

OPTIMAL DESIGN OF DEMAND-RESPONSIVE
FEEDER TRANSIT SERVICES

A Dissertation

by

XIUGANG LI

Submitted to the Office of Graduate Studies of
Texas A&M University
in partial fulfillment of the requirements for the degree of

DOCTOR OF PHILOSOPHY

August 2009

Major Subject: Civil Engineering

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ABSTRACT

Optimal Design of Demand-Responsive Feeder Transit Services.

(August 2009)

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The general public considers Fixed-Route Transit (FRT) to be inconvenient while Demand-Responsive Transit (DRT) provides much of the desired flexibility with a door-to-door type of service. However, FRT is typically more cost efficient than DRT to deploy. Therefore, there is an increased interest in flexible transit services including all types of hybrid services that combine FRT and pure DRT. The demand-responsive feeder transit, also known as Demand-Responsive Connector (DRC), is a flexible transit service because it operates in a demand-responsive fashion within a service area and moves customers to/from a transfer point that connects to a FRT network. In this research we develop analytical models, validated by simulation, to design the DRC system.

Feeder transit services are generally operated with a DRC policy which might be converted to a traditional FRT policy for higher demand. By using continuous approximations, we provide an analytical modeling framework to help planners and operators in their choice of the two policies. We compare utility functions of the two

policies to derive rigorous analytical and approximate closed-form expressions of critical demand densities. They represent the switching conditions, that are functions of the parameters of each considered scenario, such as the geometry of the service area, the vehicle speed and also the weights assigned to each term contributing to the utility function: walking time, waiting time and riding time.

We address the problem faced by planners in determining the optimal number of zones for dividing a service area. We develop analytical models representing the total cost functions balancing customer service quality and vehicle operating cost. We obtain close-form expressions for the FRT and approximation formulas for the DRC to determine the optimal number of zones.

Finally we develop a real-case application with collected customer demand data and road network data of El Cenizo, Texas. With our analytical formulas, we obtain the optimal number of zones, and the times for switching FRT and DRC policies during a day. Simulation results considering the road network of El Cenizo demonstrate that our analytical formulas provide good estimates for practical use.

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Finally, thanks to my mother and father for their encouragement and to my wife for her patience and love.

NOMENCLATURE

ADA	Americans with Disabilities Act
CAD	Computer Aided Dispatching
DRC	Demand-Responsive Connector
DRT	Demand-Responsive Transit
FRT	Fixed-Route Transit
HCPPT	High Coverage Point to Point Transit
MAST	Mobility Allowance Shuttle Transit
PDRT	Personalized Demand-Responsive Transit
TCRP	Transit Cooperative Research Program

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CHAPTER I
INTRODUCTION: THE IMPORTANCE
OF RESEARCH

Over the last decades, modern urban areas, especially within residential communities, are experiencing a steady decrease in their population density as a consequence of urban sprawl, one of the most evident phenomena of our time. In the US, from 1960 to 2000, the population density dropped by 15% despite an average overall population growth of 86% (www.demographia.com). In the majority of the rest of the world this trend is even more evident. This increasing “dispersion” of population causes conventional fixed route transit systems serving those areas to become progressively more inefficient and relegated to a marginal role, since they are designed to serve few established routes and they heavily rely on concentrated demand.

Traditionally, transit services have been divided into two broad categories: fixed route (FRT) and demand responsive (DRT). The typical cost efficiency of FRT systems is due to the predetermined schedule, the large loading capacity of the vehicles and the consolidation of many passenger trips onto a single vehicle (ridesharing). However, the general public considers them to be inconvenient because of their lack of flexibility, since often the locations of pick up and/or drop off points and/or the service’s schedule do not match the individual rider’s desires. Therefore, an increasingly larger portion

This dissertation follows the style of Transportation Research Part B: Methodological.

of the growing population relies almost exclusively on private automobiles for their transportation needs, causing many urban areas to suffer from increasing congestion and pollution problems.

DRT systems instead provide much of the desired flexibility with a door-to-door type of service, but they are generally much more costly to deploy. According to the Federal Transit Administration (2005), the national total fare revenue earned is about 9.7% of the operating expense for DRT systems, which is much less than the percentage of 27.9% for FRT systems. Therefore, DRT systems are largely limited to specialized operations such as taxicabs, shuttle vans or dial-a-ride services, other than paratransit services (mandated under the ADA). Hence, transit agencies are facing a growing demand for improved and extended DRT services.

The broad and fairly new category of “flexible” transit services includes all types of hybrid services that combine pure demand responsive and fixed route features. These services have established stop locations and/or established schedules, combined with some degree of demand responsive operation. Their characteristics have, in several cases, efficiently responded to some of the needs and wants of both the customers and the transit agency as well. However, their use has been quite limited in practice so far, as opposed to regular FRT systems.

The Demand Responsive Connector (DRC), also known as “feeder” transit line, is one type of flexible transit service. A survey conducted by Koffman (2004) for a Transit Cooperative Research Program (TCRP) project found that the DRC has been operated in quite a few cities and is one of the most often used types of flexible transit services,

especially within low density residential areas. Examples can be found in Denver (CO), Raleigh (NC), Akron (OH), Tacoma (WA), Sarasota (FL), Portland (OR) and Winnipeg (Canada). The service operates in a demand responsive fashion within a service area and move passengers from/to a transfer point that connects to a major fixed route transit network, thus closing the gap perceived as the most critical by the majority of the potential transit users.

In most cases, the service operates as a FRT service during daytime and switches to a DRC type of service during evenings, nights or early morning, when the demand is lower. The customers know the policy in effect via published schedule, telephone calls and/or internet. When designing and operating such systems, planners need to decide what type of operations, between FRT or DRC, would be the most appropriate and/or what conditions would justify a “switch” from FRT to DRC (or vice versa). The decision is not straightforward, mainly because the demand for the service is often unknown beforehand and it will depend on the established service itself. In addition, it is not clear what would be the best type of service even when assuming a known demand. This is because the service quality provided to customers is not easy to assess and might depend on external conditions, such as safety, weather, time of the day; plus, the balance between operating costs and service quality is also frequently hard to evaluate. A methodology is in needs to assist decision makers in their choice. In particular, a “critical demand density”, representing the point where the two services could reasonably be considered equivalent and where a switch from one type of service to the other would be desirable.

In designing such systems for large communities, planners may separate the whole service area into zones for an easier management of the operation, to reduce operating cost, and to provide a better level of service to customers. In each zone, an independent feeder line would provide the service to its customers. The best number of zones is hard to determine because the balance between operating costs and service quality is frequently difficult to evaluate, especially within areas with low and sparse demands. However, current trends suggest that these services will progressively increase their market share and importance within transit agencies, demanding a more rigorous and methodological design approach to the problem. Handy but powerful tools, such as analytical formulae, would aid in solving the complicated feeder transit design problem.

Therefore the research in this dissertation would include three main parts:

- A methodology to determine the critical demand density;
- Analytical models to determine the optimal number of zones;
- An application with collected data in El Cenizo, Texas.

This dissertation is divided into six chapters (including the Introduction). Chapter II contains the literature review. The subject areas reviewed include: the demand-responsive transit service, the flexible transit service and the continuous approximation approach. Chapter III contains a description of the methodology to determine the critical demand density. Analytical formulas are derived for the one-vehicle case and the two-vehicle case. Chapter IV includes analytical models to determine the optimal number of

zones of a service area. Customer cost and vehicle/bus cost are balanced to derive the minimum total cost for the optimal zone design. Chapter V contains a real-case application with collected data in El Cenizo, Texas. Theoretical results are compared with simulation results based on the real road network. Chapter VI includes conclusions and a set of recommendations for future research.

CHAPTER II

LITERATURE REVIEW

The work specifically on the DRC, which is the focus of this research, is Cayford and Yim (2004). These authors surveyed the customers' demand for DRC for the city of Millbrae, California. They also designed and implemented an automated system used for the DRC services. The service uses an automated phone in system for reservations, computerized dispatching over a wireless communication channel to the bus driver and an automated callback system for customer notifications. Khattak and Yim (2004) explored the demand for a consumer oriented personalized DRT (PDRT) service in the San Francisco Bay Area. About 60% of those surveyed were willing to consider PDRT as an option, and about 12% reported that they were "very likely" to use PDRT. Many were willing to pay for the service and highly valued the flexibility in scheduling the service.

2.1 Flexible transit service

Flexible transit services may involve checkpoints. Daganzo (1984) describes a flexible system in which the pick up and drop off points are concentrated at centralized locations called checkpoints. The related Mobility Allowance Shuttle Transit (MAST) system allows buses to deviate from the fixed path so that customers within the service area may be picked up or dropped off at their desired locations. According to Koffman (2004), this type of service is also often used and is also known as "Route Deviation".

Quadrifoglio et al. (2006) developed bounds on the maximum longitudinal velocity to evaluate the performance and help the design of MAST services by employing continuous approximations. Quadrifoglio et al. (2007) developed an insertion heuristic for scheduling MAST services by using control parameters, which properly regulate the consumption of the slack time. Finally, Quadrifoglio et al. (2008b) formulated the scheduling of the MAST services as a mixed integer programming with added logic constraints.

Analytical modeling and/or simulation have often been used to analyze flexible transit services. For example, Cortés and Jayakrishnan (2002) proposed and simulated one type of flexible transit called High Coverage Point to Point Transit (HCPPT), which requires the availability of a large number of transit vehicles. Pagès et al. (2006) identified the problem called real time mass transport vehicle routing problem and developed a global solution algorithm for the mass transport network design problem. Aldaihani et al. (2004) developed an analytical model that aids decision makers in designing a hybrid grid network that integrates a flexible demand responsive service with a fixed route service. Their model is to determine the optimal number of zones in an area, where each zone is served by a number of on demand vehicles.

2.2 DRT

Although research on DRC and flexible transit services is quite limited, purely DRT systems have been extensively investigated. Savelsbergh and Sol (1995), Desaulniers et al. (2000) and Cordeau and Laporte (2003) provide comprehensive

reviews on the proposed methodologies and solutions to deal with these very difficult problems.

Some recent examples of research on DRT are as follows. Dessouky et al. (2003) demonstrated through simulation that it is possible to reduce environmental impact substantially, while increasing operating costs and service delays only slightly for the joint optimization of cost, service, and life cycle environmental consequences in vehicle routing and scheduling of a DRT system. Dessouky et al. (2005) used computer simulation methods to investigate the effect of using a zoning vs. a no zoning strategy and time window settings on performance measures such as total trip miles, deadhead miles and fleet size. They identified quasi linear relationships between the performance measures and the independent variable, either the time-window size or the zoning policy. Sandlin and Anderson (2004) presented a procedure for calculating a serviceability index (SI) for DRT operators based on regional socioeconomic conditions and internal operation data. The SI can be used to evaluate and compare DRT operation. Palmer et al. (2004) studied the DRT system consisting of dial-a-ride programs that transit agencies use for point to point pick up and delivery of the elderly and handicapped. Their results of a nationwide survey involving 62 transit agencies show that the use of paratransit computer aided dispatching (CAD) system and agency service delivery provide a productivity benefit.

Further, Diana et al. (2006) studied the problem of determining the number of vehicles needed to provide a DRT service with a predetermined quality for the user in terms of waiting time at the stops and maximum allowed detour. They proposed a

probabilistic model that requires only the knowledge of the distribution of the demand over the service area and the quality of the service in terms of time windows associated of pickup and delivery nodes. Quadrioglio et al. (2008a) used simulation methods to investigate the effect of using a zoning vs. a no zoning strategy and time window settings on performance measures such as total trip miles, deadhead miles and fleet size. They identified quasi linear relationships between the performance measures and the independent variable, either the time-window size or the zoning policy.

2.3 Continuous approximation

In this research, we utilize continuous approximations as part of our methodology. There is a significant body of work in the literature on continuous approximation models for transportation systems. Most of the work has been developed to provide decision support tools for strategic planning in the design process. Clarens and Hurdle (1975) utilized continuous approximation to design an operating strategy for a commuter bus system. Langevin et al. (1996) provide a detailed overview of the research performed in the field. They concentrate primarily on freight distribution systems, while in this research the focus is on public transport; but most of the issues of interest are common to both fields. Szplett (1984) provides a review of the research performed on continuous models specifically for public transport. Ho and Wong (2006) provide a detailed overview of the research performed on two-dimensional continuous models.

Some recent examples of continuous approximation models for public transport are as follows. Quadrifoglio et al. (2006) utilized continuous approximations to provide upper and lower bounds of the maximum velocity of the vehicle between pairs of consecutive checkpoints at different demand levels for Mobility Allowance Shuttle Transit Services. Diana et al. (2006) presented a continuous approximation model to forecast the number of vehicles needed to operate a demand-responsive transit service. Using this approximation model provides the possibility for planners to perform sensitivity analysis of different scenarios, and the choice of the best compromise between service quality and financial resources is more effective.

CHAPTER III

CRITICAL DEMAND DENSITY

When designing and operating a DRC system, planners need to decide what type of operations, between FRT or DRC, would be the most appropriate and/or what conditions would justify a “switch” from FRT to DRC (or vice versa). With the ultimate goal of improving the efficiency and performance of this type of services, a methodology is developed to assist decision makers in their choice by providing analytical modeling and solution of the problem, with the use of continuous approximations. As noted by Daganzo (1991), the main purpose of this type of approach is to obtain reasonable solutions with as little information as possible. Hall (1986) also pointed out that these approximate models are easier to comprehend. They may provide handy but powerful tools to help solve many complicated decision problems. In particular, in this research, analytical modeling is developed to assess the service quality of the two competing operating policies (FRT and DRC) and derive the “critical demand densities”, representing the point where the two services could reasonably be considered equivalent and where a switch from one type of service to the other would be desirable.

3.1 System definition

3.1.1 Service area and demand

The service area is a representation of a residential community modeled as a rectangle of width W and length L located on the side of a main road where a major fixed

route transit service network is in service. The terminal of the feeder transit service, connecting to the major fixed route transit network, is located in the middle of the left edge of the service area (see Fig. 1). The temporal distribution of the demand is assumed to be a Poisson process with exponentially distributed interarrival times and average rate λ passengers/hour. We assume that a fraction α of the customers need to be transferred from the service area to the connection terminal (“pick up” customers) and a fraction $1-\alpha$ of them in the opposite direction (“drop off” customers). The customers’ location, either for a pick up or for a drop off, has a uniform distribution within the service area. While assuming a temporal Poisson distribution for pick up customers is very realistic, the drop off customers would instead reasonably show up in groups according to the arrival of the vehicles serving the outside FRT network. However, with the additional assumption that the number of transit lines passing by the connection terminal is high enough and/or the headways between vehicles are low enough, a Poisson distribution for the arrivals is still a reasonable assumption.

3.1.2 Competing transit policies

We consider two competing operating policies (FRT and DRC) of the transit service. For each one of them we analyze the one vehicle case and the two vehicle case. In all considered scenarios we assume an average speed of the vehicles of v_b miles/hr. The vehicle dwelling time at each stop is s_f hr for the FRT and s hr for the DRC. Dwelling times are defined differently, since we recognize that for the FRT case more

passengers would generally be served per stop. We also assume that the same type of vehicle(s) is used in all cases.

FRT Policy

The FRT operating policy offers continuous service with the vehicle moving back and forth along the route between stop 1 (the connection terminal) and stop N (see Fig. 1). There are $N-2$ stations between 1 and N . N would be determined by the selection of the optimal distance (spacing) between adjacent stations, which we assume to be a constant d miles. In practice, transit agencies use ranges from 600ft to 2500ft in suburban areas (Texas Transportation Institute, 1996). Some other references on optimal spacing include Wirasinghe and Ghoneim (1981), Kuah and Perl (1988), Furth and Rahbee (2000) and Saka (2001).

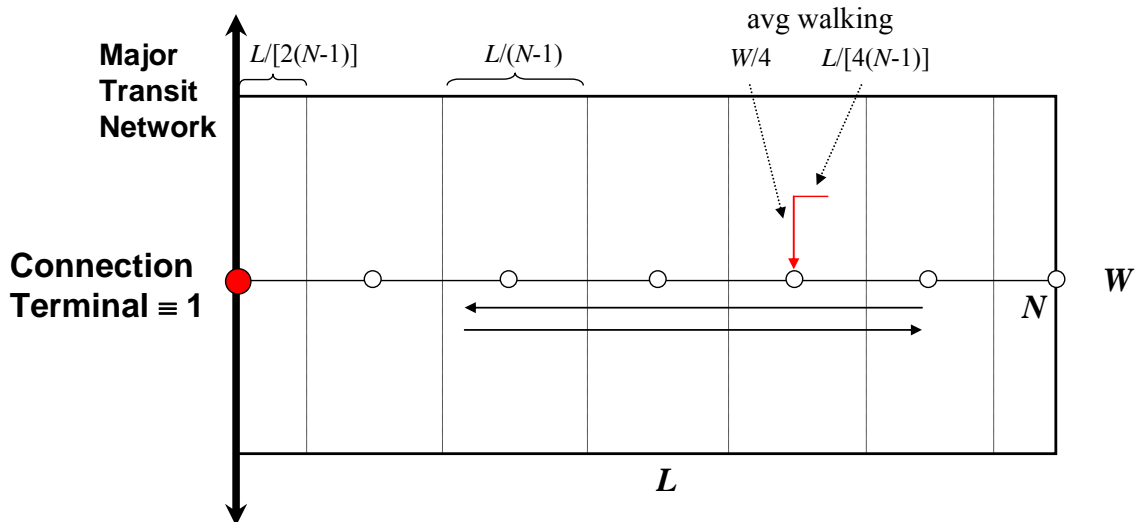


Fig. 1. Service area and FRT service.

The pick up customers show up at random within the service area and wait there until they need to walk to the nearest station to catch their bus, whose schedule is known to them. The drop off customers show up and wait at terminal 1, ride the bus to the stop nearer to their destination, and then walk to their final destination, which is located at random. There are no intra zonal trips, that is, every customer starts or ends the trip at the connection terminal.

In the one vehicle case there is only a vehicle performing the operations. In the two vehicle case we assume that the two buses begin their operations at the same time leaving from stop 1 and N respectively. At any point in time during the operations, the vehicle moving left to right performs the drop off operations (transferring customers from terminal 1 to the stops closest to their final destination) and the vehicle moving right to left performs the pick up operations (transferring customers from their stops closest to their origin to terminal 1).

