SHIFTING CRITICAL SPEEDS OUT OF THE OPERATING RANGE BY CHANGING FROM TILTING PAD TO SLEEVE BEARINGS

by

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ABSTRACT

One property of heavily to moderately loaded sleeve bearings is the resulting asymmetric stiffness and damping coefficients. These asymmetric properties often result in split first critical speeds. Since the horizontal stiffness is much softer than the vertical, a horizontal first critical speed may appear from several hundred to several thousand revolutions per minute lower than the vertical first critical. Tilting pad bearings produce more symmetric bearing properties. This symmetry usually results in a single unsplit first critical speed that is located approximately midway between the sleeve bearing split peaks.

Two different case histories are examined. An induction motor and a steam turbine were both initially designed with tilting pad bearings. Both rotor bearing systems resulted in first critical speeds in the operating speed range. Numerous attempts to shift the critical out of the operating range were explored without success. As a last attempt short of a complete rotor redesign, switching to sleeve bearings was successful. Analytical results that support the decision to switch to sleeve bearings are shown along with actual test stand response plots. Additionally, for the steam turbine, results from a high speed balance are presented along with actual field speed-amplitude plots. While a tilting pad to sleeve bearing change is not recommended for all classes of turbomachienry to shift critical speeds, it is a powerful design tool as it may produce critical speed changes of up to 1000 rpm.

INTRODUCTION

One interesting characteristic of heavily to moderately loaded sleeve bearings such as axial groove, elliptical, pressure dam, taper land, or multilobe bearings is the resulting asymmetric stiffness and damping coefficients. These asymmetric properties often result in split first critical speeds [1, 2]. That is, since the horizontal stiffness and damping is much softer than the vertical, a horizontal first critical speed may appear several hundred to several thousand revolutions per minute lower than the vertical first critical.

Conversely, tilting pad bearings produce more symmetric bearing properties, especially when loaded between pivots. This symmetry usually results in a single unsplit first critical speed that is located approximately midway between the sleeve bearing split peaks.

These two bearing properties have been used to shift critical speeds out of the operating speed range. This has been documented in the literature for an axial compressor that was operating on the higher vertical first critical with three axial groove sleeve bearings

[1]. A four pad tilting pad bearing redesign eliminated the split critical and resulted in a single first critical speed that was well below the operating speed range.

In another example, a redesign effort for an aircooled generator line considered elliptical sleeve bearings that produced split first and second critical speeds compared to tilting pad bearings with single peak criticals [2]. In order to operate between the first and second criticals with adequate separation margins, a five pad tilting pad bearing was chosen. The tilt pad bearings symmetric properties eliminated the first vertical and second horizontal peaks produced by the elliptical bearings, thereby widening the separation between the first and second criticals.

However, the literature is absent of any cases where the rotors first critical speed with tilting pad bearings is moved out of the operating speed range by changing to sleeve bearings. Usually, the designer considers preload, pad pivot offset, bearing set clearance, pivot loading, number of pads, pad arclength, etc., to move critical speeds [3, 4, 5]. While there are many parameters available for change in a tilting pad bearing, usually the resulting shift in the critical speed is of the order of a few hundred rpm, which may be less than the required separation margin.

Two different case histories are examined. An induction motor and a steam turbine were both initially designed with tilting pad bearings. Both rotor bearing systems resulted in first critical speeds in the operating speed range. Numerous attempts to shift the critical out of the operating range are explored without success. As a last attempt, short of a complete rotor redesign, switching to sleeve bearings was successful. Analytical results that support the decision to switch to sleeve bearings are shown along with actual test stand response plots. Additionally, for the steam turbine, response plots from high speed balance runs and actual field coast down runs are presented.

INDUCTION MOTOR

A two pole 5000 hp induction motor was designed and built as a rigid shaft machine. That is, it was to operate with the first critical speed located at least 20 percent above the synchronous operating speed of 3600 rpm. Thus, the first critical speed had to be above 4320 rpm.

The motors relevant mass-elastic rotor dynamic properties are summarized in Table 1. The rotor weight is 4430 lb with a bearing span of 69.4 in. The rotor mass-elastic model is depicted in Figure 1.

The resulting test stand response plot is illustrated in Figure 2 for the induction motor operating on four pad tilting pad bearings.

Table 1. Induction Motor Mass-Elastic Properties.

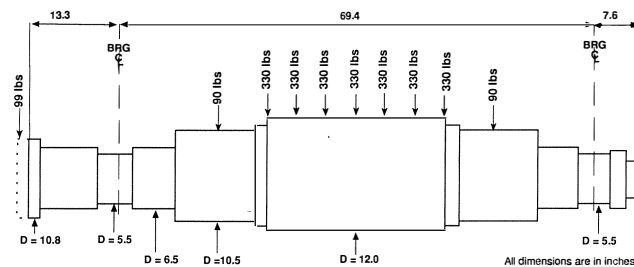| Rotor Weight (lb)	4430
Bearing Span (in)	69.4
Journal Diameter (in)	5.5

Figure 1. Induction Motor Mass-Elastic Model.

