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Application of Kinematic Wave Equations to Border Irrigation Design

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Accuracy of the kinematic wave (*KW*) approximation was tested on 31 experimental irrigation borders by computing the *KW* number and its modified version. In a majority of cases, this approximation was found to be sufficiently accurate. A *KW* model, reported previously, was used to derive dimensionless advance and recession curves for application to border irrigation design. These curves can be developed for a wide range of design variables and parameters for ready practical use. A step by step design procedure, based on this model is presented. Its validity was tested by comparing observed irrigation efficiencies with those computed by the model. A close agreement between computed and observed efficiencies suggests that the *KW* model is reasonably accurate. Its simplicity and physical basis may justify its large-scale field application.

1. Introduction

A knowledge of advance, recession, distribution of depth of water and distribution of infiltrated water is required for optimal design of border irrigation. One way to determine these design variables is by using mathematical models. There are many models of border irrigation. Most of these models can be classified in order of increasing complexity as (1) storage models, (2) kinematic models (3) zero-inertia models, and (4) hydrodynamic models. A recent study by Ram[1] presented a comprehensive survey of these models. Bassett, Fangmeier and Strelkoff[2] have discussed the current state of the art of hydraulics of surface irrigation.

This study employs a kinematic wave (*KW*) model. Sherman and Singh,[3,4] and Singh and Sherman[5] provided a comprehensive mathematical treatment of *KW* modelling of surface irrigation. Singh and Ram[6] tested this *KW* model by using data from 31 experimental borders and concluded that the model was sufficiently accurate for predicting advance and horizontal recession; the model is not capable of accommodating vertical recession. This concurred with the earlier studies by Smith,[7] and Chen, McCann and Singh.[8] However, these studies did not provide quantitative estimates regarding the accuracy of the *KW* model. Neither were irrigation efficiencies, required for irrigation design, computed. In this study we compute the kinematic wave number[9] and its modified version[10] indicating the model accuracy for all the data sets used in that study.

Although the *KW* approximation has been employed in a number of studies on surface irrigation,[3–5,7,8,11–14] its application to actual border irrigation design does not appear to have been reported. One of the objectives of designing a border irrigation system is to make optimum use of the water available for irrigating a given crop. In practice there exists a wide range of inflow stream sizes, irrigation durations and border lengths for which this objective should be achieved. This can be done more conveniently by employing dimensionless solutions of *KW* equations. We attempt to develop a design procedure based on dimensionless solutions of the *KW* model. The design procedure is tested by computing irrigation efficiencies for a typical depth of application of 0·1 m for data from 31 experimental borders, and by comparing them with observations.

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NOTATION

a	Dimensionless exponent in the Kostyakov infiltration equation (Eqn 4)
A_s	Dimensionless bulk density of soil
d_a	Depth of water applied (L)
d_d	Average absolute numerical deviation of stored depth from the average depth of water stored in root zone along the border (L)
d_n	Depth of water needed in the root zone (L)
d_s	Average depth of water stored in the root zone (L)
D_r	Depth of root zone (L)
E_a	Application efficiency (dimensionless)
E_d	Distribution efficiency (dimensionless)
E_s	Storage efficiency (dimensionless)
f	Capacity rate of infiltration (LT^{-1})
F_o	Froude number (dimensionless)
G	Normal depth of flow at the upstream end (L)
h	Depth of flow (L)
h^*	Normalized depth of flow (dimensionless)
K	Parameter in the Kostyakov infiltration equation (Eqn 4, LT^{-a})
K_1	Kinematic flow number (dimensionless)
K_2	Dimensionless number corresponding to the momentum term associated with infiltration
L	Border length (L)
n	Dimensionless exponent in the depth discharge relation (Eqn 2)
n_m	Manning's roughness coefficient ($L^{-1/3}T$)
P_w	Soil moisture deficit in percent (dimensionless)
P_1	Modified kinematic flow number (dimensionless)
P_2	Modified dimensionless number corresponding to the momentum term associated with infiltration
q	Inflow rate per unit width (L^2T^{-1})
q_o	Constant inflow rate per unit width at the upstream end (L^2T^{-1})
q^*	Normalized inflow rate (dimensionless)
Q	Discharge per unit width (L^2T^{-1})
S_f	Slope of the energy line (dimensionless)
S_o	Bed slope (dimensionless)
t	Time (T)
t^*	Normalized time (dimensionless)
T_o	Normalizing time (T)
T_1	Duration of irrigation (T)
T_1^*	Normalized duration of irrigation (dimensionless)
v	Velocity of flow (LT^{-1})
V_o	Normalizing velocity (LT^{-1})
v^*	Normalized velocity (dimensionless)
x	Distance along the border measured from the upstream end (L)
x^*	Normalized distance along the border measured from the upstream end (dimensionless)
X_o	Normalizing distance (L)
β	Kinematic friction parameter ($L^{1-n}T^{-1}$)
β_o	Kinematic friction parameter in the relation between normal depth of flow at the upstream and the corresponding inflow ($L^{1-n}T^{-1}$)
ξ	Time history of advance front (T)
ξ^*	Normalized time history of advance front (dimensionless)
τ	Infiltration opportunity time (T)