DRC Policy

The DRC policy provides a shared ride demand responsive terminal to door (and door to terminal) service to customers, by picking them up and dropping them off at their desired locations. The vehicle begins and ends each of its trips from the terminal. We assume that pick up customers are able to notify their presence by means of a phone or internet booking service. Immediately before the beginning of each trip, waiting customers (both pick up and drop off ones) are scheduled and the route for the trip in the service area is constructed. There is no planned idle time in between trips. To schedule

the requests we assume that the schedule is calculated by an insertion algorithm attempting to minimize the total distance traveled by the vehicle. An insertion heuristic approach is adopted because they are widely used in practice to solve transportation scheduling problems, as they often provide very good solutions compared to optimality, they are computationally fast and they can easily handle complicating constraints (Campbell and Savelsbergh, 2004). Rectilinear movements (as in a Manhattan network) are assumed and often chosen instead of Euclidean ones, since they better estimate distances traveled in real road networks and generally provide good approximations (see Quadrioglio et al., 2008a).

For the two vehicle case, we divide the service area into two zones with width W and length $L/2$. Zone 1 is adjacent to the terminal and Zone 2 on the right of it. Each vehicle serves a zone and operations are scheduled with an insertion heuristic algorithm as for the one vehicle case. Vehicles operate continuously and alternate their operations among zones, so their expected average cycle time is the same. This means that a vehicle would start from terminal 1, serve customers in Zone 1 (while the other vehicle is serving Zone 2), come back to terminal 1, move to serve Zone 2 (while the other vehicle is serving Zone 1) and come back to terminal 1 (see Fig. 2).

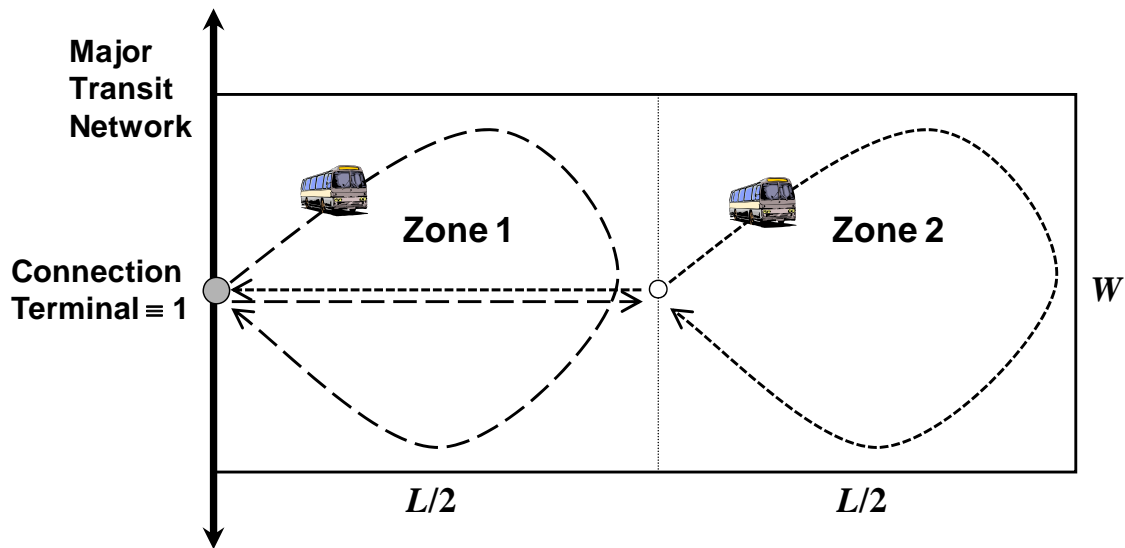


Fig. 2. DRC policy for the Two-Vehicle Case.

3.1.3 Performance measures

The performance of a transit system can roughly be considered as a combination of operating costs and service quality. The relative weight assigned to each of those two categories is a disputed matter and can differ between public transportation agencies and privately owned ones. However, in this research, we may assume the operating costs to be equivalent for the two competing transit services. The assumption is reasonable in these comparisons, because the vehicle is assumed to be the same and run continuously during the operations for both service policies at the same average speed v_b and the demand served is also the same. We recognize that the FRT may have a shorter cycle and therefore may need a slightly smaller vehicle for its operations. The bus stop infrastructure for the FRT may also bring additional cost, but it would be a small portion in the long term service operation. Thus, other than possible negligible differences, we

do not see a major disparity of the operating costs between the two cases which would cause our assumption to be unreasonable.

Thus, the comparison between the two services can be performed by considering only the service quality provided to customers. If we disregard other possible sources of noise that could influence customers' perceptions and opinions, the service quality can be considered as a combination of the following performance measures:

- $E(T_{wk})$: expected value of walking time of the passengers needed to/from their closest bus stop from/to their destination.
- $E(T_{wt})$: expected value of waiting time of the passengers from their ready time to their pick up time (subtracting the possible walking time).
- $E(T_{rd})$: expected value of ride time of the passengers from pick up to drop off.

Generally, needed transfers between vehicles to complete a trip are a major service quality factor as well, but there are none in this case. Thus, the service quality provided to customers is represented by the utility function U defined as the weighed sum of the above terms:

$$U = w_{wk} \times E(T_{wk}) + w_{wt} \times E(T_{wt}) + w_{rd} \times E(T_{rd}). \quad (3.1)$$

Lower values of U indicate a better level of service. The assessments of the weights (w_{wk} , w_{wt} , and w_{rd}) are generally difficult to make, as they are dependent upon several factors, they are not unique for all cases and they can change dynamically

depending on the circumstances. For example: the walking time could be considered more or less acceptable (thus, with a different relative weight), depending on the safety or the weather conditions of a certain area and/or the profile of the customers. However, the weight assignment is not the scope of this paper. We wish to provide decision makers with tools which will help them decide the proper service policy, once they have selected the proper weights for their scenario. A more detailed discussion for the weights can be found in two recent studies, Wardman (2004) and Guo and Wilson (2004).

In the next sections we will focus on the analytical computation of U for both competing policies, so we can make a comparison.

3.2 Analytical modeling for the One-Vehicle Case

3.2.1 FRT

In this section we calculate the expected values of the three performance measures $E(T_{wk})$, $E(T_{wt})$, $E(T_{rd})$ for the one vehicle scenario when a FRT operating policy is adopted.

Assuming that customers would walk to the nearest bus stop with a rectilinear path, the expected value of the walking time $E(T_{wk})$ is

$$E(T_{wk}) = \frac{1}{4v_{wk}} \left(\frac{L}{N-1} + W \right), \quad (3.2)$$

where v_{wk} is the average walking speed and N is number of FRT bus stations, including the connection terminal 1 (see Fig. 1).

Since the bus dwelling time at each station is s_f , the cycle time of the journey beginning at terminal 1 and back is

$$C = \frac{2L}{v_b} + 2(N-1)s_f. \quad (3.3)$$

The derivation of the expected values for the waiting time and riding time depends upon the relationship between the values assumed for the weights w_{wt} and w_{rd} . As mentioned, our scope is not to assess the weights, but to provide analytical tools given their assumed values.

A $w_{wt} < w_{rd}$ (case 1) would mean that customers would spend their time waiting rather than being on the vehicle. This is a reasonable assumption if the waiting location is a comfortable one, like at home or at a nicely built connection terminal. Since customers would walk to the nearest bus stop of the FRT, for the region (which is $\frac{1}{2(N-1)}$ in proportion to the total service area shown in Fig. 1) closest to the terminal 1, customers would walk to the terminal 1. Therefore the waiting time and ride time for these customers are 0. The expected value of the waiting time is $C/2$ for other customers.

Then the expected value of the waiting time for pick up customers, drop off customers and all customers are (superscripts p and d denote pick-up and drop-off customers respectively; subscript wt denotes waiting time):

$$E(T_{wt-1}^p) = \left[\frac{1}{2} - \frac{1}{4(N-1)} \right] C, \quad (3.4)$$

$$E(T_{wt-1}^d) = \left[\frac{1}{2} - \frac{1}{4(N-1)} \right] C, \quad (3.5)$$

$$E(T_{wt-1}) = \alpha E(T_{wt-1}^p) + (1-\alpha) E(T_{wt-1}^d) = \left[1 - \frac{1}{2(N-1)} \right] \left[\frac{L}{v_b} + (N-1)s_f \right]. \quad (3.6)$$

The expected value of the ride time for pick up customers, drop off customers and all customers are instead:

$$E(T_{rd-1}^p) = \frac{C}{4}, \quad (3.7)$$

$$E(T_{rd-1}^d) = \frac{C}{4}, \quad (3.8)$$

$$E(T_{rd-1}) = \alpha E(T_{rd-1}^p) + (1-\alpha) E(T_{rd-1}^d) = \frac{1}{2} \left[\frac{L}{v_b} + (N-1)s_f \right]. \quad (3.9)$$

A $w_{wt} > w_{rd}$ (case 2) would instead mean that customers would spend their time onboard rather than waiting. This could be the case when most of the waiting occurs at possibly unsafe locations, maybe at night and/or with adverse weather conditions. Equations (4) to (9) are then recalculated by employing conditional probability (the mathematical passages can be found in the appendix):

$$E(T_{wt-2}^p) = \left[\frac{1}{3} - \frac{1}{4(N-1)} + \frac{1}{6(N-1)^2} \right] C, \quad (3.4a)$$

$$E(T_{wt-2}^d) = E(T_{wt-1}^d) = \left[\frac{1}{2} - \frac{1}{4(N-1)} \right] C, \quad (3.5a)$$

$$\begin{aligned} E(T_{wt-2}) &= \alpha E(T_{wt-2}^p) + (1-\alpha) E(T_{wt-2}^d) = \\ &= \left[\frac{\alpha}{3} \left(\frac{1}{(N-1)^2} - 1 \right) - \frac{1}{2(N-1)} + 1 \right] \left[\frac{L}{v_b} + (N-1)s_f \right], \end{aligned} \quad (3.6a)$$

$$E(T_{rd-2}^p) = \left[\frac{5}{12} - \frac{1}{6(N-1)^2} \right] C, \quad (3.7a)$$

$$E(T_{rd-2}^d) = E(T_{rd-1}^d) = \frac{C}{4}, \quad (3.8a)$$

$$\begin{aligned} E(T_{rd-2}) &= \alpha E(T_{rd-2}^p) + (1-\alpha) E(T_{rd-2}^d) = \\ &= \left[\frac{\alpha}{3} \left(1 - \frac{1}{(N-1)^2} \right) + \frac{1}{2} \right] \left[\frac{L}{v_b} + (N-1)s_f \right]. \end{aligned} \quad (3.9a)$$

3.2.2 DRC

The calculation of the expected values of the performance measures for the demand responsive operating policy is not straightforward, due to the fact that at each cycle, the vehicle performs a different tour, to serve the demand uniformly but randomly distributed across the service area. However, it is possible to provide good estimates by following a methodology similar to the one adopted in Quadrifoglio et al. (2006). Quadrifoglio et al. (2006) proved that the distance traveled by a vehicle traveling along a corridor to serve uniformly distributed demand scheduled with an insertion heuristic algorithm (attempting to minimize the total distance traveled) is upper bounded and closely approximated (especially for lower densities) by the distance traveled by the

vehicle following a rectilinear “no backtracking policy”, which forbids backwards movements with respect to the current forward direction and, therefore, serves the customers in order of their horizontal coordinate. In this research, since the vehicle is performing a cycle to/from the terminal stop 1, we assume that the vehicle would move to the right through the upper half of the region in a no backtracking policy left-to-right, and move left through the bottom half in a no backtracking policy right to left.

Let n be the number of customers served per cycle by the DRC vehicle. Since their spatial distribution is assumed to be uniform, if x_i is the horizontal coordinate within the service area ($0 \leq x_i \leq L$) of customer i (with $i = 1, \dots, n$), the expected value of the maximum horizontal distance that the vehicle will need to travel can be derived as follows:

$$\begin{aligned}
 E[\max(x_i)|i = 1, \dots, n] &= \int_0^L \{P[\max(x_i)|i = 1, \dots, n] \geq t\} dt = \\
 &= \int_0^L \{1 - P[\max(x_i)|i = 1, \dots, n] \leq t\} dt = \int_0^L \left\{1 - \prod_{i=1}^n [P(x_i) \leq t]\right\} dt = \quad (3.10) \\
 &= \int_0^L \left(1 - \left(\frac{t}{L}\right)^n\right) dt = L \frac{n}{n+1}.
 \end{aligned}$$

Customers are uniformly distributed within the whole service area. Let y be the random variable indicating the vertical distance between any pair of customers within the upper or lower half of the service area; we have that $E(y) = W/6$. Let y' indicate the vertical distance between the terminal 1 (located at $W/2$) and the first or last customer in

the schedule; we have that $E(y') = W/4$. Finally, y'' indicate the vertical distance between the last customer served in the upper half and the first customer served in the lower half; we have that $E(y'') = W/2$. See Fig. 3.

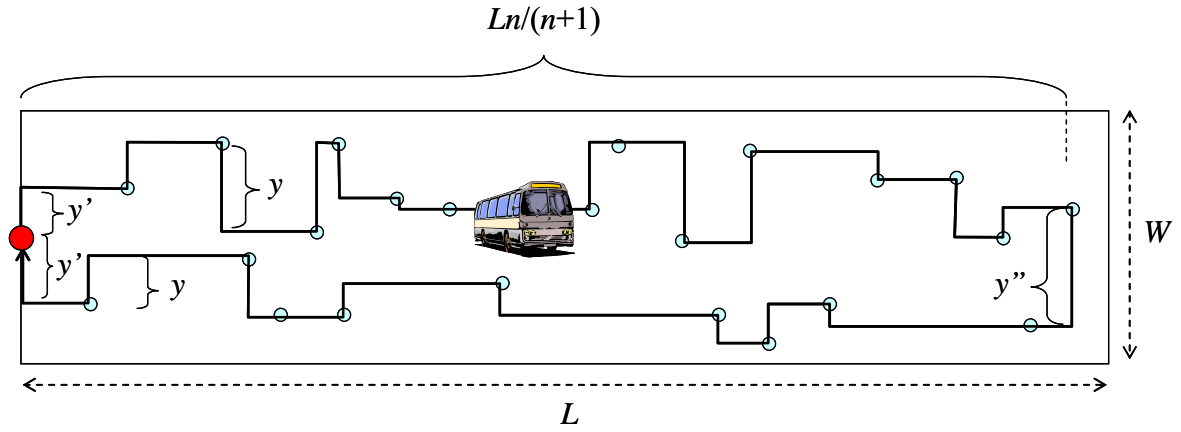


Fig. 3. No-backtracking policy.

If D represents the expected total rectilinear distance per cycle for a no-backtracking policy, C is the expected cycle time and λ is the average customer demand rate, the following relationships hold:

$$D = 2L \frac{n}{n+1} + 2 \frac{W}{4} + \frac{W}{2} + (n-2) \frac{W}{6} = 2L \frac{n}{n+1} + \frac{2W}{3} + \frac{W}{6} n, \quad (3.11)$$

$$C = \frac{D}{v_b} + (n+1)s, \quad (3.12)$$

$$n = \lambda C. \quad (3.13)$$

Drop off customers will need to wait an average of $E(T_{wt}^d) = C/2$, since they will show up and wait at the connection terminal 1 uniformly from time 0 to C of the previous cycle. They will also ride an average of $E(T_{rd}^d) = C/2$, since they can be dropped off uniformly anytime from time 0 to C of their cycle.

Pick up customers will instead need to wait $E(T_{wt}^p) = C/2 + C/2 = C$, since they will wait an average of $C/2$ from their show up time to the end of the previous cycle and an additional average of $C/2$, waiting for the vehicle to reach them. They will also ride an average of $E(T_{rd}^p) = C/2$, as for the drop off customers.

Thus, the expected values of the total waiting time and riding time are

$$E(T_{wt}) = \alpha E(T_{wt}^p) + (1 - \alpha) E(T_{wt}^d) = (1 + \alpha) \frac{C}{2}, \quad (3.14)$$

$$E(T_{rd}) = \alpha E(T_{rd}^p) + (1 - \alpha) E(T_{rd}^d) = \frac{C}{2}. \quad (3.15)$$

In order to derive C , we need to solve the system of equations composed by (3.11), (3.12) and (3.13). In doing so we obtain the following quadratic equation:

$$aC^2 + bC + c = 0, \quad (3.16)$$

where:

$$a = \lambda \left[\lambda \left(\frac{W}{6} + sv_b \right) - v_b \right], \quad (3.17)$$

$$b = \lambda \left(\frac{5W}{6} + 2L + 2sv_b \right) - v_b, \quad (3.18)$$

$$c = \frac{2W}{3} + sv_b. \quad (3.19)$$

Two obvious conditions should be satisfied: $C > 0$ and $b^2 - 4ac \geq 0$. However, a closed-form expression for C is not easy to derive.

Approximation 1:

In Equation (3.11) we could reasonably assume that

$$\frac{n}{n+1} \cong 1, \quad (3.20)$$

thus overestimating D by a factor of $\frac{2L}{n+1}$, which becomes increasingly negligible with increasing n and becomes zero for $n \rightarrow \infty$. The approximate cycle time \tilde{C} so obtained would be an upper bound of the actual cycle time C and thus still an upper bound of the actual cycle time obtainable by an insertion heuristic. After rearranging (3.11) with the above approximation (3.20) and combining it with (3.12) and (3.13), we are able to obtain a closed form expression for the approximate cycle time

$$\tilde{C} = \frac{sv_b + 2W/3 + 2L}{v_b - \lambda(W/6 + sv_b)}. \quad (3.21)$$

Approximation 2:

Still applying (3.20), we substitute $\frac{2W}{3}$ with $\frac{2W}{3} \frac{n}{n+1}$ in Equation (3.11),

underestimating D by a factor of $\frac{2W}{3(n+1)}$, and $(n+1)s$ with ns in Equation (3.12),

underestimating C by a factor of s . Then, we obtain another closed form expression for the approximate cycle time:

$$\tilde{C} = \frac{2W/3 + 2L}{v_b - \lambda(W/6 + sv_b)} - \frac{1}{\lambda}. \quad (3.21a)$$

Fig. 4 shows how the closed form values obtained by (21) and (21a) approximate the true C obtained by numerical methods. Increasing n reduces the error. Since,

generally, $\frac{2W}{3} \ll 2L$ and s is also small, \tilde{C} given by (21a) is closer to C .

