ACT-07-10, MIFPA-10-21

General Gauge and Anomaly Mediated Supersymmetry Breaking in Grand Unified Theories with Vector-Like Particles

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Abstract

In Grand Unified Theories (GUTs) from orbifold and various string constructions the generic vector-like particles do not need to form complete $SU(5)$ or $SO(10)$ representations. To realize them concretely, we present orbifold $SU(5)$ models, orbifold $SO(10)$ models where the gauge symmetry can be broken down to flipped $SU(5) \times U(1)_X$ or Pati-Salam $SU(4)_C \times SU(2)_L \times SU(2)_R$ gauge symmetries, and F-theory $SU(5)$ models. Interestingly, these vector-like particles can be at the TeV-scale so that the lightest CP-even Higgs boson mass can be lifted, or play the messenger fields in the Gauge Mediated Supersymmetry Breaking (GMSB). Considering GMSB, ultraviolet insensitive Anomaly Mediated Supersymmetry Breaking (AMSB), and the deflected AMSB, we study the general gaugino mass relations and their indices, which are valid from the GUT scale to the electroweak scale at one loop, in the $SU(5)$ models, the flipped $SU(5) \times U(1)_X$ models, and the Pati-Salam $SU(4)_C \times SU(2)_L \times SU(2)_R$ models. In the deflected AMSB, we also define the new indices for the gaugino mass relations, and calculate them as well. Using these gaugino mass relations and their indices, we may probe the messenger fields at intermediate scale in the GMSB and deflected AMSB, determine the supersymmetry breaking mediation mechanisms, and distinguish the four-dimensional GUTs, orbifold GUTs, and F-theory GUTs.

PACS numbers: 11.10.Kk, 11.25.Mj, 11.25.-w, 12.60.Jv

I. INTRODUCTION

The supersymmetric Standard Model (SM) is the most elegant extension of the SM since it solves the gauge hiearchy problem naturally. In particular, the gauge coupling unification can be achieved at about 2×10^{16} GeV [1], and the lightest supersymmetric particle (LSP) like the neutralino can be the cold dark matter candidate [2, 3]. To solve the gauge hiearchy problem in the SM, supersymmetry should be broken around the TeV scale. Thus, at the Large Hadron Collider (LHC) and future International Linear Collider (ILC), we may observe the supersymmetric particles and get information about their mass spectra and interactions. The key questions are how to determine the mediation mechanisms for supersymmetry breaking and how to probe the Grand Unified Theories (GUTs) and string derived GUTs.

In the conventional supersymmetric SMs, supersymmetry is assumed to be broken in the hidden sector, and then its breaking effects are mediated to the SM observable sector. However, the relations between the supersymmetric particle spectra and the fundamental theories can be very complicated and model dependent. Interestingly, comparing to the supersymmetry breaking soft masses for squarks and sleptons, the gaugino masses have the simplest form and appear to be the least model dependent [4, 5]. For instance, with gravity mediated supersymmetry breaking in GUTs, we have a universal gaugino mass $M_{1/2}$ at the GUT scale, which is called the minimal supergravity (mSUGRA) scenario [6]. Thus, we have the gauge coupling relation and the gaugino mass relation at the GUT scale $M_{\rm GUT}$:

$$\frac{1}{\alpha_3} ~=~ \frac{1}{\alpha_2} ~=~ \frac{1}{\alpha_1}~, \tag{1}$$

$$\frac{M_3}{\alpha_3} ~=~ \frac{M_2}{\alpha_2} ~=~ \frac{M_1}{\alpha_1}~, \tag{2}$$

where α_3, α_2, and $\alpha_1 \equiv 5\alpha_Y/3$ are gauge couplings respectively for $SU(3)_C$, $SU(2)_L$, and $U(1)_Y$ gauge symmetries, and M_3, M_2, and M_1 are the masses for $SU(3)_C$, $SU(2)_L$, and $U(1)_Y$ gauginos, respectively. Note that M_i/α_i are constant under one-loop renormalization group equation (RGE) running, thus, we obtain that the above gaugino mass relation in Eq. (2) is valid from the GUT scale to the electroweak scale at one loop. Because the two-loop RGE running effects on gaugino masses are very small, we can test this gaugino mass relation at the LHC and ILC where the gaugino masses can be measured [7, 8]. Recently, considering the GUTs with high-dimensional operators [4, 9–19] and the F-theory

GUTs with $U(1)$ fluxes [20–32], we generalized the mSUGRA scenario [33]. In particular, we studied the generic gaugino mass relations and proposed their indices [33]. As we know, there are three major supersymmetry breaking mediation schemes: gravity medidated supersymmetry breaking [6], Gauge Mediated Supersymmetry Breaking (GMSB) [34], and Anomaly Mediated Supersymmetry Breaking (AMSB) [35–37]. Thus, we shall study the generic gaugino mass relations and their indices in the general GMSB and AMSB.

On the other hand, there exists a few pecent fine-tuning to have the lightest CP-even Higgs boson mass heavier than 114 GeV in the Minimal Supersymmetric Standard Model (MSSM). One possible solution is that we introduce the TeV-scale vector-like particles [38]. The lightest CP-even Higgs boson mass can be lifted due to the large Yukawa couplings for these vector-like particles [38]. Moreover, in the GMSB [34] and deflected AMSB [37], we need messenger fields at the intermediate scale, which are also vector-like. Also, we can use the messenger fields to generate the correct neutrino masses and mixings in the mean time [39, 40]. Thus, it is interesting to study the GUTs with generic vector-like particles.

In this paper, we first point out that the generic vector-like particles do not need to form complete $SU(5)$ or $SO(10)$ representations in GUTs from the orbifold constructions [41–48], intersecting D-brane model building on Type II orientifolds [49–51], M-theory on S^1/Z_2 with Calabi-Yau compactifications [52, 53], and F-theory with $U(1)$ fluxes [20–32]. Therefore, in the GMSB and deflected AMSB, the messenger fields do not need to form complete $SU(5)$ or $SO(10)$ representations. The gauge coupling unification can be preserved by introducing the extra vector-like particles at the intermediate scale that do not mediate supersymmetry breaking. To be concrete, we present the orbifold $SU(5)$ models with additional vector-like particles, the orbifold $SO(10)$ models with extra vector-like particles where the gauge symmetry can be broken down to flipped $SU(5)\times U(1)_X$ or Pati-Salam $SU(4)_C\times SU(2)_L\times SU(2)_R$ gauge symmetries, and the F-theory $SU(5)$ models with generic vector-like particles. In short, these vector-like particles can be at the TeV scale so that we can increase the lightest CP-even Higgs boson mass in the MSSM [38], and they can be the messenger fields in the GMSB and deflected AMSB as well. By the way, if the vector-like particles are around the TeV scale, there may exist the possibility of flavour changing neutral currents even at tree level. To solve this problem, we can require that the mixings between the TeV-scale vector-like particles and the SM fermions are very small.

In addition, we shall study the general gaugino mass relations and their indices in the

GMSB and AMSB, which are valid from the GUT scale to the electroweak scale at one loop. We briefly review the gaugino mass relations and their indices in the generalization of the mSUGRA [33], and define the suitable gaugino mass relations in the GMSB and AMSB. For the GMSB, we first briefly review the gaugino masses. With various possible messenger fields, we calculate the gaugino mass relations and their indices in the $SU(5)$ models, the flipped $SU(5) \times U(1)_X$ models, and the Pati-Salam $SU(4)_C \times SU(2)_L \times SU(2)_R$ models. These kinds of models can be realized in orbifold GUTs, F-theory $SU(5)$ models with $U(1)_Y$ flux, F-theory $SO(10)$ models with $U(1)_X$ flux where the $SO(10)$ gauge symmetry is broken down to flipped $SU(5) \times U(1)_X$ gauge symmetries (we will denote them as F-theory flipped $SU(5) \times U(1)_X$ models), and F-theory $SO(10)$ models with $U(1)_{B-L}$ flux where the $SO(10)$ gauge symmetry is broken down to $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ gauge symmetries (we will denote them as F-theory $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ models). Using the gaugino mass relations and their indices, we can probe the messenger fields at the imtermediate scale. Moreover, for the AMSB, we first briefly review the gaugino masses as well. To solve the tachyonic slepton problem for the original AMSB, we consider two scenarios: the ultraviolet (UV) insensitive AMSB [36] and the deflected AMSB [37]. In the UV insensitive AMSB, we calculate the gaugino mass relations and their indices in the $SU(5)$ models with and without the TeV-scale vector-like particles that form complete $SU(5)$ multiplets, and in the flipped $SU(5) \times U(1)_X$ models with TeV-scale vector-like particles that form complete $SU(5) \times U(1)_X$ multiplets. To achieve the one-step gauge coupling unification, we emphasize that the discussions for the Pati-Salam models are similar to those in the $SU(5)$ models. In the deflected AMSB, without and with the suitable TeV-scale vector-like particles that can lift the lightest CP-even Higgs boson mass, we study the generic gaugino mass relations and their indices in the $SU(5)$ models, flipped $SU(5) \times U(1)_X$ models, and Pati-Salam $SU(4)_C \times SU(2)_L \times SU(2)_R$ models with various possible messenger fields. To probe the messenger fields at intermediate scale, we define the new indices for the gaugino mass relations, and calculate them in details. Also, we find that in most of our scenarios, the gluino can be the lightest gaugino at low energy. In particular, we propose a new kind of interesting flipped $SU(5)$ models as well.

Furthermore, using the gaugino mass relations and their indices, we explain how to determine the supersymmetry breaking mediation mechanisms, and how to probe the four-dimensional GUTs, orbifold GUTs, and F-theory GUTs. Also, in order to distinguish between the different scenarios with the same gaugino mass relations and the same indices, we need to consider the squark and slepton masses as well, which will be studied elsewhere [54].

This paper is organized as follows. In Sectin II, we discuss the vector-like particles that we are interested in, and construct orbifold GUTs and F-theory $SU(5)$ models with generic vector-like particles. We briefly discuss the gaugino mass relations and their indices in Section III. We study the gaugino mass relations and their indices for GMSB and AMSB in Section IV and V, respectively. We consider the implications of the gaugino mass relations and their indices in Section VI. Our conclusions are given in Section VII. We briefly review the del Pezzo Surfaces in Appendix A.

II. GENERIC VECTOR-LIKE PARTICLES IN THE ORBIFOLD AND F-THEORY GUTS

In the GMSB and deflected AMSB, there exist messenger fields at intermediate scales, which are vector-like particles. To realize gauge coupling unification, in the traditional GMSB and deflected AMSB, we assume that the messenger fields form complete $SU(5)$ representations, for example, $(\mathbf{5}, \overline{\mathbf{5}})$. However, we do not have vector-like particles in complete $SU(5)$ representations in quite a few kinds of model building. In the intersecting D-brane model building on Type II orientifolds where the $SU(5)$ gauge symmetry is broken down to the SM gauge symmetry by D-brane splitting [49–51], and in the M-theory on S^1/Z_2 with Calabi-Yau manifold compactifications where the $SU(5)$ and $SO(10)$ gauge symmetries are respectively broken down to the $SU(3)_C \times SU(2)_L \times U(1)_Y$ and $SU(3)_C \times SU(2)_L \times U(1)_{B-L} \times U(1)_{I3R}$ gauge symmetries by Wilson lines [52, 53], we can not have the massless vector-like particles that form complete GUT representations. For the bulk vector-like particles in the orbifold GUTs [41–48], we can not keep the zero modes for all the vector-like particles in the complete GUT representations, *i.e.*, the zero modes of some vector-like particles will be projected out. In the F-theory GUTs [20–32], we can also obtain the vector-like particles that do not form complete GUT multiplets. In fact, the $SU(5)$ models, flipped $SU(5) \times U(1)_X$ models [55–59], and $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ models with additional vector-like particles have already been constructed locally in F-theory [22, 23, 25, 26, 28, 29, 32]. Interestingly, we should emphasize that this is the reason why we can solve the doublet-triplet splitting problem in these kinds of model building. In this Section, we shall present the orbifold $SU(5)$ models with additional vector-like particles, the orbifold $SO(10)$ models with additional vector-like particles where the gauge symmetry can be broken down to flipped $SU(5) \times U(1)_X$ or Pati-Salam $SU(4)_C \times SU(2)_L \times SU(2)_R$ gauge symmetries, and the F-theory $SU(5)$ models with generic vector-like particles.

First, let us explain our convention for supersymmetric SMs. We denote the left-handed quark doublets, right-handed up-type quarks, right-handed down-type quarks, left-handed lepton doublets, right-handed neutrinos and right-handed charged leptons as Q_i, U_i^c, D_i^c, L_i, N_i^c, and E_i^c, respectively. Also, we denote one pair of Higgs doublets as H_u and H_d, which give masses to the up-type quarks/neutrinos and the down-type quark/charged leptons, respectively. In this paper, we consider the vector-like particles whose quantum numbers are the same as those of the SM fermions and their Hermitian conjugates, particles in the $SU(5)$ symmetric representation and their Hermitian conjugates, and the $SU(5)$ adjoint particles. Their quantum numbers under $SU(3)_C \times SU(2)_L \times U(1)_Y$ and their contributions to one-loop beta functions $\Delta b \equiv (\Delta b_1, \Delta b_2, \Delta b_3)$ as complete supermultiplets are given as follows

$$XQ + XQ^c = (\mathbf{3}, \mathbf{2}, \mathbf{\frac{1}{6}}) + (\mathbf{\bar{3}}, \mathbf{2}, -\mathbf{\frac{1}{6}})\,, \quad \Delta b = (\frac{1}{5}, 3, 2)\,; \tag{3}$$

$$XU + XU^c = (\mathbf{3}, \mathbf{1}, \mathbf{\frac{2}{3}}) + (\mathbf{\bar{3}}, \mathbf{1}, -\mathbf{\frac{2}{3}})\,, \quad \Delta b = (\frac{8}{5}, 0, 1)\,; \tag{4}$$

$$XD + XD^c = (\mathbf{3}, \mathbf{1}, -\mathbf{\frac{1}{3}}) + (\mathbf{\bar{3}}, \mathbf{1}, \mathbf{\frac{1}{3}})\,, \quad \Delta b = (\frac{2}{5}, 0, 1)\,; \tag{5}$$

$$XL + XL^c = (\mathbf{1}, \mathbf{2}, \mathbf{\frac{1}{2}}) + (\mathbf{1}, \mathbf{2}, -\mathbf{\frac{1}{2}})\,, \quad \Delta b = (\frac{3}{5}, 1, 0)\,; \tag{6}$$

$$XE + XE^c = (\mathbf{1}, \mathbf{1}, \mathbf{1}) + (\mathbf{1}, \mathbf{1}, -\mathbf{1})\,, \quad \Delta b = (\frac{6}{5}, 0, 0)\,; \tag{7}$$

$$XG = (\mathbf{8}, \mathbf{1}, \mathbf{0})\,, \quad \Delta b = (0, 0, 3)\,; \tag{8}$$

$$XW = (\mathbf{1}, \mathbf{3}, \mathbf{0})\,, \quad \Delta b = (0, 2, 0)\,; \tag{9}$$

$$XT + XT^c = (\mathbf{1}, \mathbf{3}, \mathbf{1}) + (\mathbf{1}, \mathbf{3}, -\mathbf{1})\,, \quad \Delta b = (\frac{18}{5}, 4, 0)\,; \tag{10}$$

$$XS + XS^c = (\mathbf{6}, \mathbf{1}, -\mathbf{\frac{2}{3}}) + (\mathbf{\bar{6}}, \mathbf{1}, \mathbf{\frac{2}{3}})\,, \quad \Delta b = (\frac{16}{5}, 0, 5)\,; \tag{11}$$

$$XY + XY^c = (\mathbf{3}, \mathbf{2}, -\mathbf{\frac{5}{6}}) + (\mathbf{\bar{3}}, \mathbf{2}, \mathbf{\frac{5}{6}})\,, \quad \Delta b = (5, 3, 2)\,. \tag{12}$$