2. Kinematic wave model

The KW model, developed by Sherman and Singh,[3,4] can be expressed for flow over a plane with a small slope and porous bed on a unit width basis as

$$\frac{\partial h}{\partial t}+\frac{\partial Q}{\partial x}=-f[t-\xi(x)] \qquad \ldots(1)$$

$$Q=v(x,t)\,h(x,t)=\beta h^n \qquad \ldots(2)$$

$$\frac{d\xi(x)}{dx}=\{\beta h^{n-1}\,[x,\xi(x)]\}^{-1} \qquad \ldots(3)$$

in which $h(x,t)$ is depth of flow, $Q(x,t)$ is discharge, v is velocity of flow, $f(\tau)$ is infiltration rate, $\tau=t-\xi(x)$, infiltration opportunity time, and n and $\beta>0$ are KW parameters; n varies from 1 to 3 inclusive. From now on, a symbol will be defined when it appears for the first time. For easy referencing, all the symbols are given in the Notation. Note that $t=\xi(x)$ denotes the time history of the advance front or the advance function. The infiltration rate $f(\tau)$ is assumed to depend only on the difference τ between the total elapsed time and the advance time, that is, it is time-dependent but independent of x for $x>0$. It can be determined by the Kostyakov equation[15] as

$$f(\tau)=\begin{cases} aK\tau_c^{a-1}, & 0\leqslant\tau\leqslant\tau_c \\ aK\tau^{a-1}, & \tau\geqslant\tau_c \end{cases} \qquad \ldots(4)$$

τ_c is a constant and can be specified for a given soil. It was taken to be 0·1 min in this study. The exponent a varies between 0 and 1, and $K>0$ is a parameter.

The initial conditions can be expressed as

$$h(0,t)=h_o(t), \quad 0\leqslant t\leqslant T_1 \qquad \ldots(5a)$$

$$h(0,t)=0, \quad t\geqslant T_1 \qquad \ldots(5b)$$

$$\xi(0)=0 \qquad \ldots(5c)$$

Where T_1 is duration of irrigation. One can also define initial conditions in terms of discharge at the upstream end. $Q(0,t)=q(t)$, $0\leqslant t\leqslant T_1$; $Q(0,t)=0$, $t\geqslant T_1$. $q(t)$ specifies the time-varying rate of inflow.

2.1. *Dimensionless solutions*

A principal advantage of dimensionless solutions is the reduced number of parameters contained in them. Further, the dimensionless solutions are independent of the system of units used. The dimensionless variables appear as ratios with respect to normalizing quantities. Therefore, it is easy to interpret the effect of proportional variations of one variable on the other. Because of the reduced size of the variables, dimensionless solutions are easy for graphical representation. To reduce Eqns 1–3, the following normalizing quantities are defined:

$q_o=$ constant inflow at the upstream end
$V_o=$ normal velocity at the upstream end
$G=$ normal depth of flow at the upstream end corresponding to the constant inflow q_o, and can be determined as

$$G=(\beta_o q_o)^{0\cdot6} \qquad \ldots(6)$$

$$\beta_o=n_m/S_o^{0\cdot5} \qquad \ldots(7)$$

where n_m is Manning's roughness coefficient and S_o bed slope.

T_o = normalizing distance defined as

$$T_o = (G/K)^{1/a} \qquad \ldots(8)$$

X_o = normalizing distance defined as

$$X_o = V_o T_o \qquad \ldots(9)$$

By using the normalizing quantities, the dimensionless variables can be defined as

$$q^* = \frac{q}{q_o},\ h^* = \frac{h}{G},\ v^* = \frac{v}{V_o},\ x^* = \frac{x}{X_o},\ t^* = \frac{t}{T_o},\ \tau^* = \frac{\tau}{T_o},\ T_1^* = \frac{T_1}{T_o} \qquad \ldots(10)$$

By substituting dimensionless variables in Eqns 1 and 3 and coupling Eqns 1 and 2, we obtain

$$\frac{\partial h^*}{\partial t^*} + nh^{*n-1}\frac{\partial h^*}{\partial x^*} + a(\tau^*)^{a-1} = 0 \qquad \ldots(11)$$

$$\frac{d\xi^*(x^*)}{dx^*} = \frac{1}{h^{*n-1}},\ \xi^*(0) = 0 \qquad \ldots(12)$$