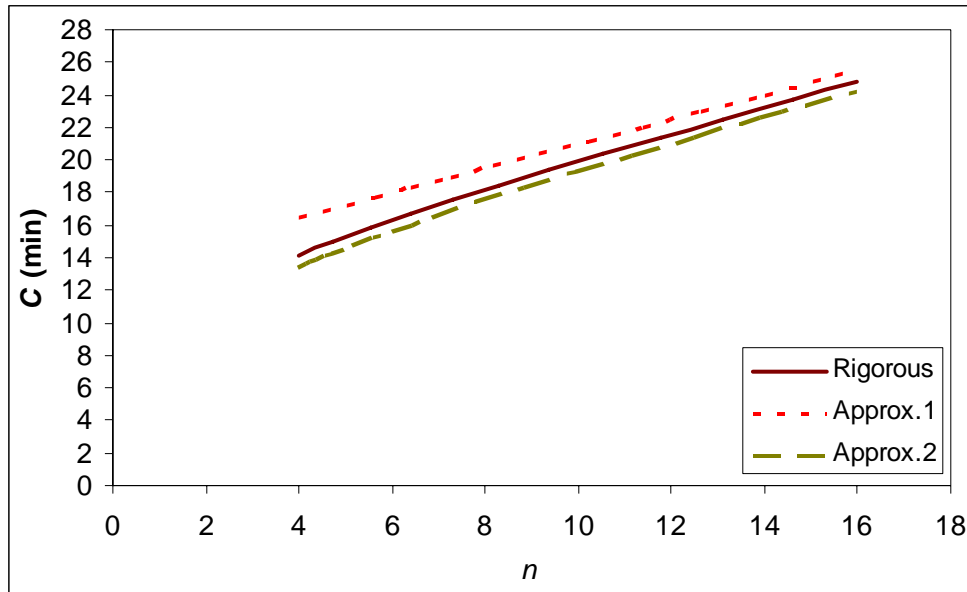


Fig. 4. C vs. \tilde{C} .

The approximate values $E(\tilde{T}_{wt})$ and $E(\tilde{T}_{rd})$ can be obtained by substituting C with \tilde{C} in (3.14) and (3.15).

$E(T_{wk})$ and $E(\tilde{T}_{wk})$ are zero, since the DRC offer a terminal-to-door (and vice versa) service and no walking is necessary.

3.2.3 Critical demand

For case 1 ($w_{wt} < w_{rd}$), we obtain the utility function for the FRT policy by substituting (3.2), (3.6) and (3.9) in (1); similarly, by substituting (3.14) and (3.15) in (1) we obtain the utility function for the DRC policy. We can now equate these two expressions and solve for λ . The resulting value λ_c represents the critical demand rate at

which the two services would be equivalent in terms of service quality provided to customers.

C does not have a closed form expression and so does not λ_c , but solutions can be obtained with numerical methods. However, if we use \tilde{C} in Equation (3.21), a closed form expression for the approximation of λ_c can be derived and is

$$\tilde{\lambda}_c = \frac{1}{\frac{W}{6v_b} + s} - \frac{[(1 + \alpha)w_{wt} + w_{rd}] \left[\frac{3v_b s + 2W + 6L}{6v_b s + W} \right]}{\frac{w_{wk}}{4v_{wk}} \left(\frac{L}{N-1} + W \right) + \left[\frac{L}{v_b} + s_f (N-1) \right] \left[\frac{w_{rd}}{2} + \frac{w_{wt}}{2N-2} \right] \left[\frac{w_{rd}}{2} + \frac{w_{wt}}{2N-3} \right]}. \quad (3.22)$$

An analogous equation is similarly calculated for case 2 ($w_{wt} > w_{rd}$) and is:

$$\tilde{\lambda}_c = \frac{1}{\frac{W}{6v_b} + s} - \frac{[(1 + \alpha)w_{wt} + w_{rd}] \left[\frac{3v_b s + 2W + 6L}{6v_b s + W} \right]}{\frac{w_{wk}}{4v_{wk}} \left(\frac{L}{N-1} + W \right) + \left[\frac{L}{v_b} + s_f (N-1) \right] \left[\frac{\alpha(w_{wt} - w_{rd})}{3(N-1)^2} + \frac{w_{rd}}{2} + \frac{w_{wt}}{2N-2} \right] \left[\frac{w_{rd}}{2} + \frac{w_{wt}}{2N-3} \right]}. \quad (3.22a)$$

Finally, the critical demand density (customers/hr/mile²) is defined as

$$\rho_c = \frac{\lambda_c}{WL}, \quad (3.23)$$

and its approximation is $\tilde{\rho}_c = \frac{\tilde{\lambda}_c}{WL}$.

The ρ_c represents the point where the two services could reasonably be considered equivalent and where a switch from one type of service to the other would be desirable. For expected demand densities lower than ρ_c , the DRC is the preferred operating policy; for expected demand densities higher than ρ_c , the FRT is the preferred operating policy.

3.3 Analytical modeling for the Two-Vehicle Case

3.3.1 FRT

For the two vehicle FRT case, the expected value of customer walking time $E(T_{wk})$ is the same as the one vehicle case and represented by Equation (3.2).

Assuming that the two vehicles have the same average speed v_b , the 1st vehicle starts from the terminal 1, and the 2nd vehicle starts from the bus station at stop N . The cycle time is still represented by Equation (3.3).

The expected value of the waiting time is

$$\begin{aligned} E(T_{wr}) &= \alpha E(T_{wr}^p) + (1 - \alpha) E(T_{wr}^d) = \\ &= \left[\frac{1}{2} - \frac{1}{4(N-1)} \right] \frac{L}{v_b} + \left(N - \frac{3}{2} \right) \frac{s_f}{2}, \end{aligned} \quad (3.24)$$

where $E(T_{wt}^p) = E(T_{wt}^d) = \left[1 - \frac{1}{2(N-1)}\right] \frac{C}{4}$, calculated similarly to the one vehicle case.

The expected value of the riding time is

$$E(T_{rd}) = \alpha E(T_{rd}^p) + (1 - \alpha) E(T_{rd}^d) = \frac{C}{4} = \frac{L}{2v_b} + (N-1) \frac{s_f}{2}, \quad (3.25)$$

where $E(T_{wt}^p) = E(T_{wt}^d) = C/4$, calculated similarly to the one vehicle case.

3.3.2 DRC

As for the one vehicle case, we approximate the insertion heuristic operations with a no-backtracking policy left-to-right on the top half and right-to-left on the bottom half of each zone. The following equations are similar to (3.11), (3.12) and (3.13), with $n/2$ instead of n in (3.26) and (3.27), since half customers are served in each zone, and $L/2$ added in (3.26), since vehicles alternate their operations between zones by driving half length of the whole service area, so their expected average cycle time is the same:

$$\begin{aligned} D &= \frac{L}{2} + 2 \frac{L}{2} \frac{\frac{n}{2}}{\frac{n}{2} + 1} + 2 \frac{W}{4} + \frac{W}{2} + \left(\frac{n}{2} - 2\right) \frac{W}{6} = \\ &= L \frac{n}{n+2} + \frac{W}{6} \frac{n}{2} + \frac{L}{2} + \frac{2W}{3}, \end{aligned} \quad (3.26)$$

$$C = \frac{D}{v_b} + \left(\frac{n}{2} + 1\right)s, \quad (3.27)$$

$$n = \lambda C. \quad (3.28)$$

Drop off customers will need to wait an average of $E(T_{wr}^d) = C/2$, since they will show up and wait at the connection terminal 1 uniformly from time 0 to C of the previous cycle. The expected ride time for drop off customers in Zone 1 is

$$E(T_{rd}^{d-1}) = \frac{1}{2} \left(C - \frac{L}{2v_b} \right), \text{ since we need to subtract from } C \text{ the vehicle transfer time}$$

$L/(2v_b)$ to Zone 2. The expected ride time for drop off customers in Zone 2 is instead

$$E(T_{rd}^{d-2}) = \frac{1}{2} \left(C - \frac{L}{2v_b} \right) + \frac{L}{2v_b}, \text{ since these customers need to spend the transfer time}$$

$L/(2v_b)$ onboard to reach Zone 2. Thus, drop off customers will ride an average of

$$E(T_{rd}^d) = \left(E(T_{rd}^{d-1}) + E(T_{rd}^{d-2}) \right) / 2 = C/2.$$

$$\text{Pick-up customers will wait } E(T_{wr}^p) = \frac{C}{2} + \frac{1}{2} \left(C - \frac{L}{2v_b} \right) = C - \frac{L}{4v_b}, \text{ since they will}$$

wait an average of $C/2$ from their show-up time to the end of the previous cycle and an

additional average of $\frac{1}{2} \left(C - \frac{L}{2v_b} \right)$, waiting for the vehicle to reach them. They will also

ride an average of $E(T_{rd}^p) = C/2$, as for the drop-off customers.

Thus, the expected values of the total waiting time and riding time are

$$E(T_{wt}) = \alpha E(T_{wt}^p) + (1 - \alpha) E(T_{wt}^d) = (1 + \alpha) \frac{C}{2} - \alpha \frac{L}{4v_b}, \quad (3.29)$$

$$E(T_{rd}) = \alpha E(T_{rd}^p) + (1 - \alpha) E(T_{rd}^d) = \frac{C}{2}. \quad (3.30)$$

Solving the system of equations composed by (3.26), (3.27) and (3.28), we obtain the quadratic equation (3.16), where:

$$a = \frac{\lambda}{2} \left[\frac{\lambda}{2} \left(\frac{W}{6} + sv_b \right) - v_b \right], \quad (3.31)$$

$$b = \frac{\lambda}{2} \left(\frac{5W}{6} + \frac{3}{2}L + 2sv_b \right) - v_b, \quad (3.32)$$

$$c = \frac{L}{2} + \frac{2W}{3} + sv_b. \quad (3.33)$$

A closed-form expression for C is not easy to derive. But, as for Approximation 1 in the one-vehicle case, we can say that

$$\frac{n}{n+2} \cong 1. \quad (3.34)$$

With this approximation we are able to obtain a closed-form expression for the approximate cycle time for the two-vehicle case:

$$\tilde{C} = \frac{sv_b + 2W/3 + 3L/2}{v_b - (\lambda/2)(W/6 + sv_b)}. \quad (3.35)$$

The approximate values $E(\tilde{T}_{wt})$ and $E(\tilde{T}_{rd})$ can be obtained by substituting C with \tilde{C} in (3.35) and (3.36).

3.3.3 Critical demand

By substituting (3.2), (3.24) and (3.25) in (3.1) we obtain the utility function for the two-vehicle FRT policy; similarly, by substituting (3.29) and (3.30) in (3.1) we obtain the utility function for the two vehicle DRC policy. We can now equate the two expressions and solve for λ , to obtain the critical demand rate λ_c for the two-vehicle case. Using \tilde{C} in Equation (3.35), a closed-form expression for the approximation of λ_c is

$$\tilde{\lambda}_c = \frac{2}{\frac{W}{6v_b} + s} - \frac{[(1 + \alpha)w_{wt} + w_{rd}] \left[\frac{6v_b s + 4W + 9L}{6v_b s + W} \right]}{\frac{w_{wk}}{4v_{wk}} \left(\frac{L}{N-1} + W \right) + \frac{1}{2} \left[\frac{L}{v_b} + s_f (N-1) \right] \left(w_{rd} + \frac{w_{wt}}{2N-2} \right) + \frac{\alpha L}{4v_b} w_{wt}}, \quad (3.36)$$

and ρ_c is derived as for (3.23).

3.4 Results

In this section we provide numerical results to validate the analytical modeling (rigorous and approximate) vs. simulation. We performed simulations according to the demand distributions assumed in Section 3.1. χ^2 statistical tests show that the simulated data have the assumed distributions. We performed 30 simulation replications. The resulting 95% confidence half-intervals are about 0.7% of the mean for the simulated utility function values (U). The DRC vehicle serves the demand following a schedule calculated with an insertion heuristic algorithm attempting to minimize the vehicle's total travel distance in each cycle.

3.4.1 Values of parameters

To represent a residential area, the values of the parameters assumed for analyses are as follows:

- FRT bus station distance $d = 0.25$ mile.
- pedestrian walking speed $v_{wk} = 2$ mile/hr.
- bus running speed $v_b = 20$ mile/hr.
- bus dwell time at each station or customer location $s_f = s = 30$ second.
- The service area $L \times W = 1$ mile². However, we considered three different L/W ratios: with the length L equal to 4, 2, 1 mile and the width W to 0.25, 0.5, 1 mile respectively.
- We considered a range of different customer demand densities: from 0 up to 90 customers/mile²/hr.

- We assume $\alpha = 0.5$, meaning that 50% of the demand are pick-up customers and 50% are drop-off customers.
- We assume $w_{wt} = 1$ and $w_{rd} = 2$. As mentioned, the value of w_{wk} is the most susceptible to variation, due to weather and changing safety conditions; therefore, we consider $w_{wk} = 3$ as a “base case”, but we also perform sensitivity analyses.

3.4.2 One-Vehicle Case for $L=2/W=0.5$

We calculated the utility function values for the FRT policy using Equations (3.1) for different demand densities and four different values for w_{wk} (2, 3, 4 and 5). To compute the three terms in (3.1), Equations (3.2), (3.6) and (3.9) have been used.

We calculated the utility function values for the no-backtracking DRC policy using Equation (3.1) for different demand densities. The rigorous analytical values of the three terms in (3.1) were computed by solving Equation (3.16) by numerical methods. The approximate analytical values of the three terms in (3.1) were instead computed with Equations (3.21) and (3.21a).

Fig. 5 graphically shows the computed utility function values.

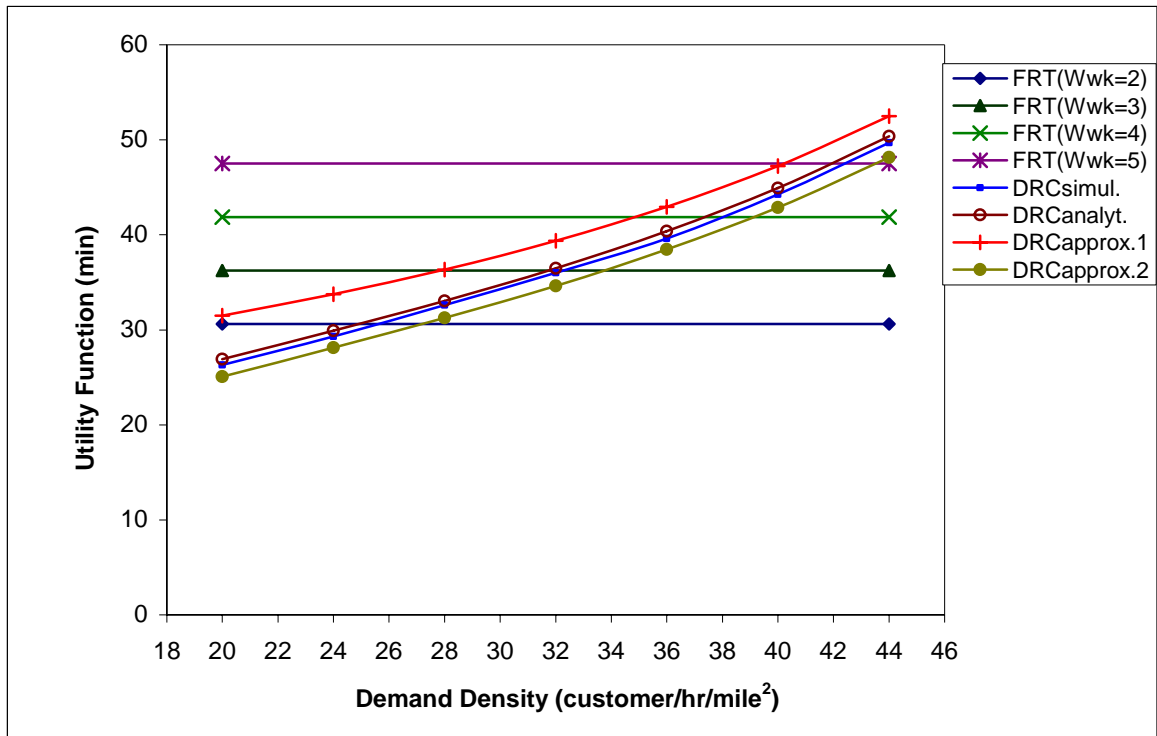


Fig. 5. Utility functions for the One-Vehicle Case with $L=2$, $W=0.5$.

FRT utility functions have four different flat values as the weight w_{wk} changes from 2 to 5, since they do not depend on the demand. DRC utility functions (rigorous analytical, approximate analytical 1, approximate analytical 2, and simulation) increase with the demand and do not depend on w_{wk} since there is no walking. While we did not assume any capacity constraint in developing our methodology, in all our simulated cases we observed a maximum loading capacity of 25 passengers within our considered range of demand rates. Thus, all our scenarios could have been performed comfortably by a 30-seat bus (for example). Clearly, for higher demand densities, capacity

constraints must be taken in consideration, as well as alternative scheduling policies, especially for the DRC.

From the above chart the following observation can be made with regards to the DRC curves:

- The rigorous analytical values are upper bounds for the corresponding simulated values. This is expected, since the no-backtracking policy provides an upper bound of the insertion heuristic algorithm in terms of the distance traveled and consequently in terms of the utility function as well. However, the error is reasonably small (in the range of 1%-3% for the considered scenarios), confirming the good approximations provided by the no-backtracking policy.
- The values of approximate analytical 1 are an upper bound for the corresponding rigorous values, since our approximate models overestimate the total distance traveled and the gap gets smaller with increasing demand densities, as expected, because of assumption (3.20).
- The values of approximate analytical 2 are a lower bound for the corresponding rigorous values, since our approximate models underestimate the total distance traveled.
- In general, the four curves are fairly close to each other, which would allow using the developed approximate but handy analytical formulas to estimate the actual utility function values.

The intersections between the DRC curves and the FRT curves represent the critical demand densities at which the FRT policy and DRC policy have the same utility function values and thus equal performance. For demand densities lower than the critical one, the DRC would be the preferred choice and vice versa. Equation (3.22) provides a closed-form expression for these critical demand densities for the approximate 1 case ($n \cong n+1$). The critical demand densities are listed in Table 1, along with the corresponding cycle times C and number of customer served n , and shown in Fig. 6.

Table 1
 ρ_c , C and n for $L=2$, $W=0.5$; One-Vehicle Case.

