A MATRIX BASED APPROACH FOR COLOR TRANSFORMATIONS IN REFLECTIONS

A Thesis

by

CARL PHILLIPS VAN HUYCK

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Chair of Committee,	Ergun Akleman
Committee Members,	Dylan Shell
	Richard R. Davison
	Richard Furuta
Head of Department,	Tim McLaughlin

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ABSTRACT

In this thesis, I demonstrate the feasibility of linear regression with 4×4 matrices to perform color transformations, specifically looking at the case of color transformations in reflections. I compare and analyze the power and performance linear regression models based on 3×3 and 4×4 matrices. I conclude that using 4×4 matrices in linear regression is more advantageous in power and performance over using 3×3 matrices in linear regressions, as 4×4 matrices allow for categorically more transformations by including the possibility of translation. This provides more general affine transformations to a color space, rather than being restricted to passing through the origin. I examine the benefits of allowing for negative elements in color transformation matrices. I also touch on the possible differences in application between filled 4×4 matrices and diagonal 4×4 matrices, and discuss the limitations inherent to linear regression used in any type of matrix operations.

DEDICATION

To my mother, my brothers, and my family.

CONTRIBUTORS

Contributors

This work was supported by a thesis committee consisting of Dr. Ergun Akleman and Professor Richard Davison from the Department of Visualization, and Dr. Richard Furuta and Dr. Dylan Shell from the Department of Computer Science and Engineering. The methodology in chapter 3 and the results, thereof, were employed by the student, with the input of Dr. Ergun Akleman.

Images from the Dreamkeepers series of graphic novels used with permission from Dave and Liz Lillie. All other images used are either in the Public Domain or were taken personally by the student. Further information about image sources can be found in the appendix. Application of Multi-Variate Analysis of Variance to create filled $4x4$ matrices was done by Dr. Derya Akleman. All other work conducted for the thesis was completed by the student independently.

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1. INTRODUCTION AND MOTIVATION

To obtain interesting imagery, artists freely choose colors when rendering reflected and refracted objects. These free transformations are difficult to replicate using physically-based computer graphics rendering approaches, due to their non-physically based nature.

In order to capture the visual effect of a reflection in an image created by an artist, we must create a special system of color transformation such that for every input color of a reflected object, there is a consistent output reflection color that corresponds to it, based on the given source image. Unfortunately, physical transformations do not have the power to represent such unusual color transformations as exist in all kinds of art, both photorealistic and non-photorealistic. Therefore, there is a need for a general method to construct color transformations for any given image.

In this thesis, I propose to use 4×4 matrix transformations for color transformations in projective alpha colors [1]. I obtain these matrices based on linear regression, which is one of the most widely used methods to obtain matrix transformations that can effectively minimize error [2]. Linear regression with 3×3 matrix transformations has previously been used for color transformations [3], but the power of 3×3 matrices is limited since they can only provide linear transformations. In linear transformations, the resulting plane or line has to go through the origin, which may not accurately model the color transformation from a reflection. Using 4×4 matrix transformations allows for more general affine forms, by providing a fourth column which can be used for translation. With 4×4 matrices, the resulting planes do not have to pass through the origin. Since 4×4 matrix transformations are widely used in computer graphics [4], this approach is easy to include in any rendering system by employing existing vector and matrix classes.

In this thesis, I visually compare the effects of four categories of matrices for color transformation due to reflections. These categories are as follows:

- 3×3 scaling matrices – matrices limited to only the diagonal of a 3×3 matrix, allowing for only the slope of a calculated line of best fit to be input.

- 4×4 scaling and translation matrices (all scaling values positive) – A basic 4×4 matrix, which provides scaling and translation of color space, and can use values for its elements that are easily determined through linear regression of sampled points.

- 4×4 scaling and translation matrices (allowing negative scaling values) – these matrices provide utility for certain non-photorealistic transformations, and even very particular photoreal cases, but also provide the possibility of returning negative color values, which must be accounted for.

- 4×4 matrices (all elements utilized) – these matrices would provide all possible transformation to color spaces, but determining the values of each element is complicated.

The biggest difference between these four matrix types is between the 3×3 matrix and the remaining 4×4 matrices. The inability of 3×3 matrices to provide translation transformations both reduces the accuracy that can be expected out of any replicated color transformation, and also means that the linear regression involved requires the use of lines of best fit that are restricted to having a y intercept of 0. This is an unusual requirement, and may in fact involve additional work on the part of the artist attempting to use this method to calculate such restricted linear regression.

The next biggest difference is between the fully utilized 4×4 matrix and the two partial matrices that make use of only scaling and translation. While a 4×4 matrix with all elements utilized should provide the most accurate replications of the sampled reflection transformation, determining which values should be applied to each element requires Multivariant Analysis of Variance (MANOVA), which can be exceptionally complicated and is not as easily available to the average artist as linear regression might be. The biggest advantage of a completely filled matrix is the unique ability this feature provides by allow the values of one color channel to have an effect on the output of another color channel, making this a cross-channel operation.

Finally, there is the difference between the two partial 4×4 matrices, one allowing negative slopes on the line of best fit, and the other constraining it to be greater than or equal to 0. In most cases that I tested, I found that the slopes of the lines of best fit, and thus the scaling values in the generated transformation matrix, were positive. In such a case, there is no difference between the two methods. However, there are some instances where some or all of these values can be negative. In these cases, a system which does not allow for negative values for the scaling elements will not provide a satisfactory color transformation.

I examine each of these types of matrix methods and compare them to each other. In particular, I look at situations such as imitating the reflections in paintings compared to the reflections in photographs, and see under which conditions each method does or does not produce reasonable results. By providing examples, I also demonstrate that this process produces transformations that are believable and similar to the ones that are sampled from to create them. However, the error inherent in linear regression prevents the results from being indistinguishable from the original reflections by human vision, except in situations where the alteration of color caused by reflections is minimal.

2. BACKGROUND AND LITERATURE REVIEW

The concept of color spaces is an integral part of computer graphics, being used in such basic concepts as displaying color on a computer monitor. In their article "Color spaces for computer graphics," Joblove and Greenberg explain how mapping a color position from one space to another can be useful in transferring between different systems of measurement, like the subtractive system of color used for paints and inks, and the additive system of color used in computer screens [5].

Transforming color spaces has been used for a variety of purposes, such as Martinzez-Verdu et al.'s method of evaluating and correcting the colors spaces of digital cameras in their âĂIJ-Calculation of the Color Matching Functions of Digital Cameras from Their Complete spectral SensitivitiesâĂİ [6]. These kinds of uses, while beneficial, serve only to clean up images that are created through other means, and do little to assist in creating artistic assets on their own merit. In this thesis, I demonstrate a method of using color space transformations to achieve believable colored reflections of provided object images, something which can be implemented into an artistic work to create rather than just to correct.

Another utility for transforming color spaces is demonstrated in Reinhard et al.'s "Color transfer between images." They show that transformation of a color space can be used to create visual effects, in their case, making the colors of one image take on the appearance of the colors in another image [3]. To do this, Reinhard et al. make use of a 3×3 matrix, with values being assigned to each cell of the matrix such that, when the color values of any individual pixel are multiplied by the values of the matrix, the resulting 3x1 vector describes a color that has been altered in the desired manner by transforming the color into a different color space and manipulating it there. In essence, the matrix is used as a function for transforming color between color spaces, in order to obtain a desired effect by working within that color space. In their paper, Reinhard et al. were able to achieve good examples of color correction and color matching. To determine the values used in their matrices, they make use of known and standard matrices for moving between one known color space to another, which is useful when the intent is to eventually return to RGB space for display, but is ultimately limited if one wishes to alter an image by altering the color space directly, rather than performing operations within a separate color space. The problem with such a method, however, is finding a way to create a matrix to achieve a desired effect.

The use of Linear Regression to generate color space transforming matrices has been examined and critiqued by Graham Finlayson, both on his own [7] and working together with Mark Drew [8]. In both papers, the authors identify the benefits of Linear Regression, being that it is simple and minimizes the error between modeled and actual color transformations, as well as its flaws, being that there is no way of knowing which particular colors will have larger or smaller errors than any of the others. In spite of this flaw, I have made use of Linear Regression for the purposes of this thesis, under the assumption that sufficient accuracy would be achieved by the simpler method. However, making use of Finlayson and Drew's White-point Preserving Linear Regression method would make for an excellent future modification for my program, beyond this thesis.

In order to apply Linear Regression to the concept of altering color spaces, the data obtained from the Linear Regression must be put into a matrix, in what is known as the Bradford chromatic adaptation transform (Bradford CAT). The Bradford CAT is a method of converting colors as perceived under one lighting setup into an objective color space, and from there into any other color space that corresponds to a new lighting setup. Finlayson and Süsstrunk discuss this in their "Performance of a Chromatic Adaptation Transform based on Spectral Sharpening," where they state that the method is good for accomplishing this task, even though it can still be optimized [9]. In this thesis, I test a variation on a linear Bradford CAT to see if it is effective in capturing color transformations caused specifically by reflections, and find that it performs well in this area.

An effort has been made to find a linear color transform that can approximate the non-linear Bradford transform to an acceptable degree, or which can even provide better results. Costantini et al. in their âĂIJA numerical minimization of perceptual error for the linear chromatic adaptation transformâĂİ discuss the concept, and provide experimental data on a number of possible matrices which can be applied to the transformation, some of which did indeed match or outperform the non-linear transform despite being linear themselves [10]. All of these matrices attempt to address the transformation of color spaces by using combinations of 3×3 matrices.

In these examples, the matrices used for color transformation have all been 3×3 matrices. The transformations possible to enact on a three-dimensional vector, such as a color, are limited if one uses only a three dimensional matrix. 3×3 matrices are capable of rotation, scaling and shearing, but in order to achieve three dimensional translation, a fourth dimensional matrix is required. In this thesis, I explore the results of different kinds of matrices in achieving color transformations, by looking at them through the lens of achieving the transformations a color goes through as it is reflected off of various surfaces.

Central to the topic of my thesis is Philip Willis' "Projective Alpha Color" [1]. In this article, Willis introduces the concept of treating pre-multiplied alpha colors in a similar way to how homogenous coordinates are treated in geometry, forming what are known as projective spaces. In projective spaces, some number of dimensions are scaled according to the value of a special additional dimension. Willis demonstrates a number of uses for this method of dealing with color, but of particular interest to this thesis is how using a homogenous color space allows us to apply matrix operations to colors. Willis demonstrates that it is possible to represent the visual properties of a material as a matrix. He claimed that it is essential to treat materials as a projective transformation over the projective alpha colours [1].

Willis uses these material matrices, in combination with "lights" with colors represented by vectors, to calculate what color of light will be reflected from a given surface. This has clear similarities to the subject of this thesis, with the method Willis describes being very similar to the one I employed. My major contribution, compared to Willis, is the addition of a method for creating the matrices that will be used for reflection, given a situation where the material values are unknown or non-existent, but an image of the result is available for sampling.

Willis also notes the fact that, in what he deems "typical" cases, materials will be represented by largely diagonal matrices, with only the diagonal and the translation elements having non-zero values [1]. His definition of a typical material assumes physically realistic objects such as those that might be recorded in a photograph. In this thesis, I test this assertion, and also see if the statement holds true for non-photographic artistic reflections as well.

3. APPLICATION OF LINEAR REGRESSION WITH COLOR MATRICES ON REFLECTION DATA

In this chapter, I provide demonstrations of the capabilities of different types of color manipulation techniques, including both diagonal matrices using Linear Regression and filled matrices using Multi-variate Analysis of Variance (MANOVA). Working in the context of color change due to reflection, I compare both 3×3 with 4×4 matrices and diagonal matrices with filled matrices in their practicality and ability to accomplish the task of modeling change in color. I also share observations on the problem of negative values in these matrices, and how different methods of handling negative values cause different results.

3.1 Process

Each of the color transformation models I examine here make use of matrices. To determine the appropriate values for the elements of these matrices, I first take a collection of data from an image, this data being the color of pairs of pixels, with one of the two pixels being the object color, and the other pixel being that same color as reflected by some object in the image. The number of pixel pairs I used to create the matrices differed between images, varying between eight at minimum to twenty-one at maximum, but for the majority of the images, I settled on fifteen pairs, seeing that as a reasonable number to employ. The method by which I used this data to create the respective matrices differed according to the type of matrix to be created.

