Archives

CTP-TAMU-97/90
ACT-19
CERN-TH.5943/90

# PROBING THE DESERT USING GAUGE COUPLING UNIFICATION

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## ABSTRACT

We present analytic one-loop expressions for $sin^2\theta_W$, the unification scale $M_X$, and the coupling at the unification scale $\alpha(M_X)$, in supersymmetric grand-unified models with arbitrary intermediate scales. We correct these expressions to agree with a two-loop calculation for central values of the inputs. Our general results quickly determine whether a particular model has hope of compatibility with the low energy couplings. We then apply these results to traditional supersymmetric $SU(5)$ and to supersymmetric flipped $SU(5) \times U(1)$. These results translate into a narrow bound on a function of the extra intermediate scales. In particular, we conclude that even allowing for the experimental uncertainties in low-energy couplings and effects of supersymmetric and Higgs thresholds, traditional supersymmetric $SU(5)$ grand unification without extra thresholds is about one standard deviation away from the measured value of $sin^2\theta_W$. We also calculate the range of proton decay in minimal flipped $SU(5) \times U(1)$ allowing for uncertainties in the low-energy couplings and the effects of supersymmetric and Higgs thresholds. Non-observation of proton decay gives a bound on another function of additional intermediate scales beyond the minimal model.

(a) Supported in part by DOE grant DE-AS05-81ER40039.

CTP-TAMU-97/90
ACT-19
CERN-TH.5943/90
November, 1990

## 1. Introduction

Gauge coupling unification has always led to attractive predictions of Grand Unified Theories, GUTs [1]. Minimal non-supersymmetric GUTs predict $sin^2\theta_W$ about 8 to 5 percent lower than the present LEP value [2], while minimal supersymmetric GUTs predict $sin^2\theta_W$ tantalizingly close to the LEP value [3]. However, as we shall see later, improving bounds on the QCD coupling $\alpha_3$ [4,5] begin to disfavour the minimal supersymmetric GUT on the basis of its prediction for $sin^2\theta_W$.

The above remarks assume that the region between $m_Z$ and $M_X$ is a complete desert, with no intermediate scales. But the evolution of the gauge couplings are sensitive to any fields with quantum numbers under the gauge group. The top mass almost certainly lies in the desert [6], and maybe many supersymmetric masses as well [7]. In addition, many unified models, in particular string theories, have whole armies of fields camped in the desert [8], and there could be significant threshold effects at the unification scale.

The present precision of experimental measurements of low energy gauge couplings warrants a two-loop treatment in any model with a specific spectrum. However, a one-loop treatment gives $sin^2\theta_W$ only about 1 percent below the two-loop result and $M_X$ about 20 percent above the two-loop result. Therefore, by fitting a general one-loop analytic calculation to a two-loop calculation for central values of the inputs, we can quickly determine which models have hopes of hitting the quickly narrowing experimental range of $sin^2\theta_W$, saving model builders many hours of numerically searching parameter space in a two-loop calculation.

If we regard $\alpha_{em}$ at $m_Z$ as fixed, requiring gauge coupling unification predicts $M_X$ and $sin^2\theta_W$ in terms of $\alpha_3$ for a model with a given spectrum. Two possible ways of viewing this are:

1. For a model with a given spectrum, the experimental range of $\alpha_3$ predicts a range of $sin^2\theta_W$ to be compared with LEP.

2. Consistency with the experimental ranges of both $sin^2\theta_W$ and $\alpha_3$ constrains the spectrum of the model.



The first point of view was taken in reference [3]. We also examine the second view in this paper.

In models such as flipped $SU(5)\times U(1)$ [9,10] where the standard model gauge group is not unified within a simple group, the prediction for $sin^2\theta_W$ becomes an upper bound on $sin^2\theta_W$ [11] and there is much more freedom in the spectrum of the model. It is likely that string threshold effects may modify the gauge coupling unification conditions such that the $U(1)$ coupling need not be equal to the other couplings at the unification scale. This significantly alters the constraints on the spectrum of a model.

For completeness, we include in our analysis another restriction on the spectrum, in any model with proton decay, by requiring that the extra representations not lower $M_X$ to a value in conflict with limits on the proton lifetime.

