MOTION PLANNING ALGORITHMS FOR A GROUP OF MOBILE AGENTS

A Dissertation

by

MAYANK LAL

August 2008

Major Subject: Mechanical Engineering

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ABSTRACT

Motion Planning Algorithms for a Group of Mobile Agents. (August 2008)

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Building autonomous mobile agents has been a major research effort for a while with cooperative mobile robotics receiving a lot of attention in recent times. Motion planning is a critical problem in deploying autonomous agents. In this research we have developed two novel global motion planning schemes for a group of mobile agents which eliminate some of the disadvantages of the current methods available. The first is the homotopy method in which the planning is done in polynomial space. In this method the position in local frame of each mobile agent is mapped to a complex number and a time varying polynomial contains information regarding the current positions of all mobile agents, the degree of the polynomial being the number of mobile agents and the roots of the polynomial representing the position in local frame of the mobile agents at a given time. This polynomial is constructed by finding a path parameterized in time from the initial to the goal polynomial (represent the initial and goal positions in local frame of the mobile agents) so that the discriminant variety or the set of polynomials with multiple roots is avoided in polynomial space. This is equivalent to saying that there is no collision between any two agents in going from initial position to goal position. The second is the homogeneous deformation method. It is based on continuum theory for motion of deformable bodies. In this method a swarm of vehicles is considered at rest in an initial configuration with no restrictions on the initial shape or the locations of the vehicles within that shape. A motion plan is developed to move this swarm of vehicles from the initial configuration

to a new configuration such that there are no collisions between any vehicles at any time instant. It is achieved via a linear map between the initial and desired final configuration such that the map is invertible at all times. Both the methods proposed are computationally attractive. Also they facilitate motion coordination between groups of mobile agents with limited or no sensing and communication.

To my parents and wife Smita

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CHAPTER I

INTRODUCTION

Autonomous mobile agents have many potential applications, with a lot of research [1, 2, 3] being done to equip them with better capabilities. Groups of mobile agents which engage in collective behavior have been of interest lately. There are several reasons for this. The complexity of the task may make it infeasible or impossible for a single mobile agent to accomplish or the performance of a single agent may be much worse than a system of multiple agents. Also it would be much more economically viable to build many cheap mobile agents to achieve sub tasks than a single sophisticated agent for the overall task. Multiple mobile agents are advantageous in terms of flexibility and fault tolerance too. Also multiple mobile agents which exhibit cooperative behavior can yield insights into social and life sciences. Hence multiple mobile agents can achieve tasks which cannot be done by single mobile agents, however powerful they are because of the inherent spatial limitation of single mobile agents.

A. Cooperative Mobile Robotics and Motion Planning

Cooperative mobile robotics as the name indicates is the field of engineering in which groups of mobile agents engage in accomplishing a common task. Motion planning is one of the aspects of cooperative robotics. One of the key driving forces for the development of cooperative robotics is the need to reduce human intervention in dangerous applications. There is an element of danger in applications such as toxic waste cleanup, fire fighting, search and rescue, border surveillance, decommissioning of nuclear plants, emergency management etc.. Groups of robots with sensors

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The journal model is *IEEE Transactions on Automatic Control.*

mounted on them can be dispatched to do these tasks autonomously. Thus with the use of cooperative robotics the risk to humans can be considerably reduced. Other potential applications include automated highway systems [12], [13], box pushing [14, 15, 16, 17, 18, 19, 20, 21], foraging[22, 23, 24, 25, 26, 27, 28, 29, 30], ,scientific data collection and industrial automation. Automated highway systems will help reduce traffic congestion and accident rates while applications such as industrial automation eliminate fatigue to humans caused by monotonous and repetitive jobs. It is also envisaged that autonomous mobile agents will play a big role in future warfare.

Cooperative robotics has it's roots in some of the early works in 1940's when Grey Walter, Wiener and Shannon experimented with turtle-like robots having touch and light sensors. These robots showed complex social behavior in response to each other's movement [4]. There was active interest in the study of coordination of multiple intelligent agents in the field of distributed artificial intelligence in the 1970's [5] though it involved software agents. Towards the latter part of 1980's there was a lot of activity in the field of cooperative robotics. Numerous projects such as CEBOT [6], SWARM [7], ACTRESS [8] and GOFER [9] were conducted. Most of these projects were done mainly in simulation though more recent work [10], [11] have done experimental work towards establishing cooperative robotic systems.

While cooperative robotics has a number of aspects such as group architecture, resolving resource conflicts, origins of cooperation and learning, we cover the geometric aspect in our work, i.e. motion planning for multiple mobile agents in two or three dimensional world. In other words, fundamental to the creation of a cooperative robotic system is the ability to move a group of mobile agents from a reference configuration A to a goal configuration B. This requires many local processes and decisions to be made. A mobile agent must be able to process data from onboard sensors for motion planning through different, dynamic terrains by detecting and

avoiding obstacles as required. Global motion planning is the first level of achieving such a mission and we address this problem in this dissertation.

B. The Problem and Motivation

1. The Problem

We consider the problem of planning the motion of a group of mobile agents from an initial configuration to a final configuration such that there is no collision between the mobile agents and the mobile agents with stationary obstacles in the environment. We have considered both two dimensional and three dimensional cases. In practical situations there could be instances when there are moving obstacles in the environment but is outside the scope of our this work.

2. Motivation

The main motivation behind this work has been the development of algorithms that are computationally attractive and require limited or no communication and sensing between the mobile agents for their implementation. A reduction in computational complexity helps minimize the computational cost. Minimizing communication and sensing on the other hand helps reduce the cost involved to facilitate it and also the issue of uncertainty and loss of communication. The developed methods in their current form though have some limitations in that they do not incorporate mobile agent failures and uncertainty in controlling the agents along the trajectories generated by the methods. There is potential for removing these limitations with a certain amount of communication between the agents and sensing of the current positions of the agents for feedback. Despite the limitations the developed methods are very promising from a computational point of view and require limited or no communication and

sensing between agents for their implementation.

C. Existing Work and Their Deficiencies

The motion planning problem of multiple mobile agents or the multimover's problem arose initially in an automated factory where mobile agents moved parts from the warehouse to assembly stations. The mobile agents needed to avoid each other and stationary obstacles in the factory. Developing an exact and efficient algorithm for the generalized multimover's problem is unlikely and several algorithms are available for special cases. Centralized methods like the cell decomposition and potential field methods were used initially to solve this problem. Centralized method is an approach in which the group of mobile agents is treated as a composite robot and the motion plan of this composite robot is planned through obstacles. In [38] this problem is solved for an arbitrary number of moving objects using the cell decomposition method. The algorithm proposed in the paper has computational complexity which is polynomial in the number of smooth surfaces of obstacles and exponential in the number of degrees of freedom of the mobile agents. In [39] the problem was restricted and solved in two dimension considering circular mobile agents while [40] analyzed the same problem using the retraction method. Later in [41] the cell decomposition approach was used for planning the coordinated motion of convex polygonal mobile agents among polygonal obstacles. In [42] cell decomposition along with dynamic programming was used to solve the problem. In [43, 44] a potential field approach is taken to solve the problem. In [45] a combination of randomized searching and potential fields is used to plan the motion of multiple translating robots. In recent years motion planning algorithms which use random sampling, like the probabilistic roadmap (PRM) planners [46, 47] have gained popularity due to their efficiency

and simplicity. Centralized methods using PRM have been developed for solving the motion planning problem for multiple agents [48, 49]. The issue with centralized planning is that the time complexity of the algorithms are exponential in the dimension of the composite configuration space. It's been shown in [50, 51] that even the supposedly simpler problem of motion planning of multiple rectangles is P-SPACE complete. To resolve this issue methods were developed in which the path of each robot was planned more or less separately and then interactions between the paths were considered [52]. This approach is called the decoupled planning approach which was introduced in [53] to solve the problem involving multiple moving objects. There are two decoupled planning approaches, prioritized planning and path coordination. In prioritized planning [54, 55] the motion planning of each robot is done one at a time with the order of planning according to priority. Each robot needs to be assigned a priority which may be done randomly or using motion constraints. In [56] the priorities are assigned to the mobile agents in a way that the number of mobile agents moving in straight lines from the initial to the final configuration is maximized. The other decoupled approach, path coordination was proposed in [57, 58]. This method is based on a scheduling technique for dealing with limited resources[59]. The notion of coordination diagram was used in [57] and later analyzed for manipulators in [60, 61]. On similar lines roadmap coordination [62, 63] was used to plan the motion of multiple robots. The problem with decoupled approaches is that despite gains in terms of computational complexity compared to centralized methods, there is loss of completeness. We have developed two approaches for solving the motion planning problem for multiple agents. The first is the homotopy approach and the second is the continuum approach. Both these approaches are promising from a computational point of view. The homotopy approach is based on finding roots of a polynomial and since Newton Raphson method can you used to calculate the roots, it is promising

from a computational point of view. The continuum approach on the other hand is based on the idea of finding a motion map between the initial and final configuration which is a homeomorphism. The time complexity of the continuum approach is the complexity of calculating values of the polynomial and trigonometric functions which are the elements of this map which transform the agents from one position to another. The continuum approach has the added advantage of being completely scalable. Apart from the potential advantage in terms of complexity, the homotopy and the continuum approaches are attractive because of their inherent simplicity. The planning using the homotopy approach is done in polynomial space rather than complex composite configuration spaces. The trajectory planning of each agent is done simply by calculating the roots of a time varying polynomial. Similarly the continuum approach generates trajectories of the mobile agents simply through time varying linear maps of the initial positions of the agents.

Another approach in literature for the motion coordination of multiple agents is the distributed approach [65, 64, 66, 67, 68]. In this approach the mobile agents sense and communicate with their neighbors and compute their paths individually. There exist good techniques for modeling individual behavior within a group in virtual environments such as Reynolds boids [69], based on each agent solely observing its local environment. This concept is called flocking. Reynolds extended the idea so as to include autonomous reactive behaviour [70]. Also there is the social potential field technique in which the desired behavior of groups of mobile agents is created by defining certain force fields between the agents [71]. The issue with the distributed approach is that since agents are not assumed to have global information about the environment, only very simple navigation and planning tasks can be handled, i.e., these methods cannot be used if complex navigation is required, such as in cities. On the other hand global motion planning algorithms such as the methods we have

developed in our this work can handle complex environments. Also since only local information is available in the approaches developed in [69, 70, 71] the agents may get stuck in cluttered environments and split up. In other words the agents do not move as a coherent group. The homotopy and continuum approaches do not present issues such as incoherence that is seen in distributed approaches such as flocking in which the agents split when they encounter an obstacle. The agents move as a coherent group. The distributed approaches also require the agents to sense and communicate with neighboring agents with the computational load of the agents increasing with increase in the number of neighbors. If each agent is given the polynomial and the initial positions in the homotopy approach, they can generate and track their paths in a distributed fashion with no communication and sensing .The continuum approach has similar properties with the agents generating and tracking their paths with no communication and sensing, given the homogeneous maps.

D. Our Contribution

Our contribution in this dissertation has been the development of two novel global motion planning methods. These methods are novel as they require no communication and sensing for their implementation and are very promising from a computational point of view. The first is based on homotopy of polynomials while the second is based on continuum theory. Both these methods are formation to formation motion planning methods with deformation and translation of a group of mobile agents such that collision between any two agents is avoided in going from an initial configuration A to a final desired configuration B. In the developed methods it is assumed that the global plan can be segmented in such a way that the required motion plan from a start position to a goal position consists of a number of well defined configurations (position

and possibly shape). For example, this can mean a set of intermediate configurations C_i defining the motion $AC_1C_2C_3.....C_k....B$ (effectively combined spatial and temporal waypoints). This necessarily means that all we have to be concerned is with the motion from one given reference configuration A_0 to a final configuration B_0 with minimal or no intermediate path constraints other than the motion plan being collision free. We have also extended the methods to handle stationary obstacles. There has been research related to motion planning in a road network environment [31, 32, 33, 34] which is essentially 1-D. In our work though we consider both 2-D plane and the 3-D space.

1.   Homotopy Approach

This approach is based on homotopy of polynomials. The motion plan consists of deformation and translation. The deformation of the group is brought about by finding roots of a polynomial which does not have multiple roots. The translation of the group is brought about by bounding the group by a disc and planning the motion of the disc as a single agent through stationary obstacles. In [35] too polynomials are used to represent configurations of robots in formations and the straight line polynomial path has been used as a local planner for generating paths for robots. In our work though we generate trajectories for robots by ensuring that the polynomial stays away from the space of polynomials having multiple roots. We solve the cases in which the mobile agents are moving in two dimensional and three dimensional workspace using this approach. We also implement the motion plan generated using the approach on a group of non holonomic agents using the controller proposed in [36].

2. Continuum Approach

This approach is based on continuum theory according to which if we can find a motion map between the start and goal configurations which is a homeomorphism, the agents will occupy unique positions at all times. This will ensure that there is no collision between the agents at any time. We extend the approach to the case in which there are stationary obstacles in the environment by bounding the group by a rectangular box which can deform, translate and rotate as has been done in [37]. Since we assume initially that the mobile agents are point objects, we develop a way to handle finite sized agents using this approach. The motion plan is implemented on a group of non holonomic agents using the controller proposed in [36].

E. Organization of the Dissertation

The rest of this dissertation is organized as follows. In Chapter II we pose the motion planning problem of moving a group of agents from an initial configuration to a final configuration. In Chapter III we develop a homotopy approach for it's solution. We first consider a straight line path in polynomial space for planning the change of shape of the group and show a way to check whether the polynomial path has multiple roots. We then develop a potential field approach of finding this polynomial path. Next we have developed a way to plan the motion of the group through stationary obstacles. We have also implemented the motion plan on a group of non holonomic mobile agents and extended the homotopy approach to handle agents moving in three dimension.

In Chapter III we solve the problem posed in Chapter II for moving a swarm of agents from an initial configuration to a final configuration using the continuum approach. The method is based on continuum theory which suggests that as long the motion map between the initial to the final configuration is a homeomorphism,

each agent will occupy a unique position in all the intermediate configurations. We initially assume that the agents are point objects but we have also developed a way to handle finite sized agents. We have also shown a way to handle stationary obstacles in the environment and implemented the motion plan on a group of non holonomic agents.