	Case	$w_{wk} = 2$	$w_{wk} = 3$	$w_{wk} = 4$	$w_{wk} = 5$
ρ_c (cust./hr/ml ²)	Simulation	25.5	32.1	38.1	42.3
	Analytical	24.8	31.6	37.3	42
	Approx. 1	18.3	27.9	34.9	40.1
	Approx. 2	27.1	33.7	39.1	43.6
C (min)	Simulation	17.2	20.4	23.8	26.4
	Analytical	17.8	20.9	24.4	27.4
	Approx. 1	19.8	22.5	25.8	28.6
	Approx. 2	16.7	19.8	23.3	26.2
N	Simulation	7.3	10.9	15.1	18.6
	Analytical	7.6	11.2	15.5	19.3
	Approx. 1	8.4	12.1	16.4	20.2
	Approx. 2	7.1	10.6	14.8	18.4

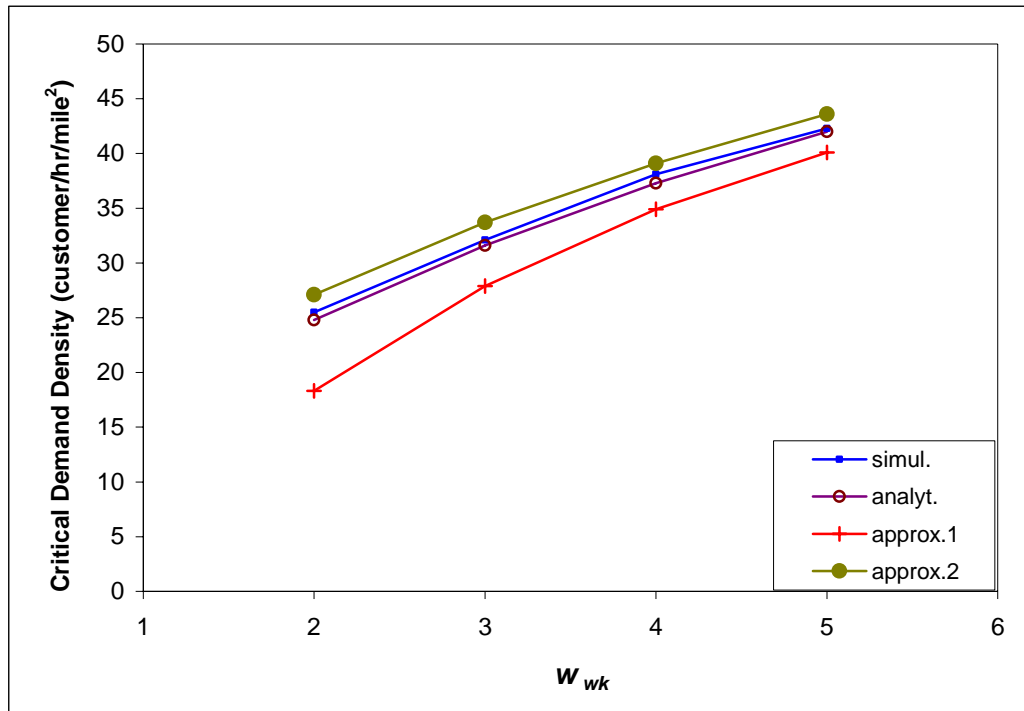


Fig. 6. Critical demand densities for $L=2$, $W=0.5$; One-Vehicle Case.

The above results show that approximate analytical values taken from Approximation 1 for the critical demand densities underestimate the rigorous analytical and simulated ones. This would mean that the critical “switching point” from DRC to FRT predicted by Equation (3.22) would be slightly anticipated with increasing demand (and vice versa). Values taken from Approximation 2 would instead do the opposite.

As an illustrative example, consider the scenario where estimated values for the weights are $w_{wk} = 4$, $w_{wt} = 1$, $w_{rd} = 2$. The approximate value of the critical demand density given by Equation (3.22) is 34.9 customers/hr/mile². As soon as the demand is expected to drop below this value a switch from a FRT to DRC operating policy would be desirable to maximize the service quality provided to customers. While this

procedure clearly has intrinsic approximations built in it, it certainly provides a good justifiable estimate. As a validation of our results, for example, the transit operator for a DRC service operated in the city of Winnipeg, Canada, where the service area is close to 1 mile², estimated that the DRC policy would be best to be operated for up to a maximum of approximately 20 customers/hr/mile², which is close to the critical demand rate we estimated for $w_{wk}=2$ (18.3 customers/hr/mile² by Approximation 1).

3.4.3 Effect of L/W ratio

In addition to the $L=2, W=0.5$ scenario, we produced the critical customer demand densities, shown in Table 2 and Fig. 7, for $L=4, W=0.25$ and $L=1, W=1$ scenarios to analyze the effects of various L/W ratios.

Table 2
Critical demand densities for various L/W ratios; One-Vehicle Case (customer/hr/mile²).

L/W	Case	$w_{wk}=2$	$w_{wk}=3$	$w_{wk}=4$	$w_{wk}=5$
1/1	Simulation	31.8	39.7	45.1	49.6
	Analytical	30.9	37.5	41.4	43.9
	Approx. 1	28.4	36.4	40.8	43.5
	Approx. 2	33.1	39.2	42.4	44.9
4/0.25	Simulation	17.5	21.1	24.9	28.7
	Analytical	16.7	20.3	24.1	27.9
	Approx. 1	6.4	13	18.8	23.7
	Approx. 2	18	21.9	25.6	29.5

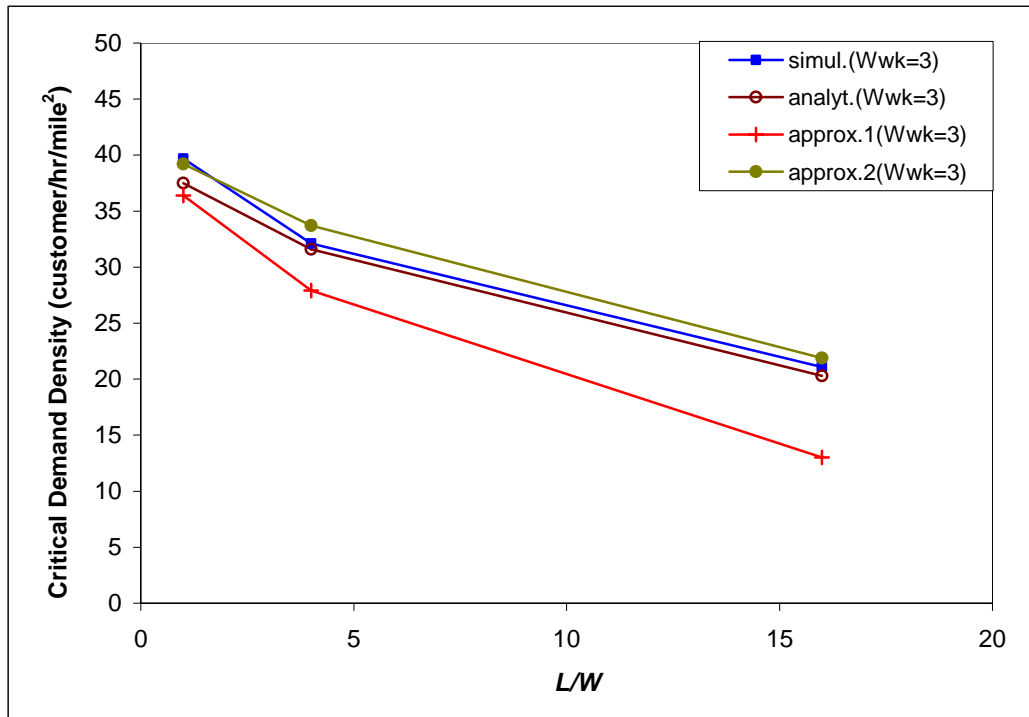


Fig. 7. Effects of various L/W ratios; One-Vehicle Case.

The critical demand densities decrease with the increase of L/W ratio as walking to stations becomes less relevant. For most of the scenarios the rigorous analytical value is very close to the simulated one except that the difference is about 10% for the scenario of $L=1$, $W=1$ and $w_{wk}=5$. We note that for larger L/W ratio and lower w_{wk} value, such as $L=4$, $W=0.25$ and $w_{wk}=2$, the approximation-1 values may have significant differences from the simulated one. However, for such scenario the approximation-2 and analytical rigorous critical demands are very close to the simulated ones. Therefore for this situation (large L/W ratio) the approximation-2 or rigorous analytical formulae should be adopted instead of the approximation.

3.4.4 Two-Vehicle Case

We briefly present the results obtained for the two-vehicle case. For the FRT policy the utility function values were computed with Equations (3.1), (3.24) and (3.25). For the DRC policy the rigorous analytical values of utility function were computed with Equations (3.1), (3.29) and (3.30), deriving the cycle times C with Equations (3.16), (3.31), (3.32) and (3.33); the approximate values were computed by using Equations (3.35) to estimate C . As for the one-vehicle case, we developed simulations to compute the utility function values for two-vehicle DRC policy. Fig. 8 shows the computed utility function values for $L=2$, $W=0.5$. The intersection points between the DRC curves and the FRT curves show the critical demand densities which are listed in Table 3.

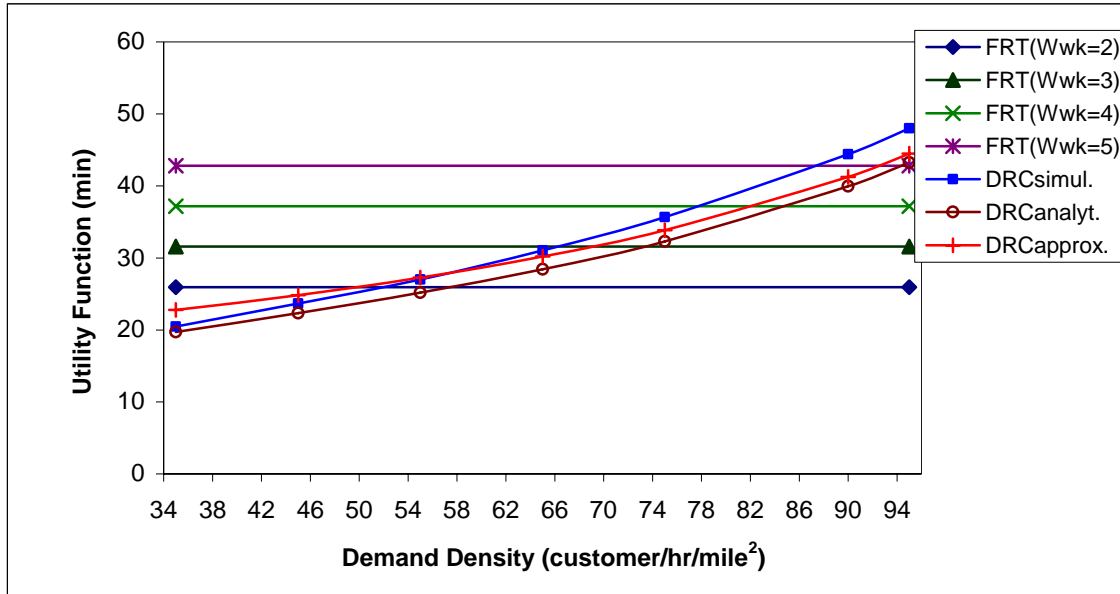


Fig. 8. Values of utility function for $L=2$, $W=0.5$; Two-Vehicle Case.

Table 3Critical demand densities for $L=2$, $W=0.5$; Two-Vehicle Case (customer/hr/mile²).

Case	$w_{wk}=2$	$w_{wk}=3$	$w_{wk}=4$	$w_{wk}=5$
Simulation	52	66.2	77.9	87.6
Analytical	57.5	73.5	84.9	94.5
Approx.	50.2	69.1	82.1	92.7

As for the one-vehicle case, the approximate values provide an upper bound to the analytical values. As opposed to the one-vehicle case, the simulated values are slightly larger than the analytical values; this is caused by the existing correlations between the vehicles' operational cycles, which are not captured by our two-vehicle analytical modeling, in which we assumed independency. While the differences are more noticeable than the one-vehicle case, they are still acceptably within a 10% maximum deviation. In addition, since the handy approximated values obtain by Equation (3.36) are too an upper bound, they are closer to the simulation values than the rigorous ones, which is a good result in practice.

3.5 Summary

Proper design and operations of feeder transit services within the modern sprawled residential areas are becoming increasingly more important to enhance the performance of the public transportation system network. Feeders are generally operated with a demand responsive policy which might be converted to a traditional fixed-route policy for higher demand.

Utility functions for the fixed-route and demand responsive operating policy are derived to determine the critical demand density, representing the condition for the switch. For the one-vehicle and two-vehicle cases we derived closed-form expressions, function of the parameters of each scenario, such as the geometry of the service area, the vehicle speed and especially the weights assigned to each term contributing to the utility function: walking time, waiting time and riding time. The weight's assessments are left to the decision makers, which might select them depending on the circumstances and the changing conditions of each scenario.

Analytical results compared to simulation outcomes show a good match and a validation of our methodological approach. Estimated critical demand densities for the one-vehicle case and a service area with $L=2$ and $W=0.5$ range from 18 to 40 customers/hr/mile² slightly underestimating the simulated values, as predicted, however, by our approximation procedure. Similar results are obtained for the two-vehicle case. We also performed sensitivity analysis over different L/W ratios. As a validation of our results, the transit operator for a DRC service operated in the city of Winnipeg, Canada, where the service area is close to 1 mile², estimated that the DRC service would be best to be operated for up to a maximum of approximately 20 customers/hr/mile², which is close to the critical demand rate we estimated for $w_{wk}=2$ (see Table 1).

CHAPTER IV

OPTIMAL NUMBER OF ZONES

In this part of research, analytical models are developed to aid planners in determining the optimal number of zones while balancing customer service quality and operating cost. Simulations are developed to validate the results of the analytical model. The main purpose is to develop simple analytical equations to guide planners in their decisions with as little information as possible. These are handy but powerful tools which would aid in solving the complicated feeder transit design problem.

4.1 System description

4.1.1 Service area and demand

The service area is a representation of residential communities and is modeled as a rectangle of width W and length L (see Fig. 9). The service area is divided into n zones with width W/n and length L . Within each zone the terminal connecting with the outside fixed-route major transit network is located at the half width on the far left of the zone. The temporal distribution of the demand is assumed to be a Poisson process with constant average arrival rate λ for the whole service area. We assume that a fraction α of the customers need to be transferred from the service area to a major attraction destination (such as a city's downtown) through the terminals (pick-up customers) and a fraction $1-\alpha$ of them in the opposite direction (drop-off customers). The customers'

location, either for a pick-up or for a drop-off, has a uniform distribution within the service area.

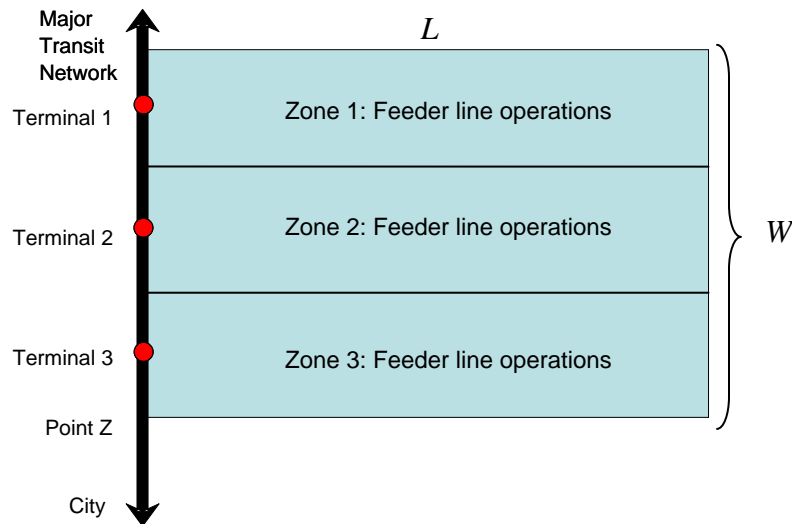


Fig. 9. Feeder line service area with three zones.

4.1.2 Transit operation policies

As shown in Fig. 9, the major fixed-route transit service connects terminals and transfers customers from the service area to the city or vice versa. While the average headway of the major transit can be slightly dependent on the number of zones, we reasonably assume it to be a constant.

Within each service zone, a FRT policy or a DRC policy would be adopted to operate the feeder service. For each operating policy we consider only one vehicle moving at an average speed v_b miles/hr and stopping at each station for a period of s hours. The operating of each policy is the same as described in Section 3.1.2.

4.2 Analytical model

We next describe the development of the analytical model needed to determine the optimal number of zones, n . For the FRT policy a customer can be in the following states: walking between the destination and the nearest bus station, waiting for the FRT, riding the FRT, waiting for the major transit, and riding the major transit. For the DRC policy a customer can be in states of waiting for an on-demand vehicle, riding an on-demand vehicle, waiting for the major transit, and riding the major transit. We assume that the different states of a customer may have a different cost to a customer.

4.2.1 Parameters and notation

The following are the parameters of the model:

- λ average demand in the whole residential area (customer/hour)
- α fraction of customers traveling from the residential area to the city; $1-\alpha$ is the fraction of customers traveling from the city to the residential area
- L length of the residential service area (mile)
- W width of the residential service area (mile)
- d distance between FRT bus stations within a zone (mile)
- a_k cost of customer walking between a FRT bus station and a house within a zone (\$/customer/hour)
- a_w cost of customer waiting at terminals (\$/customer/hour)
- a_w^h cost of customer waiting at houses (\$/customer/hour)
- a_v cost of customer traveling in an on-demand vehicle (\$/customer/hour)

a_b	cost of customer traveling in a fixed route bus in the zones (\$/customer/hour)
a_B	cost of customer traveling in a major transit vehicle between the city and terminals (\$/customer/hour)
F_v	total cost of an on-demand vehicle (\$/vehicle/hour)
F_b	total cost of a fixed route bus (\$/bus/hour)
v_{wk}	average speed of customer walking (mile/hour)
v_b	average speed of an on-demand vehicle or a fixed route bus (mile/hour)
v_B	average speed of a major transit vehicle (mile/hour)
s	dwelling time of a fixed route bus or an on-demand vehicle (hour)
S	dwelling time of a major transit vehicle at a terminal (hour)

The computed variables in the model, that are a function of n , are

$E(T_{wk})$	expected walking time for pick-up or drop-off customers in a zone
$E(T_{wt}^p)$	expected waiting time for pick-up customers in a zone
$E(T_{rd}^p)$	expected ride time for pick-up customers in a zone
$E(T_{rd-B}^p)$	expected ride time for pick-up customers in a major transit vehicle
$E(T_{wt}^d)$	expected waiting time for drop-off customers at a terminal
$E(T_{rd}^d)$	expected ride time for drop-off customers in a zone
$E(T_{rd-B}^d)$	expected ride time for drop-off customers in a major transit vehicle

4.2.2 Total cost function definition

The total cost of the designed system includes customer and vehicle cost. The vehicle cost of the major transit is dependent on the number of vehicles determined by the headway and customer demand. They are not a function of n , and are independent of the FRT or DRC policy in a zone. Therefore this vehicle cost of the major transit is not counted to determine the optimal number of zones.

