ROBUST MANUFACTURING SYSTEM DESIGN USING PETRI NETS AND BAYESIAN METHODS

A Dissertation

by

BIKRAM SHARDA

Submitted to the Office of Graduate Studies of
Texas A&M University
in partial fulfillment of the requirements for the degree of

DOCTOR OF PHILOSOPHY

May 2008

Major Subject: Industrial Engineering

ROBUST MANUFACTURING SYSTEM DESIGN USING PETRI NETS AND BAYESIAN METHODS

A Dissertation

by

BIKRAM SHARDA

Submitted to the Office of Graduate Studies of
Texas A&M University
in partial fulfillment of the requirements for the degree of

DOCTOR OF PHILOSOPHY

Approved by:

Chair of Committee,	Amarnath Banerjee
Committee Members,	Brett A. Peters
	Yu Ding
	Faming Liang
Head of Department,	Brett A. Peters

May 2008

Major Subject: Industrial Engineering

ABSTRACT

Robust Manufacturing System Design Using Petri Nets and Bayesian Methods.

(May 2008)

Bikram Sharda, B.E., Thapar Institute, Patiala, India;

M.S., Texas A&M University

Chair of Advisory Committee: Dr. Amarnath Banerjee

Manufacturing system design decisions are costly and involve significant investment in terms of allocation of resources. These decisions are complex, due to uncertainties related to *uncontrollable factors* such as processing times and part demands. Designers often need to find a robust manufacturing system design that meets certain objectives under these uncertainties. Failure to find a robust design can lead to expensive consequences in terms of lost sales and high production costs. In order to find a robust design configuration, designers need accurate methods to model various uncertainties and efficient ways to search for feasible configurations.

The dissertation work uses a multi-objective Genetic Algorithm (GA) and Petri net based modeling framework for a robust manufacturing system design. The Petri nets are coupled with Bayesian Model Averaging (BMA) to capture uncertainties associated with *uncontrollable factors*. BMA provides a unified framework to capture *model, parameter* and *stochastic* uncertainties associated with representation of various manufacturing activities. The BMA based approach overcomes limitations associated with uncertainty

representation using classical methods presented in literature. Petri net based modeling is used to capture interactions among various subsystems, operation precedence and to identify bottleneck or conflicting situations. When coupled with Bayesian methods, Petri nets provide accurate assessment of manufacturing system dynamics and performance in presence of uncertainties. A multi-objective Genetic Algorithm (GA) is used to search manufacturing system designs, allowing designers to consider multiple objectives. The dissertation work provides algorithms for integrating Bayesian methods with Petri nets. Two manufacturing system design examples are presented to demonstrate the proposed approach. The results obtained using Bayesian methods are compared with classical methods and the effect of choosing different types of priors is evaluated.

In summary, the dissertation provides a new, integrated Petri net based modeling framework coupled with BMA based approach for modeling and performance analysis of manufacturing system designs. The dissertation work allows designers to obtain accurate performance estimates of design configurations by considering *model, parameter and stochastic uncertainties* associated with representation of *uncontrollable factors*. Multi-objective GA coupled with Petri nets provide a flexible and time saving approach for searching and evaluating alternative manufacturing system designs.

DEDICATION

To my Dad

ACKNOWLEDGEMENTS

I would like to thank my advisor, Dr. Amarnath Banerjee, for his continuous guidance and support throughout my graduate studies. His valuable suggestions have helped me to not only succeed in academics but also become a better person in life.

I would also like to thank my committee members, Dr. Brett A. Peters, Dr. Yu Ding and Dr. Faming Liang, for their suggestions and for serving as members of my advisory committee.

This dissertation would not have been possible without the good wishes and blessings of my mother Sushma. She has always been a continuous source of motivation, and her sacrifices and blessings have kept me going. I would like to mention the continuous love and affection of my brother Robin and sister-in-law Chantelle. I would also like to thank my father-in-law Naresh Dutta, my mother-in-law Uma Dutta, and my sister-in-law Deepty for their love and affection.

I am also thankful to Prof. Michael R. Golla for his support and mentoring. My friends Surajit, Naren, Deepak, Deb, Dilip, Sanjay and Atul have always stood by me in difficult times.

In the end, I would like to thank my wife Shilpy for being a blessing in my life. She has always stood by me in my rough times, and this dissertation would not have been possible without her encouragement, support and patience.

TABLE OF CONTENTS

CHAPTER Page

LIST OF FIGURES

LIST OF TABLES

TABLE Page

CHAPTER I

INTRODUCTION

1.1. Overview

Robust manufacturing system design involves finding a manufacturing system configuration that yields better performance measures such as throughput and WIP under manufacturing system uncertainties. Manufacturing system uncertainties can be classified in terms of *controllable factors,* such as number of resources and type of operating policy, and *uncontrollable factors* such as processing times, product mix and demand patterns. The design process involves finding a combination of *controllable factors* within available constraints that consistently yields better performance measures in the presence of *uncontrollable factors*. For example, the design process might involve finding the minimal number of resources and operating policies such that the makespan for manufacturing different products is minimized. The *uncontrollable factors* for such a problem can be uncertainty in product demand, processing and failure times of different resources.

Decisions involving robust manufacturing system configuration design are costly and involve long term allocation of resources. These decisions typically remain fixed for

---

This dissertation follows the style of *IIE Transactions*.

a planning period and failure to design a robust manufacturing system can lead to drastic effects in terms of high production and inventory costs, and lost sales. The design decisions become more complex when new systems are being designed or new products are being launched as sufficient information about underlying processes is not available and uncertainty exists in future product demands.

The robust design process involves representation of uncertainties related to *uncontrollable factors* and candidate design configuration search. The design approaches should be able to search for a broader set of design solutions while providing designers insight into relative advantages and disadvantages of each design solution. In addition, the designers need to accurately capture uncertainties associated with *uncontrollable factors* in order to gather reliable performance estimates for a design configuration.

Table 1.1 summarizes the up-to-date work related to robust manufacturing system design. Current literature demonstrates use of Experimental design, Genetic algorithm (GA) and Risk analysis based approaches for robust manufacturing system design. Experimental design based approaches involve use of approaches such as factorial design and response surface methodology (RSM). The uncertainties related to *uncontrollable factors* are represented using classical distribution fitting methods. These methods involve fitting the available data with certain distribution models. To check the validity of these distribution models, certain goodness-of-fit tests such as Chi square are used. The selected distribution model and its parameters are then used in simulation models to represent uncertainty related to an uncontrollable factor.

Table 1.1. Comparison of different robust design approaches

Approach	Relevant work	Uncertainty representation	Limitations
Experimental design	Bulgak *et al.* (1999), Chen and Chen (1996), Lim *et al.* (1996), Madu and Madu (1999), Mezgar *et al.* (1997), Sanchez *et al.* (1996), Shang (1995), Tsai (2002)	*Stochastic uncertainty:* classical methods *Parameter uncertainty:* Orthogonal arrays *Model uncertainty:* None	• Use of same probability for each discrete level • Difficult to consider multiple objectives • Does not provide insight into relative advantages and disadvantages of each configuration • Becomes difficult to use when number of uncontrollable and controllable factors are large • Model uncertainty is not considered
Genetic algorithms	Saitou *et al.* (2002), Kazancioglu and Saitou (2004), Hamza *et al.* (2003) Al-Aomar (2002)	*Stochastic uncertainty:* classical methods *Parameter and model uncertainty:* None	• Parameter and model uncertainties not considered • Difficult to incorporate designers' risk attitude in decision making process
Risk analysis	Gaury and Kleijnen (1998), Kleijnen and Gaury (2003), Pierreval and Durieux-Paris (2002), Pierreval and Durieux-Paris (2007)	*Stochastic uncertainty:* Classical methods *Parameter uncertainty:* Sensitivity analysis, Latin hypercube sampling *Model uncertainty:* None	• Sensitivity analysis based approaches assign equal probabilities to variation in distribution models • Model uncertainties not considered • Lack of efficient search methods for large scale problems • Difficulty in generating reference curves • Difficult to consider multiple objectives

The classical methods for uncertainty representation have been challenged because of several limitations. These methods work on assumption that the selected distribution model and its parameters represent the uncertainty associated with an *uncontrollable factor* and do not take into consideration that the distribution model and its parameters might have variations. The designers rely on sensitivity analysis (Genetic algorithms, Risk analysis approaches) or factorial design (Experimental design) based approaches to consider parameter uncertainties with classical methods. The factorial design based approaches for representing parameter uncertainty consider a small number of discrete parameter variations (usually 2-3) at extreme points. The methods used to choose these discrete points are not well described in literature. The literature does not suggest any classical methods based approach for consideration of model uncertainties in robust manufacturing system design.

These model and parameter variations can arise from several reasons: lack of information about uncertainties, inherent nature of process or measurement errors. More discussion on the limitations of these methods can be found in Draper (1995), Chick (1997), Andradóttir and Bier (2000), Zouaoui and Wilson (2001a, 2001b, 2004) and Henderson (2003). The accurate representation of uncertainties in a robust design problem is critical because these decisions involve significant investments and a wrong decision can lead to excessive production or lost sales costs. In addition, the designers do not always have sufficient information about the uncertainties. Under such conditions, it becomes imperative to consider *parameter* and *model* uncertainties as they can have a significant impact on the overall design decisions.

1.2. Scope and contributions

The dissertation addresses issues related to representation of uncertainties in a robust manufacturing system design. A Bayesian Model Averaging (BMA) based approach has been considered to represent such uncertainties. It provides a unified framework to incorporate *model, parameter and stochastic* uncertainties related to representation of *uncontrollable factors*. The dissertation work shows the effects of ignoring *model* and *parameter* uncertainties for a robust manufacturing system design (in Chapter IV). The results show that ignoring these uncertainties severely underestimate the performance measures such as makespan and mean WIP, that can lead to improper design decisions. Results reveal that BMA based approach provides a better assessment of underlying performance parameters as compared to classical methods and classical methods using sensitivity analysis (Chapter IV).

The applicability of BMA in robust manufacturing system design problems has been limited for several reasons. First, the commercially available software does not allow consideration of Bayesian framework for uncertainty representation. For certain classes of distributions, the designers' have to rely on sampling/approximation based methods for analysis. This limits automatic calculation of model probabilities. In addition, the computational complexity associated with BMA approaches was higher than classical methods. However, recent advances in computational speeds have overcome this limitation.

The BMA approach has been integrated with Petri nets for modeling and performance analysis of manufacturing systems. Petri nets not only incorporate

properties of discrete event simulation but their formalism can be translated for manufacturing system monitoring and control. The integration of BMA based approach with Petri nets extends the scope of BMA based uncertainty representation to manufacturing system problems involving modeling, performance analysis, monitoring and control. The dissertation provides new algorithms for integrating BMA based approach with Petri nets. These algorithms can have potential impact wherever Petri nets are used for performance evaluation and control. To automate sampling from prior distributions for non conjugate priors, rejection sampling based algorithms have been used. These algorithms do not require fine tuning to parameters such as proposal density as in Markov Chain Monte Carlo (MCMC) based methods presented in literature.

The dissertation work provides flexibility in evaluating configurations against multiple objectives (such as number of resources and mean WIP) by using multi-objective GAs. While the approaches presented in literature aggregate objective functions and limit designers' freedom in evaluating alternative designs against each objective, multi objective GAs provide a useful way to evaluate candidate configuration designs against multiple criteria (for example, number of resources against makespan). The mean and variance components of each performance measure are transformed into a signal to noise ratio. The multi objective GA uses these ratios to evaluate alternative design configurations. In this way, both mean and variance components of each performance measure are considered in evaluation of alternative design configurations. Signal to noise ratios are an integral part of Taguchi's orthogonal array based Experimental design procedures and they provide a useful way to consider both mean

and variance components of a performance measure. Interested readers can refer to literature presented in background section for more details about signal to noise ratios.

1.3. Organization

The dissertation is organized as follows. Chapter II provides background of uncertainty modeling using BMA, Petri nets and multi-objective Genetic Algorithms. Chapter III provides a literature review of different techniques used for robust manufacturing system design. Chapter IV presents the proposed multi-objective Genetic Algorithm, Petri net and Bayesian methodology based approach. In addition, new token game simulation algorithms for integrating Bayesian methods with Petri nets are presented. The proposed approach is illustrated with a manufacturing system example in which processing time uncertainties are represented using a Bayesian framework. The performance estimates obtained using Bayesian methods are compared with classical methods and the effects of choosing different types of priors are considered. Chapter V presents a more detailed manufacturing system example in which the uncertainties related to product demand, part arrivals, processing, failure and repair times are considered. The effect of ignoring model, parameter and demand uncertainties on overall design decisions is shown. Finally, Chapter VI provides a summary and contributions of the dissertation.

CHAPTER II

BACKGROUND

This chapter provides a brief introduction to Bayesian Model Averaging (BMA), Petri nets and multi-objective Genetic Algorithms, that form the basis for later discussion in the dissertation.

2.1. Uncertainty modeling using Bayesian Model Averaging (BMA)

Manufacturing systems modeling is complex and designers must accurately capture uncertainties related to *uncontrollable factors* such as product demand, processing, failure and repair times in order to get a reliable assessment of system performance. In discrete event simulation modeling, designers use classical methods for representing such uncertainties. These methods typically involve fitting the available data with certain distributions and then testing the fitness of distribution using goodness-of-fit tests. Classical methods have been challenged because they consider only *stochastic* uncertainties and fail to incorporate *model* and *parameter* uncertainties. *Model uncertainty* refers to uncertainty associated with selection of a correct distribution model to represent an *uncontrollable factor*. Classical methods based techniques use a pass /fail criteria to select a distribution model and completely ignore the possibility that the selected model might not perfectly represent the *uncontrollable factor* (Chick, 1997). Interested readers may refer to Zouaoui (2001) for a more detailed discussion on limitations of Classical methods. *Model* uncertainty can be quite significant when sufficient information about *uncontrollable factors* is not available.

Parameter uncertainty refers to uncertainty in parameters of the selected distribution model (for example, uncertainty in parameter (λ) of exponential distribution) and *stochastic uncertainty* refers to the way the random variables are drawn from the distribution model and its selected parameters (for example, uncertainty associated with the way random variables are drawn from exponential distribution (λ)). These uncertainties may arise due to several factors, such as the inherent nature of the process, measurement/bias error and lack of available information.

The Bayesian Model Averaging (BMA) approach provides a unified framework to incorporate *model, parameter* and *stochastic uncertainties* (Zouaoui and Wilson, 2004). Chick (1997, 1999, 2001) provided a formal approach to incorporate Bayesian methods in discrete event simulation. Chick discussed the advantages of using Bayesian methods for discrete event simulation and provided steps by which this approach can be applied in practice. The BMA based approach can be explained as follows (Chick, 2001):

Let, $M= (M_1,\cdots, M_q)$ be a set of q distribution models under consideration

$X=(X_1, X_2, X_3,\cdots, X_n)$ are the dataset, where X_i are independent and identically distributed (*i.i.d.*)

$\pi(M_k)$ = prior probability of selecting model M_k, where $k = 1,\ldots. q$

$f(X|M_k)$= likelihood of data X given model M_k, where $k=1,\ldots. q$

The likelihood of data X given model M_k can be found by (2.1), where, $f(X|M_k, \theta_k)$ is the likelihood of data X given model M_k and its parameters θ_k, and $\pi(\theta_k)$ is the prior probability of selecting θ_k.

$$f(X|M_k) = \int_{\theta_k} f(X|M_k, \theta_k)\pi(\theta_k)d\theta_k \quad (2.1)$$

Some commonly used methods for assigning prior distributions $\pi(\theta_k)$ for model parameters involve the use of "non-informative priors" , "Jeffrey's prior" and "conjugate priors" (Gelman *et al.*, 2003). For certain classes of distributions (for example, Weibull), $f(X|M_k)$ cannot be easily computed as the integral (2.1) is not a closed form expression. Under such conditions, methods such as rejection sampling, importance sampling, Markov Chain Monte Carlo (MCMC) or asymptotic methods such as Laplace transform can be used. The parameters for prior distribution can be found by using "data driven methods" or "moment matching methods". Interested readers may refer to Gelman *et al.* (2003) and Gilks *et al.* (1995) for more details about these methods.

After computing $f(X|M_k)$ for each candidate model, the probability of selecting model M_k can be found by using Baye's rule (2.2). In the absence of any information about the correct model, the prior probability for selecting a model M_k, $\pi(M_k)$ can be assigned equally among all the candidate models. That is, $\pi(M_k) = 1/q$, where q is the total number of candidate models.

$$P(M_k|X) = \frac{f(X|M_k)\pi(M_k)}{\sum_{k=1}^{q} f(X|M_k)\pi(M_k)} \quad (2.2)$$

Zouaoui and Wilson (2001a, 2001b, 2004) developed simulation replication algorithms that incorporate partial and full BMA algorithms for discrete event simulation. These methods provide separate assessment of uncertainty related to distribution *models,* their *parameters* and *stochastic* uncertainty related to random variates drawn from a fixed distribution and its parameters. In partial Baye's methods,

the distribution model remains fixed but the parameters change within the simulation. In full Baye's methods, both distribution models and their parameters change in the simulation. Zouaoui and Wilson (2001a, 2001b, 2004) also suggested proportional allocation and optimal allocation procedures to find the optimal number of replications for a simulation experiment in order to capture *model, parameter* and *stochastic* uncertainties.

2.2. Petri nets

Petri nets were developed by Carl A. Petri as a part of his doctoral dissertation (Petri, 1962). Since then, there has been extensive research on the application of Petri nets in modeling, analysis, real time control and monitoring of different types of systems. Over the years, several variants of Petri nets have been developed. Some commonly used variants are Colored Petri Nets (CPN) (Jensen, 1997) and stochastic timed Petri nets (Malloy, 1982).

A Petri net can be described as a 5- tuple (Murata, 1989), $PN= (P, T, F, W, M_o)$, where:

$P= (p_1, p_2, \cdots\cdots, p_m)$ is a finite set of places

$T= (t_1, t_2, \cdots\cdots, t_n)$ is a finite set of transitions

$F \subseteq (P \times T) \cup (T \times P)$ is a set of arcs

$W: F \rightarrow (0, 1, 2, 3, \cdots\cdots)$ is a weight function

$M_o: P \rightarrow (0, 1, 2, 3, \cdots\cdots)$ is the initial marking of places

In stochastic timed Petri nets, there is a stochastic time delay (d_i) associated with transitions or places. A Petri net model can be represented as a stochastic Timed

Transition Petri net (TTPN) or a stochastic Timed Place Petri net (TPPN) depending upon whether a stochastic time delay is associated with a Transition (TTPN) or a Place (TPPN). Colored Petri nets are higher level modeling tools that use colored tokens to enhance the capability of Petri nets to capture complex models without state space explosion problem.

Besides being a good graphical modeling tool, Petri nets provide a useful way to analyze and understand key manufacturing control issues such as liveliness, deadlocks and conflicts. They provide a unified framework for modeling, performance analysis and control of manufacturing systems. In addition, Petri nets automatically encapsulate properties of discrete event simulation modeling and their formalism can be directly translated to system level controller operations for real time control and monitoring (Zhou and Venkatesh, 1999). Higher level Petri net software, such as CPN tools (Jensen, 1997), allows users to develop models at both abstract and detailed levels. At the abstract level, designers can visualize the overall structure of the problem, and at the detailed level, they can design operational details for each sub-part and understand interactions among various sub systems. These tools also allow designers to develop modules for specific tasks that can be interchanged among different problems. For example, the designer can create a module representing operations of a manufacturing cell and subsequently use the same module whenever a different part is processed through the manufacturing cell. Another example is the design of failure and maintenance modules for different manufacturing equipment.

Currently, there has been a concerted effort to standardize the way Petri net models are shared and created among different software tools (Billington *et al.*, 2003, Lee *et al.*, 2007). This will help a great deal in combining strengths of different software tools and aid in model sharing among different user groups.

