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Brany Liouville Inflation

J. Ellis[a], **N.E. Mavromatos**[b], **D. V. Nanopoulos**[c]
and
A. Sakharov[a,d]

[a] *Theory Division, Physics Department, CERN, CH-1211 Geneva 23, Switzerland*
[b] *King's College London, University of London, Department of Physics, Strand WC2R 2LS, London, U.K.*
[c] *George P. and Cynthia W. Mitchell Institute for Fundamental Physics, Texas A&M University, College Station, TX 77843, USA;*
Astroparticle Physics Group, Houston Advanced Research Center (HARC), Mitchell Campus, Woodlands, TX 77381, USA;
Academy of Athens, Academy of Athens, Division of Natural Sciences, 28 Panepistimiou Avenue, Athens 10679, Greece
[d] *Swiss Institute of Technology, ETH-Zürich, CH-8093 Zürich, Switzerland*

Abstract

We present a specific model for cosmological inflation driven by the Liouville field in a non-critical supersymmetric string framework, in which the departure from criticality is due to open strings stretched between two moving Type-II 5-branes. We use WMAP and other data on fluctuations in the cosmic microwave background to fix parameters of the model, such as the relative separation and velocity of the 5-branes, respecting also the constraints imposed by data on light propagation from distant gamma-ray bursters. The model also suggests a small, relaxing component in the present vacuum energy that may accommodate the breaking of supersymmetry.

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1 Introduction

A plethora of recent astrophysical data, ranging from measurements of the cosmic microwave background (CMB) with an unprecedented precision by WMAP [1] to direct evidence for the acceleration of the Universe from observations of high-redshift Type-Ia supernovae [2], support strongly two important characteristics of our observable Universe [3]. (i) It seems to have undergone *cosmological inflation* [4], i.e., a phase with a near-exponentially expanding scale factor in an approximately Robertson-Walker space-time seems to have been an essential component of the early evolution of our Universe, and (ii) 70% of the Universe's present energy content does not seem to be associated with any form of matter, and is termed *dark energy*.

Inflationary dynamics is supported by the spatial flatness of the Universe, and many of its aspects have been corroborated by the CMB data [1]. In the standard field-theoretic implementation, an inflationary epoch requires the presence of a scalar mode, the *inflaton* field, whose nature is still unknown. Moreover, the precise shape of its potential is not yet determined by the data from WMAP [5] and other CMB experiments.

The flatness of the Universe, as well as other astrophysical observations, requires the presence of dark energy to balance the energy budget of the current Universe, as well as to explain the data on supernovae [2]. The dark energy may be either a strict *cosmological constant* or a component of the vacuum energy that is relaxing to zero, via some non-equilibrium process as in quintessence models [6], possibly following some excitation of our Universe due to an initial catastrophic event. WMAP data have constrained the present-day equation of state $p = w\rho$, where p is the pressure and ρ the energy density of such a quintessence field, and found $w \lesssim -0.8$ [1]. This is in agreement with the cosmological constant model, which has $w = -1$, but does not require it.

Any non-trivial but constant vacuum energy density in Friedman-Robertson-Walker cosmology would eventually dominate the evolution of the Universe, causing it to re-enter an accelerating inflationary phase. In the modern context of string theory [7], such a de-Sitter-like Universe is an unwelcome feature. This because it implies the existence of an event horizon, which impedes the definition of conventional asymptotic states, and thus an S-matrix [8]. Since string theory is conventionally formulated in terms of S-matrix elements, such a background would appear to be problematic. On the other hand, relaxing quintessential scenarios, although suffering fine-tuning drawbacks related to the shape of the scalar-mode potential, may allow the definition of an asymptotic S-matrix, and hence may be easier to stomach as solutions of some versions of string theory.

This situation has been discussed in the context of string theory [9, 10], in the modern context of brane cosmology [11], and in a model involving colliding brane worlds, one of which is considered as our observable Universe [12]. Other colliding-world scenaria of the 'ekpyrotic' type have been discussed extensively in the recent literature [13], where it was suggested that an inflationary phase was absent and unnecessary. However, this point of view may be difficult to reconcile with the above-mentioned recent evidence for inflation. Moreover, this approach has been criticized in a stringy context [14], on the grounds that classical string equations of motion (conformal invariance conditions) do not lead to expanding Universes but rather to contracting ones [1].

A different point of view was advocated in [12], where the collision of the brane worlds has been viewed as a *non-equilibrium stringy process*, formulated within a *non-critical* (Liouville) string theory [16, 17] and exploiting the identification of target time with the zero mode of the Liouville mode [18, 17, 19]. In this scenario, the catastrophic cosmic event due to the collision of the brane worlds leads to a central charge deficit in the world-sheet σ model that describes the stringy excitations of our (brane) Universe. In the context of the identification of the Liouville mode with target time, this central deficit provides a starting-point for cosmic time.

An important consequence of this departure from critical string theory, and thus from the standard conformal invariance conditions used in [14], is the presence of an exponentially-expanding *inflationary* phase for the four-dimensional cosmological scale factor. Moreover, such models lead naturally to an asymptotic quintessential dark energy component of the Universe which is currently relaxing to zero, depending on the cosmic time as $1/t^2$, computed using logarithmic conformal field theory [20] methods [2]. The early inflationary and late accelerating phases of the Universe are thus correlated and occur dynamically in such models, without the introduction of extra scalar fields such as an inflaton or a scalar quintessence field. Instead, in non-critical

[1] This last point has been questioned, however, in [15], assuming the existence of a hypothetical (non-perturbative) stringy phase transition.

