NEW BOUNDING TECHNIQUES FOR CHANNEL CODES OVER QUASI-STATIC FADING CHANNELS

A Thesis

by

JINGYU HU

Submitted to the Office of Graduate Studies of
Texas A&M University
in partial fulfillment of the requirements for the degree of

MASTER OF SCIENCE

August 2003

Major Subject: Electrical Engineering

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ABSTRACT

New Bounding Techniques for Channel Codes over Quasi-static Fading Channels.

(August 2003)

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Chair of Advisory Committee: Dr. Scott L. Miller

This thesis is intended to provide several new bounding techniques for channel codes over quasi-static fading channels (QSFC). This type of channel has drawn more and more attention recently with the demanding need for higher capacity and more reliable wireless communication systems. Although there have been some published results on analyzing the performance of channel codes over QSFCs, most of them produced quite loose performance upper bounds.

In this thesis, the general Gallager bounding approach which provides convergent upper bounds of coded systems over QSFCs is addressed first. It is shown that previous Gallager bounds employing trivial low SNR bounds tended to be quite loose. Then improved low instantaneous SNR bounds are derived for two classes of convolutional codes including turbo codes. Consequently, they are combined with the classical Union-Chernoff bound to produce new performance upper bounds for simple convolutional and turbo codes over single-input single-output (SISO) QSFCs. The new bound provides a much improved alternative to characterizing the performance of channel codes over QSFCs over the existing ones.

Next the new bounding approach is extended to cases of serially concatenated space-time block codes, which show equivalence with SISO QSFCs. Tighter performance bounds are derived for this coding scheme for two specific cases: first a convolutional code, and later a turbo code. Finally, the more challenging cases of multiple-

input multiple-output (MIMO) QSFCs are investigated. Several performance upper bounds are derived for the bit error probability of different cases of space-time trellis codes (STTC) over QSFCs using a new and tight low SNR bound. Also included in this work is an algorithm for computing the unusual information eigenvalue spectrum of STTCs.

To my parents, Yunfeng Hu and Mingjing Li and my wife Ruyun Gao

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CHAPTER I

INTRODUCTION

Fading is known as a common phenomenon in modern wireless systems. It happens when communication over the wireless channel suffers from multiple superimposed delayed versions of the transmitted signal. A fading channel is said to be either frequency flat or selective due to multipath time delay spread. If the channel possesses a constant gain and linear phase response over a bandwidth that is smaller than the signal bandwidth, then the fading is frequency selective, if not the fading is frequency flat. Depending on how rapidly the transmitted baseband signal changes as compared to the rate of change of the channel, a channel is classified as either a fast fading or slow fading channel. A fading is said to be fast or time-selective if the channel gain changes rapidly within the symbol duration of the transmitted signal.

The channel of particular interest in this thesis is frequency flat and very slow whose fading has little variations in time to the point where it can be considered constant over a transmission frame. This type of channel is known in the literature as the quasi-static fading channel (QSFC). Increasing data rates along with demand for higher capacities and more reliable connections in today's wireless communication systems have motivated a large interest in this type of channel. For example, in a classical time-division multiuser system over a fading channel, each user has a very short period of time (slot) to transmit signals over which the channel fading gain remains relatively constant. However, the fading process may have great variations within a frame. The type of fading here can safely be characterized as being static. When the data rate is quite high, this is especially true. Practical communications

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The journal model is *IEEE Transactions on Automatic Control.*

systems include GSM and IS-136.

Communication over this kind of channel has proven quite challenging especially for channel code design. Unlike in the case of fast fading where the use of interleaving allows recovering from the loss of information occurring during deep fades, there is no time or frequency from which to earn a diversity gain. Thus, the performance of a single code such as a convolutional or turbo code with no transmit and receive diversity on a QSFC would be severely degraded as compared to the cases of the AWGN channel and interleaved fading channels. The main reason that a QSFC affects communications is that in such a channel the instantaneous signal-to-noise ratio (SNR) varies independently from one frame to the next, some of the frames sent over it may have very low channel gain causing low instantaneous SNR over the length of the frames. Consequently, the transmitted signal will be lost in the ambient gaussian noise and the channel will suffer poor performance even at very high average SNRs. In addition, all code designs relying on maximizing the free distance will be undermined for the QSFC because they assume that at a high enough SNR, the error rate performance of a code can be approximated by a single term in the sum of all possible error events while in a QSFC, many frames experience very low SNRs thus the performance of a code over QSFCs cannot be characterized by the worst pairwise error probability (PWEP) event.

However, The quality of the communications over QSFCs can be substantially improved by using diversity which means the transmission of redundant signals over preferably independent channels. Two common ways to achieve diversity are time diversity and space (antenna) diversity. Time diversity is achieved mostly by using channel coding or error correction codes. Space diversity means employing multiple transmit or received antennas. Codes combining space and time diversity are known as space-time codes which has gained more and more interest in both the industry

and academic community in recent years. Although there has been much research work done to develop new channel codes for QSFCs, there is relatively little done with the analysis of their performance. There is a great need to synthesize reliable analytical tools for characterrizing the performance of channel codes over QSFCs.

This thesis is set out to propose several novel performance upper bounds for various coded systems over QSFCs. The main part of the thesis is preceded by a brief overview of past research done in this area. In this part, the so-called Gallager bound is discussed to show that the looseness of the previous bounds comes from the trivial form of the low SNR part of the Gallager bound.

Chapter III is dedicated to introducing a new approach to classify convolutional codes and comparing their performance over single-input single-output (SISO) QSFCs. The results given in this chapter allow later chapters to focus attentions on developing improved upper bounds for two specific classes of convolutional codes - systemtic codes and differential detectable (DD) codes. It is noted that turbo codes also belong to systematic codes. Chapter IV looks into deriving two new upper bounds correspondingly for the bit error rates of these two classes of convolutional codes in the low instantaneous SNR region. Then it is shown that all the new bounds represent improvement over the trivial low SNR bound.

Consequently, these new low SNR upper bounds are applied to cases involving convolutional and turbo codes over SISO QSFCs in Chapter V. In that chapter, upper bounds using a combination of a classical union bound when the fading channel is in a high SNR state with the new upper bounds in Chapter IV for the low SNR state are presented. This new bounding technique produces bounds which are at least 1dB tighter than the existing ones. The following chapter investigates the cases involving space-time block codes which is shown to be equivalent to SISO QSFCs. The new bounding approach is applied to derive improved upper bounds for the serial

concatenations of convolutional/turbo and space-time block codes (STBC).

In Chapter VII, the performance of space-time trellis codes (STTC) over multiple-input multiple-output (MIMO) QSFCs is addressed. A spherical upper bound on the bit error rates of systematic 4-state STTCs employing single receive antenna and an cubical upper bound for multiple receive antennas cases are derived using a low SNR bounding approach. Additionally, a brief procedure for computing the information eigenvalue spectrum similar to the one introduced formerly for computing the eigenvalue spectrum of STTCs is described. Finally, Chapter VII concludes the thesis.

CHAPTER II

PRESENT STATUS OF THE RESEARCH

In this chapter, we present a summary review of the past research done in performance analysis of channel codes over QSFCs. First a simple method which uses the direct extension of the union bound is described. Following this is a brief review of the Gallager bound along with its applications to various coded systems.

A. Modified union bound for QSFCs

If the union bound approach is simply extended to the QSFC case, it can be applied in a conditional form of the pairwise error probability (PWEP). The conditional PWEP is then a function of the channel gain, which needs to be integrated out. Let h be the magnitude of the complex channel gain, which will be Rayleigh distributed. For example, for a simple convolutional code over a QSFC, the probability of bit error can be written as:

$$P_b \leq \sum_d N_d \int_0^{+\infty} Q\left(\sqrt{\frac{2dh^2E_s}{N_0}}\right) f(h)dh, \tag{2.1}$$

where N_d is the information weight spectrum of the code and $f(h)$ the probability density function (pdf) of the fading gain magnitude. The equations beyond are derived assuming that the distance spectrum is the same regardless of the transmitted codeword and perfect channel state information is available at the receiver.

Using the Chernoff bound on the Q function [1], the integration in (2.1) is performed without difficulties resulting in:

$$P_b \leq \sum_d N_d \frac{1}{2 + 2dE_s/N_0}. \tag{2.2}$$

It is obvious that the PWEP in (2.2) does not decrease exponentially with in-

creasing distance of d while it is known that N_d increases exponentially with d. Hence the overall bound is not convergent for any SNRs and is thus invalid for such codes over QSFCs. The reason underlying is that for a QSFC the received instantaneous SNR for a given frame can be quite low and it is well established that the union bound diverges for low SNRs, hence the use of the union bound over a QSFC is questionable.

B. Gallager bound for QSFCs

Past research provided valid upper bounds on codes performance over QSFCs. For example, the so-called Gallager bound which is somewhat similar to the one introduced by Gallager in [2] has been used extensively. In this novel approach, the error probability (P_e) is conditioned on an instantaneous channel gain region R and its complement $\overline{R}$. R is a region such that when the channel gain h falls within its boundaries the union bound is assumed to diverge and the probability of making an error will be upper bounded by one for frame error rates and one half for bit error rates. The general form of the Gallager bound is as follows:

$$P_e = P(e|h \in R)P(h \in R) + P(e|h \in \overline{R})P(h \in \overline{R}), \tag{2.3}$$

$$\leq \frac{1}{2}P(h \in R) + P(e|h \in \overline{R})P(h \in \overline{R}). \tag{2.4}$$

Although this approach seems quite simplistic, it will still yield quite good upper bounds. One of its nice features is that the conditional PWEP decreases exponentially with the increase of distance while the distance spectrum of codes of interest increases exponentially.

1. Gallager bounds for convolutional and turbo codes

For a single-input single-output (SISO) QSFC, the region R is simply a segment of the real axis. Hence the error probability P_e is conditioned on a low/high instantaneous SNR ($h^2 E_s/N_0$ where h is the complex channel gain magnitude) region, which is translated into the following equation:

$$P_e = P(e|h < h_0)P(h < h_0) + P(e|h > h_0)P(h > h_0). \tag{2.5}$$

The classical Union-Chernoff bound [1] is generally used for the high instantaneous SNR region ($h > h_0$), while for the low instantaneous SNR region, a trivial bound which takes the form of

$$P(e|h < h_0) \leq \begin{cases} 1/2 & \text{for bit error rate,} \\ 1 & \text{for frame error rate,} \end{cases} \tag{2.6}$$

is commonly used. Then the final form of the bit error rate bound is expressed as:

$$P_b \leq \frac{1}{2}(1 - e^{-h_0^2}) + \sum_d N_d \frac{e^{-(1+\gamma d)h_0^2}}{2 + 2\gamma d}. \tag{2.7}$$

Where $\gamma = E_s/N_0$ is the signal-to-noise ratio of the coded symbol, d is the Hamming distance between two output codewords and N_d its corresponding multiplicity, taking into account the number of bit errors. The minimization of the final expressions for the derived bounds is performed with respect to h_0^2.

Several published studies have applied this general approach to bounding convolutional code performance over QSFCs such as [9]-[12], and turbo codes performance such as [11],[15]. Although those bounds were convergent for all SNRs, they still tended to be quite loose. In some cases, they were almost $4 \sim 5$dB away from the actual simulated performance. One of the reason is that the trivial low SNR bound is not tight enough, although the high SNR term in (2.5) can be tightened by using

bounds other than the union bound proposed in [6]-[8]. Hence (2.7) is not a reliable analytical tool to characterize the performance of such codes over QSFCs. This has motivated the search for a tighter upper bound for the low instantaneous SNR region to improve the tightness of the Gallager bound in [20].

2. Gallager bounds for transmit diversity schemes

a. Space-time block code design in QSFC

A simple coding scheme that provides both transmit diversity and coding gain is a serial concatenation. In this serial concatenation, the outer code provides a trade-off between desired coding gain and rate loss and the inner code is an orthogonal space-time block code (STBC) providing diversity. STBCs were firstly introduced in [3] and latter generalized to more than two transmit antenna diversity in [5] using orthogonal designs.

Performance of serially concatenated STBCs over fading channels has been studied in [26],[27],[28],[29]. The Gallager bound was used in the performance analysis of such diversity schemes in several published works. For example, one concatenated scheme involving TCM codes as outer codes was proposed in [4] and then discussed in details in [30] which gave a tight upper bound on the frame error rate (FER) of its performance over QSFCs. Besides, in [11] and [15], an upper bound on the bit error rate of concatenation schemes employing turbo codes as outer codes and an inner Alamouti code proposed in [3] was presented. However, that bound was not quite tight (around $4 \sim 5$dB loose). In [20], tighter upper bounds for concatenated schemes over QSFCs were given.

b. Space-time trellis code design in QSFC

Space-time trellis codes (STTC) have the ability to provide both diversity and coding gain at the expense of higher complexity. It has proved a challenging and popular research orientation among the academic community. Several famous STTCs were presented in [22], [23],[24] and their performance results over QSFCs given. In [11],[23] and [25], it was pointed out that the overall performance of STTCs over QSFCs cannot be characterized by the worst case PWEP, hence the determinant criterion proposed set forth in [22] is a poor criterion by which to design good STTCs for QSFCs.

The performance analysis of STTCs using Gallager bounds over QSFCs was first studied in [12], [13], later in [14]-[18]. In [13], a so-called minimum upper bound using the Gallager approach gave the tightest bound on FER, but it was not in closed form, hence gave little insight into code design. In [14] and [17], the region R had two choices: spherical and cubical as described and the resulting bounds are in closed forms and quite effective in predicting performance of STTCs. Additionally, a general procedure along with several complexity reduction techniques for computing the eigenvalue spectrum of different constraint length and frame length STTCs was proposed in [11], [17] and [19].

However, there has been little analytical study of the bit error rates (BER) of STTCs over QSFCs. In [21], several approximations of the BER of STTCs over QSFCs were presented, but they didn't provide valid upper bounds. Tight performance upper bounds for the bit error probability of STTCs are still needed.

C. Summary

This chapter was dedicated to reviewing the past work done in bounding performance of channel codes over quasi-static fading channels. The extension of the union bound to the QSFC was performed and then the Gallager bounding approach described. The next chapter will deal with the first step for developing tighter performance upper bounds – classifying convolutional codes.

CHAPTER III

CLASSIFICATION OF CONVOLUTIONAL CODES

A. Introduction

Although few studies have been focused on the design of single convolutional codes in a QSFC due to their poor performance, it is rather worthy to spend some time studying them to generalize a more reliable performance analytical tool for the QSFC from the simplest case.