Clearly, the first critical speed, N_1, is located at 3900 rpm with an associated amplification factor of $A_1 = 5.7$. With 10 oz-in of unbalance placed in phase at each fan inboard of the bearings, the resulting vibration is 2.5 mils peak to peak (pk-pk).
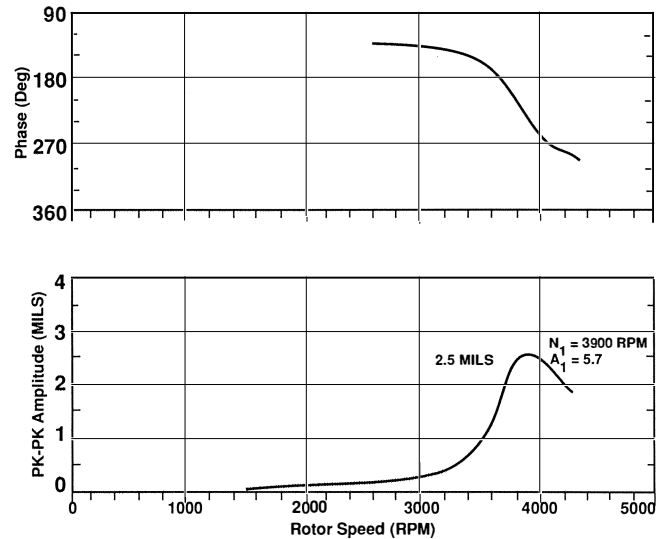

Figure 2. Induction Motor Test Stand Coast Down, Tilting Pad Bearings.

The corresponding analytical response plot is shown in Figure 3 with 10 oz-in of unbalance in phase at each fan. An approximate support stiffness value of 4.0E6 lb/in is used in the analysis [6, 7]. The resulting predicted first critical speed is 3900 rpm with an amplification factor of 6.4 and 2.6 mils of pk-pk vibration. Unfortunately, the initial analysis assumed an unreasonably high support stiffness. Furthermore, it did not consider tilting pad flexibilities [8] or tilting pad pivot flexibilities [9]. For these reasons, the first critical was predicted above 4320 rpm.
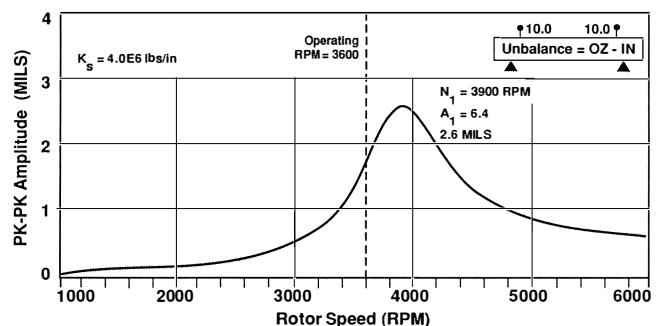

Figure 3. Analytical Speed-Amplitude, Induction Motor, Tilting Pad Bearings.

Numerous attempts were made to raise the first critical speed by stiffening the bearing support. For example, stiffeners were added to the bearing brackets and the motor was moved around the test floor seeking the least flexible location. However, the first critical speed was still well below the 4320 rpm separation margin. With two motor rotors already built, the rotor bearing design with the stiffened bearing brackets remained unacceptable to the customer.

After all other design efforts were exhausted short of a complete rotor redesign, an analysis was conducted replacing the original tilt

pad bearings with elliptical sleeve bearings. The resulting response plot is shown in Figure 4. Again with 10 oz-in of in phase unbalance at the fan locations, two distinct first criticals are evident. The lower or horizontal first critical, N_h, is located at 2350 rpm with A_h = 2.2 and with 1.6 mils of pk-pk vibration. The higher, or vertical, first critical, N_v, is at 4550 rpm with A_v = 3.8. Both criticals now meet the required separation margin.
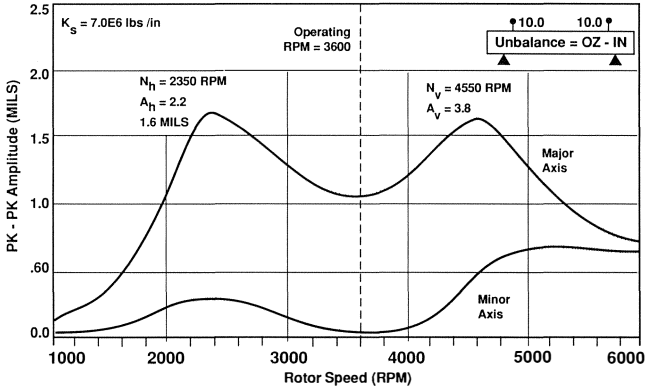

Figure 4. Analytical Speed-Amplitude, Induction Motor, Elliptical Sleeve Bearings.

