

INVENTORY MANAGEMENT OF PERISHABLE GOODS UNDER DEMAND
VARIABILITY

A Thesis

by

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Submitted to the Office of Graduate Studies of
Texas A&M University
in partial fulfillment of the requirements for the degree of

MASTER OF SCIENCE

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August 2013

Major Subject: Agricultural Economics

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ABSTRACT

Perishability of fluid milk presents challenges for efficient distribution and limits market expansion for dairy when temperatures cannot be fully controlled during transportation. This research develops a modeling framework that integrates food science and economic parameters examining the impacts of different demand specifications on the cost minimization and profit maximization problem of fluid milk. The square root model from the food science literature is used to estimate the shelf-life of fluid milk at retail level. The shelf-life parameter is then used as input to the fixed-order quantity inventory model from the business economic literature. Additionally two demand specifications, the own-price elasticity and the negative binomial distribution, are used to calculate the total cost of managing inventory and resulting profit.

Modeling results confirm that fluctuations in temperature and time dramatically increase the percentage of perishability cost and decrease profitability. Specifications of retail demand directly impact outcomes of the inventory model. Under the demand model based on price elasticity, simulated total costs are lower and profits are higher than under the negative binomial specification. The negative binomial distribution approach provides a simulated outcome where sales losses are minimized and customer satisfaction is higher. This thesis proposes, presents and uses a working model that can be extended and directly applied for fluid milk as well as other perishable food supplies.

DEDICATION

I dedicate this work to my parents and younger brother for their constant source of strength, wisdom, love, support, prayers and encouragement they provided me.

ACKNOWLEDGEMENTS

I am ever grateful to the Lord, to whom I owe my very existence. “By His grace I am what I am, and His grace toward me was not in vain” (1 Corinthians 15:10).

I would like to express my deep appreciation and sincere gratitude to Dr. Victoria Salin, who has the attitude and the substance of genius, for her thoughtful guidance pursuing my degree. Without her persistent help, critical comments, and corrections this thesis would not have been possible.

A very special thanks goes out to Dr. David Anderson, and Dr. Gary Acuff, my committee members, for their efforts, comments, and inspiring conversations that help me to accomplish my work. I would like to express my sincere thanks to Dr. James Richardson for his help and thoughtful guidance in modeling.

My deep sense of gratitude to Dr. Abdullah A. Mudhi, for his tremendous support and encouragement in pursuing my best interest. I also thank my friends Eskander Hussein, Dhurgham Salman, Ali Slah , Awf Abd Al-Rhman, Kais Tami, Osama Khalil, Haydar Al-Byati and all others for providing support and friendship. I would like to thank specially Shayma’a Ali for being supportive throughout all the times.

I am thankful to Dr. Ma’ad Rawendoozi, Eric Brenner, Dr. Edwin Price, Jeff Raymond, Morgan Sowell, and all other people at the Norman Borlaug Institute for International Agriculture for the financial support and their huge efforts in directing and managing my scholarship.

Finally, I will ever be thankful and grateful to Thomas Rodgers and his family for the support, love, encouragement, prayers, and advice. It is a blessing from God to know you. I also would like to thank Wisam Shamshoon, Catherine McIntyre, Roger Fay, Jim Miller, Aron Collins, and all other friends for their friendship and support.

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CHAPTER I

INTRODUCTION

Milk is an essential component of U.S. consumer diets. It consists of essential nutrients, proteins, minerals, and vitamins necessary for healthier bodies, especially for children. The shelf life of milk is subject to many factors, with temperature being the most important. Refrigeration is used to maintain quality and prolong shelf life. Temperature control helps maintain milk quality and safety during storage and distribution. Abusing temperature by allowing it to rise during loading, transit, and unloading phases may cause quality loss or spoilage before milk reaches the final destination.

Gustavsson et al (2011) of the Swedish Institute for Food and Biotechnology (SIK), on a request from the Food and Agriculture Organization of the United Nations (FAO), studied losses that occur throughout the food supply chain in developed and developing countries. The study indicates that about one-third of food produced is wasted, globally. In developing countries, contributing factors in food waste are: the lack of infrastructure, poor storage and processing facilities, and outdated marketing systems. On the other hand, in developed countries consumers' attitude, the wide range of brands, and high appearance quality standards at the retail level are the main reasons that about 40% of food is wasted. Hence, finding an effective way to estimate consumer's demand and utilize demand information in efficient inventory management will reduce the food waste.

In fact, the importance of retail inventory management has increased because market competition has become more intense. Retailers seek to provide a high level of customer service with the lowest cost. Adopting appropriate inventory policies and choosing an accurate method to estimate customer's demand and service level is significantly important. An effective inventory policy promotes the level of customer service, reduces inventory costs, and reduces food waste.

This thesis examines the impacts of different demand specifications on the cost minimization and profit maximization problem of fluid milk using an example of fluid milk sold at retail store. The fixed-order quantity model is the basis for this research. Retail demand is modeled using two methods: (1) negative binomial distribution model, and (2) own price elasticity of fluid milk model. A dynamic model from the food science literature is used to represent the relationship of quality decay to temperature of fluid milk, incorporating time and temperature history information, into the inventory problem.

This thesis is organized as follows. Chapter II contains a literature review of demand at the retailer level, food safety, and inventory policies. Chapter III describes an inventory model linking costs of the fluid milk to a perishability model from the food science literature. Chapter IV presents and summarizes the results. Finally, chapter V contains research conclusions.

CHAPTER II

LITERATURE REVIEW

Inventory

Inventory is a short term investment in goods that are stored in order to become part of a business outcome. Inventory is observed in almost any form of business entity in the balance sheet, under current assets. Business management systems help maintain inventory at a level which balances the cost of holding unsold goods against the chance that goods are not available when customers wish to buy.

Researchers have studied different inventory models to determine factors managers should consider to maximize profits or minimize costs at the retail level. Inventory models use different analytical and mathematical approaches.

Arrow, Harris, and Marschak (1951) developed inventory models for finished products to determine the optimal level of inventory which maximizes profit. The maximized profit depends on controlled conditions, like rules and strategies, and noncontrolled conditions such as the rate of demand of the finished goods, inventory replenishment, prices, and the relation to size of an order and gross revenue obtained.

Arrow, Harris, and Marschak developed three models to determine the optimal inventory policy for maximizing profit. The first was a simple model where demand and other factors are assumed to be known with certainty; the second was a static model with uncertainty; and the third was a dynamic model with uncertainty.

From this seminal study, several important components of inventory costs are obtained. Inventory costs include purchase costs, handling costs, carrying costs, and a “depletion penalty” which represents the cost of unsatisfied demand. The depletion penalty is the most important cost component and could be very high in a competitive retail sector. The size of the depletion penalty is the reason why managers try to optimize stock levels.

Inventory models are dynamic in nature. The choice variables reflect quantity of stock(s) and time of orders. An order of size S^* is made at a point where the inventory level reaches to zero. Orders are assumed to be filled immediately over intervals (θ_i) . In other words, the stock levels change from maximum $S^* = \alpha\theta^*$ to minimum $y = 0$ at each period as illustrated in Figure (1) below. Arrow, Harris, and Marschak concluded that the optimal stock(s) and best reordering point are a function of the demand distribution, the cost of making an order, and the penalty of stock depletion.

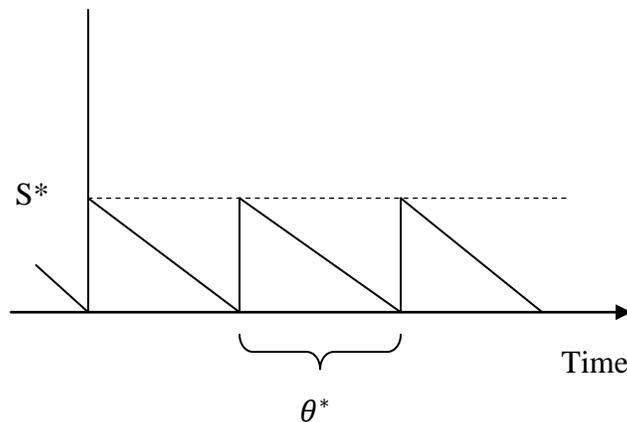


Figure 1. Basic Inventory Model Stock Level S^* , Interval Time θ_i and the Stock Level before y_i and after s_i Replenishment Respectively.

Chase, Aquilano, and Jacobs (1998) present two standard inventory models: the fixed-order quantity model, and fixed-time period model. Both are designed to identify the minimum cost of holding inventory while still meeting demand. The Fixed-Order Quantity Model is also known as the Economic Order Quantity “EOQ” model. In an EOC model the total annual cost of holding an inventory consists of the annual costs of purchasing, ordering, and holding:

$$TC = DC + DQ^{-1}S_c + 0.5 QH \quad (2.1)$$

where TC is total annual cost, D is annual demand, C is cost per unit, Q is quantity to be ordered, S_c is setup cost or cost of placing an order, and H is annual holding and storage cost per unit of average inventory. On the right hand side of the equation, $(D \cdot C)$ is the annual purchase cost, $((D/Q) S_c)$ is the annual ordering cost, and $((Q/2) \cdot H)$ is the annual holding cost.