[2] We note in passing that, in the model of [12], such a relaxing dark energy component in the current era can be made compatible with standard supersymmetry-breaking models, with the symmetry breaking scale in the TeV range.

string theory, such inflationary phases may be obtained [21] as a result of identifying the target time with the zero mode of the Liouville world-sheet σ-model field [18, 17]. The consistency of this procedure has been checked in several models.

Here this approach is revisited in some detail for the colliding brane-world scenario of [12], which is improved to incorporate space-time supersymmetry [22], as may be motivated by the stability of the underlying brane configurations as well as particle-physics considerations. We then discuss the cosmological parameters of this model, taking into account the motion of the D-branes. Since the collision of branes, assumed to take place adiabatically, induces the inflationary phase, we can constrain the recoil velocity of the branes after the collision by CMB measurements. We also constrain the distance between the branes and the string coupling in this scenario, exhibiting a region of parameter space that is compatible also with limits on deviations from naive Lorentz invariance in the propagation of high-energy photons from astrophysical sources such as gamma-ray bursters (GRBs) [23, 24, 25]. We also discuss the prospects for dark energy and supersymmetry breaking in this scenario.

2 Inflation as a Liouville String σ Model

2.1 Inflation from Generic Liouville String Models

Before presenting our specific model, we first discuss briefly how an inflationary space-time may be derived generically as a consistent background in a non-critical string theory [21, 26]. The approach could be applied to a wide range of non-critical string models, so we summarize its general features [21] before applying it to the concrete brane model constructed in [22].

As discussed in [21, 12, 26], a constant central-charge deficit Q^2 in a stringy σ model may be associated with an initial inflationary phase [27], with

$$Q^2 = 9H^2 > 0 \ , \tag{1}$$

where the Hubble parameter H can be fixed in terms of other parameters of the model. One may consider various scenaria for such a departure from criticality. For example, in the model of [12] this was due to a 'catastrophic' cosmic event, namely the collision of two brane worlds. In such a scenario, as we now review briefly, it is possible to obtain an initial *supercritical* central charge deficit, and hence a time-like Liouville mode in the theory. For instance, in the specific colliding-brane model of [28], Q (and thus H) is proportional to the square of the relative velocity of the colliding branes, $Q \propto u^2$ during the inflationary era. As is evident from (1) and discussed in more detail below, in a phase of constant Q one obtains an inflationary de Sitter Universe.

However, cosmically catastrophic non-critical string scenaria, such as that in [12], allow in general for a time-dependent deficit $Q^2(t)$ that relaxes to zero. This may occur in such a way that, although during the inflationary era Q^2 is (for all practical purposes) constant, as in (1), eventually Q^2 decreases with time so that, at the present era, one obtains compatibility with a new accelerating phase of the Universe. As already mentioned, such relaxing quintessential scenaria [12, 10] have the advantage of asymptotic states that can be defined properly as $t \to \infty$, as well as a string scattering S-matrix [3].

The specific normalization in (1) is imposed because one may identify the time t with the zero mode of the Liouville field $-\varphi$ of the *supercritical* σ model. The minus sign may be understood both mathematically, as due to properties of the Liouville mode, and physically by the requirement of the relaxation of the deformation of the space-time following the distortion induced by the recoil. With this identification, the general equation of motion for the couplings $\{g_i\}$ of the σ-model background modes is [17]:

$$\ddot{g}^i + Q\dot{g}^i = -\beta^i(g) = -\mathcal{G}^{ij}\partial C[g]/\partial g^j \ , \qquad (2)$$

where the dot denotes a derivative with respect to the Liouville world-sheet zero mode φ, and $\mathcal{G}^{ij}$ is an inverse Zamolodchikov metric in the space of string theory couplings $\{g^i\}$ [30]. When applied to scalar, inflaton-like, string modes, (2) would yield standard field equations for scalar fields in de Sitter (inflationary) space-times, provided the normalization (1) is valid, implying a 'Hubble' expansion parameter $H = -Q/3$ [4]. The minus sign in $Q = -3H$ is due to the fact that, as we discuss below, one identifies the target time t with the world-sheet zero mode of $-\varphi$ [17].

[3]Another string scenario for inducing a de Sitter Universe envisages generating the inflation space-time from string loops (dilaton tadpoles) [29], but in such models a string S-matrix cannot be properly defined.

[4]The gradient-flow property of the β functions makes the analogy with the inflationary case even more profound, with the running central charge $C[g]$ [30] playing the rôle of the inflaton potential in conventional inflationary field theory.

The relations (2) replace the conformal invariance conditions $\beta^i = 0$ of the critical string theory, and express the conditions necessary for the restoration of conformal invariance by the Liouville mode [16]. Interpreting the latter as an extra target dimension, the conditions (2) may also be viewed as conformal invariance conditions of a *critical* σ model in (D+1) target space-time dimensions, where D is the target dimension of the non-critical σ model before Liouville dressing. In most Liouville approaches, one treats the Liouville mode φ and time t as independent coordinates. In our approach [17, 10, 12], however, we take a further step, basing ourselves on dynamical arguments which restrict this extended (D+1)-dimensional space-time to a hypersurface determined by the identification $\varphi = -t$. This means that, as time flows, one is restricted to this D-dimensional subspace of the full (D+1)-dimensional Liouville space-time.

