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Strings and D-Branes with Boundaries

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Abstract

The covariant field equations of ten-dimensional super D-branes are obtained by considering fundamental strings whose ends lie in the superworldsurface of the D-brane. By considering in a similar fashion Dp -branes ending on $D(p+2)$ -branes we derive equations describing D-branes with dual potentials, as well as the vector potentials.

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1 Introduction

In a recent paper [1], an open supermembrane ending on an M-fivebrane was studied. The world-volume of the M-fivebrane was taken to be a supersubmanifold, M , of the eleven-dimensional target superspace, \underline{M} . The supermembrane action is an integral over a bosonic three dimensional worldvolume Σ , with its boundary $\partial\Sigma$ embedded in the supermanifold M , such that

$$\partial\Sigma \subset M \subset \underline{M} \tag{1}$$

It was shown that the κ -symmetry of the total action implies (1) the eleven-dimensional supergravity equations, (2) a constraint on the embedding of M in \underline{M} and (3) a constraint on a modified super 3-form field strength, H , on the superfivebrane worldvolume. The superembedding constraint and the H -constraint completely determine the superfivebrane equations of motion, although the H -constraint can be derived from the embedding constraint in this case.

In this paper, this result is extended to the following configurations of branes:

1. Fundamental type II strings ending on D-branes
2. Type II Dp -branes ending on $D(p+2)$ -branes

The target space is (10|32) dimensional type IIA or type IIB superspaces. We use the notation $(D|D')$, where D is the real bosonic dimension and D' is the real fermionic dimension of a supermanifold. The embedded supermanifold M has dimension (2|16) in case (1) and $(p+1|16)$ for case (2).

In all cases we find that the κ -symmetry of the total system implies the ten-dimensional type IIA or type IIB supergravity equations and a constraint on the embedding of M in \underline{M} . In addition, in case (1) we find a constraint on a modified super 2-form field strength on M defined as

$$\mathcal{F} = dA - B \ , \tag{2}$$

where A is the super 1-form potential on M , and B is the pullback of the target space NS-NS super 2-form M . We will use the same letter to denote the target space and worldsurface superforms, since it should be clear from the context whether a pullback is required. The superembedding constraint and the \mathcal{F} -constraint determine completely the dynamics of the D-brane on which the fundamental string ends. At the linearised level, these constraints are shown to be precisely the dimensional reduction of the ten-dimensional Maxwell superspace constraints.

In case (2) the construction leads naturally to the introduction of a p -form potential on M in addition to the usual one-form potential. The field strength forms corresponding to these potentials are essentially dual to one another, so that the dual versions of D-branes are automatically generated by this method. Again, constraints on the field strengths are derived and imply the equations of motion for the $D(p+2)$ -brane when the embedding condition is taken into account.

In the case of M-fivebrane, the 3-form H was introduced in [2, 3] for convenience in describing the field equations and it was shown that the H -constraint is a consequence of the superembedding condition [3, 4]. In [2], it was observed that the analogue of the H -constraint arises in the description of various superbranes and, in particular, the super D -branes naturally accommodate an \mathcal{F} -constraint, where \mathcal{F} is a modified two-form field strength. It was also noted in [2] that for

certain superbranes, e.g. the $D8$ -brane, the \mathcal{F} -constraint is needed to put the theory on-shell [5].

In the approach presented in [1], both the superembedding condition and the H -constraint arise naturally from the requirement of κ -symmetry. Similarly, here we will show that both the superembedding condition and the \mathcal{F} -constraint arise naturally from the considerations of κ -symmetry of suitable open branes ending on D -branes.

2 Fundamental Type II Strings Ending on D -Branes

In this section, we consider the fundamental type II strings in ten dimensions with boundary on a Dp -brane. The string worldsheet is bosonic. We will take its boundary, however, to lie in a bosonic submanifold of a supermanifold M of dimension $(p+1|16)$, which in turn is a submanifold of a target space \underline{M} of dimension $(10|32)$. We use the notations and conventions of [2]. In particular, we denote by $z^{\underline{M}} = (x^m, \theta^\mu)$ the local coordinates on \underline{M} , and by $\underline{A} = (\underline{a}, \underline{\alpha})$ the target tangent space indices. We use the ununderlined version of these indices to label the corresponding quantities on the worldsurface. The embedded submanifold M , with local coordinates $z^M = (x^m, \theta^\mu)$, is given as $z^{\underline{M}}(z)$.