The stiffness and damping properties for both tilting pad and elliptical bearings are summarized at 3600 rpm in Table 2. Note that the four pad tilting pad bearing with between pivot loading produces equal horizontal and vertical bearing properties. However, the elliptical bearings vertical stiffness is about five times larger than the horizontal stiffness and its vertical damping is a factor of 6.4 times greater than the horizontal. This stiffness and damping asymmetry for the elliptical bearing produces the split first critical speed of Figure 4.

Table 2. Induction Motor - Comparison of Tilting Pad to Elliptical Bearing Stiffness and Damping Properties at N=3600 RPM.

	Tilting Pad*	Elliptical**
D (in)	5.5	5.5
L (in)	1.75	2.5
W (lb)	2215	2215
PSI = W/(LxD)(psi)	230	161
K_{xx} (lb/in)	2.70E6	6.32E5
K_{yy} (lb/in)	2.70E6	3.12E6
C_{xx} (lb-s/in)	3150	1570
C_{yy} (lb-s/in)	3150	9980

*Four Pad, Between Pivot Loading, C_d= 7.0 mils

**C_{dh} = 11.5 mils, C_{dv} = 5.5 mils

Note that the support stiffness used to obtain the Figure 4 results is 7.0E6 lb/in. The increase in support stiffness from the 4.0E6 lb/in used previously for the motor with tilting pad bearings to 7.0E6 lb/in results from the bearing bracket stiffening discussed above. Also, since the original tilting pads are no longer present, the additional felxibilities from the tilting pads and tilting pad pivots are eliminated.

The customer initially objected to losing the tilting pad bearings and the rigid shaft design, but soon agreed to attempt the bearing change. It was clear to all concerned that changing to elliptical sleeve bearings was the only viable option.

The resulting test stand plot for the motor with elliptical bearings and 10 oz-in of in phase unbalance at the fans is shown in Figure 5. The vibration level is 1.6 mils pk-pk with N_h at 2740 rpm and A_h = 1.5. Compared to the predicted results, the vibration magnitude and amplification factor are quite close, but the actual location of the critical is 390 rpm higher than predicted. These results are summarized in Table 3.

Table 3. Induction Motor, Elliptical Bearings - Comparison of Predicted to Actual Test Stand Results.

	Predicted**	Actual
Location (rpm)*	2350	2740
Amplification Factor	2.2	1.5
Vibration Magnitude (mils pk-pk)	1.6	1.6

*First Horizontal Critical Speed

**K_s = 4.0E6 lb/in

Note that the higher vertical critical is not evident in Figure 5. The predicted location of N_v is at 4550 rpm. Since the actual N_h is higher than the predicted N_h, the actual N_v should be somewhat higher than 4550 rpm. Thus, the test stand maximum speed of 4250 rpm is insufficient to reveal the second peak, N_v, of the predicted split first critical speed.
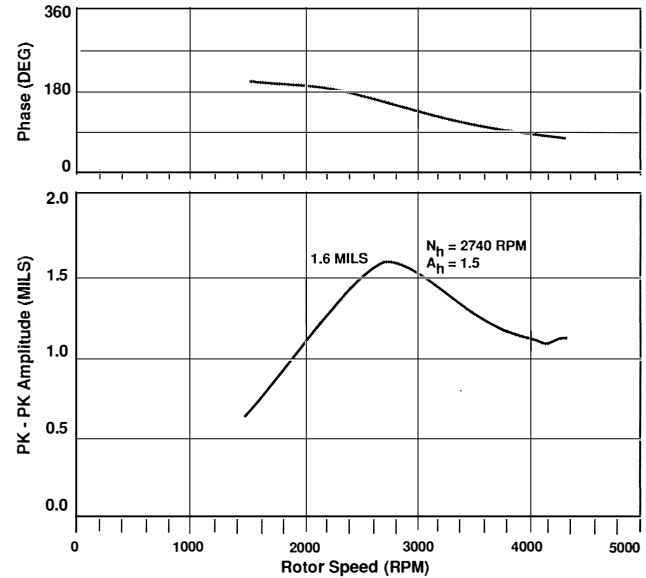

Figure 5. Induction Motor Test Stand Coast Down, Elliptical Sleeve Bearings.

The customer accepted the induction motors with the elliptical bearings and both units have been operating for about three years free of vibration problems.

