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Submitted to the Office of Graduate and Professional Studies of Texas A\&M University
in partial fulfillment of the requirements for the degree of MASTER OF SCIENCE

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May 2016

Major Subject: Electrical Engineering

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#### Abstract

Both deoxyribonucleic acid (DNA) and ribonucleic acid (RNA) play a crucial role in the existence and proper development of all living organisms. In addition, it is through these molecules that the genetic information is passed from parent to offspring. It is no surprise that, over the last decades, a lot of efforts have been put into developing technology that help us better understand their underlying mechanisms.

Similarly to how computers work using only ones and zeros, DNA and RNA only need four different characters to encrypt all the genetic information. Thanks to the sequencing technology development over the past decades, it is possible nowadays to sequence these molecules in a relatively fast and inexpensive way. However, as in any measurement, there is noise involved and this needs to be addressed if one is to reach conclusions based on these kind of data.

The hidden Markov model (HMM) is a perfect fit for this case. Through a Markov chain, the model can capture genetic patterns, while, by introducing the emission probabilities, the noise involved in the process can be taken into account. In addition, previous knowledge can be used by training the model to fit, for instance, a given organism or sequencing technology.

In this thesis, the HMM theory is applied for two purposes, (1) to assess the reliability of sequencing data, and (2) to correct potential errors in the sequences observed. The results show that the HMM model is capable of identifying genetic patterns in the sequence and to repair potential errors, thus improving the reliability of the data before any downstream analysis is performed. For these purposes, HiMMe has been developed and is publicly available on https://github.com/jordiabante/HiMMe.


## DEDICATION

Before getting into matter, I would like to acknowledge those who have supported me during my tenure as a Master of Science student at Texas A\&M University.

Firstly, I should thank Dr. Enjeti for convincing me to come to Aggieland as well as for all his support over the first weeks I spent in College Station.

Secondly, I should thank everyone in the Center for Bioinformatics and Genomic Signal Engineering (CBGSE) for their kindness and help. Specially, I would like to thank Dr. Johnson and Dr. Ghaffari for believing in me and giving me the opportunity of carrying out my research at the center while being involved in really exciting projects.

Thirdly, I want to thank Dr. Datta, Dr. Braga-Neto and Dr. Dabney for their support in the "la Caixa" fellowship application as well as my PhD applications. I believe that all that support has been instrumental for me to succeed and be granted with the fellowship and with the opportunity of going to one of the best schools in the world for my PhD.

I also want to thank my friends both from College Station and Barcelona, who have supported me unconditionally and helped me go through these two years of graduate school. Thanks Jordan and Crystal; moltes gràcies Lluís i Marc.

And last but not least, I want to thank my family for being there whenever I needed them. Although we might be 5,250 miles away, none of this would have been possible without their unconditional support. Moltes gràcies per ser com sou i per recolzar-me una vegada més en una de les meves aventures.

## NOMENCLATURE

| A | Adenine |
| :---: | :---: |
| bp | Base pair |
| C | Cytosine |
| cDNA | Complementary deoxyribonucleic acid |
| DNA | Deoxyribonucleic acid |
| G | Guanine |
| HMM | Hidden Markov model |
| LTP | Law of Total Probability |
| MC | Markov chain |
| MNase-seq | Micrococcal nuclease sequencing |
| mRNA | Messenger ribonucleic acid |
| NGS | Next-generation sequencing |
| NCBI | National Center for Biotechnology Information |
| PCR | Polymerase chain reaction |
| SEQC | Sequencing Quality Control |
| SNP | Single nucleotide polymorphism |
| RAM | Random-access memory |
| RNA | Ribonucleic acid |
| rRNA | Ribosomal ribonucleic acid |
| T | Thymine |
| U | Uracil |
| VCF | Variant Call Format |

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## 1. INTRODUCTION AND LITERATURE REVIEW

Back in 1977, gel electrophoresis was first used to separate deoxyribonucleic acid (DNA) fragments at single-base resolution. Over the next decade, electrophoresis sequencing became very popular and different groups adopted and improved it [13]. Some ambitious group of scientists believed that by 2005 the entire human genome could be sequenced. Nevertheless, a lot of skepticism arose given the high cost of the state of the art technology. At that time, the cost associated with sequencing a single base was approximately $\$ 10$.

However, a lot of effort has been put into the improvement of sequencing technology. Consequently, the cost has been drastically reduced since then. This cost reduction has been so prominent that it has even surpassed Moore's Law's trend.


Figure 1.1: Cost per megabase of DNA sequence - the cost of determining one megabase (million bases) of DNA sequence of a specified quality [16].

Figure 1.1 illustrates the cost reduction of DNA Sequencing over the past fifteen years. Note the sharp decrease around 2008, when the sequencing centers began the transition from Sanger-based technology [10] to next-generation sequencing. This sharp improvement of the technology marked the sudden and profound out-pacing of Moore's Law [16].


Figure 1.2: DNA-sequencing significant increase in the output per instrument run plotted on a logarithmic scale along some of the major breakthroughs in sequencing technology [7].

Moreover, not only has the cost associated with sequencing been significantly reduced, but the speed with which one can sequence biological samples has dramatically increased as well. Note in Figure 1.2 how there is a sudden increase in the output density per run when next-generation sequencing technology took over Sanger sequencing, reflecting again the huge impact that this sequencing technology had in
the field ten years ago. And, in addition to an increase in the output density per run, there has been a fierce competition between sequencing technology manufacturers that has led to a progressive increase in the read length as well as the base-calling accuracy [7].

However, such amount of information needs a place to be stored. Furthermore, extremely powerful hardware and software are required as well for it to be processed. Following Moore's Law in this case, computer technology has enabled scientists to deal with an important portion of these data in order to distill part of the biological insight in it. As a consequence, biological and medical research has lead to numerous discoveries during these last decades.

### 1.1 Central dogma of molecular biology

It is well known that all the living organisms store their genetic information in the DNA [15]. This huge double stranded molecule can be regarded as a sequence of nucleotides, which can be of four different kinds. These four types of nucleotides are: Adenine (A), Cytosine (C), Guanine (G) and Thymine (T). Given DNA's doublestranded nature, these four bases are found in pairs in the DNA (Adenine with Thymine, Guanine with Cytosine) [14]. Just using these four different letters, living organisms are capable of encrypting all the genetic instructions for the proper functioning of their cells. However, it is not the DNA itself that carries out all the necessary reactions in the cell, there are some other molecules that play an important role in this process as well. These two molecules are (1) ribonucleic acid (RNA) and (2) proteins.

Through a process called transcription, the DNA is transcribed into RNA which will eventually leave the nucleus of the cell after undergoing certain intermediate processes in it. The RNA is similar to the DNA in that it is a molecule that can
be regarded as a sequence of nucleotides. However, there are two main differences between the RNA and the DNA. Firstly, the RNA is not a double-stranded molecule, which in turn makes it unstable compared to the DNA. Secondly, the four bases that the RNA uses to transfer the genetic information differ in one when compared to the DNA. Instead of Thymine, RNA relies on another base called Uracil (U).


Figure 1.3: Central dogma of molecular biology.

RNA molecules, once they leave the nucleus of the cell and reach the ribosome, undergo a process called translation. Through this process, RNA molecules provide the necessary information to the ribosome to generate any needed protein. This whole process is usually referred to as the central dogma of molecular biology, and is illustrated in Figure 1.3. This dogma explains how the genetic information flows in living organisms. As Crick [2] stated:

The central dogma of molecular biology deals with the detailed residue-byresidue transfer of sequential information. It states that such information
cannot be transferred back from protein to either protein or nucleic acid.

However, not all the nucleotides in the DNA are transcribed. The parts of the genome that undergo the mentioned process are commonly referred to as genes. These are made of two different kinds of sequences, mainly exons and introns. The former usually reach the ribosome to be translated into proteins. The latter, instead, are removed in an intermediate process called splicing.


Figure 1.4: RNA splicing process example.

Through this process, the cell selects the exons that are required and gets rid of the introns in the pre-mature RNA to produce mRNA. Figure 1.4 illustrates this process in a simplified fashion. In this case, the gene contains two exons and an intermediate intron. The figure depicts how, through splicing, the cell gets rid of the
intron to produce the mRNA. Note that this process can lead to the production of different proteins coming from the same gene.

### 1.2 Next-generation sequencing (NGS) and some applications

As of 2016, the state of the art sequencing technology is usually referred to as next-generation sequencing (NGS).


Figure 1.5: Next-generation sequencing chemistry overview [4].

The main workflow of the current dominant NGS platform, Illumina, is depicted in Figure 1.5 and its steps are described as follows:

1. Library Preparation: the DNA or the cDNA is randomly fragmented and adapters are attached to both the $5^{\prime}$ and the 3 ' end of the strands. The product is then amplified by a polymerase chain reaction (PCR) and purified.
2. Cluster Generation: the products of the library preparation step bind to some oligos, which are complementary to the library adapters used in the previous step. Through bridge amplification, each fragment is amplified into clonal clusters.
3. Sequencing: fluorescent-labeled nucleotides are used to sequentially determine the nature of each nucleotide. The flow cell is imaged at each iteration, and the emission wavelength and intensity is later on used to identify the base. This process is repeated as many times as bases there are in the reads.
4. Data Analysis: once the reads are obtained, one can align them to the corresponding reference genome. Many different analysis are possible at this point such as read counting for RNA methods or studying single nucleotide polymorphisms (SNP) among others.

NGS data have been found to have several applications. For instance, through RNA-seq, one can study which genes are associated with a given condition by comparing the gene expression levels between the control and the condition subjects. To do so, RNA samples from multiple replicates are sequenced for both conditions. Once the reads are obtained, usually in FASTQ format, reads are aligned to the reference genome of the organism being studied. The number of reads that fall in each annotated region of interest, are compared between conditions. By doing so, gene
expression and allele frequency differences can be found between conditions in case such differences existed. This type of analysis is usually referred to as differential gene expression analysis. While it was already possible with the so-called microarray technology ten years ago, by taking advantage of NGS, one can speed up the process significantly given the amount of genes that can be studied in a single run [8].

Another crucial topic in the computational biology field that relies on NGS data is the study of epigenetic structures. Scientists have come to believe that the DNA does not have the last say in how living organisms develop. Different external factors to the DNA, mainly proteins, play an important role in this process by binding to the promoter region and modifying the expression level of the corresponding gene in turn [11]. Other examples of epigenetic processes discovered so far are changes in the chromatin structure [6] or DNA methylation [5]. For instance, one can study the position of nucleosomes and the effect that these have in the expression level of a given gene through micrococcal nuclease sequencing (MNase-seq) data analysis. Nucleosomes are protein complexes that the DNA uses to compact its structure in a rather efficient way. DNA wraps around nucleosomes and these are organized in a very compacted fashion to form the chromatin structure. Using the micrococcal nuclease, researchers get rid of the parts of the DNA that are not binded to any nucleosome and only those sequences wrapped around nucleosomes are sequenced. By mapping these sequences back to the reference genome, one can study the position of these protein complexes and look for potential correlations with gene expression levels.

From the moment that biologists extract DNA or RNA samples to the point in which one analyzes the data, there are multiple intermediate steps that can affect the final result. For instance, due to the inherent random sampling of the sequencing technology, RNA-seq has been found to have measurement noise [12]. This has
led to different outcomes in previous differential gene expression analysis studies. Therefore, it is of high relevance to develop tools that allow researchers to make sure that the reads that they use for their downstream analysis are as error-free and reliable as possible. This makes the hidden Markov model (HMM) approach very useful, since it allows one to deal with observing noisy signals, i.e. the sequences obtained, which are used as hints to infer the real signal, i.e. the real sequences. This model will be formally introduced in section 2 and applied to genomic sequences in section 3.

### 1.3 Thesis layout

The previous sections have underlined the relevance of the recent sequencing technology breakthroughs and some applications have been described. The relevance of sequencing data reliability to biological and medical research has also been highlighted. In section 2, the theoretical foundations of hidden Markov models (HMM) are introduced as well as the notation used throughout the thesis. In section 3, this theory is applied to genomic sequences by modeling the observed data as hints that can help one to find the real sequences and to evaluate the reliability of the observed sequences. Once the theory is adapted to NGS data, section 4 describes the way the model is implemented in a computational efficient fashion. Following, in section 5 the two main results of this thesis are presented (1) scoring sequences reliability and (2) repairing potential errors. Finally, in section 6, conclusions are provided based on the results obtained.

## 2. HIDDEN MARKOV MODELS (HMM)

Hidden Markov models (HMM) are used to describe the behavior of a system based on hints, usually referred to as observable events. For instance, one could try to infer the position of the upstairs neighbor based on the sound of his or her steps. The observable events, the sounds produced by the steps in this case, are usually called 'symbols'. On the other hand, the underlying or invisible factor one is trying to understand, in this case the position of the neighbor, is usually referred to as 'state'.

More technically, a HMM can be regarded as the interaction between two stochastic processes. That is why it is usually referred to as a doubly-embedded stochastic process [9]. The probability distribution of the symbols depend on the underlying states, while the latter form a Markov chain. Thus, given that the present state is known, the future states are conditionally independent of the past.

