\mathbf{x}) - N_D(\mathbf{x},t)\right]$. The probability that $N_D \geq N_C$ also referred as the probability of failure P_F, can then be mathematically defined as

$$P_F = P\left[g\left(\mathbf{x},t\right) \leq 0\right] \tag{1}$$

where, $P[\cdot]$ represents probability that the event $g(\mathbf{x},t) \leq 0$ will occur. Conversely, the reliability Rel, which in this context is defined as a probability that the pavement will perform its intended function, can be defined as follows

$$\mathrm{Rel} = 1 - P_f = P\left[g\left(\mathbf{x},t\right) > 0\right] \tag{2}$$

Standard reliability techniques including MCS and the first- and second-order reliability methods (FORM and SORM) [12, 13, 14] can be used for the solution of Eq. 1 when a closed-form in not available.

1.2 Repair Models and Effects of Pavement Rehabilitation

While typical reliability models do not consider the effects of repairs and rehabilitation, a general class of stochastic repair models is able to account for them. The basic assumption in these models is the efficiency of repair actions. Two extremes in modeling this efficiency exist: minimal repair, and perfect repair. These effects can be observed on their impact on a rate of occurrence of failures (ROCOF) function, which is similar to the failure, or hazard, rate function in reliability theory. Minimal repairs, or the actions that leave the system in an "As Bad as Old" condition, do not change the ROCOF failure function, or in other words, do not reduce the hazard rate. On the other hand, perfect repairs, or the actions that leave the system state in an “As Good as New” condition, change the ROCOF function. After the application of a perfect repair action, the system is effectively in the initial “As Good as New” state. While the concept of minimal repairs is tied to a description of the non-homogeneous Poisson process (NHPP), perfect repairs are modeled through application of a renewal process (RP). However, in reality, the effects of repairs and rehabilitation actions on a pavement system are neither minimal, nor perfect.

Literature on stochastic repair models reports a class of imperfect models that are able to handle situation in which the effect of repair is neither minimal, nor perfect. Brown and Proschan [15], Lin et al. [16], and Doyen and Gaudoin [17] have proposed different types of imperfect repair models. In general there are two approaches to model the effect of imperfect repairs: 1) the approach used in the Brown-Proschan model assumes that a system after a repair attains an “As Good as New” state with probability p, and an “As Bad as Old” state with probability $1-p$, and 2) an approach that considers a direct effect of repair actions on the ROCOF function.

Doyen and Gaudoin [17] developed two arithmetic reduction models that take into account direct effect of repair actions on the ROCOF function. Arithmetic Reduction of Intensity model (ARI) considers a one-time reduction of the failure intensity (ROCOF), while the rate of ROCOF stays the same as before the failure. In contrast, imperfect repairs in Arithmetic Reduction of Age model (ARA) reduce the failure intensity to its initial value, and also change the rate of ROCOF.

Even though these models are widely applied in modeling repairs of complex mechanical and electrical systems, their applicability to civil infrastructure is limited. First, these models do not consider the case when a repair action leaves the system in a "Better than New" state. For example, some rehabilitation actions, such as construction of a structural overlay can leave the pavement condition with a structural capacity that is greater than the initial one. Second, these repair models do not consider the failure mechanism of the system, which is an important consideration in the mechanistic-empirical approach to pavement design.

The effect of rehabilitation actions on pavement reliability has not been extensively studied. In deterministic settings, Abaza [3] proposed a model to take into account the impact of overlays on pavement's structural number (an indicator of pavement strength), while Ouyang and Madanat [18] proposed roughness improvement functions. Paterson [19] developed a model that considers the effectiveness of pavement rehabilitation under various conditions. This model is based on a rehabilitation intensity function that estimates the roughness before and after the application of a resurfacing action. In probabilistic terms, Damnjanovic [20] developed an analytical model that can take into account the effects of planned rehabilitation actions. This model is able to capture the stochastic nature of the pavement performance after the application of rehabilitation, but it does not consider mechanistic responses of a pavement structure in the limit state function. The next section presents a framework for modeling the effect on rehabilitation actions on pavement's responses and their impact on a component and system-level reliability.

CHAPTER II

MODELING THE EFFECTS OF REHABILITATION ACTIONS ON THE RELIABILITY OF FLEXIBLE PAVEMENTS

2.1 Model Formulation

The objective of the research is to develop a model that takes into account the effects of planned rehabilitation actions on the reliability of flexible pavements. The model considers multiple failure criteria (fatigue cracking and rutting) at the component level, and their combined effects at a system-level reliability. The performance of flexible pavements can be described as a series system, where the failure of the system occurs if any of its components fails. Determining the system reliability requires the mathematical formulation of the limit state functions for each component. Let g_f and g_r represent the limit state functions for the fatigue cracking and rutting failure criteria, respectively. The limit state function g_{sys} for the flexible pavement system can then be written such that

$$\left[g_{sys}(t) \leq 0\right] = \left[g_f(\mathbf{x}, t) \leq 0 \cup g_r(\mathbf{x}, t) \leq 0\right] \tag{3}$$

2.1.1 Modeling Component Level Demand and Capacity

The capacity and demand in the limit state functions for each failure criterion can be modeled in terms of the load applications or yearly number of 18-kip equivalent single-axle load, *ESAL* and the corresponding accumulated *ESAL*. With specified yearly traffic growth rate, ω, and $ESAL$ at $t=0$, the accumulated $ESAL$ (demand) at any time t, $N_D(\mathbf{x},t)$, can be obtained as

$$N_D(\mathbf{x},t) = N_D(\mathbf{x},t-1) + ESAL(t) \qquad t = 1,2,3,\ldots \tag{4}$$

where $ESAL(t) = (1+\omega)^t \times ESAL(0)$ is the $ESAL$ in year t.

2.1.1.1 Fatigue Cracking

In a mechanistic-empirical approach to pavement design, the maximum tensile strain, ε_t, at the bottom of the asphalt layer is considered to control the allowable number of repetitions for fatigue cracking. This critical strain is used in transfer functions to predict the performance of flexible pavement for fatigue cracking [21]

$$N_{C_f}(\mathbf{x}) = f_1(\varepsilon_t)^{-f_2}(E)^{-f_3} \tag{5}$$

where, N_{C_f} is the allowable number of load repetitions (capacity) before the fatigue cracking occurs, E is the modulus of surface asphalt layer, and f_1, f_2, and f_3 are empirical coefficients determined from tests and modified to reflect in-situ performance. Once the allowable load repetitions are defined, the limit state function for fatigue cracking can be formulated as

$$g_f(\mathbf{x},t) = N_{C_f}(\mathbf{x}) - N_D(\mathbf{x},t) \tag{6}$$

2.1.1.2 Rutting

The design methodology for flexible pavement commonly considers a maximum compressive strain ε_c at the top of a subgrade layer as the controlling response for rutting. Based on empirical equations developed using laboratory tests and field performance data, the allowable load repetitions for rutting can be expressed as [21]

$$N_{C_r}\left(\mathbf{x}\right) = f_4\left(\varepsilon_c\right)^{-f_5} \tag{7}$$

where, N_{C_r} is the allowable number of load repetitions (capacity) for rutting, and f_4 and f_5 are coefficients determined from tests and modified to reflect in-situ performance. Finally the limit state function for rutting can be formulated as

$$g_r\left(\mathbf{x},t\right) = N_{C_r}\left(\mathbf{x}\right) - N_D\left(\mathbf{x},t\right) \tag{8}$$

The quantities in Eq. 5 and 7 are random and are not readily available. They are functions of the basic variables $\mathbf{x}$ and can be computed using pavement response models that are based on the theory of linear elasticity.

2.1.1.3 Pavement Response Model

Figure 1 shows the typical flexible pavement section for which the critical responses are the functions of

$$\mathbf{x} = \{\mathbf{h}, \mathbf{E}, \mathbf{v}\}$$

where, $\mathbf{h}$, $\mathbf{E}$, $\mathbf{v}$ are the corresponding vectors of layer thicknesses, layer moduli and layer Poisson's ratios, respectively, while, q and a represents the intensity and the radius of the applied circular load (e.g., single axle load). Pavement responses can be determined with an assumption that the pavement structure behaves as linear elastic layered system. The linear elastic theory is based on the following assumptions [21]: 1) each layer i is homogeneous, isotropic, and linearly elastic with modulus E_i and

Poisson ratio v_i, 2) each layer has a finite thickness h_i, except the bottom layer that has no lower bound, 3) continuity conditions are satisfied at each layer interface in terms of vertical stresses, shear stresses, and vertical displacements.

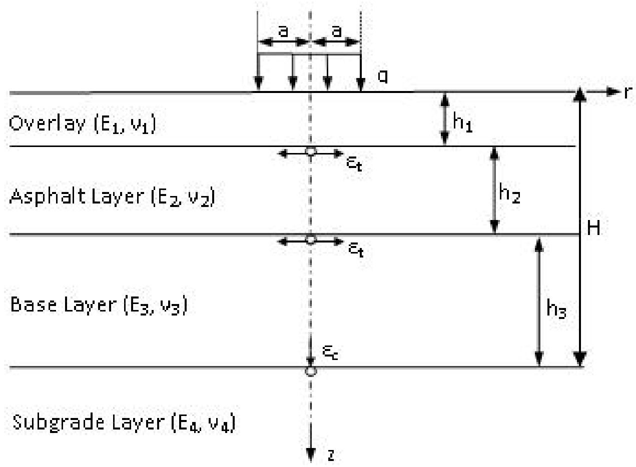

Figure 1: Flexible pavement section

Based on the assumptions of linear elastic theory, directional stresses can be obtained by assuming a stress function ϕ for each layer that satisfies the following 4th order differential equation

$$\left(\frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r}+\frac{\partial^2}{\partial z^2}\right)\left(\frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r}+\frac{\partial^2}{\partial z^2}\right)\phi=0 \tag{9}$$

where, r and z represents cylindrical coordinates in radial and vertical directions respectively. Using the stress function, the directional stresses can be computed as

$$\sigma_z = \frac{\partial}{\partial z}\left[(2-\nu)\nabla^2\phi - \frac{\partial^2\phi}{\partial z^2}\right] \tag{10}$$

$$\sigma_r = \frac{\partial}{\partial z}\left[(\nu)\nabla^2\phi - \frac{\partial^2\phi}{\partial r^2}\right] \tag{11}$$

$$\sigma_t = \frac{\partial}{\partial z}\left[(\nu)\nabla^2\phi - \frac{1}{r}\frac{\partial\phi}{\partial r}\right] \tag{12}$$

where, σ_z, σ_r, and σ_t are the stresses at the points under consideration in the vertical, radial and tangential directions, respectively.

Even though the differential equation, presented in Eq. 9, cannot be solved analytically, it can be solved numerically for specified boundary conditions. Appendix A describes the approach used in the research to solve Eq. 9, 10, 11, 12. Once the stresses are computed, the strains required for capacity modeling can be computed as

$$\varepsilon_t = \frac{1}{E}\left[\sigma_z - \nu(\sigma_r + \sigma_t)\right] \tag{13}$$

$$\varepsilon_c = \frac{1}{E}\left[\sigma_t - \nu(\sigma_z + \sigma_r)\right] \tag{14}$$

where, E is the modulus of the layer at which the strains are computed.

The effects of rehabilitation actions are incorporated in the capacity model by assuming that an overlay of certain thickness is to be constructed over the existing pavement at the time of rehabilitation. After the application of the overlay, the pavement structural system is changed. Therefore, the developed model recalculates the pavement responses to reflect its new structural specification, and determines the new level of allowable number of $ESAL$ for each failure criteria, N_{C_f} and N_{C_r}.

When recalculating pavement responses, two important assumptions are made: 1) after the application of an overlay, the tensile strain at the bottom of the overlay is considered to be the controlling response for determining the allowable repetitions for fatigue cracking, and 2) the modulus of the asphalt layer is updated to reflect its new value. The first assumption can be generalized to include any specification of the controlling tensile strain. The current assumption conforms to the case when thicker overlays are considered. The second assumption represents a reasonable assumption since with the utilization and aging the modulus of the asphalt layer decreases. Therefore, for the model to capture the true effects of rehabilitation actions, it is important to accurately predict the modulus of the asphalt layer before a rehabilitation action is undertaken.

2.1.2 Deterioration of the Asphalt Modulus

The modulus deterioration process of the asphalt material is regarded as a fatigue damage process caused by repetitive loading. Stiffness ratio (SR) is typically used to quantify the fatigue damage in the asphalt layer. Stiffness ratio is a normalized quantity that normalizes the stiffness value relative to its initial value. Figure 2 shows a change in asphalt modulus of a top layer with utilization over a period of time. As illustrated in Figure 2, in general, decrease in SR is nonlinear and similar to change in reliability over time/utilization. Since layers moduli, together with thicknesses of layers fully define behavior of a pavement system in term of its responses, to obtain pavement responses after application of rehabilitation actions, modulus of top layer at the time of application of rehabilitation actions needs to be estimated.

Researchers have developed a number of models to predict the deterioration of the modulus of asphalt layers. Attoh-Okine and Roddis [22] developed a deterioration model based on data obtained from ground penetrating radar (GPR). Ullidtz [23] developed an incremental-recursive model based on a mechanistic-empirical approach. This incremental-recursive model works in time increments and uses output from one season recursively as input for the next. Tsai *et al*. [24] suggested the application of the Weibull theory for developing the incremental-recursive model.

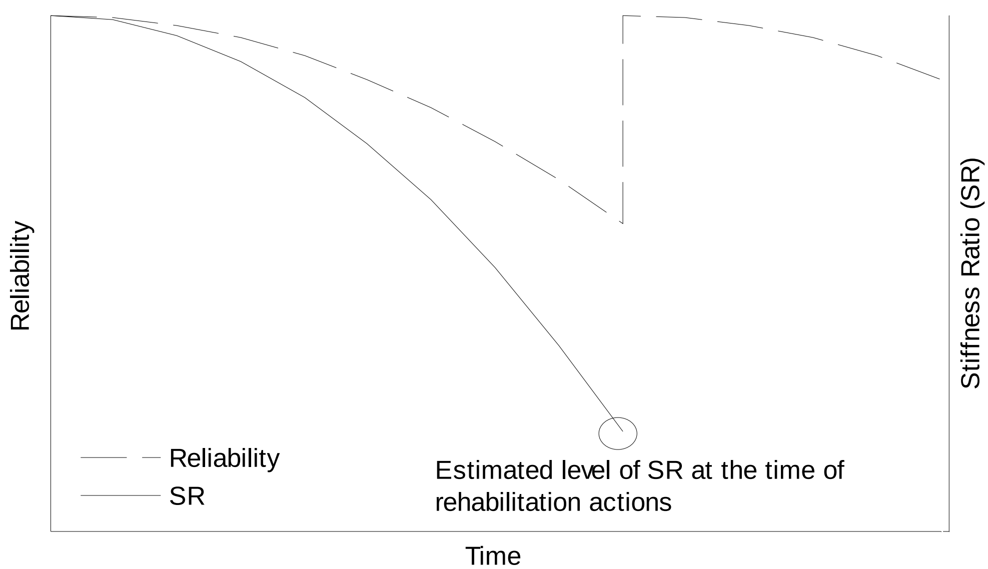

Figure 2: Typical system reliability and asphalt modulus behavior

Without loss of generality, we adopted a Weibull approach to model nonlinear accumulation of damage. The function $SR(t)$ is used to indicate the change in the stiffness ratio with utilization and can be written as

$$SR(t) = \frac{E_2(t)}{E_2(t=0)} = \exp\left\{-\lambda\left[N_D(t)\right]^o\right\} \tag{15}$$

where, λ and o are the scale and shape parameters, respectively.

With utilization, a crack initiates in an asphalt layer and propagates from micro scale to macro scale. When cracking reaches certain level, water may infiltrate the pavement system, further reducing the modulus. The effect of this excessive cracking and water infiltration can be accounted for by multiplying Eq. 15 by a constant (≤ 1) that depends on the anticipated condition of the damaged system at the time of rehabilitation. With an updated structural system and recalculated responses, the limit state functions for the rehabilitated system can be formulated. Once the limit state functions (g_f, g_r, and g_{sys}) are defined, the component and system reliability can be determined using standard structural reliability techniques [12, 13, 14].

