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Evidence for the classical integrability of the complete $AdS_4 \times CP^3$ superstring

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Abstract

We construct a zero–curvature Lax connection in a sub–sector of the superstring theory on $AdS_4 \times CP^3$ which is not described by the $OSp(6|4)/U(3) \times SO(1,3)$ supercoset sigma–model. In this sub–sector worldsheet fermions associated to eight broken supersymmetries of the type IIA background are physical fields. As such, the prescription for the construction of the Lax connection based on the Z_4–automorphism of the isometry superalgebra $OSp(6|4)$ does not do the job. So, to construct the Lax connection we have used an alternative method which nevertheless relies on the isometry of the target superspace and kappa–symmetry of the Green–Schwarz superstring.

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1 Introduction

The $AdS_4 \times CP^3$ background of type IIA superstring theory is not maximally supersymmetric. It preserves 24 supersymmetries (out of the maximum number of 32) which together with the bosonic isometries of $AdS_4 \times CP^3$ form the supergroup $OSp(6|4)$. It turns out that the type IIA superspace associated with the $AdS_4 \times CP^3$ background which has 32 Grassmann–odd directions is not a coset superspace of $OSp(6|4)$ [1]. So the complete Green–Schwarz superstring theory on this superspace is not a coset–superspace sigma–model, in contrast *e.g.* to the maximally supersymmetric type IIB superstring on $AdS_5 \times S^5$ described by the $PSU(2,2|4)/(SO(1,4) \times SO(5))$ sigma–model [2]. The worldsheet $AdS_4 \times CP^3$ superstring action can be reduced to an $OSp(6|4)/U(3) \times SO(1,3)$ sigma–model constructed in [3, 4, 5, 6, 7] in those sub–sectors of the classical configuration space of the theory in which the kappa–symmetry can be used to eliminate eight fermionic modes of the string associated with the broken supersymmetries. However, this is not always possible. For instance such a gauge choice is inadmissible when the classical string moves entirely in AdS_4 [3, 1] or forms a worldsheet instanton wrapping a 2–cycle inside CP^3 [8]. In these cases the 'broken supersymmetry' fermions are physical modes, so one should start the analysis of the theory in these sectors from the complete $AdS_4 \times CP^3$ superstring action [1] and, if required, make an alternative choice of the kappa–symmetry gauge (see e.g. [9, 10, 11, 12]).

The classical integrability of the $OSp(6|4)/U(3) \times SO(1,3)$ σ–model sub–sector of the theory was demonstrated in [3, 4] by constructing a zero–curvature Lax connection using the same techniques as for the $AdS_5 \times S^5$ superstring [13]. Such a construction is based on the Z_4–automorphism of the isometry superalgebra and can be applied to any G/H supercoset two–dimensional sigma–model that admits a Z_4–grading. Basically, the prescription is as follows. Take a left–invariant Cartan form $K^{-1}dK$ (with $K \in G/H$ being a supercoset element) which are used to build the supercoset sigma–model action [2, 13, 3, 4]. The Cartan form takes values in the isometry superalgebra $\mathcal{G}$ of G and thus can be expanded in the bosonic generators M_0 and P_2, and the fermionic generators Q_1 and Q_3 of $\mathcal{G}$

$$K^{-1}dK = \Omega_0\, M_0 + E_2\, P_2 + E_1\, Q_1 + E_3\, Q_3\,. \tag{1.1}$$

The building blocks of the G/H supercoset sigma–model action are the G/H supervielbeins E_2, E_1 and E_3, while Ω_0 is the H–valued spin connection on G/H.

The bosonic generators M_0 of the stability subgroup H have zero grading under the Z_4–automorphism and the bosonic coset–space translation generators P_2 carry grading two. The fermionic generators Q_1 and Q_3 have the Z_4–grading one and three, respectively. In terms of these generators the superalgebra $\mathcal{G}$ has the following schematic Z_4–grading structure

$$\begin{gathered}
[M_0, M_0] \sim M_0, \quad [M_0, P_2] \sim P_2, \quad [P_2, P_2] \sim M_0, \\
[M_0, Q_1] \sim Q_1, \quad [M_0, Q_3] \sim Q_3, \quad [P_2, Q_1] \sim Q_3, \quad [P_2, Q_3] \sim Q_1, \\
\{Q_1, Q_1\} \sim P_2, \quad \{Q_3, Q_3\} \sim P_2, \quad \{Q_1, Q_3\} \sim M_0.
\end{gathered} \tag{1.2}$$

In the case of the $AdS_4 \times CP^3$ superstring $M_0 \in so(1,3) \times u(3)$, $P_2 \in \frac{so(2,3)\times su(4)}{so(1,3)\times u(3)}$ and Q_1 and Q_3 are the 24 fermionic generators of $OSp(6|4)$, see Appendix A.4.

The worldsheet Lax connection one–form which takes values in $\mathcal{G}$ is constructed by taking the sum of the components of the Cartan form (1.1) and their worldsheet Hodge–duals with some arbitrary coefficients, namely

$$L = \Omega_0\, M_0 + (l_1 E_2 + l_2 * E_2)\, P_2 + l_3\, E_1\, Q_1 + l_4\, E_3 Q_3. \tag{1.3}$$

Then one imposes the requirement that the curvature associated with the connection L vanishes

$$dL - L \wedge L = 0 \tag{1.4}$$

(the exterior derivative acts from the right, and in what follows we shall not explicitly write the wedge–product). The sigma–model equations of motion and the Z_4–grading structure of the superalgebra (1.2) ensure that the coefficients in the definition of the zero–curvature

Lax connection (1.3) are expressed in terms of a single independent *spectral* parameter, *e.g.* $l_1 = \frac{1+z^2}{1-z^2}$.

By performing a gauge transformation of (1.3) one can get another form of the Lax connection [13] associated with right–invariant Cartan forms dKK^{-1}

$$\mathcal{L} = KLK^{-1} - dKK^{-1}\,, \qquad d\mathcal{L} - \mathcal{L}\,\mathcal{L} = 0\,. \tag{1.5}$$

Having at hand the Lax connection, one can then derive an infinite set of conserved charges of the integrable model from the holonomy of the Lax connection by constructing a corresponding monodromy matrix and the algebraic curve (see *e.g.* [13, 14] for more details and references therein).

In the case of the complete Green–Schwarz theory (*i.e.* when the kappa–symmetry is not fixed at all) the superstring moves in $AdS_4 \times CP^3$ superspace with thirty two Grassmann–odd directions and the eight worldsheet fermionic fields associated to the broken supersymmetry contribute to the structure of the supervielbeins E_2, E_1 and E_3 and to the connection Ω_0 thus spoiling their nature as the G/H Cartan forms. As a result, as one can check by direct calculations, the $OSp(6|4)$ Lax connection of the form (1.3) or (1.5) constructed from Ω_0, E_2, E_1 and E_3 which include the dependence on these eight fermions will not have zero curvature for any non–trivial choice of the coefficients. Therefore, a modification of the form of (1.3) or (1.5) by additional terms depending on the extra eight fermions is required for restoring the zero curvature condition (1.4). The goal of this paper is to reveal the structure of these terms.

To construct the Lax connection which includes broken supersymmetry fermions we have found helpful to look at the form of conserved Noether currents associated with the $OSp(6|4)$ isometry. In this respect it is more convenient to consider the Lax connection in the form (1.5) which, in a certain sense, has closer relation to a G/H sigma–model conserved current having the form [13, 3]

$$J_{coset} = K\left(E_2\,P_2 + \frac{1}{2} * (E_1 Q_1 - E_3 Q_3)\right)K^{-1}\,. \tag{1.6}$$

The paper is organized as follows. In Section 2 we consider the $AdS_4 \times CP^3$ superstring action truncated to the second order in fermions and show that there exist different forms of the Lax connection, related to each other by local $OSp(6|4)$ transformations, which have zero curvature at least to the second order. When the eight broken supersymmetry fermions are put to zero the Lax connection reduces (modulo a gauge transformation) to the supercoset Lax connection of [3, 4]. The reconstruction of higher order fermionic terms in the Lax connection becomes technically more and more complicated with each order and we have not been able to accomplish the construction in the complete theory with 32 fermions. So in Section 3 we consider a simpler sub–sector of the theory in which the superstring moves only in an AdS_4 superspace with eight fermionic directions associated with broken supersymmetries. This sub–sector of the theory is not reachable by the $OSp(6|4)$ supercoset sigma–model and can be regarded as a model of an $\mathcal{N} = 2$, $D = 4$ superstring in the AdS_4 background with completely broken supersymmetries [1]. Nevertheless, this model is invariant under the four–parameter kappa–symmetry, in addition to the purely bosonic isometry $SO(2,3)$ of AdS_4 and $SO(2)$ transformations of the two Majorana fermions. So, surprisingly, the integrability of its fermionic sector is not related to target space supersymmetry. To simplify the construction of the full Lax connection in this model, in Section 4 we gauge fix kappa–symmetry and perform worldsheet T–duality transformations along the AdS_4 Minkowski boundary following the results of [9]. In Subsection 4.2 we give the explicit form of the kappa–symmetry gauge–fixed Lax connection of the AdS_4 superstring to all orders in fermions thus giving more evidence for the classical integrability of the complete $AdS_4\times CP^3$ superstring itself. Section 5 is devoted to a summary of the obtained results and discussion of the possibility of their generalization and application to strings in other supergravity backgrounds. Our notation and conventions are given in Appendices A and B, and in Appendices C and D we have collected various formulas and relations which have been used to construct the Lax connections.

2 $AdS_4\times CP^3$ superstring in the quadratic approximation in fermions

2.1 The action and equations of motion

We first check that a zero–curvature Lax connection does exist in the complete $AdS_4 \times CP^3$ superstring theory at least up to the second order in the fermionic fields. To this end we start with the $AdS_4 \times CP^3$ superstring action truncated to the second order in fermions as in [15]. In the notation and conventions of [8] the action has the following form

$$\begin{aligned} S &= -\frac{e^{\frac{2}{3}\phi_0}}{4\pi\alpha'}\int d^2\xi\,\sqrt{-h}\,h^{IJ}\,e_I{}^A e_J{}^B\eta_{AB} \\ &- \frac{e^{\frac{2}{3}\phi_0}}{2\pi\alpha'}\int d^2\xi\,\Theta(\sqrt{-h}\,h^{IJ}-\varepsilon^{IJ}\Gamma_{11})\big[i\,e_I{}^A\Gamma_A\nabla_J\Theta-\frac{1}{R}e_I{}^Ae_J{}^B\Gamma_A\mathcal{P}_{24}\gamma^5\Gamma_B\Theta\big] \qquad (2.1)\end{aligned}$$

where $h_{IJ}(\xi)$ $(I,J=0,1)$ is the intrinsic (auxiliary) worldsheet metric, $e_I{}^A=\partial_I X^M e_M{}^A(X)$ are the worldsheet pullbacks of the $AdS_4\times CP^3$ vielbeins ($M=0,1,\cdots,9$ are the $D=10$ space-time indices and $A=0,1,\cdots,9$ are the tangent space indices). $X^M=(x^{\hat m},y^{m'})$ are $AdS_4\times CP^3$ coordinates ($\hat m=0,1,2,3$; $m'=1',\cdots 6'$), $\nabla\Theta=(d-\frac14\,\omega^{AB}\,\Gamma_{AB})\Theta$ is the worldsheet pullback of the conventional $AdS_4\times CP^3$ covariant derivative and $\mathcal{P}_{24}$ is the projector which splits the 32 fermionic coordinates $\Theta^{\underline{\alpha}}$ ($\underline{\alpha}=1,\cdots,32$) into 24 fermionic coordinates ϑ corresponding to the 24 unbroken supersymmetries of the $AdS_4\times CP^3$ background and 8 ‘broken supersymmetry’ coordinates υ

$$\mathcal{P}_{24}=\frac18(6+iJ_{a'b'}\,\Gamma^{a'b'}\,\gamma^7)\,,\qquad \vartheta\equiv\mathcal{P}_{24}\,\Theta\,,\qquad \upsilon\equiv(1-\mathcal{P}_{24})\,\Theta. \qquad (2.2)$$

In (2.2) $J_{a'b'}=-J_{b'a'}$ is the Kähler form on CP^3, $\Gamma^{a'}$ are $D=10$ Dirac matrices along the six CP^3 directions ($a'=1',\cdots,6'$) and $\gamma^7=i\Gamma^{1'}\cdots\Gamma^{6'}$ is the product of all of them. The presence in the action (2.1) of the projector $\mathcal{P}_{24}$ is due to the interaction of the string with the constant Ramond–Ramond $F_4\sim dx^0dx^1dx^2dx^3$ and $F_2\sim dy^{a'}dy^{b'}J_{a'b'}$ fluxes of type IIA supergravity on $AdS_4\times CP^3$. $\gamma^5=i\Gamma^{0123}$ is the product of the four gamma–matrices with AdS_4 indices. Finally, ϕ_0 is the vacuum expectation value of the dilaton and R is related to the CP^3 radius in the string frame $R_{CP^3}=e^{\frac{\phi_0}{3}}R$. See Appendix A for more details of our notation and conventions.