Concluding remarks and future work are in Chapter V.

CHAPTER II

GLOBAL MOTION PLAN FOR SWARMS

Moving a group of mobile agents from an initial configuration to a final configuration without collisions is fundamental to building cooperative robotic systems. In this chapter we formulate the problem of moving a group or a swarm of mobile agents from an initial configuration to a final configuration.

As stated in Chapter I there are a number of applications of cooperative robotic systems such as foraging, box pushing, target tracking etc. Many of these applications are inspired by biological systems such as swarms of birds, bees and herds of bisons which are quiet prevalent in nature. They all demonstrate swarming behavior to maximize their chances of finding food and to avoid predators. To achieve the applications stated above we need to plan the motion of swarms or groups of mobile agents from an initial to final configuration such that there are no collisions between agents. Also in a real world scenario, apart from avoiding collisions with each other the mobile agents should avoid collisions with stationary obstacles in the environment. These obstacles could be like rocks, trees, cliffs, walls in the environment. Let us consider a scenario in which a group of autonomous tanks needs to change from a triangular formation to a line formation as illustrated in Fig. 1 to move through a tunnel which is wide enough for just one tank. The tanks must sense and communicate with it's neighbors, avoid rocks and other obstacles in the way to achieve this objective. The tanks will have actuator constraints, in other words they have limits on their velocities and accelerations and also will encounter disturbances such as uneven ground, wind etc..Apart from this there will be uncertainty, time delay and loss in communication, sensing and control of the tanks and the tanks will have limited computational capabilities. Hence under all these limitations the desired objective is
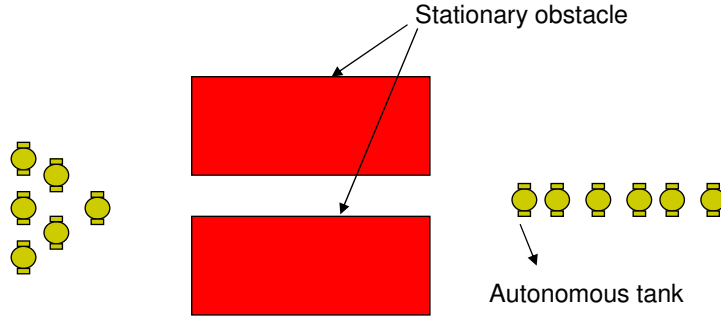
Fig. 1. Typical scenario for motion planning of a group of agents

to be achieved. We are motivated by real world scenarios such as this and formulate the problem below:

A. Problem Statement

Given n mobile agents: $R_1, R_2, ..R_n$, plan the motion of the n agents from an initial configuration A to the final configuration B such that there is no collision between the agents at any time instant and between the agents and stationary obstacles in the workspace.

We develop two approaches for solving this problem, the homotopy and continuum approaches. Both these approaches require no communication and sensing for their implementation except for sensing the positions of the mobile agents in the initial and final configurations. Hence we minimize the high cost associated with communication and sensing and the uncertainties, loss and time delays associated with it. Also the developed methods are computationally promising and will help reduce the high computational cost required for implementation of the existing methods. The limitation of these approaches is that we assume that there is no uncertainty in control of the mobile agents along their planned trajectories and there is no agent failure.

Despite the limitations the developed methods are very promising from a computational point of view and require no communication and sensing between agents for their implementation. In the next two chapters we describe the approaches in detail.

CHAPTER III

HOMOTOPY APPROACH

Motion planning of multiple mobile agents in the past has been done primarily in the configuration space(the field of all possible robot locations) of the robots [46, 47]. In this chapter we develop an approach to solve the problem formulated in Chapter II for a group of mobile agents moving in two dimensional workspace. The planning is done using homotopy of a polynomial. In Section A we give an overview of the approach. In Section B we develop the algorithm for changing the shape of the group. The approach consists of finding a time varying polynomial which has no multiple roots at all times. We analyze a polynomial path which is a straight line in the space of polynomials and the potential field approach of finding a polynomial path in this section. It is to be noted that the straight line polynomial path does not imply that the mobile agent trajectories are straight lines. In Section C we develop a way to handle stationary obstacles in the environment and also the velocity and acceleration constraints. In Section D we implement the approach on a group of non holonomic agents and in Section E we extend the approach to the three dimensional case.

A. The Approach

The approach is based on homotopy of polynomials for solving the problem formulated in Chapter II. The motion plan consists of deformation and translation. The deformation of the group is brought about by finding roots of a polynomial which lies in the complement of the discriminant variety space, the discriminant variety space being the space of polynomials with multiple roots. The translation of the group is brought about by bounding the group by a disc and planning the motion of the disc as a single agent through stationary obstacles.

The roots of the polynomial used for deformation, map to the positions of the mobile agents in local frame. The local frame is a frame which translates(pure translation) along with the group. In the case in which the agents are moving in two dimension, the real part of the roots represent the x positions of the agents and the imaginary part represent the y positions w.r.t. the local frame. The polynomial itself is a path parameterized in time between the initial and final polynomials. The roots of the initial and final polynomials represent the initial and final positions of the agents respectively. One such polynomial path is the straight line polynomial path between the initial and final polynomials. We present a way to verify if the straight line path intersects the discriminant variety. Apart from the straight line path, we also generate paths in polynomial space using a potential field like approach. In this approach a potential function is created such that as the time varying polynomial moves towards the discriminant variety, the potential function increases in value and as it moves away from the goal polynomial it again increases. This way a polynomial path is generated which reaches the goal by avoiding the discriminant variety by moving in the direction of negative gradient of the potential function. Once the planning for deformation is done, we find a disc which bounds the agents at all times and use this disc to plan the translational motion of the group through stationary obstacles using any of the standard methods for motion planning of a single agent. Since the positions, velocities and accelerations of the mobile agents can be found owing to the polynomial being differentiable, we use a non-linear controller proposed in [36] to implement the algorithm on a group of non holonomic agents moving in two dimension. We also extend the approach to 3-D by mapping the 3-D coordinates to 2-D. The idea is that we use a linear transformation for two of the coordinates to map the 3-D coordinates to 2-D such that each map is unique. We then use these new coordinates to generate a time varying polynomial which avoids the discriminant

variety. Once we have this polynomial, we find out the new coordinates as a function of time from the roots of this polynomial. The 3-D coordinates are then found out from these new coordinates by joining the initial and final values of one of the transformed coordinates by a straight line and using an inverse map.

1. Assumptions

The key assumption for the 2-D case of the problem described in Chapter I is:

- The agents are represented as point masses.

It is to be noted that even though the mobile agents are assumed to be point objects, the algorithm proposed can deal with finite sized agents. In particular finite sized agents can be handled by using the potential field approach for homotopy of polynomials developed in this work. This can be done by choosing an appropriate value for the distance of influence of the discriminant variety.

B. Algorithm for Group Shape Change

We present the homotopy approach for deforming the group to the final shape in this section. We first present the straight line path approach as this path is easy to analyze. We show a way to verify if this path is feasible. Then we present a potential field like homotopy approach.

1. Polynomial Construction

We construct a polynomial, the roots of which represent the current positions of the mobile agents in local frame. The real part of the roots represent the x coordinate of the mobile agents and the imaginary part represent the y coordinate. We construct, initial and goal polynomials, the roots of which represent the initial and goal positions
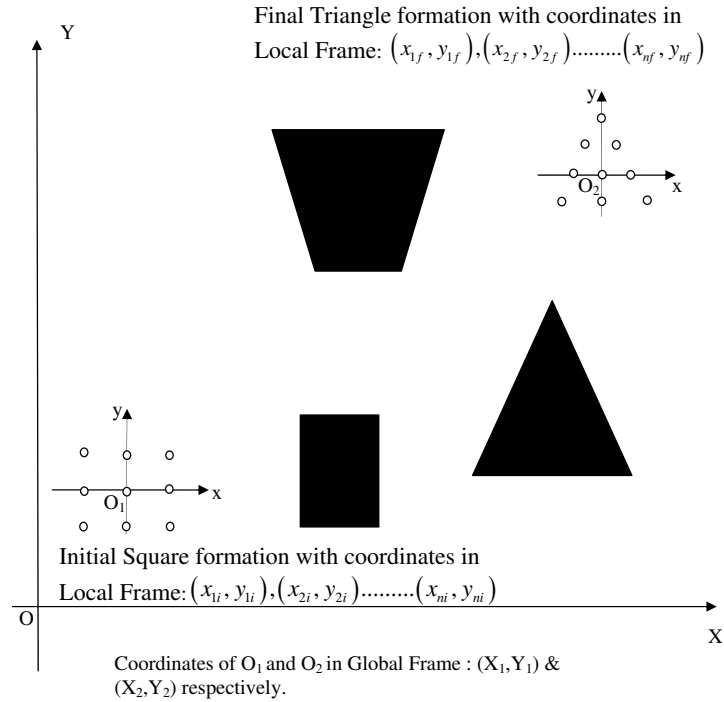
Fig. 2. Change of formation

of the mobile agents in local frame. Then we use these polynomials to construct a time varying polynomial path which represents the current positions of the mobile agents such that no two mobile agents are at the same position at a given time. Let us define the discriminant variety space. The discriminant variety space $\sum_n$, is the set of all complex polynomials of degree n with multiple roots. If we represent the set of all polynomials of degree n by P_n then $P_n - \sum_n$ represents the set of all polynomials of degree n with distinct roots which we call the complement of the discriminant variety space. There is a proposition in [78] according to which the discriminant variety is connected. Also a method is described in [79] to parameterize curves in $\sum_n$. Utilizing these ideas if we can find parametric curves connecting the initial and final polynomials, in $P_n - \sum_n$ we ensure that no two mobile agents are at the same

coordinates at any instant. The straight line polynomial path $(1-\lambda)P_i+\lambda P_g, \lambda \in [0,1]$ could be one such path. P_i and P_g are the initial and goal polynomials which have distinct roots $,\lambda = \frac{t}{T}$ and time $t \in [0,T]$. As an example, for the initial and final configurations shown in Fig. 2, $P_i(x) = (x-(x_{1i}+jy_{1i}))(x-(x_{2i}+jy_{2i}))......(x-(x_{ni}+jy_{ni}))$ and $P_g(x) = (x-(x_{1f}+jy_{1f}))(x-(x_{2f}+jy_{2f}))......(x-(x_{nf}+jy_{nf}))$. Results in [79] can be used to verify whether the straight line $(1-\lambda)P_i + \lambda P_g , \lambda \in [0,1]$ lies in $P_n - \sum_n$.

This method is developed below.

2. Verification of Intersection of Straight Line Path with Discriminant Variety.

We now show a way to parameterize the discriminant variety which has been developed in [79]. We use this parametrization to formulate a way to check if the straight line path intersects the discriminant variety. The idea is to first find the spanning set of the discriminant variety space using a matrix which is a function of the degree of the polynomial n and the elements of $(\mathbb{C}^*)^2$. $\mathbb{C}$ is the set of complex numbers. We then find two vectors which are orthogonal to the discriminant variety space from this spanning set. The check for the intersection of the straight line polynomial path with the discriminant variety is done by checking for the orthogonality of this path with these two vectors.

First, we form a matrix A as described below and determine its kernel. The result in [79] tells us that the kernel of A and elements of $(\mathbb{C}^*)^2$ parameterize $\sum_n$ in the manner described below.

$$A = \begin{pmatrix} 1 & 1 & 1 & 1 & ..... & 1 \\ 0 & 1 & 2 & 3 & ..... & n \end{pmatrix}$$

Let the kernel be:
$$\begin{pmatrix} x_0 \\ x_1 \\ . \\ . \\ . \\ x_n \end{pmatrix}$$

Therefore :

$$\begin{pmatrix} 1 & 1 & 1 & 1 & ..... & 1 \\ 0 & 1 & 2 & 3 & ..... & n \end{pmatrix} \begin{pmatrix} x_0 \\ x_1 \\ . \\ . \\ . \\ x_n \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

or equivalently,

$$x_0 + x_1 + ... + x_n = 0$$

$$x_1 + 2x_2 + ... + nx_n = 0$$

Using the above two equations we can say that the kernel is :

$$x_2 \times \begin{pmatrix} 1 \\ -2 \\ 1 \\ 0 \\ . \\ . \\ . \\ 0 \end{pmatrix} + x_3 \times \begin{pmatrix} 2 \\ -3 \\ 0 \\ 1 \\ 0 \\ . \\ . \\ 0 \end{pmatrix} + ...x_n \times \begin{pmatrix} n-1 \\ -n \\ 0 \\ . \\ . \\ . \\ 0 \\ 1 \end{pmatrix}$$

The discriminant variety is then constructed out of the Kernel:

$$\sum\nolimits_n = [\tau_1(x_2 + 2x_3 + ...(n-1)x_n) : \tau_1\tau_2(-2x_2 - 3x_3..... - nx_n) : \tau_1\tau_2^2 x_2 : \tau_1\tau_2^3 x_3 :$$

..... $: \tau_1 \tau_2^n x_n]$

$$= x_2 \tau_1 \times \begin{pmatrix} 1 \\ -2\tau_2 \\ \tau_2^2 \\ 0 \\ . \\ . \\ . \\ 0 \end{pmatrix} + x_3 \tau_1 \times \begin{pmatrix} 2 \\ -3\tau_2 \\ 0 \\ \tau_2^3 \\ 0 \\ . \\ . \\ 0 \end{pmatrix} + ... + x_n \tau_1 \times \begin{pmatrix} n-1 \\ -n\tau_2 \\ 0 \\ . \\ . \\ . \\ 0 \\ \tau_2^n \end{pmatrix}$$

where $x_2, x_3, ..., x_n$ are arbitrary complex numbers and τ_1 and τ_2 are nonzero complex numbers. Let us denote,

$$w_1 = \begin{pmatrix} 1 \\ -2\tau_2 \\ \tau_2^2 \\ 0 \\ . \\ . \\ . \\ 0 \end{pmatrix}, w_2 = \begin{pmatrix} 2 \\ -3\tau_2 \\ 0 \\ \tau_2^3 \\ 0 \\ . \\ . \\ 0 \end{pmatrix}, ...$$

$$w_{n-1} = \begin{pmatrix} n-1 \\ -n\tau_2 \\ 0 \\ . \\ . \\ . \\ 0 \\ \tau_2^n \end{pmatrix}$$

From this parametrization we see that $\sum_n$ is the union of the spans of $w_1, ..., w_{n-1}$ for all nonzero complex numbers τ_2. To check if a particular polynomial lies in $\sum_n$, for each value of τ_2, we construct two vectors s_1 and s_2 which are orthogonal to $w_1, ..., w_{n-1}$. These vectors will lie in the complement of the discriminant variety space. Hence if the polynomial path is not orthogonal to both these vectors, it will not lie in the discriminant variety space. In other words, a polynomial, P lies in $\sum_n$ if and only if $< P, s_1 > = 0$ and $< P, s_2 > = 0$ for some τ_2.