A. Traditional Four-dimensional Grand Unified Theories

First, let us briefly review the $SU(5)$ models and explain the convention. We define the $U(1)_Y$ hypercharge generator in $SU(5)$ as follows

$$T_{\rm U(1)_Y} = {\rm diag}\left(-\frac{1}{3}, -\frac{1}{3}, -\frac{1}{3}, \frac{1}{2}, \frac{1}{2}\right) ~. \tag{13}$$

Under $SU(3)_C \times SU(2)_L \times U(1)_Y$ gauge symmetry, the $SU(5)$ representations are decomposed as follows

$$\mathbf{5} ~=~ (\mathbf{3}, \mathbf{1}, \mathbf{-1/3}) \oplus (\mathbf{1}, \mathbf{2}, \mathbf{1/2}) ~, \tag{14}$$

$$\overline{\mathbf{5}} ~=~ (\overline{\mathbf{3}}, \mathbf{1}, \mathbf{1/3}) \oplus (\mathbf{1}, \mathbf{2}, \mathbf{-1/2}) ~, \tag{15}$$

$$\mathbf{10} ~=~ (\mathbf{3}, \mathbf{2}, \mathbf{1/6}) \oplus (\overline{\mathbf{3}}, \mathbf{1}, \mathbf{-2/3}) \oplus (\mathbf{1}, \mathbf{1}, \mathbf{1}) ~, \tag{16}$$

$$\overline{\mathbf{10}} ~=~ (\overline{\mathbf{3}}, \mathbf{2}, \mathbf{-1/6}) \oplus (\mathbf{3}, \mathbf{1}, \mathbf{2/3}) \oplus (\mathbf{1}, \mathbf{1}, \mathbf{-1}) ~, \tag{17}$$

$$\mathbf{24} ~=~ (\mathbf{8}, \mathbf{1}, \mathbf{0}) \oplus (\mathbf{1}, \mathbf{3}, \mathbf{0}) \oplus (\mathbf{1}, \mathbf{1}, \mathbf{0}) \oplus (\mathbf{3}, \mathbf{2}, \mathbf{-5/6}) \oplus (\overline{\mathbf{3}}, \mathbf{2}, \mathbf{5/6}) ~. \tag{18}$$

There are three families of the SM fermions whose quantum numbers under $SU(5)$ are

$$F'_i = \mathbf{10},~ \overline{f}'_i = \bar{\mathbf{5}},~ N^c_i = \mathbf{1} ~, \tag{19}$$

where $i = 1, 2, 3$ for three families. The SM particle assignments in F'_i and $\bar{f}'_i$ are

$$F'_i = (Q_i, U^c_i, E^c_i) ~,~ \overline{f}'_i = (D^c_i, L_i) ~. \tag{20}$$

To break the $SU(5)$ gauge symmetry and electroweak gauge symmetry, we introduce the adjoint Higgs field and one pair of Higgs fields whose quantum numbers under $SU(5)$ are

$$\Phi' ~=~ \mathbf{24} ~, \quad h' ~=~ \mathbf{5} ~, \quad \overline{h}' ~=~ \bar{\mathbf{5}} ~, \tag{21}$$

where h' and $\overline{h}'$ contain the Higgs doublets H_u and H_d, respectively.

Second, we would like to briefly review the flipped $SU(5) \times U(1)_X$ models [55–57]. The gauge group $SU(5) \times U(1)_X$ can be embedded into $SO(10)$. We define the generator $U(1)_{Y'}$ in $SU(5)$ as

$$T_{\rm U(1)_{Y'}} = {\rm diag}\left(-\frac{1}{3}, -\frac{1}{3}, -\frac{1}{3}, \frac{1}{2}, \frac{1}{2}\right). \tag{22}$$

The hypercharge is given by

$$Q_Y = \frac{1}{5}\left(Q_X - Q_{Y'}\right). \tag{23}$$

There are three families of the SM fermions whose quantum numbers under $SU(5)\times U(1)_X$ are

$$F_i = (\mathbf{10}, \mathbf{1}),\ \bar{f}_i = (\bar{\mathbf{5}}, -\mathbf{3}),\ \bar{l}_i = (\mathbf{1}, \mathbf{5}), \tag{24}$$

where $i = 1, 2, 3$. The particle assignments for the SM fermions are

$$F_i = (Q_i, D_i^c, N_i^c)\ ,\ \ \overline{f}_i = (U_i^c, L_i)\ ,\ \ \bar{l}_i = E_i^c\ . \tag{25}$$

To break the GUT and electroweak gauge symmetries, we introduce two pairs of Higgs fields whose quantum numbers under $SU(5) \times U(1)_X$ are

$$H = (\mathbf{10}, \mathbf{1})\ ,\ \ \overline{H} = (\overline{\mathbf{10}}, -\mathbf{1})\ ,\ \ h = (\mathbf{5}, -\mathbf{2})\ ,\ \ \overline{h} = (\bar{\mathbf{5}}, \mathbf{2})\ , \tag{26}$$

where h and $\overline{h}$ contain the Higgs doublets H_d and H_u, respectively.

Moreover, the flipped $SU(5) \times U(1)_X$ models can be embedded into $SO(10)$. Under $SU(5) \times U(1)_X$ gauge symmetry, the $SO(10)$ representations are decomposed as follows

$$\mathbf{10} \ = \ (\mathbf{5}, -\mathbf{2}) \oplus (\bar{\mathbf{5}}, \mathbf{2})\ , \tag{27}$$

$$\mathbf{16} \ = \ (\mathbf{10}, \mathbf{1}) \oplus (\bar{\mathbf{5}}, -\mathbf{3}) \oplus (\mathbf{1}, \mathbf{5})\ , \tag{28}$$

$$\mathbf{45} \ = \ (\mathbf{24}, \mathbf{0}) \oplus (\mathbf{1}, \mathbf{0}) \oplus (\mathbf{10}, -\mathbf{4}) \oplus (\overline{\mathbf{10}}, \mathbf{4})\ . \tag{29}$$

Let us consider the vector-like particles which form complete flipped $SU(5) \times U(1)_X$ multiplets. The quantum numbers for these additional vector-like particles under the $SU(5) \times U(1)_X$ gauge symmetry are

$$XF \ = \ (\mathbf{10}, \mathbf{1})\ ,\ \ \overline{XF} \ = \ (\overline{\mathbf{10}}, -\mathbf{1})\ , \tag{30}$$

$$Xf \ = \ (\mathbf{5}, \mathbf{3})\ ,\ \ \overline{Xf} \ = \ (\bar{\mathbf{5}}, -\mathbf{3})\ , \tag{31}$$

$$Xl \ = \ (\mathbf{1}, -\mathbf{5})\ ,\ \ \overline{Xl} \ = \ (\mathbf{1}, \mathbf{5})\ , \tag{32}$$

$$Xh \ = (\mathbf{5}, -\mathbf{2})\ ,\ \ \overline{Xh} \ = \ (\bar{\mathbf{5}}, \mathbf{2}) \tag{33}$$

$$XGW \ = \ (\mathbf{24}, \mathbf{0})\ ,\ \ XN \ = \ (\mathbf{1}, \mathbf{0})\ , \tag{34}$$

$$XX \ = \ (\mathbf{10}, -\mathbf{4})\ ,\ \ \overline{XX} \ = \ (\overline{\mathbf{10}}, \mathbf{4})\ . \tag{35}$$

Moreover, the particle contents for the decompositions of XF, $\overline{XF}$, Xf, $\overline{Xf}$, Xl, $\overline{Xl}$,

Xh, $\overline{Xh}$, XGW, XX, and $\overline{XX}$ under the SM gauge symmetries are

$$XF = (XQ, XD^c, XN^c) \ , \quad \overline{XF} = (XQ^c, XD, XN) \ , \tag{36}$$

$$Xf = (XU, XL^c) \ , \ \overline{Xf} = (XU^c, XL) \ , \tag{37}$$

$$Xl = XE \ , \ \overline{Xl} = XE^c \ , \tag{38}$$

$$Xh = (XD, XL) \ , \ \overline{Xh} = (XD^c, XL^c) \ , \tag{39}$$

$$XGW = (XG, XW, XQ, XQ^c) \ , \tag{40}$$

$$XX = (XY, XU^c, XE) \ , \quad \overline{XX} = (XY^c, XU, XE^c) \ . \tag{41}$$

In flipped $SU(5) \times U(1)_X$ models of $SO(10)$ origin, there are two steps for gauge coupling unification: the $SU(3)_C \times SU(2)_L$ gauge symmeties are unified first at the scale M_{32}, and then the $SU(5) \times U(1)_X$ gauge symmetries are unified at the higher scale M_U, where M_{32} is about the usual GUT scale around 2×10^{16} GeV. Thus, the condition for gauge coupling unification in the flipped $SU(5) \times U(1)_X$ models can be relaxed elegantly. To realize the string-scale gauge coupling unification in the free fermionic string constructions [58] or the decoupling scenario in the F-theory model building [26, 28], we introduce the TeV-scale vector-like particles which form the complete flipped $SU(5) \times U(1)_X$ multiplets [59]. To avoid the Landau pole problem for the strong coupling, we show that at the TeV scale, we can only introduce the vector-like particles (XF, $\overline{XF}$) or (XF, $\overline{XF}$) $\oplus$ (Xl, $\overline{Xl}$) [59]. The flipped $SU(5) \times U(1)_X$ models with these vector-like particles are dubbed as the testable flipped $SU(5) \times U(1)_X$ models since they can solve the monopole problem, realize the hybrid inflation, lift the lightest CP-even Higgs boson mass, and predict the proton decay within the reach of the future proton decay experiments, etc [28, 59].

Third, we would like to briefly review the Pati-Salam models. The gauge group is $SU(4)_C \times SU(2)_L \times SU(2)_R$, which can also be embedded into $SO(10)$. There are three families of the SM fermions whose quantum numbers under $SU(4)_C \times SU(2)_L \times SU(2)_R$ are

$$F_i^L = (\mathbf{4}, \mathbf{2}, \mathbf{1}) \ , \quad F_i^{Rc} = (\overline{\mathbf{4}}, \mathbf{1}, \mathbf{2}) \ , \tag{42}$$

where $i = 1, 2, 3$. Also, the particle assignments for the SM fermions are

$$F_i^L = (Q_i, L_i) \ , \quad F_i^{Rc} = (U_i^c, D_i^c, E_i^c, N_i^c) \ . \tag{43}$$

To break the Pati-Salam and electroweak gauge symmetries, we introduce one pair of Higgs fields and one bidoublet Higgs field whose quantum numbers under $SU(4)_C \times SU(2)_L \times$

$SU(2)_R$ are

$$\Phi = (\mathbf{4}, \mathbf{1}, \mathbf{2}) \ , \quad \overline{\Phi} = (\overline{\mathbf{4}}, \mathbf{1}, \mathbf{2}) \ , \quad H' = (\mathbf{1}, \mathbf{2}, \mathbf{2}) \ , \tag{44}$$

where H' contains one pair of the Higgs doublets H_d and H_u.

Moreover, the Pati-Salam models can be embedded into $SO(10)$ models. Under $SU(4)_C \times SU(2)_L \times SU(2)_R$ gauge symmetry, the $SO(10)$ representations are decomposed as follows

$$\mathbf{10} = (\mathbf{6}, \mathbf{1}, \mathbf{1}) \oplus (\mathbf{1}, \mathbf{2}, \mathbf{2}) \ , \tag{45}$$

$$\mathbf{16} = (\mathbf{4}, \mathbf{2}, \mathbf{1}) \oplus (\overline{\mathbf{4}}, \mathbf{1}, \mathbf{2}) \ , \tag{46}$$

$$\mathbf{45} = (\mathbf{15}, \mathbf{1}, \mathbf{1}) \oplus (\mathbf{1}, \mathbf{3}, \mathbf{1}) \oplus (\mathbf{1}, \mathbf{1}, \mathbf{3}) \oplus (\mathbf{6}, \mathbf{2}, \mathbf{2}) \ . \tag{47}$$

Let us consider the vector-like particles which form complete $SU(4)_C \times SU(2)_L \times SU(2)_R$ representations. The quantum numbers for the vector-like particles under the $SU(4)_C \times SU(2)_L \times SU(2)_R$ gauge symmetry are

$$XFL = (\mathbf{4}, \mathbf{2}, \mathbf{1}) \ , \quad \overline{XFL} = (\overline{\mathbf{4}}, \mathbf{2}, \mathbf{1}) \ , \tag{48}$$

$$XFR = (\mathbf{4}, \mathbf{1}, \mathbf{2}) \ , \quad \overline{XFR} = (\overline{\mathbf{4}}, \mathbf{1}, \mathbf{2}) \ , \tag{49}$$

$$XD\overline{D} = (\mathbf{6}, \mathbf{1}, \mathbf{1}) \ , \quad XL\overline{L} = (\mathbf{1}, \mathbf{2}, \mathbf{2}) \ , \tag{50}$$

$$XG4 = (\mathbf{15}, \mathbf{1}, \mathbf{1}) \ , \quad XWL = (\mathbf{1}, \mathbf{3}, \mathbf{1}) \ , \tag{51}$$

$$XWR = (\mathbf{1}, \mathbf{1}, \mathbf{3}) \ , \quad XZ = (\mathbf{6}, \mathbf{2}, \mathbf{2}) \ . \tag{52}$$

Also, the particle contents for the decompositions of XFL, $\overline{XFL}$, XFR, $\overline{XFR}$, $XD\overline{D}$, $XL\overline{L}$, $XG4$, XWL, XWR and XZ under the SM gauge symmetries are

$$XFL = (XQ, XL) \ , \quad \overline{XFL} = (XQ^c, XL^c) \ , \tag{53}$$

$$XFR = (XU, XD, XE, XN) \ , \quad \overline{XFR} = (XU^c, XD^c, XE^c, XN^c) \ , \tag{54}$$

$$XD\overline{D} = (XD, XD^c) \ , \quad XL\overline{L} = (XL, XL^c) \ , \tag{55}$$

$$XG4 = (XG, XU, XU^c) \ , \quad XWL = XW \tag{56}$$

$$XWR = (XE, XE^c, XN) \ , \quad XZ = (XQ, XQ^c, XY, XY^c) \ . \tag{57}$$

B. Obifold Grand Unified Theories with Generic Vector-Like Particles

In the five-dimensional orbifold supersymmetric GUTs [41–48], the five-dimensional manifold is factorized into the product of ordinary four-dimensional Minkowski space-time M^4

and the orbifold $S^1/(Z_2 \times Z_2')$. The corresponding coordinates are x^μ ($\mu = 0, 1, 2, 3$) and $y \equiv x^5$. The radius for the fifth dimension is R. The orbifold $S^1/(Z_2 \times Z_2')$ is obtained by S^1 moduloing the equivalent class

$$P : \quad y \sim -y \ , \ P' : \quad y' \sim -y' \ , \tag{58}$$

where $y' \equiv y - \pi R/2$. There are two fixed points, $y = 0$ and $y = \pi R/2$.

The $N = 1$ supersymmetric theory in five dimensions have 8 real supercharges, corresponding to $N = 2$ supersymmetry in four dimensions. In terms of the physical degrees of freedom, the vector multiplet contains a vector boson A_M with $M = 0, 1, 2, 3, 5$, two Weyl gauginos $\lambda_{1,2}$, and a real scalar σ. In the four-dimensional $N = 1$ supersymmetry language, it contains a vector multiplet $V \equiv (A_\mu, \lambda_1)$ and a chiral multiplet $\Sigma \equiv ((\sigma + iA_5)/\sqrt{2}, \lambda_2)$ which transform in the adjoint representation of group G. The five-dimensional hypermultiplet consists of two complex scalars ϕ and ϕ^c, and a Dirac fermion Ψ. It can be decomposed into two chiral mupltiplets $\Phi(\phi, \psi \equiv \Psi_R)$ and $\Phi^c(\phi^c, \psi^c \equiv \Psi_L)$, which are in the conjugate representations of each other under the gauge group.