STEAM TURBINE

A 1940s vintage 12 stage main air blower steam turbine driver in refinery operation was performance rerated by replacing the last four stages with three new stages. At the same time, the three axial

groove cylindrical sleeve bearings were upgraded to five pad tilting pad bearings. This type of steam turbine was typically utilized as a blast furnace blower driver. The turbine bearing span was 105 in with a rotor weight of 12,204 lb and a midshaft diameter of 14.0 in (Table 4 and Figure 6). The turbines speed ranged from 2450 to 2800 rpm with normal operation at 2800 rpm. This rerated rotor will be referred to as the *original* rotor.
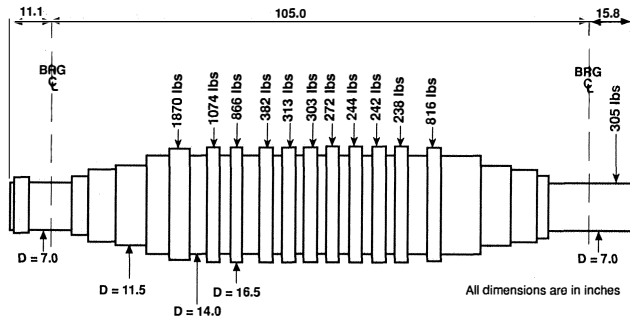

Figure 6. Steam Turbine, Original Rotor, Mass-Elastic Model.

Table 4. Steam Turbine Mass-Elastic Properties, Original Rotor and Redesigned Rotor.

	Original	Redesign
Rotor Weight (lb)	12,204	11,570
Journal Diameter (in)	7.0	7.0
Bearing Span (in)	105.0	100.0
Midshaft Diameter (in)	14.0	16.5

The analytically generated speed-amplitude curve for the original rotor with tilting pad bearings is illustrated in Figure 7. The first critical speed location is predicted at 2300 rpm with an amplification factor of 10.5.
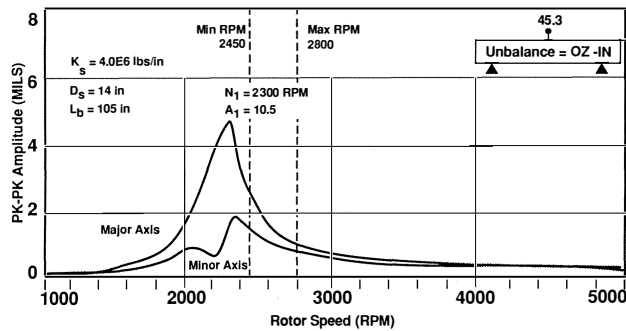

Figure 7. Analytical Speed-Amplitude, Steam Turbine Original Rotor, Tilting Pad Bearings.

Coast down plots from an uncoupled on site field run are shown in Figure 8. The actual first critical speed appears to be located between 2400 to 2800 rpm with vibration levels ranging up to 3.0 mils pk-pk.

Results from two coupled runs are shown in Figure 9. Now the first critical appears to be located closer to 2750 rpm with $A_1 = 11.0$ and vibration levels as high as 7.0 mils pk-pk. The turbine operated
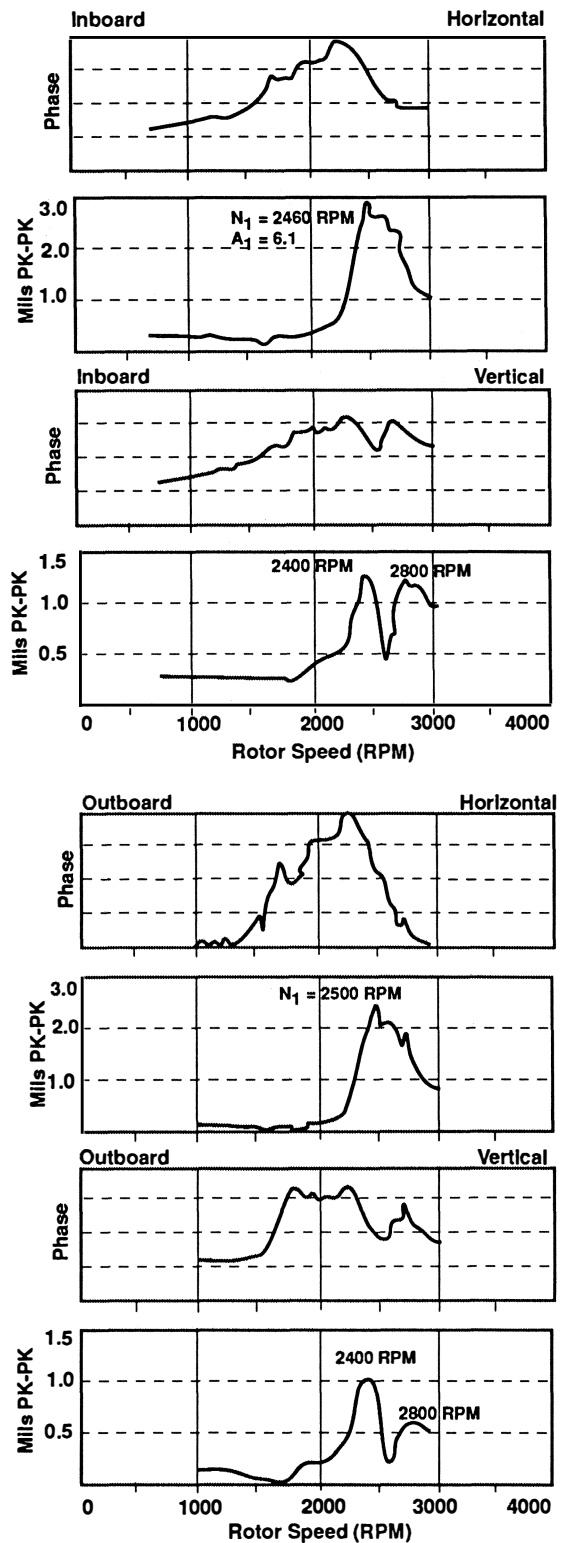

Figure 8. Steam Turbine Field Coast Down, Original Rotor, Tilting Pad Bearings—Uncoupled.

continuously for over three years in this condition with vibration readings that were typically in the 7.0 mil pk-pk range.