B. Different classes of convolutional codes

A rate $1/n$ constraint length K feed-forward convolutional code can be represented by n $K-$tap connection vectors

$$g^{(j)} = (g_0^{(j)}, g_1^{(j)}, ..., g_{K-1}^{(j)}), \quad j = 1, ..., n, \tag{3.1}$$

where $g_m^{(j)} \in \{0, 1\} \quad (0 \leq m \leq K-1)$ are the tap connections. Then, for each input bit, $x_i \in \{0, 1\}$, n bits are output according to

$$y_i^{(j)} = \sum_{m=0}^{K-1} x_{i-m} g_m^{(j)}. \tag{3.2}$$

When studying the performance of convolutional codes over QSFCs, it is convenient to classify them as follows:

- Catastrophic codes,
- Systematic codes,
- Differentially detectable (DD) codes,
- Other codes.

The definitions for the first two classes are already well known. A differentially

Table I. Classification of rate=1/2, $K = 3$ convolutional codes

octal	$g^{(1)}$	$g^{(2)}$	class
53	101	011	catastrophic
55	101	101	catastrophic
63	110	011	catastrophic
77	111	111	catastrophic
41	100	001	systematic
45	100	101	systematic
46	100	110	systematic
47	100	111	systematic
75	111	101	DD
76	111	110	DD

detectable (DD) convolutional code is defined as a non-systematic code whose tap vectors have the property that at least one pair of them differ in exactly one position.

The significance of the classification described above is illustrated in the following tables. Table I, Table II and Table III list all of the distinct feed-forward codes and their classifications for $n = 2, K = 3$; $n = 2, K = 4$ and $n = 3, K = 3$, respectively. Furthermore, in Table IV, Table V and Table VI, the size and proportion of each class of codes among all distinct codes are listed for each specific set of rate and constraint length. It seems from these tables that most of the non-catastrophic convolutional codes are either systematic or differential detectable especially for lower code rates.

Table II. Classification of rate=1/2, $K = 4$ convolutional codes

$g^{(1)}$	$g^{(2)}$	type	$g^{(1)}$	$g^{(2)}$	class
1100	1001	catastrophic	1100	0101	catastrophic
1100	0011	catastrophic	1100	1111	catastrophic
1010	1001	catastrophic	1010	0101	catastrophic
1010	1111	catastrophic	1110	1001	catastrophic
1110	0111	catastrophic	1001	1001	catastrophic
1001	1111	catastrophic	1101	1101	catastrophic
1011	1011	catastrophic	1111	1111	catastrophic
1000	0001	systematic	1000	1001	systematic
1000	0101	systematic	1000	1101	systematic
1000	0011	systematic	1000	1011	systematic
1000	0111	systematic	1000	1111	systematic
1100	1101	DD	0011	1011	DD
0011	0111	DD	0101	1101	DD
1010	1011	DD	0101	0111	DD
1110	1111	DD	1001	1101	DD
1001	1011	DD	1101	1111	DD
1011	1111	DD	1110	1101	other
1110	1011	other	1101	1011	other

Table III. Classification of rate=$1/3, K = 3$ convolutional codes

$g^{(1)}$	$g^{(2)}$	$g^{(3)}$	type	$g^{(1)}$	$g^{(2)}$	$g^{(3)}$	class
011	011	011	catastrophic	011	011	101	catastrophic
011	101	101	catastrophic	101	101	101	catastrophic
111	111	111	catastrophic	001	001	001	systematic
001	001	011	systematic	001	001	101	systematic
001	001	111	systematic	001	011	011	systematic
001	011	101	systematic	001	011	111	systematic
001	101	101	systematic	001	111	111	systematic
001	101	111	systematic	011	011	111	DD
011	101	111	DD	011	111	111	DD
101	101	111	DD	101	111	111	DD

Table IV. Properties of each class of rate=$1/2, K = 3$ convolutional codes

Class	Size	Proportion
Catastrophic	4	40%
Systematic	4	40%
DD	2	20%

Table V. Properties of each class of rate=$1/2, K = 4$ convolutional codes

Class	Size	Proportion
Catastrophic	14	38.9%
Systematic	8	22.2%
DD	11	30.6%
Other	3	8.3%

Table VI. Properties of each class of rate=$1/2, K = 3$ convolutional codes

Class	Size	Proportion
Catastrophic	5	25%
Systematic	10	50%
DD	5	25%

C. Performance comparison over QSFCs

Further analysis of convolutional codes can be seen from their performance over QSFCs. In Figure 1, simulated performance of each code in Table I over a single-input, single-output quasi-static Rayleigh fading channel is shown. It is clear, from this figure, that the codes form into three groups based on their performance. The upper four curves are the catastrophic codes, the middle two curves correspond to DD codes, while the last four curves are the systematic codes. This is further illustrated in Figure 2 and 3. Once again, it is seen that the codes naturally divide themselves according to the classification above when viewed in terms of performance over a SISO QSFC.

By investigating results for more codes rates and constraint lengths, we have found that: *For a given code rate and contraint length, the systematic codes tend to offer the best performance, followed by the differential detectable codes and other codes, and then naturally the catastrophic codes are the worst performing.*

However, the above trends are only true for the quasi-static fading channels. In AWGN and interleaved fading channels, the free distance of a code is the main feature to chracterize performance. The observation that the systematic codes perform best over QSFCs is not new and can be found in [33]. However, the observation that the DD codes offer the next best performance seems to be new. Therefore, in the later chapters, we will focus attentions on developing performance upper bounds for these two classes of convolutional codes.

D. Summary

In this chapter, one possible classification of convolutional codes was introduced. Codes with various rates and constraint lengths were classified into three or four

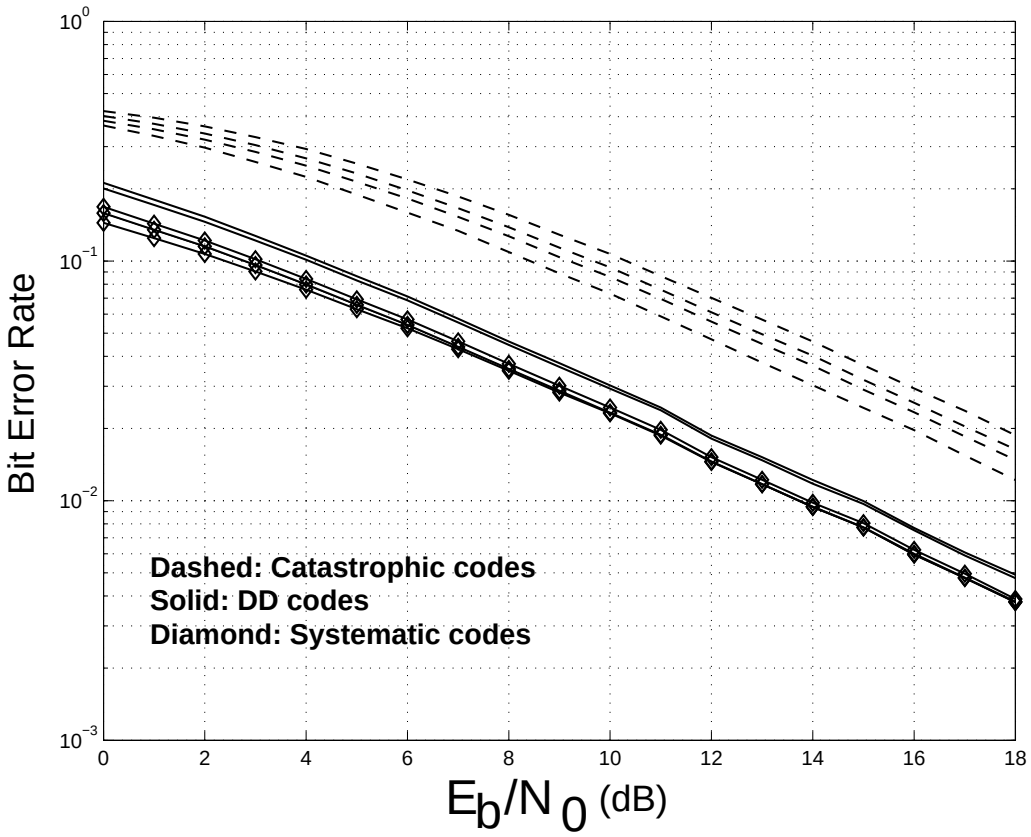

Fig. 1. Performance of rate=$1/2, K = 3$ convolutional codes listed in Table I

classes. Each class of codes was defined according to the code structure. In addition, performance of each of them over QSFCs was shown. Finally a general conclusion was made for code performance comparison over QSFCs. It was quite opposite to the AWGN case. The next chapter tackles the low SNR bounds for the two classes of codes that tend to offer the best performance.

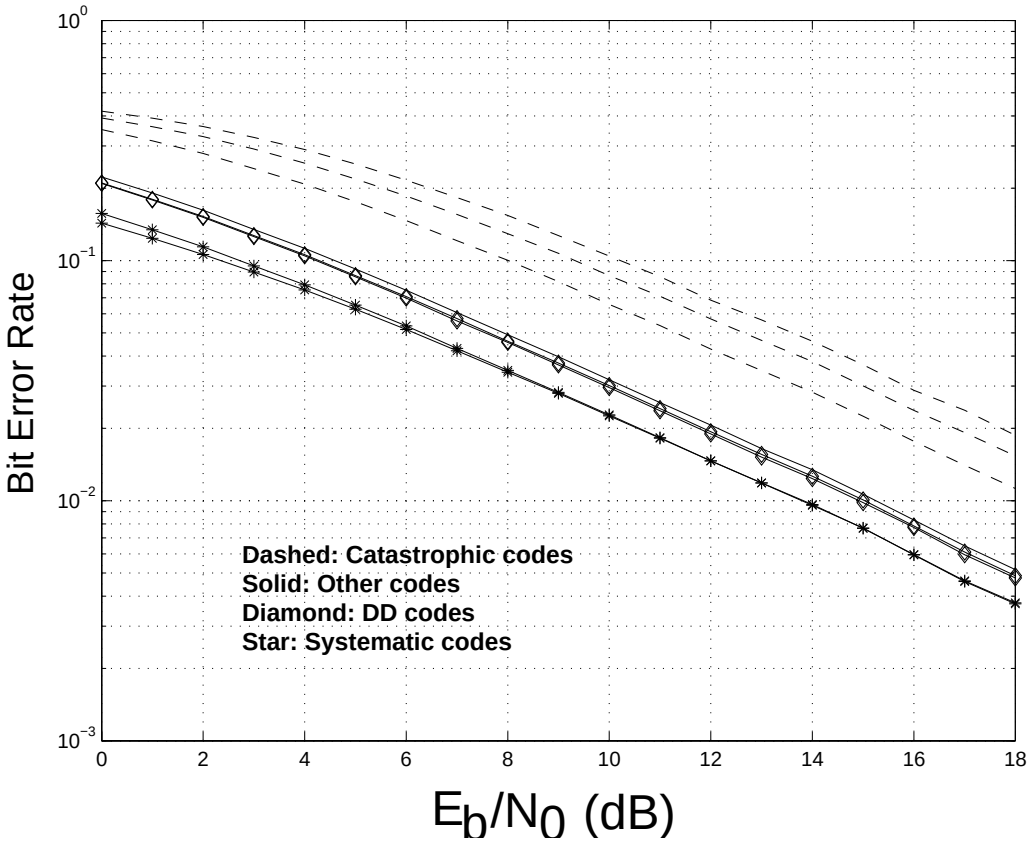

Fig. 2. Performance of some rate=$1/2, K = 4$ convolutional codes listed in Table II

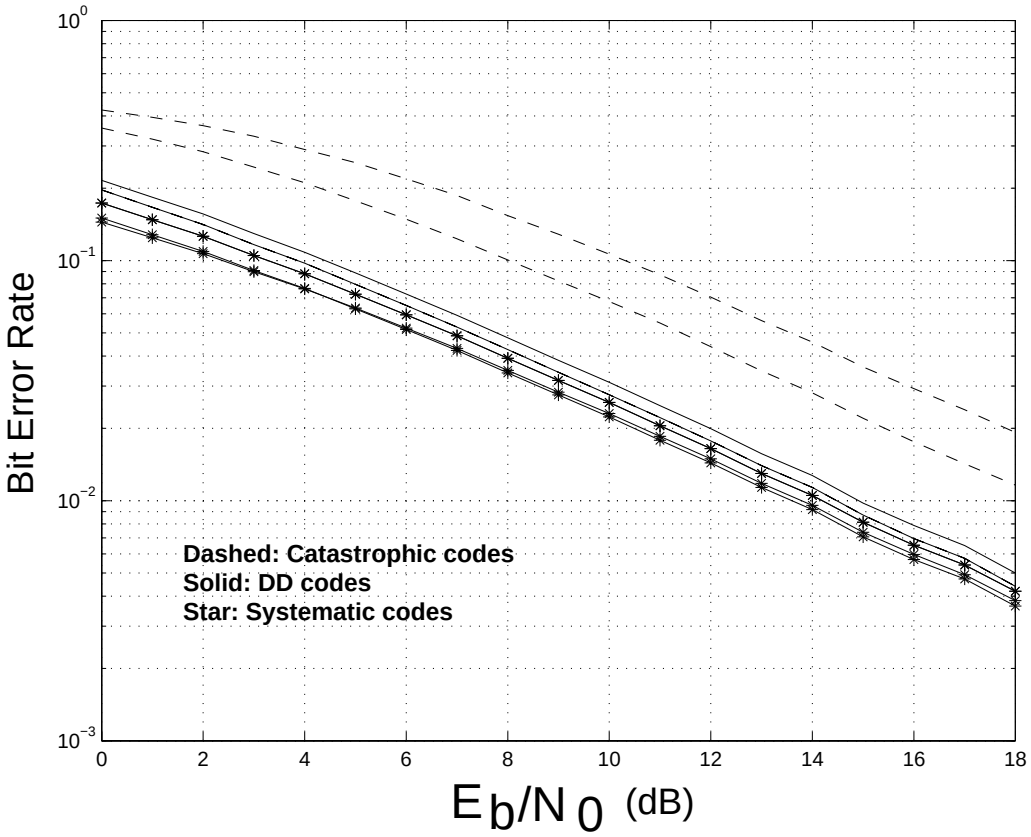

Fig. 3. Performance of some rate=$1/3, K = 3$ convolutional codes listed in Table III

CHAPTER IV

IMPROVED BOUNDS FOR THE LOW INSTANTANEOUS SNR REGION

A. Introduction

As was indicated in Chapter II, for most coded systems over QSFCs, one source of the looseness of the existing bounds is the trivial form of the low SNR bound employed. In this chapter, we will look into finding improved upper bounds for the error rate of convolutional codes in the low instantaneous SNR region.