Readers may refer to Murata (1989) for a comprehensive introductory tutorial on the history, application areas, properties, analysis methods and different types of Petri net models. Some good examples of Petri nets in manufacturing systems can be found in Zhou and Venkatesh (1999) and Al-Jaar and Desrochers (1990).

2.3. Multi-objective Genetic Algorithms

Multi-objective Genetic Algorithms are a class of Genetic Algorithms (GA) that have gained momentum because of their ability to handle multiple objective functions simultaneously. These algorithms have several advantages: they eliminate the need to specify a single objective function, and they generate a set of candidate solutions against the relative merits of each objective function. This eliminates the need to rerun the simulation, if the relative weights of each objective change.

Coello (2000) provided an excellent survey of GA based techniques used for multi-objective optimization. He classified multi-objective GA based optimization techniques into Pareto based, non Pareto based and naïve approaches. Naïve approaches include techniques such as weighted sum approach, goal programming, goal attainment and ε-constraint methods. Non-Pareto based approaches include techniques such as Vector Evaluated Genetic Algorithm (VEGA), lexicographic ordering and game theory.

Commonly used Pareto based approaches include Non dominated Sorted Genetic Algorithm (NSGA) (Srinivas and Deb, 1995), Niched Pareto Genetic Algorithm (NPGA) (Horn *et al.*, 1994), Multiple Objective Genetic Algorithm (MOGA) (Fonseca and Fleming, 1993) and NSGA-II (Deb *et al.*, 2002). These algorithms generate a Pareto optimal set that contains a set of non dominated solutions. The plot of these non dominated solutions is called a Pareto front.

The concept of Pareto optimality was proposed by Vilfredo Pareto in the nineteenth century and it forms the basis for research in multi-objective optimization (Coello, 2000). Let F be a feasible region for a multi-objective problem (minimization) defined by a set of constraints. Then, a point $x^*\epsilon F$ is called *Pareto optimal* if for every $x'\epsilon F$, either $\wedge_{i\epsilon I}\,(f_i(x') = f_i(x^*))$ or there is at least one $i\epsilon I$ such that $(f_i(x') > f_i(x^*))$ (Coello, 2000). The vector I is the number of objectives considered for the multi-objective problem (Coello, 2000). A Pareto front depicts a set of strongly and weakly non dominated solutions. A point $x^*\epsilon F$ is called a *weakly non dominated* solution if there is no $x'\epsilon F$ such that $(f_i(x') < f_i(x^*))$, for i=1,…, I. Similarly, a point $x^*\epsilon F$ is called a *strongly non dominated* solution if there is no $x'\epsilon F$ such that $(f_i(x') \leq f_i(x^*))$ and for at least one value of i, $(f_i(x') < f_i(x^*))$(Coello, 2000).

The NSGA-II algorithm is a recently proposed Pareto based multi-objective GA that is computationally less expensive and uses elitism properties to find a Pareto front of candidate solutions. The algorithm uses non-dominating sorting procedure and crowding distance measures to generate candidate solutions.

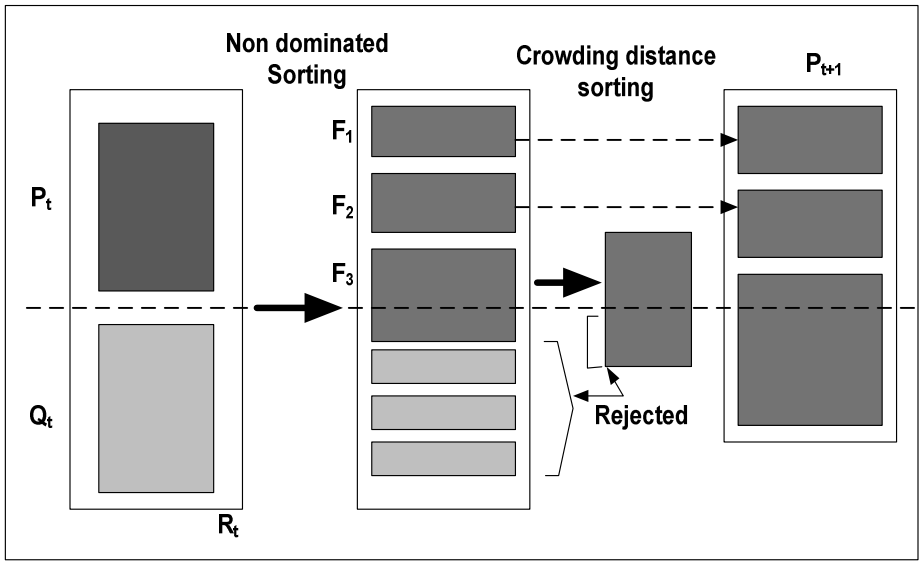

Fig. 2.1. NSGA-II procedure (Deb *et al*., 2002)

Fig. 2.1 shows the procedure employed in NSGA-II. In the beginning, an initial random population P of size N is created and their fitness values are evaluated. The population is then sorted based on a non domination procedure and each solution is assigned a fitness (or rank) according to its non domination level (for example, 1 being the best, 2 being second best and so on). In this case, minimization of objective function is assumed. The offspring population Q is then created through binary tournament selection, recombination and mutation operators. During each iteration, parent (P_t) and offspring populations (Q_t) are combined to generate another population R_t. The new population composed of parents and offsprings is then sorted according to a non domination procedure which results in a set of solutions within each non domination level (F_1, F_2, …..). The *crowding distance* is computed for each solution within a non domination level. A solution with smaller value of *crowding distance* has more solutions in its neighborhood. Combining parents and offspring, and then performing non

domination, incorporates an elitism property in these algorithms. For the next iteration, a population of size *N* is again selected from candidate solutions using a *crowding distance operator*. The *crowding distance operator* selects candidate solutions using the following rules: if there is a choice among solutions of different non domination rank, pick the solution with highest rank and if there is choice among solutions of the same rank, pick the solution with least crowding distance. This procedure is continued for the required number of iterations (Deb *et al.*, 2002).

Interested readers may find more details about non domination sorting and crowding distance sorting algorithms in Deb *et al.* (2002). The software implementation of this algorithm can be found at http://www.mathworks.com/ matlabcentral/ fileexchange/ loadFile.do?objectId=10429. The proposed work uses NSGA-II multi-objective GA to search candidate configurations against multiple objectives.

CHAPTER III

RELEVANT WORK

Over the years, several techniques have been suggested for the robust design of manufacturing systems. Major literature in this area can be classified into experimental design, risk analysis and Genetic Algorithms (GA) based approaches.

Taguchi's (1986, 1987) signal to noise ratio based methods are among the most widely used approaches for robust manufacturing system design. These methods find design solutions that are more robust against *uncontrollable* variations. The approach involves the use of orthogonal arrays and signal to noise ratios for finding a robust configuration. The signal to noise ratio takes into account both variability in response data and the closeness of average response to target value (Mezgar *et al.*, 1997).

Madu and Madu (1999) demonstrated an application of Taguchi's methods using orthogonal arrays and signal noise ratio to maximize equipment utilization for a maintenance cell. The approach provided the best design point from a limited number of design points and with minimal experimentation time. The authors considered uncertainties in machine failure and repair times as noise factors. Tsai (2002) used Taguchi's methods for solving decision making problems in integrated manufacturing systems. Lim *et al.* (1996) used Taguchi's methods for finding the optimal configuration of operating policies for a manufacturing system to maximize throughput and minimize flow time. They considered demand rates and processing time uncertainties as noise factors at two different levels. Bulgak *et al.* (1999) used orthogonal arrays and normal

probability plots for finding robust design with considerations of variation in uncontrollable factors such as jam rates and jam clear times in a assembly line. Mezgar *et al.* (1997) used design of experiments and an artificial neural network based technique for design and real time reconfiguration of manufacturing systems. The approach used Taguchi's signal to noise ratio concepts for designing and artificial neural network based technique to predict the performance of a system for a given configuration setting. The neural network based approach provided a fast approximate solution for predicting the performance of a manufacturing system as compared to simulation based techniques. Sanchez *et al.* (1996) established a framework for designing, analyzing and improving systems by combining discrete event simulation and response surface meta modeling. The authors used orthogonal arrays for experiments and considered product mix uncertainties as noise factors.

Chen and Chen (1996) presented a Taguchi's method and Response Surface Methodology (RSM) based approach for designing a robust manufacturing system configuration. The authors presented a nine step procedure that used weighted design measures as performance evaluation criteria. The improvement process was performed using a steepest descent search starting from an initial condition and then a second order RSM was fitted to approximate system behavior. The proposed approach however required determination of vector direction of steepest descent and became computationally expensive for large scale problems. Shang (1995) used Taguchi's methods and RSM methodology based approach for finding a robust design of a material handling system. Their method used Taguchi's methods for finding initial configuration

based on discrete uncontrollable factors and then used RSM based methodologies to fine tune the results. To demonstrate their proposed approach, they maximized throughput of a manufacturing system against design parameters such as the number of AGVs and number of pallets and treated mean time to failure and repair as noise factors.

Uncontrollable uncertainties such as demand rate, product mix, failure and repair times are considered as noise factors in Taguchi's methods. These factors are usually considered at 2-3 discrete levels (for example, high-medium-low) and they have equal weight for computing signal to noise ratios (Pierreval and Durieux-Paris, 2007). Computing the performance of a design solution at these (extreme) discrete levels can severely underestimate or overestimate system performance. In addition, there are very few examples that demonstrate the use of Taguchi's methods for robust design against multiple objectives. Some other limitations of these approaches can be found in Pierreval and Durieux-Paris (2007).

In addition to experimental design based approaches, some risk analysis based approaches for robust system design have also been proposed. Gaury and Kleijnen (1998) presented such an approach, in which risk was evaluated by simulating the system over a sample of environmental scenarios. The authors demonstrated the approach to compare various pull production control systems. They used Latin hypercube sampling to generate a sample of factors such as processing, failure and repair times by uniformly varying them around their base values to compare robustness of pull systems. The performance of these systems was then evaluated against variation in these factors. Kleijnan and Gaury (2003) presented a methodology that used simulation,

optimization, uncertainty analysis and bootstrapping for robustness modeling. The proposed approach provided a more practical and straightforward method to find robust policies and analyze these policies against critical factors. Pierreval and Durieux (2002) presented a two stage optimization technique in which heuristic search methods were first used to determine near best solutions and their performance. In the second stage, several possible environmental scenarios were considered and evaluated according to their performance using reference curves. Pierreval and Durieux-Paris (2007) suggested a heuristic method to measure and compare the robustness of solutions using simulation against a base environment. The proposed approach used decision theoretic methods and involved the decision maker's knowledge in the decision making process. The main problem with using such risk analysis based approaches is use of reference curves, that need to be defined in order to make decisions. In addition, different people have different risk attitudes (risk averse, risk neutral) and when multiple decision makers are involved, they might have different attitudes toward the problem at hand.

When the number of *controllable* and *uncontrollable* factors is large, finding a robust design solution becomes more complex and time consuming. For such cases, heuristic methods such as Genetic Algorithms (GA) or Simulated Annealing (SA) are used. Saitou *et al.* (2002) used a GA and Petri net based approach for finding a robust manufacturing system configuration that underwent forecasted production plan variations. In order to find a robust configuration, the authors used multi-objective optimization criteria that was an aggregated function of makespan, facility and reconfiguration cost. Their approach did not consider uncertainty in operating times of different resources and used

an aggregate objective function. Kazancioglu and Saitou (2004) presented a methodology for allocating production capacity among flexible and dedicated machines under uncertain demand forecasts by quantifying the expected values of product quality and cost. They used a multi-objective GA to generate a Pareto front of quality and total cost against different demand scenarios. Hamza *et al.* (2003) used an NSGA-II based multi-objective GA approach to optimize an assembly sequence plan and to find the type and size of assembly stations for a production shop that produced wind propelled ventilators.

Classical methods are commonly used to represent uncertainty related to *uncontrollable factors* such as demand rates, processing times and failure rates. These methods generally use a fixed distribution to represent uncertainty related to these factors. The designers typically use *sensitivity analysis* to consider uncertainty in parameters of distribution models in which the distribution parameters are varied uniformly about their mean value. This approach is, however, subjective based upon the designer's opinion of how much the parameters need to be varied. Experimental design approaches use factorial designs in which the uncertainty in distribution parameters is considered at certain discrete points. The literature does not demonstrate consideration of *model uncertainty* in robust design problems. In addition, under limited information availability there can be significant *model* and *parameter* uncertainty associated with representation of *uncontrollable factors.*

In addition to classical methods, some other approaches for representing demand uncertainties have been presented. Eppen *et al.* (1989) presented a scenario based

approach to represent demand uncertainties in which the designers assign probabilities to different events that can occur in the future. The authors also considered Expected Disaster Ratio (EDR) that provided insight into the worst case performance of a robust system. This approach is useful for a small number of scenarios however for a large number of possibilities it becomes complex. Zhang *et al.* (2004) provided a novel approach in which demand was approximated by a distribution whose support was a collection of rays emanating from a point and contained in real multi dimensional space.

Henderson (2003) provides an excellent discussion on the importance of considering *model*, *parameter* and *stochastic* uncertainties and the approaches that can be used to represent such uncertainties. Some recently proposed formal approaches for representing these uncertainties include Delta method approaches and Bayesian methods (Henderson, 2003).

Delta method based approaches were mainly presented by Cheng (1994) and then by Cheng and Holland (1997, 1998, 2004). Cheng and Holland (1997) discussed about sensitivity of simulation experiments to input data and how the variance in overall simulation output is dependent upon *stochastic* and *parameter* uncertainties. They proposed classical differential analysis and bootstrapping based approaches for assessing such variability. The problem with their approach is that sensitivity coefficients need to be estimated and the effort needed to do this increases linearly with the number of input models. Cheng and Holland (1998) proposed a δ-method and a two point-method based approach for assessing the variability in simulation. The two point method based approach provides a quick yet conservative way to compute confidence intervals against

parameter and stochastic uncertainties. Cheng and Holland (2004) provided an asymptotic normality based approach, bootstrapping approach and a conservative two point approach for assessing confidence intervals of simulation output against *parameter* and *stochastic* uncertainties. The approaches put forth by Cheng, and Cheng and Holland mainly focus on *parameter* and *stochastic* uncertainties.

Bayesian methods for uncertainty representation were mainly presented by Chick (1997). In his initial work, Chick discussed about how Bayesian formalism can be applied for modeling simulation output as an unknown function of input parameters. Chick (1999) discussed steps for implementing Bayesian methods for input distribution selection mainly emphasizing the Bayesian Model Averaging (BMA) based approach for input distribution selection. In addition, Chick (2001) addressed issues relating to implementation of the BMA approach and provided simulation replication algorithms for implementing such approach. Ng and Chick (2001) considered the problem of allocating resources for additional data collection for reducing input uncertainties so as to reduce output variability. Zouaoui and Wilson (2001a, 2001b, 2004) extended the approaches presented by Chick and developed new simulation replication algorithms that provide separate assessment of *model, parameter* and *stochastic* uncertainties. Their algorithms overcome certain limitations associated with Chick's simulation replication algorithm. They presented an example of a communication network and showed that results of Bayesian methods were much better than classical techniques. Andradóttir and Bier (2000) provide an excellent discussion on the scope of Bayesian methods in discrete event simulation.

The dissertation uses a BMA approach to represent uncertainties associated with *uncontrollable factors*. The approach provides a unified framework to incorporate *model, parameter* and *stochastic* uncertainties associated with representation of *uncontrollable factors.* While the classical methods based approaches lack a better way to represent mixture distribution models, the BMA approach automatically provides a way to represent such models. In addition, the approach encapsulates information availability about *uncontrollable factors* in the form of prior distributions. It can be argued that the *sensitivity analysis* based approaches are a special case of BMA with *model uncertainty* ignored. By using these methods for representing uncertainties in a robust design problem, designers can obtain more accurate performance estimates under different configurations and hence identify better robust design configurations.

A Petri net based modeling approach is used to generate performance estimates (for example, makespan and mean WIP) for different configurations in presence of such uncertainties. A NSGA-II multi-objective GA based approach is employed to generate a set of candidate configurations against multiple objectives. This approach provides a Pareto front of candidate solutions and eliminates the need to rerun the simulation if costs associated with each objective change. This provides designers more insight and flexibility in selecting alternative manufacturing system designs and may save considerable time in the design process.

A detailed discussion of the approach is presented in the next chapter.

CHAPTER IV

MANUFACTURING SYSTEM MODELING USING BAYESIAN METHODS AND PETRI NETS

4.1. Overview

In this chapter, the NSGA-II multi-objective GA and Petri net based approach for robust manufacturing system design is presented. The uncertainties related to *uncontrollable factors* are represented using a BMA based approach. To demonstrate the approach, a manufacturing system example that produces multiple types of parts that undergo different processing sequences is considered. The uncertainties in manufacturing system arise due to variation in processing times and product mix. The NSGA-II algorithm is first used to generate a Pareto front of candidate configurations against number of resources, makespan and mean WIP with fixed product mix. These configurations are then evaluated with respect to variation in product mix. In the final phase, the designers assign costs associated with each objective and select the configuration that results in minimal total costs against variation in product mix. New full and partial Baye's token game simulation algorithms for integrating BMA with Petri nets are presented. The performance of the manufacturing system under different configurations is compared against Bayesian and classical methods of uncertainty representation. In addition, the effect of choosing different prior distributions on full and partial Baye's based methods is evaluated. To sample prior parameters from non conjugate class of distributions, rejection sampling based methods were used. The final

results obtained show that BMA based uncertainty representation provided more accurate estimates of manufacturing system performance against classical methods and classical methods using sensitivity analysis.

4.2. Manufacturing system design problem

A manufacturing system design problem is considered in which multiple types of parts are produced that undergo different manufacturing sequences. Each part's operation is carried out in a manufacturing cell that contains multiple machines of the same type. The overall production requirement (total number of parts produced) for a planning period remains fixed; however, there can be uncertainty in the product mix. The *uncontrollable factors* for the design problem are uncertainties in processing times and product mix, and the *controllable factors* are the number of machines to be employed in each manufacturing cell. It is assumed that reconfiguration costs are high, so the design configuration obtained at the beginning of the planning horizon will remain fixed for the entire planning horizon. The objective function for the problem is formulated as a cost minimization function as defined in (4.1). It is dependent upon makespan, WIP, stock out costs and resource costs.

$$A = \left\{ \sum_{i=1}^{Z} (C_i Q_i + W_i T_i + S_i \max(0, D_i - N_i)) + \sum_{j=1}^{L} R_j V_j \right\} \tag{4.1}$$

where:

- $i=1,\ldots, Z$ are the different types of parts produced within the system
- $j=1,\ldots, L$ are the types of resources used within the system

- C_i, W_i, S_i = production cost *($/hr),* inventory cost *($/unit)* and stock out costs *($/unit)* of part type *i,* respectively
- Q_i, T_i, D_i, N_i= makespan time (*hours*), mean WIP (*units)*, demand and number of units produced of part type *i,* respectively
- V_j, R_j = number of resources and cost *($/resource*) of resource type *j*

To simplify the problem, the following assumptions are made:

- Production sequence of each part remains fixed during a planning period
- Resources do not fail
- Production, inventory and stock out costs for each part are equal, that is, $C_1 = C_2 = \ldots = C_Z$, $W_1 = W_2 = \ldots = W_Z$, $S_1 = S_2 = \ldots = S_Z$
- Cost for each resource type is equal: $R_1 = R_2 = \ldots = R_L$
- Resource constraints: $0 \leq V_j \leq K_j$ for j=1, ……, L, where K_j is maximum number of resources of type *j* allowed in the system, it is assumed that $K_1 = K_2 = \ldots = K_L$.