The bosonic field equations which follow from (2.1) are

$$\nabla_I \left[\sqrt{-h}\, h^{IJ} e_J{}^A + i\Theta(\sqrt{-h}\, h^{IJ} - \varepsilon^{IJ}\Gamma_{11})\big(\Gamma^A\nabla_J\Theta + \tfrac{2i}{R}\, e_J{}^B\Gamma^A\mathcal{P}_{24}\gamma^5\Gamma_B\Theta\big)\right] \tag{2.3}$$
$$-\tfrac{i}{4}\Theta\,(\sqrt{-h}\, h^{IJ} - \varepsilon^{IJ}\Gamma_{11})\Gamma_D{}^{BC}\Theta\, R_{BCE}{}^A\, e_I{}^D e_J{}^E = 0$$

where $R_{BCE}{}^A$ is the curvature of $AdS_4 \times CP^3$ (see Appendix A).

The Virasoro constraints are

$$e_I{}^A e_J{}^B\eta_{AB} - 2i\Theta\big(e_{(I}{}^A\Gamma_A\nabla_{J)}\Theta + \tfrac{i}{R}e_{(I}{}^A e_{J)}{}^B\Gamma_A\mathcal{P}_{24}\gamma^5\Gamma_B\Theta\big) \tag{2.4}$$
$$= \tfrac{1}{2}h_{IJ}\, h^{KL}\, \left[e_K{}^A\, e_L{}^B\eta_{AB} - 2i\Theta\big(e_K{}^A\Gamma_A\nabla_L\Theta + \tfrac{i}{R}e_K{}^A e_L{}^B\Gamma_A\mathcal{P}_{24}\gamma^5\Gamma_B\Theta\big)\right],$$

where the round brackets embracing the indices denote symmetrization $X_{(I}Y_{J)} = \frac{1}{2}\,(X_IY_J + X_JY_I)$.

The fermionic equations are

$$(\sqrt{-h}\, h^{IJ} - \varepsilon^{IJ}\Gamma_{11})\big(e_I{}^A\Gamma_A\nabla_J\Theta + \frac{i}{R}e_I{}^A e_J{}^B\Gamma_A\mathcal{P}_{24}\gamma^5\Gamma_B\Theta\big) - \frac{1}{2}\,\nabla_I(\sqrt{-h}\, h^{IJ}e_J{}^A)\Gamma_A\Theta = 0\,. \tag{2.5}$$

In virtue of the bosonic equations (2.3), the last term in (2.5) is of the third order in fermions and can be skipped in the linear approximation.

2.1.1 Comment on the relation to the supercoset sigma–model

When the fermionic fields υ are zero the superstring equations of motion reduce to the bosonic equation

$$\nabla_I \left[\sqrt{-h}\, h^{IJ} e_J{}^A + i\vartheta(\sqrt{-h}\, h^{IJ} - \varepsilon^{IJ}\Gamma_{11})\big(\Gamma^A\nabla_J\vartheta + \tfrac{2i}{R}\, e_J{}^B\Gamma^A\mathcal{P}_{24}\gamma^5\Gamma_B\vartheta\big)\right]$$
$$-\tfrac{i}{4}\vartheta\,(\sqrt{-h}\, h^{IJ} - \varepsilon^{IJ}\Gamma_{11})\Gamma_D{}^{BC}\vartheta\, R_{BCE}{}^A\, e_I{}^D e_J{}^E = 0 \tag{2.6}$$

and the fermionic equations

$$(\sqrt{-h}\, h^{IJ} - \varepsilon^{IJ}\Gamma_{11})\, e_I{}^A\mathcal{P}_{24}\Gamma_A\mathcal{P}_{24}\big(\nabla_J\vartheta + \tfrac{i}{R}e_J{}^B\gamma^5\Gamma_B\vartheta\big) = 0\,, \tag{2.7}$$

$$(\sqrt{-h}\, h^{IJ} - \varepsilon^{IJ}\Gamma_{11})\, e_I{}^A(1-\mathcal{P}_{24})\Gamma_A\mathcal{P}_{24}\big(\nabla_J\vartheta + \tfrac{i}{R}e_J{}^B\gamma^5\Gamma_B\vartheta\big) = 0. \tag{2.8}$$

Eqs. (2.6) and (2.7) are the equations of motion of the $OSp(6|4)$ supercoset sigma–model in the quadratic approximation in fermions. However, the complete Green–Schwarz superstring action gives one more fermionic equation of motion which (when $\upsilon = 0$) produces an additional equation for the 24 fermions ϑ (2.8). This eight–component equation does not directly follow from the supercoset action, but it should not be independent of (2.7) and just manifests the fact that, when the partial kappa–symmetry gauge $\upsilon = 0$ is admissible, the residual kappa–symmetry of the supercoset model has eight independent components, such that the number of physical fermionic modes of ϑ is sixteen.

To show that the fermionic equations (2.7) and (2.8) are linearly dependent let us rewrite them in an equivalent form as follows

$$\mathcal{P}_{24}(1-\Gamma)\, h^{IJ} e_I{}^A \Gamma_A \mathcal{P}_{24}\big(\nabla_J\vartheta + \tfrac{i}{R} e_J{}^B \gamma^5 \Gamma_B \vartheta\big) = 0\,, \tag{2.9}$$

$$(1-\mathcal{P}_{24})(1-\Gamma)\, h^{IJ}\, e_I{}^A \Gamma_A \mathcal{P}_{24}\big(\nabla_J\vartheta + \tfrac{i}{R} e_J{}^B \gamma^5 \Gamma_B \vartheta\big) = 0\,, \tag{2.10}$$

where $\Gamma = \frac{1}{2\sqrt{-h}}\varepsilon^{IJ} e_I^A\, e_J^B\, \Gamma_{AB}\Gamma_{11}$, $(\Gamma)^2 = 1$ and $\frac{1}{2}(1-\Gamma)$ is the canonical kappa–symmetry projector of the type IIA superstring. The two equations can, therefore, be combined into

$$(1-\Gamma)\, h^{IJ} e_I{}^A \Gamma_A \mathcal{P}_{24}\big(\nabla_J\vartheta + \frac{i}{R} e_J{}^B \gamma^5 \Gamma_B \vartheta\big) = 0\,. \tag{2.11}$$

We shall now show that eq. (2.11) actually follows from eq. (2.9). To this end let us note that in the sector of classical string solutions in which the kappa–symmetry gauge $\upsilon = 0$ is admissible, the projectors $\mathcal{P}_{24}$ and $\frac{1}{2}(1\pm\Gamma)$ do not commute [1], their commutator $[\Gamma, \mathcal{P}_{24}]$ being a non degenerate matrix. Therefore, multiplying eq. (2.9) by $(1+\Gamma)$ we have

$$[\Gamma, \mathcal{P}_{24}]\,(1-\Gamma)\, h^{IJ} e_I{}^A \Gamma_A \mathcal{P}_{24}\big(\nabla_J\vartheta + \frac{i}{R} e_J{}^B \gamma^5 \Gamma_B \vartheta\big) = 0\,. \tag{2.12}$$

Since $[\Gamma, \mathcal{P}_{24}]$ is invertible we can multiply the above equation by the inverse of $[\Gamma, \mathcal{P}_{24}]$ and get eq. (2.11) from which the equation (2.10) follows.

On the other hand, in the sub–sector in which the classical string moves in AdS_4 only (i.e. the CP^3 embedding coordinates $y^{m'}$ are constants), this kappa–gauge is not admissible ($[\Gamma, \mathcal{P}_{24}] = 0$) and putting υ to zero results in loosing four physical fermionic modes associated with υ [3, 1]. This can be seen from the structure of the fermionic equations (2.7) and (2.8) (or (2.9) and (2.10)). Since $y^{m'}$ are constants and if υ is set to zero, eq. (2.8) (or (2.10)) vanishes identically and one is left with eq. (2.7) (or (2.9)) which, since the projector $\mathcal{P}_{24}$ commutes with the $\Gamma^{\hat a}$ along the AdS_4 directions, reduces to the fermionic equation in AdS_4

$$(1-\Gamma)\, h^{IJ}\, e_I{}^{\hat a}(x) \Gamma_{\hat a}\big(\nabla_J\vartheta + \frac{i}{R} e_J{}^{\hat b} \gamma^5 \Gamma_{\hat b} \vartheta\big) = 0\,. \tag{2.13}$$

where now $\Gamma = \frac{1}{2\sqrt{-h}}\, \varepsilon^{IJ} e_I{}^{\hat a} e_J{}^{\hat b}\, \Gamma_{\hat a \hat b}\Gamma_{11}$, $(\Gamma)^2 = 1$. The projector $\frac{1}{2}(1-\Gamma)$ (which now commutes with $\mathcal{P}_{24}$) implies that among 24 equations (2.13) only 12 are independent. Hence ϑ contain only 12 physical modes while the total number must be sixteen. The missing four physical fermions are half of υ which were put to zero 'by hand', while another half of υ can be gauged away by kappa–symmetry.

2.2 Noether currents

Under the $OSp(6|4)$ isometries the Type IIA superspace coordinates X^M and Θ transform as follows (up to the second order in fermions)

$$\delta X^M\, e_M{}^A(X) = K^A(X) + i\Theta\Gamma^A\, \Xi(X),$$

$$\delta\vartheta = \mathcal{P}_{24}\delta\Theta = \Xi(X) + \tfrac{1}{4}(K^M\, \omega_M{}^{AB}(X) - \nabla^A\, K^B)\, \mathcal{P}_{24}\Gamma_{AB}\mathcal{P}_{24}\Theta\,, \tag{2.14}$$

$$\delta\upsilon = (1-\mathcal{P}_{24})\,\delta\Theta = \tfrac{1}{4}(K^M\, \omega_M{}^{AB}(X) - \nabla^A\, K^B)\,(1-\mathcal{P}_{24})\,\Gamma_{AB}\,(1-\mathcal{P}_{24})\,\Theta,$$

where $K^A(X) = K^M(X)\, e_M{}^A(X)$ are the $AdS_4 \times CP^3$ Killing vectors. More precisely, $K^A(X)$ are the Killing vectors $K^A_{\mathcal{I}}(X)$ contracted with constant $SO(2,3) \times SU(4)$ transformation parameters $\Lambda^{\mathcal{I}}$, *i.e.* $K^A(X) = K^A_{\mathcal{I}}(X)\,\Lambda^{\mathcal{I}}$, where $\mathcal{I}$ is associated with the 25 generators of the $SO(2,3) \times SU(4)$ isometries. Note that, like the spin connection ω^{AB}, $\nabla^A K^B = -\nabla^B K^A$ takes values in the stability subalgebra $so(1,3) \times u(3)$ of the $AdS_4 \times CP^3$ isometry. Properties of the Killing vectors of symmetric spaces G/H are given in Appendix D .
Ξ are 24 supersymmetry parameters of $OSp(6|4)$ satisfying the $AdS_4 \times CP^3$ Killing spinor equation

$$\nabla\Xi + \frac{i}{R} e^A\, \mathcal{P}_{24}\gamma^5\Gamma_A\Xi = 0\,, \qquad \Xi^{\underline{\alpha}}(X) = \epsilon^{\underline{\mu}}\; \Xi_{\underline{\mu}}{}^{\underline{\alpha}}(X), \qquad \Xi \equiv \mathcal{P}_{24}\,\Xi(X)\,, \tag{2.15}$$

$\Xi_{\underline{\mu}}{}^{\underline{\alpha}}(X)$ are $AdS_4 \times CP^3$ Killing spinors and $\epsilon^{\underline{\mu}} = (\mathcal{P}_{24}\epsilon)^{\underline{\mu}}$ are 24 constant Grassmann–odd parameters.

Note that the terms in the variation of the fermions which are proportional to Γ_{AB} are the compensating $SO(1,3) \times U(3)$ stability group transformations induced by the isometries in the (co)tangent space of $AdS_4 \times CP^3$. Note also that in the linear order in fermions the eight spinor fields υ are not transformed by supersymmetry. The action of the isometry group $OSp(6|4)$ on these fermions is such that it takes the form of induced $SO(1,3) \times U(1)$ rotations with parameters depending on X, ϑ and the $OSp(6|4)$ parameters

$$\delta\upsilon = \frac{1}{4}\,\Lambda_{AB}(\epsilon, X, \vartheta)\,\Gamma^{AB}\,\upsilon\,. \tag{2.16}$$

Therefore, the first nontrivial term in the supersymmetry variation of υ is quadratic in fermionic fields.

To avoid possible confusion, let us note that in the expressions for the conserved currents and in the Lax connections considered below, $K^A(X)$ and $\Xi(X)$ stand for the Killing vectors and spinors contracted with the corresponding bosonic and fermionic generators of the $OSp(6|4)$ isometry (see Appendix A.4) and not with constant parameters like in eqs. (2.14) and (2.15).