Explicitly we find s_1 and s_2 as follows to check the orthogonality of the polynomial path with them. Let this vector be

$$\begin{pmatrix} v_0 \\ v_1 \\ . \\ . \\ v_n \end{pmatrix}$$

Therefore

$$v_0 - 2\tau_2 v_1 + \tau_2^2 v_2 = 0$$

$$2v_0 - 3\tau_2 v_1 + \tau_2^3 v_3 = 0.$$

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$$(n-1)v_0 - n\tau_2 v_1 + \tau_2^n v_n = 0$$

Assigning two sets of values to the vectors v_0 and v_1 we get the vectors:

$$s_1 = \begin{pmatrix} 0 \\ 1 \\ \frac{2}{\tau_2} \\ \frac{3}{\tau_2^2} \\ . \\ . \\ \frac{n}{\tau_2^{n-1}} \end{pmatrix}, s_2 = \begin{pmatrix} 1 \\ 0 \\ -\frac{1}{\tau_2^2} \\ -\frac{2}{\tau_2^3} \\ . \\ . \\ -\frac{n-1}{\tau_2^n} \end{pmatrix}$$

Using the above two vectors we construct two polynomial equations, corresponding to $< P(\lambda), s_1 >= 0$ and $< P(\lambda), s_2 >= 0$.

$$(a_n + \lambda(b_n - a_n)) \times 0 + (a_{n-1} + \lambda(b_{n-1} - a_{n-1})) \times 1 + \ldots (a_0 + \lambda(b_0 - a_0)) \times (\frac{n}{\tau_2^{n-1}}) = 0$$

$$(a_n + \lambda(b_n - a_n)) \times 1 + (a_{n-1} + \lambda(b_{n-1} - a_{n-1})) \times 0 + \ldots (a_0 + \lambda(b_0 - a_0)) \times (\frac{-(n-1)}{\tau_2^n}) = 0$$

where $a_0, a_1, \ldots a_n$ are the coefficients of P_i and $b_0, b_1, \ldots b_n$ are the coefficients of P_g.

We can solve the above two equations for values of τ_2 and λ and check whether λ is a real number between 0 and 1. If it is not, that means the above two equations are satisfied for no values of λ between 0 and 1 and the straight line connecting the two polynomials does not intersect the discriminant variety. We also know that we can eliminate λ from the two equations and obtain a polynomial in τ_2. This polynomial has only finite number of roots and corresponding to each root there is a value of λ. This means that $P(\lambda)$ lies in the discriminant variety $\sum_n$ only for finitely many $\lambda \in F$, say. Hence even if some value of λ lies in $[0, 1]$ we can always find a path in C, as F is finite. C is the complex space. In this case we will need to parameterize the path in $u(\lambda)$, a complex variable such that $u(\lambda) \in C$.

3. Straight Line Path Algorithm

As developed in the previous section we can verify if the straight line path between the initial and final polynomial avoids the discriminant variety space. The roots of this polynomial can then be found out at each step to find the position of each mobile agent in local frame with the current local frame (frame which translates with the group) having undergone a translation from the initial frame. In other words each mobile agent is translated by the same amount with deformation caused by homotopy of the polynomial. The planning for translation can be done as described in Section D. Given the initial and final polynomial to each mobile agent, it's initial position and the velocity of translation, using Newton Raphson algorithm the mobile agents can calculate their position in the next time step in a distributed manner. Newton Raphson method can be used as we have a good initial guess at each time step for calculating the roots.

a. Example

We consider a scenario in which four mobile agents are initially arranged so that they are at four corners of a square with the group objective of transforming the square configuration to a straight line configuration. The initial coordinates of the mobile agents in local frame are $(-10, -10), (10, -10), (-10, 10), (10, 10)$ and the final coordinates in local frame are $(-11.25, -19.49), (-3.75, -6.50), (3.75, 6.50), (11.25, 19.49)$. Therefore $P_i = (x + 10 + 10i) \times (x - 10 + 10i) \times (x + 10 - 10i) \times (x - 10 - 10i)$ and $P_g = (x + 11.25 + 19.49i) \times (x + 3.75 + 6.50i) \times (x - 3.75 - 6.50i) \times (x - 11.25 - 19.50i)$. Now $P = (1 - \lambda) \times P_i + \lambda \times P_g$. Following the approach outlined above.

$$A = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 2 & 3 & 4 \end{pmatrix}$$

We let the kernel be $l = \begin{pmatrix} x_0 \\ x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix}$

Using $< A, l >= 0$ we get the kernel as:

$$x_2 \times \begin{pmatrix} 1 \\ -2 \\ 1 \\ 0 \\ 0 \end{pmatrix} + x_3 \times \begin{pmatrix} 2 \\ -3 \\ 0 \\ 1 \\ 0 \end{pmatrix} + x_4 \times \begin{pmatrix} 3 \\ -4 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$

Therefore the discriminant variety

$$\sum_n = x_2\tau_1 \times \begin{pmatrix} 1 \\ -2\tau_2 \\ \tau_2^2 \\ 0 \\ 0 \end{pmatrix} + x_3\tau_1 \times \begin{pmatrix} 2 \\ -3\tau_2 \\ 0 \\ \tau_2^3 \\ 0 \end{pmatrix} + x_4\tau_1 \times \begin{pmatrix} 3 \\ -4\tau_2 \\ 0 \\ 0 \\ \tau_2^n \end{pmatrix}$$

Hence

$$w_1 = \begin{pmatrix} 1 \\ -2\tau_2 \\ \tau_2^2 \\ 0 \\ 0 \end{pmatrix}, w_2 = \begin{pmatrix} 2 \\ -3\tau_2 \\ 0 \\ \tau_2^3 \\ 0 \end{pmatrix},$$
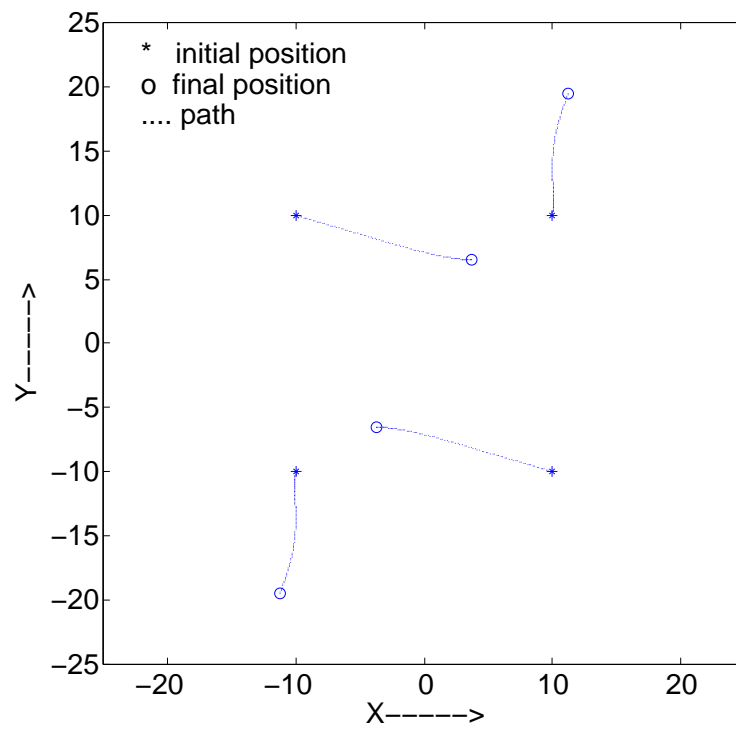
Fig. 3. Square to line formation

$$w_3 = \begin{pmatrix} 3 \\ -4\tau_2 \\ 0 \\ 0 \\ \tau_2^4 \end{pmatrix}$$

and

$$s_1 = \begin{pmatrix} 0 \\ 1 \\ \frac{2}{\tau_2} \\ \frac{3}{\tau_2^2} \\ \frac{n}{\tau_2^3} \end{pmatrix}, s_2 = \begin{pmatrix} 1 \\ 0 \\ -\frac{1}{\tau_2^2} \\ -\frac{2}{\tau_2^3} \\ -\frac{n-1}{\tau_2^4} \end{pmatrix}$$

Using $< P, s_1 >= 0$ and $< P, s_2 >= 0$ and solving for λ we get

$\lambda = -0.0114 - 0.2896i, -3.8787 + 1.9637i, 0.2216 + 0.1532i - 0.0114 - 0.2896i, -3.8787 + 1.9637i, 0.2216 + 0.1532i, 1.7729e - 034 + 7.4219e - 050i$

These values do not lie on the real line between 0 and 1 and hence the polynomial $P(\lambda)$ does not intersect the discriminant variety. Simulation of the example above in which a group of robots move from a square configuration to a line configuration is shown in Fig. 3. As expected the agents do not collide while moving from the initial to the final configuration. Fig. 4 and Fig. 5 show results for other initial and final shapes of the group of mobile agents.

4. Potential Field Approach for Homotopy

a. Potential Field Approach

In the potential field approach [72], the mobile agent is represented as a point in configuration space under the influence of an artificial potential field U. This potential field comprises of an attractive and a repulsive component. The potential field
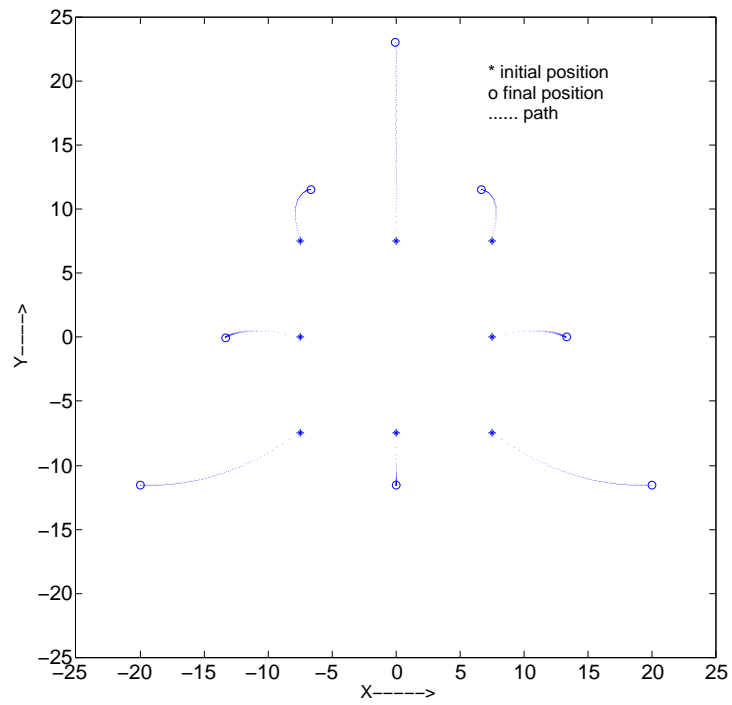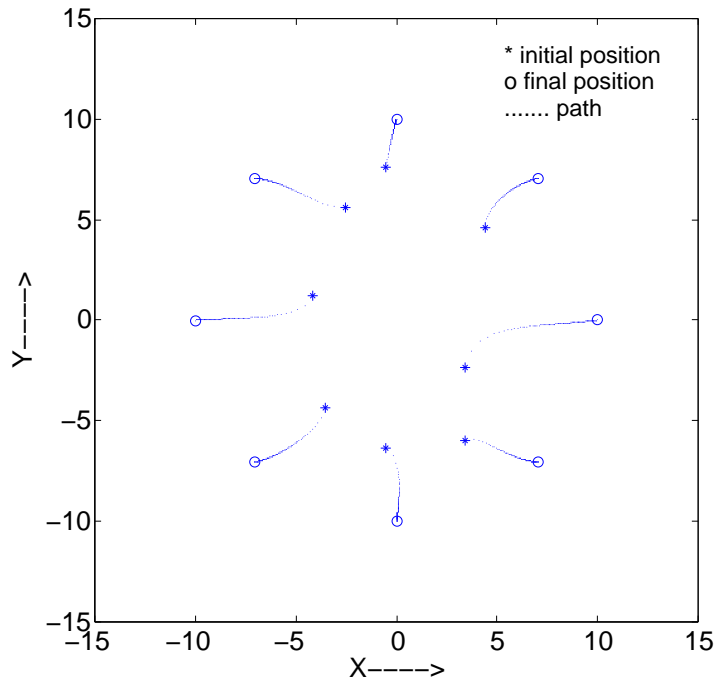
Fig. 4. Square to triangle formation

Fig. 5. Arbitrary formation to circle

is created in a manner that the mobile agent moves towards the goal configuration and moves away from obstacles. Fig. 6 illustrates this. It can be seen in the figure that where the obstacle is located, the potential field is high and where the goal is located, there is a well. Hence the agent will move away from the obstacle and towards the goal as it moves towards lower potential areas. The attractive potential is a function of the distance from the goal while the repulsive potential is a function of the minimum distance from the configuration space obstacle or C-Obstacle which is the space of all configurations for which the mobile agents collide. The artificial potential force is computed as $\overrightarrow{F}(q) = -\nabla U(q)$ where q is the current configuration. The agent takes a step in the direction of the force and repeats till the goal is reached. Following is an example of a potential function:
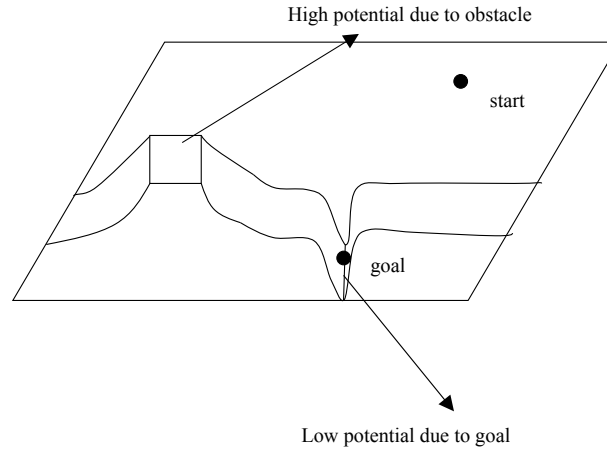
High potential due to obstacle

start

goal

Low potential due to goal

Fig. 6. Potential field

The attractive potential, $U_{att} = \frac{1}{2}\xi\rho^2(q, q_{goal})$

where $\rho(q, q_{goal}) = \|q - q_{goal}\|$

ξ is a positive scaling factor,

$\|q - q_{goal}\|$ is the euclidean distance and q_{goal} is the goal configuration. The repulsive potential

$$U_{rep} = \begin{cases} \{\frac{1}{2}\eta\left(\frac{1}{\rho(q,q_{goal})} - \frac{1}{\rho_0}\right)^2 \, if \, \rho(q, q_{goal}) \leq \rho_0 \\ \\ 0 \, if \, \rho(q, q_{goal}) > \rho_0 \end{cases}$$

where (q, q_{goal}) is the minimum distance from the C-obstacle, ρ_0 is the distance of influence and ξ, η are positive scaling factors.