Uncertainties related to processing times are defined using a Bayesian framework, which is explained as follows:

- Let X_{ij} be the time taken for processing part i on resource j, then $X_{ij} \sim F\ [(B_{ij1}, \Theta_{ij1}), (B_{ij2}, \Theta_{ij2}), \ldots\ldots, (B_{ijs}, \Theta_{ijs})]$, where:
  - B_{ijs} = candidate distribution model s used to represent resource j processing time for part i, and
  - Θ_{ijs} = parameters for candidate distribution model s, where $\Theta_{ijs} \sim \pi(\Theta_{ijs})$. $\pi(\Theta_{ijs})$ is the prior distribution used to represent uncertainty in parameters Θ_{ijs} of distribution model B_{ijs}

- It is assumed that designers have data points about processing times in order to find the parameters of prior distributions

The last point is illustrated with the following example: one of the resource's (j) processing times follows an unknown distribution. Several candidate distribution models B_{ijs} (for example, normal and exponential) can be considered to represent this processing time in a simulation model. Prior distributions $\pi(\theta_{ijs})$ are used to consider uncertainties in parameters of these distribution models. For example, for exponential distribution model with parameter λ, uncertainty in parameter λ may be represented using a conjugate distribution, $\pi(\lambda) \sim$ gamma (α, β). For gamma prior distribution, the parameters α and β may be obtained using moment matching or data driven methods. After the distribution models and their prior information are assigned, the probability that a distribution model B_{ijs} represents the resource processing time can then be computed using (2.2).

4.3. NSGA-II multi-objective GA and Petri net based approach

Fig. 4.1 shows the proposed NSGA-II and Petri net based approach. The design process is formulated as a two step procedure in which the designers first generate a Pareto front of feasible design solutions against the number of resources, makespan and mean WIP using the NSGA-II algorithm. The NSGA-II algorithm based approach makes the decision process more generalized and flexible. Instead of obtaining a single objective function solution, the algorithm generates multiple candidate solutions that reflect the relative advantages and disadvantages against each objective (for example, makespan against number of resources). Designers can then identify feasible

configurations based upon their assessment of weight (costs) associated with each objective and overall cost constraints. With a single objective function based GA there are two major limitations. First, it is difficult to analyze advantages and advantages of design solutions against multiple objectives and second, the simulation process needs to be carried out again if the relative weights associated with each objective change. In the proposed approach, a Pareto front of design solutions is obtained and these design solutions are further reduced by designer specified weights and constraints for each objective.

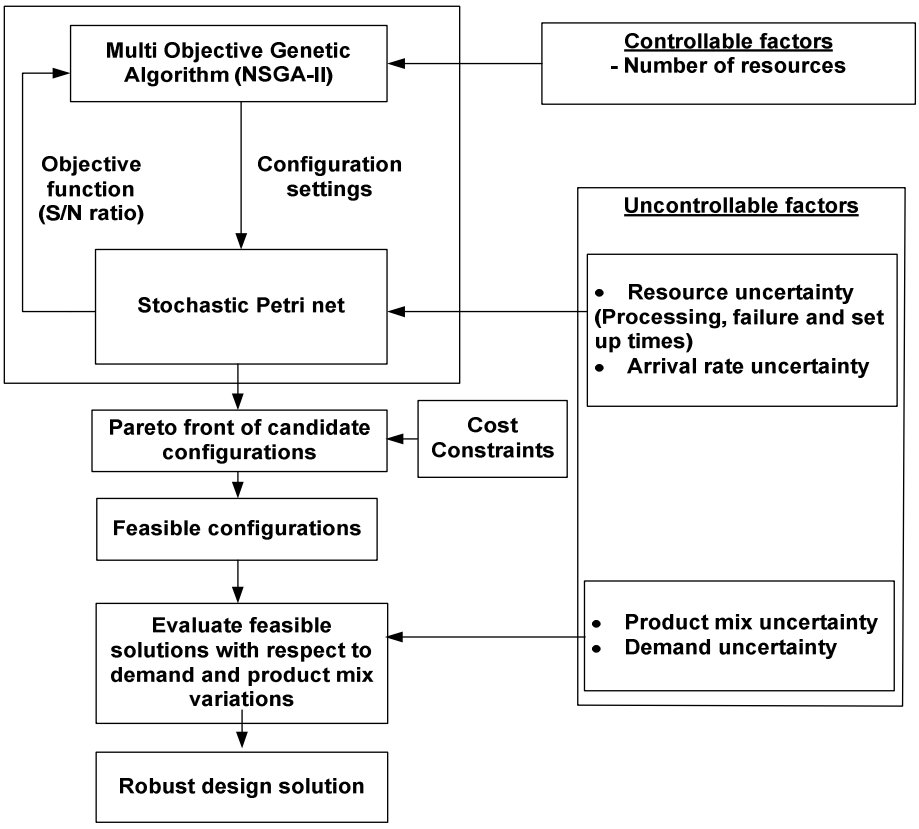

Fig. 4.1. NSGA-II multi-objective GA and Petri net based approach

The Petri net model receives a configuration setting (for example, number of resources) from the NSGA-II algorithm and evaluates performance metrics (for example, mean WIP, makespan) for the candidate configuration. The configuration settings that do not meet the designer's objectives (such as demand not met, excess WIP, deadlocks) are assigned a high objective function value. This causes these configurations to be eliminated from the set of candidate solutions. The Petri net model uses Bayesian methods of uncertainty representation to find performance measures. Currently, there is no available Petri net simulator that uses Bayesian methods to evaluate performance measures. So, new partial and full BMA based Petri net token game simulation algorithms were developed in MATLAB® by modifying algorithms presented by Zhou and Venkatesh (1999). These algorithms will be discussed in the next section.

The performance measures computed by the Petri net model are returned to the NSGA-II algorithm in the form of a "smaller the better" signal to noise (S/N) ratio for each objective function. This approach has been adopted from Al-Aomar (2002). For each objective function F_j, the signal to noise ratio is computed using (4.2), where n is the number of replications and Y_{ij} is the value of objective function j in replication i.

$$F_j = -10\log\left(\frac{1}{n}\sum_{i=1}^{n} Y_{ij}^2\right) \tag{4.2}$$

This function provides a built in trade off between mean and variance components of a performance measure (Al-Aomar, 2002) as illustrated in (4.3).

$$F_j = -10\log\left\{\left(E\left(Y_j\right)\right)^2 + \left(\frac{(n-1)}{n}\left(\mathrm{Var}(Y_j)\right)\right)\right\} \tag{4.3}$$

After the first phase of the design process, the designers have candidate configurations that can be compared using a Pareto front containing makespan, mean WIP and number of resources used. This approach gives designers more freedom to evaluate and select alternative configurations based upon relative merits (for example, low makespan) and demerits (for example, more resources used) of each design solution. The next phase involves a decision making process through which the designers define their weights (or costs) for each objective, and the configurations that do not meet budgetary constraints are eliminated. It must be pointed out that the initial NSGA-II multi-objective GA and Petri net based simulation generates a set of feasible solutions against varying objectives (for example, high makespan, less resources or low makespan, high resources), and the second part deals with reducing the feasible set. For example, if the designer specifies the weight of makespan, WIP and resource cost to be $100/hr, $50/unit, and $500/resource respectively, and assigns a maximum total cost of say $50,000, then all the solutions with cost higher than the maximum total cost will be eliminated. In case none of the design solutions meet the required total costs, the designers can either re-access their constraints or stop the design process. The remaining configurations are then evaluated against variation in demand and product mix. The configuration that yields minimal overall cost, while meeting the demand and product mix requirements is selected as a robust design solution.

New token game simulation algorithms were developed to integrate the BMA based approach with Petri nets. These algorithms were aimed at enhancing the capability of

stochastic Petri nets to accurately capture system dynamics, identify conflicting situations and provide accurate performance measures under presence of uncertainties.

The next section provides a detailed overview of the new Petri net token game simulation algorithm integrated with Bayesian methods.

4.3.1. BMA based Petri net token game simulation

Fig. 4.2 shows the algorithm developed for partial and full Baye's Petri net token game simulations. The Petri net token game simulation is divided into *initialization* and *simulation* steps. During the *initialization* phase, Petri net model information such as incidence matrix, pre and post matrix, and marking information is gathered. This information can be easily extracted after creating the Petri net model in commercially available Petri net software such as STPNPLAYER®.

In the *initialization* phase, the designers specify information available about the transition firing times. The candidate distributions and their parameters are then assigned for transitions with stochastic firing times. For example, the designers can specify that transition x time can follow either exponential (M_{1x}) or gamma distribution (M_{2x}). The prior distribution $\pi(\lambda)$ for exponential distribution parameter λ can be assigned as a gamma distribution with parameters (α, β). For gamma distribution, a non informative prior or prior specified by Miller (1980) can be considered. So in this case, two candidate distributions were considered. The choice of candidate distributions depends on the data characteristics and designers can visualize the distribution of data to see which distributions might be suitable to consider.

Initialization:

Import the Petri net model from Petri net simulator

For $k=1 \ldots K$ transitions

Assign the prior information A_k

Assign p candidate models and prior distributions

$(M_k, \pi(\Theta_k)) = [(M_{1k}, \pi(\Theta_{1k})),(M_{2k}, \pi(\Theta_{2k})), \cdots,(M_{pk}, \pi(\Theta_{pk}))]$

Find parameters of prior distributions $\pi(\Theta_{pk})$ from prior information A_k

For the available dataset $(X_{1k}, X_{2k}, X_{3k}, \cdots, X_{nk})$

Compute $f(X| M_{ik})$ for $i=1,..., p$

Compute $A_{ik} = \frac{f(X|M_{ik})\pi(M_{ik})}{\sum_{i=1}^{p} f(X|M_{ik})\pi(M_{ik})}$

Define the transition time model for each transition, where

$T_k=\sum_{i=1}^{p} A_{ik}.(M_{ik}, \pi(\Theta_{ik}))$ for full Baye's

$T_k=(M_{ck}, \pi(\Theta_{ck}))$, where $M_{ck} = max(A_{ik})$ for partial Baye's

End

Initialize Petri net token simulator parameters such as simulation time, number of replications (*model_par_replications* and *stoc_replications*) and burn-in period

Run simulation:

For number of *model_par_replications*

For each transition

Full Baye's:

With probability A_{ik}, select model $M_{ck} \leftarrow M_{ik}$ and $\Theta_{ck} \leftarrow \pi(\Theta_{ik})$

Partial Baye's:

Select $\Theta_{ck} \leftarrow \pi(\Theta_{ck})$

End

For number of *stoc_replications*

Set timer=0

While timer < simulation time

If deadlock- stop, else

Find set of enabled transitions

For enabled transitions, find transition firing time, $C_k \leftarrow f(M_{ck}|\Theta_{ck})$

Select the transition to fire as $C = \min(C_1, C_2, \ldots, C_n)$

Update the current marking , Timer = timer + C

Record statistics (after ignoring burn-in period)

End if

While loop

End *stoc_replications* loop and record mean simulation statistics

End *model_par_replications* loop and record final statistics

Fig. 4.2. Full and partial Baye's based token game simulation algorithm

The parameters α and β of prior distribution are found using the information specified by the designer. In the absence of such information, the designer can obtain subjective assessment about the processes (such as mean and deviation) from end users (such as production engineers) and use moment matching methods to obtain prior distribution parameters. Use of such methods is demonstrated in Chick (2001). In this algorithm, it was assumed that the probability of selecting each distribution model is equal. That is, $\pi(M_{1k})= \pi(M_{2k})= \ldots = \pi(M_{pk})$. After specifying the candidate distribution models and their prior distributions, the next step involves computation of $f(X|M_{ik})$, which is the likelihood of data given model M_{ik}. For conjugate family of distributions, this function is closed form and easy to evaluate. For other distributions, the designers can use rejection sampling or MCMC based methods to generate samples from prior distributions and then use the sampled values to compute likelihood. The dissertation work uses rejection sampling procedure for sampling from prior distributions. This algorithm is outlined in Appendix A. Rejection sampling algorithm was preferred because it does not require user intervention as compared to MCMC based methods in which the proposal distribution and its parameters need to be explicitly defined. Uniform distribution was used as a proposal density in order to automate the sampling process. To compute the limits of proposal density, samples from prior distribution were used.

After computing $f(X|M_{ik})$, the next step involves computing probability A_{ik}, which is the probability of data being represented by a distribution model M_{ik}. In addition, the designers also specify simulation specific information such as number of replications, warm up period and total simulation time. In this algorithm, two types of replications:

model_par_replications and *stoc_replications* need to be provided. *model_par_replications* are used to capture variability in simulation output due to *model* and *parameter* uncertainties, while *stoc_replications* are used to capture variability in simulation output due to *stochastic* uncertainties. This approach for computing separate sources of uncertainties was adopted from Zouaoui and Wilson (2004).

During the *simulation phase*, the *model_par_replications* loop is first invoked. For the full Baye's approach, the candidate distribution model (for example, exponential) is first selected with probability A_{ik} and then the parameters of candidate distribution model are sampled from their posterior distributions. In partial Baye's, the distribution model remains fixed and the parameters of the distribution model are sampled from their posterior distributions. Hence, in partial Baye's only *parameter* uncertainties are considered, whereas in full Baye's both *parameter* and *model* uncertainties are considered.

After the distribution models and their parameters are assigned, the *stoc_replication*s loop, which computes the effects of *stochastic* uncertainties with distribution model and parameters being fixed is invoked. During each step of the simulation, enabled transitions are found and their firing times are computed from the selected distribution model and its parameters. When multiple transitions are enabled, the transition with minimal time to fire is selected. This process is continued until no enabled transition exists (deadlock) or simulation time is over. At the end of the simulation, different statistics, such as total makespan and mean WIP, can be computed.

4.4. Manufacturing system design example

Table 4.1 shows the processing sequence and times for the manufacturing system example. The example was adapted from Saitou *et al.* (2002). In their approach, the authors considered multiple objectives but aggregated them into a single function. In addition, the authors used constant times to represent various operations. In this example, a multi-objective GA based approach is considered and the uncertainty in processing times is represented using a BMA approach.

As shown in Table 4.1, Part A's first operation can be carried out on either Machine 2 or Machine 3. The second operation of Part A can be done on either Machine 1 or Machine 3. The numbers in parenthesis represent mean and standard deviation of processing times for each operation on a machine. For instance, M2 (0.28,0.08) indicates that time for completing operation on Machine 2 will take a mean of 0.28 hours with a standard deviation of 0.08 hours.

Table 4.1. Processing sequence and times (in hours) for manufacturing system example (Saitou *et al.*, 2002)

Operation	Part A	Part B
1	M2(0.28,0.08)* or M3(0.18,0.05)	M1(0.17,0.05) or M2(0.24,0.07)
2	M1(0.17,0.05) or M3(0.30,0.08)	M2(0.34,0.08) or M3(0.17,0.05)

**Note: The numbers in parenthesis indicate mean and standard deviation (hours)*

Fig. 4.3 shows the Petri net model for the manufacturing system showing 1 machine of types M_1, M_2 and M_3. Places (P_a, P_{a1}, P_{a2})/ (P_b, P_{b1}, P_{b2}) represent the number of parts available at input buffer, the number of parts after first operation and the number of parts

produced for Part A/ Part B respectively. N_a and N_b represent the initial number of part types A and B available for production. The first operation of Part A (B) on Machine 2 (1) and Machine 3 (2) is represented by transitions $P_aOp_1M_2$ ($P_bOp_1M_1$) and $P_aOp_1M_3$ ($P_bOp_1M_2$) respectively. Similarly, the second operation of Part A (B) on Machine 1 (2) and Machine 3 (3) is represented by transitions $P_aOp_2M_1$ ($P_bOp_2M_2$) and $P_aOp_2M_3$ ($P_bOp_1M_3$) respectively. The Petri net uses the Shortest Imminent Operation (SIO) time rule to select candidate transitions to fire. This is a well known rule to minimize makespan for producing different parts. The bi-directional arcs in the Petri net model represent the resources being used and returned back for different operations. For instance, the bi-directional arc between place M_1 and transition $P_bOp_1M_1$ represents that a token from place M_1 is taken for firing of transition $P_bOp_1M_1$ and after transition $P_bOp_1M_1$ firing is complete, a token is placed back at place M_1. In this example, it means that machine M_1 is unavailable during transition $P_bOp_1M_1$ firing and after the transition firing is complete, the machine is available for subsequent operations.

A basic Petri net model containing the maximum allowable number of machines of each type was created in order to integrate the Petri net model with NSGA-II algorithm. The places in each machine group were designated as M_{1L}, M_{2L}, M_{3L},…, M_{KL}, where *K* is the maximum number of machines allowed and *L* is the machine group. In addition, additional transitions $P_aOp_1M_{1L}$, $P_aOp_1M_{2L}$,…, $P_aOp_1M_{KL}$ were created to represent operations performed on each of these machines. Depending upon the configuration setting received from the multi-objective GA, a specified number of machines were activated by putting tokens at places M_{1L}, M_{2L}, M_{3L}, …, M_{KL}. This approach eliminated

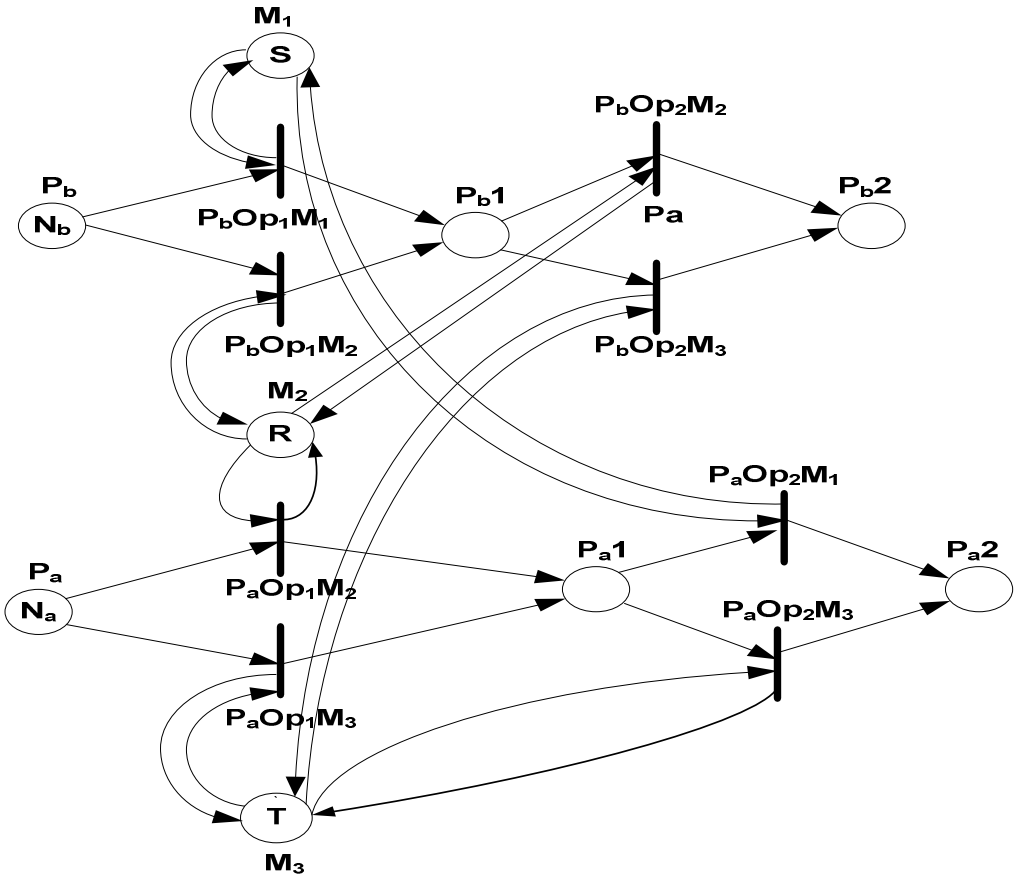

Fig. 4.3. Petri net model for the manufacturing system

the need to create different Petri net models for each configuration. The basic Petri net model was manually created using STPNPLAYER® software. The Petri net model attributes, such as incidence matrix, initial markings, and pre and post matrix were then exported and used in a Petri net token game simulation algorithm.