2.1.3 Accounting for Correlation in the Basic Random Variables

Generally, the information on basic random variables is available in the form of marginal distributions and correlation coefficients. However, in addition to marginal distributions, reliability analysis requires evaluation of the joint probability density function (PDF) of the basic random variables. Most of the pavement reliability models assume independence between random variables and this reduces the joint PDF to the product of marginal distributions. To evaluate the joint PDF of non-negative (as those considered here) and hence non-normal basic random variables accounting for their correlation, a multivariate distribution model with known marginal distributions and correlation matrix needs to be constructed. This join PDF can be constructed using either Rosenblatt [25] or Nataf transformations [26].

However, due to the limitation in the range of applicability of the Rosenblatt transformation, in the research, the Nataf transformation is used to evaluate joint probability. The Nataf transformation is applicable to a wider range of the correlation coefficients. With known marginal distributions of the basic random variables in $\mathbf{x}$ and correlation matrix $\mathbf{R}=[\rho_{ij}]$, the joint PDF is written as

$$f(\mathbf{x}) = f(x_1)\ldots f(x_n)\frac{\varphi(\mathbf{z},\mathbf{R}_o)}{\varphi(z_1)\ldots\varphi(z_n)} \tag{16}$$

where $\varphi(\cdot)$ is the standard normal PDF, the transformation to the correlated standard normal variables $\mathbf{z}$ can be obtained as

$$z_i = \Phi^{-1}\left[F_{X_i}(x_i)\right] \tag{17}$$

where, $\Phi(\cdot)$ is the standard normal cumulative distribution function (CDF) and $\mathbf{R}_o=[\rho_{o,ij}]$ is such that

$$\rho_{ij} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\left(\frac{x_i-\mu_i}{\sigma_i}\right)\left(\frac{x_j-\mu_j}{\sigma_j}\right)\varphi_2(z_i,z_j,\rho_{o,ij})dz_i dz_j \tag{18}$$

where, μ_i, μ_j, σ_i, and σ_j are the means and standard deviations of x_i and x_j, and $\varphi_2(\cdot)$ is the 2 dimensional normal PDF with zero means, unit standard deviations and correlation coefficient $\rho_{o,ij}$.

Modified correlation coefficients ρ'_{ij} are obtained by solving Eq. 18 iteratively for each pair of marginal distributions and known ρ_{ij}. Alternatively, ρ'_{ij} can be computed using following relation [26]

$$\rho'_{ij} = F \times \rho_{ij} \tag{19}$$

where, F is a function of ρ_{ij} and the marginal distributions of x_i and x_j and variables and is available in Liu and Kiureghian [26] for different combinations of marginal distributions.

2.2 Solution Approach

With the defined limit state function, the probability of failure for the system and each failure criteria can be obtained by solving the following multi-dimensional integral

$$P_{F_k} = P\left[g_k(\mathbf{x}) \leq 0\right] = \int \ldots\ldots \int_{g_k(\mathbf{x})\leq 0} f\left(\mathbf{x}\right) d\mathbf{x} \tag{20}$$

where, k corresponds to the system, fatigue cracking and rutting limit states. In this research, the probability integral in the Eq. 20 is evaluated using MCS.

2.2.1 Sensitivity Analysis and Importance Measures

Sensitivity and importance measures can be computed to assess what the effects of changes in the parameters and the random variables are on the fatigue and rutting reliability.

2.2.1.1 Sensitivity Analysis

Sensitivity analysis is used to determine to which parameter(s) the reliability is most susceptible. Let $f(\mathbf{x}, \mathbf{\Theta}_f)$ be the probability density function of the basic random variables in $\mathbf{x}$, where $\mathbf{\Theta}_f$ is a set of distribution parameters (e.g. mean, standard deviation, correlation coefficient or other parameters describing the distribution of variables in $\mathbf{x}$). The sensitivity measure for each parameter is given by computing the gradient of the reliability index, β, for each failure criteria with respect to each parameter and can be expressed as [27]

$$\nabla_{\mathbf{\Theta}_f}\beta = \mathbf{J}^T_{\mathbf{u}^*,\mathbf{\Theta}_f}\boldsymbol{\alpha} \tag{21}$$

where $\boldsymbol{\alpha}$ is the vector defined as

$$\boldsymbol{\alpha} = \nabla_{\mathbf{u}^*}\beta = \operatorname{sgn}(\beta)\frac{\mathbf{u}^*}{\|\mathbf{u}^*\|} \tag{22}$$

where, $\mathbf{u}^*$ is the most likely failure point (design point) in standard normal space, $\operatorname{sgn}(\cdot)$ is the algebraic sign of β, $\nabla_{\mathbf{u}^*}\beta$ is the gradient vector of β with respect to $\mathbf{u}^*$, $\|\cdot\|$ is the Euclidian norm of the given function, $\mathbf{J}_{\mathbf{u}^*,\mathbf{\Theta}_f}$ is the Jacobian of the probability transformation from the original space $\mathbf{x}$ to the standard normal space $\mathbf{u}$ with respect to the parameters $\mathbf{\Theta}_f$ and computed at $\mathbf{u}^*$.

To make the elements in $\nabla_{\mathbf{\Theta}_f}\beta$ comparable, $\nabla_{\Theta_f}\beta$ is multiplied by the diagonal matrix $\mathbf{D}$ of the standard deviations of the variables in $\mathbf{x}$ to obtain the sensitivity vector $\boldsymbol{\delta}$:

$$\boldsymbol{\delta} = \boldsymbol{\sigma} \nabla_{\Theta_f} \beta \tag{23}$$

The vector $\boldsymbol{\delta}$ is dimensionless and makes the parameter variations proportional to the corresponding standard deviations, which are measures of the underlying uncertainties.

2.2.1.2 Importance Measures

The limit state function is defined by the probabilistic capacity and demand models of ESAL's. Each random variable in $\mathbf{x}$ has a different contribution to the variability of the fatigue and rutting limit state functions. Important random variables have a larger effect on the variability of the limit state function than less important random variables. Knowledge of the importance of the random variables can be helpful while optimizing the performance of pavement structures. In addition, a reliability problem can only consider the uncertainty of the important variables thus simplifying the process for engineering applications.

The importance vector ($\boldsymbol{\gamma}$) for the basic random variables in original space can be obtained as [28]

$$\boldsymbol{\gamma}^T = \frac{\boldsymbol{\alpha}^T \mathbf{J}_{\mathbf{u}^*,\mathbf{x}^*} \mathbf{D}'}{\| \boldsymbol{\alpha}^T J_{\mathbf{u}^*,\mathbf{x}^*} \mathbf{D}' \|} \tag{24}$$

where $\mathbf{D}'$ is the standard deviation diagonal matrix of the equivalent normal variables $\mathbf{x}'$, defined by the linearized inverse transformation $\mathbf{x}' = \mathbf{x}^* + \mathbf{J}_{\mathbf{x}^*,\mathbf{u}^*}(\mathbf{u} - \mathbf{u}^*)$ at the design point. Each element in $\mathbf{D}'$ is the square root of the corresponding diagonal element of the covariance matrix $\Sigma' = \mathbf{J}_{\mathbf{x}^*,\mathbf{u}^*} \mathbf{J}^T_{\mathbf{x}^*,\mathbf{u}^*}$ of the variables in $\mathbf{x}'$.

2.3 Numerical Example

To illustrate the developed model, a numerical study is conducted for a typical flexible pavement section. The flexible pavement section at the time of construction consists of three layers over which an overlay was constructed at the time of rehabilitation. A MCS technique was used to estimate the failure probability of the pavement system, considering the basic random variables in the limit state functions. Table 1 lists all the basic variables $\mathbf{x}$ that enter into the models described above, along with the values of the parameters $\mathbf{\Theta}_f$. Based on physical and geometrical constrains, all the variables are assumed to follow a lognormal distribution. The probability of failure, and the sensitivity and importance measures are estimated at each time t $(1 \le t \le 11$ years). To determine the effects of the correlation between the random variables on the performance of the pavement, estimates are obtained considering both correlated and uncorrelated variables. Being a more realistic scenario, the sensitivity and importance measures are estimated only for the case with correlated variables.

Figure 3 shows the reliability estimates for uncorrelated variables, before and after the rehabilitation obtained for the pavement system (solid line) and the two individual failure criteria (fatigue cracking, dotted line, and rutting, dashed line). It is observed that shortly after construction and the rehabilitation action, the reliability of pavement is more vulnerable to rutting, due to plastic deformations of the layers. Fatigue cracking becomes more prominent with time as the accumulated traffic increases. Figure 4 shows the comparison of the reliability estimates of the system failure for the correlated and uncorrelated variables. It is observed that the system reliability increases for the correlated variables indicating that accounting for the correlation between variables improves the performance of pavement. Given that the variables in real pavements are likely to be correlated, it is important to consider their correlations to accurately predict the performance of pavement and avoid underestimating the pavement reliability which might lead to an unnecessary early repair.

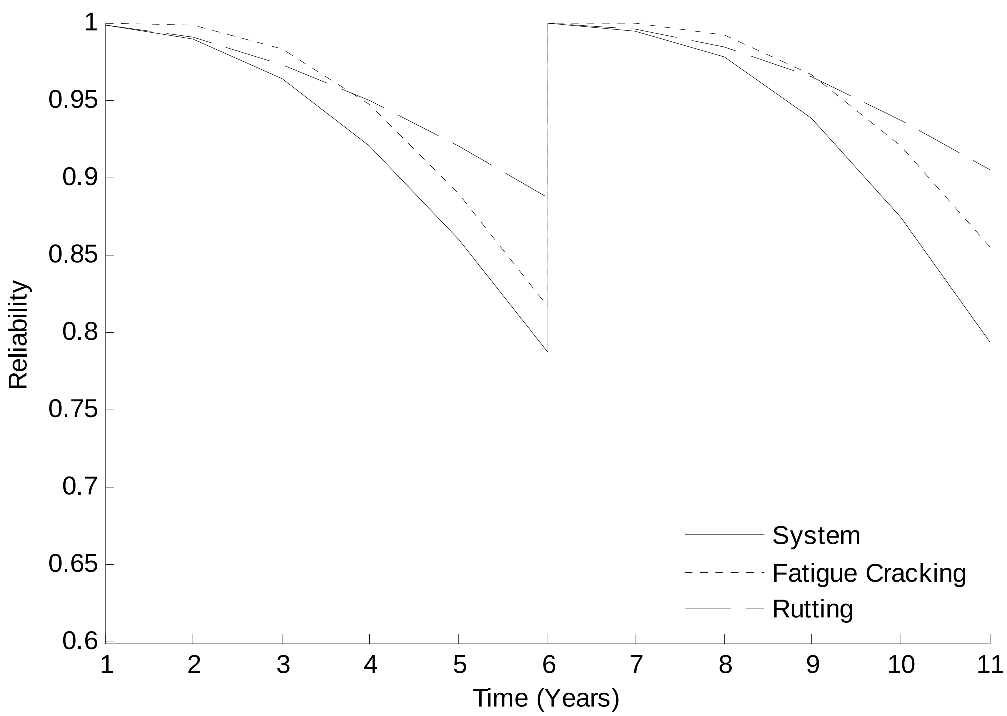

Figure 3: Reliability estimates for pavement system and individual failure modes (fatigue cracking and rutting) obtained from the numerical study considering uncorrelated variables

Table 1: Variables considered in the numerical study (Zhang and Damnjanovic [7])

Variable Description	**Distribution type**	**Mean**	**Coefficient of variation (%)**
Overlay thickness (h_1)	Lognormal	2.2 inches	15 %
Overlay modulus (E_1)	Lognormal	400,000 psi	20 %
Asphalt layer thickness (h_2)	Lognormal	4.5 inches	15 %
Asphalt layer modulus (E_2)	Lognormal	400,000 psi	20 %
Base layer thickness (h_3)	Lognormal	8 inches	15 %
Base layer modulus (E_3)	Lognormal	20,000 psi	20 %
Subgrade layer modulus (E_4)	Lognormal	10,000 psi	20 %
Yearly ESAL growth rate (gr)	Lognormal	0.08	20 %
Initial ESAL ($ESAL_{t=0}$)	Lognormal	100,000 ESAL	20 %
Poisson's ratio			
Overlay (v_1)	Deterministic	0.35	-
Asphalt layer (v_2)	Deterministic	0.35	-
Base layer (v_3)	Deterministic	0.3	-
Subgrade layer (v_4)	Deterministic	0.4	-
Limit state function parameters			
f_1	Deterministic	0.0796	-
f_2	Deterministic	3.291	-
f_3	Deterministic	0.854	-
f_4	Deterministic	$1.365\text{x}10^{-9}$	-
f_5	Deterministic	4.477	-
Loading			
Loading radius (a)	Deterministic	3.78 inches	-
Tire pressure (q)	Deterministic	100 psi	-

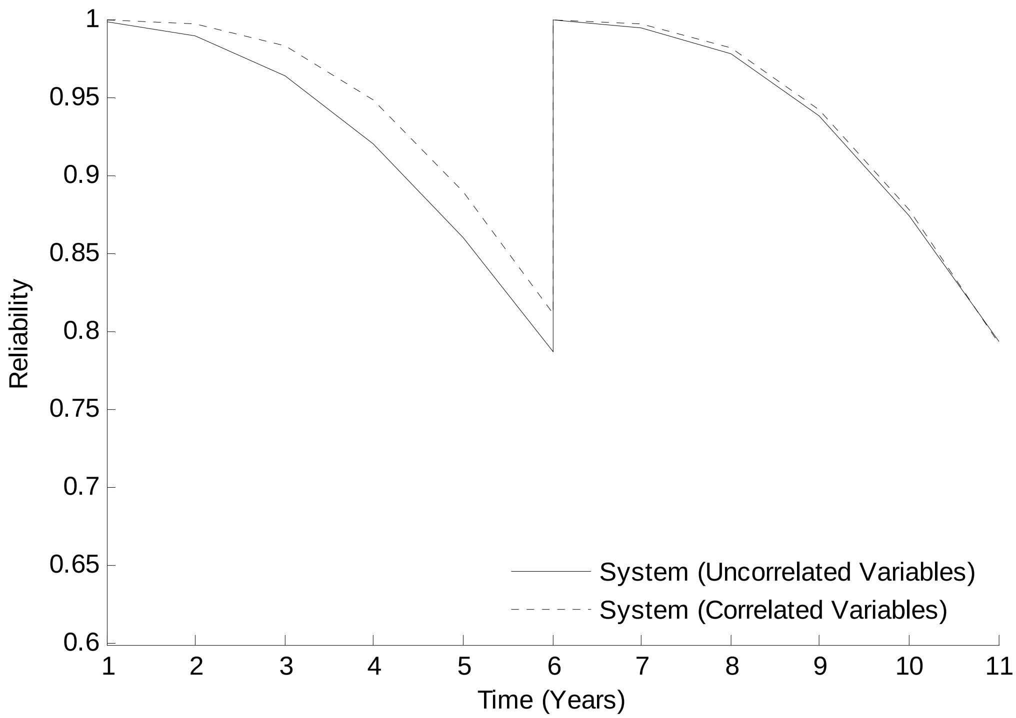

Figure 4: Comparison of system reliability estimates obtained for correlated variables and uncorrelated variables

The results of sensitivity analysis and importance measures are presented for the case of correlated variables. Figure 5 shows the sensitivity measures for the fatigue cracking to the means of the random variables used in this example. The positive value of a sensitivity measure indicates that the variable serves as a "resistance" (capacity) variable. Conversely, negative value indicates a "load" (demand) variable. Before rehabilitation actions, it is observed the means of thickness of asphalt layer, $\mu(h_2)$ and initial traffic, $\mu[ESAL(0)]$ are the variables to which the reliability is most sensitive (positively and negatively, respectively), where $\mu(\cdot)$ indicates the mean of the random variable. Whereas, after rehabilitation actions, it is observed that the fatigue cracking becomes most sensitive to the mean of modulus of asphalt layer, $\mu(E_2)$ than $\mu(h_2)$.