From equation 2.1 the Q_{opt} that minimizes the total cost can be derived. First, the first order condition, that is, the derivative of the total cost equation with respect to quantity Q is derived (Equation 2.2) and set equal to zero (Equation 2.3 and 2.4). The total cost is minimized at the point where the slope of is zero.

$$TC = DC + \frac{D}{Q}S_c + \frac{Q}{2}H \quad (2.2)$$

$$\frac{dTC}{dQ} = 0 + \left(\frac{-D}{Q^2} S_c\right) + \frac{H}{2} = 0 \quad (2.3)$$

$$Q_{opt} = \sqrt{\frac{2DS_c}{H}} \quad (2.4)$$

The first derivative for the Q_{opt} with respect to the annual demand is:

$$\frac{\partial Q_{opt}}{\partial D} = \frac{1}{2\sqrt{\frac{2S_c}{HD}}} \quad (2.5)$$

As the annual demand changes the optimal level of stock changes in the same direction. In other words, there is a direct relationship between the optimal level of stock Q_{opt} and annual demand. For example, if the annual demand is 100 units, ordering cost is \$2.5 per order, and holding cost is \$0.5 per unit; the optimal level of stock will be 31.62 units. If the annual demand increases from 100 units to 150 units, the optimal level of inventory stock increases 38.73 units.

To verify that the total cost of inventory is minimized, the second derivative should be non-negative (Equation 2.6).

$$\frac{d^2TC}{dQ^2} = \frac{2DS_c}{Q^3} \quad (2.6)$$

A manager who adopts this model has to place an order of size “Q” when the inventory is exhausted. Under this simple situation, there is a chance of a stock out, or running out of inventory on the shelf. Thus the EOQ model includes a predetermined reorder point R, which is the stock level that triggers re-ordering. The order will be received at the end of the lead time, which the EOQ assumes constant. This reorder process, illustrated in Figure 2, may take place at any time, depending on the demand for a product.

In this research, the EOQ model is used because of the model's perpetual feature. The perpetual re-order feature of the EOQ model updates the stocked level fluid milk frequently, and provides quicker response to the potential stock-out of fluid milk. Hence, the model reduces the loss in sales due to stock-out and provides better customer service.

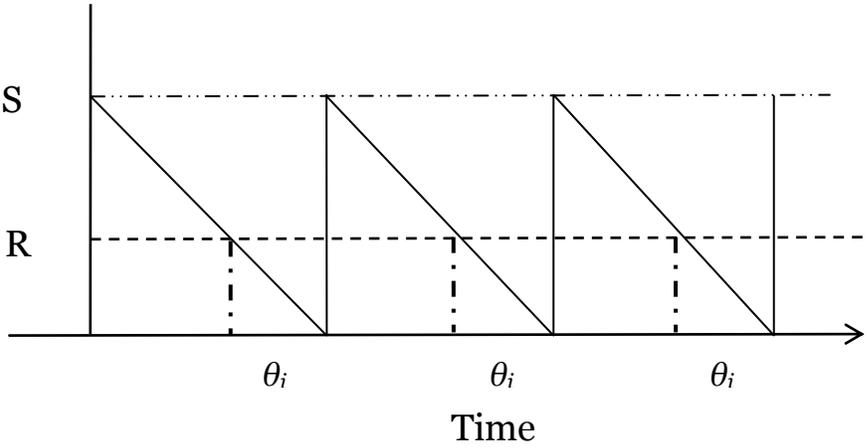


Figure 2. Basic Fixed-Order Quantity Model Lead Time θ_i , the Stock Level S , and Reorder Point R.

Solution for s and S vary when alternative specifications of demand are used. Ehrhardt introduced uncertainty about demand rates into inventory modeling in 1979. The main difference between the EOQ model and the (s, S) inventory policy is that the EOQ model assumes constant and uniform demand for a product throughout any given period, whereas the (s,S) inventory policy utilizes the mean and variance of demand.

Ehrhardt presents the Power Approximation as an alternative approximation for computing inventory policies when the mean and variance of demand are accurately specified. The Power Approximation drops the normality specification. The (s,S) inventory policy is optimized when the inventory on hand plus the amount on order y is less than, or equal to, the reorder level s . At s , an order of size $S-y$ is placed, where S is the maximum level of inventory.

Agrawal and Smith (1996) compared the Poisson, Normal, and Negative Binomial distributions, which are used in estimating demand at the retail level, in order to calculate the cost of stock-out correctly. The Poisson and Normal distributions are the most common methods used to estimate demand. The normal distribution is preferred when demand is relatively large, whereas the Poisson distribution is used when we have low demand on items.

Agrawal and Smith (1996) presented both theoretical and empirical evidence that indicate the negative binomial distribution is the appropriate approach for estimating demand at the retail level. The authors developed a parameter estimation methodology that compensates for the effects of unobservable lost sales, which are prevalent in retailing, but are omitted from most demand models.

Sales and inventory data were used in Agrawal and Smith empirical study to compare the Negative Binomial, Poisson and Normal distributions. Among the three distributions, the Negative Binomial distribution describes retail demand much better than the Normal and Poisson distributions. The Poisson distribution understates the

demand in the right tail of the distribution, and the mode of the Normal distribution is shifted to the right, relative to the Negative Binomial distribution.

Using the p-value of the Chi-Square statistic to compare the fits of the three distributions to the actual frequency distribution of the data, Agrawal and Smith found that the negative binomial is sufficient to provide at least a 90% customer service level.

Agrawal and Smith concluded that the Negative Binomial distribution is the best approach to forecast the demand at the retail level. A negative binomial distribution provides a single discrete distribution for all stock keeping units, removing the need for the Normal and Poisson distributions. In addition, the Negative Binomial distribution is an appropriate approach for representing high variability in demand at the retail level. The negative binomial is defined over non-negative integer values for the quantity demanded, which is appropriate for the situation modeled in this research.

Demand

From the previous inventory models that we presented above, the literature demonstrates that maintaining an additional amount of a product is the best way to avoid stock-out and provide satisfactory customer service, as defined as not running out of stock on the shelf. As the inventory stock criteria rely on the tail of the distribution of demand, it is important to use the appropriate distribution to represent demand.

Many factors influence the willingness of consumers to buy a particular product, including price, income, preferences, taxes, expectations, substitutes, and so on. However, it is important to understand that demand is not just about what consumers

want to buy, but it is also about what consumers are able to buy. We can conclude that the price of a product is the main factor influencing consumer quantity demanded (Hubbard and O'Brien 2007).

The inverse relationship between the price of a product and the quantity of the product demanded is known as the Law of Demand. In other words, a fall in product price will increase the quantity demanded of the product, where other factors are assumed held constant. The graphical representation (Figure 3) of the Law of Demand is called the demand curve which has a downward slope (Nicholson and Snyder 2010).

The consumer's demand function, which is also known as the Marshallian demand function, is a mathematical construct to aid in understanding how a consumer responds to changes in prices and income. This function is derived from maximizing the utility function subject to a budget and income constraint (Nicholson and Snyder 2010).

$$Q = f(P_Q, P_Y, I) \quad (2.7)$$

Equation 2.7 is the demand function where Q is the quantity demanded, P_Q is the price of good Q , P_Y is the price of good Y , and I is the consumer's income.

A business manager needs to know how a change in the prices of their products will affect the quantity that consumers are willing to buy. But, the most important question is how much will the quantity demanded change as a result of price increase or decrease? Economists use the concept of elasticity to answer the question of change in quantity demanded. The own-price elasticity of demand measures the responsiveness of

the quantity demanded to a change in its price (Hubbard and O'Brien 2007). The price elasticity of demand is defined as:

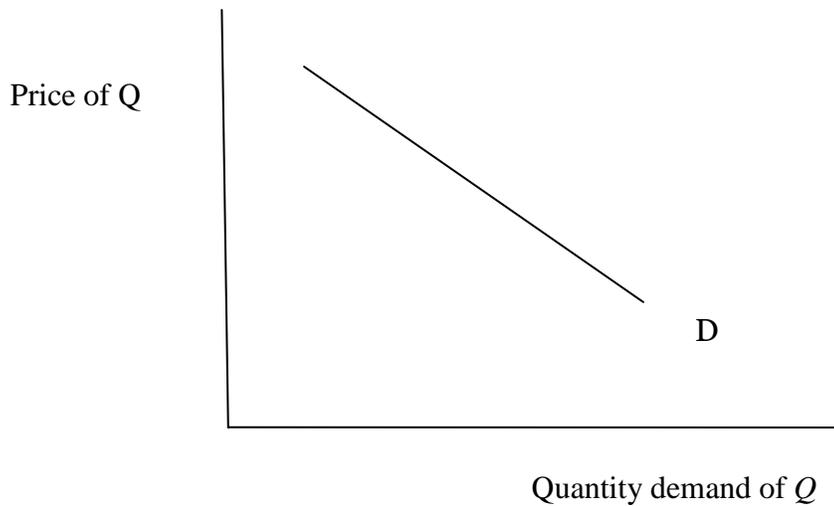


Figure 3. Demand Curve.

$$\eta = \frac{\partial Q}{\partial P_Q} * \frac{P_Q}{Q} , \quad (2.8)$$

where η is the own-price elasticity of demand, ∂Q is the change in the quantity demanded, ∂P_Q is the change in the price, p_Q is the price of product, and Q is the quantity demanded.