These equations are subject to

$$h^*(0,t^*) = h_o^*(t^*), \qquad 0 \leqslant t^* \leqslant T_1^* \qquad \ldots(13a)$$

$$h^*(0,t^*) = 0, \qquad t^* \geqslant T_1^* \qquad \ldots(13b)$$

$$\xi^*(0) = 0$$

The solution of Eqns 11 and 12 subject to Eqn 13 consists of two parts. The first part, representing the advance and storage phases, is for $0 \leqslant t^* \leqslant T_1^*$. The second part, representing the recession phase, is for $t^* \leqslant T_1^*$. The solution for the first part was obtained numerically by the kinematic wave train (*KWT*) method. This is described by Ram, Singh and Prasad[14] and Singh and Ram.[6] This requires specification of grid spacing which was taken as 1·524 m and ratio of advance tip depth to normal depth of flow which was taken as 0·05. The solution for the second part was obtained explicitly in a sequential manner, and is described by Ram, Singh and Prasad.[14] This method of solution of Eqns 11–13 is part numerical and part analytical, and is simpler and more efficient than the numerical method proposed by Sherman and Singh,[4] and Singh and Sherman.[5]

2.2. *Experimental data*

Thirty-one sets of data, as given in Tables 1–3, were used in this study. Four sets of data, designated as Roth-8–Roth-11, are due to Roth,[16] and Roth *et al.*[17] These data were collected on non-vegetated borders (soil classified as sandy loam, bulk density 1·4). Nine sets of data, referred to as K-1–K-9, were collected for irrigations on vegetated borders (bromegrass, bromegrass alfalfa, grain sorghum, barley), and are due to Kincaid[18]. Eighteen sets of data, designated as R-1–R-18, are due to Ram.[19–20] The data sets R-1–R-9 were collected on non-vegetated borders, and R-10–R-18 on vegetated (wheat crop) borders. For complete details on these data, see the cited references.

TABLE 1

Irrigation data sets for freely draining borders[16–18]

Parameters	*Roth-8*	*Roth-9*	*Roth-10*	*Roth-11*	*K-1*	*K-2*	*K-3*	*K-4*	*K-5*	*K-6*	*K-7*	*K-8*	*K-9*
Inflow rate, q_o ($m^2\ min^{-1}$)	0·105	0·145	0·201	0·146	0·301	0·206	0·307	0·223	0·323	0·457	0·346	0·329	0·379
Infiltration constant, K ($m\ min^{-a}$)	0·009	0·015	0·024	0·0178	0·012	0·016	0·020	0·014	0·008	0·008	0·0189	0·022	0·027
Infiltration exponent, a	0·441	0·336	0·110	0·257	0·242	0·259	0·151	0·306	0·428	0·334	0·224	0·161	0·280
Death at the upstream end, $G(m)$	0·015	0·029	0·028	0·026	9·114	0·108	0·135	0·137	0·117	0·102	0·131	0·087	0·087
Manning's roughness coefficient, n_m ($m^{-1/3}\ s$)	0·017	0·035	0·026	0·029	0·070	0·100	0·136	0·191	0·132	0·080	0·182	0·220	0·190
Chezy's roughness coefficient, C_h ($m^{-1/2}\ s^{-1}$)	29·32	15·78	21·23	18·74	9·96	6·89	5·27	3·76	5·30	8·55	3·92	3·03	3·51
Border bed slope, S_o	0·0010	0·0010	0·0011	0·0010	0·0002	0·0002	0·0004	0·0004	0·0006	0·0008	0·001	0·005	0·005
Kinematic friction, ($m^{1/3}\ min^{-1}$)	111·6	52·4	76·5	65·4	11·2	8·5	8·6	6·1	11·5	20·5	10·3	19·2	22·2
Border length, $L(m)$	91·46	91·46	91·46	91·46	121·95	182·93	213·41	167·68	213·41	213·41	182·93	213·41	213·41
Length of one reach (m)	9·15	9·15	9·15	9·15	15·24	15·24	15·24	15·24	15·24	15·24	15·24	30·49	30·49
Duration of irrigation (min)	181·4	179·9	179·0	179·3	30·0	82·0	55·0	110·0	48·0	43·0	51·0	65·0	105·0
Number of stations	11	11	11	11	9	13	15	12	15	15	13	8	8

TABLE 2

Irrigation data sets for non-vegetated borders with bund at the downstream end[19,20]

Parameters	*R-1*	*R-2*	*R-3*	*R-4*	*R-5*	*R-6*	*R-7*	*R-8*	*R-9*
Inflow rate, q_o ($m^2\ min^{-1}$)	0·160	0·120	0·080	0·160	0·120	0·080	0·160	0·120	0·080
Infiltration constant, K ($m\ min^{-a}$)	0·004	0·005	0·005	0·005	0·005	0·004	0·004	0·003	0·006
Infiltration exponent, a	0·567	0·574	0·590	0·605	0·588	0·615	0·690	0·690	0·527
Depth at the upstream end, $G(m)$	0·026	0·023	0·015	0·035	0·033	0·037	0·050	0·039	0·031
Manning's roughness coefficient, n_m ($m^{-1/3}\ s$)	0·059	0·066	0·048	0·077	0·092	0·100	0·080	0·071	0·073
Chezy's roughness coefficient, C_h ($m^{-1/2}\ s^{-1}$)	9·26	8·11	10·26	7·44	6·15	5·49	7·54	8·21	7·72
Border bed slope, S_o	0·005	0·005	0·005	0·003	0·003	0·003	0·001	0·001	0·001
Kinematic friction, ($m^{-1/3}\ min^{-1}$)	72·4	64·5	87·7	47·7	35·7	32·9	23·6	26·8	26·2
Border length, $L(m)$	100·0	100·0	100·0	100·0	100·0	100·0	100·0	100·0	100·0
Length of one reach (m)	10·0	10·0	10·0	10·0	10·0	10·0	10·0	10·0	10·0
Length from the upstream end where impounding starts	80·0	82·0	88·0	70·0	70·0	74·5	35·0	38·0	40·5
Duration of irrigation (min)	22·5	37·0	59·0	35·5	50·0	74·0	50·0	59·0	95·0
Number of stations	11	11	11	11	11	11	11	11	11