Thus, with respect to the fatigue cracking failure mode, it is desirable to keep asphalt layer as thick as possible in the initial design. Furthermore, because the post-reliability is most sensitive to $\mu(E_2)$, it is very important to evaluate the damaged condition of the modulus of the asphalt layer at the time of rehabilitation actions and any error in doing so can significantly affect accuracy of the estimated reliability of the system. In Figure 5, it is also observed that overlay layer modulus, E_1 act as a "load" variable. This is in conformance with behavior of flexible pavements with thin to moderate thickness asphalt layers where an increase in modulus of asphalt layer increases tensile strains; thus increases failure probability for fatigue cracking. It is also observed that the sensitivity to the mean of all the variables, except for $\mu(h_2)$, decreases with time after initial load application and rehabilitation actions. The sensitivity of $\mu(h_2)$ increases with time following the application of rehabilitation action.

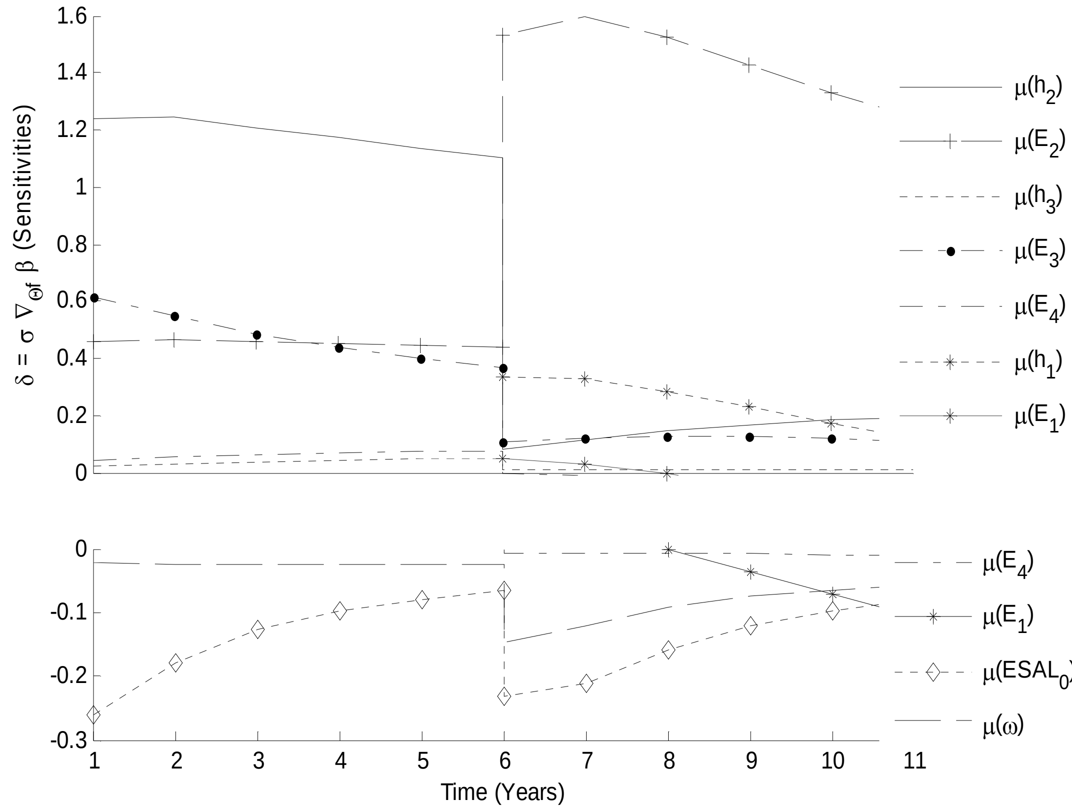

Figure 5: Sensitivities of the means of random variables for fatigue cracking estimates

Thus, with respect to the fatigue cracking failure mode, it is desirable to keep asphalt layer as thick as possible in the initial design. Furthermore, because the post-reliability is most sensitive to $\mu(E_2)$, it is very important to evaluate the damaged condition of the modulus of the asphalt layer at the time of rehabilitation actions and any error in doing so can significantly affect accuracy of the estimated reliability of the system. In Figure 5, it is also observed that overlay layer modulus, E_1 act as a “load” variable. This is in conformance with behavior of flexible pavements with thin to moderate thickness asphalt layers where an increase in modulus of asphalt layer increases tensile strains; thus increases failure probability for fatigue cracking. It is also observed that the sensitivity to the mean of all the variables, except for $\mu(h_2)$, decreases with time after initial load application and rehabilitation actions. The sensitivity of $\mu(h_2)$ increases with time following the application of rehabilitation action.

Similarly, Figure 6 shows the sensitivity measures for rutting to the means of the random variables used in this example. Before the rehabilitation action, it is observed that the rutting is most sensitive to the means of the thickness of asphalt layer, $\mu(h_2)$ and the initial traffic, $\mu[ESAL(0)]$. Similar to the fatigue cracking, in the initial design it is desirable to keep the asphalt layer as thick as possible also for rutting. Furthermore, it is observed that the sensitivity to the mean of the subgrade layer $\mu(E_4)$ is high indicating the importance of improving the stiffness of subgrade layer. After the rehabilitation, it is seen that the rutting is most sensitive to, $\mu\left(E_4\right)$. Thus improving stiffness of the subgrade layer can be helpful in the long run when considering the performance of the pavement against rutting.

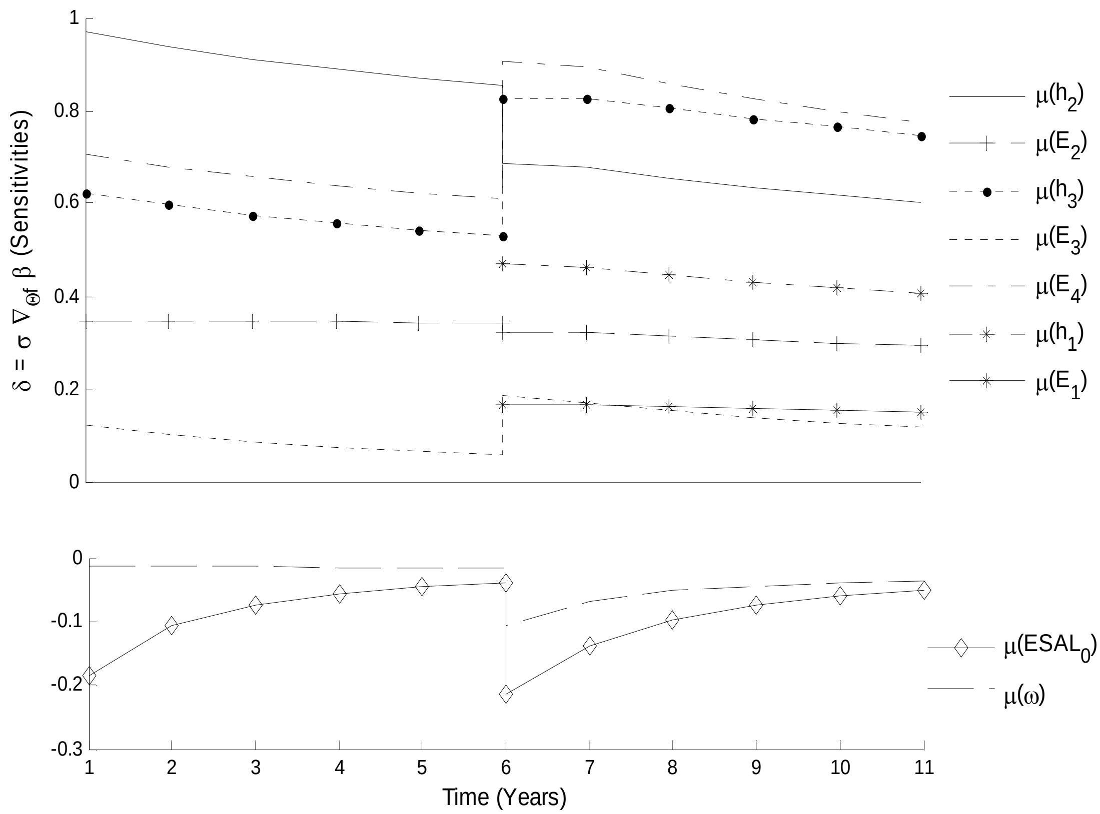

Figure 6: Sensitivities of the means of random variables for rutting estimates

Figure 7, shows the importance measures of the random variables for the fatigue cracking. For the importance measures, a negative value indicates a "resistance" variable and a positive value indicates a "load" variable. Before the rehabilitation action, it is observed that h_2 and $ESAL(0)$ are the most important "resistance" and "load" variables, respectively. Whereas, after the rehabilitation, the random variables E_2 and $ESAL(0)$ are the most important. This is in conformance with results from the sensitivity analysis. It can be said that the behavior of the asphalt layer is critical for the performance of the pavement against fatigue cracking.

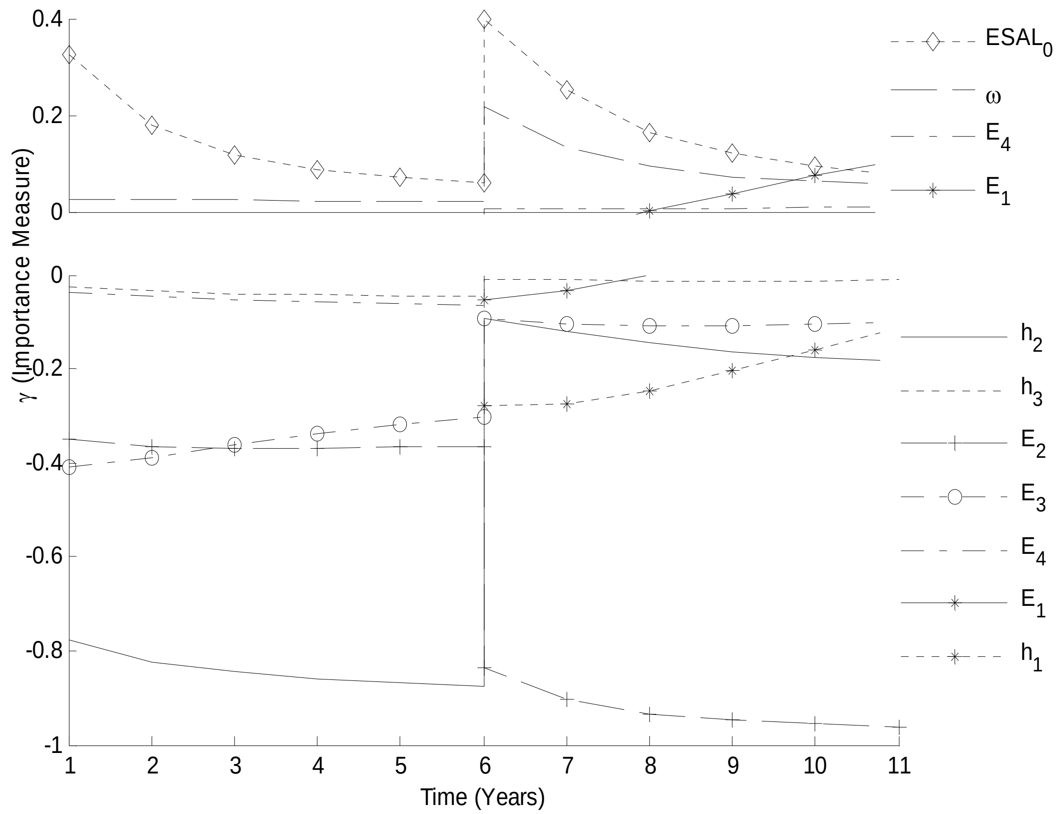

Figure 7: Importance measures of the random variables for fatigue cracking estimates

Similarly, Figure 8 shows the importance measures of the random variables for rutting. It is seen that before the rehabilitation, the thickness of the asphalt layer, h_2, is an important resistance variable. Whereas, after the rehabilitation, the thickness of the base layer, h_3, and the subgrade modulus, E_4 become equally important variables. For rutting, it is observed that along with the asphalt layer thickness, it is critical to improve the stiffness of the subgrade by means of proper compaction or any suitable practice to improve reliability.

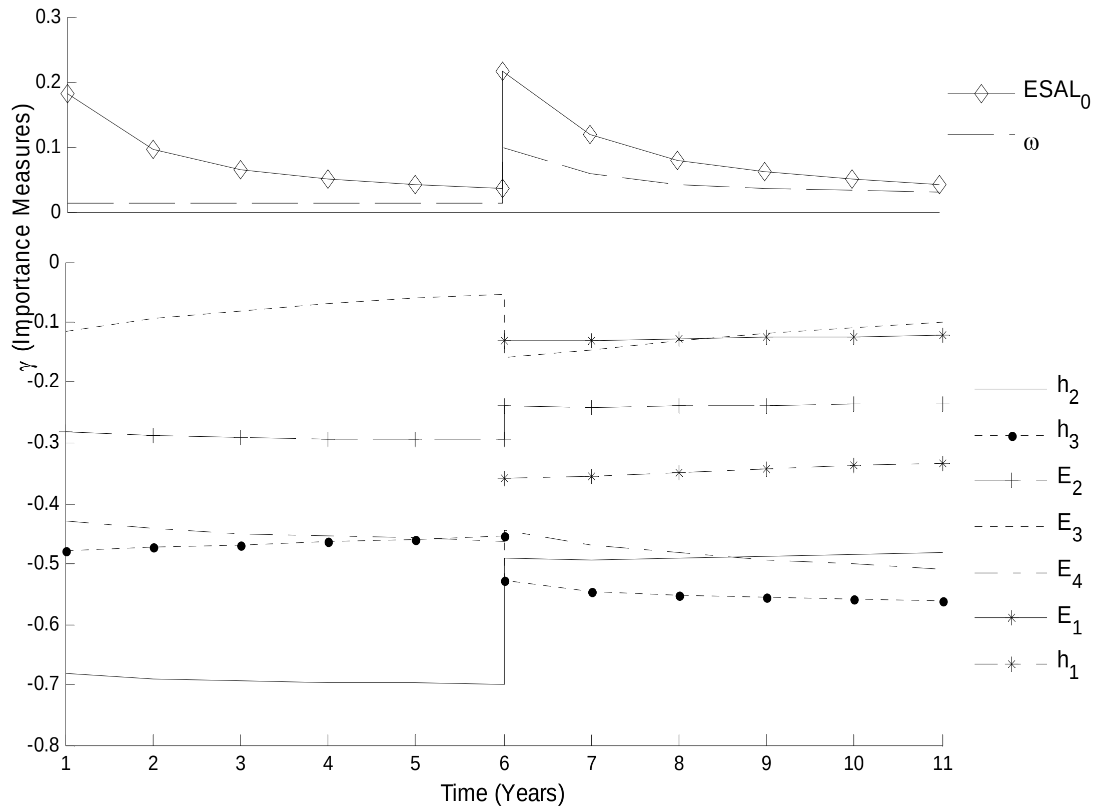

Figure 8: Importance measures of the random variables for rutting estimates

From the results obtained, it is seen that initial traffic, $ESAL(0)$, is a critical "load" variable. It is observed that the sensitivity to the mean of and importance of $ESAL(0)$ increase after the rehabilitation actions for both failure modes. Thus, decreasing the uncertainty in predicting the initial traffic can improve the accuracy of the estimated performance of pavements against fatigue cracking and rutting both before and after rehabilitation. Also it is observed that the sensitivity to the mean of and importance of $ESAL(0)$ is high during the initial period of load application and immediately after rehabilitation but they diminish rapidly with time. This might be because of the fact that, as the accumulated traffic increases, the contribution of the initial traffic to the total demand becomes less significant.

CHAPTER III

USE OF RESPONSE SURFACE METHODOLOGY AND PARAMETRIC REGRESSION FOR MODELING THE FRAGILITIES

3.1 Response Surface Modeling

The pavement responses required for capacity modeling can be computed using pavement response model that is based on the theory of linear elasticity. In the linear elastic theory, directional stresses and strains are obtained by solving the 4th order differential equation. The differential equation for the layered system cannot be solved analytically and is solved numerically for specified boundary conditions. Therefore, the relation between pavement responses and input decision variables that controls responses are implicit and pavement response model can be termed as black-box model.