If the percentage change in the quantity demanded is greater than the percentage change in the price, then the elasticity of demand is greater than 1 in absolute value,

indicating that demand is own-price elastic. In contrast, if the percent change in the quantity demanded is smaller than the percent change in price, then the elasticity of demand is less than 1 in absolute value which is known as own-price inelastic demand. Finally, if the percent change in quantity demanded is equal to the percent change in the price, then elasticity of demand equals 1; which implies that demand is unit-elastic (Nicholson and Snyder 2010, and Hubbard and O'Brien 2007).

Treatment of Demand in Inventory Management Model

The specification of the demand rate in the inventory model is the focus of this thesis. At the retail level, shopping patterns vary within a week and the demand for some products is subject to seasonal variation. Therefore, a stochastic demand model is used to compare with a constant demand rate model. From the field of statistics, various representations of random demand models are available. One can simply draw from a particular distribution range, or one can employ a statistical distribution. The primary focus of this research is to utilize a price-dependent representation of demand in this inventory simulation. Under that structure, we can represent the decision of a manager to reduce prices to clear older inventory.

Food Science

The effect of temperature on milk spoilage in cold-chain challenged areas further complicates the inventory problem of the milk case manager. Several approaches in the food science literature have been tested to model milk decay. These food science approaches demonstrate the crucial role of temperature in predicting the shelf- life of a product. As there is a positive and nonlinear relationship between microbial level and temperature, it is expected that spoilage occurs in a short time period when the product is held at higher temperature. The literature referring specifically to fluid milk is described here.

The expiration date on pasteurized milk is an assurance tool for quality and safety of milk. However, this assurance is valid only if pasteurized milk is held at a constant temperature throughout all supply chain phases. Temperature abuse will stimulate the growth of spoilage bacteria (Grisius et al 1987). In the food science literature studies milk is experimentally contaminated more than 2×10^7 CFU/mL¹ (Fu et al 1991).

Grisius et al (1987) concluded that there is no single indicator that can account for the differences in the quality and type of contaminants in the milk, nor describe the complexity of microbial growth patterns. However, the full-history of temperature over time is suitable for estimating the growth of the total microbial population in pasteurized milk.

¹ CFU/mL: colony-forming units per milliliter

Fu, Taoukis, and Labuza (1991) studied the growth of a selected psychotropic spoilage microorganism in milk to determine the relationship between temperature and growth rate of microorganisms. They contrasted the Arrhenius Theory in comparison to the Square Root Model as a representation of microbial growth. The Arrhenius equation describes the effect of temperature on the growth of microorganisms, using the equation:

$$\ln (N/N_0) = k t = k_0 \exp [E_A / (RT)]t, \quad (2.9)$$

where N is the number of microorganisms after time t , N_0 is the initial population of microorganisms, k_0 is the collision factor, T is the absolute temperature, R is the universal gas constant (8.314 Joules per mole), and E_A (J/mol) is the activation energy.

The Square Root Model is an alternative representative of microorganism growth. It indicates that the square root of the growth rate of a microorganism is a function of temperature.

$$\sqrt{K} = b(T - T_{min}), \quad (2.10)$$

where K is the specific growth rate, b is the slope of the regression line below the optimum temperature; T is the temperature, and T_{min} is the point where the square root plot intersects the temperature axis at $\sqrt{K} = 0$.

Fu, Taoukis, and Labuza concluded that both Arrhenius and Square Root models fit the lag phase and the growth rate data of microbes at constant temperature in fluid milk. However, the square root equation was a better model of deterioration under the situation of a temporary loss in control of temperature. For this reason, this thesis will use the square root model in modeling the shelf-life of fluid milk under temperature fluctuation in the inventory problem.

CHAPTER III

METHODOLOGY

The total cost of the fluid milk inventory at the retail level is based on two models. The first is a dynamic model from the food science literature, the Square Root Model, which represents the relationship of quality decay to temperature. It is needed to estimate shelf life of the fluid milk. Given shelf life information, the inventory cost is estimated with the Fixed-Order Quantity model using a simulation approach. Each model will be discussed more in the rest of this section.

Perishability Model

We use a dynamic model to represent the relationship of quality decay to temperature for fluid milk. The quality deterioration of the milk is measured by the population of the microbe *Pseudomonas fragi*. The square root model parameters from Fu, Taoukis, and Labuza (1991) are used.

Growth rates of microbial populations have two phases, a lag phase and a log (or exponential growth) phase. The lag phase immediately follows pasteurization, when initial microbe populations are low and growth is relatively slow. Later, the microbe population grows at a faster rate. The rates of growth in either phase depend on temperature. Thus, there are different parameters to be used in determining shelf-life of

fluid milk for a given temperature. They are represented in the following model of microbial population growth:

$$A = A_0 \exp \left(\sum_{i=0}^j (k_i * t_i) \right) \quad (3.1)$$

where A is the microbe population in CFU/mL, k_i is the growth rate constant for period i , A_0 is the initial level of contamination, and t_i is the duration of time period i (Labuza 1979 equation 24). The product is sellable as long as $A < 3 \times 10^7$ CFU/milliliter (Chandler 1990).

Rate constants (k_i) are a function of temperature. In this research, temperatures are represented at three stages in the supply chain with normal probability distributions. The stages are processing, transportation to retail, and retail storage.

Model Assumptions

The initial levels of contamination, temperature, and time are the input variables that are used to estimate the shelf-life of the fluid milk at the retail level. The initial level of contamination A_0 is drawn from a uniformly distributed (0.01 – 10) CFU/milliliter.

The first temperature distribution T_1 is for processing and initial storage where temperature is closely controlled. The temperature distribution is specified as normally distributed $N(3, 1)^\circ\text{C}$. The kinetic parameter for phase 1, k_1 , is a function of the random temperature and is based on the lag phase of microbial growth (Equation 3.2).

$$k_1 = [0.0172 (T+7.32)]^2 \quad (3.2)$$

After processing, the distribution phase is divided into two components: (1) a 1.5 – 8 hours period of warmer temperature (for loading or trucking, for example) and (2) a refrigerated storage phase at retail. Temperature during the brief random temperature abuse phase is specified as N (15, 4). Under refrigeration at the retail store, the temperature distribution is also random but on average is an appropriate temperature, drawn from a density N (5, 2). The growth rate of microorganisms in both of these phases are calculated using the following equation.

$$K_{2 \& 3} = [0.0306 (T+7.85)]^2 \quad (3.3)$$

The kinetic decay parameters for the phases of the distribution channel are obtained from the formula for the log (exponential) phase of microbial growth found in Fu, Taoukis, and Labuza (for the square root model, $k_i = (0.0306(T+7.85))^2$ where T is temperature in degrees Celsius).

Remaining shelf-life at retail is the key outcome variable from the simulation of random temperature and time. The derivation of retail shelf-life (t_3) after the phase of distribution is as follows:

$$\ln A = \ln A_0 + k_1 t_1 + k_2 t_2 + k_3 t_3, \text{ and} \quad (3.4)$$

$$t_3 = (\ln A - k_1 t_1 - k_2 t_2 - \ln A_0) / k_3, \quad (3.5)$$

where A is set to the threshold level of 3×10^7 .

Estimation the Shelf-Life of Fluid Milk

In a well-controlled supply chain, it will be known how long the product was held at each phase. Thus, it is reasonable to model an inventory management policy under the assumption of full information. Given an estimated retail shelf life, retail managers will be able to pull inventory according to condition of the product and safeguard consumers against spoiled goods.

Table (1) presents an example of one realization from 500 iterations of the simulated variables from the perishability model. The total shelf-life of fluid milk is about 98 hours, and 71.73 hours at the retail level; when the initial level of contamination is 2.74 CFU/mL. The storage temperature in this example at processing, distribution, and retailer is 2.06, 11.64, and 6.50 °C, respectively; for 24 hours at manufacture, and 2.39 hours during distribution. This information is an input into the inventory model, which is explained in the next section.