### 2.1 Markov chain

Let us formally define an HMM, and for that purpose, let us first define the Markov chain that the underlying states follow. The state space of the Markov chain is denoted as $E$. We denote a general sequence of states as follows:

$$
\begin{equation*}
X=X_{1} \cdots X_{m} \tag{2.1}
\end{equation*}
$$

where $m$ is the number of states in the sequence. The stochastic process $X=$ $\left\{X_{n} ; n \in \mathbb{N}\right\}$ is called a Markov chain provided that

$$
\begin{equation*}
P\left\{X_{n+1}=j \mid X_{0}, \ldots, X_{n}\right\}=P\left\{X_{n+1}=j \mid X_{n}\right\} \tag{2.2}
\end{equation*}
$$

for all $j \in E$ and $n \in \mathbb{N}[1]$. This conditional probability can be expressed also using the following notation:

$$
\begin{equation*}
P\left\{X_{n+1}=j \mid X_{n}=i\right\}=P(i, j) \tag{2.3}
\end{equation*}
$$

The probabilities $P(i, j)$ are called transition probabilities and can be arranged in a matrix P called transition matrix or Markov matrix. Thus, if $E=\{0,1,2, \ldots\}$ this matrix has the following form:

$$
P=\left[\begin{array}{cccc}
P(0,0) & P(0,1) & P(0,2) & \cdots  \tag{2.4}\\
P(1,0) & P(1,1) & P(1,2) & \cdots \\
P(2,0) & P(2,1) & P(2,2) & \cdots \\
\cdot & \cdot & \cdot & \cdots \\
\cdot & \cdot & \cdot & \cdots
\end{array}\right]
$$

For any square matrix to be a transition matrix defined over $E$ the following conditions have to be met:

1. $P(i, j) \geq 0 \forall i, j \in E$, and
2. $\sum_{j \in E} P(i, j)=1, \forall i \in E$

Therefore, each row of the transition matrix has to add up to one and all the entries have to be non-negative. In addition, when this matrix is symmetric, the associated Markov chain has some special properties that simplify the problem. Such a discrete stochastic process is usually referred to as a double chain Markov model. However, this will not be the case for the intended application of the model as described in this thesis. For instance, the following could be considered a transition
matrix for state space $E=\{1,2,3\}$ :

$$
P=\left[\begin{array}{ccc}
\frac{1}{2} & \frac{1}{4} & \frac{1}{4}  \tag{2.5}\\
\frac{2}{3} & 0 & \frac{1}{3} \\
\frac{3}{5} & \frac{2}{5} & 0
\end{array}\right]
$$

Such a transition matrix represents mathematically the following Markov chain:


Note that all the edges leaving each node in the graph add up to one, since each row in the corresponding transition matrix fulfills that condition as well. From the graph it is usually easier to understand the dynamics of the Markov chain. Consider now the following sequence of states:

$$
\begin{equation*}
X_{1}=1, X_{2}=1, X_{3}=3, X_{1}=2 \tag{2.6}
\end{equation*}
$$

For instance, considering that $X_{0}=1$, the probability of such a path can be computed as follows:

$$
P\left\{X_{1}=1, X_{2}=1, X_{3}=3, X_{4}=2 \mid X_{0}=1\right\}=P(1,1) P(1,1) P(1,3) P(3,2)
$$

$$
\begin{align*}
& =\frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{4} \cdot \frac{2}{5} \\
& =\frac{1}{40} \tag{2.7}
\end{align*}
$$

It is of interest as well the following characteristic of a Markov chain. The probability that the chain moves from state $i$ to state $j$ in $r$ steps is the $(i, j)$ entry of the $r$ th power of the transition matrix [1].

$$
\begin{equation*}
P\left\{X_{n+r}=j \mid X_{n}=i\right\}=P^{r}(i, j) \tag{2.8}
\end{equation*}
$$

Thus, one can compute the probability of going from state $i$ to state $j$ in $r$ steps, taking into account all the possible states in between, by using the preceding formula.

### 2.2 Emission probabilities

Once introduced the Markov chain that the underlying states follow, let us define the emission probabilities, which bridge the gap between the observed symbols and the underlying sequence of states. Each symbol $Y_{i}$ is a random variable that takes on a set of possible observations $O=\left\{O_{1}, \ldots, O_{L}\right\}$ based on a probability distribution conditional on the current underlying state. We denote a sequence of symbols as follows:

$$
\begin{equation*}
Y=Y_{1} \cdots Y_{m} \tag{2.9}
\end{equation*}
$$

Then, since the random variable $Y_{i}$ takes on $O$ based on the current state only, we have:

$$
\begin{equation*}
P\left\{Y_{n}=y \mid Y_{1}, \ldots, Y_{n-1}, X_{1}, \ldots, X_{n-1}, X_{n}=i\right\}=P\left\{Y_{n}=y \mid X_{n}=i\right\} \tag{2.10}
\end{equation*}
$$

for all $y \in O$ and all $i \in E$. This conditional probability is referred as emission probability of $y$ at state $i$ and represented by $e(\cdot \mid \cdot)$.

$$
\begin{equation*}
P\left\{Y_{n}=y \mid X_{n}=i\right\}=e(y \mid i) \tag{2.11}
\end{equation*}
$$

### 2.3 Complete model

The initial state probability of the Markov chain $X$ can be denoted as:

$$
\begin{equation*}
\pi_{0}(i)=P\left\{X_{1}=i\right\} \tag{2.12}
\end{equation*}
$$

Note that the HMM is completely specified by the three probability measures $\pi_{0}(i), P(i, j)$ and $e(y \mid i)$. This set of probabilities is going to be denoted as $\Theta$ for convenience from now on. For a realization Y and X we have:

$$
\begin{align*}
P\{Y, X \mid \Theta\} & =\frac{P\{Y, X, \Theta\}}{P\{\Theta\}} \\
& =\frac{P\{Y \mid X, \Theta\} P\{X, \Theta\}\}}{P\{\Theta\}} \\
& =\frac{P\{Y \mid X, \Theta\} P\{X \mid \Theta\} P\{\Theta\}}{P\{\Theta\}} \\
& =P\{Y \mid X, \Theta\} P\{X \mid \Theta\} \tag{2.13}
\end{align*}
$$

where

$$
\begin{equation*}
P\{Y \mid X, \Theta\}=e\left(Y_{1} \mid X_{1}\right) e\left(Y_{2} \mid X_{2}\right) \cdots e\left(Y_{m} \mid X_{m}\right) \tag{2.14}
\end{equation*}
$$

and

$$
\begin{equation*}
P\{X \mid \Theta\}=\pi_{0}\left(X_{1}\right) P\left(X_{1}, X_{2}\right) \cdots P\left(X_{m-1}, X_{m}\right) \tag{2.15}
\end{equation*}
$$

Thus, when the underlying state sequence $X=X_{1} \cdots X_{m}$ is known, it is easy to compute the observation probability of a given realization $Y=Y_{1} \cdots Y_{m}$. Note as well that by the Law of Total Probability (LTP):

$$
\begin{equation*}
P\{Y \mid \Theta\}=\sum_{x \in \Omega_{m}} P\{Y, X=x \mid \Theta\} \tag{2.16}
\end{equation*}
$$

where $\Omega_{m}$ is the set of all possible sequences of states of length $m$. Note that one could potentially go through every single possible hidden state sequence to find the marginal distribution of the observation based on the model used. In other words, without knowing the actual hidden state sequence, one can compute the probability of observing $Y$ given the model, i.e. $\Theta$. Depending on how large the state space is and the number of elements in the chain, this might not even be feasible. However, by taking advantage of the Bayes' rule, equation 2.16 can be expanded in the following way:

$$
\begin{align*}
\sum_{x \in \Omega_{m}} P\{Y, X=x \mid \Theta\} & =\sum_{x \in \Omega_{m}} P\{Y \mid X=x, \Theta\} P\{X=x \mid \Theta\} \\
& =\sum_{x \in \Omega_{m}} \pi_{0}\left(x_{1}\right) e\left(Y_{1} \mid x_{1}\right) \prod_{i=2}^{m} e\left(Y_{i} \mid x_{i}\right) P\left(x_{i-1}, x_{i}\right) \tag{2.17}
\end{align*}
$$

Note that if the distributions $\{Y \mid X, \Theta\}$ and $\{X \mid \Theta\}$ are known, i.e. $\Theta$ is known, then some sort of sampling technique could be used in order to find the marginal distribution of the symbol sequence independent on the hidden states sequence. For instance, a hierarchical modeling approach could be used to sample first $X^{*}$ from $P\{X \mid \Theta\}$ and then sample $Y^{*}$ from $P\left\{Y \mid X^{*}, \Theta\right\}$ and iterate as many times as needed. By doing so, one could eventually find the empirical marginal distribution of the observation $Y$ given the model $\Theta$.

## 3. SEQUENCING DATA MODELING

In section 1, NGS data and multiple applications of this sequencing technology have been introduced. In section 2, the theory behind a hidden Markov model has been introduced. The aim of this section is to apply this theory to assess the reliability of this type of data and to correct potential errors in it as well.

As introduced in the previous section, an HMM is a doubly-embedded stochastic process. Such a process is composed of an observed sequence of symbols and a sequence of states which, on the other hand, is not observed. Here, each sequence obtained through NGS is going to be regarded as a sequence of symbols. These observed sequences are going to be scored without knowing the corresponding underlying sequence of states. Then, an error-free version, i.e. the most likely underlying state sequence, is going to be provided for each of the observed sequences.

### 3.1 Amount of information in a $k$-mer

A sequence of $k$ nucleotides is usually referred to as a $k$-mer. When breaking down a DNA or RNA sequence into substrings, one can use different $k$-mer sizes. The number of possible states in the Markov chain that defines the behavior of the underlying state sequence, clearly depends on the choice of $k$-mer size, since there would be as many as $|N|^{k}$ different states, where $|N|$ is the cardinality of the set of characters used at each position in the $k$-mer and $k$ is the $k$-mer size. Therefore, following the notation introduced in section 2, the cardinality of the set of states for the hidden layer would be:

$$
\begin{equation*}
|E|=|N|^{k} \tag{3.1}
\end{equation*}
$$

For instance, if the set of characters used at each position in the $k$-mer contained the four possible nucleotides that exist in DNA, i.e. $N=\{A, C, G, T\}$, then the cardinality of the space state of the Markov chain would be $4^{k}$.


Figure 3.1: Number of states and size of the transition matrix as a function of the $k$-mer size.

Note that the size of the transition matrix, which defines the underlying sequences of states, would grow quadratically with the number of states as Figure 3.1 suggests. Therefore, the memory requirements also grow quadratically with the size of $k$-mer used. However, one can benefit from using larger $k$-mers. For instance, one could break the sequence down into individual nucleotides, thus using 1-mers, or one could break it down into groups of three nucleotides or 3-mers. When choosing the $k$-mer
size, one has to be aware that, as it becomes smaller, information is lost. In the previous example, the second choice of $k$-mer size would have more memory and computational requirements, but intuitively, the model would be able to detect more patterns in the data.

Looking at it from an Information Theory standpoint, the amount of information obtained from observing an event $A$ with probability $p_{A}$ is:

$$
\begin{equation*}
I(A)=-\log \left(p_{A}\right) \tag{3.2}
\end{equation*}
$$

Therefore, as the cardinality of the state space grows, more information can be obtained from an observation on average.


Figure 3.2: Amount of information given the probability of event A.

Note in Figure 3.2 how the amount of information obtained when observing an event A increases as the probability of this event decreases. For instance, consider the 1-mer case, where each of the outcomes has the same probability of $1 / 4$ of occurring. The amount of information obtained when observing a given 1-mer is:

$$
\begin{equation*}
I(1-m e r)=-\log \left(\frac{1}{4}\right) \approx 0.6 \tag{3.3}
\end{equation*}
$$

On the other hand, consider the case where 3-mers are used. In this case, the state space is significantly large since for every position in the 3-mer, there are four different choices. The cardinality of the state space in this case is 64 . Again, assuming that all the outcomes have the same probability, the probability of observing any 3mer is $1 / 64$ and the amount of information obtained in a single observation is:

$$
\begin{equation*}
I(3-m e r)=-\log \left(\frac{1}{64}\right) \approx 1.8 \tag{3.4}
\end{equation*}
$$

Which is larger than then amount of information obtained in a single observation of a 1-mer.

The average amount of information is usually referred to as entropy. Here, the entropy is the expected amount of information obtained in a single observation. Therefore, the entropy can be expressed as:

$$
\begin{equation*}
S=-\sum_{A \in E} p_{A} \log \left(p_{A}\right) \tag{3.5}
\end{equation*}
$$

However, since all the events were equally likely in the previous examples, their entropies are going to have the same value as the information function for any event.

$$
S=-\sum_{A \in E} p_{A} \log \left(p_{A}\right)
$$

$$
\begin{align*}
& =-\sum_{A \in E} p \log (p) \\
& =-|E| p \log (p) \\
& =-|E| \frac{1}{|E|} \log (p) \\
& =-\log (p) \\
& =I(A) \tag{3.6}
\end{align*}
$$

In addition, it is of interest to look at the behavior of the entropy function as one increases the $k$-mer size. Assuming again that all events are equally likely, we have that:

$$
\begin{align*}
I(A) & =-\log \left(p_{A}\right) \\
& =-\log \left(\frac{1}{|E|}\right) \\
& =-\log \left(\frac{1}{|N|^{k}}\right) \\
& =\log \left(|N|^{k}\right) \\
& =k \log (|N|) \\
& \propto k \tag{3.7}
\end{align*}
$$

Therefore, the amount of information is going to be proportional to the $k$-mer size, thus proving, from an Information Theory standpoint, that in fact one can learn more from the sequence the larger the choice of $k$-mer is. However, using large $k$-mers is usually associated with both higher memory and processing requirements as Figure 3.1 reflects. Thus, a compromise between the amount of biological information and computational requirements is often required.

### 3.2 Modeling the state sequences

Using the notation introduced in section 2, the hidden state sequence is represented as follows:

$$
\begin{equation*}
X=X_{1} \cdots X_{m} \tag{3.8}
\end{equation*}
$$

and the probability of such a path would be:

$$
\begin{align*}
P\{X \mid \Theta\} & =\pi_{0}\left(X_{1}\right) P\left(X_{1}, X_{2}\right) \cdots P\left(X_{m-1}, X_{m}\right) \\
& =\pi_{0}\left(X_{1}\right) \prod_{i=2}^{m} P\left(X_{i-1}, X_{i}\right) \tag{3.9}
\end{align*}
$$

Let us consider the following sequence:

$$
\begin{equation*}
S=A C T A G A C A G A T G A C A \tag{3.10}
\end{equation*}
$$

Let us assume now that the $k$-mer size chosen to break down the sequence is three. Then, the sequence of states would be:

$$
\begin{equation*}
A C T \rightarrow A G A \rightarrow C A G \rightarrow A T G \rightarrow A C A \tag{3.11}
\end{equation*}
$$

Thus, if $X=S$, then

$$
\begin{aligned}
& X_{1}=A C T \\
& X_{2}=A G A \\
& X_{3}=C A G \\
& X_{4}=A T G \\
& X_{5}=A C A
\end{aligned}
$$

and the probability of observing that sequence of states would be:
$P\{X=S \mid \Theta\}=\pi_{0}(A C T) P(A C T \mid A G A) P(A G A \mid C A G) P(C A G \mid A T G) P(A T G \mid A C A)$

These probabilities would be found in the transition matrix. The probability can be computed in the same way for any path once the sequence of states is defined.

