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ABSTRACT

Agricultural production and associated economic effects of irrigation on the Texas High Plains are seriously threatened by a rapidly declining groundwater supply and a swift upward trend in energy costs.

To optimize the amount of irrigation water to be applied during specified periods of the production process, a stochastic open-loop feedback control policy was built into a grain sorghum growth simulation model. The control policy operated under the basis of constant revision of the expectations generated at every starting point for each of the production periods. If discrepancies between the expected and the realized values existed, then, based on current conditions a reevaluation of the control variable, irrigation water, was made and the decision for the first period adopted. This process continued throughout each period of the growing season. Within the stochastic policy designed, the values for the control variable were obtained by numerical search.

The model was applied to estimate optimal irrigation strategies and the impact of fuel curtailments on them. Initially, optimal irrigation strategies were developed under the assumption of perfect knowledge. Under this assumption, the results indicated there was not a unique strategy to be applied at all times. The quantities of irrigation water to apply at each period depended on the initial or starting conditions. Since one of the purposes of building the

model was to make it perform under stochastic or real world conditions, the assumption of complete knowledge was relaxed to consider the case where the climatic environment was unknown. As in the deterministic case, the optimal amounts of irrigation water, by period. It was also observed, that with the open-loop feedback control, the results obtained for yields did not differ substantially from those obtained in the perfect knowledge case. The discrepancies among the two cases were primarily in the optimal amount of water applied and therefore in net returns. In the stochastic case, the use of irrigation water had a mean value approximately 25 percent more than in the case of perfect knowledge.

The effect of a fuel or irrigation curtailment was estimated for alternative time spans. When curtailments had a length of 10 days, there were no perceptible changes in the amount of net returns or yields, as compared to the no-curtailment case. The implication drawn was that by having frequent irrigation periods and applying optimal amounts of water, the adverse effects of 10-day curtailment periods were buffered.

The cases of twenty and thirty-day periods were found to have highly negative effects on the outcomes, especially net revenues, which decreased about 50 percent (from $99 to $50) in the curtailment case of 40 - 70 days after plant emergence compared to the no-curtailment value. The effects were not only on a decreased amount of returns perceived but also on an increased spectrum of relative fluctuation (from 18 percent to 68 percent for the same situations mentioned above).

It was also found that for the same time-span type of curtailments the effects were conditioned to the period in which they occurred. However, the 20 or 30 day curtailment period might be applicable to much shorter actual fuel curtailment periods. Producers lose not only the time of fuel curtailments, but also, they must cover many acres with a limited number of wells. As a result, a 10-day fuel curtailment could easily result in a 20 to 30 day delayed irrigation.

To summarize, improved irrigation distribution technology could result in increased yields and less irrigation water by simply having very close control on timing and quantity of water applied.

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CHAPTER I

INTRODUCTION

Since 1939 the development of irrigation has expanded the agricultural output of several regions in Texas. This expansion has been reflected in the increased volume of production in established farming areas as well as in the incorporation of regions not previously under cultivation. At the present time, from the different agricultural regions of Texas, more than 60 percent of the cash value of crops is generated on irrigated land (Knutson, et al.).

The High Plains constitutes the largest irrigated area of the state. Much of the economic growth in this region is associated with the introduction and development of irrigation on crops such as cotton, wheat and grain sorghum. The latter is the principal irrigated crop grown in the High Plains, constituting about 2 million acres or 36 percent of the total irrigated acreage (Shipley and Regier).

Besides the increase in production, irrigation on the High Plains has generated an increased demand for fertilizer, labor, chemicals for insect and weed control, steel, petroleum products and agricultural market facilities (Lacewell, Jones and Osborn).

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[a]The basis of this report is a dissertation by Dr. Luis Zavaleta. The research spanned 1976-78 and was conducted through the Texas Water Resources Institute by the Department of Agricultural Economics.

this interdependence indicates irrigation has a significant impact not only on the regional but also on the state economy.

The development of irrigation on the High Plains, however, has brought with it the threat of the economic depletion of the Ogallala aquifer. This aquifer constitutes the source for the irrigation water used in the region. The foreseen economic drainage is due to the physical characteristics of the underground source. The High Plains is essentially a plateau, hydrologically isolated by erosion from external sources of underground water recharge. Consequently, moisture precipitated on the land surface, for all practical purposes, is the only source of recharge (Harman). Annual precipitation for the region is approximately 18 inches with maximum precipitation usually occurring during May, June and July (U.S. Dept. of Commerce). The average recharge rate oscillates between 280 and 933 thousand acre feet per year (Lacewell and Pearce) while annual withdrawals are estimated to be between 3 and 5 million acre feet in the Northern and Southern High Plains, respectively (Hughes and Harman; Mapp et al.). Since natural recharge constitutes only a small percentage of the use rate, a significant net withdrawal of water from the underground basin is occurring. The average decline in the static water level varied between a minimum of 0.02 feet to a maximum of 1.96 feet for the 1977-1978 season, depending upon the specific area. These declines amount to an average drop of 19 feet in the water table for the 10 year period between 1969-1978 (High Plains

Water Conservation District). Although physical exhaustion of the stocks of groundwater is not foreseeable, water table levels may become depleted to such an extent that the economics of continuous irrigation is not feasible. This has important implications for the economy of the region which is primarily supported by the agricultural sector.

Two important factors causing increasing costs of production in this region are the declining water table and increasing energy costs. The declining water table emphasizes the need for more effective irrigation strategies.

Considering the energy issue, prior to 1973 a virtually unrestricted amount of natural gas was available at relatively low prices. Albeit the price for intrastate gas in Texas has risen rapidly in the last five years, supply has remained in adequate amounts and curtailments of fuel have not been a problem. Recent energy policy, however, may affect fuel supplies in Texas since federal authorities may reallocate natural gas for emergency use to other regions; therefore, the possibility and impact of curtailments must be recognized.

Farming under conditions of low and unstable rainfall is characterized by a large degree of risk and uncertainty. The magnitude of such risk and uncertainty is increased if, as a result of external causes, an energy shortage is also considered. With rainfall variability in timing and quantity and uncertainty associated with fuel curtailment, farmers react differently than where a complete knowledge of the environment exists. This results in altered production practices that depart from the optimal deterministic input-output combinations.

Because of (1) the high percentage of grain sorghum in the total irrigated acreage of the region, (2) the inefficiencies in applying sufficient quantities of water at the right times in each season--involving, in the past, a decision-making method based on a trial and error process--and (3) the limited quantity of groundwater available for irrigation, there is a large incentive to develop better irrigation strategies for grain sorghum. Therefore, there is a serious need to (1) estimate optimum irrigation strategies (timing and quantity) given alternative crop and fuel prices and subject to variability and risk associated with rainfall and (2) project expected effects of stochastic irrigation curtailments.

Objectives

The overall objective of this study is to estimate optimal or profit maximizing strategies of resource application--particularly, irrigation water--for grain sorghum production in a stochastic or real world environment. Specific objectives are to:

1. Modify and extend a computer grain sorghum growth model to estimate approximately optimal irrigation patterns based on an economic decision criterion.
2. Apply the grain sorghum growth model to estimate optimal

scheduling and application rates for irrigation on grain sorghum with stochastic weather patterns and alternative product and water price levels.

3. Estimate the expected impact of curtailments of irrigation water in selected periods on crop yield and producer's net returns.

Related Research and Literature

Sprinkler and gravity flow or furrow methods of irrigation are both employed on the Texas High Plains depending on soil type, slope and availability of irrigation water. Furrow irrigation predominates in the hardlands, whereas sandylands tend to dictate the use of sprinklers. The number of irrigations depends on the rainfall distribution pattern and groundwater availability. Three or four irrigations on grain sorghum are normally applied to meet the water use requirements during the growing season, but they may vary from only one preplant watering to as many as five or six postplant waterings (Shipley and Regier). In areas with very limited or no groundwater, grain sorghum is produced with no irrigation.

Jensen and Musick indicate that 22 to 24 inches of effective water in the root zone are required for high yields of grain sorghum in the Southern Great Plains area. They concluded that for the High Plains, water requirements to assure high yields of grain sorghum include typically the following: first irrigation about 3 to 5 weeks after planting; second irrigation at boot to flowering stage; and third irrigation during grain development.

Swanson and Thaxton concluded that "stretching" the irrigation supply over more acres may guarantee a crop; but unless rainfall in conjunction with limited irrigation actually meets the optimum water requirement of the crop, a reduced return per acre-inch of water may result; i.e., less grain across the entire farm from a specified quantity of water may result.

In an experiment conducted at the North Plains Research Field Station, a study was conducted to test the extent to which periods of moisture depletion and high variability of available moisture affect the yield response of irrigated grain (Shipley, Regier and Wehrly). The study was done on a Pullman silty clay loam soil and used electrical resistance blocks to measure available soil moisture. A predetermined quantity of irrigation water was applied to each treatment when a specified level of soil moisture depletion at the 12-inch depth had been reached. Data were analyzed by regression techniques to obtain a generalized mathematical expression of the yield relationship. The relationship between soil moisture depletion level and yield of grain sorghum indicates a curvilinear, diminishing returns pattern. From the estimated equation, maximum yield is obtained when irrigations are scheduled at approximately 35 percent soil moisture depletion at the one-foot depth.

In summary, the results of water use studies emphasize the need for coordinating irrigation with water requirements of the plant during critical stages of development that affect grain production. The yield response attributed to a specific irrigation depends upon several factors, including (1) amount of soil moisture available at time of irrigation or amount of rainfall immediately following irrigation, (2) stage of plant development, where moisture requirements and the effects of moisture stress differ, and (3) interaction effect from previous or subsequent irrigations, or both, which reduce or eliminate moisture stress conditions.

Importance of irrigation in the region has been estimated in several studies. For example, Casey, Jones and Lacewell estimated the regional effects on output and producer net returns for varying levels of fuel restriction by means of a profit maximizing linear programming model and the use of parametric procedures. In all cases an arbitrary shortage at a fixed fuel price was imposed. The effect would be primarily a reduction in irrigated acres and producer net returns. Due to this result, the importance and critical role of fuel to irrigated agriculture in the Texas High Plains has been emphasized. The authors also mention the different alternatives that farmers follow according to when the shortages of natural gas happen; i.e., by first reducing irrigation on grain sorghum before making adjustments in cotton production practices. A serious limitation of the Casey, Jones and Lacewell study is that the producer knew, before planting, when and for how long the irrigation water curtailment would last. Thus, the producer planned his production based on this information. By using the grain sorghum growth model this limitation will be eliminated and curtailments will be in an unexpected and stochastic framework.

Lacewell indicated that short run adjustments due to price increases in natural gas would not be significant. In a later study, Condra and Lacewell concluded increases in the price of natural gas above $2.00 in the long run will result in reductions in irrigated acreage and crop output on the Texas High Plains.

Condra, Lacewell, Sprott and Adams used a linear programming technique to allocate a given quantity of land, water and other inputs so as to maximize net returns to the producer in a subregion of the Texas High Plains. A short run demand for irrigation water was found to exceed the long run demand at all prices of water above the $11.08 per acre-foot. Natural gas price increases had no effect on irrigated acreage levels, water usage, or cropping patterns under conditions of average crop prices until the price of natural gas reached $2.15 per thousand cubic feet. At that price, derived demand for irrigation water was severely reduced. They concluded in the short run, effects of natural gas price increases will be reflected only in reduced net returns, but an abrupt shift to reduced irrigated acreages and water usage will occur as producers adjust to long run conditions.

In most of these studies, static growing conditions and deterministic environments were assumed. In addition, energy shortages were considered to be known at the beginning of the growing season. These assumptions constitute a severe limitation to the studies and the conclusions drawn from them.

Alternative approaches for considering production processes in stochastic environments have been addressed in the economic literature. The concept and treatment of risk and uncertainty, however, have varied throughout the years. Classical authors like Smith, Say, von Thünen, Mill and Marshall dealt with them - without a clear distinction - as being an explanation for the size of profits in different industries, the choice of occupations, and as a reward to entrepreneurial activities. It was not until 1921 with Knight's seminal work that the distinction among these two concepts was established.

It is occasionally found in the literature related to stochastic environments the terms risk and uncertainty are sometimes used synonymously (Magnuson; Sandmo). In this study, however, the approach followed by Knight, Malinvaud and Nemhauser among others, will be adopted - the distinction among the two concepts is based on the existing knowledge about the probability of occurrence of the events. At the lowest level of knowledge, when there is a complete ignorance about the probability distribution, the case is defined as one of decision making under uncertainty. At the other extreme, in the presence of complete knowledge about the probability distribution, the case is considered as one where the decision making process is under the influence of risk, sometimes known as probabilistic, stochastic or risky decision making (Nemhauser). It should be realized that in the whole spectrum from certainty to uncertainty the assumption kept is that, functionally, there is a single decision maker and the amount of risk decreases as the production process evolves.

As mentioned above, different methodologies have been suggested to deal with production processes affected by stochastic elements. Game theory approaches have been used as a way to face problems where uncertainty exists in competitive situations. Much of the research on this subject has involved two-person zero-sum games. The primary objective is to develop a rational criteria for selecting a strategy under the assumption that both players are rational and each will try to do as well as possible, relative to his opponent (cf. Hillier and Lieberman, ch. 7; Agrawal and Heady, ch. 6). Decision theory, in contrast, assumes the decision maker plays against a passive opponent, nature, for which its strategies are given in a random fashion. In these contexts, several criteria or methods have been developed. Wald's Maximin criterion involves the selection of the maximum of the minimum outcomes which might be realized by the decision maker. Laplace's criterion is based on the principle of insufficient reason. The argument for its use being the decision maker should consider the outcomes from the various strategies of nature or the opposing player to be equally likely to occur.

While Wald's criteria may be suitable for the case of a maximum security level decision maker, it is not best suited for games with nature since it assigns a probability of one to the worst outcome for a given strategy. In the latter cases, Laplace's criterion is better qualified than Wald's but has the disadvantage of choosing the strategy with highest expected value disregarding its variance.

Alternative criteria have been proposed by Hurwicz and Savage. Hurwicz' index of optimism-pessimism overcomes the limitations of the previous two models. It amounts to maximizing the weighted average of the highest and lowest payoffs associated with each of the strategies. In contrast to the Wald and Laplace approach, extreme values are not neglected in the first case and supposes the decision maker may study a bit, assess available information and make some judgements about the other player's--or nature's--strategies that is more likely, objective or subjectively, to occur than others. Savage proposed to minimize the amount of disappointment or regret caused by large losses or large foregone gains. This criterion, though in many cases gives more sensible results than Wald's criterion, is subject to questioning in its assumption of minimizing the economic agent's regret.

In connection with sequential decisions, the developments in the area of statistical decision theory have been most influential. The so-called minimax decision rule is "a principle for laying down a decision rule which minimizes the 'suprema' or 'least upper bounds' of a risk function, which is defined as the expected value of loss (plus the cost of experiment or survey), where the loss is associated with the choice of a decision under a given distribution function of the random variables."[1] Though utility theory was cre-

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[1] Magnusson, p. 40. Also, a fine discussion and geometric interpretation of decision making theory can be found in Halter and Dean.

ated as a pillar for game theory, the former can stand apart and has applicability in other context (Luce and Raifa, ch. 2). In the applicability of utility theory, however, the essential aspect if to fit the elicited utility points satisfactorily over the relevant range of gains and losses. For this task, a variety of different functional forms may be utilized; i.e., polynomial, logarithmic or exponential.