The total potential function is $U = U_{att} + U_{rep}$. Propagating q such that $\dot{q} =$

$-\nabla U$. Hence the differential equation governing the variation in q with time is:

$$\dot{q} = \begin{cases} \xi\rho(q,q_{goal}) + \eta(\frac{1}{\rho(q,q_{goal})} - \frac{1}{\rho_0})\frac{1}{\rho(q,q_{goal})^2}\nabla_q\rho(q,q_{goal}), \; if \; \rho(q,q_{goal}) \leq \rho_0 \\ \\ \xi\rho(q,q_{goal}), \; if \; \rho(q,q_{goal}) > \rho_0 \end{cases}$$

We develop a method to find paths in polynomial space using the potential approach below.

5.   Homotopy Using Potential Field

We apply the potential field approach described above in an analogous fashion to plan the parametric path in polynomial space [77]. By letting $P(t) = a_n(t)x^n + a_{n-1}(t)x^{n-1} + \cdots + a_1(t)x + a_0$ be a time varying polynomial and

$$a(t) = \begin{pmatrix} a_n \\ a_{n-1} \\ . \\ . \\ . \\ a_1 \\ a_0 \end{pmatrix}$$

we can interpret $a(t)$ as analogous to the mobile agent configuration $q(t)$. We intend to generate the polynomial path $P(t)$ by propagating $a(t)$ in the polynomial coefficient space in a manner that it has no multiple roots at any time. $a(t)$ represents a point on the polynomial path $P(t)$ which in turn represents the positions of the mobile agents w.r.t. a local frame in 2-D at time t. The discriminant of the polynomial Δ which signifies the distance from the discriminant variety or in other

words the distance between the roots of the polynomial, can be treated as analogous to the minimum distance from the C-Obstacle. As described above $q(t)$ propagates such that it moves away from the C-obstacle and towards the goal configuration q_{goal}. In a similar fashion, the polynomial vector $a(t)$ is made to propagate such that it moves away from the discriminant variety and towards the goal polynomial vector a_{goal}. This is shown below.

An artificial potential field is created such that the change in the polynomial coefficient vector $a(t)$ is in the direction of the negative gradient of the potential. The goal polynomial coefficients create an attractive potential while the discriminant variety creates a repulsive potential. The attractive potential U_{att} is constructed such that it increases as $a(t)$ moves away from a_{goal}, the goal polynomial coefficient vector. The repulsive potential U_{rep} is constructed in such a way that $a(t)$ moves away from the discriminant variety and is unaffected when it is far from it. An example of the potential field and the differential equation governing the change in $a(t)$ is given below. We selected this potential field as it is the most commonly used one in literature. The attractive potential is parabolic in shape while the repulsive potential is a function of the inverse of the discriminant :

$$U_{att} = \tfrac{1}{2}\xi\rho^2(a, a_{goal})$$

where $\rho(a, a_{goal}) = \|a - a_{goal}\|$

ξ is a positive scaling factor and

$\|a - a_{goal}\|$ is the two norm in polynomial coefficient space.

$$U_{rep} = \begin{cases} \{\frac{1}{2}\eta(\frac{1}{\Delta} - \frac{1}{\Delta_0})^2 \, if \, \Delta \leq \Delta_0 \\ \\ 0 \, if \, \Delta > \Delta_0 \end{cases}$$

The total potential function is $U = U_{att} + U_{rep}$. We propogate a such that $\dot{a} = -\nabla U$. Hence the differential equation governing the variation in a with time is:

$$\dot{a} = \begin{cases} \xi\rho(a, a_{goal}) + \eta(\frac{1}{\Delta} - \frac{1}{\Delta_0})\frac{1}{\Delta^2}\nabla_a\Delta, \, if \, \Delta \leq \Delta_0 \\ \\ \xi\rho(a, a_{goal}), \, if \, \Delta > \Delta_0 \end{cases}$$

where,

Δ_0 is the distance of influence of the discriminant variety and ξ, η are positive constants

Potential field approaches have an issue with the agent configuration getting stuck in local minima. A lot of work has been done to counter this problem [72]. The problem of local minima for the potential field based homotopy approach can be handled in a similar fashion.

a. Example.

We solve an example in which there are four mobile agents, using the method we developed above. The agents move from an initial arbitrary configuration to a final arbitrary configuration. Since there are four mobile agents, the polynomial is of fourth order. We take the initial polynomial as $P_i = x^4 + 4x^3 + x^2 + x + 2$ and the final polynomial as $P_g = x^4 + 5x^3 + x^2 + 4$. The choice of the polynomial coefficients for the initial and final polynomials has been on the basis of their simplicity and no other particular

reason. For a fourth order polynomial equation $a_4x^4 + a_3x^3 + a_2x^2 + a_1x + a_0 = 0$, the discriminant is :

$$[(a_1^2a_2^2a_3^2 - 4a_1^3a_3^3 - 4a_1^2a_2^3a_4 + 18a_1^3a_2a_3a_4 - 27a_1^4a_4^2 + 256a_0^3a_4^3) + a_0(-4a_2^3a_3^3 + 18a_1a_2a_3^3 +$$
$$16a_2^4a_4 - 80a_1a_2^2a_3a_4 - 6a_1^2a_3^2a_4 + 144a_1^2a_2a_4^2) + a_0^2(-27a_3^4 + 144a_2a_3^2a_4 - 128a_2^2a_4^2 -$$
$$192a_1a_3a_4^2)].$$

With the above discriminant and the values of the initial and final coefficient vectors we construct the differential equations for the polynomial coefficient vector using :

$$\dot{a} = \begin{cases} \xi\rho(a, a_{goal}) + \eta(\frac{1}{\Delta} - \frac{1}{\Delta_0})\frac{1}{\Delta^2}\nabla_a\Delta, \; if \; \Delta \leq \Delta_0 \\ \\ \xi\rho(a, a_{goal}), \; if \; \Delta > \Delta_0 \end{cases}$$

Figure 7 shows the results for this example. Each of the coefficients of the polynomial path $P(t)$ are shown as functions of time.

6. Other Polynomial Paths

Apart from the straight line polynomial path and the path generated by the potential field approach we can also use other parametric paths $P(t)$. These paths should be such that at $t = 0, P(0) = P_i$, the initial polynomial and at $t = T, P(T) = P_g$, the final polynomial. Some examples of such paths are $(1 - (\frac{t}{T})^n)P_i + (\frac{t}{T})^nP_g, n = 1, 2....,$ $(1 - (\frac{t}{T})^n + (\frac{t}{T})^{2n} - (\frac{t}{T})^{4n})P_i + ((\frac{t}{T})^n - (\frac{t}{T})^{2n} + (\frac{t}{T})^{4n})P_g, n = 1, 2.....$ It is to be noted that with the change in polynomial paths, velocities and accelerations of the mobile agents change in addition to change in their positions. This is because the differential of the polynomial paths change. The change in the positions of the mobile agents is

illustrated in Fig. 8,9 and 10 for an example in which the agents go from a square configuration to a line configuration and translate by 100 units in both X and Y directions .
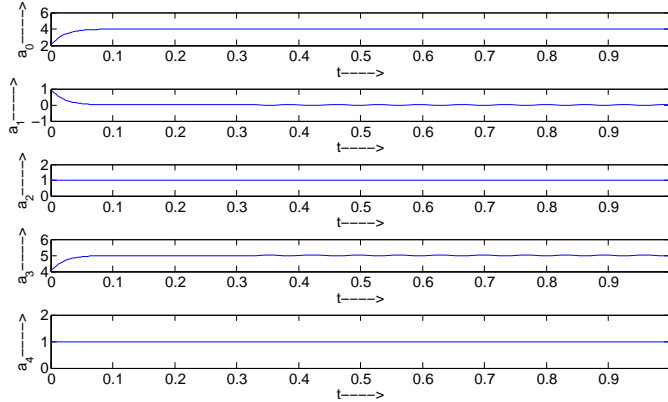

Fig. 7. Potential field approach to homotopy

C.   Planning the Translational Motion

In the previous section we developed the motion plan for the group shape change. In this section we develop the translational motion plan of the group of mobile agents. The translational motion is planned by finding a bounding disc for the group and planning the motion of this disc as a single agent through stationary obstacles. This bounding disc encloses the agents at all times. In [73] the following result is stated: The roots of a polynomial can be bounded within a disc of radius:

$$r = 2max\{|\frac{a_{n-k}}{a_n}|^{\frac{1}{k}} : k = 1, ..., n\}$$

where a_k are the coefficients of the polynomial $P(t)$. We consider the straight line polynomial path [74] in polynomial space with $P(t) = (1 - t)P_i + tP_g$ where P_i, P_g
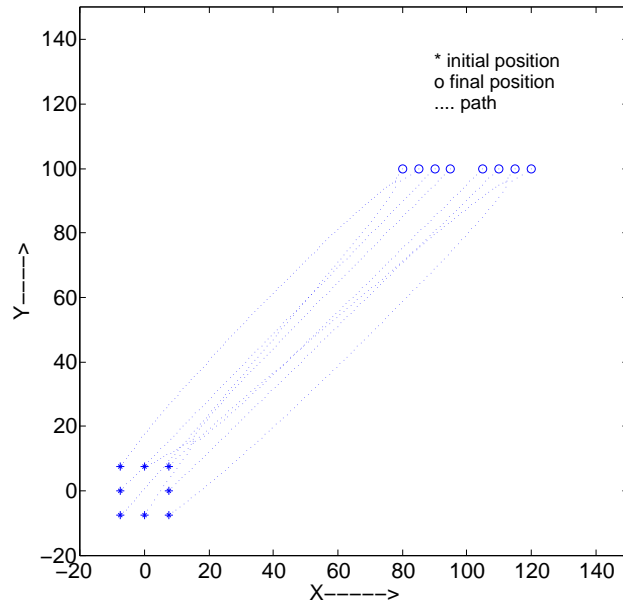
Fig. 8. Motion plan for the straight line polynomial path
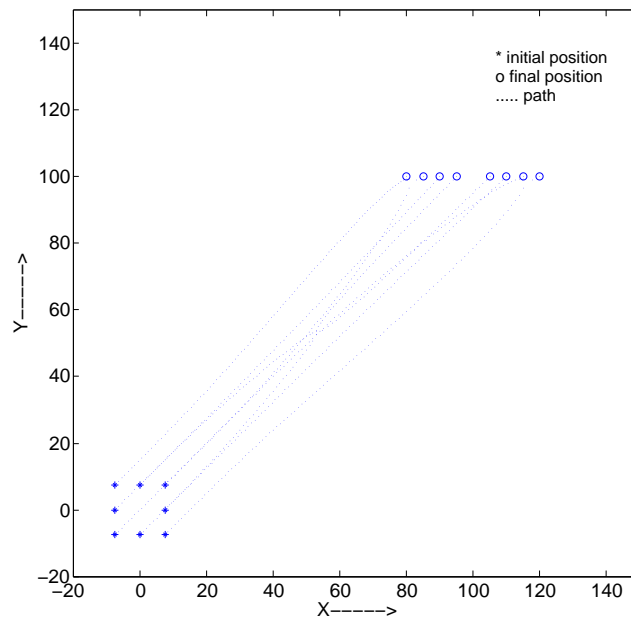

Fig. 9. Motion plan for the path $(1 - \lambda^3)P_i + \lambda^3 P_g$
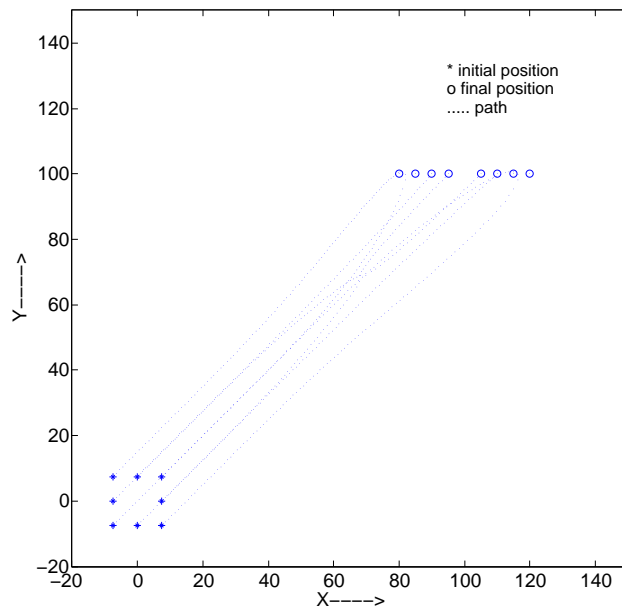
Fig. 10. Motion plan for the path $(1 - \lambda^5)P_i + \lambda^5 P_g$

are the initial and final polynomials respectively. If,

$P_i = a_{ni}x^n + a_{ni-1}x^{n-1} + ...a_{0i}$ and

$P_g = a_{ng}x^n + a_{ng-1}x^{n-1} + ...a_{0g}$ then

$r = 2max\{|\frac{(1-t)a_{ni-k}+ta_{ng-k}}{(1-t)a_{ni}+ta_{ng}}|^{\frac{1}{k}} : k = 1, ..., n\}$.

