Single Atom and Two Atom Ramsey Interferometry with Quantized Fields

G. S. Agarwal,[1] P. K. Pathak [1] and M. O. Scully [2,3]

1. Physical Research Laboratory, Navrangpura, Ahmedabad 380 009, India

2. Department of Physics, Texas A & M University, College Station, Texas 77843

3.Max-Planck Institut für Quantenoptik, Garching, Germany

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Abstract

Implications of field quantization on Ramsey interferometry are discussed and general conditions for the occurrence of interference are obtained. Interferences do not occur if the fields in two Ramsey zones have precise number of photons. However in this case it is shown how an analog of Hanbury-Brown Twiss photon-photon correlation interferometry can be used to discern a variety of interference effects as the two independent Ramsey zones get entangled by the passage of first atom. Interferences are restored by working with fields at a single photon level. Generation of entangled states including states like $|2,0\rangle + e^{i\theta}|0,2\rangle$ and $|\alpha,\beta\rangle + |-\alpha,-\beta\rangle$ is discussed.

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I. INTRODUCTION

The method of using spatially separated fields as proposed by Ramsey has been proved to be very useful for high resolution work. It was originally proposed as a technique in the microwave domain [1] which was then extended to studies in optical domain [2]. More recently, this technique has been used very successfully in the studies of quantum entanglement resulting from the interaction of atoms with radiation in a high quality cavity. In this context Ramsey technique enables one to study the entanglement in different basis of states [3]. Haroche and coworkers [3–5] detected a variety of cavity quantum electrodynamics effects [6] by monitoring changes in Ramsey fringe pattern when a cavity was placed in region between two Ramsey zones. Ramsey technique has also been suggested for the measurement of phase diffusion in a micromaser [7]. The existence of fringes in Ramsey technique has been interpreted as due to quantum interferences in the transition amplitudes and thus Ramsey technique is a way of doing atomic interferometry [8]. All these studies consider the field in each Ramsey zone as a coherent field which is prescribed and which does not evolve even though it is interacting with the atom. Now that the interference effects at a single photon or few photon levels are becoming quite common [9–13], it is natural to enquire how the results of Ramsey interferometry would be modified if the coherent field in each Ramsey zone is replaced by a quantized field [14].

The outline of the paper is as follows. In Sec.II, we examine the theory of Ramsey interferometry with quantized fields. In Sec.III, we bring out the role of the quantum statistics of the fields in Ramsey interferometry and examine the conditions on the fields so that interference fringes are obtained. If the fields in the two Ramsey zones are in a state with fixed number of photons, then interferences disappear [15]. Though fields with exactly one photon do not yield interferences, a small coherent field even at a single photon level can lead to a well-defined interference pattern. Further, we show photon-photon interaction can be mediated by a single atom passing through the two cavities. In Sec.IV, we compare this work to our previous work [14] and recent experiment of Bertet etal. [5] on complementarity and quantum eraser. In Sec.V, we demonstrate an analog of Hanbury-BrownTwiss experiment in the context of Ramsey interferometry with quantized fields by calculating atom-atom correlations [16] in atoms passing through two successive Ramsey zones containing quantized field [12]. In the case of fixed number of photons in the cavities, interference does not occur in the excitation probability of a single atom. We show how the interference can be observed by passing successively two atoms through the cavities and by detecting both atoms in the excited states. Finally, we consider the effect of decoherence in Ramsey interferometry. We demonstrate the entanglement of the fields in the two Ramsey zones. Such an entanglement results from the passage of an atom [13]. We generate various entangled states by passing a single or two atoms through cavities and show entanglement can be transfered from fields to atoms. In Sec.VI, we consider the atom-cavity interaction in the dispersive limit and show the generation of the entangled state of two coherent fields of the form $|\alpha, \beta\rangle + | -\alpha, -\beta\rangle$.

II. RAMSEY INTERFEROMETRY IN QUANTIZED FIELDS

We consider a high quality cavity [6] as the Ramsey zone of quantized field. If the number of photons in the cavity is large and the field has a well-defined phase, then it would approach the classical Ramsey interferometry. We thus consider the situation shown in the Fig.1. An atom with two levels $|e\rangle$ and $|g\rangle$ interacts with two single mode cavities with identical frequencies. Let the annihilation and creation operators in the i-th cavity be denoted by a_i and $a_i^\dagger$, respectively. For the situation shown in the Fig.1, the Hamiltonian in the interaction picture is

$$\begin{aligned} H_1 &= \hbar g_1(|e\rangle\langle g|a_1 e^{i\Delta t} + a_1^\dagger|g\rangle\langle e|e^{-i\Delta t}) && 0 < t \le \tau_1 \ , \\ H_1 &= 0 && \tau_1 < t \le T + \tau_1 \ , \\ H_1 &= \hbar g_2(|e\rangle\langle g|a_2 e^{i\Delta t} + a_2^\dagger|g\rangle\langle e|e^{-i\Delta t}) && T + \tau_1 < t \le T + \tau_1 + \tau_2 \ . \end{aligned} \tag{1}$$

Here $\Delta = \omega_0 - \omega_1$ and g_i is the coupling constant of the atom with the vacuum in the i-th cavity. Let us consider an initial state with atom in the lower state $|g\rangle$ and the fields characterized by the state $\sum_{n,\mu} F_{n,\mu}|n,\mu\rangle$. Here $|n\rangle(|\mu\rangle)$ represents the Fock state in first(second) cavity. Let ϕ_e, ϕ_g be the phase shifts in $|e\rangle$ and $|g\rangle$ which we might introduce using same external perturbation between the cavities. Using the interaction Hamiltonian (1) the time evolution of the state can be calculated. The state of the atom and cavity fields is found to be

$$\begin{aligned} |\psi(\tau_1 + T + \tau_2)\rangle = \sum_{n,\mu} &[F_{n,\mu} C_{n-1}(\tau_1) C_{\mu-1}(\tau_2) \exp\left(-i\Delta(\tau_1+\tau_2)/2 - i\phi_g\right) \\ &+ F_{n+1,\mu-1} S_n(\tau_1) S_{\mu-1}(\tau_2) \exp\left(-i\Delta(\tau_1+\tau_2+2T)/2 - i\phi_e\right)] \, |g,n,\mu\rangle \\ &+ \sum_{n,\mu} \Big[F_{n+1,\mu} S_n^*(\tau_1) C_\mu^*(\tau_2) \exp\left(i\Delta(\tau_1+\tau_2)/2 - i\phi_e\right) \\ &\quad + F_{n,\mu+1} C_{n-1}(\tau_1) S_\mu^*(\tau_2) \exp\left(i\Delta(\tau_1+\tau_2+2T)/2 - i\phi_g\right)\Big] \, |e,n,\mu\rangle \ , \end{aligned} \tag{2}$$

where

$$\begin{aligned} C_\alpha(\tau) &= \cos\left(\Omega_\alpha \tau/2\right) + \frac{i\Delta}{\Omega_\alpha} \sin\left(\Omega_\alpha \tau/2\right) \\ S_\alpha(\tau) &= \frac{2ig_\alpha\sqrt{\alpha+1}}{\Omega_\alpha} \sin\left(\Omega_\alpha \tau/2\right) \, , \\ \Omega_\alpha &\equiv \sqrt{(\Delta^2 + 4g_\alpha^2(\alpha+1)} \ , \end{aligned} \tag{3}$$

$$\alpha = n, \mu \text{ and } g_n = g_1, \ g_\mu = g_2.$$

The functions C_α and S_α describe the dynamics of the atom interacting with a single mode cavity with initial state as a Fock state . Note that $C_\alpha(S_\alpha)$ gives the probability amplitude of finding the atom in the excited(ground) state given that it was in the excited state at time $t = 0$.