3.1.1 3 x 3 Diagonal Matrices

The 3×3 matrices used linear regression to fill the elements along the diagonal. By using the object and reflection data points I had collected and graphing them out, with the object's color value for a single color channel along the x axis and the reflected color value for that same color channel along the y axis, I obtained three graphs, one for each color channel. Then, taking a linear regression of each graph, constrained such that the line must pass through the origin, I was able to take the slope of those three lines of best fit and put them into a matrix along the diagonal, as demonstrated below. In the example, "r," "g" and "b" are the slopes of the respective lines of best fit for the red, green and blue channels.

It is necessary that the linear regression be constrained, because there is no room during matrix multiplication with 3×3 matrices to contain information for a y-intercept. For that purpose, a fourth column is required.

$$\begin{bmatrix} r & 0 & 0 \\ 0 & g & 0 \\ 0 & 0 & b \end{bmatrix}$$

3.1.2 4 X 4 Diagonal Matrices

Filling a 4×4 diagonal matrix is similar to filling a 3×3 matrix. In the case of the 4×4 matrix, however, the linear regression is not constrained. Taking these new, unconstrained lines of best fit, the slopes are once again put into the matrix along the diagonal, while the y-intercepts are inserted along the new fourth column. The new fourth row of the matrix is filled up with zeroes, except for the element along the diagonal, which is a one. This has the effect that, during matrix multiplication, the y-intercept in the fourth column will be added to the product of the color value and its respective slope. This allows for some objective benefits over the 3×3 matrix, which will be covered later in the chapter.

As shown below, the slopes of the lines of best fit found by linear regression go along the diagonal as "r," "g" and "b," while the respective y-intercepts now go along the fourth column as "rw," "gw" and "bw."

$$\begin{bmatrix} r & 0 & 0 & rw \\ 0 & g & 0 & gw \\ 0 & 0 & b & bw \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

3.1.3 Four by Four Filled Matrices

The method for determining the proper values for a multi-channel color transformation is slightly different. Rather than treat each color channel individually, it is required to take the linear regression of all three color channels at once, to capture their combined effect on each other. This requires much more advanced mathematics, involving Multi-Variate Analysis of Variance (MANOVA) instead of the basic Linear Regression which could be used when dealing with each color channel individually.

$$\begin{bmatrix} rr & rg & rb & rw \\ gr & gg & gb & gw \\ br & bg & bb & bw \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Here, the values of each color channel affect every other color channel, to some extent. This allows for more complicated color relationships than a diagonal 4×4 matrix. The fourth column still provides the benefits it does in a diagonal 4×4 matrix.

3.2 Implementation

To implement this process, I created a number of programs for this thesis, using the Processing programming language.

3.2.1 Sampling

To gather the pixel pairs needed to create the matrices, I created a simple program that displays an image, and allows the user to click on points in that image, alternating between object color and corresponding reflection color. On each click, the RGB values of the pixel that is clicked on are stored in memory. Once all samples have been collected, the user pressed a key to save out the data as a human-readable text file, which would then be read by my other programs.

3.2.2 Matrix Construction

Taking the data from that text file, I then used a variety of programs to perform the variant of linear regression that corresponds to the desired transformation matrix format.

For 3×3 matrices, my program each color channel pair and assigning the object color values as positions of points on the x axis, and the reflected color values as the corresponding positions on the y axis. Solving for the slope of the line that runs through the average point and the origin, we obtain the value to be used for that color channel. This step is repeated for the remaining color channels.

For 4×4 diagonal matrices, my program uses basic linear regression for each of the three color channels separately, assigning the object values to the x positions of the points to apply linear regression to, and the sampled reflected values to the y positions. Once the line of best fit is found, the program can take the slope of that line and place it along the diagonal of the matrix for the specific color channel it belongs to. It also takes the y-intercept of the line and places that value in the corresponding translation element of the matrix, in the fourth column. The program repeats these steps for all color channels.

For 4×4 full matrices, MANOVA is required. In this instance, I submitted the data I had collected to Doctor Derya Akleman, a Statistician, who had access to the equations necessary to perform MANOVA across all three color channels at once. This process provides values for every element of the matrix, which can then be used as normal.

3.3 Matrix Application

In order to apply the matrix to an image, I created another program which could do a variety of things that were useful for the purposes of this thesis.

The program allows input of a matrix either cell-by-cell, or through loading in a prepared text file. The program can also save out matrices that are entered into it through either method.

To achieve its main purpose, the program can apply the matrix to any given color value by multiplying the input color with the matrix to return an output, "reflected" color. By manipulating sliders built into the program, I constructed it such that it would assign a horizontal line at a point in the space of the image, such that everything above the line would remain as in the original image, while everything below the line was flipped vertically, and had the matrix applied to it. These modifications to the image were done so as to create output images that were intuitive as to which parts of the image were reflected, and also so that the images would be aesthetically appealing.

Other capabilities of the program include the possibility to use both a foreground image, which would be reflected and have the filter applied to it, and a background image, which would remain untouched in the background of the resulting reflection. This was especially useful when dealing with foreground images that had areas of transparency.

The program can also apply a simple blur filter to the reflected segments of the image. This can provide a texture or glossiness to the reflection.

The program can also displace the reflection so that the reflection slides underneath the reflected image. This is another feature that is particularly useful when dealing with foreground images that have transparency, such as images of characters being reflected in a pool. By placing the reflection underneath the character, the sense of space is improved in the output image.

Once the output image has been edited to satisfaction inside the in-program display window, the result can be saved out under a variety of file formats with a name specified by the user.

3.4 Comparisons

3.4.1 Three by Three Matrices vs. Four by Four Matrices

While 3×3 Matrices can return passable results when modeling reflections off of colorless surfaces, like water or a mirror, they have a fundamental difficulty when dealing with colored reflective surfaces. Because a 3×3 Matrix lacks translation, its model is forced to run through the origin. As demonstrated in figure 3.1, this means that a 3×3 Matrix is entirely unsuitable for imitating reflections such as the ones off of the surfaces of cars. The ability of the 4×4 Matrix method to compress its color space through the use of translation allows 4×4 Matrices , both diagonal and full, to be objectively better suited to the modeling of colored reflections than the

(a) *An unaltered original image* (b) 3×3 *Matrix* (c) 4×4 *Matrix*

Figure 3.1: 3×3 vs 4×4 Matrices

3×3 Matrix. The 3×3 Matrix applied in figure 3.1b fails to imitate the material of the car, instead appearing like slightly tinted glass, while the 4×4 Matrix in figure 3.1c does a much more convincing job.

3.4.2 Diagonal vs Filled

Due to the complexity of creating a filled matrix, which necessitated outsourcing it to an expert, the number of times I was able to make use of the filled matrix method was extremely limited. Figure 3.2 demonstrates the performance of a diagonal matrix compared to a filled matrix. The reflection of the leftmost character in the water is unaltered in the first image, 3.2a, but in the other two, the reflection is imitated by vertically mirroring the character and applying either a diagonal matrix (3.2b) or a filled matrix (3.2c). The results, in this instance, are comparable.

3.4.3 Negative Numbers

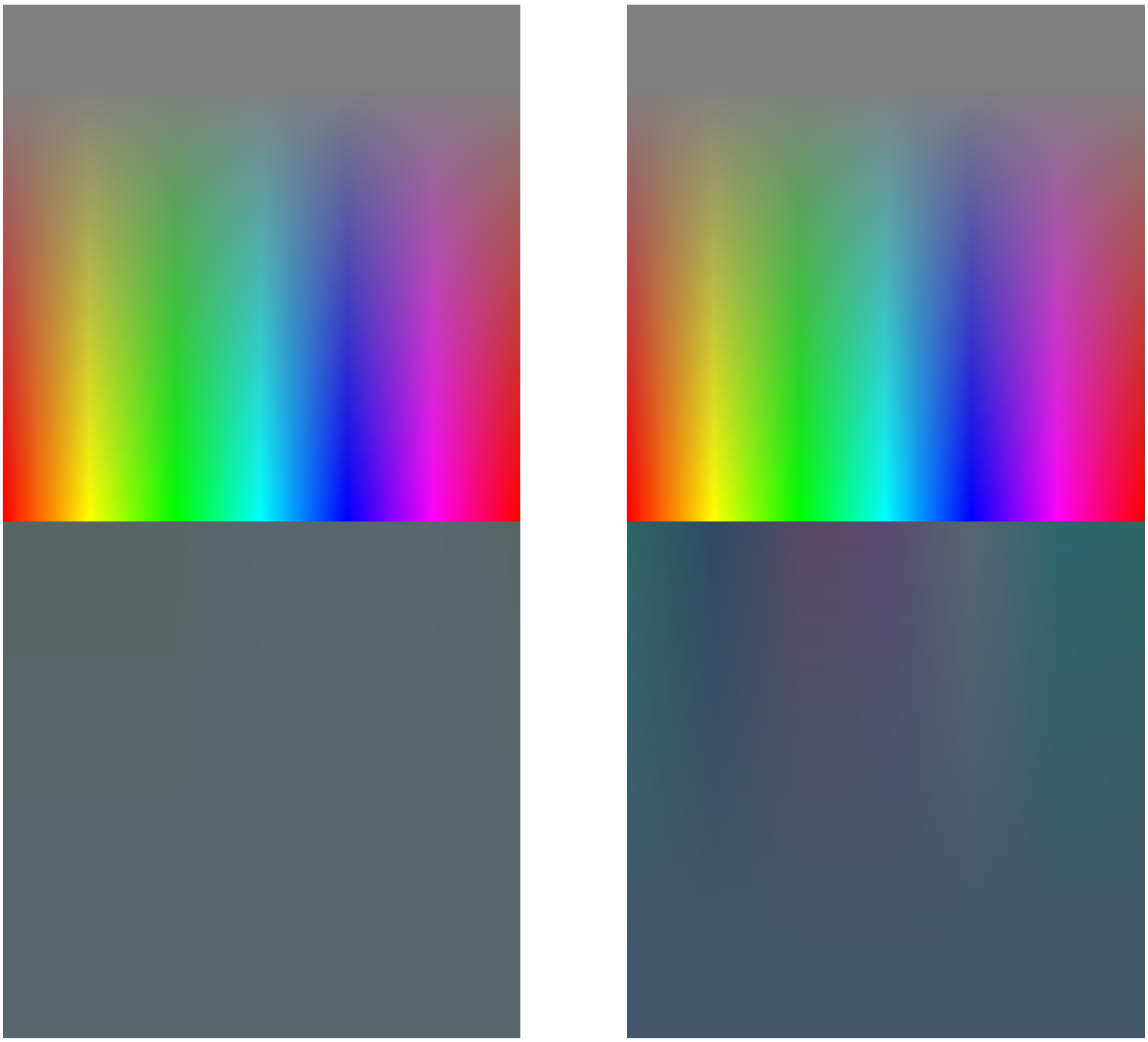
It is possible that, for given sets of input and output color values, the slope of the best fitting line may be negative. In practice, over the course of preparing this thesis, I have not found this to be common, and when it has occurred, it has usually indicated an error in the sampling process that

(a) *Original image, from Dreamkeepers graphic novels* (b) *Diagonal Matrix Reflection* (c) *Filled Matrix Reflection*

Figure 3.2: A demonstration of the use of a full 4×4 matrix

requires resampling. More commonly, the translational value for one or more color channels may be negative. In the event that elements of a matrix are negative, the possibility that the colors a matrix produces could be negative must be dealt with at some point, as traditional models of color assume color values to be positive. However, it makes a difference at what point negative values are accounted for. Figure 3.3 demonstrates this, by showing the difference between a matrix where negative numbers are prohibited and where negative numbers are allowed, but negative results are constrained to 0.

In the case of figure 3.3a, any negative slope in the matrix is reduced to zero, resulting in the transformation shown in the image. The Howetransfvormed colors ar and dull.e flaer, in the caste of figure 3.3b, negative slopes are allowed in the matrix, and the transformed colors are much more interesting. When modeling color transformations with negative slopes, allowing negative elements in the modeling matrix provides objective benefits to the resulting image.



(a) *Matrix with no negative elements allowed* (b) *Matrix with output constrained*

Figure 3.3: Simple vs Complex

4. EXAMPLES

In this chapter, I go over some more interesting individual examples of images to which I applied the 4×4 diagonal matrix method. These include images where the results were particularly nice, ones where the method erred in some way, and ones in which limitations of the method were made apparent.