The objective function defined in (4.1) was considered to compare alternative layout designs. The total demand for both parts was assumed to be 2000 with mean product mix of 50% in base period and total time available for production was 30 days with a single 8

hour shift. In addition, the total demand was assumed to be fixed, no stock outs were allowed, and the maximum mean WIP allowed in buffer stations (P_{a1} and P_{b1}) was assumed to be 100 units. The product mix of part A was assumed to vary between 30-60% of total demand (2000 units). It was assumed that the maximum number of resources allowed in the system was four and each manufacturing cell could not have more than three resources.

4.4.1. Frequentist and Bayesian analysis of processing times

The actual processing times T_j for each operation j were assumed to follow a mixture distribution as shown in equation (4.4). To generate random samples from (4.4), the parameters of exponential (λ) and gamma (α, β) distribution were obtained by moment matching from mean time and standard deviation estimates listed in Table 4.1.

$$T_j = 0.8 * \text{exponential}(\lambda) + 0.2 * \text{gamma}(\alpha, \beta) \quad (4.4)$$

The processing time distribution (4.4) was considered to be unknown for a robust design problem, and a random dataset of 300 points was considered available for analysis. This dataset was assumed to be "known" information about the processing times and was used to compute the likelihood of processing times for each distribution model. In addition, it was assumed that the designer has additional 100 data points as the "prior information" about these processes. These data points were sampled from (4.4). The "prior information" was used to find parameters of prior distributions. Exponential, gamma, lognormal and normal distributions were considered to represent uncertainties related to processing times. It is important to point here that although the "actual" distribution was a mixture distribution, the distribution plot of the "actual" data did not

look like a mixture model. This presents an example where it is very difficult to know which distribution model is correct and whether the data follows a mixture distribution or not.

For the classical analysis, Chi square goodness-of-fit test was used to evaluate which distribution model represents the "known" processing time data. Based on the distribution shape, it was decided to consider exponential, gamma, lognormal and normal distributions for classical analysis. Gamma, normal and lognormal distributions failed the Chi square goodness-of-fit test for all the processing times considered. Only exponential distribution passed the Chi square goodness-of-fit test for some of the processing times. Table 4.2 shows the mean values of parameters using data fitting and results obtained using Chi square goodness-of-fit test. The shaded cells represent the processing times where exponential distribution passed the Chi square goodness-of-fit test. The results for gamma, normal and lognormal distributions were omitted as these distributions did not pass these tests. For classical simulation analysis, exponential distribution was used to represent uncertainties in processing times.

The BMA based approach demonstrated by Zouaoui and Wilson (2001a, 2001b, 2004) was used. Non-informative priors were used for exponential, gamma, lognormal and normal distributions to represent uncertainty in distribution model parameters. Details about these priors and marginal distribution calculation are outlined in Appendix A. For finding the parameters of prior distributions, the "prior information" generated for each of the processing times was used.

Table 4.2. Mean parameter values and Chi square test results for processing times

Actual processing times	Exponential distribution				Gamma distribution				
	λ	χ^2	$\chi^2_{critical}$	LL	α	β	χ^2	$\chi^2_{critical}$	LL
(0.17, 0.05)	0.18	**17.2**	18.3	214.5	1.21	0.15	22.3	19.7	217.9
(0.18, 0.05)	0.18	**11.8**	18.3	212.4	1.19	0.15	19.8	19.7	215.3
(0.23, 0.07)	0.23	30.9	18.3	144.4	1.21	0.19	45.1	19.7	144.7
(0.29, 0.08)	0.30	21.8	18.3	60.8	1.28	0.23	30.8	19.7	66.1
(0.30, 0.08)	0.28	27.7	18.3	87.3	1.14	0.24	36.4	19.7	89.0
(0.34, 0.08)	0.34	**14.7**	18.3	21.5	1.23	0.28	27.5	19.7	25.3

*LL: Log likelihood

Table 4.3 shows the BMA results and mean estimates of candidate distributions for each processing time. Occam's window method (Madigan and Raftery, 1994) was used to remove the least likely candidate distribution models. Occam's window is based on the principle that if a distribution model represents the data far less than the model that provides the best representation, then it should be discredited and should no longer be considered (Hoeting *et al.*, 1999). Thus, any model that does not belong to the set (4.5) can be excluded from (2.2), where *C* is appropriately chosen by the designer (Hoeting *et al.*, 1999). A higher value of C allows the distribution models with lower likelihood to be considered in the analysis, while a smaller value eliminates the distribution models with lower likelihood. For this example, the value of *C* was chosen as 20. This implies that the ratio of best model to the given model should be less than 20 for the model to be considered. For example, consider that for a given dataset, the likelihood of data following a normal, gamma and lognormal distribution is 0.85, 0.17 and 0.03. According to Occam's window rule, lognormal distribution will then be eliminated for BMA

analysis. In case, C was chosen as 4, both gamma and lognormal distributions would have been eliminated for BMA analysis. After the distribution models are eliminated, the BMA values are re-computed for further analysis.

$$L = \left\{ \frac{\max_k \left(f(M_k|X) \right)}{f(M_k|X)} \leq C \right\} \tag{4.5}$$

Table 4.3. BMA results and posterior distribution parameters for processing times

Actual (mean, std)	Posterior model probabilities *f(M|X)*			
	Exponential	Gamma	Lognormal	Normal
(0.17, 0.05)	0.17	**0.80**	0.03	0.00
(0.18, 0.05)	0.35	**0.63**	0.02	0.00
(0.23, 0.07)	**1.00**	0.00	0.00	0.00
(0.29, 0.08)	0.21	**0.79**	0.00	0.00
(0.30, 0.08)	**0.53**	0.45	0.02	0.00
(0.34, 0.08)	0.48	**0.51**	0.01	0.00

Actual (mean, std)	Mean posterior parameters for each candidate distribution						
	Exponential	Gamma		Lognormal		Normal	
	$\hat{\lambda}$	$\hat{\alpha}$	$\hat{\beta}$	$\hat{\mu}$	$\hat{\sigma}$	$\hat{\mu}$	$\hat{\sigma}$
(0.17, 0.05)	0.181	1.227	0.147	-2.180	1.269	0.180	0.027
(0.18, 0.05)	0.182	1.377	0.132	-2.182	1.406	0.181	0.024
(0.23, 0.07)	0.230	1.280	0.180	-1.937	1.549	0.230	0.041
(0.29, 0.08)	0.301	1.376	0.219	-1.643	1.264	0.300	0.066
(0.30, 0.08)	0.276	1.258	0.219	-1.788	1.578	0.275	0.061
(0.34, 0.08)	0.343	1.310	0.263	-1.531	1.391	0.342	0.090

Using Occam's window method, normal and lognormal distributions were eliminated as candidate distributions for all the processing times. The BMA values were then re-computed for the simulation. During each run in the simulation phase, each of

the candidate distribution models were selected with a probability mentioned in Table 4.3. The parameters of candidate distribution models were then sampled from posterior distributions v. The parameters for these posterior distributions are outlined in Table 4.3. The selected distribution models and sampled parameters were then used for the Petri net token game simulation.

4.4.2. Candidate configurations using NSGA-II multi-objective GA and Petri net

After finding the processing time distributions, the NSGA-II algorithm was run to find the Pareto front of candidate configurations. In this problem, binary encoding was used to represent a chromosome. Since there are three resources (M_1, M_2 and M_3) in the given problem and each manufacturing cell cannot have more than three resources, two chromosomes ($2^1 2^0$) were needed to represent maximum number of machines allocated in each manufacturing cell. The total chromosome length was 6 units with 2 units for each resource. The offspring population was generated using a single point crossover with 70% crossover probability and 5% mutation probability. The NSGA-II algorithm was run for 3 iterations with a population size of 20. It was observed that increasing the number of iterations or population size did not improve the final results. The initial population was randomly generated by assigning 0-1 values to each chromosome. Based upon the configuration setting received from the NSGA-II algorithm, the Petri net model evaluated mean WIP and makespan estimates. The configurations that did not meet demand and WIP requirements were assigned a high objective function value. These estimates were returned in the form of "smaller the better" signal noise ratio to NSGA-II algorithm. For each configuration, a total of 100 iterations were carried out with 20

model_par_replications and 5 *stoc_replications.* Total simulation time was 30 days with one 8 hour shift/day.

Fig. 4.4 shows the Pareto front of candidate configurations obtained from the NSGA-II algorithm. The numbers in parenthesis show configuration, mean makespan, and mean WIP estimates for the candidate solutions. For instance, [2, 0, 2]: [104.45, 17.98] represents a configuration with 2 units of Machine 1, 0 unit of Machine 2 and 2 units of Machine 3. The makespan and mean WIP estimates of configuration [2, 0, 2] were 104.45 hours and 17.98 units respectively. It is important to point out here that the candidate configurations are evaluated using signal to noise ratio, but to facilitate understanding, mean estimates were shown in Fig. 4.4.

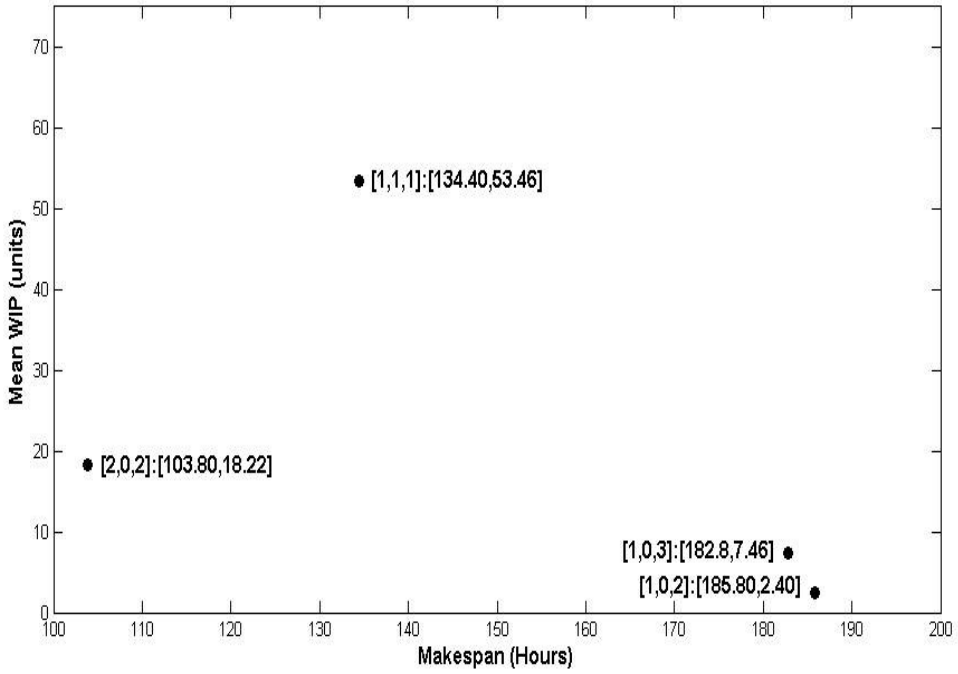

Fig. 4.4. Pareto front solution using NSGA-II multi-objective GA

The Pareto front clearly demonstrates relative advantages and disadvantages of candidate configurations. In order to simplify the analysis, the third axis involving number of resources has been omitted. Configuration [2, 0, 2] had the least makespan, but required a greater number of resources. On the other hand, configuration [1, 1, 1] had a lower number of resources, but makespan and mean WIP were higher. There was not a significant difference in results obtained using configurations [1, 0, 3] and [1, 0, 2]. So, configuration [1, 0, 2] was preferred over configuration [1, 0, 3] as it used one less resource.

Tables 4.4-4.6 show the mean, 95% half width, Mean Absolute Error (MAE), Mean Absolute Percentage Error (MAPE) and Mean Square Error (MSE) estimates using classical and Bayesian methods. Paired t-tests were conducted to test whether there was a significant difference in results obtained using the actual values and the approach (for instance, classical methods) used. The highlighted cells in the tables represent the cases where significant difference exists among actual values and the approach used. For instance in Table 4.4A, classical methods were not able to capture the true makespan for configurations [1, 0, 2], [1, 1, 1] and [2, 0, 2].

For classical methods, exponential distribution was used to represent uncertainty in processing times. In order to consider parameter uncertainty, it was assumed that the mean (λ) varied 10% around its mean value. Table 4.4 outlines the results obtained with classical methods, and classical methods considering 10% variation in mean. T-tests revealed that both of these approaches were not able to capture the true makespan.

Table 4.4. Comparison of results obtained using classical methods

A. Makespan: Mean (*hours)*, 95% Half Width (HW)

Configuration	Actual		Classical		Classical (Sensitivity)	
	Mean	HW	Mean	HW	Mean	HW
[1, 0, 2]	186.08	1.15	179.54	2.34	165.22	2.51
[1, 1, 1]	131.39	1.69	118.84	1.36	119.58	1.38
[2, 0, 2]	103.24	0.87	93.23	0.88	106.38	1.32

B. Makespan: Mean Absolute Error (MAE), Mean Absolute Percentage Error (MAPE) and Mean Square Error (MSE) comparison

Configuration	MAE		MAPE		MSE	
	C	CS	C	CS	C	CS
[1, 0, 2]	7.13	20.86	3.83%	11.21%	73.61	463.22
[1, 1, 1]	10.59	9.85	8.18%	7.61%	122.60	105.56
[2, 0, 2]	10.01	3.49	9.69%	3.38%	104.51	17.69
Overall	9.24	11.40	7.24%	7.40%	100.24	195.52

C. Mean WIP: Mean (*units)*, 95% Half Width (HW)

Configuration	Actual		Classical		Classical (Sensitivity)	
	Mean	HW	Mean	HW	Mean	HW
[1, 0, 2]	2.32	0.53	2.24	0.25	2.29	0.34
[1, 1, 1]	56.51	5.31	47.09	5.21	53.22	5.41
[2, 0, 2]	17.10	1.84	14.06	3.04	8.58	1.86

D. Mean WIP: Mean Absolute Error (MAE), Mean Absolute Percentage Error (MAPE) and Mean Square Error (MSE) comparison

Configuration	MAE		MAPE		MSE	
	C	CS	C	CS	C	CS
[1, 0, 2]	0.45	0.54	19.60%	23.08%	0.36	0.52
[1, 1, 1]	13.0	8.27	23.01%	14.63%	241.52	141.13
[2, 0, 2]	7.12	14.82	41.65%	86.63%	61.49	220.10
Overall	6.86	7.87	28.09%	41.45%	101.12	120.58

Note: C- Classical method, CS-Classical method with sensitivity analysis

Table 4.5. Comparison of results obtained using BMA (non-informative priors)

A. Makespan: Mean (*hours)*, 95% Half Width (HW)

Configuration	Actual		Partial Baye's		Full Baye's	
	Mean	HW	Mean	HW	Mean	HW
[1, 0, 2]	186.08	1.15	185.95	3.94	185.80	3.89
[1, 1, 1]	131.39	1.69	137.08	1.84	134.40	2.45
[2, 0, 2]	103.24	0.87	105.83	2.30	103.80	2.16

B. Makespan: Mean Absolute Error (MAE), Mean Absolute Percentage Error (MAPE) and Mean Square Error (MSE) comparison

Configuration	MAE		MAPE		MSE	
	P	F	P	F	P	F
[1, 0, 2]	7.39	7.77	3.97%	4.18%	85.31	85.34
[1, 1, 1]	5.67	5.21	4.31%	3.97%	45.21	42.96
[2, 0, 2]	4.44	4.10	4.30%	3.98%	36.40	26.72
Overall	5.83	5.69	4.19%	4.04%	55.64	51.67

C. Mean WIP: Mean (*units)*, 95% Half Width (HW)

Configuration	Actual		Partial Baye's		Full Baye's	
	Mean	HW	Mean	HW	Mean	HW
[1, 0, 2]	2.32	0.53	1.61	0.14	2.40	0.50
[1, 1, 1]	56.51	5.31	57.44	3.12	53.46	4.89
[2, 0, 2]	17.10	1.84	17.71	1.73	18.22	1.63

D. Mean WIP: Mean Absolute Error (MAE), Mean Absolute Percentage Error (MAPE) and Mean Square Error (MSE) comparison

Configuration	MAE		MAPE		MSE	
	P	F	P	F	P	F
[1, 0, 2]	0.73	0.86	31.53%	37.06%	0.63	1.41
[1, 1, 1]	6.14	9.06	10.86%	16.04%	55.64	144.17
[2, 0, 2]	3.44	3.28	20.12%	19.20%	17.30	16.24
Overall	3.44	4.40	20.84%	24.10%	24.52	53.94

Note: P-Partial Baye's method, F- Full Baye's method using non informative priors

Table 4.5 outlines the results obtained using BMA based approach with non informative priors. The highlighted cells represent the case where the results obtained were significantly different from the actual values. Both partial and full Baye's

approaches provided better MAE, MAPE and MSE estimates as compared to the classical methods.

Conjugate prior for exponential distribution and conjugate prior specified by Miller (1980) for gamma distribution (α, β) were used to evaluate the effect of using different prior distributions. The parameters for prior distributions were found by moment matching methods. For the gamma distribution, α was first sampled from its distribution using rejection sampling with uniform proposal distribution (Appendix A).

Fig. 4.5 shows a plot of α obtained using the distribution specified by Miller (1980) against the true distribution of α. It can be clearly seen that the algorithm provided a fairly good estimate of true density. Once α is sampled, both β and marginal likelihood $f(X)$ can be easily computed.

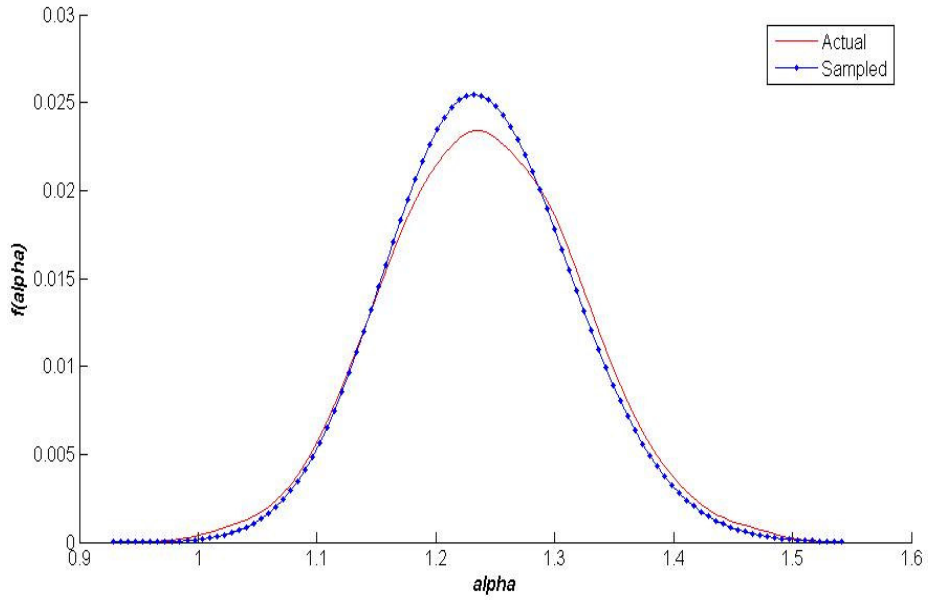

Fig. 4.5. Actual versus rejection sampling comparison for α

Table 4.6. Comparison of results obtained using BMA (conjugate priors)

A. Makespan: Mean (*hours*), 95% Half Width (HW)

Configuration	Actual		Partial Baye's		Full Baye's	
	Mean	HW	Mean	HW	Mean	HW
[1,0,2]	186.08	1.15	186.39	1.15	186.42	1.27
[1,1,1]	131.39	1.69	138.37	1.10	137.66	1.29
[2,0,2]	103.24	0.87	106.13	1.04	104.45	1.58

B. Makespan: Mean Absolute Error (MAE), Mean Absolute Percentage Error (MAPE) and Mean Square Error (MSE) comparison

Configuration	MAE		MAPE		MSE	
	P*	F*	P*	F*	P*	F*
[1,0,2]	2.08	2.30	1.12%	1.24%	7.61	9.19
[1,1,1]	8.95	8.23	6.91%	6.36%	86.79	77.17
[2,0,2]	3.12	3.21	3.02%	3.11%	14.50	15.48
Overall	4.71	4.58	3.68%	3.57%	36.30	33.94

C. Mean WIP: Mean (*units*), 95% Half Width (HW)

Configuration	Actual		Partial Baye's		Full Baye's	
	Mean	HW	Mean	HW	Mean	HW
[1,0,2]	2.32	0.53	2.08	0.16	2.52	0.65
[1,1,1]	56.51	5.31	51.14	2.03	58.96	3.73
[2,0,2]	17.10	1.84	17.46	1.58	17.98	1.76

D. Mean WIP: Mean Absolute Error (MAE), Mean Absolute Percentage Error (MAPE) and Mean Square Error (MSE) comparison

Configuration	MAE		MAPE		MSE	
	P*	F*	P*	F*	P*	F*
[1,0,2]	0.40	0.45	17.08%	19.60%	0.20	0.36
[1,1,1]	6.39	6.41	11.31%	11.35%	52.10	54.28
[2,0,2]	3.11	3.11	18.17%	19.84%	14.14	14.14
Overall	3.30	3.33	15.52%	16.37%	22.15	22.93

Note: P*-Partial Baye's method using conjugate priors, F*- Full Baye's method using conjugate priors

Table 4.6 shows a comparison of results obtained using conjugate priors based on full and partial Baye's methods. The highlighted cells represent the cases where the results obtained using partial and full Baye's methods were significantly different from actual results. Conjugate priors resulted in smaller confidence interval lengths of parameters as compared to non-conjugate prior methods. In addition, the overall MAE, MAPE and MSE estimates obtained using conjugate priors were better than BMA approach using non informative priors and classical methods. Based on these results, conjugate priors based on a full Baye's approach were used for further analysis.