2.3. *Parameter estimation*

The *KW* model, expressed by Eqns 1–3, contains two unknown infiltration parameters K and a of the Kostyakov equation and two unknown *KW* parameters n and β. The values of these parameters for each data set are given in Tables 1–3. The infiltration parameters were estimated by using a volume balance method. The *KW* parameters were estimated by representing Eqn 2 by Manning's equation. This yields $n = 5/3$ and β as expressed by

$$\beta = S_f^{0\cdot5}/n_m \qquad \ldots(14)$$

where S_f is slope of the energy line or friction slope.

3. Validation of the *KW* model

Criteria for assessing accuracy of the *KW* model can be derived from dimensionless forms of St Venant equations of shallow flow over a border as proposed by Katopodes and Strelkoff.[10] They, following the work of Woolhiser and Liggett,[9] derived the following parameters:

$$P_1 = K_1 F_o^2 \qquad \ldots(15)$$

$$P_2 = K_2 F_o^2 \qquad \ldots(16)$$

TABLE 3

Irrigation data sets for vegetated (wheat crop) borders with bund at the downstream end[19,20]

Parameters	*R-10*	*R-11*	*R-12*	*R-13*	*R-14*	*R-15*	*R-16*	*R-17*	*R-18*
Inflow rate, q_o	0·160	0·120	0·080	0·160	0·120	0·080	0·160	0·120	0·080
Infiltration constant, K (m min^{-a})	0·004	0·004	0·005	0·004	0·004	0·006	0·004	0·003	0·005
Infiltration exponent, a	0·620	0·630	0·533	0·674	0·600	0·533	0·640	0·690	0·585
Depth at the upstream end, $G(m)$	0·038	0·035	0·030	0·045	0·043	0·040	0·072	0·053	0·044
Manning's roughness coefficient, n_m (m$^{-1/3}$ s)	0·114	0·132	0·154	0·117	0·145	0·189	0·146	0·116	0·130
Chezy's roughness coefficient, C_h (m$^{-1/2}$ s^{-1})	5·07	4·32	0·363	5·10	4·10	3·10	4·41	5·26	4·57
Border bed slope, S_o	0·005	0·005	0·005	0·003	0·003	0·003	0·001	0·001	0·001
Kinematic friction, (m$^{-1/3}$ min^{-1})	37·1	32·0	27·6	28·1	22·7	17·5	13·0	16·3	14·6
Border length, $L(m)$	100·0	100·0	100·0	100·0	100·0	100·0	100·0	100·0	100·0
Length of one reach (m)	10·0	10·0	10·0	10·0	10·0	10·0	10·0	10·0	10·0
Length from the upstream end where impounding starts	70·0	71·5	76·0	57·5	60·0	65·0	10·0	15·5	35·0
Duration of irrigation (min)	41·0	51·0	75·0	50·0	60·0	96·0	60·0	77·0	105·0
Number of stations	11	11	11	11	11	11	11	11	11

in which

$$F_o^2 = \frac{V_o^2}{gG} \qquad \ldots(17)$$

$$K_1 = \frac{S_o X_o}{G F_o^2} \qquad \ldots(18)$$

$$K_2 = \frac{a K^*}{2} \qquad \ldots(19)$$

$$K^* = \frac{K T_o^2}{G} \qquad \ldots(20)$$

Here F_o is Froude number, g is acceleration due to gravity, K^* is dimensionless infiltration constant, K_1 is kinematic flow number, K_2 is dimensionless number corresponding to the momentum term associated with infiltration, P_1 is modified kinematic flow number, and P_2 is modified dimensionless number corresponding to the momentum term associated with infiltration.

