PHYSICAL REVIEW C VOLUME 9, NUMBER 4 APRIL 1974

Core coupling in ^{99}Nb†

P. K. Bindal and D. H. Youngblood
Cyclotron Institute and Physics Department, Texas A&M University, College Station, Texas 77843
(Received 21 December 1973)

The ^{100}Mo(d, ^{3}He)^{99}Nb reaction at 40.7-MeV bombarding energy was used to obtain excitation energies, spectroscopic factors, and l-value assignments for levels in ^{99}Nb. A core-coupling model is used to predict these properties and agreement with experiment is found to be good.

[NUCLEAR REACTIONS, NUCLEAR STRUCTURE ^{100}Mo(d,^{3}He), $E=40.7$ MeV, measured $\sigma(\theta)$ ^{99}Nb levels deduced S. Calculated levels, J, π, S ^{99}Nb, particle-core-coupling model.]

The proton configurations of nuclei in the Zr-Mo region have been the subject of much experimental interest.[1-3] The only information available on ^{99}Nb, however, was obtained by measurement of γ decay following ^{99}Zr β decay,[4] and is rather limited. We have studied the levels of ^{99}Nb with the ^{100}Mo(d, ^{3}He) reaction at 40.7-MeV bombarding energy. A 600-μg/cm^2 target enriched to 95.9% in ^{100}Mo was used. Two ΔE-E solid-state detector telescopes with conventional electronics permitted an over-all energy resolution of 50 keV covering an excitation energy region up to 10 MeV. Outgoing helions (^{3}He nuclei) and tritons were detected simultaneously with energy calibrations obtained using ^{96}Mo and ^{98}Mo targets and from ^{16}O and ^{12}C impurities. Angular distributions were obtained for laboratory angles from 8 to 40° for 10 levels kinematically identified with this reaction. By comparison with distorted-wave Born-approximation (DWBA) calculations performed with the computer code DWUCK,[5] l assignments could be made for eight of them. The information obtained is shown in Fig. 1. Our data acquisition, analysis, and DWBA fitting procedures have been described previously.[6] The experimental details and the data for the other Mo isotopes are the subject of another communication.[7]

In addition to the $l=4$ ground state (assumed $g_{9/2}$) five definite $l=1$ ($p_{1/2}$ and $p_{3/2}$) and two definite $l=3$ (assumed $f_{5/2}$) states are seen which account for virtually all of the available $2p_{1/2}$, $2p_{3/2}$, and $1f_{5/2}$ strength. Two additional weak states are observed for which no definite l assignment is possible. In the β-decay study of ^{99}Nb three excited states were identified, two of which are populated by ^{99}Zr β decay. The logft values[4] (~4) imply an allowed transition which, since the ^{99}Zr ground state almost certainly has positive parity (no negative-parity shell-model levels are available for the odd neutron), indicates positive parity for these levels. None of these levels were populated with the (d, ^{3}He) reaction.

The splitting of the negative-parity $2p_{1/2}$, $2p_{3/2}$, and $1f_{5/2}$ states into just a few levels suggests that this splitting might be due to coupling of the single-hole states with the 0^+ ground state and the 2^+ first excited state of the ^{100}Mo core. As this structure is all below 2 MeV in excitation, the 3^- octupole state at 1.9 MeV in ^{100}Mo probably does not represent an important contribution. Briefly, the Hamiltonian for the interaction between the core and the particle can be represented as[8,9]

$$H_{\text{int}}=-\xi(J_c^{(1)}J_p^{(1)})-\eta(Q_c^{(2)}Q_p^{(2)})\,, \tag{1}$$

where J and Q are the angular momentum and the quadrupole moment operators, respectively. The subscripts p and c refer to the particle and the core. In our study the parameters of the model are ξ, the strength of the dipole-dipole interaction; χ_1 defined as $\eta(0||Q_c^{(2)}||2)$ and χ_2 defined as $\eta(2||Q_c^{(2)}||2)$. The value of the oscillator parameter ν, which occurs in the evaluation of the radial integrals, is taken to be 0.212 fm^{-2}.

In a simpleminded shell-model prescription, one would expect that for Mo isotopes ($Z=42$), the proton $2p_{1/2}$, $2p_{3/2}$, and $1f_{5/2}$ subshells would be completely filled and there would be two protons in the $1g_{9/2}$ shell. Spectroscopic information obtained from several experiments,[1,2] however, indicates that πp and πf orbits are not completely filled. Thus, a quasiparticle formalism is used for ^{99}Nb in the present calculation, where $1g_{9/2}$, $2p_{1/2}$, $2p_{3/2}$, and $1f_{5/2}$ orbits are assumed to be 30, 70, 80, and 90% full, respectively. The negative-parity levels of ^{99}Nb are assumed to be given by the coupling of proton holes in the $2p_{1/2}$, $2p_{3/2}$, and $1f_{5/2}$ subshells to the 0^+ ground state and the 2^+ first excited state at 0.54 MeV in ^{100}Mo. The single-hole energies were estimated from the