The following relations between the Killing vectors and spinors contracted with the $OSp(6|4)$ generators reflect the structure of the $OSp(6|4)$ superalgebra (A.8)–(A.10)

$$\begin{gathered} K_A(X) \doteq k(X) P_A k^{-1}(X)\,, \qquad \gamma^5\Xi(X) \doteq k(X) Q k^{-1}(X)\,, \\ \nabla_A K_B \doteq -\tfrac{1}{2} R_{AB}{}^{CD}\, k(X) M_{CD} k^{-1}(X)\,, \end{gathered} \tag{2.17}$$

where $k(X)$ is an $\frac{SO(2,3)\times SU(4)}{SO(1,3)\times U(3)}$ coset element of the bosonic isometry, and

$$\begin{gathered} [K_A, \Xi] = -\tfrac{i}{R}\Xi\Gamma_A\gamma^5\mathcal{P}_{24}\,, \\ [\nabla_A K_B, \Xi] = -\tfrac{1}{4} R_{AB}{}^{CD}\Xi\Gamma_{CD}\mathcal{P}_{24}\,, \\ \{\Xi, \Xi\} = 2i\,\mathcal{P}_{24}\gamma^5\Gamma^A\gamma^5\mathcal{P}_{24}\,K_A - \tfrac{R}{2}\mathcal{P}_{24}\Gamma^{AB}\gamma^5\mathcal{P}_{24}\,\nabla_A K_B\,. \end{gathered} \tag{2.18}$$

The conserved Noether current associated with the $SO(2,3) \times SU(4)$ invariance of the action (2.1) is

$$\begin{aligned} J^I_B \;=\;& \sqrt{-h}\,h^{IJ}\,e_J{}^A\,K_A + i\Theta(\sqrt{-h}\,h^{IJ} - \varepsilon^{IJ}\Gamma_{11})\left[\Gamma^A\nabla_J\Theta + \frac{2i}{R} e_J{}^B\Gamma^A\mathcal{P}_{24}\gamma^5\Gamma_B\Theta\right] K_A \\ & -\frac{i}{4}\,\Theta(\sqrt{-h}\,h^{IJ} + \varepsilon^{IJ}\Gamma_{11})\,e_J{}^A\,\Gamma_A{}^{BC}\Theta\,\nabla_B\,K_C \end{aligned} \tag{2.19}$$

and the conserved (fermionic) supersymmetry current (up to the leading order in fermions) is

$$
\begin{aligned}
J^I_F &= \frac{i}{2R}\Big(\sqrt{-h}\,h^{IJ}\,e_J{}^A\,\Theta\Gamma_A\Xi + \Theta(\sqrt{-h}\,h^{IJ} + 2\varepsilon^{IJ}\Gamma_{11})\,e_J{}^A\,\Gamma_A\Xi(X)\Big) \\
&= \frac{i}{R}\,\Theta(\sqrt{-h}\,h^{IJ} + \varepsilon^{IJ}\Gamma_{11})\,e_J{}^A\,\Gamma_A\Xi(X)\,, \qquad (2.20)
\end{aligned}
$$

where the factor of 2 in the last term of the first line appears because the action is invariant under supersymmetry only up to a boundary term which must therefore be subtracted from the current to make it conserved. The currents are normalized to be dimensionless (the dimensions of Ξ and K_A are $1/\sqrt{R}$ and $1/R$ respectively). The sum of J_B and J_F is the conserved current taking values in the $OSp(6|4)$ superalgebra

$$
J = J_B + J_F. \qquad (2.21)
$$

Let us now compare this current with the conserved current of the $OSp(6|4)$ supercoset sigma–model which describes the string with $\upsilon = (1-\mathcal{P}_{24})\Theta = 0$. As we have mentioned in the Introduction, the supercoset model conserved current has the following form (in our conventions)

$$
J_{coset}(X,\vartheta) = K(X,\vartheta)\,\Lambda\,K^{-1}(X,\vartheta) \doteq K(X,\vartheta)\left(E^A P_A + \frac{1}{2}Q\Gamma_{11} * E\right)K^{-1}(X,\vartheta)\,, \qquad (2.22)
$$

where $E^A(X,\vartheta)$ and $E^{\underline{\alpha}}(X,\vartheta)$ are components of the $OSp(6|4)$–valued Cartan form

$$
K^{-1}dK(X,\vartheta) = E^A P_A + E^{\underline{\alpha}} Q_{\underline{\alpha}} + \frac{1}{2}\Omega^{AB} M_{AB}, \qquad (2.23)
$$

$\Omega^{AB}(X,\vartheta)$ is the spin connection on $\frac{OSp(6|4)}{SO(1,3)\times U(3)}$ and $K(X,\vartheta)$ is a coset representative. Up to the second order in fermions the supervielbeins and spin connection of the $OSp(6|4)$ supercoset are given by

$$
\begin{aligned}
E^A &= e^A(X) + i\vartheta\Gamma^A\,E\,, \qquad (2.24)\\
E^{\underline{\alpha}} &= \nabla\vartheta^{\underline{\alpha}} + \frac{i}{R}e^B\,(\mathcal{P}_{24}\gamma^5\Gamma_B\,\vartheta)^{\underline{\alpha}}\,,\\
\Omega^{AB} &= \omega^{AB}(X) - \frac{2}{R}\vartheta\Gamma^{[A}\mathcal{P}_{24}\gamma_5\Gamma^{B]}E\,.
\end{aligned}
$$

The current (2.22) is conserved ($d * J_{coset} = 0$) as a consequence of the sigma–model equations of motion

$$
d * \Lambda - [K^{-1}dK, *\Lambda] = 0. \qquad (2.25)
$$

Using a supercoset element of the form $K = k(X)\,e^{\vartheta Q}$, the $OSp(6|4)$ superalgebra (Appendix A.4) and eqs. (2.17) we then have

$$
\begin{aligned}
J_{coset} &= E^A K_A + e^A k\vartheta[Q,P_A]k^{-1} + \frac{1}{2}e^A k[\vartheta Q,[\vartheta Q,P_A]]k^{-1} + \frac{1}{2}kQk^{-1}\Gamma_{11} * E \\
&\quad + \frac{1}{2}k\vartheta\{Q,Q\}\Gamma_{11} * Ek^{-1} \\
&= J|_{\upsilon=0} - \frac{R}{8} * d(\vartheta\Gamma^{AB}\gamma^7\vartheta\,\nabla_A K_B) - \frac{1}{2} * d(\Xi\gamma^7\vartheta)\,, \qquad (2.26)
\end{aligned}
$$

where $J = J_B + J_F$ (2.21) is the Noether current directly derived from the quadratic Green-Schwarz action. The two conserved currents therefore differ only by total derivative terms, as should be the case. A useful relation in checking eq. (2.26) is

$$\mathcal{P}_{24}\Gamma_{[A}\mathcal{P}_{24}\gamma^5\Gamma_{B]}\mathcal{P}_{24} = -\frac{R^2}{8}R_{AB}{}^{CD}\mathcal{P}_{24}\Gamma_{CD}\gamma^5\mathcal{P}_{24}\,. \qquad (2.27)$$

2.3 Lax connections to the second order in fermions

2.3.1 The supercoset sigma–model Lax connection

The Lax connection (1.5) of the $OSp(6|4)$ supercoset sigma–model can be written in the following form in terms of the conserved current (2.22) and components of the Cartan form (2.23)

$$\begin{aligned}\mathcal{L}_{coset} &= K\Big(\alpha_1 E^A P_A + \alpha_2 * E^A P_A + \beta_1 Q\Gamma_{11}E + (1+\beta_2)QE\Big)K^{-1} \\ &= K\Big(\alpha_1\, E^A\, P_A + (1+\beta_2)\, QE + (\beta_1 - \frac{\alpha_2}{2})\, Q\Gamma_{11}E\Big)K^{-1} + \alpha_2\, * J_{coset}\,, \qquad (2.28)\end{aligned}$$

where

$$\begin{aligned}\alpha_1 &= \frac{2z^2}{1-z^2}\,,\\ \alpha_2^2 &= \alpha_1^2 + 2\alpha_1\,,\\ \beta_1 &= \mp\sqrt{\frac{\alpha_1}{2}}\,,\\ \beta_2 &= \pm\frac{\alpha_2}{\sqrt{2\alpha_1}}\,. \qquad (2.29)\end{aligned}$$

The specific dependence of the coefficients on the spectral parameter z ensures the zero curvature of the Lax connection[1] [13, 3, 4]. Note that the Z_4–automorphism splitting of the fermionic $OSp(6|4)$ generators Q and the corresponding fermionic components of the Cartan form is simply made by the $D = 10$ chirality projectors $\frac{1}{2}(1 \mp \Gamma_{11})$ (see Appendix A.4).

2.3.2 Lax connection of the complete $AdS_4 \times CP^3$ superstring

When the extra eight fermionic degrees of freedom υ are switched on, they contribute to the supervielbeins, superconnection, conserved current and equations of motion and, as a consequence, the form of the Lax connection should be modified to account for this. In contrast to the case of the $OSp(6|4)$ supercoset Z_4–grading, it is not obvious which is the group–theoretical structure that would allow one to guess the dependence of $\mathcal{L}$ on υ. So, to find this dependence we shall use a brute–force method, i.e. we will try to build the Lax connection out of components of the conserved currents J_B (2.19) and J_F (2.20), which depend on the extra fermions υ, by introducing them with arbitrary coefficients in the Lax connection.

[1]The numerical coefficients in eq. (2.28) are related to those in eq. (1.3) and those of [3] (eq.(4.1) therein) as follows $\alpha_1 = l_1 - 1$, $\alpha_2 = l_2$, $\beta_1 = \frac{l_3-l_4}{2}$ and $\beta_2 = -\frac{l_3+l_4}{2}$.

The dependence of these coefficients on the spectral parameter is then determined by the zero–curvature condition. This procedure is akin to the construction of Lax connections for two–dimensional supersymmetric non–linear sigma–models considered in [16]. The Lax connection constructed in this way has the following form

$$L = L_B + L_F \tag{2.30}$$

where the bosonic isometry part is

$$L_B = \alpha_1 e^A K_A + \alpha_2 * J_B + \alpha_2^2 J^{AB} \nabla_A K_B + \alpha_1 \alpha_2 * J^{AB} \nabla_A K_B \,, \tag{2.31}$$

and the supersymmetry part is

$$L_F = -\alpha_2 \beta_1 J_F + \alpha_2 \beta_2 * J_F \,. \tag{2.32}$$

J^{AB} stands for the term in the bosonic isometry current (2.19) which is contracted with $\nabla_A K_B$, namely $J_B = J^A K_A + J^{AB} \nabla_A K_B$, and α_1, α_2, β_1 and β_2 are the same as in (2.29).

It is not very difficult to verify that this Lax connection indeed has zero curvature. To check the zero–curvature condition one should use the conservation of the Noether current, the equations of motion as well as the relations

$$\begin{aligned}
\nabla J^{AB} &= -e^{[A}(J^{B]} - e^{B]}) - \frac{1}{2R} e^C e^D \, \Theta \Gamma_C \mathcal{P}_{24} \Gamma^{AB} \gamma^5 \mathcal{P}_{24} \Gamma_D \Theta \\
&\quad + \frac{1}{2R} e^C * e^D \, \Theta \Gamma_C \mathcal{P}_{24} \Gamma^{AB} \gamma^5 \mathcal{P}_{24} \Gamma_D \Gamma_{11} \Theta \,, \\
dJ_F &= \frac{i}{R} d(e^A \, \Theta \Gamma_A \Xi - *e^A \, \Theta \Gamma_A \Gamma_{11} \Xi) \\
&= \frac{2}{R^2} e^A e^B \, \Theta \Gamma_A \mathcal{P}_{24} \gamma^5 \Gamma_B \Xi - \frac{2}{R^2} e^A * e^B \, \Theta \Gamma_A \mathcal{P}_{24} \gamma^5 \Gamma_B \Gamma_{11} \Xi
\end{aligned} \tag{2.33}$$

and the symmetry properties of the Γ–matrices.

Note that the construction of this Lax connection does not make use (at least directly) of the Z_4–grading of the $OSp(6|4)$ superalgebra but only the Z_2–grading of its bosonic subalgebra. Its form is different from the υ–fermion extension of the supercoset Lax connection (2.28) (*e.g.* the former does not have terms linear in $d\Theta$, while such terms are present in the latter). We will now show that the two Lax connections are related by an $OSp(6|4)$ gauge transformation.