Since $a_{ni} = a_{ng} = 1$

$\Rightarrow r = 2max\{|(1-t)a_{ni-k} + ta_{ng-k}|^{\frac{1}{k}} : k = 1, ..., n, t \in [0,1]\}$

$\Rightarrow r = 2max\{| a_p |^{\frac{1}{k}}: (p,k) = (0i, n), (1i, n-1), \cdots, (ni-1, 1), (0g, n), (1g, n-$

$1), \cdots , (ng-1, 1)\}$ We support the theory developed in this section with an example below.

1.   Example for Finding Bounding Disc.

We consider an initial arbitrary formation in which the coordinates of the mobile agents are:

$(-9, -7), (-2, -3), (10, -8), (10, 0), (5, 4), (9, 7), (-8, 7)$
$, (-5, 2)$

and a final square formation in which the mobile agents are at coordinates:

$(-10, -10), (0, -10), (10, -10), (-10, 0), (10, 0), (-10, 10),$
$(0, 10), (10, 10).$

We find a bounding disc for this initial and final configuration so that the disc can be used to plan the motion of the mobile agents through stationary obstacles. The initial and final polynomials using the above coordinates are: $P_i = x^8 + (-10 + 2i)x^7 + (-72 - 14i)x^6 + (838 - 374i)x^5 + (1.87 \times 10^4 + 3.22 \times 10^3 i)x^4 + (-1.625 \times 10^5 + 5.58 \times 10^4 i)x^3 + (-6.38 \times 10^5 - 1.66 \times 10^5 i)x^2 + (1.95 \times 10^6 - 1.9 \times 10^6 i)x + 7.86 \times 10^6 - 2 \times 10^7 i$ and,

$P_g = x^8 + 30000x^4 - 400000000.$

Using, $r = 2max\{| a_p |^{\frac{1}{k}} : (p, k) = (0i, n), (1i, n-1), \cdots , (ni - 1, 1), (0g, n), (1g, n - 1), \cdots , (ng - 1, 1)\}$ we get the bounding disc radius $r = 26.32$. Figure 11 shows the

result for this example.

Once r is found, we can use any of the standard methods for motion planning of a single agent through obstacles in 2-D for planning the motion of the bounding disc. In this work we use the roadmap method [72]. The disc enclosing the mobile agents is shrunk to a point and the C-obstacles grown accordingly. The roadmap is constructed and the shortest path found between the initial and final configuration.

Figure 12 illustrates the result for an example in which a group of agents in square formation need to change to a triangular formation while avoiding polygonal obstacles on the way. The straight line polynomial path was used for group shape change planning while a bounding disc was found to plan the motion of the group through the polygonal obstacles. Some intermediate configurations have been shown and it is seen that the agents do not collide with each other or with the obstacles. The roadmap method was used for planning the motion of the bounding disc in this example. Next we develop a way to handle velocity and acceleration constraints which need to be incorporated due to the actuation limitations of real mobile agents.

2.   Imposing Velocity and Acceleration Constraint

Since the actuators of the mobile agents have actuation limits, we must impose velocity and acceleration constraints on the motion plan. The velocities and accelerations of the mobile agents have two components. One is the shape change component and the other is the group translation component. The shape change components are the velocities and accelerations of the mobile agents due to the deformation of the group while the translational component is due to translation of the group as a whole. We can impose velocity and acceleration constraints on each mobile agent by reparameterizing P, the polynomial path to keep the shape change component within bounds and by planning the translational velocity and acceleration component in a
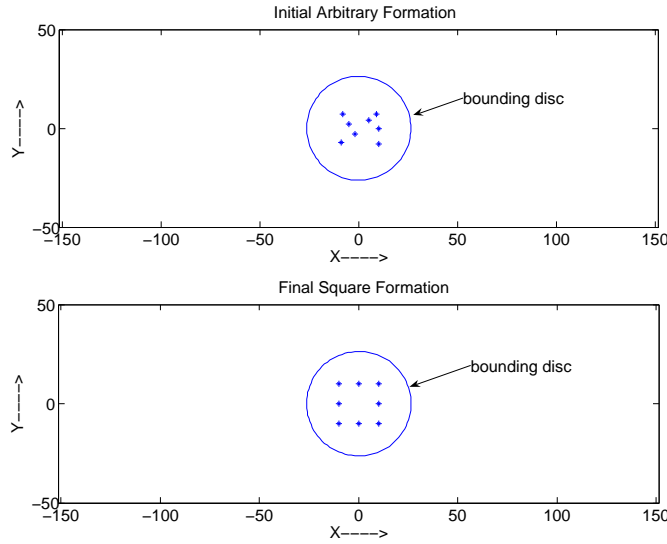
Fig. 11. Disc bounding the agents at all times

way that the resultant velocity and acceleration bounds are not violated. We define
$P'(\lambda) = P(f(\lambda)) = (1 - f(\lambda))P_i + f(\lambda)P_g, f \in [0, 1]$. This produces the same geometric curves for shape change but the rate at which these curves are traversed by each agent as λ varies will depend on the function f, a strictly increasing function. The function f should be a strictly increasing function so that the polynomial path does not retrace back to a previous state. For example since the speed $\frac{dx}{dt} = \frac{dx}{df} \times \frac{df}{dt}$, we can chose f in a way that $\frac{df}{dt}$ is small when $\frac{dx}{df}$ is large to keep the modulus of the velocity within bounds. Figure 13 and Fig. 14 show simulations with and without velocity constraints for shape change of the form $\frac{dx}{dt} = \frac{dx}{df} \times \frac{df}{dt} < \gamma, \gamma$ being a constant, for the deformation of a square shaped group to a circle. The sparsely spaced dots in Fig. 13 near the square shape indicate high velocities which are in violation of the velocity bounds. By reparameterization of the polynomial path we were able to satisfy the velocity constraint as indicated in Fig. 14 by the somewhat uniformly spaced dots. Once the shape change velocities and accelerations are planned to satisfy the
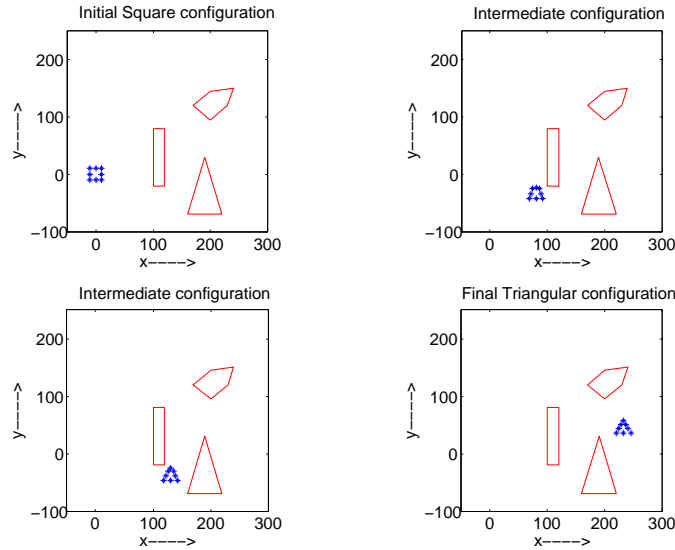
Fig. 12. Obstacle avoidance

constraints, we plan the translational velocities and accelerations of the entire group such that the resultant velocities and accelerations are within bounds. In the next section we implement the motion plan generated using the homotopy approach on a group of non holonomic vehicles.

D. Implementation on a Group of Non Holonomic Vehicles

The roots of the polynomial and the plan for translational motion of the group gives us the x and y positions of the mobile agents and their angle $\theta = \arctan(\frac{\dot{y}(t)}{\dot{x}(t)})$ at each time instant. Hence the desired pose $p_r = \begin{pmatrix} x_r \\ y_r \\ \theta_r \end{pmatrix}$ can be found at each time instant. Similarly by differentiation of the polynomial path generated through the group shape change algorithm and usage of the translational motion plan of the group, the vector
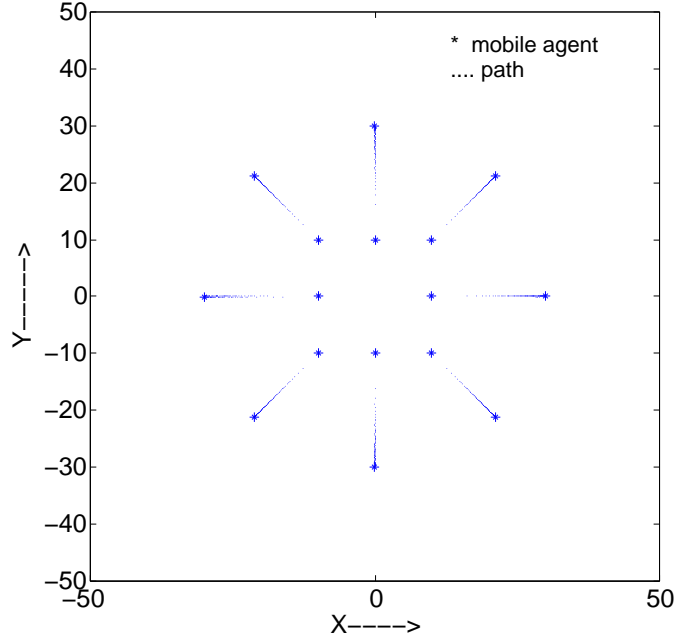
Fig. 13. Without velocity constraint

$q_r = \begin{pmatrix} v_r \\ \omega_r \end{pmatrix}$ can be found out at each time instant where v is the linear velocity and ω is the angular velocity. Once we have the vectors $p_r(t)$ and $q_r(t)$ we can use the non linear controller proposed in [36] to control any group of non holonomic [75] vehicles along the trajectories generated by the homotopy algorithm proposed in this work. The control law proposed in [36] is $q = \begin{pmatrix} v \\ \omega \end{pmatrix} = \begin{pmatrix} v_r cos\theta_e + K_x x_e \\ \omega_r + v_r(K_y y_e + K_\theta sin\theta_e) \end{pmatrix}$.

$p_e = \begin{pmatrix} x_e \\ y_e \\ \theta_e \end{pmatrix} = p_r - p_c$ is the error pose, p_c being the current pose. $q_e = \begin{pmatrix} v_e \\ \omega_e \end{pmatrix} = q_r - q_c$ is the error velocity vector, q_c being the current velocity vector. K_x, K_y, K_θ are the controller gains. We assume that we can measure the current positions of the mobile agents using dead reckoning etc..
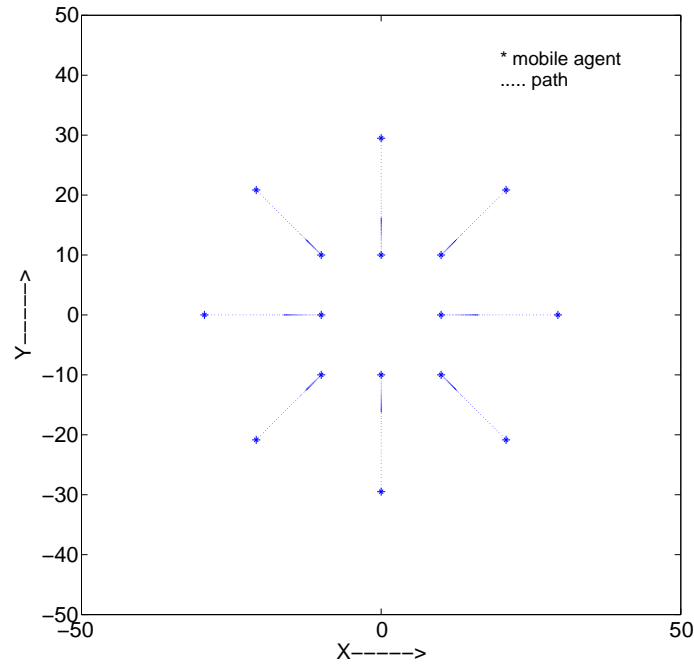
Fig. 14. With implementation of velocity constraint

Figure 15 illustrates an example in which there are eight non holonomic agents in an initial square formation and a final circular formation. In this example we assume there are no obstacles in the environment and that the entire group is translated by 100 units in the X and Y directions. As can be seen from the figure, the paths that are planned using the homotopy approach are tracked though there is some deviation from the reference path initially. In the next section we formulate and solve the 3-D version of a global motion planning problem using the homotopy approach.

E.   3-D Problem

In the previous sections we have assumed that the mobile agents are moving in 2-D. In this section we solve the 3-D version of the problem formulated in Chapter II for the case in which there are no obstacles in the environment.   There are a
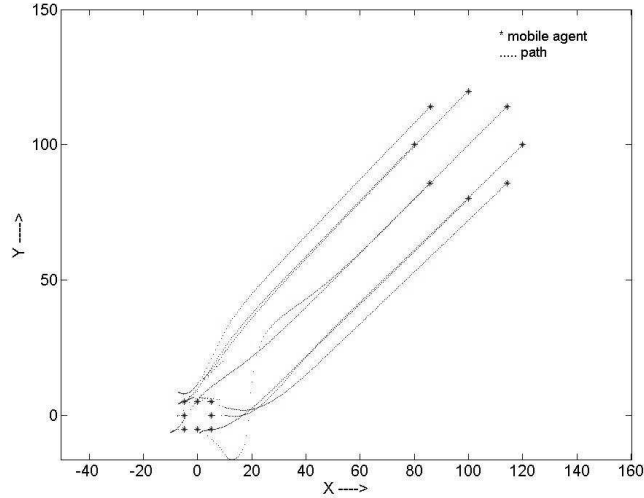
Fig. 15. Trajectory tracking

number of practical situations in which mobile agents move in the three dimensional real world. As an example, a group of Unmanned Air Vehicles(UAVs) may need to change formation to avoid being tracked by the enemy radar or for better surveillance of the enemy installations. We are motivated by the need to develop an algorithm which facilitates the mobile agents in situations like this to change formation with no communication and sensing. Other applications could be change in the position of satellites in deep space.