4.1 Color Paintings

(a) *Original image*

(b) *Image mirrored.*

(c) *Reflected color samples*

Figure 4.1: Buffalo.jpg

Figure 4.1 is an excellent example of a successful color transformation result. The mirrored image 4.1b fits in well with the original colors, even with the basic compositing performed here, and the result is aesthetically quite pleasing.

In figure 4.2, some of the limitations of the 4×4 diagonal method are made clear. In the original image, 4.2a, the distant mountains are affected differently by the reflection than the nearer green hills, with the mountains showing up more blue in the lake below them. The 4×4 method, as implemented in this thesis, has no way of dealing with depth as a factor in the alteration of color, nor does it have any way of applying multiple different color transformations to separate objects

(a) *Original image* (b) *Image mirrored.* (c) *Reflected color samples*

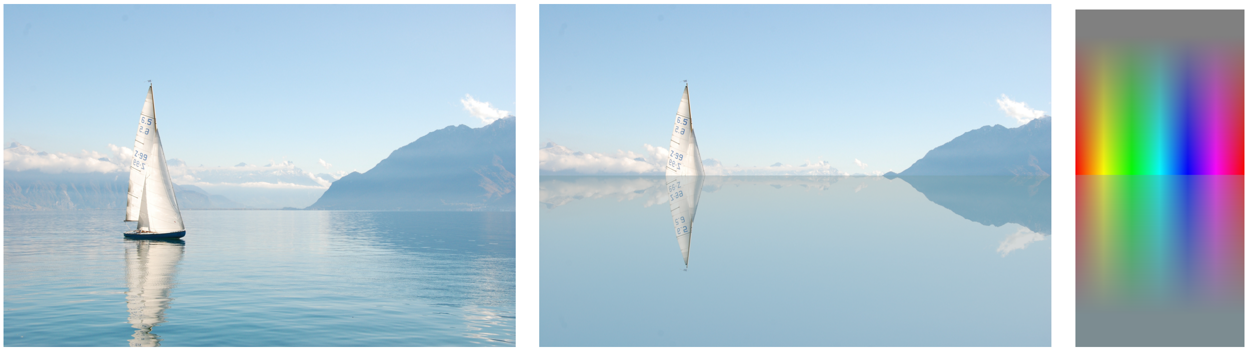
Figure 4.2: LakeAlbano.jpg

with a single matrix. In spite of these drawbacks, the resulting image, 4.2b, is still aesthetically pleasing, although it lacks the depth of the original.

(a) *Original image* (b) *Image mirrored.* (c) *Reflected color samples*

Figure 4.3: Procession.jpg, an example of sampling error

Figure 4.3 provides an example of what the result looks like when the sampling method fails to accurately capture the color transformation of an image. The mirrored result, 4.3b, looks nothing like the original, 4.3a. Of note in this example is that the lines provided by linear regression (shown in Appendix B, B.22) all have negative slopes. This causes the notable inversion of dark and light colors demonstrated in the mirrored image. Of the images which I processed over the course of this thesis, almost all of the lines found through linear regression had positive slopes, and the majority of the ones that did not produced transformations that did not reflect the source image.



(a) *Original image* (b) *Image mirrored* (c) *Reflected color samples*

Figure 4.4: Sailboat.jpg

Figure 4.4 demonstrates how the 4×4 diagonal matrix method is well suited to the subtle color transformations of photorealistic reflections. Although the application of the transformation is crude, the color transformation itself works well with the colors in the image, and provides a convincing approximation of color reflecting off of water. The setup used in this thesis has no method of imitating the texture of waves, but applying the color transformation in combination with some texture generator could theoretically provide excellent results.

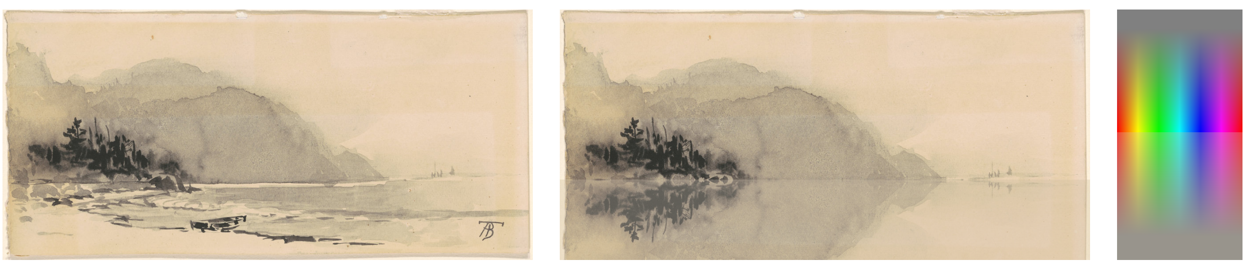
4.2 Color Photographs

In figure 4.5, we see an example of another situation where the 4×4 diagonal matrix method can have difficulties. The original image, 4.5a, has large areas of blown-out color, meaning that its color space is already compressed. This results in the transformed image, 4.5b, having a heavier effect on the reflection than the original image did, with the mirrored image turning the white mountaintops blue. The fact that multiple areas of the original image with the same color, having been blown out to pure white, return different reflection colors causes the calculated model to have more than usual error, leading to noticeable color discrepancy.

(a) *Original image* (b) *Image mirrored* (c) *Reflected color samples*

Figure 4.5: GgPhoto02.jpg

4.3 Monocolor and Greyscale images



(a) *Original image* (b) *Image mirrored left to right.* (c) *Reflected color samples*

Figure 4.6: Beach.jpg

Figure 4.6 shows what happens when the principals of the 4×4 diagonal matrix are applied to a greyscale or low-color image. Using the same method of application as with color images, excellent results can still be obtained even with fewer colors to work with. The process can actually be made simpler with the reduction in the number of color channels.

5. EVALUATION

5.1 Color Difference

Given a cursory visual examination, it is apparent that the method of using linear regression to estimate reflection color transformations provides results that are generally close to the reflections in the source image. However, it also becomes apparent that there is a difference between the source and result colors, a degree of error that occurs because of the nature of linear regression. To accurately judge the success of the method, an objective measure is required.

One method of evaluation presents itself as possible using only the data gathered in the process of performing the linear transformation. The data collected includes pixel data of color from the source and color from the reflection, as included in the image. By comparing the recorded transformation data with the calculated transformation data of the same color as the source color, a strong indication of the success of the process can be provided. To this end, I took a selection of images and created diagrams with the source colors, calculated reflection colors (using single-channel linear regression), sampled reflection colors and the difference between the two reflections. The result of this method is demonstrated in 5.1a and 5.2a.

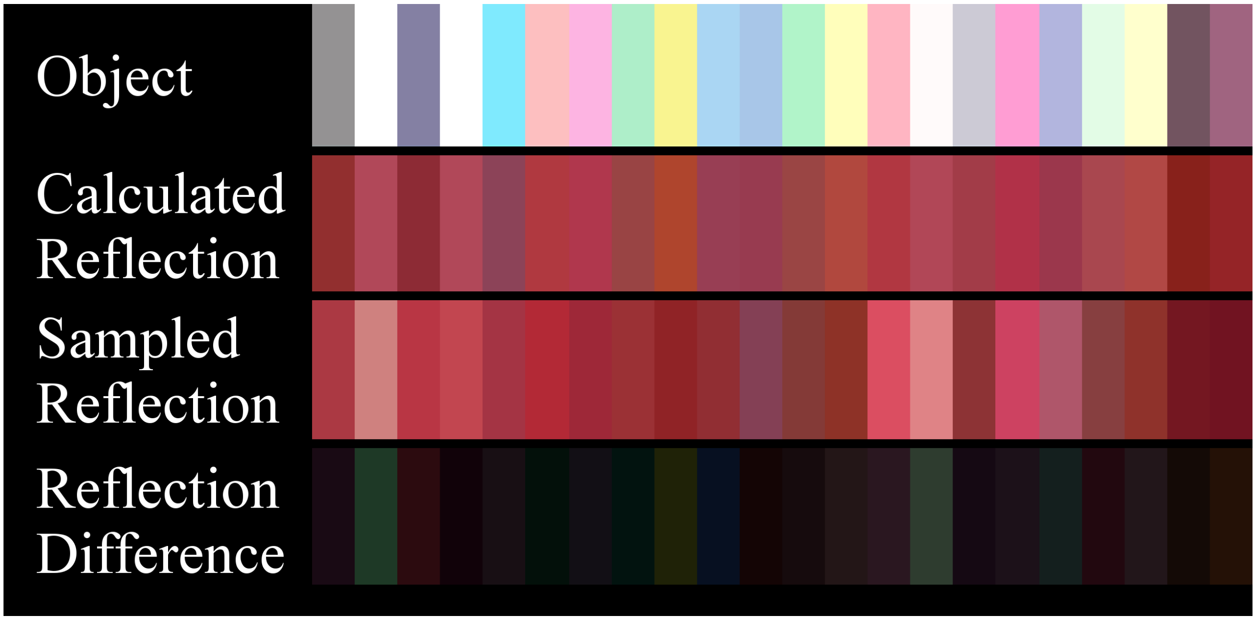
As these evaluations make clear, the differences between the calculated reflection and the sampled reflection vary, being in some cases very small, but often noticeable, and sometimes even quite large.

One possible reason for this discrepancy would be limitations in the ability of a diagonal matrix to transform the color space. With only the diagonal and the fourth column, the possible transformations are scaling and translation. Using only these two transformations, mapping one color space onto another is difficult and prone to error, much in the same way as a linear regression's line of best fit may differ greatly from the individual points used to make it.

It is important to note that this method of evaluation for the process does not necessarily reflect the similarity of the output reflection colors with those found in the original image. Because of the

(a) *Evaluation of image CarRef.jpg*

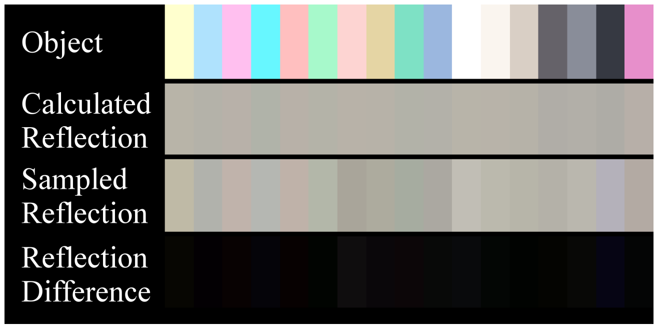
Figure 5.1: Sample Evaluation 1



(a) *Evaluation of image HydrantRef.jpg*

Figure 5.2: Sample Evaluation 2

(a) *carRef.jpg*

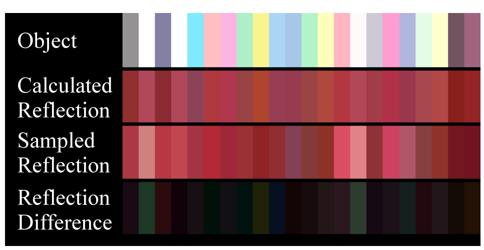


(b) *Evaluation of carRef.jpg*

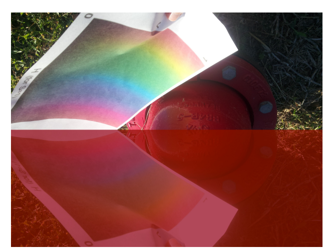
(c) *carRef.jpg, with "reflection" applied*

Figure 5.3: An example of a good transformation resulting in a poor reflection.

(a) *Painting 09*



(b) *Evaluation of hydrantRef.jpg*



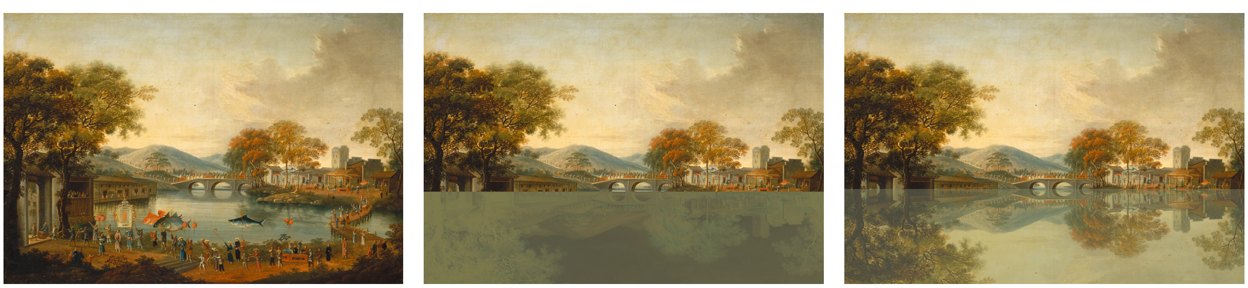
(c) *hydrantRef.jpg, with "reflection" applied*

Figure 5.4: An example of a poor transformation resulting in a good reflection.

method and the evaluation both depending on the sampled data, if there is an error in the data itself, rather than the fit of the linear regression to that data, this method of evaluation will not detect that error. As an example, the evaluation in 5.3 indicates an accurate transformation according to the input data, but the resulting reflection is somewhat inaccurate, being unable to handle blown-out colors that appear as white. In comparison, the evaluation in 5.4b indicates a relatively poor fitting transformation matrix, but the resulting reflection is visually fairly convincing.