Since the major transit has a constant headway, the customer waiting time for the major transit is independent of the number of zones. For drop-off customers, this waiting time is also independent of the FRT or DRC policy in a zone. We assume no coordination between the major transit headway and the FRT headway in a zone. Hence, the expected waiting time of pick-up customers at a terminal for the FRT policy is approximately the same as that for the DRC policy. Therefore, the customer waiting time for the major transit is not included in the total cost definition.

Then the total cost of the system for FRT policy and DRC policy are as follows.

FRT Total Cost = Customer Cost + FRT Bus Cost

$$\begin{aligned}
 &= n \frac{\lambda}{n} \alpha \left\{ a_k E(T_{wk}) + a_w E(T_{wt}^p) + a_b E(T_{rd}^p) + a_B E(T_{rd-B}^p) \right\} \\
 &\quad + n \frac{\lambda}{n} (1 - \alpha) \left\{ a_B E(T_{rd-B}^d) + a_w E(T_{wt}^d) + a_b E(T_{rd}^d) + a_k E(T_{wk}) \right\} + nF_b \quad (4.1)
 \end{aligned}$$

DRC Total Cost = Customer Cost + DRC Vehicle Cost

$$\begin{aligned}
&= n \frac{\lambda}{n} \alpha \left\{ a_w^h E(T_{wt}^p) + a_v E(T_{rd}^p) + a_B E(T_{rd-B}^p) \right\} \\
&\quad + n \frac{\lambda}{n} (1 - \alpha) \left\{ a_B E(T_{rd-B}^d) + a_w E(T_{wt}^d) + a_v E(T_{rd}^d) \right\} + n F_v
\end{aligned} \tag{4.2}$$

4.2.3 Derivation of the computed variables in the total cost function

For the FRT and DRC policies the customers have the same ride time on the major transit. Shown in Fig. 9, in the service area, Z is the point nearest to the city. We define the ride time as the vehicle dwelling time plus vehicle running time between a terminal and Point Z. For customers transferring at terminal $k \in \{1, 2, \dots, n\}$, the ride time

is $\left(k - \frac{1}{2}\right) \frac{W}{nV_B} + kS$. Then we have the following results for customer ride time on the

major transit:

$$E(T_{rd-B}^p) = E(T_{rd-B}^d) = \frac{1}{n} \sum_{k=1}^n \left[\left(k - \frac{1}{2}\right) \frac{W}{nV_B} + kS \right] = \frac{W}{2nV_B} + \frac{n+1}{2} S. \tag{4.3}$$

FRT Policy

The width of each zone is W/n . According to Quadrifoglio and Li (3), we have the following results for the FRT policy. In one zone, the expected walking time to the nearest bus stop $E(T_{wk})$ is

$$E(T_{wk}) = \frac{1}{4v_{wk}} \left(\frac{L}{N-1} + \frac{W}{n} \right); \quad (4.4)$$

the expected ride time of all customers is

$$\begin{aligned} \alpha E(T_{rd}^p) + (1-\alpha) E(T_{rd}^d) &= \\ &= \begin{cases} \frac{1}{2} \left[\frac{L}{v_b} + (N-1)s \right], & \text{for } a_w \leq a_b \\ \left[\frac{\alpha}{3} \left(1 - \frac{1}{(N-1)^2} \right) + \frac{1}{2} \right] \left[\frac{L}{v_b} + (N-1)s \right], & \text{for } a_w > a_b \end{cases} \end{aligned} \quad (4.5)$$

and the expected waiting time of all customers is

$$\begin{aligned} \alpha E(T_{wt}^p) + (1-\alpha) E(T_{wt}^d) &= \\ &= \begin{cases} \left[1 - \frac{1}{2(N-1)} \right] \left[\frac{L}{v_b} + (N-1)s \right], & \text{for } a_w \leq a_b \\ \left[\frac{\alpha}{3} \left(\frac{1}{(N-1)^2} - 1 \right) - \frac{1}{2(N-1)} + 1 \right] \left[\frac{L}{v_b} + (N-1)s \right], & \text{for } a_w > a_b \end{cases} \end{aligned} \quad (4.6)$$

DRC Policy

Let C represent the average cycle time of a DRC vehicle leaving and returning to a terminal. For the DRC policy, pick-up customers will ride an average of

$E(T_{rd}^d) = C/2$, since they can be dropped off uniformly anytime from time 0 to C of their

cycle. They will need to wait $E(T_{wt}^p) = C/2 + C/2 = C$, since they will wait an average of $C/2$ from their show up time to the end of the previous cycle and an additional average of $C/2$, waiting for the vehicle to reach them. Drop-off customers will need to wait an average of $E(T_{wt}^d) = C/2$, since they will show up and wait at the terminal uniformly from time 0 to C of the previous cycle. They will also ride an average of $E(T_{rd}^d) = C/2$, like the pick-up customers.

Since the scheduling of customers is a vehicle routing problem, it is difficult to derive C analytically. Approximating the commonly used insertion heuristic scheduling procedure with a non-backtracking policy, Quadrifoglio and Li (3) derived an analytical solution of C for the case of one zone. For each zone with demand λ/n and width W/n , C is the solution of the following equation:

$$aC^2 + bC + c = 0, \quad (4.7)$$

where:

$$a = \frac{\lambda}{n} \left[\frac{\lambda}{n} \left(\frac{W}{6n} + sv_b \right) - v_b \right], \quad (4.8)$$

$$b = \frac{\lambda}{n} \left(\frac{5W}{6n} + 2L + 2sv_b \right) - v_b, \quad (4.9)$$

$$c = \frac{2W}{3n} + sv_b. \quad (4.10)$$

Two conditions should be satisfied: $C > 0$ and $b^2 - 4ac \geq 0$. Obviously $c > 0$; if $a > 0$, then $b > 0$, and both solutions of $C < 0$. When $a < 0$, only one solution of $C > 0$, and the cycle time C is

$$C = \frac{-b - (b^2 - 4ac)^{1/2}}{2a}. \quad (4.11)$$

Since $a < 0$ the following condition should be satisfied:

$$n > \frac{1}{2} \left\{ \lambda s + \frac{[(\lambda s v_b)^2 + 2\lambda W v_b / 3]^{1/2}}{v_b} \right\}. \quad (4.12)$$

However, a closed-form expression for C is not easy to derive. Let k represent the average number of customers for a cycle time. Assume $k/(k+1)=1$ which is true when $k \rightarrow \infty$. According to Quadrifoglio and Li (2008) we obtain a closed-form expression for the approximate cycle time, \tilde{C} , for each zone with demand λ/n and width W/n :

$$\tilde{C} = \frac{sv_b + \frac{2W}{3n} + 2L}{v_b - \frac{\lambda}{n} \left(\frac{W}{6n} + sv_b \right)}, \quad (4.13)$$

where n should satisfy Expression (12) to guarantee $\tilde{C} > 0$.

4.2.4 Optimal number of zones

FRT Policy

We substitute the computed variables in Equation (1) and obtain the FRT Total Cost $f(n)$ as

$$\begin{aligned}
 f(n) = & \frac{\lambda a_k}{4v_{wk}} \left(\frac{L}{N-1} + \frac{W}{n} \right) \\
 & + \lambda \left[\frac{L}{v_b} + (N-1)s \right] \left[\frac{1}{2} a_b + \left(1 - \frac{1}{2(N-1)} \right) a_w + I(a_w - a_b) \right] \\
 & + \lambda a_B \left(\frac{W}{2v_B} + \frac{n+1}{2} S \right) + nF_b, \tag{4.14}
 \end{aligned}$$

where $I = \frac{\alpha}{3} \left(\frac{1}{(N-1)^2} - 1 \right)$ if $a_w > a_b$, or $I = 0$ if $a_w \leq a_b$.

Although n is a discrete variable, we assume it is a continuous variable to derive the optimal n . The derivative and the second derivative of function $f(n)$ with respect to n are

$$\frac{df(n)}{dn} = -\frac{\lambda a_k W}{4v_{wk} n^2} + \frac{1}{2} \lambda a_B S + F_b, \tag{4.15}$$

$$\frac{d^2 f(n)}{dn^2} = \frac{\lambda a_k W}{2v_{wk} n^3} > 0. \quad (4.16)$$

Since $\frac{d^2 f(n)}{dn^2} > 0$, the FRT total cost $f(n)$ is a convex function for $n > 0$.

Thus, $f(n)$ has a global minimum when $\frac{df(n)}{dn} = 0$. The optimal n value is

$$n = \left[\frac{\lambda a_k W}{2v_{wk} (\lambda a_B S + 2F_b)} \right]^{1/2}. \quad (4.17)$$

If optimal n is not an integer, the optimal integer number of zones is, because of convexity, either $\lceil n \rceil$ or $\lfloor n \rfloor$, whichever has the minimum total cost.

DRC Policy

We substitute the computed variables in Equation (2) and obtain the analytical rigorous DRC total cost $r(n)$ and its derivative as

$$r(n) = \lambda \left[\alpha a_w^h + (1-\alpha) \frac{a_w}{2} + \frac{a_v}{2} \right] C + \lambda a_B \left(\frac{W}{2v_B} + \frac{n+1}{2} S \right) + nF_v, \quad (4.18)$$

$$\begin{aligned} \frac{dr(n)}{dn} = & \frac{1}{2} \lambda a_B S + F_v + \lambda \left[\alpha a_w^h + (1-\alpha) \frac{a_w}{2} + \frac{a_v}{2} \right] \times \\ & \times \frac{\lambda \left[\frac{W}{n} \left(\frac{W}{2n} + 2sv_b \right) - v_b \right] C^2 + \lambda \left[\frac{5W}{3n} + 2(L + sv_b) \right] C + \frac{2W}{3}}{(2aC + b)n^2}, \end{aligned} \quad (4.19)$$

where C , a , b are obtained from Equations (8) to (11).

In Equation (18) we substitute C with the approximation \tilde{C} from Equation (13) and we obtain the approximate analytical DRC total cost $p(n)$ and its derivative as

$$p(n) = \lambda \left[\alpha a_w^h + (1-\alpha) \frac{a_w}{2} + \frac{a_v}{2} \right] \left[\frac{sv_b + \frac{2W}{3n} + 2L}{v_b - \frac{\lambda}{n} \left(\frac{W}{6n} + sv_b \right)} \right] + \lambda a_B \left(\frac{W}{2v_B} + \frac{n+1}{2} S \right) + nF_v, \quad (4.20)$$

$$\frac{dp(n)}{dn} = \frac{1}{2} \lambda a_B S + F_v - \lambda \left[\alpha a_w^h + (1-\alpha) \frac{a_w}{2} + \frac{a_v}{2} \right] \times \frac{n^2 v_b (\lambda s^2 v_b + 2W/3 + 2\lambda Ls) + n\lambda W (sv_b + 2L)/3 + \lambda W^2/9}{\left[n^2 v_b - \lambda (W/6 + nsv_b) \right]^2}. \quad (4.21)$$

Because of their convexity, when $\frac{dr(n)}{dn} = 0$ or $\frac{dp(n)}{dn} = 0$ the rigorous DRC total

cost or the approximated DRC total cost have global minimum values. The corresponding optimal n has no closed-form expression, but it is possible to obtain a numerical solution and derive the optimal integer n as for the FRT policy.

4.3 Simulation development

We developed a simulation model to validate the DRC analytical modeling results. The simulation replicates the operations of the insertion heuristic algorithm

described below, which is a widely used scheduling algorithm for demand responsive services.

Let $P_1, P_2, P_3, \dots, P_m$, denote m customers. The insertion algorithm creates the customer sequence choosing the minimum additional distance at each insertion step in an $O(m^2)$ fashion, as follows:

- (1) Insert P_1 : AP_1A is the only possible route.
- (2) Insert P_2 : Possible routes include AP_2P_1A , and AP_1P_2A ; Find the route R_2 with the minimum DRC running distance among the two possible routes. Suppose R_2 is Route AP_1P_2A .
- (3) Insert P_3 : Possible routes include $AP_3P_1P_2A$, $AP_1P_3P_2A$, and $AP_1P_2P_3A$; Find the route R_3 with the minimum DRC running distance among the three possible routes.
- (4) ...
- (m) Insert P_m : Suppose the route R_{m-1} is generated by inserting P_{m-1} ; Insert P_m to the route R_{m-1} ; Find the route R_m with the minimum DRC running distance among the m possible routes.

If we were to consider the insertion heuristic, the analytical derivation of the terms of the DRC total cost function described in the previous section would be very difficult to perform because of the embedded vehicle routing problem. Therefore, the analytical modeling of DRC assumes that vehicles follow a non-backtracking policy (vehicles are not allowed to backtrack with respect to their primary forward direction to

serve customers), which is a good approximation of the above insertion heuristic, especially for long and narrow service areas (Quadrioglio et al., 2006).

4.4 Computational experiment

In this section, we perform computational experiment to test our analytical modeling. Three cases are analyzed. We assume the parameter values are as follows and as those listed in Table 4:

1. a relatively large service area ($L=2$ miles, $W=6$ miles), with demand λ of 80 customers/hr (density 6.67 customers/hr/mile²), a relatively high walking cost of \$40/hr, a relatively high FRT bus cost of \$100/hr;
2. a relatively large service area ($L=2$ miles, $W=6$ miles), with relatively high demand λ of 200 customers/hr (density 16.67 customers/hr/mile²), a relatively low walking cost of \$20/hr, a relatively high FRT bus cost of \$50/hr;
3. a relatively small service area ($L=2$ miles, $W=2$ miles), with relatively low demand λ of 10 customers/hr (density 2.5 customers/hr/mile²), a relatively high walking cost of \$40/hr, a relatively high FRT bus cost of \$100/hr.

Table 4
Parameter values for One-Vehicle Zone Case.

Case	Parameter	Value	Unit
Case 1	W	6	Miles
	λ	80	Customers/hr
	a_k	40	\$/customer/hr
	F_b	100	\$/veh/hr
Case 2	W	6	Miles
	λ	200	Customers/hr
	a_k	20	\$/customer/hr
	F_b	50	\$/veh/hr
Case 3	W	2	Miles
	λ	10	Customers/hr
	a_k	40	\$/customer/hr
	F_b	100	\$/veh/hr
Cases 1, 2, and 3	L	2	Miles
	α	0.6	
	a_w^h	10	\$/customer/hr
	a_w	20	\$/customer/hr
	a_v	10	\$/customer/hr
	a_b	10	\$/customer/hr
	a_B	10	\$/customer/hr
	F_v	100	\$/veh/hr
	v_{wk}	2	Miles/hr
	v_b	20	Miles/hr
	v_B	30	Miles/hr
	s	0.008333	Hrs
	S	0.025	Hrs
	d	0.25	Miles

4.4.1 Case 1

For the FRT policy we obtained $n = 4.7$ from Equation (4.17). By using Equation (4.14), the total cost (\$1766/hr) for $n = 5$ is less than the total cost (\$1782.7/hr) for $n = 4$. So the integer optimal number of zones for the FRT policy is 5.

For the DRC policy, $n > 2.4$ with Expression (4.12). By using the rigorous formulas, with Equation (4.19) we obtained $n = 4.6$ when $\frac{dr(n)}{dn} = 0$; with Equation (4.18), the total cost is \$1016.2/hr for $n = 4$ and is \$1003.6/hr for $n = 5$. So the optimal integer number of zones is 5.

By using the approximation formulas, with Equation (4.21), we obtained $n = 4.4$ when $\frac{dp(n)}{dn} = 0$; with Equation (4.20), the total cost is \$1061.7/hr for $n = 4$ and is \$1061.4/hr for $n = 5$. So the optimal integer number of zones is 5, the same as that with rigorous formulas.

The simulations show that the minimum total cost for the DRC policy with the insertion heuristic algorithm is \$1074.1/hr. The optimal number of zones is 5, which is the same as those from analytical rigorous and approximation formulas.

The total costs for various numbers of zones are shown in Fig. 10. We have the following observations:

- The minimum DRC total cost is less than that of the FRT policy, suggesting that the optimal configuration for this case would be a 5-zone DRC feeder policy.
- For the DRC policy, the total costs obtained from the approximation formulas, the rigorous formulas and the simulation are very close, validating the assumptions in our modeling approach.
- For the DRC policy, the total cost obtained from simulations is less than that from rigorous formulas when $n < 4$. This shows, as expected

(Quadrifoglio et. al, 2006), that the non-backtracking policy progressively worsens its effectiveness in approximating the insertion heuristic algorithm when the shape of the zone widens as a consequence of the reduction of the number of zones.