1. Assumptions

The key assumption for the 3-D problem is:

- The agents are represented as point masses.

2. Shape Change Algorithm

We approach the 3-D problem just like the 2-D problem by mapping the 3-D coordinates of the mobile agents in local frame to 2-D.

a. Mapping

If (x, y, z) are the coordinates of an agent in local frame then we map it to 2-D through a linear map of two of the coordinates. This is done so that we can use the algorithm that we used for the 2-D case. For example if we select the x and y coordinates for the linear map, the 2-D map is $(l(x, y), z)$ where $l(x, y) = ax + by$, a and b are constants. We select the map $l(x, y)$ in a way that the pair $(l(x, y), z)$ corresponding to each robot's coordinates in the initial configuration is unique. We do the same for each robot's coordinates in the final configuration. There are a maximum of $2n_{C_2}$ lines in the XY plane such that atleast 2 mobile agents lie on them when their coordinates in the initial and final configurations are mapped onto the XY plane through a linear map, $l(x, y)$. This is because there are 2n sets of coordinates in the initial and final configurations which yield $2n_{C_2}$ lines. If $l(x, y) = 0$ is such that it is not parallel to any of these $2n_{C_2}$ lines, the mapping $l(x, y)$ will be unique for each agent. Hence we can always find an $(l(x, y), z)$ such that the pair is unique for each robot in the initial and final configuration. Figure 16 shows an example of a feasible map l.

Once we have found the mapping l, we construct the polynomial as in the 2-D case so that we can follow a similar approach. Therefore if $(x_{1i}, y_{1i}, z_{1i}), (x_{2i}, y_{2i}, z_{2i}), ...,$ (x_{ni}, y_{ni}, z_{ni}) are the coordinates of the mobile agents in the initial configuration and $(x_{1f}, y_{1f}, z_{1f}), (x_{2f}, y_{2f}, z_{2f}), ..., (x_{nf}, y_{nf}, , z_{nf})$ are the coordinates of the mobile agents in the final configuration, the initial and final polynomials are constructed as below:

$$P_i(x) = (x - (l(x_{1i}, y_{1i}) + jz_{1i}))(x - (l(x_{2i}, y_{2i}) + jz_{2i}))......(x - (l(x_{ni}, y_{ni}) + jz_{ni}))$$
and $P_g(x) = (x - (l(x_{1f}, y_{1f}) + jz_{1f}))(x - (l(x_{2f}, y_{2f}) + jz_{2f}))......(x - (l(x_{2f}, y_{2f}) + jz_{nf}))$

Using the initial and final polynomials we find a path in polynomial space which avoids the discriminant variety. The path can be the straight line polynomial path
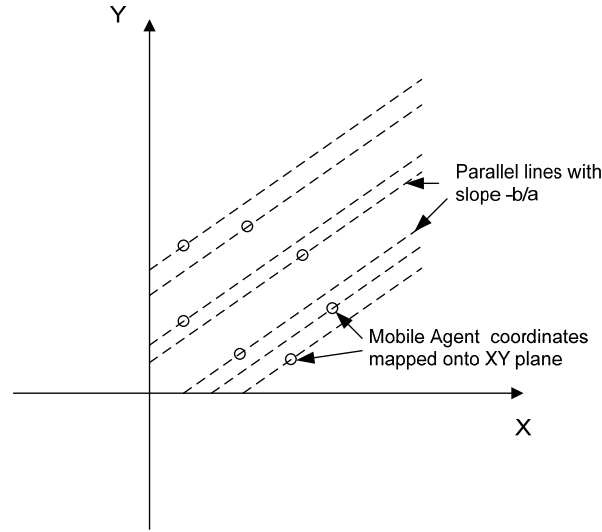
Fig. 16. Feasible mapping

$P(t) = (1 - \lambda)P_i + \lambda P_g, \lambda \in [0, 1]$ where $\lambda = \frac{t}{T}$ and time $t \in [0, T]$ or can be found out using the potential field approach. This path generates $l(t)$, the linear map and $z(t)$ for each robot. One of the coordinate paths $x(t)$ or $y(t)$ can be generated by joining the corresponding initial and final coordinates by straight lines. Hence if we chose $y(t)$ then for the first robot, $y(t) = (1 - \lambda)y_1 i + \lambda y_1 g, \lambda \in [0, 1]$ where $\lambda = \frac{t}{T}$ and time $t \in [0, T]$ and so on for all robots. Please note that $y_1 g$ is the y coordinate corresponding to the final values that $(l(x_{1i}, y_{1i}), z_{1i})$ end up at. The coordinate path $x(t)$ is then generated from $l(t)$ and $y(t)$ for each robot.

b. Example

We solve an example which has six mobile agents. The initial coordinates of the mobile agents are $(1, 0, 0), (-1, 0, 0), (0, 1, 0), (0, -1, 0), (0, 0, 1), (0, 0, -1)$ and the final coordinates are $(0, \frac{1}{\sqrt{3}}, 0), (\frac{1}{2}, -\frac{1}{2\sqrt{3}}, 0), (-\frac{1}{2}, -\frac{1}{2\sqrt{3}}, 0), (0, 0, 0), (0, 0, 3), (0, 0, -3)$. We take $l(x, y) = 3x + 2y$. Therefore $P_i = (x-3) \times (x+3) \times (x-2) \times (x+2) \times (x-i) \times (x+i)$
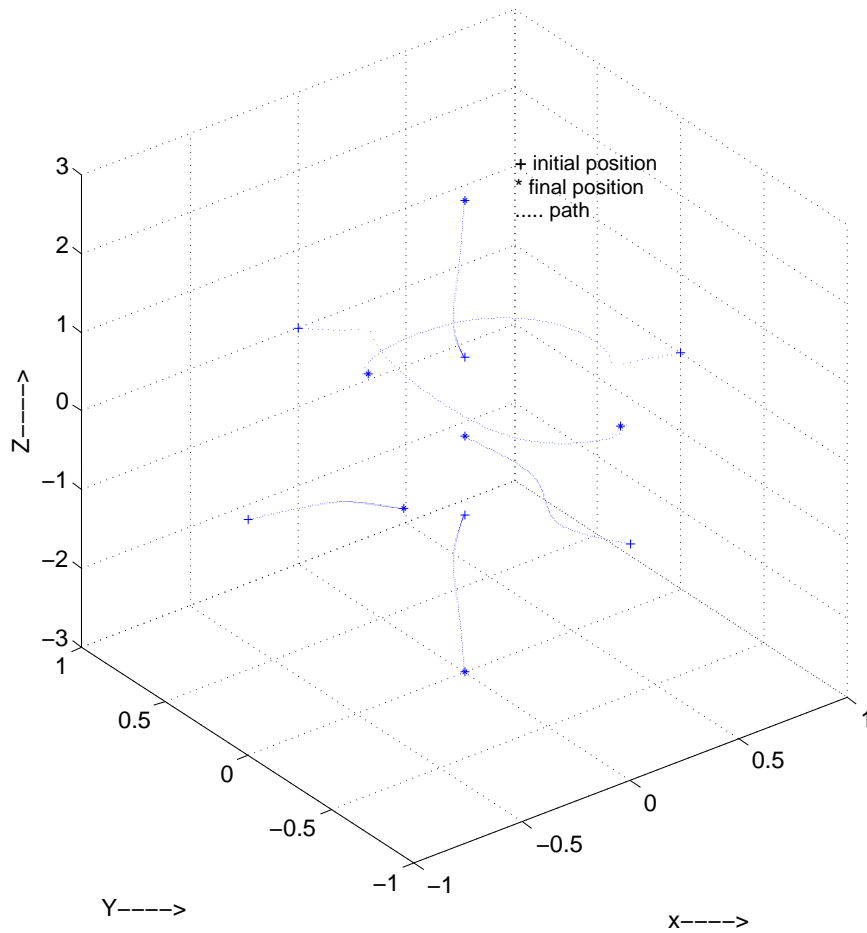
Fig. 17. Motion plan in 3-D

and $P_g = (x - \frac{2}{\sqrt{3}}) \times (x - (\frac{3}{2} - \frac{1}{\sqrt{3}})) \times (x - (-\frac{3}{2} - \frac{1}{\sqrt{3}})) \times (x - 0) \times (x - 3i) \times (x + 3i)$.

We use the straight line polynomial path $P(t) = (1 - \lambda) \times P_i + \lambda \times P_g$ as it avoids the discriminant variety. Once we have $l(t)$ and $z(t)$ corresponding to each robot by finding the roots of $P(t)$, we generate $y(t)$ by joining the corresponding initial and final y coordinates of the robots by straight lines and then find $x(t)$ for each robot using $x(t) = (l(t) - 2y(t))/3$.Figure 17 illustrates the path of each robot for this example.

The translational planning can be done just as in the 2-D case to avoid stationary obstacles by bounding the group by a sphere. Velocity and acceleration constraints can be imposed by reparameterization as in the 2-D case.

In this chapter we have developed the homotopy approach for motion planning of mobile agents. We first solved the two dimensional case and then the three dimensional case. We have developed an algorithm to deal with stationary obstacles for the 2-D case and implemented the algorithm on a group of non holonomic vehicles. We have imposed velocity and acceleration constraints using reparameterization of the planned polynomial path.

CHAPTER IV

CONTINUUM APPROACH

In this chapter we develop a novel motion planning approach to solve the problem formulated in Chapter II for a swarm of mobile agents based on continuum theory. In Section A we give an overview of the approach. In Section B we develop the continuum approach. We have initially assumed that the agents are point sized but in Section C we have developed a way to handle finite sized agents. We have implemented the algorithm on a group of non holonomic agents in Section D. Finally in Section E we develop a way to handle stationary obstacles in the workspace.

A. The Approach

The continuum approach consists of finding a motion map between the initial and final configuration which is a homeomorphism. This will ensure no collision between agents. We solve the case in which there are stationary obstacles in the environment by bounding the group by a rectangular box which can deform, translate and rotate as has been done in [37]. Since we assume initially that the mobile agents are point objects, we show a way to handle finite sized agents using this approach. We also implement the motion plan on a group of non holonomic agents using the controller proposed in [36].

1. Assumptions

The main assumptions are :

- There are no restrictions on the initial shape or the locations of vehicles within that shape provided no two vehicles occupy the same location at any given

instant.

- Each vehicle knows it's position with reference to a global coordinate system.

B. Continuum Approach

We consider a swarm of vehicles at rest in an initial configuration B_0 at time $t = t_0$ as shown in Fig. 18. We are interested in moving this swarm of vehicles from the initial configuration B_0 to a new configuration, termed the current configuration B_t . The current configuration differs from the initial configuration and we say that the swarm has undergone a deformation from B_0 to B_t. Again we assume that each vehicle knows its current location with respect to a global reference frame which may be inferred by data fusion, onboard sensing,inter-agent communications etc. It is further assumed that the current configuration may be prescribed apriori so that the desired goal configuration is achieved. For example, a typical scenario might be where a vehicle swarm is originally in a square configuration of side a at $t = t_0$, that may be required to deform into an elongated rectangular shape with a different orientation and location so that the swarm may travel through a narrow passage. As long as the motion map (path) is a homeomorphism between the reference and the current configuration, each mobile agent is guaranteed to occupy a unique position in any of the configurations, implying no two mobile agents can occupy the same place. One particular class of feasible deformations is homogeneous deformations. A homogeneous deformation is one that can be decomposed into a rigid body rotation and a special deformation following the classic polar decomposition theorem of matrices according to which the matrix can be decomposed into a unitary matrix and a positive semidefinite Hermitian matrix.

We introduce a fixed Cartesian reference frame with origin O and basis vectors e_i.
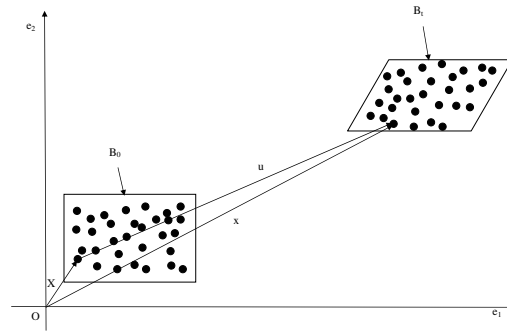
Fig. 18. Formation change

All motion will be relative to this fixed frame and all vector and tensor components are with respect to the base vectors e_i. Let $\mathbf{X}$ be the position vector, relative to O, of a typical vehicle V_o within B_0 . Then the components X_J of $\mathbf{X}$, in the chosen coordinate system, are the coordinates of the position occupied by the vehicle in B_0 at $t = t_0$. Now suppose that the vehicle that occupies a position $\mathbf{X}$ at time $t = t_0$ in the reference configuration moves so that at a subsequent time it occupies a new position x at time t. Let us now denote the position x of the vehicle at time t with respect to its reference position $\mathbf{X}$ at time $t = t_0$, by an equation of the form X=x($\mathbf{X}$,t. We can think of this relationship specifying the locations of agents in a given reference configuration with respect to a current configuration. The idea here is that once we know the reference position of each agent and the mapping between the reference configuration B_0 and the current configuration B_t , the current locations of the agents can be immediately determined. So the key idea then becomes to see if the map defining the resulting motion can be determined in a meaningful way. This is exactly what we propose to do.

To facilitate the above process we define the following quantities. The displacement vector $\mathbf{u}$ of a typical vehicle from its position $\mathbf{X}$ in the reference configuration

B_0 to its position x at t in the current configuration B_t is $\mathbf{u}=\mathbf{x}\text{-}\mathbf{X}$. In the reference co-ordinates $\mathbf{u}$ is regarded as a function of $\mathbf{X}$ and t so that, $\mathbf{u}(\mathbf{X},t)=\mathbf{x}(\mathbf{X},t)\text{-}\mathbf{X}$ and in the current configuration $\mathbf{u}$ is regarded as a function of x and t, so that $\mathbf{u}(x,t)=\mathbf{x}\text{-}\mathbf{X}(x,t)$.