B. Improved low SNR bound for systematic codes

Consider a rate $r = k/n$ systematic convolutional code. There are two coded bits (systematic bit and parity bit) out of the encoder in each information bit interval. Suppose we have a sub-optimal receiver that ignores the parity bits and only demodulate/decode the systematic bits on a symbol-by-symbol basis. While it is not the best way to decode the data, the bit error rate of such a receiver (conditioned on an instantaneous channel gain of h) must provide an upper bound for that of the optimal receiver at all practical SNRs. It is not hard to write the bound as

$$P(e|h) \leq Q\Big(\sqrt{\frac{2rh^2E_b}{N_0}}\Big), \tag{4.1}$$

where r is the rate of the code and E_b/N_0 is the signal-to-noise ratio of the uncoded information symbol (bit).

Figure 4 illustrates the comparison of this bound and the union bound with the true bit error rate of the optimum receiver for a typical systematic code whose taps are described by the octal number 45 over the AWGN channel. It is no doubt that the new bound in (4.1) represents an improvement over the trivial bound $P(e|h) \leq 1/2$

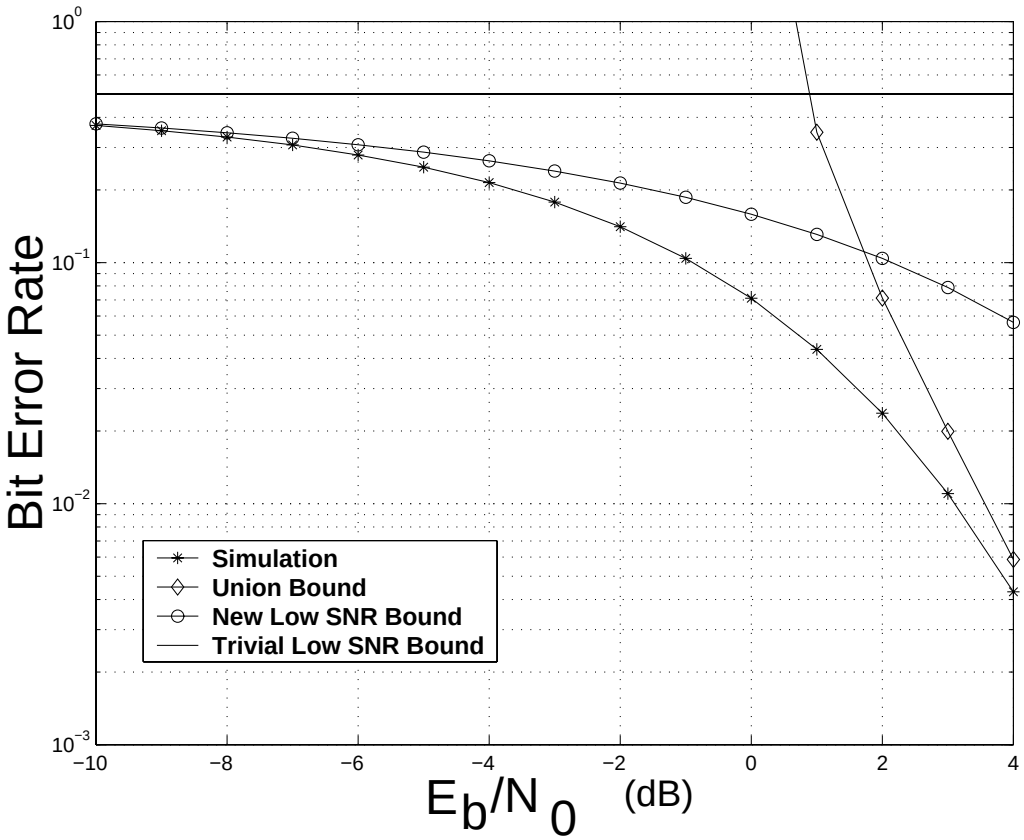

Fig. 4. Simulated performance and upper bounds for code 45 over the AWGN channel

for all SNRs. It is especially tight for low SNRs while at high SNRs, the union bound is tighter.

C. Improved low SNR bound for DD codes

A similar approach can also be used for the DD codes. The structures of DD codes allow a simple sub-optimal differential detector to extract the input data bits from the coded bits. That is, if as defined, two tap vectors $g^{(j_1)}$ and $g^{(j_2)}$ differ only in the mth position, then $x_{i-m} = y_i^{(j_1)} \oplus y_i^{(j_2)}$. We note that once the coded symbols are transmitted over a channel with additive Gaussian noise, using such a

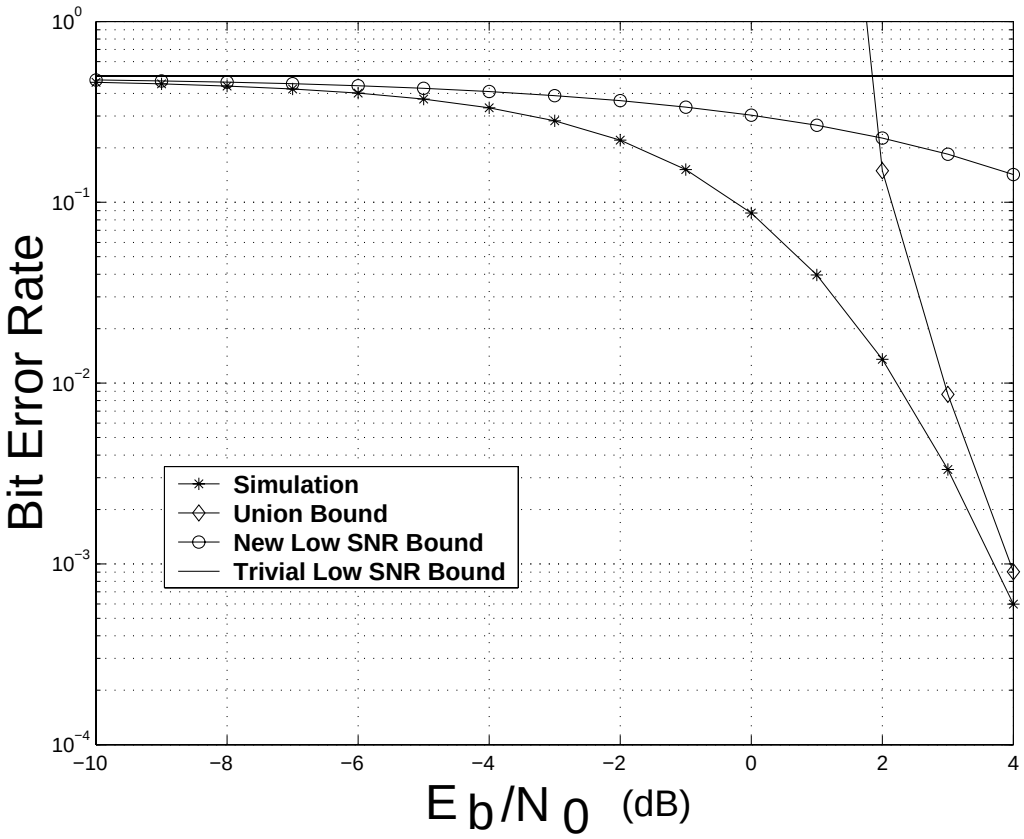

Fig. 5. Simulated performance and upper bounds for code 75 over the AWGN channel

differential detection scheme may not be the most intelligent way to decode tha data. Therefore, the bit error rate of the optimal receiver can be upper bounded by that of the differential detector. Once again, the bound is written as

$$P(e|h) \leq \frac{1}{2} exp\Big(- \frac{rh^2 E_b}{N_0} \Big). \tag{4.2}$$

Figure 5 gives the bound in (4.2) for the one-half rate DD code whose taps are described by the octal number 75 over an AWGN channel. The union bound, trivial low SNR bound and simulated performance are also plotted to show that the new bound is quite tight for low SNRs.

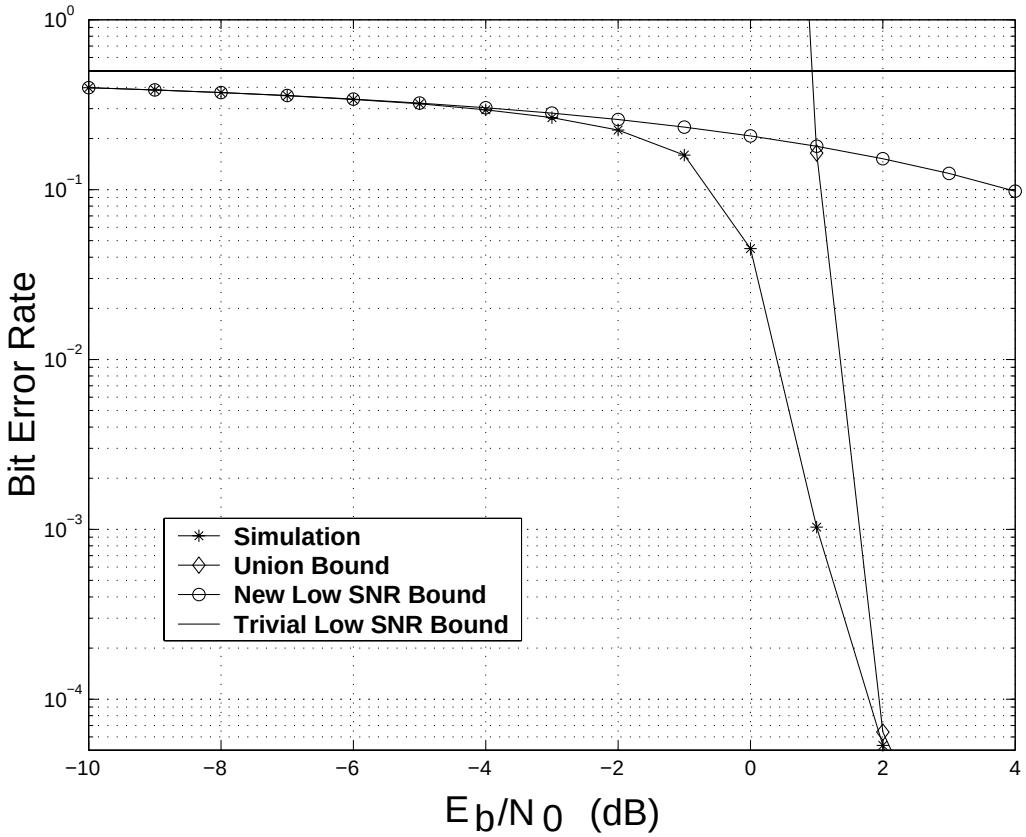

Fig. 6. Simulated performance and upper bounds for a rate 1/3 turbo code over the AWGN channel

D. Improved low SNR bound for turbo codes

A rate r turbo code can be also viewed as a systematic convolutional code. Hence a similar low SNR bounding approach in (4.1) for the systematic codes can be used on it. In Figure 6, we select the one with generator polynomials $g_0 = 05, g_1 = 07$ and plot its correponding bounds and simulated performance over an AWGN channel. The information weight spectrum can be found in [32]. As can be seen from the figure, this new bounding technique seems to work particularly well for the turbo codes.

E. Summary

In this chapter, several improved low SNR bounds for convolutional codes were presented. The improved bounds are derived using a sub-optimal receiver which provides an upper bound on the performance of the optimal receiver at all SNRs. First the bounds were derived for systematic codes, then for DD codes and finally for turbo codes. These bounds will be used in the next chapter to present bounds for convolutional and turbo codes over a single-input, single-output QSFC.

CHAPTER V

NEW PERFORMANCE BOUNDS FOR SINGLE-INPUT SINGLE-OUTPUT QUASI-STATIC FADING CHANNELS

A. Introduction

In this chapter, the new low SNR bounds proposed in Chapter IV will be applied to derive tighter upper bounds on the performance of some codes that can be used on single-input single-output (SISO) quasi-static fading channels. First the simple case of convolutional code performance over SISO QSFCs will be studied. An improved upper bound on the bit error rates is derived and an alternative for free distance as a measure of performance is proposed. In addition, different bounds on the frame error rates are mentioned. The more interesting case of turbo codes is investigated next.

B. Convolutional codes over SISO QSFCs

Convolutional codes are extensively used in communications systems today. The performance of convolutional codes over QSFCs has been studied so far by [9], [10] and [12]. While they gave quite good bounds, they still had a several dB looseness. In this chapter, the bounds derived in Chapter IV will be combined with the Gallager bounding approach presented in Chapter II to derive tighter upper bounds on the performance of different classes of convolutional codes over QSFCs. The derived upper bound will give a much improved analytical tool for understanding the performance of convolutional codes over QSFCs.

1. System model and notation

The system under consideration is a BPSK convolutional code over a single-input single-output fading channel. The fading channel under consideration is a QSFC, its complex magnitude will be denoted by h and it is constant over a frame and independently changes from one frame to the next. The convolutional codes have rate r and their constraint length and coding gains are captured in their information weight spectrum N_d.