2.3.3 Relation to the supercoset Lax connection

When $\upsilon = 0$ the Lax connection (2.30) constructed above should be related to the supercoset Lax connection in eq. (2.28) by a gauge transformation, so that

$$\mathcal{L}_{coset} = g^{-1} L|_{\upsilon=0} \, g + g^{-1} dg \,. \tag{2.34}$$

for some $g \in OSp(6|4)$. It is possible to show that this is indeed the case and with a bit of algebra one finds that the supergroup element

$$\begin{aligned}
g(X, \vartheta; \alpha_2, \beta_1, \beta_2) &= k(X) \, e^{\frac{\alpha_2 R}{16} \vartheta \Gamma^{AB} \gamma^7 \vartheta \, R_{AB}{}^{CD} M_{CD}} \, e^{-\beta_1 \vartheta \Gamma_{11} Q} \, e^{-(1+\beta_2) \vartheta Q} \, k^{-1}(X) \\
&= e^{-\frac{\alpha_2 R}{8} \vartheta \Gamma^{AB} \gamma^7 \vartheta \, \nabla_A K_B} \, e^{\beta_1 \vartheta \gamma^7 \Xi} \, e^{-(1+\beta_2) \vartheta \gamma^5 \Xi}
\end{aligned} \tag{2.35}$$

does the job. If we apply this gauge transformation to the Lax connection L (2.30) without setting υ to zero we obtain the supercoset Lax connection extended with the terms up to quadratic order in υ

$$\mathcal{L} = g^{-1}L\,g + g^{-1}dg\,. \tag{2.36}$$

The Lax connections constructed above have zero curvatures only up to the quadratic order in fermions. To get zero curvature also at quartic and higher orders in fermions one should add to the Lax connection (2.30) or (2.36) corresponding higher–order fermionic υ–terms with appropriate coefficients at each order. We have not been able to find a generic prescription for the construction of such terms from the components of the conserved currents of the complete $AdS_4 \times CP^3$ superstring, and the brute force computation becomes technically more and more involved with each new order in fermions. So to simplify the analysis we shall turn to the consideration of a simpler AdS_4 sub–sector of the theory in which the problem of the construction of the Lax connection can be completely solved at least in a particular kappa–symmetry gauge.

3 String in $\mathcal{N}=2$ AdS_4 superspace

As has been shown in [1] the structure of the $AdS_4 \times CP^3$ superstring action and equations of motion allows one to consistently truncate this theory to a model describing a string propagating in a four–dimensional superbackground with eight fermionic directions parameterized by $\upsilon = (1-\mathcal{P}_{24})\Theta$. The bosonic subspace of this superbackground is AdS_4 but it does not preserve any supersymmetry[2]. This model is obtained by putting to zero the 24 supersymmetric fermionic fields $\vartheta = \mathcal{P}_{24}\,\Theta = 0$ and restricting the string to move entirely in AdS_4 (*i.e.* the CP^3 embedding coordinates are worldsheet constants). It is, therefore, not described by the supercoset sigma–model of [3, 4, 5, 6, 7]. Lacking supersymmetry this model is also *not* the $\frac{OSp(2|4)}{SO(1,3)\times SO(2)}$ supercoset sigma–model [1]. Nevertheless, it possesses the four–parameter kappa–symmetry in addition to the purely bosonic isometry $SO(2,3)$ of AdS_4 and the $SO(2)$ symmetry rotating the two $D=4$ Majorana fermions. So it is somewhat surprising that this model turns out to be integrable, and the integrability of its fermionic sector is not at all related to target space supersymmetry which is lacking.

Let us consider this model in more detail. It is convenient to represent the eight–component spinors $\upsilon = (1-\mathcal{P}_{24})\Theta$ as four–component Majorana spinors in AdS_4, $\upsilon^{\alpha i}$ ($\alpha = 1,2,3,4$), carrying the internal $SO(2)$ index $i = 1,2$. This $SO(2)$ is a relic of the $U(1)$ gauge symmetry associated with the RR one–form field of $D=10$ type IIA supergravity. The Green–Schwarz action for the superstring moving in this AdS_4 superspace has the following form [1]

$$S = -\frac{1}{4\pi\alpha'}\int d^2\xi\,\sqrt{-h}\,h^{IJ}\,\mathcal{E}_I{}^{\hat a}\mathcal{E}_J{}^{\hat b}\eta_{\hat a\hat b} - \frac{1}{2\pi\alpha'}\int B_2\,, \tag{3.1}$$

where the vector supervielbeins $\mathcal{E}^{\hat a} = dx^{\hat m}\,\mathcal{E}_{\hat m}{}^{\hat a} + d\upsilon^{\alpha i}\mathcal{E}_{\alpha i}{}^{\hat a}$ along the AdS_4 directions of the

[2] A somewhat analogous non–supersymmetric AdS_4 vacuum was found in a matter–coupled $\mathcal{N}=2$, $D=4$ supergravity in [17].

target superspace are

$$\mathcal{E}^{\hat{a}}(x,\upsilon) = e^{\frac{1}{3}\phi(\upsilon)}\left(e^{\hat{b}}(x) + 4i\upsilon\gamma^{\hat{b}}\frac{\sinh^2\mathcal{M}/2}{\mathcal{M}^2}D\upsilon\right)\Lambda_{\hat{b}}{}^{\hat{a}}(\upsilon) + e^{-\frac{1}{3}\phi(\upsilon)}\frac{4R}{kl_p}\upsilon\,\varepsilon\gamma^5\frac{\sinh^2\mathcal{M}/2}{\mathcal{M}^2}D\upsilon\,V^{\hat{a}}(\upsilon)\,, \tag{3.2}$$

and the NS–NS superform B_2 is expressed through components of its field strength $H_3 = dB_2$ as follows

$$B_2 = \int_0^1 dt\, i_\upsilon H_3(x,t\upsilon)\ . \tag{3.3}$$

$$H_3 = dB_2 = -\frac{1}{3!}\mathcal{E}^{\hat{c}}\mathcal{E}^{\hat{b}}\mathcal{E}^{\hat{a}}\left(\frac{6}{kl_p}e^{-\phi}\varepsilon_{\hat{a}\hat{b}\hat{c}\hat{d}}V^{\hat{d}}\right) + \mathcal{E}^{\hat{a}}\mathcal{E}^{\beta j}\mathcal{E}^{\alpha i}(\gamma_{\hat{a}}\gamma^5)_{\alpha\beta}\,\varepsilon_{ij} - \mathcal{E}^{\hat{b}}\mathcal{E}^{\hat{a}}\mathcal{E}^{\alpha i}(\gamma_{\hat{a}\hat{b}}\gamma^5\,\varepsilon\lambda)_{\alpha i}\,, \tag{3.4}$$

where $\mathcal{E}^{\alpha i}(x,\upsilon)$ are the fermionic supervielbeins

$$\mathcal{E}^{\alpha i}(x,\upsilon) = e^{\frac{1}{6}\phi(\upsilon)}\left(\frac{\sinh\mathcal{M}}{\mathcal{M}}D\upsilon\right)^{\beta j}S_{\beta j}{}^{\alpha i}(\upsilon) - ie^{\phi(\upsilon)}\mathcal{A}_1(x,\upsilon)\,(\gamma^5\varepsilon\lambda(\upsilon))^{\alpha i} \tag{3.5}$$

and $\mathcal{A}_1(x,\upsilon)$ is a relic of the type IIA RR one–form

$$\mathcal{A}_1(x,\upsilon) = \frac{R}{kl_p}e^{-\frac{4}{3}\phi(\upsilon)}\left[\left(e^{\hat{a}}(x) + 4i\upsilon\gamma^{\hat{a}}\frac{\sinh^2\mathcal{M}/2}{\mathcal{M}^2}D\upsilon\right)V_{\hat{a}}(\upsilon) - 4\upsilon\,\varepsilon\gamma^5\frac{\sinh^2\mathcal{M}/2}{\mathcal{M}^2}D\upsilon\,\Phi(\upsilon)\right]. \tag{3.6}$$

Note that $\mathcal{A}_1$ is zero when $\upsilon = 0$.

The AdS_4 covariant derivative D is defined as

$$D\upsilon = \left(\nabla + \frac{i}{R}e^{\hat{a}}(x)\,\gamma^5\gamma_{\hat{a}}\right)\upsilon = \left(d - \frac{1}{4}\omega^{\hat{a}\hat{b}}(x)\,\gamma_{\hat{a}\hat{b}} + \frac{i}{R}e^{\hat{a}}(x)\,\gamma^5\gamma_{\hat{a}}\right)\upsilon \tag{3.7}$$

and $\gamma^{\hat{a}}$, γ^5 are the four–dimensional gamma–matrices in the Majorana representation.

The dilaton superfield $\phi(\upsilon)$, which depends only on the eight fermionic coordinates, has the following form in terms of the quantities $V^{\hat{a}}(\upsilon)$ and $\Phi(\upsilon)$

$$e^{\frac{2}{3}\phi(\upsilon)} = \frac{R}{kl_p}\sqrt{\Phi^2 + V^{\hat{a}}\,V^{\hat{b}}\,\eta_{\hat{a}\hat{b}}}\,. \tag{3.8}$$

The value of the dilaton at $\upsilon = 0$ is

$$e^{\frac{2}{3}\phi(\upsilon)}|_{\upsilon=0} = e^{\frac{2}{3}\phi_0} = \frac{R}{kl_p} \tag{3.9}$$

(l_p is the Plank's length and k corresponds to the Chern–Simons level in the ABJM model). The fermionic field $\lambda^{\alpha i}(\upsilon)$ describes the non–zero components of the dilatino superfield which is related to the dilaton superfield by the equation [18]

$$\lambda_{\alpha i} = -\frac{i}{3}D_{\alpha i}\,\phi(\upsilon)\,. \tag{3.10}$$

The new objects appearing in these expressions, $\mathcal{M}$, $\Lambda_{\hat{a}}{}^{\hat{b}}$, Φ, $V^{\hat{a}}$ and $S_{\alpha i}{}^{\beta j}$, are functions of υ and their explicit forms are given in Appendix B. Contracted spinor indices have been suppressed, *e.g.* $(\upsilon\varepsilon\gamma^5)_{\alpha i} = \upsilon^{\beta j}\varepsilon_{ji}\gamma^5_{\beta\alpha}$, where $\varepsilon_{ij} = -\varepsilon_{ji}$, $\varepsilon_{12} = 1$ is the $SO(2)$ invariant tensor.

As we have already noted, in the AdS_4 superspace under consideration all supersymmetries are broken and it only has the bosonic AdS_4 isometry $SO(2,3)$. The superstring action (3.1) is thus invariant under the $SO(2,3)$ variations of the coordinates

$$\delta x^{\hat{m}}\, e_{\hat{m}}{}^{\hat{a}}(x) = K^{\hat{a}}(x) = K^{\hat{m}}(x)\, e_{\hat{m}}{}^{\hat{a}}\,, \qquad \delta\upsilon = \frac{1}{4}(K^{\hat{m}}\,\omega_{\hat{m}}{}^{\hat{a}\hat{b}}(x) - \nabla^{\hat{a}}\,K^{\hat{b}})\,\gamma_{\hat{a}\hat{b}}\,\upsilon. \tag{3.11}$$

The associated conserved $SO(2,3)$ current has the following form

$$J^I = \sqrt{-h}\,h^{IJ}\,\mathcal{E}_J{}^{\hat{a}}\,(i_{\delta x}\,\mathcal{E}^{\hat{b}} + i_{\delta\upsilon}\,\mathcal{E}^{\hat{b}})\,\eta_{\hat{a}\hat{b}} - \varepsilon^{IJ}\,(i_{\delta x}\,B_2 + i_{\delta\upsilon}\,B_2)_J\,. \tag{3.12}$$

Due to the complicated form of the supervielbein and B_2, the explicit dependence of this current on υ is still a bit too involved to try to construct a Lax connection. So we shall further simplify things by gauge fixing kappa–symmetry in a way considered in [9].

4 Gauge fixed superstring action in AdS_4 superspace

Let us choose the AdS_4 metric in the conformally flat form

$$ds^2_{{}_{AdS_4}} = \frac{1}{u^2}(dx^a\eta_{ab}dx^b + \frac{R^2_{CP^3}}{4}\,du^2)\,, \qquad u = \left(\frac{R_{CP^3}}{r}\right)^2\,, \tag{4.1}$$

where x^a $(a = 0,1,2)$ are the coordinates of the $D = 3$ Minkowski boundary and u (or r) is the AdS_4 radial coordinate. If the components of the AdS_4 vielbein associated with the metric (4.1) are chosen to be[3]

$$e^{\frac{\phi_0}{3}}\,e^a = \frac{r^2}{R^2_{CP^3}}\,dx^a = u^{-1}\,dx^a\,, \qquad e^{\frac{\phi_0}{3}}\,e^3 = \frac{R_{CP^3}}{r}\,dr = -\frac{R_{CP^3}}{2u}\,du, \tag{4.2}$$

the components of the $SO(1,3)$ spin connection are

$$\omega^{a3} = -\frac{2}{R}\,e^a\,, \tag{4.3}$$

and

$$\omega^{ab} = 0\,, \tag{4.4}$$

where the index 3 stands for the 3rd (radial) direction in AdS_4.