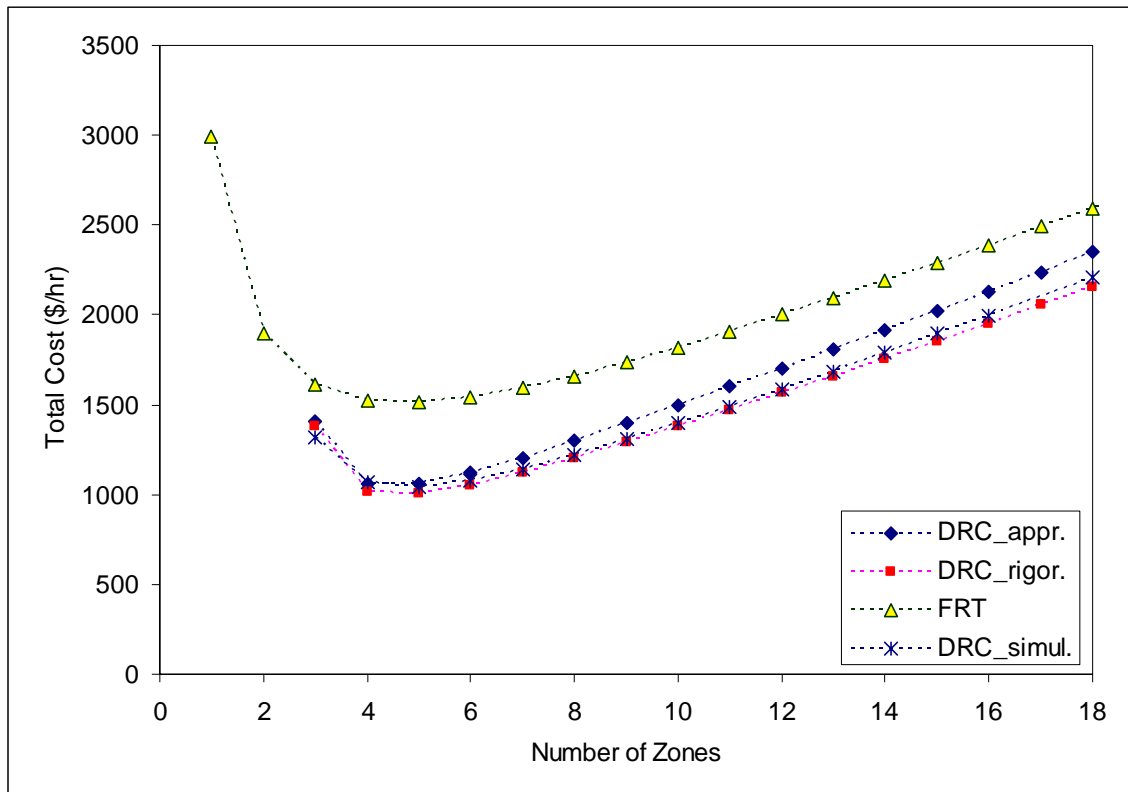


Fig. 10. Total cost functions for Case 1.

4.4.2 Case 2

This is a case with relatively high demand, low walking cost and low FRT bus cost. Table 4 shows the input parameter values to the model. Fig. 11 shows the values of total cost functions.

For the FRT policy, the minimum total cost is \$2026/hr for 6 zones. For the DRC policy, with the rigorous formulas, the minimum total cost is \$2225.5/hr for 8 zones. With the approximation formulas, the minimum total cost is \$2327/hr for 8 zones. From simulations the minimum total cost is \$2363/hr for 9 zones. As for case 1, the optimal number of zones obtained from the approximation formula is very close to those from rigorous formulas and simulations. In this case, the minimum cost of the

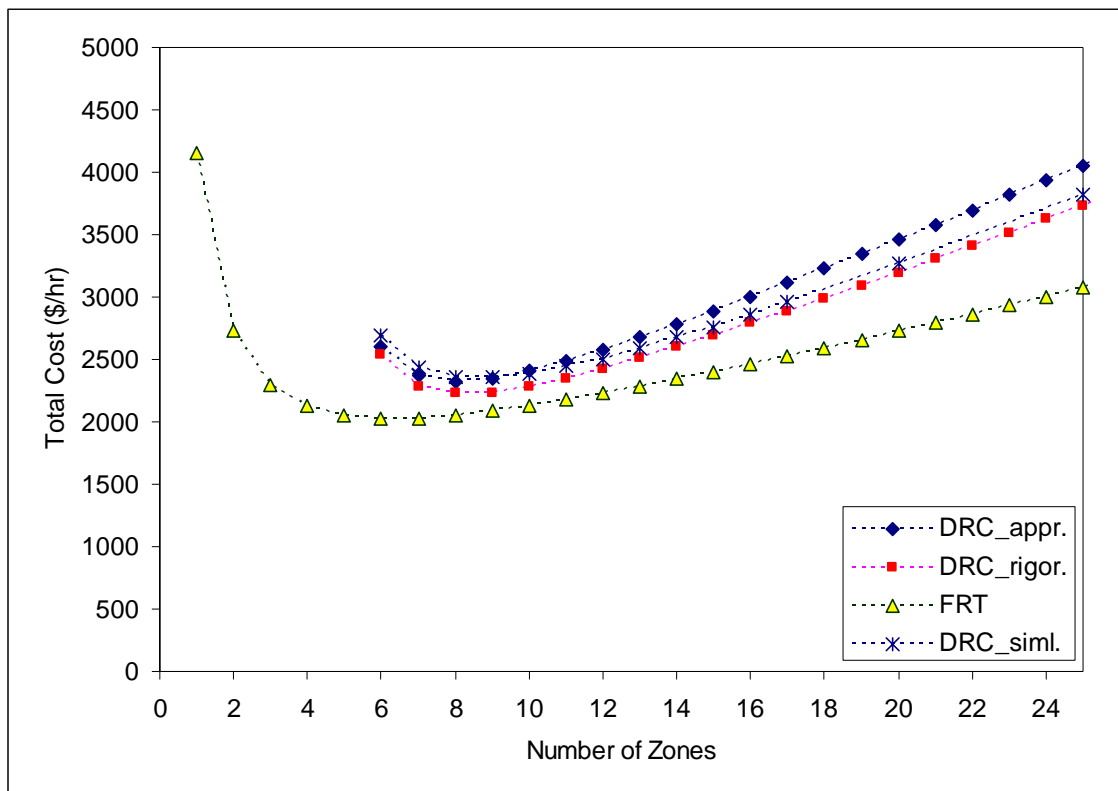


Fig. 11. Total cost functions for Case 2.

FRT is less than that of the DRC, suggesting that this configuration would require a 6-zone FRT feeder service.

4.4.3 Case 3

This is a case with a relatively small area. Table 4 shows the input parameter values. Fig. 12 shows the values of total cost functions. For the FRT policy, the minimum cost is \$255/hr for 1 zone. For the DRC policy, the minimum costs are \$161/hr and \$151/hr, respectively, for 1 zone, with the approximation and rigorous formulas. Simulations also show that one zone is optimal with the minimum cost of \$154/hr.

As for the previous two cases, the approximate optimal number of zones and the minimum total cost are very close to those from the rigorous formulas and the simulations for the DRC policy. Both service policies suggest a single zone optimal design, but with lower cost for the DRC policy.

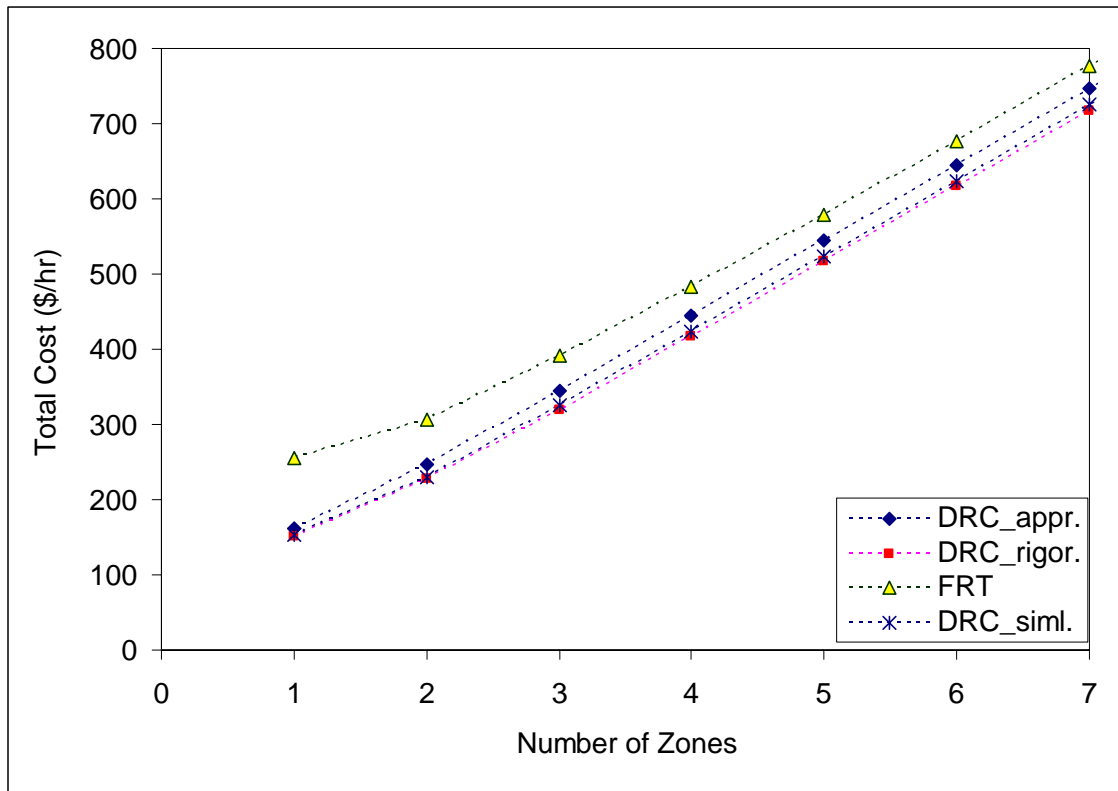


Fig. 12. Total cost functions for Case 3.

4.5 Further study for the Two-Vehicle Zone

In this section, we expand our previous research to address the optimal zone design problem faced by planners for feeder transit services with high demands and long length of service area, where a two-vehicle operation is adopted in each zone.

The system service area and demand are the same as described in section 4.1.1. Within each service zone shown in Fig. 9, a 2-vehicle FRT policy or a 2-vehicle DRC policy would be adopted to operate the feeder service. For each operating policy we consider only two vehicles moving at an average speed v_b miles/hr and stopping at each

station for a period of s . Within each zone, the 2-vehicle operation policies are the same as described in section 3.1.2.

4.5.1 Analytical model

We next describe the development of the analytical model needed to determine the optimal number of bus stations N for the FRT policy and the optimal number of zones n for both FRT and DRC policies.

4.5.1.1 Total cost function

Analogous to Equations (4.1) and (4.2), the total costs of the system for FRT policy and DRC policy are as follows.

FRT Total Cost = Customer Cost + FRT Bus Cost

$$\begin{aligned}
 &= n \frac{\lambda}{n} \alpha \left\{ a_k E(T_{wk}) + a_w E(T_{wt}^p) + a_b E(T_{rd}^p) + a_B E(T_{rd-B}^p) \right\} \\
 &+ n \frac{\lambda}{n} (1 - \alpha) \left\{ a_B E(T_{rd-B}^d) + a_w E(T_{wt}^d) + a_b E(T_{rd}^d) + a_k E(T_{wk}) \right\} + 2nF_b \quad (4.22)
 \end{aligned}$$

DRC Total Cost = Customer Cost + DRC Vehicle Cost

$$\begin{aligned}
 &= n \frac{\lambda}{n} \alpha \left\{ a_w^h E(T_{wt}^p) + a_v E(T_{rd}^p) + a_B E(T_{rd-B}^p) \right\} \\
 &+ n \frac{\lambda}{n} (1 - \alpha) \left\{ a_B E(T_{rd-B}^d) + a_w E(T_{wt}^d) + a_v E(T_{rd}^d) \right\} + 2nF_v \quad (4.23)
 \end{aligned}$$

For the FRT policy, the expressions of computed variables $E(T_{rd-B}^p)$, $E(T_{rd-B}^d)$, $E(T_{wk})$, $E(T_{wt}^p)$ and $E(T_{wt}^d)$ are the same as those for the 1-vehicle zone case, while average ride times $E(T_{rd}^p)$ and $E(T_{rd}^d)$ are computed with Equation (3.25).

For the DRC policy, C is the expected cycle time for each vehicle to serve both subzones of the whole service area. From section 3.2.2 we have that

$E(T_{wt}^p) = C/2 - L/(4v_b)$, $E(T_{wt}^d) = C/4$, and $E(T_{rd}^p) = E(T_{rd}^d) = C/4$, where C is obtained from Equation (4.11) with

$$a = \frac{\lambda}{4n} \left[\frac{\lambda}{n} \left(\frac{W}{6n} + sv_b \right) - 2v_b \right], \quad (4.24)$$

$$b = \frac{\lambda}{n} \left(\frac{5W}{6n} + \frac{3L}{2} + 2sv_b \right) - 2v_b, \quad (4.25)$$

$$c = 2L + \frac{8W}{3n} + 4sv_b. \quad (4.26)$$

Analogous to Equation (4.13), the closed-form expression for the approximate cycle time \tilde{C} is

$$\tilde{C} = \frac{2sv_b + \frac{4W}{3n} + 3L}{v_b - \frac{\lambda}{2n} \left(\frac{W}{6n} + sv_b \right)}. \quad (4.27)$$

4.5.1.2 Optimal number of zones

FRT Policy

Substitute the computed variables in Equation (4.22) and obtain the FRT Total

Cost $f(n, N)$ as

$$\begin{aligned}
 f(n, N) &= \frac{\lambda a_k}{4v_{wk}} \left(\frac{L}{N-1} + \frac{W}{n} \right) \\
 &+ \frac{\lambda a_w}{2} \left\{ \left[1 - \frac{1}{2(N-1)} \right] \frac{L}{v_b} + \left(N - \frac{3}{2} \right) s_f \right\} + \frac{\lambda a_b}{2} \left[\frac{L}{v_b} + (N-1) s_f \right] \\
 &+ \lambda a_B \left(\frac{W}{2v_B} + \frac{n+1}{2} S \right) + 2nF_b. \tag{4.28}
 \end{aligned}$$

Assume n and N are continuous variables, and then the partial derivative and the second order partial derivative of function $f(n, N)$ are

$$\frac{\partial f(n, N)}{\partial n} = -\frac{\lambda a_k W}{4v_{wk} n^2} + \frac{1}{2} \lambda a_B S + 2F_b, \tag{4.29}$$

$$\frac{\partial^2 f(n, N)}{\partial n^2} = \frac{\lambda a_k W}{2v_{wk} n^3} > 0. \tag{4.30}$$

$$\frac{\partial f(n, N)}{\partial N} = -\frac{\lambda a_k L}{4v_{wk} (N-1)^2} + \frac{\lambda a_w}{2} \left[\frac{L}{2v_b (N-1)^2} + s_f \right] + \frac{\lambda a_b s_f}{2}, \tag{4.31}$$

$$\frac{\partial^2 f(n, N)}{\partial N^2} = \frac{\lambda L}{2(N-1)^3} \left[\frac{a_k}{v_{wk}} - \frac{a_w}{v_b} \right] > 0. \tag{4.32}$$

Since $\frac{\partial^2 f(n, N)}{\partial n^2} > 0$ and $\frac{\partial^2 f(n, N)}{\partial N^2} > 0$, the FRT total cost $f(n, N)$ is a convex

function, and has a global minimum. When $\frac{\partial f(n, N)}{\partial n} = 0$, the optimal n value is

$$n = \left[\frac{\lambda a_k W}{2v_{wk} (\lambda a_b S + 4F_b)} \right]^{1/2}. \quad (4.33)$$

When $\frac{\partial f(n, N)}{\partial N} = 0$, the optimal N value is

$$N = 1 + \left[\frac{L}{2s_f (a_b + a_w)} \left(\frac{a_k}{v_{wk}} - \frac{a_w}{v_b} \right) \right]^{1/2}. \quad (4.34)$$

The optimal integer number of zones and the optimal integer number of bus stations are $\lceil n \rceil$ or $\lfloor n \rfloor$, and $\lceil N \rceil$ or $\lfloor N \rfloor$, whichever has the minimum total cost.

DRC Policy

Substitute the computed variables in Equation (4.23) and obtain the analytical rigorous DRC total cost $r(n)$ and its derivative as

$$r(n) = \frac{\lambda}{2} \left[\alpha a_w^h + (1-\alpha) \frac{a_w}{2} + \frac{a_v}{2} \right] C + \lambda \left[a_B \left(\frac{W}{2v_B} + \frac{n+1}{2} S \right) - \frac{\alpha a_w^h L}{4v_b} \right] + 2nF_v, \quad (4.35)$$

$$\begin{aligned} \frac{dr(n)}{dn} &= \frac{1}{2} \lambda a_B S + 2F_v + \frac{\lambda}{2} \left[\alpha a_w^h + (1-\alpha) \frac{a_w}{2} + \frac{a_v}{2} \right] \times \\ &\times \frac{\frac{\lambda}{2} \left[\frac{\lambda}{n} \left(\frac{W}{4n} + sv_b \right) - v_b \right] C^2 + \lambda \left[\frac{5W}{3n} + \frac{3L}{2} + 2sv_b \right] C + \frac{8W}{3}}{(2aC+b)n^2}, \end{aligned} \quad (4.36)$$

where C, a, b are obtained from Equations (4.24) to (4.26).

In Equation (4.35) we substitute C with the approximation \tilde{C} from Equation (4.27) and we obtain the approximate analytical DRC total cost $p(n)$ and its derivative as

$$\begin{aligned} p(n) &= \frac{\lambda}{2} \left[\alpha a_w^h + (1-\alpha) \frac{a_w}{2} + \frac{a_v}{2} \right] \left[\frac{2sv_b + \frac{4W}{3n} + 3L}{v_b - \frac{\lambda}{2n} \left(\frac{W}{6n} + sv_b \right)} \right] \\ &+ \lambda \left[a_B \left(\frac{W}{2v_B} + \frac{n+1}{2} S \right) - \frac{\alpha a_w^h L}{4v_b} \right] + 2nF_v, \end{aligned} \quad (4.37)$$

$$\begin{aligned} \frac{dp(n)}{dn} &= \frac{1}{2} \lambda a_B S + 2F_v - \frac{\lambda}{2} \left[\alpha a_w^h + (1-\alpha) \frac{a_w}{2} + \frac{a_v}{2} \right] \times \\ &\times \frac{n^2 v_b (\lambda s^2 v_b + 4W/3 + 3\lambda Ls/2) + n\lambda W (sv_b/3 + L/2) + \lambda W^2/9}{[n^2 v_b - (\lambda/2)(W/6 + nsv_b)]^2}. \end{aligned} \quad (4.38)$$

Because of their convexity, when $\frac{dr(n)}{dn} = 0$ or $\frac{dp(n)}{dn} = 0$ the rigorous DRC total

cost or the approximated DRC total cost have global minimum values.

4.5.2 Computational experiment

As the 1-vehicle zone operation, we developed a simulation model of the 2-vehicle zone operation to validate the DRC analytical modeling results. The simulation replicates the operations of the insertion heuristic algorithm attempting to minimize the vehicle's total travel distance in each cycle. We assume the parameter values are as those listed in Table 5.