The general action for the group G gauge fields and their couplings to the bulk hypermultiplet Φ is [60]

$$\begin{aligned} S \ = \ & \int d^5x \frac{1}{kg^2} \mathrm{Tr} \left[\frac{1}{4} \int d^2\theta \left(W^\alpha W_\alpha + \mathrm{H.C.} \right) \right. \\ & \left. + \int d^4\theta \left((\sqrt{2}\partial_5 + \bar{\Sigma}) e^{-V} (-\sqrt{2}\partial_5 + \Sigma) e^V + \partial_5 e^{-V} \partial_5 e^V \right) \right] \\ & + \int d^5x \left[\int d^4\theta \left(\Phi^c e^V \bar{\Phi}^c + \bar{\Phi} e^{-V} \Phi \right) \right. \\ & \left. + \int d^2\theta \left(\Phi^c (\partial_5 - \frac{1}{\sqrt{2}} \Sigma) \Phi + \mathrm{H.C.} \right) \right] \ . \end{aligned} \tag{59}$$

Under the parity operator P, the vector multiplet transforms as

$$V(x^\mu, y) \ \rightarrow V(x^\mu, -y) = PV(x^\mu, y)P^{-1} \ , \tag{60}$$

$$\Sigma(x^\mu, y) \ \rightarrow \Sigma(x^\mu, -y) = -P\Sigma(x^\mu, y)P^{-1} \ . \tag{61}$$

For the hypermultiplet Φ and Φ^c, we have

$$\Phi(x^\mu, y) \ \rightarrow \Phi(x^\mu, -y) = \eta_\Phi P^{l_\Phi} \Phi(x^\mu, y) (P^{-1})^{m_\Phi} \ , \tag{62}$$

$$\Phi^c(x^\mu, y) \ \to \Phi^c(x^\mu, -y) = -\eta_\Phi P^{l_\Phi} \Phi^c(x^\mu, y)(P^{-1})^{m_\Phi} \ , \tag{63}$$

where η_Φ is $\pm$, l_Φ and m_Φ are respectively the numbers of the fundamental index and anti-fundamental index for the bulk multiplet Φ under the bulk gauge group G. For example, if G is an $SU(N)$ group, for a fundamental representation, we have $l_\Phi = 1$ and $m_\Phi = 0$, and for an adjoint representation, we have $l_\Phi = 1$ and $m_\Phi = 1$. Moreover, the transformation properties for the vector multiplet and hypermultiplets under P' are the same as those under P.

For $G = SU(5)$, to break the $SU(5)$ gauge symmetry, we choose the following 5×5 matrix representations for the parity operators P and P'

$$P = \mathrm{diag}(+1, +1, +1, +1, +1) \ , \ P' = \mathrm{diag}(+1, +1, +1, -1, -1) \ . \tag{64}$$

Under the P' parity, the gauge generators T^α ($\alpha = 1$, 2, ..., 24) for $SU(5)$ are separated into two sets: T^a are the generators for the SM gauge group, and $T^{\hat{a}}$ are the generators for the broken gauge group

$$P \ T^a \ P^{-1} = T^a \ , \ P \ T^{\hat{a}} \ P^{-1} = T^{\hat{a}} \ , \tag{65}$$

$$P' \ T^a \ P'^{-1} = T^a \ , \ P' \ T^{\hat{a}} \ P'^{-1} = -T^{\hat{a}} \ . \tag{66}$$

The zero modes of the $SU(5)/SM$ gauge bosons are projected out, thus, the five-dimensional $N = 1$ supersymmetric $SU(5)$ gauge symmetry is broken down to the four-dimensional $N = 1$ supersymmetric SM gauge symmetry for the zero modes. For the zero modes and KK modes, the four-dimensional $N = 1$ supersymmetry is preserved on the 3-branes at both fixed points, and only the SM gauge symmetry is preserved on the 3-brane at $y = \pi R/2$ [47].

For $G = SO(10)$, the generators T^α of $SO(10)$ are imaginary antisymmetric 10×10 matrices. In terms of the 2×2 identity matrix σ_0 and the Pauli matrices σ_i, they can be written as tensor products of 2×2 and 5×5 matrices, $(\sigma_0, \sigma_1, \sigma_3) \otimes A_5$ and $\sigma_2 \otimes S_5$ as a complete set, where A_5 and S_5 are the 5×5 real anti-symmetric and symmetric matrices. The generators of the $SU(5) \times U(1)$ gauge symmetries are

$$\begin{aligned} &\sigma_0 \otimes A_3 \, , \quad \sigma_0 \otimes A_2 \, , \quad \sigma_0 \otimes A_X \\ &\sigma_2 \otimes S_3 \, , \quad \sigma_2 \otimes S_2 \, , \quad \sigma_2 \otimes S_X \, , \end{aligned} \tag{67}$$

the generators for flipped $SU(5) \times U(1)_X$ gauge symmetries are

$$\begin{aligned} &\sigma_0 \otimes A_3 \, , \quad \sigma_0 \otimes A_2 \, , \quad \sigma_1 \otimes A_X \\ &\sigma_2 \otimes S_3 \, , \quad \sigma_2 \otimes S_2 \, , \quad \sigma_3 \otimes A_X \, , \end{aligned} \tag{68}$$

and the generators for Pati-Salam $SU(4)_C \times SU(2)_L \times SU(2)_R$ gauge symmetries are

$$\begin{array}{ll} (\sigma_0, \sigma_1, \sigma_3) \otimes A_3 ~, & (\sigma_0, \sigma_1, \sigma_3) \otimes A_2 ~, \\ \sigma_2 \otimes S_3 ~, & \sigma_2 \otimes S_2 ~, \end{array} \tag{69}$$

where A_3 and S_3 are respectively the diagonal blocks of A_5 and S_5 that have indices 1, 2, and 3, while the diagonal blocks A_2 and S_2 have indices 4 and 5. A_X and S_X are the off-diagonal blocks of A_5 and S_5.

We choose the 10×10 matrix for P as

$$P ~=~ \sigma_0 \otimes \mathrm{diag}(1, 1, 1, 1, 1) ~. \tag{70}$$

To break the $SO(10)$ down to $SU(5) \times U(1)$, we choose

$$P' ~=~ \sigma_2 \otimes \mathrm{diag}(1, 1, 1, 1, 1) ~, \tag{71}$$

to break the $SO(10)$ down to flipped $SU(5) \times U(1)_X$, we choose

$$P' ~=~ \sigma_2 \otimes \mathrm{diag}(1, 1, 1, -1, -1) ~, \tag{72}$$

and to break the $SO(10)$ down to the Pati-Salam gauge symmetries, we choose

$$P' ~=~ \sigma_0 \otimes \mathrm{diag}(1, 1, 1, -1, -1) ~. \tag{73}$$

For the zero modes, the five-dimensional $N = 1$ supersymmetric $SO(10)$ gauge symmetry is broken down to the four-dimensional $N = 1$ supersymmetric $SU(5) \times U(1)$, flipped $SU(5) \times U(1)_X$ and Pati-Salam $SU(4)_C \times SU(2)_L \times SU(2)_R$ gauge symmetries. Including the KK modes, the 3-branes at both fixed points preserve the four-dimensional $N = 1$ supersymmetry, and the gauge symmetry on the 3-brane at $y = \pi R/2$ is $SU(5) \times U(1)$, flipped $SU(5) \times U(1)_X$ and Pati-Salam gauge symmetries, for different choices of P' [47].

In Table I, Table II, and Table III, we present the possible vector-like particles, which remain as zero modes after orbifold projections, in the orbifold $SU(5)$ models, in the orbifold $SO(10)$ models whose gauge symmetry is broken down to the flipped $SU(5) \times U(1)_X$ gauge symmetry by orbifold projections, and the orbifold $SO(10)$ models whose gauge symmetry is broken down to the Pati-Salam $SU(4)_C \times SU(2)_L \times SU(2)_R$ gauge symmetry by orbifold projections, respectively.

Representation	η_Φ	Zero Modes	Representation	η_Φ	Zero Modes
$(\mathbf{5},\ \overline{\mathbf{5}})$	$+1$	$(XD,\ XD^c)$	$(\mathbf{5},\ \overline{\mathbf{5}})$	-1	$(XL,\ XL^c)$
$(\mathbf{10},\ \overline{\mathbf{10}})$	$+1$	$(XU,\ XU^c),\ (XE,\ XE^c)$	$(\mathbf{10},\ \overline{\mathbf{10}})$	-1	$(XQ,\ XQ^c)$
$(\mathbf{15},\ \overline{\mathbf{15}})$	$+1$	$(XT,\ XT^c),\ (XS,\ XS^c)$	$(\mathbf{15},\ \overline{\mathbf{15}})$	-1	$(XQ,\ XQ^c)$
$\mathbf{24}$	$+1$	$XG,\ XW$	$\mathbf{24}$	-1	$(XY,\ XY^c)$

TABLE I: The possible vector-like particles which remain as zero modes after orbifold projections in the orbifold $SU(5)$ models.

Representation	η_Φ	Zero Modes	Representation	η_Φ	Zero Modes
$\mathbf{10}$	$+1$	Xh	$\mathbf{10}$	-1	$\overline{Xh}$
$(\mathbf{16},\ \overline{\mathbf{16}})$	$+1$	$(XF,\ \overline{XF})$	$(\mathbf{16},\ \overline{\mathbf{16}})$	-1	$(Xf,\ \overline{Xf}),\ (Xl,\ \overline{Xl})$
$\mathbf{45}$	$+1$	$XGW,\ XN$	$\mathbf{45}$	-1	$(XX,\ \overline{XX})$

TABLE II: The possible vector-like particles which remain as zero modes after orbifold projections in the orbifold $SO(10)$ models where the gauge symmetry is broken down to the flipped $SU(5) \times U(1)_X$ gauge symmetries.

C. F-Theory $SU(5)$ Models with Generic Vector-Like Particles

We first briefly review the F-theory model building [20–24]. The twelve-dimensional F theory is a convenient way to describe Type IIB vacua with varying axion-dilaton $\tau = a + ie^{-\phi}$. We compactify F-theory on a Calabi-Yau fourfold, which is elliptically fibered

Representation	η_Φ	Zero Modes	Representation	η_Φ	Zero Modes
$\mathbf{10}$	$+1$	$XD\overline{D}$	$\mathbf{10}$	-1	$XL\overline{L}$
$(\mathbf{16},\ \overline{\mathbf{16}})$	$+1$	$(XFL,\ \overline{XFL})$	$(\mathbf{16},\ \overline{\mathbf{16}})$	-1	$(XFR,\ \overline{XFR})$
$\mathbf{45}$	$+1$	$XG4,\ XWL,\ XWR$	$\mathbf{45}$	-1	XZ

TABLE III: The possible vector-like particles which remain as zero modes after orbifold projections in the orbifold $SO(10)$ models where the gauge symmetry is broken down to the Pati-Salam $SU(4)_C \times SU(2)_L \times SU(2)_R$ gauge symmetries.

$\pi : Y_4 \to B_3$ with a section $\sigma : B_3 \to Y_4$. The base B_3 is the internal space dimensions in Type IIB string theory, and the complex structure of the T^2 fibre encodes τ at each point of B_3. The SM or GUT gauge theories are on the worldvolume of the observable seven-branes that wrap a complex codimension-one suface in B_3. Denoting the complex coordinate transverse to these seven-branes in B_3 as z, we can write the elliptic fibration in Weierstrass form

$$y^2 = x^3 + f(z)x + g(z) \ , \tag{74}$$

where $f(z)$ and $g(z)$ are sections of $K_{B_3}^{-4}$ and $K_{B_3}^{-6}$, respectively. The complex structure of the fibre is

$$j(\tau) \ = \ \frac{4(24f)^3}{\Delta} \ , \quad \Delta \ = \ 4f^3 + 27g^2 \ . \tag{75}$$

At the discriminant locus $\{\Delta = 0\} \subset B_3$, the torus T^2 degenerates by pinching one of its cycles and becomes singular. For a generic pinching one-cycle $(p,q) = p\alpha + q\beta$ where α and β are one-cylces for the torus T^2, we obtain a (p,q) seven-brane in the locus where the (p,q) string can end. The singularity types of the ellitically fibres fall into the familiar ADE classifications, and we identify the corresponding ADE gauge groups on the seven-brane world-volume. This is one of the most important advantages for the F-theory model building: the exceptional gauge groups appear rather naturally, which is absent in perturbative Type II string theory. And then all the SM fermion Yuakwa couplings in the GUTs can be generated.

We assume that the observable seven-branes with GUTs on its worldvolume wrap a complex codimension-one suface S in B_3, and the observable gauge symmetry is G_S. When $h^{1,0}(S) \neq 0$, the low energy spectrum may contain the extra states obtained by reduction of the bulk supergravity modes of compactification. So we require that $\pi_1(S)$ be a finite group. In order to decouple gravity and construct models locally, the extension of the local metric on S to a local Calabi-Yau fourfold must have a limit where the surface S can be shrunk to zero size. This implies that the anti-canonical bundle on S must be ample. Therefore, S is a del Pezzo n surface dP_n with $n \geq 2$ in which $h^{2,0}(S) = 0$ (for a brief review of del Pezzo surfaces, see Appendix A). By the way, the Hirzebruch surfaces with degree larger than 2 satisfy $h^{2,0}(S) = 0$ but do not define the fully consistent decoupled models [22, 23].

To describe the spectrum, we have to study the gauge theory of the worldvolume on the seven-branes. We start from the maximal supersymmetric gauge theory on $\mathbb{R}^{3,1} \times \mathbb{C}^2$

and then replace $\mathbb{C}^2$ with the Kähler surface S. In order to have four-dimensional $\mathcal{N}=1$ supersymmetry, the maximal supersymmetric gauge theory on $\mathbb{R}^{3,1}\times\mathbb{C}^2$ should be twisted. It was shown that there exists a unique twist preserving $\mathcal{N}=1$ supersymmetry in four dimensions, and chiral matters can arise from the bulk S or the codimension-one curve Σ in S which is the intersection between the observable seven-branes and the other seven-brane(s) [22, 23].

In order to have the matter fields on S, we consider a non-trivial vector bundle on S with a structure group H_S which is a subgroup of G_S. Then the gauge group G_S is broken down to $\Gamma_S\times H_S$, and the adjoint representation $\mathrm{ad}(G_S)$ of the G_S is decomposed as

$$\mathrm{ad}(G_S)\to\mathrm{ad}(\Gamma_S)\bigoplus\mathrm{ad}(H_S)\bigoplus_j(\tau_j,T_j)\ . \tag{76}$$

Employing the vanishing theorem of the del Pezzo surfaces, we obtain the numbers of the generations and anti-generations by calculating the zero modes of the Dirac operator on S

$$n_{\tau_j}\ =\ -\chi(S,\mathbf{T_j})\ ,\quad n_{\tau_j^*}\ =\ -\chi(S,\mathbf{T_j}^*)\ , \tag{77}$$

where $\mathbf{T_j}$ is the vector bundle on S whose sections transform in the representation T_j of H_S, and $\mathbf{T_j}^*$ is the dual bundle of $\mathbf{T_j}$. In particular, when the H_S bundle is a line bundle L, we have

$$n_{\tau_j}\ =\ -\chi(S,L^j)\ =\ -\Big[1+\frac{1}{2}\Big(\int_S c_1(L^j)c_1(S)+\int_S c_1(L^j)^2\Big)\Big]\ . \tag{78}$$

In order to preserve supersymmetry, the line bundle L should satisfy the BPS equation [22]

$$J_S\wedge c_1(L)=0, \tag{79}$$

where J_S is the Kähler form on S. Moreover, the admissible supersymmetric line bundles on del Pezzo surfaces must satisfy $c_1(L)c_1(S)=0$, thus, $n_{\tau_j}=n_{\tau_j^*}$ and only the vector-like particles can be obtained. In short, we can not have the chiral matter fields on the worldvolume of the observable seven-branes.