We shall consider type IIA and type IIB superspaces. The fermionic coordinates consist of two Majorana-Weyl spinors. In type IIA superspace, these spinors carry opposite chiralities which can be combined into a single 32 component Majorana spinor, while in type IIB superspace they are of the same chirality. We will use the fermionic index $\underline{\alpha}$ in both cases, though in type IIB superspace it is understood to be a composite index of Majorana-Weyl spinor index and an $SO(2)$ doublet index, acted on by a direct product of chirally projected Γ -matrices and $SO(2)$ matrices. Further details of our notation and conventions are given in the Appendix.

2.1 Constraints from κ -Symmetry of the Open String

We consider the following action for the total system of a type II open string ending on a D -brane (with the target metric taken to be in the Einstein frame),

$$S = - \int_{\Sigma} d^2\xi \left(\sqrt{-g} + \epsilon^{ij} B_{ij} \right) + \int_{\partial\Sigma} d\tau A, \quad (3)$$

where ξ^i ($i = 0, 1$) are the coordinates of the string worldsheet Σ , τ is the coordinate on the boundary $\partial\Sigma$. We will take both ends of the string to lie on a Dp -brane supermanifold M of dimension $(p+1|16)$.¹ A is the pullback to $\partial\Sigma$ of a super one-form defined on M . The induced metric g_{ij} , and the pullbacks B_{ij}, A are defined as:

$$\begin{aligned} B_{ij} &= E_j^{\underline{B}} E_i^{\underline{A}} B_{\underline{A}\underline{B}}, \\ A &= E_{\tau}^{\underline{C}} A_{\underline{C}}, \\ g_{ij} &= E_i^{\underline{a}} E_j^{\underline{b}} \eta_{\underline{a}\underline{b}}, \end{aligned} \quad (4)$$

¹In general, the end points may lie on two different Dp -branes or one end-point of a semi-infinite open string may lie on a Dp -brane while the other end is freely moving. It is sufficient to consider the case where both end-points are ending on a single Dp -brane for the purpose of deriving the constraints that govern the dynamics of the Dp -brane. It is straightforward to generalize the discussion for the other two cases.

where η_{ab} is the Minkowski metric in ten dimensions, and

$$\begin{aligned} E_i^{\underline{A}} &= \partial_i z^{\underline{M}} E_{\underline{M}}^{\underline{A}} , \\ E_\tau^{\underline{A}} &= \partial_\tau z^{\underline{M}} E_{\underline{M}}^{\underline{A}} , \end{aligned} \quad (5)$$

where $E_{\underline{M}}^{\underline{A}}$ is the target space supervielbein and $E_M^{\underline{A}}$ is the worldsurface supervielbein. We note the useful relation

$$d\xi^i E_i^{\underline{A}}|_{\partial\Sigma} = d\tau E_\tau^{\underline{A}} E_A^{\underline{A}}|_{\partial\Sigma} . \quad (6)$$

where $E_A^{\underline{A}}$ is the embedding matrix which plays an important rôle in the description of the model. It is defined by

$$E_A^{\underline{A}} = E_A^{\underline{M}} \partial_M z^{\underline{M}} E_{\underline{M}}^{\underline{A}} , \quad (7)$$

We consider a κ -symmetry transformation of the form

$$\begin{aligned} \delta_\kappa z^{\underline{a}} &= 0 , \\ \delta_\kappa z^{\underline{\alpha}} &= \frac{1}{2} \kappa^{\underline{\alpha}}(\xi) (1 + \Gamma)_{\underline{\alpha}}^{\underline{\alpha}} , \end{aligned} \quad (8)$$

on the string worldsheet Σ , where

$$(\Gamma)_{\underline{\alpha}}^{\underline{\beta}} = \frac{1}{2\sqrt{-g}} \epsilon^{ij} (\gamma_{ij} P)_{\underline{\alpha}}^{\underline{\beta}} , \quad (9)$$

where

$$P = \begin{cases} \Gamma_{11} & \text{(IIA)} \\ \sigma_3 & \text{(IIB)} \end{cases} \quad (10)$$

Note that the Γ_{11} acts on a 32 component Majorana index, while σ_3 acts on the $SO(2)$ doublet index of the two 16-component Majorana-Weyl spinors of same chirality.