The structure of the state clearly suggests the elementary process responsible for the final state. A given final state is reached in two different ways. Consider a measurement in which the outgoing atom is found in the excited state. The probability of excitation P_{eg} defined by

$$P_{eg} = Tr_{field}\langle e|\psi(T+\tau_1+\tau_2)\rangle\langle\psi(T+\tau_1+\tau_2)|e\rangle \tag{4}$$

can be calculated using Eq.(2). We find the result

$$P_{eg} = \sum_{n,\mu} |F_{n+1,\mu}X_{n+1,\mu} + e^{i\Delta T + i\phi}F_{n,\mu+1}Y_{n,\mu+1}|^2 \ , \tag{5}$$

where

$$\begin{aligned} \phi &= \phi_e - \phi_g \\ X_{n+1,\mu} &= S_n^*(\tau_1)C_\mu^*(\tau_2) \\ Y_{n,\mu+1} &= C_{n-1}(\tau_1)S_\mu^*(\tau_2) \ . \end{aligned} \tag{6}$$

A similar result is obtained for P_{ge} i.e. the probability of finding the atom in the ground state if initially the atom is in the excited state,

$$P_{ge} = \sum_{n,\mu} |F_{n-1,\mu}X^*_{n,\mu-1} + e^{-i\Delta T + i\phi}F_{n,\mu-1}Y^*_{n+1,\mu}|^2 \ . \tag{7}$$

In particular if each cavity is in vacuum state and $g_1 = g_2, \tau_1 = \tau_2$ then we get much simpler result,

$$P_{ge} = \frac{4g^2}{\Omega_0^2}\sin^2\frac{\Omega_0\tau_1}{2}\left(2 - \frac{4g^2}{\Omega_0^2}\sin^2\frac{\Omega_0\tau_1}{2}\right), \tag{8}$$

which exhibits no interferences as it is phase independent.

The results (5) and (7) are important for understanding Ramsey interferometry with quantized fields. These give rise to a number of important consequences as far as the fundamentals of atom-field interaction are concerned. For classical fields result(7) can be modified, as probability amplitude functions $F_{n,\mu}$ is peaked around average number of photons $\bar{n}$ and $\bar{\mu}$. So in the summation we can replace

$$\begin{aligned} X_{n,\mu-1} &\rightarrow X_{\bar{n},\bar{\mu}}, \\ Y_{n+1,\mu} &\rightarrow Y_{\bar{n},\bar{\mu}}. \end{aligned} \tag{9}$$

Further for large n and μ, we replace,

$$\begin{aligned} F_{n-1,\mu+1} &\rightarrow F_{n,\mu} \\ F_{n,\mu-1} &\rightarrow F_{n,\mu}. \end{aligned} \tag{10}$$

For normalised photon probability amplitude functions $F_{n,\mu}$ Eq.(7) reduces to,

$$P_{ge} = |X^*_{\bar{n},\bar{\mu}} + e^{-i\Delta T + i\phi}Y^*_{\bar{n},\bar{\mu}}|^2 \ . \tag{11}$$

Equation (11) is the result for classical fields.

III. DEPENDENCE OF THE FRINGES ON QUANTUM STATISTICS OF THE FIELDS

We now examine the consequences of the quantized nature of the field, and in particular investigate when the interferences are most pronounced. From the result(5), we see there are two paths which contribute to the amplitude for detecting the atom in excited state

$$\begin{aligned} &|g, n, \mu\rangle \rightarrow |e, n-1, \mu\rangle \rightarrow |e, n-1, \mu\rangle \ , \\ &|g, n, \mu\rangle \rightarrow |g, n, \mu\rangle \rightarrow |e, n, \mu-1\rangle \ . \end{aligned} \tag{12}$$

The interference between these two paths depends on the nature of the photon statistics i.e. on the functions $F_{n,\mu}$. Clearly if the field in each cavity is in a Fock state $|n_0, \mu_0\rangle$

$$F_{n,\mu} = \delta_{n,n_0}\delta_{\mu,\mu_0} \ , \tag{13}$$

then the interference terms in (5) drop out and the two paths (7) become independent. This happens even for Fock states with large number of photons. Interferences are obtained as long as the photon statistics is such that

$$\sum_{n,\mu} F_{n+1,\mu}F^*_{n,\mu+1}X_{n+1,\mu}Y^*_{n,\mu+1} \neq 0 \ . \tag{14}$$

In order to understand the meaning of Eq.(9), consider a situation where detuning Δ can be ignored while considering evolution in Ramsey zone i.e. in each cavity. The condition (9) can be reduced to a very interesting form

$$\langle a_1^\dagger \frac{1}{\sqrt{a_1 a_1^\dagger}} \sin(g_1\tau_1\sqrt{a_1 a_1^\dagger}) \cos(g_1\tau_1\sqrt{a_1^\dagger a_1}) \cos(g_2\tau_2\sqrt{a_2 a_2^\dagger}) \frac{1}{\sqrt{a_2 a_2^\dagger}} \sin(g_2\tau_2\sqrt{a_2 a_2^\dagger}) a_2\rangle \neq 0 \ , \tag{15}$$

which for small interaction times reduces to

$$\langle a_1^\dagger a_2\rangle \neq 0 \ . \tag{16}$$

Thus the nature of interference depends on the quantum statistics of the fields in the two Ramsey zones. The conditions (15) and (16) imply that if the cavities are independent, then the field in each cavity must have a well defined phase for interference to occur. The interference would also not occur if one cavity has a definite number of photons and the other has a field in coherent state. However, interference is obtained if fields in the two cavities are entangled even though the field in each cavity does not have a well-defined phase. In Fig.2, results for classical as well as quantized fields are plotted when each Ramsey zone has a coherent field with average number of photons ($|\alpha|^2 = 5$). Interference fringes for classical fields show higher visibility than in the case of quantized fields.