Interestingly, the chiral superfields can come from the intersections between the observable seven-branes and the other seven-brane(s) [22, 23]. Let us consider a stack of seven-branes with gauge group $G_{S'}$ that wrap a codimension-one surface S' in B_3. The intersection of S and S' is a codimenion-one curve (Riemann surface) Σ in S and S', and the gauge symmetry on Σ will be enhanced to G_Σ where $G_\Sigma\supset G_S\times G_{S'}$. On this curve, there exist chiral matters from the decomposition of the adjoint representation $\mathrm{ad}G_\Sigma$ of G_Σ as follows

$$\mathrm{ad}G_\Sigma = \mathrm{ad}G_S \oplus \mathrm{ad}G_{S'} \oplus_k (U_k \otimes U'_k) \ . \tag{80}$$

Turning on the non-trivial gauge bundles on S and S' respectively with structure groups H_S and $H_{S'}$, we break the gauge group $G_S \times G_{S'}$ down to the commutant subgroup $\Gamma_S \times \Gamma_{S'}$. Defining $\Gamma \equiv \Gamma_S \times \Gamma_{S'}$ and $H \equiv H_S \times H_{S'}$, we can decompose $U \otimes U'$ into the irreducible representations as follows

$$U \otimes U' = \bigoplus_k (r_k, V_k), \tag{81}$$

where r_k and V_k are the representations of Γ and H, respectively. The light chiral fermions in the representation r_k are determined by the zero modes of the Dirac operator on Σ. The net number of chiral superfields is given by

$$N_{r_k} - N_{r_k^*} = \chi(\Sigma, K_\Sigma^{1/2} \otimes \mathbf{V}_k), \tag{82}$$

where K_Σ is the restriction of canonical bundle on the curve Σ, and $\mathbf{V}_k$ is the vector bundle whose sections transform in the representation V_k of the structure group H.

In the F-theory model building, we are interested in the models where $G_{S'}$ is $U(1)'$, and H_S and $H_{S'}$ are respectively $U(1)$ and $U(1)'$. Then the vector bundles on S and S' are line bundles L and L'. The adjoint representation $\mathrm{ad}G_\Sigma$ of G_Σ is decomposed into a direct sum of the irreducible representations under the group $\Gamma_S \times U(1) \times U(1)'$ that can be denoted as $(\mathbf{r_j}, \mathbf{q_j}, \mathbf{q'_j})$

$$\mathrm{ad}G_\Sigma = \mathrm{ad}(\Gamma_S) \oplus \mathrm{ad}G_{S'} \oplus_j (\mathbf{r_j}, \mathbf{q_j}, \mathbf{q'_j}) \ . \tag{83}$$

The numbers of chiral supefields in the representation $(\mathbf{r_j}, \mathbf{q_j}, \mathbf{q'_j})$ and their Hermitian conjugates on the curve Σ are given by

$$N_{(\mathbf{r_j},\mathbf{q_j},\mathbf{q'_j})} \ = \ h^0(\Sigma, \mathbf{V}_j) \ , \quad N_{(\bar{\mathbf{r}}_\mathbf{j},-\mathbf{q_j},-\mathbf{q'_j})} \ = \ h^1(\Sigma, \mathbf{V}_j) \ , \tag{84}$$

where

$$\mathbf{V}_j \ = \ K_\Sigma^{1/2} \otimes L_\Sigma^{q_j} \otimes L'^{q'_j}_\Sigma \ , \tag{85}$$

where $K_\Sigma^{1/2}$, $L_\Sigma^{r_j}$ and $L'^{q'_j}_\Sigma$ are the restrictions of canonical bundle K_S, line bundles L and L' on the curve Σ, respectively. In particular, if the volume of S' is infinite, $G_{S'} = U(1)'$ is decoupled. And then the index $\mathbf{q'_j}$ can be ignored.

Using Riemann-Roch theorem, we obtain the net number of chiral supefields in the representation $(\mathbf{r_j}, \mathbf{q_j}, \mathbf{q'_j})$

$$N_{(\mathbf{r_j},\mathbf{q_j},\mathbf{q'_j})} - N_{(\bar{\mathbf{r}}_\mathbf{j},-\mathbf{q_j},-\mathbf{q'_j})} \ = \ 1 - g + c_1(\mathbf{V}_j) \ , \tag{86}$$

where g is the genus of the curve Σ, and c_1 means the first Chern class.

Moreover, we can obtain the Yukawa couplings at the triple intersection of three curves Σ_i, Σ_j and Σ_k where the gauge group or the singularity type is enhanced further. To have the triple intersections, the corresponding homology classes $[\Sigma_i]$, $[\Sigma_j]$ and $[\Sigma_k]$ of the curves Σ_i, Σ_j and Σ_k must satisfy the following conditions

$$[\Sigma_i] \cdot [\Sigma_j] > 0 \ , \quad [\Sigma_i] \cdot [\Sigma_k] > 0 \ , \quad [\Sigma_j] \cdot [\Sigma_k] > 0 \ . \tag{87}$$

The $SU(5)$ models, flipped $SU(5) \times U(1)_X$ models, and $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ models with additional vector-like particles have been constructed previously [22, 23, 25, 26, 28, 29, 32]. However, the $SU(5)$ models with generic vector-like particles have not been studied systematically yet. Thus, we shall construct the $SU(5)$ models with additional vector-like particles in general here. In such $SU(5)$ models, we introduce the vector-like particles YF' and $\overline{YF}'$, and Yf' and $\overline{Yf}'$, whose quantum numbers under $SU(5)$ are

$$YF' = \mathbf{10} \ , \ \overline{YF}' = \overline{\mathbf{10}} \ ; \ Yf' = \mathbf{5} \ , \ \overline{Yf}' = \overline{\mathbf{5}} \ . \tag{88}$$

Moreover, the particle contents from the decompositions of YF', $\overline{YF}'$, Yf', and $\overline{Yf}'$ under the SM gauge symmetry are

$$YF' = (XQ, XU^c, XE^c) \ , \ \overline{YF}' = (XQ^c, XU, XE) \ , \tag{89}$$

$$Yf' = (XD, XL^c) \ , \ \overline{Yf}' = (XD^c, XL) \ . \tag{90}$$

Assuming that S is a dP_8 surface, we consider the observable gauge group $SU(5)$. On codimension-one curves that are the intersections of the observable seven-branes and other seven-branes, we obtain the SM fermions, Higgs fields, and extra vector-like particles. To break the $SU(5)$ gauge symmetry down to the $SU(3)_C \times SU(2)_L \times U(1)_Y$ gauge symmetries, we turn on the $U(1)_Y$ flux on S specified by the line bundle L. To obtain the SM fermions, Higgs fields and additional vector-like particles, we also turn on the $U(1)$ fluxes on the other seven-branes that intersect with the observable seven-branes, and we specify these fluxes by the line bundle L'^n.

We take the line bundle $L = \mathcal{O}_S(E_1 - E_2)^{6/5}$. Note that $\chi(S, L^{5/6}) = 0$, we do not have the vector-like particles on the bulk S. Moreover, the curves with homology classes for the matter fields, Higgs fields and vector-like particles, and the gauge bundle assignments for each curve in the $SU(5)$ models are given in Table IV. From this table, we obtain: all the SM fermions are localized on the matter curves $\Sigma_{F'}$ and $\Sigma_{\overline{f}'}$; the Higgs fields H_u and H_d are localized on the curves Σ_{Hu}, and Σ_{Hd}, respectively; and the vector-like particles YF', $\overline{YF}'$, Yf', $\overline{Yf}'$, $(XQ,\ XQ^c)$, $(XU,\ XU^c)$, $(XD,\ XD^c)$, $(XL,\ XL^c)$, and $(XE,\ XE^c)$ are localized on the curves $\Sigma_{F'}$, $\Sigma_{\overline{F'}}$, $\Sigma_{f'}$, $\Sigma_{\overline{f'}}$, Σ_{XQ}, Σ_{XU}, Σ_{XD}, Σ_{XL}, and Σ_{XE}, respectively. In addition, there exist singlets from the intersections of the other seven-branes. It is easy to check that we can realize the SM fermion Yukawa coupling terms in our models. All the vector-like particles can obtain masses by giving vacuum expectation values (VEVs) to the SM singlets at the intersections of the other seven-branes. Furthermore, if we take the line bundle $L = \mathcal{O}_S(E_1 - E_2 + E_7 - E_8)^{6/5}$. we shall have one pair of vector-like particles $(XY,\ XY^c)$ on the bulk S because $\chi(S, L^{5/6}) = -1$.

Fields	Curves	Class	g_Σ	L_Σ	$L_\Sigma^{\prime n}$
H_u	Σ_{Hu}	$2H - E_1 - E_3$	0	$\mathcal{O}(1)^{6/5}$	$\mathcal{O}(1)^{2/5}$
H_d	Σ_{Hd}	$2H - E_2 - E_3$	0	$\mathcal{O}(-1)^{6/5}$	$\mathcal{O}(-1)^{2/5}$
$10_i + n \times XF'$	$\Sigma_{F'}$	$2H - E_4 - E_6$	0	$\mathcal{O}(0)$	$\mathcal{O}(3+n)$
$n \times \overline{XF}'$	$\Sigma_{\overline{F}'}$	$2H - E_5 - E_6$	0	$\mathcal{O}(0)$	$\mathcal{O}(-n)$
$\overline{5}_i + m \times \overline{Xf}'$	$\Sigma_{\overline{f}'}$	$H - E_7$	0	$\mathcal{O}(0)$	$\mathcal{O}(-3-m)$
$m \times Xf'$	$\Sigma_{f'}$	$H - E_8$	0	$\mathcal{O}(0)$	$\mathcal{O}(m)$
$(XQ,\ XQ^c)$	Σ_{XQ}	$3H - E_1 - E_2$ (pinched)	1	$\mathcal{O}(p_{12})^{6/5}$	$\mathcal{O}(p_{12})^{-1/5}$
$(XU,\ XU^c)$	Σ_{XU}	$3H - E_1 - E_2 - E_3$ (pinched)	1	$\mathcal{O}(p_{12}^3)^{6/5}$	$\mathcal{O}(p_{12}^3)^{4/5}$
$(XD,\ XD^c)$	Σ_{XD}	$3H - E_1 - E_2 - E_4$ (pinched)	1	$\mathcal{O}(p_{12}^4)^{6/5}$	$\mathcal{O}(p_{12}^4)^{2/5}$
$(XL,\ XL^c)$	Σ_{XL}	$3H - E_1 - E_5$ (pinched)	1	$\mathcal{O}(p_{12}^5)^{6/5}$	$\mathcal{O}(p_{12}^5)^{-3/5}$
$(XE,\ XE^c)$	Σ_{XE}	$3H - E_1 - E_2 - E_6$ (pinched)	1	$\mathcal{O}(p_{12}^6)^{6/5}$	$\mathcal{O}(p_{12}^6)^{-6/5}$

TABLE IV: The particle curves and gauge bundle assignments for each curve in the $SU(5)$ models from F-theory. Here $i = 1,\ 2,\ 3$. Moreover, $p_{12} = p_1 - p_2$, $p_{12}^l = p_1^l - P_2^l$ for $l = 3,\ 4,\ 5,\ 6$, and we denote the corresponding blowing up points as p_1, p_2, p_1^l, and p_2^l.

III. GAUGINO MASS RELATIONS AND THEIR INDICES

First, let us briefly review the generalization of mSUGRA. In four-dimensional GUTs with high-dimensional operators [4, 9–12], and F-theory $SU(5)$ models [24, 27] and $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ models [29], the SM gauge kinetic functions are not unified at the GUT scale. In general, the gaugino masses at the GUT scales can be parametrized as follows [33]

$$\frac{M_i}{\alpha_i} = M_{1/2}^U + a_i M_{1/2}^{NU} , \tag{91}$$

where $M_{1/2}^U$ and $M_{1/2}^{NU}$ are the universal and non-universal gaugino masses at the GUT scale. Thus, we define the index k of the gaugino mass relation by the following equation [33]

$$\frac{M_2}{\alpha_2} - \frac{M_3}{\alpha_3} = k \left(\frac{M_1}{\alpha_1} - \frac{M_3}{\alpha_3} \right) , \tag{92}$$

where

$$k \equiv \frac{a_2 - a_3}{a_1 - a_3} . \tag{93}$$

Because M_i/α_i are renormalization scale invariant under one-loop RGE running and the two-loop RGE running effects are very small [31], the gaugino mass relation in Eq. (92) can be preserved very well at low energy. Note that the gaugino masses can be measured from the LHC and ILC experiments [7, 8], we can determine k at low energy. In addition, we have the following gauge coupling relation at the GUT scale

$$\frac{1}{\alpha_2} - \frac{1}{\alpha_3} = k \left(\frac{1}{\alpha_1} - \frac{1}{\alpha_3} \right) . \tag{94}$$

Thus, we can define the GUT scale via the above gauge coupling relation. In short, the index k describes not only the gauge coupling relation in Eq. (94) at the GUT scale, but also the gaugino mass relation in Eq. (92) which is exact from the GUT scale to the electroweak scale at one loop. Although k is not well defined in the mSUGRA, in this paper, we symbolically define the index k for the mSUGRA gaugino mass relation as $0/0$, *i.e.*, $k = 0/0$ means the mSUGRA gaugino mass relation.

Interestingly, in the GMSB and AMSB, the gaugino masses are given by Eq. (91) with $M_{1/2}^U = 0$. Thus, $M_i/(a_i\alpha_i)$ are proportional to the same constant. And then we can define their gaugino mass relations as follows

$$\frac{M_3}{a_3\alpha_3} = \frac{M_2}{a_2\alpha_2} = \frac{M_1}{a_1\alpha_1} . \tag{95}$$

Therefore, to present the gaugino mass relations in the GMSB and AMSB, we only need to calculate a_i in the following.