TABLE 4

Dimensionless parameters in Eqns 15–20

Data set	*Dimensionless parameters*						
	a	F_o	K_1	P_1	K^*	K_2	P_2
Roth-8	0·44	0·30	15·79	1·38	1·00	0·22	0·019
Roth-9	0·34	0·16	45·53	1·16	1·00	0·17	0·004
Roth-10	0·11	0·23	21·46	1·08	1·00	0·06	0·003
Roth-11	0·25	0·19	27·01	0·96	1·00	0·13	0·005
K-1	0·24	0·04	29920·00	51·35	1·00	0·12	0·000
K-2	0·26	0·03	6214·50	6·03	1·00	0·13	0·000
K-3	0·16	0·03	97438·00	1061·10	1·00	0·08	0·000
K-4	0·31	0·02	14963·00	8·18	1·00	0·15	0·000
K-5	0·43	0·04	4258·00	7·80	1·00	0·21	0·000
K-6	0·33	0·08	10927·00	60·70	1·00	0·11	0·001
K-7	0·22	0·04	79567·50	120·50	1·00	0·11	0·000
K-8	0·16	0·07	254974·00	1179·00	1·00	0·08	0·000
K-9	0·28	0·08	2637·00	16·36	1·00	0·14	0·000
R-1	0·57	0·21	771·90	33·70	1·00	0·28	0·012
R-2	0·57	0·18	530·80	19·40	1·00	0·28	0·009
R-3	0·59	0·23	245·50	13·18	1·00	0·29	0·016
R-4	0·61	0·13	620·30	10·49	1·00	0·30	0·005
R-5	0·59	0·11	802·50	9·28	1·00	0·29	0·003
R-6	0·62	0·10	942·30	8·68	1·00	0·30	0·003
R-7	0·69	0·08	425·90	2·47	1·00	0·35	0·002
R-8	0·69	0·08	411·30	2·83	1·00	0·35	0·002
R-9	0·53	0·08	281·41	1·71	1·00	0·26	0·002
R-10	0·62	0·11	1367·10	17·92	1·00	0·31	0·004
R-11	0·63	0·10	1904·00	18·10	1·00	0·32	0·003
R-12	0·53	0·08	1711·77	11·48	1·00	0·27	0·002
R-13	0·67	0·09	1120·90	8·94	1·00	0·34	0·003
R-14	0·60	0·07	1947·20	9·98	1·00	0·30	0·002
R-15	0·53	0·05	1745·80	5·13	1·00	0·27	0·001
R-16	0·64	0·05	1348·30	2·67	1·00	0·32	0·001
R-17	0·69	0·05	815·90	2·30	1·00	0·35	0·001
R-18	0·59	0·05	921·90	1·96	1·00	0·29	0·001

In border irrigation the Froude number is usually very small.[10] The acceleration terms in the momentum equation can therefore be neglected and the zero-inertia approximation thereof is sufficiently accurate. If X_o is large then the zero-inertia approximation reduces to the kinematic wave approximation. Morris and Woolhiser[21] found that the *KW* approximation was sufficiently accurate for $P_1 \geqslant 5$ where X_o was considered as the length of the overland flow plane. Katopodes and Strelkoff[10] suggested $P_1\ x^* \geqslant 100$ for the *KW* model to be accurate for border irrigation.

To evaluate the accuracy of the *KW* model employed here, P_1 and P_2 were computed for each of 31 data sets. Their values along with those of other relevant parameters are given in Table 4. The two parameters K_2 and P_2 which are associated with the infiltration term in the momentum equation are very small ($0{\cdot}06 \leqslant K_2 \leqslant 0{\cdot}345$, $0{\cdot}000 \leqslant P_2 \leqslant 0{\cdot}019$). Therefore, their effect on the flow phenomenon is negligible. The values of the Froude number F_o are small ($0{\cdot}023 \leqslant F_o \leqslant 0{\cdot}296$), $P_1 \geqslant 5$ and $K_1 > 100$ for a majority of the data sets. However, there are some data sets where the criteria for accuracy of the *KW* approximation[9,21] are not satisfied. It is not clear if the model would be acceptable on these borders from a design standpoint.

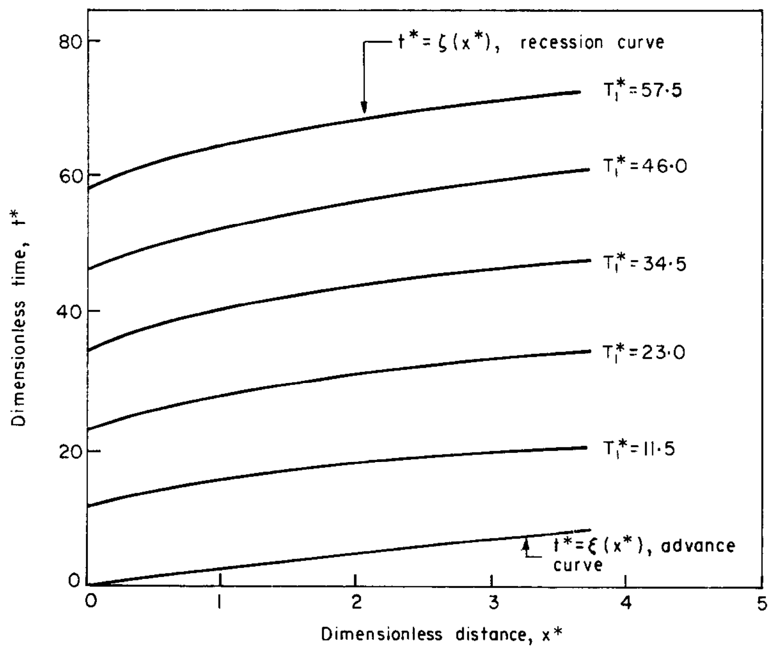

Fig. 1. Dimensionless advance and recession curves for a constant value of a = *0·2 in the Kostyakov equation and various durations of irrigation. The advance is computed by the* KWT *method and the recession by the sequential method*