A. Ramsey fringes with fields at single photon level

Having shown that Ramsey fringes vanish if each cavity contains one photon, the next question arises what happens if the field in each cavity is at single photon level [10]. For this purpose, we consider a case where each cavity is pumped by a weak coherent state so that the initial state of the cavities is

$$|\psi_{cavities}\rangle \cong \frac{1}{(1+|\alpha|^2)}(|0\rangle + \alpha|1\rangle)(|0\rangle + \alpha e^{i\theta}|1\rangle). \tag{17}$$

In this case, the result (5) leads to

$$\begin{aligned} P_{eg} = \frac{4|\alpha|^2}{(1+|\alpha|^2)^2} \Big| \frac{g_1}{\Omega_0} \sin(\Omega_0\tau_1/2) \left(\cos(\Omega_0\tau_2/2) - \frac{i\Delta}{\Omega_0}\sin(\Omega_0\tau_2/2)\right) \\ + \frac{g_2}{\Omega_0}\sin(\Omega_0\tau_2/2)\exp[i\{\Delta(T+\tau_1/2)+\theta+\phi\}]\Big|^2 + O(|\alpha|^4) \ , \end{aligned} \tag{18}$$

which for $\Delta = 0$ and $g_1\tau_1 = \sqrt{2}g_2\tau_2 = \pi/2$ reduces to

$$P_{eg} = \frac{|\alpha|^2}{(1+|\alpha|^2)^2}\left(1 + \sin(\pi/\sqrt{2})\cos(\theta+\phi)\right). \tag{19}$$

This leads to high visibility for the fringes(about80%). It is clear that the interference in (18) arises from the cross terms in (17) as such cross terms lead to the same final state via two different pathways

$$\begin{aligned} |g,1,0\rangle \to |e,0,0\rangle \to |e,0,0\rangle \ , \\ |g,0,1\rangle \to |g,0,1\rangle \to |e,0,0\rangle. \end{aligned} \tag{20}$$

The other terms in (17) do not result in interference as $|1,1\rangle$ leads to different final states and $|0,0\rangle$ can not produce excitation. It should be borne in mind that (18) is different from the result obtained for classical fields. For classical fields on resonance $\Delta = 0$, P_{eg} is given by,

$$P_{eg} = |\sin(\Omega_R\tau_1/2)\cos(\Omega_R\tau_2/2) + \sin(\Omega_R\tau_2/2)\cos(\Omega_R\tau_1/2)\exp\{i(\theta+\phi)\}|^2 \ . \tag{21}$$

Here Ω_R is Rabi frequency, θ is phase difference between the two fields . To compare the result for classical fields with the result for quantized fields, one can use approximation $\Omega_R = 2g|\alpha|$. For comparing Eq.(19) to the result for classical fields we put $g\tau_1 = g\sqrt{2}\tau_2 = \pi/2$ in (21). The result (21) reduces to

$$P_{eg} = \left|\sin\left(\frac{\pi|\alpha|}{2}\right)\cos\left(\frac{\pi|\alpha|}{2\sqrt{2}}\right) + \sin\left(\frac{\pi|\alpha|}{2\sqrt{2}}\right)\cos\left(\frac{\pi|\alpha|}{2}\right)\exp\{i(\theta+\phi)\}\right|^2 \ . \tag{22}$$

For small $|\alpha|$ the above result takes the form,

$$P_{eg} = \frac{|\alpha|^2\pi^2}{4}\left(\frac{3}{2} + \sqrt{2}\cos(\theta+\phi)\right). \tag{23}$$

The Eq.(19) shows nearly 80% visibility while the result (23) gives nearly 95% visibility. For weak classical fields Ramsey interference pattern shows higher visibility than the results derived with quantized fields. The result for classical fields can have 100% visibility if both cavities have the same coherent fields and $\Omega_R\tau_1 = \Omega_R\tau_2$.

B. Ramsey fringes with fields in nonclassical state

Recently an arbitrary superposition of $|0\rangle$ and $|1\rangle$ states $\frac{1}{\sqrt{1+|\alpha|^2}}(|0\rangle + \alpha|1\rangle)$ has been realised [11,12], here the parameter α need not be small. Such a state is highly nonclassical and is quite distinct from a coherent state with very small excitation. This nonclassical state also has the important characteristics that the average value of the field is nonzero and thus the off-diagonal elements of the density matrix are nonzero. For such a nonclassical state and for $\Delta = 0$, the Ramsey fringes are given by,

$$\begin{aligned} P_{eg} &= \frac{|\alpha|^2}{(1+|\alpha|^2)^2} \left|\sin(g_1\tau_1)\cos(g_2\tau_2) + \sin(g_2\tau_2)\exp\{i\phi\}\right|^2 \\ &+ \frac{|\alpha|^4}{(1+|\alpha|^2)^2} \left\{\sin^2(g_1\tau_1)\cos^2(g_2\sqrt{2}\tau_2) + \cos^2(g_1\tau_1)\sin^2(g_2\tau_2)\right\} \end{aligned} \tag{24}$$

For $g_1\tau_1 = g_2\sqrt{2}\tau_2 = \pi/2$, it reduces to the previously derived result (19). Remarkably the visibility from the result(24) does not depend on α which is in contrast to the result (22) . We show in the Fig(3) a comparison of the visibility in the case of coherent field and a nonclassical field. It may be noted that for the state $|0\rangle + \alpha|1\rangle$, the P-distribution is quite singular.

C. Photon-photon interaction mediated by a single atom and quantum entanglement of two cavities

It is well known in nonlinear optics in a macroscopic system that the fields effectively interact and one knows many examples of three wave and four wave interactions in a medium. Such interactions are significant at macroscopic densities of atoms. In this section, we demonstrate a rather remarkable result that a single atom in a high quality cavity can produce photon-photon interaction. For this purpose consider an atom in the ground state passing through the two cavity system. We calculate the state of the two cavity system subject to the condition that the atom at the output is detected in the ground state. Such a conditional field state is found to be,

$$\begin{aligned} |\psi_{c,g}\rangle &= \langle g|\psi(T+\tau_1+\tau_2)\rangle \ , \\ &= C_{n-1}(\tau_1)C_{\mu-1}(\tau_2)\exp\left(-i\Delta(\tau_1+\tau_2)/2 - i\phi_g\right)|n,\mu\rangle \\ &+ S_{n-1}(\tau_1)S_{\mu}(\tau_2)\exp\left(-i\Delta(\tau_1+\tau_2+2T)/2 - i\phi_e\right)|n-1,\mu+1\rangle \ . \end{aligned} \tag{25}$$

This involves a linear combination of states $|n,\mu\rangle$ and $|n-1,\mu+1\rangle$ leading to the entanglement of two cavities. In addition, the passage of one atom transfers one photon from the first cavity to the second cavity. The transfer from one cavity to the other will be complete if $C_{n-1}(\tau_1) = C_{\mu-1}(\tau_2) = 0$. Other entangled states are possible, for example, if the atom was initially in the ground state and if it was detected in the excited state, then the conditional state of the cavities is,

$$\begin{aligned} |\psi_{c,e}\rangle &= \langle e|\psi(T+\tau_1+\tau_2)\rangle \ , \\ &= S^*_{n-1}(\tau_1)C^*_{\mu}(\tau_2)\exp\left(i\Delta(\tau_1+\tau_2)/2 - i\phi_e\right)|n-1,\mu\rangle \\ &+ C_{n-1}(\tau_1)S^*_{\mu-1}(\tau_2)\exp\left(i\Delta(\tau_1+\tau_2+2T)/2 - i\phi_g\right)|n,\mu-1\rangle \ . \end{aligned} \tag{26}$$

IV. RELATION TO THE EARLIER WORKS

Before we proceed further we compare the above with our previous work [14] and a recent experiment of Bertet etal [5]. In our earlier work we used a perturbative approach valid for short interaction times.For most part we also assumed that the two regions of Ramsey interaction were identical in all respects. Thus the atoms interacted with the same quantized field in each region. In such a situation, the interference always occurs as the condition (16) is replaced by $\langle a^\dagger a \rangle \neq 0$. Thus in our previous investigation, the effects of quantized field did not have any effect on Ramsey fringes. However, the statistics of quantized field affects the quantum noise in the measured signal. In the current paper, we deal with the case of strong atom-cavity interaction; besides the two cavities do not necessarily have identical fields. Thus the quantum statistics is very relevant even for Ramsey fringes in the excitation probability as is borne out by the examples given in the Sect.III.