IV. GAUGE MEDIATED SUPERSYMMETRY BREAKING

First, let us consider the gaugino mass relations and their indices in the GMSB [34]. In the messenger sector, we introduce a set of the SM vector-like particles Φ_j and $\overline{\Phi}_j$. To break supersymmetry, we introduce a chiral superfield X, whose F-term breaks supersymmetry. The messenger fields couple to X via the following superpotential

$$W \supset \lambda_j X \overline{\Phi}_j \Phi_j \ , \tag{96}$$

where λ_i are Yukawa couplings. For simplicity, we assume that the scalar and auxiliary components of X obtain VEVs

$$\langle X \rangle \ = \ M + \theta^2 F \ . \tag{97}$$

Thus, the fermionic components of Φ_j and $\overline{\Phi}_j$ form Dirac fermions with masses $\lambda_j M$. Denoting the superfields and their scalar components of Φ_j and $\overline{\Phi}_j$ in the same symbols, we obtain that their scalar components have the following squared-mass matrix in the basis $(\Phi_j, \ \overline{\Phi}_j^\dagger)$

$$M^2 \ = \ \begin{pmatrix} |\lambda_j M|^2 & -(\lambda_j F)^\dagger \\ -(\lambda_j F) & |\lambda_j M|^2 \end{pmatrix} \ . \tag{98}$$

Therefore, the scalar messenger mass eigenvetors are $(\Phi_j + \overline{\Phi}_j^\dagger)/\sqrt{2}$ and $(\Phi_j - \overline{\Phi}_j^\dagger)/\sqrt{2}$, and the corresponding squared-mass eigenvalues are $(\lambda_j M)^2 \pm \lambda_j F$. The supersymmetry breaking, which is obvious in the messenger spectrum, is communicated to the SM sector via the gauge interactions of Φ_j and $\overline{\Phi}_j$. And then we obtain the gaugino masses at one loop as follows

$$\frac{M_i}{\alpha_i} \ = \ \frac{1}{4\pi} \frac{F}{M} \sum_j n_i(\Phi_j) g(x_j) \ , \tag{99}$$

where $n_i(\Phi_j)$ is the sum of Dynkin indices for the vector-like particles Φ_j and $\overline{\Phi}_j$, $x_j = |F/(\lambda_j M^2)|$, and

$$g(x) \ = \ \frac{1}{x^2} \left[(1+x)\ln(1+x) + (1-x)\ln(1-x)\right] \ . \tag{100}$$

Approximately, we have the expansion of $g(x)$ as follows

$$g(x) \;=\; 1+\frac{x^2}{6}+\frac{x^4}{15}+\frac{x^6}{28}+\cdots \;. \tag{101}$$

Because the squared-masses for the messenger fields must be positive, we obtain $0 \leq x_j \leq 1$. Also, $g(x)$ is a monotonically increasing function from $g(0) = 1$ to $g(1) = 1.386$. Therefore, in the GMSB, we have

$$a_i \;=\; \sum_j n_i(\Phi_j) g(x_j) \;. \tag{102}$$

In particular, if all the messenger fields have the same Yukawa couplings to X, $i.e$, λ_j are the same, we have

$$a_i \;=\; \sum_j n_i(\Phi_j) \;. \tag{103}$$

Moreover, if the messenger fields are heavier than 10^7 GeV and their Yukawa couplings to X are about order one for naturalness, we obtain $x_j \leq 0.1$, and then $g(x_j) \simeq 1$. So we have

$$a_i \;\simeq\; \sum_j n_i(\Phi_j) \;. \tag{104}$$

To preserve the gauge coupling unification in GUTs, we usually assume that the vector-like messengers form complete $SU(5)$ multiplets, for example, $(\mathbf{5},\ \overline{\mathbf{5}})$. In general, the messengers do not need to form complete $SU(5)$ multiplets. To achieve the gauge coupling unification, we can introduce extra vector-like particles around the same messeger scale, which do not couple to supersymmetry breaking chiral superfield X. For example, assuming that we have the vector-like messenger fields $(XD,\ XD^c)$ (or $(XL,\ XL^c)$), we introduce the vector-like particles $(XL,\ XL^c)$ (or $(XD,\ XD^c)$) at the messenger scale so that the gauge coupling unification can be preserved. In GUTs from orbifold constructions, intersecting D-brane model building on Type II orientifolds, M-theory on S^1/Z_2 with Calabi-Yau compactifications, and F-theory model building, $(XD,\ XD^c)$ and $(XL,\ XL^c)$ do not need to arise from the same GUT multiplets since the zero modes of their triplet partners and doublet partners can be projected out, respectively. Thus, we can realize such scenarios with some fine-tuning. Interestingly, in the flipped $SU(5) \times U(1)_X$ models, we do not need to fine-tune the mass scales for the vector-like particles due to the two-step gauge coupling unification.

Cases	Messengers	(n_1, n_2, n_3)	k	Cases	Messengers	(n_1, n_2, n_3)	k
(1)	$(XQ,\ XQ^c)$	$(1/5, 3, 2)$	$-5/9$	(2)	$(XU,\ XU^c)$	$(8/5, 0, 1)$	$-5/3$
(3)	$(XD,\ XD^c)$	$(2/5, 0, 1)$	$5/3$	(4)	$(XL,\ XL^c)$	$(3/5, 1, 0)$	$5/3$
(5)	$(XE,\ XE^c)$	$(6/5, 0, 0)$	0	(6)	$(XY,\ XY^c)$	$(5, 3, 2)$	$1/3$
(7)	XG	$(0, 0, 3)$	1	(8)	XW	$(0, 2, 0)$	∞
(9)	$(XT,\ XT^c)$	$(18/5, 4, 0)$	$10/9$	(10)	$(XS,\ XS^c)$	$(16/5, 0, 5)$	$25/9$
(11)	$(XQ,\ XQ^c)$ $(XE,\ XE^c)$	$(7/5, 3, 2)$	$-5/3$	(12)	$(XU,\ XU^c)$ $(XE,\ XE^c)$	$(14/5, 0, 1)$	$-5/9$
(13)	XG XW	$(0, 2, 3)$	$1/3$	(14)	$(XT,\ XT^c)$ $(XS,\ XS^c)$	$(34/5, 4, 5)$	$-5/9$
(15)	$(\mathbf{5}, \overline{\mathbf{5}})$	$(1, 1, 1)$	$0/0$	(16)	$(\mathbf{10}, \overline{\mathbf{10}})$	$(3, 3, 3)$	$0/0$
(17)	$(\mathbf{15}, \overline{\mathbf{15}})$	$(7, 7, 7)$	$0/0$	(18)	$\mathbf{24}$	$(5, 5, 5)$	$0/0$

TABLE V: The $n_i(\Phi)$ for the messenger fields and the corresponding indices k of the gaugino mass relations in $SU(5)$ models.

To calculate the parameters a_i and indices k for the gaugino mass relations, we assume for simplicity that either all the messenger fields have the same Yukawa couplings to X, or the messenger fields are heavier than 10^7 GeV, and then, the parameters a_i are given by Eq. (103). Thus, we only need to present the Dynkin indices n_i for the messenger fields. We emphasize that with the gaugino mass relations and their indices k, we may probe the messenger fields at the intermediate scale. With various messenger fields, we shall consider $SU(5)$ models, flipped $SU(5) \times U(1)_X$ models with $SO(10)$ origin, and Pati-Salam Models with $SO(10)$ origin in the following:

(i) $SU(5)$ Models

In Table V, we present the $n_i(\Phi)$ for the messenger fields and the corresponding indices k of the gaugino mass relations in $SU(5)$ models. We can construct orbifold $SU(5)$ models with vector-like particles in the Cases (1), (3), (4), (6), (12), (13), (14), (15), (16), (17), and (18) in Table V. Here, the Cases (15), (16), (17), and (18) can be considered as the combinations of two Cases, Cases (3) and (4), Cases (1) and (12), Cases (1) and (14), and Cases (6) and (13), respectively. Assuming the superpotential between the messenger fields and X is on the D3-brane at $y = \pi R/2$ where only the SM gauge symmetries is preserved, we can construct orbifold $SU(5)$ models with vector-like particles in the rest Cases in Table V, *i.e.*, the Cases (2), (5), (7), (8), (9), (10), and (11). Moreover, in the F-theory $SU(5)$ models, we can construct the $SU(5)$ models with vector-like particles in the Cases (1), (2), (3), (4), (5), (6), (11), (12), (15), and (16) in Table V. In addition, for the Cases (2), (3), (4), (9), (10), (12), and (13), there are one massless gaugino, and in the Cases (5), (7), and (8), there are two massless gauginos. Thus, each of these Cases can not be consistent with the low-energy phenomenological constraints. To give masses to all the SM gauginos, we can combine the different Cases, and the corresponding indices can be calculated similarly. For example, we can add the messenger fields $(\mathbf{5}, \overline{\mathbf{5}})$ for each of these Cases. Then the Dynkin indices for the messenger fields increase by one, *i.e.*, we change n_i to $n_i + 1$ for each of these Cases in Table V. Interestingly, the indices k are the same as those in Table V since $(\mathbf{5}, \overline{\mathbf{5}})$ form complete $SU(5)$ representations. Also, some interesting combinations of the different Cases will be studied in the flipped $SU(5) \times U(1)_X$ models and the Pati-Salam $SU(4)_C \times SU(2)_L \times SU(2)_R$ models in the following. Furthermore, we emphasize that we do have the mSUGRA gaugino mass relation if the messenger fields form the complete $SU(5)$ representations. Also, if two sets of the messenger fields form complete $SU(5)$ representations, we can show that the indices k for these two sets of the messenger fields are the same. For example, the messenger fields $(XD,\ XD^c)$ and (XL, XL^c) have the same index $k = 5/3$.

(ii) Flipped $SU(5) \times U(1)_X$ Models

In Table VI, we present the $n_i(\Phi)$ for the messenger fields and the corresponding indices k of the gaugino mass relations in flipped $SU(5) \times U(1)_X$ models. We can construct the orbifold $SO(10)$ models with vector-like particles in the Cases (1), (4), (5), (6), (8), and (11) in Table VI where the $SO(10)$ gauge symmetry is broken down to the flipped $SU(5) \times U(1)_X$ gauge symmetries. Assuming the superpotential between the messenger fields and X is on the D3-brane at $y = \pi R/2$ where only the $SU(5) \times U(1)_X$ gauge symmetries is preserved, we can construct the orbifold $SO(10)$ models with vector-like particles in the rest Cases in Table VI, *i.e.*, the Cases (2), (3), (7), (9), (10), and (12).

Moreover, in the F-theory $SO(10)$ models where the gauge symmetry is broken down to the flipped $SU(5) \times U(1)_X$ gauge symmetries by turning on the $U(1)_X$ flux, we can construct the flipped $SU(5) \times U(1)_X$ models with vector-like particles in all the Cases in the Table VI except the Case (5) [26, 28]. Interestingly, the indices k for the gaugino mass relations are zero for all the Cases except the Case (4) with messenger fields $(Xh, \overline{Xh})$. For the Case (4), we obtain the mSUGRA gaugino mass relation. In addition, we have two massless gauginos in the Case (3), so it can not be consistent with the low-energy phenomenological constraints by itself. Furthermore, for the Cases (1), (4), (5), (7), (10), and (11), we can realize the gauge coupling unification naturally. While for the Cases (2), (3), (6), (8), (9), and (12), we can achieve the gauge coupling unification in the testable flipped $SU(5)\times U(1)_X$ models due to the two-step gauge coupling unification.

Cases	Messengers	(n_1, n_2, n_3)	k	Cases	Messengers	(n_1, n_2, n_3)	k
(1)	$(XF, \overline{XF})$	$(3/5, 3, 3)$	0	(2)	$(Xf, \overline{Xf})$	$(11/5, 1, 1)$	0
(3)	$(Xl, \overline{Xl})$	$(6/5, 0, 0)$	0	(4)	$(Xh, \overline{Xh})$	$(1, 1, 1)$	0/0
(5)	(XGW, XN)	$(1/5, 5, 5)$	0	(6)	$(XX, \overline{XX})$	$(39/5, 3, 3)$	0
(7)	$(XF, \overline{XF})$ $(Xl, \overline{Xl})$	$(9/5, 3, 3)$	0	(8)	$(Xf, \overline{Xf})$ $(Xl, \overline{Xl})$	$(17/5, 1, 1)$	0
(9)	$(Xh, \overline{Xh})$ $(Xl, \overline{Xl})$	$(11/5, 1, 1)$	0	(10)	$(XF, \overline{XF})$ $(Xf, \overline{Xf})$	$(14/5, 4, 4)$	0
(11)	$(XF, \overline{XF})$ $(Xh, \overline{Xh})$	$(8/5, 4, 4)$	0	(12)	$(Xf, \overline{Xf})$ $(Xh, \overline{Xh})$	$(16/5, 2, 2)$	0

TABLE VI: The $n_i(\Phi)$ for the messenger fields and the corresponding indices k of the gaugino mass relations in flipped $SU(5) \times U(1)_X$ models.

(iii) Pati-Salam $SU(4)_C \times SU(2)_L \times SU(2)_R$ Models

In Table VII, we present the $n_i(\Phi)$ for the messenger fields and the corresponding indices k of the gaugino mass relations in Pati-Salam $SU(4)_C \times SU(2)_L \times SU(2)_R$ models. We can construct the orbifold $SO(10)$ models with vector-like particles in all the Cases in Table VII where the $SO(10)$ gauge symmetry is broken down to the Pati-Salam $SU(4)_C \times SU(2)_L \times$

$SU(2)_R$ gauge symmetries. Moreover, in F-theory $SO(10)$ models where the $SO(10)$ gauge symmetry is broken down to the $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ gauge symmetries by turning on the $U(1)_{B-L}$ flux [25, 29], we can construct the $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ models with vector-like particles in all the Cases in the Table VII except the Case (5) [29]. In addition, in the Cases (2), (3), (4), and (8), there are one massless gaugino, and then each of them is not consistent with the low-energy phenomenological constraints by itself. We can solve the problem by combining the different Cases, and some combinations of the different simple Cases are given in Table VII as well.

Cases	Messengers	(n_1, n_2, n_3)	k	Cases	Messengers	(n_1, n_2, n_3)	k
(1)	$(XFL,\ \overline{XFL})$	$(4/5, 4, 2)$	$-5/3$	(2)	$(XFR,\ \overline{XFR})$	$(16/5, 0, 2)$	$-5/3$
(3)	$XD\overline{D}$	$(2/5, 0, 1)$	$5/3$	(4)	$XL\overline{L}$	$(3/5, 1, 0)$	$5/3$
(5)	$(XG4, XWL)$ XWR	$(14/5, 2, 4)$	$5/3$	(6)	XZ	$(26/5, 6, 4)$	$5/3$
(7)	$(XFL,\ \overline{XFL})$ $XD\overline{D}$	$(6/5, 4, 3)$	$-5/9$	(8)	$(XFR,\ \overline{XFR})$ $XD\overline{D}$	$(18/5, 0, 3)$	-5
(9)	$(XFL,\ \overline{XFL})$ $XL\overline{L}$	$(7/5, 5, 2)$	-5	(10)	$(XFR,\ \overline{XFR})$ $XL\overline{L}$	$(19/5, 1, 2)$	$-5/9$

TABLE VII: The $n_i(\Phi)$ for the messenger fields and the corresponding indices k of the gaugino mass relations in Pati-Salam $SU(4)_C \times SU(2)_L \times SU(2)_R$ models.

V. ANOMALY MEDIATED SUPERSYMMETRY BREAKING

We first briefly review the AMSB [35–37]. The supergravity Lagrangian can be obtained from a local superconformal field theory by a gauge fixing of extra symmetries, which can be done by setting the values of the components of a chiral compensator field C. Thus, C couples to the conformal symmetry violation, *i.e.*, all the dimensionful parameters including the renormalization scale μ. To have the canonical normalization for the gravity kinetic terms, we determine the scalar component of C. To cancel the cosmological constant after supersymmetry breaking in the hidden sector, we give a non-zero VEV to the auxiliary component F^C of C, which is the only source of supersymmetry breaking. With $\langle C \rangle =$

$M_C + \theta^2 F^C$, we obtain the gravitino mass $m_{3/2} = F^C/M_C$. To suppress the supergravity contributions to the supersymmetry breaking soft terms, we assume the sequestering between the observable and hidden sectors for simplicity. This can be realized naturally in the five-dimensional brane world scenario where the observable and hidden sectors are confined on the different branes [61], or in the models where the contact terms between the observable and hidden sectors are suppressed dynamically by a conformal sector [62].

In this paper, we concentrate on the gaugino masses. The relevant Lagrangian is

$$\mathcal{L} \supset \int d^2\theta \frac{1}{2g^2} \mathrm{Tr}\left[W^\alpha W_\alpha\right] + \mathrm{H.C.} \;, \tag{105}$$

where W^α is the field strength of the vector superfield. Because the compensator C couples to the renormalization scale μ, there are additional contributions at quantum level. Then we have

$$\mathcal{L} \supset \int d^2\theta \frac{1}{2g^2\left(\frac{\mu}{C}\right)} \mathrm{Tr}\left[W^\alpha W_\alpha\right] + \mathrm{H.C.} \;. \tag{106}$$

Thus, we obtain the SM gaugino masses

$$\frac{M_i}{\alpha_i} \;=\; \frac{b_i}{4\pi}\frac{F^C}{M_C} \;, \tag{107}$$

where b_3, b_2, and b_1 are the one-loop beta functions for $SU(3)_C$, $SU(2)_L$, and $U(1)_Y$, respectively. In particular, if there are vector-like particles at the intermediate scales which do not mediate supersymmetry breaking, we emphasize that these vector-like particles will not affect the low-energy gaugino masses in the AMSB after they are integrated out [5].