In the work of [12] which invoked a brane collision as a source of departure from criticality, this restriction arose because the potential between massive particles, in an effective field theory context, was found to be proportional to $\cosh(t + \varphi)$, which is minimized when $\varphi = -t$. However, the flow of the Liouville mode opposite to that of target time may be given a deeper mathematical interpretation. It may be viewed as a consequence of a specific treatment of the area constraint in non-critical (Liouville) σ models [18, 17], which involves the evaluation of the Liouville-mode path integral via an appropriate steepest-descent contour. In this way, one obtains a 'breathing' world-sheet evolution, in which the world-sheet area starts from a very large value (serving as an infrared cutoff), shrinks to a very small one (serving as an ultraviolet cutoff), and then inflates again towards very large values (returning to an infrared cutoff). Such a situation may then be interpreted as a world-sheet 'bounce' back to the infrared, implying, following the reasoning of [31], that the physical flow of target time is opposite to that of the world-sheet scale (Liouville zero mode).

We now become more specific. We consider a non-critical σ model in metric ($G_{\mu\nu}$), antisymmetric tensor ($B_{\mu\nu}$), and dilaton (Φ) backgrounds. These have the following $\mathcal{O}(\alpha')$ β functions, where α' is the Regge slope [32]:

$$\beta^G_{\mu\nu} = \alpha' \left(R_{\mu\nu} + 2\nabla_\mu \partial_\nu \Phi - \frac{1}{4} H_{\mu\rho\sigma} H_\nu^{\rho\sigma} \right) ,$$
$$\beta^B_{\mu\nu} = \alpha' \left(-\frac{1}{2} \nabla_\rho H^\rho_{\mu\nu} + H^\rho_{\mu\nu} \partial_\rho \Phi \right) ,$$
$$\tilde{\beta}^\Phi = \beta^\Phi - \frac{1}{4} G^{\rho\sigma} \beta^G_{\rho\sigma} = \frac{1}{6} (C - 26) . \qquad (3)$$

The Greek indices are four-dimensional, including target-time components $\mu, \nu, ... = 0, 1, 2, 3$ on the D3-branes of [12], and $H_{\mu\nu\rho} = \partial_{[\mu} B_{\nu\rho]}$ is the field strength.

We consider the following representation of the four-dimensional field strength in terms of a pseudoscalar (axion-like) field b:

$$H_{\mu\nu\rho} = \epsilon_{\mu\nu\rho\sigma} \partial^\sigma b \tag{4}$$

where $\epsilon_{\mu\nu\rho\sigma}$ is the four-dimensional antisymmetric symbol. Next, we choose an axion background that is linear in the time t [27]:

$$b = b(t) = \beta t \ , \quad \beta = \text{constant}, \tag{5}$$

which yields a constant field strength with spatial indices only: $H_{ijk} = \epsilon_{ijk}\beta$, $H_{0jk} = 0$. This implies that such a background is a conformal solution of the full $\mathcal{O}(\alpha')$ β-function for the four-dimensional antisymmetric tensor. We also consider a dilaton background that is linear in the time t [27]:

$$\Phi(t, X) = \text{const} + (\text{const})' t. \tag{6}$$

This background does not contribute to the β functions for the antisymmetric tensor and metric.

Suppose now that only the metric is a non-conformal background, due to some initial quantum fluctuation or catastrophic event, such as the collision of two branes discussed above and in [28], which results in an initial central charge deficit Q^2 (1) that is constant at early stages after the collision. Let

$$G_{ij} = e^{\kappa\varphi + Hct} \eta_{ij} \ , \quad G_{00} = e^{\kappa'\varphi + Hct} \eta_{00}, \tag{7}$$

where t is the target time, φ is the Liouville mode, $\eta_{\mu\nu}$ is the four-dimensional Minkowski metric, and κ, κ' and c are constants to be determined. As already discussed, the standard inflationary scenario in four-dimensional physics requires $Q = -3H$, which partially stems from [18, 17], and

$$\varphi = -t. \tag{8}$$

This latter restriction is imposed dynamically [28, 17] at the end of our computations. Initially, one should treat φ, t as independent target-space components.

The Liouville dressing induces [16] σ-model terms of the form $\int_\Sigma R^{(2)} Q\varphi$, where $R^{(2)}$ is the world-sheet curvature. Such terms provide non-trivial contributions to the dilaton background in the (D+1)-dimensional space-time (φ, t, X^i):

$$\Phi(\varphi, t, X^i) = Q\,\varphi + (\text{const})' t + \text{const}. \tag{9}$$

If we choose $(\text{const})' = Q$, (9) implies a constant dilaton background.