### 3.3 Modeling the symbol sequences

As previously introduced, each DNA sequence present in the NGS dataset is going to be regarded as a sequence of symbols. Using the notation introduced in section 2 , the symbol sequence is represented as follows:

$$
\begin{equation*}
Y=Y_{1} \cdots Y_{m} \tag{3.13}
\end{equation*}
$$

Considering the model introduced in section 2, the HMM uses the emission probabilities to evaluate how likely is to observe a given symbol given the hidden state. Recall that:

$$
\begin{equation*}
P\left\{Y_{n}=y \mid X_{n}=i\right\}=e(y \mid i) \tag{3.14}
\end{equation*}
$$

For instance, if one wants to model the probability that the $i t h$ nucleotide is observed to be Adenine given that the hidden state is Guanine, a value between zero and one can be assigned to that emission probability:

$$
\begin{equation*}
P\left\{Y_{i}=A \mid X_{i}=G\right\}=e(A \mid G)=0.05 \tag{3.15}
\end{equation*}
$$

In addition, recall that by the LTP:

$$
\begin{equation*}
\sum_{y \in E} e\left(Y_{i}=y \mid X_{i}=x\right)=1, \quad \forall y \in E, j=1, \ldots, m \tag{3.16}
\end{equation*}
$$

Note that the state space $E$ is shared in this case between the symbols and the states. This is usually preferred for convenience when working with HMM. Nevertheless, in case one wanted to work with larger $k$-mers in order to capture more biological information, it would be convenient to compute the emission probabilities for $k$-mers with $k$ larger than one.

Let us consider $k$-mers $R_{1}=R_{11} \ldots R_{1 k}$ and $R_{2}=R_{21} \ldots R_{2 k}$, which both have length $k$. The emission probability of observing $R_{2}$ provided that $R_{1}$ is the hidden state would be:

$$
\begin{equation*}
P\left\{R_{2}=R_{21} \ldots R_{2 k} \mid R_{1}=R_{11} \ldots R_{1 k}\right\}=e\left(R_{21} \ldots R_{2 k} \mid R_{11} \ldots R_{1 k}\right) \tag{3.17}
\end{equation*}
$$

However, since a SNP database is going to be used to learn the emission probabilities, the method proposed is based on the assumption that the emission probabilities of adjacent nucleotides are independent. This assumption allows one to express the emission probability of observing $k$-mer $R_{2}=R_{21} \ldots R_{2 k}$ given $k$-mer $R_{1}=R_{11} \ldots R_{1 k}$ as a product of the different emission probabilities at a single base resolution:

$$
\begin{align*}
e\left(R_{2} \mid R_{1}\right) & =e\left(R_{21} \mid R_{11}\right) e\left(R_{22} \mid R_{12}\right) \cdots e\left(R_{2 k} \mid R_{1 k}\right) \\
& =\prod_{i=1}^{k} e\left(R_{2 i} \mid R_{1 i}\right) \tag{3.18}
\end{align*}
$$

where $k$ would be the length of the $k$-mer. For instance, if we observe the symbol $Y_{i}=A C T A G$ and we know that the corresponding hidden state is $X_{i}=A C T T G$,
the emission probability would be:

$$
\begin{equation*}
P\left\{Y_{i}=A C T A G \mid X_{i}=A C T T G\right\}=e(A C T A G \mid A C T T G) \tag{3.19}
\end{equation*}
$$

Then, assuming independence, this emission probability could be computed in the following way:

$$
\begin{equation*}
e(A C T A G \mid A C T T G)=e(A \mid A) e(C \mid C) e(T \mid T) e(A \mid T) e(G \mid G) \tag{3.20}
\end{equation*}
$$

Each single emission probability would be obtained from a matrix learned from the SNP database previously mentioned. The exact way this matrix is constructed is explained in section 3.5.

### 3.4 Learning the transition matrix $P$

The dynamics of the Markov chain are totally defined by the transition matrix, and this can be deduced from the reference genome when working with DNA or from the reference transcriptome if one was dealing with RNA data. These references are based on multiple sequencing datasets from the same species that the scientific community has put together creating a consensus sequence. Therefore, they can be considered a good foundation for the transition matrix. That is, one can deduce the conditional probability of having a $k$-mer $X_{n}$ after $k$-mer $X_{n-1}$ from the reference. For instance, a transition matrix using 1-mers, thus for state space $E=\{A, C, G, T\}$, could look like:

$$
P=\left[\begin{array}{llll}
P(A, A) & P(A, C) & P(A, G) & P(A, T)  \tag{3.21}\\
P(C, A) & P(C, C) & P(C, G) & P(C, T) \\
P(G, A) & P(G, C) & P(G, G) & P(G, T) \\
P(T, A) & P(T, C) & P(T, G) & P(T, T)
\end{array}\right]
$$

A transition matrix has to fulfill the properties described in section 2.1. Therefore, the method suggested in this thesis to learn the transition matrix is to, based on a $k$-mer size choice, scan the reference of the organism being studied and count the transitions from any $k$-mer to any other $k$-mer, including itself. For instance, the probability:

$$
\begin{equation*}
P\left\{X_{n}=A C T G T \mid X_{n-1}=A A G T A\right\}=P(A A G T A, A C T G T) \tag{3.22}
\end{equation*}
$$

can be estimated from the reference by counting how many times the 5-mer AAGTA is followed by ACTGT and dividing by the number of times that AAGTA is present in it so that the row corresponding to that 5 -mer adds up to one as any transition matrix or Markov matrix requires. This process would have to be repeated for all the states in the state space $E$.

As stated at the beginning of this section, the choice of $k$-mer size has both memory and processing requirements. Table 3.1 illustrates the approximate amount of memory that the transition matrix would require for different $k$-mer sizes. Note that if one wanted to store the whole matrix in the random-access memory (RAM), this would require a RAM capacity that few machines have when the $k$-mer size choice gets large. For example, it looks feasible to store a transition matrix built based on 6 -mers in a regular machine. If one wanted to store larger transition matrices, then it would not probably be feasible in terms of memory requirements to run the algorithm in a regular desktop and something more powerful would be required. Note as well that there are going to be other matrices and variables to store in the RAM, hence the memory available for the transition matrix will be very limited depending on the capacity of the machine used.

| Transition matrix sizes |  |  |
| :---: | :---: | :---: |
| $k$-mer size | Elements in matrix | Memory required |
| 1 | 16 | 150 B |
| 2 | 256 | 2.2 KB |
| 3 | 4,096 | 33 KB |
| 4 | 65,536 | 515 KB |
| 5 | $1,048,576$ | 8.1 MB |
| 6 | $16,777,216$ | 129 MB |
| 7 | $268,435,456$ | 2.1 GB |

Table 3.1: Memory required by the transition matrix for different $k$-mer sizes.

Nevertheless, storing this matrix in the RAM would be ideal in order to accelerate the lookup process, since this would be an operation repeated continuously in the method proposed. By storing it in the RAM, the lookup process would be much faster rather than having it saved in the hard drive and having the algorithm open that file, look for the number of interest and then close it again every time a transition probability was needed. In addition, when working with larger $k$-mer sizes, even the mere fact of storing the matrix in the hard drive could potentially be an issue.

Another issue to take into account, from the computational standpoint, is the time required to build this matrix. However, once the matrix is built for a given organism and $k$-mer size, the user would not need to go through this process again. The perl code developed can be found in Appendix A.1.1 with some comments that will help the reader to better understand the structure.

### 3.5 Learning the emission probabilities e $(\cdot \mid \cdot)$

As discussed in section 3.3, the emission probabilities of $k$-mers with $k$ larger than one will be computed as the product of individual emission probabilities. Therefore, the matrix containing the emission probabilities is going to be the equivalent to a transition matrix based on 1-mers. Note that this is a very different case when compared to the transition matrix scenario given that, in that case, we are interested in building matrices with the largest $k$-mer size as possible. The matrix containing the emission probabilities will be referred to as $V$ and will contain only 16 elements:

$$
V=\left[\begin{array}{llll}
e(A \mid A) & e(C \mid A) & e(G \mid A) & e(T \mid A)  \tag{3.23}\\
e(A \mid C) & e(C \mid C) & e(G \mid C) & e(T \mid C) \\
e(A \mid G) & e(C \mid G) & e(G \mid G) & e(T \mid G) \\
e(A \mid T) & e(C \mid T) & e(G \mid T) & e(T \mid T)
\end{array}\right]
$$

For testing purposes, these emission probabilities are going to be learned from a Human Variation database generated with human data by the National Center for Biotechnology Information (NCBI). The common format for variant calling is the so called Variant Call Format (VCF). In such a file, the third and fourth columns contain the information relevant to us, the sequence information of the variants found in the dataset. By counting the frequencies of each variant, the emission probability of a given nucleotide being observed given that the hidden state is the same nucleotide can be found then by subtracting to one the summation all the other possible outcomes:

$$
\begin{equation*}
e(i \mid i)=1-\sum_{j \in E \backslash i} e(j \mid i), \tag{3.24}
\end{equation*}
$$

for all $i \in E$. Therefore, by parsing the training VCF file, the matrix containing the emission probabilities can be built in a relative simple and fast fashion. This matrix would probably be specific to each organism and updated as new variants were discovered. Again, the software suite developed will allow the user to generate his or her own emission probabilities matrix based on whatever SNP database he or she wants to use to train the HMM. The perl code developed can be found in Appendix A.1.2 with some comments that will help the reader to better understand the structure.

## 4. COMPUTATIONAL APPROACHES

In section 3, the HMM model has been applied to genomic sequencing data. However, it is still not clear how to compute the probabilities that the method presented in this thesis seeks to find. This section will introduce the way these are to be found from a computational standpoint. By taking advantage of dynamic programming, the complexity of the algorithms will be significantly reduced when compared to an exhaustive search approach.

First, the forward algorithm will be adapted to allow us to compute the probability of observing a given sequence of symbols given the model $\Theta$. By doing so, the probability of observing a given sequence of symbols $\{Y=y\}$, without knowing the corresponding underlying sequence of states $X$, can be computed. This will serve as a score for the sequence in that the higher the probability, the more confident one can be about it. In addition, one can compare the scores obtained to that of randomly generated sequences to get a sense of how significant these are.

Second, an adaptation of the well-known Viterbi algorithm will be introduced. This algorithm will allow us to find the optimal path $X^{*}$, i.e. the sequence of states that better explain the observed sequence of symbols $\{Y=y\}$. In addition, the probability of this path will also be computed, thus obtaining a second score for the sequence. Again, the higher this probability is, the more confident that one can be about the sequence observed.

Third, another scoring approach is suggested by using the optimal path $X^{*}$ found through the Viterbi algorithm to compute the conditional probability of observing $\{Y=y\}$ given the optimal path and the model, i.e. $\Theta$.

### 4.1 Adaptation of the forward algorithm

It is of interest to compute the probability of observing a given sequence of symbols based on a HMM. In other words, to compute the probability:

$$
\begin{equation*}
P\{Y=y \mid \Theta\} \tag{4.1}
\end{equation*}
$$

This problem is usually referred to as the scoring problem [17]. In this case, the corresponding underlying sequence of states is not known. However, recall that, as discussed previously, by the LTP:

$$
\begin{equation*}
P\{Y=y \mid \Theta\}=\sum_{x \in \Omega_{m}} P\{Y=y, X=x \mid \Theta\} \tag{4.2}
\end{equation*}
$$

Therefore, by considering all the possible underlying sequences of states, one could eventually find the marginal probability of a given observation $\{Y=y \mid \Theta\}$. Nevertheless, this would not be feasible for somewhat elaborated hidden Markov models. Recall, that there would be as many as $|E|^{m}$ possible combinations, where $|E|$ is the cardinality of the state space and $m$ the number of elements in a given symbol sequence. Therefore, this number grow exponentially with the length of the observation being studied.

The forward algorithm is an algorithm that allows us to deal with this issue. It is based on dynamic programming, and can compute the probability of interest in a rather efficient way [9]. This computational approach consists in solving a complex problem by breaking it down into smaller problems that are much simpler, solving these and storing the solutions to finally find the answer to the larger problem by combining all these solutions. In this case, the following recursive variable is defined:

$$
\begin{equation*}
\alpha(n, i)=P\left\{Y_{1}=y_{1}, \ldots, Y_{n}=y_{n}, X_{n}=i \mid \Theta\right\} \tag{4.3}
\end{equation*}
$$

and this variable is recursively computed:

$$
\begin{equation*}
\alpha(n, i)=\sum_{s \in E} \alpha(n-1, s) P(s, i) e\left(y_{n} \mid i\right), \quad n=2, \ldots, m \tag{4.4}
\end{equation*}
$$

Therefore, the first three iterations would be:

1. Set $n=1$ and, for all $i$ in $E$ compute and store all the $\alpha(1, \cdot)$

$$
\begin{align*}
\alpha(1, i) & =P\left\{Y_{1}=y_{1}, X_{1}=i \mid \Theta\right\} \\
& =\pi_{0}(i) e\left(y_{1} \mid i\right) \tag{4.5}
\end{align*}
$$

2. Set $n=2$ and, for all $i$ in $E$ compute and store all the $\alpha(2, \cdot)$

$$
\begin{align*}
\alpha(2, i) & =P\left\{Y_{1}=y_{1}, Y_{2}=y_{2}, X_{2}=i \mid \Theta\right\} \\
& =\sum_{s \in E} \alpha(1, s) P(s, i) e\left(y_{2} \mid i\right) \tag{4.6}
\end{align*}
$$

3. Set $n=3$ and, for all $i$ in $E$ compute and store all the $\alpha(3, \cdot)$

$$
\begin{align*}
\alpha(3, i) & =P\left\{Y_{1}=y_{1}, Y_{2}=y_{2}, Y_{3}=y_{3}, X_{3}=i \mid \Theta\right\} \\
& =\sum_{s \in E} \alpha(2, s) P(s, i) e\left(y_{3} \mid i\right) \tag{4.7}
\end{align*}
$$

Note that there are going to be as many recursive variables $\alpha(n, \cdot)$ as the size of the cardinality of the state space $|E|$. In addition, we would need as many iterations as $k$-mers there are in the symbol sequence of interest. In order to improve the
performance of the algorithm, since each recursive variable will be independent, this calculation could be multi-threaded.

Once the recursions are completed, then the probability of interest can be computed in the following way:

$$
\begin{equation*}
P\{Y=y \mid \Theta\}=\sum_{s \in E} \alpha(m, s) \tag{4.8}
\end{equation*}
$$

As a result, the probability $\{Y=y \mid \Theta\}$ is obtained with a complexity of $O\left(m|E|^{2}\right)$. Note that the complexity of the algorithm is linear with respect to the number of symbols $m$ in the sequence observed and quadratic with respect to the cardinality of the space state $E$. This makes a huge difference when compared to the initial guess, which grows exponentially with the length of the sequence of symbols.