4.4.3. Effect of product mix

The candidate configurations generated using the NSGA-II algorithm were analyzed with respect to variation in product mix. In this example, it was assumed that overall demand remains fixed, however the product mix in future planning horizons can vary.

Fig. 4.6 shows the effect of product mix on makespan and mean WIP. Although for this problem, product mix variation was between 30-60%, the performance of each configuration under extreme cases (only Part A or Part B) is also shown. At this moment, there was a tradeoff between number of resources and system performance in terms of makespan and mean WIP. Configuration [1, 1, 1] had on average 33 hours more makespan and 40 units more mean WIP as compared to configuration [2, 0, 2], but it used one less resource than configuration [2, 0, 2]. On the other hand, configurations [1, 0, 2] and [1, 1, 1] used the same number of resources, but there was a tradeoff between makespan and WIP.

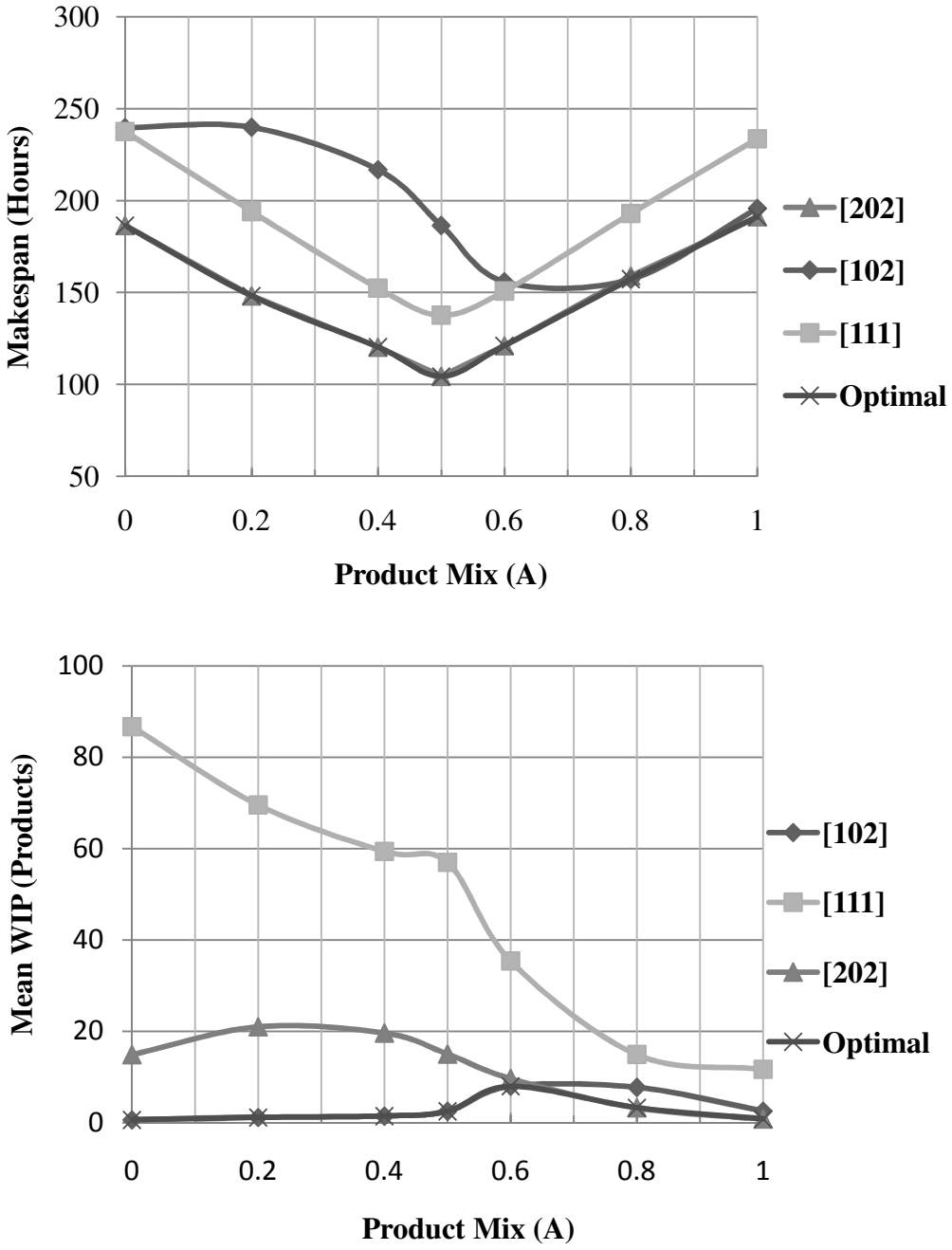

Fig. 4.6. Effect of product mix on makespan and mean WIP

Final configuration selection at this point is dependent upon relative weights of production, WIP and resource costs. If the resource costs are significantly higher than

production costs, and inventory costs are significantly low, then a reasonable choice will be configuration [1, 1, 1]. For this problem, the production cost, WIP cost and resource costs were assumed to be $25/hr, $5/unit and $250/resource respectively.

Fig. 4.7 shows total cost against variation in product mix under these costs. The figure clearly shows that configuration [2, 0, 2] had consistently less total cost as compared to other configurations under product mix variation of 30-60%. Hence, configuration [2, 0, 2] was chosen as a final configuration for this problem.

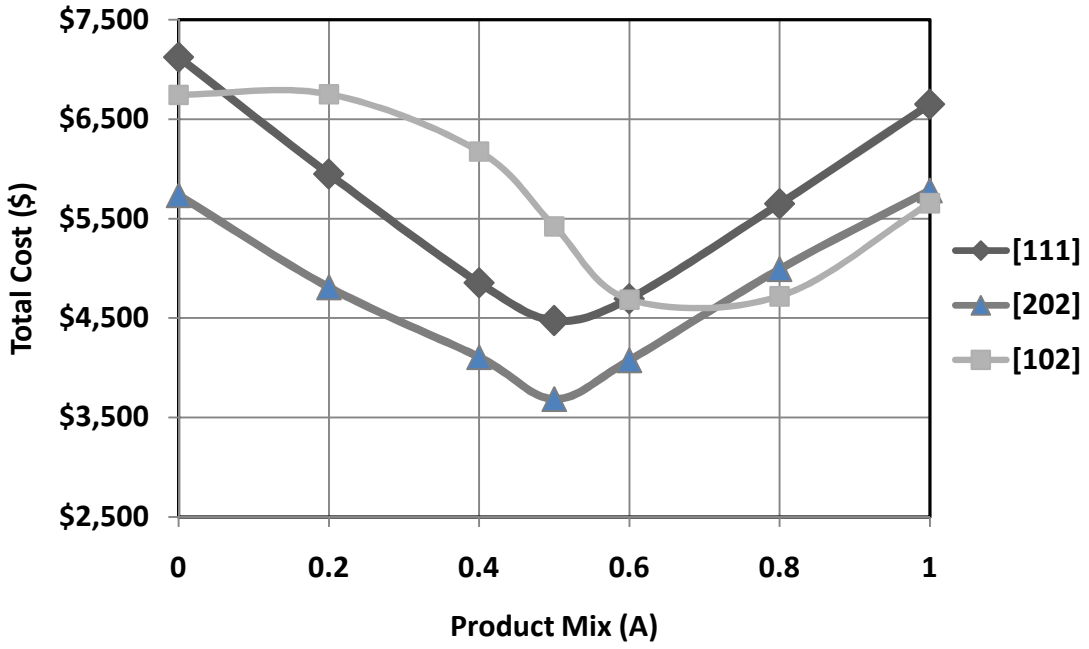

Fig. 4.7. Total cost versus product mix variation

4.5. Summary and conclusions

This chapter presented an NSGA-II multi-objective GA and Petri net based approach coupled with Bayesian methods of uncertainty representation for robust manufacturing system design. The approach provides a systematic way to evaluate candidate configurations by using formal approaches of uncertainty representation (Bayesian methods) and model evaluation (Petri nets). In addition, instead of giving a single point assessment of objective function, the approach generates a Pareto front of candidate solutions that can be evaluated against their relative merits and demerits.

A manufacturing system design example that produced multiple types of parts that undergo different processing sequences was considered. The problem was formulated as a cost minimization function that depended upon resource, production and inventory holding costs. The NSGA-II algorithm coupled with Petri net model provided a Pareto front of candidate configurations. These candidate configurations were then evaluated with respect to variation in product mix, with overall demand assumed constant.

Furthermore, this chapter provided Petri net token game simulation algorithms for integrating Bayesian methods with Petri nets. Current literature shows a void in using Bayesian methods with Petri nets, and these algorithms may have significant application potential where uncertainty modeling is critical. For non conjugate distributions, a rejection sampling algorithm was used to sample from prior and posterior distributions. The “actual” performance estimates obtained for each configuration were evaluated against classical, partial and full Baye’s methods. Both conjugate and non-informative priors were considered to see the effect of different types of priors.

In summary, this chapter introduced the NSGA-II algorithm and Petri net based methodology using BMA for robust design of manufacturing systems. The chapter introduced new token game simulation algorithms for integrating BMA with Petri nets. These algorithms were used with an NSGA-II algorithm to generate a set of configurations against multiple objectives. The candidate configurations were then evaluated with respect to variation in product mix. The results reveal that BMA based approach using conjugate priors provided better estimates of underlying estimates as compared to BMA methods using non informative priors and classical methods.

In this chapter, it was assumed that overall demand of products remains fixed for future planning horizons, but there can be a variation in product mix. In addition, the effect of uncertainties related to part arrivals, machine failures and repairs and product demand were not considered. In the next chapter, a more rigorous manufacturing system example is presented, in which abovementioned uncertainties are also considered.

CHAPTER V

ROBUST MANUFACTURING SYSTEM DESIGN: CONSIDERATION OF DEMAND UNCERTAINTIES

5.1. Introduction

In the previous chapter, the NSGA-II multi-objective GA and Petri net based approach for robust manufacturing system design was presented. In this chapter, a more rigorous manufacturing system example is considered in which there are uncertainties related to processing times, part arrivals, machine failures and repairs. These uncertainties are represented in a Bayesian framework. In addition, there is uncertainty in part demand for future planning periods. The problem is formulated as a two step cost minimization, in which a Pareto front of alternative configurations is first generated using an NSGA-II algorithm. In the next step, the designers identify potential configurations that meet budgetary constraints under weights (costs) associated with makespan, WIP and number of resources. The potential configurations are then analyzed with respect to demand variation that can occur in future planning periods. The uncertainty in part demands is represented using a Bayesian approach. The configuration that results in least overall costs while meeting demand requirements is selected as a robust configuration. In the next section, the problem formulation for a manufacturing system example is presented.

5.2. Problem formulation

A manufacturing system design problem is considered in which multiple types of parts that undergo different manufacturing sequences are produced. There is uncertainty in processing times, arrival rate of parts and the machines are subject to stochastic failures and repairs. The demand for each part in the base period is known with certainty; however, the demand in future planning periods is uncertain. It is assumed that production sequence of each part is known and there is information available about failure, repair and processing times as well as arrival rate of parts. This information is used to obtain the parameters of prior distributions and to compute the likelihood against different distributions. The objective function for the problem is formulated as a cost minimization function as defined in (5.1). It is dependent upon makespan, WIP, resource costs and initial setup cost. To simplify the problem, it is assumed that stock outs are not allowed.

$$A = \left[\sum_{p=1}^{P} \delta_p \left[\left\{\sum_{i=1}^{Z} (C_{ip}Q_{ip} + W_{ip}T_{ip})\right\} + \left\{\sum_{j=1}^{L} R_{jp}V_{jp}\right\}\right] + \left\{\sum_{j=1}^{L} S_{j1}(R_{j1} - N_{j1})\right\}\right] \quad (5.1)$$

where:

- $i=1,\ldots, Z$ are the different types of parts produced within the system
- $j=1,\ldots., L$ are the types of resources used within the system
- $p=1,\ldots., P$ are the planning horizons considered for this problem, where $p=1$ is the

base period

- C_{ip}, W_{ip} = production cost *($/hr),* inventory cost *($/unit/planning horizon)* of part type *i* in planning period *p*
- Q_{ip}, T_{ip} = makespan time (*hr*) and mean WIP (*units)* of part type *i* in planning period *p*
- V_{jp}, R_{jp} = number of resources and cost *($/resource/planning horizon*) of using resource type *j* in planning period *p*
- S_{j1},R_{j1},N_{j1} = initial set up cost (*$/resource*), number of resources of type *j* needed in base period and number of resources of type *j* already existing before base period
- δ_p is the idiscount factor to compute the present value of costs in planning period *p,* where η is the interest rate. The value of the discou*nt* factor in each planning period is computed usi*ng* (5.2):

$$\delta_p = \frac{1}{(1+\eta)^{p-1}} \tag{5.2}$$

subject to:

- *Budgetary constraints*: F_p<TC_p, w*he*re F_p is the total cost in planning period *p* and TC_p is the maximum allowable total cost in planning p*e*riod *p*
- *Resource constraints*: $V_{jp} \leq K_j$, where V_{jp} is the number of resource type *j* allocated in period *p* and K_j is the maximum number of resources of type *j* available for allocation in each planning period

- *Demand constraints:* $U_{ip}=D_{ip,}$ *t*hat is, the number of units (U_{ip}) of part type *i* produced in planning period *p* should be equal to demand (D_{ip}) of part type *i* in planning period *p*
- Processing times, arrival rates, repair and failure times are functions of *model distribution*s and their parameters as defined in the Bayesian framework. For example, let X_{ij} be the time taken for processing part i on resource *j*, then $X_{ij} \sim F[(B_{ij1}, \Theta_{ij1}), (B_{ij2}, \Theta_{ij2}), \cdots\cdots, (B_{ijs}, \Theta_{ijs})]$, where:
  - B_{ijs} = candidate distribution model *s* to represent resource *j* processing time for part *i*, and
  - Θ_{ijs} = parameters for candidate distribution model *s*, where $\Theta_{ijs} \sim \pi(\Theta_{ijs})$. $\pi(\Theta_{ijs})$ is the prior distribution to represent uncertainty in parameters Θ_{ijs} of distribution model B_{ijs}
  - It is assumed that designers have data points about these uncertainties in order to find the parameters of prior distributions

The last point is illustrated with an example: one resource's (*j*) processing time follows some unknown distribution. Several distributions models B_{ijs} (for example, normal, exponential) can represent these processing times. The uncertainty associated with selection of parameters for these distribution models is represented using prior distributions. For example, for exponential distribution model with parameter λ, the uncertainty in parameter λ can be represented using a conjugate distribution $\pi(\lambda) \sim$ gamma (α, β). To find the parameters α and β of prior distribution, moment matching or data driven methods can be used. After the distribution models and their prior

information is assigned, the probability that distribution model B_{ijs} represents the resource processing time can then be computed using (2.2).

To simplify the problem, the following assumptions are made:

- Production and inventory costs for each part are equal: C_{1p}=… C_{2p}=…=C and W_{1p}=… W_{2p}=…=W, for $i=1,….,Z$ and $p=1,….,P$
- Resource cost is equal for all resources: $R_{1P}= R_{2P}= … = R_{LP}$ for $p=1,….,P$
- Initial set up cost is equal for different types of resources: $S_{11}= S_{21}= … = S_{L1}$
- Reconfigurations are not allowed from one planning period to another: $V_{j1}= V_{j2}=…= V_{jp}$

If the costs are different, the NSGA-II algorithm proceeds in a similar fashion except that the Pareto front is generated against production, inventory and resource costs, instead of makespan, mean WIP and number of resources. This requires that the weights associated with each factor (for example, resource cost associated with each resource) be defined prior to running the multi-objective GA. With the budgetary constraint, the designers can then find a set of configurations that meet the budget requirements and then evaluate them with respect to demand variations.

5.3. NSGA-II and Petri net based approach

The NSGA-II multi-objective GA and Petri net based approach presented in Chapter IV was used to find a robust design solution. The Petri net model was first generated based on processing sequences for each part. The model was generated using STPNPLAYER® software and Petri net attributes such as incidence matrix, pre and post

matrices were then exported to a token game simulation algorithm. The Petri net attributes and information about *uncontrollable uncertainties* (for instance, processing times) were used by the token game simulation algorithm to generate makespan and mean WIP estimates for candidate configurations provided by the NSGA-II algorithm. These estimates were provided to the NSGA-II algorithm in the form of signal to noise ratios. The final candidate configurations obtained from the NSGA-II algorithm were reduced to a feasible set that met designer's maximum investment constraint under production, resource and WIP costs. These configurations were then analyzed with respect to demand variations in future planning horizons. The configuration that yielded the least overall costs at the end of planning horizon was selected as a robust configuration.

5.4. Manufacturing system design example

Table 5.1 shows the processing sequence of each part along with mean and standard deviation of processing times (in parenthesis) for the manufacturing system example. This example was presented by Chen and Chen (1996), however in this case, a different problem formulation is used and the uncertainties are represented using a Bayesian framework. The manufacturing system produces parts A, B and C that undergo operations in different manufacturing cells. Each manufacturing cell contains certain number of identical machines that are subject to stochastic failures and repairs.

Table 5.1. Operation sequence and processing times (Chen and Chen, 1996)

	Operation step				
Part	I	II	III	IV	V
A	**2*** (1.2, 0.096)**	**5** (0.25,0.020)	**1** (0.70,0.056)	**4** (0.90,0.072)	**3** (1.0,0.08)
B	**3** (0.50,0.045)	**1** (0.60,0.054)	**2** (0.85,0.077)	**5** (0.50,0.045)	-
C	**4** (1.10,0.100)	**1**(0.80,0.080)	**3** (0.75,0.075)	-	-

*Note: * The number indicates the machine type required for an operational step,*
***The numbers in parenthesis indicate mean and standard deviation (hours)*

The true processing times were assumed to follow a normal distribution (μ, σ^2). In addition, the mean parameter (μ) was assumed to uniformly vary 10% around the mean values specified in Table 5.1. For example, for Part A operation I, the mean processing time varied between 1.08-1.32 hours. Similar explanation holds for other processing times. The true mean inter-arrival times for parts A, B and C was assumed to follow an exponential distribution (λ) with a mean of 0.5 hours. It was assumed that the mean (λ) uniformly varied by 10% around its mean value of 0.5. For example, part A inter-arrival time mean (λ) varied between 0.45-0.55 and followed an exponential distribution. The maximum WIP allowed in system was 100 and stock outs were not allowed. Any configuration that did not meet these criteria was assigned a high objective function value and was eliminated from the set of candidate configurations.