Conventionally, the limit state function is evaluated using MCS technique. However, the MCS technique typically requires a relatively large number of simulations in order to obtain sufficiently accurate estimates of failure probabilities and it becomes impractical to simulate black-box model thousands of times. Under these circumstances, variance reduction techniques can improve the efficiency of MCS and significantly reduce the number of simulations. But even after using the variance reduction techniques and availability of advanced computers, the computation time is very large, then the black-box model can be categorized as very complex. Figure 9 shows the general categorization of analytical models based on computational time and structural reliability methods that can be most suitably applied. Since the use of MCS technique becomes impractical for very complex models, use of alternative approaches that can provide accurate results seem to be justifiable. Based on the computational time, the pavement response model presented in the research can be categorized as very complex. In the research, an alternative approach of response surface methodology (RSM) is used to approximate the black-box model into a closed form function.

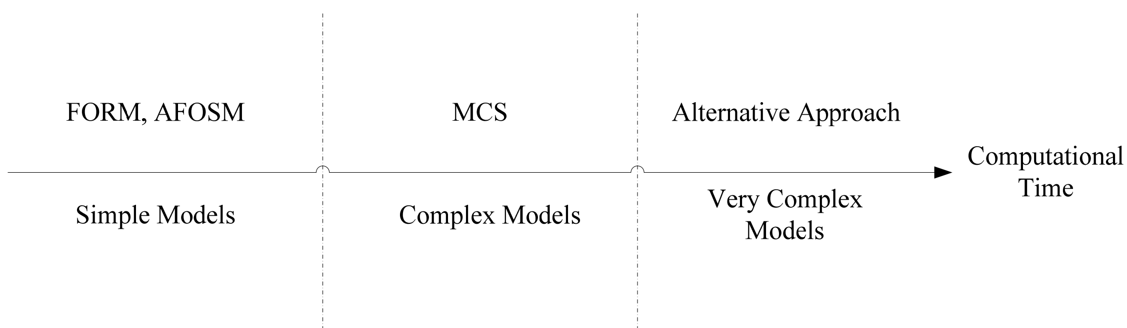

Figure 9: Computational time for reliability analysis and suitable reliability methods

3.1.1 Response Surface Methodology (RSM)

The Response surface methodology has already been widely used in the field of reliability analysis [29-34]. The primary objective of RSM in reliability analysis is to approximate the implicit responses into a closed-form function of decision variables. The approximated function will be computationally simple and can be easily simulated to obtain reliability estimates. Typically, the approximated response model can be expressed as

$$\begin{aligned} y &= \hat{y} + \pi \\ \hat{y} &= f(\mathbf{x}) \end{aligned} \tag{25}$$

where, y is the actual response, $\hat{y}$ is the estimated response, $\mathbf{x}$ is the vector or matrix of decision variables, π is the model error or residual and function f can be a polynomial of any order. Since the pavement responses are non-linear, initially it is assumed that second order (quadratic) polynomial will fit appropriately. The general form of the second order polynomial can be expressed as

$$y = \eta_0 + \sum_{i=1}^{n} \eta_i x_i + \sum_{i=1}^{n} \eta_{ii} x_i^{\,2} + \sum_{i=1}^{n-1} \sum_{j>1}^{n} \eta_{ij} x_i x_j + \pi \qquad (26)$$

where, η_0, η_i, η_{ii}, η_{ij} are the unknown coefficients to be estimated, n is the number of decision variables. In the above polynomial, even though there are higher order terms, it is still a linear combination of variables in $\mathbf{x}$ and can be expressed as

$$y = \eta_0 + \sum_{i=1}^{l} \eta_i z_i + \pi \qquad (27)$$

where, z represents variables, squares of variables and interactions between variables, l is the total number of parameters in the polynomial. In quadratic polynomial for n variables there are $l = (n+1)(n+2)/2$ parameters. Suppose there are k observations, the Eq. 27 can be expressed in matrix notation as

$$\mathbf{y} = \mathbf{z}\boldsymbol{\eta} + \boldsymbol{\pi} \qquad (28)$$

where,

$$\mathbf{y} = \begin{bmatrix} y_1 \\ y_2 \\ . \\ . \\ . \\ y_k \end{bmatrix} \quad \mathbf{z} = \begin{bmatrix} 1 & z_{11} & z_{12} & \dots & z_{1l} \\ 1 & z_{21} & z_{22} & \dots & z_{2l} \\ . & . & . & \dots & . \\ . & . & . & \dots & . \\ . & . & . & \dots & . \\ 1 & z_{k1} & z_{k2} & \dots & z_{kl} \end{bmatrix} \quad \boldsymbol{\eta} = \begin{bmatrix} \eta_0 \\ \eta_1 \\ . \\ . \\ . \\ \eta_k \end{bmatrix} \quad \boldsymbol{\pi} = \begin{bmatrix} \pi_0 \\ \pi_1 \\ . \\ . \\ . \\ \pi_k \end{bmatrix}$$

3.1.2 Least Square Estimation (LSE)

The estimates of unknown coefficients $\boldsymbol{\eta}$ in the quadratic polynomial can be evaluated using the least squares estimation technique. In the least square method, unknown estimates are obtained by minimizing the sum of the square of errors, SS_E

$$SS_E = \sum_{i=1}^{k} \pi_i^{\,2} \tag{29}$$

Therefore, the estimators $\hat{\boldsymbol{\eta}}$ of $\boldsymbol{\eta}$ can be obtained by solving following equation

$$\frac{\partial SS_E}{\partial \eta} = 0 \tag{30}$$

The solution to the Eq. 30 in the matrix notation is

$$\hat{\boldsymbol{\eta}} = \left(\mathbf{z}'\mathbf{z}\right)^{-1} \mathbf{z}'\mathbf{y} \tag{31}$$

Once the parameters are estimated, the fitted response surface model can be expressed as

$$\hat{\mathbf{y}} = \mathbf{z}\hat{\boldsymbol{\eta}} \tag{32}$$

In least squares estimation, the estimates of coefficients are unbiased estimators under the assumption that the errors π_i are normally distributed and statistically independent with zero mean and constant variance ς^2. The next step is to validate the fitted model.

3.1.3 Statistical Validation of Fitted Model

There are number of measures that can be used to statistically validate the model. Some of the very common measures that are used for statistical validation are discussed. One of the most common and simple measure to determine significance of the model is the coefficient of determination, R^2 which is obtained as [35].

$$R^2 = \frac{SS_R}{SS_T} \tag{33}$$

where, SS_R is the sum of the square due to regression and SS_T is the total sum of squares and can be computed as

$$SS_R = \hat{\boldsymbol{\eta}}'\mathbf{z}'\mathbf{y} - \frac{\left(\sum_{i=1}^{k} y_i\right)^2}{k} \tag{34}$$

$$SS_T = SS_R + SS_E \tag{35}$$

where, SS_E is the sum of squares due to error defined in Eq. 29. Value of R^2 is between 0 and 1 where 1 represents the best fit. However, one of the problems with R^2 is that it increases with the addition of variables in the model without giving information about usefulness of the new variable in the model. Adjusted $R_{adj}{}^2$ is more preferable as it has the advantage that it only increases if the added variable reduces the mean square error in the model. The adjusted $R_{adj}{}^2$ can be computed as

$$R_{adj}{}^2 = 1 - \frac{SS_E / (k-p)}{SS_T / (k-1)} \tag{36}$$

Often root means square error ($\%RMSE$) is used to determine overall accuracy of the fitted model. The $\%RMSE$ defined by prediction error sum of squares ($PRESS$) has advantage that it does not provide overly optimistic behavior of the model [36]. The $PRESS$ and $\%RMSE$ statistics can be computed as

$$PRESS = \sum_{i=1}^{k} \pi_i{}^2 = \sum_{i=1}^{k} (y_i - \hat{y}_{(i)})^2 \tag{37}$$

$$\%RMSE = 100\frac{\sqrt{\frac{PRESS}{k}}}{\frac{1}{k}\sum_{i=1}^{k} y_i} \tag{38}$$

In addition to the above discussed statistics, residual plots can be efficiently used to validate the accuracy of the model.

Coefficient of determination and $\%RMSE$ can be used as the global statistics to validate the overall accuracy of the model. But in addition to overall accuracy, it is necessary to test whether linear relationship exists between response and design variables. This is usually tested using F_0 statistics that depend on sum of square of regression coefficients and error and degrees of freedom for the model and can be obtained as [35]

$$F_0 = \frac{SS_R / n}{SS_E / (k - l)} \tag{39}$$

If the F_0 statistic is greater than desired value, it signifies the linear relationship between response and decision variables.

3.1.4 Model Selection

One of challenge in multiple regression analysis is to select important variables to be used in the model. In the quadratic polynomial, for n variables there are $l = (n+1)(n+2)/2$ variables and there is the always the possibility that some of the variables may not contribute significantly to the change in response. These variables can be removed from the developed response model without affecting the accuracy of predicted response. Sometimes the presence of unwanted variables can also increase the error in the model. Therefore it is necessary to select a model that includes all the important variables.

In the research, backward elimination process is used for model selection. In backward elimination, model development starts with all the parameters i.e. l. The model with l parameters will have certain $R_{adj}{}^2$. Since the $R_{adj}{}^2$ only increases with addition of significant variable, the elimination of significant variable from the model will cause significant reduction in the value of $R_{adj}{}^2$. In multiple linear regression, t_0 is used to determine the significance of individual regression coefficient in the model and can be obtained as [35]

$$t_0 = \frac{\hat{\eta}_i}{\sqrt{\varsigma^2 C_{ii}}} \tag{40}$$

where, ς^2 is the estimate of the variance in the error term in the model and is computed as $\varsigma^2 = SS_E / (k-l)$ and C_{ii} is the variance of the ith coefficient obtained from covariance matrix $\mathbf{C} = (\mathbf{z}'\mathbf{z})^{-1}$. Once the model with l parameters is developed and t_0 statistics is obtained, elimination process is started wherein the variable with t_0 statistics closest to 0 is removed and the reduced model is checked for $R_{adj}{}^2$. This process is continued till there is a significant decrease in the value of $R_{adj}{}^2$. Final model will be one of best fitted model and can be validated for different tests as already discussed.

3.2 Modeling the Pavement Fragilities

Typically in performance based design, the performance is modeled in terms of fragilities. Advantage of expressing performance in terms of fragilities is that the fragilities can be easily defined for different performance requirements. For instance, fragilities can be developed for performance measures like fatigue cracking, rutting, thermal cracking and other performance measures. Even within each performance measure, the fragilities can be developed for different performance indices like for instance, the fragilities for 10% and 45% cracking in fatigue. Developed fragilities then can be used as performance measures for different loading conditions like high traffic demand, low traffic demand, loading due to snow, etc. One of the most important uses of fragilities is that the fragilities expressed in terms of decision variables can be efficiently used in optimization formulations.

The fragility in the simple words can be defined as the conditional probability of failure given the level of demand and can be expressed as

$$P_{F/D} = P\left[g(\mathbf{x}) \le 0 / N_D\right] \tag{41}$$

where, the form $P\left[g(\mathbf{x}) \le 0 / N_D\right]$ is the conditional probability of event $g(\mathbf{x}) \le 0$ given the values of N_D. From the definition of conditional probabilities, the fragilities can be obtained by evaluating the limit state function for the deterministic demand. The uncertainty in the event $g(\mathbf{x}) \le 0$ for given N_D arises from the inherent randomness in the capacity variables in $\mathbf{x}$. Once the fragility is obtained, it can be used to compute failure probability of the system by accounting for uncertainties in the demand as follows

$$P_F = \int_0^\infty P\left[g(\mathbf{x}) \le 0 / N_D\right] P\left[N_D\right] dN_D \tag{42}$$

where, $P[g(\mathbf{x}) \le 0 / N_D]$ is the fragility for given performance measure and $P[N_D]$ is the distribution for the demand or hazard function. However, the fragilities are functions of decision variables (layer thickness, layer modulus of elasticity) in the sense that stronger the pavement lesser is the failure probability and vice versa. To express fragilities in terms of decision variables, in the research, a parametric regression model is developed for defining a closed-from function for fragilities.

3.2.1 Parametric Regression Modeling

Parametric modeling for failure probabilities is already a popular area in the field of lifetime data analysis [37]. The basic concept in parametric modeling for failure probabilities is to fit an appropriate model using the available failure data. The most common models used for parametric modeling are lognormal, extreme-type I, Weibull and logistic distribution models. Parametric modeling involves simply determining the distribution parameters that best fits the available failure data. However, the relation between decision variables and fragilities is of interest. The effect of decision variables can be incorporated in parametric model by specifying a relationship between distribution parameters and decision variables. Generally, for modeling the fragilities, use of two parameter lognormal distribution is very common [38] and parametric model for lognormal distribution can be expressed as

$$F(N_D, \mathbf{x}) = \Phi\left[\frac{\log(N_D) - \psi(\mathbf{x})}{\xi}\right] \tag{43}$$

where, ψ and ξ are the lognormal distribution parameters i.e. mean and standard deviation respectively, Φ is the standard normal cumulative distribution function. In the above equation, the mean of lognormal distribution is made a function of decision variables in $\mathbf{x}$. A linear specification is assumed between distribution parameter and decision variables and can be expressed as

$$\psi(\mathbf{x}) = \mathbf{c}'\mathbf{x} \tag{44}$$

where, $\mathbf{c}$ is the vector of regression parameters to be estimated. Estimation of regression parameters falls in the category of non-linear regression and can be estimated efficiently using maximum likelihood estimation technique.

3.2.2 Maximum Likelihood Estimation (MLE)

The basic behind MLE is to determine the parameters that maximize the likelihood of the available observations. For the fragilities obtained using MCS technique, the limit state function is evaluated using binary numbers i.e. 1 for the failure event $g \le 0$ and 0 otherwise and the likelihood function can be expressed as [38]

$$L(\mathbf{c}, \xi) = \prod_{i=i}^{M} \left[F(N_{D_i}, \mathbf{x}_i)\right]^{e_i} \left[1 - F(N_{D_i}, \mathbf{x}_i)\right]^{1-e_i} \tag{45}$$

where, $F(\cdot)$ is the fragility curve, M is the total number of pavement sections simulated, N_{D_i} is the demand to which pavement i is subjected, $e_i = 1$ or 0 depends on the state of limit state. The likelihood function defined in Eq. 45 is maximized to obtain parameter estimates and can be computed easily using standard optimization algorithms.

Once the parameters are estimated using MLE, the next step is to validate the developed model for its accuracy.

3.2.3 Model Validation

The parametric regression model is developed with the assumption of linear specification between model parameter ψ and decision variables in $\mathbf{x}$. Therefore it is necessary to validate the developed model for its accuracy and assumptions. In the research, primarily the different kinds of plots are used to verify the model. To check the accuracy of the developed model, the actual probabilities are plotted against the predicted probabilities. If all the points in the plot are scattered over 1:1 line, then the model is validated for accuracy. Next the residual plots against predicted probabilities and decision variables can be used to validate the model. Any trend is residual plot indicates that some transformation or higher order term might be needed in the model else it signifies that the included parameters are significant. In addition to the plots, mean absolute percentage error $MAPE$ can be used to validate the accuracy of the model. $MAPE$ can be obtained as

$$MAPE = \frac{1}{k}\sum_{i=1}^{k}\left|\frac{P_{actual} - P_{predicted}}{P_{actual}}\right| \tag{46}$$

3.2.4 Model Selection

In maximum likelihood estimation technique, each estimated regression parameters will be characterized by the corresponding standard deviation. Best fit model will have standard deviation of all the estimated regression parameters low as compared to their mean value i.e. coefficient of variation will be very low. Also, while specifying the relationship between distribution parameter and decision variables, there is always the possibility that some of the variables might not contribute to the model. Therefore it is necessary to remove the variables that are not significant in the model. In the research, backward elimination is used for the model selection process. In backward elimination, selection process starts with developing a model with all the possible variables in the linear specification. The regression parameters for the model are estimated by maximizing the likelihood function. The process of elimination is started with the variable corresponding to regression parameter with highest coefficient of variation. As the removed variable is assumed to be insignificant in the model, elimination of the same will not significantly affect the maximum value of likelihood function of the reduced model. The process of elimination is continued till there is a significant decrease in the maximum likelihood function value. The model in the step previous to significant decrease in the maximum likelihood value can be chosen as the best possible combination of decision variables.

