CODE DESIGN FOR MULTIPLE-INPUT MULTIPLE-OUTPUT BROADCAST CHANNELS

A Thesis

by

MOMIN AYUB UPPAL

Submitted to the Office of Graduate Studies of
Texas A&M University
in partial fulfillment of the requirements for the degree of

MASTER OF SCIENCE

August 2006

Major Subject: Electrical Engineering

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ABSTRACT

Code Design for Multiple-Input Multiple-Output
Broadcast Channels. (August 2006)

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Recent information theoretical results indicate that dirty-paper coding (DPC) achieves the entire capacity region of the Gaussian multiple-input multiple-output (MIMO) broadcast channel (BC). This thesis presents practical code designs for Gaussian BCs based on DPC. To simplify our designs, we assume constraints on the individual rates for each user instead of the customary constraint on transmitter power. The objective therefore is to minimize the transmitter power such that the practical decoders of all users are able to operate at the given rate constraints. The enabling element of our code designs is a practical DPC scheme based on nested turbo codes. We start with Cover's simplest two-user Gaussian BC as a toy example and present a code design that operates 1.44 dB away from the capacity region boundary at the transmission rate of 1 bit per sample per dimension for each user. Then we consider the case of the multiple-input multiple-output BC and develop a practical limit-approaching code design under the assumption that the channel state information is available perfectly at the receivers as well as at the transmitter. The optimal precoding strategy in this case can be derived by invoking duality between the MIMO BC and MIMO multiple access channel (MAC). However, this approach requires transformation of the optimal MAC covariances to their corresponding counterparts in the BC domain. To avoid these computationally complex transformations, we derive a closed-form expression for the optimal precoding matrix for the two-user case and use it to determine the optimal precoding strategy. For more than two users

we propose a low-complexity suboptimal strategy, which, for three transmit antennas at the base station and three users (each with a single receive antenna), performs only 0.2 dB worse than the optimal scheme.

Our obtained results are only 1.5 dB away from the capacity limit. Moreover simulations indicate that our practical DPC based scheme significantly outperforms the prevalent suboptimal strategies such as time division multiplexing and zero forcing beamforming. The drawback of DPC based designs is the requirement of channel state information at the transmitter. However, if the channel state information can be communicated back to the transmitter effectively, DPC does indeed have a promising future in code designs for MIMO BCs.

To the victims of the October 2005 South Asian earthquake disaster

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CHAPTER I

INTRODUCTION

Since the formulation of Shannon's classical point to point information theory, several practical schemes have been developed which achieve the performance promised by theory. This is not true however for the case of multi-terminal communication networks, where there exists no unified network information theory. The partially developed theory in this case promises performance gains over the conventional point to point scenario, at the cost of increased complexity. An interesting case of a multi-terminal communication network is the broadcast channel (BC), also sometimes referred to as the downlink channel, where a single transmitter (base station) transmits messages to many users. Whereas one can argue that a great deal of progress has been made recently on the underlying information theory of a BC, the same cannot be said about practical coding strategies. Thus this thesis makes contributions to this area by developing practical coding schemes for multiple-input multiple-output (MIMO) BCs.

A. Degraded Gaussian BC

The simplest setup for a Gaussian BC is where the transmitter is equipped with only one transmit antenna. If the users receive their signals at different signal-to-noise ratios (SNRs), the channel is known as a degraded Gaussian BC. This can be true in many practical situations, e.g., users farther away from the base station will receive a weaker signal than the ones near it. Such a degraded Gaussian BC with two users was considered by Cover [1] in 1972, for which he provided an achievable rate region.

The journal model is *IEEE Transactions on Automatic Control*.

Cover's scheme was based on the principle of superposition coding where the message for one user is embedded in that for the other. Bergman [2] showed that Cover's rate region is in fact the capacity by proving the converse.

B. Non-Degraded MIMO BC

It is well known that the presence of multiple transmit/receive antennas provides gains in both diversity and multiplexing. Hence, a BC where both the base station and the users can have multiple antennas is of great practical interest. Unfortunately, the channels in this setup might not necessarily be degraded, and therefore Cover's superposition scheme is no longer capacity achieving. In fact, the capacity region for such a MIMO BC had been an open problem until only very recently. A rate region for the MIMO Gaussian BC was found in [3] and was shown to achieve the sum-rate capacity [3]–[6]. The same region was later shown to characterize the whole capacity region in [7].

C. Dirty-Paper Coding and BCs

The core of the capacity-achieving scheme [3] for a Gaussian MIMO BC is a non-linear precoding technique which involves channel coding with encoder side information, called dirty-paper coding (DPC) [8]. According to the somewhat surprising result of [8], in a Gaussian interference channel, if the interfering signal is known non-causally at the transmitter, there is no loss in capacity due to the interference. This scenario is typical in the Gaussian BC, where the signal received at each user includes interference from signals meant for other users, but which are available non-causally at the transmitter – thus one can employ DPC at the base station to mitigate their effect.

DPC is the only optimal, i.e., capacity-achieving technique for the non-degraded Gaussian MIMO BC. Moreover, besides superposition coding, DPC can also be shown to achieve the capacity of a degraded Gaussian BC. However, it has not found wide-spread use in code designs for practical applications – perhaps because its complexity might be too high for applications that require encoding and decoding in real time. So far mostly suboptimal techniques e.g. time division multiple-access (TDMA) and beamforming have found their way into the existing practical designs for the MIMO BC. Recent theoretical comparisons of achievable rate regions [3, 9] indicate significant coding gains of DPC over TDMA and beamforming strategies for the MIMO fading BC in many setups, especially when the SNR is high and the number of transmit antennas large. Practical DPC involves both source and channel coding [10] and near-capacity code designs have appeared recently [11]–[14]. Thus the motivation to develop practical DPC-based designs for the MIMO fading BC and compare their performance and complexity with others based on suboptimal strategies. We point out that our scheme is the first practical DPC-based limit approaching design[1].

D. Summary of Our Work

In this thesis [2], we start with Cover's simplest, yet most celebrated, two-user degraded Gaussian BC [1], and develop a DPC-based design using advanced nested turbo codes [11]. Due to efficient code nesting and high turbo coding gain, our design is superior to previously reported DPC-based schemes of [16, 17] and comes within only 1.44

[1]We note that besides our design, the scheme of [15] involving Tomlinson-Harashima precoding (THP) is the only practical DPC-based design for the MIMO BC. However, since THP is a scalar scheme which does not have any channel coding, it incurs a large practical coding loss

[2]Preliminary results of this work appeared in [18]. Some parts of this thesis have been taken from the journal version submitted for publication [19].

dB of the capacity region boundary at the transmission rate of 1.0 bit per sample per dimension (b/s) for each user. Note that, in this simple setup, besides DPC, superposition coding [1] also achieves the capacity. A practical coding scheme that exploits superposition coding reported in [20] performs only 1 dB from the capacity at 1 b/s. However, the advantage of DPC is that it guarantees the privacy of users, since it ensures that the stronger user (the one with the better channel) decodes its message without knowing the codebook of the weaker user (the one with the worse channel). This is in contrast to the superposition coding scheme [1, 20], where the only way the stronger user can decode its message is by decoding the message intended for the weaker user first.

Another problem of the superposition coding scheme of [20] is that it is not clear how it can be extended to handle MIMO fading BC, where the channels are not necessarily degraded. On the other hand, since in contrast to superposition, DPC achieves the capacity of both degraded and non-degraded BCs, our DPC-based design for Cover's setup naturally applies to the MIMO BC. Thus, we continue with the MIMO Rayleigh slow flat-fading BC and develop *practical capacity-approaching DPC-based designs* for the cases where the total number of transmit antennas and the total number of users are either two or three. We assume that both the transmitter and the receivers have non-causal knowledge of the channel state information. [3]

Most of the information-theoretical works [3]–[6] have focused on maximizing the sum-rate, i.e., the sum of the transmission rates for different users under a fixed total transmission power constraint, which is equivalent to minimizing the total transmission power for a fixed sum-rate constraint. This implies all possible combinations of different transmission rates at different users (as long as the sum-rate is fixed). In

[3]This is the only scenario where the capacity region is fully known.

practice, however, to simplify the implementation of our DPC-based design, we limit the rate of the employed code at each user to a few (e.g., integer) choices. Thus, our design objective is to minimize the total transmission power under fixed individual transmission rate constraints (with the sum-rate constraint being implicit). Because we cannot optimally allocate transmission rates to different users, we incur a small loss in the minimum total transmission power when compared to the case with only the sum-rate constraint.

To determine a precoding scheme at the transmitter that minimizes the total power, the duality [4, 5, 21] between the BC and the multiple access channel (MAC) can be invoked. However, such a duality-based approach is computationally expensive. To reduce the complexity, we directly compute the optimal precoding matrix and give a closed-form expression for the two-user case. For more than two users, we were not able to find a closed-form expression for the optimal precoding matrix; we thus propose a suboptimal strategy, which avoids the complex BC-to-MAC transformation needed in the duality-based approach. Our suboptimal design is of the same order of complexity as that of the two suboptimal strategies proposed in [3], namely, zero-forcing DPC and zero-forcing linear beamforming, yet it outperforms the latter two by a significant margin.

Our simulations indicate that for both the two and three-user cases, our practical DPC-based designs perform only 1.5 dB worse than the theoretical limits computed from our precoding schemes. Although the focus of this thesis has been on code design for two or three users, we point out that our DPC-based design philosophy extends to the case with many more users and that the practical performance loss in terms of the total transmission power will stay at the same 1.5 dB regardless of the number of users.

E. Notation

Notation-wise, all logarithms are of base two unless otherwise stated; vectors and matrices are represented by boldface small and capital letters, respectively; $\mathbf{I}_k$ is the $k \times k$ identity matrix; $|\cdot|$ denotes magnitude of a complex number and $||\cdot||$ represents norm of a vector; $(\cdot)^H$ means Hermitian and $(\cdot)^*$ denotes conjugation, and $tr(\cdot)$ trace of a matrix.

F. Thesis Organization

Our objective is to develop code designs for MIMO Gaussian BCs using DPC. Therefore, we will present the basics of channel coding with side information (CCSI) in Chapter II, where we will describe DPC as a subclass of CCSI. Although we provide basic theoretical aspects of CCSI in this chapter, our emphasis will be to examine approaches to developing practical DPC schemes. It is in this chapter that we will present the practical DPC scheme which we will use later in our code designs.

Chapter III will then provide background on Gaussian BCs, along with the channel capacities and more importantly the role played by DPC in achieving the capacities. We will also introduce the duality of MIMO BCs with MIMO multiple access channels, since it serves as a helpful tool in the code designs which will be presented in Chapter IV. Before presenting the overall coding scheme, Chapter IV will discuss various approaches to evaluating the precoding matrix at the transmitter such that the individual rate constraints at the users are satisfied. Simulation results will be given at the end of Chapter IV. Finally conclusions and areas of possible future research will be presented in Chapter V.

CHAPTER II

CHANNEL CODING WITH SIDE INFORMATION

Channel coding with side information (CCSI) is just one of the problems faced in multi-terminal communication networks. CCSI refers to the problem of communicating over a noisy channel with some knowledge of the channel state available as side information at the encoder, but not at the decoder. Gelfand and Pinsker [22] obtained the capacity for the problem involving a discrete memoryless channel in 1980. Three years later Costa [8] used Gelfand's and Pinsker's result to formulate the theory for the special case of Gaussian channel. Costa's work, also referred to as "writing on dirty paper" [1] , did not address the relevance of its results to communication networks and hence did not draw much attention at first. However, we now know that besides broadcast, several situations in communication networks can be modelled as a CCSI problem e.g. ISI channels, cross talk interference pre-subtraction in vectored digital subscriber line, and cooperative networks to name a few. Moreover it finds widely celebrated applications in covert operations such as data hiding and watermarking.

Since we want to design practical schemes for a Gaussian broadcast channel (BC), we will mostly discuss dirty-paper coding (DPC) as a special case of CCSI. The objective of this chapter is to review both theoretical and practical aspects of DPC, with greater emphasis on discussing practical approaches to solving the problem. The organization of this chapter is as follows. In Section A we will introduce the Gelfand-Pinsker coding problem and discuss how it applies to the special case of Costa coding. Section B discusses practical approaches to solving the DPC problem, which will highlight the importance of source coding in the apparent channel coding

[1] We will use the terms "Costa Coding" and "Dirty-paper Coding" interchangeably throughout this chapter.

problem of DPC. Section C will present an information theoretic perspective to the requirement of a source code in DPC. We will finally present a few sophisticated practical DPC schemes in Section D.

A. Gelfand-Pinkser Coding and Costa Coding

Gelfand and Pinsker [22] considered the case of CCSI in a discrete memorlyless channel. The channel model is shown in Fig. 1.

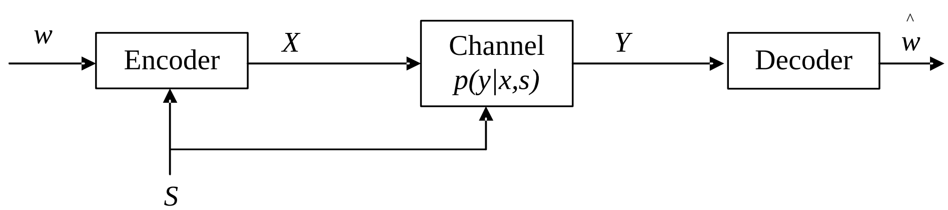

Fig. 1. Gelfand-Pinsker Channel

The input to the channel is denoted by X, the output by Y, and the side information by S which is known non-causally at the encoder but not at the decoder. The encoder is to transmit message w over a discrete memoryless channel characterized by the transition probability $p(y|x,s)$. [22] showed that the capacity of this channel is given by

$$C = \max_{p(v,x|s)} (I(V;Y) - I(V;S)), \tag{2.1}$$

where V is an auxiliary random variable. The proof of Gelfand-Pinsker capacity is based on random coding and binning. For the general CCSI, Gelfand-Pinsker coding suffers a loss compared to the situation when the side information is available at both the encoder and the decoder.

Costa [8] used the general formula in (2.1) to prove the capacity of a Gaussian channel, where the signal is corrupted by an additive Gaussian noise as well as Gaussian interference. Costa's channel is shown in Fig. 2. Costa drew an analogy of this channel to the problem of writing on a sheet of paper covered with dirt, where the writer knows the location and intensity of the dirt particles but the reader does not. Thus the whimsical title of "dirty-paper coding".

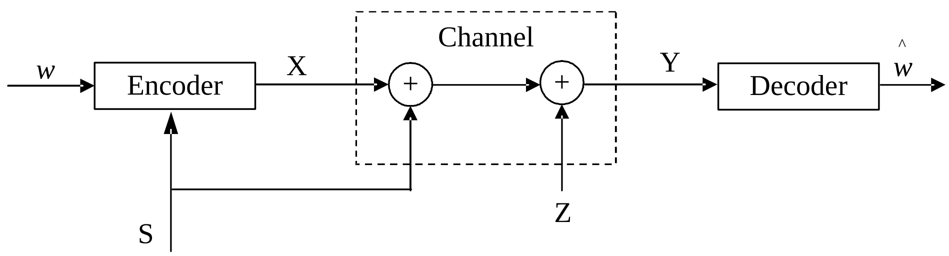

Fig. 2. Costa channel

The transmitter wishes to send the message such that a power constraint $E[|X|^2] \leq P_t$ is satisfied. The output of the channel is given by $Y = X + S + Z$, where the interference $S \sim \mathcal{N}(0, P_Q)$ is known non-causally at the transmitter but not at the receiver, and $Z \sim \mathcal{N}(0, P_Z)$ is the additive noise. If the auxiliary random variable is chosen as $V = X + \alpha S$, with $\alpha = \frac{P_t}{P_t+N}$, Costa proved the surprising result that the capacity of the channel in Fig. 2 is the same as if the interfering signal S were not present at all. This capacity is given by

$$C = \frac{1}{2} \log \left(1 + \frac{P_t}{P_Z} \right). \tag{2.2}$$

Costa's proof is once again based on random coding and binning arguments. Although Costa proved this result for a Gaussian interference, it was later generalized for any arbitrary distribution on S in [23].

B. Approaches to Practical Dirty-Paper Coding

Since Costa's proof is based on random coding and binning, its practical implementation is not possible. However, it does provide a very visible clue of "binning". Not surprisingly, many recent works on practical schemes for DPC have utilized the concept of structured binning.

In this section we will introduce Tomlinson-Harashima precoding (THP) which can be seen as a one dimensional implementation of DPC. We will then draw parallels between THP and scalar quantizers, and thus show the need of a source code in solving the DPC problem. Finally, we will introduce a structured binning strategy based on nested lattices [10].

1. Tomlinson-Harashima Precoding

THP [24, 25] shown in Fig. 3 was originally designed to counter the interference in ISI channels. Consider a message codeword U to be transmitter over a channel characterized by an additive interference S and an additive noise Z, with powers P_Q and P_Z, respectively. The interference S is available non-causally to the encoder but not to the decoder. One can immediately see the equivalence of this problem to DPC if the noise Z were Gaussian. At first glance one would consider pre-subtracting the side information from the transmitted signal in order to cancel the interference, i.e., transmitting $X^s = U - S$. Indeed, the received signal will now be $Y^s = X^s + S + Z = U + Z$, and hence interference free. A closer look at this approach however reveals that this pre-subtraction would have to pay a severe power penalty. Assuming that U and S are independent, the transmitter power will be $E[|X^s|^2] = E[|U|^2] + E[|S|^2]$. Since the side information can have an arbitrarily high power, $E[|X^s|^2]$ can be much higher than $E[|U|^2]$, which will result in a severely reduced transmission rate than

(2.2). In order to avoid this power penalty, THP uses *modulo* arithmetic in order to constrain the transmitted signal to a finite interval.

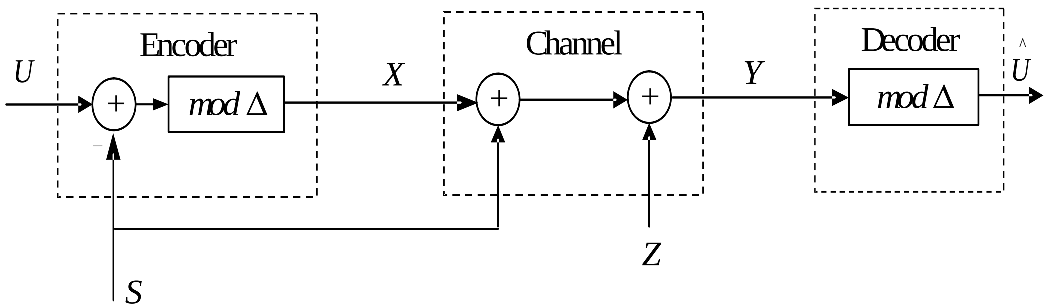

Fig. 3. Tomlinson-Harashima precoding

Let the codeword to be transmitted U be constrained to a finite interval of length Δ, i.e., $U \in [0, \Delta]$. The signal transmitted to the channel is $X = (U - S)\ mod\ \Delta$. Because of the *mod* operation, X is now limited to the same finite interval as U and hence it does not suffer the power penalty which a simple pre-subtraction would. At the decoder, a same *mod* operation is performed to get an estimate of U. In the absence of noise, THP guarantees that U is recovered without error at the decoder. This can be shown as follows. The recovered codeword $\widehat{U}$ is given by

$$\begin{aligned}
\widehat{U} &= Y\ mod\ \Delta \\
&= (X + S)\ mod\ \Delta \\
&= ((U - S)\ mod\ \Delta + S)\ mod\ \Delta \\
&= (U - S + S)\ mod\ \Delta \\
&= U\ mod\ \Delta \\
&\overset{(a)}{=} U,
\end{aligned}$$

where (a) follows from the fact that $U \in [0, \Delta]$.