### 4.2 Adaptation of the Viterbi algorithm

It is of interest as well to find the optimal sequence of states that can better explain the observed sequence of symbols. More technically, it is of interest to find:

$$
\begin{equation*}
x^{*}=\arg \max _{x \in \Omega_{m}} P\{X=x \mid Y=y, \Theta\} \tag{4.9}
\end{equation*}
$$

Given Bayes' rule,

$$
\begin{align*}
x^{*} & =\arg \max _{x \in \Omega_{m}} P\{X=x \mid Y=y, \Theta\} \\
& =\arg \max _{x \in \Omega_{m}} \frac{P\{X=x, Y=y, \Theta\}}{P\{Y=y, \Theta\}} \\
& =\arg \max _{x \in \Omega_{m}} \frac{P\{X=x, Y=y \mid \Theta\} P\{\Theta\}}{P\{Y=y \mid \Theta\} P\{\Theta\}} \\
& =\arg \max _{x \in \Omega_{m}} \frac{P\{X=x, Y=y \mid \Theta\}}{P\{Y=y \mid \Theta\}} \tag{4.10}
\end{align*}
$$

Note that the denominator is going to be a constant that will effect every possible candidate path in the same way. Therefore, maximizing equation 4.9 is the same as maximizing:

$$
\begin{equation*}
x^{*}=\arg \max _{x \in \Omega_{m}} P\{X=x, Y=y \mid \Theta\} \tag{4.11}
\end{equation*}
$$

As in section 4.2, enumerating all possible states might be unfeasible depending on the length of the sequence of observations and the cardinality of the space state. Therefore, we can benefit again from using dynamic programming to find an efficient way to find the optimal path as well as its conditional probability based on the observations. The well-known Viterbi algorithm fits this scenario perfectly [3]. In this case, we define the variable:

$$
\begin{equation*}
\gamma(n, i)=\max _{x_{1}, \ldots, x_{n-1}} P\left\{Y_{1}=y_{1}, \ldots, Y_{n}=y_{n}, X_{1}=x_{1}, \ldots, X_{n-1}=x_{n-1}, X_{n}=i \mid \Theta\right\} \tag{4.12}
\end{equation*}
$$

for $n=2, \ldots, m$ and we compute it recursively in the following way:

$$
\begin{equation*}
\gamma(n, i)=\max _{s \in E} \gamma(n-1, s) P(s, i) e\left(y_{n} \mid i\right) \tag{4.13}
\end{equation*}
$$

Therefore, the first three iterations would be:

1. Set $n=1$ and, for all $i$ in $E$ compute and store all the $\gamma(1, \cdot)$

$$
\begin{align*}
\gamma(1, i) & =P\left\{Y_{1}=y_{1}, X_{1}=i \mid \Theta\right\} \\
& =\pi_{0}(i) e\left(y_{1} \mid i\right) \tag{4.14}
\end{align*}
$$

2. Set $n=2$ and, for all $i$ in $E$ compute and store all the $\gamma(2, \cdot)$

$$
\begin{align*}
\gamma(2, i) & =\arg \max _{s \in E} P\left\{Y_{1}=y_{1}, Y_{2}=y_{2}, X_{1}=s, X_{2}=i \mid \Theta\right\} \\
& =\arg \max _{s \in E} \gamma(1, s) P(s, i) e\left(y_{2} \mid i\right) \tag{4.15}
\end{align*}
$$

3. Set $n=3$ and, for all $i$ in $E$ compute and store all the $\gamma(3, \cdot)$

$$
\begin{align*}
\gamma(3, i) & =\arg \max _{s \in E} P\left\{Y_{1}=y_{1}, Y_{2}=y_{2}, Y_{3}=y_{3}, X_{2}=s, X_{3}=i \mid \Theta\right\} \\
& =\arg \max _{s \in E} \gamma(2, s) P(s, i) e\left(y_{3} \mid i\right) \tag{4.16}
\end{align*}
$$

As in the forward algorithm case, note that there are going to be as many recursive variables $\gamma(n, \cdot)$ as the size of the cardinality of the state space $|E|$. In addition, we would need as many iterations as $k$-mers present in the symbol sequence of interest.

Once equation 4.13 is computed, the maximum observation probability can be calculated as:

$$
\begin{align*}
P^{*} & =\max _{x \in \Omega_{m}} P\{Y=y, X=x \mid \Theta\} \\
& =\max _{s \in E} \gamma(m, s) \tag{4.17}
\end{align*}
$$

Finally, the optimal path $x^{*}$ can be found by tracing back the states that led tot $P^{*}$. As in the forward algorithm, the Viterbi algorithm has an complexity of $O\left(m|E|^{2}\right)$, thus the time required will be proportional to the length of the sequence of symbols and quadratic to respect the cardinality of the space state.
4.3 Using the optimal path $X^{*}$ to score an observation

Once the optimal path $X^{*}$ has been obtained by using the Viterbi algorithm, it is possible to compute the following probability as well:

$$
\begin{equation*}
P\left\{Y=y \mid X^{*}, \Theta\right\} \tag{4.18}
\end{equation*}
$$

This is pretty straightforward since this probability is simply the product of the different emission probabilities.

$$
\begin{equation*}
P\left\{Y=y \mid X^{*}, \Theta\right\}=\prod_{n=1}^{m} e\left(y_{n} \mid x_{n}^{*}\right) \tag{4.19}
\end{equation*}
$$

## 5. RESULTS

In order to evaluate the performance of the algorithms introduced in sections 4.1 and 4.2 , a set of data has been generated. Firstly, six files have been generated containing each 250 random sequences of the same length. For instance, the first file contains 250 sequences of length ten. The second file contains 250 sequences of length twenty, and so forth. On the other hand, six files have been generated by randomly drawing sequences from the reference from which the transition matrix has been generated. Again, the first file contains 250 sequences of length ten, the second one contains 250 sequences of length twenty, and so forth. In total, the dataset generated to test the methods described in this thesis consist of twelve FASTA files with 250 sequences each. In addition to this dataset, data from [12] have been used as well to test HiMMe with real NGS data.

In this section, two main results are presented. Firstly, all the sequences have been scored based on the HMM model built. The results suggest that the algorithm is able to recognize those sequences that belong to the reference and, in turn, score them higher than those that have been randomly generated. Secondly, the randomly generated sequences are corrected using the algorithm described in section 4.2 to achieve a higher score.

### 5.1 Scoring sequences

The algorithm described in section 4.1 has been used to score randomly generated sequences as well as sequences derived from the reference used to learn the transition matrix. The purpose of this comparison is to see whether the algorithm scores higher those sequences that belong to the reference from which the transition matrix was learned. Intuitively, the larger the sequences are, the larger the difference between
scores should be given the amount of information the sequences would contain. In addition, one would expect to encounter more differences between scores as the $k$-mer size increases, since the algorithm should be able to identify more genomic patterns in the sequences.


Figure 5.1: Logarithm of the score ratio between the random and true sequences as function of the sequence length.

In Figure 5.1 the logarithm of the ratio of the average scores for different sequence lengths and $k$-mer sizes has been plotted. Note that as the length of the sequences increases, the ratio between true and random sequences increases. Therefore, the hypothesis that the longer the sequences, the larger the discriminant power is, is confirmed. In addition, as the $k$-mer size increases, this ratio seems to be more prominent. That is consistent again with the fact that the larger the $k$-mers are, the more biological information they are able to carry.

In order to statistically determine whether there is a significant difference between the random and true scores obtained, a t-test has been performed for each possible combination.

| t-test summary |  |  |  |
| :---: | :---: | :---: | :---: |
| Sequence length | p-value (1-mers) | p-value (3-mers) | p-value (5-mers) |
| 10 bp | $1 \mathrm{e}-10$ | $1.1 \mathrm{e}-07$ | $1.3 \mathrm{e}-09$ |
| 20 bp | $4.4 \mathrm{e}-23$ | $1.3 \mathrm{e}-17$ | $8.6 \mathrm{e}-03$ |
| 30 bp | $8.9 \mathrm{e}-17$ | $1 . \mathrm{e}-13$ | $4.7 \mathrm{e}-02$ |
| 40 bp | $4.8 \mathrm{e}-23$ | $2.6 \mathrm{e}-07$ | $9.9 \mathrm{e}-05$ |
| 50 bp | $4.5 \mathrm{e}-15$ | $2.5 \mathrm{e}-09$ | 0.1 |
| 60 bp | $9.9 \mathrm{e}-16$ | $6.3 \mathrm{e}-10$ | 0.05 |

Table 5.1: Statistics for each comparison.

All the p-values, except for one, conclude that the differences between score populations were different with a $95 \%$ confidence level. As the $k$-mer size increases, the magnitude of the p-values, however, seems to increase. This is due to the fact
that as the $k$-mer size increases, the difference in score magnitude decreases faster than the standard deviation does. Therefore, the t-test loses power as the $k$-mer size increases. However, as depicted in Figure 5.1, the algorithm is capable of scoring higher those sequences that were sampled from the reference and this power increases with the length of the sequences as well as with the $k$-mer size.


Figure 5.2: Boxplots of the logarithm of the score ratios between the sequences sampled from SEQC and the random sequences for $k$-mer sizes 1,3 and 5 respectively.

HiMMe has been used as well to score part of the RNA-seq data from [12]. This dataset consists of NGS data that come from six different sequencing centers: the Australian Genome Research Facility, the Beijing Genomics Institute, City of Hope, Cornell, Mayo Clinic and Novartis, the pharmaceutical company. For each sequencing center present in the dataset, 250 sequences of length 100 have been randomly extracted and scored. Note in Figure 5.2 that the sequences extracted from their dataset score significantly higher than the randomly generated ones. Furthermore, the ratio grows as the $k$-mer size used grows as in Figure 5.1, suggesting again that the larger the choice of $k$-mer is, the more genetic information and patterns the algorithm is able to identify in the data. The code used to generate the random sequences, learn the transition matrix, learn the emission probabilities and score FASTA files, can be found in Appendix A.1.

### 5.2 Correcting sequences

The algorithm described in section 4.2 has been applied to the random sequences. As a result, these sequences have been corrected so that the joint probability

$$
\begin{equation*}
P\{X=x, Y=y \mid \Theta\} \tag{5.1}
\end{equation*}
$$

is maximized. The results are illustrated in Figure 5.3. Note that for 1-mers, the score of the corrected versions does not improve significantly compared to that of the original random reads. However, for 3-mers and 5-mers, the algorithm is capable of improving the scores significantly. For instance, when using 3-mers, there is a twofold increase when the length of the sequence is above forty nucleotides. When using 5mers, the improvement is even more prominent, reaching an eightfold increase when the length of the sequence is fifty nucleotides.


Figure 5.3: Score ratio between the corrected and original sequences as function of the sequence length.

### 5.3 Speed assessment

In order to increase the efficiency, since both algorithms share some common steps, they have been implemented together.

| Time elapsed running HiMMe |  |  |  |
| :---: | :---: | :---: | :---: |
| Sequence length | 1 -mers | 3 -mers | 5 -mers |
| 10 bp | $<1.0 \mathrm{~s}$ | 3.0 s | 721.0 s |
| 20 bp | $<1.0 \mathrm{~s}$ | 7.5 s | 2057.0 s |
| 30 bp | $<1.0 \mathrm{~s}$ | 13.0 s | 3497.0 s |
| 40 bp | $<1.0 \mathrm{~s}$ | 16.5 s | 4860.0 s |
| 50 bp | $<1.0 \mathrm{~s}$ | 20.5 s | 5340.0 s |
| 60 bp | $<1.0 \mathrm{~s}$ | 25.5 s | 7100.0 s |

Table 5.2: Time elapsed when running HiMMe on the generated dataset.

Note in Table 5.2 how the sequence length, as explained before, has a somewhat linear relationship with the time elapsed. However, the $k$-mer choice ends up having a huge impact on it and drastically affects the processing time. Recall that the complexity of the algorithms introduced in section 4 grows quadratically with the cardinality of the space state. In addition, the latter grows exponentially with the $k$-mer size.

| Average speed HiMMe |  |  |
| :---: | :---: | :---: |
| 1 -mers | 3 -mers | 5 -mers |
| $10,000 \mathrm{bp} / \mathrm{s}$ | $646 \mathrm{bp} / \mathrm{s}$ | $2.42 \mathrm{bp} / \mathrm{s}$ |

Table 5.3: Average number of nucleotides processed per second by HiMMe.

In Table 5.3 the average number of nucleotides processed by HiMMe is summarized. As the $k$-mer size gets larger, more memory is required to store not only the
transition matrix but also all the recursive variables involved in the algorithm and the search space grows exponentially. Combined, this has a huge impact in terms of performance.

## 6. CONCLUSIONS

The sequence scoring method proposed in this thesis is capable of scoring higher those sequences that were used to construct the hidden Markov model than those that were randomly generated. In other words, it is able to identify those patterns in the sequencing data that are more likely to happen based on previous knowledge about the organism being studied. This method can be applied in multiple cases when analyzing NGS data.

One application could be to evaluate NGS reads quality. Although FASTQ files do contain such information, this would serve as another metric to quantify how confident one can be about the reads that the sequencing machine produced. A threshold could be used to determine which reads are kept for the downstream analysis for instance.

Another application that the scoring method proposed here could have, would be to evaluate the reliability of assemblies, either genomes or transcriptomes. One could generate different assemblies with different parameters and pick the one with the highest score. Also, by using these scores as a benchmark, this method would allow researchers in the field of assemblers to assess the performance of their algorithms and improve based on that.

Finally, in a metagenomics scenario, where one wants to identify the organisms present in a given environment, this tool could be used to classify the reads obtained into the different organisms present in the genetic material recovered. Different models should be learned for the species being studied and the algorithm would allow the researcher to find the model, and thus organism, that better explains the observed reads. These are only some of the applications that the method proposed
in this thesis has.
As for the repairing algorithm proposed, it is capable of identifying potential errors and repairing them. The scores obtained once the sequences are repaired are higher, thus improving the reliability of the data. This has potentially a plethora of applications as well. For instance, one could run the algorithm on NGS data, before the downstream analysis is performed, to ensure that the dataset used in the analysis is as reliable as possible. The same could be done with assemblies to correct errors induced throughout the whole process, specially in rather complex organisms with a lot of polymorphism in their genes.

A clear advantage of the method proposed in this thesis is that there is no alignment involved and that, in turn, it could be faster than some of the state of the art scoring and error-correcting tools. In a nutshell, the HMM model, with the appropriate $k$-mer choice, seems to be able to capture the biological information and detect potential errors in NGS data. In this thesis, two applications of this stochastic model have been described and implemented leading, in both cases, to positive results.