The velocity vector $\mathbf{v}$ of an agent is the rate of change of its displacement. Since $\mathbf{X}$ is constant for an agent in its reference position, it is convenient to employ the reference description so that $v(X,t) = \frac{\partial u(X,t)}{\partial t} = \frac{\partial x(X,t)}{\partial t}$, where the differentiations are performed with $\mathbf{X}$ held constant. In component form the latter can be written as $v_i(X_J,t) = \frac{\partial x_i(X_J,t)}{\partial t}$. The result of performing the latter differentiation is to express the velocity components as functions of X_J and t; that is, they give the velocity at time t of the agent that was at $\mathbf{X}$ at time $t = t_0$. Similarly, we can describe the acceleration in component form as $a_i(X_J,t) = \frac{\partial v_i(X_J,t)}{\partial t} = \frac{\partial^2 x_i(X_J,t)}{\partial t^2}$ or in vector form as $a = \dot{v}(X,t) = \ddot{x}(X,t)$.

1. Dynamic Constraints

With expressions for velocity and acceleration available in terms of the reference configuration, we need to find acceptable maps that satisfy the various dynamic con-straints the vehicles must satisfy. These maps could be of the form $x_k = \sum_{i=0}^{2} \alpha_{ik}(X_J)t^i, x_i = \sum_{i=0}^{2} \alpha_i(X_J)sin(w_i t)$ or $x_i = \sum_{i=0}^{2} \alpha_i(X_J)e^{-\lambda_i t}$. Then we have a way of imposing bounds on forces acting on each agent so that they do not grow with time. This facilitates handling of kinematic constraints in a rather nice way. As an example, if agents move according to the map given by the first form, then the accelerations become $\ddot{x}_k = 2\alpha_{k2}(X_J)$, with velocity $\dot{x}_k = \alpha_{k1}(X_J) + 2\alpha_{k2}(X_J)t$. With this setup the forces acting on an agent can be represented as $F_k = m_k\ddot{x}_k = 2\alpha_{k2}(X_J)$, where the vehicle is simply treated as a particle. Now suppose each of the forces is constrained to be $|F_k| \leq \gamma_k$, which then leads to the requirements $|F_k| =| m_k\ddot{x}_k |=| 2\alpha_{k2}(X_J) |\leq \gamma_k$,

from which constraints on $\alpha_{k2}(X_J)$ can be obtained. For example, when the initial configuration is known we can explicitly write down $\alpha_{k2}(X_J)$. Similarly, velocity constraints may also be employed in defining the suitable motion map. In general the maps $\alpha_k(X_J)$ can be nonlinear to handle various constraints. In general maps could be of the form $\alpha_{ki}(X_J) = \sum_{i=0}^{3} \beta_i X_i$ with β_i constant parameters to be chosen or it is even feasible to make the β_i's functions of time. The challenge of course is to find a single map that will satisfy all the important constraints of each of the agents.

2. Example

We solve an example with B_0 a square of side 4 units and B_T a rectangle of sides 8 and 2 units where T is the final time. The whole group has to be translated by a distance of 10 in the x direction and -20 in the y-direction The entries of Q and b are quadratic functions of time t. This way the acceleration turns out to be independent of time.

$$x(t) = Q(t)X + b(t)x, X \in R^2$$

$$\text{Let } x(t) = \begin{pmatrix} x_1(t) \\ x_2(t) \end{pmatrix}, X = \begin{pmatrix} X_1 \\ X_2 \end{pmatrix},$$

$$\text{Let } Q(t) = \begin{pmatrix} q_{11}(t) & q_{12}(t) \\ q_{21}(t) & q_{22}(t) \end{pmatrix}$$

$$= \begin{pmatrix} \alpha_{111} + \alpha_{112}t + \alpha_{113}t^2 & \alpha_{121} + \alpha_{122}t + \alpha_{123}t^2 \\ \alpha_{211} + \alpha_{212}t + \alpha_{213}t^2 & \alpha_{221} + \alpha_{222}t + \alpha_{223}t^2 \end{pmatrix} \text{ Let}$$

$$b(t) = \begin{pmatrix} b_1(t) \\ b_2(t) \end{pmatrix}$$

$$= \begin{pmatrix} b_{11} + b_{12}t + b_{13}t^2 \\ b_{21} + b_{22}t + b_{23}t^2 \end{pmatrix}, t \in [0, T]$$

Since at time $t = 0$ the mobile agents are inside B_0, Q is an identity matrix. Also since the agents have had no translation $b = 0$.

Initial conditions :

$$q_{11}(0) = 1 \Rightarrow \alpha_{111} = 1 \tag{4.1}$$

$$q_{22}(0) = 1 \Rightarrow \alpha_{221} = 1 \tag{4.2}$$

$$q_{12}(0) = 0 \Rightarrow \alpha_{121} = 0 \tag{4.3}$$

$$q_{21}(0) = 0 \Rightarrow \alpha_{211} = 0 \tag{4.4}$$

$$b_1(0) = 0 \Rightarrow b_{11} = 0 \tag{4.5}$$

$$b_2(0) = 0 \Rightarrow b_{21} = 0 \tag{4.6}$$

The required translation and final formation shape determines the final conditions. Therefore

Final conditions :

$$q_{11}(T) = 2 \Rightarrow \alpha_{111} + \alpha_{112}T + \alpha_{113}T^2 = 2 \tag{4.7}$$

$$q_{22}(T) = \frac{1}{2} \Rightarrow \alpha_{221} + \alpha_{222}T + \alpha_{223}T^2 = \frac{1}{2} \tag{4.8}$$

$$q_{12}(T) = 0 \Rightarrow \alpha_{121} + \alpha_{122}T + \alpha_{123}T^2 = 0 \tag{4.9}$$

$$q_{21}(T) = 2 \Rightarrow \alpha_{211} + \alpha_{212}T + \alpha_{213}T^2 = 0 \tag{4.10}$$

$$b_1(T) = 10 \Rightarrow b_{11} + b_{12}T + b_{13}T^2 = 10 \tag{4.11}$$

$$b_2(T) = -20 \Rightarrow b_{21} + b_{22}T + b_{23}T^2 = -20 \tag{4.12}$$

Since the entries in Q and b are quadratic functions of time t the acceleration of the mobile agents are independent of time. The constraint is determined as follows.

Acceleration Constraint :

$$\begin{pmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{pmatrix} = \begin{pmatrix} 2\alpha_{113} & 2\alpha_{123} \\ 2\alpha_{213} & 2\alpha_{223} \end{pmatrix} \begin{pmatrix} X_1 \\ X_2 \end{pmatrix} + \begin{pmatrix} 2b_{13} \\ 2b_{23} \end{pmatrix} = \begin{pmatrix} 2\alpha_{113}X_1 + 2\alpha_{123}X_2 + 2b_{13} \\ 2\alpha_{213}X_1 + 2\alpha_{223}X_2 + 2b_{23} \end{pmatrix}$$

Therefore acceleration constraint :

$$\sqrt{Y_1^2 + Y_2^2} \le \gamma \tag{4.13}$$

where $Y_1 = (2\alpha_{113}X_1 + 2\alpha_{123}X_2 + 2b_{13})$ & $Y_2 = (2\alpha_{213}X_1 + 2\alpha_{223}X_2 + 2b_{23})$

To avoid two agents being at the same location at a given time instant we impose the non-singularity condition. This ensures that the map is invertible at all times and the locations of the agents are unique.The condition is determined as follows:

The Non-Singularity condition:

$\det(Q(t)) \neq 0 \forall t \in [0, T]$ (4.14)

$\Rightarrow (\alpha_{111} + \alpha_{112}t + \alpha_{113}t^2) \times (\alpha_{221} + \alpha_{222}t + \alpha_{223}t^2) -$

$(\alpha_{121} + \alpha_{122}t + \alpha_{123}t^2) \times (\alpha_{211} + \alpha_{212}t + \alpha_{213}t^2) \neq 0 \forall t \in [0, T]$

There are infinite solutions and we present two of the possible solutions below:

Solutions:

1)$b_{13} = b_{23} = 0, \alpha_{ij3} = \frac{\gamma}{4\sqrt{2}}$ satisfies condition 4.13

Using Eqns. $4.1 - 4.12$

$\alpha_{112} = (\frac{1}{T} - \frac{\gamma T}{4\sqrt{2}})$

$\alpha_{222} = (-\frac{1}{2T} - \frac{\gamma T}{4\sqrt{2}})$

$\alpha_{122} = -\frac{\gamma T}{4\sqrt{2}}$

$\alpha_{212} = -\frac{\gamma T}{4\sqrt{2}}$

$b_{12} = \frac{10}{T}$

$b_{22} = -\frac{20}{T}$

Condition 4.14 turns out to be :

$\frac{\gamma^2}{32}t^4 + (-\frac{\gamma}{8\sqrt{2}T} - \frac{\gamma^2 T}{16})t^3 + (-\frac{1}{2T^2} + \frac{3\gamma}{8\sqrt{2}} + \frac{\gamma^2 T^2}{32})t^2 + (\frac{1}{2T} - \frac{\gamma T}{2\sqrt{2}})t + 1 \neq 0, t \in [0, T]$

If $\gamma = 1$ then condition 4.14 is satisfied for T<5. Similarly if $\gamma = 2$ then condition

4.14 is satisfied for T<2

2) $b_{13} = b_{23} = -\frac{\gamma}{4\sqrt{2}}, \alpha_{ij3} = \frac{\gamma}{4\sqrt{2}}$ satisfies condition 4.13

Using Eqns. $4.1 - 4.12$

$\alpha_{112} = (\frac{1}{T} - \frac{\gamma T}{4\sqrt{2}})$

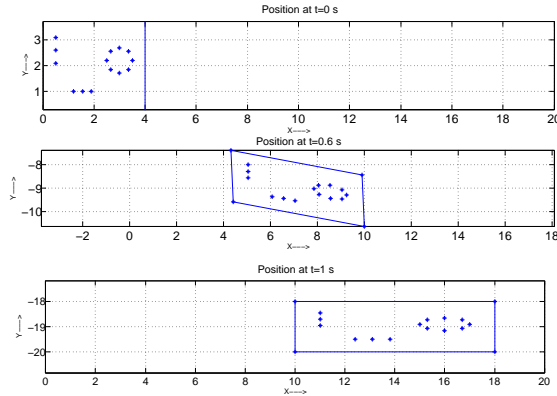$\alpha_{222} = (-\frac{1}{2T} - \frac{\gamma T}{4\sqrt{2}})$

Fig. 19. Agents in circles and lines

$\alpha_{122} = -\frac{\gamma T}{4\sqrt{2}}$

$\alpha_{212} = -\frac{\gamma T}{4\sqrt{2}}$

$b_{12} = \frac{10}{T} + \frac{\gamma T}{4\sqrt{2}}$

$b_{22} = -\frac{20}{T} + \frac{\gamma T}{4\sqrt{2}}$

Condition 4.14 turns out to be :

$\frac{\gamma^2}{32}t^4 + (-\frac{\gamma}{8\sqrt{2}T} - \frac{\gamma^2 T}{16})t^3 + (-\frac{1}{2T^2} + \frac{3\gamma}{8\sqrt{2}} + \frac{\gamma^2 T^2}{32})t^2 + (\frac{1}{2T} - \frac{\gamma T}{2\sqrt{2}})t + 1 \neq 0, t \in [0,T]$

If $\gamma = 1$ then condition 4.14 is satisfied for T<5. Similarly if $\gamma = 2$ then condition 4.14 is satisfied for T<2

We support the above theory with results shown in Fig. 19 and Fig. 20. The motion plan is done for a team of agents occupying a square shape in its reference configuration and a rectangular shape in the goal configuration. The particles undergo the homogeneous motion prescribed by the homeomorphism , $Q : X \longrightarrow x$, given by $x = Q(t)X + b(t)$ where $x, X \in R^2$. Notably, due to the properties of the mapping, a circle topology is mapped to an ellipse, and a line to a line, enabling motion planning with topology control.
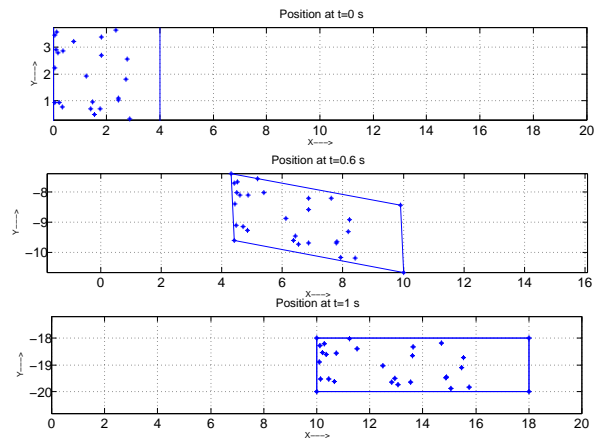
Fig. 20. Agents in random positions

C. Finite Sized Agents for the 2-D Case

In the previous sections we had assume that the agents are point objects. On the other hand all real world mobile agents are finite sized. Hence we develop a way to handle finite sized agents using the continuum approach of motion planning in this section. We illustrate the approach we have developed with the two dimensional case. It is to be noted that the effect of multiplying a vector in two dimension, by a matrix is a change in length of the vector and a rotation as illustrated in Fig. 21 . Hence if we consider the vector joining the center of two mobile agents, it will contract and change angles as the mobile agents move using the motion plan generated by the continuum approach. This is because the motion plan generated using the continuum approach is by the multiplication of a time varying matrix and addition of a time varying vector. There is a relationship between the change in length of the vector joining the centers of two agents and the eigenvalues of the matrix being multiplied. The minimum length to which a vector can contract is the magnitude of the minimum eigenvalue of the matrix times the original length of the vector. Hence if we ensure
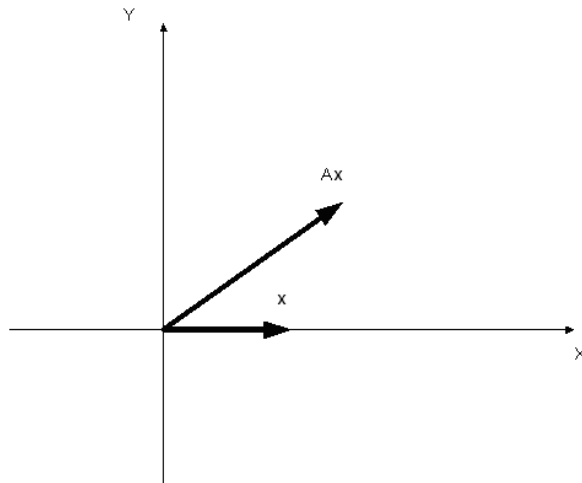
Fig. 21. Effect of matrix multiplication with a vector

that the minimum eigenvalue of the linear map used for planning the motion is at all times such that the length of the vectors joining the centers of the mobile agents are greater than the sum of the radii of the agents (each agent can be made to lie within a disc of certain radius), the mobile agents will not collide with each other.