## 2. One-loop analytic results.

The one-loop renormalization group equations for gauge couplings may be written as

$$\frac{d\alpha_i}{dt} = \frac{b_i^{(1)}}{2\pi}\alpha_i^2, \quad i = 1,2,3 \tag{1}$$

with $dt = ln(\mu)$. This equation can be solved exactly to give

$$\alpha_i(\mu) = \frac{\alpha_i(\mu')}{1-\alpha_i(\mu)\frac{b_i^{(1)}}{2\pi}ln(\frac{\mu}{\mu'})}. \tag{2}$$

The boundary conditions at $m_Z$ are the $SU(3)\times SU(2)\times U(1)$ gauge couplings determined from the experimental values of

$$\frac{1}{\alpha_{em}} = 127.9 \pm .3\ [12], \quad sin^2\theta_W = .2329 \pm .0013\ [3], \quad \alpha_3 = .111 \pm .003^*,$$

where we use the $\overline{MS}$ prescription for all these inputs.

* This is the average of results from deep inelastic processes [4] and LEP [5]. Sceptical readers might wish to increase the systematic errors in these determinations.

We assume three light generations of quarks and leptons, all except the top with masses less than $m_Z$, and one Higgs doublet with mass $m_Z$. In addition, we assume the following nine light mass scales: $m_t$, the mass of the top quark; $m_a$, the mass of a second Higgs doublet; $m_{\tilde{t}_l}$, the mass of the spartner of the left-handed top quark; $m_{\tilde{t}_r}$, the mass of the spartner of the right-handed top quark; $m_{\tilde{W}}$, the masses of the four neutralinos and two charginos; $m_{\tilde{q}}$, the masses of five squarks, all except the stop; $m_{\tilde{l}_l}$, the masses of the three slepton doublets; $m_{\tilde{l}_r}$, the masses of the three slepton singlets; $m_{\tilde{g}}$, the masses of the eight gluinos. Our simplified choice of Higgs spectrum needs some comment. There are eight degrees of freedom in the two Higgs doublets. Three of these become longitudinal degrees of freedom of massive weak bosons through the Higgs mechanism. The minimal supersymmetric extension of the Standard Model gives sum rules among the other five degrees of freedom in terms of $m_h$, the lightest neutral Higgs mass, and $m_a$, the pseudoscalar Higgs mass [13]:

$$m_+^2 = m_a^2 + m_W^2, \quad m_H^2 = m_a^2 + m_Z^2 - m_h^2, \quad m_h < m_Z \tag{3}$$

Experimental lower bounds on the lightest Higgs boson mass [14] prompt us to make the simplifying assumption that $m_h \approx m_Z$, giving half of the Higgs degrees of freedom, namely one doublet, at $m_Z$. The masses of the other four degrees of freedom are parameterized by 1 TeV $> m_a > m_Z$. The upper bound comes from naturalness [15]. Since each Higgs degree of freedom contributes the same to the running of the gauge couplings, neglecting small splittings has no effect on our results.

Our inclusion of the separate scales $m_{\tilde{t}_l}$, $m_{\tilde{t}_r}$ and neglect of the mass splittings between different charginos and between different neutralinos will be justified later in this section.

As well, we allow for general extra heavy supersymmetric $Q, D^c, U^c, L, E^c$ representations between $m_Z$ and $M_X$. The constants in the renormalization group



equations (RGE's) ,$b_i^{(1)}$, for this field content are:

$$\begin{aligned}
b_y^{(1)} =& \frac{109}{30} + \frac{17}{30}\delta_t + \frac{49}{60}\delta_{\tilde{q}} + \frac{4}{15}\delta_{\tilde{t}_r} + \frac{1}{60}\delta_{\tilde{t}_l} + \frac{3}{10}\delta_{\tilde{l}_l} + \frac{3}{5}\delta_{\tilde{l}_r} + \frac{2}{5}\delta_{\tilde{W}} + \frac{1}{10}\delta_a \\
&+ \frac{4}{5}n_{U^c} + \frac{1}{5}n_{D^c} + \frac{1}{10}n_Q + \frac{3}{10}n_L + \frac{3}{5}n_{E^c} \\
b_2^{(1)} =& -\frac{7}{2} + \frac{1}{2}\delta_t + \frac{5}{4}\delta_{\tilde{q}} + \frac{1}{2}\delta_{\tilde{l}_l} + 2\delta_{\tilde{W}} + \frac{1}{6}\delta_a + \frac{3}{2}n_Q + \frac{1}{2}n_L \\
b_3^{(1)} =& \frac{-23}{3} + \frac{2}{3}\delta_t + \frac{5}{3}\delta_{\tilde{q}} + \frac{1}{6}\delta_{\tilde{t}_r} + \frac{1}{6}\delta_{\tilde{t}_l} + 2\delta_{\tilde{g}} + \frac{1}{2}n_{U^c} + \frac{1}{2}n_{D^c} + n_Q.
\end{aligned} \tag{4}$$

We consider boundary conditions at $M_X$ for two types of models:
(a) Unification of $SU(3) \times SU(2) \times U(1)$ within a larger group at the same scale $M_X$. Possible string threshold effects modifying the unification relation are parameterized by the factor c. The boundary condition is the equality:

$$\frac{1}{c}\alpha_y = \alpha_2 = \alpha_3 \tag{5}$$

(b) Unification of $SU(3) \times SU(2)$ within a larger group at $M_X$ and subsequent unification of the remaining $U(1)$ at a scale greater than or equal to $M_X$. An example of this class of model is the flipped $SU(5) \times U(1)$ model [9,10]. Possible string threshold effects modifying the unification relation are again parameterized by the factor c. The boundary condition is the inequality:

$$\frac{1}{c}\alpha_y \leq \alpha_2 = \alpha_3 \tag{6}$$

Solving the RGE's with the boundary conditions for model (b) gives the following constraints on $M_X$, $sin^2\theta_W$, and $\alpha(M_X)$:

$$\begin{aligned}
ln(\frac{M_X}{m_Z}) \leq& \frac{a_c}{f_c(s)} - \Sigma\frac{f_c(h)}{f_c(s)}ln(\frac{M_X}{M_h}) + \Sigma\frac{f_c(l)}{f_c(s)}ln(\frac{M_l}{m_Z}) \\
sin^2\theta_W \leq& \alpha_{em}[\frac{1}{\alpha_3} + \frac{2a_c}{\pi f_c(s)} + \Sigma f_s(h)ln(\frac{M_X}{M_h}) - \Sigma f_s(l)ln(\frac{M_l}{m_Z})] \\
\frac{1}{\alpha(M_X)} \geq& \frac{1}{\alpha_3} - \frac{1}{2\pi}[-3\frac{a_c}{f_c(s)} + \Sigma f_a(h)ln(\frac{M_X}{M_h}) - \Sigma f_a(l)ln(\frac{M_l}{m_Z})],
\end{aligned} \tag{7}$$

where the first sum is over the extra heavy representations 'h' and the second sum is over the eight extra light scales 'l' in the supersymmetric Standard Model. In model (a) these bounds become equalities. The various constants are defined by:

$$\begin{aligned} a_c &= 2\pi(\frac{3c}{5\alpha_{em}} - \frac{5+3c}{5\alpha_3}) \\ f_c(b) &= cb_y^{(1)} + \frac{3c}{5}b_2^{(1)} - \frac{5+3c}{5}b_3^{(1)} \\ f_s(b) &= \frac{1}{2\pi}[b_2^{(1)} - b_3^{(1)} - \frac{4f_c(b)}{f_c(s)}] \\ f_a(b) &= b_3^{(1)} + 3\frac{f_c(b)}{f_c(s)}, \end{aligned} \qquad (8)$$

where 'b' indicates the contribution of a particular particle representation to the one-loop beta functions and 's' indicates the contribution of the standard supersymmetric three generations and two Higgs doublets.