5.1.1 Sampling Concerns

The process of matrix color operations as described relies on being able to utilize accurate color reflection samples. It is also important that these samples be diverse, covering a wide area of the color space, or else the transformations provide by the matrix for colors outside of the sampled range may be inaccurate. As an example, an early sampling attempt for the image procession.jpg 5.5a provided results that were wildly different from the reflection as show in the image, although the transformation fit the sample quite well. This bad result is shown at 5.5b. By going back and re-sampling the image afterwards, and making sure to include a variety of colors in the sampled pixels, a more artistically pleasing result was achieved. 5.5c



(a) *Original procession.jpg* (b) *Faulty Sampling.* (c) *Corrected Sampling*

Figure 5.5: A demonstration of the impact that proper sampling can have on the results of the matrix operations.

Example HSVs									
	Sampled Obj.			Sampled Ref.			Calc. Ref.		
name	H	S	V	H	S	V	H	S	V
hydrant	189	50	99	351	68	64	342	52	54
bcPhoto	249	97	59	252	89	75	257	72	50
boatmen	29	45	62	33	54	54	26	40	55

Table 5.1

5.2 Change in Hue

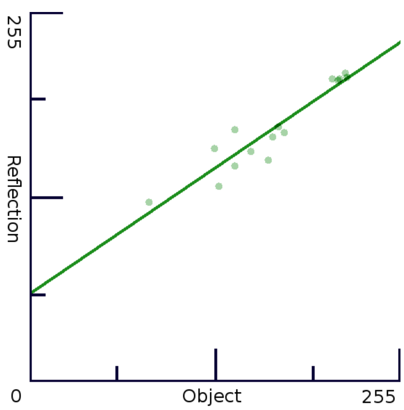
Another method of evaluating the change in color is to compare the Hue of the HSV values of the sampled and calculated reflected image, and see if any difference in hue present in the sample is present in the calculation.

Taking the sampled object and reflection colors and the calculated reflection color and translating all of these RGB values into HSV, the results vary depending on the type of image examined. In images with colored reflective surfaces and in paintings, the hue of the sampled reflection differed to varying extents from the sampled source image. In photographs of non-reflective surfaces, however, the hue of the sampled reflection differed very little from the sampled object, if at all. In both cases, the calculated reflection approximated the hue of the sampled reflection in much the same way as it did the RGB values. Table 5.1 is a selection of sample HSVs, taken from multiple images.

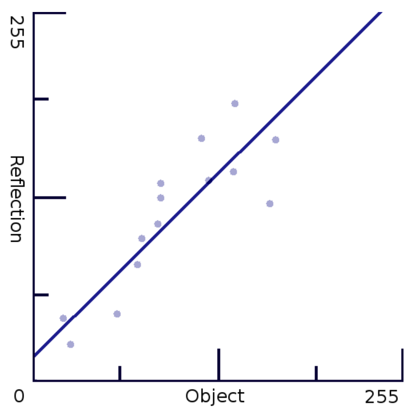
5.3 Linearity

It is necessary to determine whether or not an assumption of linearity is justified for this purpose. The literature on the subject seems to support that a linear model of transformation can be used effectively [10]. To double-check this understanding, I reviewed the graphs of the data which I collected for the sampled object and reflection colors. In most cases, the graph of the points was either highly linear 5.6a, scattered in a vaguely linear fashion 5.6b, or else scattered wildly with very little pattern 5.6c.

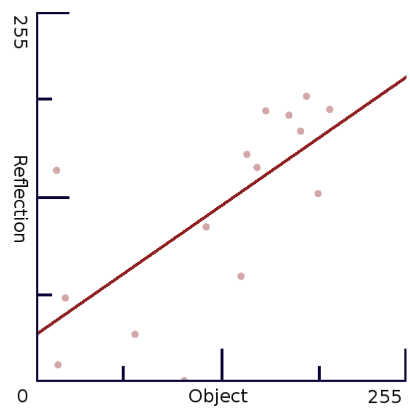
The wide degree of variance of the sampled points from the line of best fit indicates that a considerable amount of error is taking place in the matrix-based approximation of the color transformation. This indicates that some improvement could be made either in sampling methodology or in the method by which the transformation is modeled. Either of these would make excellent subjects for future work.



(a) *beach.jpg G*



(b) *colorMountains.jpg B*



(c) *geese.jpg R*

Figure 5.6: Sample linear regression graphs

6. CONCLUSION AND FUTURE WORK

6.1 Conclusion

By using linear regressions of color samples, it is possible to create matrices that, when applied to the colors of an image, result in believable reflected colors for a variety of surfaces. While these colors are not perfect matches for the original image reflection as hoped, they are still often aesthetically appealing and could be utilized in a variety of artistic applications.

6.2 Future Work

Considering the tendency of the linear regression used with matrix color operations to provide approximations of sampled color transformations rather than interpolating from precise samples, Professor Akleman noted that a different approach to the issue may provide better results. By using a warping transformation on the color space rather than matrix operations, any sampled color transforms should be returned exactly as sampled by the program, with the remaining colors being interpolated so as to provide a coherent color space. In theory, this will provide more accurate results to sampled reflection transformations, as a warping transformation would be less limited than the scaling and translation transformations provided by diagonal matrices, or even the additional rotation, skew, and perspective transformations provided by a fully utilized matrix. While matrix color transformations provide believable reflection imagery for artistic use, a warp transformation would hopefully provide a better fit for the purposes of imitation and compositing.

My ability to investigate the possiblities of full 4×4 matrices was limited over the course of this thesis. Observing more examples of full 4×4 matrices in use would help improve understanding on how full matrices differ from diagonal matrices, and to what extent.

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APPENDIX A

EXAMPLE IMAGES

In this Appendix are all the examples of diagonal 4x4 matrices applied to images created for this thesis, as described in the thesis itself.

A.1 Student-taken Photographs

(a) *Original image* (b) *Image mirrored left to right.* (c) *Reflected color samples*

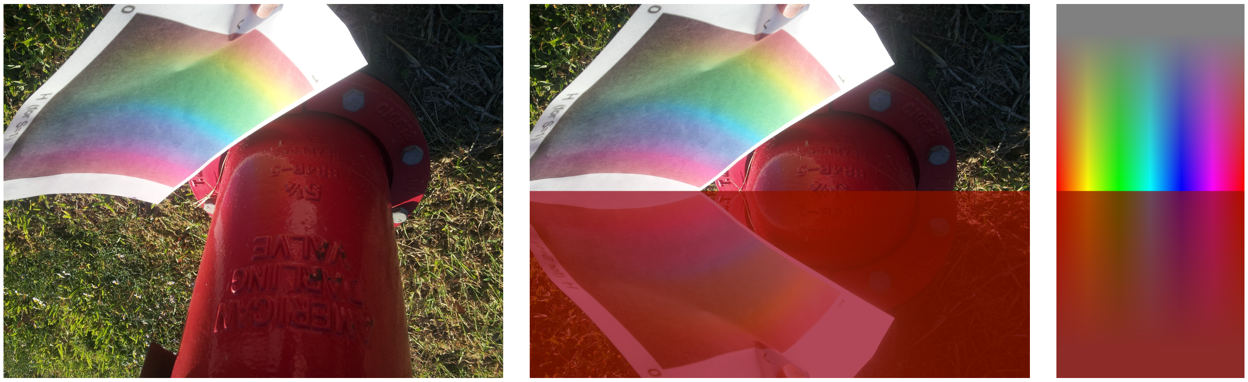
Figure A.1: Car Reflection

(a) *Original image* (b) *Image mirrored vertically.* (c) *Reflected color samples*

Figure A.2: Chair Reflection

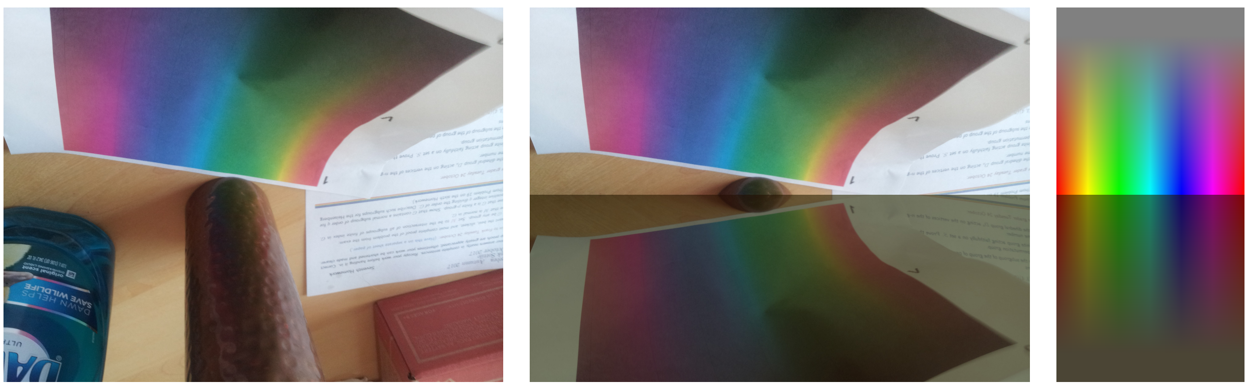
(a) *Original image* (b) *Image mirrored left to right.* (c) *Reflected color samples*

Figure A.3: Break Light Reflection



(a) *Original image* (b) *Image mirrored vertically.* (c) *Reflected color samples*

Figure A.4: Fire Hydrant Reflection



(a) *Original image* (b) *Image mirrored vertically.* (c) *Reflected color samples*

Figure A.5: Flask Reflection

(a) *Original image* (b) *Image mirrored vertically.* (c) *Reflected color samples*

Figure A.6: Soap Reflection

A.2 Public Domain Images

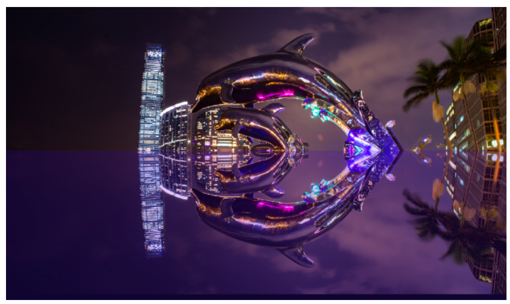

(a) *Original image*


(b) *Image mirrored*


(c) *Reflected color samples*

Figure A.7: Abbey


(a) *Original image*


(b) *Image mirrored*

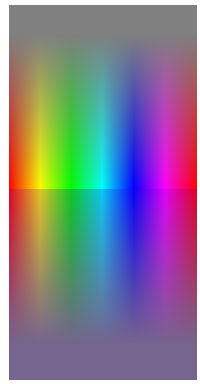
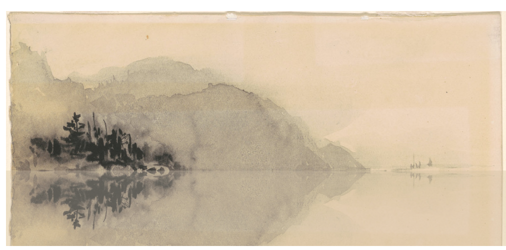

(c) *Reflected color samples*

Figure A.8: Bcphoto


(a) *Original image*


(b) *Image mirrored*

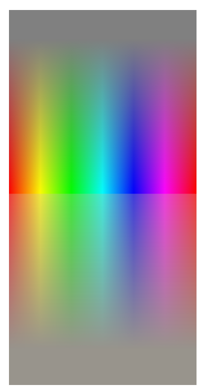
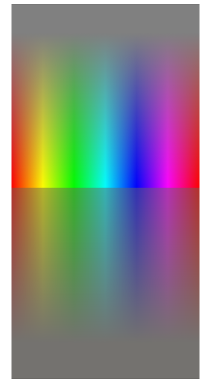