The recent experiment of Bertet etal [5] on complementarity and quantum eraser is a realization of Ramsey interferometry with quantized fields. These authors create two Ramsey zones in the same cavity Fig.4, by having a region in which atom undergoes a phase shift. The fringes are restored for this situation as the two Ramsey zones are located in the same cavity, a case for which condition (16) is always fulfilled. Let us assume that the ground state of atom $|g\rangle$ undergoes phase change after time τ_1 by some internal perturbation but excited state $|e\rangle$ remains same and total interaction time for an atom is $\tau_1 + \tau_2$. The field inside the cavity is in state $\sum_n F_n|n\rangle$. A single atom initially in the excited $|e\rangle$ passes through the cavity and detected in the ground state $|g\rangle$. The probability P_{ge} is given by,

$$P_{ge} = \sum_n \left| F_n S_n(\tau_1) C_n(\tau_2) \exp(-i\phi) + F_n C_n^*(\tau_1) S_n(\tau_2) \right|^2 . \qquad (27)$$

If the cavity has coherent field with average number of photons equal to $\bar{n}$ and $\tau_1 = \tau_2$, $\Delta = 0$, then the above expression leads to the simple result,

$$P_{ge} = \sum_n \frac{\bar{n}^n e^{-\bar{n}}}{n!} \sin^2(2g\sqrt{n+1}\tau_1) \cos^2(\phi/2). \qquad (28)$$

The phase factor appears as an overall multiplication factor. Thus interferences always occur even if the field in the cavity is in Fock state. This situation is quite different from the one involving two cavities (7).

V. TWO ATOM INTERFEROMETRY

In Sect.III, we considered the possibility of producing entanglement between the two cavities by conditional detection of the atomic state. We next examine how such entanglement (26) can be detected. From our previous discussion leading to (14), (15) it is clear that if we send a second atom and measure its excitation probability then such a probability would exhibit interference fringes.

At the outset, we mention that a large body of literature exists on atom-atom correlations and entanglement, when such atoms fly through a high quality cavity. The cavity could be a closed one as in Garching experiments [16] or an open one as in Paris experiments [3]. For a second atom coming in the ground state $|g\rangle$ and detected in the excited state following are the possible pathways,

$$\begin{aligned}
&|n-1,\mu\rangle|g\rangle \rightarrow |n-1,\mu\rangle|g\rangle \rightarrow |n-1,\mu-1\rangle|e\rangle \ , \\
&|n,\mu-1\rangle|g\rangle \rightarrow |n-1,\mu-1\rangle|e\rangle \rightarrow |n-1,\mu-1\rangle|e\rangle \ , \\
&|n-1,\mu\rangle|g\rangle \rightarrow |n-2,\mu\rangle|e\rangle \rightarrow |n-2,\mu\rangle|e\rangle \ , \\
&|n,\mu-1\rangle|g\rangle \rightarrow |n,\mu-1\rangle|g\rangle \rightarrow |n,\mu-2\rangle|e\rangle \ .
\end{aligned} \tag{29}$$

In summary, the system as a whole starting with an initial state $|n,\mu,g_1,g_2\rangle$ has two different pathways leading to the detection of the atom-cavity system in state $|n-1,\mu-1,e_1,e_2\rangle$.

$$\begin{aligned}
&|n,\mu,g_1,g_2\rangle \rightarrow |n-1,\mu,e_1,g_2\rangle \rightarrow |n-1,\mu-1,e_1,e_2\rangle \ , \\
&|n,\mu,g_1,g_2\rangle \rightarrow |n,\mu-1,e_1,g_2\rangle \rightarrow |n-1,\mu-1,e_1,e_2\rangle \ .
\end{aligned} \tag{30}$$

The joint probability of detecting both atoms in the excited state $P^{e_1e_2}_{g_1g_2}$ can be used for doing atomic interferometry even if each cavity is in Fock state. This is reminiscent of photon-photon correlation measurements with light produced in the process of down conversion. Mandel and coworkers [19] carried out a series of measurements with photons from a down converted source where they reported no interferences in the measurement of mean intensities whereas photon-photon correlation exhibited a variety of interference phenomena. In the context of Ramsey interferometry with quantized fields we suggest a measurement of the atom-atom correlation. An explicit form of the joint detection probability can be obtained following Jaynes-Cummings dynamics. A long calculation leads to the following expression for the joint probability if the initial state of the cavities is $|n,\mu\rangle$

$$\begin{aligned}
P^{e_1e_2}_{g_1g_2} = &\left|S_{n-1}(\tau_1)S_{n-2}(\tau_1^{'})C^*_{\mu}(\tau_2)C^*_{\mu}(\tau_2^{'})\right|^2 + \left|C_{n-1}(\tau_1)C_{n-1}(\tau_1^{'})S_{\mu-1}(\tau_2)S_{\mu-2}(\tau_2^{'})\right|^2 \\
&+ \Big|S_{n-1}(\tau_1^{'})C_{n-1}(\tau_1)S_{\mu-1}(\tau_2)C^*_{\mu-1}(\tau_2^{'}) + S_{n-1}(\tau_1)C_{n-2}(\tau_1^{'})S_{\mu-1}(\tau_2^{'})C^*_{\mu}(\tau_2) \\
&\qquad \exp[i(\Delta(T^{'}-T)+\phi^{'}-\phi)]\Big|^2 \ .
\end{aligned} \tag{31}$$

Here we allow the possibility of different interaction times and phases(denoted by dash) for the second atom. In the special case when $\Delta=0, g_1=g_2, \tau_1=\tau_2=\tau_1^{'}=\tau_2^{'}$ and $g\tau=\pi/4$ and when initially cavities are in state $|1,1\rangle$, the joint detection probability has the form

$$\begin{aligned}
P^{e_1e_2}_{g_1g_2} &= \frac{1}{16} + \frac{1}{4}\cos^2(\pi/2\sqrt{2}) + \frac{1}{4}\cos(\pi/2\sqrt{2})\cos(\phi^{'}-\phi) \\
&= 0.1118 + 0.1110\cos(\phi^{'}-\phi).
\end{aligned} \tag{32}$$

Interference fringes with almost 100% visibility are obtained. Thus two atom interferometry could produce perfect visibility in the situations where single atom interferometry exhibits no interferences. Other joint detection probabilities like finding one atom in the excited state and the other in the ground state also display interference fringes. An interesting situation also corresponds to sending both atoms in the excited state and measuring the final states of the two atoms. In the case when initially cavities are in the state $|0,0\rangle$ and $\Delta=0$, the expression for the probability of detecting both the atoms in their ground states has the form

$$P_{e_1e_2}^{g_1g_2}=\sin^2(g_1\tau_1)\sin^2(g_1\sqrt{2}\tau_1^{'})+\sin^2(g_2\tau_2)\sin^2(g_2\sqrt{2}\tau_2^{'})\cos^2(g_1\tau_1)\cos^2(g_1\tau_1^{'})$$
$$+\left|\cos(g_1\tau_1)\sin(g_1\tau_1^{'})\sin(g_2\tau_2)\cos(g_2\tau_2^{'})+\sin(g_1\tau_1)\cos(g_1\sqrt{2}\tau_1^{'})\sin(g_2\tau_2^{'})e^{i(\phi-\phi^{'})}\right|^2. \quad (33)$$