Henceforth we will assume $c = 1$. In this case, the above bounds become

$$\begin{aligned} ln(\frac{M_X}{m_Z}) &\leq \frac{\pi}{6}(\frac{3}{5\alpha_{em}} - \frac{8}{5\alpha_3}) + \frac{1}{20}\Sigma[L_Q + L_{D^c} - L_L - L_{E^c}] \\ &+ \frac{1}{120}[2L_t + 2L_a - L_{\tilde{l}_l} - 11L_{\tilde{q}} + 6L_{\tilde{l}_l} + 6L_{\tilde{l}_r} - 32L_{\tilde{g}} + 16L_{\tilde{W}}] \\ sin^2\theta_W &\leq \frac{1}{5} + \frac{7\alpha_{em}}{15\alpha_3} + \frac{\alpha_{em}}{20\pi}\Sigma[7L_Q - 3L_{D^c} - 5L_{U^c} + 3L_L - 2L_{E^c}] \\ &+ \frac{\alpha_{em}}{20\pi}[L_t - L_a + \frac{5}{3}L_{\tilde{l}_r} - \frac{7}{6}L_{\tilde{l}_l} + \frac{1}{2}L_{\tilde{q}} - 3L_{\tilde{l}_l} + 2L_{\tilde{l}_r} + \frac{28}{3}L_{\tilde{g}} - \frac{44}{3}L_{\tilde{W}}], \end{aligned} \qquad (9)$$

where the inequalities hold for model (b) and the equalities apply to model (a). The L's are defined by

$$L_h = log(\frac{M_X}{M_h}), \quad L_l = log(\frac{M_l}{m_Z}). \qquad (10)$$

The prediction for $sin^2\theta_W$ is where the most precision is needed. Since the sign of $f_s(b)$ is the same for the winos as for the higgsinos, splittings in the charginos



or neutralinos may be accounted for by taking an appropriate universal mass for all the charginos and neutralinos. However, the sign of $f_s(b)$ is different for the $\tilde{t}_l$ and $\tilde{t}_r$, so we need to include both mass scales. This completes the explanation of the choice of the nine mass scales.

In model (b), $sin^2\theta_W$ can take any value below its upper bound. For an arbitrary value of $sin^2\theta_W$ below its upper bound, $M_X$ and $\alpha(M_X)$ (defined by where $\alpha_3 = \alpha_2$) are given by

$$\begin{aligned} ln(\frac{M_X}{m_Z}) &= \frac{\pi}{2}(\frac{sin^2\theta_W}{\alpha_{em}} - \frac{1}{\alpha_3}) - \Sigma f_{23}(h) ln(\frac{M_X}{M_h}) + \Sigma f_{23}(l) ln(\frac{M_l}{m_Z}) \\ \frac{1}{\alpha(M_X)} &= \frac{1}{4\alpha_3} + \frac{3sin^2\theta_W}{4\alpha_{em}} + \frac{1}{2\pi}[\Sigma f_{3a}(l) ln(\frac{m_l}{m_Z}) - \Sigma f_{3a}(h) ln(\frac{M_X}{M_h})], \end{aligned} \qquad (11)$$

where

$$\begin{aligned} f_{23}(b) &= \frac{b_2^{(1)} - b_3^{(1)}}{4} \\ f_{3a}(b) &= b_3^{(1)} + 3\frac{f_{23}(b)}{f_{23}(s)}. \end{aligned} \qquad (12)$$

Notice that these bounds do not involve the $U(1)$ coupling and are therefore independent of c.

## 3. Analysis.

We first analyze $sin^2\theta_W$ in model (a), i.e. the minimal supersymmetric SU(5) GUT. A two-loop numerical calculation assuming three supersymmetric generations and two supersymmetric Higgs doublets, all with masses less than or equal to $m_Z$, and taking the central values $\alpha_3 = .111$ and $\frac{1}{\alpha_{em}} = 127.9$ gives

$$sin^2\theta_W = .2358, \quad M_X = 1.10 \times 10^{16}, \quad \alpha(M_X) = .0418 \qquad (13)$$