In cases where utility depends on a single attribute, the utility function may be specified as an expected value in terms of (1) the n moments of the probability distribution of the stochastic variable, the first four being: mean, variance, skewness and Kurtosis and (2) the first n derivatives of the utility function. This approximation is better for distributions with relative small variances and improves with the number of "information" or terms included. The number of terms necessary to retain depends on the type of problem at hand, and the closeness of the outcomes (Anderson, Dillon and Hardaker). The utility function type of approach is the most used technique of analysis of the theory of the firm in stochastic environments. Specifically, utility functions defined by their first and second moments are the ones commonly used to account for risk.[2]

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[2] Markowitz cites six measures of risk:
(1) the standard deviation
(2) the semi-variance
(3) the expected value of loss
(4) the expected absolute deviation
(5) the probability of loss
(6) the maximum loss (In Magnusson, p. 26).

Albeit the long recognition of the existence of risk, the major contributions to the theory of economic behavior with unknown conditions had not been made until recent years. Magnusson's (1969) pioneer work extended the neo-classical production theory to account for risk. Sandmo (1971) analyzed a competitive firm under price uncertainty and risk aversion. On one hand, the firm's attitude toward risk was assumed to be represented by a von Neuman-Morgenstern utility function. On the other hand, the existence of price uncertainty led to the questions of (1) how does the optimal output compare to the optimal solution under certainty, and (2) what is the marginal effect of a mean preserving spread in risk.[3]

The concluding answer, inter alia, was a reduction in the level of output but could not be proved categorically. In a more general framework of analysis, Leland analyzed the case of a firm facing uncertain demand. Sandmo's results feature, as a particular case, in the spectrum considered in the aforementioned study. The recent papers by Batra, Ullah and Coes--in 1974 and 1977, respectively--dealt with the subject of behavior of the firm under risk. Although there may be differences in the ways in which risk is introduced in the analysis; with no exception the studies reviewed considered as a common tool the expected utility approach. The use of it dwells on the analytical appeal to simplify an otherwise

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[3] Briefly stated a mean preserving spread in risk is a stretching of the conditional probability distribution around a constant mean; cf. Rothschild & Stiglitz.

complicated and unexplicit set of preferences. However, a serious handicap emanates when, rather than the analytical, the empirical field is considered: utility functions for most cases are not available, and if so, there is no reason to believe the same parameters will apply to different agents, or that at least, the same family of equations will hold.

In view of these considerations and after revising methods of optimization in a stochastic or risky environment, this study focused on the maximization of expected net revenues received by the producer within the framework of optimal control theory. This approach and its application in developing optimum irrigation strategies for grain sorghum is discussed in the following chapter. With the methodology developed, the debate turns to the description and procedure followed to develop optimum irrigation strategies and to estimate the impact of stochastic fuel curtailments. A discussion of the results is presented in Chapter IV which is followed by a summary and conclusions section.

CHAPTER II

THE FIRM UNDER A DYNAMIC AND STOCHASTIC ENVIRONMENT

The static economic problem can be described as the allocation of scarce resources among competing ends at a given point in time. In mathematical terms, the problem is that of choosing values for certain variables, called instruments, so as to maximize a given objective function. The dynamic or intertemporal economic problem is the allocation of scarce resources among competing ends over a period of time from "initial time to terminal time" (Aoki; Intriligator).

The latter problem is one of choosing time paths for a set of "control variables" so as to maximize a given measure of performance. Each time path selected for the control variable implies a time path or sequence of movements of the system, from one stage to another, throughout the entire horizon. The system's description or value, at each point in time, is given by a set of variables known as state variables. The laws governing the rate of change of these variables are described by a set of differential equations called the equations of motion.

Mathematically stated, the dynamic economic problem is that of choosing a time path for the control variable that will maximize the value of a given objective functional.[4] This value depends on

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[4] Intriligator refers to a functional as a real-valued function defined on a set of functions, that is, the domain is a set of functions.

the time path of the control and state variables. The presentation of the dynamic economic problem in this form, is referred in the literature as a control problem (Intriligator, p. 292).

In this chapter, because optimal allocation of irrigation water in the production process is a problem of the dynamic economic type, it will be presented in the context of control theory. Of special interest is the part of control theory denoted as adaptive stochastic control. It recognizes that as the system progresses through time, more information is available which can be used to modify or reestimate the influence of alternative control variable settings on various performance measures. The formulation of a strategy in the use of the decision variable, irrigation water, at the beginning of the growing season is based on the expectations held by the decision maker on relevant variables such as rain and temperature. As the season evolves, he is not committed to keep his initially formulated plan, but rather, it can be altered. In this case, a revision and reformulation procedure takes place in which the newly observed data are taken into account to reevaluate the effectiveness of the control variable. As a result, a new strategy is formulated for the remaining production periods.

The discussion is initiated with a presentation of the theory of production and statement of some of the reasons for the applicability of optimal control theory. Initially, there is a description of the case where known conditions exist which is then expanded to consider the decision procedure where stochastic elements are present. The last section of this chapter will consider the numerical technique to be used for solving the control program.

On the Theory of Production

Production rarely takes place in an instantaneous fashion. In most cases a series of stages must occur in time before the production process is terminated. As a result, a T-stage system with one product and one resource defines the present value of profits to be given by:

$$\Pi^* = pq\ \xi^{-T} - \sum_{t=0}^{T-1} r_t\ X_t\ \xi^{-t}$$

subject to

$$F(q, X_1, \ldots, X_{T-1}) = 0$$

$$g_t(X_1, \ldots, X_{T-1}) \leq b_t$$

or:

$$\text{(1)} \quad \Pi^* = pq\ \xi^{-T} - \sum_{t=0}^{T-1} r_t X_t\ \xi^{-t} - \lambda[F(q, X_1, \ldots, X_{T-1})] - \gamma[g_t(X) - b_t]$$

where p, r, q and X are the price and quantity of output and resource used, ξ^{-t} is the discounting factor, and the terms λ_t $[F(q, X_1, \ldots, X_{T-1})]$ and $\gamma[g_t(X) - b_t]$ are the constraints imposed on profits by the production function and by the limited values that the input can have at each period. The profit maximizing conditions for this system are defined by:

$$(2)\quad \partial\Pi^{*}/\partial X_t = p\partial q/\partial X_t\xi^{-t} - \lambda\partial F/\partial X_t - \gamma\partial g/\partial X_t = 0 \quad t=1,2,\ldots,T$$

$$(3)\quad \partial\Pi^{*}/\partial\lambda = F(q,X_1,\ldots,X_{T-1}) = 0$$

$$(4a)\quad \partial\Pi^{*}/\partial\lambda = g_t(X_1,\ldots,X_{T-1}) - b_t \leq 0$$

$$(4b)\quad \text{and } \gamma\cdot\partial\Pi^{*}/\partial\gamma = 0$$

From (2) it follows that,

$$\frac{p(\partial q/X_t)\xi^{-T} - r_t\xi^{-t}}{p(\partial q/\partial X_\tau)\xi^{-T} - r_\tau\xi^{-t}} = \frac{\lambda(\partial F/\partial X_t) + \gamma(\partial g/\partial X_t)}{\lambda(\partial F/\partial X_\tau) + \gamma(\partial g/\partial X_\tau)} \quad t \neq \tau$$

to preserve equilibrium, the ratio of marginal additions to profits in any two periods should equate the ratio of the imputed costs required to relax the constraints imposed. Equations (3) and (4) are a restatement of the restrictions imposed derived from applying the Kuhn-Tucker theorem.

These conditions will lead to the maximization of net returns under some restrictive assumptions. Considerations such that (1) the non-existence of random elements; i.e., those recognized as additive disturbances in econometric models, (2) perfect knowledge on the value of the parameters, (3) the non-existence of measurement errors as well as (4) certainty about the scenario and existence and availability of resources in the future are assumptions that make this model unfitted in its application as a framework of reference for decisions to be made. Producers, and specifically, agricultural producers are typically faced with situations where these premises do not hold. In those cases, recognition and consideration of the "actual decision process" to allocate resources is necessary for economically sound system analysis and model building. Such recognition and consideration is provided by optimal control theory.

Optimal Control Theory

The problem of optimal allocation of irrigation water in the production process can be represented by the following set of equations:

$$(5) \quad J(X_t, U_t, t) = \sum_{t=0}^{T=1} I(X_t, U_t, t) + F(X_T)$$

J is assumed to be a convex functional made up of the summation of the discounted net returns from the T-stage system operating under a deterministic environment: the intermediate function I(...) represents the discounted values for each period up to T-1 and $F(\cdot)$ is the ending value or terminal state function. The vector of the state variables X represents the values obtained by the system at each period t, therefore, it includes measures of output as well as input units. The control variables at each instant t are completely described by the values of the vector U.

The maximization of the objective functional (5) is subject to

$$(6) \quad X_{t+1} = f_t(X_t, U_t, t) \text{ and}$$

$$(7) \quad g(X_t, U_t) \leq b_t$$

where the planting date (t_o), the state of the system at $t_o(X_o)$, and the date at which the physiological maturity of the plant is reached (T) are characteristics determined within the system.

Equation (6) represents the dynamic behavior of the system indicating that the change in the level of the state variables at any instant is a function of its present state, the decision taken, and the time period. Together, the equation of motion, equation 6, and the intermediate function, $I(X_t, U_t, t)$, reflect the problem of making decisions in a dynamic context (Dorfman) and imply that the decisions do, in fact, influence the level of production and returns in each time period.

The solution to the defined system can be approached by the optimality principle (Bellman) based on a first order difference equation. Though this method offers a solution technique, it provides no analytical solution for all but the simplest cases.

Other methods such as the Newton-Raphson, quazi Newton-Raphson, Golden Section, False Position and other gradient techniques bear the same kind of limitation faced by Bellman's principle in that analytical solutions cannot be obtained.

A third type of approach is obtained by the use of the maximum principle. This approach, though initially developed by Pontryagin for the case of continuous-time type of problems, has been expanded by Holmes, among other authors, to include the discrete-time case. This method overcomes the limitation faced by the previously mentioned approaches providing an analytical solution. Since the concern at the moment is one of interpretation and not of numerical answer, Pontryagin's principle is developed in the following lines.

The maximum principle can be derived by introducing a set of costate variables to the system defined by equations (5) through (7). The resulting Lagrangian expression is

$$(8) \quad L = \sum_{t=0}^{T=1} [H_t(X_t,U_t,\lambda_t,t) - \lambda_t X_{t+1} + \gamma_t(q_t(X_t,U_t)-b_t)] + F(X_T)$$

H(...) being the Hamiltonian function defined as the sum of the intermediate function of the objective functional plus the inner product of the costate variables and the function defining the rate of change of the state variables,

$$H_t(X_t,U_t,\lambda_t,t) = I(X_t,U_t,t) + \lambda_t f_t(X_t,U_t,t)$$

Because the Hamiltonian in period t is the total net revenue accruing to the economic agent in period t, made up of both the net revenue $I_t(X_t,U_t,t)$ and future net revenues $\lambda_t f_t(X_t,U_t,t)$ by means of the structural interconnections represented by the equation of motion it follows, as shown below, that the constrained maximization of total net revenues implies the maximization of H in each period. Thus, employing the Kuhn-Tucker theorem, the first order necessary conditions for the maximum of the Lagrangian expression (8) are:

$$(9) \quad \partial L/\partial X_t = \partial H_t/\partial X_t - \lambda_{t-1} = 0 \qquad t = 1, \ldots, T-1$$

(10) $\partial L/\partial U_t = \partial H_t/\partial U_t - \gamma_t \partial g_t(X_t,U_t)/\partial U_t = 0 \quad t = 1, \ldots, T-1$

(11) $\partial L/\partial \lambda_t = \partial H_t/\partial \lambda_t - X_{t+1} = 0 \quad t = 1, \ldots, T-1$

(12a) $\partial L/\partial \lambda_t = g_t(X_t,U_t) - b_t \leq 0 \quad t = 1, \ldots, T-1$

(12b) $\gamma_t \cdot \partial L/\partial \gamma_t = 0, \ \gamma_t \geq 0$

The firm cannot maximize L with respect to X, as indicated in expression (9), since this is not a control variable. However, this equation states the conditions that the firm should follow with respect to the time path of U and λ to make the value of total net returns, J, as great as possible at every stage.

The costate variables λ in dynamic processes are the equivalent of the Lagrange multipliers in the static problem of maximization subject to constraints, and their interpretation (Benavie) can be obtained by considering the equation of motion as being

$$X_{t+1} = f_t(X_t,U_t,t) + a_t$$

where a_t represents exogenous variables, variations in which will affect X_{t+1} as well as all the state variables from t+2 to T via the f(...). Also, from the Lagrange-Kuhn-Tucker theory, we know that

$$\lambda_t^* = \partial L/\partial a_t = \partial J^*/\partial a_t$$

the asterisk indicating the derivatives are computed at the optimum. Therefore, the costate variable λ_t^* may be interpreted as the rate at which the optimal value of the objective functional changes with respect to X_{t+1} which in turn has changed due to an autonomous change in a_t. Economically speaking, λ_t^* is the marginal net revenue or the shadow price of $(X_{t+1}, \ldots, X_t)$ at the optimum; i.e., the shadow price of all future optimal states of the system. In the same context,

$$\partial L/\partial b_t = \partial J^*/\partial b_t = \gamma_t^*$$

implying that, at the optimum γ_t^* is the value to the economic agent of a marginal relaxation on the constraints to the control variable, that is, how much net returns would be augmented by loosening up the limitations on irrigation water that can be applied at each period.

Finally, equation (10) defines the use of the control variable. The input use should be expanded until the contribution to present and future net returns are exactly offset by its opportunity cost; i.e., irrigation water at each period should be applied until the change in profits is exactly equal to the cost of its application. Also, for any two periods in time:

$$\frac{\partial H_t/\partial U_t}{\partial H_\tau/\partial U_\tau} = \frac{\gamma_t(\partial g_t(X_t,U_t)/\partial U_t)}{\gamma_\tau(\partial g_\tau(X_\tau,U_\tau)/\partial U_\tau)}$$

the ratio of marginal net returns in any two periods should be equal to the ratio of the imputed marginal cost for the same periods.

The fulfillment of these conditions leads to the optimal allocation of resources through time under deterministic conditions. However, when the equation of motion is modeled for the stochastic case as

$$X_{t+1} = f_t(X_t, U_t, V_t, t)$$

to include a vector of random variables (V_t), an analog of the maximum principle does not usually lead to optimal policies since the multi-stage decision taken under non-deterministic environments are prevented from having the exactly desired effects on the production process. As shown by Holmes, the policy obtained from the maximum principle (an open-loop control rule) is not optimal because it does not make use of the information resulting from the multi-stage decision process.

In those cases where uncertainties can be adequately modeled on stochastic processes, the problem of decision making can be approached by using adaptive stochastic control theory.

Adaptive Stochastic Control

Similar to the deterministic case, the problem of optimal allocation of irrigation water in a production process when stochastic conditions exist can be modeled as a control problem, specifically, as an adaptive stochastic control problem.

In this context, the objective function is represented by the maximization of expected net returns given by

$$(13) \quad J = E[R(X^T, U^{T-1}, T)]$$

and the state of the system at time t is presumed to evolve according to

$$(14) \quad X_{t+1} = f(X_t, U_t, V_t, t) \qquad t = 0, 1, \ldots, T\text{-}1$$

with X_t and U_t being the vectors of the state and control variables at time t and V_t being the process noise. In equation (13), R is a real valued function and the expectation is taken over all the underlying random variables.

In this case, the choice of the controls, which is a multistage decision process, constitutes the stochastic control problem and the ways of choosing them determines the class of control policies to be applied.

Alternative Control Policies

The various classes of stochastic control policies are defined according to the information on past and anticipated future observations available to the controller. This knowledge about the probabilities of future observations allows the controller to statistically anticipate the information to be obtained from subsequent observations and to be used in deciding the most desirable present action.