We now consider the Liouville-dressing equations [16] (2) for the β functions of the metric and antisymmetric tensor fields (3). For a constant dilaton field, the dilaton equation yields no independent information, apart from expressing the dilaton β function in terms of the central charge deficit as usual. For the axion background (5), only the metric yields a non-trivial constraint (we work in units with $\alpha' = 1$ for convenience):

$$\ddot{G}_{ij} + Q\dot{G}_{ij} = -R_{ij} + \frac{1}{2}\beta^2 G_{ij}, \tag{10}$$

where the dot indicates differentiation with respect to the (world-sheet zero mode of the) Liouville mode φ, and R_{ij} is the (non-vanishing) Ricci tensor of the (non-critical) σ model with coordinates $(t, \vec{x})$: $R_{00} = 0\ , R_{ij} = \frac{c^2H^2}{2} e^{(\kappa-\kappa')\varphi}\eta_{ij}$. One should also take into account the temporal (t) equation for the metric tensor (for the antisymmetric backgrounds this is identically zero):

$$\ddot{G}_{00} + Q\dot{G}_{00} = -R_{00} = 0, \tag{11}$$

where the vanishing of the Ricci tensor stems from the specific form of the background (7). We seek metric backgrounds of Robertson-Walker inflationary (de Sitter) form:

$$G_{00} = -1\ , \quad G_{ij} = e^{2Ht}\eta_{ij}. \tag{12}$$

Then, from (12), (7), (6) and (5), and imposing (8) at the end, we observe that there indeed is a consistent solution with:

$$Q = -3H = -\kappa',\ c = 3,\ \kappa = H,\ \beta^2 = 5H^2, \tag{13}$$

corresponding to the conventional form of inflationary equations for scalar fields.

2.2 A Concrete Non-critical String Example: Colliding Branes

We now concentrate on one particular example of the previous general scenario [21], in which the non-criticality is induced by the collision of two branes, as seen in Fig. 1. We first discuss the basic features of this scenario, and then proceed to demonstrate explicitly the emergence of inflationary space-times from such situations.

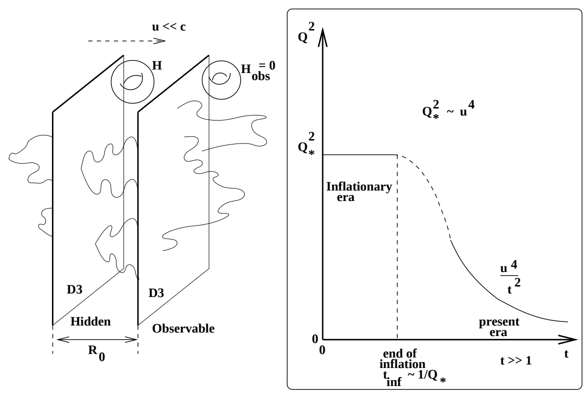

Figure 1: *A scenario in which the collision of two Type-II 5-branes provides inflation and a relaxation model for cosmological vacuum energy.*

Following [12], we consider two 5-branes of Type-II string theory, in which the extra two dimensions have been compactified on tori. On one of the branes (assumed to be the hidden world), the torus is magnetized with a field intensity $\mathcal{H}$. Initially our world is compactified on a normal torus, without a magnetic field, and the two branes are assumed to be on a collision course with a small relative velocity $v \ll 1$ in the bulk, as illustrated in Fig. 1. The collision produces a non-equilibrium situation, which results in electric current transfer from the hidden brane to the visible one. This causes the (adiabatic) emergence of a magnetic field in our world.

The instabilities associated with such magnetized-tori compactifications are not a problem in the context of the cosmological scenario discussed here. In fact, as discussed in [12], the collision may also produce decompactification of the extra toroidal dimensions at a rate much slower than any other rate in the problem. As discussed in [12], this guarantees asymptotic equilibrium and a proper definition of an S-matrix for the stringy excitations on the observable world.

The collision of the two branes implies, for a short period afterwards, while the branes are at most a few string scales apart, the exchange of open-string excitations stretching between the branes, whose ends are attached on them. As argued in [12], the exchanges of such pairs of open strings in Type-II string theory result in an excitation energy in the visible world. The latter may be estimated by computing the corresponding scattering amplitude of the two branes, using string-theory world-sheet methods [33]: the time integral for the relevant potential yields the scattering amplitude. Such estimates involve the computation of appropriate world-sheet annulus diagrams, due to the existence of open string pairs in Type-II string theory. This implies the presence of 'spin factors' as proportionality constants in the scattering amplitudes, which are expressed in terms of Jacobi Θ functions. For the small brane velocities $v \ll 1$ we are considering here, the appropriate spin structures start at *quartic order* in v, as a result of the mathematical properties of the Jacobi functions [33]. This in turn implies [12] that the resulting excitation energy on the brane world is of order $V = \mathcal{O}(v^4)$, which may be thought of as an initial (approximately constant) value of a *supercritical* central-charge deficit for the non-critical σ model that describes stringy excitations in the observable world after the collision:

$$Q^2 = \mathcal{O}\left(v^4\right) > 0. \tag{14}$$

The supercriticality of the model is essential [27] for a time-like signature of the Liouville mode, and hence its interpretation as target time.

At times long after the collision, the branes slow down and the central charge deficit is no longer constant but relaxes with time t. In the approach of [12], this relaxation has been computed by using world-sheet logarithmic conformal field theory methods [20, 19], taking into account recoil (in the bulk) of the observable-world brane and the identification of target time with the (zero mode of the) Liouville field. This late-time varying deficit $Q^2(t)$ has been identified [12] with a 'quintessential' dark energy density component:

$$\Lambda(t) \sim \frac{R^2(\mathcal{H}^2 + v^2)^2}{t^2}\left(\frac{M_s}{M_P}\right)^4 M_P^4 \ , \tag{15}$$

where R is the compactification radius. This yields a present era dark energy compatible in order of magnitude with the WMAP observations [1].