Consider the case when $g_1\sqrt{2}\tau_1^{'}=g_2\sqrt{2}\tau_2^{'}=\pi$ and $g_1\tau_1=g_2\tau_2=\pi/4$. The probability of detecting both atoms in the ground states is given by

$$P_{e_1e_2}^{g_1g_2}=\frac{1}{2}\left|\frac{1}{\sqrt{2}}\sin\frac{\pi}{\sqrt{2}}\cos\frac{\pi}{\sqrt{2}}-\sin\frac{\pi}{\sqrt{2}}\exp[i(\phi-\phi^{'})]\right|^2,$$
$$=0.4327+0.3835\cos(\phi-\phi^{'}). \quad (34)$$

The visibility of fringes in two atom interferometry is quite significant. We next show how two atom interferometry can be used to produce a variety of entangled states.

A. Preparation of the entangled state $\alpha|2,0\rangle+\beta|0,2\rangle$

Consider the situation when two identical atoms are coming in their excited states and each cavity is in vacuum state. The mode of each cavity is in resonance with the atomic transition frequency. If after passing through the cavities both atoms are detected in their ground states, the state of the field inside the cavities is given by

$$|\Phi(\tau_1+T+\tau_2)\rangle=\sin(g_1\tau_1)\sin(g_1\sqrt{2}\tau_1^{'})\exp\{-i(\phi_g+\phi_g^{'})\}|2,0\rangle$$
$$+\cos(g_1\tau_1)\cos(g_1\tau_1^{'})\sin(g_2\tau_2)\sin(g_2\sqrt{2}\tau_2^{'})\exp\{-i(\phi_e+\phi_e^{'})\}|0,2\rangle$$
$$+\Big[\cos(g_1\tau_1)\sin(g_1\tau_1^{'})\sin(g_2\tau_2)\cos(g_2\tau_2^{'})\exp\{-i(\phi_e+\phi_g^{'})\}$$
$$+\sin(g_1\tau_1)\cos(g_1\sqrt{2}\tau_1^{'})\sin(g_2\tau_2^{'})\exp\{-i(\phi_g+\phi_e^{'})\}\Big]|1,1\rangle\,. \quad (35)$$

The $|1,1\rangle$ component drops out for $g_1\tau_1^{'}=g_2\tau_2^{'}=\pi$ and the cavities will be in the entangled state

$$|\Phi(\tau_1+T+\tau_2)\rangle=\sin(g_1\tau_1)\sin(\pi\sqrt{2})\exp[-i(\phi_g+\phi_g^{'})]|2,0\rangle$$
$$-\cos(g_1\tau_1)\sin(g_2\tau_2)\sin(\pi\sqrt{2})\exp[-i(\phi_e+\phi_e^{'})]|0,2\rangle. \quad (36)$$

This entangled state is very interesting, we can change the degree of entanglement by changing the value of $g_1\tau_1$ and $g_2\tau_2$. The state will be maximally entangled for $g_1\tau_1=\pi/4$ and $g_2\tau_2=\pi/2$. This can be seen as first atom comes in excited state $|e\rangle$ interacts with the first cavity for a time such that $g_1\tau_1=\pi/4$. The interaction is like an interaction with a $\pi/2$ pulse the state of the system evolves into

$$|e,0,0\rangle\rightarrow\frac{1}{\sqrt{2}}(|e,0,0\rangle-i|g,1,0\rangle). \quad (37)$$

In the second cavity the interaction is a π pulse interaction and then the state of the total system becomes,

$$\frac{1}{\sqrt{2}}(|e,0,0\rangle - i|g,1,0\rangle) \rightarrow -\frac{i}{\sqrt{2}}(|g,1,0\rangle + |g,0,1\rangle). \tag{38}$$

Thus after passing the first atom the state of the fields in the cavities is,

$$|\psi_1\rangle = -\frac{i}{\sqrt{2}}(|1,0\rangle + |0,1\rangle). \tag{39}$$

The second atom comes in the excited state $|e\rangle$ interacts with the fields inside the cavities for times $\tau_1^{'}$ and $\tau_2^{'}$ such that $g_1\tau_1^{'} = g_2\tau_2^{'} = \pi$ and after passing through the cavities the atom is detected in the ground state $|g\rangle$, so the atom can follow the two paths. The first path is,

$$|e\rangle|1,0\rangle \rightarrow \cos(\pi\sqrt{2})|e,1,0\rangle - i\sin(\pi\sqrt{2})|g,2,0\rangle \rightarrow -i\sin(\pi\sqrt{2})|g,2,0\rangle - \cos(\pi\sqrt{2})|e,1,0\rangle, \tag{40}$$

The second path is,

$$|e\rangle|0,1\rangle \rightarrow -|e,0,1\rangle \rightarrow i\sin(\pi\sqrt{2})|g,0,2\rangle - \cos(\pi\sqrt{2})|e,0,1\rangle. \tag{41}$$

Thus passing both the atoms initially in the excited states and subsequently detecting them in their ground states, we obtain a maximally entangled state

$$|\psi_2\rangle = \frac{1}{\sqrt{2}}\sin(\pi\sqrt{2})\left(\exp[-i(\phi_g + \phi_g^{'})]|2,0\rangle - \exp[-i(\phi_e + \phi_e^{'})]|0,2\rangle\right), \tag{42}$$

of the fields inside the cavities. The phase terms in (42) come from the phase change in the region between the cavities. Now if we pass another atom initially in the excited state $|e\rangle$ through the cavities having field in state (42) and the atom is detected in its ground state $|g\rangle$ after passing through the cavities, entangled state of three photons is generated. The degree of entanglement is controlled by the selection of interaction times in the cavities. For a special case a three photons maximally entangled state,

$$|\psi_3\rangle = -\frac{i}{\sqrt{2}}\sin(\pi\sqrt{2})\sin(\pi\sqrt{3})\left[e^{-i(\phi_g+\phi_g^{'}+\phi_g^{''})}|3,0\rangle + e^{-i(\phi_e+\phi_e^{'}+\phi_e^{''})}|0,3\rangle\right], \tag{43}$$

is generated if we choose interaction times $\tau_1^{''}$ and $\tau_2^{''}$ for third atom such that $g_1\tau_1^{''} = g_2\tau_2^{''} = \pi$.