According to the amount of information used, four classes of stochastic control policies--as defined by Bar-Shalom and Tse--can be distinguished:

1. Open-loop (OL) policy. In this case the controller does not use any real-time information, therefore, the control is defined

$$U^{OL}(t) = U^{OL}(t, S^{o}) \qquad t = 0, 1, \ldots, T\text{-}1$$

S^{o} indicates that no measurement knowledge is available for the controller. No information on past as well as future values of the decision variables is included in the determination of this control.

2. Open-loop feedback (F) policy. The inclusion of the history of the decision and state variables constitutes the added feature--with respect to OL. The controller considers all past knowledge but anticipated future information does not influence current decisions

$$U^{F}(t) = U^{F}[t, Y^{t}, U^{t-1};\ M^{t}, S^{t}], \qquad t = 0, 1, \ldots, T\text{-}1$$

with Y^{t} being the set of observations up to time t after the sequence U^{t-1} has been applied; M^{t} the knowledge about the measurement system between time 0 and time t, called the observation program; and S^{t} is the joint density distribution of the random variables.

3. The m-measurement Feedback (mF) policy. Similarly to the previous policy, it incorporates the currently available information and in addition the subsequent anticipated m-measurements with their statistics available to the controller

$$U^{mF}(t) = U_{mF}[t, Y^t, U^{t-1}; M^{t+m}, S^{t+m}],\ t = 0, 1, \ldots, T-1$$

4. Closed loop (CL) policy. In this type of policy the current information as well as all future expected information is taken into account. In the present policy, the control rule will have the form

$$U^{CL}(t) = U^{CL}[t, Y^t, U^{t-1}; M^{T-1}, S^{T-1}],\ t = 0, 1, \ldots, T-1$$

It should be noted that F, mF and CL have the same information about the past, and the only difference among them resides in the anticipation of future knowledge. Of these four policies, the optimal stochastic control belongs, in general, to the closed loop class (Bar-Shalom and Tse; Intriligator; Rauser). The optimal stochastic control is obtained by using Bellman's principle of optimality. Since the policy has to be optimal for each remaining period, the last control is obtained by maximizing the expected net revenue conditioned on the information state $[Y^{T-2}, U^{T-2}]$ available at time T-1, and independent of past decision

$$\max_{U(T-1)} E[j(T, X^T, U^{T-1}) | Y^{T-1}, U^{T-2}] \quad .$$

Proceeding backwards in time, the control policy can be expressed as:

$$J^{CLO}(T) = \text{Max } E[\ldots \max E[J(T, X^T, U^{T-1})\ Y^{T-1}, U^{T-2}]\ Y^{T-2} U^{T-3}] \ldots]$$

where the nested terms show the property of the closed loop control that it anticipates subsequent feedback (Bar-Shalom and Tse, pp.

118-9).

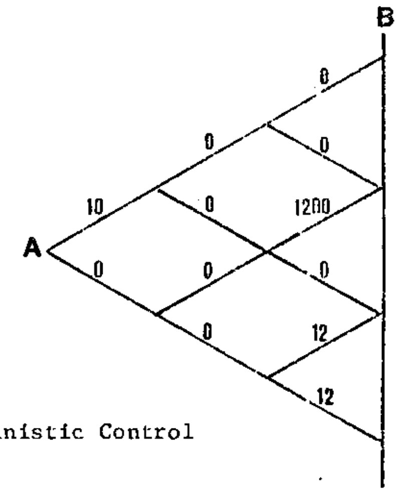
The inherent analytical difficulties in deriving the closed loop control rule, though, suggest other policy classes should be used as an approximation. The analytical hardships can be reduced by decreasing the amount of information available to the decision maker. Other types of stochastic controls are possible, and one frequently used has been the so-called certainty equivalent (CE) controller which--heuristically--assumes the classical certainty equivalent theorem holds: the optimal decision in a risky situation is the same as in some associated riskless situation (Malinvaud, p. 706). Simon and Theil considered a linear, quadratic, Gaussian control problem and suggest it be solved by replacing all random variables by their expected values. It was shown by Theil that under these conditions, the first-period action of the strategy which maximizes the objective function is identical with that of the strategy which neglects the risk by using the expected values of the stochastic elements. More precisely, when the assumptions given above are met, and when the decision maker does not lose information, the initial optimal values of the control variables are the same as if stochastic elements were not present.

Control Policies: An Illustrative Approach

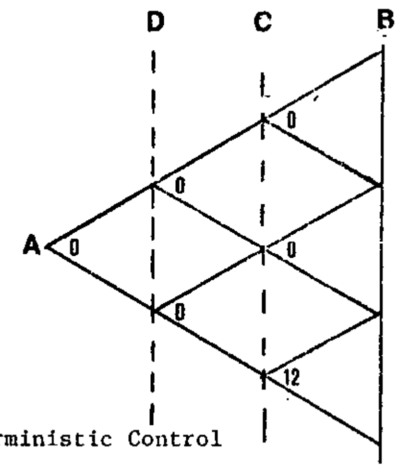
The form or choice of the optimal control, as seen above, depends much on the assumptions made about the amount and nature of the information which is available to the controller. From the four classes of control described before, the open and closed loops are of special interest since they represent the two extreme cases. To point out their differences, a presentation given by Sorenson is paraphrased here.

In figure 1, a simple three-stage deterministic control problem is shown. The objective is to move from point A to the line B with the minimum sum of the values (costs) associated with each branch of the network. For example, the decision of going Up-Down-Up (U-D-U), has a cost of 10 + 0 + 1200 = 1210. The open-loop policy will evaluate all eight possible paths and obtain the policy D-U-D which generates the minimum cost (0 + 0 + 0). The closed-loop control is obtained by choosing, at each node or state, the optimal path to be followed from that state to the end, regardless of the previous decisions made. This policy is calculated most efficiently by proceeding from line B to the point A. First, the cost involved in going from all nodes in stage C to the final line is calculated. Once these values are obtained, the cost of advancing from each node in stage D to the final line is also estimated. Finally, the optimal decision is made by choosing the minimum cost from the sum of (1) the branches departing from A, to each node in stage D, and (2) the cost of the optimal path to follow from this stage. Thus estimated, the optimal closed-loop policy is seen to be the same as the one generated by the open-loop control, i.e., D-U-D.

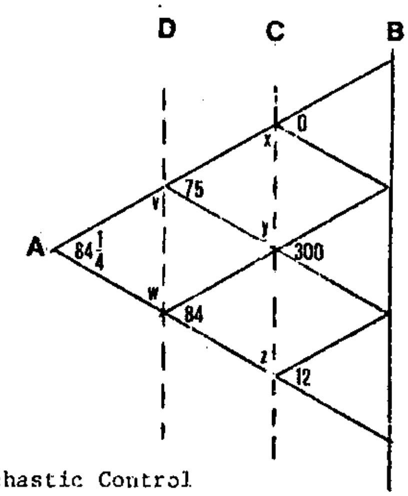
Under the probabilistic case, it is assumed the outcome of a



A. Open-loop Deterministic Control



B. Closed-loop Deterministic Control



C. Closed-loop Stochastic Control

Figure 1. An Illustration of Alternative Control Policies.

decision at each state is a random event. Consider the case where, after the decision D is made, the probability it will occur is 3/4 and the probability that it will not happen is 1/4. Likewise, when the decision U is made, there is a 75 percent chance that it will materialize and a 25 percent chance that D will occur.

Under these conditions, the open-loop policy is evaluated for all eight possible outcomes. The expected costs of the decision U-U-U is obtained by:

$$E[U\text{-}U\text{-}U] = (27/64)10 + 9/64(1200 + 1210 + 10) + 3/64(12 + 0 + 10) + (1/64)12 = 346$$

In this evaluation, the other possible sequences lead to the choosing of U-U-D as the one yielding the minimum cost (120). The stochastic closed-loop policy is computed similarly to the method followed in the deterministic case. At each state on stage C, the costs to arrive at the line B is estimated so that after arriving at each node the optimal decision to be made will be known, e.g., the optimal decision to be made at X is either U or D, if at point Y is D and if at Z' is either U or D. The same procedure is repeated for the states V and W. If at V, the decision would be U (a cost of 75) and if the decision is to be made at W, it would be D (minimum cost is 84). Finally, at A the decision that minimizes the expected total cost is D resulting in a cost of 84.25. The comparatively lower expected cost obtained with the closed-loop control results because the decision made at each state is based on the knowledge that you are located at the node (Sorenson, p. 11).

The feedback policy is considered to be an intermediate between the open-loop and the closed-loop policies. The reason for this consideration is that the open-loop policy is used to make the initial decision at A. The result of this decision is observed and the open-loop control is reestimated for this state and the initial decision is taken. The result obtained from the open-loop feedback policy is different from the one obtained by the open-loop or closed-loop policy. This decision strategy is similar to the closed-loop everywhere except for the first decision. Though this policy seems to be superior to the open-loop, it is inferior to the closed-loop since at the initial point it does not consider the knowledge that future measurements are to be made.

The intrinsic hardships and extensive information requirements involved in the modeling of the closed-loop control make it unfitted for use in this study. A viable alternative left is the use of the open-feedback control that generates approximately optimal values for the control variable.

Numerical Optimization Algorithms

A final step towards the solution of control problems is iterative numerical technique based on iterative algorithms. With regard to iterative procedures there are two critical issues: (1) whether or not a given algorithm in some sence yields, at least in the limit, a solution to the original problem; and (2) how fast the algorithm converges to the solution.

If for any arbitrary starting points, the algorithm generates a sequence of points that will always converge to a solution, then it is said that it has the property of being globally convergent. However, this is not the case for all algorithms though it is possible to modify them as to assure global convergence.

Addressing the second question, the speed of convergence cannot be cast in the form of a single theorem as could be possible in the case of global convergence (Nemhauser). There are, however, some basic ways to predict with confidence the relative effectiveness of a wide class of algorithms for which the definition of the order of convergence is required.

In a sequence of real numbers $\{r_k\}_{k=0}^{\infty}$ convergent to the limit r^*, the order of convergency is defined as the supremum of the non-negative numbers p satisfying [5]

$$0 \leq \lim_{k\to\infty} \frac{|r_{k+1} - r^*|}{|r_k - r^*|^p} < \infty$$

Within this context two major approaches can be distinguished:

(1) Step-wise convergence. In this case, the bounds on the progress made are defined by going a single step: from K to K+1.

If the sequence $\{r_k\}$ converges to r^* such that

[5] Luenberger, p. 127.

$$\lim_{k\to\infty} \frac{|r_{k+1} - r^*|}{|r_k - r^*|} = B < 1,$$ the convergence is defined to be linear with convergence ratio B (sometimes referred to as geometric convergence). The lower the ratio among two competing algorithms with linear convergence, the faster the rate. The case where B=0 is commonly referred as superlinear convergence.

(2) Average Rates. This approach is defined in relation to the average progress per step over a large number of steps: the average order of convergence is the infimum of the numbers $p > 1$ such:

$$\lim_{k\to\infty} |r_k - r^*|^{1/p^k} = 1$$

Iterative Solution Techniques

The practical importance of iterative numerical solution techniques should by no means be overlooked. They often offer the simplest, most direct alternatives for obtaining solutions. Maybe their greatest achievement relies in establishing trade-off points between the difficulty of implementation and the speed of convergence. Though there exist many algorithms to obtain optimum points and many more may be envisioned, there is a fundamental underlying structure for almost all of them: beginning at an initial point determines, according to a fixed rule, the direction of movement and proceeds to locate, in the predetermined direction, a (relative) optimum of the objective function on that line. Once located at the new point, a new direction is determined, and the procedure is repeated. The line search allows the finding of the optimal point for those nonlinear functions that cannot be analytically optimized. An extended number of approaches to this important phase of optimization can be cited: Fibonacci, the Golden Section, Newton and quasi-Newton methods, False Position, cubic and quadratic fit, steepest ascent, coordinate ascent methods, conjugate direction methods, etc., in which the sophistication of implementation varies directly to the speed of convergence. The selection of the method depends on the particularities of each problem analyzed. In the specific case of the problem faced in this study, professional judgment and ciphering facilities lead to the use of the theory involved in quasi-Newton methods.

Quasi-Newton Methods

These methods can be pictured to reside in an intermediate place between the steepest ascent and Newton's method. The common background in quasi-Newton methods underlies the fact that the evaluation and inversion of the Hessian[6] matrix that is required

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[6]The Hessian of a function f at x is defined to be the nxn matrix denoted

$\nabla^2 f(x)$ or $F(X)$ such $F(X) = \frac{[\partial^2 f(x)]}{\partial x_i \partial x_j}$. Also, since

$$\frac{\partial^2 f}{\partial x_i \partial x_j} = \frac{\partial^2 f}{\partial x_j \partial x_i}$$

it is implied that the Hessian is symmetric.

for steepest ascent methods is impractical or costly to obtain; hence, it is replaced by an approximation. The type or form of the approximation varies according to the method used. The spectrum covers the simplest cases where the type of approximation remains fixed, to the more sophisticated cases where improved approximations are built up in the light of new information gathered during the optimizing process. Ideally, the approximation will converge to the inverse of the Hessian at the solution point.

The scheme developed by Fletcher and Powell has the property of simultaneously generating the directions by selecting the successive direction vectors as a conjugate version of the successive gradients obtained as the method progresses--while constructing the inverse Hessian. At each step the inverse Hessian is updated by the sum of two symmetric matrices of rank one. For the general case of nonquadratic objective function, the method requires only that the gradient be obtainable, and the directions generated are thus always guaranteed to be the directions of optimum. A third attractive feature of this method is the convergence to the inverse Hessian to the solution in a super-linear fashion.

Albeit the outstanding analytical characteristics of this method, in its pure form the convergence characteristics may not be that appealing. It has not been possible to prove the global convergence of the method, and sometimes it may converge to a locally optimal point. However, the advantages reflected by this method seemed to outweigh the difficulties found.

Summary

In the present chapter, the subject of allocating different amounts of a resource in a dynamic environment was presented and the conditions for an optimal solution were derived. Since the stochastic maximum principle is not information optimal, the different classes of stochastic control policies were addressed as alternative means to overcome the deficiency found. Because of the complexity involved in the modeling and analytical solution of the stochastic closed-loop policy, the stochastic open-loop feedback control was considered as an alternative. Albeit the limitation of finding an analytical solution could not be overcome for this control, the possibility of being modeled so as to obtain a numerical solution leaves it as the best viable alternative. To this extent, different concepts involved in numerical optimization algorithms were briefly introduced and later expanded for the Quasi-Newton methods case.

CHAPTER III

MODEL DESCRIPTION AND PROCEDURE

The basis of this study is the computerized grain sorghum growth model developed by Arkin, Vanderlip and Ritchie. The core model is a system representation of the soil-plant-atmosphere continuum. In it, the modeling of physical and physiological processes of water use, light interception, appearance of leaves, etc., permits the calculation of the daily plant growth under field conditions.

The objective of this study was to modify the plant growth model to develop optimum irrigation strategies and estimate impact of stochastic irrigation fuel curtailments. Since the model has an adequate mathematical representation, optimal values for the control variable, irrigation water, can be estimated within the framework of open-loop feedback policy and the use of numerical searching techniques.

The use of this model to generate statistics describing the mean and variance as well as irrigation water applied at each period, demands a large set of climatological data. Information requirements for each variable, however, are not available for lengthy periods of time. Therefore statistical parameters that would generate consistent values are needed. The methodology required to satisfy such demand has been developed by Rockwell and its application was incorporated into this model. Finally, the decision process likely to be followed to optimally allocate irrigation water was added to the simulation model by means of considering output prices, expected yields and resource costs.

The Open-Loop Feedback Model

The grain sorghum plant growth model was modified to include economic components and optimize timing and quantity of irrigation water. In general terms, it can be described as consisting of three major stages as shown by figure 2. First, there is input data supplied by the user to initialize model use. Second, is the dynamic grain sorghum growth section which includes physical relationships. Lastly is the feedback routine incorporated into the physical relationships.