Table 5
Parameter values for the Two-Vehicle Zone Case.

Parameter	Value	Unit
W	6	Miles
λ	200	Customers/hr
a_k	30	\$/customer/hr
F_b	100	\$/veh/hr
L	4	Miles
α	0.5	
a_w^h	10	\$/customer/hr
a_w	20	\$/customer/hr
a_v	10	\$/customer/hr
a_b	10	\$/customer/hr
a_B	10	\$/customer/hr
F_v	100	\$/veh/hr
v_{wk}	2	Miles/hr
v_b	20	Miles/hr
v_B	30	Miles/hr
s	0.008333	Hrs
S	0.025	Hrs

For the FRT policy we obtain $n = 4.5$ from Equation (4.33), and $N=11.6$ from Equation (4.34). By using Equation (4.28), the total cost (\$3371.2/hr) for $n = 4$ and $N=12$ is the smallest among total costs for $n = 4$ or 5 and $N=11$ or 12. So the integer optimal number of zones is 5 and the integer optimal number of bus stations is 12. The corresponding distance of adjacent stations is $4/(12-1) = 0.364$ miles = 1,920 ft, which is within the range [600÷2,500 ft] adopted by transit agencies in suburban areas (Texas Transportation Institute, 1996). Fig. 13 shows total FRT costs for various n and N values. The total cost is sensitive to the number of zones.

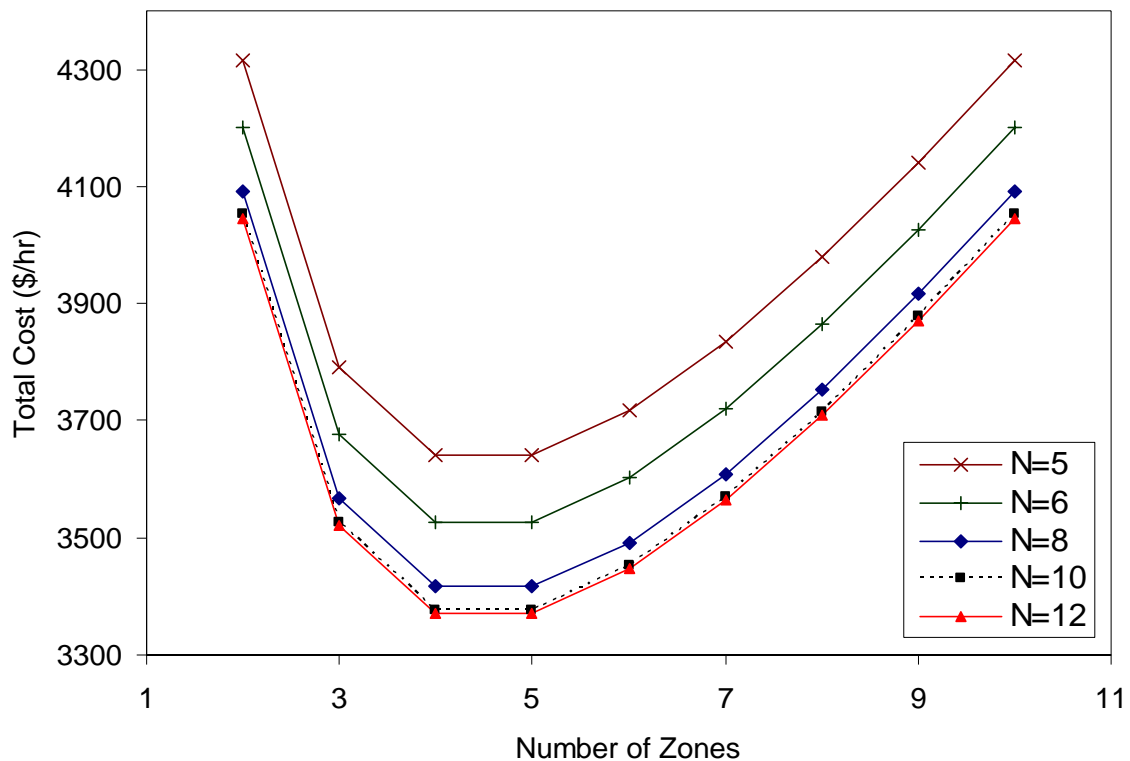


Fig. 13. FRT total costs for various n and N values.

For the DRC policy, we use the rigorous formulas, Equation (4.36), to obtain

$n = 5.6$ when $\frac{dr(n)}{dn} = 0$; with Equation (4.35), the total cost is \$2867.2/hr for $n = 5$ and is \$2844.5/hr for $n = 6$. So the integer optimal number of zones is 6.

By using the approximation formulas, with Equation (4.38), we obtain $n = 5.5$

when $\frac{dp(n)}{dn} = 0$; with Equation (4.37), the total cost is \$2950/hr for $n = 5$ and is \$2944.2/hr for $n = 6$. So the integer optimal number of zones is 6, the same as that with rigorous formulas.

The simulations show that the minimum total cost for the DRC policy with the insertion heuristic algorithm is \$3060.7/hr. The optimal number of zones is 6, which is the same as those from analytical rigorous and approximation formulas.

The total costs for various numbers of zones are shown in Fig. 14. We have the following observations:

- The minimum DRC total cost is less than that of the FRT policy, suggesting that the optimal configuration for this case would be a 6-zone DRC feeder policy.
- For the DRC policy, the total costs obtained from the approximation formulas, the rigorous formulas and the simulation are very close, validating the assumptions in our modeling approach.
- For the DRC policy, the total cost obtained from simulations is slightly larger than that from rigorous formulas when $n < 9$. As expected (Quadrifoglio and Li, 2009), this is caused by the existing correlations between the vehicles'

operational cycles, which are not captured by our two-vehicle modeling, in which we assume independently.

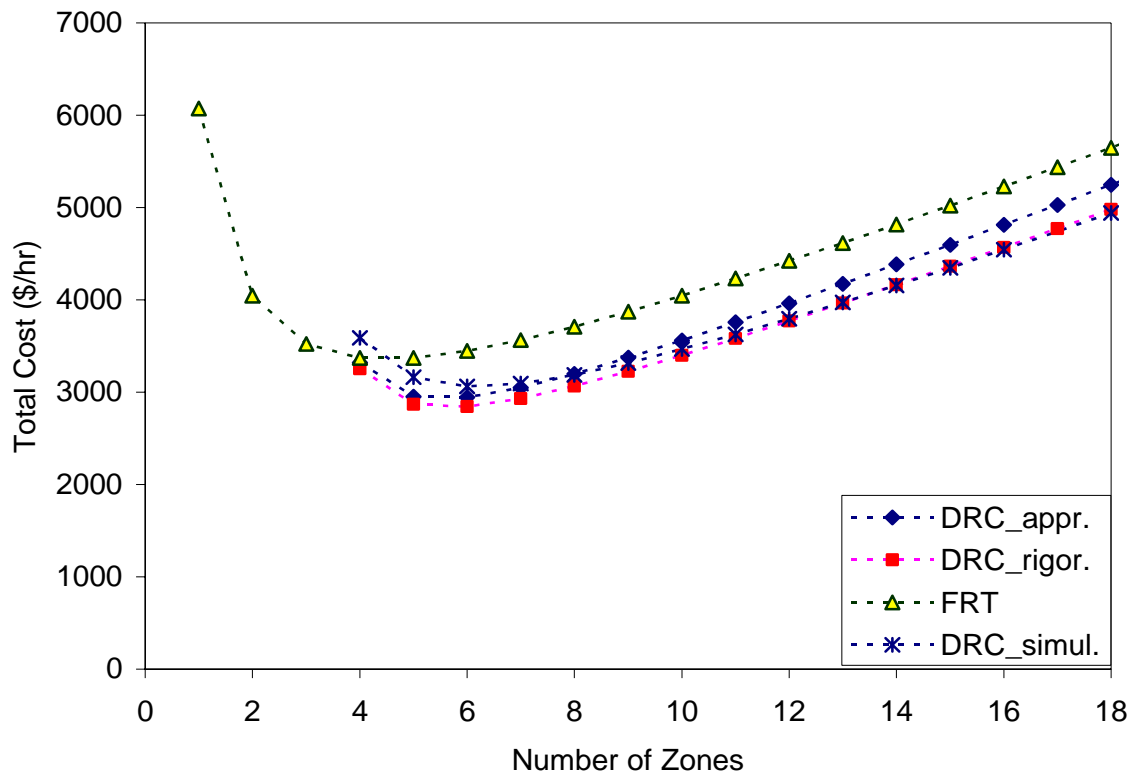


Fig. 14. Total cost functions for FRT and DRC policies.

4.6 Summary

In this research, we address the problem of the optimal number of zones to divide a service area, faced by planners in designing feeder transit services. We have developed an analytical model representing the total cost functions balancing customer service quality and vehicle operating cost. By analytical derivation, we obtain closed-form expressions for the FRT and approximation formulas for the DRC to

determine the optimal number of zones. For the DRC, simulations are used to validate the results of the analytical formulas. All our case studies show that the optimal number of zones and the total cost obtained from our approximation formulas are very close to those obtained from simulations.

Our analytical formulation leads to a strictly convex optimization problem to minimize the cost by controlling the number of zones. This formulation provides evidence of the existence and uniqueness of the problem solution.

Limitations of results come from our simplified system configuration model. Our formulation might only be useful for service areas close to a rectangle and service area with a uniform land-use pattern, which are, however, the majority of residential housing areas. A practical implementation of the partition of the whole service area in an optimal number of zones might be affected by possible street network constraints. However, in a planning/design phase of a new residential area, this can be taken into consideration before constructing the road network as well. In addition, our modeling assumes rectilinear movements of the vehicles among demand points which might not be realistic within some of the residential service areas with complex road network and land use patterns.

CHAPTER V

APPLICATIONS

In this chapter we apply the models developed in the previous chapters to El Cenizo, Texas. El Cenizo is one of the colonias, which are unincorporated settlements along the U.S. – Mexico border. Because the residents' income is relatively low and there is no adequate public transportation, the residents' needs for transportation services are not properly met. With the travel demand data collected in El Cenizo, we design a feeder transit service as a real case application of our approaches.

5.1 Travel demand

Quadrifoglio et. al (2009) conducted a survey on the travel demand characteristics in El Cenizo. They found that approximately one-fourth of the households do not own any vehicle, and nearly half (47%) of them have only one vehicle. In contrast to the national average 2.5, the household size is 4.25 in El Cenizo. However, each household has only 1.13 vehicles on an average.

From the survey, Quadrifoglio et. al (2009) found that the number of trips is approximately 1250 per day in El Cenizo, whose area is about 0.5 mile². The distribution of trips during a day is shown in Fig. 15. The morning peak hours are from 6am to 8am during which 99% of trips leave home. The afternoon peak hours are from 2pm to 6pm during which 98% of trips return home.

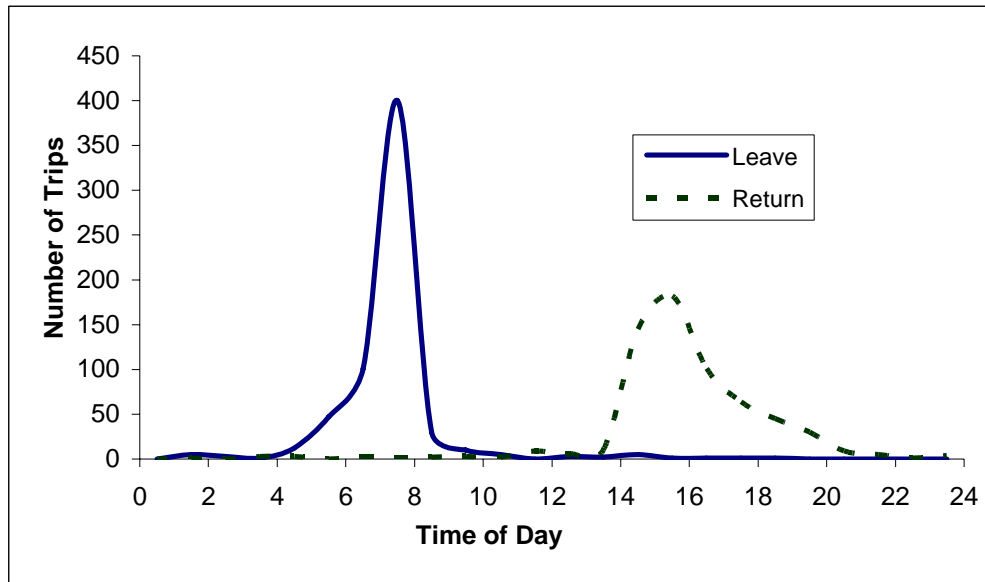


Fig. 15. Distributions of trips during a day.

Quadrifoglio et. al (2009) also found that approximately 65% of the respondents are definitely willing to use the proposed demand-responsive transit service. Further, safety is the most essential factor in using the new service. So in the following design, we assume the transit service takes 65% of the travel demand except for school trips. Since school buses take about 46% of school trips, we assume transit service takes $[(1-46\%) \times 65\%] = 35\%$ of school trips.

5.2 Modeling approaches

We design feeder transit services in El Cenizo by utilizing the analytical and simulation approaches described in Chapters III and IV to determine the optimal number of service zones and the critical demands to switch between FRT and DRC services. For

these approaches we do not consider the road network and assume rectilinear travel distances.

In this section we further describe a simulation approach considering the road network in El Cenizo. Fig. 16 shows the straight line representations of roads in El Cenizo. The terminal is located in the middle of the right edge of the service area (see Fig. 16).

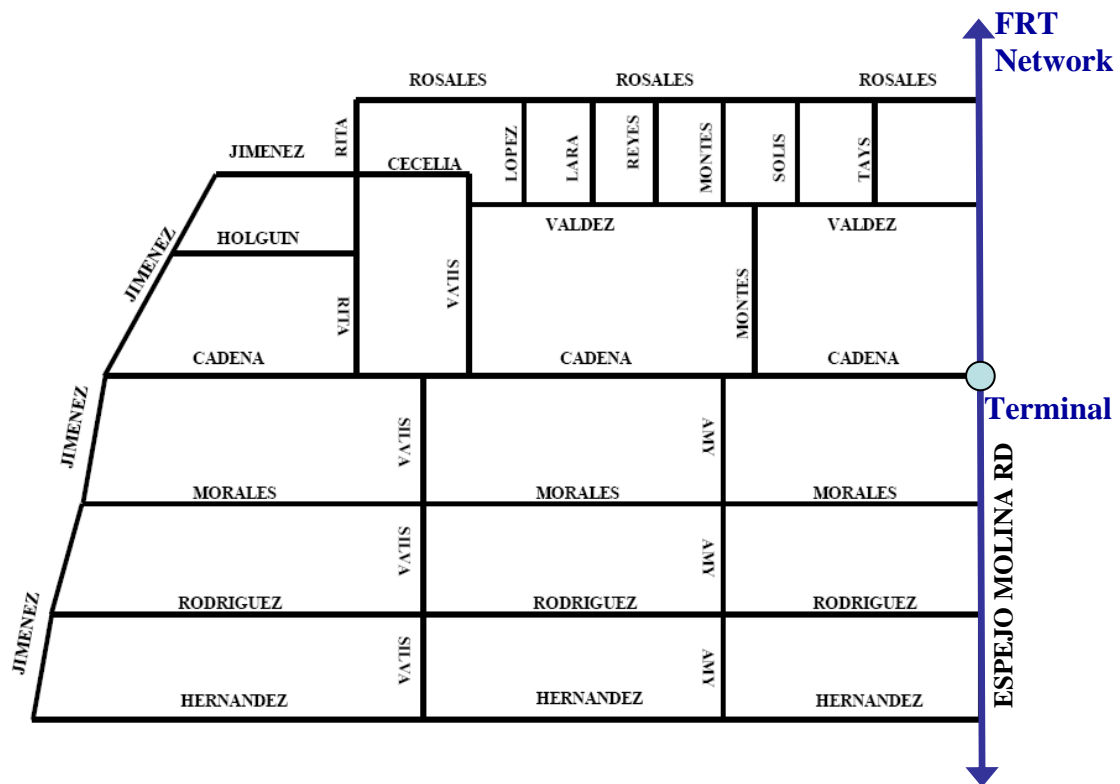


Fig. 16. Straight line representations of roads in El Cenizo.

This simulation model for the DRC operation is the same as the one described in Chapters III and IV, except for the following assumptions:

- We assume the DRC vehicle travels only on the road network and customers walk to the nearest roads to ride the DRC vehicle.
- The insertion algorithm is the same as the one described in Section 4.3 except for the calculation of travel distances. In Section 4.3, we do not consider the road network; when inserting a customer, we use rectilinear distances from this customer to adjacent customers. But in this simulation model, we use the Dijkstra's shortest-path algorithm to compute these travel distances on the road network. The detail of the Dijkstra's algorithm is in Cormen et al. (2001).

We developed two simulation models (one assumes rectilinear movements without a road network, and one assumes the vehicle moves on the road network). The comparison of simulation results with these two types of models is described in the following section.

5.3 Results

5.3.1 Optimal zone design

We approximate the service area as a rectangle. The parameter values are listed in Table 6. Then we obtain the optimal number of zones with the approach described in Chapter IV.

Table 6
Parameter values for El Cenizo.

Parameter	Value	Unit
L	0.85	Miles
W	0.5	Miles
λ	changeable	
α	0.99 (morning peak hours); 0.02 (afternoon peak hours)	
a_w^h	10	\$/customer/hr
a_w	10	\$/customer/hr
a_v	20	\$/customer/hr
a_b	20	\$/customer/hr
a_B	20	\$/customer/hr
a_k	30	\$/customer/hr
F_v	100	\$/veh/hr
F_b	100	\$/veh/hr
v_{wk}	2	Miles/hr
v_b	20	Miles/hr
v_B	30	Miles/hr
s	0.008333	Hrs
S	0.025	Hrs
d	0.25	Miles

For morning hours, we take α as 0.99, obtained from the survey. Since the maximum demand to ride this system is about 165 customers/hour, we change the demand rate λ from 20 to 170. For FRT policy, the optimal number of zones is 1 when $\lambda < 150$. When $150 \leq \lambda \leq 170$, the total costs for two-zone design are less than, but close to, the costs for one-zone design. Considering the additional cost of bus-station construction, the one-zone design is the best for FRT policy. For DRC policy, when $\lambda > 50$, the optimal numbers of zones are greater than 1, but the minimum total costs are greater than the cost of one-zone FRT policy.