We note that in any interference experiment involving superposition of quantum states, the decoherence of the state is an important issue. The well defined interferences occur provided that experimental time scales are much smaller compared to the decoherence time. Thus the preparation of the entangled state(42) would be sensitive to the decoherence of the state(39) in the interval τ_0 between the detection of the first atom and the arrival of the second atom. Such issues have been experimentally studied by Brune etal and Raimond etal [21]. The decoherence of the state(39) can be studied by solving the master equation describing the decay of field in each cavity,

$$\dot{\rho} \equiv -\sum_{i=1}^{2} \kappa_i \left(a_i^\dagger a_i \rho - 2a_i \rho a_i^\dagger + \rho a_i^\dagger a_i \right), \tag{44}$$

where $2\kappa_i$ gives the rate of loss of photons from the ith cavity. Under the initial condition(39) a long calculation shows that,

$$\begin{aligned} |\psi_1\rangle\langle\psi_1| \quad &\rightarrow |\psi_{1t}\rangle\langle\psi_{1t}| + \left(1 - \frac{1}{2}(e^{-2\kappa_1 t} + e^{-2\kappa_2 t})\right) |0,0\rangle\langle 0,0|, \\ |\psi_{1t}\rangle &= -\frac{i}{\sqrt{2}} \left(|1,0\rangle e^{-\kappa_1 t} + |0,1\rangle e^{-\kappa_2 t} \right). \end{aligned} \tag{45}$$

Thus decoherence will not be a serious issue as long as $\kappa_i \tau_0 << 1$.

B. Entanglement Transfer from Fields to Atoms

Here we show how entanglement of fields is transfered to the atoms. For this purpose consider the fields inside the cavities are in an entangled state,

$$|\psi_{cf}\rangle = \alpha|0,1\rangle + \beta|1,0\rangle. \tag{46}$$

An atom initially in the ground state $|g\rangle$ is passed through the cavities and the fields inside the cavities are in resonance with atomic transition frequency, then the state of the cavity-atom system is,

$$\begin{aligned} |\psi_4\rangle = &\left\{ \alpha \cos(g_2\tau_2) e^{-i\phi_g} - \beta \sin(g_1\tau_1) \sin(g_2\tau_2) e^{-i\phi_e} \right\} |g,0,1\rangle + \beta \cos(g_1\tau_1) e^{-i\phi_e} |g,1,0\rangle \\ &- i \left(\alpha \sin(g_2\tau_2) e^{-i\phi_g} + \beta \sin(g_1\tau_1) \cos(g_2\tau_2) e^{-i\phi_e} \right) |e,0,0\rangle. \end{aligned} \tag{47}$$

If another atom coming in the ground state $|g\rangle$, interacts with the fields in both the cavities for the times $\tau_1^{'}$ and $\tau_2^{'}$ such that $g_1\tau_1^{'} = g_2\tau_2^{'} = \pi/2$, then the state of the cavity-atom system is,

$$\begin{aligned} |\psi_5\rangle = &-i \left(\alpha \sin(g_2\tau_2) e^{-i(\phi_g + \phi_g^{'})} + \beta \sin(g_1\tau_1) \cos(g_2\tau_2) e^{-i(\phi_e + \phi_g^{'})} \right) |e,g\rangle|0,0\rangle \\ &- i \left(\alpha \cos(g_2\tau_2) e^{-i(\phi_g + \phi_g^{'})} + \beta \sin(g_1\tau_1) \sin(g_2\tau_2) e^{-i(\phi_e + \phi_g^{'})} \right) |g,e\rangle|0,0\rangle \\ &- \beta \cos(g_1\tau_1) e^{-i(\phi_e + \phi_g^{'})} |g,g\rangle|1,0\rangle. \end{aligned} \tag{48}$$

If we choose the interaction time for first atom in first cavity such that $g_1\tau_1 = \pi/2$ the state (48) becomes,

$$\begin{aligned} |\psi_6\rangle = &-i \left(\alpha \sin(g_2\tau_2) e^{-i(\phi_g + \phi_g^{'})} + \beta \cos(g_2\tau_2) e^{-i(\phi_e + \phi_g^{'})} \right) |e,g\rangle|0,0\rangle \\ &- i \left(\alpha \cos(g_2\tau_2) e^{-i(\phi_g + \phi_g^{'})} + \beta \sin(g_2\tau_2) e^{-i(\phi_e + \phi_g^{'})} \right) |g,e\rangle|0,0\rangle. \end{aligned} \tag{49}$$

The state (49) shows that the atoms are now in entangled state and fields are in independent states so the entanglement of fields has been transfered to the atoms.

VI. CREATION OF ENTANGLED STATES OF COHERENT STATES

Here we discuss the case when atom field coupling in the cavities is dispersive in nature. This is the case when atom-field detuning is very large compared to coupling so that fields inside the cavities can produce phase changes only [21] without any photon absorption or emission. In such a case, a simple perturbative analysis shows that the state $|e,n\rangle(|g,n\rangle)$ experiences the energy shift $\frac{\hbar g^2(n+1)}{\Delta}(-\frac{\hbar g^2 n}{\Delta})$. The effective Hamiltonian of atom-cavity system can be written as

$$\begin{aligned}
H =& \sum_{n,\mu} \hbar \left[(n+\mu)\omega + \frac{\omega_0}{2} + \frac{g^2(n+1)}{\Delta} \right] |e,n,\mu\rangle\langle e,n,\mu| \\
&+ \hbar \left[(n+\mu)\omega - \frac{\omega_0}{2} - \frac{g^2 n}{\Delta} \right] |g,n,\mu\rangle\langle g,n,\mu| \quad 0 < t \leq \tau_1 \\
H =& \sum_{n,\mu} \hbar \left[(n+\mu)\omega + \frac{\omega_0}{2} \right] |e,n,\mu\rangle\langle e,n,\mu| \\
&+ \hbar \left[(n+\mu)\omega - \frac{\omega_0}{2} \right] |g,n,\mu\rangle\langle g,n,\mu| \qquad \tau_1 < t \leq \tau_1 + T \\
H =& \sum_{n,\mu} \hbar \left[(n+\mu)\omega + \frac{\omega_0}{2} + \frac{g_2^2(\mu+1)}{\Delta} \right] |e,n,\mu\rangle\langle e,n,\mu| \\
&+ \hbar \left[(n+\mu)\omega - \frac{\omega_0}{2} - \frac{g^2 \mu}{\Delta} \right] |g,n,\mu\rangle\langle g,n,\mu| \qquad \tau_1 + T < t \leq \tau_1 + T + \tau_2. \qquad (50)
\end{aligned}$$

A single atom initially in the superposition state $|\psi_{atom}\rangle = c_e|e\rangle + c_g|g\rangle$ is passed through the cavities and the cavities have the fields in the coherent states $|\alpha\rangle$ and $\beta\rangle$, respectively. The state of the atom-cavity system at any time t is found to be,

$$|\psi_{ac}(t)\rangle = \sum_{n,\mu} A_{n,\mu}(t)|e,n,\mu\rangle + B_{n,\mu}(t)|g,n,\mu\rangle, \qquad (51)$$

where the coefficients acquire a time dependent phase as we are dealing with an interaction which is diagonal and where

$$\frac{A_{n,\mu}(0)}{c_e} = \frac{B_{n,\mu}(0)}{c_g} = \frac{\alpha^n \beta^\mu}{\sqrt{n!\mu!}} \exp\left[-\left(\frac{|\alpha|^2 + |\beta|^2}{2} \right) \right]. \qquad (52)$$