2. THP with Scalar Quantizers

The encoding process in THP reduces the signal $U' = U - S$ to one of the equivalent representatives of the symbol given as $n\Delta$, where $n = \lfloor \frac{U'}{\Delta} \rfloor$. The difference $X = U' - n\Delta$ is then transmitted to the channel. One can draw parallels between the output of the *mod* operation in THP and the quantization error in a scalar quantizer. Consider a scalar uniform quanitzer whose quantization points are given by $n\Delta$ with $n \in \mathbb{Z}$. If U is distributed on the interval $[0, \Delta)$, then the *mod* operation in THP is related to the quantizer by

$$U' \; mod \; \Delta = U' - Q(U' - \frac{\Delta}{2}), \tag{2.3}$$

where $Q(\cdot)$ represents uniform quantization. It can be shown that the *mod* operations in THP can be replaced by the scalar quantizer by making sure that the input signal is distributed on the interval $[-\frac{\Delta}{2}, \frac{\Delta}{2})$ instead of on $[0, \Delta)$. Fig. 4 shows equivalent THP with scalar quantizers. When the interference power P_S is large, the quantization error X is approximately uniformly distributed on the interval $[-\frac{\Delta}{2}, \frac{\Delta}{2})$ and hence the power of the transmitted signal is independent of P_S and is approximately given by $\frac{\Delta^2}{12}$.

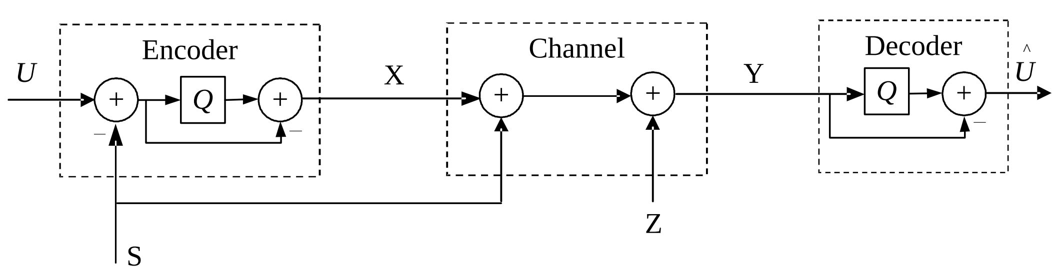

Fig. 4. Tomlinson-Harashima precoding with scalar quantizers

3. Generalization of THP to Vector Quantizers

As pointed out in [23, 26] THP suffers a signigicant loss from Shannon's capacity limit, especially at low signal to noise ratios (SNRs). The main drawback of THP is that it only uses the current value of the side information S and does not consider the future values. The *mod* operation is performed on a symbol by symbol basis resulting in an output which is uniformly distributed on $[-\frac{\Delta}{2}, \frac{\Delta}{2})$. This is equivalent to performing a *mod* operation over a high dimensional cuboid, which suffers a *shaping loss*. An optimal quantizer however should be equivalent to performing a *mod* operation over a high dimensional sphere, resulting in Gaussian quantization error in each dimension. Thus instead of using the side information on a symbol by symbol basis, one needs to consider an entire sequence. The solution to recovering the shaping loss therefore lies in performing a high dimensional *mod* operation, or equivalently in *vector quantization*.

4. Binning Based on Nested Lattices

So far, we have only discussed the source coding (quantization) portion of the DPC problem, which is essential to satisfy the power constraint. We found that one can accurately retrieve the coded message in the absence of noise. However, in practice one needs to add error protection to the transmission in order to combat the channel's additive Gaussian noise. This therefore introduces an additional channel coding aspect to the problem. The question here is: How do we view the joint source and channel code design under a similar framework? Zamir *et al* [10] proposed a practical binning scheme based on nested codes. Hence the solution to the Gelfand-Pinsker problem lies in nested parity check codes, and in nested lattice codes for the Costa coding problem.

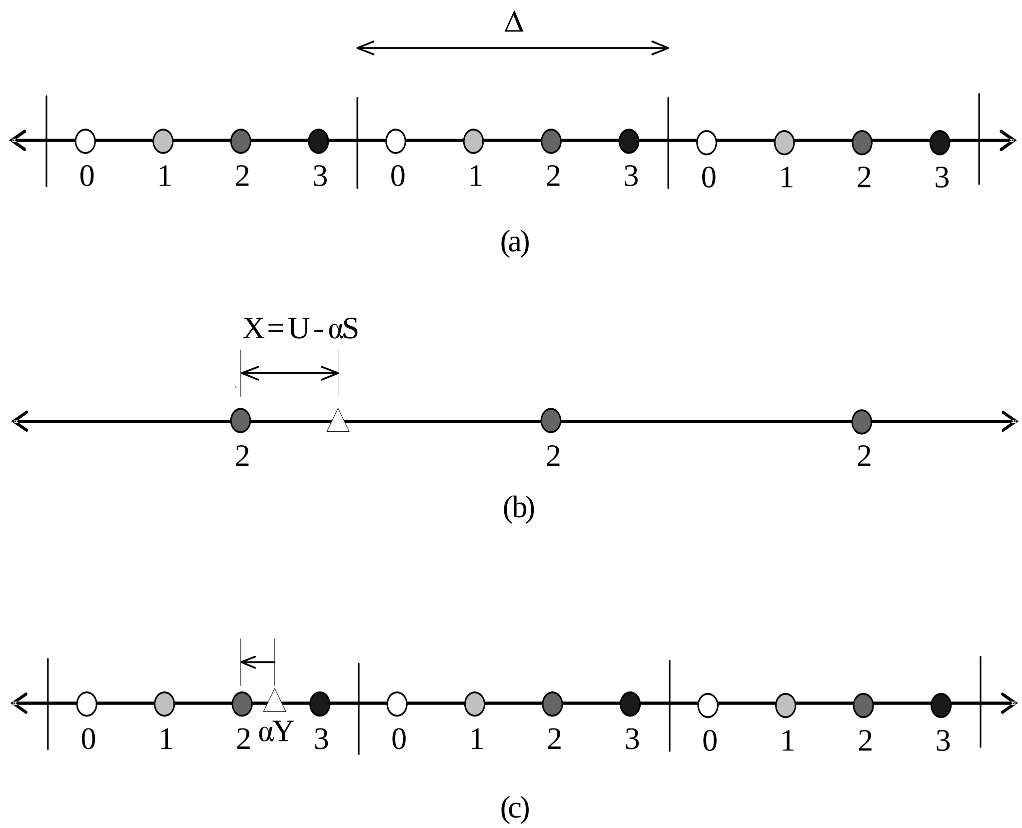

Fig. 5. Binning scheme using a 1-D nested lattice (a) Nested lattice (b) Encoding (c) Decoding

A nested lattice code comprises of a *coarse lattice code* nested inside a *fine channel code*, i.e., every codeword of the coarse lattice code is also a codeword of the fine lattice code but every codeword of the fine lattice is not a codeword of the coarse lattice. According to [10], for a good dirty-paper code design, the fine code should be a good channel code whereas the coarse code should be a good source code. Hence the source code is nested within the channel code. The concept of binning can be derived from this nesting approach. The group of channel codewords nested within a single source codeword are said to belong to the same *bin*, where the bin is indexed by that particular source codeword.

Let us illustrate how binning based on nesting works by considering a one dimensional nested lattice as an example. Note that we select a one dimensional lattice for illustrative purposes only – in practice a high dimensional lattice should be used in order to achieve good performance. Fig. 5 demonstrates a binning strategy based on a 1-D nested integer lattice. The points on the lattice indexed by a 0 correspond to the channel codewords in the basic coset. Similarly the points indexed by the other numbers correspond to the other cosets. The message to be transmitted (which in this case will be a two bit message) selects one of these cosets. In this example, coset 2 is indexed by the message. The message is first scaled by a factor α (the necessity of this scaling comes from Costa's original proof in [8]). This scaled side information is then quantized to the nearest codeword in the coset 2 and the quantization error is sent to the channel. At the decoder the nearest codeword to the scaled received signal is found to get an estimate of the transmitted signal. The decoded message therefore is the index of this estimate. THP with scalar quantizers can be viewed as a binning scheme based on nested lattices. The input U in Fig. 4 can be thought of as a channel codeword selected by the message. Quantizing the difference $U - \alpha S$ to an infinite integer lattice $n\Delta$ is the same as quantizing αS to a lattice where the channel codeword U has been infinitely replicated.

As mentioned earlier, the lattice codes in practice should be of higher dimensions, as opposed to the 1-D lattice we used for the illustration. However we point out that nested lattice codes require a joint source-channel code design with the same dimensional source and channel lattice code, which becomes difficult to implement at higher dimensions. Let Λ_c denote the L dimensional channel/fine lattice code which is nested within the source/coarse lattice code Λ_s, and let Λ_w be the coset code indexed by a length L message sequence w^L. Then a general block diagram of a DPC scheme is presented in Fig. 6.

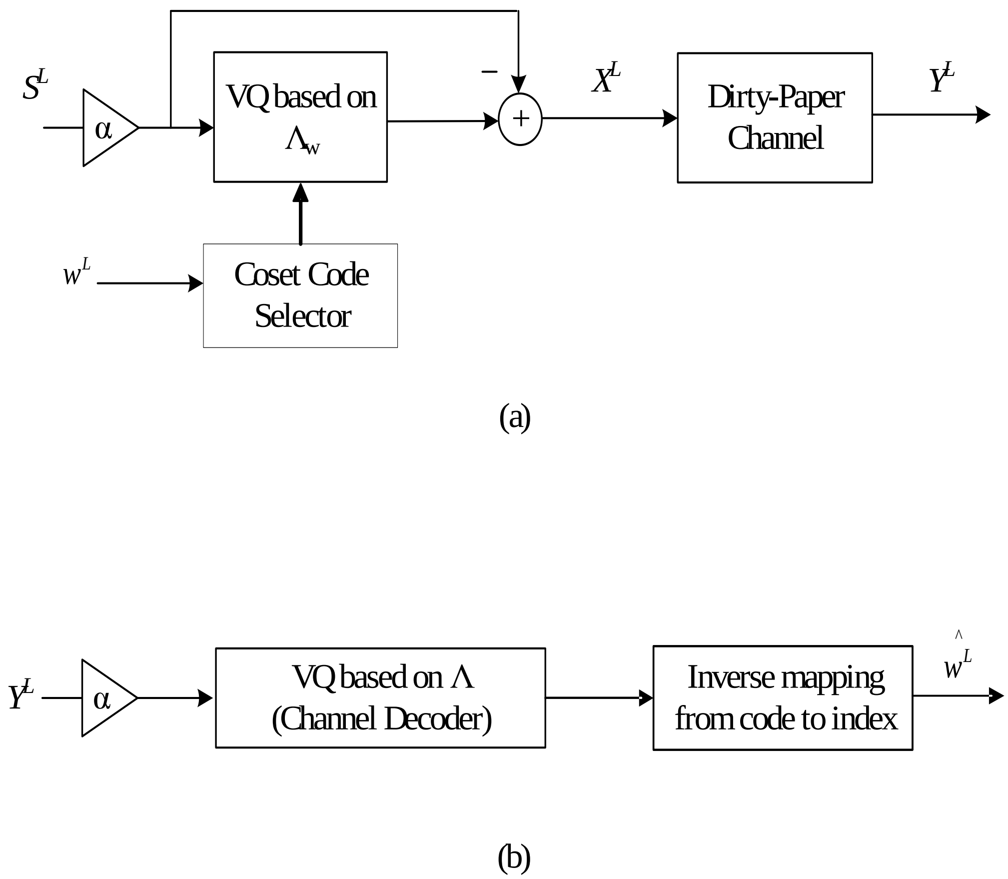

Fig. 6. A DPC scheme based on nested lattices (a) Encoder (b) Decoder

C. Information Theoretic Perspective

An information theoretic framework for studying the Costa coding problem was presented in [23]. Costa coding is inherently a channel coding problem. According to [27], there are packing and shaping gain in channel coding. The shaping gain has to do with the shape of the Voronoi region of the lattice, which ideally has to be a sphere. The packing gain has to do with the way the code regions are packed against each other. Costa coding problem as explained earlier can be split into a source coding and channel coding component. The source coding becomes necessary to satisfy the power constraint and is hence required to reduce the scaled side information modulo the Voronoi region. The constellation therefore needs to be replicated infinitely so that one can quantize the side information to satisfy the power constraint. This source coding therefore is not conventional in the sense that it only has the granular gain and no boundary gain. One can easily draw equivalence between the granular gain in source coding and the shaping gain in channel coding. Hence in channel coding with side information problem, the shaping gain is achieved through source coding and the packing gain through channel coding. In order to get close to the Costa capacity limit, the source coder should be designed such that its Voronoi region is almost a spherical region in high dimensional Euclidean space (such as trellis coded quantization). Similarly the channel code should also be near capacity (such as Turbo codes or LDPC). Erez et al [12] proved that the capacity limit for a dirty paper channel with its source coder having a shaping gain of $g_s(\Lambda)$ is given by

$$C^* = \frac{1}{2}\log\left(1 + \frac{P_t}{P_Z}\right) - \frac{1}{2}\log\left(2\pi e G(\Lambda)\right), \tag{2.4}$$

where $G(\Lambda)$ is the normalized second moment of the quantizer lattice Λ. $G(\Lambda)$ is upper bounded by $\frac{1}{12}$ for a uniform quantizer whose Voronoi region is a high dimensional cuboid, and asymptotically approaches $\frac{1}{2\pi e}$ with the dimensionality of Λ going to infinity for a quantizer lattice whose Voronoi region is a high dimensional sphere [28]. We can see that with a lattice that achieves the lowest normalized second moment (ideal quantizer), the capacity limit of the nested lattice DPC scheme is equivalent to Costa's capacity in (2.2). This necessitates the use of a strong source code, along with a capacity achieving channel code in order to get close to Costa's capacity limit.

D. Practical DPC Schemes

A practical nested lattice coding scheme was presented in [29], which uses trellis coded quantization TCQ as the source code and turbo trellis coded modulation (TTCM) as the channel code. An improved design involving nested turbo codes was described in [11], which uses a stronger source code referred to as Turbo TCQ (TTCQ)[2] nested within a channel code based on TTCM. At a transmission rate of 1.0 bit per sample per dimension (b/s), the DPC design of [11] outperforms the one in [29] by 0.54 dB. Hence we will utilize the scheme of [11] which we refer to as TTCQ/TTCM for our BC code design. However, before describing the nested lattice coding scheme based on TTCQ/TTCM, we will first introduce the TCQ/TTCM scheme of [29].

1. The TCQ/TTCM Scheme [29]

The TCQ/TTCM scheme of [29, 30] is shown in Fig. 7. The trellis structure is constructed via a rate-$k/n/m$ concatenated code (denoted by C_1+C_2, with C_1 being a non-systematic rate-k/n convolutional code characterized by trellis Γ_1 and C_2 being a systematic rate-n/m convolutional code characterized by trellis Γ_2). The message to be transmitted w is input to an inverse syndrome mapper $\mathbf{H}^{-1}$, where $\mathbf{H}$ is the parity check matrix for code C_1. We point out that the inverse syndrome mapper $\mathbf{H}^{-1}$ is in fact the pseudo-inverse of the parity check matrix $\mathbf{H}$. The output of this inverse syndrome mapper is used to shift the codewords of C_1 by a fixed amount and select a source code for quantization. In other words, w selects the coset to be used for quantization. The channel code is a punctured TTCM [32] which consists of a parallel concatenated code with convolutional code C_2 in both parallel branches. C_2

[2]TTCQ was shown to underperform TCQ in [31] as the number of encoding iterations increases. The quantization in [11] is different from that of [31] in the sense that it uses only one iteration. Despite this difference we still call it TTCQ.

in the bottom branch is preceded by an n-bit symbol interleaver and followed by an m-bit symbol deinterleaver. The two branches are multiplexed by taking half of the samples from the top branch, and the other half from the bottom (hence a punctured TTCM). The times indices at which the final output sequence is taken from the top branch is referred to as the even indices, whereas the ones at which it is taken from the bottom branch are referred to as odd indices.

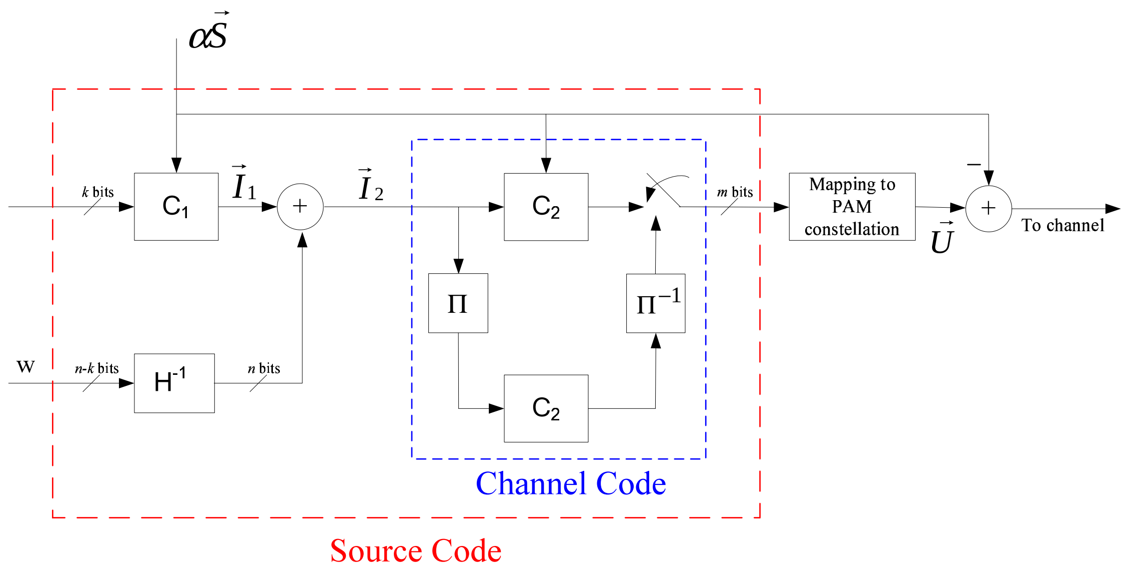

Fig. 7. Nested turbo construction with TCQ as the source code and punctured TTCM as the channel code.

The objective of the encoder is to quantize the side information sequence $\alpha\vec{S} = \alpha[S(0), \ldots, S(L-1)]$ using the coset selected by the message sequence $\vec{w}$, where L is the sequence length (or trellis size) and $S(t)$ is the t-th value of the sequence. Let $\vec{I_1}$ and $\vec{I_2}$ be the n-bit output sequence of C_1, and the n-bit input sequence of C_2, respectively. If instead of the parallel concatenated turbo structure, we had a simple convolutional code C_2 as the channel code, the best way to perform this quantization would have been to use Viterbi algorithm on the trellis $\Gamma_{12}(\vec{w})$[3] of the serially concatenated code C_1+C_2 such that the mean squared quantization error (MSQE) was minimized. However, using the Viterbi algorithm to minimize the MSQE for the current turbo setup is no longer a practical option, since presence of the interleaver greatly increases the number of paths that need to be searched. Therefore [29] proposes a suboptimal solution which involves first ignoring the bottom path and using the trellis $\Gamma_{12}(\vec{w})$ of the serially concatenated code C_1+C_2 to minimize MSQE. The sequence $\vec{I_2}$ thus determined is input to the bottom branch. The output sequence of the top branch is the one which minimizes MSQE. However, the output of the bottom branch is randomly different from the top branch because of the interleaver. Since the final output takes half of the samples from the top branch, and half from the bottom, the resulting sequence will result in a higher distortion than if only the sequence from the top branch were used. Consequently, the performance of the source code suffers.