The following kappa–symmetry gauge fixing condition on υ drastically simplifies the form of the superstring action

$$\upsilon = \frac{1}{2}(1 + \gamma^{012})\,\upsilon\,, \qquad \gamma^{012} \equiv \gamma\,, \tag{4.5}$$

[3]Note that the vielbeins e^a and e^3 appearing in eq. (4.2) correspond to the AdS_4 metric of the $D = 11$ $AdS_4 \times S^7$ solution characterized by the radius R which is related to the CP^3 radius in the string frame as follows $R_{CP^3} = e^{\frac{1}{3}\phi_0}R = \left(\frac{R^3}{kl_p}\right)^{1/2}$. These bosonic vielbeins appear in our explicit expressions for the AdS_4 supergeometry.

where γ^{012} is the product of the gamma matrices along the 3d Minkowski boundary slice of AdS_4.

In this gauge the supervielbeins take the following simple form [9]

$$\mathcal{E}^a(x,\upsilon) = (\frac{R}{kl_p})^{1/2}(e^a(x) + i\upsilon\gamma^a D\,\upsilon)\,(1-\frac{1}{R^2}(\upsilon\upsilon)^2)\,,$$
$$\mathcal{E}^3(x,\upsilon) = (\frac{R}{kl_p})^{1/2}e^3(x)\,(1-\frac{3}{R^2}(\upsilon\upsilon)^2)\,, \tag{4.6}$$

and the covariant derivative becomes

$$D\upsilon = \left(d - \frac{1}{R}e^3(x) - \frac{1}{4}\omega^{ab}(x)\,\gamma_{ab}\right)\upsilon\,. \tag{4.7}$$

Actually, the $SO(1,2)$ Lorentz connection ω^{ab} is zero when the AdS_4 supervielbeins are taken in the form (4.2).

The NS-NS two form becomes

$$B_2 \;=\; -\frac{i}{kl_p}\Big[(e^b + i\upsilon\gamma^b D\upsilon)\,(e^a + i\upsilon\gamma^a D\upsilon)\,\upsilon\gamma^c\varepsilon\upsilon\,\varepsilon_{abc} - R\,e^3\,\upsilon\varepsilon D\upsilon\Big]\,. \tag{4.8}$$

The kappa–symmetry gauge–fixed superstring action reduces to

$$\begin{aligned} S \;=\; & -\frac{1}{4\pi\alpha'}\frac{R}{kl_p}\int d^2\xi\,\sqrt{-h}\,h^{IJ}\Big[e_I{}^3 e_J{}^3\,(1-\frac{6}{R^2}(\upsilon\upsilon)^2) \\ & \qquad + (e_I{}^a + i\upsilon\gamma^a D_I\upsilon)\,(e_J{}^b + i\upsilon\gamma^b D_J\upsilon)\,\eta_{ab}\,(1-\frac{2}{R^2}(\upsilon\upsilon)^2)\Big] \\ & + \frac{1}{2\pi\alpha'}\frac{i}{kl_p}\int\Big[(e^b + i\upsilon\gamma^b D\upsilon)\,(e^a + i\upsilon\gamma^a D\upsilon)\,\upsilon\gamma_{ab}\varepsilon\upsilon - R\,e^3\,\upsilon\varepsilon D\upsilon\Big]. \end{aligned} \tag{4.9}$$

This action is slightly more complicated than the action for the $AdS_5\times S^5$ superstring in the analogous kappa–symmetry gauge [19]. The latter contains fermions only up to the fourth order.

In this kappa–symmetry gauge, the conserved $SO(2,3)$ current (3.12) has the following explicit form

$$\begin{aligned} J \;=\; & \sqrt{-h}\,h^{IJ}\,e_J{}^{\hat a}\,K_{\hat a} + i\upsilon(\sqrt{-h}\,h^{IJ} - i\varepsilon^{IJ}\,\gamma^5\varepsilon)\gamma^{\hat a}\nabla_J\upsilon\,K_{\hat a} \\ & - \frac{i}{4}\upsilon(\sqrt{-h}\,h^{IJ} + i\varepsilon^{IJ}\,\gamma^5\varepsilon)\gamma_{\hat a}{}^{\hat b\hat c}\,\upsilon\,e_J{}^{\hat a}\nabla_{\hat b}\,K_{\hat c} \\ & -\sqrt{-h}\,h^{IJ}\left[\frac{(\upsilon\upsilon)^2}{2R^2}(e_J{}^a\,K_a + 12\,e_J{}^3\,K_3) + \frac{3\,(\upsilon\upsilon)^2}{8R}e_J{}^a\,(\nabla_3\,K_a - \nabla_a\,K_3) - \frac{\upsilon\upsilon}{4}\varepsilon^{abc}\,\upsilon\gamma_a\nabla_J\upsilon\,\nabla_b K_c\right] \\ & - \frac{3}{2R}\,\varepsilon^{IJ}\,\upsilon\gamma_a\nabla_J\upsilon\,\upsilon\gamma^{ab}\varepsilon\upsilon\,K_b - \frac{1}{8}\varepsilon^{IJ}\,\upsilon\gamma_a\nabla_J\upsilon\,\upsilon\gamma^{ab}\varepsilon\upsilon(\nabla_3\,K_b - \nabla_b\,K_3)\,, \end{aligned} \tag{4.10}$$

where remember that ε without indices implies $\varepsilon^{ij} = -\varepsilon^{ji}$ with $i,j = 1,2$ labeling the two $D=4$ Majorana fermions $\upsilon^{\alpha i}$. The first two lines in (4.10) are the same as in the quadratic current (2.19) reduced to AdS_4 and with $\vartheta = 0$. The third and the fourth line are quartic in υ and its derivative.

The problem of the construction of the Lax connection thus becomes more treatable, but we would like to simplify things even further.

4.1 Worldsheet T–dual action for the AdS_4 superstring

Upon a T–duality transformation on the worldsheet [9], similar to that described in [19] [4], the action (4.9) takes an even simpler form

$$\begin{aligned} S &= -\frac{1}{4\pi\alpha'}\frac{R}{kl_p}\int d^2\xi\,\sqrt{-h}\,h^{IJ}\left(\tilde{e}_I{}^a\,\tilde{e}_J{}^b\,\eta_{ab}+e_I{}^3 e_J{}^3\right)(1-\frac{6}{R^2}(\upsilon\upsilon)^2) \\ &- \frac{1}{2\pi\alpha'}\frac{iR}{kl_p}\int\Big(e^3\,\upsilon\varepsilon D\upsilon+\tilde{e}^a\,\upsilon\gamma_a D\upsilon-\frac{1}{R}\,\tilde{e}^a\,\tilde{e}^b\,\upsilon\gamma_{ab}\varepsilon\upsilon\Big), \end{aligned} \tag{4.11}$$

where $e^3(r)$ and $\tilde{e}^a(\tilde{x},r)$ are the vielbeins of the *dual* AdS_4 space. The dual vielbeins $\tilde{e}^a(\tilde{x},r)$ along the Minkowski directions are related to the initial quantities as follows (see [9] for more details)

$$\partial_I\,(\frac{r^2}{R^2}\,P^I_a)=0 \qquad \Rightarrow \qquad P^I_a=\frac{R^2}{r^2}\,\varepsilon^{IJ}\partial_J\,\tilde{x}_a\equiv\varepsilon^{IJ}\,\tilde{e}_{Ja} \tag{4.12}$$

where

$$P^I_a=-\sqrt{-h}\,(1-\frac{2}{R^2}(\upsilon\upsilon)^2)\Big(h^{IJ}\eta_{ab}+\frac{2i}{R\sqrt{-h}}\,\varepsilon^{IJ}\,\upsilon\gamma_{ab}\varepsilon\upsilon\Big)\,(e_J{}^b+i\upsilon\gamma^b D_J\upsilon)\,. \tag{4.13}$$

The quantities (4.13) (up to a rescaling) are the conserved currents of the $d=3$ translation part of the $SO(2,3)$ isometries

$$\delta\,\tilde{x}^a=c^a, \qquad \delta\,r=\delta\,\upsilon=0\,. \tag{4.14}$$

The action (4.11) can be cast in the manifestly $SO(1,3)$ covariant form

$$S=-\frac{1}{2\pi\alpha'}\frac{R}{kl_p}\int d^2\xi\,\left(\frac{1}{2}\sqrt{-h}\,h^{IJ}\tilde{e}_I{}^{\hat{a}}\,\tilde{e}_J{}^{\hat{b}}\,\eta_{\hat{a}\hat{b}}\,(1-\frac{6}{R^2}(\upsilon\upsilon)^2)+i\varepsilon^{IJ}\,\tilde{e}_I{}^{\hat{a}}\,\upsilon(1-\Gamma_{11})\,\gamma_{\hat{a}}\nabla_J\upsilon\right), \tag{4.15}$$

where Γ_{11} stands for $(1-\mathcal{P}_{24})\gamma^5\,\gamma^7(1-\mathcal{P}_{24})\equiv i\gamma^5\,\varepsilon$ which indicates its origin from $D=10$ and $\nabla=d-\frac{1}{4}\,\tilde{\omega}^{\hat{a}\hat{b}}\,\gamma_{\hat{a}\hat{b}}$.

[4] Note that in contrast to the $AdS_5\times S^5$ superstring where this bosonic T–duality can be accompanied by a fermionic one [20, 21, 22] which brings the superstring action to itself but in a different kappa–symmetry gauge, in the $AdS_4\times CP^3$ case the fermionic T–duality is not possible [23, 9], at least in the same fashion and in application to the broken supersymmetry fermions υ. For an alternative suggestion to perform bosonic and fermionic worldsheet T–duality of the $AdS_4\times CP^3$ supercoset model see [24, 25, 26] and for problems with its realization see [27].

The bosonic and fermionic equations of motion which follow from (4.15) are, respectively,

$$\nabla_I\left(\sqrt{-h}\,h^{IJ}\,\tilde{e}_J{}^{\hat{a}}\,(1-\frac{6}{R^2}(\upsilon\upsilon)^2)+i\varepsilon^{IJ}\,\upsilon(1-\Gamma_{11})\,\gamma^{\hat{a}}\nabla_J\upsilon\right)-\frac{2i}{R^2}\,\varepsilon^{IJ}\,\tilde{e}_I{}^{\hat{b}}\,\tilde{e}_J{}^{\hat{c}}\,\upsilon(1-\Gamma_{11})\,\gamma^{\hat{a}}{}_{\hat{b}\hat{c}}\upsilon=0 \tag{4.16}$$

and

$$\frac{i}{2}\,(1+\gamma)(1-\Gamma^{11})\,\varepsilon^{IJ}\,\tilde{e}_I{}^{\hat{a}}\,\gamma_{\hat{a}}\,\nabla_J\,\upsilon-\frac{6}{R^2}\,\upsilon\,(\upsilon\upsilon)\,\sqrt{-h}\,h^{IJ}\,\tilde{e}_I{}^{\hat{a}}\,\tilde{e}_J{}^{\hat{b}}\,\eta_{\hat{a}\hat{b}}=0\,, \tag{4.17}$$

The fermionic equation can also be rewritten in the following form

$$i\sqrt{-h}h^{IJ}\tilde{e}_J{}^{\hat{a}}\,(1-\Gamma_{11})\,\gamma_{\hat{a}}\,\nabla_I\upsilon-\frac{6}{R^2}\,\varepsilon^{IJ}\tilde{e}_I{}^{\hat{a}}\,\tilde{e}_J{}^{\hat{b}}\,(1-\Gamma_{11})\gamma_{\hat{a}\hat{b}}\upsilon\,(\upsilon\upsilon)=0\,, \tag{4.18}$$

The conserved current of the $SO(2,3)$ isometry is

$$J^I=\left(\sqrt{-h}\,h^{IJ}\,\tilde{e}_J{}^{\hat{a}}\,(1-\frac{6}{R^2}(\upsilon\upsilon)^2)+i\varepsilon^{IJ}\,\upsilon(1-\Gamma_{11})\,\gamma^{\hat{a}}\nabla_J\upsilon\right)\,K_{\hat{a}}+\frac{i}{4}\,\varepsilon^{IJ}\,\tilde{e}_J{}^{\hat{a}}\,\upsilon\,(1-\Gamma_{11})\,\gamma_{\hat{a}}{}^{\hat{b}\hat{c}}\,\upsilon\,\nabla_{\hat{b}}\,K_{\hat{c}}\,. \tag{4.19}$$