The state of the system after passing the atom through the cavities at time $\tau(\tau > \tau_1 + T + \tau_2)$ is given by,

$$\begin{aligned}
|\psi_{ac}(\tau)\rangle =& \sum_{n,\mu} A_{n,\mu}(0) \exp\left[-i\left\{ (n+\mu)\omega + \frac{\omega_0}{2} \right\}\tau - \frac{ig_1^2(n+1)\tau_1}{\Delta} - \frac{ig_2^2(\mu+1)\tau_2}{\Delta} \right] |e,n,\mu\rangle \\
&+ \sum_{n,\mu} B_{n,\mu}(0) \exp\left[-i\left\{ (n+\mu)\omega - \frac{\omega_0}{2} \right\}\tau + \frac{ig_1^2 n\tau_1}{\Delta} + \frac{ig_2^2 \mu\tau_2}{\Delta} \right] |g,n,\mu\rangle. \qquad (53)
\end{aligned}$$

Which on using the values of $A_{n,\mu}(0)$ and $B_{n,\mu}(0)$ leads to the compact result

$$
\begin{aligned}
|\psi_{ac}(\tau)\rangle &= c_e \exp\left\{-\frac{i}{\Delta}(g_1^2\tau_1 + g_2^2\tau_2)\right\}|e, \alpha_0 e^{-\frac{ig_1^2\tau_1}{\Delta}}, \beta_0 e^{-\frac{ig_2^2\tau_2}{\Delta}}\rangle \\
&\quad + c_g \exp(i\omega_0\tau)|g, \alpha_0 e^{\frac{ig_1^2\tau_1}{\Delta}}, \beta_0 e^{\frac{ig_2^2\tau_2}{\Delta}}\rangle, \\
\alpha_0 &= \alpha e^{-i\omega\tau}, \beta_0 = \beta e^{-i\omega\tau}.
\end{aligned} \tag{54}
$$

If we choose the interaction times such that $g_1^2\tau_1/\Delta = g_2^2\tau_2/\Delta = \pi/2$, and $c_g = c_e = 1/\sqrt{2}$ and setting $i\alpha_0 = \alpha', i\beta_0 = \beta'$, the state (54) becomes

$$
|\psi_{ac}(\tau)\rangle = \frac{1}{\sqrt{2}}\left\{|g, \alpha', \beta'\rangle e^{i\omega_0\tau} - |e, -\alpha', -\beta'\rangle\right\}. \tag{55}
$$

If we detect the atom after passing through the cavities in the state $c_g'|g\rangle + c_e'|e\rangle$, then the state of the field is reduced to,

$$
|\psi_{cavities}\rangle = \frac{c_g'^*}{\sqrt{2}}|\alpha', \beta'\rangle e^{i\omega_0\tau} - \frac{c_e'^*}{\sqrt{2}}| - \alpha', -\beta'\rangle. \tag{56}
$$

The state (56) is the entangled state of two coherent fields.

VII. CONCLUSIONS

Before we conclude, we mention that cavities are not absolutely essential for doing Ramsey interferometry with quantized fields, though the usage of cavities results in the enhancement of atom-photon interaction. We could, for example, imagine the usage of the correlated photons produced by a down converter for doing Ramsey interferometry though such an interaction would be quite weak. A possible arrangement is shown in the Fig.5. The final results for the interference pattern can be obtained in a way similar to the derivation of the result (5). The visibility of the interference pattern depends on the characteristics of the beam splitter and correlations between the down converted photons. Note that the arrangement of the Fig.5 generally makes $\langle a_1^\dagger a_2\rangle \neq 0$. For the input state $|1, 1\rangle$ the output state is [20],

$$
|\psi_{out}\rangle = (|t|^2 - |r|^2)|1, 1\rangle + i\sqrt{2}|r||t|(|2, 0\rangle + |0, 2\rangle), \quad |t|^2 = 1 - |r|^2, \tag{57}
$$

where $|r|^2$ is the reflectivity of the beam splitter. For this state output modes are correlated i.e.

$$
\langle a_1^\dagger a_2\rangle = -2i(|t|^2 - |r|^2)|r||t| \neq 0 \ , \tag{58}
$$

as long as $|r| \neq |t|$. We note in passing that the problem of weak interaction between the atom and single photon could possibly be overcome by considering a collective system such as a Bose condensate [22]. The two Ramsey zones can be realised in the free expansion of a condensate. The interaction in such a case would be enhanced by the square root of the density of atoms.

In conclusion we have discussed in detail the theory of Ramsey interferometry with quantized fields. The interference is very sensitive to the quantum statistics of the fields in the two Ramsey zones. We have derived general conditions for interference to occur. We have shown how an analog of Hanbury-Brown Twiss photon-photon correlation interferometry can be used to discern a variety of interference effects even in situations where the single atom detection probabilities do not exhibit interferences. We have demonstrated atoms acting as a mediator for photon-photon interaction between two cavities and entanglement can be transfered from fields to atoms. We have generated entangled state of two and three photons by passing two and three atoms through the cavities. Further we have shown the generation of the entangled state of two coherent fields $|\alpha, \beta\rangle + |-\alpha, -\beta\rangle$ by using cavities in dispersive limit.

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FIGURES

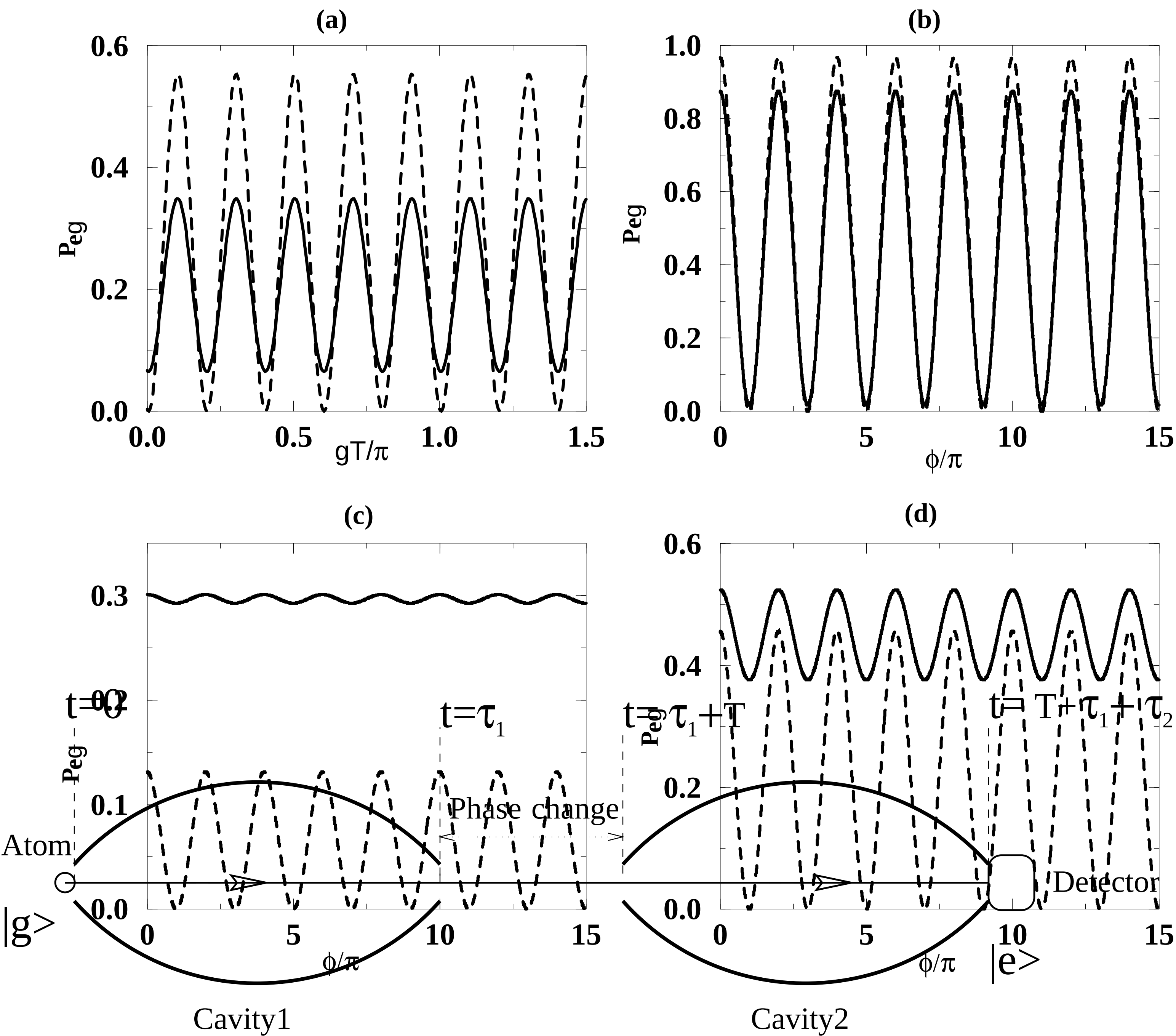
FIG. 1. A schematic arrangement for Ramsey interferometry with quantized fields. Each classical Ramsey zone is replaced by a cavity. There is a phase change between two cavities as $|e\rangle \rightarrow e^{-i\phi_e}|e\rangle$ and $|g\rangle \rightarrow e^{-i\phi_g}|g\rangle$.