The magnetic field $\mathcal{H}$ in the extra dimensions [12] breaks target-space supersymmetry [34], due to the fact that bosons and fermions on the brane worlds couple differently to $\mathcal{H}$. In our problem, where the magnetic field is turned on adiabatically, the resulting mass difference between bosonic and fermionic string excitations is found to be [12]:

$$\Delta m^2_{\rm string} \sim 2\mathcal{H}\cosh\left(\epsilon\varphi + \epsilon t\right)\Sigma_{45}, \tag{16}$$

where Σ_{45} is a standard spin operator in the plane of the torus, and $\epsilon \to 0^+$ is the regulating parameter of the Heaviside operator $\Theta_\epsilon(t) = -i\int_{-\infty}^{\infty}\frac{d\omega}{\omega - i\epsilon}e^{i\omega t}$ appearing in the D-brane recoil formalism [19]. The dependence in (16) implies that the formalism selects dynamically a Liouville mode which flows opposite to the target time $\varphi = -t$, as discussed earlier, as a result of minimization of the effective field-theoretic potential of the various stringy excitations. By choosing appropriately $\mathcal{H}$, we may arrange for the supersymmetry-breaking scale to be of the order of a few TeV. Such a magnetic field contribution would be subdominant, compared with the velocity contribution, in the expression (15) for the present dark energy density.

The model is capable in principle of reproducing naturally a value of the present dark energy density (i.e., for $t \sim 10^{60}t_P$) that is compatible with observations [2, 1], provided one chooses relatively large compactification radii $R \sim 10^{17}\ell_P \sim 10^{-18}$ m, which are common in modern string theories. In models where the compactification involves higher-dimensional manifolds than tori, a volume factor $R^n : n > 2$ is the number of extra dimensions, appears in (15). In such cases, the compactification radii are significantly smaller.

However, this (toy) model suffers from fine tuning, since the final asymptotic value of the central charge deficit has been arranged to vanish, by an appropriate choice of various constants appearing in the problem [12]. This is required by the assumption that our non-critical string system relaxes asymptotically in time to a critical string. In the complete model, the identification of the Liouville field with target time [17, 19] would define the appropriate renormalization-group trajectory, which hopefully would pick up the appropriate asymptotic critical string state dynamically. This still remains to be seen analytically in realistic models, although it has been demonstrated numerically for some stringy models in [10]. Nevertheless, the current toy example is sufficient to provide a non-trivial, and physically relevant, concrete example of an inflationary Universe in the context of Liouville strings.

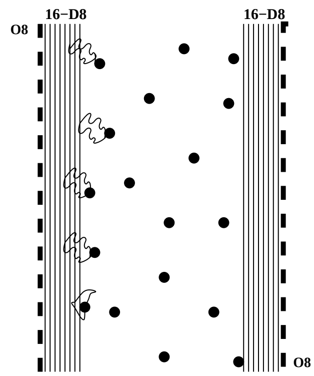

Figure 2: *A model for supersymmetric D-particle foam consisting of two stacks each of sixteen parallel coincident D8-branes, with orientifold planes (thick dashed lines) attached to them. The space does not extend beyond the orientifold planes. The bulk region of ten-dimensional space in which the D8-branes are embedded is punctured by D0-particles (dark blobs). The two parallel stacks are sufficiently far from each other that any Casimir contribution to the vacuum energy is negligible. Open-string interactions between D0-particles and D8-branes are also depicted (wavy lines). If the D0-particles are stationary, there is zero vacuum energy on the D8-branes, and the configuration is a consistent supersymmetric string vacuum.*

This type of model can be extended to incorporate supersymmetry, as shown in a recent paper [22]. As illustrated in Fig. 2, this consists of two stacks of D8-branes with the same tension, separated by a distance R. The transverse bulk space is restricted to lie between two orientifold planes, and is populated by D-particles. It was shown in [22] that, in the limit of static branes and D-particles, this configuration constitutes a zero vacuum-energy supersymmetric ground state of this brane theory. Bulk motion of either the D-branes or the D-particles [5] results in non-zero vacuum energy and hence the breaking of target supersymmetry, proportional to some power of the average (recoil) velocity squared, which depends on the precise string model used to described the (open) stringy matter excitations on the branes.

The colliding-brane scenario introduced earlier can be realized in this framework by allowing (at least one of) the D-branes to move, keeping the

[5] The latter could arise from recoil following scattering with closed string states propagating in the bulk.

orientifold planes static. One may envisage a situation in which the two branes collide, at a certain moment in time corresponding to the Big Bang - a catastrophic cosmological event setting the beginning of observable time - and then bounce back. The width of the bulk region is assumed to be long enough that, after a sufficiently long time following the collision, the excitation energy on the observable brane world - which corresponds to the conformal charge deficit in a σ-model framework [12, 22] - relaxes to tiny values. It is expected that a ground state configuration will be achieved when the branes reach the orientifold planes again (within stringy length uncertainties of order $\ell_s = 1/M_s$, the string scale). In this picture, since observable time starts ticking after the collision, the question how the brane worlds started to move is merely philosophical or metaphysical. The collision results in a kind of phase transition, during which the system passes through a non-equilibrium phase, in which one loses the conformal symmetry of the stringy σ model that describes perturbatively string excitations on the branes. At long times after the collision, the central charge deficit relaxes to zero [12], indicating that the system approaches equilibrium again. The dark energy observed today may be the result of the fact that our world has not yet relaxed to this equilibrium value. Since the asymptotic ground state configuration has static D-branes and D-particles, and hence has zero vacuum energy as guaranteed by the exact conformal field theory construction of [22], it avoids the fine tuning problems in the model of [12].