The decoder uses BCJR to decode the received signal to the closest to the codeword, and the n-bit input sequence of C_2, $\hat{\vec{I_2}}$ is recovered. Since $\vec{I_2} = \vec{I_1} + \vec{w}\mathbf{H}^{-1}$, the message sequence can be recovered by calculating the syndrome of the recovered sequence $\hat{\vec{I_2}}$. If $\hat{\vec{I_2}}$ is decoded without any errors, then the recovered message $\hat{w}(t)$ at time t is given by

$$\begin{aligned}\hat{w}(t) &= I_2(t)\mathbf{H} = (I_1(t) + w(t)\mathbf{H}^{-1})\mathbf{H} \\ &= I_1(t)\mathbf{H} + w(t)\mathbf{H}^{-1}\mathbf{H} \\ &\overset{(b)}{=} w\end{aligned}$$

where (b) follows from the fact that $I_1(t)\mathbf{H} = 0$. Hence the original message can be

[3]This indicates that the trellis Γ is not only a function of the two constituent trellises but also of the message sequence $\vec{w}$

recovered perfectly.

One can immediately see a venue for improving on this TCQ/TTCM design by employing a stronger source code. Sun *et al.* [11] view the performance loss of the source code as a consequence of the mismatch between dimensionalities of the equivalent lattice source and channel codes. The presence of the interleaver increases the dimensionality (and hence performance) of the equivalent lattice channel code. However, this results in a dimensional mismatch between the high dimensional channel code and the relatively lower dimensional source lattice code. The work in [11] improves upon the design of [29, 30] by attempting to reduce this dimensional mismatch.

2. The TTCQ/TTCM Scheme [11]

In order to reduce the dimensional mismatch mentioned above, Sun *et al.* [11] propose a stronger source code. The construction in [11] follows the same principles as the one in Fig. 7 except that the calculation of $\vec{I}_1$ is realized via a single iteration of TTCQ [31]. A block diagram of the quantization procedure is shown in Fig. 8, a brief description of which follows.

Before calculating the sequence $\vec{I}_1$, its soft version $\vec{I}_{S1}$ is evaluated using a soft-output Viterbi algorithm (SOVA) [33]. Let $\vec{I}_1^d$ be a dummy sequence over which the search operation will be performed. Corresponding to this dummy sequence is a dummy codeword sequence $\vec{U}^d = [U^d(0), \ldots, U^d(L-1)]$ which is generated by the turbo encoder of the right portion of Fig. 7 with the sequence $\vec{I}_1^d + \vec{w}$ as the input. Let $\mathcal{C} = \{0, 1, \ldots, 2^n - 1\}$ be the set of all possible n-tuples. Then the soft value $I_{S1}(t, c)$ for a fixed time t and a fixed n-bit symbol c is given by the minimum total quantization error over all possible input sequences $\vec{I}_1^d \in \mathcal{C}^L$, provided that all such sequences at time t are equal to c, i.e., $I_1^d(t) = c$. This soft value is given mathematically as

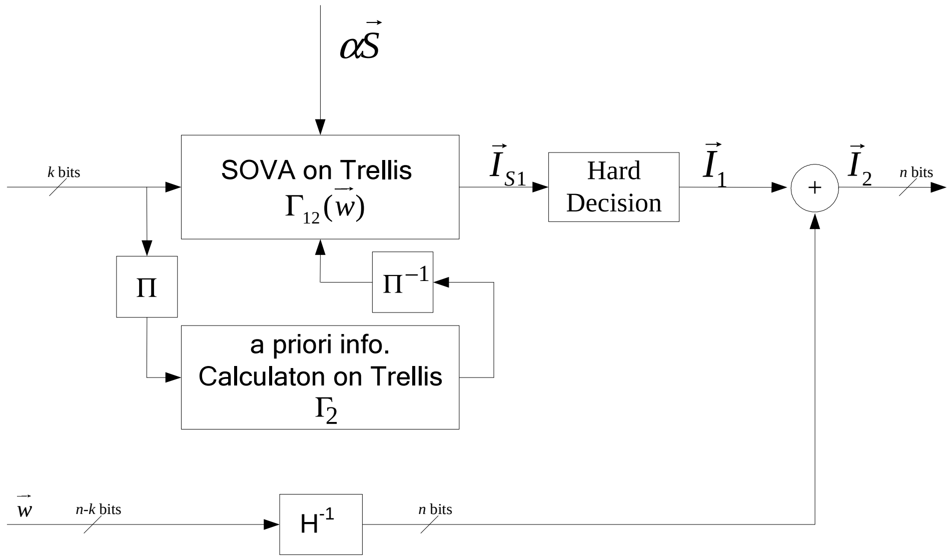

Fig. 8. Turbo-Trellis coded quantization for the Costa problem.

$$I_{S1}(t,c) = \min_{\substack{\vec{I}_1^d \in \mathcal{C}^L; \\ I_1^d(t) = c \in \mathcal{C}}} \sum_{l=0}^{L-1} \left\{ \left| U^d(l) - \alpha S(l) \right|^2 \right\}, \tag{2.5}$$

where $\vec{S} = [S(0), \ldots, S(L-1)]$ is the length-L sequence of side information. The soft values need to be calculated for all $t = 0, 1, \ldots, L-1$ and all $c \in \mathcal{C}$. Therefore, $\vec{I}_{S1}$ can be written in matrix form as

$$\vec{I}_{S1} = \begin{bmatrix} I_{S1}(0,0) & \cdots & I_{S1}(L-1,0) \\ \vdots & \ddots & \vdots \\ I_{S1}(0,2^n-1) & \cdots & I_{S1}(L-1,2^n-1) \end{bmatrix}. \tag{2.6}$$

Once the soft values have been calculated, the sequence $\vec{I}_1$ can be found by performing a hard thresholding operation on $\vec{I}_{S1}$, i.e., $I_1(t) = \arg\min_{c \in \mathcal{C} = \{0,1,\ldots,2^n-1\}} I_{S1}(t,c)$. Let us now see how the soft values are calculated.

The calculation of $\vec{I}_{S1}$ is based on two parallel trellises as shown in Fig. 8. The trellis of the top branch, $\Gamma_{12}(\vec{w})$ is constructed by the serial concatenation of codes C_1 and C_2. The argument $\vec{w}$ indicates that this trellis is a function of the message sequence (Recall that the message sequence should select the coset for quantization). The trellis of the bottom branch is constructed by the trellis Γ_2 of code C_2. Because of the even odd multiplexing of the TTCM encoder of Fig. 7, the branch metrics in trellis Γ_{12} is set to the quantization error for even indices only, i.e., $\rho_e(t) = \left|U^d(t) - \alpha S(t)\right|^2$ with t taken from the set of even indices. The metrics at odd indices are provided by trellis Γ_2 as *a priori* information. Similar to the idea of the initialization step in TTCM decoding, for a systematic C_2, the *a priori* information at time t, denoted by $A_o(t,c)$ as the minimum distortion corresponding to the bits $I_1^d(t) = c$ and all possible parity bits $B(t) \in \mathcal{B} = \{0, 1, \ldots, 2^{m-n}-1\}$, i.e., $A_o(t,c) = \min_{I_1^d(t)=c; B(t)\in\mathcal{B}} |U^d(\Pi(t)) - \alpha S(\Pi(t))|^2$ with t taken from the set of odd time indices, and where $\Pi(t)$ denotes the same symbol interleaver as used in the TTCM encoder. This *a priori* information is then deinterleaved and fed into the top trellis code. If $\rho_e(t) = 0$ for odd t, and $A_o(t,c) = 0$ for even t, then the computation in (2.5) can be modified to

$$I_{S1}(t,c) = \min_{\substack{\vec{I}_1^d \in \mathcal{C}^L; \\ I_1^d(t) = c \in \mathcal{C}}} \sum_{l=0}^{L-1} \Big\{ \rho_e(l) + A_o(\Pi^{-1}(l), c) \Big\}, \tag{2.7}$$

with $\Pi^{-1}(l)$ denoting the symbol deinterleaver as the TTCM encoder. The minimization in (2.7) can be performed by using SOVA on trellis $\Gamma_{12}(\vec{w})$.

Since the turbo-like TTCQ source code in this case has a similar parallel concatenated code structure as that of TTCM, the dimensionality of the source code is higher than that of simple TCQ, and thus it facilitates better nesting of the source code inside the channel code. An additional means of alleviating the dimensional mismatch can be achieved by varying the percentage T of the total number of samples

L that are selected from the top branch in the parallel-branch structure. By default for both TCQ/TTCM and TTCQ/TTCM, $T = 50\%$. Increasing T from 50% reduces the effect of the interleaver in the bottom branch causing degradation in the channel code performance. However, at the same time, it guarantees improved performance of the source code (Note that when $T = 100\%$, no interleaving is performed. Hence the source code becomes regular TCQ based on trellis $\Gamma_{12}(\vec{w})$, whereas the channel code becomes regular TCM). Increasing T can be viewed as increasing the dimensionality

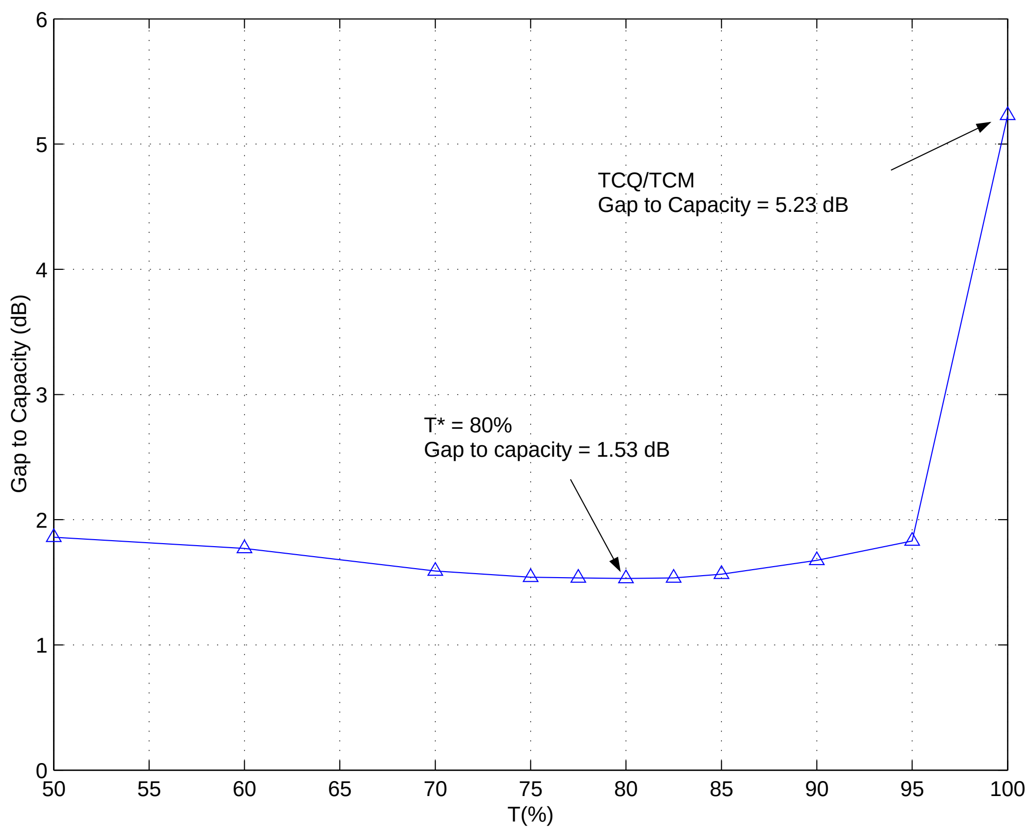

Fig. 9. Effect of T on the performance of TTCQ/TTCM scheme at a transmission rate of 1.0 b/s and block length of $L = 10,000$.

of the equivalent source lattice code, while decreasing that of the channel lattice code, thus providing a way of reducing the dimensional mismatch. An optimal T^* can be searched between 50% and 100% to achieve the best performance in terms of the bit error rate (BER) of the decoded message. Fig. 9 shows the gap to Costa's capacity (2.2) of the TTCQ/TTCM scheme of [11] as a function of T. It can be seen that the optimal percentage is $T^* = 80\%$ for which the gap to capacity is 1.53 dB. We point out that the process of dimensionality balancing by varying T is equally applicable to the TCQ/TTCM scheme of [30]. The effect of this dimensionality balancing for TCQ/TTCM scheme was studied in [11] and have been provided here in Table 2. We also provide the performance of TCQ/TCM scheme as a benchmark. Note that at $T = 50\%$ TTCQ/TTCM outperforms TCQ/TTCM, thus corroborating the fact that the former facilitates better nesting of the source and channel lattice codes than the latter. Although at the optimum percentages T^*, the gap between the two is decreased, yet TTCQ/TTCM still performs better than TCQ/TTCM.

Table I. A performance comparison of the practical DPC schemes of [30] and [11] in terms of the gap (in dB) to Costa's capacity.

TCQ/TCM [30]	TCQ/TTCM [30]		TTCQ/TTCM [11]	
	T=50%	T^*=82.5%	T=50%	T^*=80%
5.23	2.07	1.63 [11]	1.86	1.53

CHAPTER III

GAUSSIAN BROADCAST CHANNELS

In this chapter we will introduce the theoretical aspects of Gaussian BCs and present the channel capacities. We will start in Section A with the simplest example of a BC with one transmit antenna at the base station sending messages to two users each with a single antenna. Under these conditions we will specifically consider the scenario where the two users receive their signals at different SNRs, which is referred to as degraded Gaussian BC. We will then introduce a MIMO Gaussian BC in Section B where the channels might not necessarily be degraded. Finally we will introduce duality of the MIMO Gaussian BC with the MIMO multiple access channel (MAC) in Section C, which will serve as a helpful tool in our code designs.

A. Degraded Gaussian BC

1. Channel Model

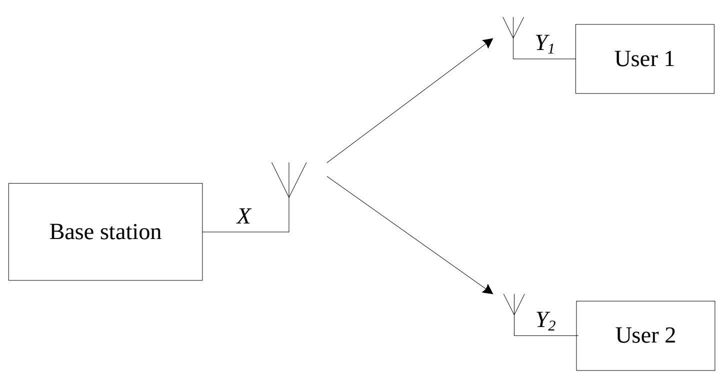

Fig. 10. A simple broadcast channel with one transmit antenna at the base station and two users each with a single antenna.

A two-user Gaussian BC is shown in Fig. 10. The base station wishes to send messages w_1 and w_2 to user 1 and user 2, respectively. The single antenna at the base station transmits the baseband signal $X(w_1, w_2)$ indicating that it will be a function of the messages for both users. If P_t is the maximum allowable transmitter power, the transmitter signal X should satisfy $E[|X|^2] \leq P_t$. The received signals at the two users are given by

$$\begin{aligned} Y_1 &= X + Z_1 \\ Y_2 &= X + Z_2, \end{aligned} \tag{3.1}$$

where Z_1 and Z_2 are independent, identically distributed (i.i.d.) zero mean Gaussian noises with variances N_1 and N_2, respectively, independent of X. Since we consider the case of a degraded Gaussian BC, the received SNRs at the two users will be different. The user with the better channel is referred to as the "strong" user, and the one with the worse channel as the "weak" user. Without loss of generality, we assume that user 2 is strong and user 1 is weak, i.e., $N_2 < N_1$

2. Channel Capacity

Let the messages w_1 be coded to U_1 and w_2 be coded to U_2, such that both the codebooks are of unit power i.e., $E[|U_1|^2] \leq 1$ and $E[|U_2|^2] \leq 1$. The total transmission power can be allocated to the two users through precoding, i.e., by selecting the transmitted signal as $X = \mathbf{Bu}$, where $\mathbf{u} = [U_1,\ U_2]^T$ and B is the precoding matrix given by $\mathbf{B} = [\sqrt{(1-\gamma)P_t}, \sqrt{\gamma P_t}]$. γ $(0 \leq \gamma \leq 1)$ is a parameter that controls the power allocation between the two users. Cover [1] obtained the capacity region for this setup by using superposition coding, where the codebooks U_1 and U_2 are Gaussian and uncorrelated. The received signals at the two users are given by

$$\begin{aligned} Y_1 &= \sqrt{(1-\gamma)P_t}U_1 + \sqrt{\gamma P_t}U_2 + Z_1 \\ Y_2 &= \sqrt{(1-\gamma)P_t}U_1 + \sqrt{\gamma P_t}U_2 + Z_2. \end{aligned} \tag{3.2}$$

Consider the following decoding scheme. User 1 treats U_2 as an unknown interference, which means the unwanted noise term for user 1 is $\sqrt{\gamma P_t}U_2 + Z_1$. Since U_1 is Gaussian and is independent of Z_1, the effective noise is Gaussian with variance $\gamma P_t + N_1$. The achievable rates at user 1 therefore satisfy

$$R_1 \leq \frac{1}{2}\log\left(1 + \frac{(1-\gamma)P_t}{\gamma P_t + N_1}\right). \tag{3.3}$$

Since the noise at user 2 is weaker than that at user 1, i.e., $N_2 < N_1$, user 2 can also decode U_1 with arbitrarily low probability of error. It can then subtract the decoded U_1 term to obtain $Y_2' = \sqrt{\gamma P_t}U_2 + Z_2$, which implies that w_2 can be transmitted at rates satisfying

$$R_2 \leq \frac{1}{2}\log\left(1 + \frac{\gamma P_t}{N_2}\right). \tag{3.4}$$

Since user 2 also decodes w_1 correctly, the effective upper bound on the achievable rates at user 2 is $R_1 + R_2$. Cover proved that the maximum achievable rates in (3.3) and (3.4) are in fact the maximum rates over all encoding schemes. Bergman [2] completed the capacity result by proving that any rates above these bounds are not achievable. The achievable rate region characterized by (3.3) and (3.4) is shown in Fig. 11. The rates along the dashed line are achievable with time-sharing. The concavity of the capacity region indicates that one can always do better than time sharing by employing Cover's superposition scheme (or a DPC scheme as will be shown later). An interesting case however is when $N_2 = N_1$, for which the time sharing region is the same as the rate region described by (3.3) and (3.4).