Table 1. Statistical Summary from 500 Iterations of the Simulated Variables of the Perishability Model.

| Input Variables | Input parameters | Simulation results |
|-----------------------------------------------------|-------------------------------|---------------------------|
| Initial quality (A_0) | Uniform (0.01, 10) CFU/ mL | 2.74 CFU/mL |
| Temperature in processing (T_1) | Normal(3,1) | 2.06 degrees Celsius. |
| Time in processing | 24 hours | 24 |
| Temperature in trucking (T_2) | Normal (5,1.5) | 11.64 degrees Celsius. |
| Time in trucking | Uniform (1.5 – 8) hours | 2.39 |
| Temperature at retail (T_3) | Normal (5,2) | 6.50 degrees Celsius. |
| Time at retail | Equation (3) | 71.73 hours |
| Total time (Shelf-life) | $t_1 + t_2 + t_3$ | 98.12 hours |

Source: Author's calculations

Inventory Model

The fixed-order quantity model from the management literature is used to represent the relationship between the level of inventory and the total cost. The total cost consists of five variables:

- Purchasing cost,
- Storage cost,
- Setup cost,
- Cost of lost sales, and
- The hazard cost of perished fluid milk.

Hence, the cost equation is written as:

$$TC = \sum_{t=1}^n (DC_t + H_t + S_{c_t} + Los_t + Per_t) \quad (3.6)$$

where the purchasing cost (DC_t) is the number of purchased units multiplied by the price per unit; the storage cost (H_t) is the number of units held, times the cost of holding per unit; the setup cost (S_{c_t}) is the cost of placing an order of size Q ; the loss in sales cost (Los_t) is the number of units of unmet demand multiplied by the selling price of each unit; the perishability cost (Per_t) is the number of perished units which are unsafe for consumption, times the hazard cost of perishability. In further discussion, we will refer to these cost components as the output variables, along with profits and revenues.

The storing cost for the total amount on hand is the number of units held in each day times storing cost per unit. The number of sold units equals the quantity demanded if

and only if the total supply is bigger than the quantity demanded in each day. Multiplying the number of daily units sold by the selling price results in the daily revenue. The left-over units, which have not reached the day of perishability, are held on the shelf and may be sold the next day. If milk is perished, then, the perishability cost will be the sum of storing cost and purchasing cost. In contrast, in a situation where demand is unmet, the sold amount equals the total available units on supply in that day. In this case, the manager will have to bear the cost due to stock out, which equals to the difference between the demanded units and supplied units times the margin that was foregone when units were not available for sale. By summing up the cost for each component over a year, we can calculate the total cost of holding inventory.

Model Assumptions

The fixed-order quantity (Chase et al, 1998) model assumes that replenishment time, size of inventory, prices, and demand at the retail level are known in advance. In practice, one of these inputs varies from one retailer to another, depending on many factors. The time between placing and receiving an order may take an hour or less, or more than 48 hours depending on, i.e., the distance between the warehouse and a retail store. The size of the inventory at a retail store may be subject to the total capacity of the store, which may not allow stocking enough fluid milk to meet the entire customer's demand. Also, prices of fluid milk vary from a retailer to another, i.e. geography, milk types, etc. Finally, demand also depends on many factors such as population, prices,

preferences, etc. while there are many possible ways to simulate, in this study reasonable choices have to be made to construct the baseline. We have assumed the replenishment time to be 2 days, and 100 gallons of fluid milk is the inventory size, on average, with a standard deviation of 15 gallons. Since we have been interested in fluid milk in general, not in a specific type, we set a range for the purchasing and the selling prices that consider the different varieties of fluid milk and their prices in the market. Thus, we assume that the range of purchasing cost and selling price is (\$1.50-\$2.50) and (\$2.50-\$4.00) per gallon, respectively. The setup cost is set to equal zero in this case, because it does not cost anything for a manager to place an order. The cost of storing fluid milk varies as overhead cost varies from one retailer to another. However, storage cost was assumed to be over a range from \$0.01 to \$0.50 per gallon. The cost of perished fluid milk is the sum of purchasing cost and storing cost; where the cost of unmet demand is the margin from selling one gallon of fluid milk. Purchasing cost, selling price, and storage cost were assumed to be uniformly distributed reflecting an equal probability for every possibility.

Demand Specification

The focal point of this inventory model is to examine the role of alternative specifications of demand at the retail level. We first used the own price elasticity of fluid milk in predicting the demand at the retail level.

Demand Specified with Own-Price Elasticity

Capps et al (2012) found that the own price elasticity for fluid milk is -0.0725. This means that the percentage change in the quantity demanded of fluid milk is less than the percentage change in the price, or demand is inelastic.

Demand specification is derived as follows. First consider the linear demand.

$$Q = \alpha + \beta P_Q, \quad (3.7)$$

and the first derivative of the demand function with respect to P_Q is.

$$\frac{\partial Q}{\partial P_Q} = \beta \quad (3.8)$$

where β is the slope of the demand curve, or the marginal change in quantity demanded for a unit change in price. Because elasticity η is:

$$\eta = \frac{\partial Q}{\partial P_Q} * \frac{P_Q}{Q} = \beta * \frac{P_Q}{Q}, \quad (3.9)$$

$$\beta = \eta * \frac{Q}{P_Q}. \quad (3.10)$$

Using $\eta = -0.0725$, then

$$\beta = (-0.0725) * \frac{Q}{P_Q}, \quad (3.11)$$

then, substituting β in the demand function,

$$Q = \alpha + \left[-0.0725 * \frac{Q}{P_Q} \right] P_Q. \quad (3.12)$$

By rearranging the last equation,

$$Q = \frac{\alpha}{1.0725} \quad (3.13)$$

Since we did not have data about quantity demanded of fluid milk at the retail level that we can use to estimate α , we assumed that α can be anywhere between 40-60 gallons per day, uniformly distributed. Then, for example, when α is 50 gallons, the predicted quantity demanded of fluid milk is about 46.62 gallons per day on average. This distribution is assumed to draw daily sales.

In one phase of this research, we explore inventory costs where managers select a sales price that is constant for all iteration of time and temperature. In an elaboration on inventory management decision, we consider how manager's choice of pricing affects unsold, nearly perished stocks. That model assuming pricing choices is explained in the following section.

Demand Specified with Negative Binomial Distribution

Although there are several methods to specify the demand at the retail level, we will use the negative binomial distribution to specify demand at the retail level. From the inventory control literature, Agrawal and Smith (1996) concluded that the negative binomial distribution is a preferred approach. It has the capability to predict the quantity demanded at the retail level at the tail of the demand distribution.

The negative binomial distribution is defined as the number of the N th success in a sequence of independent Bernoulli trials with probability P_r of success on each trial (Law and Kelton 1982).

The equation of the negative binomial distribution is:

$$\Pr[M = k|N, P_r] = \binom{k + N - 1}{N - 1} P_r^N (1 - P_r)^k, \quad (3.14)$$

$$0 < P_r < 1, \quad N > 0, \quad k = 0, 1, \dots, n.$$

where M is the total number of trials, N is the number of successful trials, P_r is the probability of successful trails, and k is the number of failure trials (Johnson et al 2005).

In order to generate the base case for both models, we use the predicted quantity demanded from the initial economic model. For instance, if the quantity demanded is 46.62 gallons per day, on average, with an 85-99% probability daily demand of fluid milk is 46.62 gallon per day or less. The model indicates that there is a probability of about 1-15%, assumed to be uniformly distributed, that daily demand is more than 46.62 gallon on average. By the definition of the negative binomial distribution, N would be the predicted daily demand of 46.62 gallons, and P is 85-99% probability. Thus, the predicted quantity demanded based on this approach is about 50.71 gallons or less, on average, of fluid milk per day with a 99% probability (Figure 4).

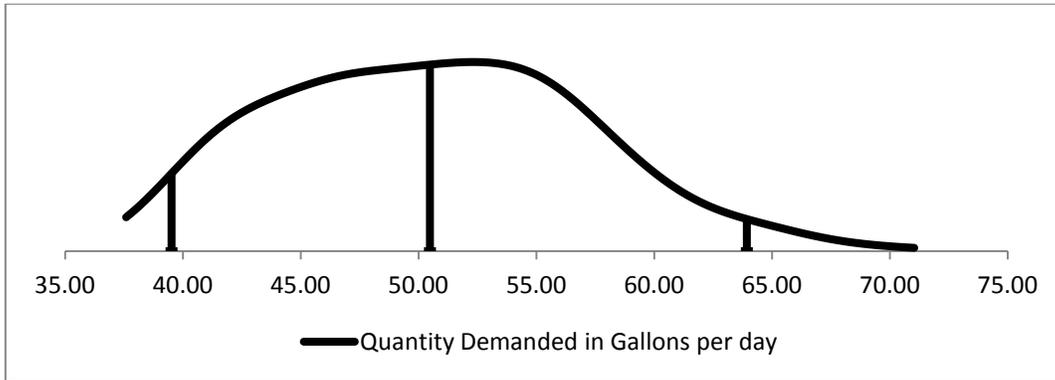


Figure 4. Probability Density Function of the Predicted Daily Demand Using the Negative Binomial Distribution.