Sub-asymptotically, there are several contributions to the excitation energy of our brane world in this picture. One comes from the interaction of the brane world with nearby D-particles, i.e., those within distances of order $\mathcal{O}(\ell_s)$, as a result of open strings stretched between them. These are of order $n_0\mathcal{V}$, where n_0 is the density of D-particles on or near the brane world, and

$$\mathcal{V}_{D0-D8} \sim \mathcal{O}(u^2) f(R) \tag{17}$$

where u the velocity of such a D-particle, which may in general be different from the recoiling velocity v of the colliding D-branes, and $f(R)$ is an appropriate function of the distance between the D-particle and the D-brane [35], which is of order unity for distances of the same order as the string scale. In the case of an isolated D-particle/D8-brane system, there are also u-independent contributions to $\mathcal{V}$, which, however, are cancelled in the construction of [22] by the addition of the other D8 branes and orientifold planes.

The other contribution to the dark energy of our brane world comes from the collision of the identical D-branes, which is of order $\mathcal{O}(v^4)$, as mentioned above [12, 22]. For a sufficiently dilute gas of nearby D-particles we may assume that this latter contribution is the dominant one. In this case, we may ignore the D-particle/D-brane contributions (17) to the vacuum energy, and hence apply the previous considerations on inflation, based on the $\mathcal{O}(v^4)$ central charge deficit, also in this scenario of colliding branes and D-particle foam.

The presence of D-particles, which inevitably cross the D-branes in such a picture, even if the D-particle defects are static initially, distorts slighlty the inflationary metric on the observable brane world at early times after the collision, during an era of approximately constant central charge deficit, without leading to significant qualitative changes [26]. Moreover, the existence of D-particles on the branes will affect the propagation of string matter on the branes, in the sense of modifying their dispersion relations by inducing local curvature in space-time, as a result of recoil following collisions with string matter. However, it was argued in [36, 22] that only photons are susceptible to such effects in this scenario, due to the specific gauge properties of the membrane theory at hand. The dispersion relations for chiral matter particles, or in general fields on the D-branes that transform non-trivially under the Standard Model gauge group, are protected by special gauge symmetries in string theory, and as such are not modified. These specifically stringy reasons were outlined in [36, 37].

One may derive stringent limits on the possible modification of photon dispersion relations using observations of gamma-ray bursters (GRBs) [23, 24, 25]. Writing the photon dispersion relation as $E^2 = p^2 + \xi \frac{p^3}{M_{\rm QG}}$, and restricting ourselves to subluminal models with $\xi < 0$ as required by string theory considerations [23, 22], we have found [25]:

$$M_{\rm QG} \gtrsim 10^{16} \text{ GeV}. \tag{18}$$

Limits on a possible modification of photon propagation stronger than (18) have been claimed in the literature, but we do not consider them secure. Some are based on time-of-flight analyses as proposed in [23] using either a single GRB [39] - with unknown redshift and with no accounting for the possible systematic uncertainty due to the possible energy-dependent souce effect that was considered in [24, 25], or a flare from an Active Galactic Nucleus [40] - where statistics is an issue as well as a possible energy-dependent souce effect. There are other constraints based on threshold analyses of absorption by the infrared (IR) diffuse extragalactic background of TeV γ-rays emitted by blazars [41, 42, 43] - which are vulnerable to assumptions on the IR background and depend, in some cases, on assumptions about the possible modifications of dispersion relations for other particle species. However, these are not generic features of models of quantum gravity, since violations of the equivalence principle appear, for instance, in a stringy model of space-time foam [36] related to the model used in this paper [6]. For all the above reasons, we retain (18) as a conservative and reliable limit for the purposes of the present work.

The relation between $M_{\rm QG}$ and the four-dimensional Planck scale M_P is a model-dependent issue. In models of D-particle foam, the quantum gravity scale responsible for the modification of the dispersion relation is the mass of the D-particle, $M_{\rm QG} = M_D = M_s/g_s$, where M_s is the string scale, and g_s is the string coupling.

The string scale M_s may or may not be the same as the four-dimensional Planck scale $M_P \sim 10^{19}$ GeV. One scenario is to identify in our case $M_s = M_P$, and then interpret the lower bound on $M_{\rm QG}$ found in the analysis of [25] as implying a real effect on photons, with $M_{\rm QG} = M_D = M_s/g_s = M_P/g_s \sim 10^{16}$ GeV. This would imply $g_s \le 10^3$. On the other hand, one may identify M_D with M_P, in which case the results of [25] may be interpreted only as providing a sensitivity limit for quantum-gravity effects, three orders of magnitude below the Planck scale. In this case, no information is obtained on g_s and M_s separately from this experiment alone.