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## APPENDIX A

## A. 1 Code

## A.1.1 Learning the transition matrix

```
1#!/usr/bin/env bash
2#
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4##
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15##copies or substantial portions of the Software.
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1 9 \# \# F I T N E S S ~ F O R ~ A ~ P A R T I C U L A R ~ P U R P O S E ~ A N D ~ N O N I N F R I N G E M E N T . ~ I N ~ N O ~ E V E N T ~ S H A L L ~ T H E ~
20##AUTHORS OR COPYRIGHT HOLDERS BE LIABLE FOR ANY CLAIM, DAMAGES OR OTHER
21 ##LIABILITY, WHETHER IN AN ACTION OF CONTRACT, TORT OR OTHERWISE, ARISING FROM,
22 ##OUT OF OR IN CONNECTION WITH THE SOFTWARE OR THE USE OR OTHER DEALINGS IN THE
23 ##SOFTWARE.
24 #
25 shopt -s extglob
26
27 abspath_script="$(readlink -f -e "$0")"
28 script_absdir="$(dirname "$abspath_script")"
29 script_name="$(basename "$0".sh)"
30
31# Find perl scripts
```

```
perl_script="${script_absdir}/ perl/${script_name}.pl"
33
34 if [ $# -eq 0 ]
        then
            cat "$script_absdir/${script_name}_help.txt"
            exit 1
fi
39
0TEMP=$(getopt -o hd:t:k: - l help,outdir:,threads:,kmer_size: -n "$script_name.sh" --
    "$@")
4 1
42 if [ $? -ne 0 ]
43 then
4 4 ~ e c h o ~ " T e r m i n a t i n g ~ . . . " ~ > \& 2 ~
4 5 ~ e x i t ~ - 1 ~
4 6 ~ f i
4 7
48 eval set -- "$TEMP"
4 9
50# Defaults
1 outdir="$PWD"
threads=2
53 kmer_size=1
54
55# Options
56 while true
57 do
58 case "$1" in
59 -h|--help)
60 cat "$script_absdir"/${script_name}_help.txt
6 1 ~ e x i t
62 ; ;
6 3 ~ - ~ d \| - - o u t d i r ) ~
64 outdir="$2"
65 shift 2
66 ; ;
67 -t|--threads)
68 threads=" $2"
69 shift 2
```



```
        kmer_size="$2"
        shift 2
        ; ;
    --)
        shift
        break
        ; ;
    *)
    echo "$script_name.sh:Internal error!"
    exit -1
        ;;
esac
done
8 5
# Print LICENSE
7cat "${script_absdir }/../../LICENSE"
88
89# Inputs
input="$1"
91
# Output
input_basename="$(basename "$input")"
prefix="${input_basename%%.*}"
tempfile="${outdir}/${prefix}"
outfile="${outdir}/${prefix}_tm${kmer_size}.txt.gz"
97
8# Output directory
mkdir -p "$outdir"
1 0 0
101 # Count number of entries in FASTA
102 n_entries="$(zcat -f "$input" | grep "^>" | wc - l)"
1 0 3
104 # Run
105zcat -f "$input" | "$perl_script" "$input" "$kmer_size" "$n_entries" "$outfile"
    1#!/usr/bin/env perl
2#
```

```
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4##
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20 ##AUTHORS OR COPYRIGHT HOLDERS BE LIABLE FOR ANY CLAIM, DAMAGES OR OTHER
21 ##LIABILITY, WHEIHER IN AN ACTION OF CONTRACT, TORT OR OTHERWISE, ARISING FROM,
22##OUT OF OR IN CONNECTION WITH THE SOFTWARE OR THE USE OR OTHER DEALINGS IN THE
23 ###SOFTWARE.
24# -
25
26# Libraries
27 use strict;
28 use Algorithm:: Combinatorics qw(combinations variations_with_repetition);
29
30# Read arguments
31 my $scriptname = $0; # Get script name
32my $fasta_file = @ARGV[0]; # Get target FASTA file name
33 my $kmer_size= @ARGV[1]; # Get user k-mer size
34 my $n_entries=@ARGV[2]; # Number of entries in the FASTA file
35my $outfile= @ARGV[3]; # Output file
36
37# Variables
38 my $FASTA; # Fasta file handler
39 my $dim=1; # Dimension P (based on k-mer size)
40 my @markov_matrix =(); # 3D array [assembly][row][column]
41 my $n_proc=0; # Number of entries processed
```

```
42 my $n_mem=0; # Number of entries stored in RAM
43 my $n_limit=2500; # Limit for number of entries in RAM
4 4
45 # Time stamps
46 my $st_time=0; # Start time
47my $end_time=0; # End time
48 my $current_time=0; # Current time
49 my $elapsed_time=0; # Time elapsed
50
51# Hashes
52 my %fasta_hash=(); # Hash containing sequence info of each sample
53my %transition_hash=(); # Hash containing duplets combinations
54
```



```
56
57# Read in fasta file
58 $st_time = localtime;
59 print STDERR "${st_time}: FASTA file: ${fasta_file}\n";
6 0
61# Markov matrices
62 $current_time = localtime;
63 print STDERR "${current_time}: Initializing Markov matrix ...\n";
64 initialize();
65 $current_time = localtime;
66 print STDERR "${current_time}: Learning Markov matrix ...\n";
67 fill_markov_matrix();
68
69# Print stuff
70 $current_time = localtime;
71 print STDERR "${current_time}: Saving Markov matrix in ${outfile}...\n";
72 print_markov_matrices() ;
7 3
74 ###################################### Subs #######################################################
75 ## Fill the markov matrix
76 sub fill_markov_matrix
77 {
78 # Process in chunks of n_limit
79 while($n_proc<$n_entries)
80 {
```

```
        # Read n_lim entries
        read_fasta();
        # Get chromosomes from fasta
        my @entries=keys %fasta_hash;
        foreach my $entry (@entries)
        {
            for(my $i=0;$i<=(scalar @{$fasta_hash{$entry }}) - 2*$kmer_size; $i++)
            {
                # Get nucleotides of the iteration
                my $seq_1=@{ $fasta_hash{$entry }}[$i];
                my $seq_2=@{ $fasta_hash {$entry }}[$i+$kmer_size];
                for(my $j=1;$j<$kmer_size; $j++)
                {
                    $seq_1=$seq_1.@{$fasta_hash{$entry }}[$i+$j];
                    $seq_2=$seq_2.@{$fasta_hash {$entry }}[$i+$kmer_size+$j];
            }
            # Get codon index form markov_matrix
            my $row=$transition_hash{$seq_1 };
            my $col=$transition_hash{$seq_2 };
            # Fill markov_matrix
            $markov_matrix [ $row ] [ $col]+=1;
            }
            # Get rid of that entry
            delete $fasta_hash{$entry };
            $n_mem--;
            # Update progress
            $n_proc++;
            my $perc=$n_proc/$n_entries * 100;
            printf STDERR "\rCurrent progress: %.2f%", $perc;
        }
}
printf STDERR "\n";
# Scale matrix
my $sum;
for(my $i=0;$i<=$dim;$i++)
{
    $sum=0;
    # Count frequencies per row
```

```
120 for (my $j=0;$j<=$dim; $j++)
121 {
122 $sum+=$markov_matrix [$i][$j];
123 }
124 # Normalize each row
125 if ($sum!=0)
126 {
127 for(my $j=0;$j<=$dim; $j++)
128 {
129 $markov_matrix [$i][ $j]/=$sum;
130 }
131 }
132 }
133}
1 3 4
135 ## Initialize markov_matrices
136 sub initialize
137{
1 3 8 ~ \# ~ N u c l e o t i d e s ~ t a k e n ~ i n t o ~ c o n s i d e r a t i o n ~
139 my @ nucleotides=('A', 'C', 'G', 'T');
140 for(my $i=1;$i<=$kmer_size; $i++)
141 {
142 $dim*=(scalar@nucleotides);
143 }
1 4 4 ~ \# ~ B e c a u s e ~ w e ~ s t a r t ~ w i t h ~ d i m = 0 ~ i n ~ t h e ~ l o o p s
145 $dim - = 1;
146 # Get all possible combinations of kmer_size nucleotides
147 my @permutations=variations_with_repetition(\@nucleotides,$kmer_size);
148 # Codify numerically each possible permutation
1 4 9 ~ m y ~ \$ ~ i = 0 ;
1 5 0 \text { foreach my \$combination (@permutations)}
151 {
152 my $length=$kmer_size - 1;
153 my $sequence = join(,',@{$combination}[0..$length]);
154 $transition_hash {$sequence}=$i;
155 $ i ++;
156}
57 # Initialize markov_matrix
158 for(my $i=0;$i<=$dim;$i++)
```

```
159 {
                for(my $j=0;$j<=$dim; $j++)
        {
            $markov_matrix [$i ] [ $j]=0;
        }
        }
165 }
166
167## Read in fasta file
168 sub read_fasta
169 {
170 my $entry;
171 while((my $line =<STDIN>) and ($n_mem<=$n_limit))
172 {
173 chomp($line);
174 if( $line =~ / >/)
175 {
176 $entry=substr($line,1); # Get rid of leading ">" character
177 $n_mem++;
178 }
179 else
180 {
181 my @array = split //, $line;
182 push @{$fasta_hash{$entry }},@array;
183 }
184 }
185}
186
187## Print fasta
188 sub print-fasta
189 {
190 # Print output
1 9 1 ~ f o r e a c h ~ m y ~ \$ k e y ~ ( s o r t ~ k e y s ~ \% f a s t a \_ h a s h )
192 {
193 my @entry=@{$fasta_hash{$key }};
194 print ">$key\n";
195 foreach my $nuc (@entry)
196 {
197 print "$nuc";
```

    print " n ";
    \}
    $201\}$
202
203\#\# Print markov_matrices
204 sub print_markov_matrices
205 \{
open(OUT, "|gzip -c $>\$\{$ outfile\}") \#\$ is our process id
or die "Can't open file '\$\{outfile\}' \$!";
\# Reverse hash
my \%reverse_hash = reverse \%transition_hash;
\# Print to OUT
print OUT " $\backslash t$ ";
for (my $\$ \mathbf{j}=0 ; \$ \mathbf{j}<=\$ \operatorname{dim} ; \$ \mathbf{j}++$ )
\{
my $\$$ key $=\$$ reverse_hash $\{\$ \mathrm{j}\}$;
print OUT "\$key $\backslash \mathrm{t}$ ";
\}
print OUT " $\backslash \mathrm{n}$ ";
for (my $\$ \mathrm{i}=0 ; \$ \mathrm{i}<=\$ \operatorname{dim} ; \$ \mathrm{i}++$ )
\{
my $\$$ key $=\$$ reverse_hash $\{\$ \mathrm{i}\}$;
print OUT " \$key $\backslash \mathrm{t}$ ";
for (my $\$ \mathrm{j}=0 ; \$ \mathrm{j}<=\$ \operatorname{dim} ; \$ \mathrm{j}++$ )
\{
printf OUT " \%.5f $\backslash \mathrm{t}$ ", \$markov_matrix [\$i][\$j];
\}
print OUT " n ";
\}
close OUT;
229 \}


## A.1.2 Learning the emission probabilities

```
1#!/usr/bin/env bash
2# \longrightarrow
3##The MIT License (MIT)
```

```
4##
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21##LIABILITY, WHETHER IN AN ACTION OF CONTRACT, TORT OR OTHERWISE, ARISING FROM,
22 ##OUT OF OR IN CONNECTION WITH THE SOFTWARE OR THE USE OR OTHER DEALINGS IN THE
23 ##SOFTWARE.
24#
25 shopt -s extglob
26
abspath_script="$(readlink -f -e "$0")"
script_absdir="$(dirname "$abspath_script")"
9 script_name="$(basename "$0".sh)"
30
31# Find perl scripts
2 perl_script="${script_absdir}/perl/${script_name}.pl"
33
4 if [ $# -eq 0 ]
        then
                cat "$script_absdir/${script_name}_help.txt"
                exit 1
f i
39
40 TEMP=$(getopt -o hd:t:b: -l help,outdir:,threads:,bases: -n "$script_name.sh"-- "$@
    ")
4 1
```

```
42 if [ $? -ne 0 ]
4 3 \text { then}
4 echo "Terminating ... " >&2
exit -1
4 6 \mathrm { fi }
4 7
48 eval set -- "$TEMP"
4 9
50# Defaults
1 outdir="$PWD"
52threads=2
53 bases =10000000
54
55# Options
56 while true
57 do
    case "$1" in
        -h|--help)
            cat "$script_absdir"/${script_name}_help.txt
            exit
                ;;
        -d|--outdir)
            outdir="$2"
            shift 2
                ; ;
        -t|--threads )
            threads="$2"
            shift 2
                ;;
        -b|--bases )
            bases="$2"
            shift 2
                ; ;
            --)
            shift
                break
                ; ;
            *)
            echo "$script_name.sh:Internal error!"
```

```
81 exit -1
        ; ;
    esac
done
85
86 # Print LICENSE
87cat "${script_absdir } /../../LICENSE"
88
89# Inputs
90 input=" $1"
91
92# Output
3 input_basename="$(basename "$input")"
4 prefix="${input_basename%%.*}"
5 tempfile="${outdir}/${prefix }"
6 outfile="${outdir}/${prefix}_ep${bases }.txt.gz"
97
98# Output directory
99 mkdir -p "$outdir"
1 0 0
101# Count number of entries in FASTA
102 n_entries="$(zcat -f "$input" | grep -v "^#" | wc -l)"
1 0 3
104 # Run
105 zcat -f "$input" | "$perl_script" "$input" "$n_entries" "$bases" "$outfile"
    1#!/usr/bin/env perl
    2 #
    3##The MIT License (MIT)
    4##
    5##Copyright (c) 2016 Jordi Abante
    6 ##
    7##Permission is hereby granted, free of charge, to any person obtaining a copy
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```

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22 ##OUT OF OR IN CONNECIION WITH THE SOFTWARE OR THE USE OR OTHER DEALINGS IN THE
23 ##SOFTWARE.
24# -
25
26 # Libraries
27 use strict;
28 use Algorithm::Combinatorics qw(combinations variations_with_repetition);
29
30# Read arguments
31my $scriptname = $0; # Get script name
32my $vcf_file =@ARGV[0]; # Get target FASTA file name
33my $n_entries=@ARGV[1]; # Number of entries in the FASTA file
3 4 \text { my \$n_bases=@ARGV[2]; \# Number of bases}
35 my $outfile=@ARGV[3]; # Output file
36
37# Variables
38 my $VCF; # Fasta file handler
39 my $dim=1; # Dimension P (based on k-mer size)
40my @emission_matrix =(); # 3D array [assembly][row][column]
41 my $n_proc=0; # Number of entries processed
42 my $n_mem=0; # Number of entries stored in RAM
43my $n_limit=2000; # Limit for number of entries in RAM
44 my $kmer_size=1;
45
4 6
47# Time stamps
48 my $st_time=0; # Start time
49 my $end_time=0; # End time
50 my $current_time=0; # Current time
51 my $elapsed_time=0; # Time elapsed
52
```

```
53# Hashes
54my %vcf_hash=(); # Hash containing sequence info of each sample
55my %transition_hash=(); # Hash containing duplets combinations
56
57 ######################################## Main #############################################################
58
59# Read in fasta file
$st_time = localtime;
61 print STDERR "${st_time}: VCF file: ${vcf_file}\n";
62# Initialize matrix
3 $urrent_time = localtime;
64 print STDERR "${current_time}: Initializing emission matrix ...\n";
65 initialize();
66 # Count frequencies
$current_time = localtime;
8 print STDERR "${current_time}: Learning emission matrix $n_bases ...\n";
fill_emission_matrix();
0# Print stuff
$current_time = localtime;
2 print STDERR "${current_time}: Saving emission matrix in ${outfile }...\n";
73 print_emission_matrices();
7 4
75 ###################################### Subs #######################################################
76 ## Fill the emission matrix
77 sub fill_emission_matrix
78 {
    # Process in chunks of n_limit
    while($n_proc<$n_entries)
        {
            # Read n_lim entries
            read_vcf();
            # Get new entries
            my @entries=keys %vcf_hash;
            # Loop through the entries
            foreach my $entry (@entries)
            {
                # Get variants
                    my @ref=@{ $vcf_hash{$entry}{'ref'}};
            my @alt=@{$vcf_hash{$entry}{'alt'}};
```