Discount Policy in Inventory Management

Economic theory predicts that quantity demanded changes inversely with price. Managers can sell the excess inventory to avoid or minimize the cost of perishing by cutting the price of fluid milk that has very short shelf-life remaining. But, by how much does a manager need to reduce the price? This question can be answered using the own price elasticity mathematical equation. For example, suppose the beginning inventory is 45 gallons, and it is on its last day of shelf-life at a selling price of \$2.66 per gallon. The sold amount by the end of the day is predicted to be 41.5 gallons. This means that there is 0.45 gallon remaining in the inventory on the next day(s). Then, the new reduced selling price is determined as follows:

$$\eta = \frac{\% \Delta Q}{\% \Delta P_q}, \quad (3.15)$$

where, the own price elasticity is -0.0725 (Capps et al 2012). The amount needed to sell to avoid perishing is $\% \Delta Q$ ($0.45/45 = 0.01$), then

$$\% \Delta P_Q = \frac{0.01}{-0.0725} = -13.793 \%$$

This means, to sell the unsold 0.45 gallon of fluid milk, a manager needs to sell the remaining quantity at a discount of 13.793 %. This example utilizes a very inelastic demand framework and results in a very steep discount at the retail level. The pricing of fluid milk at the retail level is assumed to be within management's control, but we assume that price is not reduced below \$1 per gallon in this model.

Simulation Procedure

The predicted quantity demanded of fluid milk is then utilized in the inventory model to calculate the reorder point (R), sales, unmet demand, stock out cost, and revenues per day. A manager may place an order whenever the current stock level plus any amount that has been ordered, but not yet received are equal to or less than x, where x is the minimum safety stock level. For example, assume replenishment time is two days and the predicted quantity demanded is 50 gallons per day. In that situation, the manager will place an order when the inventory of fluid milk plus the amount on order in the last two days is 100 gallons or less; otherwise no order will be placed.

Additional Considerations

The model also considers any changes in the demand patterns due to seasonality, which may occur in the foreseeable future. In this situation, the inventory model initiates

another reorder point to meet the seasonal demand with respect to the total capacity of the warehouse, considering the amount on order in the last two days based on the replenishment time, the current level of fluid milk, and the total received amount. Thus, the total amount of fluid milk supplied at the retail level in each day will be the sum of the current amount on hand, and the amount received.

Simulated Variables

Simulated outcome variables for 500 iterations over a year are obtained using Simetar, a simulation software package, to determine the approach minimizes the total cost. Perishability and stock out cost are key output variables of interest. Overall, it is expected, a priori, that the negative binomial distribution specifying demand will provide different results from a model in which inventory managers can utilize pricing strategies to reduce stocks.

CHAPTER IV

RESULTS AND ANALYSIS

In this chapter we will present the results of the perishability model and the inventory model using tables and graphs of the simulated outcome variables and ranks of scenarios for each model. Also, we will measure the sensitivity of simulated outcome variables to small changes in the input variables. The sensitivity analysis focuses on conditions of temperature and the lead time of ordering.

Perishability Model

The simulated variables of the perishability model are: the shelf-life of fluid milk at the retail level; and the shelf-life of product during the entire supply chain, expressed in days. Table (2), contains the results for 500 iterations.

The basic scenario shows that, when fluid milk is held at, on average, 3, 15, and 5 °C degrees throughout the three phases, the total shelf-life of the fluid milk is about 5 days, and 4 days at the retail level. In other words, there is a 50% probability that the total shelf-life of the fluid milk is 5 days, and 4 days or less at the retail level. The results also show that there is low probability, less than 5%, that fluid milk may perish within 2 days of retail holding. In contrast, there is a high probability that the shelf-life of the product is about 12 days or less (Figure 5).

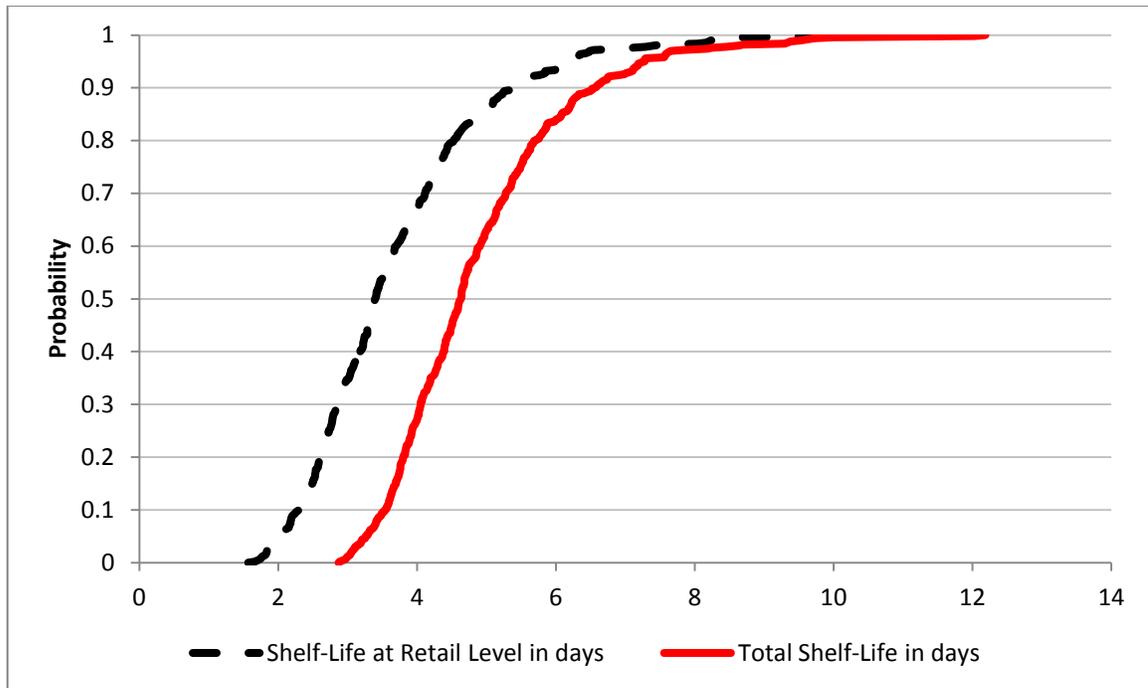


Figure 5. Cumulative Distribution Function of the Total Shelf-Life, and Shelf-Life of Milk at the Retail Level.

An alternative scenario was also specified where fluid milk experiences temperature abuse in distribution, for any reason that may cause temperature and time to rise, such as an accident on the road or consumers opening refrigerator door at a store more frequently during a busy day. An example of a temperature abuse scenario is holding temperature at 6 °C for 24 hours in a manufacturer, 20 °C in distribution for 6 hours, and 10 °C at retail level. The results for the alternative scenario show that the expected shelf-life of fluid milk, on average, falls dramatically by about 2 days in comparison with the basic scenario (Table 2).

Table 2. Statistical Summary of 500 Iterations for the Simulated Variables from the Milk Perishability Model.

| | Shelf-Life at Retail Level in days | Total Shelf-Life in days |
|-----------------------------------|-------------------------------------------|---------------------------------|
| Base Scenario | | |
| Mean | 3.68 | 4.88 |
| St. Dev. | 1.34 | 1.33 |
| Min | 1.57 | 2.87 |
| Max | 10.97 | 12.19 |
| Temperature Abuse Scenario | | |
| Mean | 1.42 | 2.70 |
| St. Dev. | 0.82 | 0.83 |
| Min | 0 | 0.77 |
| Max | 6.66 | 6.80 |

Note: Base Scenario: temperature 3, 15, and 5 °C , and time 24, and 4.75 hours, on average.

Temperature Abuse Scenario: temperature 6, 20, and 10 °C, and time: 24, and 6 hours, on average.

Inventory Model

Cost Minimization: Baseline

Table 3 contains the results of 500 iterations of the outcome variables, in a comparison between approaches for the specification of demand; where fluid milk is held with a good control of temperature during distribution and retailing. The demand specifications compare a demand that is responsive to price with a stochastic demand that follows the negative binomial distribution. The results show that using the own price elasticity of fluid milk in predicting the daily demand at the retail level will, in general, have lower total cost in comparison with the negative binomial approach. This is mainly attributed to the lower purchasing cost. Figures 6 and 7 contain a breakdown of costs indicating that the purchasing cost is the dominant component of the total cost, accounting for 81%, and 84% of the total cost when specifying demand using the own price elasticity, and negative binomial, respectively. Other components, together, compose less than 20% of total cost. In addition, the results show that using negative binomial distribution to predict demand will reduce the cost of stock out, however, it increases the perishability cost as it is shown in Table 3. Under the negative binomial distribution, stocks of fluid milk at a retail store are greater, hence there will be more gallons of fluid milk purchased, stocked and spoiled in comparison to the other demand approach. On the other hand, stocking more gallons will reduce the probability and cost of running out of milk.