4.2 The Lax connection

As in the quadratic approximation of Section 2.3, we construct an $SO(2,3)$–valued zero–curvature Lax connection $\mathcal{L}$

$$\mathcal{R}=d\mathcal{L}-\mathcal{L}\mathcal{L}=0\qquad\Longrightarrow\qquad\varepsilon^{IJ}(\partial_I\,\mathcal{L}_J+\mathcal{L}_I\,\mathcal{L}_J)=0 \tag{4.20}$$

using the pieces of the conserved current (4.19) which enter the Lax connection with arbitrary coefficients. The problem has a non–trivial solution if the zero–curvature condition allows for expressing the coefficients in terms of a single spectral parameter. In the case under consideration the zero–curvature Lax connection has the following form

$$\begin{aligned}\mathcal{L}_I\;&=\;\alpha_1\,\tilde{e}_I{}^{\hat{a}}\,K_{\hat{a}}+\alpha_2\,\frac{\varepsilon_{IJ}}{-h}\,J^J+\frac{\alpha_2^2}{\sqrt{-h}}\,F_I+\alpha_1\alpha_2\,\frac{\varepsilon_{IJ}}{-h}\,F^J\\&\quad-\frac{\alpha_2^2}{4R^2}\,\upsilon\,(1-\Gamma_{11})\,\gamma_{\hat{a}}\,\nabla_I\,\upsilon\,\upsilon\,(1-\Gamma_{11})\,\gamma^{\hat{a}\hat{b}\hat{c}}\,\upsilon\,K_{\hat{b}}\,K_{\hat{c}}\\&\quad+\frac{3\alpha_2^2}{2R^2}(\upsilon\upsilon)^2\,\tilde{e}_I{}^{\hat{a}}\,K_{\hat{a}}+\frac{3\alpha_2(\alpha_1+2)}{8}\,\partial_I\left(\frac{(\upsilon\upsilon)^2}{\sqrt{-G}}\,\varepsilon^{JK}\,\tilde{e}_J{}^{\hat{a}}\,\tilde{e}_K{}^{\hat{b}}\,K_{\hat{a}}K_{\hat{b}}\right)\,,\end{aligned} \tag{4.21}$$

where $G=\det(\tilde{e}_I{}^{\hat{a}}\,\tilde{e}_J{}^{\hat{b}}\,\eta_{\hat{a}\hat{b}})$,

$$F^I=\frac{i}{4}\,\varepsilon^{IJ}\,\tilde{e}_J{}^{\hat{a}}\,\upsilon\,(1-\Gamma_{11})\,\gamma_{\hat{a}}{}^{\hat{b}\hat{c}}\,\upsilon\,\nabla_{\hat{b}}\,K_{\hat{c}}\;=\frac{i}{2}\,\varepsilon^{IJ}\,\tilde{e}_J{}^{\hat{a}}\,\upsilon\,(1-\Gamma_{11})\,\gamma_{\hat{a}}{}^{\hat{b}\hat{c}}\,\upsilon\,K_{\hat{b}}\,K_{\hat{c}} \tag{4.22}$$

and (as in Section 2.3)

$$\alpha_2^2=\alpha_1^2+2\alpha_1. \tag{4.23}$$

So the Lax connection contains one independent (spectral) parameter $\alpha_1=\frac{2z^2}{1-z^2}$.

To check that (4.21) has zero curvature one should use the string equations of motion (4.16)– (4.18), the Killing vector relations (Appendix D) and the Fierz identities (Appendix C).

Note that in the kappa–symmetry gauge under consideration the Lax connection is of the fourth order in fermions (as is the action (4.15) and the conserved current (4.19)).

Applying the inverse duality transformation (4.12) to (4.21) one gets the Lax connection for the original model (4.9) which is non–local in the coordinates x^a of the Minkowski boundary of the AdS_4 space (4.1), the non–local quantities being the Killing vectors $K_A(\tilde{x}(x), r)$ and their derivatives $\nabla_A K_B(\tilde{x}(x), r) = [K_A, K_B]$ expressed in terms of the original AdS_4 coordinates. With some more technical effort, it should be possible to construct an alternative local Lax connection of the model (4.9) directly from the conserved current (4.10). We leave this exercise for future consideration.

5 Conclusion and Discussion

In this paper we have constructed the full Lax connection for the AdS_4 sub–sector of the $AdS_4 \times CP^3$ superstring with eight 'broken supersymmetry' fermionic modes which is not described by the supercoset sigma–model. Because of the technical complexity of the problem, the construction has been carried out for the kappa–symmetry gauge fixed and worldsheet T–dualized action of the theory. For a generic (semi)classical configuration of the $AdS_4 \times CP^3$ superstring with 32 fermionic fields (which are not subject to a kappa–symmetry gauge fixing) we have constructed the Lax connections up to the second order in the fermionic fields. These results provide a direct evidence for the classical integrability of the complete $AdS_4 \times CP^3$ superstring theory.

It would be useful, though, to find a procedure for the construction of a Lax connection of the complete theory to all orders in the thirty two fermions. A hint at a possible method to achive this goal may come from the construction of Lax connections in two–dimensional supersymmetric $O(N)$ and CP^N sigma–models. When these sigma–models are formulated in components of corresponding $d = 2$ supermultiplets, a prescription for constructing the Lax connection was proposed in [16] which, as we have already mentioned, has prompted the techniques used in this paper. A more systematic way of constructing the Lax connections for these supersymmetric sigma–models is in the framework of their worldsheet superfield description which allows one to operate with a corresponding Cartan superform or a conserved super–current in the worldsheet superspace rather than with their components [28, 29, 30].

One can try to develop similar methods for studying the classical integrability of Green–Schwarz superstrings in the framework of the superembedding approach (see [31, 32, 33] for review and references). The superembedding description of superparticles, superstrings and superbranes is based on the fact that the worldsheet kappa–symmetry is a somewhat weird realization of the conventional extended worldsheet supersymmetry [34]. The dynamics of p–branes is described by an embedding of a worldsheet supersurface into a target superspace subject to a certain superembedding condition. The embedding super–coordinates X^M and $\Theta^{\underline{\alpha}}$ of a superstring in this formulation are therefore worldsheet superfields, as in the case of two–dimensional supersymmetric sigma–models and the Ramond–Neveu–Schwarz strings. A difference is that in the latter the component (bosonic and fermionic) worldsheet fields are in the same supermultiplet, while in the superembedding approach X and Θ are (a priori) in different supermultiplets and corresponding superfields. However, these superfields are related to each other by the superembedding constraint which (at least in some cases) can be solved in terms of a single 'prepotential'. The superembedding formulation is intrinsically related to super–twistors [34, 35, 36] and pure–spinors [37, 38]. It has proved to be extremely useful *e.g.* for the derivation of the M5–brane equations of motion [39, 40] and for making progress in the covariant description of multiple coincident branes [41, 42, 43, 44, 45].

To construct a Lax connection for a superstring in the superembedding approach one should first derive a conserved worldsheet supercurrent associated with the superisometry of the supergravity background under consideration and then try to use it in combination with a spectral parameter for building the worldsheet superfield Lax connection. The expansion of this Lax connection in worldsheet superfield components should then reproduce the form of Lax connections considered in this paper to all orders in fermions. We hope to address this problem in the near future.

Other possible applications and development of the results of this paper can be the generalization to the complete $AdS_4 \times CP^3$ superstring of the algebraic curve constructed in [46] and the study of the integrability of type IIB superstrings compactified on $AdS_3 \times S^3 \times S^3 \times S^1$ and on $AdS_3 \times S^3 \times T^4$ (with 16 preserved supersymmetries) in those sectors which are not described by corresponding supercoset sigma–models (see [47, 14] and references therein). An even more interesting case is type II superstrings in an $AdS_2 \times S^2 \times T^6$ superbackground which preserves only eight supersymmetries and is related to the near horizon geometry of $D = 4$ black holes [48]. In this case 16 independent kappa–symmetries are not enough to eliminate 24 'broken supersymmetry' fermions and hence the $\frac{PSU(1,1|2)}{S0(1,1)\times U(1)}$ supercoset sigma–model [49] cannot be regarded as a kappa–gauge fixed description of this theory.

One may also look for other examples of integrable superstrings in superbackgrounds with less or no supersymmetry, whose purely bosonic sub–sector is integrable. As we have seen in Section 3, the superstring in the $\mathcal{N} = 2$ AdS_4 superspace is integrable in spite of the fact that all the eight supersymmetries are broken. If we did not know that this non–supersymmetric model is a truncation of the $AdS_4 \times CP^3$ superstring, we would wonder what might be the reason for its integrability. An obvious further example to check for integrability is the superstring in the $AdS_4 \times CP^3$ background with all supersymmetries broken. This superbackground is obtained from the 24–supersymmetric solution by changing the sign of the F_2 flux [50]. We have not been able to construct a zero–curvature Lax connection for this case using the technique developed in this paper. So it still remains to be understood what is the deep reason for the integrability of the $AdS_4 \times CP^3$ superstring in the fermionic sub–sector corresponding to the broken supersymmetries. Does this indicate that the superstring in $AdS_4 \times CP^3$ remembers that it is obtained by the dimensional reduction of the maximally supersymmetric $AdS_4 \times S^7$ superbackground of $D = 11$ supergravity [50, 51, 52]?

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Appendix A. Main notation and conventions

The convention for the ten–dimensional metric is the ‘almost plus’ signature $(-,+,\cdots,+)$. Generically, the tangent space vector indices are labeled by letters from the beginning of the Latin alphabet, while letters from the middle of the Latin alphabet stand for curved (world) indices. The spinor indices are labeled by Greek letters.

A.1 AdS_4 space

AdS_4 is parametrized by the coordinates $x^{\hat m}$ and its vielbeins are $e^{\hat a} = dx^{\hat m}\, e_{\hat m}{}^{\hat a}(x)$, $\hat m = 0,1,2,3$; $\hat a = 0,1,2,3$. The $D=4$ gamma–matrices satisfy:

$$\{\gamma^{\hat a},\gamma^{\hat b}\} = 2\,\eta^{\hat a\hat b}\,, \qquad \eta^{\hat a\hat b} = \mathrm{diag}\,(-,+,+,+)\,, \tag{A.1}$$

$$\gamma^5 = i\gamma^0\,\gamma^1\,\gamma^2\,\gamma^3, \qquad \gamma^5\,\gamma^5 = 1\,. \tag{A.2}$$

The charge conjugation matrix C is antisymmetric, the matrices $(\gamma^{\hat a})_{\alpha\beta} \equiv (C\,\gamma^{\hat a})_{\alpha\beta}$ and $(\gamma^{\hat a\hat b})_{\alpha\beta} \equiv (C\,\gamma^{\hat a\hat b})_{\alpha\beta}$ are symmetric and $\gamma^5_{\alpha\beta} \equiv (C\gamma^5)_{\alpha\beta}$ is antisymmetric, with $\alpha,\beta = 1,2,3,4$ being the indices of a 4–dimensional spinor representation of $SO(1,3)$ or $SO(2,3)$.

The AdS_4 curvature is

$$R_{\hat a\hat b\hat c}{}^{\hat d} = \frac{8}{R^2}\,\eta_{\hat c[\hat a}\,\delta^{\hat d}_{\hat b]}\,, \qquad R^{\hat a\hat b} = -\frac{4}{R^2}\,e^{\hat a}\,e^{\hat b}\,, \tag{A.3}$$

where $\frac{R}{2}$ is the AdS_4 radius.

A.2 CP^3 space

CP^3 is parametrized by the coordinates $y^{m'}$ and its vielbeins are $e^{a'} = dy^{m'} e_{m'}{}^{a'}(y)$, $m' = 1,\cdots,6$; $a' = 1,\cdots,6$. The $D=6$ gamma–matrices satisfy:

$$\{\gamma^{a'},\gamma^{b'}\} = 2\,\delta^{a'b'}\,, \qquad \delta^{a'b'} = \mathrm{diag}\,(+,+,+,+,+,+)\,, \tag{A.4}$$

$$\gamma^7 = \frac{i}{6!}\,\varepsilon_{\,a'_1a'_2a'_3a'_4a'_5a'_6}\,\gamma^{a'_1}\cdots\gamma^{a'_6} \qquad \gamma^7\,\gamma^7 = 1\,. \tag{A.5}$$

The charge conjugation matrix C' is symmetric and the matrices $(\gamma^{a'})_{\alpha'\beta'} \equiv (C\,\gamma^{a'})_{\alpha'\beta'}$ and $(\gamma^{a'b'})_{\alpha'\beta'} \equiv (C'\,\gamma^{a'b'})_{\alpha'\beta'}$ are antisymmetric, with $\alpha',\beta' = 1,\cdots,8$ being the indices of an 8–dimensional spinor representation of $SO(6)$.

The CP^3 curvature is

$$R_{a'b'c'}{}^{d'} = -\frac{2}{R^2}\left(\delta_{c'[a'}\,\delta^{d'}_{b']} - J_{c'[a'}\,J_{b']}{}^{d'} + J_{a'b'}\,J_{c'}{}^{d'}\right). \tag{A.6}$$

A.3 The $D = 10$ gamma–matrices Γ^A

$$\{\Gamma^A,\, \Gamma^B\} = 2\eta^{AB}, \qquad \Gamma^A = (\Gamma^{\hat a},\, \Gamma^{a'})\,,$$
$$\Gamma^{\hat a} = \gamma^{\hat a} \otimes \mathbf{1}, \qquad \Gamma^{a'} = \gamma^5 \otimes \gamma^{a'}, \qquad \Gamma^{11} = \gamma^5 \otimes \gamma^7, \qquad \hat a = 0,1,2,3; \quad a' = 1,\cdots,6\,. \tag{A.7}$$

The charge conjugation matrix is $\mathcal{C} = C \otimes C'$.