The boundary κ -transformations will be taken to be of the form as in [1], namely

$$\begin{aligned} \delta_\kappa z^{\underline{a}} &= 0 , \\ \delta_\kappa z^{\underline{\alpha}} &= \frac{1}{2} \kappa^{\underline{\alpha}}(\sigma) \left(1 + \Gamma_{(p+1)}\right)_{\underline{\alpha}}^{\underline{\alpha}} \quad \text{on } \partial\Sigma , \end{aligned} \quad (11)$$

where the matrix $\Gamma_{(p+1)}$ is defined by

$$E_{\underline{\alpha}}^{\underline{\alpha}} E_{\alpha}^{\underline{\alpha}} = \frac{1}{2} \left(1 + \Gamma_{(p+1)}\right)_{\underline{\alpha}}^{\underline{\alpha}} . \quad (12)$$

The matrix $E_{\alpha}^{\underline{\alpha}}$ is obtained from $E_{\underline{A}}^{\underline{A}}$ which is the inverse of $E_A^{\underline{A}}$. For more details, see [3, 4].

The vanishing of the terms on Σ imposes constraints on the torsion super two-form T , and the super three-form $H = dB$, such that they are consistent with the equations of motion of the ten-dimensional type II supergravities [6].

The constraints which follow from κ -symmetry on Σ are

$$\begin{aligned} T_{\underline{\alpha}\underline{\beta}}^{\underline{a}} &= -i(\Gamma^{\underline{a}})_{\underline{\alpha}\underline{\beta}} , \\ T_{\underline{a}\underline{b}}^{\underline{c}} &= \delta_{\underline{b}}^{\underline{c}} \chi_{\underline{a}} , \end{aligned} \quad (13)$$

and

$$\begin{aligned} H_{\underline{\alpha}\underline{\beta}\underline{\gamma}} &= 0 , \\ H_{\underline{\alpha}\underline{\beta}\underline{a}} &= i(\Gamma_{\underline{a}} Q)_{\underline{\alpha}\underline{\beta}} , \\ H_{\underline{a}\underline{b}\underline{c}} &= (\Gamma_{\underline{a}\underline{b}} Q \chi)_{\underline{c}} \end{aligned} \quad (14)$$

where

$$Q = \begin{cases} \Gamma_{11} & \text{(IIA)} \\ \sigma_1 & \text{(IIB)} \end{cases} \quad (15)$$

and $\chi_{\underline{\alpha}}$ is a spinor superfield proportional to the dilaton superfield of the supergravity background.

The remaining variations are on the boundary. Proceeding exactly as in [1], we learn that they vanish provided that the following two constraints are satisfied:

$$E_{\alpha}{}^{\underline{a}} = 0, \quad (16)$$

$$\mathcal{F}_{\alpha B} = 0. \quad (17)$$

Here,

$$\mathcal{F} = dA - B, \quad (18)$$

is the modified 2-form superfield strength which satisfies the Bianchi identity

$$d\mathcal{F} = -H. \quad (19)$$

There will also be a mixture of Dirichlet and Neumann boundary conditions from the requirement that the action be stationary when the string field equations hold. These can be derived straightforwardly as in the case of the open supermembrane which has been discussed in detail in [1].

2.2 Solution of the Linearised Constraints

In this section, we shall analyse the embedding condition (16) and the the \mathcal{F} -constraints (17) in order to extract the equation of motion for the D-brane worldvolume fields. To determine the field content, it is sufficient to study the linearised constraints in flat target space limit.

The supervielbein for the flat target superspace is,

$$\begin{aligned} E^{\underline{a}} &= dx^{\underline{a}} - \frac{i}{2} d\theta^{\underline{\alpha}} (\Gamma^{\underline{a}})_{\underline{\alpha}\underline{\beta}} \theta^{\underline{\beta}} \\ E^{\underline{\alpha}} &= d\theta^{\underline{\alpha}}. \end{aligned} \quad (20)$$

Let us choose the physical gauge,

$$\begin{aligned} x^{\underline{a}} &= \begin{cases} x^a \\ x^{a'}(x, \theta) \end{cases} \\ \theta^{\underline{\alpha}} &= \begin{cases} \theta^{\alpha} \\ \theta^{\alpha'}(x, \theta) \end{cases} \end{aligned} \quad (21)$$

and take the embedding to be infinitesimal so that $E_A{}^M \partial_M$ can be replaced by $D_A = (\partial_a, D_{\alpha})$ where D_{α} is the flat superspace covariant derivative on the worldsurface, provided that the embedding constraint holds. In this limit the embedding matrix is:

$$\begin{aligned} E_a{}^{\underline{b}} &\rightarrow \begin{cases} \delta_a{}^b \\ \partial_a X^{b'} \end{cases} \\ E_{\alpha}{}^{\underline{b}} &\rightarrow \begin{cases} 0 \\ D_{\alpha} X^{a'} - i(\Gamma^{a'})_{\alpha\beta'} \theta^{\beta'} \end{cases} \\ E_a{}^{\underline{\beta}} &\rightarrow \begin{cases} 0 \\ \partial_a \theta^{\beta'} \end{cases} \\ E_{\alpha}{}^{\underline{\beta}} &\rightarrow \begin{cases} \delta_{\alpha}{}^{\beta} \\ D_{\alpha} \theta^{\beta'} \end{cases} \end{aligned} \quad (22)$$

where

$$X^{a'} := x^{a'} + \frac{i}{2} \theta^\alpha (\Gamma^{a'})_{\alpha\beta'} \theta^{\beta'} . \quad (23)$$

Using this in the embedding condition (16) we find, at the linearised level,

$$D_\alpha X^{a'} = i(\Gamma^{a'})_{\alpha\beta'} \theta^{\beta'} . \quad (24)$$

The Bianchi identity (19) in component form is,

$$D_{[A} \mathcal{F}_{BC]} + T_{[AB}{}^E \mathcal{F}_{|E|C]} = -\frac{1}{3} E_C{}^C E_B{}^B E_A{}^A H_{ABC} . \quad (25)$$

Linearising this equation using (22), we find that the $ABC = (\alpha\beta\gamma)$ component of this identity is satisfied automatically, while the $(ab\gamma)$, (abc) and $(\alpha\beta c)$ components give

$$D_\gamma \mathcal{F}_{ab} = -2i \partial_{[a} \theta^{\alpha'} (\Gamma_{b]} Q)_{\gamma\alpha'} \quad (26)$$

$$\partial_{[c} \mathcal{F}_{ab]} = 0 , \quad (27)$$

$$(\gamma^b)_{\alpha\beta} \mathcal{F}_{bc} = i(\Gamma_c Q)_{\alpha\beta} + (\Gamma_{a'} Q)_{\alpha\beta} \partial_c X^{a'} + 2D_{(\alpha} \theta^{\beta')} (\Gamma_c Q)_{\beta)\beta'} \quad (28)$$

Our strategy is to interpret these equations, together with (24), as the dimensional reduction of the $(N = 1)$ ten-dimensional super Maxwell system to $(p + 1)$ dimensions. The relation between \mathcal{F} and the non-covariant $F = dA$ follows from (18). In component form, (18) reads

$$\mathcal{F}_{AB} = F_{AB} - E_B{}^B E_A{}^A \underline{B}_{AB} . \quad (29)$$

These relations are:

$$\mathcal{F}_{ab} = F_{ab} , \quad (30)$$

$$\mathcal{F}_{\alpha\beta} = F_{\alpha\beta} , \quad (31)$$

$$\mathcal{F}_{a\alpha} = F_{a\alpha} - i(\Gamma_a Q)_{\alpha\beta'} \theta^{\beta'} . \quad (32)$$

Using (17), the constraints (26), (27) and (30)-(32) can be summarized as

$$F_{\alpha\beta} = 0 , \quad (33)$$

$$F_{a\alpha} = i(\Gamma_a Q)_{\alpha\beta'} \theta^{\beta'} , \quad (34)$$

$$D_\gamma F_{ab} = -2i \partial_{[a} \theta^{\alpha'} (\Gamma_{b]} Q)_{\gamma\alpha'} \quad (35)$$

$$\partial_{[c} F_{ab]} = 0 . \quad (36)$$

It is now easy to combine these and (24) to obtain ten dimensional master constraints. To do this, we first define a ten dimensional vector superfield $A^{\underline{a}}$, and a spinor superfield λ :²

$$\begin{aligned} A^{\underline{a}}, iX^{a'} &\rightarrow A^{\underline{a}} , \\ \theta^{\alpha'} &\rightarrow \lambda^\alpha , \end{aligned} \quad (37)$$

where the index α now labels a sixteen component Majorana-Weyl spinor in ten dimensions. With these definitions, the constraints (24) and (34) combine to

$$F_{\underline{a}\alpha} = (\sigma_{\underline{a}})_{\alpha\beta} \lambda^\beta , \quad (38)$$

² A is imaginary due to our choice (13) of the torsion.