                    # Get index form emission_matrix
                my $row=$transition_hash{@ref [0]};
            my $col=$transition_hash{@alt [0]};
            # Fill emission_matrix
            $emission_matrix [ $row ] [ $col]+=1;
            # Get rid of that entry
            delete $vcf_hash{$entry };
            # Update progress
            $n_mem--;
            $n_proc++;
            my $perc=$n_proc/$n_entries * 100;
            printf STDERR "\rCurrent progress: %.2f%", $perc;
        }
    }
    printf STDERR "\n";
    
# Add e(i|i) and scale matrix

    for (my $i=0; $i<=$dim;$i++)
    {
        my $sum=0;
        $emission_matrix [ $i][ $i]=$n_bases / 4;
    # Count frequencies per row
    for (my $j=0; $j<=$dim ; $j++)
    {
            $sum+=$emission_matrix [ $i ] [ $j];
    }
    # Normalize each row
    if ($sum!=0)
    {
        for (my $j=0; $j<=$dim ; $j++)
        {
            $emission_matrix [ $i ] [ $j]/=$sum;
        }
    }
    }
    2 }
127
128\#\# Initialize emission_matrices
129 sub initialize
130{

```
```

1 3 1 ~ \# ~ N u c l e o t i d e s ~ t a k e n ~ i n t o ~ c o n s i d e r a t i o n ~
132 my @nucleotides=('A', 'C', 'G', 'T');
133 for(my $i=1;$i<=\$kmer_size ; \$i++)
134 {
135 \$dim*=(scalar @nucleotides);
}

# Because we start with dim=0 in the loops

\$dim-=1;

# Get all possible combinations of kmer_size nucleotides

my @permutations=variations_with_repetition(\@nucleotides,\$kmer_size);

# Codify numerically each possible permutation

my \$ i=0;
foreach my \$combination (@permutations)
{
my $length=$kmer_size - 1;
my $sequence = join(',',@{$combination }[0.. \$length]);
$transition_hash {$sequence }=\$i;
\$ i++;
}

# Initialize emission_matrix

for(my $i=0;$i<=\$dim; \$i++)
{
for (my \$j=0; $j<=$dim ; \$j++)
{
\$emission_matrix [ \$i ] [ \$j]=0;
}
}
158}
1 5 9
160\#\# Read in vcf file
161 sub read_vcf
162{
163 my $entry;
164 while(($n_mem<\$n_limit) and (my $line =<STDIN>))
165 {
    chomp($line);
if( \$line =~ /\#/)
{
\# nothing

```
```

170 }
171 else
172 {
173 my @array = split('\t',\$line);
174 my \$id = $array[2];
    my @ref = split(//,$array[3]);
my @alt = split(//,\$array[4]);
if((scalar @ref eq 1) and (scalar @alt eq 1))
{
$vcf_hash{$id}{'ref'}=[@ref ];
$vcf_hash{$id}{'alt '}=[@alt ];
\$n_mem++;
}
else
{
\$n_entries--;
}
}
}
189 }
190
191\#\# Print emission_matrices
192 sub print_emission_matrices
193 {

194 open(OUT, "|gzip -c > \${outfile}") \# \$\$ is our process id
195 or die "Can't open file '\${outfile}' \$!";
196 \# Reverse hash
197 my %reverse_hash = reverse %transition_hash;
198 \# Print to OUT
199 print OUT "\t";
200 for(my $j=0;$j<=$dim;$j++)
201 {
202 my \$key = $reverse_hash{$j};
203 print OUT "\$key\t";
204 }
205 print OUT "\n";
206 for (my $i=0;$i<=$dim;$i++)
207 {
208 my \$key = $reverse_hash{$i};
``` ```
209 print OUT "\$key\t";
for(my $j=0;$j<=$dim;$j++)
{
printf OUT "%.5f\t", $emission_matrix[$i][\$j];
}
print OUT "\n";
}
close OUT;
217}
218 \#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#
```

## A.1.3 HiMMe algorithm

```
1#!/ usr/bin/env bash
2 #
3##The MIT License (MIT)
4##
5##Copyright (c) 2016 Jordi Abante
6 ##
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21##LIABILITY, WHETHER IN AN ACTION OF CONTRACT, TORT OR OTHERWISE, ARISING FROM,
22 ##OUT OF OR IN CONNECTION WITH THE SOFTWARE OR THE USE OR OTHER DEALINGS IN THE
23 ##SOFTWARE.
```

```
24 #
```

24 \#
25 shopt -s extglob
25 shopt -s extglob
26

```
```

7 abspath_script="\$(readlink -f -e "$0")"
script_absdir="$(dirname "$abspath_script")"
29 script_name="$(basename "$0".sh)"
30
1# Find perl scripts
perl_script="${script_absdir}/perl/\${script_name}.pl"
3 3
4 if [ $# -eq 0 ]
            then
                cat "$script_absdir/${script_name}_help.txt"
            exit 1
f i
39
TEMP=$(getopt -o hd:t:k: -l help,outdir:,threads:,kmer_size: -n "$script_name.sh" --
            "$@")
4 1
2 if [ $? -ne 0}
then
echo "Terminating ..." >&2
exit -1
fi
4 7
eval set -- "$TEMP"
4 9
0\# Defaults
outdir="\$PWD"
threads=2
kmer_size=1

# Options

while true
do
case "$1" in
        -h|-- help)
            cat "$script_absdir"/\${script_name}_help.txt
exit
;;
-d|--outdir)
outdir="\$2"

```
```

65 shift 2
; ;
-t|--threads )
threads="\$2"
shift 2
; ;
-k|--kmer_size)
kmer_size="$2"
    shift 2
    ; ;
    --)
    shift
    break
    ;;
    *)
    echo "$script_name.sh:Internal error!"
exit -1
;;
esac
done
85
86\# Inputs
tm_file="\$1"
ep_file="\$2"
fasta_file="\$3"
90

## Output

fasta_basename="$(basename "$fasta_file")"
prefix="${fasta_basename%%.*}"
outfile_sum="${outdir}/${prefix}_summary_${kmer_size}.txt"
outfile_fasta="${outdir}/${prefix}_corrected_${kmer_size}.fa"
96
97# Output directory
8mkdir -p "$outdir"
99
100\# Count number of entries in FASTA
101 n_entries="$(zcat -f "$fasta_file" | grep "^>" | wc - l)"
102
103 \# Run

```
```

104zcat -f "$fasta_file" | "$perl_script" "$tm_file" "$ep_file" \
"$fasta_file" "$kmer_size"\
"$n_entries" "$outfile_sum" "\$outfile_fasta"
1\#!/usr/bin/env perl
2 \#
3\#\#The MIT License (MIT)
4 \#\#
5\#\#Copyright (c) 2016 Jordi Abante
6 \#\#
7\#\#Permission is hereby granted, free of charge, to any person obtaining a copy
8\#\#of this software and associated documentation files (the "Software"), to deal
9\#\#in the Software without restriction, including without limitation the rights
10\#\#to use, copy, modify, merge, publish, distribute, sublicense, and/or sell
11\#\#copies of the Software, and to permit persons to whom the Software is
12\#\#furnished to do so, subject to the following conditions:
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14\#\#The above copyright notice and this permission notice shall be included in all
15\#\#copies or substantial portions of the Software.
16 \#\#
17 \#\#THE SOFTWARE IS PROVIDED "AS IS", WITHOUT WARRANTY OF ANY KIND, EXPRESS OR
18 \#\#IMPLIED, INCLUDING BUT NOT LIMITED TO THE WARRANTIES OF MERCHANTABILITY,
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21\#\#LIABILITY, WHETHER IN AN ACTION OF CONTRACT, TORT OR OTHERWISE, ARISING FROM,
22\#\#OUT OF OR IN CONNECTION WITH THE SOFTWARE OR THE USE OR OTHER DEALINGS IN THE
23 \#\#SOFTWARE.
24\# -
25
26 \# Libraries
27 use strict;
28 use Algorithm::Combinatorics qw(combinations variations_with_repetition);
29 use POSIX;
30
31\# Read arguments
32 my \$scriptname = \$0; \# Get script name
33my \$tm_file = @ARGV[0]; \# Transition matrix file name
34my \$ep_file = @ARGV[1]; \# Emission probabilities file name
35my \$fasta_file = @ARGV[2]; \# Get target FASTA file name

```
```

36my \$kmer_size= @ARGV[3]; \# Get user k-mer size
37my \$n_entries=@ARGV[4]; \# Number of entries in the FASTA file
38 my \$outfile_sum= @ARGV[5]; \# Output file
39my \$outfile_fasta= @ARGV[6]; \# Output file
4 0
41 \# For testing purposes
42 my \$tm_outfile="out/tm_test.txt.gz"; \# File to check the transition matrix
43my \$ep_outfile="out/ep_test.txt.gz" ; \# File to check the emission probs
44
45 \# Handlers
4 6 my \$FASTA; \# Fasta file handler
47 my \$TM; \# Transition matrix handler
48 my \$EP; \# Emission probabilities handler
4 9
50\# Variables
51 my \$dim=1;
52 my @markov_matrix=(); \# Transition Matrix [row][column]
53my @emission_matrix =(); \# Emission Matrix [row][column]
54 my \$n_proc=0; \# Number of entries processed
55 my \$n_mem=0; \# Number of entries stored in RAM
56my \$n_limit=250; \# Limit for number of entries in RAM
57
58\# Time stamps
59 my \$st_time=0; \# Start time
60 my Send_time=0; \# End time
61 my \$current_time=0; \# Current time
62 my \$elapsed_time=0; \# Time elapsed
6 3
64 \# Hashes
65my %fasta_hash=(); \# Hash containing sequence info of each sample
66my %transition_hash=(); \# Hash containing transition matrix
6 7 my \%emission_hash=(); \# Hash containing emission probs
68my %emission_all_hash=(); \# Hash containing all comb emission probs
69my %score_hash_1=(); \# Hash containing scores iter n-1
70 my %gamma_1=(); \# Recursive variable gamma for opt. path -1
71 my %score_hash_2=(); \# Hash containing scores iter n
72 my %gamma=(); \# Recursive variable gamma for opt. path curr.
73my %results_hash=(); \# Hash containing scores of all sequences
74my %viterbi_out=(); \# Hash containing Viterbi's output

```
```

75 my %template_hash=(); \# Hash generated based on all the combinations
76
77 \#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\# Main \#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#
78
79 \#\# Read input files
80 $st_time = localtime;
81 print STDERR "${st_time}: ${n_entries} entries to process...\n";
82 print STDERR "${st_time}: Reading in transition matrix file: \${tm_file }...\n";
83 read_tm ();
84\#print_markov_matrix();
85 $current_time = localtime;
86 print STDERR "${current_time}: Reading in emission probabilities file: \${ep_file
}...\n";
87read_ep ();
88\#print_emission_matrix();
89 $current_time = localtime;
90 print STDERR "${current_time}: FASTA file: \${fasta_file}\n";
91
92\#\# Initialize state hash and emission all hash
93 $current_time = localtime;
94 print STDERR "${current_time}: Initializing ...\n";
95 initialize();
96
97\#\# Run algorithm
8 $current_time = localtime;
99 print STDERR "${current_time}: Computing scores \& optimal path .. \n";
100 run_algorithm();
1 0 1
102\# Save stuff
103 $current_time = localtime;
104 print STDERR "${current_time}: Saving in \${outfile_sum} \& ${outfile_fasta }...\n";
105 save_results ();
106
107 ####################################### Subs ############################################################
108## Run forward and Viterbi algorithms
109 sub run_algorithm
110 {
1 1 1 ~ \# ~ P r o c e s s ~ i n ~ c h u n k s ~ o f ~ n \_ l i m i t ~
112 while($n_proc<\$n_entries)

```
```