2. A new upper bound for systematic convolutional codes

In this section, we will use the general approach in (2.5) to present an improved upper bound for the bit error rate of a systematic convolutional code over a SISO QSFC. The low SNR term in (2.5) can be expressed as:

$$P(e|h < h_0)P(h < h_0) = \int_0^{h_0} P(e|h)f(h)dh. \tag{5.1}$$

Instead of using the trivial bound for $P(e|h)$, we use the bound in (4.1) with the Rayleigh distribution for the instantaneous channel gain given by $f(h) = 2he^{-h^2}$, then (5.1) is bounded by:

$$\begin{aligned} P(e|h < h_0)P(h < h_0) &\leq \int_0^{h_0^2} Q\left(\sqrt{2\gamma h}\right)e^{-h}dh, && (5.2)\\ &= \frac{1}{2}\left(1 - \sqrt{\frac{\gamma}{1+\gamma}}\right) + \sqrt{\frac{\gamma}{1+\gamma}}Q\left(h_0\sqrt{2+2\gamma}\right) \\ &\quad -e^{-h_0^2}Q\left(\sqrt{2\gamma}h_0\right), && (5.3) \end{aligned}$$

where $\gamma = E_s/N_0 = rE_b/N_0$ is the signal-to-noise ratio of the coded symbol. A standard Union bound is used for bounding the high SNR term,

$$P(e|h > h_0)P(h > h_0) \leq \sum_d N_d \int_{h_0^2}^{+\infty} Q\left(\sqrt{2d\gamma h}\right)e^{-h}dh, \tag{5.4}$$

where d is the Hamming distance between two output codewords and N_d its corresponding multiplicity, taking into account the number of bit errors. Using the Chernoff bound on the Q function [1]

$$Q(x) \leq A_0 e^{-\frac{x^2}{2}}, \tag{5.5}$$

the parameter A_0 is generally taken to be 1/2 but in this case in order to tighten the bound, it can be chosen as

$$A_0 = Q\Big(\sqrt{2d_f\gamma}h_0\Big)e^{d_f\gamma h_0^2}, \tag{5.6}$$

because the bound on the Q-function only needs to be valid over the range $h > h_0^2$. In the above equation, d_f is the free distance of the code. By using this modified Chernoff bound on the Q-function, the integral in (5.4) can be easily evaluated in closed form. Hence, the overall bound for systematic convolutional codes, operating over a SISO QSFC, the bit error probability is upper bounded by:

$$\begin{aligned} P_b &= P(e|h < h_0)P(h < h_0) + P(e|h > h_0)P(h > h_0), && (5.7) \\ &\leq \frac{1}{2}\left(1 - \sqrt{\frac{\gamma}{1+\gamma}}\right) + \sqrt{\frac{\gamma}{1+\gamma}}Q\left(h_0\sqrt{2+2\gamma}\right) - e^{-h_0^2}Q\left(\sqrt{2\gamma}h_0\right) \\ &\quad + A_0 \sum_{d=d_f}^{\infty} N_d \frac{e^{-(1+\gamma d)h_0^2}}{1+\gamma d}. && (5.8) \end{aligned}$$

The minimization of the final expression for the derived bound is performed with respect to h_0. From a practical view of point, for most SNRs of interest, the series in (5.8) can be truncated after only a few spectrum terms without affecting its accuracy. This is shown in Fig. 7.

Simulated performance and the new upper bound is plotted for a systematic code (octal 45) over a SISO QSFC in Fig. 8. For comparison purposes, an old upper bound which employed the trivial bound for the low SNR region is also plotted. As can be

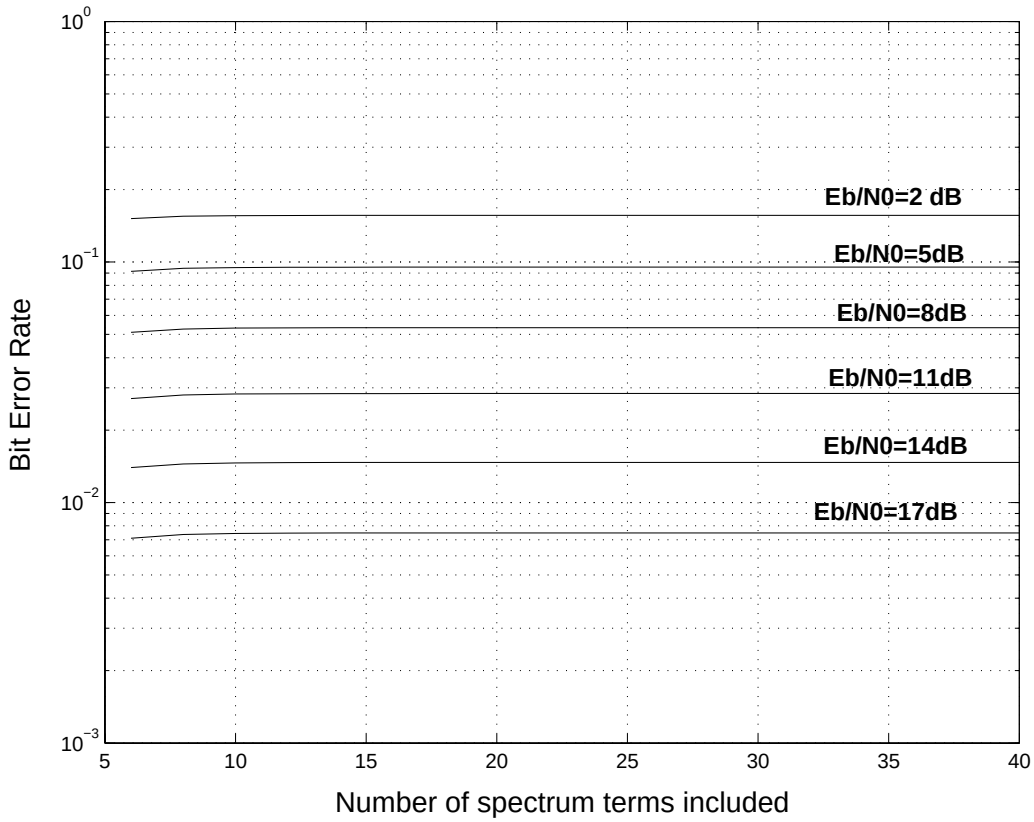

Fig. 7. New upper bound vs number of spectrum terms for code 45 over QSFC

seen from the figure, the derived upper bound is at least two full decibels tighter than the existing bound(s) and is only $1 \sim 2$dB away from the simulation results.

3. A new upper bound for DD convolutional codes

Now we develop a slightly looser bound for the DD codes by starting with the low SNR bound in (4.2). In which case, the low SNR term in (2.5) can be written as:

$$P(e|h < h_0)P(h < h_0) \leq \int_0^{h_0^2} \frac{1}{2} e^{-\gamma h} e^{-h} dh, \tag{5.9}$$

$$= \frac{1}{2+2\gamma}(1 - e^{-h_0^2(1+\gamma)}), \tag{5.10}$$

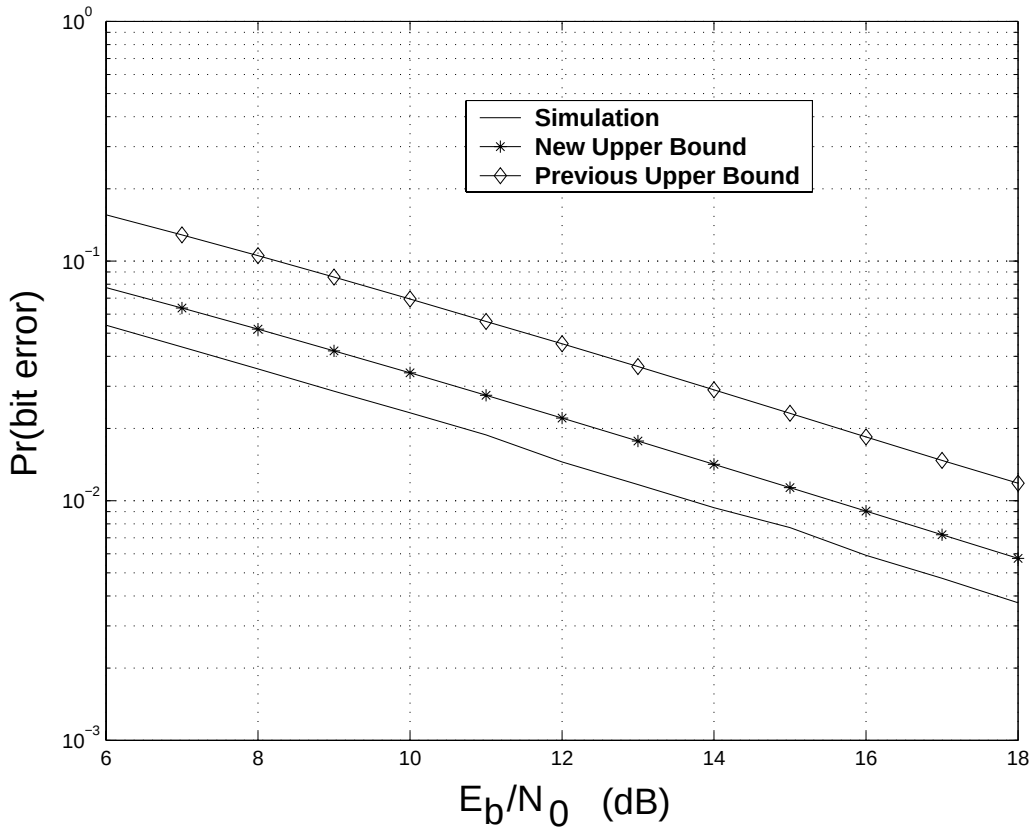

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Fig. 8. Simulated performance and upper bounds for code 45 over QSFC

the resulting upper bound for the performance of a DD convolutional code over a SISO QSFC can then be given by the sum of (5.9) and the last term in (5.8):

$$P_b \leq \frac{1}{2+2\gamma}\left(1 - e^{-h_0^2(1+\gamma)}\right) + A_0 \sum_{d=d_f}^{\infty} N_d \frac{e^{-(1+\gamma d)h_0^2}}{1+\gamma d}. \tag{5.11}$$

Again, this new bound is illustrated in Fig. 9 for DD code 75 as an example.

4. Improved performance indicator

It is not hard to find that the bound in (5.11) can also serve as an upper bound on the performance of systematic codes as well since (4.2) serves as a bound for (4.1).

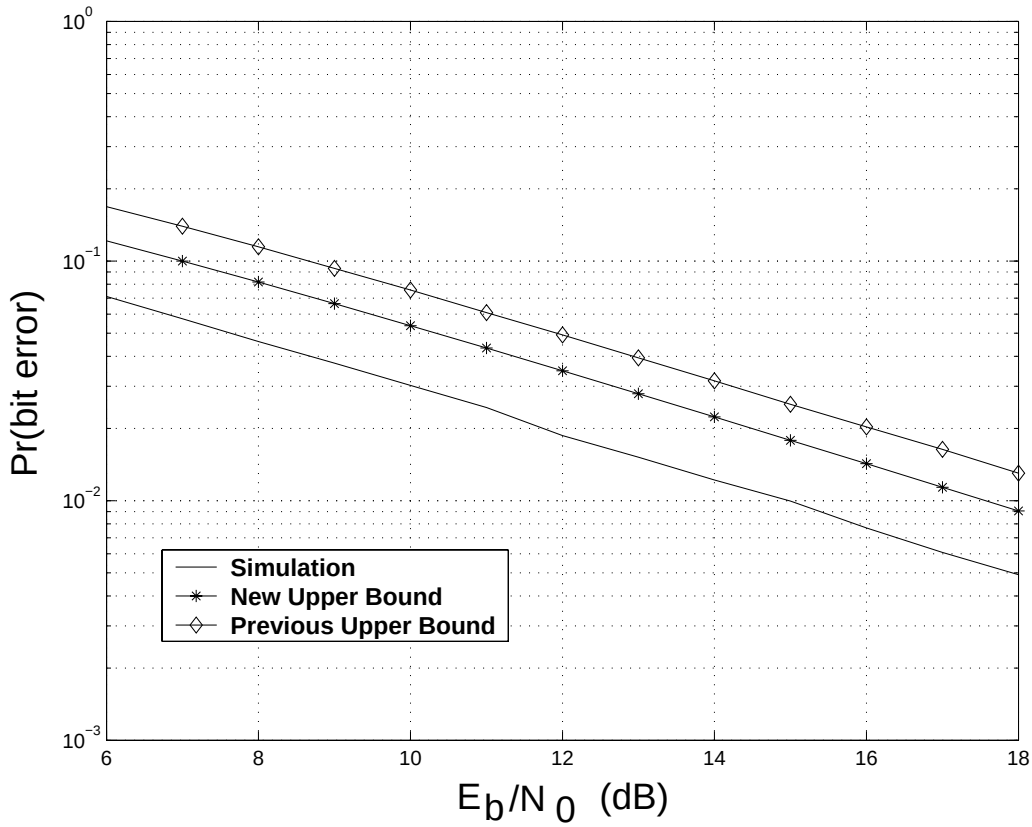

Fig. 9. Simulated performance and upper bounds for code 75 over QSFC

The value of h_0 which minimizes the new upper bound in (5.11), h_{0min}, is a function of the spectrum of the code. By definition, h_{0min}^2 is a root of the following equation:

$$\frac{dP_b}{dh_0^2} = 0, \tag{5.12}$$

which can be in turn expressed as:

$$\frac{1}{2}e^{-(1+\gamma)h_0^2} - A_0 \sum_d N_d \frac{1}{1+\gamma d}(1+\gamma d)e^{-(1+\gamma d)h_0^2} = 0, \tag{5.13}$$

$$\sum_d N_d e^{-\gamma h_0^2(d-1)} = \frac{1}{2A_0}, \tag{5.14}$$

let $T(x)$ be the information weight enumerating function of the code:

$$\frac{T(x)}{x}\Big|_{x=e^{-\gamma h_0^2}=x_0} = \frac{1}{2A_0}, \tag{5.15}$$

the constant A_0 can be set to 1/2 or more generally:

$$A_0 = x_0^{-d_f} Q\Big(\sqrt{-2d_f ln x_0}\Big). \tag{5.16}$$

Therefore, x_0 has a relationship only with the distance spectrum of the code and it can be found by solving numerically (5.15). Table VII gives the parameter x_0 for each distinct convolutional code in Table I.

Then the bound in (5.11) can be rewritten through x_0:

$$P_b \leq \frac{1}{2+2\gamma}\Big[1 - e^{lnx_0(1+\frac{1}{\gamma})}\Big] + A_0 \sum_d N_d \frac{x_0^{d+1/\gamma}}{1+\gamma d}, \tag{5.17}$$

since the first term in (5.17) tends to dominate the whole bound which is shown in Table VIII and IX, (5.17) can be approximated by:

$$P_b \leq \frac{1}{2+2\gamma}\Big[1 - e^{lnx_0(1+\frac{1}{\gamma})}\Big] \approx \frac{ln(1/x_0)}{2\gamma}. \tag{5.18}$$

The above equation shows that the new upper bound in (5.11) for convolutional codes depends on the particular code only through the parameter x_0. Codes with larger x_0 have tighter analytical bounds and are predicted to give better performance over QSFCs. Relating Table VII, it is obvious that systematic and DD codes have the largest x_0 while catastrophic codes have the smallest ones. This seems to agree with the simulation results in Chapter III which states systematic and DD codes to be the best codes over QSFCs. The parameter x_0 can thus serve as an improved indicator of the performance of convolutional codes over QSFCs. We can search for good codes by evaluating the parameter x_0 for each candidate code and this procedure will produce better codes than searches based on the free distance.