where the σ matrices are the ten dimensional chirally projected γ - matrices. This constraint, together with (28), (33), (35), and (36) are precisely the superspace constraints of ten dimensional super Maxwell system that satisfy the ten dimensional Bianchi identity $dF = 0$. In particular, the constraint (28) is the dimensional reduction of the ten dimensional Bianchi identity

$$D_{(\alpha} F_{\beta)\underline{c}} + \frac{i}{2} (\sigma^{\underline{b}})_{\alpha\beta} F_{\underline{bc}} = 0 , \quad (39)$$

for $\underline{c} = c$. The other component of this equation, i.e. for $\underline{c} = c'$, is also satisfied, thanks to the supersymmetry algebra,

$$\{D_{\alpha}, D_{\beta}\} = i (\sigma^{\underline{a}})_{\alpha\beta} \partial_{\underline{a}} . \quad (40)$$

In doing these calculations, we have used the properties of the Γ -matrices, given in the Appendix, to set the first term on the right hand side of (28) equal to zero.

We conclude that the linearised versions of our two master constraints (17) and (16) describe precisely the dimensional reduction of the ten-dimensional super Maxwell system to Dp -brane worldvolume. We expect [2, 5] that the full constraints (16) and (17) imply the full field equations that follow from the super D-brane actions of [7, 8, 9, 10, 11].

3 Dp-brane ending on a D($p + 2$)-brane

In this section we study an open Dp -brane ending on a $D(p+2)$ -brane and show that it naturally gives rise to a dual potential on the worldsurface of the $(p+2)$ -brane M .

3.1 The Action and κ -symmetry Constraints

The action for a type II κ symmetric D-brane has been constructed in [7, 8, 9, 10, 11]. κ symmetry implies constraints for the target superspace torsion, the NS-NS three form field strength and the RR field strength. For an open Dp -brane ending on a $D(p+2)$ -brane, we propose the action (here we take the target metric to be in the string frame)

$$S = \int_{\Sigma} \left(-e^{-\phi} \sqrt{-\det(g_{ij} + \mathcal{F}_{ij})} + C e^{\mathcal{F}} + m \omega_{p+1} \right) + \int_{\partial\Sigma} A_p , \quad (41)$$

where

$$\mathcal{F} = dA - B , \quad (42)$$

and $\omega_{p+1}(A, dA)$ is the Chern-Simons form present for even p in a massive IIA background, with m being the mass parameter. We define this form by the relation

$$d\omega_{p+1}(A, dA) = (e^{dA})_{p+2} . \quad (43)$$

In (42), B represents the pullback of the target space super two-form to the *bosonic* Dp -brane worldvolume. The potential A_p is identified with the pullback onto the bosonic boundary $\partial\Sigma$ of a p -form potential living on the $D(p+2)$ -brane superworldvolume. Furthermore, it is assumed that the pullback of the field strength $\mathcal{F}_2 = dA_1 - B$ defined on the superworldvolume of the $D(p+2)$ -brane onto $\partial\Sigma$ coincides with \mathcal{F} for the Dp -brane restricted to the (bosonic) boundary. Thus we have both a one-form potential A_1 and the dual p -form potential A_p on the $(p+2)$ -brane. For simplicity, we will use in the following the same symbol \mathcal{F} to denote both the two-form field

strengths on the Dp -brane and on the $D(p+2)$ -brane. It should be clear from the context which one is being referred to.

For later reference, we record here the definitions of the target space RR field strengths:

$$G = dC - CH + me^B , \quad (44)$$

where

$$H = dB . \quad (45)$$

We also record the Bianchi identity

$$dG = GH , \quad (46)$$

Note that the m -dependent terms have cancelled. We use the superspace conventions of [12] according to which the exterior derivative acts from the right.

The field strengths \mathcal{F}, G, H are invariant under the gauge transformations

$$\delta A = \lambda , \quad \delta B = d\lambda , \quad \delta C = -m\lambda e^B , \quad (47)$$

where λ is a target space super one-form gauge parameter. The field strength G is also invariant under the gauge transformation

$$\delta A = 0 , \quad \delta B = 0 , \quad \delta C = e^B d\mu . \quad (48)$$

where μ is a target space superform of appropriate rank.

Since the gauge variation of the Chern-Simons form has the form

$$\delta\omega_{p+1} = \lambda e^{dA} + dX_p^1 , \quad (49)$$

for some p -form $X_p^1(\lambda, A, dA)$ defined by this equation, the action (41) is invariant under the gauge transformations (47) and (48), provided that A_p transforms as

$$\delta A_p = -m X_p^1 - \mu e^{dA} . \quad (50)$$

Next, we turn to the discussion of κ -symmetry. One can verify that, under a κ -symmetry transformation, the vanishing of the variations on Σ impose constraints on the supertorsion T , the NS-NS field strength H and the RR field strengths G [7, 8, 9, 10, 11] such that they are consistent with the field equations of the type II supergravities.