Due to the high vibration levels and close proximity of the actual critical speed to the maximum operating speed, a rotor redesign
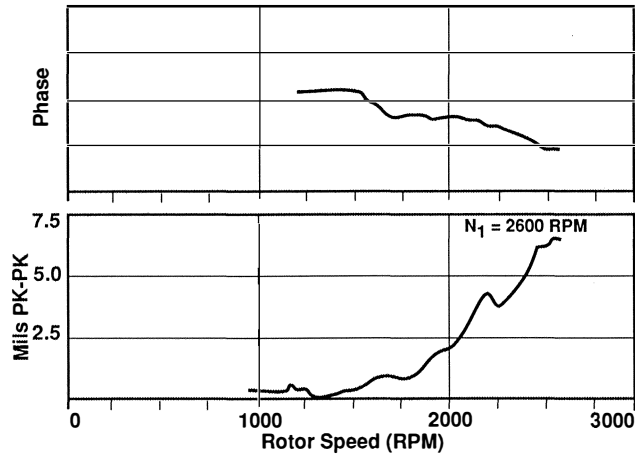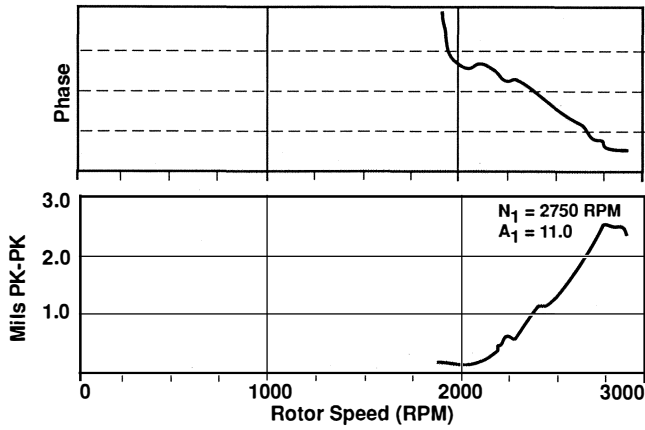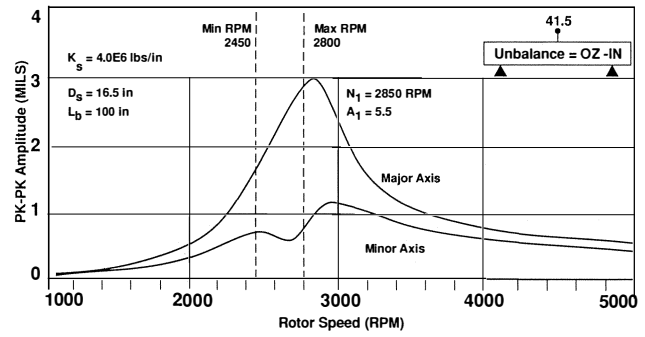
Figure 11. Analytical Speed-Amplitude, Steam Turbine Redesigned Rotor, Tilting Pad Bearings.

3.6, and the higher vertical critical at 3150 rpm, A_v = 7.9. The turbines operating speed range of 2450 to 2800 rpm is between the two first critical peaks.
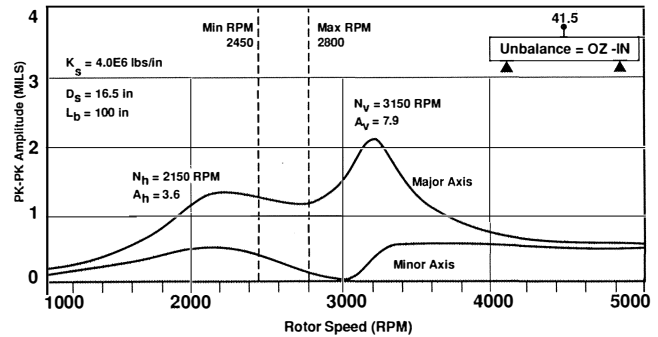

Figure 12.Analytical Speed-Amplitude, Steam Turbine Redesigned Rotor, Three Axial Groove Bearings.

Figure 9. Steam Turbine Field Coast Down, Original Rotor, Tilting Pad Bearings—Coupled.

was undertaken. It was convenient to increase the midshaft diameter from 14.0 to 16.5 in and to decrease the bearing span from 105 to 100 in (Table 4 and Figure 10).