3.3 Numerical Example

To illustrate the proposed methodology, fatigue cracking failure for flexible pavement is considered. Typical three-layer flexible pavement system is considered for numerical study. For fatigue cracking, maximum tensile strain at the bottom of the asphalt layer controls the allowable number of repetitions and the response model is developed for the critical tensile strain. To account for the effects of rehabilitation actions, it is assumed that an overlay will be constructed at the time of rehabilitation actions and the system will behave as four layered system. After rehabilitation actions, tensile strain at the bottom of overlay is considered critical and another response model is developed to account for the pavement responses after rehabilitation actions. Using developed response models, the fragilities are computed for the pavement system before and after rehabilitation actions. Once the fragilities are obtained, the reliability estimates are estimated by accounting for uncertainties in the demand variables.

Table 1 lists all the decision variables $\mathbf{x}$ that enter into the model. Based on physical and geometrical constrains, all the random variables are assumed to follow a Lognormal distribution. For developing the response model, decision variables are normalized to obtain dimensionless decision variables so that the developed response model can be used irrespective of the measuring units. Table 2 shows the typical upper and lower limits that are used to normalize the decision variables.

Table 2: Typical upper and lower limits values considered for modeling pavement response model

Variable Description	Symbol	Unit	Lower Limit	Upper Limit
Overlay thickness	h_1	Inches	2.5	5.0
Overlay modulus	E_1	Psi	300,000	600,000
Asphalt layer thickness	h_2	Inches	5.0	9.5
Asphalt layer modulus	E_2	Psi	300,000	600,000
Base layer thickness	h_3	Inches	9.5	14
Base layer modulus	E_3	Psi	10,000	30,000
Subgrade modulus	E_4	Psi	5,000	15,000

3.3.1 Pavement Response Model for Critical Tensile Strain

The set of observations for critical tensile strain at the bottom of asphalt layer before rehabilitation actions and at the bottom of overlay after rehabilitation actions are obtained from analytical pavement response model. Table 3 shows the final response models along with the statistical validation of the developed models. All the statistics show that the developed response models are able to describe the actual responses obtained from the analytical model. Residual plots are shown in Figure 10 and it is seen that the assumption of constant variance for residuals is validated and there is no trend in the residuals.

Table 3: Results for developed response models for critical tensile strain

Description	Model	Model Validation Statistics
Before rehabilitation actions	$\varepsilon_t = 0.001 - 0.0011 \times h_2 - 0.0005 \times E_2 - 0.0002 \times E_3$ $+ 0.0004 \times h_2^{\ 2} + 0.0001 \times E_2^{\ 2} + 1.733e^{-5} \times E_3^{\ 2}$ $+ 0.0002 \times h_2 \times E_2 + 0.0001 \times h_2 \times E_3 + 6.403e^{-5} \times E_2 * E_3$	$R^2 = 0.9991$ $R_{adj}^{\ 2} = 0.9988$ $\%RMSE = 4.22\%$ $F_0 = 3740.36 \gg 3.07$
After rehabilitation actions	$\varepsilon_t = 0.0008 - 0.0003 \times h_1 - 0.0002 \times E_1 - 5.095e^{-5} \times h_2 - 0.0038 \times E_2$ $- 5.299e^{-5} \times E_3 + 0.0037 \times E_2^{\ 2} - 3.503e^{-5} \times E_1 \times h_1 + 0.0015 \times$ $+ 0.0009 \times E_2 \times E_1 + 3.163e^{-5} \times E_3 \times h_2 + 0.0001 \times E_3 \times E_2$	$R^2 = 0.9911$ $R_{adj}^{\ 2} = 0.9909$ $\%RMSE = 3.88\%$ $F_0 = 4112.39 \gg 2.4$

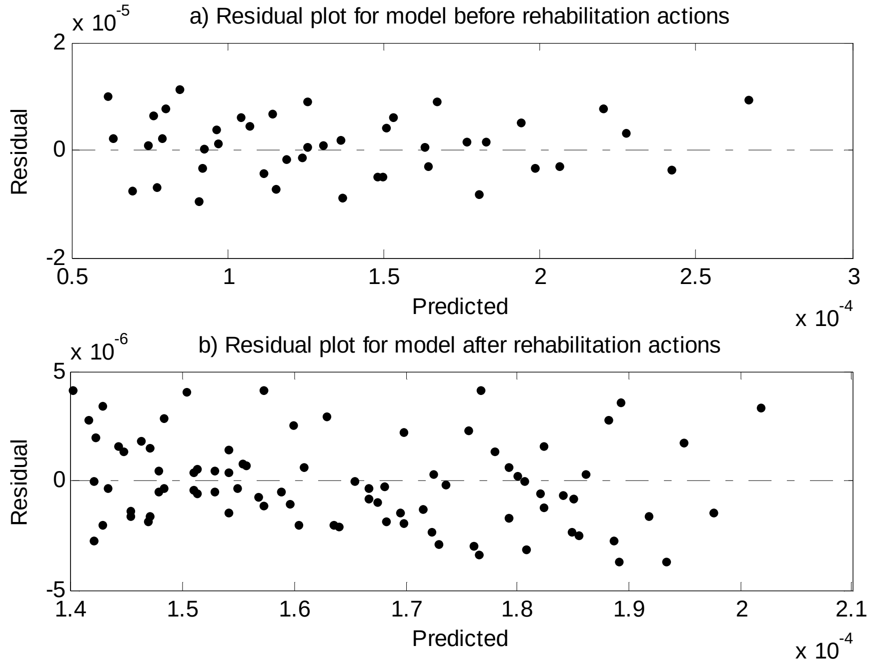

Figure 10: Residual plots for developed response model

3.3.2 Fragility Model for Fatigue Cracking Failure

The response surface model for tensile strains before and after rehabilitation actions in conjunction with MCS is used to simulate the failure data for fatigue cracking. The parameters $\mathbf{c}$ and ξ are estimated using MLE. Once the model is developed, it is validated for accuracy and made assumptions.

3.3.2.1 Before Rehabilitation Actions (Three-layer System)

Figure 11 shows the model selection process with maximum likelihood function value computed at each step of the backward elimination. In the Figure 11, it is seen that the maximum likelihood value decreases significantly after step 6 and therefore model at step 6 is chosen as the final model. The linear specification for the model in step 6 is of the form

$$\psi = c_0 + c_1 \times h_2 + c_2 \times E_2 + c_3 \times E_3 - c_4 \times h_2^{\,2} \tag{47}$$

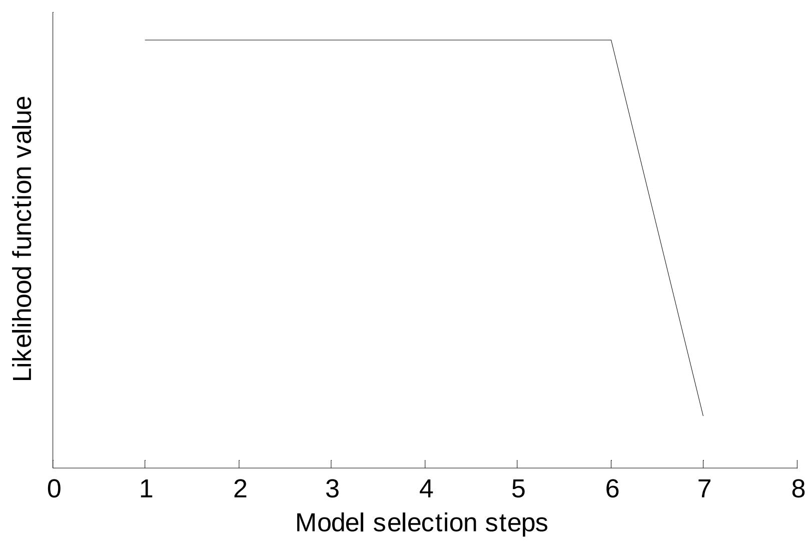

Figure 11: Model selection process for modeling fatigue fragilities for pavement system before rehabilitation actions

Parameters are estimated using MLE and Table 4 gives the details about parameter estimates along with parameter standard deviations and corresponding correlation matrix. Figure 12 shows the plots used for model validation. All the plots in Figure 12 validate the developed model for accuracy and made assumptions. The mean absolute percentage error for the developed model is $MAPE = 5.36\%$ which is very low and further validates the model.

Table 4: Details about parameter estimates obtained from fragility modeling for pavement system before rehabilitation actions

Symbol	Mean	Std. Dev.	Correlation Matrix					
			c_0	c_1	c_2	c_3	c_4	ξ
c_0	6.47	0.99	1.00	-0.96	-0.22	-0.05	0.93	-0.11
c_1	12.74	2.68	-0.96	1.00	0.00	-0.11	-0.99	0.05
c_2	1.87	0.26	-0.22	0.00	1.00	0.07	0.02	0.15
c_3	1.02	0.19	-0.05	-0.11	0.07	1.00	0.14	0.11
c_4	-5.05	1.78	0.93	-0.99	0.02	0.14	1.00	-0.03
ξ	0.67	0.05	-0.11	0.05	0.15	0.11	-0.03	1.00

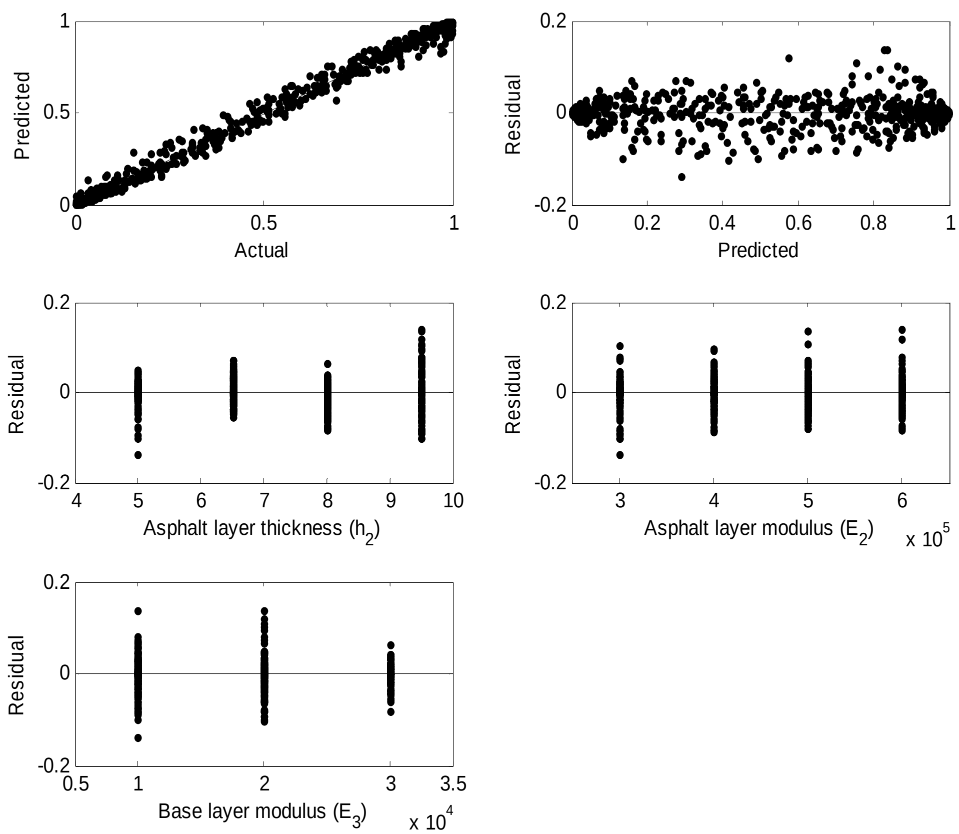

Figure 12: Plots used for validating fatigue cracking parametric regression model for pavement system before rehabilitation actions

3.3.2.2 After Rehabilitation Actions (Four-layer System)

Similarly for the pavement system after rehabilitation actions, the linear specification of the final model obtained through selection process is

$$\begin{aligned}\psi = c_0 &+ c_1 \times h_1 + c_2 \times E_1 + c_3 \times h_2 + c_4 \times E_2 + c_5 \times E_3 \\ &+ c_6 \times h_1^2 + c_7 \times E_1^2 + c_8 \times E_2^2 + c_9 \times h_1 \times E_1 + c_{10} \times h_1 \times h_2 \\ &+ c_{11} \times h_1 \times E_2 + c_{12} \times E_1 \times h_2 + c_{13} \times E_1 \times E_2 + c_{14} \times h_2 \times E_2 \\ &+ c_{15} \times h_2 \times E_3 + c_{16} \times E_2 \times E_3\end{aligned} \tag{48}$$

Table 5 gives the details about parameter estimates along with parameter standard deviations and corresponding correlation matrix. Figure 13 shows the plots used for model validation. All the plots in Figure 13 validate the developed model for accuracy and made assumptions. The mean absolute percentage error for the developed model is $MAPE = 9.36\%$ which is low and further validates the model.

3.3.3 Pavement Performance

Using the developed response surface model and parametric regression model for fragilities, the reliability estimates were obtained for fatigue cracking failure by solving the integral in the Eq. 42. The reliability estimates obtained from response model and fragilities are compared to the reliability estimates obtained by simulating analytical pavement response model. Figure 14 shows the reliability estimates for the flexible pavement system before as well as after rehabilitation actions. In the Figure 14, the rehabilitation actions are carried in the year 6 and an overlay is constructed at the time of rehabilitation actions. It is observed that the developed response surface and fragility models can be used efficiently to predict the performance before as well as after rehabilitation actions.

Table 5: Details about parameter estimates obtained from fragility modeling for four-layer system

Symbol	Mean	Std. Dev.	Correlation Matrix																	
			c_0	c_1	c_2	c_3	c_4	c_5	c_6	c_7	c_8	c_9	c_{10}	c_{11}	c_{12}	c_{13}	c_{14}	c_{15}	c_{16}	ξ
c_0	9.02	0.0018	1.00	-0.71	-0.30	0.69	-0.64	0.00	0.04	0.48	0.27	-0.53	0.48	-0.51	-0.56	-0.60	-0.68	-0.42	-0.53	0.52
c_1	1.71	0.0031	-0.71	1.00	0.12	-0.86	0.53	0.00	-0.15	-0.81	-0.04	0.82	-0.50	0.59	0.70	0.69	0.72	0.56	0.78	-0.83
c_2	-1.57	0.0027	-0.30	0.12	1.00	-0.34	0.55	0.00	-0.38	-0.15	-0.18	0.42	-0.61	0.57	0.49	0.47	0.44	-0.39	0.35	-0.06
c_3	0.19	0.0029	0.69	-0.86	-0.34	1.00	-0.71	0.00	0.02	0.92	0.01	-0.95	0.79	-0.85	-0.92	-0.90	-0.88	-0.50	-0.94	0.92
c_4	47.84	0.0059	-0.64	0.53	0.55	-0.71	1.00	0.00	0.20	-0.71	-0.60	0.62	-0.90	0.91	0.83	0.90	0.91	0.33	0.68	-0.62
c_5	0.88	4.4E-14	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
c_6	1.13	0.0022	0.04	-0.15	-0.38	0.02	0.20	0.00	1.00	-0.15	-0.19	-0.17	-0.21	0.06	-0.05	0.06	0.04	0.46	-0.15	-0.11
c_7	1.14	0.0068	0.48	-0.81	-0.15	0.92	-0.71	0.00	-0.15	1.00	0.15	-0.90	0.79	-0.85	-0.90	-0.89	-0.84	-0.57	-0.91	0.94
c_8	-39.82	0.0017	0.27	-0.04	-0.18	0.01	-0.60	0.00	-0.19	0.15	1.00	0.09	0.30	-0.31	-0.23	-0.29	-0.32	-0.14	-0.05	0.04
c_9	1.92	0.0064	-0.53	0.82	0.42	-0.95	0.62	0.00	-0.17	-0.90	0.09	1.00	-0.78	0.83	0.92	0.86	0.80	0.29	0.94	-0.85
c_{10}	0.44	0.0047	0.48	-0.50	-0.61	0.79	-0.90	0.00	-0.21	0.79	0.30	-0.78	1.00	-0.95	-0.89	-0.91	-0.84	-0.27	-0.75	0.68
c_{11}	-18.32	0.0058	-0.51	0.59	0.57	-0.85	0.91	0.00	0.06	-0.85	-0.31	0.83	-0.95	1.00	0.96	0.98	0.94	0.26	0.88	-0.75
c_{12}	0.22	0.0055	-0.56	0.70	0.49	-0.92	0.83	0.00	-0.05	-0.90	-0.23	0.92	-0.89	0.96	1.00	0.97	0.92	0.31	0.96	-0.83
c_{13}	-11.73	0.0037	-0.60	0.69	0.47	-0.90	0.90	0.00	0.06	-0.89	-0.29	0.86	-0.91	0.98	0.97	1.00	0.98	0.38	0.91	-0.83
c_{14}	2.83	0.0039	-0.68	0.72	0.44	-0.88	0.91	0.00	0.04	-0.84	-0.32	0.80	-0.84	0.94	0.92	0.98	1.00	0.39	0.88	-0.80
c_{15}	-0.64	0.0017	-0.42	0.56	-0.39	-0.50	0.33	0.00	0.46	-0.57	-0.14	0.29	-0.27	0.26	0.31	0.38	0.39	1.00	0.34	-0.67
c_{16}	-1.24	0.0040	-0.53	0.78	0.35	-0.94	0.68	0.00	-0.15	-0.91	-0.05	0.94	-0.75	0.88	0.96	0.91	0.88	0.34	1.00	-0.89
ξ	0.61	0.0015	0.52	-0.83	-0.06	0.92	-0.62	0.00	-0.11	0.94	0.04	-0.85	0.68	-0.75	-0.83	-0.83	-0.80	-0.67	-0.89	1.00