One of the issues with Cover's superposition scheme is that in order to achieve capacity, user 2 besides decoding its own message, also has to decode the message intended for user 1 which does not bode good from the privacy point of view. This problem can be overcome by using dirty-paper coding (DPC) which also achieves all

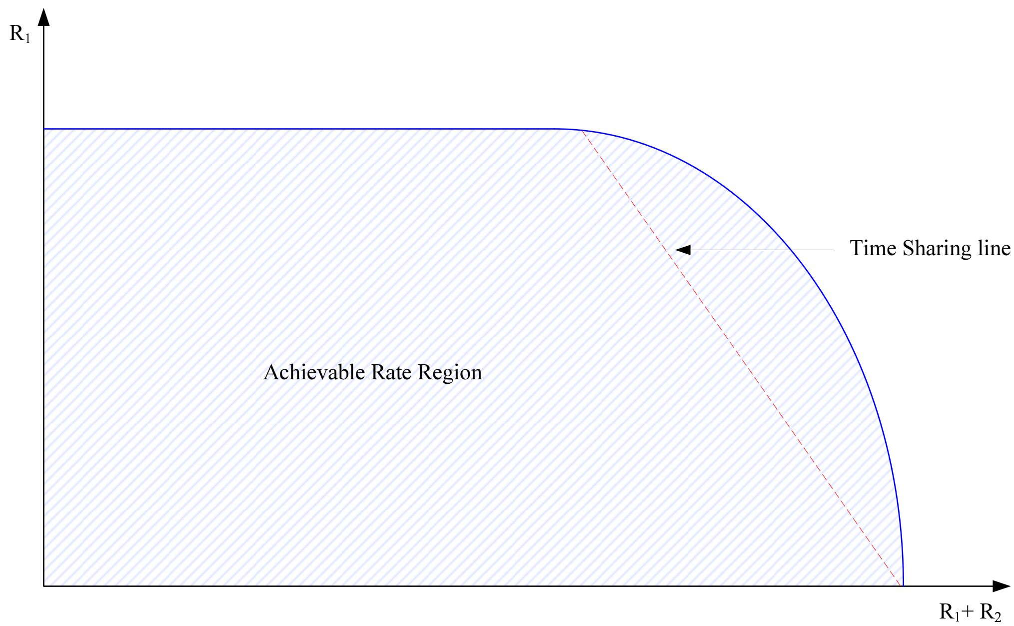

Fig. 11. Achievable rate region of a degraded Gaussian BC with $N_2 < N_1$.

the points in the capacity region described by (3.3) and (3.4). Indeed, if U_1 is drawn from a standard Gaussian codebook and U_2 is dirty-paper coded with $\sqrt{(1-\gamma)P_t}U_1$ as the encoder side information (ideal DPC codebook is Gaussian), user 1 still treats the U_2 term as unknown interference and therefore the achievable rates at user 1 satisfy (3.3). User 2, because of DPC, achieves the same rate as if the interference from U_1 were not present, and therefore the achievable rates at user 2 satisfy (3.4). Note that if the encoding order is reversed, i.e., if U_1 is dirty-paper coded with U_2 as the encoder side-information with U_2 drawn from a standard Gaussian codebook, another rate pair is also achievable which is given by

$$\begin{aligned} R_1 &\leq \frac{1}{2}\log\left(1+\frac{(1-\gamma)P_t}{N_1}\right), \\ R_2 &\leq \frac{1}{2}\log\left(1+\frac{\gamma P_t}{(1-\gamma)P_t+N_2}\right). \end{aligned} \tag{3.5}$$

However, it can be shown that when $N_2 < N_1$, this rate region is smaller than the true capacity region characterized by (3.3) and (3.4).

B. Non-Degraded MIMO BC

1. Channel Model

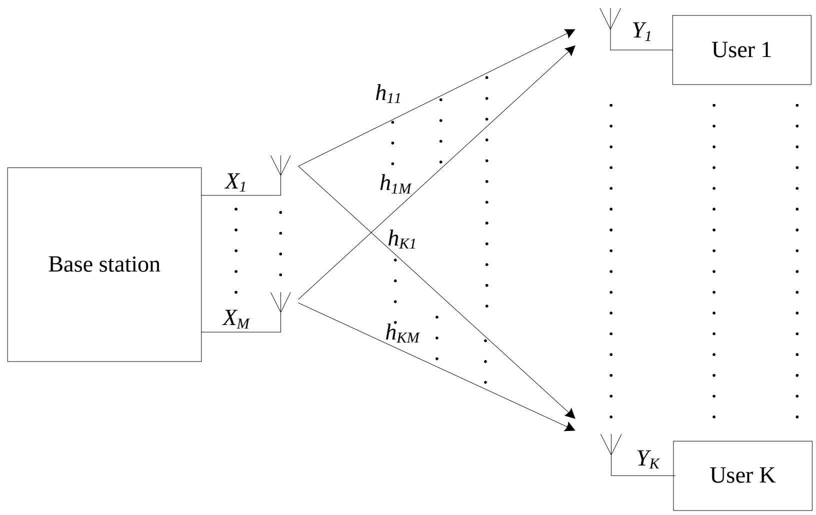

Fig. 12. A broadcast channel with M transmit antennas at the base station, and K users each with a single antenna. The channel between antenna j and user i experiences a fading coefficient of h_{ij}.

The previous section was limited to the case where the number of users and the transmit antennas at the base station are limited to two. However, the demands on modern day communications require the base station to service several users simultaneously. Moreover, additional transmit antennas at the base station promise gains in data rate as well as in quality of service. Therefore, in this section we consider a BC which can have multiple transmit antennas at the base station sending messages possibly to more than two users. Note that the aforementioned gains in data rate and quality of service can also be attained by employing multiple receive antennas at the users. These antennas should be physically located significantly far apart in order to attain any noteworthy gains. However, in applications where the users are mobile, the receiving units are limited by their size and power, and therefore having more than one antenna at the receiver becomes unfeasible. Therefore, we will only consider the scenario where the users have a single receive antenna each. Moreover, in many practical wireless channels, the transmitted signal undergoes fading in addition to an additive noise. Therefore, we model the channel as a quasi-static flat fading channel with additive white Gaussian noise. We are now ready to mathematically define the channel model.

A MIMO fading BC is shown in Fig. 12. Let the number of transmit antennas at the base station be M, which sends messages to K users each with a single antenna. If h_{ij} is a complex channel gain between user i $(1 \leq i \leq K)$ and transmit antenna j $(1 \leq j \leq M)$, then

$$Y_i = \sum_{j=1}^{M} h_{ij} X_j + Z_i \tag{3.6}$$

is the complex baseband equivalent of the signal received by user i, X_j is the complex baseband equivalent of the transmitted signal at antenna j, and the Z_i's are i.i.d. complex zero-mean Gaussian noises with unit variances, independent of the X_j's. The transmitter should satisfy the following power constraint:

$$\sum_{j=1}^{M} E[|X_j|^2] \leq P_t \tag{3.7}$$

Let $\mathbf{y} = [Y_1, Y_2, \ldots, Y_K]^T$, $\mathbf{x} = [X_1, X_2, \ldots, X_M]^T$, and $\mathbf{z} = [Z_1, Z_2, \ldots, Z_K]^T$ and $\mathbf{H}$ be a size $K \times M$ channel matrix whose element at the i^{th} row and j^{th} column is given by the channel coefficient h_{ij}; then (3.6) in the matrix form becomes

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{z}. \tag{3.8}$$

2. Channel Capacity

An achievable rate region using successive DPC was presented in [3]. We first present a description of this successive DPC scheme along with its achievable rate region. We will then see how this rate region relates to the capacity of the MIMO BC.

Let w_i be the message intended for user i, then the transmitter sends $\mathbf{X} = \mathbf{BU}$, where $\mathbf{B}$ is an $M \times K$ precoding matrix, and

$$\begin{aligned}\mathbf{u} &= [U_1, U_2, U_3, \ldots, U_K]^T \\ &= [U_1(w_1), U_2(w_2; U_1), U_3(w_3; U_1, U_2), \ldots, U_K(w_K; U_1, \ldots, U_{K-1})]^T\end{aligned}$$

is generated using successive DPC with all K codebooks being uncorrelated and Gaussian with unit power. Here, $U_i = U_i(w_i; U_1, \ldots, U_{i-1})$, $2 \leq i \leq K$, indicates that w_i is coded to the codeword U_i using DPC with the linear combination of $U_1, U_2, \ldots, U_{i-1}$ as the encoder side information (i.e., known interference).

Let $\mathbf{b}_i$ be the i^{th} column of the precoding matrix $\mathbf{B}$, i.e., $\mathbf{B} = [\mathbf{b}_1, \ldots, \mathbf{b}_K]$. Then $\mathbf{x} = \mathbf{Bu}$ translates to $\mathbf{x} = \sum_{i=1}^{K} \mathbf{b}_i U_i$. The covariance matrix of the vector $\mathbf{x}$ can now be written as a function of $\mathbf{b}_i$'s as

$$\begin{aligned}\mathbf{S}_{xx} &= E[\mathbf{x}\mathbf{x}^H] \\ &= E[\textstyle\sum_{i=1}^{K} \mathbf{b}_i U_i \sum_{j=1}^{K} \mathbf{b}_j^H U_j^H] \\ &= \textstyle\sum_{i=1}^{K} \sum_{j=1}^{K} \mathbf{b}_i E[U_i U_j^H] \mathbf{b}_j^H \\ &\overset{(a)}{=} \textstyle\sum_{i=1}^{K} \mathbf{b}_i \mathbf{b}_i^H\end{aligned}$$

where (a) follows from the fact that the codebooks are of unit power and uncorrelated, i.e., $E[U_i U_j^H]$ is equal to 1 for $i = j$ and equal to 0 otherwise. Let $\mathbf{S}_i = \mathbf{b}_i \mathbf{b}_i^H$ be a size $M \times M$ positive semi-definite transmitter covariance matrix for each user which indicates how the codebook U_i is correlated across the M transmit antennas. The power constraint of (3.7) in terms of these transmit covariance matrices translates into

$$\sum_{j=1}^{M} E[|X_j|^2] = E[\mathbf{x}^H \mathbf{x}] = \sum_{i=1}^{K} \mathbf{b}_i^H \mathbf{b}_i = \sum_{i=1}^{K} tr(\mathbf{S}_i) \leq P_t. \qquad (3.9)$$

In order to compute the achievable rates with this precoding scheme, let us analyse the received signal at the users. Let $\mathbf{h}_i$ represent the i^{th} row of the channel matrix $\mathbf{H}$, i.e., $\mathbf{H} = [\mathbf{h}_1^T, \ldots, \mathbf{h}_K^T]^T$. The received vector $\mathbf{y}$ in (3.8) thus becomes

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{z} = \begin{bmatrix} \mathbf{h}_1 \\ \vdots \\ \mathbf{h}_K \end{bmatrix} [\mathbf{b}_1, \ldots, \mathbf{b}_K] \begin{bmatrix} U_1 \\ \vdots \\ U_K \end{bmatrix} + \mathbf{z}.$$

The received signal Y_i at user i is given by

$$\begin{aligned} Y_i &= \mathbf{h}_i \textstyle\sum_{k=1}^{K} \mathbf{b}_k U_k + Z_i \\ &= \mathbf{h}_i \textstyle\sum_{k=1}^{i-1} \mathbf{b}_k U_k + \mathbf{h}_i \mathbf{b}_i U_i + \mathbf{h}_i \textstyle\sum_{k=i+1}^{K} \mathbf{b}_k U_k + Z_i. \end{aligned} \tag{3.10}$$

Note that due to DPC, user i can cancel out the first term, whereas the second term is the useful signal, and the third treated as Gaussian interference. Since U_i's are Gaussian codebooks, with the noise being white Gaussian, the maximum achievable rate for this Gaussian channel is given by $\frac{1}{2}\log(1 + \text{SNR})$. The signal power can be evaluated from (3.10) as

$$\begin{aligned} E[(\mathbf{h}_i \mathbf{b}_i U_i)(\mathbf{h}_i \mathbf{b}_i U_i)^H] &= \mathbf{h}_i \mathbf{b}_i E[U_i U_i^H] \mathbf{b}_i^H \mathbf{h}_i^H \\ &= \mathbf{h}_i \mathbf{S}_i \mathbf{h}_i^H. \end{aligned}$$

Similarly the power of the unknown interference in (3.10) is given by

$$\begin{aligned} E[(\mathbf{h}_i \textstyle\sum_{k=i+1}^{K} \mathbf{b}_k U_k)(\mathbf{h}_i \textstyle\sum_{j=i+1}^{K} \mathbf{b}_j U_j)^H] &= \mathbf{h}_i \textstyle\sum_{k=i+1} \sum_{j=i+1} \mathbf{b}_k E[U_k U_j^H] \mathbf{b}_j^H \mathbf{h}_i \\ &\overset{(b)}{=} \mathbf{h}_i \textstyle\sum_{k=i+1}^{K} \mathbf{S}_k \mathbf{h}_i^H, \end{aligned}$$

where (b) once again is due to the codebooks being uncorrelated. The achievable rate at user i therefore satisfies

$$R_i^{BC} \leq \frac{1}{2} \log \left(1 + \frac{\mathbf{h}_i \mathbf{S}_i \mathbf{h}_i^H}{1 + \mathbf{h}_i \sum_{k=i+1}^{K} \mathbf{S}_k \mathbf{h}_i^H} \right), \quad i = 1, \ldots, K. \tag{3.11}$$

Note that the rate vector $\mathbf{R}^{BC} = [R_1^{BC}, \ldots, R_K^{BC}]$ is achievable under a fixed encoding order, where the message of user i is dirty-paper coded by treating signals for previously encoded users as known interference. One can therefore obtain $K!$ different achievable rate vectors, one for each distinct encoding order. The DPC achievable rate region of the MIMO BC for a fixed channel matrix $\mathbf{H}$ and a power constraint P_t is the union of all rate vectors obtained over all possible encoding orders and all covariance matrices $\mathbf{S}_i$ satisfying the power constraint $\sum_{i=1}^{K} tr(\mathbf{S}_i) \leq P_t$. This region turns out to be convex, and hence this region $\mathcal{R}_{DPC}(P_t, \mathbf{H})$ is given by

$$\mathcal{R}_{DPC}(P_t, \mathbf{H}) = \text{Co} \left(\bigcup_{\pi, \mathbf{S}_i, \ldots \mathbf{S}_K, \mathbf{S}_i \geq 0; \sum_{i=1}^{K} tr(\mathbf{S}_i) \leq P_t} \mathbf{R}^{BC}(\pi, \mathbf{S}_i, \ldots \mathbf{S}_K, \mathbf{H}) \right), \tag{3.12}$$

where Co$(\cdot)$ denotes the "convex hull" operation.

As mentioned above, the rate region in (3.12) was presented in [3]. A number of works [3]–[6] have focused on maximizing the achievable sum-rate with the successive DPC scheme just described. The maximum achievable sum-rate for the fixed encoding order discussed in this section can be written in terms of the following maximization

$$\mathcal{R}_{DPC}^{SR}(\mathbf{H}, P_t) = \max_{\mathbf{S}_1, \ldots, \mathbf{S}_K; \mathbf{S}_i \geq 0, \sum_{j=1}^{K} tr(\mathbf{S}_j) \leq P_t} \sum_{i=1}^{K} \frac{1}{2} \log \left(1 + \frac{\mathbf{h}_i \mathbf{S}_i \mathbf{h}_i^H}{1 + \mathbf{h}_i \sum_{k=i+1}^{K} \mathbf{S}_k \mathbf{h}_i^H} \right) \tag{3.13}$$

The maximization is to be performed over the positive semi-definite transmitter covariance matrices $\mathbf{S}_1, \ldots \mathbf{S}_K$. However, it can be seen that the objective function (3.13) is not a concave function of $\mathbf{S}_1, \ldots \mathbf{S}_K$. Thus, numerically finding the maximum is not easy. However, the authors in [3] were able to derive closed-form expression for $\mathcal{R}_{DPC}^{SR}$ for any number of transmit antennas M but where the total number of users is two, i.e., $K = 2$. Their calculations are based on performing a QR decomposition on the channel matrix, i.e., $\mathbf{H} = \mathbf{GQ}$, where $\mathbf{G}$ is a lower diagonal matrix, and $\mathbf{Q}$ satisfies $\mathbf{QQ}^H = \mathbf{I}_2$. By selecting the precoding matrix $\mathbf{B}$ as $\mathbf{B} = \mathbf{Q}^H\mathbf{R}$, where $\mathbf{R}$ is an upper diagonal matrix, they were able express the sum-rate as a function of the non-zero elements of the matrix $\mathbf{R}$ and evaluate the expressions for this elements which yield the maximum achievable sum-rate over this DPC scheme. This maximum sum-rate was calculated to be

$$\mathcal{R}_{DPC}^{SR} = \begin{cases} \frac{1}{2}\log(1 + ||\mathbf{h}_1||^2), & P_t \leq A, \\ \\ \frac{1}{2}\log\frac{\{P_t \det(\mathbf{HH}^H) + tr(\mathbf{HH}^H)\}^2 - 4||\mathbf{h}_2(\mathbf{h}_1)^H||^2}{4\det(\mathbf{HH}^H)}, & P_t > A, \end{cases} \tag{3.14}$$

where $A = \frac{||\mathbf{h}_1||^2 - ||\mathbf{h}_2||^2}{\det(\mathbf{HH}^H)}$, and it is assumed without loss of generality that $||\mathbf{h}_1|| \geq ||\mathbf{h}_2||$. In addition, they showed that an outer bound on the maximum achievable sum-rate using Sato's technique [34] coincides with the maximum achievable sum-rate of (3.14) indicating that the precoding scheme in fact achieves sum-rate capacity. The biggest disadvantage with the capacity result of [3] however, is that it is not clear how it can be extended to a case with more than two users, and neither does it give an insight into the structure of the optimal covariance matrices $\mathbf{S}_i$'s.

The sum-rate capacity result of [3] has been generalized to more than two users, with each user possibly having more than one receive antennas, separately in [6] and [4]. The former uses the idea of a generalized decision feedback encoder (GDFE), and the latter uses duality of the MIMO BC with MIMO MAC to prove the achievability of the sum-rate capacity. As for the converse part of the capacity result, both works use Sato's upper bound [34] on the achievable rates of a BC. We will separately review the duality result [4] between MIMO BC and MIMO MAC in Section C, since it will be helpful in calculating the precoding matrix for our practical DPC based code design. Let us therefore briefly review how [6] arrives at the capacity result.

The problem in calculating the capacity of a BC lies in the fact that the information is spread across several users, which are not allowed to cooperate amongst each other. Therefore, the capacity cannot be calculated as the mutual information between the input vector $\mathbf{x}$ and the output vector $\mathbf{y}$. The main idea behind the work of [6] is that an equivalent of jointly processing the received signal vectors at the users (receiver cooperation) can be implemented using precoding (using DPC and the precoding matrix $\mathbf{B}$) at the encoder. Consider a Gaussian vector channel given by $\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{z}$. This channel differs from the BC of (3.8) because the vector channel assumes a single user with K receive antennas (thus the received signals can be jointly processed) as opposed to the BC where there are K users each with a single receive antenna which are not allowed to cooperate. The GDFE of [6] is a generalization of a decision-feedback equalizer which is widely used to mitigate the effect of inter-symbol interference (ISI) in linear dispersive channels, where each input symbol is decoded sequentially, the effect of which is subtracted before decoding the next symbol. This DFE is generalized to the vector Gaussian channel, the advantage of which is that it decomposes the vector channel into sub-channels for which encoding and decoding of the elements vector $\mathbf{x}$ can be performed independently. As long as the decision-feedback operation is error free, the achievable sum-rate of these sub-channels is the same as the achievable rate of the original vector channel. Since the encoding of each element of the vector $\mathbf{x}$ can be done independently, transmitter cooperation is not necessary to achieve the capacity. However, receiver cooperation is required since in a GDFE the entire received vector $\mathbf{y}$ should be input to a feedforward filter, and secondly the feedback operation requires correct codeword from one sub-channel to be available for correct decoding of the codewords for other sub-channels. Thus GDFE is naturally suited to MACs. The main result of [6] is that this GDFE structure can be moved to the transmitter, and is equivalent to a DPC based precoder. The sum-rate capacity of the BC can be upper bounded by the capacity of the vector channel $I(\mathbf{x};\mathbf{y})$, where the $Y_1,\ldots,Y_K$ are allowed to cooperate. Since in a BC, the users do not cooperate, the BC capacity only depends on the marginal statistics of the noise vector, and not on the joint statistics. For Gaussian vector $\mathbf{x}$, $I(\mathbf{x};\mathbf{x}) = \frac{1}{2}\log\left(\frac{|\mathbf{H}\mathbf{S}_{xx}\mathbf{H}^H+\mathbf{S}_{zz}|}{|\mathbf{S}_{zz}|}\right)$, where $\mathbf{S}_{zz}$ is the covariance matrix for the noise vector $\mathbf{z}$. An outer bound can be found by the following optimization problem

$$\min_{\mathbf{S}_{zz}} \frac{1}{2}\log\left(\frac{|\mathbf{H}\mathbf{S}_{xx}\mathbf{H}^H+\mathbf{S}_{zz}|}{|\mathbf{S}_{zz}|}\right) \quad \text{subject to} \quad \mathbf{S}_{zz}(i,i)=1, \quad i=1,\ldots,K. \tag{3.15}$$

Thus the optimization is over all off-diagonal elements of $\mathbf{S}_{zz}$. The noise vector whose statistics minimize the objective function of (3.15) is referred to as the "least favorable noise". The outer bound should be maximized over the input covariance matrix $\mathbf{S}_{xx}$, and can be written as

$$\mathcal{R}^{SR} = \min_{\mathbf{S}_{zz}} \max_{\mathbf{S}_{xx}} \frac{1}{2}\log\left(\frac{|\mathbf{H}\mathbf{S}_{xx}\mathbf{H}^H+\mathbf{S}_{zz}|}{|\mathbf{S}_{zz}|}\right) \quad \text{subject to} \quad \begin{array}{c}\mathbf{S}_{zz}(i,i)=1, \quad i=1,\ldots,K, \\ tr(\mathbf{S}_{xx}) \le P_t\end{array} \tag{3.16}$$

Thus the sum capacity is a saddle point of a Gaussian mutual information game where the signal player chooses the transmit covariance matrix to maximize the mutual information, while an imaginary noise player chooses the noise covariance matrix to minimize the mutual information. The achievability of the outer bound is proved by showing an existence of the precoding matrix $\mathbf{B}$ such that when the GDFE designed for this $\mathbf{B}$ and the least favorable noise $\mathbf{S}_{zz}$ is moved to the transmitter to form the equivalent DPC based precoder, it results in a diagonal feedforward filter at the receiver indicating that no user cooperation is required, and thus the receiver is not concerned about the cross correlation between the noises. Since the marginal distributions of the least favorable noise are the same as those of the actual noise, the DPC precoder with the precoding matrix **B** achieves the outer bound. This saddle point corresponding to the sum-rate capacity can be found iteratively by first computing the best input covariance matrix $\mathbf{S}_{xx}$ for a given noise covariance, and then computing the least favorable noise covariance matrix $\mathbf{S}_{zz}$ for the given input covariance, until the iterative process converges.