The remaining variations are on the boundary, and they take the form

$$\delta S = \int_{\partial\Sigma} i_\kappa \mathcal{F}_{p+1} , \quad (51)$$

where the modified field strengths \mathcal{F}_{p+1} for the worldvolume potentials A_p are

$$\mathcal{F}_{p+1} := dA_p + (Ce^{\mathcal{F}})_{p+1} + m\omega_{p+1} , \quad (52)$$

They satisfy the Bianchi identity

$$d\mathcal{F}_{p+1} = (Ge^{\mathcal{F}})_{p+2} . \quad (53)$$

Observe that the m -dependent terms have cancelled. Using $i_\kappa \mathcal{F} = 0$, the vanishing of (51) implies that

$$\mathcal{F}_{\alpha B_1 \dots B_p} = 0 . \quad (54)$$

In addition, we must have the usual embedding constraint

$$E_\alpha^a = 0 , \quad (55)$$

by similar argument to the one given in the preceding discussion of string ending on branes.

3.2 An Example: The D2-brane ending on a D4-brane

To illustrate the general formalism introduced above we consider an open D2-brane ending on a D4-brane. According to the results of the preceding section there will be two potentials, the usual one-form potential A_1 , and the dual two-form potential A_2 , on the D4-brane. The Bianchi identities are (although the mass parameter m does not arise in the Bianchi identities (46) and (53), we will nonetheless set it to zero for simplicity)

$$d\mathcal{F} = -H , \quad (56)$$

and

$$d\mathcal{F}_3 = G_4 + G_2\mathcal{F} . \quad (57)$$

In addition we are required to take all the components of both \mathcal{F} and \mathcal{F}_3 to be zero, except for those which have solely vectorial indices.

In the case of the D4-brane the embedding constraint is enough to force the equations of motion [2, 5] which are usually written in terms of the one-form potential A_1 . However, there is also a dual GS version in which one replaces A_1 with a two-form A_2 . Since in the superembedding formalism the brane is on-shell due to the basic constraint (55) it follows that we should be able to construct either, or indeed both, versions, and the open brane set-up naturally gives both potentials.

To analyse the above Bianchi identities we set

$$E_\alpha{}^\alpha = u_\alpha{}^\alpha + h_\alpha{}^{\beta'} u_{\beta'}{}^\alpha , \quad (58)$$

and

$$E_a{}^a = u_a{}^a , \quad (59)$$

where u denotes an element of $Spin(1,9)$, in either the spin representation or the vector representation according to the indices. The basic constraint (55) implies that

$$E_\alpha{}^\alpha E_\beta{}^\beta T_{\alpha\beta}{}^c = T_{\alpha\beta}{}^c E_c{}^c , \quad (60)$$

from which one finds that [5]

$$\begin{aligned} h_\alpha{}^{\beta'} \rightarrow h_{\alpha i}{}^{\beta' j} &= i\delta_i{}^j (\gamma^{ab})_\alpha{}^{\beta'} h_{ab} , \\ T_{\alpha i \beta j}{}^c &= -i\eta_{ij} \left((\gamma^b)_{\alpha\beta} m_b{}^c + C_{\alpha\beta} m^c \right) . \end{aligned} \quad (61)$$

Here we have introduced the two-step notation for the spinor indices on M . The index α , running from 1 to 16, is rewritten as the pair αi , where α is a five-dimensional spinor index (running from 1 to 4) and i is an $Sp(4)$ index, also running from 1 to 4. The m -tensors are given by [5]

$$\begin{aligned} m_{ab} &= (1 - 2y_1)\eta_{ab} + 8(h^2)_{ab} , \\ m^a &= -i\epsilon^{abcde} h_{bc} h_{de} , \end{aligned} \quad (62)$$

where y_1 and y_2 denote the two invariants

$$\begin{aligned} y_1 &= \text{tr } h^2 , \\ y_2 &= \text{tr } h^4 . \end{aligned} \quad (63)$$