Figure 10. Steam Turbine, Redesigned Rotor, Mass-Elastic Model.

The resulting speed-amplitude plot with tilting pad bearings is illustrated in Figure 11. While the amplification factor decreased considerably form 10.5 (Figure 7) to 5.5, the predicted location of the first critical speed is 2850 rpm which is unacceptably close to the maximum operating speed of 2800 rpm.

A response plot for the redesigned rotor with three axial groove sleeve bearings is presented in Figure 12. Now the first critical speed is split with the lower horizontal critical at 2150 rpm, A_h =

Note that the support stiffness used for both the original and redesigned rotor analyses is 4.0E6 lb/in. Furthermore, K_s = 4.0E6 lb/in is used for the three axial groove analysis of Figure 12. This is a conservative value since it should increase as the additional flexibilities of the tilting pad and pad pivot have been eliminated. Thus, it is likely that the N_v = 3150 rpm critical peak will actually be higher than predicted. Of course, this means that the N_h = 2150 rpm peak may also be higher than predicted but concern regarding this critical is secondary to N_v for two reasons. First, the amplification factor is much lower (A_h = 3.6 compared to A_v = 7.9). Second, the normal operating speed is 2800 rpm and operation at lower speeds is rare. Thus, the N_v = 3150 rpm peak is the critical of concern.

A summary of bearing characteristics for the tilting pad and the three axial groove bearings is shown in Table 5. Even though the principle stiffness asymmetry for the axial groove bearings is not as great compared to the tilting pad bearing, the damping asymmetry increases from about a factor of two for the tilting pad design to a factor of four for the axial groove design. This damping asymmetry causes the split first critical peaks presented in Figure 12.

The redesigned rotor was manufactured with the intent to use the three axial groove bearings. The customer requested a high speed balance for a balance check and verification of the locations of the critical speed peaks. Problems locating the bearings correctly in the high speed balance facility caused the bearing span to temporarily increase from 100.0 to 103.0 in.

Table 5. Steam Turbine - Comparison of Tilting Pad to Three Axial Groove Bearing Stiffness and Damping Properties at N=2800 RPM.

	Tilting Pad *	3 Axial Groove**
D (in)	7.0	7.0
L (in)	5.25	7.0
W (lb)	5785	5785
PSI = W/LxD) (psi)	157	118
K_{xx} (lb/in)	2.47E6	2.41E6
K_{yy} (lb/in)	4.65E6	3.19E6
C_{xx} (lb-s/in)	4720	9.67E3
C_{yy} (lb-s/in)	8410	3.88E4

*Five Pad, Between Pivot Loading, C_d=14.0 mils

**C_d = 11.5 mils

The analytical speed-amplitude curve for the high speed balance is shown in Figure 13. The support stiffness of K_s = 4.7E6 lb/in is used in the analysis to simulate the balance machine hydraulic pedestals with the hydraulic stiffeners in the off position. The analysis predicts N_v = 3250 rpm, A_v = 7.7. The lower horizontal peak is nearly critically damped.
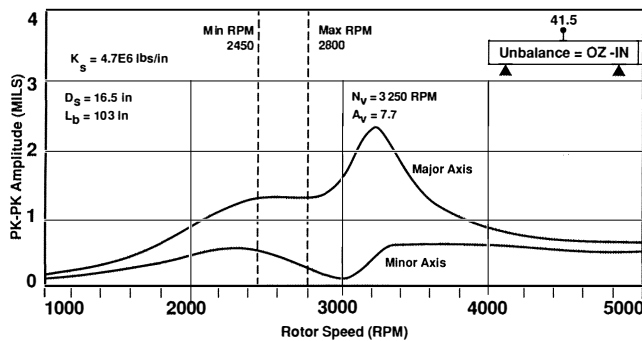

Figure 13. Analytical Speed-Amplitude, Steam Turbine Redesigned Rotor, Three Axial Groove Bearings, High Speed Balance.

Actual coast down response plots from the high speed balance are shown in Figure 14. The predicted first critical split peak is not evident as the lower horizontal first critical is indeed critically damped and can not be seen in Figure 14. The higher vertical first critical is located at 3000 rpm with A_v = 4.1. These results are summarized in Table 6.

Uncoupled on site response plots for the redesigned rotor with three axial groove bearings are shown in Figures 15 and 16. There are no indications of critical peaks anywhere up to the trip speed

Table 6. Steam Turbine, Three Axial Groove Bearings - Comparison of Predicted to Actual High Speed Balance Results.