Despite the fact that the negative binomial approach generates more revenues than using the own price elasticity approach, using the own price elasticity in specifying demand generates more profits than the other approach. Figure 8 contains the model results indicating that there is 2.90% probability that using the own price elasticity in predicting demand will generate negative profits, where the probability of generating negative profits using the negative binomial distribution is 3.23%.

Minimizing purchasing cost leads to minimizing perishability cost. However, it may not be in the best interest of managers, because it may result in a decline in customer service levels and increase the loss in sales cost. In contrast, it will reduce the cost of spoiled milk. Therefore, managers who tend to minimize the perishability cost along with providing high levels of customer service will be in gray area.

Table 3. Average Total Cost, Perishability Cost, Loss in Sales Cost, Purchasing Cost, Revenues, and Profits in US Dollars in Milk Demand Resulting from 500 Model Iterations.

| Simulated Outcome | Demand Specification | | | |
|---------------------------|-----------------------------|----------------------------|--------------------------|----------------------------|
| | Own Price Elasticity | | Negative Binomial | |
| | Mean | % of Total cost | Mean | % of Total cost |
| Total Cost | 34,021.29 | | 36,026.14 | |
| Perishability Cost | 201.49 | 0.59% | 227.22 | 0.63% |
| Loss in Sales | 3,867.42 | 11.37% | 3,250.99 | 9.02% |
| Purchasing Cost | 27,712.38 | 81.46% | 30,370.40 | 84.30% |
| Storing Cost | 2,239.99 | 6.58% | 2,178.19 | 6.05% |
| Revenue | 48,601.31 | | 49,209.14 | |
| Profit | 14,580.02 | | 13,182.33 | |

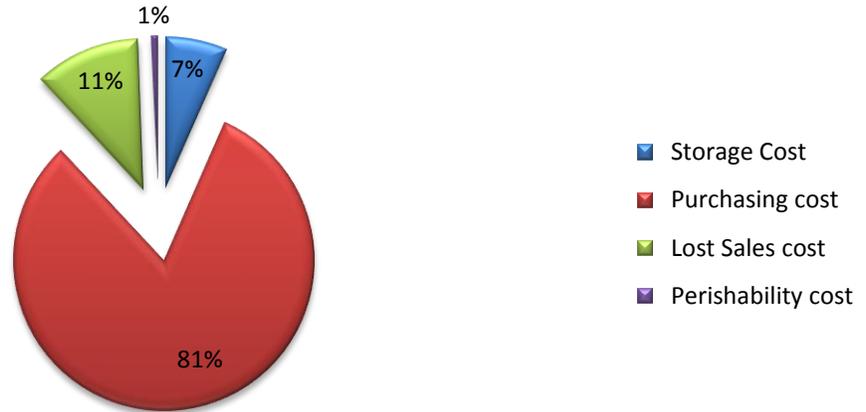


Figure 6. The Percentage of Cost Components in the Total Cost Using the Own Price Elasticity in Predicting Demand for Baseline.

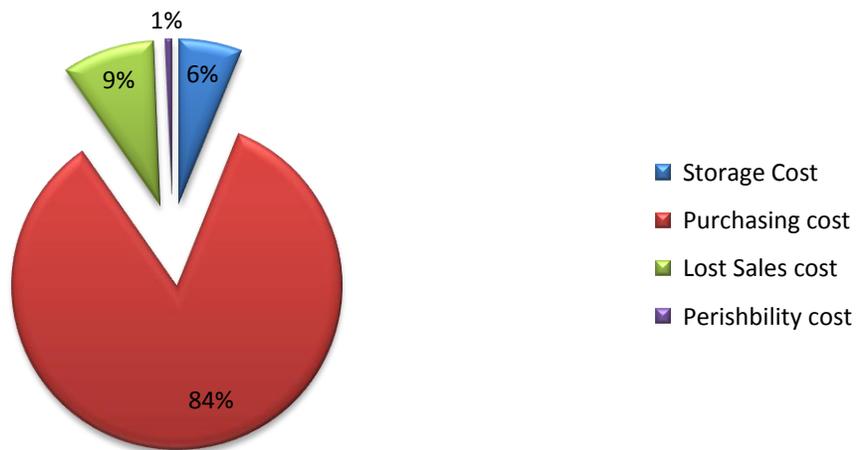


Figure 7. The Percentage of Cost Components in the Total Cost Using the Negative Binomial Distribution in Predicting Demand for Baseline.

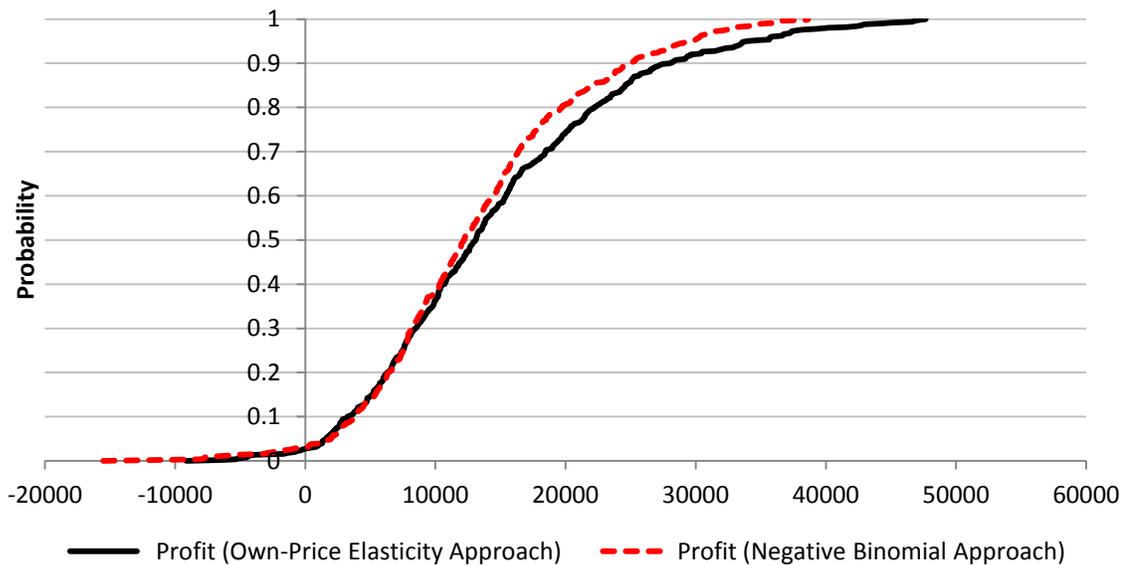


Figure 8. Cumulative Distribution Functions of Baseline Milk Profits.

Cost Minimization: Temperature Abuse Scenario

Table 4 contains the statistical summary of 500 model iterations of the simulated outcome variables from the inventory where microorganisms in the fluid milk grow faster because there is higher temperature in the distribution system. The results show that there is a substantial increase in total cost overall and perishability cost specifically due to the short shelf-life of fluid milk. More spoiled gallons of fluid milk reduce the available amount in store, which increases the loss in sales (Figures 9 and 10). Less milk quantity available at the retail level to meet demand, due to perishability, reduces the revenue from selling fluid milk, which eventually reflects on profits. Figure 11 is a breakdown of costs showing that there is 58.78% possibility that holding the current

level of inventory will generate negative profits when using an own price elasticity of demand model to predict demand in comparison to an 83.17% probability of negative profits, using the negative binomial distribution. Assuming persistent temperature abuse, the fluid milk distribution system is not profitable. However, the use of the price elasticity model to forecast demand results in lower losses than if managers forecast based on the negative binomial demand.

Table 4. Average Total Cost, Perishability Cost, Loss in Sales Cost, Purchasing Cost, Revenues, and Profits in US Dollars from 500 Iterations Temperature Abuse Scenario.

| Simulated Outcome Variables | Demand Specification | | | |
|--------------------------------|----------------------|------------------------|-------------------|------------------------|
| | Own Price Elasticity | | Negative Binomial | |
| | Mean | % to the Total cost | Mean | % to the Total cost |
| Total Cost | 46,249.98 | | 57,938.00 | |
| Perishability Cost | 8,128.92 | 17.58% | 14,316.28 | 24.71% |
| Loss in Sales | 7,393.20 | 15.99% | 7,812.10 | 13.48% |
| Purchasing Cost | 29,144.28 | 63.01% | 33,629.32 | 58.04% |
| Storing Cost | 1,583.58 | 3.42% | 2,180.29 | 3.76% |
| Revenue | 38,547.22 | | 34,272.05 | |
| Profit | (7,702.26) | | (23,665.95) | |

Note: Any value between parentheses is a negative value.