(c) *Reflected color samples*

Figure A.9: Beach

(a) *Original image*

(b) *Image mirrored*



(c) *Reflected color samples*

Figure A.10: BnwBoat

(a) *Original image*

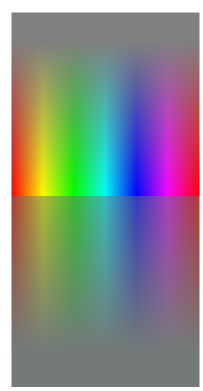
(b) *Image mirrored*

(c) *Reflected color samples*

Figure A.11: Boatmen

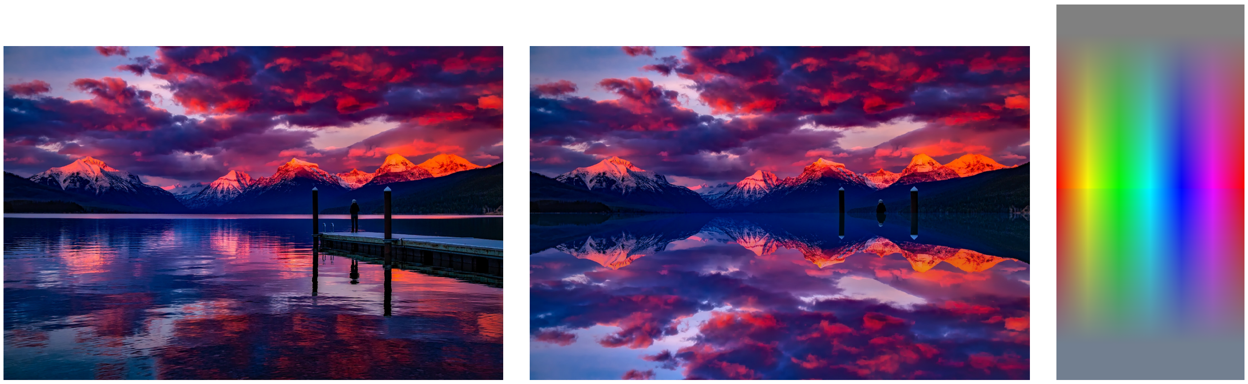
(a) *Original image*

(b) *Image mirrored*



(c) *Reflected color samples*

Figure A.12: Buffalo



(a) *Original image* (b) *Image mirrored* (c) *Reflected color samples*

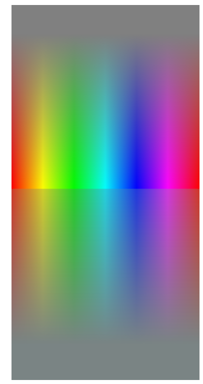
Figure A.13: ColorMountain

(a) *Original image* (b) *Image mirrored* (c) *Reflected color samples*

Figure A.14: Dock

(a) *Original image*

(b) *Image mirrored*

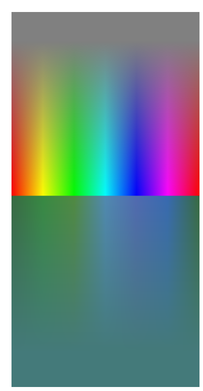


(c) *Reflected color samples*

Figure A.15: Geese

(a) *Original image*

(b) *Image mirrored*



(c) *Reflected color samples*

Figure A.16: GgPhoto01

(a) *Original image* (b) *Image mirrored* (c) *Reflected color samples*

Figure A.17: GgPhoto02

(a) *Original image* (b) *Image mirrored* (c) *Reflected color samples*

Figure A.18: LakeAlbano

(a) *Original image* (b) *Image mirrored* (c) *Reflected color samples*

Figure A.19: LakeGeorge


(a) *Original image*


(b) *Image mirrored*


(c) *Reflected color samples*

Figure A.20: Mailboat


(a) *Original image*


(b) *Image mirrored*

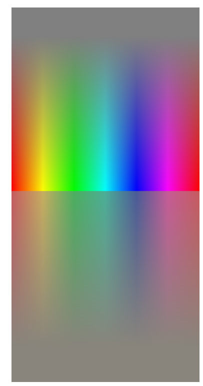

(c) *Reflected color samples*

Figure A.21: MountainLake


(a) *Original image*


(b) *Image mirrored*

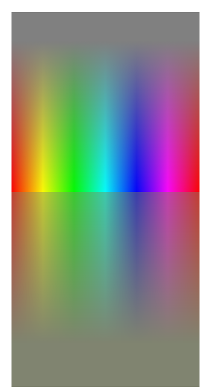
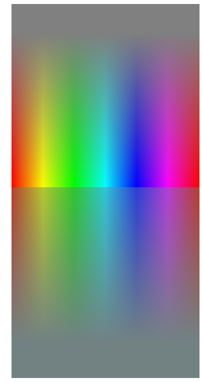

(c) *Reflected color samples*

Figure A.22: Procession

(a) *Original image*

(b) *Image mirrored*

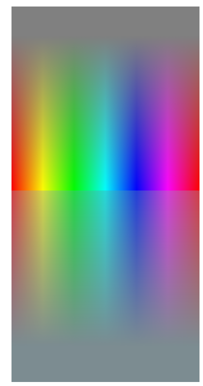


(c) *Reflected color samples*

Figure A.23: Riverland

(a) *Original image*

(b) *Image mirrored*

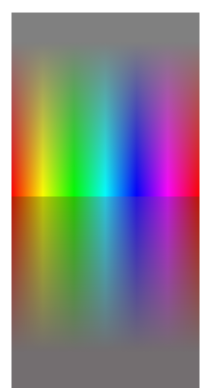


(c) *Reflected color samples*

Figure A.24: Sailboat

(a) *Original image*

(b) *Image mirrored*



(c) *Reflected color samples*

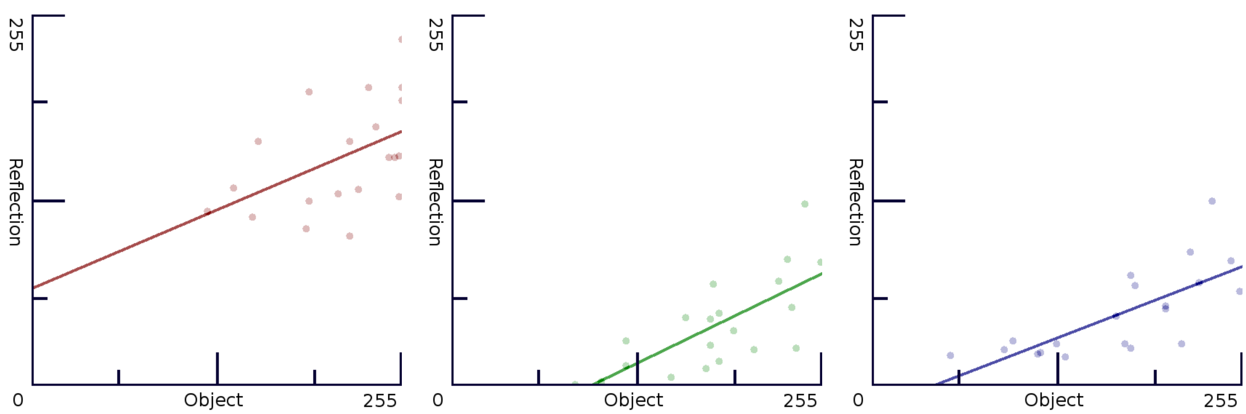
Figure A.25: Village

APPENDIX B

LINEAR REGRESSION GRAPHS

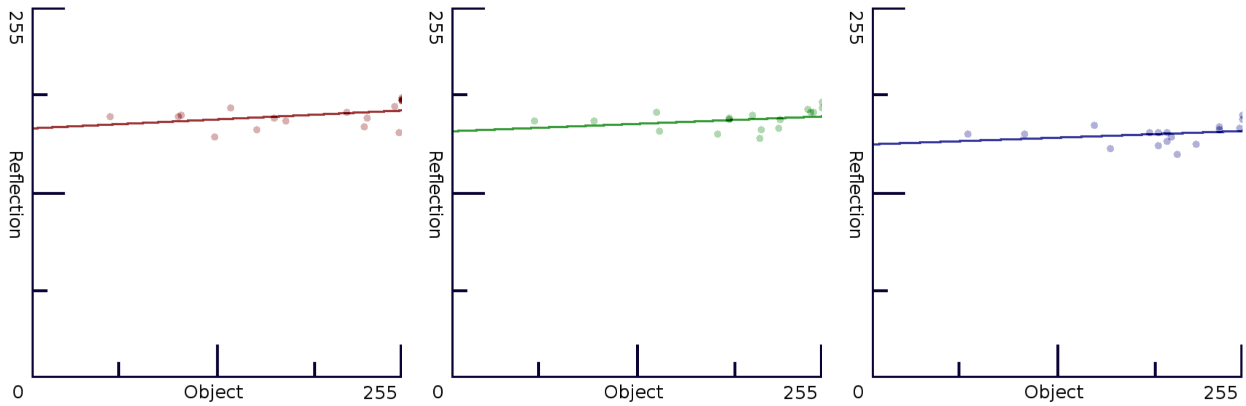
The following are graphs of the input and output colors (corresponding to object color and reflected color), intended for the purpose of examining the true linearity of the transformation. The color data for each image is split into red, green and blue channels. Input color is placed along the X axis, and output color is placed along the Y axis. The data points are shown along with the line of best fit determined by linear regression.

In general, the points on the graphs either appear fairly linear, making linear regression a good fit, or else scattered, with high amounts of potential error.



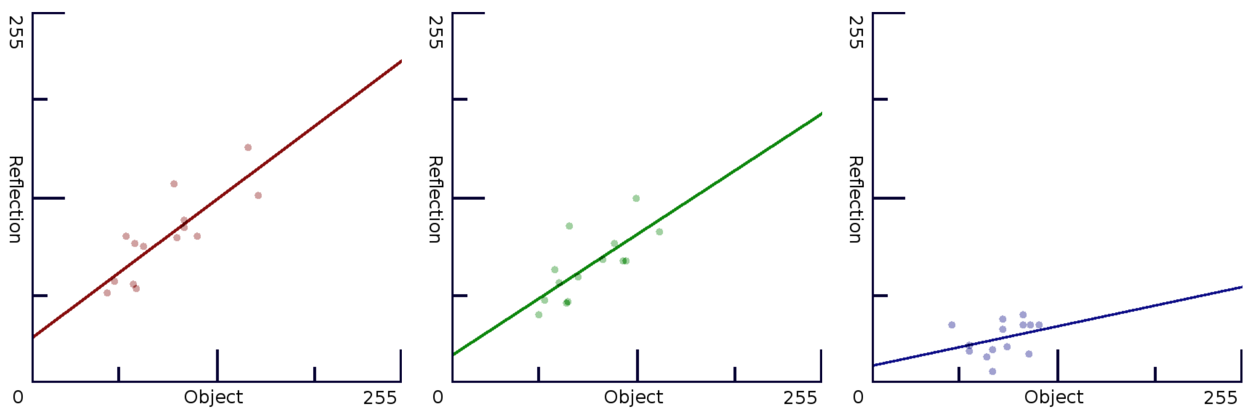
(a) *Red channel* (b) *Green Channel* (c) *Blue Channel*

Figure B.1: breakLightRef.png linear regression graphs



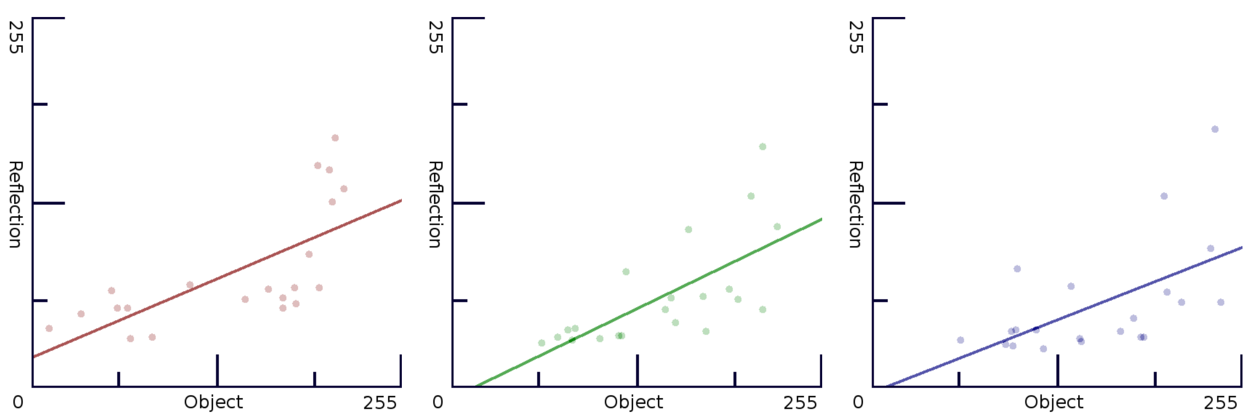
(a) *Red channel* (b) *Green Channel* (c) *Blue Channel*