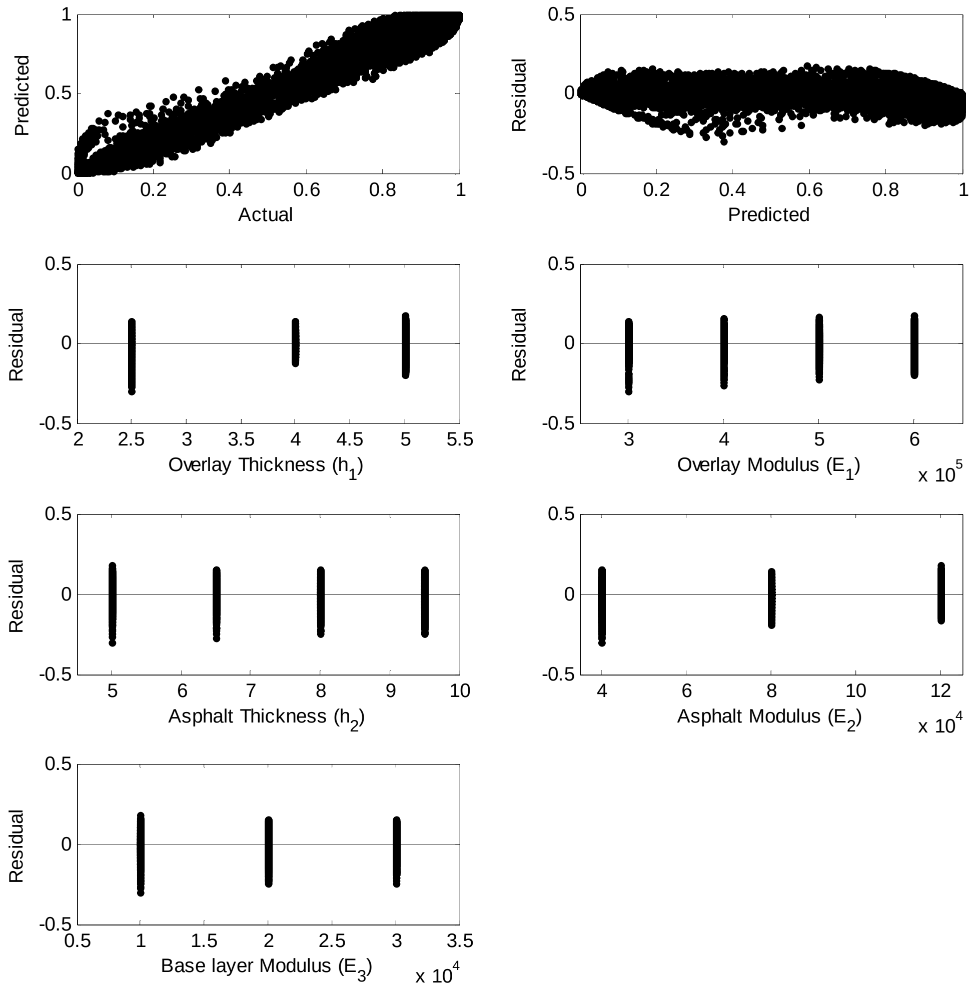

Figure 13: Plots used for validating fatigue cracking parametric regression model for pavement system after rehabilitation actions

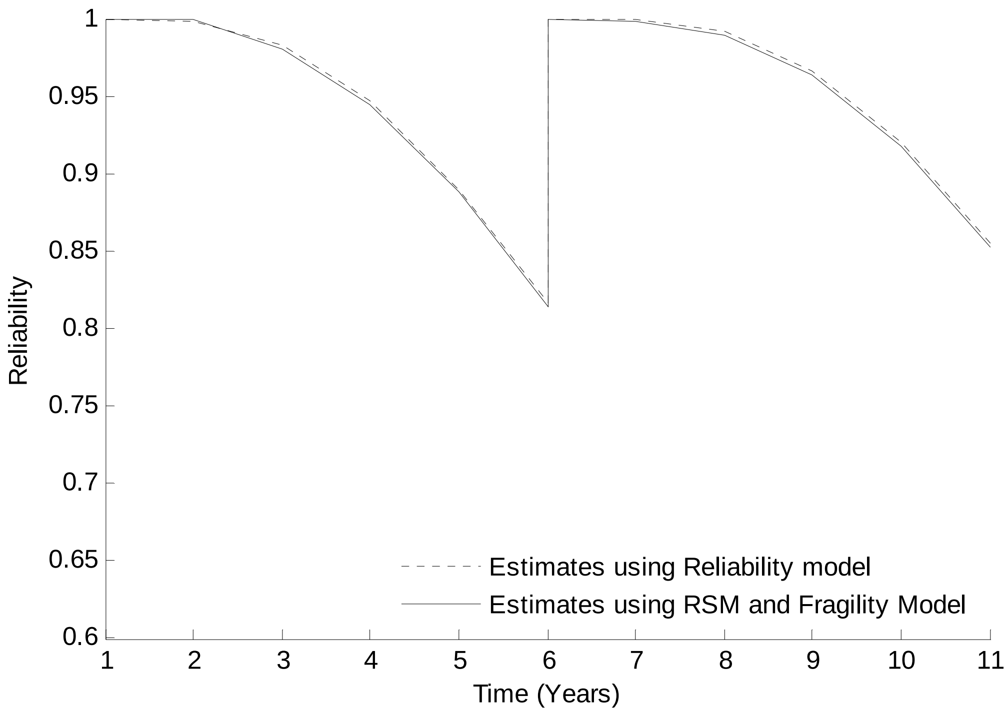

Figure 14: Fatigue cracking reliability estimates obtained using developed response surface model and parametric regression model for fragilities

3.4 Summary

The research presents a reliability model that is able to account for the effects of rehabilitation actions on the reliability of flexible pavements. A mechanistic-empirical approach is used to define limit state functions based on the pavement responses before and after the application of rehabilitation actions. Conventionally, the limit state function is evaluated using MCS technique. However, the MCS technique typically requires a relatively large number of simulations in order to obtain sufficiently accurate estimates of failure probabilities and it becomes impractical to simulate the black-box model thousands of times. In the research, an alternative approach of response surface methodology is explored for obtaining reliability estimates. Typically, in reliability analysis, fragilities are used to express the performance of the system. In the research, the parametric regression model is developed to express the fragilities in terms of decision variables.

Statistical validation of pavement response model shows that the developed response models are good fit to the responses obtained from analytical model and can be used efficiently for predicting pavement responses. Similarly the statistical validation of parametric regression model shows the accuracy of the developed model. The reliability estimates obtained using developed response surface models and parametric regression models describe the behavior of new and rehabilitated flexible pavement systems. The developed models can be effectively implemented in all the applications that require the estimation of the performance of flexible pavement systems before and/or after rehabilitation actions. Most importantly, the advantage of a closed-form function of fragility is its suitability for implementation in optimization models.

CHAPTER IV

RELIABILITY-BASED OPTIMIZATION FOR FLEXIBLE PAVEMENTS

As discussed, the probabilistic optimization technique that can account for uncertainties in pavement performance is essential. One of the probabilistic optimization techniques is reliability-based optimization (RBO). The RBO can be efficiently used in balancing the needs between safety in performance and economy in design. In RBO, pavement reliability which is the probability that pavement will perform its intended function under a given set of conditions over a specified period of time is used as a performance measure One of the most important advantage of using reliability as a performance measure is that the reliability models can take into account pavement characteristics and utilization patterns in the specification of propensity functions.

Typically, RBO is a type of probabilistic optimization technique that accounts for uncertainties in the performance of the structure. The performance is measured in terms of probability of failure, P_F or reliability, Rel of the structure. In RBO, the cost function can be considered deterministic or probabilistic based on the needs of design strategies. The obtained performance measures and cost function can be formulated in optimization problem as an objective function or a constraint based on decision policies to be implemented.

4.1 Decision Policies in Reliability-based Optimization

One of the main advantages of RBO is that it balances the needs between safety against performance and economy in design. The decision policies in RBO that balances the need for flexible pavements can formulated as:

1. Minimize rehabilitation cost by keeping reliability within desired limits
2. Maximize reliability by constraining the budget for rehabilitation actions
3. Trade-off between minimizing cost and maximizing reliability

Decision policy# 1 is best suited in the situations when the desired performance requirements are known and there is no constraint on budget for rehabilitation actions. With the knowledge of desired performance, only option is to find minimum cost that can keep the performance within desired limits. Whereas, decision policy# 2 is suited for the situation when there is constraint on budget for the application of rehabilitation actions. In such situations, the quantity of interest will be the maximum reliability that can be obtained within the budget constraints. However, generally there is always a conflict between cost and performance and trade-off between two is preferred as a possible solution. In such situation decision policy# 3 is preferred, wherein a trade-off decision strategy that can minimize cost and maximize performance is possible.

4.1.1 Problem Formulation

Based on the decision polices, the optimization problem formulation for each decision policy will be different. If $RC(\mathbf{x})$ is the rehabilitation cost that is the function of decision variables, the optimization problem for decision policy# 1 can be formulated as

$$\begin{aligned} \min \quad & RC(\mathbf{x}) \\ s.t. \quad & \mathrm{Rel}(\mathbf{x}) \geq \mathrm{Rel}_t \\ & x_i^l \leq x_i \leq x_i^u \end{aligned} \tag{49}$$

where, Rel_t is the target reliability, l and u are the lower and upper limits of the decision variables respectively. The formulation in Eq. 49 can be used to minimize the rehabilitation cost by constraining the reliability within desired limit. Though the cost is minimized in the above formulation, the optimization search will have tendency to find the solution with active performance constraint i.e. estimated Rel will be equal or very close to Rel_t.

Decision policy# 2 can be used in the situations where budget is constrained and performance is to be maximized. Optimization problem for such situation can be formulated as

$$\begin{aligned} \max \quad & \mathrm{Rel}(\mathbf{x}) \\ s.t. \quad & \mathrm{RC}(\mathbf{x}) \leq \mathrm{RC}_B \\ & x_i^l \leq x_i \leq x_i^u \end{aligned} \tag{50}$$

where, RC_B is the budget constraint on rehabilitation actions. The formulation in the Eq. 50 can be used to obtain decision parameters that maximize the reliability of flexible pavement keeping the cost for rehabilitation actions within the budget. The trade-off between reliability and cost can be taken care by optimizing both the objective functions in Eq. 49 and 50 and the problem can be formulated as

$$\begin{aligned} \min \quad & RC(\mathbf{x}) \quad \& \quad \max \quad \mathrm{Rel}(\mathbf{x}) \\ s.t. \quad & x_i^l \leq x_i \leq x_i^u \end{aligned} \tag{51}$$

Reliability-based optimization formulations are complex and require a robust optimization technique that can provide a global optimal solution. Traditional optimization techniques which include gradient projection algorithms are robust in finding a single local optimal solution. However, complex domain like in RBO can have more than one optimal solutions and therefore more robust technique is required that can find a near-global solution. In the research, Genetic Algorithm (GA) is used because of its efficiency in finding a near-global solution. The GA performs a global and probabilistic search thus increasing the likelihood of obtaining a near-global solution.

4.2 Genetic Algorithm (GA)

A GA is a stochastic optimization tool that is based on mechanics of natural evolution and genetics. [39]. In GA, the search algorithm reproduces and creates new population of chromosomes at each generation and competes for survival to stay in the next generation. Beginning with randomly generated population of chromosomes from the solution space, the process of evolution and survival is controlled by operators such as selection, crossover, and mutation.

As already discussed, the selection operator is based on the mechanics of natural selection and survival. At every generation, the population that shows the improvement in fitness of the objective function has better chance to survive and reproduce. Common methods used for the selection process are tournament selection, proportionate selection, and ranking selection [40]. The survived population of chromosomes is termed as parent solution. During each generation, total population of chromosomes is maintained same and to fill the space created by eliminated chromosomes, a crossover operator merges two parent solutions to generate offspring. On the other hand, a mutation operator randomly modifies the parent or offspring solutions and helps in speeding up the convergence towards global optima. Typically, the chromosomes in population are encoded in the form of bit strings using binary integers 0 and 1. Figure 15 shows the representation of chromosomes in binary form and process of crossover and mutation that are typically used in GA.

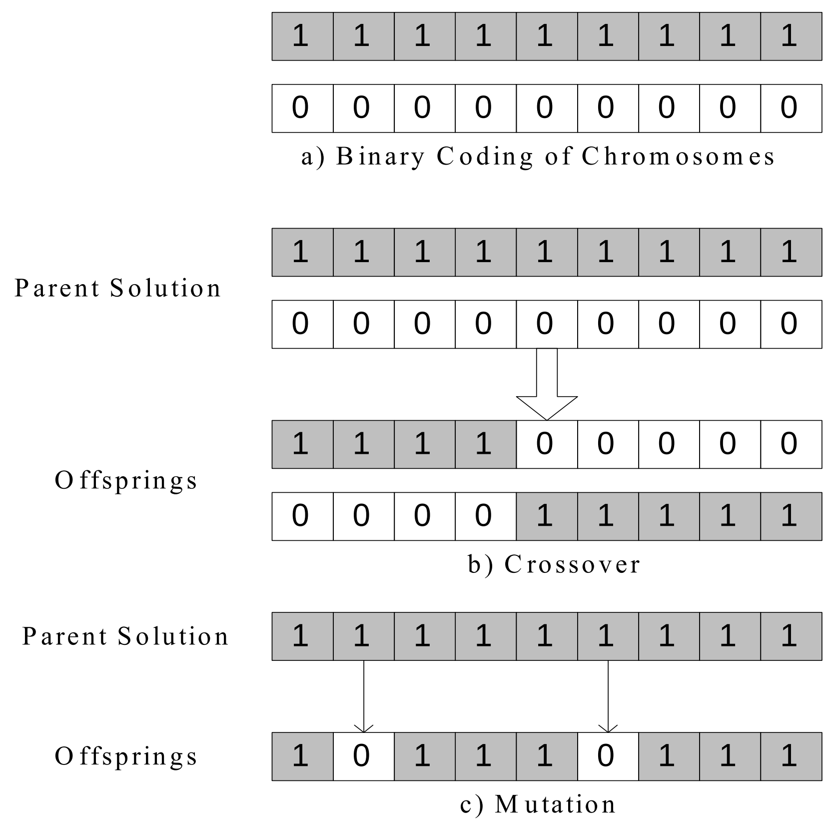

Figure 15: Binary coding of chromosomes, crossover and mutation process in GA

In GA, the process is initiated by randomly encoding a solution. Once a solution is encoded in the form of bit string, the selection operator identifies the parent solutions that improve fitness of objective function and survive for the next generation. After identifying the parent solutions, the crossover and mutation operators are used to reproduce offspring from parent solutions as shown in Figure 15. The process is continued through continuous improvement in fitness of objective function until a global or near-global solution is reached.