Mean failure and repair times of each machine are shown in Table 5.2. It was assumed that there cannot be more than 5 machines in each manufacturing cell. So, at maximum there can be 25 machines in the system. The failure and repair times were assumed to follow an exponential mixture distribution with 95% of failures occurring in times specified in Table 5.2 and 5% of the failures occurring in a short time with mean of 5 hours and standard deviation of 0.5 hours. In addition, the mean value of Mean

Time to Fail (MTTF) and Mean Time to Repair (MTTR) was assumed to uniformly vary by 10%. For instance, the MTTF of machine 1 followed an exponential mixture distribution with 95% failures following exponential distribution with means varying between 18-22 hours and 5% failure following exponential distributions with means varying between 4.5-5.5 hours.

Table 5.2. Machine failure and repair times

	Machine				
	1	2	3	4	5
MTTF[1]	(20, 3)*	(25,4)	(20,3)	(30, 5)	(20, 3)
MTTR[2]	(3,0.5)	(2.5,0.5)	(3,0.5)	(3,0.5)	(2.5,0.5)

[1]*MTTF:* Mean Time to Fail (hours), [2]*MTTR:* Mean Time to Repair (hours)
**Note: The numbers in parenthesis indicate mean and standard deviation (hours)*

The demand of parts A, B and C in base period was assumed to be 30, 50 and 20 units Table 5.3 shows the expected demand for parts A, B and C in future planning periods. The numbers in parenthesis indicate mean and standard deviation of part demand. Demand uncertainty for each part increases from period 1 to period 3, and it is represented with an increased standard deviation. The demand for part B decreases over the planning periods. The demand for part C increases from base period and then stabilizes in periods 2 and 3. The true demand for each part was assumed to follow a normal distribution and was assumed to uniformly vary about 10% about their mean value. For example, the demand for part A in period 1 followed normal distribution with the mean varying between 23-28 units with a standard deviation of 3.

Table 5.3. Expected demand in future planning periods

Part	Period 1	Period 2	Period 3
A	(25,3)*	(13, 4)	(27, 5)
B	(55, 3)	(52, 4)	(46, 5)
C	(25, 3)	(28,4)	(28, 5)
Total expected demand	105	92	100

**Numbers in parenthesis represent mean and standard deviation of demand*

Fig. 5.1 shows the Petri net model for the manufacturing system. The Petri model was manually developed using STPNPLAYER® software. The Petri net model attributes such as incidence matrix, initial markings, pre and post matrix were then exported and used in Petri net token game simulation algorithm. Modules for each machine were created (shown in dotted boxes) to represent machine failure and repair. A token N_{M1} at place M_1 represents availability of machine 1. The firing of transition T_{M1F} represents failure of machine 1. When this transition is enabled, the machine is taken out of service and a repair is performed. To simplify the problem, it is assumed that failure does not occur during the time a machine is processing a part. The repair of machine 1 is represented by transition T_{M1R}. The machine is available for the next operation after transition T_{M1R} fires. Similar explanations for other machine modules follow.

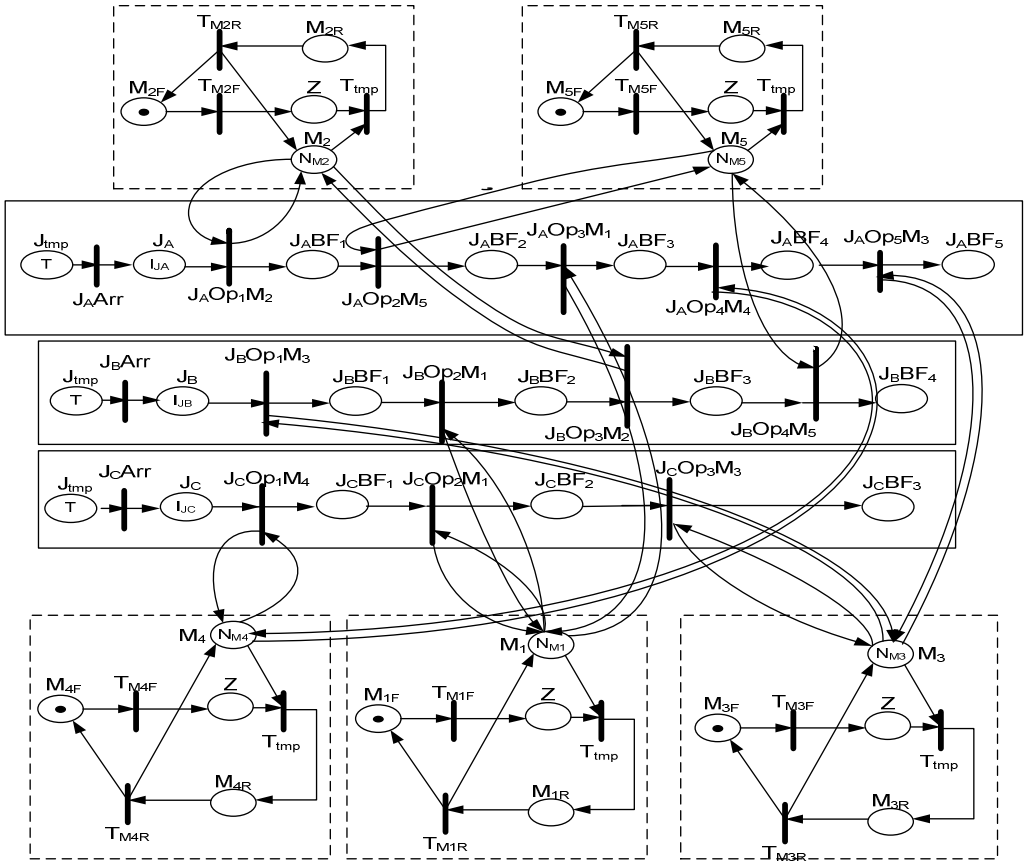

Fig. 5.1. Petri net model for the manufacturing system

Transitions J_AArr, J_BArr and J_CArr invoke arrival of parts A, B and C into the manufacturing system. After the arrival, the parts are stored in inventory storage, which is represented by places J_A, J_B and J_C. There is an initial inventory of parts for parts A, B and C, which is represented by initial tokens (I_{JA}, I_{JB} and I_{JC}) at places J_A, J_B and J_C. After arriving in the manufacturing system, the parts go through different operations that are represented by transitions with nomenclature (Part-Operation Sequence-Machine). For example, $J_AOP_1M_2$, represents Part A first operation on Machine 2. Similarly, places

J_XBF_N represent N^{th} buffer or inventory storage for Part X. Places J_ABF_5, J_BBF_4 and J_CBF_3 represent the final storage for parts A, B and C.

The Petri net model shown here represents one machine of each type. Since, there can be a maximum of 5 machines in each manufacturing cell, a basic Petri net model was first created with 25 machines. Depending upon the configuration setting received from the NSGA-II algorithm, certain numbers of machines in a machine group were invoked by placing tokens at places $\{N_{MX1}, N_{MX2},\ldots, N_{MX5}\}$, where X is type of machine group $\{1,\ldots., 5\}$. For example, to invoke 3 machines in machine group 1, the places $\{N_{M11}, N_{M12}, N_{M13}, N_{M14}, N_{M15}\}$ were assigned tokens 1, 1, 1, 0, 0.

After developing the Petri net model in STPNPLAYER® software, Petri net model information such as pre matrix, post matrix, incidence matrix and number of markings were exported to token game simulator. The token game simulator used this information to evaluate manufacturing system performance against different configurations.

The objective function described in (5.1) was used to evaluate alternative configuration designs. The total demand of parts A, B and C in base period was assumed to be 30, 50 and 20 units. It was assumed that there were 9 machines available in the beginning of base period and a total of 4 planning periods (including base period) were considered. The manufacturing system was assumed to work for 2 shifts/ day with 8 hours/shift and 7 days/ week. The NSGA-II algorithm was first used to find candidate configurations with fixed demand assigned in base period.

5.4.1. Bayesian analysis of processing, failure, repair times and arrival rates

A random dataset of "actual" and "prior" data points of size 250 and 50 was generated from "true" processing, failure, repair and arrival rate distributions to demonstrate the proposed approach. The "prior" sample was used to find parameters of prior distributions, and "actual" dataset was used to compute likelihood that a distribution model represents the actual time for an event (for example, processing time). Table 5.4 enlists the distributions that were considered for Bayesian representation of different stochastic events in this manufacturing system.

Table 5.4. Distributions considered for different events in a manufacturing system

Type	Distributions
Processing times	Exponential, Normal, Gamma
Failure times	Exponential, Gamma
Repair times	Exponential, Gamma
Inter-arrival times	Exponential, Gamma

Table 5.5 shows the results obtained from Bayesian analysis of failure, repair and inter-arrival times. The uncertainty in parameter λ of exponential distribution and parameters α and β of gamma distribution was represented using conjugate priors. A detailed discussion of these priors is provided in Appendix A. $f(M|X)$ shows the probability that data follows an exponential or gamma distribution. For inter-arrival times, the BMA results showed a high probability for exponential distribution. In addition, α value obtained for gamma distribution was close to 1, indicating closeness to exponential distribution.

Table 5.5. BMA results for repair, failure and inter-arrival times

A. Repair and failure times (hours)

Actual time		Exponential		Gamma		
Mean	*Std*	*f(M|X)*	$\hat{\lambda}$	*f(M|X)*	$\hat{\alpha}$	$\hat{\beta}$
2.50	0.50	0.48	2.31	0.52	1.34	1.72
3.00	0.50	0.46	2.46	0.54	1.15	2.14
3.50	0.50	0.79	3.91	0.21	0.69	5.62
20.00	3.00	0.62	20.39	0.38	0.92	22.43
25.00	4.00	0.18	29.21	0.82	2.03	14.21
30.00	5.00	0.55	29.01	0.45	1.30	22.18

B. Inter-arrival time (hours)

Actual time	Exponential		Gamma		
Mean	*f(M|X)*	$\hat{\lambda}$	*f(M|X)*	$\hat{\alpha}$	$\hat{\beta}$
0.5	0.80	0.52	0.20	0.97	0.53

5.4.2. Candidate configurations using NSGA-II algorithm and Petri net based approach

The NSGA-II algorithm coupled with Petri net model was used to find a Pareto front of configurations against makespan, mean WIP and number of resources in base period. The demand for each part was known with certainty during the base period; however, there was uncertainty related to processing, failure, repair and inter-arrival times.

Binary encoding was used to represent a chromosome in the NSGA-II algorithm. The chromosome length was 15 units with 3 units for each machine group. Since a binary encoding of 3 units ($2^2 2^1 2^0$) can produce 7 machines , the encodings that resulted in a value greater than 5 were ignored. The NSGA-II algorithm was run using a single point crossover, 70% crossover probability and 5% mutation probability. The algorithm was run for 5 iterations with a population size of 25. The initial population was generated by assigning a random 0-1 value to each chromosome. The configuration

setting generated by the NSGA-II algorithm was sent to the Petri net model, which computed makespan and mean WIP estimates and returned them as signal to noise (S/N) ratio. For each candidate configuration, 100 replications were conducted with 20 *model_par_replications* and 5 *stoc_replications.* The total simulation time for each run was 112 hours (2 shifts/ day of 8 hours each and 7 days/ week) and a warm up period of 32 hours was considered.

Fig. 5.2 shows the Pareto front of candidate configurations for base period using the NSGA-II algorithm. The details of configuration numbers listed in Fig. 5.2 are provided in Table 5.6. For example, configuration number 1 represents configuration [2, 2, 3, 1, 1]. A configuration represents the number of machines used for each type. For instance, configuration [2, 2, 3, 1, 1] represents a configuration with 2 machines of type 1, 2 machines of type 2, 3 machines of type 3, 1 machine of type 4 and 1 machine of type 5.

The Pareto front clearly shows the advantages and disadvantages of using each configuration against number of resources, makespan and mean WIP. In the next phase, the designers evaluate these configurations with their assessment of production, resource and WIP costs, and find candidate configurations that meet budgetary constraints. Once the candidate configurations are found, they are then evaluated with respect to demand variations that can occur in future planning periods.

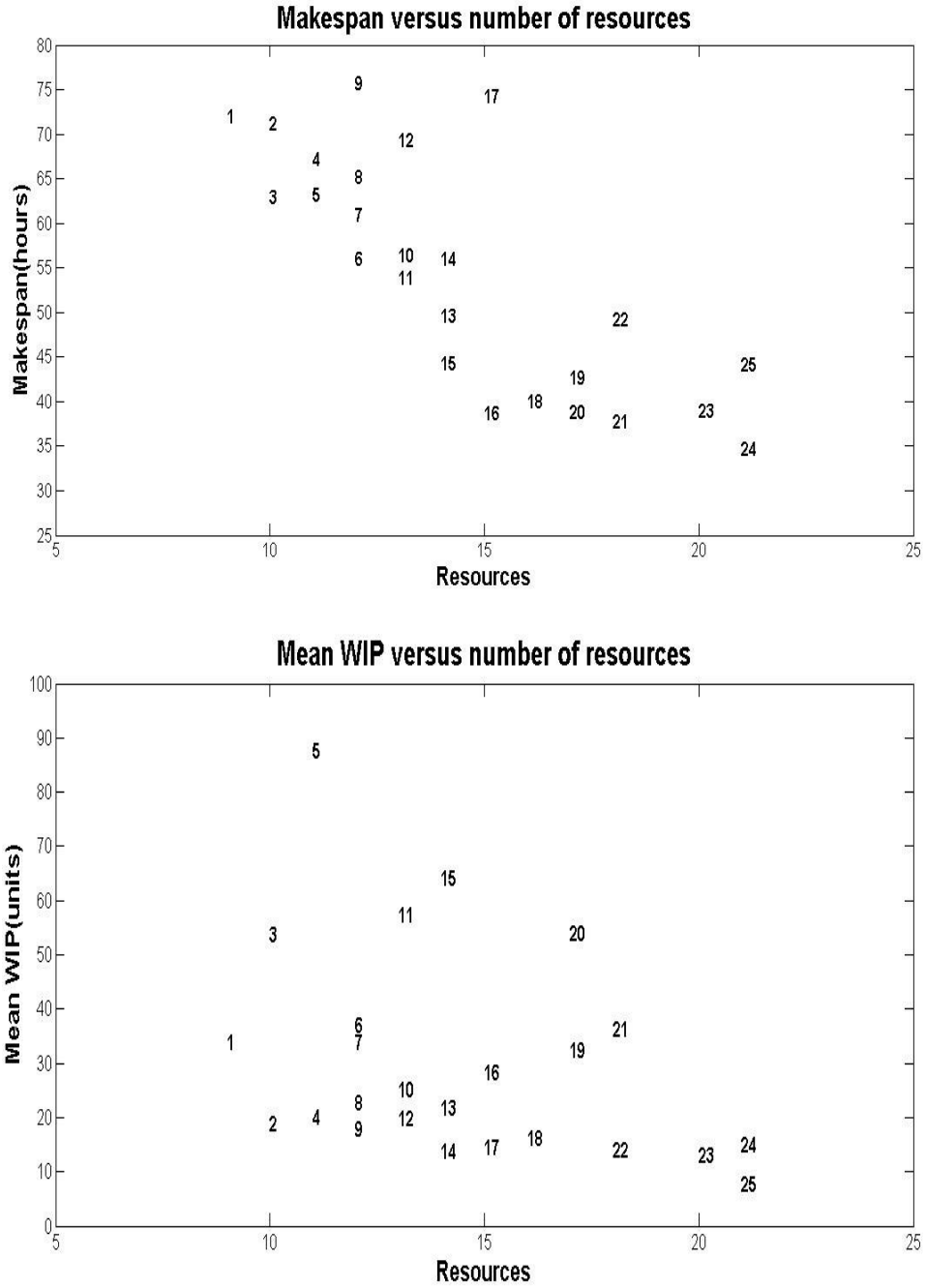

Fig. 5.2. Pareto front of candidate configurations

Table 5.6. Makespan and mean WIP details for each configuration

Configuration number	Configuration	Total number of Resources	Makespan (*hours)*		Mean WIP (*units)*	
			Mean	Std	Mean	Std
1	[2, 2, 3, 1, 1]	9	71.90	3.52	33.80	5.67
2	[3, 2, 2, 1, 2]	10	71.21	3.91	18.83	2.71
3	[2, 3, 3, 1, 1]	10	62.91	3.07	54.87	9.10
4	[4, 2, 2, 1, 2]	11	67.15	2.37	20.00	3.70
5	[3, 3, 2, 2, 1]	11	63.16	2.15	87.43	3.29
6	[3, 3, 3, 2, 1]	12	56.01	6.69	36.87	8.99
7	[2, 2, 4, 2, 2]	12	60.90	5.10	35.59	6.95
8	[3, 3, 2, 1, 3]	12	65.16	5.22	22.56	2.94
9	[3, 2, 4, 1, 2]	12	75.66	4.53	17.80	3.30
10	[4, 2, 3, 2, 2]	13	56.31	1.75	25.07	3.07
11	[3, 3, 3, 3, 1]	13	53.79	4.66	57.16	12.35
12	[4, 2, 4, 1, 2]	13	69.34	2.29	19.80	3.26
13	[3, 3, 3, 2, 3]	14	49.52	6.71	21.78	5.32
14	[4, 2, 4, 2, 2]	14	55.90	1.76	13.64	3.20
15	[5, 3, 3, 2, 1]	14	44.29	1.05	64.12	4.12
16	[4, 4, 3, 2, 2]	15	38.63	4.02	28.12	4.49
17	[5, 3, 4, 1, 2]	15	74.20	5.40	14.30	2.79
18	[4, 3, 5, 2, 2]	16	39.92	1.59	16.20	2.77
19	[3, 3, 4, 2, 5]	17	42.70	2.92	32.41	3.42
20	[3, 3, 3, 5, 3]	17	38.83	1.53	53.84	3.53
21	[3, 3, 5, 2, 5]	18	37.78	1.07	36.29	7.65
22	[5, 4, 3, 2, 4]	18	49.13	7.36	14.00	2.70
23	[5, 3, 4, 5, 3]	20	38.85	4.02	13.07	3.17
24	[4, 4, 5, 3, 5]	21	34.69	5.51	15.01	3.13
25	[5, 4, 5, 2, 5]	21	44.08	5.96	7.63	0.98

5.4.3. Effect of demand and product uncertainty

After a set of candidate configurations was generated using the NSGA-II algorithm, the next phase eliminated the configurations that exceeded budgetary constraints. At this stage, the designers assigned costs associated with makespan, WIP and number of resources. For this problem, resource, production and WIP costs were assumed to be $1,000/resource/planning horizon, $500/hr and $250/unit/planning horizon, respectively.

The maximum investment allowed in each planning horizon was assumed to be $65,000 and for installing more machines, an initial investment of $2,000/resource was required.

Based on the budgetary constraints, the feasible configurations were identified and analyzed with respect to demand variations. For each of the demand scenarios outlined in table 5.3, normal and gamma distributions were considered for BMA analysis. For gamma distribution, both α and β were assumed to be unknown and their uncertainties were represented using conjugate priors. For normal distribution, both μ and σ^2 were considered unknown and their uncertainties were represented using conjugate priors. The details about these priors are presented in Appendix A.