It is straightforward to check the Bianchi identities for the \mathcal{F} 's using this information. If we take the target space to be flat for simplicity, the only non-vanishing components of the RR tensors G_4 and G_2 are

$$G_{\underline{\alpha}\underline{\beta}cd} = -i(\Gamma_{cd})_{\underline{\alpha}\underline{\beta}} , \quad (64)$$

and

$$G_{\underline{\alpha}\underline{\beta}} = -i(\Gamma_{11})_{\underline{\alpha}\underline{\beta}} , \quad (65)$$

while the non-vanishing component of the NS tensor H is

$$H_{\underline{\alpha}\underline{\beta}c} = -i(\Gamma_{\underline{c}}\Gamma_{11})_{\underline{\alpha}\underline{\beta}} . \quad (66)$$

The dimension zero component of the \mathcal{F} Bianchi (56) is found to be satisfied if [5]

$$m_a{}^c \mathcal{F}_{cb} = 4h_{ab} , \quad (67)$$

which can be rewritten as

$$\mathcal{F}_{ab} = \frac{4}{(1 + 4y_1^2 - 16y_2)}((1 + 2y_1)h_{ab} - 8(h^3)_{ab}) . \quad (68)$$

In obtaining this result, it is useful to note the identity $X^5 = \frac{1}{2}X^3\text{tr}X^2 - \frac{1}{8}X(\text{tr}X^2)^2 + \frac{1}{4}X\text{tr}X^4$, which holds for any 5×5 matrix X . One also finds that the dimension zero component of the F_3 Bianchi identity (57) is satisfied if

$$\tilde{\mathcal{F}}_{ab} = \frac{4}{(1 + 4y_1^2 - 16y_2)}((1 - 2y_1)h_{ab} + 8(h^3)_{ab}) , \quad (69)$$

where $\tilde{\mathcal{F}}_{ab}$ is the dual of \mathcal{F}_{abc} ,

$$\tilde{\mathcal{F}}_{ab} = -\frac{1}{3!}\epsilon_{abcde}\mathcal{F}^{cde} . \quad (70)$$

It is in fact enough to show that the dimension zero components of these identities are satisfied to show that that the complete identities are. Furthermore, we know from [2, 5] and from the string discussion that the worldsurface multiplet for a D4-brane with a one-form potential satisfying the standard constraints is on-shell. It is easy to confirm that this is still the case here by considering the linearised limit in which it becomes clear that \mathcal{F}_{abc} is the dual of \mathcal{F}_{ab} . Since the three-form field strength is not a new independent field the version we have derived here is also on-shell.

4 Comments

In this paper we have shown that the equations describing various branes in superspace can be derived by considering the κ -symmetry of an open brane of appropriate dimension ending on the brane of interest. It is remarkable that the κ -symmetry considerations for open superbranes, within the framework of superembedding approach, give rise to dual formulations of D -branes automatically. Traditional methods to derive the dual D -brane actions have been considered in [14]. Here, we find that the potential dual to the usual Born-Infeld vector is furnished in a natural fashion by a suitable p -form that lives on the boundary of an open D -brane which supports its own Maxwell field.

The results presented here, in our opinion, also furnish further evidence for the power of superembedding approach to a geometrical and elegant description of all superbranes. Indeed,

this approach should be applicable to the description of type IIA/B solitonic fivebranes and type I strings/fivebranes as well. It would also be interesting to consider a limit of the model considered here to extract an action for self-dual string in six dimensions. Yet another possible application would be a description of longer superembedding chains or brane networks. Results in this direction will be reported elsewhere [15].

The formalism described here is a hybrid one involving bosonic worldsurface of the first brane but the superworldsurface of the brane to be investigated. One may envisage an approach in which the open brane worldsurface is also elevated into a superspace. This would make the target space and worldsurface supersymmetry manifest and moreover in this approach the geometrical meaning of κ -symmetry as odd diffeomorphisms of the superworldsurface would become more transparent. Indeed a purely superspace description of open superbranes ending on other superbranes is possible, as we will be shown elsewhere [15].

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Appendix

Here we collect the properties of the various Γ matrices in diverse dimensions.