4.3 Multi Objective Genetic Algorithm (MOGA)

The optimization problem formulation in Eq. 51 involves two objective functions and the Multi Objective Genetic Algorithm (MOGA) is required for evaluating such formulations. The MOGA primarily involves finding a set of solutions each of which satisfies the objectives and are non-dominant with respect to each other [41]. For minimization problem, the feasible solution $\mathbf{x}^*$ is said to be non-dominant if there exists no feasible solution $\mathbf{x}$ such that [42]

$$f_o(\mathbf{x}) \leq f_o(\mathbf{x}^*) \qquad \textit{for all } o \in \{1,2,.....\} \tag{52}$$

$$f_o(\mathbf{x}) < f_o(\mathbf{x}^*) \qquad \textit{for atleast one } o \in \{1,2,.....\} \tag{53}$$

where, f_o is the objective function, o represents the set of number of objective functions. The optimal solution that satisfies the conditions in Eq. 52 and 53 is termed as Pareto optimal. The set of all non-dominant solutions that satisfies objectives is termed as Pareto optimal solution set and the corresponding set of objective values is termed as Pareto front.

The Pareto optimal set is determined in MOGA using the ranking approach in conjunction with GA operators [41]. In ranking approach, the population of chromosomes is ranked based on the dominance criteria and are assigned a fitness value based on the rank in population. For instance, if all the objectives are minimized, lower rank corresponds to better solution. The process of ranking is continued till all the chromosomes in the population are categorized into different ranks. Once the entire population is ranked, tournament selection is performed to identify the chromosomes with lowest rank. Crossover and mutation is performed over the identified chromosomes to create new population for next generation. The process is continued till the convergence is obtained.

4.4 Numerical Example

Typical three-layer flexible pavement system is considered for numerical study. To account for the effects of rehabilitation actions, it is assumed that an overlay will be constructed at the time of rehabilitation actions and the system will behave as four-layered system. To illustrate the proposed models, fatigue cracking failure for flexible pavement is considered. For fatigue cracking, maximum tensile strain at the bottom of the asphalt layer controls the allowable number of repetitions before rehabilitation actions, whereas, after rehabilitation actions, tensile strain at the bottom of overlay is considered critical. The critical strains are computed using theory of linear elasticity. Once the critical strains are computed, limit state function is evaluated using Monte Carlo simulation to obtain fragility data before and after rehabilitation actions. Using the obtained fragility data, the parametric regression model that expresses fragilities in terms of decision variable is developed. Then the reliability estimates that are required in optimization formulations can be estimated by solving the integral shown in Eq. 42.

Though the developed fragilities are functions of all the variables in $\mathbf{x}$, to simplify the understanding and since fatigue cracking is considered, only the overlay thickness, h_1 is considered as a decision variable in $\mathbf{x}$. The study can be easily extended to include other decision variables in $\mathbf{x}$. It is assumed that the initial design is fixed and optimal decision policies for only rehabilitation actions are determined. Deterioration of asphalt modulus is accounted while determining the performance after rehabilitation actions. Typical lower and upper limits of h_1 that used to normalize the quantity are 2.5 inches and 5.0 inches respectively. The formulations in Eq. 49 and 50 are evaluated using GA and Eq. 51 using MOGA for determining the near-global optimum solution. The objective function for rehabilitation cost considered for the study is

$$RC(\mathbf{x}) = \frac{100 \times x_1}{(1+i)^t} \tag{54}$$

where, x_1 is the normalized quantity of the overlay thickness h_1, i is the interest rate at which the cost is discounted to present value, t is the time at which rehabilitation actions are applied.

4.4.1 Minimizing Rehabilitation Cost

For minimizing the rehabilitation costs, the formulation in Eq. 49 is evaluated using GA. The target reliability Rel_t is considered to be 75% and it is assumed that rehabilitation actions are planned in such a manner that the estimated reliability is always greater than the target reliability. Figure 16 shows the result of decision policy where the application of rehabilitation actions is delayed till the estimated reliability before rehabilitation actions reaches the target reliability.

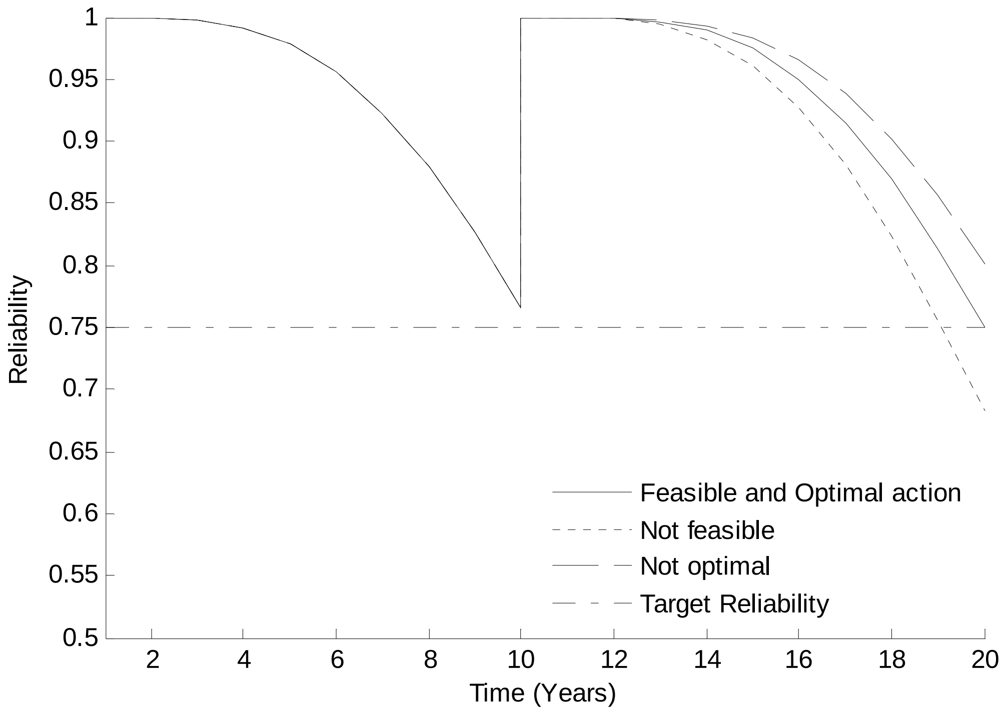

Figure 16: Optimization results for minimizing cost where rehabilitation actions are delayed till the estimated reliability reaches the target reliability

In the Figure 16, the optimal solution shown by solid line corresponds to optimal overlay thickness of 3.73 inches. It is observed that decreasing thickness makes reliability cross target reliability thus making solution not feasible. On the other side, though increasing thickness beyond optimal value improves reliability, the rehabilitation cost increases thereby making it a non optimal solution. In the Figure 16, the application of rehabilitation actions is delayed till the reliability before rehabilitation actions reaches the target reliability i.e. 10 years. Delaying the rehabilitation actions can increase the deterioration of asphalt layer thus making the system weak and thereby requiring stronger overlay to satisfy the desired performance over the design life. There is always the possibility that the early application rehabilitation actions when deterioration of asphalt layer is comparatively less can further reduce the rehabilitation costs. Therefore it is necessary to determine the value of early application of rehabilitation actions. Figure 17 shows the optimal rehabilitation costs for early application of rehabilitation actions between years 1 to 10. It is observed in the Figure 17 that the early application of rehabilitation actions reduces the cost thereby adding the value. Also it is seen that the interest rate i also plays a significant role while making decision policies.

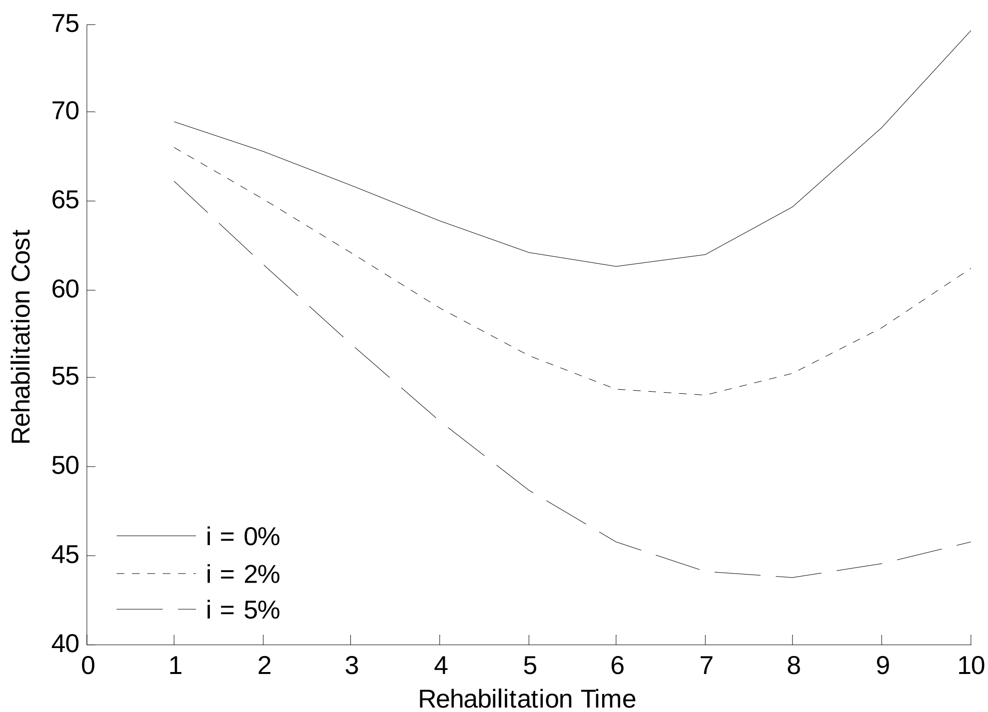

Figure 17: Optimization results for minimizing cost when rehabilitation actions are applied in different years

4.4.2 Maximizing the Reliability

To maximize the reliability, the formulation in Eq. 50 is evaluated using GA. To validate the optimization formulations, the results from cost minimization are used to obtain optimal actions while maximizing the reliability. For instance, the minimum rehabilitation cost at the year 10 is used as budget constraint. At the design life, the maximum reliability obtained by constraining rehabilitation budget is 0.75 which is same as the target reliability for the cost minimization problem. The optimum overlay thickness for both the cases is 3.73 inches. This validates both the formulations and any formulation can be used based on the requirements. Further, to determine the value of early application of rehabilitation actions, the maximum reliability is evaluated for the rehabilitation actions applied from years 1 to 10. Figure 18 shows the optimal reliability for early application of rehabilitation actions between years 1 to 10. The budget for rehabilitation actions is constrained to 70 units. It is observed in the Figure 18 that the early application of rehabilitation actions maximizes the reliability thereby adding the value. Both the formulations i.e. maximizing reliability and minimizing cost indicates that early application of rehabilitation actions can be more beneficial and there is an optimal time that can optimize the overall design strategy.

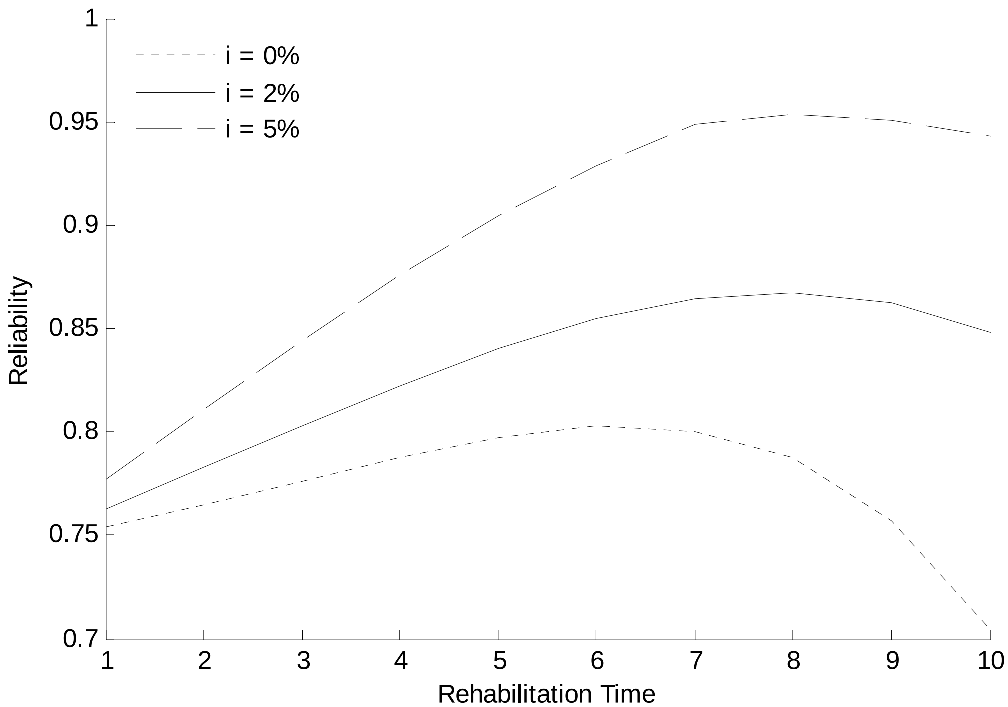

Figure 18: Optimization results for maximizing reliability when rehabilitation actions are applied in different years

4.4.3 Trade-off Between Performance And Rehabilitation Cost (Pareto)

In the Eq. 51, there are two objective functions i.e. minimize cost and maximize reliability and the trade-off between two objectives is necessary for avoiding the conflict between two decision policies. As already discussed, the trade-off between two objectives can obtained in the form of Pareto optimal solution set. Using MOGA, the formulation in Eq. 51 that has two objective functions is evaluated to obtain Pareto optimal set and Pareto front. Figure 19 shows the Pareto front for the rehabilitation actions applied at the year 10 and for different interest rate. It is observed that if the reliability is increased the rehabilitation cost increases and vice versa. The behavior of

Pareto front seems reasonable and can be used while making decision policies that require trade-off between cost and performance.

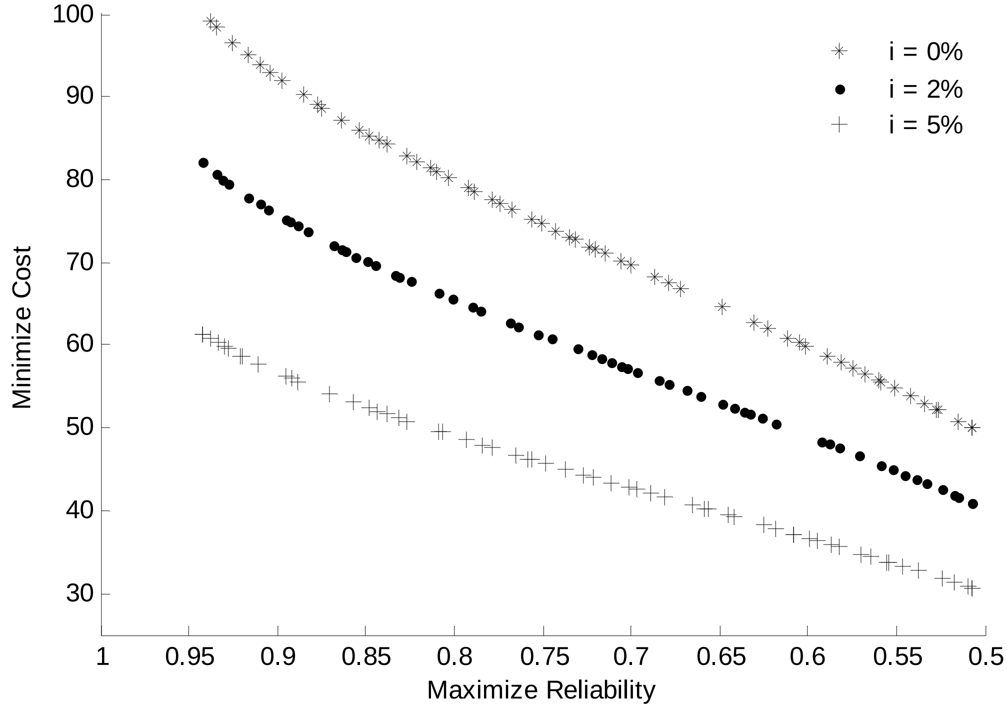

Figure 19: Pareto front obtained from numerical study

4.5 Summary

The research presents an optimization model for flexible pavement that is able to account for the optimal rehabilitation design strategies. The reliability-based optimization technique is used to balance the needs between performance and cost and account for uncertainties in pavement performance. For pavement reliability, a mechanistic-empirical approach is used to define limit state functions based on the pavement responses (tensile and compressive strains) before and after the application of rehabilitation actions. Fatigue cracking failure criteria for flexible pavement is considered. To express pavement fragilities in terms of decision variables, a parametric regression model with two parameter lognormal distribution CDF is used. The GA and MOGA are used to evaluate optimization formulations. Three rehabilitation decision policies for flexible pavements are discussed. A numerical example is presented to illustrate the developed optimization formulations.