Using these results we can beef up our one-loop analytic expressions by adding constants so that they give the same results for the same initial assumptions. For example, the new and improved result for $sin^2\theta_W$ is:

$$sin^2\theta_W = \frac{1}{5} + \frac{7\alpha_{em}}{15\alpha_3} + .0029 + \delta_s(l) + \delta_s(h). \tag{14}$$

where $\delta_s(l)$ and $\delta_s(h)$ are the contributions of the light and heavy particles to $sin^2\theta_W$ in Eq. (9) . Assuming universal supersymmetry breaking at the unification scale, we can approximately parameterize five of the nine light scales as follows [16]:

$$\begin{aligned} m_{\tilde{q}} &= \sqrt{7m_{\frac{1}{2}}^2 + m_0^2} \\ m_{\tilde{l}_l} &= \sqrt{.5m_{\frac{1}{2}}^2 + m_0^2} \\ m_{\tilde{l}_r} &= \sqrt{.15m_{\frac{1}{2}}^2 + m_0^2} \\ m_{\tilde{W}} &= m_{\frac{1}{2}} \\ m_{\tilde{g}} &= 3m_{\frac{1}{2}}. \end{aligned} \tag{15}$$

Bounding $m_{\frac{1}{2}}$ and $m_0$ from above by requiring that no supersymmetric masses exceed 1 TeV and bounding $m_{\frac{1}{2}}$ from below by requiring no supersymmetric masses are below 40 GeV, the above relations between supersymmetric thresholds further restrict $sin^2\theta_W$ as a function of $\alpha_3$ to the band in fig. 1. The lower bound is obtained for $\frac{1}{\alpha_{em}} = 128.2$, $m_{\frac{1}{2}} = 40\,\text{GeV}$, $m_0, m_a, m_{\tilde{l}_l} = 1\,\text{TeV}$, and $m_t, m_{\tilde{l}_r} < m_Z$. There could possibly be some extra excursion due to the thresholds for heavy particles around the unification scale. Also plotted is the $1-\sigma$ error ellipse allowed by the LEP determination [3] of $sin^2\theta_W$ and determinations [4,5] of the strong QCD coupling which are essentially uncorrelated. For comparison, we show the prediction for $sin^2\theta_W$ in non-supersymmetric $SU(5)$ models, which are many standard deviations away from the LEP value.

Our last example of the use of these analytic unification results is proton decay in model (b). A calculation of proton decay from the dominant dimension-six operators in the flipped $SU(5)\times U(1)$ model gives [11]

$$\tau(p\rightarrow e^+\pi^0) \approx 3.3\times 10^{31}(\frac{M_X}{10^{15}\,\text{GeV}})^4(\frac{.042}{\alpha(M_X)})^2 y \tag{16}$$

where we have used a lattice calculation of the nucleon wave function [17]. Taking



into account the experimental ranges of the low-energy couplings and the variation of the light scales, assuming the relations implied by universal soft supersymmetry breaking, the model predicts a lifetime of

$$1.2\times 10^{36} y > \tau(p\rightarrow e^+\pi^0) > 8.8\times 10^{31} y \tag{17}$$

The effects of the light scales on the proton lifetime through varying $\alpha(M_X)$ are less than ten percent and will be ignored. The only significant effect of varying the parameters defining the light scales was to lower the minimum value of $M_X$ by a factor of two which resulted in the lower bound above being sixteen times less than if the light scales had been neglected. Using the experimental bound on proton decay $\tau(p\rightarrow e^+\pi^0) > 5.5\times 10^{32} y$ [18] we can put another restriction on the heavy representations which is independent of c, by requiring that the extra particles do not push the upper bound in Eq. (17) below present limits on the proton lifetime:

$$\frac{1}{8}\Sigma[L_Q + L_L - L_{U^c} - L_{D^c}] > -1.9 \tag{18}$$

Clearly proton decay in minimal flipped $SU(5)\times U(1)$ already restricts the parameter space of low energy couplings and light scales, and extra representations could pull the upper bound into the reach of present experiments. However, they could equally well push the lower bound on the proton lifetime out of the reach of future experiments.