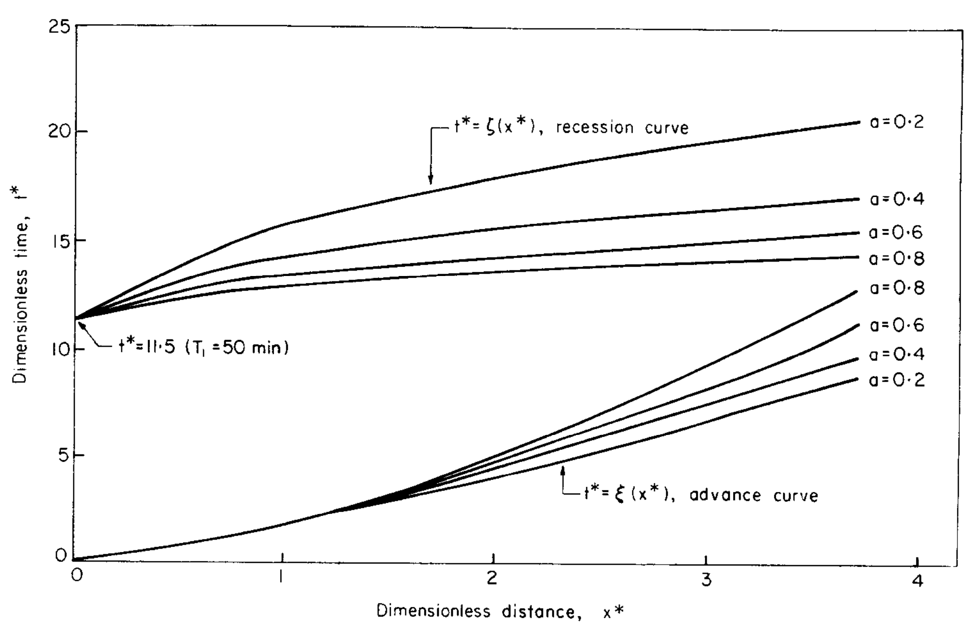

Fig. 2. Dimensionless advance and recession curves for a constant duration of irrigation t* = *11·5 and various values of* a *in the Kosryakov equation. The advance is computed by the* KWT *method and the recession by the sequential method*

TABLE 5

Advance and recession times from dimensionless curves

Data set	*Dimensionless distance*	*Dimensionless time*			
		Advance		*Recession*	
		From Fig. 2	*Observed*	*From Fig. 2*	*Observed*
Roth-8	3·04	7·75	7·75	63·7	60·5
Roth-9	2·75	6·37	6·34	33·5	32·7
Roth-11	3·70	8·75	9·00	48·8	49·7
K-5	0·15	0·21	0·17	—	—
K-9	0·74	1·20	0·95	—	—
R-1	0·58	1·04	1·06	1·82	1·39
R-2	1·12	2·25	2·50	4·30	3·21
R-3	2·30	5·37	6·60	10·90	12·30
R-4	0·57	0·85	0·77	2·32	2·08
R-5	0·69	1·20	1·04	2·80	2·52
R-6	0·90	1·60	1·55	4·30	3·62
R-7	0·24	0·30	0·23	1·79	1·79
R-8	0·36	0·48	0·46	2·30	2·26
R-9	0·75	1·30	1·38	6·30	5·64
R-10	0·51	0·75	0·71	2·26	1·89
R-11	0·55	0·82	0·78	2·37	1·92
R-12	1·01	1·90	1·68	5·00	3·72
R-13	0·37	0·50	0·45	2·32	1·86
R-14	0·42	0·56	0·51	1·66	1·72
R-15	0·89	1·55	1·35	4·80	3·81
R-18	0·35	0·50	0·40	2·70	2·74

4. Dimensionless advance and recession

For a fixed value of n ($n = 5/3$ for Manning's equation and $n = 3/2$ for Chezy's equation) the only unknown parameter appearing explicitly in Eqns 11 and 12 is a if we seek the solution for advance. However, if the solutions are extended to calculate recession, the duration of irrigation will also enter as another parameter in solutions for the complete irrigation cycle.