Table VII. Correlation between connecting vector, free distance, class and x_0 of convolutional codes

octal	free distance	class	x_0
53	4	catastrophic	0.2135
55	4	catastrophic	0.1184
63	4	catastrophic	0.0981
77	4	catastrophic	0.1126
41	2	systematic	0.5229
45	3	systematic	0.4986
46	3	systematic	0.5352
47	4	systematic	0.5058
75	5	DD	0.4491
76	4	DD	0.4807

Table VIII. Domination of the first term in the new bound for code 75

SNR(dB)	first term value	percentage	second term value	percentage
3dB	0.2001	97.3%	0.0055	2.7%
6dB	0.1171	96.5%	0.0042	3.5%
9dB	0.0637	95.9%	0.0027	4.1%
12dB	0.0333	95.6%	0.0015	4.4%

Table IX. Domination of the first term in the new bound for code 45

SNR(dB)	first term value	percentage	second term value	percentage
3dB	0.1838	89.5%	0.0217	10.5%
6dB	0.1056	86.2%	0.0169	13.8%
9dB	0.0568	84.1%	0.0108	15.9%
12dB	0.0296	83.0%	0.0060	17.0%

5. Bounds on the frame error rates

The previous bounds are valid for the bit error rates (BER) only. For the frame error rates (FER) of convolutional codes, we can tighten the trivial low SNR bound $P(e|h) < 1$ using (4.1) and (4.2). Let L be the frame length and $P(f)$ be the frame error rate,

For systematic codes:

$$P(f|h) \leq 1 - \left[1 - Q\Big(\sqrt{\frac{2rh^2E_b}{N_0}}\Big)\right]^L. \tag{5.19}$$

For DD codes:

$$P(f|h) \leq 1 - \left[1 - \frac{1}{2}exp\Big(-\frac{rh^2E_b}{N_0}\Big)\right]^L. \tag{5.20}$$

Upper bounds can be similarly derived for the frame error rates of convolutional codes over SISO QSFCs except that N_d is replaced by the total number of simple and compound error paths of weight d. Numerial evaluations of (5.19) and (5.20) show that for practical frame lengths, they are very close to one at low SNRs. Therefore, it is expected that the corresponding bounds for QSFCs will only improve little over the previous bounds employing the trivial low SNR bound. This is confirmed by Fig.

10 and 11.

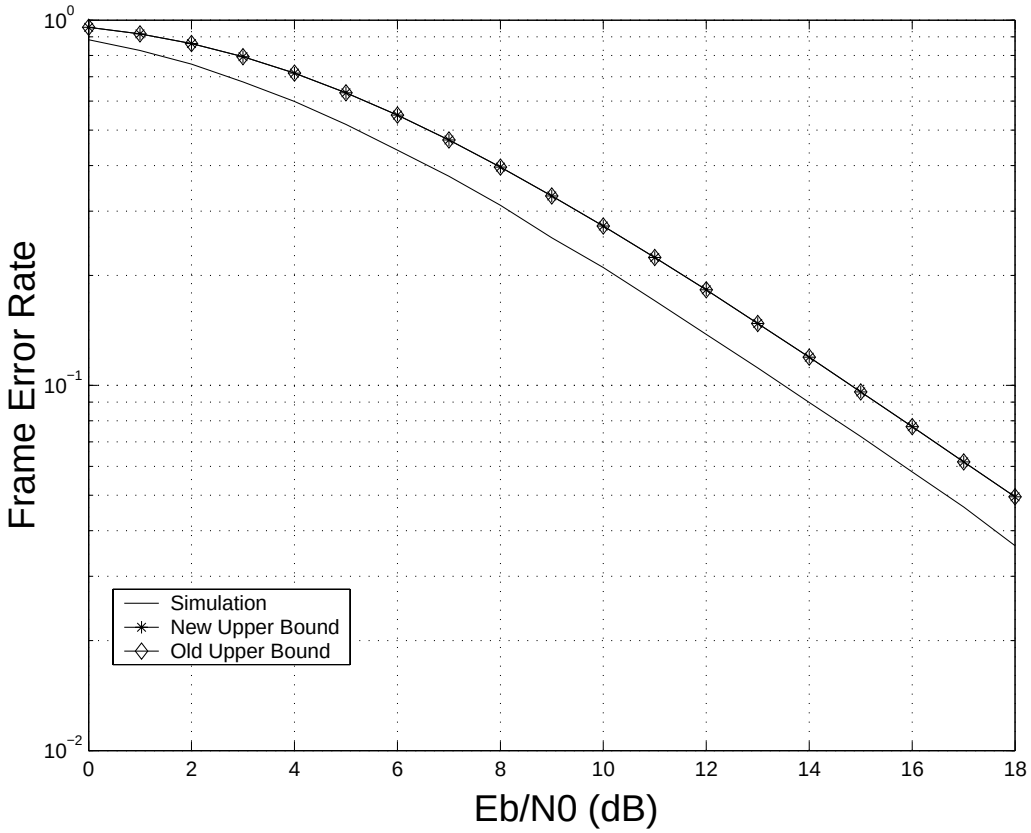

Fig. 10. Simulated performance and FER upper bounds for code 45 over QSFC

C. Turbo codes over SISO QSFCs

In this section an upper bound on turbo codes performance over single-input single-output QSFCs is derived. As in the case of systematic convolutional codes, the improved low SNR bound in (4.1) is used. The upper bound obtained is shown to be much closer to the actual simulated performance compared to the previous bound.

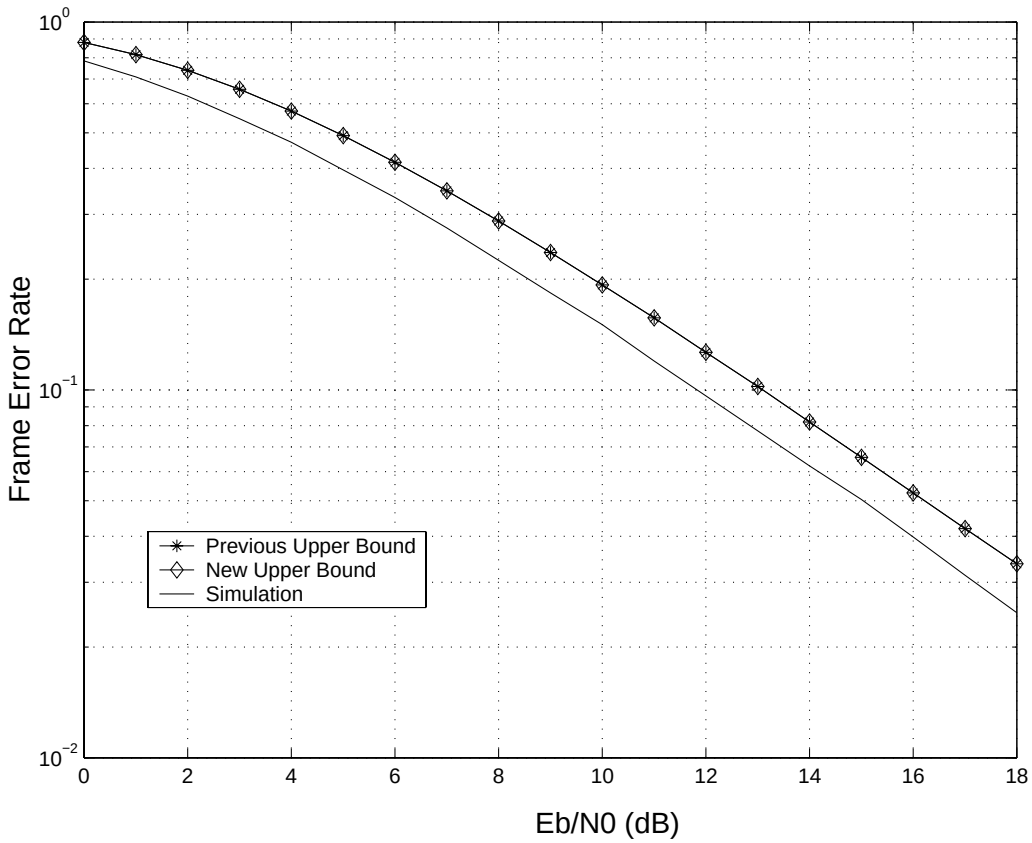

Fig. 11. Simulated performance and FER upper bounds for code 75 over QSFC

1. System model and notation

The system model of interest here is a simple rate 1/3 BPSK turbo code over a SISO QSFC. The decoder used is an iterative decoder with each iteration using two map decoders, similar to the one proposed in [31]. The information weight spectrum N_d to compute the upper bound is found in [32] and d_f is the minimum hamming distance of the code. This structure is retained in the third generation (3G) celluar systems.

2. New upper bound

It is obvious that equation (5.8) for systematic codes can also serve as an upper bound for turbo codes. For turbo codes over a SISO QSFC, the bit error rate is upper bounded by:

$$P_b \leq \frac{1}{2}\left(1-\sqrt{\frac{\gamma}{1+\gamma}}\right)+\sqrt{\frac{\gamma}{1+\gamma}}Q\left(h_0\sqrt{2+2\gamma}\right)-e^{-h_0^2}Q\left(\sqrt{2\gamma}h_0\right)+A_0\sum_{d=d_f}^{\infty}N_d\frac{e^{-(1+\gamma d)h_0^2}}{1+\gamma d}. \tag{5.21}$$

Where γ, A_0 and h_0 are as previously defined.

Fig. 12 shows the derived upper bound along with the simulated performance for the rate 1/3 turbo code proposed in [32]. The new bound is compared with the previous bound proposed in [11] and [15]. As can be seen from the figure, the derived upper bound is particularly tight for turbo codes and provides a more useful analytical expression to depict their performance.

D. Summary

In this chapter the issue of channel code performance over single-input single-output quasi-static fading channels was investigated. First the results of Chapter IV was used to derive upper bounds for systematic and DD convolutional codes. The resulting bounds are much tighter than previous bounds employing the trivial low SNR bound. It was then shown that the new bounds give an idea about the relative performance of different classes of convolutional codes in QSFCs and some insight into code design for QSFCs. The same approach was successfully reproduced for the more interesting case of turbo codes. The derived upper bound also gave at least 2dB addidtional tightness. The case of transmit diversity over QSFCs is the main subject of the next chapter.

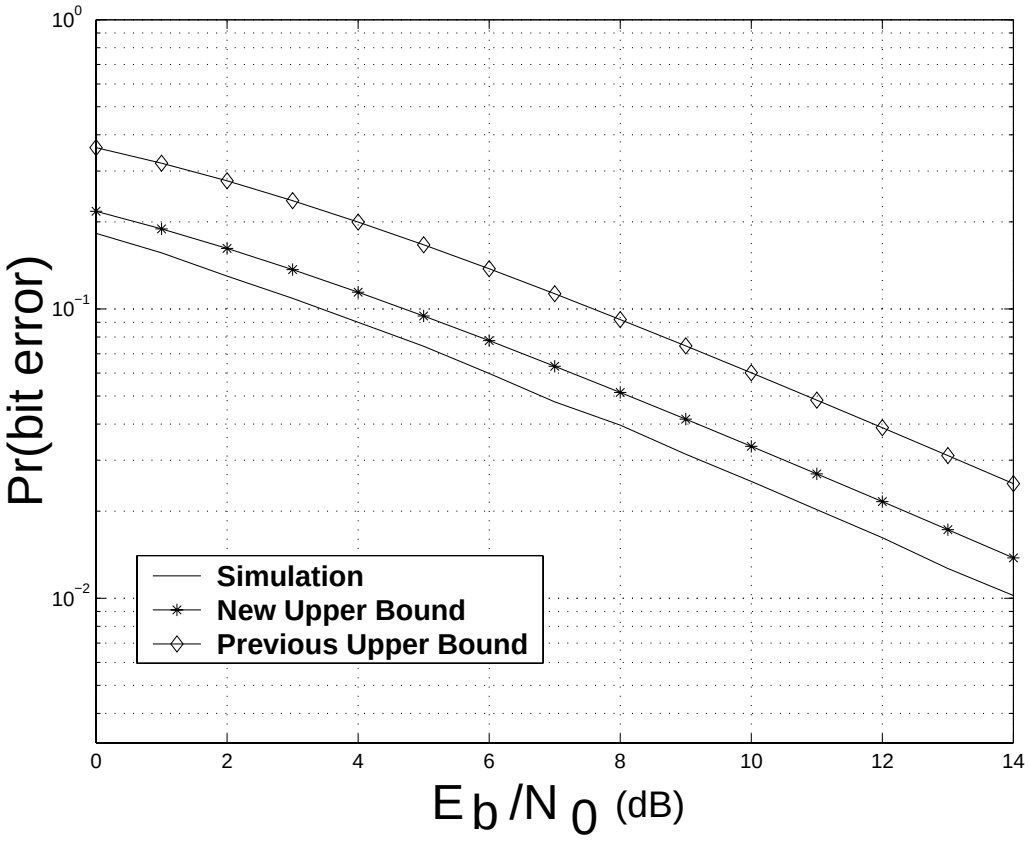

Fig. 12. Simulated performance and upper bounds for turbo code over QSFC

CHAPTER VI

NEW PERFORMANCE BOUNDS FOR EQUIVALENT SINGLE-INPUT SINGLE-OUTPUT QUASI-STATIC FADING CHANNELS

A. Introduction

Space-time block codes (STBC) is one category of codes that are able to provide transmit diversity. The best known STBC scheme for two transmit antennas is that proposed by Alamouti in [3]. This scheme has an advantage that it is a rate one code, i.e. there is no loss of transmission rate by using it. Such a scheme is adopted in several wireless communications standards such as 3G and the broadband wireless access standard (802.16).