Moreover, although AMSB can solve the flavour changing neutral current problem, the minimal AMSB is excluded since the squared slepton masses are negative and then the electromagnetism will be broken. In this paper, we consider two solutions: (1) UV insensitive anomaly mediation [36]; (2) Deflected anomaly mediation [37].

A. UV Insensitive Anomaly Mediation

In the UV insensitive anomaly mediation [36], the $U(1)$ D-terms contribute to the slepton masses, and then the squared slepton masses can be positive. In particular, the $U(1)$ symmetries can be $U(1)_Y$ and $U(1)_{B-L}$ so that we only need to introduce three right-handed neutrinos to cancel the $U(1)_{B-L}$ gauge anomalies. Interestingly, the gaugino masses are still given by Eq. (107). Thus, we obtain

$$a_i \;=\; b_i \;. \tag{108}$$

We shall consider the $SU(5)$ and flipped $SU(5)\times U(1)_X$ models with TeV-scale vector-like particles. To achieve the one-step gauge coupling unification, we emphasize that the discussions for the Pati-Salam $SU(4)_C \times SU(2)_L \times SU(2)_R$ models are similar to those in the $SU(5)$ models. Thus, we will not consider the Pati-Salam models here for simplicity. In $SU(5)$ models, to achieve the gauge coupling unification, we consider the TeV-scale vector-like particles that form complete $SU(5)$ representations. In Table VIII, we present the parameters a_i and the indices k of the gaugino mass relations in the $SU(5)$ models without and with TeV-scale vector-like particles. Especially, the indices k are equal to 5/12 for all these Cases. In addition, we present the parameters a_i and the indices k of the gaugino mass relations in Table IX in the flipped $SU(5)\times U(1)_X$ models with TeV-scale vector-like particles. These vector-like particles also form complete $SU(5)\times U(1)_X$ representations. For the Cases (1), (4), (5), (8), and (9), we can have the gauge coupling unification naturally. However, for the Cases (2), (3), (6), and (7), we should introduce the vector-like particles $(XF,\ \overline{XF})$ at the intermediate scale 10^8 GeV or smaller so that we can obtain the gauge coupling unification.

Furthermore, for the Cases (4) and (6) in the $SU(5)$ models and the Cases (1) and (5) in the flipped $SU(5)\times U(1)_X$ models, gluino is massless. This problem can be solved elegantly in the deflected AMSB in the next subsection. Also, for the Cases (5) and (7) in the $SU(5)$ models and the Cases (8) and (9) in the flipped $SU(5)\times U(1)_X$ models, we emphasize that the masses of the vector-like particles may need to be about 20 TeV or larger so that we can avoid the Landau pole problem for the strong coupling [28, 59]. Thus, we can not test these models at the LHC since we may have 10 TeV scale supersymmetry breaking.

B. Deflected Anomaly Mediation

In the deflected anomaly mediation [37], similar to the GMSB, we introduce a chiral superfield X and a set of the SM vector-like particles Φ_j and $\overline{\Phi}_j$. The superpotential is

$$W \supset \lambda_j X \overline{\Phi}_j \Phi_j + M_*^{3-p} X^p \;, \tag{109}$$

Case	New Particles	$(a_1,\, a_2,\, a_3)$	k	Case	New Particles	$(a_1,\, a_2,\, a_3)$	k
(1)	No	$(33/5, 1, -3)$	$5/12$	(2)	$(\mathbf{5}, \overline{\mathbf{5}})$	$(38/5, 2, -2)$	$5/12$
(3)	$2 \times (\mathbf{5}, \overline{\mathbf{5}})$	$(43/5, 3, -1)$	$5/12$	(4)	$3 \times (\mathbf{5}, \overline{\mathbf{5}})$	$(48/5, 4, 0)$	$5/12$
(5)	$4 \times (\mathbf{5}, \overline{\mathbf{5}})$	$(53/5, 5, 1)$	$5/12$	(6)	$(\mathbf{10}, \overline{\mathbf{10}})$	$(48/5, 4, 0)$	$5/12$
(7)	$(\mathbf{5}, \overline{\mathbf{5}})$, $(\mathbf{10}, \overline{\mathbf{10}})$	$(53/5, 5, 1)$	$5/12$				

TABLE VIII: The parameters a_i and the indices k for the UV insensitive AMSB in the $SU(5)$ models without and with additional vector-like particles.

Case	New Particles	$(a_1,\, a_2,\, a_3)$	k	Case	New Particles	$(a_1,\, a_2,\, a_3)$	k
(1)	$(XF, \overline{XF})$	$(36/5, 4, 0)$	$5/9$	(2)	$(Xf, \overline{Xf})$	$(44/5, 2, -2)$	$10/27$
(3)	$(Xl, \overline{Xl})$	$(39/5, 1, -3)$	$10/27$	(4)	$(Xh, \overline{Xh})$	$(38/5, 2, -2)$	$5/12$
(5)	$(XF, \overline{XF})$ $(Xl, \overline{Xl})$	$(42/5, 4, 0)$	$10/21$	(6)	$(Xf, \overline{Xf})$ $(Xh, \overline{Xh})$	$(49/5, 3, -1)$	$10/27$
(7)	$(Xl, \overline{Xl})$ $(Xh, \overline{Xh})$	$(44/5, 2, -2)$	$10/27$	(8)	$(XF, \overline{XF})$ $(Xf, \overline{Xf})$	$(47/5, 5, 1)$	$10/21$
(9)	$(XF, \overline{XF})$ $(Xh, \overline{Xh})$	$(41/5, 5, 1)$	$5/9$				

TABLE IX: The parameters a_i and the indices k for the UV insensitive AMSB in the flipped $SU(5) \times U(1)_X$ models with additional vector-like particles.

where $p \neq 3$, and M_* is a model-dependent mass parameter. The chiral compensator C couples to X at tree level by the scale non-invariant term $M_*^{3-p} X^p$, and then the VEVs of X can be fixed. It was shown that X is stabilized at $\langle X \rangle >> m_{3/2}$ for $M_* >> m_{3/2}$ if $p > 3$ or $p < 0$ as follows

$$\langle X \rangle \ = \ M_X + \theta^2 F^X \ , \tag{110}$$

where

$$M_X \simeq m_{3/2}^{1/(p-2)} M_*^{(p-3)/(p-2)} \ , \quad \frac{F^X}{M_X} \ = \ -\frac{2}{p-1} \frac{F^C}{M_C} \ . \tag{111}$$

In addition, even without the term $M_*^{3-p}X^p$ in the superpotential, X can still be stabilized by the radiative corrections to its Kähler potential, and then we have

$$\frac{F^X}{M_X} \simeq -\frac{F^C}{M_C} . \quad (112)$$

Thus, the contributions to the supersymmetry breaking soft masses from gauge mediation are comparable to those from anomaly mediation, and then we can solve the tachyonic slepton problem in the AMSB. Moreover, we obtain the gaugino masses at the TeV scale

$$\frac{M_i}{\alpha_i} = \frac{1}{4\pi}\left(b_i + \frac{2}{p-1}\sum_j n_i(\Phi_j)g(x_j)\right)\frac{F^C}{M_C} . \quad (113)$$

Thus, we have

$$a_i = b_i + \frac{2}{p-1}\sum_j n_i(\Phi_j)g(x_j) . \quad (114)$$

If the messenger fields are heavier than 10^7 GeV and their Yukawa couplings to X are about order one, we obtain

$$a_i \simeq b_i + \frac{2}{p-1}\sum_j n_i(\Phi_j) . \quad (115)$$

Thus, choosing the possible value for p and introducing the TeV-scale vector-like particles, we can calculate the parameters a_i and the indices k of the gaugino mass relations.

To probe the messenger fields in the deflected anomaly mediation, we should define a new index k' for the gaugino mass relations. In the supersymmetric SM, we have

$$b_1 = \frac{33}{5} , \quad b_2 = 1 , \quad b_3 = -3 . \quad (116)$$

Thus, b_1 and b_2 will aways be positive even if we introduce the vector-like particles at the TeV scale. Therefore, for $b_3 \neq 0$, we define the new index k' as follows

$$k' \equiv \frac{b_1 b_3 \frac{M_2}{\alpha_2} - b_1 b_2 \frac{M_3}{\alpha_3}}{b_2 b_3 \frac{M_1}{\alpha_1} - b_1 b_2 \frac{M_3}{\alpha_3}} = \frac{b_1 b_3 \sum_j n_2(\Phi_j)g(x_j) - b_1 b_2 \sum_j n_3(\Phi_j)g(x_j)}{b_2 b_3 \sum_j n_1(\Phi_j)g(x_j) - b_1 b_2 \sum_j n_3(\Phi_j)g(x_j)} . \quad (117)$$

And for $b_3 = 0$, we define the new index k' as follows

$$k' \equiv \frac{b_1 \frac{M_2}{\alpha_2} - b_2 \frac{M_1}{\alpha_1}}{\frac{M_3}{\alpha_3}} = \frac{b_1 \sum_j n_2(\Phi_j)g(x_j) - b_2 \sum_j n_1(\Phi_j)g(x_j)}{\sum_j n_3(\Phi_j)g(x_j)} . \quad (118)$$

Assuming that the messenger fields are heavier than 10^7 GeV and their Yukawa couplings to X are about order one, we consider the $SU(5)$ models, the flipped $SU(5)$ models, the Pati-Salam Models, and the other possible models in the following:

Cases	Messengers	$(a_1^0,\ a_2^0,\ a_3^0)$	k_0	$(a_1^1,\ a_2^1,\ a_3^1)$	k_1	$(a_1^2,\ a_2^2,\ a_3^2)$	k_2
(1)	$(XQ,\ XQ^c)$	$(\frac{101}{15}, 3, -\frac{5}{3})$	$\frac{5}{9}$	$(\frac{116}{15}, 4, -\frac{2}{3})$	$\frac{5}{9}$	$(\frac{146}{15}, 6, \frac{4}{3})$	$\frac{5}{9}$
(2)	$(XU,\ XU^c)$	$(\frac{23}{3}, 1, -\frac{7}{3})$	$\frac{1}{3}$	$(\frac{26}{3}, 2, -\frac{4}{3})$	$\frac{1}{3}$	$(\frac{32}{3}, 4, \frac{2}{3})$	$\frac{1}{3}$
(3)	$(XD,\ XD^c)$	$(\frac{103}{15}, 1, -\frac{7}{3})$	$\frac{25}{69}$	$(\frac{118}{15}, 2, -\frac{4}{3})$	$\frac{25}{69}$	$(\frac{148}{15}, 4, \frac{2}{3})$	$\frac{25}{69}$
(4)	$(XL,\ XL^c)$	$(7, \frac{5}{3}, -3)$	$\frac{7}{15}$	$(8, \frac{8}{3}, -2)$	$\frac{7}{15}$	$(10, \frac{14}{3}, 0)$	$\frac{7}{15}$
(5)	$(XE,\ XE^c)$	$(\frac{37}{5}, 1, -3)$	$\frac{5}{13}$	$(\frac{42}{5}, 2, -2)$	$\frac{5}{13}$	$(\frac{52}{5}, 4, 0)$	$\frac{5}{13}$
(6)	$(XY,\ XY^c)$	$(\frac{149}{15}, 3, -\frac{5}{3})$	$\frac{35}{87}$	$(\frac{164}{15}, 4, -\frac{2}{3})$	$\frac{35}{87}$	$(\frac{194}{15}, 6, \frac{4}{3})$	$\frac{35}{87}$
(7)	XG	$(\frac{33}{5}, 1, -1)$	$\frac{5}{19}$	$(\frac{38}{5}, 2, 0)$	$\frac{5}{19}$	$(\frac{48}{5}, 4, 2)$	$\frac{5}{19}$
(8)	XW	$(\frac{33}{5}, \frac{7}{3}, -3)$	$\frac{5}{9}$	$(\frac{38}{5}, \frac{10}{3}, -2)$	$\frac{5}{9}$	$(\frac{48}{5}, \frac{16}{3}, 0)$	$\frac{5}{9}$
(9)	$(XT,\ XT^c)$	$(9, \frac{11}{3}, -3)$	$\frac{5}{9}$	$(10, \frac{14}{3}, -2)$	$\frac{5}{9}$	$(12, \frac{20}{3}, 0)$	$\frac{5}{9}$
(10)	$(XS,\ XS^c)$	$(\frac{131}{15}, 1, \frac{1}{3})$	$\frac{5}{63}$	$(\frac{146}{15}, 2, \frac{4}{3})$	$\frac{5}{63}$	$(\frac{176}{15}, 4, \frac{10}{3})$	$\frac{5}{63}$
(11)	$(XQ,\ XQ^c)$ $(XE,\ XE^c)$	$(\frac{113}{15}, 3, -\frac{5}{3})$	$\frac{35}{69}$	$(\frac{128}{15}, 4, -\frac{2}{3})$	$\frac{35}{69}$	$(\frac{158}{15}, 6, \frac{4}{3})$	$\frac{35}{69}$
(12)	$(XU,\ XU^c)$ $(XE,\ XE^c)$	$(\frac{127}{15}, 1, -\frac{7}{3})$	$\frac{25}{81}$	$(\frac{142}{15}, 2, -\frac{4}{3})$	$\frac{25}{81}$	$(\frac{172}{15}, 4, \frac{2}{3})$	$\frac{25}{81}$
(13)	$(XG,\ XW)$	$(\frac{33}{5}, \frac{7}{3}, -1)$	$\frac{25}{57}$	$(\frac{38}{5}, \frac{10}{3}, 0)$	$\frac{25}{57}$	$(\frac{48}{5}, \frac{16}{3}, 2)$	$\frac{25}{57}$
(14)	$(XT,\ XT^c)$ $(XS,\ XS^c)$	$(\frac{167}{15}, \frac{11}{3}, \frac{1}{3})$	$\frac{25}{81}$	$(\frac{182}{15}, \frac{14}{3}, \frac{4}{3})$	$\frac{25}{81}$	$(\frac{212}{15}, \frac{20}{3}, \frac{10}{3})$	$\frac{25}{81}$
(15)	$(\mathbf{5}, \overline{\mathbf{5}})$	$(\frac{109}{15}, \frac{5}{3}, -\frac{7}{3})$	$\frac{5}{12}$	$(\frac{124}{15}, \frac{8}{3}, -\frac{4}{3})$	$\frac{5}{12}$	$(\frac{154}{15}, \frac{14}{3}, \frac{2}{3})$	$\frac{5}{12}$
(16)	$(\mathbf{10}, \overline{\mathbf{10}})$	$(\frac{43}{5}, 3, -1)$	$\frac{5}{12}$	$(\frac{48}{5}, 4, 0)$	$\frac{5}{12}$	$(\frac{58}{5}, 6, 2)$	$\frac{5}{12}$
(17)	$(\mathbf{15}, \overline{\mathbf{15}})$	$(\frac{169}{15}, \frac{17}{3}, \frac{5}{3})$	$\frac{5}{12}$	$(\frac{184}{15}, \frac{20}{3}, \frac{8}{3})$	$\frac{5}{12}$	$(\frac{214}{15}, \frac{26}{3}, \frac{14}{3})$	$\frac{5}{12}$
(18)	$\mathbf{24}$	$(\frac{149}{15}, \frac{13}{3}, \frac{1}{3})$	$\frac{5}{12}$	$(\frac{164}{15}, \frac{16}{3}, \frac{4}{3})$	$\frac{5}{12}$	$(\frac{194}{15}, \frac{22}{3}, \frac{10}{3})$	$\frac{5}{12}$

TABLE X: The parameters a_i^0, a_i^1, and a_i^2, and the indices k_0, k_1, and k_2 of the gaugino mass relations in the $SU(5)$ models with various messenger fields.