Figure B.2: car.png linear regression graphs



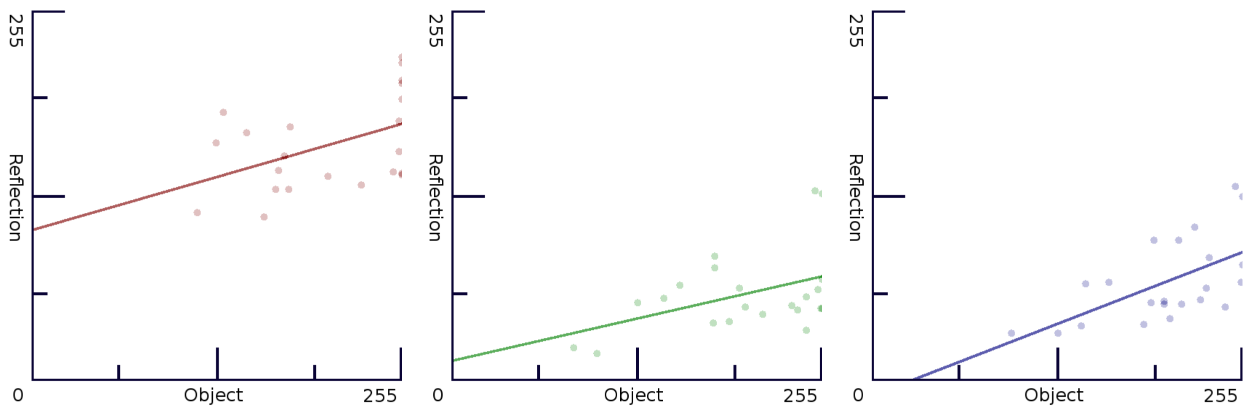
(a) *Red channel* (b) *Green Channel* (c) *Blue Channel*

Figure B.3: chair.png linear regression graphs



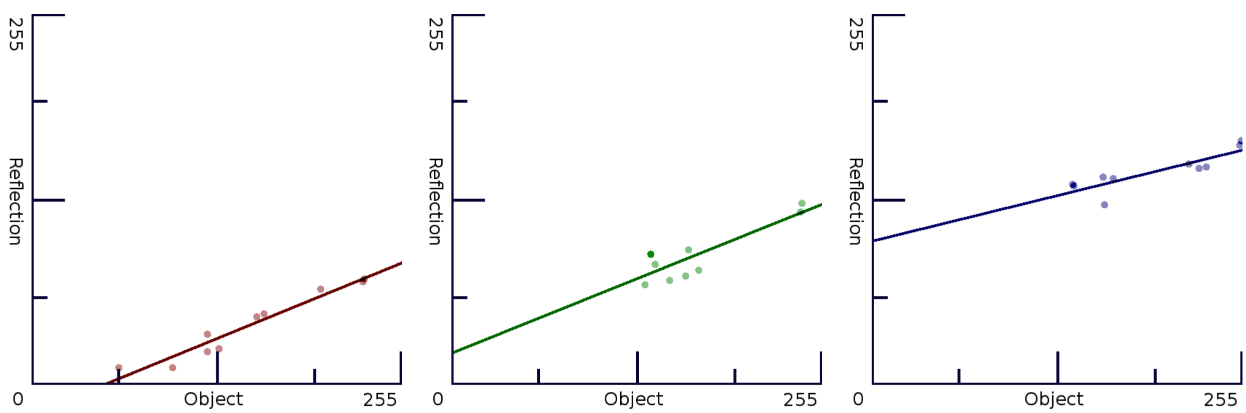
(a) *Red channel* (b) *Green Channel* (c) *Blue Channel*

Figure B.4: flask.png linear regression graphs



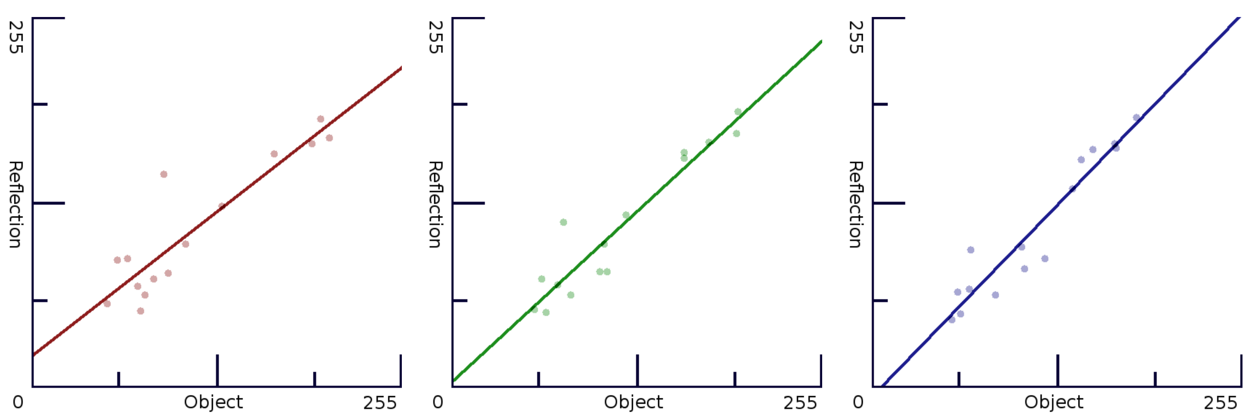
(a) *Red channel* (b) *Green Channel* (c) *Blue Channel*

Figure B.5: hydrant.png linear regression graphs



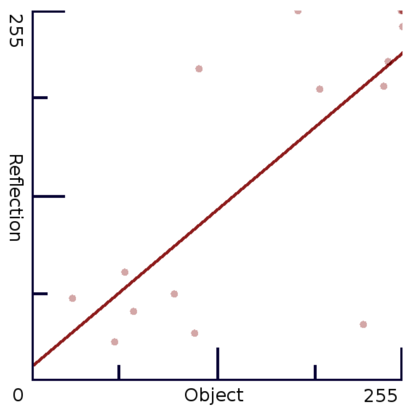
(a) *Red channel* (b) *Green Channel* (c) *Blue Channel*

Figure B.6: soap.png linear regression graphs

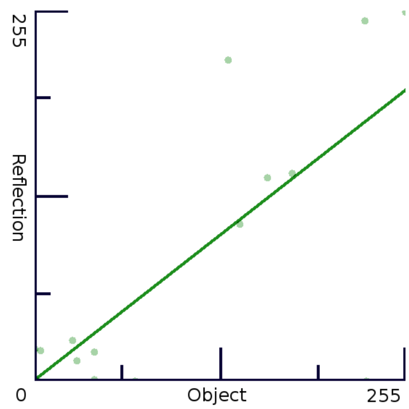


(a) *Red channel* (b) *Green Channel* (c) *Blue Channel*

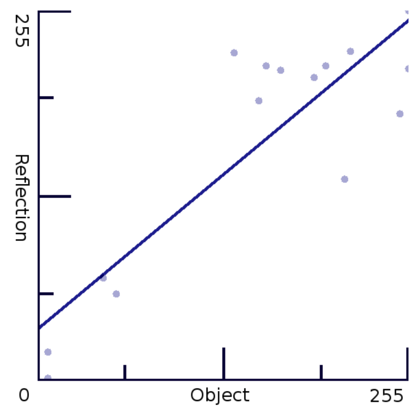
Figure B.7: abbey.jpg linear regression graphs



(a) *Red channel*

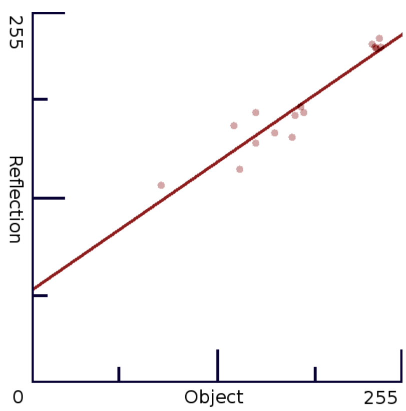


(b) *Green Channel*

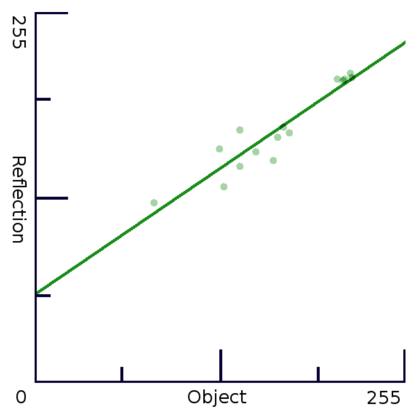


(c) *Blue Channel*

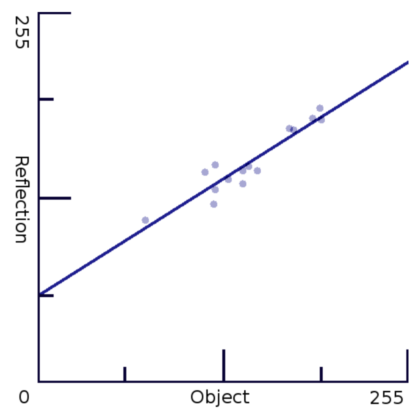
Figure B.8: bcPhoto.jpg linear regression graphs



(a) *Red channel*

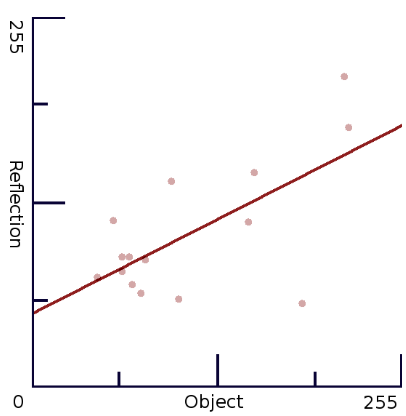


(b) *Green Channel*

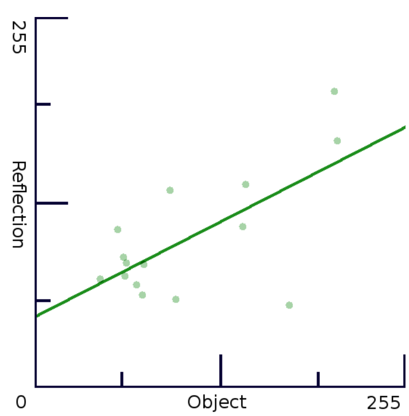


(c) *Blue Channel*

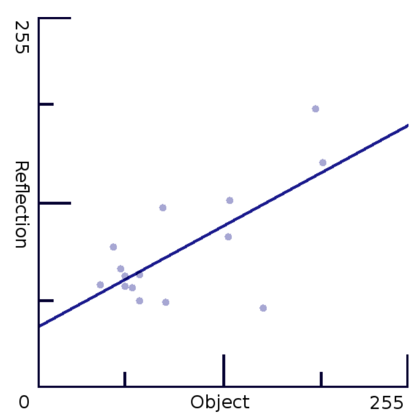
Figure B.9: beach.jpg linear regression graphs