Initialization

The initialization step involves inputing data to the program which is stored for use when repeated simulations are made. The data supplied consist of morphological and planting characteristics, climatic, soil water and soil physical properties necessary to initialize the grain sorghum simulation model. The number of leaves produced by the plant, the maximum area capable of being developed by each leaf, the plant population, the row spacing, the planting date, daily climatic information, the study area's latitude, and the extractable soil water content on planting date as well as the

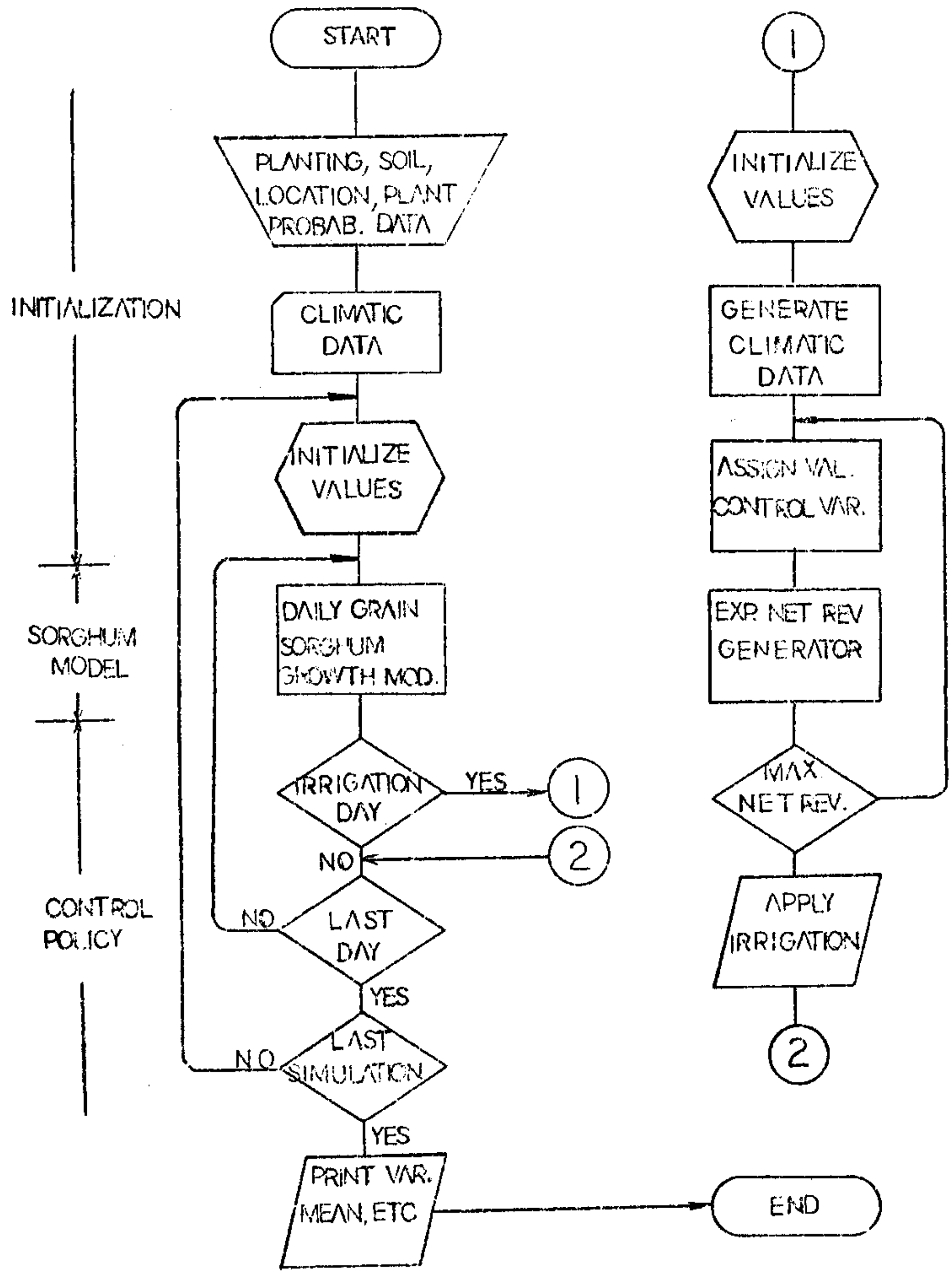

Figure 2. Open-loop feedback model

extractable soil water capacity are characteristics required to set the initial values for the sorghum growth model. Once these values are set, information about the probabilities that define the distribution of the different climatic variables is read and stored. In this stage also, all economic information relevant to the problem is introduced including prices of natural gas, grain sorghum and the costs of all other resources used in the production process. With this information, the program sets, internally, the initial values for all the variables of the grain sorghum model. By using the relevant information from this model, it also sets up the dates in which planned irrigation would take place. After the initialization period is completed, the control of the central processing unit is turned to the grain sorghum model.

Grain Sorghum Growth Model

A detailed description of the dynamic plant growth model was developed by Arkin, Vanderlip and Ritchie. Therefore, this discussion is limited to a brief account of the major and relevant highlights.

Throughout the model there are a series of efficiency functions reflecting the effects of non-optimal conditions. An example is net photosynthesis. Potential net photosynthesis (Po) is defined as the net CO_2 fixed on a ground area basis for non-limiting water and temperature conditions (Arkin, Vanderlip and Ritchie, p. 7). When limiting factors exist, the potential value is reduced by some fraction that reflects the stress imposed on the plant due to the presence of a non-optimum environment; i.e., extremely high or low temperatures and/or lack of water availability. Whenever water or temperature conditions are limiting, net photosynthesis is considered to be given by:

$$(15) \quad P = (Po\ \xi_1\ \xi_2) - N$$

P is the net photosynthetic rate of the crop, Po remains as defined before, ξ_1 and ξ_2 are efficiency parameters corresponding to temperature and soil water and N is the value of the nighttime respiration losses. The parameters ξ_1 and ξ_2 are dimensionless fractions whose values oscillate between 0 and 1. Expression (15) was based on the assumption that limiting variables in the environment proportionately reduce the photosynthetic rate regardless of the value of the other limiting variables. Each efficiency parameter represents a particular environmental constraint on the photosynthetic rate.

The temperature efficiency parameter, ξ_1, forces a complete inactivity in photosynthesis for values below 5°C. and above 45°C. An optimal range is considered to be between 25°C. and 40°C. Limitations in soil moisture is reflected by the coefficient ξ_2. Here, reductions in net photosynthesis are considered to be proportionate to the reduction in plant evaporation resulting from a limited supply of water. As reported by the study, plant evaporation is not affected until a threshold of extractable soil water is reached. When approximately 80 percent of the extractable soil water is depleted by evapotranspiration, the value of the soil water parameter, ξ_2, becomes less than one indicating net photosynthesis is affected. This relationship is speculative, and net photosynthesis may be affected to a larger degree if it is limited more by plant water status than it is by evapotranspiration (Arkin, Vanderlip and Ritchie, p. 9).

By how much and/or how long should net photosynthesis be affected due to a stress in the availability of soil water, if it should be stressed at all, is a question that needs to be answered in economic rather than physiological terms.

The Feedback Model

To optimize the amount of irrigation water needed, it was necessary to separate the growing season into specific periods. Particularly, eight periods were established including 20 days from the day of the plant emergence and 10 days per period until harvest.

The determination of how much irrigation water would be applied in each period of the growing season, was achieved by aseembling a stochastic open-loop feedback policy into the grain sorghum growth model and numerically searching for the optimal expected values. At every planned irrigation date, the feedback sub-model is used to provide the decision maker with an optimal set of values for the control variable. The process to estimate optimum irrigation schedules is started with the subroutine MAXIMA.

Whenever an irrigation date is reached, the model calls this subroutine to estimate expected net returns, expected yields and the optimal irrigation schedules that are required to generate these values. By means of a numerical searching technique, the irrigation schedule producing the highest expected net revenue is selected among all other schedules. The chosen plan is then considered to be followed by the decision maker and the first period decision is adopted.

The selection of the optimal strategy to follow is based on the expectations held by the producer about future climatic, economic and institutional conditions. If any of these expectations are not realized, then a process of reevaluation is started on the next predetermined date. Optimal values are, again, estimated and the first period decision is adopted. The process is continued in the same fashion until the last period evaluation is completed. The procedure to measure whether the expectations formulated were correct was by returning to the actual model and to use "real" information. If institutional changes occurred (i.e., natural gas curtailments) or climatic expectations did not hold (i.e., a longer than expected drought period, the occurrence of precipitation in different amount or timing than was anticipated or extreme temperatures not foreseen) then, these shocks to the system, would be reflected in expected values, different to those previously calculated. Making use of this information, the proper values for the control variable were reestimated to maximize the objective function subject to the new starting conditions obtained.

Expected Weather Pattern. At each period, the values for weather variables in the future are not known with certainty, however, some expectations can be constructed. The generation of a possible climatic pattern is obtained through the subroutine WEATHER. This subroutine is called by MAXIMA and requires the maximum temperature and rainfall that occurred the last day of the preceding period be supplied as a starting point. With this basis, the subroutine WEATHER will generate for every day until harvest, the climatic values for rainfall, maximum and minimum temperature and solar radiation.

The values assigned to each of the weather variables were based on research by Rockwell. In the Rockwell study, several procedures were used to develop a probability distribution of the atmospheric variables. The concluding results from a research state that the presence of rainfall could be considered as a binomial random variate and the amount by which it occurs can be determined by an empirical distribution, modeled for each month. The distribution obtained was assumed to satisfy

$$(16) \quad X = (e^{F(X)n/\hat{b}} - 1)/\hat{a}$$

where $\hat{a}$ and $\hat{b}$ are the parameters estimated by the nonlinear regression

$$(17) \quad RANK = b\ Ln(a(X) + 1)$$

RANKis the number of observations for each level of rainfall, X is the amount of rainfall observed and n is the maximum value for RANK.

The actual value to be assigned to the variable rainfall was obtained by generating two uniform randomly distributed variates in the interval 0 and 1 [U(0,1)]. The value of the first variate was compared against the probability of raining that had already been supplied by the user. If the value generated was smaller (greater) than the value supplied, then it was considered rain occurred (did not occur) that day. If the occurrence of rain was determined, the amount in which it happened was estimated with equation (16). Because the dependent variable RANK was scaled down to the range 0,1 when divided by N, the value of F(X) in equation (16) was allowed to be equated to the second uniform variate generated. Thus, with all parameters known in equation (16), the value of X (rainfall) could be estimated.

In the case of solar radiation, the final model was based on a three way empirical frequency table conditioned by both precipitation and maximum temperature for the day. Two more uniform variates were generated to assign a value to this variable. The first value generated was used to locate in which range the value of solar radiation (SOLRA) could be placed. Taking into account precipitation, and maximum temperature that occurred in that day and comparing the random variate against the probability of occurrence the "base" value for SOLRA was determined. The second variate was scaled to the range 0,10 and this value added to the base. The result was considered to be the value for SOLRA for that day.

The same as for solar radiation, a three way empirical frequency table was built for maximum temperature (TEMPMX). The value of this variable was conditioned on that day's amount of rainfall and the maximum temperature during the previous day. Again, two uniform random variates were originated. The first was used to determine the base value for TEMPMX. Considering the amount of precipitation occurring that day and the maximum temperature observed the day before, the base value was determined in similar fashion to the one described for SOLRA. The second random variate was scaled up to the range 0,10 and added to the initial value. The outcome was considered to be the value adopted by TEMPMX for that day.

Finally, once maximum temperature was determined, minimal temperature (TEMPMIN) was estimated by means of a transformation under the assumption that these two variables were approximately bivariate normal distributed.

As reported by Rockwell, whenever

$$E(\text{TEMPMX}) = \mu_1, \quad E(\text{TEMPMN}) = \mu_2 \text{ and}$$

$$\Sigma = \begin{pmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{12} & \sigma_{22} \end{pmatrix}$$

there exists a linear transformation of the variable TEMPMX and TEMPMN that generate the values MX and MN which are independent and normally distributed random variables. Making

(18) $MX = TEMPMX$ and

(19) $MN = TEMPMN - (\sigma_{12}/\sigma_{11})\ TEMPMX$

Rearranging terms in the last expression, the value of TEMPMN is given by

(20) $TEMPMN = MN + (\sigma_{12}/\sigma_{11})\ TEMPMX$

from (19) the $\mathrm{var}(MN) = \sigma_{22}(1-\rho^2)$
with $\rho = \sigma_{12}/\sigma_1\sigma_2$. If a random value from the distribution of MN is required, it could be obtained by

(21) $MN = \mu_2 - (\sigma_{12}/\sigma_{11})\mu_1 + \sigma_2(\sqrt{1-\rho^2})\eta$

η being a standard normal variate. Substituting (21) in (20) and rearranging terms, the value of TEMPMN would be given by

(22) $TEMPMN = (\mu_2 - (\sigma_{12}/\sigma_{11})\mu_1) + (\sigma_{12}/\sigma_{11})TEMPMX + \sigma_2(\sqrt{1-\rho^2})\eta$

Expression (22) can be simplified to

(23) $TEMPMN = a + b*TEMPMX + \varepsilon$

The values of a and b were, thus, previously estimated for each month of the growing season and inputted to the program. The random error term was obtained from the product between the standard deviation of the residuals obtained by means of the least squared estimates of equation (23) and a random normal variate η. Therefore, the value of TEMPMN was given by

$$\text{TEMPMN} = \hat{a} + \hat{b}*\text{TEMPMX} + \sigma_{RES} * \eta$$

After the spectrum of the required climatic data is satisfied, the conditions required to generate maximum expected net returns are partially fulfilled.

Optimal Values of Irrigation Water. With a complete pattern of weather data generated with the above procedure, the control of the central processing unit is returned to MAXIMA. The next step within this subroutine is to call the use of the subroutine ZXMIN.

The ZXMIN algorithm, available through the IMS Library, searches numerically for those values of the control variable which will make the value of the objective function as great as possible. This algorithm uses the subroutine PRODFN to estimate grain sorghum yields based on the expected pattern for the variables generated by WEATHER and a set of values for the control variable. PRODFN is a replica of the grain sorghum model that has been modified to take into account the prices of the inputs required in the production process as well as the price of grain sorghum and the amount of irrigation water used. Once the expected yield from the given weather pattern is estimated the values of the net revenue is used as the measure to judge whether the value of the function is optimal or not.

After the first evaluation performed by ZXMIN, a second set of values for irrigation water is generated. Again, the whole procedure is repeated until a maximum value for net returns is encountered. After this, ZXMIN returns to control of the central processing unit to MAXIMA. As output, MAXIMA provides a listing of the optimal values for irrigation water and the value of the net revenues that could be expected.

The subrouting MAXIMA inputs to the grain sorghum growth model the irrigation level for the growth period being analyzed. This value is considered to be the optimal decision to make under risk with respect to the weather for the present and remainder of the growing season. All other irrigation rates for the following production periods are considered to be "an optimal strategy"; that is, they are the values the economic agent would foresee as necessary to be applied if his expectations about the climatic variables were correct. In case those expectations are not compatible with the "actual weather" pattern, a new evaluation process is started in the next irrigation period considering the latest information available and the previous optimal strategy as the starting point to find the new values. The decision process is stopped once physiological maturity is reached by the sorghum plant.

Model Validation and Assumptions of the Analysis

Simulation consists of a high number of elements, rules and logical linkages. Therefore, even when the individual components have been carefully tested, numerous small approximations can still cummulate into gross distortions in the output of the overall model

(Hillier and Lieberman, p. 633). Consequently, it is important to test the validity of the model for reasonableness of predicted values.