The general system under study in this chapter is a serial concatenation of an outer convolutional or turbo code which provides coding gain and an inner orthogonal STBC offering space diversity. It is shown in Fig. 13. The system employs the Alamouti STBC scheme with two transmit antennas and N_r receive antennas. The received symbol at the jth receive antenna in the kth time interval is given by:

$$r_{j,k} = h_{1,j}s_{1,k} + h_{2,j}s_{2,k} + n_{j,k}, \quad j = 1, ..., N_r, \tag{6.1}$$

where $s_{1,k}$ and $s_{2,k}$ are the symbols transmitted from the first and second transmit antennas at time instant k, respectively. More specifically, $(s_{1,2i-1}, s_{1,2i}) = (x_{2i-1}, -x^*_{2i})$ and $(s_{2,2i-1}, s_{2,2i}) = (x_{2i}, x^*_{2i-1})$. In (6.1), $h_{1,j}$ and $h_{2,j}$ are the two zero-mean complex Gaussian channel gains from the two transmit antennas to the jth receive antenna and $n_{j,k}$ is the zero-mean complex Gaussian noise received at the jth receive antenna during the kth time instant. The space-time block decoder uses linear processing techniques according to $y_{2i-1} = (h^*_1 r_{2i-1} + h_2 r^*_{2i})/h$ and $y_{2i} = (h^*_2 r_{2i-1} - h_1 r^*_{2i})/h$

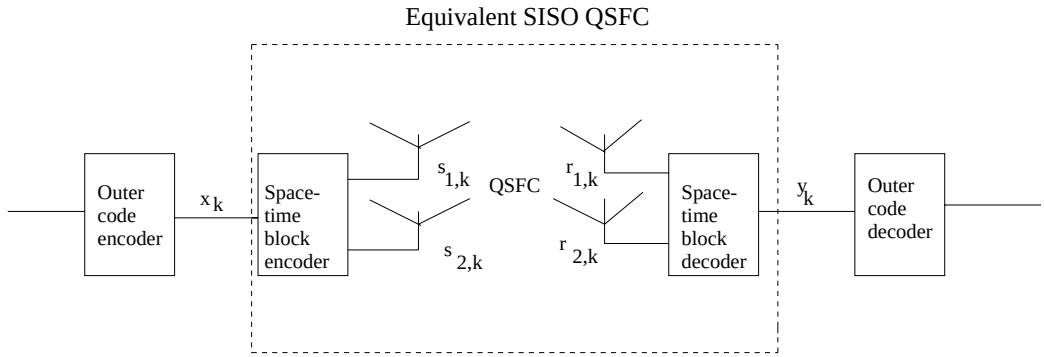

Fig. 13. System model using serially concatenated Alamouti STBC

where $h = \sqrt{|h_1|^2 + |h_2|^2}$, and $|h_i|^2$ is defined as: $|h_i|^2 = |h_{i,1}|^2 + |h_{i,2}|^2 + \ldots + |h_{i,N_r}|^2$ where $i \in \{1, 2\}$. Hence, h^2 is the sum of the squared magnitude of all the channel gains.

Therefore, the outer code sees an equivalent single-input single-output (SISO) QSFC which is described by the following input/output relationship:

$$y_k = hx_k + n_k, \tag{6.2}$$

where k refers to the time instant and n_k is a complex Gaussian noise sample with the same variance as $n_{j,k}$ which is $N_0/2$ in each complex dimension. The equivalent channel gain h^2 is a Chi-square random variable with $2N_r$ degrees of freedom ($\chi^2_{2N_r}$). The probability density function (pdf) of h has a Nakagami distribution given by:

$$f(h) = \frac{2}{(2N_r - 1)!} h^{4N_r - 1} e^{-h^2} U(h). \tag{6.3}$$

This newly defined equivalent SISO QSFC will facilitate the use of the approach adopted in Chapter V to derive an upper bound on the performance of a serially concatenated scheme. With this equivalent model, the concatenated space-time code can be viewed as an outer code operating over a SISO QSFC where the fading happens

to follow a distribution given by (6.3). The remainder of this chapter focuses on developing improved analytical tools for the performance of this coding scheme for two specific outer codes: first a convolutional code and later a turbo code. The upper bounds will be derived for the bit error rates (BER) in both cases.

B. Convolutional codes over equivalent SISO QSFCs

Using the system description in Fig. 13, we now present a serially concatenated convolutional code and STBC scheme over QSFCs in this section. For a rate $r = k/n$ outer code, k information bits with energy E_b are encoded through the convolutional encoder into n BPSK symbols. Only systematic and DD codes will be dealt with as we have the improved low SNR bounds for them in Chapter IV. At the decoder end, a Viterbi algorithm is used to extract the maximum likelihood transmitted codeword. The next sections will provide improved performance bounds using similar approaches proposed in the previous chapter for these concatenated schemes.

1. Performance analysis

a. Upper bound employing trivial low SNR bound

As was stated in [11], the BER of a convolutional code concatenated with an Alamouti space-time block code can be upper bounded by

$$P_b \leq \frac{1}{2}[1-(1+h_0^2)e^{-h_0^2}] + A_0 \sum_{d=d_f}^{\infty} N_d \frac{1+(1+\gamma d)h_0^2}{1+\gamma d^2} e^{-(1+\gamma d)h_0^2}. \tag{6.4}$$

Where $\gamma = rE_b/N_0$, and A_0 is as defined in (5.6). For bit error rates, N_d denotes the information distance spectrum of the convolutional code which is easily obtained from the transfer function of the code [1]. As in previous cases, h_0 is chosen properly to ensure the tightest possible form of this bound.

b. Upper bound employing new low SNR bound

Given the defined equivalent SISO QSFC in (6.2), the same approach in upper bounding convolutional codes over SISO QSFCs in Chapter V can be reproduced. The fading pdf $f(h)$ in (5.1) is now replaced by (6.3). Starting with the DD codes defined in Chapter III and using the bound in (4.2), the low SNR term is bounded by:

$$\begin{aligned} P(e|h<h_0)P(h<h_0) &\leq \int_0^{h_0} \frac{1}{2} exp\Big(-\frac{rh^2E_b}{N_0}\Big)\Big(\frac{2}{(2N_r-1)!}h^{4N_r-1}e^{-h^2}\Big)dh, \\ &= \frac{\overline{\gamma}(n,(1+\gamma)h_0^2)}{2(n-1)!(1+\gamma)^n}, \end{aligned} \tag{6.5}$$

where $\gamma = rE_b/N_0$, and $\overline{\gamma}(A,B) = \int_0^B x^{A-1}e^{-x}dx$ is the incomplete gamma function. The parameter $n = 2N_r$ is the total diversity offered by the inner space-time code. The high SNR term in the Gallager bound is still bounded using a standard Union-Chernoff bound technique resulting in:

$$P(e|h>h_0)P(h>h_0) \leq \sum_d N_d \int_{h_0}^{+\infty} Q\left(\sqrt{2d\gamma}h\right) f(h)dh, \tag{6.6}$$

$$\leq A_0 \sum_{d=d_f}^{\infty} \frac{N_d}{(1+d\gamma)^n}\Big[1 - \frac{\overline{\gamma}(n,(1+d\gamma)h_0^2)}{(n-1)!}\Big], \tag{6.7}$$

where d_f is the free distance of the outer code, and N_d, A_0 are as defined previously. Hence, the bit error rate of a DD convolutional code serially concatenated with an Alamouti code over a QSFC is upper bounded by:

$$P_b \leq \frac{\overline{\gamma}(n,(1+\gamma)h_0^2)}{2(n-1)!(1+\gamma)^n} + A_0 \sum_{d=d_f}^{\infty} \frac{N_d}{(1+d\gamma)^n}\Big[1 - \frac{\overline{\gamma}(n,(1+d\gamma)h_0^2)}{(n-1)!}\Big]. \tag{6.8}$$

This bound is then tightened by numerically optimizing with respect to the parameter h_0. Also, for most SNRs of interest, only the first few terms in the series in (6.8) need to be kept in computing the bound without losing accuracy.

By starting with the low SNR bound in (4.1), we now develop a slightly tighter bounds for the cases when the outer code is systematic. In which case,

$$P(e|h<h_0)P(h<h_0) \leq \int_0^{h_0} Q(\sqrt{2\gamma h^2})\Big(\frac{2}{(2N_r-1)!}h^{4N_r-1}e^{-h^2}\Big)dh, \quad (6.9)$$

$$= I_0 - \sum_{m=1}^{n-1} B_m, \quad (6.10)$$

where

$$I_0 = \frac{1}{2}\Big[1-\sqrt{\frac{\gamma}{\gamma+1}}\Big] + \sqrt{\frac{\gamma}{\gamma+1}}Q(\sqrt{2h_0^2(1+\gamma)}) - e^{-h_0^2}Q(\sqrt{2\gamma h_0^2}), \quad (6.11)$$

$$B_m = \frac{h_0^{2m}e^{-h_0^2}}{m!}Q(\sqrt{2\gamma h_0^2}) + \frac{\sqrt{\gamma/\pi}\Big[\overline{\gamma}\Big(m+1/2,(1+\gamma)h_0^2\Big)\Big]}{2(m!)(1+\gamma)^{m+1/2}}. \quad (6.12)$$

The resulting upper bound for the performance of a systematic convolutional code serially concatenated with an Alamouti code over a SISO QSFC is then given by the sum of (6.10) and the second term in (6.8).

2. Numerical results

These new upper bounds are illustrated in Fig. 14 and 15 for the $r = 1/2, K = 3$, systematic outer code (45) and DD outer code (75), respectively. BPSK modulation is used. For each type of code, the new bound is compared with the previous bound employing the trivial low SNR bound as well as the simulated performance. Each figure represents the results for two cases:

- $N_r = 1, n = 2$, a transmit diversity system with no receive diversity,
- $N_r = 2, n = 4$, a transmit/receive diversity system.

The upper three curves show for $n = 2$ while the lower three ones for $n = 4$. In all cases, the fading coefficients are independent in space (from each transmit antenna to receive antenna) and in time from one code block to the next, however they remain constant over the duration of the codeword length of the outer code.

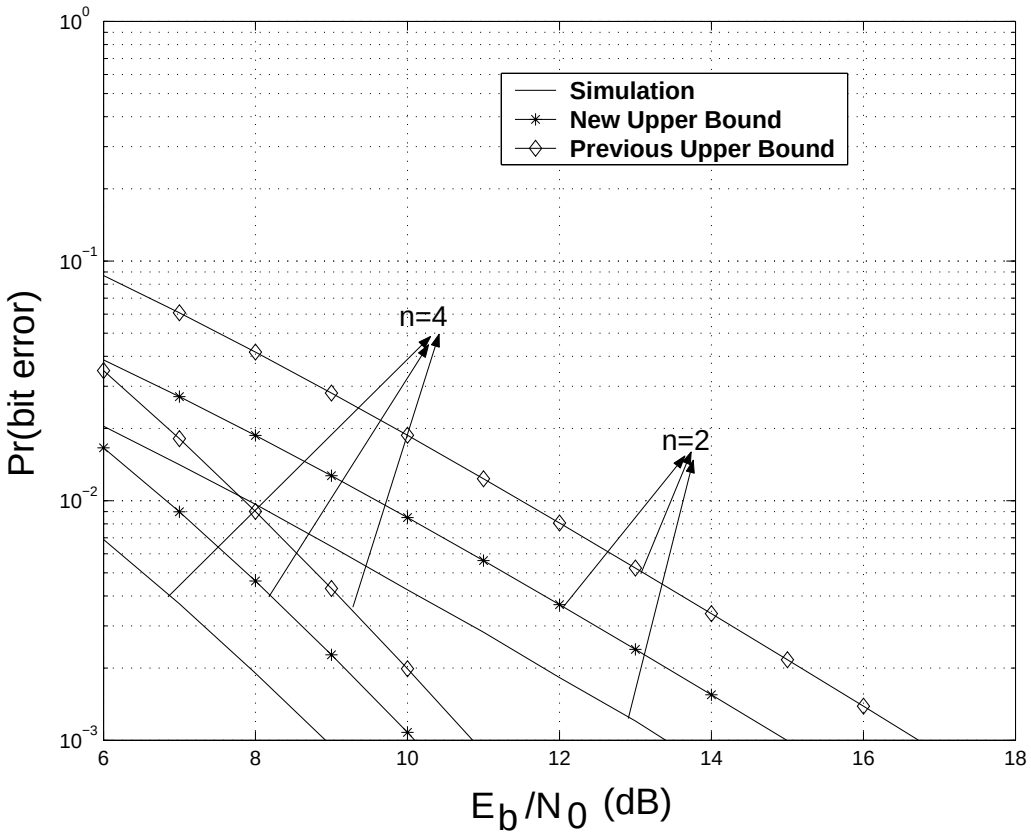

Fig. 14. Simulated performance and upper bounds for serially concatenated systematic code and STBC over QSFCs

As can be seen from the figures, in all cases, the new upper bound is at least 1dB tighter than the existing bounds. This new analytical tool provides a more reliable prediction of the performance of these codes over QSFCs. It works better for systematic outer codes than for DD outer codes.

C. Turbo codes over equivalent SISO QSFCs

In the second part of this chapter, an improved upper bound on the performance of serially concatenated turbo codes and STBCs is proposed.

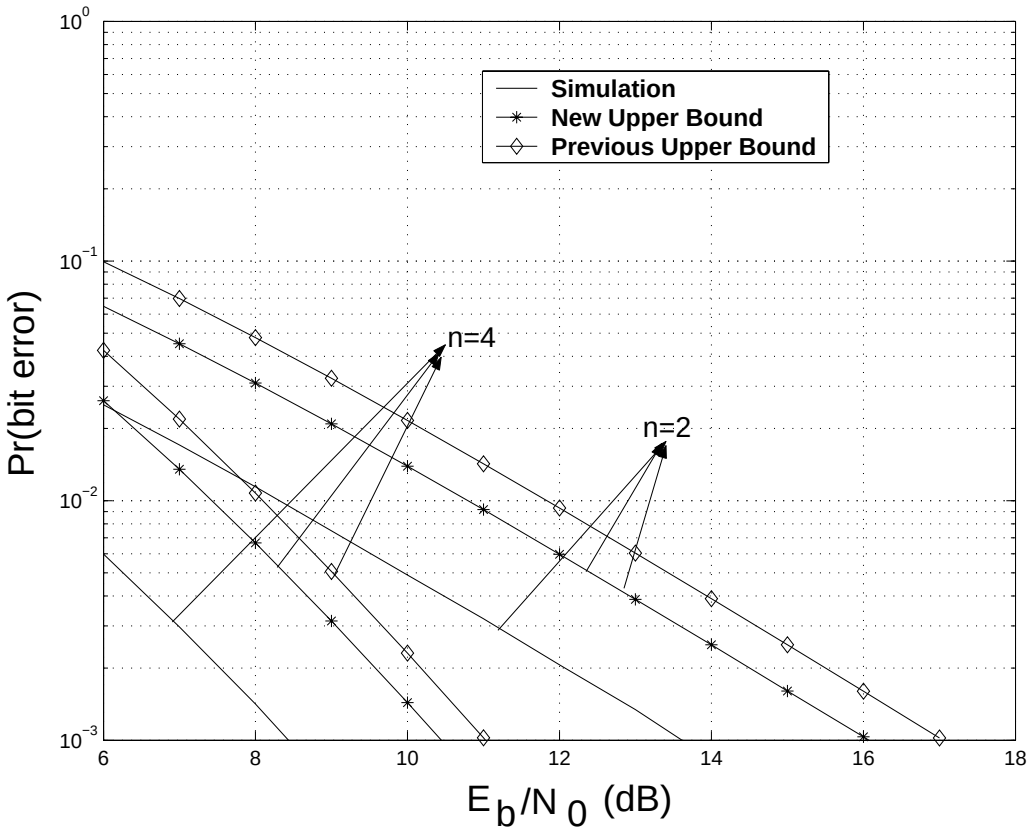

Fig. 15. Simulated performance and upper bounds for serially concatenated DD code and STBC over QSFCs

1. Performance analysis

The same derivation used for computing upper bounds of systematic codes over equivalent SISO QSFCs in the previous section is reiterated for this concatenation case. The resulting upper bound is the sum of (6.10) and (6.6).