The fermionic variables $\Theta^{\underline{\alpha}}$ of IIA supergravity carrying 32–component spinor indices of $Spin(1,9)$, in the $AdS_4 \times CP^3$ background and for the above choice of the $D = 10$ gamma–matrices, naturally split into 4–dimensional $Spin(1,3)$ indices and 8–dimensional spinor indices of $Spin(6)$, i.e. $\Theta^{\underline{\alpha}} = \Theta^{\alpha\alpha'}$ ($\alpha = 1,2,3,4$; $\alpha' = 1,\cdots,8$).

A.4 $OSp(6|4)$ superalgebra

The bosonic part of the $OSp(6|4)$ algebra is generated by translations and Lorentz-transformations which split into AdS_4 and CP^3 parts as $P_A = (P_{\hat a}, P_{a'})$ and $M_{AB} = (M_{\hat a\hat b}, M_{a'b'})$ respectively. These satisfy the commutation relations

$$[P_A, P_B] = -\frac{1}{2}R_{AB}{}^{CD}M_{CD}, \qquad [M_{AB}, P_C] = \eta_{AC}\,P_B - \eta_{BC}\,P_A\,, \tag{A.8}$$

$$[M_{AB}, M_{CD}] = \eta_{AC}\,M_{BD} + \eta_{BD}\,M_{AC} - \eta_{BC}\,M_{AD} - \eta_{AD}\,M_{BC}\,, \tag{A.9}$$

where the curvature $R_{AB}{}^{CD} = (R_{\hat a\hat b}{}^{\hat c\hat d}, R_{a'b'}{}^{c'd'})$, and the AdS_4 and CP^3 curvature are given in (A.3) and (A.6) respectively. The fermionic part of the algebra consists of 24 supersymmetry generators which can be described by 32–component Majorana spinor generators subject to the projection $Q_{\underline{\alpha}} = (\mathcal{P}_{24}\,Q)_{\underline{\alpha}}$ (see eq. (2.2)). Their commutation relations are as follows

$$[P_A,\, Q] = \tfrac{i}{R}\,Q\gamma^5\Gamma_A\mathcal{P}_{24}\,, \quad [M_{AB},\, Q] = -\tfrac{1}{2}\,Q\,\Gamma_{AB}\mathcal{P}_{24}\,, \tag{A.10}$$

$$\{Q, Q\} = 2i\,(\mathcal{P}_{24}\Gamma^A\mathcal{P}_{24})\,P_A + \tfrac{R}{4}(\mathcal{P}_{24}\gamma^5\Gamma^{AB}\mathcal{P}_{24})R_{AB}{}^{CD}M_{CD}\,,$$

where $\gamma_5 = i\Gamma^0\Gamma^1\Gamma^2\Gamma^3$. Note that the splitting of the fermionic generators Q into Q_1 and Q_3 by the Z_4–grading of $OSp(6|4)$ is simply achieved by splitting the $D = 10$ Majorana spinor Q into the left– and right Majorana–Weyl spinors

$$Q_1 = \frac{1}{2}\,Q\,(1 - \Gamma_{11})\,, \qquad Q_3 = \frac{1}{2}\,Q\,(1 + \Gamma_{11})\,. \tag{A.11}$$

Appendix B. Quantities appearing in the definition of the $AdS_4 \times CP^3$ superspace of Section 3

$$R\,(\mathcal{M}^2)^{\alpha i}{}_{\beta j} = 4(\varepsilon\upsilon)^{\alpha i}(\upsilon\varepsilon\gamma^5)_{\beta j} - 2(\gamma^5\gamma^{\hat a}\upsilon)^{\alpha i}(\upsilon\gamma_{\hat a})_{\beta j} - (\gamma^{\hat a\hat b}\upsilon)^{\alpha i}(\upsilon\gamma_{\hat a\hat b}\gamma^5)_{\beta j}\,, \tag{B.1}$$

$$\Lambda_{\hat a}{}^{\hat b} = \delta_{\hat a}{}^{\hat b} - \frac{R^2}{k^2 l_p^2} \cdot \frac{e^{-\frac{2}{3}\phi}}{e^{\frac{2}{3}\phi} + \frac{R}{kl_p}\,\Phi}\, V_{\hat a}\, V^{\hat b}\,,$$

$$S_{\beta j}{}^{\alpha i} = \frac{e^{-\frac{1}{3}\phi}}{\sqrt{2}} \left[(1 - \mathcal{P}_{24}) \left(\sqrt{e^{\frac{2}{3}\phi} + \frac{R}{kl_p}\,\Phi}\,\mathbf{1} - \frac{R}{kl_p}\, \frac{V^{\hat a}\,\Gamma_{\hat a}\Gamma_{11}}{\sqrt{e^{\frac{2}{3}\phi} + \frac{R}{kl_p}\,\Phi}} \right) (1 - \mathcal{P}_{24}) \right]_{\beta j}{}^{\alpha i} \tag{B.2}$$

$$\begin{aligned} V^{\hat a}(\upsilon) &= -\frac{8i}{R}\, \upsilon\gamma^{\hat a}\, \frac{\sinh^2 \mathcal{M}/2}{\mathcal{M}^2}\, \varepsilon\, \upsilon\,, \\ \Phi(\upsilon) &= 1 + \frac{8}{R}\, \upsilon\, \varepsilon\gamma^5\, \frac{\sinh^2 \mathcal{M}/2}{\mathcal{M}^2}\, \varepsilon\upsilon\,. \end{aligned} \tag{B.3}$$

Let us emphasise that the $SO(2)$ indices $i, j = 1, 2$ are raised and lowered with the unit matrices δ^{ij} and δ_{ij} so that there is actually no difference between the upper and the lower $SO(2)$ indices, $\varepsilon_{ij} = -\varepsilon_{ji}$, $\varepsilon^{ij} = -\varepsilon^{ji}$ and $\varepsilon^{12} = \varepsilon_{12} = 1$.

Appendix C. Identities for the kappa-projected fermions

When the fermionic variables $\upsilon^{\alpha i}$ are subject to the constraint (4.5), the following identities hold.

$$\upsilon^i\gamma^5\upsilon^j = \upsilon^i\gamma^3\upsilon^j = 0\,, \qquad \upsilon^{\alpha i}\upsilon^{\beta j}\delta_{ij} = -\frac{1}{4}((1+\gamma)C^{-1})^{\alpha\beta}\upsilon\upsilon\,, \tag{C.1}$$

where $\gamma = \gamma^{012}$ and $\upsilon\upsilon = \delta_{ij}\upsilon^{\alpha i}C_{\alpha\beta}\upsilon^{\beta j}$.

Another useful relation is ($\varepsilon^{012} = -\varepsilon_{012} = 1$)

$$\upsilon\gamma_{ab}d\upsilon = \pm\varepsilon_{abc}\upsilon\gamma^c d\upsilon\,. \tag{C.2}$$

Using eqs. (C.1) and (C.2) we find that

$$\upsilon\varepsilon\gamma^a\upsilon\,\upsilon\varepsilon\gamma_b\upsilon = \delta^a_b(\upsilon\upsilon)^2\,, \qquad \upsilon\varepsilon\gamma^{ac}\upsilon\,\upsilon\varepsilon\gamma_{cb}\upsilon = 2\delta^a_b(\upsilon\upsilon)^2\,, \tag{C.3}$$

and

$$(\mathcal{M}^2\varepsilon\upsilon)^{\alpha i} = 0\,. \tag{C.4}$$

A similar computation shows that

$$\upsilon\varepsilon\gamma^5\mathcal{M}^2 = 0. \tag{C.5}$$

It is also true in general (i.e. without fixing κ–symmetry) that

$$\mathcal{M}^2\upsilon = 0\,, \qquad \upsilon\gamma^5\mathcal{M}^2 = 0. \tag{C.6}$$

Using the above identities we find that for υ satisfying (4.5)

$$\mathcal{M}^2 D\upsilon = \frac{6i}{R^2}(e^a + \frac{R}{2}\omega^{a3})(\gamma_a\upsilon)\,\upsilon\upsilon \tag{C.7}$$

which results in

$$4\upsilon\gamma^a\frac{\sinh^2(\mathcal{M}/2)}{\mathcal{M}^2}D\upsilon=\upsilon\gamma^a(1+\frac{1}{12}\mathcal{M}^2)D\upsilon=\upsilon\gamma^a\,(d-\frac{1}{4}\omega^{bc}\gamma_{bc})\upsilon+\frac{i}{2R^2}(e^a+\frac{R}{2}\omega^{a3})(\upsilon\upsilon)^2\,, \quad \text{(C.8)}$$

where e^a, e^3 ω^{bc} and ω^{a3} are AdS_4 vielbeins and connection defined in eqs. (4.2)–(4.4) and the matrix $\mathcal{M}^2$ is defined in eq. (B.1).
We also find that

$$4\upsilon\varepsilon\gamma^5\frac{\sinh^2\mathcal{M}/2}{\mathcal{M}^2}D\upsilon=\upsilon\varepsilon\gamma^5D\upsilon=\frac{i}{R}(e^a+\frac{R}{2}\omega^{a3})\upsilon\varepsilon\gamma_a\upsilon\,. \quad \text{(C.9)}$$

Other $D=4$ covariant Fierz identities ($\hat{a}=(a,3)$) used in the construction of the Lax connection in Section 4.2 are

$$\varepsilon^{IJ}\,\nabla_I\upsilon\,(1-\Gamma_{11})\,\gamma_{\hat{a}}\,\nabla_J\,\upsilon\,\upsilon\,(1-\Gamma_{11})\,\gamma^{\hat{a}\hat{b}\hat{c}}\,\upsilon-2\,\varepsilon^{IJ}\,\upsilon\,(1-\Gamma_{11})\,\gamma^{\hat{b}}\,\nabla_I\,\upsilon\,\upsilon\,(1-\Gamma_{11})\,\gamma^{\hat{c}}\,\nabla_J\,\upsilon\;= \quad \text{(C.10)}$$

$$=\frac{1}{2}\,\varepsilon^{IJ}\nabla_I\Big(\upsilon\,(1-\Gamma_{11})\,\gamma_{\hat{a}}\,\nabla_J\,\upsilon\,\upsilon\,(1-\Gamma_{11})\,\gamma^{\hat{a}\hat{b}\hat{c}}\,\upsilon\Big)-\frac{2}{R^2}\,\varepsilon^{IJ}\,e_I{}^{\hat{b}}\,e_J{}^{\hat{c}}\,(\upsilon\upsilon)^2\,,$$

$$\upsilon(1-\Gamma^{11})\gamma_{\hat{a}\hat{c}\hat{d}}\upsilon\,\upsilon(1-\Gamma^{11})\gamma^{\hat{b}\hat{c}\hat{d}}\upsilon=-6\delta^{\hat{b}}_{\hat{a}}\,(\upsilon\upsilon)^2=6\upsilon(1-\Gamma^{11})\gamma_{\hat{a}}\gamma_5\upsilon\,\upsilon(1-\Gamma^{11})\gamma^{\hat{b}}\gamma_5\upsilon\,. \quad \text{(C.11)}$$

where Γ_{11} stands for $(1-\mathcal{P}_{24})\gamma^5\,\gamma^7(1-\mathcal{P}_{24})\equiv i\gamma^5\,\varepsilon$ which indicates its origin from $D=10$.

Appendix D. Basic relations for the Killing vectors on symmetric spaces G/H

Let $K_M(X)$ or $K_A(X)=e_A{}^M(X)\,K_M(X)$ be the Killing vectors of a D–dimensional symmetric space G/H, where M are world indices and A are tangent space indices. The Killing vectors $K_M(X)$ take values in the algebra of the isometry group G and the one–forms $K=dX^M\,K_M$ satisfy the Maurer–Cartan equations

$$dK=-2K\wedge K,\qquad dK\wedge K=K\wedge dK=-2K\wedge K\wedge K. \quad \text{(D.1)}$$

The following relations also hold

$$[\nabla_A,\nabla_B]K_C=-R_{ABC}{}^D\,K_D\,,\qquad \nabla_AK_B=[K_A,K_B], \quad \text{(D.2)}$$

$$\nabla_A\nabla_BK_C=[\nabla_AK_B,\,K_C]+[K_B,\nabla_AK_C]=[\nabla_AK_B,\,K_C]-[\nabla_AK_C,\,K_B]=-2R_{A[BC]}{}^DK_D, \quad \text{(D.3)}$$

$$[\nabla_AK_B,\,K_C]=[[K_A,\,K_B],\,K_C]=-R_{ABC}{}^D\,K_D, \quad \text{(D.4)}$$

$$\Big[[K_A,K_B],[K_C,K_D]\Big]=R_{AB[C}{}^F\,[K_{D]},K_F]-R_{CD[A}{}^F\,[K_{B]},K_F]\,, \quad \text{(D.5)}$$

where $R_{ABC}{}^D$ is the curvature of the symmetric space G/H.