Fig. 1 shows a plot between dimensionless distance x^* and time t^* for various durations of irrigation T_1^* for a specified value of $a = 0{\cdot}2$. We get only one advance curve but a number of recession curves corresponding to different values of T_1^*. All the recession curves are approximately parallel and can therefore be transferred to a single recession curve for a specified parameter a. By keeping $T_1^* = 11{\cdot}5$, the dimensionless advance and recession curves were derived by varying a from 0·2 to 0·8 as shown in *Fig. 2*. For any other T_1^* the recession curve can be shifted parallel to the recession curve of *Fig. 2* corresponding to the given parameter a. To test the accuracy of these dimensionless curves, the values of t^* at some x^* for different data sets were read from *Fig. 2* and are given along with observed t^* in *Table 5*. The values of t^* read from the figure and those observed for the same values of x^* agree quite closely. This shows that these dimensionless curves are sufficiently accurate. However, their validity still remains to be tested for all possible applicable ranges of input data.

5. Application to border irrigation design

To design an irrigation border the variables assumed to be known are: (1) depth of water to be applied in the root zone; (2) soil infiltration characteristics; (3) border bed roughness, and (4) the initial and boundary conditions. The unknown variables are: (1) inflow rate; (2) duration of irrigation; (3) length of the border, and (4) infiltrated depth of water. The inflow rate, duration of irrigation and length of the border are adjusted to match the advance and recession curves. This affects the infiltrated depth of water and the depth of water stored in the root zone and in turn the irrigation efficiencies corresponding to the specified depth of water application.

The procedure for design of border irrigation is as follows.

(1) Depth of water to be applied in the root zone can be calculated by using the following relationship:

$$d_n = \frac{P_w A_s D_r}{100} \qquad \ldots(21)$$

where d_n is depth of water needed in the root zone. A_s bulk density of soil, P_w soil moisture deficit in percent and D_r depth of root zone.

(2) We assume that inflow rate q_o is known.

(3) We determine advance and recession curves as described previously.

(4) Although there are various ways of expressing irrigation efficiencies,[22–32] we compute them by using the following most commonly used equations:[22]

$$\text{Application efficiency, } E_a = 100\,\frac{d_s}{d_a} \qquad \ldots(22)$$

$$\text{Distribution efficiency, } E_d = 100\left(1 - \frac{d_d}{d_s}\right) \qquad \ldots(23)$$

$$\text{Storage efficiency, } E_s = 100\,\frac{d_s}{d_n} \qquad \ldots(24)$$

where d_s is average depth of water stored in root zone, d_a is depth of water applied, and d_d is average absolute numerical deviation of stored depth from d_s along the border. The depth of water infiltrated along the border length and stored in the root zone can be calculated from Eqn 4. The opportunity time is obtained from advance and recession curves determined in step (3).

(5) If the computed efficiencies are at least equal to the efficiencies desired, the design is completed and the length of the border and the duration of irrigation as assumed in step (3) are sufficient. However, if the efficiencies are less than desired, we change duration of irrigation and repeat steps (3)–(5).

(6) We continue steps (3)–(5) until the desired irrigation efficiencies are achieved. If efficiencies are less than desired, we change the border length and continue steps (3)–(6); otherwise the design is completed.

6. Validation of design procedure

Irrigation efficiencies were used to validate the design procedure. These efficiencies are measures of the effective utilization of the water applied to the border. The observed and computed irrigation efficiencies for all the data sets in Tables 1–3 are given in Tables 6–7. For the data sets K-1–K-9 there are no observed efficiencies. For the data sets Roth-8–Roth-11, the observed and computed distribution efficiencies compare with a difference of less than 8%. The observed and computed application efficiencies compare equally well with a difference of less than 9%. Likewise, the observed and computed storage efficiencies differ by about 11%. For the data sets R-1–R-18, the difference between observed and computed results ranges up to about

TABLE 6

Irrigation efficiencies on the basis of calculated and observed advance and recession times (min) for a total depth of application of 0·1 m for freely draining borders

Data set	*Efficiencies*					
	Observed			*Calculated kinematic model*		
	E_e	E_d	E_s	E_a	E_d	E_s
Roth-8	40·01	91·67	83·57	44·09	99·05	92·08
Roth-9	28·24	91·67	80·44	31·12	99·57	88·64
Roth-10	10·12	91·57	39·85	11·18	99·50	43·99
Roth-11	21·15	91·67	60·42	23·54	99·71	66·66
K-1	—	—	—	45·12	95·28	33·43
K-2	—	—	—	63·05	96·87	58·33
K-3	—	—	—	57·01	96·71	45·06
K-4	—	—	—	47·68	96·43	69·77
K-5	—	—	—	76·01	96·02	55·30
K-6	—	—	—	39·96	95·04	36·82
K-7	—	—	—	56·10	96·32	54·09
K-8	—	—	—	47·98	97·44	48·08
K-9	—	—	—	53·59	100·00	100·00

TABLE 7

Irrigation efficiencies on the basis of calculated and observed advance and recession times (min) for a total depth of application of 0·1 m for closed end borders