(a) *Red channel*



(b) *Green Channel*

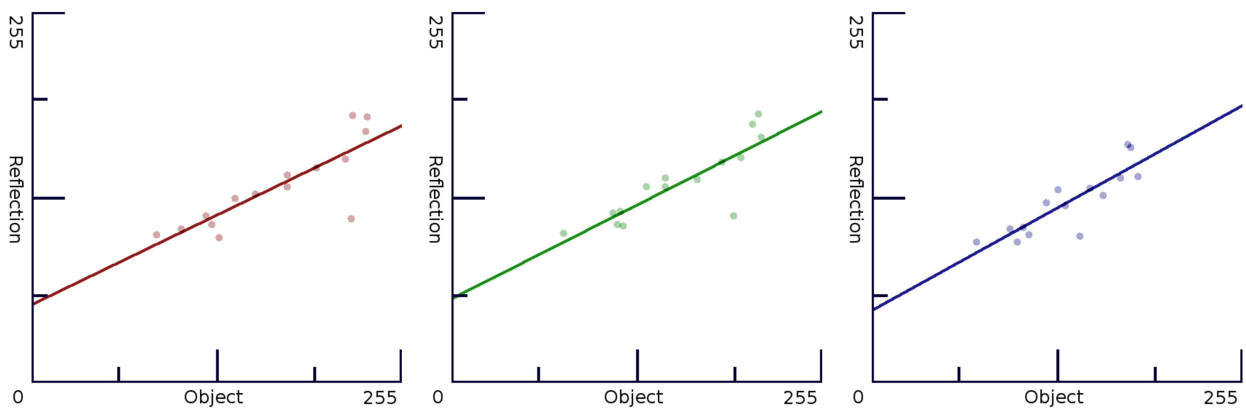


(c) *Blue Channel*

Figure B.10: bnwBoat.jpg linear regression graphs

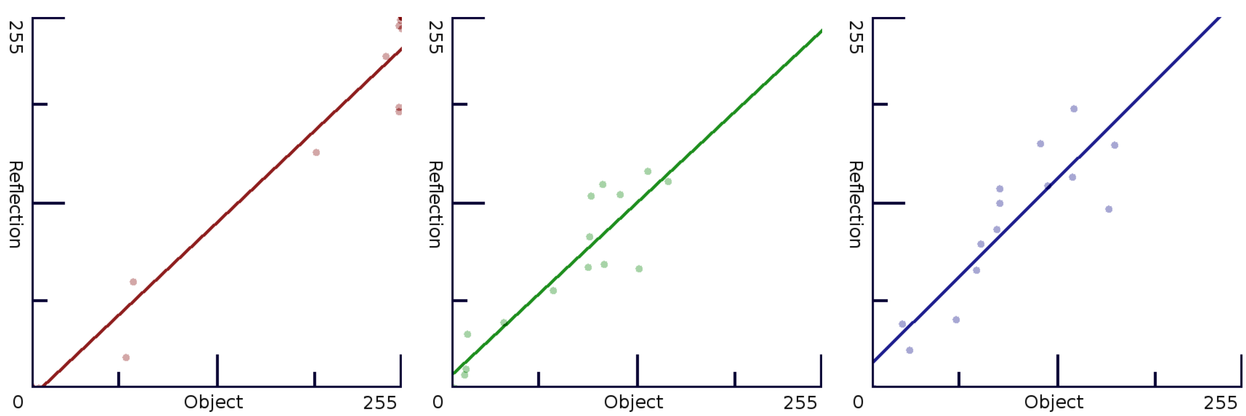
(a) *Red channel* (b) *Green Channel* (c) *Blue Channel*

Figure B.11: boatmen.jpg linear regression graphs



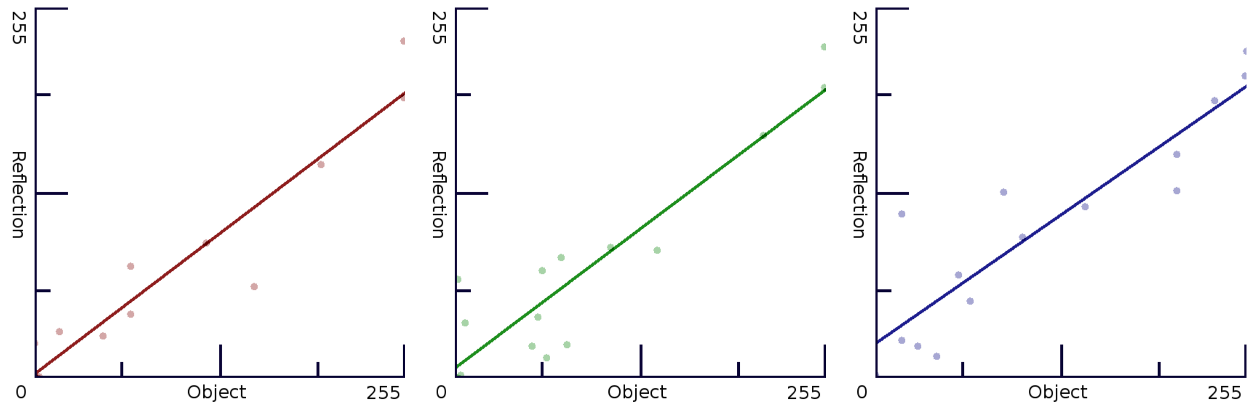
(a) *Red channel* (b) *Green Channel* (c) *Blue Channel*

Figure B.12: buffalo.jpg linear regression graphs



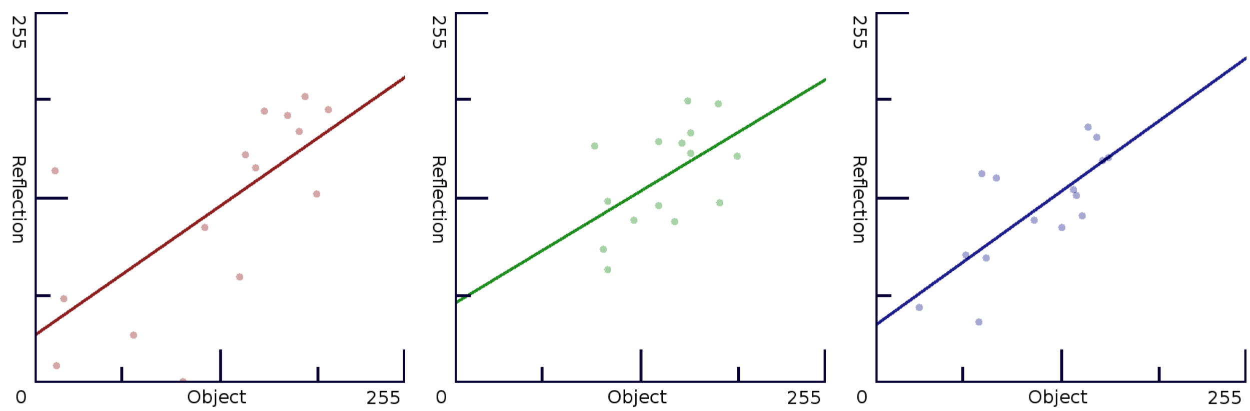
(a) *Red channel* (b) *Green Channel* (c) *Blue Channel*

Figure B.13: colorMountains.jpg linear regression graphs



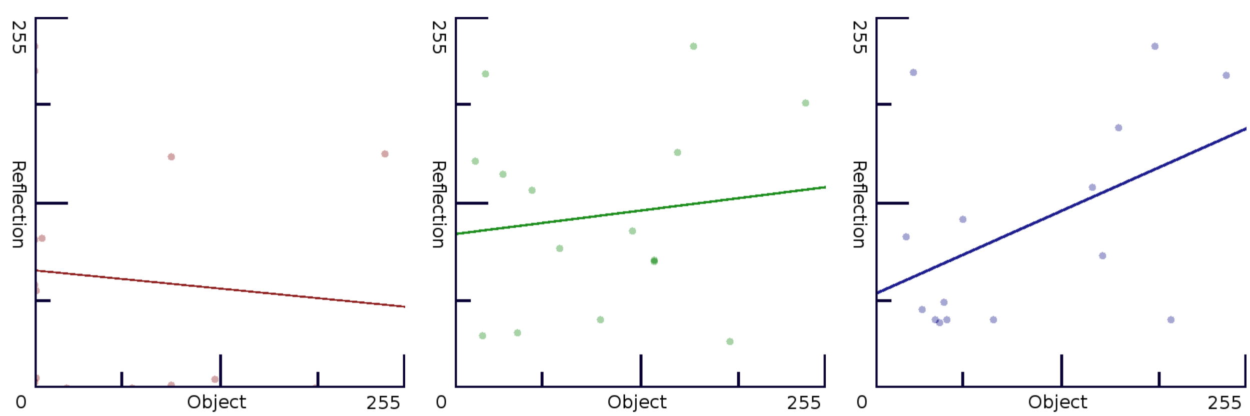
(a) *Red channel* (b) *Green Channel* (c) *Blue Channel*

Figure B.14: dock.jpg linear regression graphs



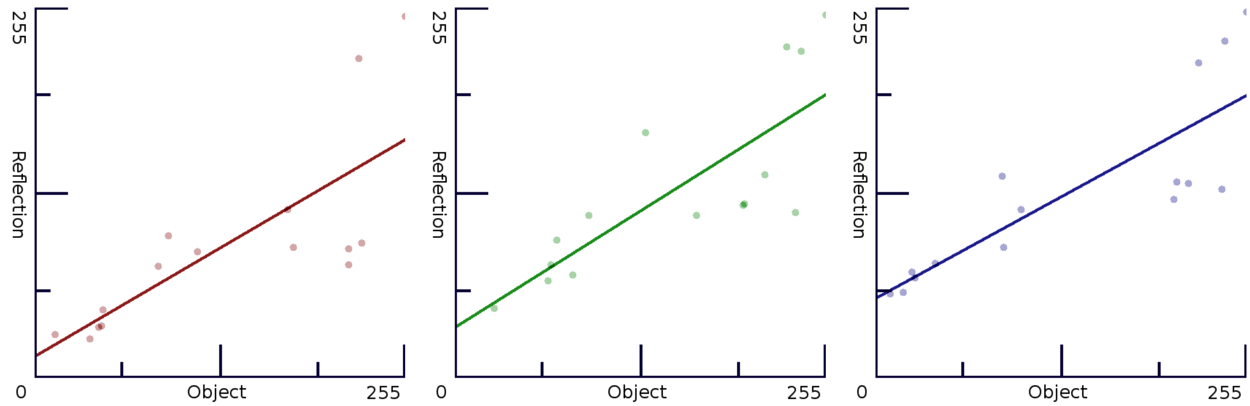
(a) *Red channel* (b) *Green Channel* (c) *Blue Channel*

Figure B.15: geese.jpg linear regression graphs



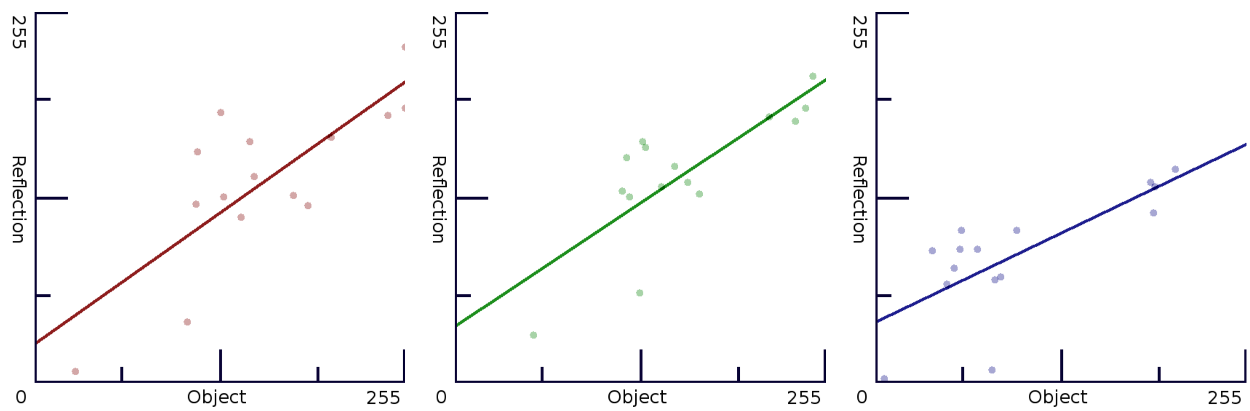
(a) *Red channel* (b) *Green Channel* (c) *Blue Channel*