The economic optimization procedure introduced into the dynamic grain sorghum growth simulation program operates on a per acre basis. Because of the size of the economic unit under analysis, it was considered that in the optimizing procedure the resulting output would not be capable of influencing the product price level. Also, it was assumed that irrigation water was available at any moment during the growing season and the price for natural gas (main source of energy to pump water in the High Plains region) was known by the producer. The cost of other resources such as seed, fertilizer, herbicide, insecticide, machinery, tractors, labor (tractor and machinery) interest on operational capital and labor used in irrigation were classified as preharvesting costs. Harvest costs included custom combine and custom haul. Machinery, tractors, irrigation machinery and land were considered as fixed cost. These costs were obtained from budgets prepared for the Texas High Plains III Region. In the land item (net rent), government payments were not included (Texas Crop Budgets).

The model was developed to satisfy the objective of estimating optimal strategies in the use of irrigation water given different grain sorghum and natural gas prices. The expected effect of energy curtailments was estimated by systematically "blocking" the periods initially defined for irrigation. At each time, the blocked period represented a stochastic event for the decision maker. The model determines optimal amounts of irrigation under the assumption that the required levels of natural gas, at each period, would be available. The specification that natural gas curtailments exist permits model application to observe how the initially optimal strategy is affected once this event occurs. Also, it provides an estimate of the expected impact on yields as well as producers' net returns. The systematic blocking in the application of irrigation water for a specified period in the growing season means that no irrigation water could be applied during the specific period for which irrigation was previously planned. A maximum of 10 irrigation periods have been set in the model. However, if a more frequent evaluation of the requirements for irrigation water is desired, the model can easily be adapted.

Determination of optimal strategies in the use of irrigation water is dependent on the water requirements of plants, Since the photosynthetic process and, therefore, the plant development is partly dependent in the row spacing and plant population, optimal strategies were computed for specific values of these parameters.

Input Data

The generation of stochastic weather patterns needed to calculate expected optimal amounts for irrigation water required estimating the probabilities of occurrence of each weather event included in the model. To estimate these values, daily data on rainfall, maximum and minimum temperature and solar radiation occurring in each day were collected. The source of information was the Texas A&M Experiment Station located in Lubbock. The daily records obtained from this source are for the years of 1945 through 1977. Though information about temperature and rainfall were available for the entire period, solar radiation is available after 1966 only. Even after this year, there are skips in the solar radiation data.

After obtaining the required probabilistic information, these values were used to generate 30 different weather patterns. Each pattern has the climatic information required for a growing season (April-November). These 30 "years" constitute the climatic data used by the modified simulation model as depicted in Figure 2.

In addition to the probabilistic information on weather variables, model application requires specification of the number of leaves produced by the plant, the maximum area capable of being developed by each leaf, the plant population, the row spacing, the planting date, the location's latitude, the extractable soil water content on planting date and the extractable soil water capacity.

Different types of grain sorghum plants exist and their field performance depends on both the specific hybrid and the environment characteristics. The time required to reach each stage of development by the plant is light dependent and therefore the grain sorghum plant canopy must be known. To estimate the amount of intercepted light, the leaf area of the grain sorghum plant must be calculated. The plant used in the simulation model was considered to have 17 leaves--each one with a maximum area of 0.88, 2.30, 7.60, 12.30, 22.80, 42.50, 69.50, 113.00, 170.80, 248.80, 287.00, 357.50, 336.50, 340.80, 272.30, 209.30 and 116.00 cm^2, respectively (Arkin).

In the process of light interception, not all the leaf area on the plant is exposed to light source. Some leaves overlap and neighboring plants shade each other. Because the sun's altitude and azimuth determine the shading the plant canopy, the latitude of location is required. The Local Climatological Data, U.S. Department of Commerce, reports the latitude for Lubbock as being 33° 33' N.

Also, the plant population as well as the row spacing are factors influencing the amount of light being intercepted along with the amount of nutrients required. A row spacing of 68.60 cm was used in equipopulated plantings of grain sorghum together with a population level of 148,200 plants per Ha.

With respect to the soil water content, it was considered to have a maximum capacity of 15 cm. The extractable soil water content on planting date was considered to be 10 cm to reflect a preplant irrigation of approximately 4 inches.

Finally, two levels for the price of natural gas were considered in the study, i.e., \$1.50/MCF and \$2.50/MCF. The cost of the natural gas was translated to be the cost of irrigation water where the well had a lift of 250 feet and 30 PSI pressure.

The evaluation of the optimal applications of irrigation water also required the knowledge of the value assigned to the grain sorghum. The loan rate (\$3.37) and the target price (\$4.07) were used in the analyses. All other costs considered are reproduced in table

1. To reflect harvesting and hauling costs, the price of grain sorghum was reduced by custom costs of combining and hauling.

Model Output

When the simulation of the grain sorghum model is stopped due to the physiological maturity of the plant, the MAIN program proceeds to print the economic information used to evaluate the performance of the model. The output is delivered under a similar format to that used to report the estimated costs and returns per acre in the enterprise budgets for the Texas High Plains.

Forecasting as well as policy decisions are made based on conditioned probabilities. Thus, whenever the user requests several simulations (years) of using the entire model, statistical information with respect to net returns, grain sorghum yields and the amounts of irrigation water applied may be obtained. To meet this objective, the computer program was made to consist of a two-step job. The first-step is to generate values corresponding to the aforementioned variables by means of replicated simulations and placing this information on an on-line storage facility. The second-step recalls the stored values and by means of the Statistical Analysis System (SAS) package obtains the mean, variance, maximum and minimum values, t values, and other statistics for each of the variables depicted above.

Table 1. Grain Sorghum, Irrigated, Texas High Plains III Region Estimated Costs per Acre (Sprinklers).

	Unit	Price or Cost/Unity	Quantity
Preharvest			
Seed	lbs.	0.45	12.50
Fert (120-40-0)	acre	27.20	1.00
Herbicide	acre	3.85	1.00
Insecticide	acre	5.00	1.00
Machinery	acre	4.10	1.00
Tractors	acre	8.69	1.00
Labor (tractor & machinery)	hour	5.00	3.75
Interest on Op. Cap.	dol.	0.10	41.55
Harvest Costs			
Custom Combine [b]	cwt.	0.30	
Custom Haul [b]	cwt.	0.25	
Fixed Costs			
Machinery	acre	5.32	1.00
Tractors	acre	9.75	1.00
Irrigation Machinery	acre	29.28	1.00
Land (net rent) [a]	acre	30.09	1.00

[a] In land (net rent) government payment is not included.
[b] Varies with the level of production.

Source: Prepared by Marvin O. Sartin, TAEX, Lubbock, Texas. Projected Texas Crop Budgets.

CHAPTER IV

MODEL APPLICATIONS AND RESULTS

Previous economic studies on estimating optimal amounts of irrigation water are primarily based on the assumption that the producer knew in advance the state of the different future environments that would surround him. Climatic, institutional and economic conditions throughout the production year were considered as known at the beginning of the year. This assumption constitutes a serious over-simplification and limits the applicability of the models as well as the conclusions that can be drawn.

To overcome the above limitations, this study adopted a computerized grain sorghum growth program to consider stochastic situations in either weather, economic conditions and/or institutional factors. The flexibility of the model to incorporate environmental changes makes it highly attractive as a farm management tool as well as a valuable research model.

The model was used to address two basic issues: (1) to estimate irrigation strategies that will maximize net returns per acre of sorghum and (2) analyze the expected effects that irrigation fuel curtailments would have on the distribution of optimum amounts of irrigation water among the remaining irrigation periods and associated impact on grain sorghum yield and net returns.

The analysis of optimum irrigation strategies and effect of fuel curtailment were both addressed under a scenario of (1) a perfect knowledge case (known weather pattern) for which the open-loop feedback control yields the optimal (or closed-loop) solution and (2) the stochastic case (random climatic values as well as uncertain curtailments) for which the open-loop feedback control is used as an alternative to the optimal control. Although the result obtained is not information optimal, as described earlier, it is the best attainable solution considering the intricate relationships involved in the model and the amount of information used by the control policy. The analysis was extended over 30 different generated sets of weather patterns. These sets could be interpreted as 30 years experience.

Perfectly Known Environments

In this section, all environments (economic, climatic and institutional) are considered to be known at the beginning of the simulation process. For this case, the open-loop feedback policy generates values that are the same as those produced by the closed-loop control and therefore no higher values for the objective function could be obtained. Also, different from the stochastic case, the values of the control variable can be, and in fact are, established in the first period. This is because in all subsequent periods--the expected values are identical to the realized or actual values, hence, no adjustments are needed.

Optimal Irrigation Strategies

With all conditions known, the optimal amounts of irrigation water to be applied were determined for each period within a growing season over the 30 years of different weather patterns. The amounts of water used in the eight irrigation periods within a specific year differed one period to another. Similarly, the levels applied in each period also differed across the years. These results indicate that, although optimal irrigation strategies are possible of being generated, there is not a "unique strategy" to be applied at all times. In fact, the determination of the strategy to follow depends on the existing conditions at the beginning of the period for which the irrigation would take place. The presence or absence of rainfall, its timing and amount in conjunction with the value of other weather factors, such as temperature and solar radiation cause different irrigation patterns every time. To illustrate table 2 presents the results from the simulation runs number 1, 2 and 3 under the perfect knowledge assumption. In all three cases reported, the amount of water applied at each period differs within and across years. The same type of results are obtained for the other 27 simulated weather patterns. Though these outcomes may prevent us from identifying one optimal strategy for all possible cases, some inferences can be made from them.

To maximize per acre net returns, under conditions where the producer has perfect knowledge about the future states of nature as well as economic and institutional conditions, irrigation occurs more frequently and involves smaller amounts of water in each application than is currently practiced in the area. Stated in a slightly different form, frequent irrigations with small applica-

Table 2. Optimal Quantity of Irrigation to Maximize Net Returns Per Acre of Grain Sorghum with Known Weather Patterns: Texas High Plains.

```
IRRIGATION SCHEDULES FOR YEAR  1 (SEASON APRIL THROUGH SEPTEMBER)

       PLANTING DATE = 121        CALENDAR DAY =     5        1
              THE OPTIMAL IRRIGATION PATTERN IS:
       THE FIRST    IRRIGATION HAS A VALUE OF        2.42 IN.
       THE SECOND   IRRIGATION HAS A VALUE OF        0.58 IN.
       THE THIRD    IRRIGATION HAS A VALUE OF        1.38 IN.
       THE FOURTH   IRRIGATION HAS A VALUE OF        1.60 IN.
       THE FIFTH    IRRIGATION HAS A VALUE OF        1.09 IN.
       THE SIXTH    IRRIGATION HAS A VALUE OF        1.36 IN.
       THE SEVENTH  IRRIGATION HAS A VALUE OF        0.69 IN.
       THE EIGHTH   IRRIGATION HAS A VALUE OF        1.60 IN.
       PHYSIOLOGICAL MATURITY OCCURRED ON JULIAN DAY  234
      TOTAL GRAIN PRODUCED = 89.00 CWT, PROFIT = $105.65

IRRIGATION SCHEDULES FOR YEAR  2  (SEASON APRIL THROUGH SEPTEMBER)

       PLANTING DATE = 121        CALENDAR DAY =     5        1
              THE OPTIMAL IRRIGATION PATTERN IS:
       THE FIRST    IRRIGATION HAS A VALUE OF        2.63 IN.
       THE SECOND   IRRIGATION HAS A VALUE OF        1.40 IN.
       THE THIRD    IRRIGATION HAS A VALUE OF        1.50 IN.
       THE FOURTH   IRRIGATION HAS A VALUE OF        0.72 IN.
       THE FIFTH    IRRIGATION HAS A VALUE OF        1.70 IN.
       THE SIXTH    IRRIGATION HAS A VALUE OF        1.36 IN.
       THE SEVENTH  IRRIGATION HAS A VALUE OF        1.42 IN.
       THE EIGHTH   IRRIGATION HAS A VALUE OF        1.09 IN.
       PHYSIOLOGICAL MATURITY OCCURRED ON JULIAN DAY  237
      TOTAL GRAIN PRODUCED = 96.67 CWT, PROFIT = $130.64

IRRIGATION SCHEDULES FOR YEAR  3  (SEASON APRIL THROUGH SEPTEMBER)

       PLANTING DATE = 121        CALENDAR DAY =     5        1
              THE OPTIMAL IRRIGATION PATTERN IS:
       THE FIRST    IRRIGATION HAS A VALUE OF        3.12 IN.
       THE SECOND   IRRIGATION HAS A VALUE OF        0.64 IN.
       THE THIRD    IRRIGATION HAS A VALUE OF        0.71 IN.
       THE FOURTH   IRRIGATION HAS A VALUE OF        1.27 IN.
       THE FIFTH    IRRIGATION HAS A VALUE OF        0.99 IN.
       THE SIXTH    IRRIGATION HAS A VALUE OF        0.88 IN.
       THE SEVENTH  IRRIGATION HAS A VALUE OF        1.59 IN.
       THE EIGHTH   IRRIGATION HAS A VALUE OF        1.15 IN.
       PHYSIOLOGICAL MATURITY OCCURRED ON JULIAN DAY 228
      TOTAL GRAIN PRODUCED = 90.82 CWT, PROFIT = $115.74

PSORG = 4.07 P NAT. GAS = 2.5 PSI = 30.0 LFT = 250.0 WCOST = 1.81
  DAYS PLANNED TO IRRIGATE:   150  160  170  180  190  200  210  220
```

[a] This is an example of the model printout for a three year period. Water applied is irrigation only and refers exclusively to water at the plant. There are no distribution system or environmental losses. The cost of water used in irrigation was adjusted using a coefficient of 1.66 to reflect the cost that would be incurred due to distribution systems and environmental losses.

PSORG Price of Sorghum per cwt., P NAT. GAS Price of Natural Gas per mcf., WCOST Cost of Water for a well with 250 feet of lift (LFT) and 30 psi. (PSI).

tion rates produce, ceteris paribus, higher net returns than in the case where the same total amount of irrigation water is applied in a less frequent bases. This statement is constraint by the limited amount of wells existing on a typical farm. Also, application rates are sensitive to acres irrigated and the type of distribution system.

In repeated samples, 95 percent of the time, the total amount of water to be applied could be expected to be in the range of 10.5 to 11.5 inches. This is irrigation water applied to the plant and does not include distribution system or environmental losses associated with applications. Plant and soil evaporation are included in the model. In addition, preplant irrigation water is not included. It is assumed that soil moisture is available at planting. Thus, to obtain total irrigation needed, the preplant application, (about 4 inches net to the plant) would have to be added.

The average distribution of the amount of irrigation required at each period is presented in table 3. The mean values were tested under the null hypothesis that they did not differ from zero. In all cases, this hypothesis was not accepted at the 95 percent level of confidence. An average amount of 1.1 to 1.4 inches would be required in all periods but the first, which needs an average amount of 2.6 inches.

Of particular importance are the maximum and minimum values of irrigation water required at each period and especially the minimum values. These quantities of irrigation water, over the 30 year weather pattern, indicate given the prices of product and re-

Table 3. Optimal Post Plant Irrigations to Maximize Net Returns Per Acre of Grain Sorghum with Perfectly Known Weather Patterns: Texas High Plains.[a]

Days After Plant Emergence	Post Plant Irrigation Period	Application Rate Characteristics				
		Mean[b]	Standard Deviation	Minimum Value	Maximum Value	Coefficient of Variation
		Inches				%
20	1	2.62	0.45	1.49	3.23	17.09
30	2	1.10	0.38	0.00	1.59	34.15
40	3	1.34	0.57	0.00	2.21	42.63
50	4	1.15	0.57	0.00	2.03	49.63
60	5	1.19	0.55	0.00	1.97	46.52
70	6	1.35	0.38	0.45	1.91	28.57
80	7	1.25	0.38	0.16	1.76	30.66
90	8	1.12	0.45	0.22	1.74	39.89

[a] Based on a price of grain sorghum of $4.07/cwt. and price of Natural Gas of $2.50/mcf.