Data set	*Efficiencies*					
	Observed			*Calculated kinematic model*		
	E_a	E_d	E_s	E_a	E_d	E_s
R-1	87·99	54·33	31·72	69·47	94·92	25·04
R-2	97·02	64·29	43·08	79·36	90·02	35·24
R-3	95·50	86·67	44·96	100·00	90·12	47·46
R-4	94·66	65·46	53·84	74·67	90·41	42·47
R-5	91·71	69·45	55·02	77·24	90·88	46·34
R-6	85·03	86·13	50·21	83·44	89·98	47·27
R-7	92·14	85·50	73·81	77·93	92·38	62·43
R-8	85·13	89·81	60·27	79·05	91·24	55·97
R-9	89·91	91·67	68·16	86·15	90·26	65·31
R-10	81·32	71·86	53·41	70·08	92·63	46·03
R-11	78·27	76·88	47·90	72·59	92·45	44·43
R-12	83·10	88·72	49·73	84·79	90·52	50·75
R-13	70·42	91·67	56·41	71·70	93·63	57·43
R-14	77·34	81·01	55·68	68·64	92·73	49·42
R-15	86·54	90·99	66·30	86·59	90·00	66·34
R-16	84·28	85·80	81·01	65·93	94·20	63·38
R-17	83·03	91·41	76·72	75·92	91·83	70·15
R-18	83·80	91·67	70·21	82·38	91·24	69·03

25% for application efficiency, up to about 80% for distribution efficiency and up to about 15% for storage efficiency. It is not surprising that the *KW* model does not predict distribution efficiency as accurately as application and storage efficiencies, for the depth, as a function of time, over the border is not modelled accurately.[6] This is partly because the depth of flow involved is quite small. This comparison suggests that the design procedure based on the *KW* model is reasonably accurate for purposes of border irrigation design if application and storage efficiencies are the governing considerations. This would not be true if the distribution efficiency is the controlling factor.

7. An example of border irrigation design

Let us assume that the infiltration characteristics of the soil, bed roughness and slope, inflow rate and the depth of water needed in the root zone are known. We want to determine the length of the border and duration of irrigation to obtain application, distribution and storage efficiencies above 90% for freely draining borders. We determine the border length and duration of irrigation as follows.

TABLE 8

Irrigation efficiencies by kinematic wave model for the data set Roth-9 for various lengths of border and durations of irrigation. Depth of water needed in root zone = 8 cm

Length of border, m	Duration of irrigation, min								
	100			120			140		
	Efficiencies percentage								
	E_a	E_d	E_s	E_a	E_d	E_s	E_a	E_d	E_s
80	40·71	98·13	92·12	35·82	99·11	97·27	31·56	100·00	100·00
100	50·21	98·95	90·90	44·52	99·21	96·71	39·45	100·00	100·00
120	60·25	98·95	90·90	53·42	99·21	96·71	37·34	100·00	100·00
140	70·29	98·95	90·90	62·32	99·21	96·71	55·24	100·00	100·00
160	80·33	98·95	90·90	71·23	99·21	96·71	63·13	100·00	100·00
180	90·37	98·95	90·90	80·13	99·21	96·71	71·02	100·00	100·00
200	100·00	98·95	90·90	89·03	99·21	96·71	78·91	100·00	100·00
	Duration of irrigation, min								
	160			180			200		
	Efficiencies percentage								
	E_a	E_d	E_s	E_a	E_d	E_s	E_a	E_d	E_s
80	27·62	100·00	100·00	24·55	100·00	100·00	22·09	100·00	100·00
100	34·52	100·00	100·00	30·69	100·00	100·00	27·62	100·00	100·00
120	41·43	100·00	100·00	36·82	100·00	100·00	33·14	100·00	100·00
140	48·33	100·00	100·00	42·96	100·00	100·00	38·66	100·00	100·00
160	55·24	100·00	100·00	49·10	100·00	100·00	44·19	100·00	100·00
180	62·14	100·00	100·00	55·24	100·00	100·00	49·71	100·00	100·00
200	69·04	100·00	100·00	61·37	100·00	100·00	55·24	100·00	100·00

For purposes of illustration, we use the data set Roth-9 and calculate irrigation efficiencies following the procedure outlined for the *KW* model. The efficiencies for 8 cm of depth of water application are given in Table 8. We see that a border length of 200 m and duration of irrigation of 100 min gives irrigation efficiencies above 90% ($E_a = 100{\cdot}0$, $E_d = 98{\cdot}95$, $E_s = 90{\cdot}90$), if the *KW* model is used. For any other sets of efficiencies, the border length and duration of irrigation can be chosen from Table 8.

8. Conclusions

(1) The *KW* model is sufficiently accurate for modelling border irrigation for a majority of the data sets used in this study.

(2) One-parameter family of curves for advance as well as for recession can be generated for ready use in irrigation design. A sample of such curves has been presented in this study.

(3) The application and storage efficiencies computed by the model agree reasonably well with those observed on experimental borders. However, this is not the case with distribution efficiency.

(4) The design procedure, based on the *KW* model, can be reasonably accurate if application and storage efficiencies are the governing considerations.

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