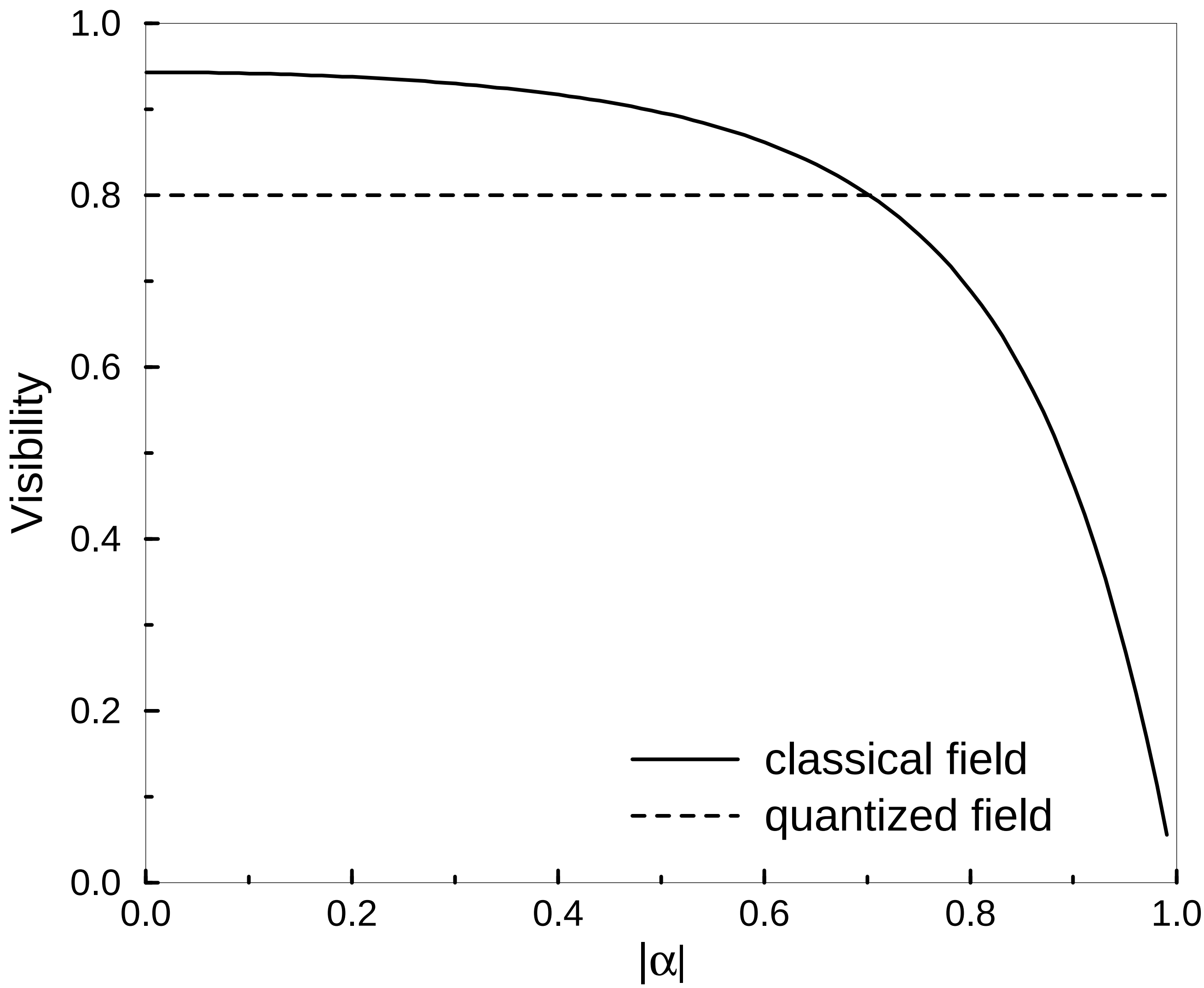
FIG. 2. Interference fringes in the probability of detecting a single atom in the excited state when the atom is initially in the ground state for quantized (solidlines) and classical (dashedlines) fields. The parameters are (a) $g\tau = \pi, \Delta/g = 10, \phi = 0, V_q = 0.68$ (b) $\Delta = 0, g\tau = \pi/8, V_q = 0.96$ (c) $\Delta = 0, g\tau = \pi/4, V_q = 0.14E{-}01$ and (d) $\Delta = 0, g\tau = \pi/2, V_q = 0.16$. The common parameters for above graphs are $|\alpha|^2 = 5$, $\tau_1 = \tau_2 = \tau$, $g_1 = g_2 = g, V_c = 1.00$. V_c, V_q are the visibilities for classical fields and quantized fields.

FIG. 3. The visibility of interference fringes vs. $|\alpha|$ for weak classical fields (solidline) and for a nonclassical state (dashedline) $\frac{1}{\sqrt{1+|\alpha|^2}}(|0\rangle + \alpha|1\rangle)$, with parameters $g\tau_1 = g\sqrt{2}\tau_2 = \pi/2$ and $\Delta = 0$.

FIG. 4. Case when both Ramsey zones are created inside a single cavity

FIG. 5. An alternate scheme for Ramsey interferometry with quantized fields. The input fields a_0 and b_0 could be the outputs of a down converter.


(a)
(b)
(c)
(d)
Peg
gT/π
φ/π
t=0
t=τ1
t=τ1+T
t= T+τ1+τ2
Phase change
Atom
Detector
|g>
|e>
Cavity1
Cavity2



1.0
0.8
0.6
0.4
0.2
0.0
Visibility
0.0
0.2
0.4
0.6
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|α|
classical field
quantized field