{
\# Read n_lim entries
read_fasta();
\# Get chromosomes from fasta
my @entries=keys %fasta_hash;
foreach my \$entry (@entries)
{
my \$n_iter=floor (( scalar @{ \$fasta_hash { $entry } })/$kmer_size ) - 1;
my \$pos=1;
\# Loop through the sequence
for(my \$i=0; $i<=$n_iter*\$kmer_size ; $i+=$kmer_size)
{
\# Get states in the iteration
my $seq_1=@{$fasta_hash {$entry }}[$i];
my \$seq_2=@{ $fasta_hash{$entry }}[$i+$kmer_size ];
for(my $j=1;$j<\$kmer_size; \$j++)
{
$seq_1=$seq_1.@{ $fasta_hash{$entry}}[$i+$j];
$seq_2=$seq_2.@{$fasta_hash{$entry}}[$i+$kmer_size+$j];
            }
            # Get all possible hidden states
            %score_hash_2=%template_hash ;
            # In case it's the first iteration, initialize alpha
            if($i eq 0)
{
\# Get all possible hidden states
%score_hash_1=%template_hash ;
\# Compute score for each hidden state
foreach my \$hidden_state (keys %score_hash_1)
{
my $pi_0=1/(4**$kmer_size);
my $emission=$emission_all_hash { $hidden_state}{$seq_1 };
my $score=$pi_ 0*\$emission;
\$score_hash_1 { $hidden_state}=$score ;
$gamma{$pos}{$hidden_state}{score}=$score;
$gamma{$pos}{\$hidden_state}{seq}="";
}
\$pos++;
}

```
```

    if(length($seq_2) eq $kmer_size)
    {
        # Consider every possible hidden state
        foreach my $hidden_state_2 (keys %score_hash_2)
            {
                my $emission=$emission_all_hash { $hidden_state_2 }{$seq_2 };
                    my $sum=0;
                    foreach my $hidden_state_1 (keys %score_hash_1)
            {
                    # Forward algorithm
                    my $row=$transition_hash{$hidden_state_1};
                    my $col=$transition_hash{$hidden_state_2 };
                    my $score=$score_hash_1{$hidden_state_1}*
                    $markov_matrix [ $row][$col]*$emission;
                    $sum+=$score;
                # Viterbi algorithm
                        my $score=$gamma{ $pos - 1}{$hidden_state_1}{score}*
                $markov_matrix [ $row ] [ $col]* $emission;
                    if($score > $gamma{$pos}{$hidden_state_2 }{score})
                    {
                $gamma{$pos}{$hidden_state_2}{score}=$score;
                    $gamma{ $pos}{$hidden_state_2}{seq}=$hidden_state_1;
                    }
            }
            $score_hash_2{$hidden_state_2}=$sum;
            }
            $pos++;
        }
        else
        {
            %score_hash_2=%score_hash_1;
        }
        %score_hash_1=%score_hash_2;
    }

## Traceback gamma

\$pos --;
foreach my $hidden_state (keys %{$gamma{$pos}})
{
    if($gamma{$pos}{$hidden_state}{score} > $viterbi_out {$entry}{\$pos}{

```
```

    score})
        {
            $viterbi_out{$entry}{$pos}{score}=$gamma{$pos}{$hidden_state}{
    score};
            $viterbi_out{$entry}{$pos}{seq}=$hidden_state;
                }
            }
            $pos--;
            for (my $i=$pos; $i>=1;$i--)
            {
            my $seq=$viterbi_out{$entry}{$i+1}{seq };
            my $seq-1=$gamma{$i+1}{$seq}{seq };
            $viterbi_out{$entry}{$i}{seq}=$seq_1;
            }
            %gamma=();
            # Total score P(X=x |HMM)
            my $total_score=0;
            foreach my $kmer (keys %score_hash_1)
            {
            $total_score+=$score_hash_1{$kmer };
            }
            # Store in results hash
            $results_hash{$entry}=$total_score;
            # CLean score hash
            %score_hash_1 =();
            %score_hash_2=();
            # Get rid of that entry
            delete $fasta_hash{$entry };
            $n_mem--;
            # Update progress
            $n_proc++;
            my $perc=$n_proc/$n_entries * 100;
            printf STDERR "\rCurrent progress: %.2f%", $perc;
        }
    }
    print STDERR "\n";
    225}
226
227\#\# Initialize stuff

```
```

228 sub initialize
229{
\# Get possible permutations of lenght k
my @nucleotides=('A','C','T','G');
my @permutations=variations_with_repetition(\@nucleotides,\$kmer_size);
\# Get all possible hidden states
foreach my \$string_1 (@permutations)
{
my $length=$kmer_size - 1;
my $sequence_1 = join(',, @{$string_1 }[0..\$length]);
$template_hash {$sequence_1}=1;
foreach my \$string_2 (@permutations)
{
my $length=$kmer_size - 1;
my $sequence_2 = join(',, @{$string_2 }[0..\$length]);
my \$emission=1;
for(my $j=0;$j<\$kmer_size; $j++)
            {
                    my @state_1=split(//,$sequence_1);
my @state_2=split(//,\$sequence_2);
my $row=$emission_hash{$state_1[$j]};
my $col=$emission_hash{$state_2[$j]};
$emission*=$emission_matrix[$row][$col];
}
$emission_all_hash {$sequence_1}{$sequence_2}=$emission;
}
}
255 }
256\#\# Save results
257 sub save_results
258 {
my \$sum=0;
my $n=0;
    # Open output file
    open(OUT,">$outfile_sum") or die "Can't open file '\${outfile_sum }' \$!";
\# Loop through all sequences
foreach my \$key (sort keys %results_hash)
{
my \$string = sprintf('%.3e', $results_hash{$key});

```
```

2 6 7
2 6 8
2 6 9
2 7 0
2 7 1
2 7 3
2 7 4
2 7 5
2 7 6
2 7 7
278
291 \#\# Read in fasta file
292 sub read_fasta
293 {
294 my \$entry;
295 \$n_mem=0;
296 while((my $line = <STDIN>) and ($n_mem<$n_limit))
{
    chomp($line);
if( \$line =~ />/)
{
$entry=substr($line,1); \# Get rid of leading ">" character
}
else
{
my @array = split //, \$line;

```
```

306 push @{$fasta_hash{$entry }},@array;
$n_mem++;
    }
    }
310 }
311
312## Print fasta
313 sub print_fasta
314 {
315 # Print output
3 1 6 ~ f o r e a c h ~ m y ~ \$ k e y ~ ( s o r t ~ k e y s ~ \% f a s t a \_ h a s h ) ,
317 {
318 my @entry=@{$fasta_hash{$key }};
319 print ">$key\n";
320 foreach my $nuc (@entry)
321 {
322 print "$nuc";
}
print "\n";
}
326 }
327\#\# Read in transition matrix
328 sub read_tm
329{
330 my @nucleotides=('A', 'C', 'T', 'G');
331 for(my $i=1;$i<=\$kmer_size; \$i++)
32 {
\$dim*=(scalar @nucleotides);
}

# Because we start with dim=0 in the loops

    $dim-=1;
        # Open file
        open($TM,"gunzip -c ${tm_file} | ")
            or die "Can't open file '${tm_file}' $!";
        my $i=0;
        # Loop through each row
        while(my $line=<$TM>)
        {
            if($i gt 0)
    ```
```

345
346
347
348
349
366
367\#\# Print transition matrix
368 sub print_markov_matrix
369 {

370 open(OUT, "|gzip -c>\${tm_outfile}") \# $$
is our process id
371 or die "Can't open file '${tm_outfile}' $!";
372 # Reverse hash
373 my %reverse_hash = reverse %transition_hash;
3 7 4 ~ \# ~ P r i n t ~ t o ~ O U T
375 print OUT "\t";
376 for (my $j=0; $j<=$dim; $j++)
377 {
378 my $key = $reverse_hash { $j };
379 print OUT "$key\t";
380 }
381 print OUT "\n";
382 for(my $i=0; $i<=$dim; $i++)
383 {
```
```
384 my $key = $reverse_hash{$i };
385 print OUT "$key\t";
386 for(my $j=0;$j<=$dim; $j++)
{
    printf OUT "%.5f\t", $markov_matrix [$i][$j];
        }
        print OUT "\n";
    }
    close OUT;
393 }
394
395## Read in emission probabilities
396 sub read_ep
397 {
    # Open file
    open($EP, "gunzip -c ${ep_file} | ")
            or die "Can't open file '${ep_file}' $!";
        my $i=0;
        # Loop through each row
        while(my $line=<$EP>)
        {
            if($i gt 0)
            {
                    chomp($line);
                    my @row=split(/\t/,$line);
                    my $j=0;
                    foreach my $col (@row)
            {
                    if($j eq 0)
                    {
                    $emission_hash{$col }=$i-1;
                    }
                    else
                    {
                        $emission_matrix [$i-1][$j-1]=$col;
                    }
                    $j++;
                    }
        }
```
```
4 2 3
4 2 4
4 2 5
426}
4 2 7
428 ## Print emission probabilities
429 sub print_emission_matrix
430 {
431 open(OUT, "|gzip -c > ${ep_outfile}") #
$$ is our process id

or die "Can't open file '\${ep_outfile}' \$!";

# Reverse hash

my %reverse_hash = reverse %emission_hash;
\# Print to OUT
print OUT "\t";
for (my \$j=0; \$j<=3; \$j++)
{
my \$key = $reverse_hash{$j};
print OUT "\$key\t";
}
print OUT "\n";
for (my \$i=0; \$i<=3; \$i++)
{
my \$key = $reverse_hash{$i };
print OUT "\$key\t";
for(my \$j=0; \$j<=3; \$j++)
{
printf OUT "%.5f\t", \$emission_matrix [ $i][$j]
}
print OUT "\n";
}
close OUT;
454}
4 5 5

```


\section*{A.1.4 Generation of random sequences}
```

1\#!/usr/bin/env bash
2 shopt -s extglob

```
```

4 abspath_script="\$(readlink -f -e "$0")"
5 script_absdir="$(dirname "$abspath_script")"
6 script_name="$(basename "$0".sh)"
7
8TEMP=$(getopt -o hl:r -l help,length:,rna -n "$script_name.sh" -_ "$@")
9
0 if [ $? -ne 0 ]
1 1 \text { then}
echo "Terminating ..." >&2
    exit -1
f i
1 5
16 eval set -- "$TEMP"
1 7
18\# Defaults
9 length=10
20 bases="ACTG"
2 1
22 while true
23 do
24 case "$1" in
25 -h|--help)
26 cat "$script_absdir"/\${script_name}_help.txt
27 exit
28 ;;
29 - l---length)
30 length="\$2"
31 shift 2
;;
-r|--rna)
bases="ACUG"
shift
;;
--)
shift
break
;;
*)

```
```

4 2
echo "$script_name.sh:Internal error!"
    exit -1
        ; ;
    esac
done
4 7
48# Run
49 cat /dev/urandom | tr -dc "$bases" | fold -w "$length" | head -n 1
1#!/ usr/bin/env bash
2 shopt -s extglob
3
4 abspath_script="$(readlink -f -e "$0")"
5 script_absdir="$(dirname "$abspath_script")"
6 script_name="$(basename "$0".sh)"
7
8 if [$\#-eq 0}
9 then
10 cat "$script_absdir/${script_name}_help.txt"
exit 1
12 fi
13
14 TEMP=\$(getopt -o hd:t:c:l:p: - l help, outdir:, threads:, chr:, length:, prefix: -n "
$script_name.sh" _- "$@")
1 5
16 if [$? -ne 0]
17 then
1 8 ~ e c h o ~ " T e r m i n a t i n g ~ . . . " ~ > \& 2 ~
19 exit -1
20 fi
2 1
22 eval set -- "$TEMP"
23
24 \# Defaults
25 outdir="\$PWD"
26 chr=5
27 length=2000
28 prefix="simulated_genome"
29 bases="ACTG"

```
```

threads=2
31

# Options

33 while true
34 do
case "$1" in
        -h|--help)
            cat "$script_absdir"/\${script_name}_help.txt
exit
;;
-d|--outdir)
outdir="\$2"
shift 2
;;
-t|--threads)
threads="\$2"
shift 2
;;
-c|--chr)
chr="\$2"
shift 2
;;
-l|--length)
length="\$2"
shift 2
;;
-p|-- prefix )
prefix="$2"
            shift 2
            ;;
        --)
            shift
            break
            ;;
        *)
            echo "$script_name.sh:Internal error!"
exit -1
;;
esac

```
```

69 done
70
71\# Start time
72 start_time="$(date +"%s%3N")"
73
74# Temp and output
75 outfile="${outdir}/${prefix }.fa"
76 tempfile="${outdir}/${prefix }"
77 export outfile
78 export tempfile
79 export length
80
81# Outdir
82 mkdir -p "$outdir"
83
84\# Generate a file for each chromosome
85 seq 1 "$chr" | xargs -I {} --max-proc "$threads" bash -c \
86 'echo ">random_seq_{}">> '${tempfile}_{}.tmp'&& random_sequence_generator.sh -
        l '$length',>> '${tempfile}_{}.tmp',
87
88# Concatenate all chromosomes and filter
89 seq 1 "$chr" | xargs -I {} --max-proc 1 bash -c \
90 'cat '${tempfile} _ {}.tmp'>> '$outfile','
91
92\# Remove temp file
93 rm -f ${tempfile}*tmp*
94
95# Time elapsed
96 end_time="$(date +"%s%3N")"
97 echo "Time elapsed: \$(( \$end_time - \$start_time )) ms"

```

\section*{A.1.5 Scoring results analysis}
```

1\#\#\# t-test
2\#\# 1-mers
3 read.table('random_seq_10_summary_1.txt'') ->r_10_1
4 read.table('random_seq_ 20_summary_1.txt ') ->r_20_1
5 read.table('random_seq_ 30_summary_1.txt'') ->r_30_1
6 read.table('random_seq_40_summary_1.txt ') ->r_40_1

```
```

7 read.table('random_seq_50_summary_1.txt') ->r_50_1
8 read.table('random_seq_60_summary_1.txt'')->r_60_1
9
10 read.table('true_seq_10_summary_1.txt')->t_10_1
11 read.table('true_seq_ 20_summary_1.txt') ->t_ 20_1
12 read.table('true_seq_30_summary_1.txt')->t_ 30_1
13 read.table('true_seq_40_summary_1.txt ') ->t_40_1
14 read.table('true_seq_50_summary_1.txt') ->t_ 50_1
15 read.table('true_seq_60_summary_1.txt') ->t_60_1
16
17 stats_ 10_1=t.test(r_10_ 1[1:250, 2], t_ 10_ 1 [1:250, 2], alternative='less')
18 stats_ 20_1=t.test(r_20_ 1 [1:250, 2], t_ 20_ 1 [1:250, 2], alternative='less')
19 stats_ 30_1=t.test(r_ < 30_ 1 [1:250, 2], t_ 30_ 1 [1:250, 2], alternative='less')
20 stats_40_1=t.test(r_40_1[1:250, 2],t_40_ 1 [1:250, 2], alternative='less')
21 stats_50_1=t.test(r_ 50_ 1 [1:250, 2],t_ 50_ 1 [1:250, 2], alternative='less')
22 stats_60_1=t.test(r_60_ 1[1:250, 2],t_60_ 1 [1:250, 2], alternative='less')
23
24 \#\# 3-mers
25 read.table('random_seq_10_summary_3.txt') -> r_10_3
26 read.table('random_seq_ 20_summary_ 3.txt') -> r_ 20_3
27 read.table('random_seq_ 30_summary_ 3.txt'')->r _ 30_3
28 read.table('random_seq_40_summary_ 3.txt') -> r_40_3
29 read.table('random_seq_ 50_summary_3.txt') -> r_ 50_3
30 read.table('random_seq_60_summary_3.txt'')->r_60_3
31
32 read.table('true_seq_10_summary_ 3.txt') ->t_10_3
33 read.table('true_seq_ 20_summary_3.txt')->t_20_3
34 read.table('true_seq_ 30_summary_ 3.txt') ->t_ 30_3
35 read.table('true_seq_40_summary_ 3.txt') ->t _ 40_3
36 read.table('true_seq_ 50_summary_ 3.txt') ->t_ _ 50_3
37 read.table('true_seq_60_summary_ 3.txt') ->t _ 60_3
38
39 stats _ 10_ 3=t.test(r_10_ 3[1:250, 2], t_ 10_ 3[1:250, 2], alternative='less')
40 stats_ 20_3=t.test(r_20_ 3[1:250, 2], t_ 20_ 3 [1:250, 2], alternative='less')
41 stats_ 30_ 3=t.test(r_ 30_ 3[1:250, 2], t_ 30_ 3[1:250, 2], alternative='less')
42 stats_ 40_ 3=t.test(r_40_ 3[1:250, 2], t_ 40_ 3[1:250, 2], alternative='less')
43 stats_ 50_3=t.test(r_ 50_ 3[1:250, 2], t_ 50_ 3[1:250, 2], alternative='less')
44 stats_60_3=t.test(r_60_ 3[1:250, 2], t_ 60_ 3[1:250, 2], alternative='less')
45