	Predicted**	Actual**
Location (rpm)*	3250	3000
Amplification Factor	7.7	4.1

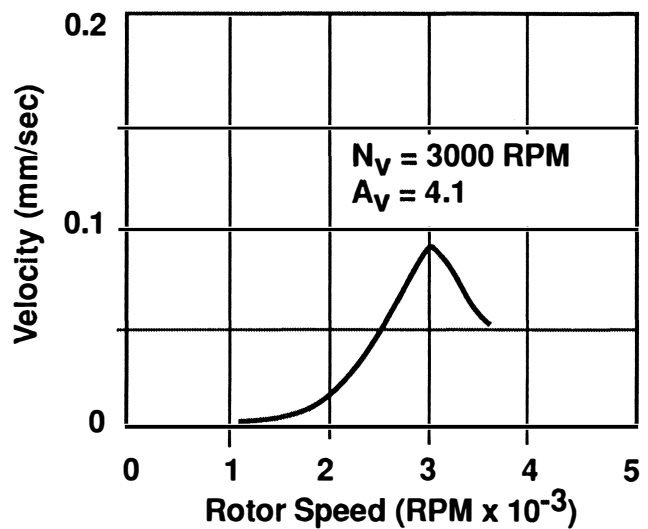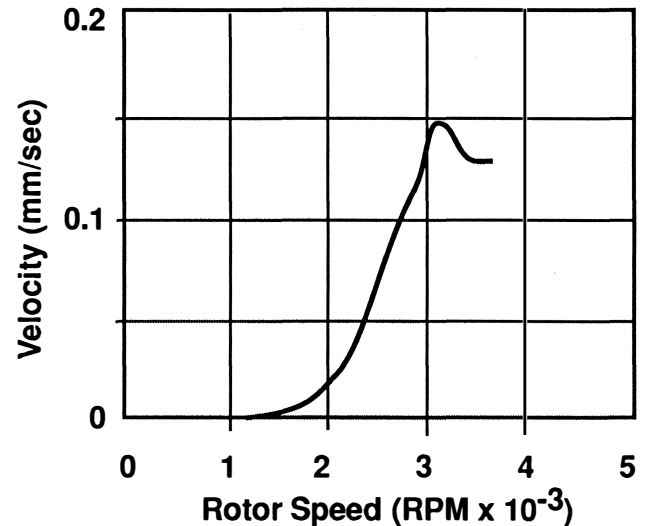*First Vertical Critical Speed

**K_s = 4.7E6 lb/in

Figure 14. Steam Turbine High Speed Balance Coast Down, Redesigned Rotor, Three Axial Groove Bearings.

of 3080 rpm confirming a nonresponsive horizontal first critical and a vertical first critical above 3080 rpm. The turbine is currently operating under 0.5 mils of pk-pk vibration at the increased maximum continuous speed of 2900 rpm.

DISCUSSION

Retrofitting or changing from tilting pad to sleeve bearings to split the first critical should be attempted only if the current rotor bearing system meets several criteria. First, the system must fit into a certain class of machine which operates at a relatively low operating speed, say below 5000 rpm, and with bearing unit loads relatively high, say above 100 psi.

Both systems discussed here meet these suggested criteria. The induction motor runs at 3600 rpm while the steam turbine runs below 2900 rpm. Also, the bearing unit loadings are above 100 psi. The induction motors elliptical bearing unit load is 161 psi, as indicated in Table 2. The three axial groove steam turbine bearing unit load is 118 psi as indicated in Table 5.

Rotors that operate above 5000 rpm may be susceptible to sleeve bearing induced instabilities from stiffness crosscoupling that are not present in tilting pad bearings. A full rotor bearing system
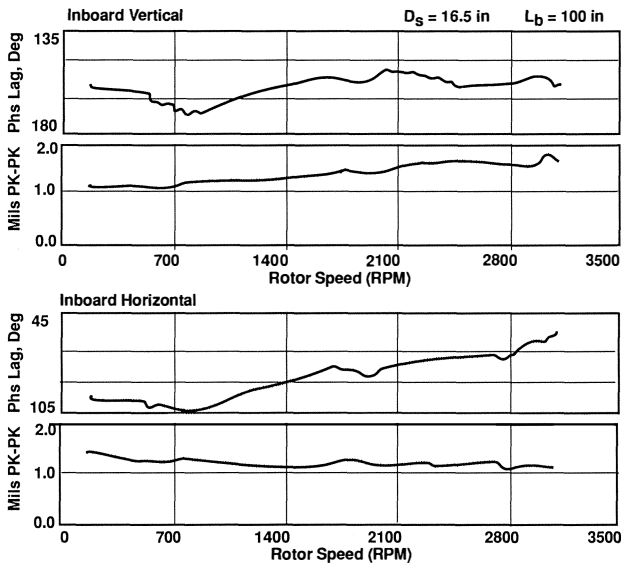
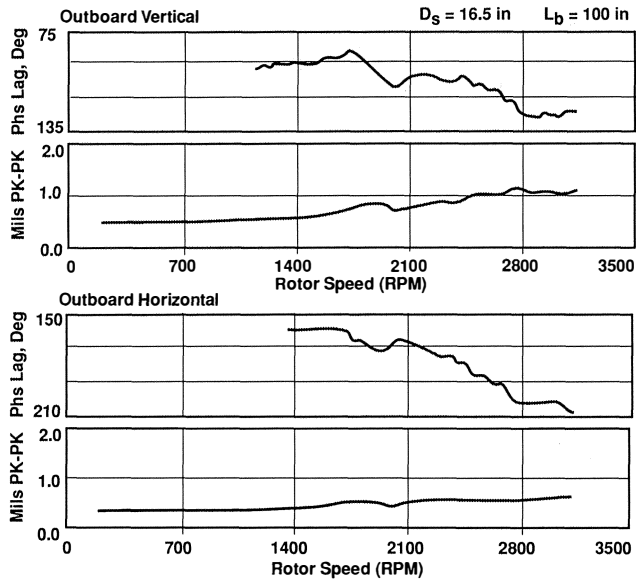*Figure 15 Steam Turbine Field Coast Down, Redesigned Rotor, Three Axial Groove Bearings, Inboard End.*