For afternoon hours, we take α as 0.02. The maximum demand to ride this system is about 70 customers/hour, so we change the demand rate λ from 20 to 70. The optimal number of zones is 1 for the FRT policy. For the DRC policy, the optimal number of zones is 1 when $\lambda < 50$. When $50 \leq \lambda \leq 70$, the optimal number of zones is 2, but the minimum total cost is close to the cost of one-zone FRT policy. From the analysis above, we suggest the one-zone design.

5.3.2 Critical demands

With the approach described in Chapter III, we can obtain the critical demand for the switch between the FRT and DRC policies. Following the parameter values in Table 6, we have $w_{wk}=3$, $w_{wt}=1$, and $w_{rd}=2$. For morning hours we have $\alpha=0.99$, and then the approximate critical demand is 38.2 customers/hour, while the analytical and simulated values are 41.5 and 41.9 customers/hour respectively. These values are shown in Fig. 17 as the cross points of utility functions of the FRT and DRC policies.

For afternoon hours we have $\alpha=0.02$, and then the approximate critical demand is 48.4 customers/hour, while the analytical and simulated values are 50.1 and 51.6 customers/hour respectively. These values are shown in Fig. 18 as the cross points of utility functions.

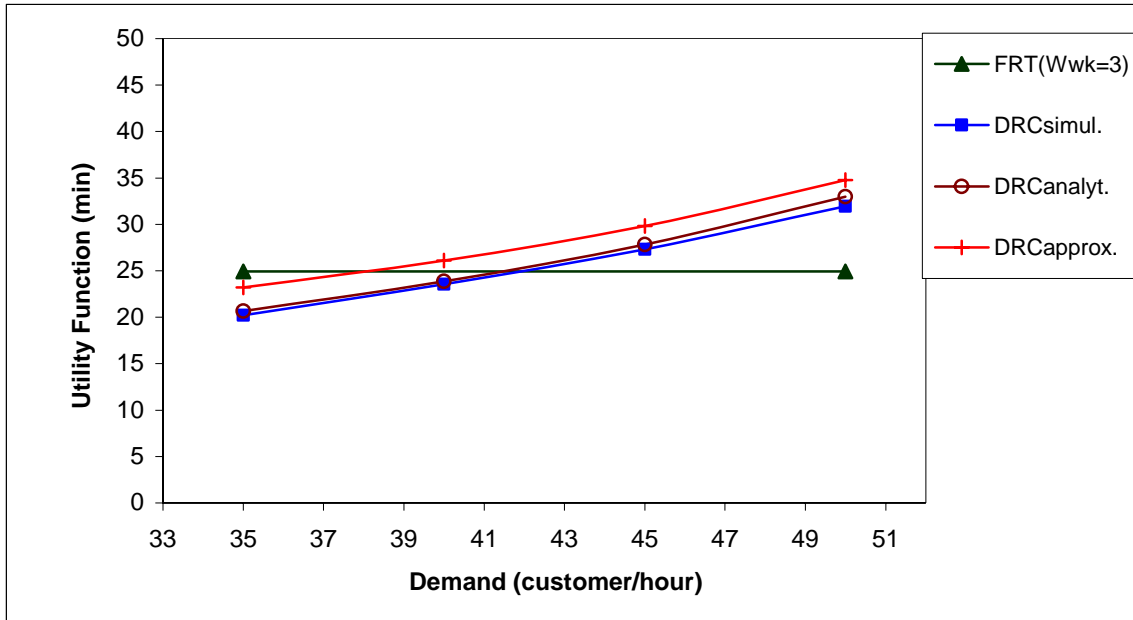


Fig. 17. Critical demands for morning hours.

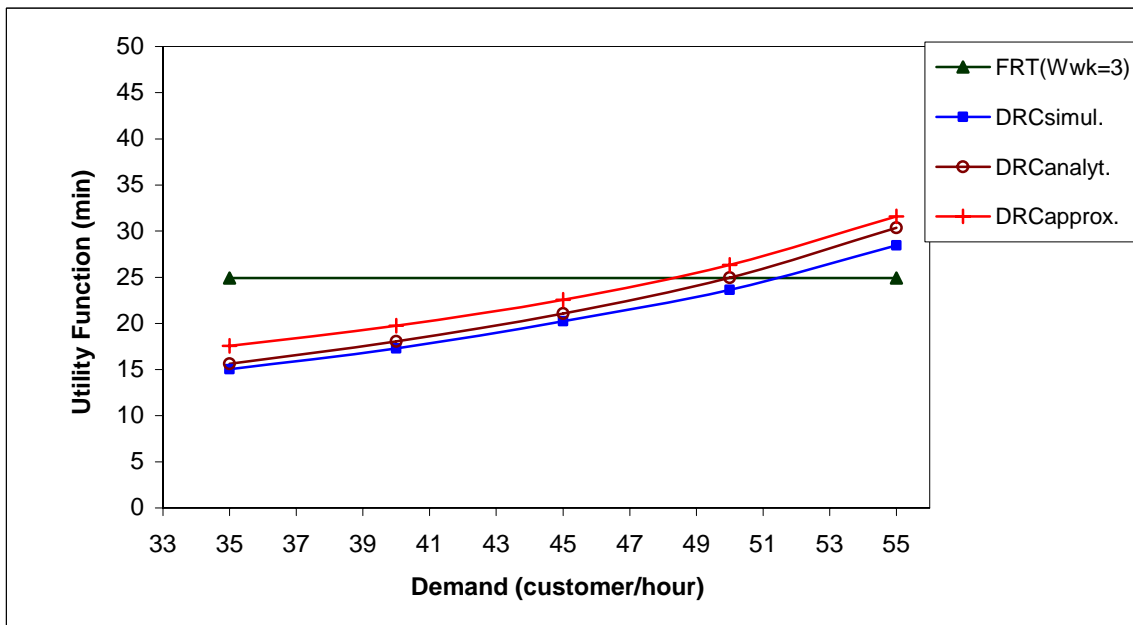


Fig. 18. Critical demands for afternoon hours.

Further, considering the road network, we use the simulation model to produce DRC utility functions. Fig. 19 and Fig. 20 show utility functions produced from simulation models with and without the road network. From these two figures we have the following observations:

- The values of utility functions with the road network are larger than those without the road network. This is reasonable because vehicular travel distance on the network is usually longer than the distance without the constraint of road network.
- When customer demand increases, the difference of utility function values with and without the road network increases too. This trend exists for different ratios of pick-up and drop-off customers; for instance, there are many more pick-up customers in morning hours and many more drop-off customers in afternoon hours.
- When considering the road network, the critical demands are 32 customers/hr in morning hours and 40 customers/hr in afternoon hours. These values are 20% to 24% less than those obtained without considering the road network.
- When customer demand is low, utility function values with and without the road network are close. Since the DRC is suitable for low demand, the rectilinear movement is a good approximation of vehicular movement on the road network for the case of El Cenizo. Dessouky et al. (2005) also

showed that a rectilinear movement of the vehicle is a good approximation of the reality.

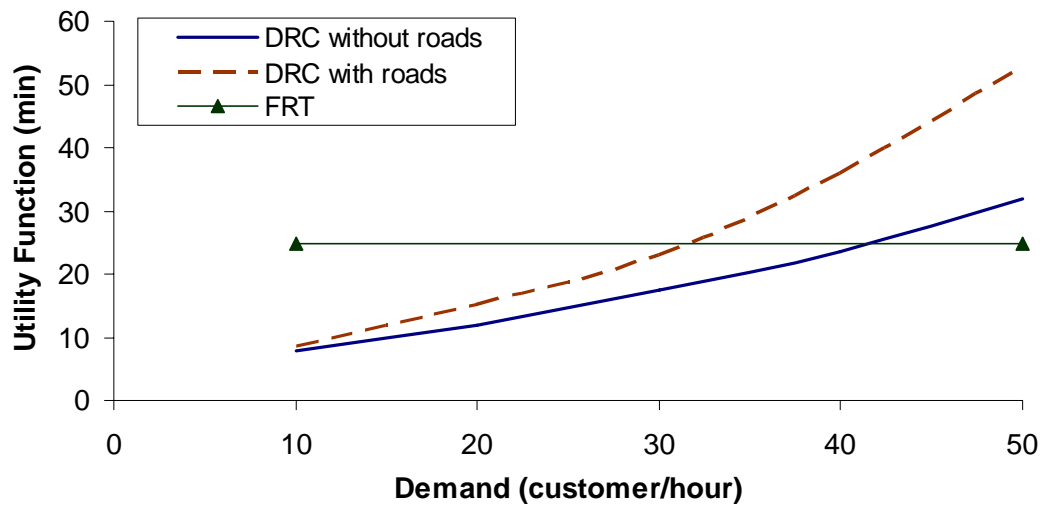


Fig. 19. Simulated utility functions of DRC for morning hours.

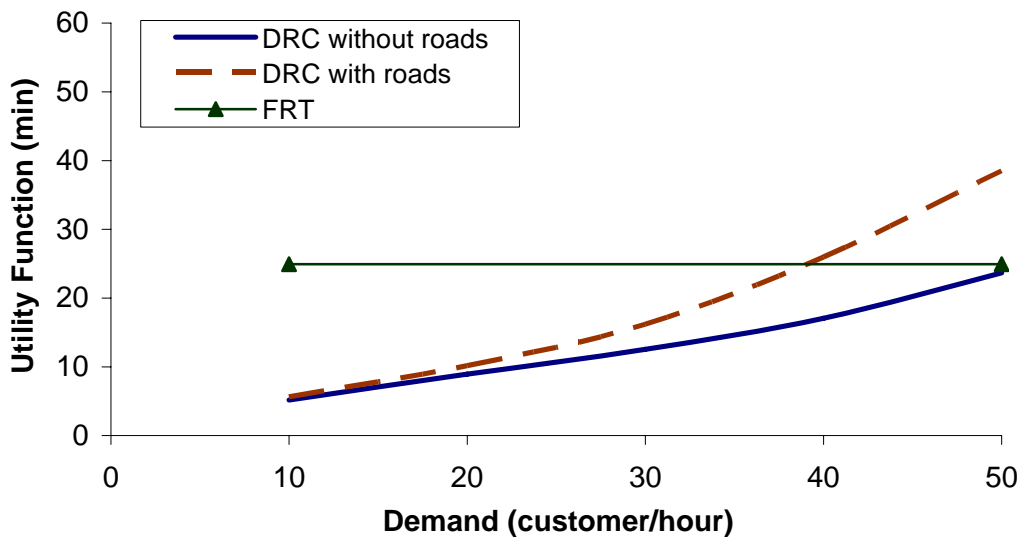


Fig. 20. Simulated utility functions of DRC for afternoon hours.

In Fig. 21, we compare the collected demands and the critical demands. The collected demands are greater than the critical values from 6:00am to 8:30am and from 2:00pm to 4:30pm, showing the FRT policy has better performance. In the other time periods during a day, the DRC policy provides better service. Without considering the road network, the estimated time periods are close to those estimated with the network.

Note that from 2:00pm to 4:30pm the collected demands are slightly greater than the critical demand without the road network. From 30 simulation replications, the resulting 95% confidence half-intervals are about 0.7% of the mean for utility function values. In this time period the FRT has better performance than the DRC, as shown in Fig. 21 that the collected demands are much greater than the critical demand with the road network.

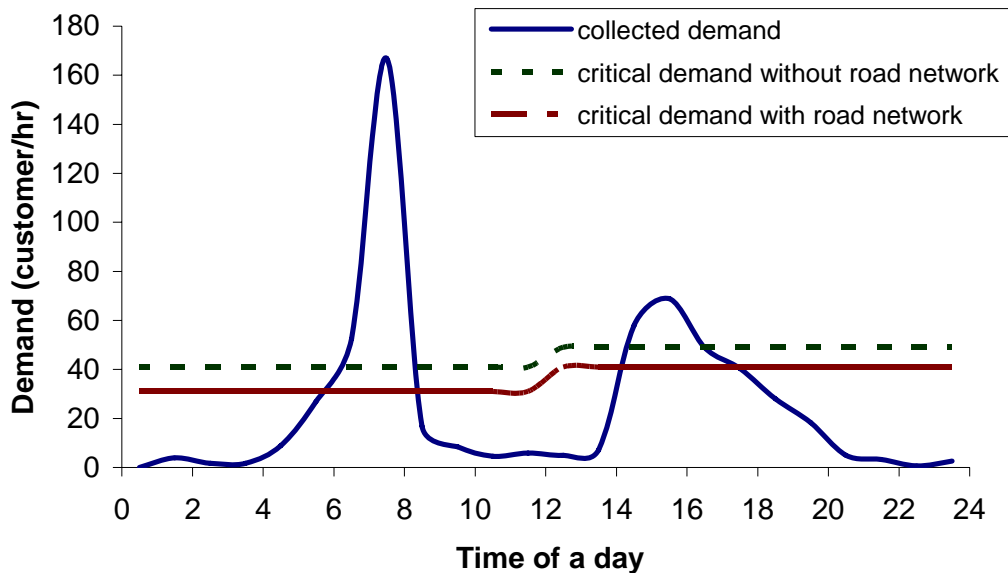


Fig. 21. Comparison of collected demands and critical demands.

5.4. Summary

In this chapter we develop a case study of feeder transit design for El Cenizo, Texas. With the approaches described in Chapters III and IV, we use the collected demand data to determine the optimal number of zones and the critical demand to switch between the FRT and DRC policies. Our study shows that one-zone service is the best for the current demands; from 6:00am to 8:30am and from 2:00pm to 4:30pm, the FRT policy would provide a better service while the DRC policy would be superior in the other time periods. We also expect that the DRC would provide better service on weekends since the demands would be smaller.

Since the road network is not considered in the approaches in Chapters III and IV, we further develop a simulation model that the vehicle only travels on the road network. Comparisons of simulation results show that critical demands are overestimated by about 25% to 30% without considering the road network for the case of El Cenizo, but the estimated times of the day to switch FRT and DRC policies are close whether we consider the road network or not. Further, when customer demand is low, utility function values with and without the road network are close. This shows that the rectilinear movement is an acceptable approximation of vehicular movement on the road network for the DRC operation.

CHAPTER VI

CONCLUSIONS AND FUTURE RESEARCH

Proper design and operations of feeder transit services within the modern sprawled residential areas are becoming increasingly more important to enhance the performance of the public transportation system network. In this research we analyzed the demand responsive feeder transit, also known as Demand Responsive Connector (DRC), which is one of the most used types of flexible transit services combining pure demand responsive and fixed-route features.

Feeder transit services are generally operated with a demand responsive policy which might be converted to a traditional fixed route policy for higher demand. We investigated the conditions that would justify the switch between the two policies. By employing continuous approximations, we provided an analytical modeling framework of the decision problem to help planners and operators in their choice. We compared the utility functions of the competing operating policies to identify the critical demand densities. They represent the switching conditions, that are functions of the parameters of each considered scenario, such as the geometry of the service area, the vehicle speed and also the weights assigned to each term contributing to the utility function: walking time, waiting time and riding time. The derived rigorous analytical values and approximate closed-form expressions of the critical demand density are validated by simulation for a range of plausible scenarios.

In designing feeder transit systems for large communities, planners may divide the whole service area into zones. A non-optimal structure is often adopted, and sometimes there is a lack of zone design. We developed analytical models representing the total cost functions balancing customer service quality and vehicle operating cost. By analytical derivation, we obtained closed-form expressions for the FRT and approximation formulas for the DRC to determine the optimal number of zones. Our analytical formulation leads to a strictly convex optimization problem to minimize the cost by controlling the number of zones.

Finally we developed a real-case application with collected customer demand data and road network data of El Cenizo, TX. With our analytical formulas, we obtained the optimal number of zones, and the times for switching FRT and DRC policies during a day. We also found that analytical formulas without road networks overestimate the critical demands by about 25% to 30% compared with the simulation results with the road network of El Cenizo. But the estimated times of a day to switch FRT and DRC policies are close whether we consider the road network or not. This demonstrates that our analytical formulas provide acceptable estimates for practical use.

This research suggests and encourages transit planners and operators to make use of this methodological approach in selecting the optimal number of zones and correct operating policy for feeders within modern sprawled urban and suburban areas. The use of our handy but powerful closed-form analytical expressions to estimate the optimal number of zones and the critical demand density may not be limited to urban residential

areas. It can also be applied to rural regions, with much larger service areas, and traditionally lower demand rates.

Future research includes applying our approaches to various case studies with different patterns of customer demands and road networks. Further, new formulas could be derived for customer demand which is not uniformly distributed and is not a Poisson process. The coordination of the demand responsive feeder transit with the major transit network is another direction for further development.

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APPENDIX

DERIVATION OF EQUATION (3.7a)

For pick-up passengers, let X denotes the nearest bus station to passengers, $x \in \{1, 2, \dots, N\}$. Let Y denotes the ride direction of pick-up passengers at the bus station, $y \in \{1, -1\}$. $Y = 1$ for direction leaving the terminal, and $Y = -1$ for direction approaching the terminal. The Probability Mass Function (pmf) of X is

$$f_X(x) = \begin{cases} \frac{1}{2(N-1)} & \text{for } x = 1 \\ \frac{1}{N-1} & \text{for } x = 2, \dots, N-1 \\ \frac{1}{2(N-1)} & \text{for } x = N-1 \end{cases}$$

The conditional pmf $f(y|x) = P(Y = y|X = x)$ is

$$f(1|x) = \frac{x-1}{N-1}, \quad f(-1|x) = \frac{N-x}{N-1}, \quad \text{for } x = 1, 2, \dots, N-1$$

$$f(1|N) = 0, \quad f(-1|N) = 1$$

The ride time of pick-up passengers $T_{rd-2}^p = g(X, Y)$. Assuming $w_{rd} < w_{wt}$ then we have

$$g(x, y) = \begin{cases} 0 & \text{for } x = 1 \\ C - \frac{(x-1)C}{2(N-1)} & \text{for } y = 1; x = 2, \dots, N \\ \frac{(x-1)C}{2(N-1)} & \text{for } y = -1; x = 2, \dots, N \end{cases}$$

Therefore

$$E(T_{rd-2}^p) = \sum_{x,y} g(x, y) f(x, y) = \sum_{x,y} g(x, y) f(y|x) f_x(x) = \left[\frac{5}{12} - \frac{1}{6(N-1)^2} \right] C$$

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