2. Numerical results

Fig. 16 shows the new upper bounds along with the previous ones and simulated results of a rate 1/3 BPSK turbo code in [32] over equivalent SISO QSFCs for $n = 2$

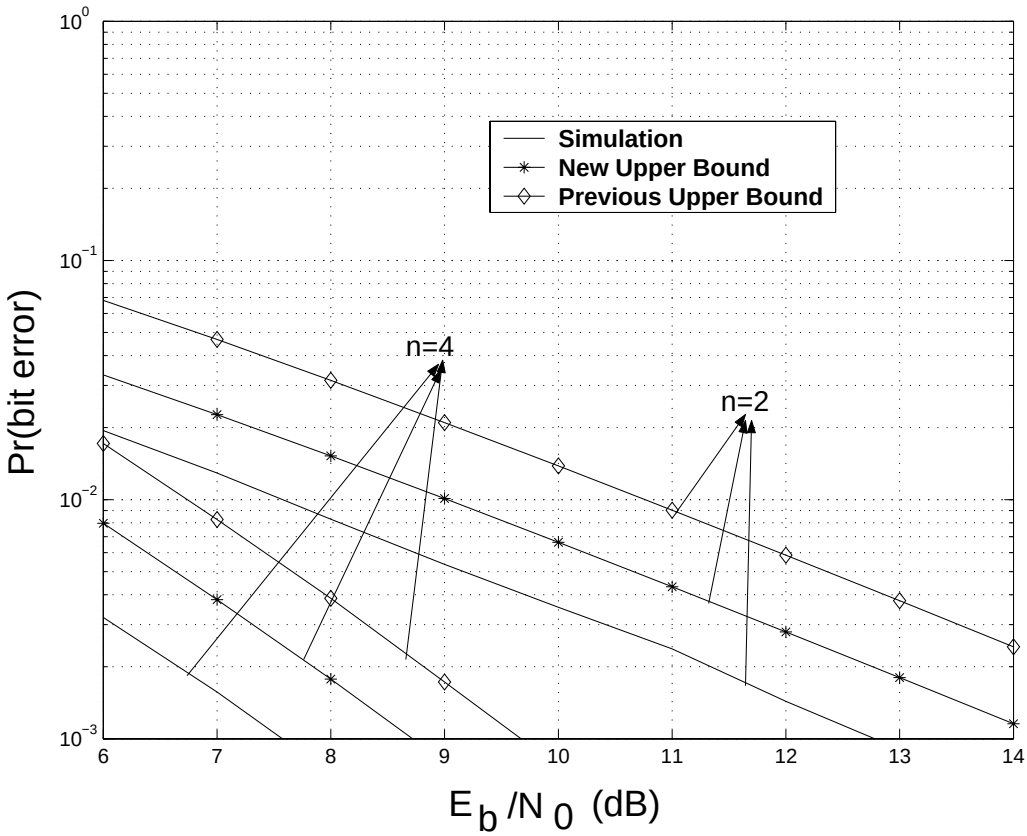

Fig. 16. Simulated performance and upper bounds for serially concatenated turbo code and STBC over QSFCs

and $n = 4$. For the turbo code results, a uniform interleaver and a block length of 1000 were used. The information distance spectrum N_d is found in [32]. The upper three curves correspond to $n = 2$ while the lower three ones are for $n = 4$. Obviously, the new bound is around 2dB tighter for $n = 2$ and 1dB tighter for $n = 4$. It works better for systems with higher orders of diversity.

D. Summary

In this chapter the performance of serially concatenated STBCs over QSFCs was studied. First a system model that defined an equivalent single-input single-output quasi-static fading channel (SISO QSFC) was proposed. This simplification enabled the extension of the bounding approaches proposed in Chapter V. Three specific cases for outer codes in the concatenation scheme were mentioned in details: systematic convolutional code, DD convolutioal code and turbo code. For each code, new upper bounds were derived for two different diversity cases. As in the SISO QSFC case, the derived upper bounds exhibit much closerness to the simulation results.

CHAPTER VII

NEW PERFORMANCE BOUNDS FOR MULTIPLE-INPUT MULTIPLE-OUTPUT QUASI-STATIC FADING CHANNELS

A. Introduction

This chapter deals with another type of space-time codes - space-time trellis codes (STTC) which can provide coding gain along with space diversity. It is organized as follows: First a multi-dimensional system model is presented. Next a tight low SNR upper bound is derived for systematic STTCs after which two tight bounds are respectively derived for the bit error probability of STTCs with single and multiple receiver antennas over QSFCs . The new bounds depend on the unusual information eigenvalue spectrum which is mentioned finally.

B. System model and notation

STTCs fall into the category of multiple-input multiple-output (MIMO) QSFCs. The general system model is as follows:

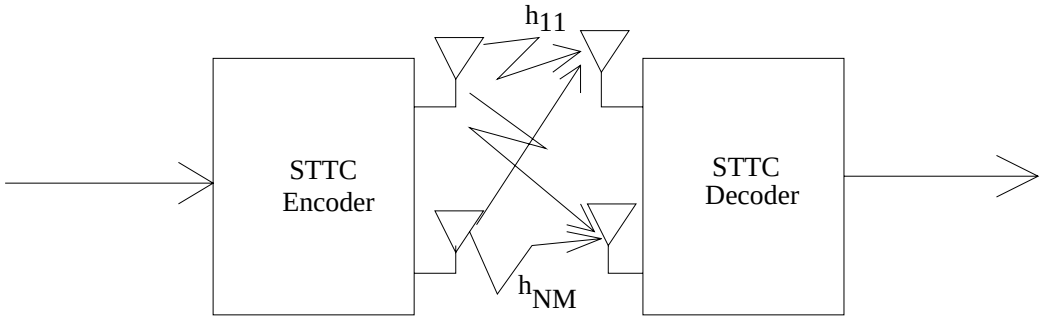

Fig. 17. STTC system model

At time instant k, the received signal at antenna j, r_j^t can be written as:

$$r_j^k = \sqrt{E_s} \sum_{i=1}^{N} h_{ij} x_i^k + n_j^k, \quad j = 1, ..., M, \tag{7.1}$$

where N and M denote the number of transmit and receive antennas, repectively. In (7.1), h_{ij} is the complex channel gain from transmit antenna i to receive antenna j. It is assumed in the rest of the thesis that those channel gains are independent from each other. The noise n_j^k is the usual additive Gaussian noise with variance $N_0/2$ in each dimension and x_i^k is the transmitted symbol with unit energy from antenna i at time instant k. The channel gains will be regrouped in a matrix $\mathbf{h}$, where the (i, j) element h_{ij} is defined as above. In the case of one receive antenna, $\mathbf{h}$ reduces to a vector. For the QSFC case, $\mathbf{h}$ is constant over a frame and independently changes from one frame to the next. A full frame of transmitted symbols x_i^k, $k \in 1, ..., T$ and $i \in 1, ..., N$, is considered to be a transmitted codeword $\mathbf{x}$ which is chosen among the ensemble of all possible valid codewords χ. Similarly, a received vector $\mathbf{r}$ regroups all r_j^k:

$$\mathbf{r} = \mathbf{h}\mathbf{x} + \mathbf{n}. \tag{7.2}$$

Where $\mathbf{n}$ groups all n_j^k in a similar way.

C. Upper bounds on the bit error probability of STTCs over QSFCs

As in the previous cases, the bit error rates (BER) of STTCs over QSFCs can be upper bounded using the Gallager bounding approach in (2.3). The following general form of the BER will be used:

$$P_b = P(e|\mathbf{h} \in R)P(\mathbf{h} \in R) + P(e|\mathbf{h} \in \overline{R})P(\mathbf{h} \in \overline{R}). \tag{7.3}$$

In the above equation, a low SNR bound tighter than 1/2 needs to be derived for $P(e|\mathbf{h} \in R)$; the high SNR term $P(e, \mathbf{h} \in \overline{R})$ is upper bounded using the classical Union-Chernoff bound; and the choices of region R can be found in details in [11], [14] and [17]. In addition, the information eigenvalue spectrum of the STTCs needs to be known for computing the overall bound.

1. Low SNR upper bound

Consider a systematic QPSK space-time trellis code with two transmit antennas and single receive antenna, the receiver sees at time instant k a signal r_k which is:

$$r_k = h_1 s_{1,k} + h_2 s_{2,k} + n_k, \tag{7.4}$$

where $s_{1,k}, s_{2,k}$ are the systematic and parity PSK symbols transmitted from antenna 1 and 2 at time instant k, respectively, and h_1,h_2 are the complex channel gains from each transmit antenna.

If h_2 is unknown to the receiver and only h_1 is used in decoding which means partial channel state information is known, a simple way to detect the received signal is to just decode the received signal on a symbol-by-symbol basis:

$$min_{s_{1,k}} |r_k - h_1 s_{1,k}|^2, \tag{7.5}$$

as h_2 can be treated as a complex Gaussian random variable with zero mean and unit variance, $h_2 s_{2,k} + n_k$ turns out to be zero-mean Gaussian with variance $N_0 + E_s$. Since the code is systematic, the two input information bits at time instant k can be easily extracted from the detected $s_{1,k}$. While this simple decoding strategy is not the most intelligent way to decode the received signal, it must provide an upper bound for the performance of the optimal receiver that has availability to perfect channel state information at all SNRs. Hence, we note that for such a systematic STTC,

the bit error probability of the optimal receiver conditioned on the two instantaneous channel gains is bounded by:

$$P(e|\mathbf{h}) \leq Q\Big(\sqrt{\frac{E_s|h_1|^2}{N_0+E_s}}\Big). \tag{7.6}$$

Obviously, it is tighter than the trivial low SNR bound $P(e|\mathbf{h}) \leq 1/2$.

2. Spherical upper bound

Now we present an upper bound for a systematic QPSK STTC over QSFCs corresponding to a choice of the region R as a N-dimensional hypershere. Only the two transmit antennas ($N = 2$) and one receive antenna case ($M = 1$) is considered.

According to [11], the spherical region can be defined as:

$$R = \{\mathbf{h}/\sum_{i=1}^{N}|h_i|^2 \leq h_0^2\}, \tag{7.7}$$

where h_0 is the radius of R. Hence, R corresponds to the low instantaneous SNR region and $\overline{R}$ vice versa. Using this region and the bound in (7.6), the low SNR term in (7.3) can be expressed as:

$$P(e|\mathbf{h} \in R)P(\mathbf{h} \in R) \leq \int_0^{h_0^2} Q\Big(\sqrt{\frac{E_s|h_1|^2}{N_0+E_s}}\Big)e^{-|h_1|^2}\int_0^{h_0^2-|h_1|^2} e^{-|h_2|^2}d|h_2|^2d|h_1|^2. \tag{7.8}$$

The Union-Chernoff bound for the high SNR term in (7.3) was derived in [14] and [17] for the frame error rates (FER). Suppose $\mathbf{x}$ is the all-zero codeword transmitted and the valid codeword corresponding to an error event is $\mathbf{x'} \in \chi$, it was shown in these two papers that the two eigenvalues λ_1 and λ_2 of matrix $A(\mathbf{x}, \mathbf{x'}) = (\mathbf{x}-\mathbf{x'})(\mathbf{x}-\mathbf{x'})^H$ uniquely define the pairwise error probability (PWEP) between $\mathbf{x}$ and $\mathbf{x'}$. For the bit error rates, all error events that contribute the same set of eigenvalues are grouped and their multiplicity taking into account the number of bit errors are kept track of to compute the union bound. Let $\gamma = E_s/4N_0$, the final form of the spherical upper

bound is as follows:

$$P_b \leq I_1 + I_2 + \frac{A_0}{b} \sum_{(\lambda_1,\lambda_2)} N_{(\lambda_1,\lambda_2)} \left[\frac{e^{-(1+\lambda_1\gamma)h_0^2}}{\gamma(1+\lambda_1\gamma)(\lambda_2-\lambda_1)} + \frac{e^{-(1+\lambda_2\gamma)h_0^2}}{\gamma(1+\lambda_2\gamma)(\lambda_1-\lambda_2)} \right], \quad (7.9)$$

where $b = 2$ is the number of input information bits per trellis transition. The parameters I_1 and I_2 are given by:

$$I_1 = \frac{1}{2}(1 - \sqrt{\frac{x}{1+x}}) - e^{-h_0^2} Q(\sqrt{2x}h_0) + \sqrt{\frac{x}{1+x}} Q(\sqrt{2+2x}h_0), \quad (7.10)$$

$$I_2 = e^{-h_0^2} h_0^2 Q(\sqrt{2x}h_0) - \frac{1}{\sqrt{4\pi x}} e^{-(1+x)h_0^2} h_0 + \frac{1}{4x} e^{-h_0^2} \Big(1 - 2Q(\sqrt{2x}h_0)\Big), \quad (7.11)$$

where $x = \gamma/(2\gamma + 1/2)$. In (7.9), $N_{(\lambda_1,\lambda_2)}$ is the information eigenvalue spectrum of the STTC, i.e. the total number of bit errors associated with error events which will produce eigenvalues (λ_1, λ_2). To tighten the bound, the constant A_0 is given by:

$$A_0 = Q\Big(\sqrt{2\lambda_{min}\gamma}h_0\Big) e^{\lambda_{min}\gamma h_0^2}. \quad (7.12)$$

where λ_{min} is the minimum eigenvalue. The final form of the upper bound is a function of h_0, which can be numerically optimized.

To illustrate the ability of our upper bound to predict the performance of STTCs, in Fig. 18, we plot our upper bound along with the simulated BER performance versus the SNR at each received antenna of the 4 state systematic STTC code from [22] for a frame length of 260 bits (130 PSK symbols transmitted out of each antenna). The information eigenvalue spectrum can be computed using a similar procedure in [11] and [17]. As can be seen from the plot, the upper bound is quite tight at all SNRs. It is to the best of our knowledge the only useful upper bound for the bit error rates of STTCs over QSFCs.