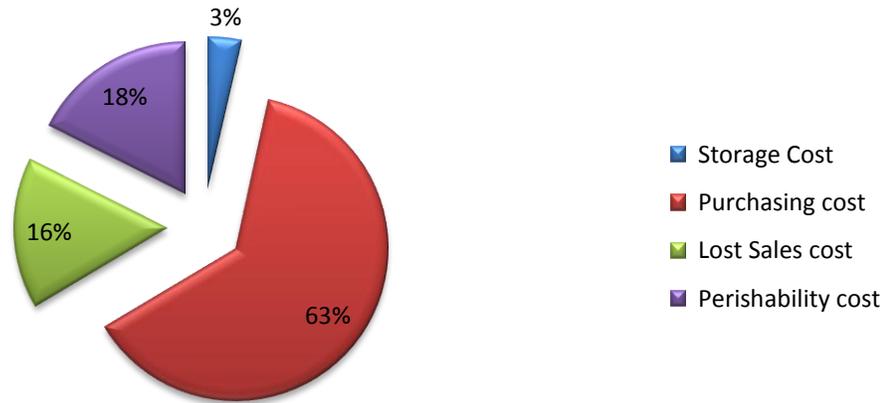


Figure 9. The Percentage of Cost Components in the Total Cost Using the Own Price Elasticity in Predicting Demand for Temperature Abuse Scenario.

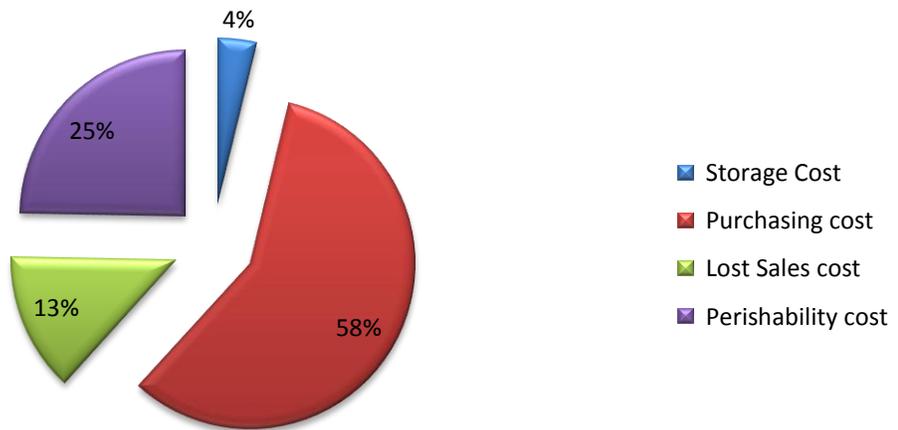


Figure 10. The Percentage of Cost Components in the Total Cost Using the Negative Binomial Distribution in Predicting Demand for Temperature Abuse Scenario.

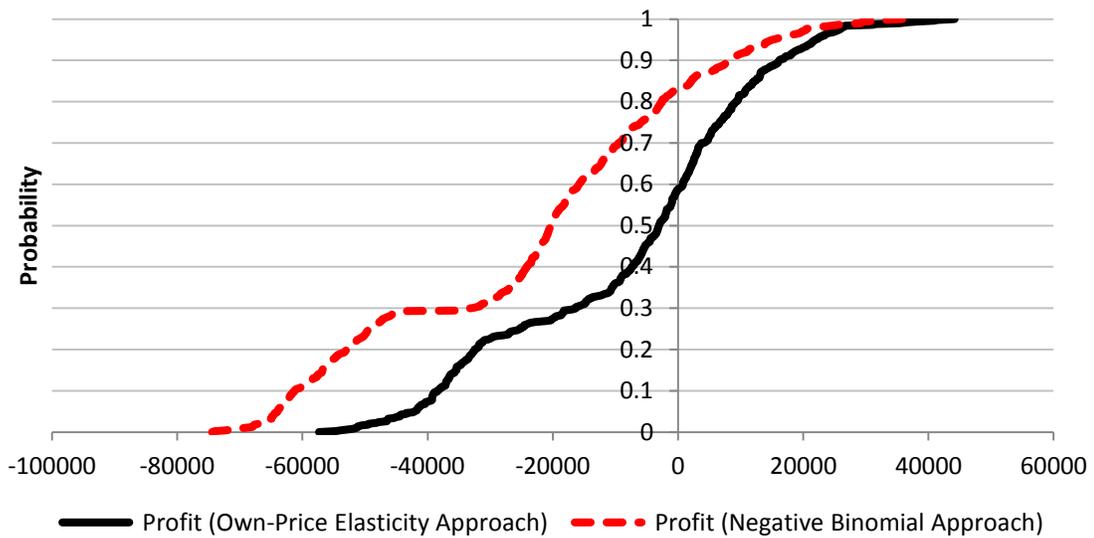


Figure 11. Cumulative Distribution Function of Profits, Temperature Abuse Scenario.

Profit Maximization

When there is a 50% probability that the daily demand for fluid milk is 50 gallons and the replenishment time is two days, raises a question about the optimal inventory size that optimizes profit. Four inventory level options were studied to analyze the profit maximizing optimal milk inventory level using two approaches to specify retail level demand. The assumed scenarios are:

Table 5. Assumed Inventory Size Options in Gallons.

| Options | Mean | Standard Deviation |
|----------------|-------------|---------------------------|
| 1 | 75 | 7.5 |
| 2 | 100 | 10 |
| 3 | 125 | 12.5 |
| 4 | 150 | 15 |

Source: Author's assumptions

The results are summarized in Table 7. Total cost and all of its components increase as the size of the inventory increases, except lost sales cost. In contrast, profit tends to decrease when the size of the inventory goes up. On average, all inventory size options generate profit; however, a manager can maximize profit by stocking 75 gallons of fluid milk (Table 8). Milk purchasing cost is greater using the negative binomial of demand model specification than the own-price elasticity specification. Specifying demand with the negative binomial leads to more purchased gallons when compared to the own-price elasticity because of the safety stock purchased. On the other hand, the level of customer service is higher under negative binomial demand specification; hence, it reduces the cost of stock-out.

Table 6. Milk Related Costs by Item for 500 Model Iterations in U.S. Dollars.

| Items | Average Inventory Size Options | Mean | | % difference Negative Binomial to Own-Price Elasticity |
|---------------------------|--------------------------------|----------------------|-------------------|--------------------------------------------------------|
| | | Own Price Elasticity | Negative Binomial | |
| Storage Cost | 75 | 1,649 | 1,584 | (4)% |
| Storage Cost | 100 | 2,232 | 2,097 | (6)% |
| Storage Cost | 125 | 3,197 | 3,008 | (6)% |
| Storage Cost | 150 | 4,134 | 4,025 | (4)% |
| Purchasing cost | 75 | 28,510 | 30,522 | 8% |
| Purchasing cost | 100 | 27,931 | 30,567 | 10% |
| Purchasing cost | 125 | 28,594 | 30,707 | 7% |
| Purchasing cost | 150 | 29,614 | 31,808 | 7% |
| Lost Sales cost | 75 | 3,394 | 3,231 | (9)% |
| Lost Sales cost | 100 | 3,765 | 3,322 | (17)% |
| Lost Sales cost | 125 | 3,625 | 3,299 | (8)% |
| Lost Sales cost | 150 | 3,517 | 3,085 | (11)% |
| Perishability cost | 75 | 98 | 121 | 3% |
| Perishability cost | 100 | 199 | 139 | (48)% |
| Perishability cost | 125 | 535 | 391 | (34)% |
| Perishability cost | 150 | 1,522 | 1,117 | (29)% |

Note: Any value between parentheses is a negative value.

Table 7. Profit, Revenue, and Total Cost for 500 Iterations in U.S. Dollars.

| Outcomes | Average Inventory Size Options | Mean | | % difference Negative Binomial to Own-Price Elasticity |
|--------------------|---------------------------------------|-----------------------------|--------------------------|---------------------------------------------------------------|
| | | Own Price Elasticity | Negative Binomial | |
| Profit | 75 | 15,984 | 13,866 | (13)% |
| Profit | 100 | 14,156 | 13,050 | (8)% |
| Profit | 125 | 13,467 | 11,708 | (13)% |
| Profit | 150 | 10,838 | 9,824 | (9)% |
| Revenue | 75 | 49,665 | 49,323 | (1)% |
| Revenue | 100 | 48,525 | 49,175 | 1% |
| Revenue | 125 | 49,445 | 49,113 | (1)% |
| Revenue | 150 | 49,852 | 49,859 | 0% |
| Total Costs | 75 | 33,681 | 35,457 | 5% |
| Total Costs | 100 | 34,369 | 36,125 | 5% |
| Total Costs | 125 | 35,978 | 37,405 | 4% |
| Total Costs | 150 | 39,014 | 40,035 | 3% |

Note: Any value between parentheses is a negative value.