It was assumed that 200 "actual" data points and 50 "prior" data points for each demand scenario were available to the designers. The "actual" data points were used to compute the likelihood of the distribution model representing the demand, whereas "prior" data points were used to find the parameters of prior distributions. The BMA results gave a high probability (>0.95) for normal distribution. Hence in this case, normal distribution was chosen as the candidate distribution model. For each period, 25 demand scenarios were sampled from parts A, B and C demand distributions. The parameters for these demand distributions were sampled from posterior distributions of μ and σ^2. Based upon these demand scenarios, the performance of each configuration was evaluated.

For each planning period, the expected risk associated with each configuration was computed in addition to the overall expected cost. Eppen *et al.* (1989) suggested

Expected Downside Risk (EDR) criteria to compute the risk associated with each configuration. The Expected Downside Risk (EDR) is defined as:

$$\text{EDR}(Z)=E[f(\pi)]=\sum_{\pi=-\infty}^{\infty} \emptyset(\pi) f(\pi) \quad (5.3)$$

where:

- $f(\pi) = \begin{cases} \pi - Z, & \text{for } \pi > Z \\ 0, & \text{otherwise} \end{cases}$
- Z is expected cost with each configuration
- $\Phi(\pi)$ is the probability that excess cost $f(\pi)$ will occur

A smaller value of EDR is preferred over a larger one because decision makers are unhappy with these additional costs (Eppen *et al.*, 1989). It was assumed that each of the demand scenarios occur with the equal probability.

Table 5.7 shows the results obtained for top 5 configurations with demand variations. The highlighted cells indicate the best configuration for each period. In this case, configurations [3, 3, 3, 2, 3], [4, 4, 3, 2, 2] and [4, 3, 5, 2, 2] resulted in less overall costs as compared to other configurations. T-tests revealed that there was not a significant difference in costs obtained using configuration [4,3,5,2,2] and configuration [4,4,3,2,2]. In this case, configuration [4, 3, 5, 2, 2] was selected as a robust design as the EDR obtained using configuration [4, 4, 3, 2, 2] was not significantly lower than configuration [4, 3, 5, 2, 2].

Table 5.7. Expected costs and EDR for different configurations under demand uncertainties

			[3,3,3,2,3]	[4,4,3,2,2]	[4,3,5,2,2]	[3,3,4,2,5]	[4,2,3,2,2]
Base Period		Mean	$54,203	$53,345	$54,009	$62,452	$55,420
		Std	$2,965	$1,463	$744	$1,111	$1,534
Period 1	Total Cost	Mean	$40,730	$40,293	$39,035	$44,111	$44,396
		Std	$745	$1,088	$1,278	$1,307	$887
	EDR		$1,286	$1,861	$2,128	$3,313	$1,604
	Max		$11,936	$17,302	$20,298	$21,441	$9,928
Period 2	Total Cost	Mean	$37,143	$37,322	$36,064	$37,334	$37,071
		Std	$1,408	$1,317	$1,619	$1,070	$1,389
	EDR		$2,486	$3,479	$3,043	$2,200	$2,266
	Max		$20,651	$20,272	$22,537	$23,618	$20,422
Period 3	Total Cost	Mean	$32,940	$32,619	$31,388	$33,516	$35,060
		Std	$865	$844	$1,043	$568	$887
	EDR		$1,294	$1,233	$1,424	$1,003	$1,638
	Max		$17,445	$16,203	$21,438	$5,053	$13,997
Overall	Total Cost	Mean	$165,017	$163,578	$160,496	$177,413	$171,947
		Std	$3,475	$2,402	$2,428	$2,100	$2,420
	EDR		$5,067	$6,573	$6,596	$6,516	$5,508
	Max		$20,651	$20,272	$22,537	$23,618	$20,422
	Percentage difference		2.82%	1.92%	-	10.54%	7.13%

5.5. Comparison of robust and non robust design solutions

In section 5.4, a design configuration was obtained by considering uncertainties in part arrivals, processing times, machine failure and repairs, and demand uncertainties for future planning periods. These uncertainties were represented using Bayesian Model Averaging (BMA) that allowed consideration of *model, parameter* and *stochastic* uncertainties. This section analyzes the effect of ignoring *model*, *parameter* and demand uncertainties on the overall design decisions. The design solution obtained by ignoring *model*, *parameter, stochastic* and demand uncertainties will be referred as design approach "A" and the design approach ignoring *model*, *parameter* and demand

uncertainties will be referred as design approach "B". The design configurations obtained by considering design approaches "A" and "B" will be called "DC_A" and "DC_B".

Based on results in section 5.4, the design configuration "DC_A" was [4, 3, 5, 2, 2]. Table 5.8 summarizes the uncertainties considered in design approaches "A" and "B". The performance of design configurations "DC_A" and "DC_B" was compared on "true" distribution (Refer section 5.4) assumed for each uncertain factor.

Table 5.8. Uncertainties considered in design approaches "A" and "B"

Design approach "A"	Design approach "B"
• Uncertainties considered: Arrival rates, processing times, machine failure and repairs, product demand • Uncertainty representation using Bayesian Model Averaging (BMA) ✓ Stochastic uncertainty ✓ Parameter uncertainty ✓ Model uncertainty • Future planning horizon demand is stochastic	• Uncertainties considered: Arrival rates, processing times, machine failure and repairs, product demand • Uncertainty representation with Classical methods ✓ Stochastic uncertainty X Parameter uncertainty X Model uncertainty • Future planning horizon demand varies but fixed

For design approach "B", the uncertainties were represented using classical methods. Based on "actual" dataset for each uncertainty, distribution fitting and Chi square methods were used to find the distribution model. Table 5.9 shows the distribution models selected to represent uncertainty associated with each factor. The distribution models allowed consideration of *stochastic* uncertainty, but did not consider *model* and *parameter* uncertainties. The demand for part A, B and C in base period was 30, 50 and

20 units. For future planning periods, the demand for each part was assumed to be fixed. However, this demand varied from one period to another. This fixed demand for each planning period was obtained by considering mean of "actual" demand data.

Table 5.9. Distribution models used in design approach "B"

Factor	Distribution model used
Processing times	Normal
Failure times	Exponential
Repair times	Exponential
Inter-arrival times	Exponential
Part demand	-

NSGA-II multi-objective GA coupled with Petri net was used to generate a set of candidate configurations. Table 5.10 shows top 5 candidate configurations and overall costs for each planning period. The results show that configuration [3, 3, 4, 2, 5] resulted in lowest total cost. So, configuration [3, 3, 4, 2, 5] was selected as a design configuration "DC_B" with an overall total cost of $153,814.

Table 5.10. Candidate configurations obtained using design approach "B"

		[3,2,2,1,3]	[4,4,3,2,2]	[4,3,5,2,2]	[3,3,4,2,5]	[4,4,5,3,2]
Base Period	Mean	$54,144	$50,713	$54,227	$60,148	$57,470
	Std	$1,476	$794	$786	$969	$1,272
Period 1	Mean	$48,313	$39,280	$46,979	$34,803	$43,709
	Std	$1,060	$891	$805	$834	$426
Period 2	Mean	$36,412	$46,622	$45,807	$28,844	$39,521
	Std	$1,096	$1,459	$736	$492	$629
Period 3	Mean	$42,686	$45,177	$31,044	$30,019	$35,021
	Std	$598	$1,274	$279	$728	$552
Total Cost	**Mean**	**$181,555**	**$181,791**	**$178,057**	**$153,814**	**$175,721**
	Std	**$2,205**	**$2,275**	**$1,373**	**$1,552**	**$1,581**

The performance of design configurations "DC_A"([4, 3, 5, 2, 2]) and "DC_B" ([3, 3, 4, 2, 5]) were evaluated on the "true" distribution models assumed for each uncertainty. The details for "true" distribution models assumed for this problem can be found in section 5.4. Table 5.11 shows the final results obtained for design configurations "DC_A" and "DC_B". Since configurations [4, 3, 5, 2, 2] and [4, 4, 3, 2, 2] did not had a significant cost difference (see Table 5.8), the results for both these configurations are shown in Table 5.11. The results show that configuration obtained using design approach "A" provided lower costs as compared to design approach "B". These results demonstrate that stochastic nature of product demand and *model* and *parameter* uncertainties can have a significant impact on design decisions. The "robust" design approach accounting for such uncertainties lead to a better design decision.

Table 5.11. Comparison of design configuration "DC_A" and "DC_B" on "true" data

		[3,3,4,2,5]	[4,4,3,2,2]	[4,3,5,2,2]
Base Period	Mean	$67,336	$ 62,290	$54,986
	Std	$1,576	$1,565	$1,426
Period 1	Mean	$48,401	$44,807	$49,002
	Std	$1,879	$1,414	$1,532
Period 2	Mean	$41,533	$38,987	$44,066
	Std	$1,346	$1,494	$1,655
Period 3	Mean	$37,104	$37,013	$37,688
	Std	$1,265	$1,471	$992
Total Cost	**Mean**	**$194,374**	**$183,097**	**$185,742**
	Std	**$3,070**	**$2,973**	**$2,848**

5.6. Summary and conclusions

This chapter presented an NSGA-II multi-objective GA and Petri net based methodology for robust manufacturing system design. The manufacturing system example presented here contained uncertainties related to processing, failure, repair, and inter-arrival times. In addition, there was uncertainty in part demands for future planning horizons.

The objective function for the problem was formulated as a cost minimization function that was dependent upon makespan, mean WIP and number of resources used. In addition, there was an initial set up cost for installing new resources. In this problem, it was assumed that reconfiguration costs were significantly high, so the configurations do not change from one planning horizon to another.

The problem was formulated as a two step procedure in which the candidate configurations were first generated using NSGA-II algorithm. The configurations that did not meet budgetary constraints were eliminated and remaining configurations were analyzed with respect to future demand variations. The manufacturing system uncertainties were represented using a Bayesian approach that allowed consideration of *model, parameter* and *stochastic uncertainties.* Configuration [4, 3, 5, 2, 2] resulted in lower overall total costs and was selected as a "robust" design solution.

The effect of ignoring *model* and *parameter* uncertainties and demand variations on the final design decisions was evaluated. The final design configuration ([3, 3, 4, 2, 5]) obtained by not considering such uncertainties produced significantly higher total costs under "true" conditions as compared to the configuration ([4, 3, 5, 2, 2]) obtained by

using a design approach considering such uncertainties. This shows the impact of ignoring *model, parameter* and demand uncertainties on the final design decisions.

In the next chapter, the scope and contributions of the dissertation are presented.

CHAPTER VI

SUMMARY AND CONTRIBUTIONS

Manufacturing system design decisions are costly and involve significant investment in terms of allocation of resources. These decisions are complex due to uncertainties related to *uncontrollable factors* such as processing times, machine failures and part demands. The design decisions typically remain fixed for a planning horizon and failure to design a robust manufacturing system can lead to drastic results in terms of high production costs and lost sales. In order to make robust design decisions, the designers need accurate ways to capture uncertainties and efficient methods to model and search alternative design configurations. The design decisions become more complex when sufficient information about underlying uncertainties is not available.

The dissertation work considers an NSGA-II multi-objective GA and Petri net based approach for robust manufacturing system design. The uncertainties in a manufacturing system are modeled using a BMA based approach. The BMA approach provides a unified framework to incorporate *model, parameter and stochastic uncertainties* associated with representation of *uncontrollable factors.* The dissertation work presents new token game simulation algorithms for integrating BMA with Petri nets. Two manufacturing system design examples demonstrated the proposed approach. In the first example, the performance of a manufacturing system was compared against classical and Bayesian methods. Both full and partial Baye's methods were considered and the effect of choosing non-informative and conjugate priors on overall performance was

evaluated. Rejection sampling algorithm was used to sample parameters from non conjugate distributions. This algorithm did not require significant user intervention and helped in automating the simulation procedure. The final results revealed that BMA based approach provided better assessment of performance measures as compared to classical methods. The results obtained using full Baye's methods with conjugate priors were found to be better than full Baye's methods using non informative priors.

The second manufacturing system design problem contained uncertainties related to processing times, part arrivals, machine failures and repairs, and product demands. Full Baye's BMA with conjugate priors was used to represent such uncertainties. Although, the importance of considering processing time, part arrival, machine failure and repair uncertainties was not directly observable, the uncertainties in product demands showed how the effects of ignoring these uncertainties can lead to excessive costs in future planning horizons.

The dissertation work provides several key contributions to research in robust design and Petri net based uncertainty analysis. These contributions are summarized below:

- The dissertation provides a mechanism to incorporate *model, parameter* and *stochastic* uncertainties associated with representation of *uncontrollable factors* by using a BMA approach for robust manufacturing system design. This approach overcomes limitations associated with uncertainty representation in the approaches presented in literature (namely, classical methods with sensitivity analysis or factorial design based approaches). The approach not only allows the designers to incorporate information availability about uncertainties using prior

distributions, but also allows them to consider model uncertainties that are not considered in existing literature. It can be argued that the sensitivity analysis based approaches are a special case of BMA approach in which model uncertainties are not considered (Partial Baye's). Sometimes, the robust design decisions are required when sufficient information about such uncertainties is not available. The proponents of Bayesian methods have shown the limitations of classical methods under limited information. BMA based approach can aid in providing reliable estimates of performance measures under such circumstances. In addition, despite being a useful approach for uncertainty representation, there have been very few application examples that demonstrate the use of BMA methods in manufacturing systems. The dissertation work demonstrates an area where such methods can have a significant contribution.

- The dissertation provides new algorithms for integrating Bayesian methods with Petri nets. Over the years, numerous publications have demonstrated the use of Petri nets for modeling, analysis, control and monitoring of manufacturing systems. Uncertainty prevails in most manufacturing systems. Bayesian methods coupled with Petri nets provide a better ability to analyze and monitor such systems. The dissertation work demonstrates examples where Petri nets are used for modeling and performance analysis of different design configurations. The dissertation work provides new token game simulation algorithms which can be directly used to implement BMA based uncertainty representation. The users can employ powerful analytical and modeling capabilities of commercial Petri net

software such as STPNPLAYER® to design Petri net models, and then use the Petri net properties (for example, Incidence matrix) in new token game simulation algorithms to evaluate performance measures. The users will need to provide distribution models and prior information about transitions (or places for a TPPN). To overcome the limitation of Bayesian approach in non conjugate distributions, rejection sampling algorithms are used which aid in automating the sampling process from non conjugate distributions.

The dissertation work uses multi-objective GA to search for potential configurations. The multi-objective GA based approach provides designers flexibility in evaluating candidate designs against their relative merits and demerits. The approach also avoids the need to re-run the entire simulation process if the relative weights associated with each objective change. Signal to noise (S/N) ratios are used to consider both mean and variance components of performance measures. The multi-objective GA used the signal to noise (S/N) ratios to rank alternative design configurations.

The following extensions can be considered for the dissertation work:

- *Consideration of system reconfiguration*: The dissertation assumed that system reconfiguration costs were high, so reconfigurations were not allowed from one planning period to another. A potential extension of current work can be the consideration of cases where reconfigurations are allowed and understanding the impact of reconfigurations on design decisions.
- *Extension of Bayesian methods to colored Petri nets:* The dissertation used stochastic Petri nets for modeling manufacturing systems. Stochastic Petri nets

suffer from the stated space explosion problem and they are difficult to analyze for large and complex models. A possible extension for this work can be integration of Bayesian methods with colored Petri nets. Colored Petri nets are higher level nets that use colored tokens to simplify the structure of Petri nets. One implementation of these nets can be found in CPN tools developed by Jensen (1997). The CPN tools allow abstract and detailed level modeling with colored Petri nets. At abstract level, the designers can create the overall manufacturing system model and use modules to represent various entities within a system. At a detailed level, the designers can create operational details for each module. The integration of BMA based approach with CPN tools is difficult because of the limited distribution model support and limited support to write BMA algorithms. Algorithm development in software tools like MATLAB® is challenging due to difficulty in implementation of token game simulation for colored Petri nets. This difficulty arises because the transition firing rules and Petri net model properties will need to be manually created. Besides these limitations, the fundamentals behind transition firings in colored and stochastic Petri nets are similar and thus a future work in this direction can address BMA integration with colored Petri nets.

- *Extension to manufacturing system monitoring and control:* The dissertation mainly focused on using Petri nets and Bayesian methods for manufacturing system modeling and design. The proposed approach can be extended for manufacturing system monitoring, control and performance analysis. One

example of such an approach was presented by Sharda and Banerjee (2007). In this work, the information obtained from manufacturing system was used to update knowledge about existing processes using Baye's rule. This updated information can be used to find key performance measures such as throughput and mean WIP. Based upon performance measures obtained, supervisory actions can then be taken to adjust the performance to the desired level. For example, based upon information obtained from a manufacturing system, the supervisors can reallocate parts to other manufacturing cells in order to reduce congestion in the system.

REFERENCES

Al-Aomar, R. (2002) A robust simulation-based multicriteria optimization methodology, in *Proceedings of the 2002 Winter Simulation Conference: Exploring New Frontiers*, Yucesan, E., Chen, C.H., Snowdon, J.L. and Charnes, J.M. (eds.), San Diego, CA, pp. 1931-1939.

Al-Jaar, R.Y. and Desrochers, A.A. (1990) Petri nets in automation and manufacturing. *Advances in Automation and Robotics*, **2**, 153-225.

Andradóttir, S. and Bier, V.M. (2000) Applying Bayesian ideas in simulation. *Simulation Practice and Theory*, **8** (3-4), 253-280.

Billington, J., Christensen, S., Hee, K.V., Kindler, E., Kummer, O., Petrucci, L., Post, R., Stehno, C. and Weber, M. (2003) The Petri net markup language: Concepts, technology, and tools, in *International Conference on Applications and Theory of Petri Nets (ICATPN)*, Eindhoven, Netherlands.

Bulgak, A.A., Tarakci, Y. and Verter, V. (1999) Robust design of asynchronous flexible assembly systems. *International Journal of Production Research*, **37** (14), 3169-3184.

Chen, L.H. and Chen, Y.H. (1996) A design procedure for a robust job shop manufacturing system under a constraint using computer simulation experiments. *Computers and Industrial Engineering*, **30** (1), 1-12.

Cheng, R.C.H. (1994) Selecting input models, in *Proceedings of the 1994 Winter Simulation Conference*, Tew, J.D., Manivannan, S., Sadowski, D.A. and Seila, A.F. (eds.), Piscataway, NJ, pp. 184-191.

Cheng, R.C.H. and Holland, W. (1997) Sensitivity of computer simulation experiments to errors in input data. *Journal of Statistical Computation and Simulation*, **57**, 219-241.

Cheng, R.C.H. and Holland, W. (1998) Two-point methods for assessing variability in simulation output. *J. Statist. Comput. Simul.*, **60**, 183-205.

Cheng, R.C.H. and Holland, W. (2004) Calculation of confidence intervals for simulation output. *ACM Transactions on Modeling and Computer Simulation*, **14** (4), 344 - 362.

Chick, S.E. (1997) Bayesian analysis for simulation input and output, in *Proceedings of the 1997 Winter Simulation Conference*, Andradóttir, S., Healy, K.J., Withers, D.H. and Nelson, B.L. (eds.), Atlanta, GA, pp. 253-260.

Chick, S.E. (1999) Steps to implement Bayesian input distribution selection, in *Proceedings of the 1999 Winter Simulation Conference: Simulation-A Bridge to the Future*, Farrington, P.A., Nembhard, H.B., Sturrock, D.T. and Evans, G.W. (eds.), Phoenix, AZ, pp. 317-324.

Chick, S.E. (2001) Input distribution selection for simulation experiments: Accounting for input uncertainty. *Operations Research*, **49** (5), 744-758.

Coello, C.A. (2000) An update survey of GA-based Multiobjective optimization techniques. *ACM Computing Surveys*, **32** (2), 109-143.

Deb, K., Pratap, A., Agarwal, S. and Meyarivan, T. (2002) A fast and elitist multiobjective genetic algorithm: NSGA-II. *IEEE Transactions on Evolutionary Computation*, **6** (2), 182-197.