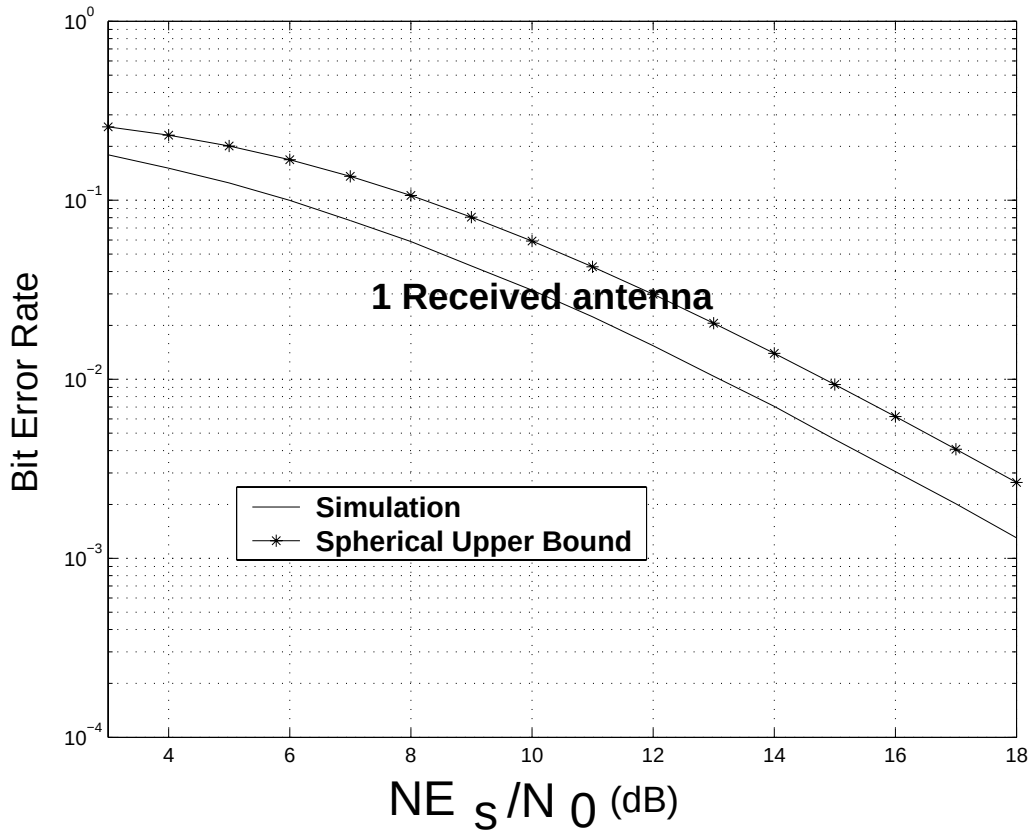

Fig. 18. Simulated performance and spherical upper bound for 4 state STTC in [22] with two transmit and one receive antennas over QSFC

3. Cubical upper bound

For systematic QPSK STTCs with two transmit ($N = 2$) and multiple receive antennas ($M > 1$), the spherical region R doesn't allow for a mathematically tracatable form. Thus a cubical region should be used:

$$R = \{\mathbf{h}/|h_1| \leq h_0, \ldots, |h_2| \leq h_0\}. \tag{7.13}$$

The advantage of this choice of R is the multiple integrals are not nested anymore. It is reasonable to sum all the channel gains out of transmit i into a single equivalent

channel gain whose magnitude squared is defined as: $|h_i|^2 = |h_{i1}|^2 + |h_{i2}|^2 + \ldots + |h_{iM}|^2$ where $1 \leq i \leq N$. These new equivalent channel gains are χ^2 distributed with $2M$ degrees of freedom. When all the equivalent channel gains are less than h_0, the conditional error probability will be upper bounded using the low SNR upper bound in (7.6). For all other cases we apply the modified Union-Chernoff bound as in the previous section which was also presented in [11],[14] for computing the FER. The resulting multiple integrals are separable allowing the straightforward computing of the final bound. Let $G_M(x)$ be the cumulative distribution function (cdf) of a χ^2_{2M} distributed random variable, the resulting upper bound is as follows:

$$\begin{aligned} P_b \leq\ & \frac{1}{(M-1)!}(I_0 - \sum_{m=1}^{M-1} B_m)\overline{\gamma}(M, h_0^2) + \frac{A_0}{b} \sum_{(\lambda_1,\lambda_2)} N_{(\lambda_1,\lambda_2)} \frac{1}{(1+\lambda_1\gamma)^M (1+\lambda_2\gamma)^M} \\ & \Big\{ [1 - G_M((1+\lambda_1\gamma)h_0^2)][1 - G_M((1+\lambda_2\gamma)h_0^2)] + \\ & \sum_{i=1}^{2} G_M((1+\lambda_{3-i}\gamma)h_0^2)[1 - G_M((1+\lambda_i\gamma)h_0^2)] \Big\}, \end{aligned} \tag{7.14}$$

where

$$I_0 = \frac{1}{2}\Big[1 - \sqrt{\frac{x}{x+1}}\Big] + \sqrt{\frac{x}{x+1}} Q(\sqrt{2h_0^2(1+x)}) - e^{-h_0^2} Q(\sqrt{2xh_0^2}), \tag{7.15}$$

$$B_m = \frac{h_0^{2m} e^{-h_0^2}}{m!} Q(\sqrt{2xh_0^2}) + \frac{\sqrt{x/\pi}\Big[\overline{\gamma}\Big(m + 1/2, (1+x)h_0^2\Big)\Big]}{2(m!)(1+x)^{m+1/2}}. \tag{7.16}$$

$$\tag{7.17}$$

In (7.14), the definitions for A_0, x, γ and $N_{(\lambda_1,\lambda_2)}$ are as defined previously in this chapter, and $\overline{\gamma}(A, B)$ is the incomplete gamma function defined in Chapter VI. The final expression is a function of h_0 which can be optimized numerically.

The cubical upper bound is used in Fig. 19 for the same STTC code as in Fig. 18 with two and four receive antennas. In these cases as well, the derived upper bound continues to show remarkable ability in predicting the performance of STTCs when

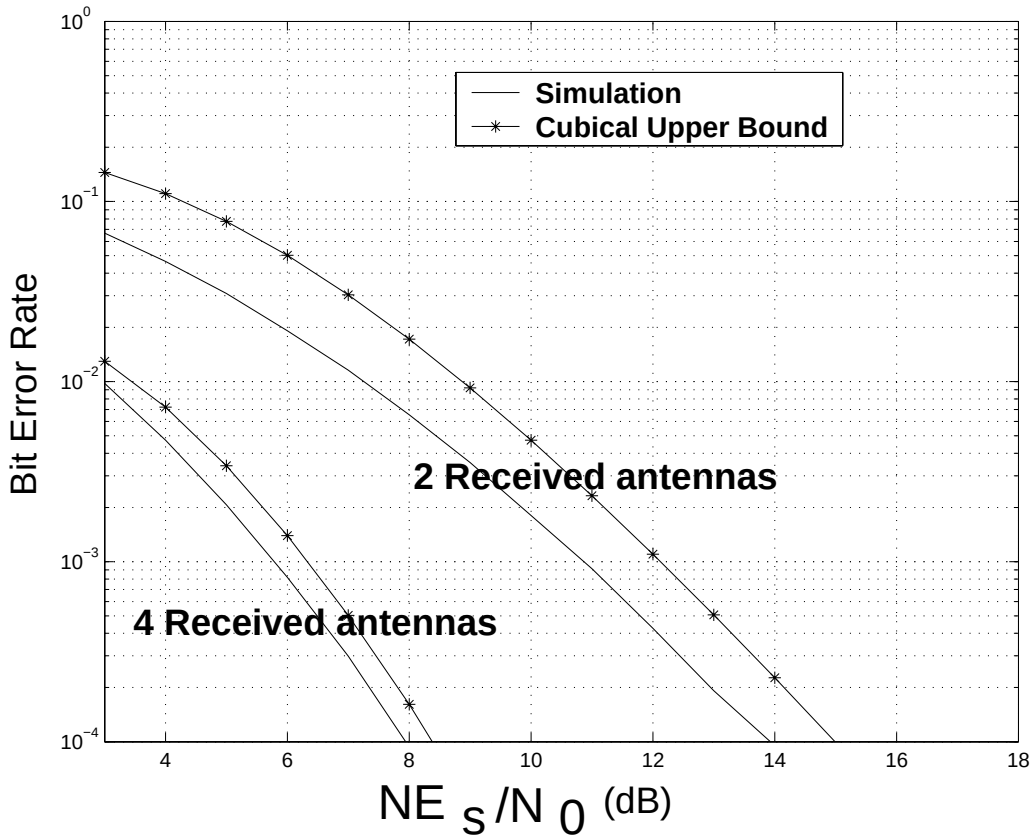

Fig. 19. Simulated performance and cubical upper bound for 4 state STTC in [22] with two transmit and mutiple receive antennas over QSFCs

a higher number of receive antennas is used.

The final section of this chapter deals with the issue of computing the unusual information eigenvalue spectrum of STTCs.

D. Information eigenvalue spectrum of STTCs

A similar procedure as in [11] and [17] can be used for computing the information eigenvalue spectrum of STTCs. A trellis approach is used to compute the information eigenvalue spectrum of linear STTCs. First, a N-tuple is defined for each trellis

transition of the STTC. This vector keeps track of the Euclidean distance out of each transmit antenna as well as the cross terms between antennas. Then the Euclidean distance N-tuple is cumulated using a trellis structure. Finally, each distinct error event is corresponding to one cumulated N-tuple which is used to compute the eigenvalues of the matrix A (previously defined in this chapter). For computing the bit error rates, all error events that contribute the same set of eigenvalues are grouped and the multiplicity of each pair of eigenvalues taking into account the number of bit errors needs to be kept track of separately.

Several complexity reduction techniques are used. First, only error events that diverge once from the all-zero codeword in the first trellis step and then merge back only once are considered, which means concatenations of error events are excluded from consideration. Also, we restrict our search to error events of up to one-half frame length [11],[19]. All these greatly simply the task to compute the information eigenvalue spectrum for low complexity STTCs with moderate frame lengths. Fig. 20 shows a flow chart of the full algorithm.

E. Summary

In this chapter, the general approach in Chapter II was combined with a low SNR bounding approach to derive performance upper bounds for the bit error rates of STTCs over QSFCs. First a tight low SNR bound was derived for systematic QPSK STTCs using similar approaches in Chapter IV. Then two possibilities corresponding to two different regions of R: cubical region and spherical region were provided to apply the general Gallager bound after which a spherical upper bound was presented for STTCs with single receive antenna and cubical upper bound for multiple receive antennas cases. All these upper bounds showed particular tightness compared to

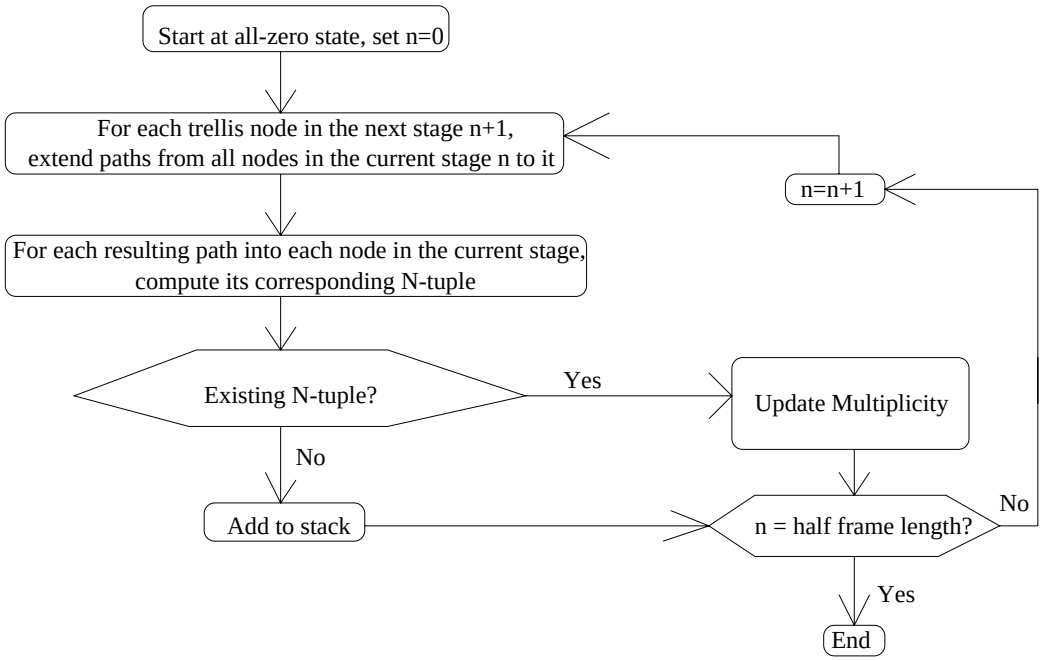

Fig. 20. Flow chart for computing the information eigenvalue spectrum of STTC codes

the simulation results. Finally, the problem of computing information eigenvalue spectrum of small contraint length STTCs was explored.

CHAPTER VIII

CONCLUSION

The work contained in this thesis gave more reliable performance analysis of channel codes over quasi-static fading channels (QSFC). Previous approaches in bounding codes performance over QSFCs used the Gallager bound and gave quite loose bounds. The reason lied in the trivial form of the low instanstaneous SNR bound employed. In this thesis, we started from finding a novel approach for classifying convolutional codes according to their performance over QSFCs. Then tighter performance upper bounds were provided for two classes of convolutional and turbo codes in the low instantaneous SNR region. Consequently, they were used with the union bound to bound codes performance over single-input single-output (SISO) QSFCs. This new analytical bounding tool was applied to both systematic and differential detectable convolutional codes as well as turbo codes. It produced new bounds at least 1 dB tighter than the existing bounds. The new approach was then extended to several more interesing cases. First two cases of serially concatenated space-time block codes (STBC) systems over QSFCs were analyzed. The first case involved a convolutional code as an outer code while the second a turbo code. As in the previous cases, the derived new upper bounds were more likely to capture the performance of the concatenated scheme than the previous bounds. Towards the end, the case of the spectrally efficient space-time trellis codes (STTC) was explored. Two upper bounds: spherical and cubical upper bound were derived for the bit error probability of STTCs over QSFCs. The upper bounds were very adequate for characterizing the performance of systematic STTCs over QSFCs. Also included was a general procedure to compute the information eigenvalue spectrum of STTCs.

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