To illustrate the constraints on extra representations in the models considered above, supersymmetric $SU(5)$ and flipped $SU(5)\times U(1)$, we graph the bounds from $sin^2\theta_W$ and proton decay in each model for two choices of the extra heavy field content: Fig. 2a, a vector $U^c$ quark triplet and a vector $L$ lepton doublet; and Fig. 2b, a vector $L$ lepton doublet and a vector $E^c$ lepton singlet. Note that for the flipped models, there is no lower bound from $sin^2\theta_W$ and that the upper bound from $sin^2\theta_W$ in the flipped model coincides with that of the $SU(5)$ model. The bound from proton decay in the $SU(5)$ model comes from considering dimension-six operators. Note that $\tau_{SU(5)}(p \to e^+\pi^0) = \frac{1}{5}\tau_{SU(5)\times U(1)}(p \to e^+\pi^0)$ for dimension-six operators [9]. It is well known that the minimal supersymmetric $SU(5)$ model has troubles with dimension-five proton decay operators that may give more restrictive bounds [19] , but these need not be a problem for flipped $SU(5) \times U(1)$ models [9].

## 4. Conclusion.

We have derived one-loop analytic expressions for $sin^2\theta_W$, the unification scale, and the gauge coupling at the unification scale for several classes of unified gauge theories with general intermediate spectra. These expressions can be corrected to agree with the two-loop calculation for central values. We have demonstrated how to use experimental constraints on the low-energy couplings and proton decay to bound functions of the intermediate scales. These bounds depend on whether there is one unification scale as in model (a) or two as in model (b), and will be modified if there is a nonstandard unification relation for the $U(1)$ coupling due to string threshold effects. In particular, we conclude that minimal supersymmetric $SU(5)$ without extra representations is about one standard deviation away from present measurements of $sin^2\theta_W$ and $\alpha_3$, which are well inside the range allowed by supersymmetric flipped $SU(5) \times U(1)$, whilst non-supersymmetric $SU(5)$ models are many standard deviations away. Our general results enable model builders quickly to check a unified model with any intermediate spectrum for its consistency with low-energy couplings and proton decay.

*Acknowledgments:* We would like to thank I. Antoniadis, J. Lopez, and H. Pois for useful discussions.



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## FIGURE CAPTIONS

1) Comparison of the experimentally allowed region in ($\alpha_3$,$sin^2\theta_W$) space and the region predicted by minimal supersymmetric $SU(5)$ without extra representations, that allowed by supersymmetric flipped $SU(5) \times U(1)$ and proton decay experiments, and that predicted by non-supersymmetric $SU(5)$ models.

2) Bounds on extra representations in supersymmetric $SU(5)$ and supersymmetric flipped $SU(5) \times U(1)$. The solid line represents the lower bound on the contribution of extra representations to $sin^2\theta_W$ applicable to both models. The dashed line represents the upper bound on the contribution of extra representations to $sin^2\theta_W$ applicable only to minimal supersymmetric $SU(5)$. The dotted line represents the bound on extra representations from non-observation of d=6 proton decay in minimal supersymmetric SU(5). The bound from non-observation of d=6 proton decay in supersymmetric flipped $SU(5) \times U(1)$ gives a less stringent bound than the one from $sin^2\theta_W$ for the extra representations considered in these figures and is not shown. The allowed areas in the space of the ratio of $M_X$ to the extra representation masses is the dotted area for supersymmetric $SU(5)$ and the area above and to the left of the solid line for supersymmetric flipped $SU(5) \times U(1)$. Two sets of extra representations are considered: (a) a vector $U^c$ quark triplet and a vector $L$ lepton doublet, and (b) a vector $L$ lepton doublet and a vector $E^c$ lepton singlet. Note that the origin of the graph corresponds to the absence of extra representations, which is allowed in the flipped model but lies on the outskirts of the region allowed in the minimal model.