The results from numerical study for optimization shows that the cost minimization and reliability maximization formulations are efficiently used in determining optimal rehabilitation policies. It is also seen that there can be added value for providing rehabilitation actions early rather than waiting until failure. Also the effect of interest rate that discounts cost to present value is significant. Pareto optimal solution obtained from MOGA shows that as the reliability increases the rehabilitation cost increases and vice versa. This behavior seems reasonable and obtained Pareto solutions can be efficiently used to obtain trade-off between cost and performance and avoid possible conflict between two decision policies.

CHAPTER V

CONCLUSIONS

The research presents a reliability model that is able to account for the effects of rehabilitation actions on the reliability of flexible pavements. A mechanistic-empirical approach is used to define limit state functions based on the pavement responses (tensile and compressive strains) before and after the application of rehabilitation actions. Two failure criteria are considered (fatigue cracking and rutting). A numerical example is presented to illustrate the developed model, and sensitivity and importance measures are computed for the parameters and the random variables included in the limit state functions. The results obtained from the numerical study describe the behavior of new and rehabilitated flexible pavement systems.

The sensitivity measures suggest that the reliability of flexible pavements before as well as after rehabilitation actions can effectively be improved by providing asphalt layer as thick as possible in the initial design, improving the stiffness for subgrade and reducing the error in predicting the asphalt modulus at the time of rehabilitation actions. The importance measures suggest that the asphalt layer modulus at the time of rehabilitation actions represent the principal uncertainty for the performance after rehabilitation actions. The results from the sensitivity analysis and importance measures can be used as directive device to plan optimal decision policies. The application of mechanistic-empirical approach and inclusion of correlations has added flexibility to the model.

Conventionally, the limit state function is evaluated using MCS technique. However, the MCS technique typically requires a relatively large number of simulations in order to obtain sufficiently accurate estimates of failure probabilities and it becomes impractical to simulate the pavement response black-box model thousands of times. In the research, an alternative approach of Response Surface Methodology is explored for obtaining reliability estimates. Statistical validation of pavement response model shows that the developed response models are good fit to the responses obtained from analytical model and can be efficiently used for predicting pavement responses. In reliability analysis, often fragilities are used to express the performance of the system. In the research, the parametric regression model is developed to express the fragilities in terms of decision variables. Maximum likelihood estimation technique is used to obtain parameter estimates. The statistical validation of parametric regression model developed in numerical study shows the accuracy of the developed model.

The developed performance models for flexible pavements that accounts for rehabilitation actions are further explored for their applications in determining optimal rehabilitation policies. To account for the uncertainties in performance and maintain a balance between performance and cost, the reliability-based optimization technique is used in the research. For reliability-based optimization, three decision policies are defined along with the optimization problem formulation for each policy. Because of its efficiency in obtaining a near-global solution, the genetic algorithm is used to evaluate the optimization formulations. For two objective functions, MOGA is used to obtain Pareto optimal solution set that provides a trade-off between cost and reliability. Using the developed parametric regression models for fragilities, a numerical study is presented to illustrate the developed optimization formulations.

The results from numerical study for optimization shows that the cost minimization and reliability maximization formulations are efficiently used in determining optimal rehabilitation policies. It is also seen that there can be added value for providing rehabilitation actions early rather than waiting until failure. Also the effect of interest rate that discounts cost to present value is significant. Pareto optimal solution obtained from MOGA shows that as the reliability increases the rehabilitation cost increases and vice versa. This behavior seems reasonable and obtained Pareto solutions can be efficiently used to obtain trade-off between cost and performance and avoid possible conflict between two decision policies.

The developed pavement reliability model in conjunction with response surface methodology and parametric regression modeling for fragilities can be effectively implemented in all the applications that require the estimation of the performance of flexible pavement systems before and/or after rehabilitation actions. Expressing fragilities in terms of decision variables has added flexibility in using them as performance measures in optimization models. Developed performance model that accounts for rehabilitation actions are efficiently used in optimizing the rehabilitation policies for flexible pavements. Different formulations for optimization problem provide flexibility in making decision policies and obtaining optimal trade-off between the pavement performance and cost.

REFERENCES

[1] Highway statistics 2005. FHWA. http://www.fhwa.dot.gov/policy/ohim/hs05/index.htm. Accessed October 28, 2007.

[2] Report card for America's infrastructure. ASCE. http://www.asce.org/reportcard/2005/page.cfm?id=30#conditions. Accessed December 04, 2007.

[3] Abaza, K.A. Optimum flexible pavement life-cycle analysis model. Journal of Transportation Engineering 2002; 128(6): 542-549.

[4] Wang, K. C. P., J. Zaniewski, and G. Way. Probabilistic behavior of pavements. Journal of Transportation Engineering 1994; 120(3): 358-375.

[5] Li, N., R. Haas, and W. Xie. Reliability-based processing of Markov chains for modeling pavement network deterioration. Transportation Research Board 1996; 1524: 203-213.

[6] Hong, H. P., and S. S. Wang. Stochastic modeling of pavement performance. The International Journal of Pavement Engineering 2003; 4(4): 235-243.

[7] Zhang, Z., and I. Damnjanovic. Applying method of moments to model reliability of pavement infrastructure. Journal of Transportation Engineering 2006; 132(5): 416-424.

[8] Alsherri, A., and K. P. George. Reliability model for pavement performance. Journal of Transportation Engineering 1988; 114(3): 294-306.

[9] Zhou, J., and A. S. Nowak. System reliability models for bridges. Structural Safety 1990; 7: 247-254.

[10] Chua, K. H., A. Der Kiureghian, and C. L. Monismith. Stochastic model for pavement design. Journal of Transportation Engineering 1992; 118(6): 769-786.

[11] Darter, M., L. Khazanovich, T. Yu, and J. Mallela. Reliability analysis of cracking and faulting prediction in the new mechanistic-empirical design procedure. Transportation Research Record: Journal of the Transportation Research Board, No. 1936. Transportation Research Board of the National Academies, Washington, D.C., 2005: 150-160.

[12] Bjerager, P. On computation methods for structural reliability analysis. Structural Safety 1990; 9: 79-96.

[13] Ditlevsen, O., and H. O. Madsen. Structural reliability methods. Chichester, NY: John Wiley and Sons; 1996.

[14] Melchers, R. E. Structural reliability analysis and prediction. Chichester, NY: Ellis Horwood Limited; 1987.

[15] Brown, M., and F. Proschan. Imperfect repair. Journal of Applied Probability 1983; 20(4): 851-859.

[16] Lin, D., M. J. Zuo, and R. C. M. Yam, Sequential imperfect preventive maintenance models with two categories of failure modes. Naval Research Logistics 2001; 48(2): 172-183.

[17] Doyen, L., and O. Gaudoin. Classes of imperfect repair models based on reduction of failure intensity or virtual age. Reliability Engineering and System Safety 2004; 84: 45-56.

[18] Ouyang, Y., and S. Madanat. Optimal scheduling of rehabilitation activities for multiple pavement facilities: Exact and approximate solutions. Transportation Research Part A 2004; 38: 347-365.

[19] Paterson, W. D. O. Quantifying the effectiveness of pavement maintenance and rehabilitation. In: Proceedings at the 6th REAAA conference, Kuala Lumpur, Malaysia, 1990.

[20] Damnjanovic, I. A risk based approach to modeling life-cycle cost associated with warranty specifications for transportation infrastructure. Ph.D. thesis, University of Texas at Austin; 2006.

[21] Huang, Y. H. Pavement analysis and design. New Jersey: Pearson Education, Inc.; 2004.

[22] Attoh-Okine, O. N., and W. M. K. Roddis. Pavement thickness variability and its effect on determination of moduli and remaining life. Transportation Research Record 1994; 1449: 39-45.

[23] Ullidtz, P. Deterioration models for managing flexible pavements. Transportation Research Record 1999; 1655: 31-34.

[24] Tsai, B., J. T. Harvey, and C. L. Monismith. The application of weibull theory in prediction of asphalt concrete fatigue performance prediction. Transportation Research Board 2003; 1832: 121-130.

[25] Hohenbichler, M., and Rackwitz, R. Non-normal dependent vectors in structural safety. J. Engrg. Mech., ASCE 1981; 107(6): 1227–1238.

[26] Liu, P., and A. Der Kiureghian. Multivariate distribution models with prescribed marginals and covariances. Probabilistic Engineering Mechanics 1986; 1(2): 105-112.

[27] Hohenbichler , M., and R. Rackwitz. First-order concepts in system reliability. Structural Safety 1983; 1(3): 177-188.

[28] Der Kiureghian, A., and Ke, J.-B.. Finite-element based reliability analysis of frame structures. Proceedings of ICOSSAR '85, 4th international conference on structural safety and reliability, Kobe, Japan 1995; 1: 395-404.

[29] Lee, S. H., B. M. Kwak. Response surface augmented moment method for efficient reliability analysis. Structural Safety 2006; 28: 261-272.

[30] Yao, T. H., Y. K. Wen. Response surface method for time-variant reliability analysis. Journal of Structural Engineering 1996; 122(2): 193-201.

[31] Wong, S. M., R. E. Hobbs and C. Onof. An adaptive response surface method for reliability analysis of structures with multiple loading sequences. Structural Safety 2005; 27: 287-308.

[32] Faravelli, L. Response surface approach for reliability analysis. Journal of Engineering Mechanics 1989; 115(12): 2763–2781

[33] Rajashekhar, M. R. and B. R. Ellingwood. A new look at the response surface approach for reliability analysis. Structural Safety 1993; 12: 205-220.

[34] Bucher, C. G. and U. Bourgund. A fast and efficient approach response surface approach for structural reliability problems. Structural Safety 1990; 7: 57-66.

[35] Montgomery, D. C. and G. C. Runger. Applied statistics and probability for engineers – 3rd edition. NY: John Wiley and Sons, Inc.; 2002.

[36] Venter, G., R. T. Haftka and M. Chirehdast. Response surface approximations for fatigue life prediction. American Institute of Aeronautics and Astronautics1997: AIAA-97-1331; 1383-1396.

[37] Lawless, J. F. Statistical models and methods for lifetime data – 2nd edition. Wiley series in probability and statistics, N.J.: John Wiley and Sons, Inc.; 2003.

[38] Shinozuka, M. and *et.al*. Statistical analysis of fragility curves. Journal of Engineering Mechanics 2000; 126(12): 1224-1231.

[39] Chatterjee, S., M. Laudato, and L. A. Lynch. Genetic algorithms and their statistical application: an introduction. Computational Statistics & Data Analysis 1996; 22: 633-651.

[40] Deb, K. Introduction to genetic algorithms. Sadhna Academy Proceedings in Engineering Sciences 1999; 24(4): 293-315.

[41] Konak, A., D. W. Coit, A. E. Smith. Multi-objective optimization using genetic algorithms: a tutorial. Reliability Engineering and System Safety 2006; 91: 992-1007.

[42] Mathakari, S. et al. Reliability-based optimal design of electrical transmission towers using multi-objective genetic algorithms. Computer-Aided Civil and Infrastructure Engineering 2007; 22: 282-292.

APPENDIX

Responses in the layered system can be evaluated based on linear elastic theory by assuming a stress function, ϕ for each layer that satisfies the 4th differential equation shown in Eq. 9. Solution to the 4th order differential equation will comprise of four constants of integration that can be determined from the boundary and continuity conditions. In the Figure 1, considering $\theta = r/H$ and $\varepsilon = z/H$, the stress function satisfying Eq. 9 can be obtained as [21]

$$\phi = \frac{H^3 Y_0(m\theta)}{m^2}\left[A_i e^{-m(\varepsilon_i-\varepsilon)} - B_i e^{-m(\varepsilon-\varepsilon_{i-1})} + C_i m\varepsilon e^{-m(\varepsilon_i-\varepsilon)} - D_i m\varepsilon e^{-m(\varepsilon-\varepsilon_{i-1})}\right] \tag{A.1}$$

where H is the distance from the surface to the upper boundary of the lowest layer as shown in the Figure 1, Y_0 is a Bessel function of the first kind and order 0, m is a parameter, A, B, C, D are constants to be determined from the boundary and continuity conditions, i corresponds to the number of the layer at which the stress function is evaluated. Substituting Eq. A.1 in the Eq. 10, 11, and 12 gives

$$(\sigma'_z)_i = -mY_0(m\theta)\left\{\begin{array}{l}\left[A_i - C_i(1-2v_i - m\varepsilon)\right]e^{-m(\varepsilon_i-\varepsilon)} \\ +\left[B_i + D_i(1-2v_i + m\varepsilon)\right]e^{-m(\varepsilon-\varepsilon_{i-1})}\end{array}\right\} \tag{A.2}$$

$$\begin{aligned}(\sigma'_r)_i = &\left[mY_0(m\theta) - \frac{Y_1(m\theta)}{\theta}\right]\left\{\begin{array}{l}\left[A_i + C_i(1+m\varepsilon)\right]e^{-m(\varepsilon_i-\varepsilon)} \\ +\left[B_i - D_i(1-m\varepsilon)\right]e^{-m(\varepsilon-\varepsilon_{i-1})}\end{array}\right\} \\ &+ 2v_i mY_0(m\theta)\left[C_i e^{-m(\varepsilon_i-\varepsilon)} - D_i e^{-m(\varepsilon-\varepsilon_{i-1})}\right]\end{aligned} \tag{A.3}$$

$$\left(\sigma_t'\right)_i = \frac{Y_1\left(m\theta\right)}{\theta}\left\{\left[A_i + C_i\left(1+m\varepsilon\right)\right]e^{-m\left(\varepsilon_i-\varepsilon\right)} + \left[B_i - D_i\left(1-m\varepsilon\right)\right]e^{-m\left(\varepsilon-\varepsilon_{i-1}\right)}\right\}$$
$$+ 2v_i m Y_0(m\theta)\left[C_i e^{-m\left(\varepsilon_i-\varepsilon\right)} - D_i e^{-m\left(\varepsilon-\varepsilon_{i-1}\right)}\right] \tag{A.4}$$

where, Y_1 is the Bessel function of the first kind and order one, superscript ' for the stresses indicates that stresses are computed for the load of $-mY_0(m\theta)$. Actual stresses, σ due to load, q over a circular area of radius, a can be obtained from the following transformation

$$\sigma = q\tau\int_0^\infty \frac{\sigma'}{m} Y_1(m\tau)dm \tag{A.5}$$

where, $\tau = a / H$.

Above system of equations can be solved by assigning values to m from 0 to some large positive number until the stresses in Eq. A.2, A.3. A.4 converges. For each value of m, constant of integrations can be determined from the boundary and continuity conditions. These constant of integrations can be used in Eq. A.2, A.3. A.4 to compute stresses (σ') due to load $-mY_0(m\theta)$. Finally, using these stresses, Eq. A.5 can be solved numerically to obtain actual stresses.

VITA

Vighnesh Prakash Deshpande received his Bachelor of Engineering degree in civil engineering from Sardar Patel College of Engineering, Mumbai, India in 2004. Upon completion, he worked for 2 years for Reliance Energy Limited, India wherein he was involved in the construction of power plants and substation across India. He entered the Construction Engineering and Management program at Texas A&M University in August 2006 and received his Master of Science degree in civil engineering in August 2008. His research interests include project risk management, project finance, reliability engineering, and optimization.

Mr. Vighnesh may be reached at CE/TTI 705, 3136 TAMU, College Station, TX 77843, USA. His email is vighnesh.deshpande@gmail.com.