We consider a case in which the agents are circular with equal radius, say r. If D_{min} is the minimum distance between the centers of any two robots in the reference configuration as shown in Fig. 22, then to ensure no collision between any two robots at all times $D_{min}|\lambda_{min}(t)| > 2r \forall t$. $\lambda_{min}(t)$ is the minimum eigenvalue of the matrix $Q(t)$ at time t. This bound for the minimum eigenvalue of the linear map is conservative as the eigenvector directions have not been taken into consideration. Also if we have already planned for $Q(t)$ we can handle mobile agents of atleast a size which fits within a disc of diameter $D_{min} \times min|\lambda_{min}(t)| \forall t \in [0, T]$ where T is the final time such that there is no collisions between agents at any time. In the next section we implement the motion plan generated using the continuum approach on a group of non holonomic agents.
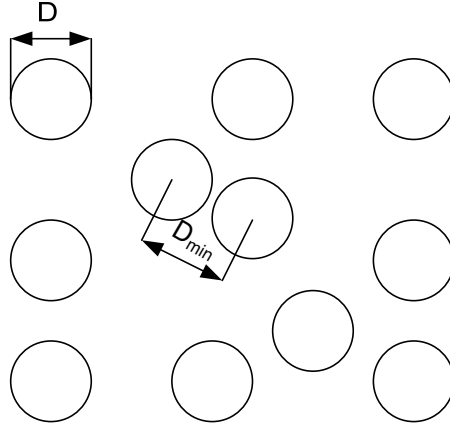
Fig. 22. Finite sized agents

D. Implementation on Non Holonomic Agents

We implement the motion plan generated using the continuum approach on a group of non holonomic agents which move on a 2-D plane in this section. The motion map gives us the x and y positions of the mobile agents and their angle $\theta = \arctan(\frac{\dot{y}(t)}{\dot{x}(t)})$ at each time instant. Hence the desired posture $p_r = \begin{pmatrix} x_r \\ y_r \\ \theta_r \end{pmatrix}$ can be found at each time instant. Similarly by differentiation of the current position the vector $q_r = \begin{pmatrix} v_r \\ \omega_r \end{pmatrix}$ can be found out at each time instant where v_r is the reference linear velocity and ω_r is the reference angular velocity. Once we have the vectors $p_r(t)$ and $q_r(t)$ we can use the non linear controller proposed in [36] to control any group of non holonomic vehicles along the trajectories generated by the homotopy algorithm proposed in [36]. The control law proposed in this paper is $q = \begin{pmatrix} v \\ \omega \end{pmatrix} = \begin{pmatrix} v_r cos\theta_e + K_x x_e \\ \omega_r + v_r(K_y y_e + K_\theta sin\theta_e) \end{pmatrix}$.
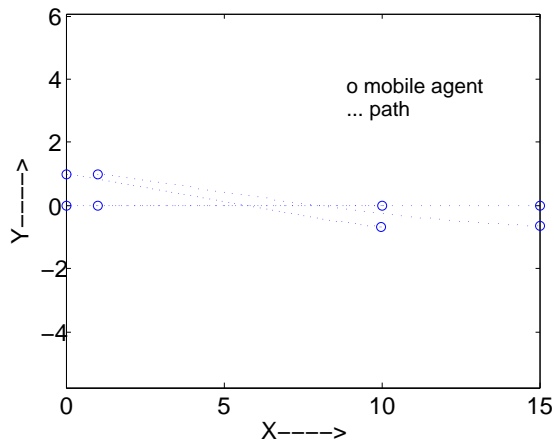
Fig. 23. Implementation on non holonomic agents

$$p_e = \begin{pmatrix} x_e \\ y_e \\ \theta_e \end{pmatrix} = p_r - p_c \text{ is the error pose, } p_c \text{ being the current pose. } q_e = \begin{pmatrix} v_e \\ \omega_e \end{pmatrix} =$$

$q_r - q_c$ is the error velocity vector, q_c being the current velocity vector. K_x, K_y, K_θ are the controller gains. We assume that we can measure the current positions of the mobile agents using dead reckoning etc..

Figure 23 illustrates an example in which there are four non holonomic agents arranged at the corners of an initial square formation and a final rectangular formation. In this example we assume there are no obstacles in the environment. As can be seen from the figure, the motion plan generated by the continuum approach is tracked satisfactorily. In the next section we solve the motion planning problem using the continuum approach for the case in which there are stationary obstacles in the environment. The agents are assumed to move in two dimension.

E.   Obstacle Avoidance

We solve the two dimensional case of the motion planning problem of moving a group of mobile agents through stationary obstacles using the continuum approach in this section. The motion map is again assumed to be linear. To facilitate obstacle avoidance we bound the agents at all times by a rectangular box as has been done in previous work [37]. The box can translate, deform and rotate. If we make sure that the box avoids collisions with the stationary obstacles, we will ensure that the mobile agents do not collide with the obstacles. We can use the PRM approach or we can simply place rectangles in the workspace to generate the intermediate configurations which avoid the obstacles. The motion of the box can then be generated by interpolating between the intermediate configurations. To use the PRM approach we need to define the configuration space of the box. The configuration space of the box is four dimensional if we assume that the rectangle is of constant area(implies that the length is dependent on the width). Two of the dimensions are for the position of the box and one each for the orientation and the width of the box. Once the intermediate collision free configurations are generated we plan the motion between the intermediate configurations using the continuum approach. Please note that the change in the position of the box is brought about by the $b(t)$ vector in $x = Q(t)X + b(t)$ while the change in orientation and the width of the box is brought about by the $Q(t)$ matrix. The $Q(t)$ matrix is a product of a rotation matrix and a diagonal matrix which causes the deformation and whose determinant remains constant. The constant determinant ensures that the area of the rectangle is constant at all times.

Figure 24 illustrates an example in which the agents are in an initial square configuration and need to move through polygonal obstacles to a final rectangular con-
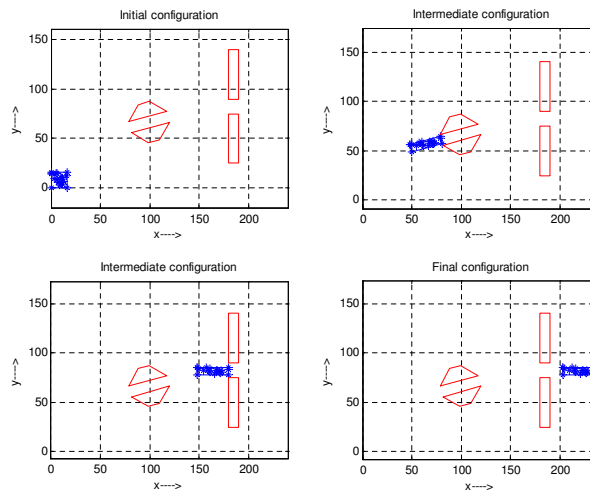
Fig. 24. Avoiding stationary obstacles

figuration. We planned the intermediate configurations by placing rectangles which do not intersect the obstacles. The continuum approach was then used to interpolate between these intermediate configurations to generate the final motion plan. A linear interpolation was used in this example to generate $Q(t)$ and $b(t)$ for motion between the intermediate configurations.

In this chapter we developed a novel global motion planning approach for a group of mobile agents. One of the important attributes of this approach is that each particle or agent has a well defined path that is based solely on its reference position. That necessarily means that an agent does not have to know the location of any other agent once the common motion map is communicated by the central command after the global motion plan is complete. We emphasize: (1) that no communication between agents is required for its implementation, and (2) the method is independent of the number of agents, meaning that it is completely scalable. These two attributes we believe are a major advantage that is not present in any presently known motion planner and we believe it is a significant breakthrough.

CHAPTER V

CONCLUSIONS

Motion planning of multiple agents has been of increased interest in recent times. There are a number of applications including emergency management, collection of scientific data, space exploration etc.. In this work we have have developed two novel motion planning approaches for multiple agents which require no communication and sensing for their implementation and are promising from a computational point of view.

A. Contributions

Our contribution in this work has been the development of two novel methods of motion planning of multiple mobile agents which are computationally attractive and require no communication and sensing for their implementation.

1. Homotopy Approach

In this dissertation we developed the homotopy approach for coordinated motion planning of groups of mobile agents. In this approach the motion plan consists of a plan for change of the group shape and a plan for the translational motion of the entire group considering it as a single agent. We initially assume that the agents are moving in two dimension and later on consider the three dimensional case. The planning for change of the group shape was transformed to the problem of finding a time varying polynomial which does not have multiple roots. This was done by mapping the positions of the mobile agents to the roots of a polynomial. The concept of discriminant variety was used to make sure the required polynomial was generated by finding paths in polynomial space which do not intersect the discriminant variety.

We developed two ways of finding the polynomial path. The first is the straight line path between the initial and final polynomials. The second is the potential field approach. We showed a way to verify whether the straight line polynomial path intersects the discriminant variety. We also presented a potential field like approach for finding parametric paths in the complement of the discriminant variety space. In this approach a potential function was created such that the discriminant variety had very high potential and the goal polynomial had minimum potential. Hence by moving along the negative gradient of the potential function, a polynomial path was generated which stayed away from the discriminant variety and reached the goal polynomial. In this work we have assumed that the mobile agents are point objects though real life robots are finite sized. It is to be noted though that using an appropriate potential function which ensures that the polynomial stays away from the discriminant variety by a certain distance, we can handle finite sized agents.

After we planned for the group shape change, we found a bounding disc for the group such that the agents are inside the disc at all times. We used this disc to plan the translational motion of the group through stationary obstacles. This way we ensured that the agents did not collide with each other and also avoided collisions with stationary obstacles in the environment. We imposed velocity and acceleration constraints on the mobile agents using reparameterization of the time varying polynomial. Also we implemented the 2-D algorithm on a group of non holonomic vehicles using a non linear controller described in [36]. We also extended the homotopy approach for planning the motion of a group of mobile agents moving in three dimensional world. We mapped the 3-D coordinates of each agent to the roots of a polynomial and used an approach similar to the 2-D approach for finding the motion plan for each mobile agent.

2. Continuum Approach

Apart from the homotopy approach we developed another novel global motion planning method, the continuum approach . In this approach if we made sure that the motion map between the reference and current configurations is a homeomorphism, we will ensure that each agent will occupy a unique position at all times. We presented an example where the motion map is linear and is of the form $x(t) = Q(t)X + b(t)x, X \in R^2$. As illustrated by this example, it is possible and beneficial to prescribe a single, common feedback law $x = x(\mathbf{X}, T)$ based on the reference states of the agents so that the network topology may propagate with minimal communication with other agents in the team or with no communication at all in the perfect scenario, where each agent's state is precisely known. We have shown that the dynamic constraints can be handled in a rather nice way by using the motion map in certain forms. We have initially assumed that the mobile agents are point objects but have shown a way to deal with finite sized agents by putting restrictions on the eigenvalues of the motion map. We have also shown that if we have decided on the motion map apriori, what the maximum size of the mobile agents can be such that there are no collisions. The bounds generated on the eigen values of the motion map though are conservative as we have not considered the eigenvector directions in this analysis. We have implemented the motion plan generated using the continuum approach on a group of non holonomic agents. The controller described in [36] was used. We have also developed a way to handle stationary obstacles using the continuum approach. We have bounded the agents by a rectangular box which can deform, rotate and translate and planned the motion of the box through obstacles. The position of the agents inside the box is determined by using motion maps between the intermediate configurations of the rectangular box.

B.   Future Work

1.   Homotopy Approach

Further research effort can go into finding paths in the complement of the discriminant variety space which guarantee the maintenance of a certain minimum distance between the roots. This will facilitate the usage of the proposed algorithm for any finite sized mobile agents. Research can be done to find out an exact correlation between the size of the discriminant and the minimum distance between the roots of a polynomial. Once this is done we can use the potential field approach for avoiding the discriminant variety by a certain amount which guarantees the maintenance of a minimum distance between the roots. Also research needs to be done to find a generalized method to parameterize all paths in the complement of the discriminant variety so that the optimal path can be found. In this work we have proposed a way to handle stationary obstacles using the homotopy approach. We need to extend the proposed approach to handle moving obstacles.

2.   Continuum Approach

In future research can be done to study homogeneous, area preserving, group deformations for mission updates under three types of exploring behaviors. The first is a covering behavior useful for the detection problem, in which each agent will explore and try to reach each point in an area. In the second type each agent will first explore a goal, the position of which is not known, and once it is found all the agents will reach the goal. The third is shepherding, where a group is steered by one or more external agents. We can study the deformation of the group shape to fit into various constrained forms dictated by external agents possessing global knowledge. An example is search and rescue, where a manned search and rescue vehicle shepherds a

mobile agent group in an optimal sensing configuration. It appears feasible to exploit the geometric aspects of the motion maps (especially homogeneous deformations) to develop new communication/formation protocols related to the mobile network topology. For example, under the correct map, any agents that initially lie on a straight line will continue to stay on a straight line that can translate and rotate, and those on a circle will lie on a precisely defined ellipse. Such topology control can be be exploited for combat and sensing configurations that are invariant to group motion. The method is amenable to analysis, and we can incorporate inter-agent communication ranges, as constraints on the maps and incorporate obstacles that are moving for computationally efficient determination of the important motion maps. Study can be done to consider nonlinear motion maps to address the same type of questions alluded to earlier. Additionally, nonlinear maps provide the ability to change the distribution of the mobile agents within an area, permitting precise control over the spatial density of the mobile agents, a feature that can be exploited in cooperative motion and distributed sensing. Also a way to counter the failure of mobile agents needs to be studied. We need to come up with a strategy to replan the motion of the group in the event of failure of some agents.

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