For instance, for the AdS_4 Killing vectors we have

$$[\nabla_{\hat{a}},\nabla_{\hat{b}}]K_{\hat{c}}=-R_{\hat{a}\hat{b}\hat{c}}{}^{\hat{d}}\,K_{\hat{d}}\,,\qquad R_{\hat{a}\hat{b}\hat{c}}{}^{\hat{d}}=\tfrac{8}{R^2}\,\eta_{\hat{c}[\hat{a}}\,\delta^{\hat{d}}_{\hat{b}]},\qquad R^{\hat{a}\hat{b}}=-\tfrac{4}{R^2}\,e^{\hat{a}}\,e^{\hat{b}}\,, \quad \text{(D.6)}$$

$$\nabla_{\hat{a}}K_{\hat{b}}=[K_{\hat{a}},K_{\hat{b}}], \quad \text{(D.7)}$$

$$\nabla_{\hat{a}}\nabla_{\hat{b}}K_{\hat{c}}=[\nabla_{\hat{a}}K_{\hat{b}},\,K_{\hat{c}}]+[K_{\hat{b}},\nabla_{\hat{a}}K_{\hat{c}}]=[\nabla_{\hat{a}}K_{\hat{b}},\,K_{\hat{c}}]-[\nabla_{\hat{a}}K_{\hat{c}},\,K_{\hat{b}}]=\tfrac{8}{R^2}\,\eta_{\hat{a}[\hat{b}}\,K_{\hat{c}]}, \quad \text{(D.8)}$$

$$[\nabla_{\hat{a}}K_{\hat{b}},\,K_{\hat{c}}]=[[K_{\hat{a}},\,K_{\hat{b}}],\,K_{\hat{c}}]=-\tfrac{8}{R^2}\,\eta_{\hat{c}[\hat{a}}\,K_{\hat{b}]} \quad \text{(D.9)}$$

$$\Big[[K_{\hat{a}},K_{\hat{b}}],[K_{\hat{c}},K_{\hat{d}}]\Big]=-\tfrac{16}{R^2}\,(K_{[\hat{c}}\,\eta_{\hat{d}][\hat{a}}\,K_{\hat{b}]}-K_{[\hat{a}}\,\eta_{\hat{b}][\hat{c}}\,K_{\hat{d}]}). \quad \text{(D.10)}$$

References

[1] J. Gomis, D. Sorokin, and L. Wulff, "The complete AdS(4) x CP(3) superspace for the type IIA superstring and D-branes," *JHEP* **03** (2009) 015, `arXiv:0811.1566 [hep-th]`.

[2] R. R. Metsaev and A. A. Tseytlin, "Type IIB superstring action in AdS(5) x S(5) background," *Nucl. Phys.* **B533** (1998) 109–126, `arXiv:hep-th/9805028`.

[3] G. Arutyunov and S. Frolov, "Superstrings on $AdS_4 \times CP^3$ as a Coset Sigma-model," *JHEP* **09** (2008) 129, `arXiv:0806.4940 [hep-th]`.

[4] J. Stefanski, B., "Green-Schwarz action for Type IIA strings on $AdS_4 \times CP^3$," *Nucl. Phys.* **B808** (2009) 80–87, `arXiv:0806.4948 [hep-th]`.

[5] P. Fré and P. A. Grassi, "Pure Spinor Formalism for $Osp(N|4)$ backgrounds," `arXiv:0807.0044 [hep-th]`.

[6] G. Bonelli, P. A. Grassi, and H. Safaai, "Exploring Pure Spinor String Theory on $AdS_4 \times \mathbb{CP}^3$," *JHEP* **10** (2008) 085, `arXiv:0808.1051 [hep-th]`.

[7] R. D'Auria, P. Fre, P. A. Grassi, and M. Trigiante, "Superstrings on AdS_4 x CP^3 from Supergravity," *Phys. Rev.* **D79** (2009) 086001, `arXiv:0808.1282 [hep-th]`.

[8] A. Cagnazzo, D. Sorokin, and L. Wulff, "String instanton in AdS(4)xCP(3)," *JHEP* **05** (2010) 009, `arXiv:0911.5228 [hep-th]`.

[9] P. A. Grassi, D. Sorokin and L. Wulff, "Simplifying superstring and D-brane actions in AdS(4) x CP(3) superbackground," JHEP **0908** (2009) 060 `arXiv:0903.5407 [hep-th]`.

[10] D. V. Uvarov, "$AdS_4 \times CP^3$ superstring in the light-cone gauge," *Nucl. Phys.* **B826** (2010) 294–312, `arXiv:0906.4699 [hep-th]`.

[11] D. V. Uvarov, "Light-cone gauge Hamiltonian for $AdS_4 \times CP^3$ superstring," *Mod. Phys. Lett.* **A25** (2010) 1251–1265, `arXiv:0912.1044 [hep-th]`.

[12] D. Bykov, "The worldsheet low-energy limit of the $AdS_4 \times CP^3$ superstring," *Nucl. Phys.* **B838** (2010) 47–74, `arXiv:1003.2199 [hep-th]`.

[13] I. Bena, J. Polchinski, and R. Roiban, "Hidden symmetries of the AdS(5) x S**5 superstring," *Phys. Rev.* **D69** (2004) 046002, `arXiv:hep-th/0305116`.

[14] K. Zarembo, "Algebraic Curves for Integrable String Backgrounds," `arXiv:1005.1342 [hep-th]`.

[15] M. Cvetic, H. Lu, C. N. Pope, and K. S. Stelle, "T-Duality in the Green-Schwarz Formalism, and the Massless/Massive IIA Duality Map," *Nucl. Phys.* **B573** (2000) 149–176, `arXiv:hep-th/9907202`.

[16] T. L. Curtright and C. K. Zachos, "Nonlocal currents for supersymmetric nonlinear models," *Phys. Rev.* **D21** (1980) 411.

[17] D. Cassani and A.-K. Kashani-Poor, "Exploiting N=2 in consistent coset reductions of type IIA," *Nucl. Phys.* **B817** (2009) 25–57, arXiv:0901.4251 [hep-th].

[18] P. S. Howe and E. Sezgin, "The supermembrane revisited," *Class. Quant. Grav.* **22** (2005) 2167–2200, arXiv:hep-th/0412245.

[19] R. Kallosh and A. A. Tseytlin, "Simplifying superstring action on $AdS_5 \times S^5$," *JHEP* **10** (1998) 016, arXiv:hep-th/9808088.

[20] R. Ricci, A. A. Tseytlin, and M. Wolf, "On T-Duality and Integrability for Strings on AdS Backgrounds," *JHEP* **12** (2007) 082, arXiv:0711.0707 [hep-th].

[21] N. Berkovits and J. Maldacena, "Fermionic T-Duality, Dual Superconformal Symmetry, and the Amplitude/Wilson Loop Connection," *JHEP* **09** (2008) 062, arXiv:0807.3196 [hep-th].

[22] N. Beisert, R. Ricci, A. A. Tseytlin, and M. Wolf, "Dual Superconformal Symmetry from AdS5 x S5 Superstring Integrability," *Phys. Rev.* **D78** (2008) 126004, arXiv:0807.3228 [hep-th].

[23] I. Adam, A. Dekel, and Y. Oz, "On Integrable Backgrounds Self-dual under Fermionic T- duality," *JHEP* **04** (2009) 120, arXiv:0902.3805 [hep-th].

[24] T. Bargheer, F. Loebbert, and C. Meneghelli, "Symmetries of Tree-level Scattering Amplitudes in N=6 Superconformal Chern-Simons Theory," arXiv:1003.6120 [hep-th].

[25] J. M. Henn, J. Plefka, and K. Wiegandt, "Light-like polygonal Wilson loops in 3d Chern-Simons and ABJM theory," arXiv:1004.0226 [hep-th].

[26] Y.-t. Huang and A. E. Lipstein, "Dual Superconformal Symmetry of N=6 Chern-Simons Theory," arXiv:1008.0041 [hep-th].

[27] I. Adam, A. Dekel, and Y. Oz, "On the fermionic T-duality of the $AdS_4 \times CP^3$ sigma-model," arXiv:1008.0649 [hep-th].

[28] L.-L. Chau, M.-l. Ge, and Y.-s. Wu, "Noether Currents And Algebraic Structure Of The Hidden Symmetry For Superchiral Fields," *Phys. Rev.* **D25** (1982) 1080.

[29] E. A. Ivanov, "Duality In D = 2 Sigma Models Of Chiral Field With Anomaly," *Theor. Math. Phys.* **71** (1987) 474–484.

[30] U. Saleem and M. Hassan, "Superfield Lax formalism of supersymmetric sigma model on symmetric spaces," *Eur. Phys. J.* **C46** (2006) 797–805, arXiv:hep-th/0605091.

[31] I. A. Bandos, D. P. Sorokin, M. Tonin, P. Pasti, and D. V. Volkov, "Superstrings and supermembranes in the doubly supersymmetric geometrical approach," *Nucl. Phys.* **B446** (1995) 79–118, arXiv:hep-th/9501113.

[32] P. S. Howe, E. Sezgin, and P. C. West, "Aspects of superembeddings," arXiv:hep-th/9705093.

[33] D. P. Sorokin, "Superbranes and superembeddings," *Phys. Rept.* **329** (2000) 1–101, arXiv:hep-th/9906142.

[34] D. P. Sorokin, V. I. Tkach, and D. V. Volkov, "Superparticles, Twistors and Siegel Symmetry," Preprint KIPT-31 04/04/1988, *Mod. Phys. Lett.* **A4** (1989) 901–908.

[35] D. V. Volkov and A. A. Zheltukhin, "Extension of the penrose representation and its use to describe supersymmetric models," *JETP Lett.* **48** (1988) 63–66.

[36] A. I. Gumenchuk and D. P. Sorokin, "Relativistic dynamics of superparticles and the twistor correspondence," *Sov. J. Nucl. Phys.* **51** (1990) 350–355.

[37] M. Tonin, "World sheet supersymmetric formulations of Green-Schwarz superstrings," *Phys. Lett.* **B266** (1991) 312–316.

[38] M. Matone, L. Mazzucato, I. Oda, D. Sorokin, and M. Tonin, "The superembedding origin of the Berkovits pure spinor covariant quantization of superstrings," *Nucl. Phys.* **B639** (2002) 182–202, arXiv:hep-th/0206104.

[39] P. S. Howe and E. Sezgin, "D = 11, p = 5," *Phys. Lett.* **B394** (1997) 62–66, arXiv:hep-th/9611008.

[40] P. S. Howe, E. Sezgin, and P. C. West, "Covariant field equations of the M-theory five-brane," *Phys. Lett.* **B399** (1997) 49–59, arXiv:hep-th/9702008.

[41] P. S. Howe, U. Lindstrom, and L. Wulff, "Superstrings with boundary fermions," *JHEP* **08** (2005) 041, arXiv:hep-th/0505067.

[42] P. S. Howe, U. Lindstrom and L. Wulff, "On the covariance of the Dirac-Born-Infeld-Myers action," *JHEP* **0702** (2007) 070, arXiv:hep-th/0607156.

[43] P. S. Howe, U. Lindstrom, and L. Wulff, "Kappa-symmetry for coincident D-branes," *JHEP* **09** (2007) 010, arXiv:0706.2494 [hep-th].

[44] I. A. Bandos, "On superembedding approach to multiple D-brane system. D0 story," *Phys. Lett.* **B680** (2009) 267–273, arXiv:0907.4681 [hep-th].

[45] I. A. Bandos, "Superembedding approach to M0-brane and multiple M0-brane system," *Phys. Lett.* **B687** (2010) 258–263, arXiv:0912.5125 [hep-th].

[46] N. Gromov and P. Vieira, "The AdS4/CFT3 algebraic curve," *JHEP* **02** (2009) 040, arXiv:0807.0437 [hep-th].

[47] A. Babichenko, B. Stefanski, Jr., and K. Zarembo, "Integrability and the AdS(3)/CFT(2) correspondence," *JHEP* **03** (2010) 058, arXiv:0912.1723 [hep-th].

[48] I. R. Klebanov and A. A. Tseytlin, "Intersecting M-branes as four-dimensional black holes," *Nucl. Phys.* **B475** (1996) 179–192, arXiv:hep-th/9604166.

[49] J.-G. Zhou, "Super 0-brane and GS superstring actions on AdS(2) x S(2)," *Nucl. Phys.* **B559** (1999) 92–102, arXiv:hep-th/9906013.

[50] B. E. W. Nilsson and C. N. Pope, "Hopf fibration of eleven-dimensional supergravity," *Class. Quant. Grav.* **1** (1984) 499.

[51] D. P. Sorokin, V. I. Tkach, and D. V. Volkov, "Kaluza-Klein theories and spontaneous compactification mechanisms of extra space dimensions,". In *Moscow 1984, Proceedings, Quantum Gravity*, 376-392.

[52] D. P. Sorokin, V. I. Tkach, and D. V. Volkov, "On the relationship between compactified vacua of D = 11 and D = 10 supergravities," *Phys. Lett.* **B161** (1985) 301–306.