It was shown recently in [7] that the achievable rate region of the successive DPC scheme characterized by (3.11) not only achieves the sum rate capacity as shown in [3]–[6] but in fact achieves the full capacity region. The main idea behind their proof was an introduction of the concept of an enhanced channel. According to [7] for every point on the boundary of the achievable rate region of a degraded MIMO BC, an enhanced channel can be found which contains the achievable rate region (due to Gaussian coding) of the original channel. The same Gaussian coding scheme which was used to obtain the points on the achievable rate region in the original channel can be used to achieve the same rate points in the enhanced channel. Using entropy power inequality (the same idea as that of Bergman [2]) it can be shown that this point lies on the capacity region boundary of the enhanced channel. As this can be repeated for every point on the achievable rate region, the achievable rate region due to Gaussian coding is indeed the capacity region.

C. Duality between the Gaussian BC and MAC

The duality between the capacity regions of the Gaussian MIMO BC and MAC was pointed out in [4, 5]. Unlike the achievable rates for the MIMO BC, given by (3.11), the rates for the MAC are concave functions of the input covariances. Therefore,

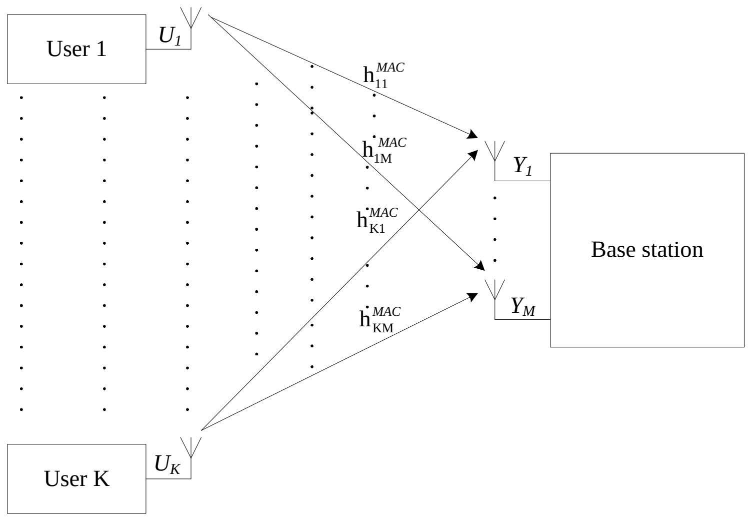

Fig. 13. A multiple access channel with K users each with a single transmit antenna sending messages simultaneously to a base station with M receive antennas. The channel between user i and antenna j of the base station experiences a fading coefficient of h_{ij}^{MAC}.

it is easier to find the boundary of the capacity region of the MAC than that of the BC. By exploiting the duality [4, 5], the achievable rates for the MIMO BC can be derived from those of its dual MAC. This fact can be helpful in evaluating the minimum transmitter power required to achieve a certain point on the capacity region boundary. Indeed we will show in the next chapter that the optimal precoding matrix **B** for our practical DPC based code design can be evaluated by invoking this duality. In the following, we first discuss the dual MIMO MAC and the corresponding channel capacity. We will then briefly review the duality principle.

1. MIMO Multiple Access Channel

a. Channel Model

A Gaussian MAC is shown in Fig. 13 where K users, each with a single antenna transmit messages simultaneously over flat Rayleigh fading channels to a single receiver with M receive antennas. The i^{th} user transmits a codeword U_i which is uncorrelated with the codewords for the other users. The received signal vector is given by

$$\mathbf{y} = \sum_{i=1}^{K} U_i \mathbf{h}_i^{MAC} + \mathbf{n}, \tag{3.17}$$

where $\mathbf{n}$ is an i.i.d. Gaussian noise vector independent of the U_i's with its covariance matrix equal to $\mathbf{I}_M$, and $\mathbf{h}_i^{MAC}$ is a column vector comprising of the fading coefficients of the channels between user i and the M receive antennas of the base station, i.e., $\mathbf{h}_i^{MAC} = [h_{i1}^{MAC}, \ldots, h_{iM}^{MAC}]^T$. The individual transmission power of each user is given by $\xi_i = E[|U_i^2|]$. The sum-power constraint therefore is given by $\sum_{i=1}^{K} \xi_i \leq P_t^{MAC}$.

b. Channel Capacity

An achievable rate region of MIMO MAC can be found using successive decoding at the receiver, where all codewords U_i, $i = 1, \ldots, K$ are drawn from independent Gaussian codebooks. Assuming that the receiver decodes the message for user K first, it treats the signals from all other users as unknown interference. Thus the useful signal vector is $U_K \mathbf{h}_i^{MAC}$, while the unwanted interference plus noise term is $\sum_{i=1}^{K-1} U_i \mathbf{h}_i^{MAC} + \mathbf{n}$. Once the message for user K is decoded without error, the term $U_K \mathbf{h}_K^{MAC}$ can be subtracted from $\mathbf{y}$ and the result used to decode message for user $K-1$. Hence for decoding U_{K-1}, only the signals from user 1 through $K-2$ contribute to the unwanted interference. Using the same line of reasoning, the unwanted interference term at any user i $(i = 2, \ldots, K)$ comprises of signals from user 1 through $i-1$ only. For user 1, there is no unwanted interference term left. Thus the useful signal vector at user i $(i = 1, \ldots, K)$ is

$$\mathbf{x}_i = U_i \mathbf{h}_i^{MAC},$$

whereas the unwanted interference plus noise vector is

$$\mathbf{z}_i = \sum_{k=1}^{i-1} U_k \mathbf{h}_k^{MAC} + \mathbf{n}.$$

The achievable rates for such a Gaussian vector channel at user i should satisfy $R^i \leq \frac{1}{2}\log\left(\frac{|\mathbf{K}_x^i + \mathbf{K}_z^i|}{|\mathbf{K}_z^i|}\right)$, where $\mathbf{K}_x^i$ and $\mathbf{K}_z^i$ are the covariance matrices of $\mathbf{x}_i$ and $\mathbf{z}_i$, respectively. These covariance matrices can be calculated as

$$\begin{aligned}\mathbf{K}_x^i &= E[\mathbf{x}_i\mathbf{x}_i^H] \\ &= \mathbf{h}_i^{MAC} E[|U_i|^2] \mathbf{h}_i^{MAC\ H} \\ &= \mathbf{h}_i^{MAC} \xi_i \mathbf{h}_i^{MAC\ H}\end{aligned}$$

and

$$\begin{aligned}\mathbf{K}_z^i &= E[\mathbf{z}_i\mathbf{z}_i^H] \\ &\stackrel{(c)}{=} \textstyle\sum_{k=1}^{i-1}\sum_{j=1}^{i-1} \mathbf{h}_k^{MAC} E[U_i U_j^*] \mathbf{h}_j^{MAC\ H} + \mathbf{I}_M \\ &\stackrel{(d)}{=} \textstyle\sum_{k=1}^{i-1} \mathbf{h}_k^{MAC} \xi_k \mathbf{h}_k^{MAC\ H} + \mathbf{I}_M\end{aligned}$$

where (c) is due to the fact that the noise vector is independent of the channel inputs, and (d) due to the codebooks being uncorrelated. Hence the achievable rate for user i should satisfy

$$R_i^{MAC} \leq \frac{1}{2}\log\left(\frac{|\mathbf{I}_M + \sum_{k=1}^{i} \xi_k \mathbf{h}_k^{MAC}\mathbf{h}_k^{MAC\ H}|}{|\mathbf{I}_M + \sum_{k=1}^{i-1} \xi_k \mathbf{h}_k^{MAC}\mathbf{h}_k^{MAC\ H}|}\right). \quad i = 1, \dots, K \tag{3.18}$$

The achievable rate region is the union of all rate vectors obtained over all possible encoding orders and over all possible input covariances $\{\xi\}_{i=1}^{K}$ which satisfy the power constraint $\sum_{i=1}^{K} \xi_i \leq P_t^{MAC}$. The rate region characterized by (3.18) is in fact the capacity region [35] of the MIMO MAC of Fig. 13.

2. Duality

According to [4] a duality exists between the MIMO BC and MIMO MAC. Specifically, if the power constraint for both are the same, i.e., $P_t = P_t^{MAC}$, and if the MAC channel vectors $\{\mathbf{h}_i^{MAC}\}_{i=1}^{K}$ are related to the BC channel vectors $\{\mathbf{h}_i\}_{i=1}^{K}$ by $\mathbf{h}_i^{MAC} = \mathbf{h}_i^{H}$, the achievable rates in (3.11) of the BC domain are exactly equal to the achievable rates in (3.18) of the MAC domain, i.e., $R_i^{MAC} = R_i^{BC}$ for $i = 1, \ldots, K$. Note that the encoding order of the BC is reverse of the decoding order of the MAC, e.g., when in BC, the message for user i is encoded by treating signals from user 1 through $i-1$ as encoder side information, in its dual MAC, when decoding the message for user i, the signals from user $i+1$ through K should be treated as known interference.

As explained earlier, it is generally much easier to find the optimum input covariances in the MAC domain which maximize the weighted sum of the achievable MAC rates (since the rate in (3.18) is a concave function of ξ_i's) than it is to do the same in the BC domain. Since $R_i^{MAC} = R_i^{BC}$, the BC optimization could be performed by performing the same optimization in the MAC domain over its corresponding covariances. [4] shows the existence of a transformation of the covariances from the MAC domain to the BC domain, and vice versa. Hence the optimal BC covariances could be evaluated by applying the MAC to BC transformation on the MAC covariances. We thus review these transformations in the proceeding subsections. The precise set of circumstances where this duality comes in handy for our code designs would become clear in Chapter IV. Although we will only require the MAC to BC transformations, for the sake of completeness, we list the BC to MAC transformation as well.

a. MAC to BC Transformation

Given a set of covariances $\{\xi_i\}_{i=1}^K$ of a MAC channel (described by (3.17) with $\mathbf{h}_i^{MAC} = \mathbf{h}_i^H$) that satisfy the power constraint $\sum_{i=1}^K \xi_i \leq P_t$, a one to one transformation from ξ_i's to the BC covariance matrices $\mathbf{S}_i$'s can be defined as a function of the $\mathbf{h}_i$'s [4] such that the $\mathbf{S}_i$'s satisfy the same power constraint $\sum_{i=1}^K tr(\mathbf{S}_i) \leq P_t$, and such that $R_i^{MAC} = R_i^{BC}$ for $i = 1, \ldots, K$. This transformation is as follows. Let

$$\begin{aligned} c_i &= 1 + \mathbf{h}_i \textstyle\sum_{k=i+1}^K \mathbf{S}_k \mathbf{h}_i^H \\ \text{and } \mathbf{D}_i &= \mathbf{I}_M + \textstyle\sum_{k=1}^{i-1} \xi_k \mathbf{h}_k^H \mathbf{h}_k \end{aligned} \qquad 1 \leq i \leq K. \tag{3.19}$$

If $\mathbf{D}_i^{-\frac{1}{2}} \mathbf{h}_i^H c_i^{-\frac{1}{2}} = g\mathbf{f}_i$ such that $\mathbf{f}_i^H \mathbf{f}_i = 1$ with some constant g, then the corresponding covariance matrix $\mathbf{S}_i$ $(1 \leq i \leq K)$ for the MIMO BC can be computed as

$$\mathbf{S}_i = c_i \xi_i \mathbf{D}_i^{-\frac{1}{2}} \mathbf{f}_i \mathbf{f}_i^H \mathbf{D}_i^{-\frac{1}{2}}. \tag{3.20}$$

Since $\mathbf{S}_i = \mathbf{b}_i \mathbf{b}_i^H$, the precoding vector $\mathbf{b}_i$ can be obtained as

$$\mathbf{b}_i = \sqrt{c_i \xi_i} \mathbf{D}_i^{-\frac{1}{2}} \mathbf{f}_i. \tag{3.21}$$

b. BC to MAC Transformation

Given a set of BC covariance matrices $\{\mathbf{S}_i\}_{i=1}^K$, a transformation can be defined to obtain the MAC covariances which satisfy the same power constraint as that of their BC counterparts, and that achieve exactly the same rates in the MAC domain as the BC covariance matrices do in the BC domain [4]. This transformation is similar to the MAC to BC transformation. This transformation is as follows. If $\mathbf{D}_i$ and c_i are defined by (3.19), and if $\mathbf{D}_i^{-\frac{1}{2}} \mathbf{h}_i^H c_i^{-\frac{1}{2}} = g\mathbf{f}_i$ such that $\mathbf{f}_i^H \mathbf{f}_i = 1$ with some constant g, then the MAC covariances can be calculated from

$$\xi_i = c_i \mathbf{f}_i^H \mathbf{D}_i^{-\frac{1}{2}} \mathbf{S}_i \mathbf{D}_i^{-\frac{1}{2}} \mathbf{f}_i \tag{3.22}$$

CHAPTER IV

CODE DESIGN FOR MIMO BCS

Practical code design for the MIMO BC not only requires designing practical dirty-paper and channel codes, but it also calls for selecting an appropriate precoding matrix $\mathbf{B}$ which caters for the practical aspects of the dirty-paper/channel codes. Previous information-theoretical works [3]–[6] have mainly focused on evaluating $\mathbf{B}$ to maximize the achievable sum-rate, in which case the encoder and decoder for each user must be able to operate at an arbitrary code rate. However, it is impractical to design good channel codes and dirty-paper codes at different rates on the fly. Therefore, we consider a more practical scenario in which each user is assumed to operate at a fixed transmission rate, for which we already have an efficient dirty-paper code design (TTCQ/TTCM scheme of Chapter II – Section D). Thus, instead of having one single sum-rate constraint, we have K separate rate constraints $-$ one for each user (As a reminder to the reader, K is the number of users). When compared to the former case, the latter will lead to performance loss because we cannot now arbitrarily allocate transmission rates to different users. Thus we will discuss approaches to selecting the precoding matrix $\mathbf{B}$ under K individual rate constraints in Section A. We will convert these rate constraints to individual SNR constraints; the two of which are equivalent. With these individual SNR constraints, it would be easier for us, as will be shown, to cater for the fact that the practical DPC and channel coding schemes require a higher SNR than if they were ideal. Besides the optimal approach to selecting the precoding matrix, we will also present a few suboptimal approaches and provide a complexity comparison of these approaches to the optimal scheme. We will then present the overall scheme involving the TTCQ/TTCM DPC scheme in Section B, followed by the simulation results in Section C.

A. Optimal Precoding under Individual Rate Constraints

In this section we discuss approaches to evaluating the precoding matrix $\mathbf{B}$, when the practical channel/dirty-paper codes are required to operate at fixed transmission rates. Let R_i be the rate of the practical channel code ($i = 1$) and the dirty-paper code ($2 \leq i \leq K$). If the rate-R_i code performs δ_i dB away (at a certain BER) from the corresponding Shannon limit of $10\log_{10}(2^{2R_i} - 1)$ dB, its operating SNR η_i must satisfy

$$\eta_i \geq \eta_i^o = 10\log_{10}(2^{2R_i} - 1) + \delta_i \text{ dB}. \tag{4.1}$$

For example, the dirty-paper code of [11], that we use in this work, performs $\delta_i = 1.53$ dB away from the Shannon limit at $R_i = 1.0$ b/s and a BER of 10^{-5}. Since there is a one-to-one correspondence between the rate R_i of the practical code and its minimum operating SNR η_i^o given by (4.1), in the subsequent sections, we exclusively speak of the SNR constraints instead of the rate constraints at different users.

Our optimal choice of the precoding matrix in practice minimizes the total transmission power P_t such that the SNR requirement (4.1) at each user is satisfied. We perform this power minimization while assuming a fixed encoding order. A search for an encoding order that minimizes the total power is then needed. In our simulations, since we deal with two or three users, we use the brute force approach. Since the total number of encoding orders is $K!$, the brute force method will not be feasible when the number of users K is larger. In this case, the iterative algorithm proposed in [15] can be adopted to find the optimal encoding order.

In the following, we discuss optimal precoding for the case of a degraded Gaussian BC [1] before moving on to the case of MIMO BCs, for which we also mention a few suboptimal precoding approaches.