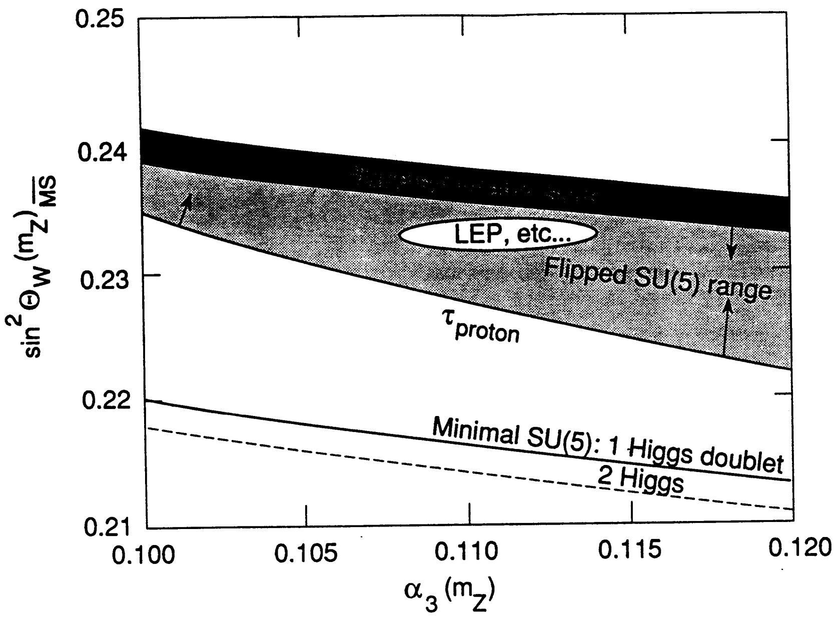


**Figure 1**

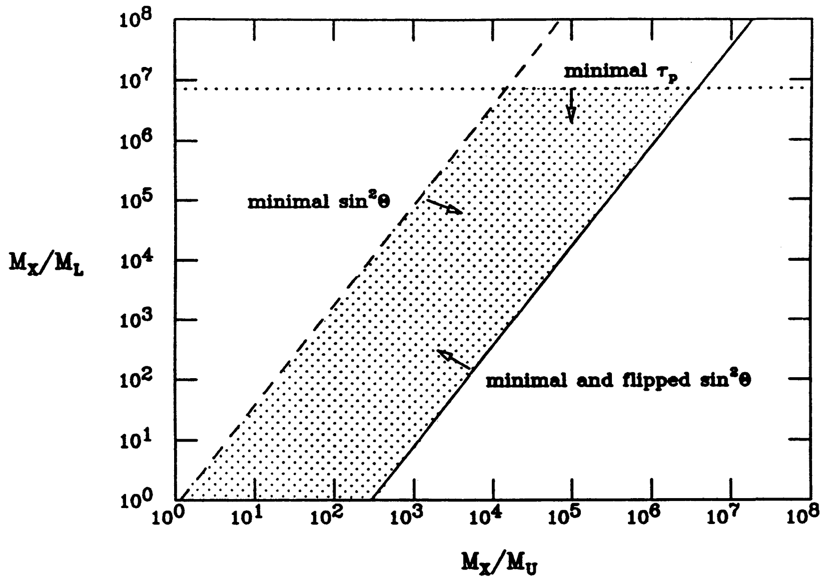


**Figure 2a**

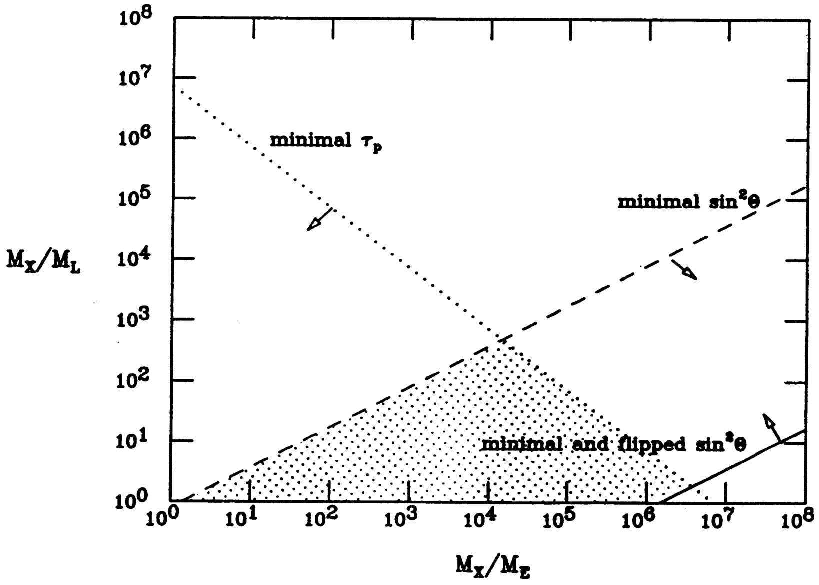


**Figure 2b**