For type IIA, we use the conjugation matrices,

$$C_{\underline{\alpha}\underline{\beta}} = \begin{cases} -i\sigma_2 \times C \times \eta, & p = 0, 4, 8 \\ 1 \times C \times \eta, & p = 2, 6 \end{cases} \quad (71)$$

and the Γ -matrices

$$(\Gamma^a)_{\underline{\alpha}}{}^{\underline{\beta}} = \begin{cases} \sigma_1 \times \gamma^a \times 1 \\ \sigma_3 \times 1 \times \gamma^{a'} \end{cases} \quad p = 0, 4, 8$$

$$(\Gamma^a)_{\underline{\alpha}}{}^{\underline{\beta}} = \begin{cases} \sigma_3 \times \gamma^a \times 1 \\ \sigma_1 \times 1 \times \gamma^{a'} \end{cases} \quad p = 2, 6 \quad (72)$$

where γ^a and $\gamma^{a'}$ are the γ -matrices, while C and η are the charge conjugation matrices of $SO(p, 1)$ and $SO(9 - p)$, respectively. The matrices γ^a and η are symmetric for $p = 0, 2, 8$ and antisymmetric for $p = 4, 6$, while the matrices C and $\gamma^{a'}$ are symmetric for $p = 0, 6, 8$ and antisymmetric for $p = 2, 4$.

The chirality matrix $\Gamma_{11} = \Gamma_0 \Gamma_1 \cdots \Gamma_9$ is given by

$$(\Gamma_{11})_{\underline{\alpha}}{}^{\underline{\beta}} = (\sigma_2 \times 1 \times 1)_{\underline{\alpha}}{}^{\underline{\beta}}. \quad (73)$$

A 32 component Majorana spinor ψ in ten dimensions decomposes as

$$\psi^{\underline{\alpha}} = \begin{pmatrix} \psi^{\alpha} \\ \psi^{\alpha'} \end{pmatrix} \quad (74)$$

where $\alpha = 1, \dots, 16$ labels the fermionic coordinates of the worldvolume, and $\alpha' = 1, \dots, 16$ labels the fermionic transverse directions. The σ -matrix factors of the Γ -matrices act on the doublet (74). Thus, we have for example,

$$\Gamma_{\alpha\beta'}^a = 0, \quad \Gamma_{\alpha'\beta'}^a = -C_{\alpha\beta}\eta_{ij}, \quad \Gamma_{\alpha\beta'}^{a'} = C_{\alpha\beta}\gamma_{ij}^{a'} \quad (75)$$

where $i = 1, \dots, 9 - p$ label the vector representation of the transverse $SO(9 - p)$.

In the case of type IIB Γ -matrices, we suppress the ten dimensional $SO(2)$ doublet index, and consider the chirally projected 16×16 Γ -matrices. The unprimed spinor index labelling the fermionic worldvolume coordinates, and the primed spinor indices labelling the transverse fermionic directions are defined by using the projection operators

$$P_{\pm} = \frac{1}{2}(1 \pm \sigma_3), \quad (76)$$

acting on the $SO(2)$ indices $I, J = 1, 2$, as follows:

$$\begin{aligned} \psi_{\alpha} &= (P_+ \psi)^{\alpha} \\ \psi_{\alpha'} &= (P_- \psi)^{\alpha'}. \end{aligned} \quad (77)$$

Now we can construct the ten dimensional Γ -matrices as:

$$\begin{aligned} p = 9 : \quad \Gamma_{\underline{\alpha}\underline{\beta}}^a &= \gamma_{\alpha\beta}^a P_+ \\ p = 7 : \quad \Gamma_{\underline{\alpha}\underline{\beta}}^a &= \begin{cases} \sigma_{\alpha\dot{\beta}}^a P_+ \\ C_{\alpha\beta} P_- \end{cases} \\ p = 5 : \quad \Gamma_{\underline{\alpha}\underline{\beta}}^a &= \begin{cases} \gamma_{\alpha\beta}^a \eta_{ij} P_+ \\ \delta_{\alpha}^{\dot{\beta}} \gamma_{ij}^{a'} P_- \end{cases} \\ p = 3 : \quad \Gamma_{\underline{\alpha}\underline{\beta}}^a &= \begin{cases} \sigma_{\alpha\dot{\beta}}^a \delta_i^j P_+ \\ C_{\alpha\beta} \gamma_{ij}^{a'} P_- \end{cases} \\ p = 1 : \quad \Gamma_{\underline{\alpha}\underline{\beta}}^a &= \begin{cases} \gamma_{\alpha\beta}^a \delta_i^j P_+ \\ C_{\alpha\beta} \gamma_{ij}^{a'} P_- \end{cases} \end{aligned} \quad (78)$$

where $\gamma_{\alpha\beta}$ and $\sigma_{\alpha\dot{\beta}}$ are the chirally projected γ -matrices appropriate to $(p + 1)$ -dimensions. For $p = 9$, there is no transverse direction, and consequently $\underline{a} = a = 0, 1, \dots, 9$. In the case of $p = 7$, the two transverse coordinates have been combined into a single complex coordinate. Further details can be found in [5].

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