1. Precoding for Two-User Degraded Gaussian BC

The problem here is to find a precoding matrix $\mathbf{B}$ that minimizes the total transmission power subject to the individual SNR constraint on η_1 and η_2. The achievable rates for the two-user degraded Gaussian BC were provided in (3.3) and (3.4), which we reproduce here for convenience.

$$\begin{aligned} R_1 &\leq \tfrac{1}{2}\log\left(1 + \tfrac{(1-\gamma)P_t}{\gamma P_t + N_1}\right), \\ R_2 &\leq \tfrac{1}{2}\log\left(1 + \tfrac{\gamma P_t}{N_2}\right). \end{aligned}$$

The received SNRs at the two users are therefore given by $\eta_1 = \frac{(1-\gamma)P_t}{\gamma P_t + N_1}$, and $\eta_2 = \frac{\gamma P_t}{N_2}$. Also recall that the precoding matrix is given by $\mathbf{B} = [\sqrt{(1-\gamma)P_t}, \sqrt{\gamma P_t}]$. Hence optimal precoding requires evaluating the parameter γ, which would minimize the transmitter power P_t subject to the SNR constraints in (4.1). It can be shown that P_t is an increasing function of both η_1 and η_2, therefore, the received SNRs at both users should be equal to the minimum required values. Thus

$$\begin{aligned} \eta_1 &= \tfrac{(1-\gamma)P_t}{\gamma P_t + N_1} = \eta_1^o \quad \text{and} \\ \eta_2 &= \tfrac{\gamma P_t}{N_2} = \eta_2^o. \end{aligned} \tag{4.2}$$

Hence, the optimum γ can be found by solving the two equations in (4.2) for it. The solution yields

$$\gamma' = \frac{\eta_2^o N_2}{\eta_2^o N_2(\eta_1^o + 1) + \eta_1^o N_1} \tag{4.3}$$

resulting in a minimum transmitter power of

$$P_t' = \eta_2^o N_2(\eta_1^o + 1) + \eta_1^o N_1 \tag{4.4}$$

2. Precoding for Non-Degraded MIMO BC

In this case, we want to find a precoding matrix $\mathbf{B}$ that minimizes the total transmission power P_t while satisfying the individual SNR constraints of (4.1). That is, if π

is a fixed encoding order and $\mathbf{H}$ the known channel matrix, we have an optimization problem of

$$\min_{\pi} \min_{\mathbf{B}} P_t(\mathbf{B}, \mathbf{H}, \pi) \text{ subject to } \eta_i \geq \eta_i^o, \quad i = 1, \ldots, K, \tag{4.5}$$

where η_i^o's are given by (4.1). Since the transmitter covariance matrices $\mathbf{S}_i$'s are directly related to $\mathbf{B}$ via $\mathbf{S}_i = \mathbf{b}_i \mathbf{b}_i^H$, (4.5) is equivalent to

$$\min_{\pi} \min_{\mathbf{S}_1, \ldots, \mathbf{S}_K} P_t(\mathbf{S}_1, \ldots, \mathbf{S}_K, \mathbf{H}, \pi) \text{ subject to } \eta_i \geq \eta_i^o, \quad i = 1, \ldots, K. \tag{4.6}$$

However, direct minimization of P_t is not easy since it is in general not a convex function of $\mathbf{S}_i's$. In the following, The calculations presented in the proceeding subsections are for a fixed encoding order. For a brute force approach to finding the best encoding order, these calculations can be repeated for $K!$ encoding orders to find the one which yields the minimum transmitter power.

a. Optimal Precoding

Instead of directly minimizing (4.6), we outline two alternative methods for determining the optimal precoding matrix $\mathbf{B}$. The first one follows [15] and is based on the duality between BC and MAC. In the second approach, we give a closed-form expression for the optimal $\mathbf{B}$ and use it to directly compute the optimal precoding matrix; this approach is less complex than the first, but it is optimal only for the two-user setup.

Duality-based Approach: Recall that the transmission power for the MIMO BC equals the transmission power for its dual MAC, i.e., $\sum_{i=1}^{K} tr(\mathbf{S}_i) = \sum_{i=1}^{K} \xi_i \leq P_t$. Moreover, the achievable rates in both domains are the same as well, which implies the equivalence of the received SNRs too. Hence the SNR constraints of the BC domain described by (4.1) are equally applicable to the dual MAC. Therefore the minimization in (4.6) is equivalent to minimizing P_t with respect to covariances ξ_i's of the dual MAC. According to (3.18), the achievable rate for user 1 in the dual MIMO MAC is $R_1 \leq \frac{1}{2}\log(1+\xi_1\mathbf{h}_1\mathbf{h}_1^H)$. Thus $\eta_1 = \xi_1\mathbf{h}_1\mathbf{h}_1^H$. In order to satisfy the SNR requirement at user 1, we must have

$$\xi_1 \geq \frac{\eta_1^o}{\mathbf{h}_1\mathbf{h}_1^H}. \tag{4.7}$$

Since the achievable rate R_i^{MAC} at user i for the dual MAC is a function of the covariances of only the preceding users (a function of users j with $j \leq i$), ξ_i's can be calculated recursively via

$$\xi_i \geq \frac{\eta_i^o}{\mathbf{h}_i(\mathbf{I}_M + \sum_{k=1}^{i-1}\xi_k\mathbf{h}_k^H\mathbf{h}_k)^{-1}\mathbf{h}_i^H}. \tag{4.8}$$

The transmission power is minimized when the SNR requirements are satisfied with equality, i.e., when equality in (4.8) holds. Once the ξ_i's that minimize the transmission power for the particular encoding order are known, we can apply the transformation outlined in Chapter III- Section C to obtain the optimal covariance matrices $\mathbf{S}_i$'s and hence the optimal precoding matrix $\mathbf{B}$.

Direct Calculations: The above duality-based approach is optimal, but computationally complex. Therefore, we derive a simpler approach by directly computing the precoding matrix $\mathbf{B}$ that minimizes the transmission power. For the case of two users, we derive the optimal solution for $\mathbf{B}$ [18], whereas for the case of three or more users we provide a suboptimal solution, which is close to the optimal one obtained from the duality-based approach.

In deriving the optimal $\mathbf{B}$, we modify the method of [3], which is developed under the sum-rate constraint, to suit our setup with individual rate/SNR constraints. Let $\mathbf{H} = \mathbf{G}_{K\times K}\mathbf{Q}_{K\times M}$ be the QR decomposition of the channel matrix obtained by Gram-Schmidt orthogonalization, where $\mathbf{G}$ is a lower diagonal matrix, i.e., $g_{ij} = 0$ for

$j > i$, and $\mathbf{Q}$ satisfies $\mathbf{Q}\mathbf{Q}^H = \mathbf{I}_K$. As before, M denotes the number of transmitter antennas, and K is the number of users (two or three in our case) each with a single antenna.

The precoding matrix is chosen as $\mathbf{B} = \mathbf{Q}^H \mathbf{R}_{K\times K}$, where without loss of generality $\mathbf{R}$ is a complex upper diagonal matrix, i.e., $r_{ij} = 0$ for $j < i$. The power constraint becomes

$$\begin{aligned} E[\mathbf{x}^H\mathbf{x}] &= E[\mathbf{u}^H\mathbf{R}^H\mathbf{Q}\mathbf{Q}^H\mathbf{R}\mathbf{u}] \\ &= E[\mathbf{u}^H\mathbf{R}^H\mathbf{R}\mathbf{u}] \\ &= tr(\mathbf{R}E[\mathbf{u}\mathbf{u}^H]\mathbf{R}^H) \\ &\overset{(a)}{=} tr(\mathbf{R}\mathbf{R}^H) \\ &\overset{(b)}{=} \textstyle\sum_{i=1}^K \sum_{j=i}^K |r_{ij}|^2 \le P_t. \end{aligned}$$

where (a) is due to the fact that $E[\mathbf{u}\mathbf{u}^H] = \mathbf{I}_K$, and (b) is due to $\mathbf{R}$ being an upper diagonal matrix. Let $\mathbf{q}_j$ be the j^{th} row ($j = 1, \ldots, K$) of the matrix $\mathbf{Q}$, i.e. $\mathbf{Q} = [\mathbf{q}_1^T, \ldots, \mathbf{q}_K^T]^T$. Then, the i^{th} row of the channel matrix $\mathbf{H}$ is given by $\mathbf{h}_i = \sum_{j=1}^i g_{ij}\mathbf{q}_j$. Similarly, the i^{th} column of the precoding matrix $\mathbf{B}$ is given by $\mathbf{b}_i = \sum_{j=1}^i r_{ji}\mathbf{q}_j^H$. The received SNR at user i from (3.11) is evaluated as $\eta_i = \frac{\mathbf{h}_i\mathbf{S}_i\mathbf{h}_i^H}{1+\mathbf{h}_i\sum_{k=i+1}^K \mathbf{S}_k\mathbf{h}_i^H}$. We need to write this received SNR as a function of the elements of the matrix $\mathbf{G}$ and $\mathbf{R}$. The signal power $\mathbf{h}_i\mathbf{S}_i\mathbf{h}_i^H$ can be evaluated as

$$\begin{aligned} \mathbf{h}_i\mathbf{S}_i\mathbf{h}_i^H &= \mathbf{h}_i\mathbf{b}_i\mathbf{b}_i^H\mathbf{h}_i^H \\ &= \textstyle\sum_{j=1}^i \sum_{k=1}^i \sum_{l=1}^i \sum_{m=1}^i g_{ij} r_{ki} g_{mj}^* r_{li}^* \mathbf{q}_j \mathbf{q}_k^H \mathbf{q}_l \mathbf{q}_m^H \\ &= \textstyle\sum_{j=1}^i \sum_{l=1}^i g_{ij} r_{ji} g_{lj}^* r_{li}^* \quad \text{Since } \mathbf{q}_j\mathbf{q}_k^H = \delta_{jk} \\ &= |\textstyle\sum_{j=1}^i g_{ij} r_{ji}|^2 \end{aligned}$$

Thus the received SNR is

$$\eta_i = \frac{\mathbf{h}_i\mathbf{S}_i\mathbf{h}_i^H}{1 + \mathbf{h}_i\sum_{k=i+1}^K \mathbf{S}_k\mathbf{h}_i^H} = \frac{|\sum_{j=1}^i g_{ij} r_{ji}|^2}{1 + \sum_{k=i+1}^K |\sum_{j=1}^i g_{ij} r_{jk}|^2}. \tag{4.9}$$

In the two-user case, the SNR constraints become

$$\frac{|g_{11}r_{11}|^2}{1+|g_{11}r_{12}|^2} \geq \eta_1^o \tag{4.10}$$

$$|g_{21}r_{12}+g_{22}r_{22}|^2 \geq \eta_2^o. \tag{4.11}$$

(We note that these are in fact our individual rate constraints expressed in terms of the SNR constraints.) One needs now to minimize the total transmtter power $P_t = |r_{11}|^2 + |r_{12}|^2 + |r_{22}|^2$ subject to the constraints in (4.10). In this optimization problem, there are six unknowns: the magnitudes and the phases of r_{11}, r_{12}, and r_{22}. The optimum choices of phases for r_{12} and r_{22} are such that $|g_{21}r_{12}+g_{22}r_{22}|^2 = (|g_{21}r_{12}|+|g_{22}r_{22}|)^2$. The phase of r_{11} is irrelevant since it is not involved in the rate equations. Thus (4.10) and (4.11) can be re-written as

$$|r_{11}|^2 \geq \frac{\eta_1^o(1+|g_{11}r_{12}|^2)}{|g_{11}|^2}$$

$$|r_{22}|^2 \geq \frac{(\sqrt{\eta_2^o}-|g_{21}r_{12}|)^2}{|g_{22}|^2}, \tag{4.12}$$

with $0 \leq |r_{12}| \leq \frac{\sqrt{\eta_2^o}}{|g_{21}|}$. Thus the total transmitter power P_t can be lower bounded by a function of $|r_{12}|$ only, via

$$P_t \geq \frac{\eta_1^o(1+|g_{11}r_{12}|^2)}{|g_{11}|^2} + |r_{12}|^2 + \frac{(\sqrt{\eta_2^o}-|g_{21}r_{12}|)^2}{|g_{22}|^2}. \tag{4.13}$$

Since the right hand side of (4.13) is a convex function of $|r_{12}|$, differentiating it with respect to $|r_{12}|$ gives a minima at

$$|r_{12}|' = \min\left(\frac{\sqrt{\eta_2^o}|g_{21}|}{|g_{22}|^2+|g_{22}|^2\eta_1^o+|g_{21}|^2}, \frac{\sqrt{\eta_2^o}}{|g_{21}|}\right). \tag{4.14}$$

Since $r_{21} \equiv 0$, (4.12) and (4.14) completely specify the optimal choice for $\mathbf{R}$, hence the optimal precoding matrix $\mathbf{B} = \mathbf{Q}^H\mathbf{R}$.

We perform a similar analysis for the three-user case. From (4.9) the individual

SNR requirements in this case are

$$\frac{|g_{11}r_{11}|^2}{1+|g_{11}r_{12}|^2+|g_{11}r_{13}|^2} \geq \eta_1^o, \quad (4.15)$$

$$\frac{|g_{21}r_{12}+g_{22}r_{22}|^2}{1+|g_{21}r_{13}+g_{22}r_{23}|^2} \geq \eta_2^o, \quad (4.16)$$

$$|g_{31}r_{13}+g_{32}r_{23}+g_{33}r_{33}|^2 \geq \eta_3^o. \quad (4.17)$$

In this problem, there are twelve unknowns: the phases and magnitudes of the six nonzero elements of the matrix $\mathbf{R}$. As means of simplifying the problem, we sub-optimally force the interference at user 2 to zero by choosing

$$r_{23} = -\frac{g_{21}}{g_{22}} r_{13}. \quad (4.18)$$

Substituting (4.18) in (4.17), we get

$$|(g_{31} - \frac{g_{32}g_{21}}{g_{22}})r_{13} + g_{33}r_{33}|^2 \geq \eta_3^o. \quad (4.19)$$

Let $t_g = g_{31} - \frac{g_{32}g_{21}}{g_{22}}$. Then, the choices for the phases of r_{13} and r_{33} should be such, such that $|t_g r_{13} + g_{33}r_{33}| = |t_g r_{13}| + |g_{33}r_{33}|$. Similarly, the phases of r_{12} and r_{22} should be chosen such that $|g_{21}r_{12} + g_{22}r_{22}| = |g_{21}r_{12}| + |g_{22}r_{22}|$. The choice of these phases in (4.15)–(4.19) yields the following constraints

$$\begin{aligned} |r_{11}|^2 &\geq \frac{\eta_1^o(1+|g_{11}r_{12}|^2+|g_{11}r_{13}|^2)}{|g_{11}|^2}, \\ |r_{22}| &\geq \frac{\sqrt{\eta_2^o} - |g_{21}r_{12}|}{|g_{22}|}, \\ |r_{33}| &\geq \frac{\sqrt{\eta_3^o} - |t_g r_{13}|}{|g_{33}|}, \\ |r_{23}| &= \frac{|g_{21}|}{|g_{22}|}|r_{13}|. \end{aligned} \quad (4.20)$$

The total transmitter power is $P_t = |r_{11}|^2 + |r_{12}|^2 + |r_{13}|^2 + |r_{22}|^2 + |r_{23}|^2 + |r_{33}|^2$. Substituting (4.20) into P_t and setting its derivative with respect to $|r_{12}|$ and $|r_{13}|$ to zero, we get the optimal $|r_{12}|'$ and $|r_{13}|'$ as

$$|r_{12}|' = \min\left(\frac{|g_{21}|\sqrt{\eta_2^o}}{|g_{22}|^2(1+\eta_1^o)+|g_{21}|^2}, \frac{\sqrt{\eta_2^o}}{|g_{21}|}\right), \tag{4.21}$$

$$|r_{13}|' = \min\left(\frac{|t_g|\sqrt{\eta_3^o}}{|g_{33}|^2(1+\eta_1^o+\frac{|g_{21}|^2}{|g_{22}|^2})+|t_g|^2}, \frac{\sqrt{\eta_3^o}}{|t_g|}\right). \tag{4.22}$$

Together with (4.20), the above two equations give a suboptimal choice for the upper diagonal matrix $\mathbf{R}$, for which the precoding matrix is evaluated as $\mathbf{B} = \mathbf{Q}^H\mathbf{R}$. We point out that, in contrast to the two-user case, where we find the closed-form expression for the optimal $\mathbf{B}$, for the three-user setup, our choice with $r_{23} = -\frac{g_{21}}{g_{22}}r_{13}$ does not guarantee power minimization and is therefore suboptimal.

When the number of users K is more than three, we provide a suboptimal yet simple extension of the above precoding strategy. Since $K > 3$, we first apply the three-user precoding technique to users K, $K-1$, and $K-2$, by performing a QR decomposition on the last three rows of the channel matrix $\mathbf{H}$, denoted by $\mathbf{h}_K$, $\mathbf{h}_{K-1}$, and $\mathbf{h}_{K-2}$. This provides us with a suboptimal choice of the last three columns $\mathbf{b}_K$, $\mathbf{b}_{K-1}$, and $\mathbf{b}_{K-2}$ of the precoding matrix $\mathbf{B}$. We now need to evaluate $\mathbf{b}_i$'s for $i \leq K-3$. Recall that the received SNR constraint at user $K-3$ is given by

$$\frac{\mathbf{h}_{K-3}\mathbf{b}_{K-3}\mathbf{b}_{K-3}^H\mathbf{h}_{K-3}^H}{1+\mathbf{h}_{K-3}\sum_{j=K-2}^{K}\mathbf{b}_j\mathbf{b}_j^H\mathbf{h}_{K-3}^H} \geq \eta_{K-3}^o. \tag{4.23}$$

Note that the denominator is a known quantity, since $\mathbf{b}_j$'s for $j \geq K-2$ have already been evaluated. Denote the denominator of (4.23) as c_{K-3}. Then we choose $\mathbf{b}_{K-3}$ such that the individual contribution of $\mathbf{S}_{K-3}$ to the total transmitter power is minimized. Hence we choose $\mathbf{b}_{K-3}$ such that $tr(\mathbf{S}_{K-3}) = \mathbf{b}_{K-3}^H\mathbf{b}_{K-3}$ is minimized (Recall that the total transmission power is $P_t = \sum_{i=1}^K tr(\mathbf{S}_i) = \sum_{i=1}^K \mathbf{b}_i^H\mathbf{b}_i$) subject to the constraint of (4.23). This optimization results in the following choice of $\mathbf{b}_{K-3}$

$$\mathbf{b}_{K-3} = \sqrt{\eta_{K-3}^o c_{K-3}}\frac{\mathbf{h}_{K-3}^H}{|\mathbf{h}_{K-3}|^2}.$$

This choice is clearly suboptimal since it does not depend on $\mathbf{b}_j$'s or $\mathbf{h}_j$'s for $j < K-3$. Similarly, the rest of $\mathbf{b}_j$'s can be chosen such that their individual contribution to the total power is minimized. This leads to the following step down recursion:

$$\mathbf{b}_j = \sqrt{\eta_j^o c_j}\frac{\mathbf{h}_j^H}{||\mathbf{h}_j||^2}, \quad j = K-3, \ldots, 1. \tag{4.24}$$

We note that with increasing number of users, we expect the performance gap of this suboptimal scheme from the optimal to increase.

b. Zero-Forcing

We briefly mention the two suboptimal approaches considered in [3], namely, zero-forcing DPC (ZFDPC) and zero-forcing beamforming (ZFBF) [36]. As the name zero-forcing suggests, the choice of the precoding matrix forces the interference at each user to be zero, and hence induces K non-interfering channels between the transmitter and the K users.

In ZFDPC, the precoding matrix is chosen as $\mathbf{B} = \mathbf{Q^H R}$, where $\mathbf{R}$ is a *diagonal* matrix, i.e., $r_{ij} = 0$ for all $i \neq j$. This choice of $\mathbf{R}$ ensures that at user i the interference from all users $j > i$ is forced to zero. Hence, the received SNR in (4.9) reduce to $\eta_i = |g_{ii} r_{ii}|$ $(1 \leq i \leq K)$. The SNR constraints of (4.1) means that the diagonal elements of the matrix $\mathbf{R}$ should satisfy

$$|r_{ii}|^2 \geq \frac{\eta_i^o}{|g_{ii}|^2}, \qquad i = 1, \ldots, K. \tag{4.25}$$

For minimizing the transmitter power, the inequality in (4.25) should be replaced with equality.

On the other hand, in ZFBF, the precoding matrix is chosen as the pseudo-inverse of the channel matrix, i.e., $\mathbf{B} = \mathbf{H}^H(\mathbf{H}\mathbf{H}^H)^{-1}\mathbf{R}$, with $\mathbf{R}$ being again a diagonal matrix. The received signal vector in this case is $\mathbf{y} = \mathbf{HBu} + \mathbf{z} = \mathbf{Ru} + \mathbf{z}$. Thus the interfering signals from all users are forced to zero, which simplifies the code design since DPC is no longer required. Since now the received SNR at user i is equal to $|r_{ii}|^2$, the diagonal elements of matrix $\mathbf{R}$ should satisfy $|r_{ii}|^2 \geq \eta_i^o$ $(1 \leq i \leq K)$.