Draper, D. (1995) Assessment and propagation of model uncertainty. *Journal of Royal Statistical Society*, **45**, 45-97.

Eppen, G.D., Martin, R.K. and Schrage, L. (1989) A scenario approach to capacity planning. *Operations Research*, **37** (4), 517-527.

Fonseca, C.M. and Fleming, P.J. (1993) Genetic algorithms for multiobjective optimization: Formulation, discussion and generalization, in *Proceedings of the Fifth International Conference on Genetic Algorithms*, Forrest, S. (ed.), San Mateo, CA, pp. 416–423.

Gaury, E.G.A. and Kleijnen, J.P.C. (1998) Risk analysis of robust system design, in *Proceedings of the 1998 Winter Simulation Conference*, Medeiros, D.J., Watson, E.F., Carson, J.S. and Manivannan, M.S. (eds.), Washington, DC, pp. 1533 - 1540.

Gelman, A., Carlin, J.B., Stern, H.S. and Rubin, D.B. (2003) *Bayesian Data Analysis*, Chapman & Hall, London, UK.

Gilks, W.R., Richardson, S. and Spiegelhalter, D.J. (1995) *Markov Chain Monte Carlo in Practice*, Chapman & Hall, London, UK.

Hamza, K., Reyes-Luna, J.F. and Saitou, K. (2003) Simultaneous assembly planning and assembly system design using multi-objective genetic algorithms, in *Genetic and Evolutionary Computation – GECCO 2003*, Springer Berlin / Heidelberg, pp. 2096-2108.

Henderson, S.G. (2003) Input modeling: input model uncertainty: why do we care and what should we do about it?, in *Proceedings of the 2003 Winter Simulation Conference: Driving Innovation*, Chick, S., Sanchez, P.J., Ferrin, D. and Morrice, D.J. (eds.), New Orleans, LA, pp. 90-100.

Hoeting, J.A., Madigan, D., Raftery, A.E. and Volinsky, C.T. (1999) Bayesian Model Averaging: A tutorial. *Statistical Science*, **14** (4), 382-417.

Horn, J., Nafploitis, N. and Goldberg, D.E. (1994) A niched Pareto genetic algorithm for multiobjective optimization, in *Proceedings of the First IEEE Conference on Evolutionary Computation*, Piscataway, NJ, pp. 82–87.

Jensen, K. (1997) *Coloured Petri Nets: Basic Concepts, Analysis Methods and Practical Use*, Springer-Verlag, London, UK.

Kazancioglu, E. and Saitou, K. (2004) Multi-period robust capacity planning based on product and process simulations, in *Proceedings of the 2004 Winter Simulation Conference*, R.G. Ingalls, Rossetti, M.D., Smith, J.S. and Peters, B.A. (eds.), Washington, DC, pp. 1781-1789.

Kleijnen, J.P.C. and Gaury, E. (2003) Short term robustness of production management systems: a case study. *European Journal of Operational Research* **148**(2), 452-465.

Lee, H., Sharda, B. and Banerjee, A. (2007) Representation and simulation of stochastic Petri net models using XPNML, in *Proceedings of the 2007 Winter Simulation Conference*, S. G. Henderson, B. Biller, M.H. Hsieh, J. Shortle, J. D. Tew and Barton, R.R. (eds.), Washington, DC, pp. 1063-1071.

Lim, J.M., Kim, K.S., Yom, B.J. and Hwang, H. (1996) Determination of an optimal configuration of operating policies for direct-input-output manufacturing systems using the Taguchi method. *Computers and Industrial Engineering*, **31** (3-4), 555-560.

Madigan, D. and Raftery, A.E. (1994) Model selection and accounting for model uncertainty in graphical models using Occam's window. *Journal of the American Statistical Association*, **89**, 1535-1546.

Madu, I.E. and Madu, C.N. (1999) Design optimization using signal-to-noise ratio. *Simulation Practice and Theory*, **7** (4), 349-372.

Malloy, M.K. (1982) Performance analysis using stochastic Petri nets. *IEEE Transactions on Computers*, **C-31**, 913-917.

Mezgar, I., Egresits, C. and Monostori, L. (1997) Design and real-time reconfiguration of robust manufacturing systems by using design of experiments and artificial neural networks. *Computers in Industry*, **33** (1), 61-70.

Miller, R.R. (1980) Bayesian analysis of the two parameter gamma distribution. *Technometrics*, **22** (1), 65-69.

Murata, T. (1989) Petri nets: properties, analysis and applications. *Proceedings of the IEEE*, **77** (4), 541-80.

Ng, S.H. and Chick, S. (2001) Reducing input parameter uncertainties for simulations, in *Proceedings of the 2001 Winter Simulation Conference*, Peters, B.A., Smith, J.S., Medeiros, D.J. and Rohrer, M.W. (eds.), IEEE Computer Society, Arlington, VA, pp. 364-371.

Petri, C.A. (1962) Kommunikation mit Automaten, Ph.D. dissertation, University of Bonn, Bonn, Germany.

Pierreval, H. and Durieux-Paris, S. (2007) Robust simulation with a base environmental scenario. *European Journal of Operational Research*, **182** (2), 783-793.

Pierreval, H. and Durieux, S. (2002) A two-stage simulation optimization heuristic for robust design, in *Proceedings of the IEEE International Conference on Systems, Man and Cybernetics*, Institute of Electrical and Electronics Engineers Inc., Yasmine Hammamet, Tunisia, pp. 477-482.

Saitou, K., Malpathak, S. and Qvam, H. (2002) Robust design of flexible manufacturing systems using colored Petri net and genetic algorithm. *Journal of Intelligent Manufacturing*, **13** (5), 339-351.

Sanchez, S.M., Sanchez, P.J., Ramberg, J.S. and Moeeni, F. (1996) Effective engineering design through simulation. *International Transactions in Operational Research*, **3** (2), 169-185.

Shang, J.S. (1995) Robust design optimization of material handling resource in an FMS. *International Journal of Production Research*, **33** (2), 2437-2454.

Sharda, B. and Banerjee, A. (2007) Bayesian Petri net based framework for performance analysis and control of manufacturing systems, in *Proceedings of the 2007 Industrial Engineering Research Conference (IERC 2007)*, Nashville, TN, (Available in CD-ROM).

Srinivas, N. and Deb, K. (1995) Multi-objective function optimization using nondominated sorting genetic algorithms. *Evolutionary Computation*, **2** (3), 221-248.

Taguchi, G. (1986) *Introduction to Quality Engineering*, UNIPUB/Krauss International, White Plains, NY.

Taguchi, G. (1987) *System of Experimental Design*, UNIPUB/Krauss International, White Plains, NY.

Tsai, C. S. (2002) Evaluation and optimization of integrated manufacturing system operations using Taguchi's experiment design in computer simulation. *Computers and Industrial Engineering*, **43**, 591-604.

Zhang, F., Roundy, R., Cakanyildirim, M. and Huh, W.T. (2004) Optimal capacity expansion for multi-product, multi-machine manufacturing systems with stochastic demand. *IIE Transactions*, **36**, 23-36.

Zhou, M. and Venkatesh, K. (1999) *Modeling, Simulation, and Control of Flexible Manufacturing Systems—A Petri Net Approach*, World Scientific Publishing Company, River Edge, NJ.

Zouaoui, F. (2001) Accounting for input uncertainty in discrete event simulation, Ph.D. dissertation, North Carolina State University, Raleigh, NC.

Zouaoui, F. and Wilson, J.R. (2001a) Accounting for parameter uncertainty in simulation input modeling, in *Proceedings of the 2001 Winter Simulation Conference*, Peters, B.A., Smith, J.S., Rohrer, M.W. and Medeiros, D.J. (eds.), IEEE Computer Society, Arlington, VA, pp. 354-363.

Zouaoui, F. and Wilson, J.R. (2001b) Accounting for input model and parameter uncertainty in simulation, in *Proceedings of the 2001 Winter Simulation Conference*,

Peters, B.A., Smith, J.S., Rohrer, M.W. and Medeiros, D.J. (eds.), IEEE Computer Society, Arlington, VA, pp. 290-299.

Zouaoui, F. and Wilson, J.R. (2004) Accounting for input-model and input-parameter uncertainties in simulation. *IIE Transactions*, **36** (11), 1135-1151.

APPENDIX A

BAYESIAN ANALYSIS OF COMMON DISTRIBUTIONS

Let, $X=x_1, x_2,\ldots.., x_N$ be the actual dataset of size (N) and $Z=z_1, z_2,\ldots.., z_T$ be the prior dataset of size (T). The prior dataset is used to compute the parameters of prior distributions and the actual dataset is considered as "known" information about an *uncontrollable factor.*

Let μ_x, σ_x be the mean and standard deviation of the actual dataset and μ_z, σ_z be the mean and standard deviation of the prior dataset.

1.1. Exponential Distribution

The likelihood for exponential distribution is defined as:

$$f(X|\lambda) = \lambda^n e^{-\lambda \sum_{i=1}^{N} x_i}, \lambda > 0.$$

1.1.1. Conjugate prior

The conjugate prior for exponential distribution is a gamma distribution (Gelman *et al.*, 2003), $\pi(\lambda|\alpha,\beta) = \text{Gamma}(\alpha,\beta),\ \alpha,\beta > 0$. The parameters for this prior can be found by moment matching mean and variance estimates of prior dataset (Z) with gamma distribution mean ($\alpha\beta$) and variance ($\alpha\beta^2$). The details of this approach can be found in Chick (2001). The resulting posterior distribution is a gamma distribution, $f(\alpha,\beta|X) = \text{Gamma}(\alpha + N, \beta + \sum_{i=1}^{N} x_i)$. The marginal distribution of X is defined as:

$$f(X) = \frac{\Gamma(\alpha+N).\beta^{\alpha}}{\Gamma(\alpha).(\beta+\sum_{i=1}^{N} x_i)^{\alpha+N}}$$

1.1.2. Non informative prior

The non-informative prior for exponential distribution parameter λ is defined as $\pi(\lambda|\alpha,\beta) = \lambda^{-1}$ (Zouaoui and Wilson, 2004). Since this prior is improper, it can be made proper using the prior data (*Z*). The resulting posterior is a gamma distribution with $\pi(\lambda|\alpha,\beta) = \text{Gamma}(T, \sum_{i=1}^{T} z_i)$. The posterior (proper) distribution in this case is again a gamma distribution with $f(\alpha,\beta|X) = \text{Gamma}(N, \sum_{i=1}^{N} x_i)$. The marginal distribution of data *X* is defined as:

$$f(X) = \frac{\Gamma(T+N).(\sum_{i=1}^{T} z_i)^{T}}{\Gamma(T).(\sum_{i=1}^{T} z_i + \sum_{i=1}^{N} x_i)^{N+T}}$$

1.2. Gamma Distribution

The likelihood of gamma distribution is defined as:

$$f(X|\alpha,\beta) = \left[\frac{\beta^{\alpha}}{\Gamma(\alpha)}\right]^{n} (\prod_{i=1}^{N} x_i^{\propto -1}) e^{-\beta(\sum_{i=1}^{N} x_i)},\ \alpha,\beta > 0$$

1.2.1. α known and β unknown (Zouaoui and Wilson, 2004):

In this case, the non-informative prior distribution for β is defined as, $\pi(\beta) \sim 1/\beta$. Since this prior is improper, it can be made proper by using prior data (*Z*). The resulting prior distribution is a gamma distribution, $\pi(\beta|\alpha) = \text{Gamma}(T\alpha, \frac{1}{\sum_{i=1}^{T} z_i})$. The value of α is obtained from training set (*Z*) using $\alpha = \frac{\mu_z^2}{\sigma_z^2}$. The marginal distribution of *X* is:

$$f(X) = \frac{\Gamma(N\alpha+T\alpha)}{\Gamma(T\alpha)\Gamma(\alpha)^{N}} \frac{(\prod_{i=1}^{N} x_i)^{\alpha-1} (\prod_{i=1}^{T} z_i)^{T\alpha}}{(\sum_{i=1}^{N} x_i + \sum_{i=1}^{T} z_i)^{N\alpha+T\alpha}}$$

The posterior distribution of β in this case is a gamma distribution, $f(\beta|\alpha', X) \sim \text{Gamma}\ (T\alpha', 1/\sum_{i=1}^{n} x_i)$, where $\alpha' = \frac{\mu_x^2}{\sigma_x^2}$.

1.2.2. Case 2: Conjugate prior (Miller, 1980)

The conjugate prior distribution for gamma distribution is defined as:

$$f(\alpha,\beta)\sim\frac{\beta^{\nu'\alpha-1}(p')^{\alpha-1}e^{-\beta s'}}{[\Gamma(\alpha)]^{\eta'}},\text{where }\alpha>0,\beta>0,\eta'>0,p'>0,s'>0\ ,$$

Such that $\eta'^{n'}\sqrt{p'/_{s'}}$<1, $s'=\sum_{i=1}^{T}z_i\ ,p'=\prod_{i=1}^{T}z_i$

The posterior distribution using such a prior is:

$$f(\alpha,\beta|X)\sim\frac{\beta^{\nu''\alpha-1}(p'')^{\alpha-1}e^{-\beta s''}}{[\Gamma(\alpha)]^{\eta''}}$$

Where, $s=\sum_{i=1}^{N}x_i\ ,p=\prod_{i=1}^{N}x_i,\ \eta''=\eta'+N,p''=p'p,s''=s'+s$

The marginal distributions for α and β are defined as:

$$f(\beta|\alpha)\sim\text{Gamma}(v''\alpha,s'')\text{ and }f(\alpha)\sim\frac{\Gamma(v''\alpha)(\sqrt[v'']{p''/s''})^{v''\alpha}}{[\Gamma(\alpha)]^{\eta''}}$$

In this case, α needs to be sampled from its marginal distribution $f(\alpha)$ and then β can be computed from $f(\beta|\alpha)$. To sample α from its marginal distribution, methods such as rejection sampling or Markov Chain Monte Carlo (MCMC) can be used. In the dissertation, rejection sampling is used because it is easy to implement and its performance does not depend upon choice of proposal distribution, starting points and proposal variances as do MCMC based methods. The algorithm used for rejection sampling is as follows (Gelman *et al.*, 2003):

a) Sample α from the proposal density $g(\alpha)$

b) With probability $f(\alpha)/(M(g(\alpha))$, accept α as a draw from $f(\alpha)$. If the draw is rejected, return to step a.

For this case, uniform distribution was considered as a proposal density g(α). The parameters for uniform distribution were set by sampling points from prior distribution $f(\alpha)$ and then observing the minimum and maximum values. Based upon the minimum and maximum values, the parameters for uniform proposal density were assigned. Once α is sampled from its marginal distribution, sampling β from $f(\beta \mid \alpha)$ is trivial. Based upon α and β values, the marginal distribution $f(X)$ can be easily computed.

1.3. Normal Distribution:

The likelihood function for normal distribution is defined as:

$$f(X,\mu,\sigma^2) \sim \left[\frac{1}{\sqrt{2\pi}\sigma}\right]^N e^{-\frac{\sum_{i=1}^{N}(x_i-\mu)^2}{2\sigma^2}} \text{ where } -\infty \le \mu \le \infty \text{ and } \sigma > 0$$

1.3.1. Both μ and σ^2 unknown, $\tau = 1/\sigma^2$ is the precision parameter

In this case, the prior distributions for μ and τ are defined as:

$$f(\mu|\tau) \sim \text{Normal}(\mu_0, \tau^{-1}\sigma_0^2) \text{ and } \tau \sim \text{Gamma}\left(\frac{\delta_0}{2}, \frac{\gamma_0}{2}\right)$$

Let, $a = \frac{(\sum_{i=1}^{N} x_i + \mu_0/\sigma_0^2)}{(n+\frac{1}{\sigma_0^2})}, c = n + \frac{1}{\sigma_0^2}$ and $b = \gamma_0 + \frac{\mu_0^2}{\sigma_0^2} + \sum_{i=1}^{N} x_i^2$

The marginal distribution of μ, $f(\mu|\ \tau, X)$ is a *t*-distribution with location parameter *a,* dispersion parameter $\left(\frac{(n+\delta_0)c}{b-ca^2}\right)$ and $n + \delta_0$ degree of freedom. The distribution of τ is given as:

$$f(\tau|X) \sim \text{Gamma}\left(\frac{n+\delta_0}{2}, \frac{1}{2}(b-ca^2)\right)$$

The marginal distribution of X, $f(X)$ is defined as:

$$f(X)=\left((2\pi)^{-n/2}\sigma_0^{-1}\frac{(\frac{\gamma_0}{2})^{\frac{\delta_0}{2}}}{\Gamma(\frac{\delta_0}{2})}\right)\left(\frac{\Gamma\left(\frac{n+\gamma_0}{2}\right)(2^{\frac{n+\delta_0}{2}})(n+\frac{1}{\sigma_0^2})^{-1/2}}{\left[b-\frac{(\sum_{i=1}^{N}x_i+\mu_0/\sigma_0^2)^2}{(n+\frac{1}{\sigma_0^2})}\right]^{\frac{n+\delta_0}{2}}}\right)$$

*1.3.2. Standard non- informative prior (*Zouaoui*, 2001)*:

The standard non-informative priors for parameters μ and σ^2 of normal distribution are defined as:

$$\pi(\mu,\sigma^2)\sim\frac{1}{\sigma^2}$$

In this case, the posterior distributions for μ and σ^2 are defined as:

$$f(\sigma^2|Z)\sim\text{Inverse Gamma}\left(\frac{T-1}{2},\frac{\sum_{i=1}^{T}(z_i-\mu_z)^2}{2}\right)$$

$$f(\mu|Z)\sim\frac{\Gamma\left(\frac{T}{2}\right)\sqrt{T}}{\Gamma\left(\frac{T-1}{2}\right)\sigma_z\sqrt{(T-1)\pi}}\left(1+\frac{T}{T-1}\left(\frac{\mu-\mu_z}{\sigma_z}\right)^2\right)^{(T-1)/2}$$

The marginal distribution of (*X*) is given as:

$$f(X)=\frac{\pi^{-\frac{n}{2}}(T-1)^{\frac{T-1}{2}}\Gamma\left(\frac{N+T-1}{2}\right)\sigma_X^{(T-1)/2}}{(T+N)^{-\frac{1}{2}}(N+T-1)^{(N+T-1)/2}\Gamma\left(\frac{T-1}{2}\right)\sigma_o^{(N+T-1)/2}}$$

Where, $\sigma_0=\frac{T-1}{N+T-1}\sigma_z^2+\frac{N-1}{N+T-1}\sigma_x^2+\frac{NT}{(N+T-1)(N+T)}(\mu_x-\mu_z)^2$

The analysis of log-normal distribution is similar to normal distribution, except *X=log(X)* and *Z=log(Z).* The marginal distribution of lognormal distribution is defined as (Zouaoui and Wilson, 2004):

$$f(X) = \frac{\pi^{-\frac{n}{2}}(T-1)^{\frac{T-1}{2}}\Gamma\left(\frac{N+T-1}{2}\right)\sigma_X^{(T-1)/2}}{\left(\prod_{i=1}^{N} x_i\right)(T+N)^{-\frac{1}{2}}(N+T-1)^{(N+T-1)/2}\Gamma\left(\frac{T-1}{2}\right)\sigma_o^{(N+T-1)/2}}$$

VITA

Bikram Sharda received his Bachelor of Engineering (B.E.) degree from Thapar Institute, Patiala, India, in 1999 and Master of Science (M.S.) degree from Texas A&M University, College Station, TX, USA, in 2003. Before joining Texas A&M University in August 2001, he worked as an industrial engineer in Swaraj Engines Limited, India, from July 1999 until July 2001. His research interests are in the areas of simulation modeling, Petri nets, heuristic optimization and Bayesian methods. His permanent address is 2048/5 Guru Teg Bahadur Nagar, Patiala, Punjab, India 147001.