3 CMB Constraints on Brany Inflation

We now use WMAP data to set limits on the central charge deficit Q, i.e., the recoil velocity v, in our model of colliding branes, and also on the separation of the 5-branes during inflation. We recall that, in our approach, the central charge deficit Q^2 of the D-particle space-time foam that is responsible for cosmological inflation is related to the Hubble expansion rate during inflation by $Q^2 \simeq 9H^2$. Since the vacuum energy is dynamical in this scenario, we have the prospect of a graceful exit from the inflationary epoch. In our Liouville

[6] We note, also, that the strong limits obtained from synchrotron radiation emission from the Crab Nebula [38, 37] apply to modifications of the dispersion relation for electrons, which are absent in this models [36].

string model, inflation can be taken as ending when the colliding branes are separated by a distance exceeding the string length l_s by a few orders of magnitude. If t_I denotes the duration of inflation, we have:

$$vt_I = x\ell_s, \tag{19}$$

where v is the relative velocity of the branes, and $x \gg 1$ is to be determined below from observations of the CMB. The amount of inflationary expansion within a given timescale is usually parametrized in terms of the number of e-foldings of the scale factor, denoted by N. This number must be larger than about 60 (see [44] for details). Assuming an approximately constant relative velocity of the branes, the inflation lasts for the right amount of e-foldings if the following condition is satisfied:

$$H_I \frac{x\ell_s}{v} = N \geq 60. \tag{20}$$

A recent analysis [45] of WMAP data provides complementary information about the energy scale during inflation, at the 2σ level:

$$\frac{H_I}{M_P} \leq 1.48 \times 10^{-5}. \tag{21}$$

Combining (21), saturating the bound so as to fix ideas, with (20), we find

$$\frac{v}{x} \simeq \frac{1.48}{N} \times 10^{-5} \frac{M_P}{M_s}. \tag{22}$$

On the other hand, from our model above, we know that $H\ell_s \sim v^2$ as an order of magnitude estimate (up to factors pertaining to the volume of compactified extra dimensions. In this section, we assume that the compactification radii are of order of the string scale ℓ_s, otherwise such factors should be taken properly into account). On account of (21), then, we have

$$v^2 \leq 1.48 \times 10^{-5} \frac{M_P}{M_s}. \tag{23}$$

One should consider the two cases mentioned in previous sections, namely that, either $M_P \simeq M_s$, or $M_P = M_s/g_s$, with g_s no less than 10^{-3}. In both cases we see that (23) corresponds to comfortably non-relativistic relative motion of the branes, as we had implicitly assumed.

Our non-critical string scenario realizes inflation dynamically, without the explicit introduction of an extra inflaton field. In our case, the Hubble parameter H depends on the relative velocity v of the branes, $H \propto v^2$. This is small and slowly-varying in cosmic time, mimicking in essential respects the behaviour of a conventional scalar inflaton field. In our stringy model, the recoil velocity v corresponds to a coupling of the underlying two-dimensional world-sheet σ model, pertaining to logarithmic deformations [19, 20]. As described in [17, 19], summation over world-sheet genera results in the quantization of these deformations, inducing quantum fluctuations of the 'coupling' v and thus also H. Therefore, the constancy of the dilaton field, based on the equality of the constants Q and (const)$'$ in (9), should be viewed only as a mean field result. The summation over world-sheet genera, which corresponds to the full quantum theory, leads to quantum fluctuations $\Delta Q \propto \Delta v^2 = \mathcal{O}(g_s^2 v^2)$, in Q [19]. These induce, in turn, quantum fluctuations of the dilaton Φ, $\Delta\Phi \sim g_s^2 v^2 t$, with $t < t_I$, which should therefore be regarded as a fully-fledged, canonically-normalized (in the so-called Einstein frame) scalar quantum field, to be integrated over in a path integral of the corresponding string field theory. The effective low-energy quantum theory is thus a quantum field theory equivalent to a conventional slow-roll inflation, with Einstein gravity at lowest order. This observation justifies our subsequent application of the well-established behaviour of quantum field fluctuations in a de Sitter background to the relative velocity of the branes, that we use as a dynamical degree of freedom driving inflation. We are therefore justified in using horizon-flow parameters [46] to analyze the predictions of this Liouville string model for inflation.

The horizon-flow functions are generalizations of the usual slow-roll parameters [47], and are defined recursively as the logarithmic derivatives of the Hubble scale with respect to the number of e-foldings:

$$\epsilon_{i+1} = \frac{d\ln|\epsilon_i|}{dN}, \qquad i \geq 0, \qquad \epsilon_0 = \frac{H_I}{H}. \tag{24}$$

where H_I denotes an initial value of the Hubble parameter. In the case of a constant-horizon-H inflation, as our case here, the spectral index for the power spectrum is given in terms of horizon-flow parameters by the exact relation [46]:

$$n_S - 1 = -2\epsilon_1 - \epsilon_2. \tag{25}$$

The weak energy condition (for a spatially flat universe) requires $\epsilon_1 > 0$, while inflation requires $\epsilon_1 < 1$. Using (24) and the relation (20), and assuming that only v varies with the number of e-foldings N, and not H_I or the parameter x, one can easily see that the first three horizon-flow functions read:

$$\epsilon_0 = \left(\frac{v_I}{v}\right)^2; \qquad \epsilon_1 = 2\frac{v}{xH_I\ell_s} = \frac{2}{N}; \qquad \epsilon_2 = -\frac{v}{xH_I\ell_s} = -\frac{1}{N}. \tag{26}$$

from which we see that the weak energy condition for a spatially flat universe, requiring $\epsilon_1 > 0$, is satisfied, and, that, due to (20), we also have $\epsilon_1 < 1$, as is the case in typical inflationary scenaria [46]. One can then estimate the spectral index using (26) and (25) as follows:

$$n_S - 1 = -\frac{3v}{H_I x\ell_s} = -\frac{3}{N}. \tag{27}$$

The WMAP data [1] are consistent with a scale-invariant power spectrum [45] at the 1-σ level [1], with

$$n_s - 1 = -4 \times 10^{-2} \tag{28}$$

From (28), (27) and (22) we obtain

$$\frac{v}{x}\frac{M_s}{M_P} \simeq 1.48 \times \frac{10^{-5}}{N} \simeq 1.97 \times 10^{-7} \tag{29}$$

from which $N \simeq 75$. This result ensures that the horizon-flow functions (26) are small, justifying *a posteriori* our assumption of a slow-roll regime [46]. This is not surprising, since the dependence of the Hubble rate on the relative velocity of the branes is akin to a single-field (dilaton) slow-roll model of inflation, for the reasons mentioned above.

We recall that, if we identify the four-dimensional quantum gravity scale $M_P \sim 10^{19}$ GeV with M_s, and interpret the quantum gravity scale limit of [25] (18) as referring to M_s/g_s (the mass of the D-particles in the foam), the string constant g_s should be no less than 10^{-3} in the framework of inflationary Liouville string scenario. From (23), (29) then, we conclude that g_s does not enter, and

$$x \geq 1.95 \times 10^4. \tag{30}$$

This means that, at the end of inflation, the two recoiling branes find themselves some $10^4\ell_s$ apart. On the other hand, if we assume $M_P/M_s = 1/g_s$,

[1] Similar sensitivity to the spectral index was reached previously by combining data from Boomerang, Maxima-1 and COBE [48].

in which case our GRB analysis does not yield a bound on g_s, a 'reasonable' perturbative value $g_s \sim 10^{-2}$ in (23), (29) would lead to

$$x \geq 1.95 \times 10^3. \tag{31}$$

Either way, this simple analysis indicates that D-particle space-time foam can accommodate an inflationary scenario that is consistent with the CMB data for a reasonable range of values of the string coupling.

4 Summary and Outlook

We have exhibited a specific brany scenario for cosmological inflation in the general framework of non-critical string theory [21]. The collision of two Type-II 5-branes [12] (or generalizations thereof to incorporate supersymmetric D-particle foam in inflationary brany scenaria [22]) causes a central-charge deficit in the world-sheet σ model related to their relative velocity. The Hubble expansion rate during inflation is directly related to this deficit, which is compensated by the Liouville field on the string world sheet, whose zero mode is identified dynamically (up to a sign) with cosmic time. Observations of the CMB by WMAP [1] and other experiments provide constraints on inflationary parameters [5] that may be interpreted as limits on the relative velocity and separation of the colliding branes, for values of the string coupling that are compatible with limits on the energy dependence of photon propagation from astrophysical sources [23, 24, 25]. This brany scenario also provides the possibility of a 'quintessential' contribution to the present-day dark energy that relaxes towards zero, as required for supersymmetry [22]. Meanwhile, in the presence of vacuum energy, supersymmetry is broken.

Many details of such a scenario remain to be worked out, such as the graceful exit from inflation and reheating of the Universe, and the magnitude of the present-day dark energy and its possible relation to supersymmetry breaking need to be understood better. Our framework offers the possibility of tackling some of these issues in unconventional ways. In the case of reheating, the dilaton mass during inflation, which is of order H, appears insufficient to reheat the Universe. However, there may be an alternative solution in our approach, using the supersymmetric brane/D-particle model of [22]. In addition to guaranteeing the vanishing of the vacuum energy when the branes are static, the D-particles of [22] may cluster and form primordial black holes (PBHs) on the branes and/or in the bulk space at the end of the inflationary period.

A possible scenario for the formation of such PBHs is the reduction of the propagation velocity of the recoiling branes as they approach the orientifold planes, where they eventually stop. Such a variation in the recoil velocity would break the scale invariance of the spectrum of primordial density perturbations, and it might be possible that n_s in (25), which in that case would get modified by higher order corrections involving (higher powers of) time dependent ϵ_i, becomes greater than 1 at small scales, in some analogy with curvaton effects in conventional cosmologies. This would lead to an increase of the initial density contrast of the D-particles nearby, i.e., within distances of order ℓ_s, and on the D-brane world, and hence increase the probability of forming small PBHs [49]. For instance, these might be formed by dust-like collapse of the D particles on the brane. The masses of the PBHs produced in this way would be controlled by the scale at which the effective bump in the initial spectrum of perturbations appears. In the above scenario, this would be during the last e-foldings of inflation, when the adiabaticity of the relative motion of the branes is violated. The subsequent Hawking evaporation of these black holes may provide a source for radiation in the Universe and the required reheating process, as advocated in the context of a hybrid inflationary scenario in [50].

There are many issues of course that should be carefully looked at in this scenario for reheating, which we reserve for a future publication. However, we are convinced that many of these and other issues related to inflation and vacuum energy can only be understood within a stringy framework, and hope that this paper may contribute to the formulation of these problems within non-critical Liouville string theory.

This is a much broader framework than critical string theory, and allows for the possibility of connecting various string vacua, including metastable ones, that may not be possible otherwise. This is because of the essential non-equilibrium nature of non-critical strings, which seems particularly appropriate for discussing the early history of the Universe. The non-equilibrium physics at this epoch appears well suited to the use of Liouville strings.

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