Indeed, the percentage difference in cost of lost sales between the two demand specifications, when the size of inventory is 75 gallons, is -9%. At the same time, perishability cost is 3% lower under the own-price elasticity of demand model.

Discount Policy in Inventory Management

Table 4 contains the model results comparing two situations: the first one, where a manager adopts a discount-price policy; and the second one where the manager does not discount prices. It appears that cutting the price of fluid milk to clear the shelf will decrease costs in general, especially the perishability cost. However, the cost of running out of stock increases because selling fluid milk at lower price increases the demand on the product while supply stays at the same level. The reduction in the selling price of fluid milk is also reflected through a dynamic effect on profit.

Table 8. Average Cost of Inventory with and without a Discount Policy, in US Dollars.

| Variables | No discount policy | Discount policy | % Change |
|---------------------------|---------------------------|------------------------|-----------------|
| Total Costs | 32,040.89 | 32,001.32 | -0.12% |
| Storage Cost | 2,496.08 | 2,475.05 | -0.84% |
| Purchasing cost | 26,036.87 | 26,012.73 | -0.09% |
| Lost Sales cost | 2,716.30 | 2,728.47 | 0.45% |
| Perishability cost | 791.63 | 785.07 | -0.83% |
| Revenue | 44,137.76 | 43,745.10 | -0.89% |
| Profit | 12,096.87 | 11,743.78 | -2.92% |

Note: Own-price elasticity is -0.0725

Sensitivity Analysis

This test measures the sensitivity of the simulated variables to a small change in the input variables. In particular, we consider the sensitivity of the effect of the selling and purchasing price of fluid milk on profits. The sensitivity model can help to determine which input variable(s) needs the most attention in our models by identifying the variables that have the biggest impact on profits. The Simulation Engine dialog box in Simetar expands to add the Estimate Sensitivity Elasticity, allowing easy specification of the percentage change to use for estimating the sensitivities (Richardson 2010).

For example, assume that quantity demanded of a product is 10 units per week at the price of \$2 per unit. When the price falls to \$1.98 per unit, the quantity demanded increases to 12 units per week. Thus a 1% change in the price leads to 20% change in the quantity demanded, which means that demand is highly sensitive to a small change in price.

The economic outcomes of interest in this study are inventory management costs and the total profit from selling milk. The key input variables that are used in the model are shelf-life, inventory safety stock, and the reorder point. Finally, demand specifications are based on assumption of the baseline parameters. In this section, we present the result of sensitivity analysis.

Sensitivity of Shelf-Life of Fluid Milk at Retail Level to Temperature and Time

Table 9 contains the results of the sensitivity analysis of the shelf-life of fluid milk given a 1% increase in the temperature and time. The results show that, on average, a 1% increase in the storing temperature at a manufacturer will decrease the shelf-life of the fluid milk by 0.037%. Shelf-life decreases by 0.27% and 0.74% for 1% increase in the storing temperature during processing and at the retail level respectively. A 1% increases in time at the manufacture and during distribution decreases the shelf-life by 0.07%, and 0.20%, respectively. Overall, shelf-life is relatively inelastic to temperature and time meaning that a small change in temperature and time results in a relatively small change in shelf-life. Thus, while the conditions on distribution are important, a marginal change is relatively unimportant when compared to shelf-life. However, greater increases in temperature and time have a greater impact on the shelf-life.

Table 9. Statistical Summary of Sensitivity Analysis of Shelf-Life for a 1% Increase in Temperature and Time.

| Factors | Percentage in shelf-life for a 1% change | | | |
|----------------------------------------------|------------------------------------------|-----------------------|--------|--------|
| | Mean | Standard Deviation | Min | Max |
| Storing temperature during processing | (0.04) | 0.02 | (0.10) | 0.00 |
| Storing temperature during transportation | (0.27) | 0.21 | (1.56) | (0.02) |
| Storing temperature at the retail level | (0.74) | 0.21 | (1.17) | 0.38 |
| Time at manufacture stage | (0.07) | 0.02 | (0.15) | (0.03) |
| Time during distribution stage | (0.20) | 0.14 | (1.06) | (0.03) |

Note: Values between parentheses are negative values

Sensitivity Analysis of the Economic Outcomes to Shelf-Life

Total cost depends on shelf-life, in a limited way. Table 10 contains the model results of the sensitivity elasticity of total cost with respect to several input variables. Total cost falls by 0.02%, on average, for a 1% increase in shelf-life. Where a 1% increases in inventory size, replenishment time, and predicted demand lead to increases in total cost of 0.03%, 2.74%, and 0.90%, on average, respectively. The results indicate that the replenishment time has the biggest impact on total cost compared to other variables.

Table 11 contains the model results for the sensitivity of profit to a 1 percent change in the shelf-life of fluid milk at the retail level, replenishment time, and predicted demand. Retail shelf-life, replenishment time, and demand increases of 1 percent increase profit by 1.66%, 1.41%, and 3.86%, on average, respectively. In contrast, a 1% increase in inventory level leads to decrease in profit by 0.44%, on average.

In conclusion, shelf-life has a relatively small impact on costs (-0.02).

Interestingly, the influence of shelf-life on profit is considerable greater, although it is still an inelastic response. The replenishment time has a highly elastic relationship to costs, where predicted demand has the higher influence on profit. The elasticity is 2.74 on costs, which indicates that 1% delay in replenishment time results in costs increasing by 2.74%. Yet, profits increase by nearly 1.14 % on average when replenishment time decreases by 1%. This can be explained by the decline in the lost sales cost, as a 1% increase in the replenishment time decreases loss in sales by 1.53%.

Table 10. Sensitivity Analysis of Total Costs for a 1% Increase in Input Variables.

| Total Costs with respect to | Mean | St. Dev. | Min | Max |
|-----------------------------------------|-------------|-----------------|------------|------------|
| Shelf-life of fluid milk in days | (0.02) | 0.53 | (11.75) | 0.21 |
| Inventory size in gallon | 0.03 | 2.28 | (19.68) | 2.22 |
| Replenishment time in hours | 2.74 | 3.65 | (14.21) | 25.59 |
| Predicted demand in gallon | 0.90 | 2.10 | (1.46) | 25.22 |

Note: Values between parentheses are negative values.

Table 11. Sensitivity Analysis of Profit for a 1% Increase in Input Variables.

| Profit with respect to | Mean | St. Dev. | Min | Max |
|-----------------------------------------|-------------|-----------------|------------|------------|
| Shelf-life of fluid milk in days | 1.66 | 29.80 | (0.00) | 650.55 |
| Inventory size in gallon | (0.44) | 10.56 | (63.86) | 27.87 |
| Replenishment time in hours | 1.41 | 8.38 | (48.88) | 57.10 |
| Predicted demand in gallon | 3.86 | 26.93 | (206.37) | 184.57 |

Note: Values between parentheses are negative values.

CHAPTER V

CONCLUSIONS

The simulation of temperature and time throughout the supply chain results in different remaining shelf-life of fluid milk at the retail level. When fluid milk is held at a constant low temperature, perishability cost does not exceed 1% of total cost. On the other hand, fluctuations in temperature and time increase the percentage of perishability cost out of total cost dramatically. Minimizing cost or maximizing profit with regard to inventory management at the retail level requires an understanding of the relationship between temperature, time, growth rate of microorganisms, total cost, and profit. This relationship can be applied to different products such as fruit, vegetables, and frozen products to minimize the total cost of managing inventory or maximize profits from selling products.

The specifications of demand at the retail level have a direct impact on the outcomes of the inventory model. The different specifications of the demand at the retail level lead to different results. The negative binomial demand specification incurs relatively higher total cost and lower level of profit in comparison to a model in which the demand is linear and the price elasticity of demand is taken into account in the management decision. However, the negative binomial model is associated with higher level of customer service and fewer lost sales. On the other hand, the own-price elasticity has lower purchasing and storing costs in comparison to the other approach.

The most important factors that affect the total cost and profits of a retailer are shelf-life at the retail level, reorder point, the size of an order, and the size of the inventory level. Any delay in the replenishment time and reorder point will cost the manager more due to loss in sales as well as underestimating the size of inventory.

This thesis covers a part of the supply chain from manufacturers to retailers assuming only two approaches to specify the demand at the retail level. Further studies can be done using data from the retail level. For example, collecting and analyzing data of temperature over time of fluid milk from the provider to the retail level will allow an empirical estimate of the shelf-life to estimate the shelf-life of the fluid milk at the retail level, and will enhance realism of this model.

It is also important to consider the implications of this research for farm-level businesses. The relatively short shelf-life of fluid milk at the retail level implies that there is product that reaches its expiration and thus removed from the market supply. If retail inventory management is improved through temperature control and improvements in the distribution system, how will the supply and therefore the prices of grade A milk at the farm level change? For instance, will the prices of milk fall? This issue remains a topic for future studies to examine the impacts of short-shelf at the retail level.

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