[b] Based on 30 simulated replicates.

sources and the states of nature, to maximize net revenues the producer can expect to irrigate in all years in at least four periods; i.e., the first, sixth, seventh and eighth. In the first period, a minimum amount of about 1.5 inches can be expected in any year.

Under the present set of economic, institutional and climatic assumptions used, the average yield and net return that could be expected are about 90 cwt and $110, respectively, as shown in table 4. In repeated samples, 95 percent of the time the values for these two variables would be expected to fall in the range of 87-81 cwt and $102-116. This provides an indication of the optimum allocation of water under a deterministic scenario and offers some insights on the process that would take place when all variables are known. Yet, the major contribution is that these results constitute a landmark or base against which the effectiveness of obtaining optimal results form applying the stochastic feedback control can be compared.

Stochastic Environments

For this section of the analysis, the complete knowledge assumption was released so the model operated in a situation of (1) stochastic climatic environments, and (2) stochastic climatic and institutional contexts. In the first case, results obtained from using the open-loop feedback control are compared to the results obtained from the deterministic (or optimal solution) to provide

Table 4. Per Acre Net Returns, Yield and Irrigation Level for 30 Years of Perfectly Known and Stochastic Weather Patterns: Texas High Plains.[a]

Item	Perfectly Known			Stochastic		
	Net Return	Yield	Water Used[b]	Net Return	Yield	Water Used[b]
	($)	(cwt)	(in.)	($)	(cwt)	(in.)
Mean	109.54	89.89	11.11	99.36	89.94	13.80
S. D.	17.99	5.02	1.18	17.81	5.06	0.80
MN. V.	75.20	79.38	8.98	61.67	79.38	11.21
MX. V.	146.82	99.28	13.58	132.43	99.28	15.05
C. V.	16.42	5.58	10.62	17.92	5.63	5.79

S. D. Standard Deviation, MN. V. Minimum Value, MX. V. Maximum Value, C. V. Coefficient of Variation.

[a]Price of Sorghum $4.07/cwt., Price of Natural Gas $2.50/mcf.

[b]Irrigation water used. Does not include preplant irrigation.

some implications of the effect of uncertainty and/or risk and overall performance of the model.

Optimal Irrigation Strategies

Due to imperfect knowledge about future weather conditions, it is necessary to formulate a scheme to generate expected values about the future climatic variables. This was accomplished by including a set of statistics defining the probability values for each of the weather variables in the model. Following the simulation of the expected weather pattern, net revenues and yields were calculated by defining a specific time path for the control variable; i.e., time and quantity of irrigation water to apply. The value of the vector for the control variable that maximizes net returns is adopted and considered to be the optimal strategy. Different to the perfect knowledge case, however, the true optimal irrigation strategy cannot be determined at the beginning period. The initial irrigation strategy is revised at each subsequent period as the model progresses. Based on the values of the state variables a new optimal irrigation strategy is defined. To illustrate, table 5 presents the results obtained for the simulated year 10. It can be observed that at the first period the value for all 8 future irrigations is estimated based on some expected pattern for the weather. Given these values (1.59, 1.44, 0.83, 0.78, 0.91, 0.39, 1.58 and 1.10 inches) for the control variable in each period, the total product that can be expected is 89.95 cwt. with a

Table 5. Optimal Quantity of Irrigation to Maximize Net Returns Per Acre of Grain Sorghum with Stochastic Weather Patterns: Texas High Plains.[a]

```
IRRIGATION SCHEDULES FOR YEAR    10 (SEASON APRIL THROUGH SEPTEMBER)

          PLANTING DATE = 121    CALENDAR DAY     5       1
                  THE OPTIMAL IRRIGATION PATTERN IS:
  THE FIRST   OF THE REMAINING IRRIGATIONS HAS A VALUE OF  1.59 IN.
  THE SECOND  OF THE REMAINING IRRIGATIONS HAS A VALUE OF  1.44 IN.
  THE THIRD   OF THE REMAINING IRRIGATIONS HAS A VALUE OF  0.83 IN.
  THE FOURTH  OF THE REMAINING IRRIGATIONS HAS A VALUE OF  0.78 IN.
  THE FIFTH   OF THE REMAINING IRRIGATIONS HAS A VALUE OF  0.91 IN.
  THE SIXTH   OF THE REMAINING IRRIGATIONS HAS A VALUE OF  0.39 IN.
  THE SEVENTH OF THE REMAINING IRRIGATIONS HAS A VALUE OF  1.58 IN.
  THE EIGHTH  OF THE REMAINING IRRIGATIONS HAS A VALUE OF  1.10 IN.
   THE EXPECTED TOTAL PHYSICAL PRODUCTIVITY IS:  89.95 CWT.  WITH
   PROFIT = $119.35.
   THE AMOUNT OF WATER USED HAS BEEN 0.0 AND IT IS EXPECTED THAT
   ANOTHER 8.61 INCHES WILL BE REQUIRED.


                  THE OPTIMAL IRRIGATION PATTERN IS:
  THE FIRST   OF THE REMAINING IRRIGATIONS HAS A VALUE OF  1.44 IN.
  THE SECOND  OF THE REMAINING IRRIGATIONS HAS A VALUE OF  0.90 IN.
  THE THIRD   OF THE REMAINING IRRIGATIONS HAS A VALUE OF  1.00 IN.
  THE FOURTH  OF THE REMAINING IRRIGATIONS HAS A VALUE OF  1.60 IN.
  THE FIFTH   OF THE REMAINING IRRIGATIONS HAS A VALUE OF  1.69 IN.
  THE SIXTH   OF THE REMAINING IRRIGATIONS HAS A VALUE OF  1.58 IN.
  THE SEVENTH OF THE REMAINING IRRIGATIONS HAS A VALUE OF  1.21 IN.
   THE EXPECTED TOTAL PHYSICAL PRODUCTIVITY IS:  89.90 CWT.  WITH
   PROFIT = $110.94.
   THE AMOUNT OF WATER USED HAS BEEN 1.59 AND IT IS EXPECTED THAT
   ANOTHER 9.41 INCHES WILL BE REQUIRED.


                  THE OPTIMAL IRRIGATION PATTERN IS:
  THE FIRST   OF THE REMAINING IRRIGATIONS HAS A VALUE OF  0.90 IN.
  THE SECOND  OF THE REMAINING IRRIGATIONS HAS A VALUE OF  1.15 IN.
  THE THIRD   OF THE REMAINING IRRIGATIONS HAS A VALUE OF  1.60 IN.
  THE FOURTH  OF THE REMAINING IRRIGATIONS HAS A VALUE OF  1.69 IN.
  THE FIFTH   OF THE REMAINING IRRIGATIONS HAS A VALUE OF  1.58 IN.
  THE SIXTH   OF THE REMAINING IRRIGATIONS HAS A VALUE OF  1.22 IN.
   THE EXPECTED TOTAL PHYSICAL PRODUCTIVITY IS:  89.25 CWT.  WITH
   PROFIT = $109.57.
   THE AMOUNT OF WATER USED HAS BEEN 3.03 AND IT IS EXPECTED THAT
   ANOTHER 8.14 INCHES WILL BE REQUIRED.
```

Table 5. (Continued)

```
                     THE OPTIMAL IRRIGATION PATTERN IS:
THE FIRST   OF THE REMAINING IRRIGATIONS HAS A VALUE OF  1.15 IN.
THE SECOND  OF THE REMAINING IRRIGATIONS HAS A VALUE OF  1.60 IN.
THE THIRD   OF THE REMAINING IRRIGATIONS HAS A VALUE OF  1.69 IN.
THE FOURTH  OF THE REMAINING IRRIGATIONS HAS A VALUE OF  1.58 IN.
THE FIFTH   OF THE REMAINING IRRIGATIONS HAS A VALUE OF  1.23 IN.
 THE EXPECTED TOTAL PHYSICAL PRODUCTIVITY IS:  89.34 CWT.  WITH
 PROFIT = $110.63.
 THE AMOUNT OF WATER USED HAS BEEN 3.93 AND IT IS EXPECTED THAT
 ANOTHER 7.25 INCHES WILL BE REQUIRED.

                     THE OPTIMAL IRRIGATION PATTERN IS:
THE FIRST   OF THE REMAINING IRRIGATIONS HAS A VALUE OF  1.62 IN.
THE SECOND  OF THE REMAINING IRRIGATIONS HAS A VALUE OF  1.69 IN.
THE THIRD   OF THE REMAINING IRRIGATIONS HAS A VALUE OF  1.58 IN.
THE FOURTH  OF THE REMAINING IRRIGATIONS HAS A VALUE OF  1.23 IN.
 THE EXPECTED TOTAL PHYSICAL PRODUCTIVITY IS:  81.39 CWT.  WITH
 PROFIT = $83.52.
 THE AMOUNT OF WATER USED HAS BEEN 5.08 AND IT IS EXPECTED THAT
 ANOTHER 6.12 INCHES WILL BE REQUIRED.

                     THE OPTIMAL IRRIGATION PATTERN IS:
THE FIRST   OF THE REMAINING IRRIGATIONS HAS A VALUE OF  1.69 IN.
THE SECOND  OF THE REMAINING IRRIGATIONS HAS A VALUE OF  1.58 IN.
THE THIRD   OF THE REMAINING IRRIGATIONS HAS A VALUE OF  1.25 IN.
 THE EXPECTED TOTAL PHYSICAL PRODUCTIVITY IS:  88.80 CWT.  WITH
 PROFIT = $110.88.
 THE AMOUNT OF WATER USED HAS BEEN 6.70 AND IT IS EXPECTED THAT
 ANOTHER 4.52 INCHES WILL BE REQUIRED.

                     THE OPTIMAL IRRIGATION PATTERN IS:
THE FIRST   OF THE REMAINING IRRIGATIONS HAS A VALUE OF  1.58 IN.
THE SECOND  OF THE REMAINING IRRIGATIONS HAS A VALUE OF  1.25 IN.
 THE EXPECTED TOTAL PHYSICAL PRODUCTIVITY IS:  82.36 CWT.  WITH
 PROFIT = $89.61.
 THE AMOUNT OF WATER USED HAS BEEN 8.38 AND IT IS EXPECTED THAT
 ANOTHER 2.83 INCHES WILL BE REQUIRED.

                     THE OPTIMAL IRRIGATION PATTERN IS:
THE FIRST   OF THE REMAINING IRRIGATIONS HAS A VALUE OF  1.25 IN.
 THE EXPECTED TOTAL PHYSICAL PRODUCTIVITY IS:  84.97 CWT.  WITH
 PROFIT = $100.13.
 THE AMOUNT OF WATER USED HAS BEEN 9.97 AND IT IS EXPECTED THAT
 ANOTHER 1.25 INCHES WILL BE REQUIRED.

 PHYSIOLOGICAL MATURITY OCCURRED ON JULIAN DAY 229
 TOTAL GRAIN PRODUCED = 79.96 CWT, PROFIT = $74.17.
```

[a] This represents a printout from the model indicating the planning process through the irrigation periods of one year.

net revenue of $119.35. It also indicates the amount of irrigation water that has been used up to that period. At the beginning period the amount was 0.0 inches which does not include a preplant irrigation. The sum of the values for all remaining irrigations (8.61 inches) is also reported. After the values for the control vector have been estimated, the first period decision e.g.: (the first of the remaining irrigations) is adopted. The first irrigation applied to the system, therefore, has a value of 1.59 inches.

At the beginning of the second irrigation period, the procedure is repeated (1) the set of expectations on weather variables is generated based on weather experienced in the first period (2) the optimal values for the remaining subset of the control variable is numerically searched (1.44, 0.90, 1.00, 1.60, 1.69, 1.58 and 1.21 inches) and (3) the first of the remaining decisions (1.44 inches) is applied. It should be noticed that the reevaluation has altered the values of irrigation water in two ways: the number of irrigation periods is not eight but seven since the first period decision was already made; the second modification is in the values of the control variable; i.e., the irrigation values for the remaining periods have changed (1.44, 0.90, 1.00, 1.60, 1.69, 1.58, and 1.21 inches). Expected yields and net revenues also show different values (89.80 cwt and $110.94, respectively), based on the new strategy, the physical development of the plant up to that period and the new expectation on weather variables. Changes in the control variable and, therefore, in yields and net revenues are detailed in the same fashion for the rest of the season. At the end, when physiological maturity of the plant occurs, the actual grain produced (79.96 cwt.) is used to obtain the corresponding net revenue ($74.17) and the value for the amount of irrigation water actually used (9.97 + 1.25 inches) is detailed. The closer the open-loop feedback solution to the values obtained for the perfect knowledge case, the more accurate and, therefore, the more valuable the control policy is to decision makers.

For this particular year, the solution for the perfect knowledge case gave the values of $82.75 for profit, 79.96 cwt. for yield, and 8.98 inches of irrigation water. The open-loop feedback control obtained the same yield. However, the amount of irrigation water used was about 25 percent higher than in the complete knowledge case. This higher value was reflected in a 10 percent lower net return also with respect to the deterministic case.

A detailed description of the values obtained for all 30 years under perfect knowledge and stochastic conditions are presented in Appendix , Tables 1 and 2. For all practical purposes, the yields obtained for both scenarios do not differ. It is in the amount of water applied where the differences are found implying there is a price to pay for the lack of knowledge. This is shown in the higher amounts of irrigation water used which in turn is reflected in smaller net revenues (table 4).

The distribution of the average amount of irrigation water required at each period is presented in table 6. The mean values obtained were statistically tested using the null hypothesis that they

Table 6. Optimal Quantity of Post Plant Irrigations to Maximize Net Returns Per Acre of Grain Sorghum with Stochastic Weather Patterns: Texas High Plains.[a]

Days After Plant Emergence	Post Plant Irrigation Period	Application Rate Characteristics				
		Mean[b]	Standard Deviation	Minimum Value	Maximum Value	Coefficient of Variation
		Inches				%
20	1	2.63	0.42	1.49	3.12	16.19
30	2	1.25	0.23	0.62	1.54	18.62
40	3	1.66	0.30	0.90	2.22	17.90
50	4	1.63	0.21	1.15	2.05	12.58
60	5	1.69	0.16	1.36	2.02	9.68
70	6	1.68	0.19	1.13	1.94	11.50
80	7	1.67	0.15	1.37	1.98	8.93
90	8	1.60	0.14	1.25	1.86	8.67

[a]Based on a price of Grain Sorghum of $4.07/cwt. and price of Natural Gas of $2.50/mcf.

[b]Based on 30 simulated replicates.

did not differ from zero. In all cases, the hypothesis was not accepted at the 95 percent level of confidence. Compared to the deterministic case, the mean values were always alightly higher for the stochastic case. This is the result of not possessing complete knowledge about the occurrence of rainfall, therefore, consistantly higher amounts are applied. The relative spectrum of variation (measured by the coefficient of variation), for net returns and yields follow an opposite pattern to the one for the average total amount of irrigation water. While the coefficient of variation for net returns and yields increased for the stochastic case compared to the deterministic values, the coefficient for irrigation water, was smaller when the assumption of complete knowledge did not hold. A possible explanation can be found in the nature of the scenarios. Complete knowledge of the weather allowed determination of maximum net revenues with lower quantities of irrigation water. Since in the perfect knowledge case, irrigation is used as an exact complement to actual or realized rainfall (i.e., whenever rainfall occurs, the necessity for artificial rain is less and vice versa) its range of fluctuation is broader than in the stochastic case where decisions about the expected rainfall are made based on the mean values of the distribution. Stated differently, partial knowledge tends, on the average, to make use of consistently higher values for irrigation water resulting in a decrease of its relative spectrum of variation, however, that increases the variation of associated net revenues (as shown in table 4).