```
```

46 \#\# 5-mers
47 read.table('random_seq_10_summary_5.txt') ->r _ 10_5
48 read.table('random_seq_ 20_summary_5.txt'')->r_ 20_5
49 read.table('random_seq_30_summary_5.txt')->r_30_5
50 read.table('random_seq_ 40_summary_ 5.txt') ->> _ 40_5
51 read.table('random_seq_ 50_summary_5.txt') -> r_ 50_5
52 read.table('random_seq_60_summary_5.txt') -> r_6 _ % _ 5
53
54 read.table('true_seq_10_summary_ 5.txt') ->t_10_5
55 read.table('true_seq_ 20_summary_5.txt')->t_20_5
56 read.table('true_seq_ 30_summary_ 5.txt') ->t_ 30_5
57 read.table('true_seq_40_summary_ 5.txt') ->t_40_5
58 read.table('true_seq_ 50_summary_ 5.txt') ->t_ 50_5
59 read.table('true_seq_60_summary_ 5.txt')->t_60_5
6 0
61 stats_ 10_5=t.test(r_10_ 5 [1:250, 2], t_ 10_ 5 [1:250, 2], alternative='less')
62 stats _ 20_5=t.test (r_20_ 5 [1:250, 2], t_ 20_ 5 [1:250, 2], alternative='less')
63 stats_ 30_ 5 = t.test (r_30_ 5 [1:250, 2], t_ 30_ 5 [1:250, 2], alternative='less')
64 stats 40_5=t.test(r_40_ 5 [1:250, 2],t_ 40_ 5 [ 1:250, 2], alternative='less')
65 stats _ 50_ 5=t.test(r_50_ 5 [1:250, 2], t_ 50_ 5 [1:250, 2], alternative='less')
66 stats_60_5=t.test(r_60_ 5 [1:250, 2], t_ 60_ 5 [ 1:250, 2], alternative='less')
6 7
68 \#\#\# Summary
69 out=as.data.frame(cbind(c(stats_10_1$p.value, stats_20_1$p.value,
stats_ 30_1\$p.value, stats_40_ 1 $p.value,
    stats_50_1$p.value, stats_60_1$p.value),
    c(stats _ 10_ 3$p.value, stats _ 20_ 3$p.value,
    stats_ 30_ 3$p.value, stats_ 40_ 3$p.value,
    stats_50_ 3$p.value, stats_60_ 3$p.value),
    c(stats_10_ 5$p.value, stats_ 20_ 5$p.value,
    stats_ 30_ 5$p.value, stats_40_ 5 $p.value,
    stats_50_ 5$p.value, stats_60_ 5\$p.value)))
78 colnames (out)=c('1-mers', '3-mers ', '5-mers')
79 write.table(out, file='pvalues.txt', quote=F, sep='\t',row.names=F)
80
81 \#\#\# Plots
82 read.table('scores_1.txt') ->s 1
83 read.table('scores_3.txt')->s3
84 read.table('scores_5.txt'))->s5

```
```

85 colnames(s1)=c('random','true')
86 colnames(s3)=c('random','true')
87 colnames( s5 ) =c ('random','true')
88
89x=seq (10,100,10)
90
91\# Ratios
92 s1_ratios=log(s1[, 2]/s1[, 1])
93 s3_ratios=log(s3[, 2]/s3[, 1])
94 s5_ratios=log(s5[, 2]/s5[, 1])
95
96 \# Plot
97 pdf("scoring.pdf")
98 par (mar=c (5, 5, 1, 1))
99 plot(x, s5_ratios , pch=16,type="b", col="blue", xlab="Sequences length",
100 ylab=expression(log(s[true]/s[random ])))
101 lines(x, s3_ratios, pch=15, type="b", col="red", xlab=" ", ylab=" ")
102 lines(x,s1_ratios, pch=14,type="b", col="green", xlab="",ylab="")
103 legend("topleft", legend=c("5-mers","3-mers","1-mers"),
104 text.col=c("blue","red","green"),
105 pch=c (16,15,14),
106 col=c("blue","red","green"))
107
108 dev.off()

```

\section*{A.1. 6 SEQC results analysis}
```

1\#!/usr/bin/env Rscript
2
3\# Read in seqc scores
4 read.table('seqc_scores.txt',sep=" ")->seqc
5
6\# Random scores for sequence length 100
7 random=c (5.442e-61,2.21893034881624e-60,2.254e-61)
8
9 ratios_1=cbind(log(seqc [, 1]/random [1]), rep (1, 6))
10 ratios _ 3=cbind(log(seqc [, 2]/random [2]),rep (3,6))
11 ratios_5=cbind( (log (seqc [, 3]/random [3]),rep (5,6))
12

```
```

13 seqc_df=as.data.frame(rbind(ratios_1, ratios_ 3, ratios_ 5))
14 colnames(seqc_df)=c("Ratio","kmer")
15 \# Plot
16 pdf("seqc _ scoring - 1.pdf")
17 par (mar=c (5, 5, 1, 1))
18 boxplot(Ratio ~ kmer, data=seqc_df,lwd = 2, col='blue', ylim=c (0, 30),
19 xlab="k-mer size",ylab= expression(log(s[SEQC]/s[random])))
20 dev.off()

```

\section*{A.1.7 Viterbi results analysis}
```

1\# Before
2 read.table('before/random_seq_10_summary_1.txt')->b_10_1
3 read.table('before/random_seq_10_summary_3.txt') ->b_10_ 3
4 read.table('before/random_seq_10_summary_5.txt ') ->b_ 10_5
5 read.table('before/random_seq_20_summary_1.txt') ->b_20_1
6 read.table('before/random_seq_20_summary_3.txt') ->b_ 20_3
7 read.table('before/random_seq_20_summary_5.txt') ->b_ 20_5
8 read.table('before/random_seq_ 30_summary_1.txt ')->b_ 30_1
9 read.table('before/random_seq_30_summary_3.txt') ->b_30_3
10 read.table('before/random_seq_ 30_summary_5.txt') ->b_ _ 30_5
11 read.table(' before/random_seq_ 40_summary_1.txt') ->b_40_1
12 read.table('before/random_seq_40_summary_3.txt') ->b_40_3
13 read.table('before/random_seq_40_summary_ 5.txt') ->b_40_5
14 read.table('before/random_seq_50_summary_1.txt') ->b_50_1
15 read.table('before/random_seq_50_summary_3.txt') ->b _ 50_3
16 read.table('before/random_seq_50_summary_5.txt')->b_50_5
17 read.table('before/random_seq_60_summary _ 1.txt'')>>b_60_1
18 read.table('before/random_seq_60_summary_3.txt ') ->b_60_3
19 read.table('before/random_seq_60_summary _ 5.txt'')->b_60_5
20
21\# After
22 read.table(', after/random_seq_10_corrected__1_summary_1.txt'')->a_10_1
23 read.table(' after/random_seq_10_corrected_3_summary_ 3.txt'')->a_10_3
24 read.table(', after/random_seq_10_corrected_5_summary_ 5.txt'') ->a_10_5
25 read.table(' after/random_seq_ 20_corrected_1_summary_1.txt'')->a_20_1
26 read.table(' after/random_seq_ 20_corrected_ 3_summary_ 3.txt'')->a_20_3
27 read.table(' after/random_seq_ 20_corrected_ 5_summary_ 5.txt'') ->a_20_5
28 read.table(' after/random_seq_ 30_corrected_1_summary_1.txt') ->a_30_1

```
```

29 read.table(', after/random_seq_ 30_corrected_3_summary_ 3.txt'') ->a_30_ 3
30 read.table(' after/random_seq_ 30_corrected_ 5_summary_5.txt') ->a_30_5
31 read.table(' after/random_seq_ 40_corrected__1_summary_1.txt') ->a_40_1
32 read.table(' after/random_seq_40_corrected_ 3_summary_ 3.txt') ->a_40_ 3
33 read.table(' after/random_seq_ 40_corrected_ 5_summary_5.txt' ) ->a_40_5

```

```

35 read.table(' after/random_seq_50_corrected_ 3_summary_ 3.txt' ) ->a_50_ 3
36 read.table(' after/random_seq_50_corrected _ 5_summary_5.txt') ->a_50_5
37 read.table(' after/random_seq_60_corrected__ _ summary_1.txt'')->a_60_1
38 read.table(' after/random_seq_60_corrected _ 3_summary_ 3.txt ' ) ->a_60_3
39 read.table(' after/random_seq_ 60_corrected_ 5_summary_5.txt'')->a_60_5
40
41\# Difference means
42 diff_1=c(a_10_1[nrow (a_10_1), 1] - b_ 10_1[nrow (b_10_1), 1],
a_20_1[nrow (a_20_1),1] - b_ 20_1[nrow (b_20_1), 1],
a_30_1[nrow (a_30_1),1] - b_ 30_1[nrow (b_30_1), 1],
a_40_1[nrow (a_40_1),1] - b_40_1[nrow (b_40_1), 1],
a_50_1[nrow (a_50_1),1] - b_ 50_1[nrow (b_50_1), 1],
a_60_1[nrow (a_60_1),1] - b_60_1[nrow (b_60_1),1])
8diff _ 3=c (a_10-3[nrow (a_10_ 3),1] - b_ 10_ 3[nrow (b_ 10_ 3), 1],
a_20_3[nrow (a_20_3),1] - b_ 20_3[nrow (b_20_3),1],
a_30_3[nrow (a_30_3),1] - b_ 30_3[nrow (b_30_3),1],
a_40_3[nrow (a_40_3),1] - b_40_3[nrow (b_40_3), 1],
a_50_3[nrow (a_50_3),1] - b_ 50_ 3[nrow (b_50_3), 1],
a_60_3[nrow (a_60_3),1] - b_60_3[nrow (b_60_3),1])
4 diff _ 5=c (a_10_5[nrow (a_10_5), 1] - b_ 10_ 5[nrow (b_ 10_ 5), 1],
a_20_5[nrow (a_20_5),1] - b _ 20_ 5[nrow (b_ 20_5),1],
a _ 30_5 [nrow (a_30_5),1] - b_ 30_5[nrow (b_30_5),1],
a_40_5[nrow (a_40_5),1] - b_40_ 5[nrow (b_40_5),1],
a _ 50_5[nrow (a_50_5),1] - b_ 50_ 5[nrow (b_50_5), 1],
a_60_5[nrow (a_60_5),1] - b_60_ 5 [nrow (b_60_5),1])
60\# Ratios means
61 ratio_1=c(a_10_1[nrow (a_10_1), 1]/b_10_1[nrow(b_10_1), 1],
62 a_20_1[nrow (a_20_1), 1]/b_ 20_1[nrow(b_20_1), 1],
a_30_1[nrow (a_30_1), 1]/b_30_1[nrow (b_30_1), 1],
64 a_40_1[nrow (a_40_1), 1]/b_40_1[nrow (b_40_1), 1],
65 a_50_1[nrow (a_50_1),1]/b_50_1[nrow (b_50_1), 1],
66 a_60_1[nrow (a_60_1),1]/b_60_1[nrow (b_60_1), 1])
67 ratio_3=c(a_10_3[nrow (a_10_3), 1]/b_10_ 3[nrow(b_10_3), 1],

```
```

68 a_20_3[nrow (a_20_3),1]/b_20_3[nrow (b_20_3), 1],
69 a_30_3[nrow (a_30_3),1]/b_30_3[nrow (b_30_3), 1],
70 a_40_3[nrow (a_40_3),1]/b_40_3[nrow (b_40_3), 1],
71 a_50_3[nrow (a_50_3),1]/b_50_3[nrow (b_50_3), 1],
72 a_60_3[nrow (a_60_3),1]/b_60_3[nrow (b_60_3), 1])
73 ratio_5=c(a_10_5[nrow (a_10_5), 1]/b_10_5[nrow (b_10_5), 1],
74 a_20_5[nrow (a_20_5),1]/b_20_5[nrow (b_20_5),1],
75 a_30_5[nrow (a_30_5),1]/b_30_5[nrow(b_30_5), 1],
76 a_40_5[nrow (a_40_5),1]/b_40_5[nrow (b_40_5),1],
77 a_50_5[nrow (a_50_5),1]/b_50_5[nrow (b_50_5), 1],
78 a_60_5[nrow (a_60_5),1]/b_60_5[nrow (b_60_5), 1])
79\# Plot
80 pdf("viterbi.pdf")
81 x=seq (10,60,10)
82 par (mar=c(5, 5, 1, 1))
83 plot(x, ratio_5, pch=16,type="b",col="blue", xlab="Sequences length",
84 ylab=expression (s[corrected]/s[random]))
85 lines(x, ratio_3, pch= 15,type="b", col="red", xlab="", ylab="")
86 lines(x, ratio_1, pch=14,type="b", col="green", xlab=" ", ylab="")
87 legend("topleft", legend=c("5-mers","3-mers","1-mers"),
88 text.col=c("blue","red","green"),pch=c(16,15,14), col=c("blue"
,"red","green"))
89
90 dev.off()

```
```