Figure 16. Steam Turbine Field Coast Down, Redesigned Rotor, Three Axial Groove Bearings, Outboard End.

stability analysis should be undertaken prior to any sleeve bearing retrofit.

Furthermore, sleeve bearings with low unit loads tend to operate at lower eccentricities. Stiffness and damping asymmetry tend to decrease with eccentricity, thereby reducing the separation between split peaks.

Both of these concerns can be somewhat alleviated by utilizing a sleeve bearing design that improves stability and increases the bearings operating eccentricity, and thus the stiffness and damping asymmetry. Examples of this type of sleeve bearing include pressure dam bearings [10], and double pocket bearings [11]. The bearing unit load and operating speed criteria can be somewhat relaxed with the use of these types of bearing designs.

Another important criterion concerns amplification factor. As the amplification factor increases, the ratio of shaft stiffness to

bearing stiffness decreases, thereby reducing the ability of the bearing stiffness and damping properties to affect the rotor bearing system. That is, the bearing damping becomes less effective in vibration attenuation. More importantly for this discussion, both the bearing stiffness and damping properties become less and less effective in locating the rotors critical speed. In the limit, for infinitely stiff bearings, the rotors critical speed is determined solely by the shaft geometry. Thus, a tilting pad to sleeve bearing retrofit to split a critical speed should not be comtemplated for tilt pad system criticals whose amplification factors are greater than say eight. As the amplification factor increases, the split between the horizontal and vertical critical peaks decreases.

For the two cases discussed here, the induction motors first critical amplification factor with tilting pad bearings is 5.7 from Figure 2 (actual test stand response curve) and 6.4 from Figure 3 (analytical response curve). The steam turbines first critical amplification factor of the redesigned rotor with tilting pad bearings is 5.5 from the analytical response plot in Figure 11.

Note that the original steam turbine rotor with tilting pad bearings had a first critical amplification factor of 10.5 (Figure 7). An attempt to split this critical with a sleeve bearing would have been unsuccessful due to the high amplification factor. Therefore, a two step approach was necessary. First, the shaft stiffness was increased by increasing the midshaft diameter and decreasing the bearing span, thereby reducing the amplification factor with tilting pad bearings to 5.5 (Figure 11). Then, with a much lower amplification factor, the journal bearings were switched from tilting pad to three axial groove and the first critical peak was successfully split (Figure 12).

CONCLUSIONS

The major conclusions from this study are summarized below:

• The first critical speed for two turbomachines initially designed with tilting pad bearings was successfully moved outside of the operating speed range by changing to sleeve type bearings.

• The elliptical and three axial groove sleeve bearings asymmetric stiffness and damping properties produced a split first critical speed with two distinct peaks, one below and one above the operating speed range.

• Switching from tilting pad to sleeve bearings is a useful design tool that can be utilized to shift critical speeds outside of the operating speed range.

• The recommended criteria that a rotor bearing system should meet prior to attempting a tilting pad to sleeve bearing retrofit are summarized below:

· The amplification factor of the interfering critical should be below 8.0.

· The operating speed range should be below 5000 rpm.

· The sleeve bearings unit loading should be above 100 psi.

NOMENCLATURE

A_1 amplification factor, first critical speed (dim)

A_h, A_v amplification factor, first horizontal, vertical critical speed (dim)

C_{xx}, C_{yy} principle bearing damping in the horizontal, vertical direction (lb-s/in)

C_d diametral bearing clearance, (in)

C_{dh}, Cdv diametral bearing clearance in the horizontal, vertical direction (in)

D journal diameter (in)

D_s midshaft diameter (in)

K_s support stiffness (lb/in)

K_{xx}, K_{yy} principle bearing stiffness in the horizontal, vertical direction (lb/in)

L bearing axial length (in)

L_b bearing span (in)

N operating or synchronous speed (rpm)

N_1 first critical speed (rpm)

N_h, N_v first horizontal, vertical critical speed (rpm)

PSI = LxD/W bearing unit loading (psi)

W bearing load (lb)

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