Thus, in summary, the performance of the model (when the assumption of known climatic conditions is relaxed) is reflected in irrigation levels slightly higher than the strictly necessary levels; i.e., approximately 25 percent higher with respect to the deterministic case. This in turn means lower net returns with wider fluctuations. However, yields obtained are the same as those in the perfect knowledge case.

Energy Curtailments

The energy issue involves not only increasing costs but also the possibility of curtailments. To evaluate the effects that potential energy curtailments may have on producer's net revenues, yields and water used, fuel curtailments were incorporated by blocking any irrigation for a predetermined period or periods. For this analysis, stochastic weather patterns were also used. Different sets of shocks (curtailments) were introduced to the system as random events. Three arbitrarily defined time spans of curtailments were examined. In all cases analyzed it was considered the producer could not foresee the future existence of energy restrictions, so optimal allocations of irrigation water were sought assuming complete availability of water. For those periods where curtailments had been defined to take place, the initial decision to irrigate was overridden forcing the model to consider rain as the only source of water. In the present context, curtailments should be considered as the number of days by which planned irrigation is delayed.

Ten-Day Curtailment. For 10-day curtailments there were no changes in yields or total amounts of irrigation water as compared to the stochastic situation presented in table 4. Only the distribution of the water used per period changed. The analysis suggests, to obtain maximum net returns, the best action to follow after a 10-day curtailment is to supply the plants with the moisture required (table 7) so no major stress is imposed in the normal growth. This alternative is subject to the condition that optimal amounts of water had been supplied in the previous periods, and, also, they will be delivered immediately after the curtailment has ceased. Implicitly assumed herein is that the producer has excellent control of irrigation water across the farm.

Twenty-Day Curtailments. Net revenues for the three different 20-day curtailments analyzed reflected a decline in their mean value. Net returns are more severely affected when curtailment takes place during the period of 40-60 days after emergence (table 8 and Appendix table 13). During this period, the head is developing and light interception approaches to its maximum. Because in this period, growth and nutrient uptake occur rapidly, adequate supplies of nutrients and water are necessary. Stressing the crop by limiting the

Table 7. Average Optimal Quantity of Post Plant Irrigation to Maximize Net Returns Per Acre of Grain Sorghum with Stochastic Weather Patterns and Irrigation Curtailments: Texas High Plains.[a]

Curtailment During Days After Emergence	Irrigation Period - Days after Emergence[b]							
	20	30	40	50	60	70	80	90
	--Inches--							
20-30	--	3.05	1.68	1.63	1.68	1.68	1.68	1.60
30-40	2.63	--	2.16	1.67	1.69	1.68	1.67	1.60
40-50	2.63	1.25	--	2.47	1.71	1.68	1.67	1.60
50-60	2.63	1.25	1.66	--	2.28	1.70	1.67	1.60
60-70	2.63	1.25	1.66	1.63	--	2.39	1.70	1.60
No curtailment	2.63	1.25	1.66	1.63	1.69	1.68	1.67	1.60

[a] Based on a price of Grain Sorghum of $4.07/cwt. and price of Natural Gas of $2.50/mcf.

[b] Based on 30 simulated replicates.

Table 8. Estimated Effect of 20 Day Curtailments on Per Acre Yield Net Returns and Water Use for Grain Sorghum: Texas High Plains.[a]

Item	Days After Emergence When Curtailment Occurred								
	20-40			40-60			60-80		
	Net Return	Yield	Water Used[b]	Net Return	Yield	Water Used[b]	Net Return	Yield	Water Used[b]
	($)	(cwt)	(in.)	($)	(cwt)	(in.)	($)	(cwt)	(in.)
Mean	94.64	86.45	11.83	71.45	80.01	11.97	83.37	83.56	12.12
Standard Deviation	19.47	5.45	0.96	29.91	7.91	0.94	25.10	6.70	0.99
Minimum Value	56.12	75.66	8.18	11.60	64.21	10.31	37.75	68.34	9.74
Maximum Value	129.84	96.27	13.42	119.22	92.43	13.36	135.28	96.21	14.06
Coefficient Variation	20.57	6.30	8.11	41.86	9.89	7.85	30.11	8.02	8.17

[a]Based on 30 simulated replicates and having price of Grain Sorghum $4.07/cwt., price of Natural Gas $2.50/mcf.

[b]Irrigation water used. Does not inlcude preplant irrigation.

amount of water has adverse consequences in the net revenues received.

In the case of curtailments during the 40-60 days after emergence period, net returns had the lowest value for the 20-day institutional constraint. They decreased in 28 percent with respect to the no-curtailment case. What is even more significant, though, is the increase in their variation: from an 18 percent (for the no-curtailment case) to a 42 percent. The minimum value dropped from $62 to $12 while maximum values declined from $132 to $119. The same trend was observed for yields obtained; from a mean of 90 cwt. without curtailment to 80 cwt. (approximately 11 percent less).

Similar consequences were observed for curtailments during the half-bloom and soft-dough stage of the plant (approximately 60 through 80 days after emergence). In these periods, grain formation begins. If any limitations occurred in plant size and leaf area, they cannot be corrected. Severe moisture stress will then be reflected in poor head filling. Net returns during these stages were observed to decrease about 16 percent (from $99 for the no-curtailment case to $83), and their variation increased from 18 percent to 30 percent. Yields for these periods, also experienced a decline of 7 percent.

Briefly stated, the existence of stochastic curtailments had, on one side, a downward effect on net revenues and yields, decreasing them up to 28 and 11 percent, respectively; while on the other side, they increased the fluctuations associated with production (measured by the coefficient of variation) by more than 100 percent, for the case of curtailment during the 40-60 period.

Thirty-Day Curtailment. Impacts similar to the 20-day curtail are obtained for the 30-day case except the effects are greater (table 9). The periods where curtailment was imposed were 20-50 and 40-70 days after plant emergence. Of the two instances, the latter reflected more severe impacts. During these periods, and different to the cases analyzed before, negative net returns are present in the solutions obtained for some years (Appendix table 4).

The net returns and yields for the 20-50 and 40-70 day after plant emergence curtailment are $67 and $50 and 77 cwt. and 73 cwt., respectively, representing a decrease of almost 50 percent and 19 percent with respect to the non-institutional constraint case.

In summary, the primary effects of irrigation curtailments are to reduce net revenues and increase the relative spectrum of fluctuation. The amount in which they are reduced is dependent upon the stage of growth of the plants and the time span of the curtailment. The impacts estimated and detailed in this study may be considered as conservative: they assume that the producer can immediately irrigate after curtailment, whereas, in reality with limited wells, it may be several days after the curtailment has ended before he can irrigate all acres.

Table 9. Estimated Effect of 30 Day Curtailments on Per Acre Yield, Net Returns and Use for Grain Sorghum: Texas High Plains.[a]

Item	Days After Emergence When Curtailments Occurred					
	20-50			40-70		
	Net Return	Yield	Water Used[b]	Net Return	Yield	Water Used[b]
	($)	(cwt)	(in.)	($)	(cwt)	(in.)
Mean	66.97	77.22	10.59	50.16	72.92	11.02
Standard Deviation	35.31	9.60	1.03	33.91	9.08	0.90
Minimum Value	-10.16	56.18	7.81	-11.73	56.52	8.77
Maximum Value	119.85	92.54	12.04	117.36	91.60	12.61
Coefficient Variation	52.73	12.43	9.73	67.60	12.45	8.71

[a]Based on 30 simulated replicates.

[b]Irrigation water used. Does not include preplant irrigation.

Parametric Analysis

A common approach to test sensitivity of a solution or results is by changing parameters in the model. The principal concern in this analysis was to estimate the effect of price changes for grain sorghum and natural gas on net revenue and yield. Four different sets of prices were considered. The price of grain sorghum was assumed to be \$4.07/cwt. and \$3.37/cwt.; while the price for natural gas was set at \$1.50/mcf and \$2.50/mcf. All four combinations were tested and the results indicated no major changes in the use of irrigation water occurred. Yields remained essentially unchanged. Net revenues adjusted directly proportional to the change of the price for grain sorghum, and inversely proportional to the changes in the price of natural gas (Appendix table 5).

The implication is that in the short run, such as one growing season, the marginal value product is so sufficiently large that it compensates the increase in costs at least within the price ranges analyzed. This suggests, within the irrigation levels and yield obtained herein, the expected yield is sensitive to reductions in water use and insensitive to additional water.

CHAPTER V

SUMMARY AND CONCLUSIONS

The High Plains region of Texas constitutes the largest cultivated area in the state. Economic development in this region has been primarily associated with the introduction and the expansion of irrigation. This practice has increased the productivity of fields previously producing under dryland conditions and, further, it has enabled irrigation farming on other land areas previously not used for crop production. As a result a multiplicative effect in the economy has been generated not only throughout the region but also for the entire state.

The agricultural production and associated economic effects of irrigation on the Texas High Plains are seriously threatened by two factors: (1) a rapidly declining groundwater supply and (2) a swift upward trend in energy costs. The continued mining of the available groundwater supply emphasizes the need for better planning and more effective irrigation strategies to provide for conservation of the resource. With respect to the energy issue, the region has not only experienced rapid increases in price, but, recent national policy allows federal authorities to reallocate fuel supplies for emergency use to other regions. Therefore, not only higher costs are to be expected but also the possibility of energy curtailments.

As a result of considering potential energy curtailments during the growing season, there is an increase in risk and uncertainty involved in agricultural production. The magnitude of such risk and uncertainty is even greater when farming occurs in regions with low and unstable rainfall as in the case of the Texas High Plains.

Methodology

Previous studies that deal with the water issue and particularly, the importance of irrigation follow a defined dichotomy in their methodology of analysis. There are the water studies which (1) emphasize the dynamic and physical aspects of the growing process in the soil-plant-atmosphere continuum and alternatively (2) the economic analyses, which view the production process under a static and deterministic environment. The first approach seeks to obtain the maximum physical output, while the economic concepts involved to optimally allocate resources are disregarded to a second level. The second approach stresses the importance of the economic variables to be optimized and basically neglects the dynamic and stochastic considerations involved in the production process.

The optimal allocation of irrigation water in the production process can be considered as a problem of the dynamic economic type. To illustrate, this study considered the problem in the context of control theory. Since the system analyzed progresses through time and is subject to the effects of stochastic variables, such as weather, the part of control theory denoted as adaptive stochastic control was used as framework of reference.

To evaluate the optimum use of irrigation and its importance, a grain sorghum growth simulation model was modified to include relevant economic variables. To optimize the amount of irrigation water to be applied during the production process, a stochastic open-loop feedback control policy was built into the plant growth model. At every pre-determined period of evaluation, the stochastic control policy establishes the optimal amounts of irrigation water to be applied; i.e., the amount that would maximize net returns to the producer based on expected weather patterns. This procedure starts at the beginning of the first period and estimates optimum irrigation for every period. The stochastic control policy operates under the basis of constant revision of the expectations generated at every starting point. If discrepancies between the expected and the realized values exist, then, based on current conditions a reevaluation of the control variable, irrigation water, is made and the first period of the growing season.

Within the stochastic open-loop feedback policy designed, the values for the control variable are obtained by numerical search. For this objective, the different concepts involved in numerical optimization algorithms were previously addressed and later expanded for the Quasi-Newton methods case. Finally, a set of 30 different weather patterns was created and used by the model to estimate its performance under alternative circumstances.

Results

The model was applied to estimate optimal irrigation strategies and the impact of fuel curtailments. Initially, optimal irrigation strategies were developed under the assumption of existing perfect knowledge. Under this assumption, the results indicated there was not a unique strategy to be applied in all years. The quantities of irrigation water applied depended on the initial or starting conditions of each period. The presence and amount of soil moisture, the expectations on the number of occurrences and quantities of rainfall, the stage of growth of the plant and its physical condition and finally, the expectations on other climatic variables such as maximum and minimum temperature and solar radiation, were determinants of the optimal strategy to be formulated. From these findings, it was concluded under the present assumptions, to maximize net returns, producers will follow a policy of frequent irrigations with smaller water applications than they actually used, at each time. The total amount to be applied was estimated to be in the range of 10.5 to 11.5 inches, not including a preplant irrigation. The average per acre net returns and yields were estimated to be $110 and 90 cwt., respectively.

Since one of the purposes of building the model was to perform under stochastic or real world conditions, the assumption of complete knowledge was relaxed to consider the case where the climatic environment was unknown. As in the deterministic case, the optimal amounts of irrigation water, by period, depended much on the existing initial conditions at each period. It was observed, also, with the open-loop feedback control, the results obtained for yields did not differ from those obtained in the deterministic case. The discrepancies among the two cases were found in the optimal amount of water to be applied and therefore in net returns. In the stochastic case, the use of irrigation water had a mean value of 13.80 inches, approximately 25 percent more than in the case of complete knowledge. Because of the higher amount of water used per acre, net returns were diminished by 10 percent (from $110 in the deterministic case to $99 in the stochastic case). This could be considered to be the cost of not possessing complete information. Lower net returns may be expected if irrigations are not made in an optimal way. A deviation from the optimal strategy would be expected to result in lower yields and/or higher levels of irrigation water which would contribute negatively to the net revenue. The decrease depends on the amount and timing of the irrigation made.

The expected effect of irrigation curtailment was estimated for alternative time spans. When curtailments had a length of 10 days there were no perceptible changes in the amount of net returns of yields, with respect to the no-curtailment case. These results were consistent with the deterministic environments analysis, i.e., a reallocation of irrigation water towards the following period when the institutional restriction had ceased. The implication drawn was that by having frequent irrigation periods and applying optimal amounts of water, the adverse effects of 10-day curtailment periods are buffered.

The cases of twenty and thirty-day periods were found to have highly negative effects on the outcomes, especially per acre net revenues; i.e., they decreased about 50 percent - from $99 to $50 - in the curtailment case of 40-70 days after emergence compared to the no-curtailment value. The effects were not only a decreased amount of net returns but also an increased spectrum of their relative fluctuation (from 18 percent to 68 percent for the same situations mentioned above).

It was also found for the same length curtailment, the effects were conditioned to the period in which they occurred. In the 20-day curtailment case, the most sensitive period was found 30-50 days after emergence. In the 30-day case, the 40-70 days after the emergence period reflected the lowest mean net revenue as well as the highest coefficient of variation out of the two situations analyzed ($50 compared to $67 and 68 percent compared to 52 percent, respectively).

These estimates should be taken as being conservative. To obtain them, the model--using the assumption of immediate delivery of water--optimizes the amount of irrigation water needed at the time that it is required. However, with the given state of the arts, this may not be the case. The 20 or 30 day curtailment period might be applicable to much shorter actual fuel curtailment periods. Producers lose not only the time of fuel curtailments but, also, they must cover many acres with a limited number of wells. As a result, a 10-day fuel curtailment could easily result in a 20 to 30 day delayed irrigation.

As a conclusion, it could be said that improved irrigation distribution technology could result in increased yields using less irrigation water applied and thereby also reduce the negative impacts of curtailment.

Limitations

This study could not cover and evaluate all possible alternatives that might arise, therefore, certain needed constraints and limitations were adopted:

To determine the optimal time trajectory for the control variable the assumption of profit maximization was made. However, this way may not reflect the objective of each producer. Other goals could be alternatively specified; i.e., secure some minimum net return or to produce to the extent where minimum variations in net returns are obtained. Which specification is chosen may alter the value of the control variable.

The process of numerical search to maximize the value of the objective functional by no means assures a global rather than a local maximum. Though parametric analyses were made to assure that no other higher values exist, at least in the vicinity, the possibility of finding higher values is still open. The main reason for this resides in the highly non-linear shape of the objective functional.