Cases	Messengers	k'_0	k'_1	k'_2	Cases	Messengers	k'_0	k'_1	k'_2
(1)	$(XQ,\ XQ^c)$	121/23	95/39	14	(2)	$(XU,\ XU^c)$	11/19	19/27	−32/5
(3)	$(XD,\ XD^c)$	11/13	19/21	−8/5	(4)	$(XL,\ XL^c)$	11	19/3	∞
(5)	$(XE,\ XE^c)$	0	0	∞	(6)	$(XY,\ XY^c)$	121/47	95/63	22/5
(7)	XG	1	1	0	(8)	XW	∞	∞	∞
(9)	$(XT,\ XT^c)$	22/3	38/9	∞	(10)	$(XS,\ XS^c)$	55/71	95/111	−64/25
(11)	$(XQ,\ XQ^c)$ $(XE,\ XE^c)$	121/29	19/9	58/5	(12)	$(XU,\ XU^c)$ $(XE,\ XE^c)$	11/25	19/33	−56/5
(13)	XG XW	3	5/3	32/5	(14)	$(XT,\ XT^c)$ $(XS,\ XS^c)$	187/89	57/43	56/25
(15)	$(\mathbf{5},\overline{\mathbf{5}})$	11/4	19/12	28/5	(16)	$(\mathbf{10},\overline{\mathbf{10}})$	11/4	19/12	28/5
(17)	$(\mathbf{15},\overline{\mathbf{15}})$	11/4	19/12	28/5	(18)	$\mathbf{24}$	11/4	19/12	28/5

TABLE XI: The indices k'_0, k'_1, and k'_2 of the gaugino mass relations in the $SU(5)$ models with various messenger fields.

(i) The $SU(5)$ Models

We consider three Types of the $SU(5)$ models with or without additional SM singlet(s): Type I $SU(5)$ models are the minimal $SU(5)$ models ; Type II $SU(5)$ models are the $SU(5)$ models with TeV-scale vector-like particles $(\mathbf{5},\ \overline{\mathbf{5}})$; Type III $SU(5)$ models are the $SU(5)$ models with TeV-scale vector-like particles $(\mathbf{10},\ \overline{\mathbf{10}})$ (or three pairs of $(\mathbf{5},\ \overline{\mathbf{5}})$). We denote the parameters a_i, and the indices k and k' for the gaugino mass relations in Type I $SU(5)$ models as a_i^0, k_0, and k'_0, in Type II $SU(5)$ models as a_i^1, k_1, and k'_1, and in Type III $SU(5)$ models as a_i^2, k_2, and k'_2, respectively. For k'_i, we have

$$k'_0 = \frac{33\sum_j n_2(\Phi_j)g(x_j) + 11\sum_j n_3(\Phi_j)g(x_j)}{5\sum_j n_1(\Phi_j)g(x_j) + 11\sum_j n_3(\Phi_j)g(x_j)} , \tag{119}$$

$$k'_1 = \frac{19\sum_j n_2(\Phi_j)g(x_j) + 19\sum_j n_3(\Phi_j)g(x_j)}{5\sum_j n_1(\Phi_j)g(x_j) + 19\sum_j n_3(\Phi_j)g(x_j)} , \tag{120}$$

$$k'_2 = \frac{48\sum_j n_2(\Phi_j)g(x_j) - 20\sum_j n_1(\Phi_j)g(x_j)}{5\sum_j n_3(\Phi_j)g(x_j)} . \tag{121}$$

Choosing $p = 4$, we present the parameters a_i^0, a_i^1, and a_i^2, and the indices k_0, k_1, and k_2 for various messenger fields in Table X. For the Cases (7), (13), and (16) in Type II $SU(5)$ models, and for the Cases (4), (5), (8), and (9) in Type III $SU(5)$ models, we have massless gluino. This problem can be solved easily by choosing the other p, for example, $p = 5$. Also, we present the indices k'_0, k'_1, and k'_2 for various messenger fields in Table XI. We emphasize that the indices k'_0, k'_1, and k'_2 will be the same if we assume that all the messenger fields have the same Yukawa couplings to X since $g(x_j)$ is the same for all the messenger fields. However, in the Cases (7), (8), and (13), we can not solve the tachyonic slepton problem since the messenger fields are not charged under $U(1)_Y$. Interestingly, the gluino is the lightest gaugino in most of our scenarios.

(ii) The flipped $SU(5) \times U(1)_X$ models

We consider three Types of the flipped $SU(5) \times U(1)_X$ models with or without additional SM singlet(s): Type I flipped $SU(5) \times U(1)_X$ models are the minimal flipped $SU(5) \times U(1)_X$ models; Type II flipped $SU(5) \times U(1)_X$ models are the flipped $SU(5) \times U(1)_X$ models with TeV-scale vector-like particles (XF, $\overline{XF}$); Type III flipped $SU(5) \times U(1)_X$ models are the flipped $SU(5) \times U(1)_X$ models with TeV-scale vector-like particles (XF, $\overline{XF}$) and (Xl, $\overline{Xl}$). Moreover, we denote the parameters a_i, and the indices k and k' for gaugino mass relations in the Type I flipped $SU(5) \times U(1)_X$ models as a_i^0, k_0, and k'_0, in the Type II flipped $SU(5) \times U(1)_X$ models as a_i^1, k_1, and k'_1, and in the Type III flipped $SU(5) \times U(1)_X$ models as a_i^2, k_2, and k'_2, respectively. In addition, k'_0 is given by Eq. (119), and we have

$$k'_1 = \frac{36\sum_j n_2(\Phi_j)g(x_j) - 20\sum_j n_1(\Phi_j)g(x_j)}{5\sum_j n_3(\Phi_j)g(x_j)} , \tag{122}$$

$$k'_2 = \frac{42\sum_j n_2(\Phi_j)g(x_j) - 20\sum_j n_1(\Phi_j)g(x_j)}{5\sum_j n_3(\Phi_j)g(x_j)} . \tag{123}$$

Cases	Messengers	$(a_1^0,\ a_2^0,\ a_3^0)$	k_0	$(a_1^1,\ a_2^1,\ a_3^1)$	k_1	$(a_1^2,\ a_2^2,\ a_3^2)$	k_2
(1)	$(XF,\ \overline{XF})$	$(7,3,-1)$	$\frac{1}{2}$	$(\frac{38}{5},6,2)$	$\frac{5}{7}$	$(\frac{44}{5},6,2)$	$\frac{10}{17}$
(2)	$(Xf,\ \overline{Xf})$	$(\frac{121}{15},\frac{5}{3},-\frac{7}{3})$	$\frac{5}{13}$	$(\frac{26}{3},\frac{14}{3},\frac{2}{3})$	$\frac{1}{2}$	$(\frac{148}{15},\frac{14}{3},\frac{2}{3})$	$\frac{10}{23}$
(3)	$(Xl,\ \overline{Xl})$	$(\frac{37}{5},1,-3)$	$\frac{5}{13}$	$(8,4,0)$	$\frac{1}{2}$	$(\frac{46}{5},4,0)$	$\frac{10}{23}$
(4)	$(Xh,\ \overline{Xh})$	$(\frac{109}{15},\frac{5}{3},-\frac{7}{3})$	$\frac{5}{12}$	$(\frac{118}{15},\frac{14}{3},\frac{2}{3})$	$\frac{5}{9}$	$(\frac{136}{15},\frac{14}{3},\frac{2}{3})$	$\frac{10}{21}$
(5)	(XGW, XN)	$(\frac{101}{15},\frac{13}{3},\frac{1}{3})$	$\frac{5}{8}$	$(\frac{22}{3},\frac{22}{3},\frac{10}{3})$	1	$(\frac{128}{15},\frac{22}{3},\frac{10}{3})$	$\frac{10}{13}$
(6)	$(XX,\ \overline{XX})$	$(\frac{59}{5},3,-1)$	$\frac{5}{16}$	$(\frac{62}{5},6,2)$	$\frac{5}{13}$	$(\frac{68}{5},6,2)$	$\frac{10}{29}$
(7)	$(XF,\ \overline{XF})$ $(Xl,\ \overline{Xl})$	$(\frac{39}{5},3,-1)$	$\frac{5}{11}$	$(\frac{42}{5},6,2)$	$\frac{5}{8}$	$(\frac{48}{5},6,2)$	$\frac{10}{19}$
(8)	$(Xf,\ \overline{Xf})$ $(Xl,\ \overline{Xl})$	$(\frac{133}{15},\frac{5}{3},-\frac{7}{3})$	$\frac{5}{14}$	$(\frac{142}{15},\frac{14}{3},\frac{2}{3})$	$\frac{5}{11}$	$(\frac{32}{3},\frac{14}{3},\frac{2}{3})$	$\frac{2}{5}$
(9)	$(Xh,\ \overline{Xh})$ $(Xl,\ \overline{Xl})$	$(\frac{121}{15},\frac{5}{3},-\frac{7}{3})$	$\frac{5}{13}$	$(\frac{26}{3},\frac{14}{3},\frac{2}{3})$	$\frac{1}{2}$	$(\frac{148}{15},\frac{14}{3},\frac{2}{3})$	$\frac{10}{23}$
(10)	$(XF,\ \overline{XF})$ $(Xf,\ \overline{Xf})$	$(\frac{127}{15},\frac{11}{3},-\frac{1}{3})$	$\frac{5}{11}$	$(\frac{136}{15},\frac{20}{3},\frac{8}{3})$	$\frac{5}{8}$	$(\frac{154}{15},\frac{20}{3},\frac{8}{3})$	$\frac{10}{19}$
(11)	$(XF,\ \overline{XF})$ $(Xh,\ \overline{Xh})$	$(\frac{23}{3},\frac{11}{3},-\frac{1}{3})$	$\frac{1}{2}$	$(\frac{124}{15},\frac{20}{3},\frac{8}{3})$	$\frac{5}{7}$	$(\frac{142}{15},\frac{20}{3},\frac{8}{3})$	$\frac{10}{17}$
(12)	$(Xf,\ \overline{Xf})$ $(Xh,\ \overline{Xh})$	$(\frac{131}{15},\frac{7}{3},-\frac{5}{3})$	$\frac{5}{13}$	$(\frac{28}{3},\frac{16}{3},\frac{4}{3})$	$\frac{1}{2}$	$(\frac{158}{15},\frac{16}{3},\frac{4}{3})$	$\frac{10}{23}$

TABLE XII: The parameters a_i^0, a_i^1, and a_i^2, and the indices k_0, k_1, and k_2 of the gaugino mass relations in the flipped $SU(5) \times U(1)_X$ models with various messenger fields.

Choosing $p = 4$, we present the parameters a_i^0, a_i^1, and a_i^2, and the indices k_0, k_1, and k_2 for various messenger fields in Table XII. For the Case (3) in Type II and Type III flipped $SU(5) \times U(1)_X$ models, we have massless gluino. This problem can be solved by choosing the other p, for example, $p = 5$. Moreover, we present the indices k_0', k_1', and k_2' for various messenger fields in Table XIII. We emphasize that the indices k_0', k_1', and k_2' will be the same if we assume that all the messenger fields have the same Yukawa couplings to X. And we have gluino as the lightest gaugino in most of our scenarios.

Cases	Messengers	k'_0	k'_1	k'_2	Cases	Messengers	k'_0	k'_1	k'_2
(1)	$(XF,\ \overline{XF})$	11/3	32/5	38/5	(2)	$(Xf,\ \overline{Xf})$	2	−8/5	−2/5
(3)	$(Xl,\ \overline{Xl})$	0	∞	∞	(4)	$(Xh,\ \overline{Xh})$	11/4	16/5	22/5
(5)	(XGW, XN)	55/14	176/25	206/25	(6)	$(XX,\ \overline{XX})$	11/6	−16/5	−2
(7)	$(XF,\ \overline{XF})$ $(Xl,\ \overline{Xl})$	22/7	24/5	6	(8)	$(Xf,\ \overline{Xf})$ $(Xl,\ \overline{Xl})$	11/7	−32/5	−26/5
(9)	$(Xh,\ \overline{Xh})$ $(Xl,\ \overline{Xl})$	2	−8/5	−2/5	(10)	$(XF,\ \overline{XF})$ $(Xf,\ \overline{Xf})$	88/29	22/5	28/5
(11)	$(XF,\ \overline{XF})$ $(Xh,\ \overline{Xh})$	44/13	28/5	34/5	(12)	$(Xf,\ \overline{Xf})$ $(Xh,\ \overline{Xh})$	44/19	4/5	2

TABLE XIII: The indices k'_0, k'_1, and k'_2 of the gaugino mass relations in the flipped $SU(5)\times U(1)_X$ models with various messenger fields.

(iii) The Pati-Salam $SU(4)_C \times SU(2)_L \times SU(2)_R$ Models

We consider three Types of the Pati-Salam $SU(4)_C \times SU(2)_L \times SU(2)_R$ models with or without additional SM singlet(s): Type I Pati-Salam models are the minimal Pati-Salam models; Type II Pati-Salam models are the Pati-Salam models with TeV-scale vector-like particles ($\mathbf{5}$, $\overline{\mathbf{5}}$) under $SU(5)$; and Type III Pati-Salam models are the Pati-Salam models with TeV-scale vector-like particles ($\mathbf{10}$, $\overline{\mathbf{10}}$) (or three pairs of ($\mathbf{5}$, $\overline{\mathbf{5}}$)) under $SU(5)$. We denote the parameters a_i, and the indices k and k' for the gaugino mass relations in Type I Pati-Salam models as a_i^0, k_0, and k'_0, in Type II Pati-Salam models as a_i^1, k_1, and k'_1, and in Type III Pati-Salam models as a_i^2, k_2, and k'_2, respectively. Also, k'_0, k'_1, and k'_2 are given by Eqs. (119), (120), and (121), respectively.