Although zero-forcing is near optimal when the sum-rate is maximized [3], our simulations in Section C show that in practice it is far from optimum in the setup with individual rate constraints. One disadvantage of zero-forcing in this setup is that it fails when there are more users than the total number of transmit antennas, i.e., when $M < K$. This is because when $M < K$, $\mathbf{H}$ is not a full rank matrix, which in ZFDPC will result in one or more of the diagonal elements of the matrix $\mathbf{G}$ to be zero, indicating that the received SNR at one or more users is always zero. Similarly in ZFBF, when $M < K$, the inverse $(\mathbf{H}\mathbf{H}^H)^{-1}$ will not exist. This problem was recently addressed in [37], where different suboptimal solutions based on partial interference cancellation are proposed.

3. Complexity Comparisons

Table 3 compares the computational complexity of different precoding strategies in terms of the number of complex multiplications. (One complex multiplication refers to multiplication of two complex numbers). We assume that inverting or taking the square root of an $M \times M$ matrix requires M^3 multiplications, whereas the QR decomposition of a $K \times M$ matrix needs a total of $\min(K, M) \times KM$ multiplications. In addition, we count each complex division as one multiplication. Note from Table 3 that for $K = M$, the duality-based approach is of complexity $O(K^4)$, whereas all other approaches (direct calculations, ZFDPC, and ZFBF) are of complexity $O(K^3)$. When $M = K = 2$, both the duality-based approach and the direct calculations yield the same optimal result, however, the number of multiplications required by the duality-based approach is significantly higher at 98 compared to 30 for the direct calculations. When $M = K = 3$, our suboptimal approach is only slightly (0.2 dB) worse in performance than the optimal duality-based approach, but it requires only 73 complex multiplications versus 354 needed with the duality-based approach.

Table II. Number of multiplications required by various techniques for evaluating the precoding matrix $\mathbf{B}$, with K and M being the number of users and transmit antennas, respectively.

Precoding technique	**Number of complex multiplications**
Duality-based approach	$K(2M^3 + 5M^2 + 6M + 1)$
Our direct calculation (for $K = 2$)	$7M + 16$
Our suboptimal calculation (for $K > 2$)	$\min(3, M)3M + 6M + 28 + (K - 3)(M^2 + 3M + 2)$
ZFDPC (for $K \leq M$)	$K^2M + KM + K$
ZFBF (for $K \leq M$)	$\frac{3}{2}K^2M + K^3 + \frac{M}{2} + KM$

B. Proposed DPC Based Design for MIMO BCs

Our analysis so far assumes the baseband equivalent of the coded messages U_i to be complex numbers. In practice this can be realized by using a two dimensional constellation such as QAM. However, note that the coded message in the DPC scheme of Fig. 7 is mapped to a PAM constellation, indicating that the baseband equivalent of the coded message is real. Moreover, the side information V in Fig. 7 is also real, as opposed to it being complex in our analysis. In order to get a complex output, we combine the outputs of two independent nested turbo codes (denoted by U_I and U_Q), in which the phase of U_Q is shifted by 90 degrees (multiplied by $j = \sqrt{-1}$) as shown in Fig. 14. The outputs of the two copies are thus analogous to the in-phase I and quadrature Q components of two dimensional signals in many digital communication system architectures. Hence we refer to the two dirty-paper encoder as the I and the Q encoders, which separately encode w_I and w_Q respectively, where w_I and w_Q are obtained by splitting the original message w (e.g., for a two bit message, w_I can be the first bit and w_Q the second). If V is a complex side information at the encoder, the side information inputs to the two encoders are $V_I = Re\{V\}$ and $V_Q = Im\{V\}$. At the decoder, the real part of the received signal Y_I can be tied to the input of one DPC decoder which gives the decoded message w'_I, while the imaginary part Y_Q to another independent decoder which decodes w'_Q. This way we effectively convert the PAM constellation of our DPC scheme to a QAM constellation.

The complex baseband equivalent of the received signal at user i is now given by (3.10). In order to keep the same constellation step size at both the encoder and the decoder, we normalize the received signal by $\mathbf{h}_i\mathbf{b}_i$. It is apparent that this normalization does not affect the received SNR. The resulting signal can be written as

$$\begin{aligned} Y'_i &= \underbrace{U_i(w_i; U_1, \ldots, U_{i-1})}_{\textit{Useful signal}} + \underbrace{\frac{\mathbf{h}_i \sum_{j=1}^{i-1} \mathbf{b}_j U_j}{\mathbf{h}_i\mathbf{b}_i}}_{\textit{Encoder side information}} \\ &+ \underbrace{\frac{\mathbf{h}_i \sum_{j=i+1}^{K} \mathbf{b}_j U_j}{\mathbf{h}_i\mathbf{b}_i}}_{\textit{Unknown interference}} + \underbrace{\frac{Z_i}{\mathbf{h}_i\mathbf{b}_i}}_{\textit{Gaussian noise}} . \end{aligned}$$

Hence the effective encoder side information at user i ($i > 1$) is

$$V_i = \frac{\mathbf{h}_i \sum_{j=1}^{i-1} \mathbf{b}_j U_j}{\mathbf{h}_i\mathbf{b}_i}. \tag{4.26}$$

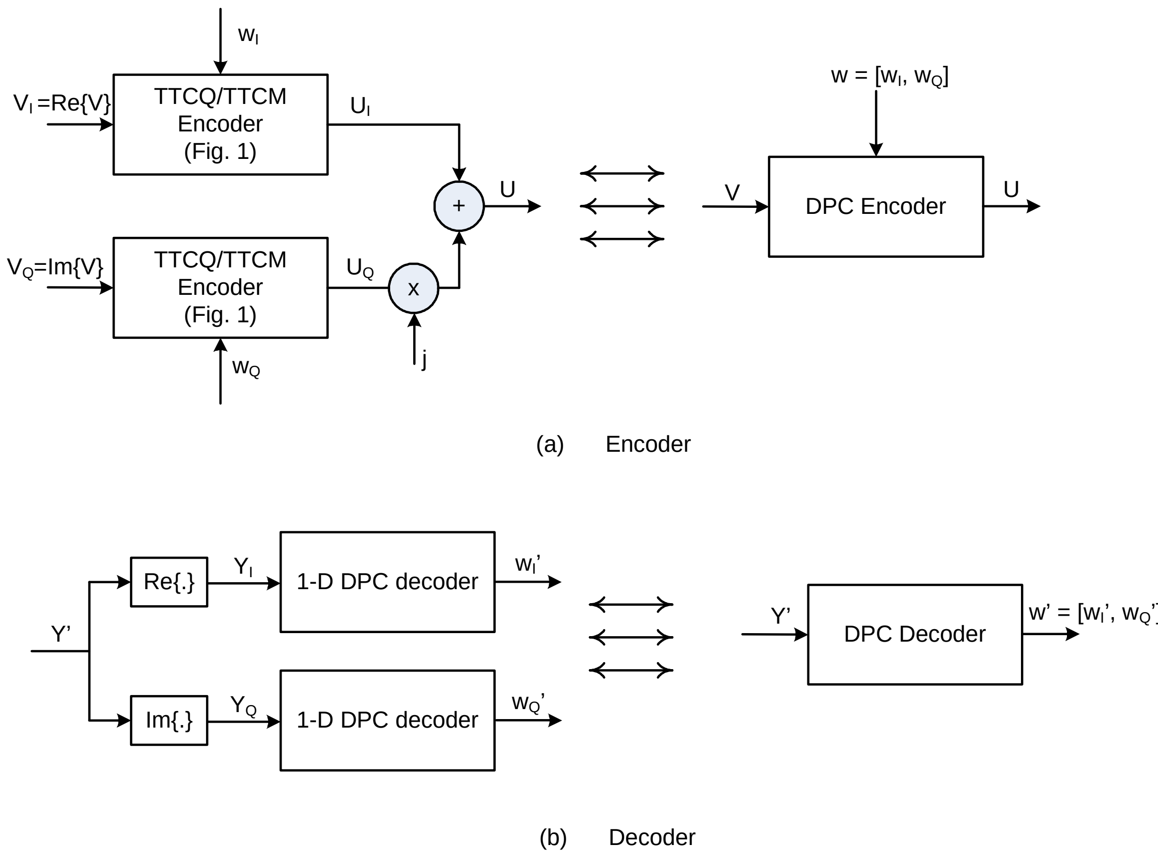

Fig. 14. Combining two 1-D TTCQ/TTCM schemes to a 2-D scheme.

In theory the unknown interference term has to be Gaussian for the rate equation (3.11) to hold. In practice it is not perfectly Gaussian. However, our decoders assume that the interference noise is Gaussian, which will lead to a small loss compared to the case when the interfering signals are from an ideal Gaussian codebook. Similarly, the side information V_i will not be Gaussian, but Costa's capacity result [8] holds also for arbitrary side information. Our simulations with our TTCQ/TTCM DPC scheme verify this.

Since user 1 does not have side information, we use a conventional TTCM code and a PAM constellation for user 1. We combine two independent copies of these codes (similar to the combination of Fig. 14) to effectively generate a QAM constellation. The remaining users exploit nested turbo code for DPC. Thus we require one conventional channel code and $K-1$ dirty-paper codes. Our overall DPC-based code design is schematically shown in Fig. 15. This design is applicable to both the degraded Gaussian BC and the MIMO Gaussian BC. As described in Chapter III – Section B, for the degraded Gaussian BC, we assume that the channels experience no fading, and hence the entries of the channel matrix in Fig. 15 can be considered as being equal to one.

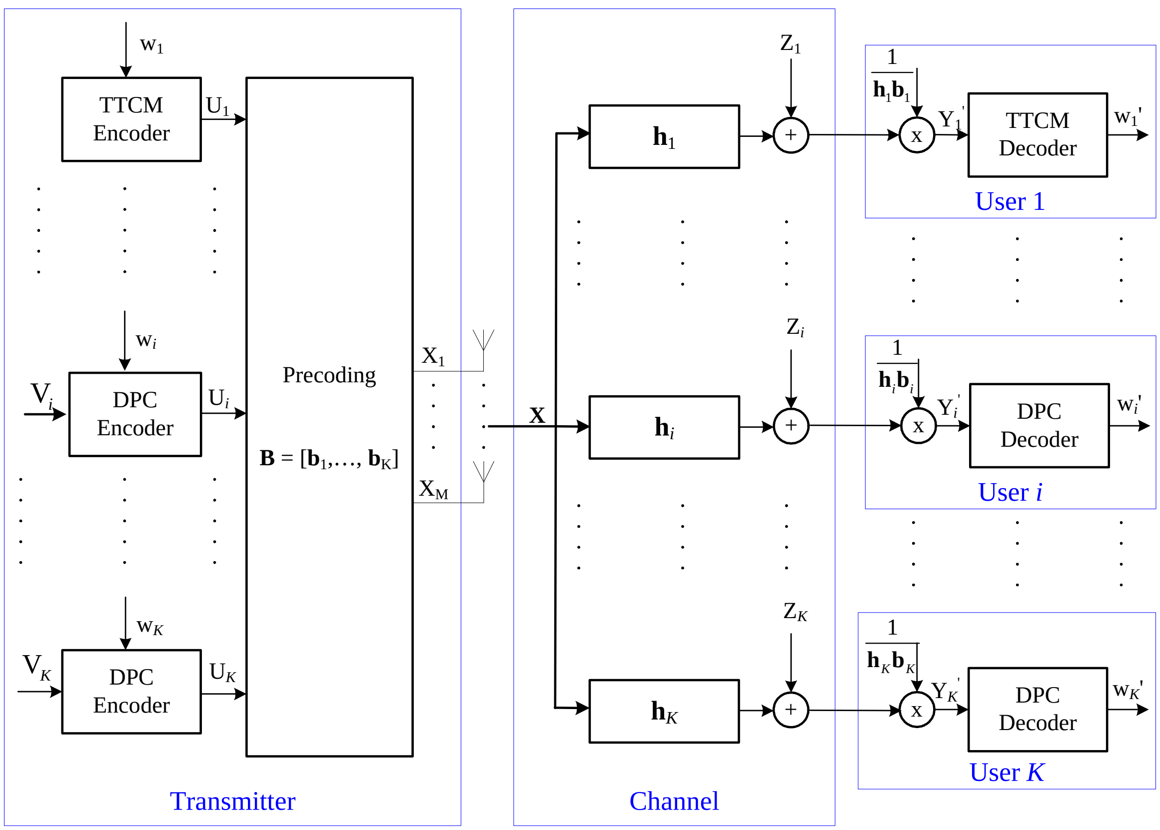

Fig. 15. Overall coding scheme requires one channel code and $K-1$ dirty-paper codes. The side information V_i for the DPC encoders is calculated from (4.26).

C. Simulation Results

In our code designs we use a 16-state, rate-$\frac{1}{2}$ systematic convolutional code for TTCM. The code polynomial is chosen as the constraint-length four Ungerboeck code [38] for the PAM constellation (suboptimally to maximize the average Euclidean distance between TCM codewords). Specifically, the parity check polynomials for this code are $h_0(D) = 23$ and $h_1(D) = 10$ in octal notation. For our practical TTCQ/TTCM DPC scheme, we choose C_1 as a 16-state, rate-$\frac{1}{2}$, non-systematic convolutional code with generator polynomials $g_0(D) = 23$ and $g_1(D) = 10$. Code C_2, on the other hand, is a 16-state, rate-$\frac{2}{3}$, systematic convolutional code with parity check polynomials $h_0(D) = 23$, $h_1(D) = 10$, and $h_2(D) = 0$. The block length for both TTCM and dirty-paper code is fixed at $10,000$ samples.

1. Degraded Gaussian BC

First we simulate our DPC-based design for the two-user degraded Gaussian BC (with $N_1 = 10$ and $N_2 = 1$) at fixed individual rates of $R_1 = R_2 = 1.0$ b/s. Our results indicate that the TTCM code for user 1 suffers a loss of $\delta_1 = 0.98$ dB at $R_1 = 1.0$ b/s. At $R_2 = 1.0$ b/s, the dirty-paper code at user 2 performs $\delta_2 = 1.53$ dB away from the Shannon limit. We use the optimal $\gamma = \frac{\eta_2^o N_2}{\eta_2^o N_2(\eta_1^o+1)+\eta_1^o N_1} = 0.0742$. The overall bit error rate (BER) of both users 1 and 2 versus the total transmission power P_t is shown in Fig. 16. With BER=10^{-5}, it is seen that the transmission power needed to achieve $R_1 = R_2 = 1.0$ b/s is 17.65 dB, which is the same as that calculated from (4.4), and is 1.44 dB away from the power required if both the channel code and dirty-paper code were ideal. This result is 1.8 dB better than that reported in [16]. Fig. 17 depicts the capacity region for $P_t = 17.65$ dB, which is the required total power for our code design to operate at $R_1 = R_2 = 1.0$ b/s. Our operating point is significantly above the time-sharing line.

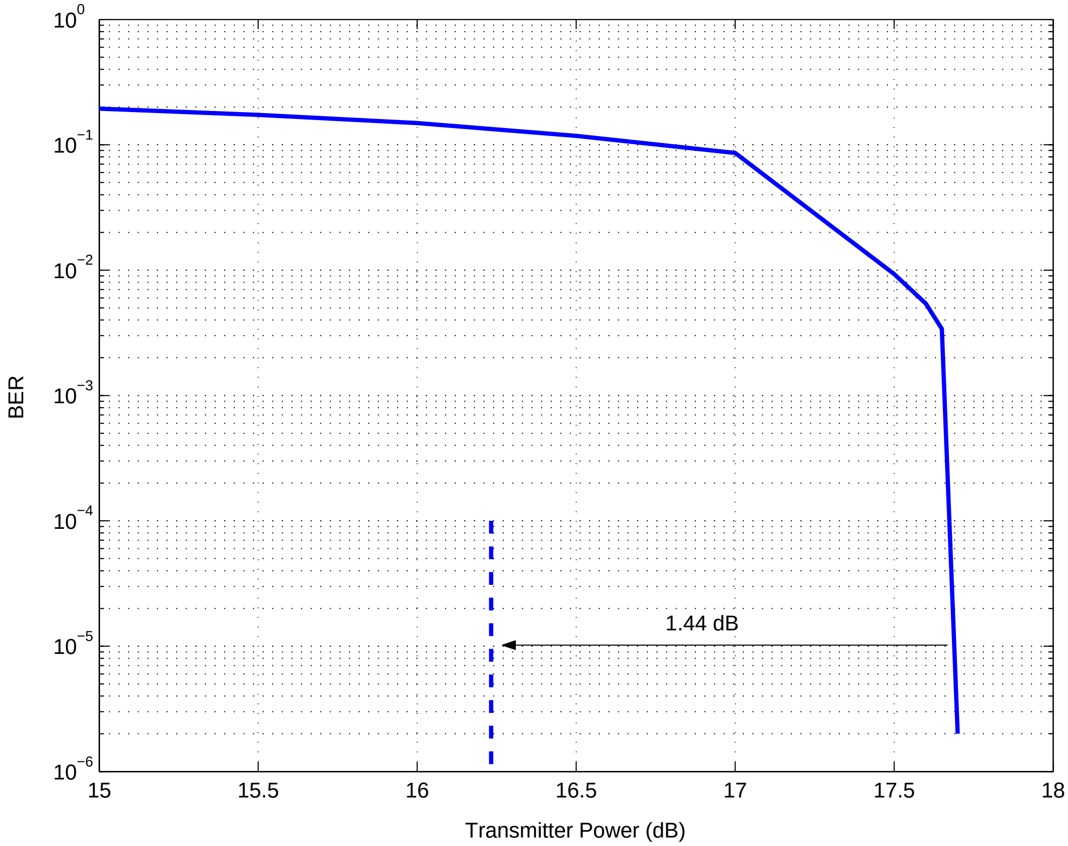

Fig. 16. BER vs. the transmission power P_t for the degraded Gaussian BC, with $R_1 = R_2 = 1$ b/s, $N_2 = 1$, $N_1 = 10$, and optimal $\gamma = 0.0742$. The dash line represents the capacity region boundary.

2. Non-Degraded MIMO Fading BC

We assume the channels undergo independent Rayleigh slow flat fading, i.e., each element of the matrix $\mathbf{H}$ is i.i.d., circularly symmetric, zero-mean, complex Gaussian with unit variance, and $\mathbf{H}$ is frame-wise constant. In all our simulations, we fix transmission rate to 1 b/s at each user. For a particular encoding order π and precoding scheme (described in Section 2), we compute the required transmission power $P_t(\mathbf{B}, \mathbf{H}, \pi)$ such that the set of SNR requirements for $\{\eta_i\}$ in (4.1) are satisfied.