The optimizing of net revenues is made under the assumption water is available at the time needed and in the amounts required. However, as mentioned before, a limited number of wells imposes the restriction of a limited amount of water that can be obtained in a given period of time.

The optimization is made with respect to the use of water and assumes the plant is affected exclusively by the climate. The inclusion of fertilizer and insect interaction would help on the predictive ability of the model and in the economic decision criterion, i.e., model sensitivity would be increased for input or product changes if other interactive inputs were included.

Grain sorghum is considered to be the only crop under production. Real farm situations grow more than one crop. In cases where water is a limited factor, the existing crops would compete for the supply of the resource determining a possible different allocation for the control variable.

Tilling of the grain sorghum plant can result in significant variations in the plant population. If this occurs, the predictive ability of the model may be seriously affected.

The eight irrigation periods used throughout the analysis are advanced technology of water distribution not available now to the average size farm.

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APPENDIX

Table 1. Per Acre Net Returns, Yield and Irrigation Level for 30 Years of Deterministic and Stochastic Weather Patterns: Texas High Plains.[a]

Year	Deterministic			Stochastic		
	Net Return	Yield	Water Used[b]	Net Return	Yield	Water Used[b]
	($)	(cwt)	(in.)	($)	(cwt)	(in.)
1	105.65	89.00	11.30	93.83	89.00	14.38
2	130.64	96.67	11.83	120.73	96.80	14.52
3	115.74	90.82	10.35	100.82	90.82	14.23
4	132.07	95.21	10.12	117.08	95.59	14.37
5	119.52	93.25	11.59	111.83	93.25	13.58
6	98.62	85.63	10.05	82.33	85.63	14.28
7	103.61	88.78	11.64	94.65	88.78	13.97
8	124.97	92.14	9.39	116.83	93.03	12.09
9	105.81	88.46	10.77	94.55	88.46	13.70
10	82.75	79.96	8.98	74.17	79.96	11.21
11	124.34	94.10	11.12	117.94	94.10	12.77
12	75.20	79.38	10.42	61.67	79.38	13.94
13	88.98	85.47	12.41	82.98	85.47	13.97
14	106.86	88.44	10.47	96.96	88.44	13.05
15	120.80	91.95	10.07	106.95	91.95	13.67
16	83.12	85.09	13.58	77.46	85.09	15.05
17	132.77	98.14	12.62	128.54	98.29	13.86
18	146.82	99.28	10.01	132.43	99.28	13.75
19	101.04	87.98	11.57	90.55	87.98	14.30
20	111.81	92.59	12.99	106.65	92.59	14.33
21	102.32	86.68	10.05	89.62	86.68	13.35
22	107.90	90.88	12.44	98.89	90.88	14.78
23	82.00	83.27	12.21	74.15	83.27	14.25
24	120.09	92.43	10.69	107.55	92.43	13.95
25	107.51	87.63	9.56	93.01	87.63	13.33
26	124.67	95.56	12.36	119.41	95.56	13.73
27	100.25	87.34	11.19	90.14	87.33	13.81
28	87.51	84.49	11.90	80.78	84.49	13.65
29	137.58	96.22	9.61	124.18	96.22	13.09
20	106.04	89.81	11.95	94.26	89.81	15.01

[a]Based on a price of Grain Sorghum of $4.07/cwt. and price of Natural Gas of $2.50

[b]Irrigation water used. Does not include preplant irrigation

Table 2. Irrigation Water Used Per Period for 30 Years of Deterministic and Stochastic Weather Patterns: Texas High Plains.[a]

Year	Deterministic								Stochastic							
	Days After Emergence															
	20	30	40	50	60	70	80	90	20	30	40	50	60	70	80	90
	Inches															
1	2.42	1.15	1.38	1.60	1.09	1.36	0.69	1.60	2.62	1.28	1.75	1.55	1.89	1.94	1.77	1.59
2	2.63	1.40	1.50	0.72	1.70	1.36	1.42	1.09	2.63	1.38	1.83	1.61	1.82	1.84	1.77	1.65
3	3.12	0.64	0.71	1.27	0.99	0.88	1.59	1.15	3.12	0.81	1.61	1.64	1.91	1.72	1.86	1.55
4	3.04	0.00	1.58	1.86	0.21	1.35	1.01	1.06	3.04	1.17	1.48	1.81	1.83	1.72	1.66	1.67
5	2.29	1.34	1.14	1.25	1.69	1.46	1.11	1.32	2.29	1.27	2.04	1.15	1.70	1.56	1.95	1.63
6	2.89	1.53	0.04	0.21	1.74	1.54	0.96	1.13	2.89	1.48	1.58	1.57	1.64	1.79	1.64	1.70
7	2.58	1.36	1.96	0.36	1.49	1.91	1.48	0.49	2.58	1.29	1.84	1.78	1.44	1.82	1.71	1.50
8	1.49	1.15	1.59	1.11	1.14	0.45	1.30	1.16	1.49	1.11	1.70	1.67	1.46	1.64	1.62	1.40
9	2.69	1.28	0.85	1.38	0.69	1.77	1.31	0.79	2.69	1.36	1.72	1.62	1.66	1.60	1.56	1.48
10	1.59	0.83	0.63	0.85	1.67	1.38	1.57	0.46	1.59	1.44	0.90	1.15	1.62	1.69	1.58	1.25
11	2.97	1.39	0.43	1.55	0.00	1.54	1.76	1.49	2.74	1.51	0.90	1.34	1.54	1.59	1.60	1.54
12	2.97	1.14	1.12	0.98	1.46	0.55	0.96	1.23	3.08	1.06	1.81	1.49	1.36	1.69	1.83	1.62
13	3.06	1.51	1.93	1.41	1.79	0.91	1.57	0.25	3.06	1.47	1.95	1.44	1.78	1.13	1.70	1.43
14	2.44	0.85	1.52	0.00	1.60	1.38	1.35	1.33	2.44	1.02	1.71	1.67	1.52	1.39	1.68	1.62
15	2.50	1.16	1.45	1.62	0.88	0.85	1.38	0.22	2.50	1.36	1.74	1.54	1.46	1.82	1.89	1.34
16	3.11	1.32	1.57	1.22	1.47	1.68	1.76	1.45	3.05	1.48	1.82	1.78	1.81	1.65	1.66	1.80
17	2.56	0.60	1.78	1.80	1.67	1.15	1.44	1.61	2.56	1.38	1.48	1.86	1.68	1.79	1.54	1.57
18	2.64	1.10	1.36	0.41	1.86	1.57	0.16	0.91	2.82	0.94	1.35	1.50	1.89	1.90	1.62	1.74
19	2.79	1.10	1.97	0.91	1.83	0.52	0.77	1.69	2.79	1.09	1.98	1.87	1.69	1.86	1.52	1.50
20	2.98	1.00	1.92	1.57	0.88	1.79	1.31	1.54	2.99	1.36	1.82	1.57	1.52	1.85	1.62	1.59
21	2.11	0.88	1.22	1.68	1.22	1.37	1.26	0.51	2.11	1.36	1.85	1.57	1.52	1.74	1.37	1.83
22	2.66	1.52	1.37	1.74	1.28	1.57	1.27	1.03	2.66	1.45	1.93	1.56	1.84	1.74	1.98	1.61
23	2.99	1.35	1.61	1.73	0.96	1.71	0.79	1.07	2.99	1.54	1.51	1.88	1.67	1.49	1.60	1.57
24	3.09	1.59	0.00	0.36	1.36	1.12	1.53	1.65	3.09	1.20	1.39	1.84	1.71	1.15	1.87	1.69
25	2.34	1.34	1.19	0.00	0.55	1.71	1.25	1.19	2.34	1.33	1.20	1.82	1.87	1.63	1.55	1.59
26	3.23	0.63	0.69	1.25	1.97	1.41	1.46	1.74	2.96	0.81	1.62	1.63	1.74	1.74	1.47	1.75
27	2.64	0.64	2.21	1.34	0.44	1.54	1.70	0.67	2.64	0.62	2.22	1.85	1.65	1.54	1.64	1.65
28	2.11	1.22	1.79	0.91	1.30	1.62	1.40	1.54	2.45	1.24	1.65	1.53	1.56	1.87	1.75	1.59
29	1.86	0.68	1.96	1.25	0.42	1.38	0.44	1.62	1.86	1.17	1.81	1.57	1.75	1.65	1.43	1.86
30	2.66	1.56	1.68	2.03	0.29	1.57	1.53	0.63	2.69	1.51	1.69	2.05	2.02	1.72	1.58	1.76

[a] Based on a price of Grain Sorghum of $4.07/cwt. and price of Natural Gas of $2.50.

Table 3. Estimated Effect of 20 Day Curtailments on Per Acre Yield, Net Returns and Water Used for Grain Sorghum: Texas High Plains.[a]

Year	20-40			40-60			60-80		
	Net Return	Yield	Water Used[b]	Net Return	Yield	Water Used[b]	Net Return	Yield	Water Used[b]
	($)	(cwt)	(in.)	($)	(cwt)	(in.)	($)	(cwt)	(in.)
1	90.74	86.07	12.50	59.82	77.49	12.69	79.05	82.62	12.38
2	114.62	93.03	12.66	91.05	86.25	12.59	95.45	87.95	13.00
3	99.06	87.71	11.84	94.29	86.47	11.94	94.44	86.56	11.99
4	116.95	93.05	12.31	96.64	87.04	11.86	123.41	94.51	11.73
5	107.74	90.03	11.71	74.87	81.45	12.40	83.15	83.64	12.25
6	78.59	81.21	11.21	93.10	85.63	11.48	54.21	76.11	12.88
7	88.25	84.83	12.01	66.33	78.87	12.26	55.75	76.81	13.12
8	117.96	91.98	10.83	106.69	88.34	10.43	120.25	91.65	9.93
9	95.74	85.92	11.06	81.39	82.52	11.69	79.46	81.89	11.61
10	85.83	79.96	8.18	62.50	76.00	10.62	38.92	68.34	9.74
11	108.32	89.43	11.01	111.62	90.04	10.70	112.22	90.78	11.23
12	56.12	75.66	11.98	61.91	77.13	11.81	62.03	77.22	11.87
13	67.40	78.91	12.01	11.60	64.21	13.07	52.74	76.04	13.20
14	96.27	86.35	11.32	106.47	88.23	10.39	70.41	80.13	12.35
15	104.24	89.97	11.74	57.12	76.50	12.48	104.41	88.92	11.56
16	68.60	80.04	12.74	52.02	76.01	13.36	37.75	72.72	14.06
17	129.84	96.27	11.67	71.77	81.25	13.03	110.44	91.70	12.53
18	123.32	95.41	12.58	112.80	92.26	12.43	113.94	91.30	11.26
19	80.71	83.30	12.57	37.21	71.25	12.85	82.68	83.83	12.55
20	99.54	88.33	12.28	40.62	72.11	12.76	87.54	85.58	12.89
21	90.15	84.65	11.36	46.82	73.24	12.18	92.30	85.26	11.35
22	90.92	86.60	12.94	46.07	74.26	13.31	73.75	81.69	12.91
23	61.18	77.65	12.48	32.84	70.13	12.97	52.57	75.81	13.04
24	109.60	89.26	10.51	119.22	92.43	10.92	90.20	86.23	12.79
25	86.72	84.48	12.09	104.63	87.63	10.31	80.64	82.19	11.58
26	117.40	92.67	11.61	90.72	85.88	12.33	94.29	86.88	12.32
27	79.90	82.94	12.49	20.31	64.76	11.31	78.99	82.25	12.06
28	74.24	81.01	12.16	46.02	73.30	12.44	55.55	75.90	12.34
29	121.25	93.89	11.73	102.34	87.19	10.51	135.28	96.21	10.20
30	79.02	83.74	13.42	44.84	72.54	12.05	89.21	86.12	12.95

Days After Emergence When Curtailments Occurred

[a]Based on a price of Grain Sorghum of $4.07/cwt. and price of Natural Gas of $2.50

[b]Irrigation water used. Does not include preplant irrigation.

Table 4. Estimated Effect of 30 Day Curtailments on Per Acre Yield, Net Returns and Water Use for Grain Sorghum: Texas High Plains.[a]

Year	Days After Emergence When Curtailments Occurred					
	20-50			40-70		
	Net Return	Yield	Water Used[b]	Net Return	Yield	Water Used[b]
	($)	(cwt)	(in.)	($)	(cwt)	(in.)
1	70.16	79.23	11.59	33.73	68.79	11.51
2	96.34	86.20	11.16	53.27	74.58	11.72
3	69.20	77.74	10.48	78.51	80.49	10.58
4	103.35	88.39	11.34	72.19	79.29	11.11
5	88.44	83.87	11.08	40.05	70.69	11.60
6	81.11	79.66	9.14	80.48	81.68	11.15
7	63.92	76.36	10.58	37.93	69.84	11.37
8	115.11	89.85	9.62	73.46	77.09	8.77
9	69.43	77.84	10.51	64.39	77.14	11.18
10	86.00	79.59	7.81	43.26	69.72	9.88
11	69.81	77.77	10.35	106.49	88.42	10.56
12	37.97	68.58	10.22	51.46	72.83	10.60
13	-2.82	57.90	11.05	-11.73	56.52	12.09
14	97.08	83.57	8.57	91.76	83.80	10.16
15	80.03	81.44	11.05	34.40	68.37	10.95
16	27.73	66.86	11.30	24.52	67.38	12.61
17	119.85	92.54	10.85	8.24	61.58	11.53
18	108.34	89.18	10.77	76.33	80.86	11.47
19	29.36	67.01	11.02	27.58	66.07	10.61
20	62.94	76.56	11.03	17.66	64.68	11.93
21	67.82	77.65	10.76	40.74	69.24	10.10
22	46.42	72.98	12.04	20.94	66.06	12.33
23	-10.16	56.18	11.38	0.88	60.31	12.29
24	114.26	88.46	8.57	117.36	91.60	10.64
25	54.75	72.20	9.17	107.41	87.24	9.24
26	85.76	82.83	10.83	68.96	78.28	11.04
27	18.40	64.39	11.46	1.21	58.71	10.74
28	45.13	71.53	11.05	27.36	67.00	11.52
29	102.25	87.46	10.78	86.36	81.91	9.83
30	11.06	62.92	12.03	29.50	67.55	11.47

[a]Based on a price of Grain Sorghum of $4.07/cwt. and price of Natural Gas of $2.50

[b]Irrigation water used. Does not include preplant irrigation

Table 5. Effect of Alternative Grain Sorghum and Natural Gas Prices on Yield, Profit and Water Use: Texas High Plains.[a]

Item	Grain Sorghum Price ($/cwt) 4.07	4.07	4.07	3.37	3.37
		Natural Gas Price ($/mcf)			
	Dryland	1.50	2.50	1.50	2.50
Yield (cwt/acre)					
Average	14.80	87.29	89.94	87.46	87.56
High	34.25	99.28	99.28	99.28	99.28
Low	4.00	77.63	79.38	77.66	77.60
C.V.[b]	59.42	5.59	5.63	5.60	5.74
Net Returns ($/acre)					
Average	-12.66	118.25	99.36	57.03	38.48
High	61.65	159.89	132.43	90.39	72.37
Low	-53.89	89.18	61.67	31.68	14.49
C.V.	-256.23	14.37	17.92	23.99	36.09
Water Applied[c] (in./acre)					
Average	0.00	11.24	13.79	11.46	11.51
High	0.00	13.55	15.05	13.47	13.98
Low	0.00	8.42	11.21	8.70	8.69
C.V.	0.00	9.78	5.82	9.88	10.65

[a]Based on 30 simulations for each set of prices

[b]Coefficient of variation is a measure of dispersion; i.e., the higher the value the more instable

[c]Water application is effective water to the root zone and does not include evaporation or percolation