Figure B.16: ggPhoto01.jpg linear regression graphs



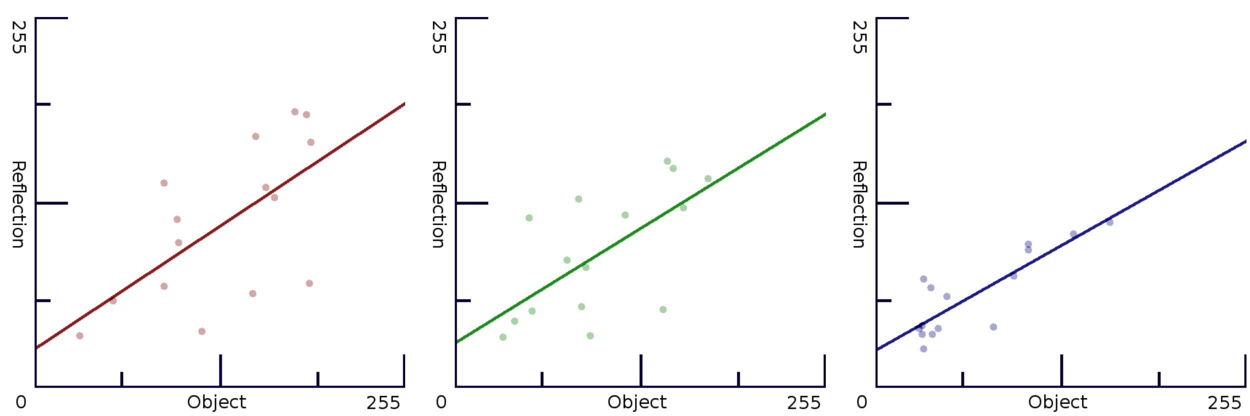
(a) *Red channel* (b) *Green Channel* (c) *Blue Channel*

Figure B.17: ggPhoto02.jpg linear regression graphs



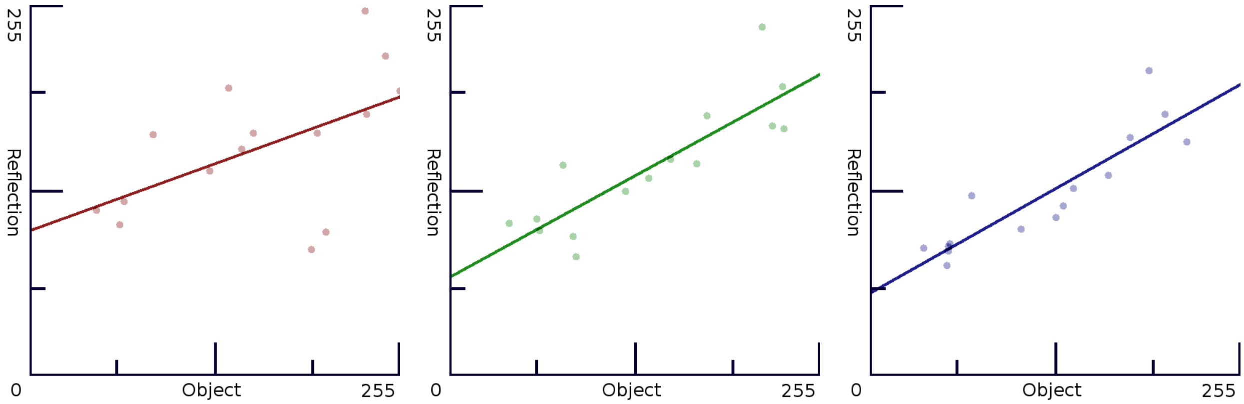
(a) *Red channel* (b) *Green Channel* (c) *Blue Channel*

Figure B.18: lakeAlbano.jpg linear regression graphs



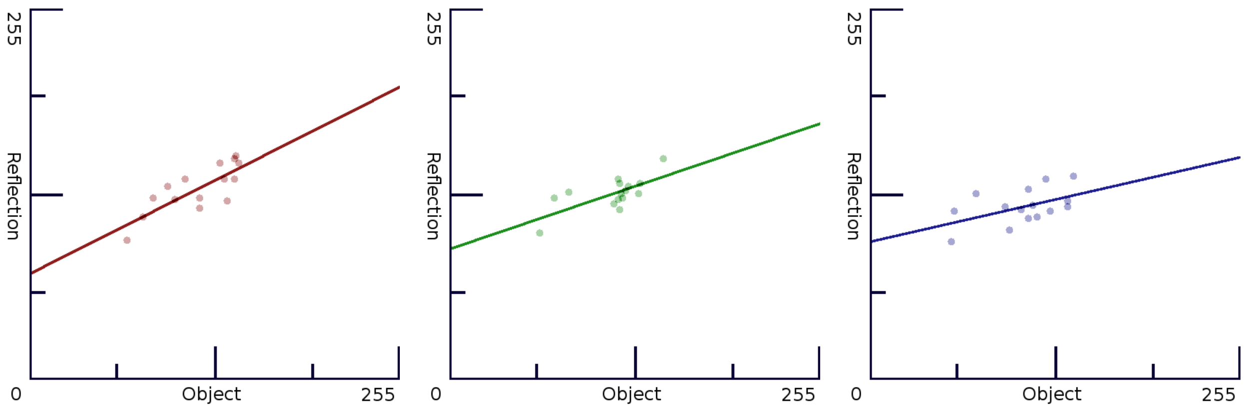
(a) *Red channel* (b) *Green Channel* (c) *Blue Channel*

Figure B.19: lakeGeorge.jpg linear regression graphs



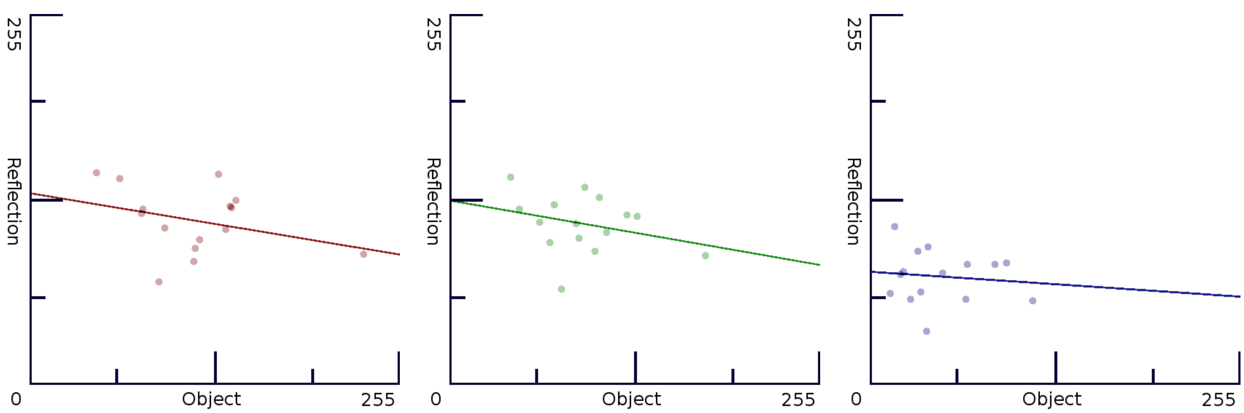
(a) *Red channel* (b) *Green Channel* (c) *Blue Channel*

Figure B.20: mailboat.jpg linear regression graphs



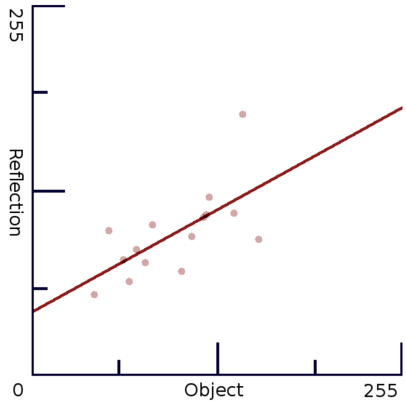
(a) *Red channel* (b) *Green Channel* (c) *Blue Channel*

Figure B.21: mountainLake.jpg linear regression graphs

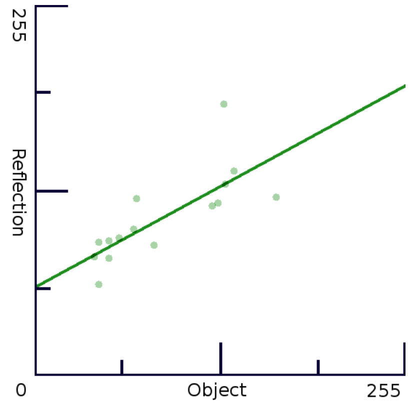


(a) *Red channel* (b) *Green Channel* (c) *Blue Channel*

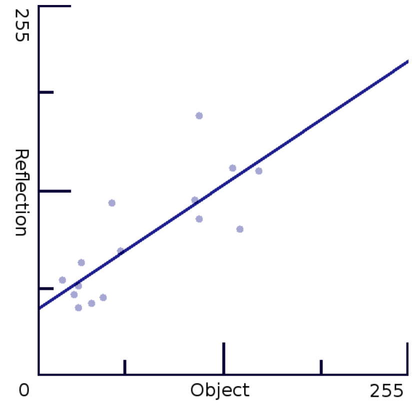
Figure B.22: procession.jpg linear regression graphs



(a) *Red channel*

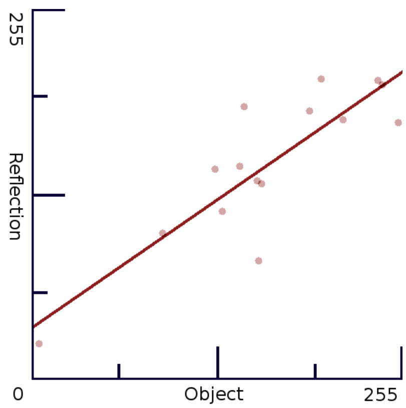


(b) *Green Channel*

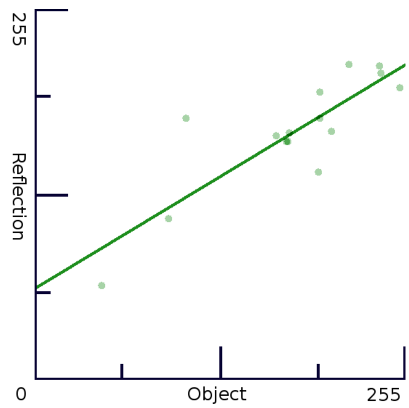


(c) *Blue Channel*

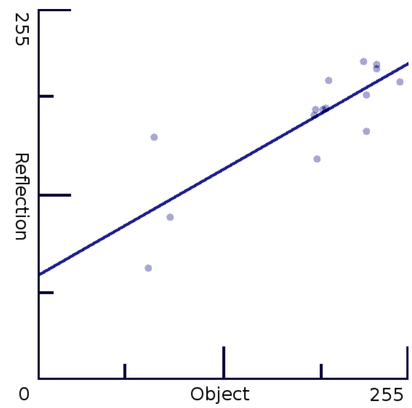
Figure B.23: riverland.jpg linear regression graphs



(a) *Red channel*

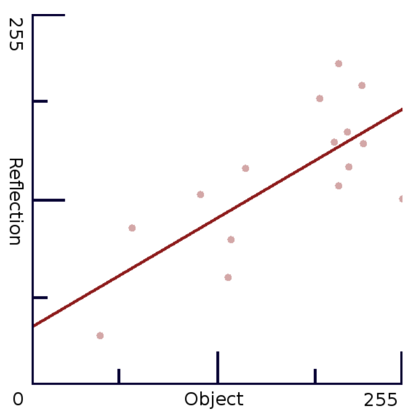


(b) *Green Channel*

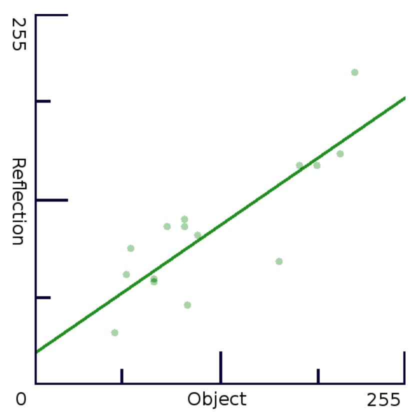


(c) *Blue Channel*

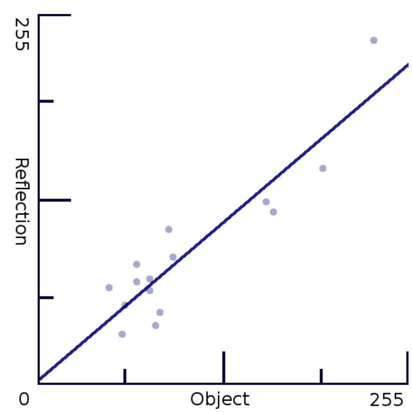
Figure B.24: sailboat.jpg linear regression graphs



(a) *Red channel*



(b) *Green Channel*



(c) *Blue Channel*

Figure B.25: village.jpg linear regression graphs