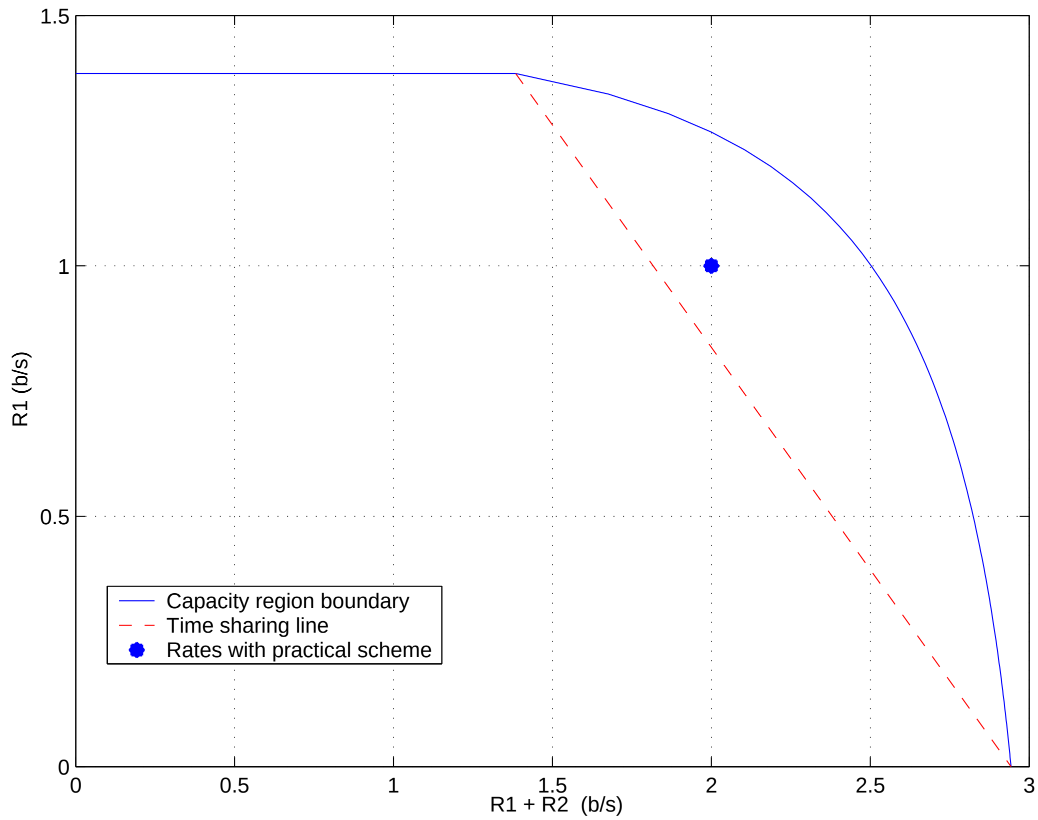

Fig. 17. The capacity region for the degraded Gaussian BC with transmission power $P_t = 17.65$ dB, $N_2 = 1$, and $N_1 = 10$.

We use the probability of frame error as the performance measure. Assuming that the transmitter is power limited, i.e., the maximum power it can transmit is P_t^{max}, the probability of frame error P_{fe} is computed as

$$P_{fe} = Pr\{\min_{\pi} P_t(\mathbf{B}, \mathbf{H}, \pi) > P_t^{max}\}. \tag{4.27}$$

Note that this probability can be thought of as the outage probability. If the system is in outage, we assume that the frames at all users are received in error. The probability in (4.27) is calculated by averaging over the entire ensemble of the channel matrix $\mathbf{H}$.

In the following we present our simulation results for cases when the number of users and transmit antennas is up to three.

a. Simulations for the Two-User Case

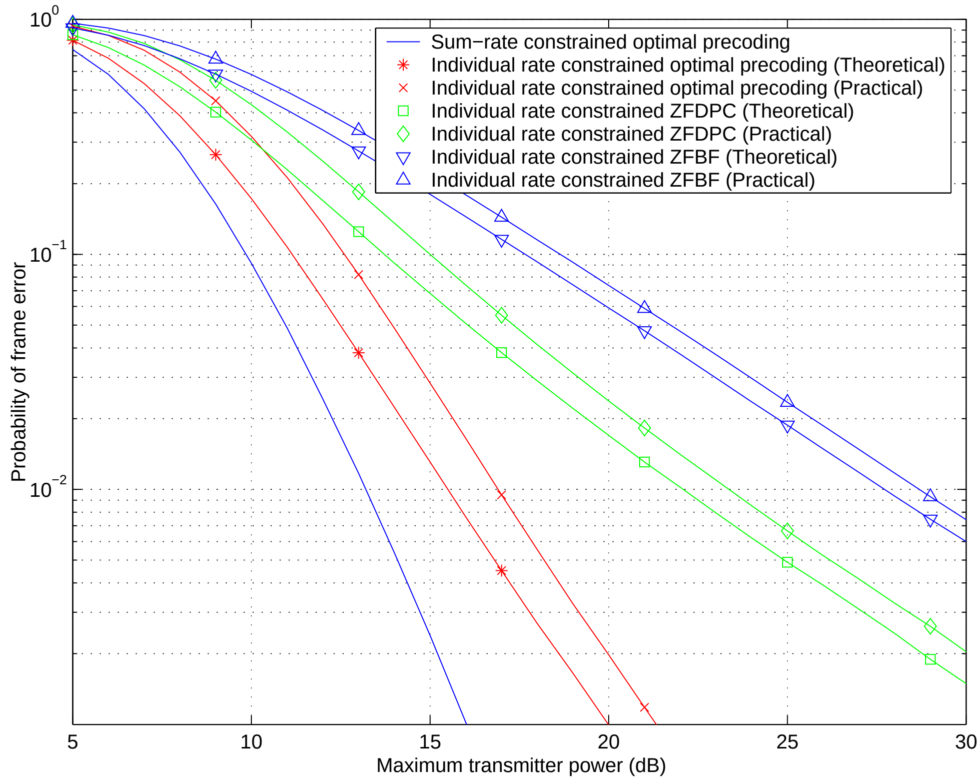

Fig. 18. Probability of frame error vs. maximum transmission power P_t^{max} for $K = 2$ and $M = 2$.

Fig. 18 compares code designs based on optimal DPC, ZFDPC, and ZFBF in terms of the probability of frame error vs. transmission power for two antennas at the transmitter. At a frame error rate of 2%, compared to the sum-rate constrained ($R_1 + R_2 = 2.0$ b/s) optimal scheme of [3], our practical DPC-based code design loses 3.7 dB in performance. About 2.3 dB of this loss is due to the individual rate constraints $R_1 = 1.0$ and $R_2 = 1.0$ b/s. Practical coding accounts for the remaining 1.4 dB loss. Compared to the optimal DPC-based design, ZFDPC is approximately 6.5 dB worse; ZFBF loses an additioal 5.5 dB. Note that for the case of two users, the duality approach and our direct calculations yield identical results. However, in order to compare their complexities from a practical point of view, we record the CPU time that Matlab needs for computing the precoding matrix $\mathbf{B}$ for each case. The duality approach takes 0.5 ms CPU time versus 0.16 ms for the direct approach.

Fig. 19 shows similar results for three transmit antennas. Note that with the increase in transmit antennas the loss due to the constraint on individual rates at a frame error rate of 2% is reduced to 1.3 dB. However, the practical coding loss remains at 1.4 dB. The performance gap between the optimal DPC scheme and zero-forcing is also reduced to 1.4 dB for ZFDPC and 5 dB for ZFBF.

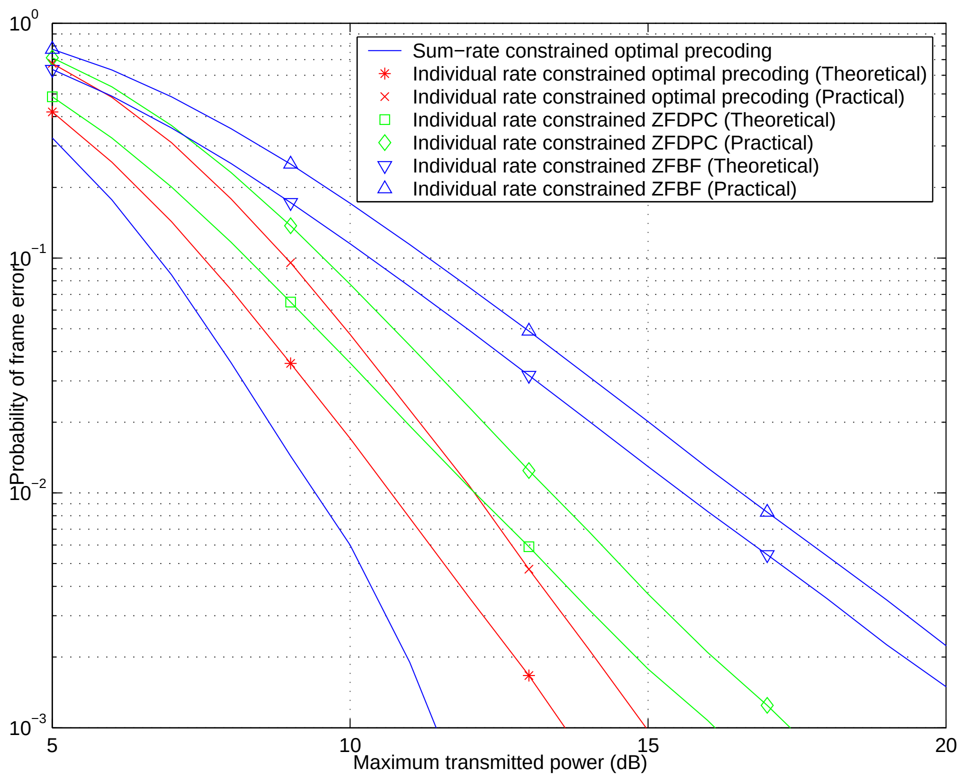

Fig. 19. Probability of frame error vs. maximum transmission power P_t^{max} for $K = 2$ and $M = 3$.

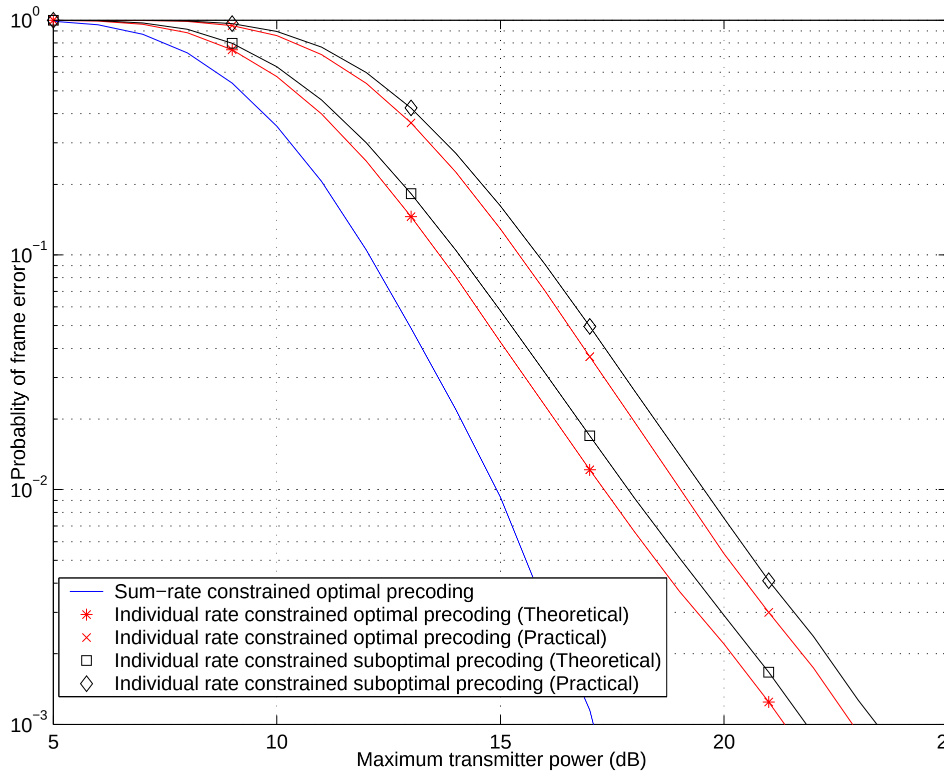

Fig. 20. Probability of frame error vs. maximum transmission power P_t^{max} for $K = 3$ and $M = 2$.

b. Simulations for the Three-User Case

The results for three users and two transmit antennas are provided in Fig. 20. Since in this case the number of transmit antennas is less than the number of users, zero-forcing (both ZFDPC and ZFBF) does not work. The optimal sum-rate constrained curve is obtained by using the iterative waterfilling algorithm of [39]. The overall practical coding loss is 1.50 dB. We also include the performance of the suboptimal scheme based on direct calculations presented in Section a, which at a frame error rate of 2% loses only 0.5 dB from the optimal precoding strategy.

Results for the case of three transmit antennas are presented in Fig. 21, where our suboptimal precoding strategy is only 0.2 dB worse than the optimal scheme, whereas complexity-wise the duality approach requires a CPU time of 1.1 ms, which is almost four times 0.28 ms required by the suboptimal approach. Moreover, there is a huge gap of 6.5 dB and 15.5 dB between the performance of our practical DPC scheme with optimal precoding and the theoretical ZFDPC and ZFBF, respectively.

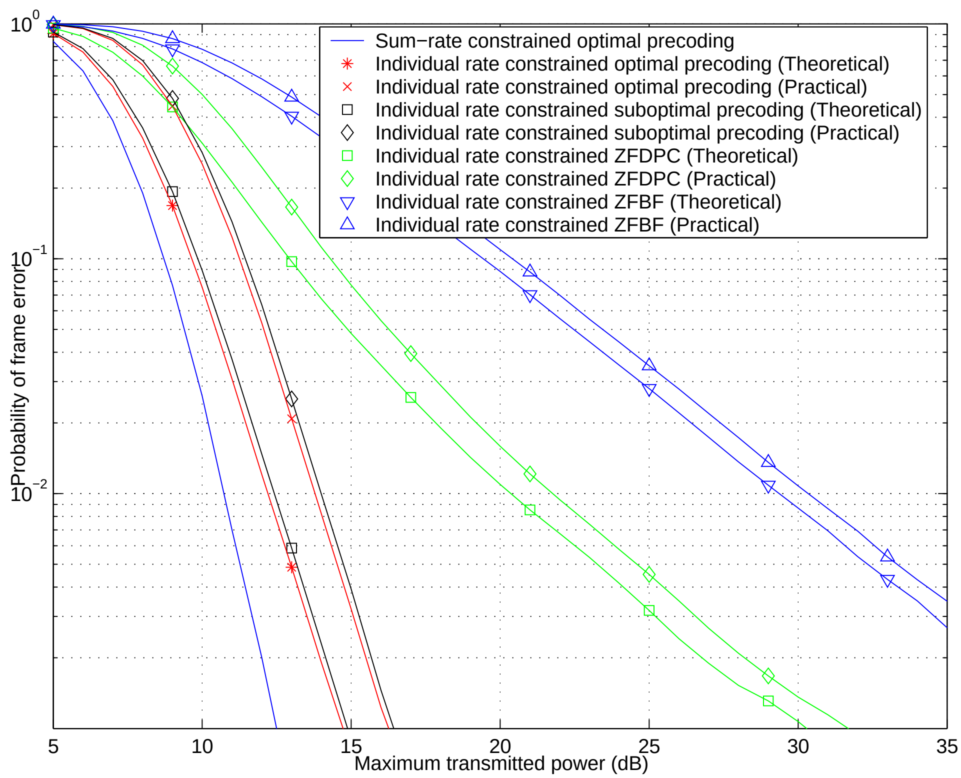

Fig. 21. Probability of frame error vs. maximum transmission power P_t^{max} for $K = 3$ and $M = 3$.

We note that regardless of the number of users, the practical coding loss of all schemes remains at roughly 1.5 dB. Upon first glance, many readers would tend to think that the coding dB loss of the overall scheme should be roughly equal to the sum of the individual coding dB losses. Upon close examination however one would realize that this is not true. We show here simple calculations which shows that the practical dB loss does not blow up with increasing number of users.

For illustrative purposes, we consider the simple example of ZFDPC, where the transmitter power is minimized when the diagonal elements of the matrix $\mathbf{R}$ should satisfy (4.25) with equality. Then the total transmitter power is given by

$$P_t = \sum_{i=1}^{K} |r_{ii}|^2 = \sum_{i=1}^{K} \frac{|\eta_i^o|}{|g_{ii}|^2}.$$

Let the minimum required SNR at user i for an ideal channel/dirty-paper code be η_i^{th}, and for the practical code with a coding loss of δ_i be $\eta_i^{pr} = \eta_i^{th}\delta_i$. Then the required transmitter power for the practical scheme is $P_t^{pr} = \sum_{i=1}^{K} \frac{|\eta_i^{th}|\delta_i}{|g_{ii}|^2}$. If the individual coding losses δ_i's are almost the same, i.e., $\delta_1 \approx \ldots \approx \delta_K \approx \delta$, then

$$P_t^{pr} \approx \sum_{i=1}^{K} \frac{\eta_i^{th}}{|g_{ii}|^2}\delta.$$

The summation term is in fact the required theoretical transmitter power P_t^{th}. Hence $\frac{P_t^{pr}}{P_t^{th}} \approx \delta$, indicating that the overall coding loss is not a function of the number of users.

CHAPTER V

CONCLUSIONS

In this thesis, we have presented practical capacity-approaching code designs for the degraded Gaussian BC and for the MIMO fading BC. Realizing the importance of DPC in achieving the full capacity region of not only the degraded Gaussian BC but also that of the MIMO Gaussian BC, we presented a few practical approaches to DPC. Starting with the simplest approach of THP for illustrative purposes, we built our case to present more sophisticated practical DPC schemes which perform close to Costa's capacity limit. Specifically, we propose using a DPC scheme which employs nested turbo codes – with TTCQ as the source code and TTCM as the channel code. Before employing this TTCQ/TTCM scheme to develop, to the best of our knowledge, the first capacity-approaching designs for the non-degraded MIMO BC, we identify the role of precoding at the transmitter. Limited by the inability of our DPC scheme to adapt to varying transmission rates, we consider the scenario where each user operates under a fixed rate constraint. Under these individual rate constraints, the optimal precoding should try to minimize the transmitter power such that the individual rate constraints are satisfied. Although duality of the MIMO BC with MIMO MAC provides an easy means of evaluating this optimal precoding strategy, it requires considerable computational complexity. Therefore, for the two user case we provide the optimal precoding approach using direct calculations which possesses significantly lower complexity than the duality approach. For more than two users, based on direct calculations, we provide a suboptimal approach which for three users and three transmit antennas performs only 0.2 dB worse than the optimal duality based approach. We also present other suboptimal precoding strategies such as ZFDPC and ZFBF.

Simulation results indicate that our schemes perform close to the capacities, with a practical coding loss of approximately 1.5 dB. Moreover, our results show a significant performance gain of optimal DPC over other suboptimal strategies (e.g., time sharing and zero-forcing linear beamforming), e.g., for the case of three users and three transmit antennas our practical DPC design with suboptimal precoding outperforms theoretical ZFBF by approximately 15.5 dB.

In short, our DPC based design beats suboptimal strategies by a significant margin, provided that the channel state information (CSI) is known perfectly at the receivers as well as at the transmitter. Whereas, CSI can be estimated quite accurately at the receivers, it is not easily available at the transmitter. An important future direction of research is to design DPC based practical schemes where the channel state information is not perfectly available at the transmitter. It would be interesting to analyze the performance of DPC in such a situation and compare it to the prevalent suboptimal strategies.

There can be several other directions of future work as well. For example, one research direction is to improve the performance of our designs by employing the stronger dirty-paper codes of [12, 13, 14]. Finding a closed-form expression for the optimal precoding matrix where the number of users is more than three is still an open problem. A research area closely related to the BCs is that of cooperative networks, where closely located network nodes are grouped together into clusters, inside which nodes cooperate when sending or receiving information. For instance, consider two closely located nodes which intend to transmit messages to two distant nodes. Instead of sending messages independently, the two nodes can cooperate by first exchanging each other's messages. After this exchange, the network effectively is a BC. Thus the two transmitting nodes can make use of spatial diversity without the need of multiple transmit antennas at a single node. Similarly, the receiving nodes can also cooperate to achieve receive diversity. The channel capacities for such cooperative networks however are not fully known. At the same time code designs for such networks also hold a healthy research potential. This is part of our ongoing research, results for which have been presented in [40] and [41].

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