Choosing $p = 4$, we present the parameters a_i^0, a_i^1, and a_i^2, and the indices k_0, k_1, and k_2 for various messenger fields in Table XIV. For the Cases (7) and (8) in Type II Pati-Salam models, and for the Case (4) in Type III Pati-Salam models, we have massless gluino. This problem can be solved by choosing the other p, for example, $p = 5$. Also, we present the indices k'_0, k'_1, and k'_2 for various messenger fields in Table XV. We emphasize that the indices k'_0, k'_1, and k'_2 will be the same if we assume that all the messenger fields have the

Cases	Messengers	$(a_1^0,\ a_2^0,\ a_3^0)$	k_0	$(a_1^1,\ a_2^1,\ a_3^1)$	k_1	$(a_1^2,\ a_2^2,\ a_3^2)$	k_2
(1)	$(XFL,\ \overline{XFL})$	$(\frac{107}{15}, \frac{11}{3}, -\frac{5}{3})$	$\frac{20}{33}$	$(\frac{122}{15}, \frac{14}{3}, -\frac{2}{3})$	$\frac{20}{33}$	$(\frac{152}{15}, \frac{20}{3}, \frac{4}{3})$	$\frac{20}{33}$
(2)	$(XFR,\ \overline{XFR})$	$(\frac{131}{15}, 1, -\frac{5}{3})$	$\frac{10}{39}$	$(\frac{146}{15}, 2, -\frac{2}{3})$	$\frac{10}{39}$	$(\frac{176}{15}, 4, \frac{4}{3})$	$\frac{10}{39}$
(3)	$XD\overline{D}$	$(\frac{103}{15}, 1, -\frac{7}{3})$	$\frac{25}{69}$	$(\frac{118}{15}, 2, -\frac{4}{3})$	$\frac{25}{69}$	$(\frac{148}{15}, 4, \frac{2}{3})$	$\frac{25}{69}$
(4)	$XL\overline{L}$	$(7, \frac{5}{3}, -3)$	$\frac{7}{15}$	$(8, \frac{8}{3}, -2)$	$\frac{7}{15}$	$(10, \frac{14}{3}, 0)$	$\frac{7}{15}$
(5)	$(XG4, XWL)$ XWR	$(\frac{127}{15}, \frac{7}{3}, -\frac{1}{3})$	$\frac{10}{33}$	$(\frac{142}{15}, \frac{10}{3}, \frac{2}{3})$	$\frac{10}{33}$	$(\frac{172}{15}, \frac{16}{3}, \frac{8}{3})$	$\frac{10}{33}$
(6)	XZ	$(\frac{151}{15}, 5, -\frac{1}{3})$	$\frac{20}{39}$	$(\frac{166}{15}, 6, \frac{2}{3})$	$\frac{20}{39}$	$(\frac{196}{15}, 8, \frac{8}{3})$	$\frac{20}{39}$
(7)	$(XFL,\ \overline{XFL})$ $XD\overline{D}$	$(\frac{37}{5}, \frac{11}{3}, -1)$	$\frac{5}{9}$	$(\frac{42}{5}, \frac{14}{3}, 0)$	$\frac{5}{9}$	$(\frac{52}{5}, \frac{20}{3}, 2)$	$\frac{5}{9}$
(8)	$(XFR,\ \overline{XFR})$ $XD\overline{D}$	$(9, 1, -1)$	$\frac{1}{5}$	$(10, 2, 0)$	$\frac{1}{5}$	$(12, 4, 2)$	$\frac{1}{5}$
(9)	$(XFL,\ \overline{XFL})$ $XL\overline{L}$	$(\frac{113}{15}, \frac{13}{3}, -\frac{5}{3})$	$\frac{15}{23}$	$(\frac{128}{15}, \frac{16}{3}, -\frac{2}{3})$	$\frac{15}{23}$	$(\frac{158}{15}, \frac{22}{3}, \frac{4}{3})$	$\frac{15}{23}$
(10)	$(XFR,\ \overline{XFR})$ $XL\overline{L}$	$(\frac{137}{15}, \frac{5}{3}, -\frac{5}{3})$	$\frac{25}{81}$	$(\frac{152}{15}, \frac{8}{3}, -\frac{2}{3})$	$\frac{25}{81}$	$(\frac{182}{15}, \frac{14}{3}, \frac{4}{3})$	$\frac{25}{81}$

TABLE XIV: The parameters a_i^0, a_i^1, and a_i^2, and the indices k_0, k_1, and k_2 of the gaugino mass relations in the Pati-Salam $SU(4)_C \times SU(2)_L \times SU(2)_R$ models with various messenger fields.

same Yukawa couplings to X. Interestingly, the gluino is the lightest gaugino in most of our scenarios.

(iv) The Other Possible Models

There are some other possible models that are consistent with gauge coupling unification. For example, in the $SU(5)$ models, we introduce one pair of the vector-like particles $(XD,\ XD^c)$ (or $(XL,\ XL^c)$) around the TeV scale, and we introduce two or three or more pairs of the vector-like particles $(XL,\ XL^c)$ (or $(XD,\ XD^c)$) at the intermediate scale. However, to obtain the gauge coupling unification, we do need to fine-tune the masses of these vector-like particles. Interestingly, in the flipped $SU(5) \times U(1)_X$ models, we can relax

Cases	Messengers	k'_0	k'_1	k'_2	Cases	Messengers	k'_0	k'_1	k'_2
(1)	$(XFL,\ \overline{XFL})$	77/13	19/7	88/5	(2)	$(XFR,\ \overline{XFR})$	11/19	19/27	−32/5
(3)	$XD\overline{D}$	11/13	19/21	−8/5	(4)	$XL\overline{L}$	11	19/3	∞
(5)	$(XG4, XWL)$ XWR	55/29	19/15	2	(6)	XZ	121/35	95/51	46/5
(7)	$(XFL,\ \overline{XFL})$ $XD\overline{D}$	55/13	19/9	56/5	(8)	$(XFR,\ \overline{XFR})$ $XD\overline{D}$	11/17	19/25	−24/5
(9)	$(XFL,\ \overline{XFL})$ $XL\overline{L}$	187/29	133/45	106/5	(10)	$(XFR,\ \overline{XFR})$ $XL\overline{L}$	55/41	1	−14/5

TABLE XV: The indices k'_0, k'_1, and k'_2 of the gaugino mass relations for various messenger fields in the Pati-Salam $SU(4)_C \times SU(2)_L \times SU(2)_R$ models.

the gauge coupling unification condition due to the two-step gauge coupling unification. Let us present a new kind of the flipped $SU(5) \times U(1)_X$ models. We introduce the vector-like particles $(Xf,\ \overline{Xf})$ around the TeV scale, and introduce the messenger vector-like particles $(XF,\ \overline{XF})$ or $(XF,\ \overline{XF}) \oplus (Xh,\ \overline{Xh})$ at the intermediate scale 10^8 GeV or smaller so that the gauge coupling unification can be realized. For the index k', we have

$$k' = \frac{22\sum_j n_2(\Phi_j)g(x_j) + 22\sum_j n_3(\Phi_j)g(x_j)}{5\sum_j n_1(\Phi_j)g(x_j) + 22\sum_j n_3(\Phi_j)g(x_j)} \ . \tag{124}$$

For the model with the intermediate-scale vector-like messenger fields $(XF,\ \overline{XF})$, we choose $p = 5$. Thus, we have $a_1 = 91/10$, $a_2 = 7/2$, and $a_3 = -1/2$, and the indices $k = 5/12$, and $k' = 44/23$. Also, for the model with the intermediate-scale vector-like messenger fields $(XF,\ \overline{XF}) \oplus (Xh,\ \overline{Xh})$, we choose $p = 4$. Thus, we have $a_1 = 148/15$, $a_2 = 14/3$, and $a_3 = 2/3$, and the indices $k = 10/23$, and $k' = 11/6$.

VI. IMPLICATIONS OF GAUGINO MASS RELATIONS AND THEIR INDICES

With the gaugino mass relations and their indices, we may distinguish the different supersymmetry breaking mediation mechanisms and probe the four-dimensional GUTs and string derived GUTs if we can measure the gaugino masses at the LHC and future ILC. In particular, we emphasize again that the gaugino mass realtions in the gravity mediated supersymmetry breaking is different from those for the gauge and anomaly mediated supersymmetry breaking, as discussed in Section III. Here, we summarize the indices k of the gaugino mass relations in the typical GUTs with gravity, gauge and anomaly mediated supersymmetry breaking:

- Gravity Mediated Supersymmetry Breaking

  In the typical four-dimensional $SU(5)$ and $SO(10)$ models, in the F-theory $SU(5)$ models with $U(1)_Y$ flux, and in the F-theory $SO(10)$ models with $U(1)_{B-L}$ flux where the gauge symmetry is broken down to the $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ gauge symmetries, the indices for the gaugino mass relations are either 0/0 or 5/3, where $k = 0/0$ means the mSUGRA gaugino mass relation [33]. However, in the F-theory $SO(10)$ models with $U(1)_X$ flux where the gauge symmetry is broken down to the flipped $SU(5) \times U(1)_X$ gauge symmetries, we only have the mSUGRA gaugino mass relation [33]. Also, in the four-dimensional minimal $SO(10)$ model [63], the Higgs field, which breaks the $SO(10)$ gauge symmetry, is in the $SO(10)$ **45** representation. Thus, only the dimension-six operators can induce the non-universal SM gauge kinetic functions at the GUT scale, and then such non-universal effects on the SM gauge kinetic functions are very small and negligible. Therefore, we only have the mSUGRA gaugino mass relation as well. In short, if we obtain $k = 5/3$ from the LHC and ILC experiments, we can rule out the F-theory $SO(10)$ models with $U(1)_X$ flux and the four-dimensional minimal $SO(10)$ model.

- Gauge Mediated Supersymmetry Breaking

  In the four-dimensional $SU(5)$ and $SO(10)$ models, we have the mSUGRA gaugino mass relation in general since it is difficult to split the complete $SU(5)$ and $SO(10)$ multiplets. However, in the orbifold GUTs and F-theory GUTs with various messenger fields, we have many new possible gaugino mass relations and their indices, as discussed in Section IV. In particular, the indices k can be 5/3 in quite a few $SU(5)$ models and Pati-Salam models. In the flipped $SU(5) \times U(1)_X$ models, we have $k = 0$ in general, which are different from the mSGURA gaugino mass relation except that the messenger fields are Xh and $\overline{Xh}$.

- UV Insensitive Anomaly Mediated Supersymmetry Breaking

  In the four-dimensional $SU(5)$ and $SO(10)$ models (or say Pati-Salam models) with or without the TeV-scale vector-like particles that form complete GUT multiplets, we generically have $k = 5/12$. In the flipped $SU(5) \times U(1)_X$ models, in addition to $k = 5/12$, we can have $k = 5/9$, $k = 10/27$, and $k = 10/21$. Especially, all the indices k are smaller than 1, and then they can not be 5/3 as in the gravity mediated supersymmetry breaking.

- Deflected Anomaly Mediated Supersymmetry Breaking

  If the messenger fields form complete $SU(5)$ or $SO(10)$ representations, we also have $k = 5/12$. For generical messenger fields, the detailed discussions are given in subsection V.B. Especially, all the indices k are smaller than 1. In addition, we would like to point out that the discussions for mirage mediation [64] are similar to those for the deflected AMSB.

Furthermore, to distinguish the different scenarios with the same gaugino mass relations and the same indices, we need to consider the squark and slepton masses as well, which will be studied elsewhere [54].

VII. CONCLUSIONS

In GUTs from orbifold constructions, intersecting D-brane model building on Type II orientifolds, M-theory on S^1/Z_2 with Calabi-Yau compactifications, and F-theory with $U(1)$ fluxes, we pointed out that the generic vector-like particles do not need to form the complete $SU(5)$ or $SO(10)$ representations. Thus, in the GMSB and deflected AMSB, the messenger fields do not need to form complete $SU(5)$ representations. We can achieve the gauge coupling unification by introducing the extra vector-like particles that do not mediate supersymmetry breaking. To be concrete, we presented the orbifold $SU(5)$ models with additional vector-like particles, the orbifold $SO(10)$ models with additional vector-like particles where the gauge symmetry can be broken down to flipped $SU(5) \times U(1)_X$ or Pati-Salam $SU(4)_C \times SU(2)_L \times SU(2)_R$ gauge symmetries, and the F-theory $SU(5)$ models with generic vector-like particles. Interestingly, these vector-like particles can be the TeV-scale vector-like particles that we need to increase the lightest CP-even Higgs boson mass in the MSSM, and they can be the messenger fields in the GMSB and deflected AMSB as well.

In addition, we have studied the general gaugino mass relations and their indices in the GMSB and AMSB, which are valid from the GUT scale to the electroweak scale at one loop. For the GMSB, we calculated the gaugino mass relations and their indices for the $SU(5)$ models, the flipped $SU(5)\times U(1)_X$ models, and the Pati-Salam $SU(4)_C\times SU(2)_L\times SU(2)_R$ models with various possible messenger fields. These kinds of GUTs can be realized in orbifold GUTs, F-theory $SU(5)$ models with $U(1)_Y$ flux, and F-theory $SO(10)$ models with $U(1)_X$ flux and $U(1)_{B-L}$ flux where the $SO(10)$ gauge symmetry is respectively broken down to the flipped $SU(5)\times U(1)_X$ gauge symmetries and the $SU(3)_C\times SU(2)_L\times SU(2)_R\times U(1)_{B-L}$ gauge symmetries. Especially, we pointed out that using gaugino mass relations and their indices, we may probe the messenger fields at the intermediate scale. Moreover, for the AMSB, we considered the UV insensitive AMSB and the deflected AMSB. In the UV insensitive AMSB, we calculated the gaugino mass relations and their indices in the $SU(5)$ models without and with TeV-scale vector-like particles that form complete $SU(5)$ multiplets, and in the flipped $SU(5)\times U(1)_X$ models with TeV-scale vector-like particles that form complete $SU(5)\times U(1)_X$ multiplets. To achieve the one-step gauge coupling unification, we emphasize that the discussions for the Pati-Salam models are similar to those in the $SU(5)$ models. In the deflected AMSB, we defined the new indices for the gaugino mass relations to probe the messenger fields at intermediate scale. Without or with the suitable TeV-scale vector-like particles that can lift the lightest CP-even Higgs boson mass, we studied the generic gaugino mass relations, and their indices k and k' in the $SU(5)$ models, the flipped $SU(5)\times U(1)_X$ models, and the Pati-Salam $SU(4)_C\times SU(2)_L\times SU(2)_R$ models with various possible messenger fields. Also, we found that in most of our scenarios, gluino can be the lightest gaugino at low energy. Especially, we proposed a new kind of interesting flipped $SU(5)\times U(1)_X$ models.

Furthermore, using the gaugino mass relations and their indices, we may not only determine the supersymmetry breaking mediation mechanisms, but also probe the four-dimensional GUTs, orbifold GUTs, and F-theory GUTs.

Acknowledgments

This research was supported in part by the Natural Science Foundation of China under grant numbers 10821504 and 11075194 (TL), and by the DOE grant DE-FG03-95-Er-40917 (TL and DVN).

Appendix A: Breifly Review of del Pezzo Surfaces

The del Pezzo surfaces dP_n, where $n = 1,\ 2,\ ...,\ 8$, are defined by blowing up n generic points of $\mathbb{P}^1 \times \mathbb{P}^1$ or $\mathbb{P}^2$. The homological group $H_2(dP_n, Z)$ has the generators

$$H,\ E_1,\ E_2,\ ...,\ E_n\ , \tag{A1}$$

where H is the hyperplane class for P^2, and E_i are the exceptional divisors at the blowing up points and are isomorphic to $\mathbb{P}^1$. The intersecting numbers of the generators are

$$H \cdot H = 1\ , \quad E_i \cdot E_j = -\delta_{ij}\ , \quad H \cdot E_i = 0\ . \tag{A2}$$

The canonical bundle on dP_n is given by

$$K_{dP_n} = -c_1(dP_n) = -3H + \sum_{i=1}^{n} E_i. \tag{A3}$$

For $n \geq 3$, we can define the generators as follows

$$\alpha_i = E_i - E_{i+1}\ , \quad \text{where}\ \ i = 1,\ 2, ...,\ n-1\ , \tag{A4}$$

$$\alpha_n = H - E_1 - E_2 - E_3\ . \tag{A5}$$

Thus, all the generators α_i is perpendicular to the canonical class K_{dP_n}. And the intersection products are equal to the negative Cartan matrix of the Lie algebra E_n, and can be considered as simple roots.

The curves Σ_i in dP_n where the particles are localized must be divisors of S. And the genus for curve Σ_i is given by

$$2g_i - 2\ =\ [\Sigma_i] \cdot ([\Sigma_i] + K_{dP_k})\ . \tag{A6}$$

For a line bundle L on the surface dP_n with

$$c_1(L) = \sum_{i=1}^{n} a_i E_i, \tag{A7}$$

where $a_i a_j < 0$ for some $i \neq j$, the Kähler form J_{dP_n} can be constructed as follows [22]

$$J_{dP_k} = b_0 H - \sum_{i=1}^{n} b_i E_i, \tag{A8}$$

where $\sum_{i=1}^{k} a_i b_i = 0$ and $b_0 \gg b_i > 0$. By the construction, it is easy to see that the line bundle L solves the BPS equation $J_{dP_k} \wedge c_1(L) = 0$.

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