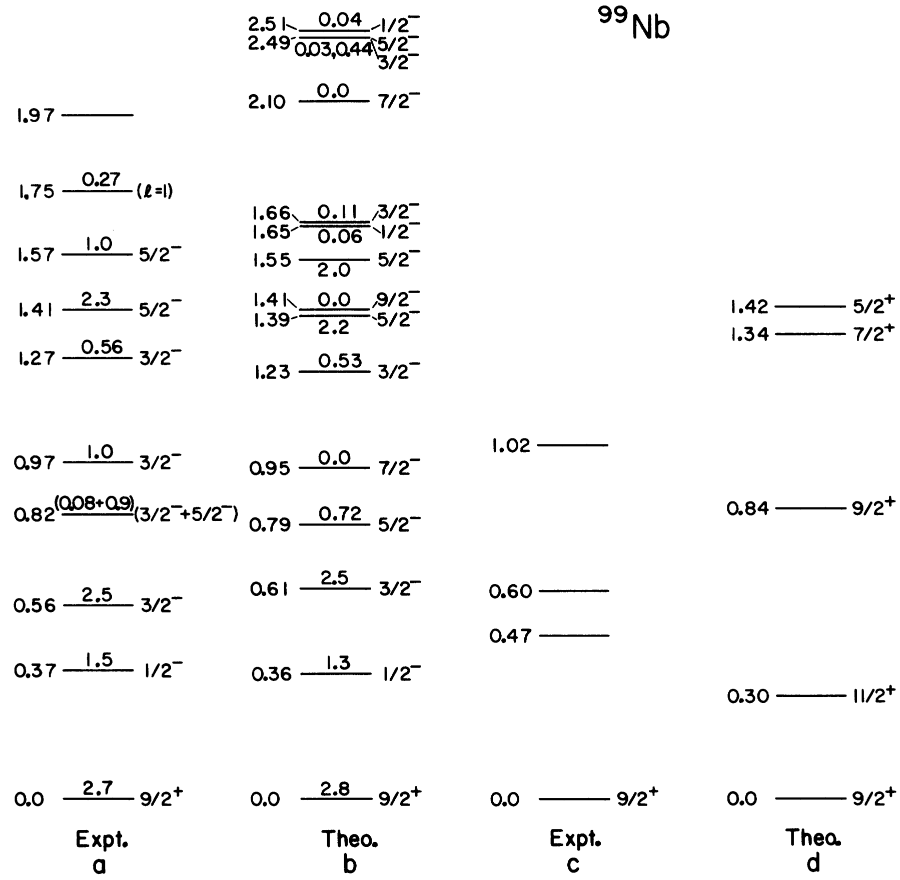

FIG. 1. The experimental energy levels are shown along with theoretical predictions for ^{99}Nb. The experimental data in column a are from the present study and in column c are taken from Ref. 4. The present calculations are shown with negative-parity excited states in column b and positive-parity excited states in column d. The excitation energies (experimentally ±10 keV) are shown to the left of the lines (in MeV), while spectroscopic factors are indicated on the line, and J^{π} assumptions (experimental) or predictions (theoretical) are indicated to the right of the line.

present experiment to be 0.47, 0.76, and 1.46 MeV for the $2p_{1/2}$, $2p_{3/2}$, and $1f_{5/2}$ orbitals, respectively, and were not varied.

The Hamiltonian matrices for $I=\frac{1}{2}^-, \frac{3}{2}^-, \frac{5}{2}^-, \frac{7}{2}^-$, and $\frac{9}{2}^-$ were diagonalized to obtain the energy eigenvalues and eigenfunctions for values of ξ, χ_1, and χ_2 ranging from 0.0 to 0.5. It was found that the best agreement with the experimental energy spectrum and spectroscopic factors was obtained with values of

$$\xi = 0.10 \text{ MeV}, \quad \chi_1 = 0.14 \text{ MeV fm}^{-2},$$

and

$$\chi_2 = 0.12 \text{ MeV fm}^{-2}.$$

In Fig. 1 the energy levels obtained from the present experiment are shown in column a with the theoretical predictions in column b. One notices that for the first two excited states, the $l=1$ (assumed $\frac{1}{2}^-$) state at 0.369 MeV and the $l=1$ (assumed $\frac{3}{2}^-$) state at 0.562 MeV, the spectroscopic factors are reproduced very well, although the predicted energy spacing between these states is somewhat large. The third excited state is predicted to be $\frac{5}{2}^-$, whereas the experimental data were not definitive, as the angular distribution for the $(d, {}^3\text{He})$ reaction leading to the 0.82-MeV state gave the best fit by using a mixture of $l=1$ and $l=3$ transfers with the spectroscopic factors shown in Fig. 1. This indicates that this may be a doublet. The $l=1$ state at 0.970 MeV ($C^2S=1.0$) is not predicted by this model if one assumes the predicted $\frac{3}{2}^-$ state at 1.23 MeV ($C^2S=0.53$) is to be identified with the $l=1$ state observed at 1.27 MeV ($C^2S=0.56$). Other predicted $\frac{3}{2}^-$ states are very weak and lie higher in excitation. This suggests that possibly our model space is too limited and that the 0.970-MeV state may have, as a strong component, coupling of an $l=1$ hole to the excited 0^+ state at 680 keV in ^{100}Mo, although this would re-

TABLE I. The wave functions for the negative-parity states of ^{99}Nb are listed. The third column contains the amplitudes for the coupling of the 0^+ ground state of ^{100}Mo to the angular momentum j of the single particle. In the next columns amplitudes of the coupling of the first excited 2^+ state of ^{100}Mo to the single-particle states of a given j value (in increasing order) giving the appropriate I^π value are listed.

E_x (MeV)	I^π	$\lvert 0^+j\rangle$	$\lvert 2^+p_{1/2}\rangle$	$\lvert 2^+p_{3/2}\rangle$	$\lvert 2^+f_{5/2}\rangle$
0.36	$\frac{1}{2}^-$	0.964		−0.215	0.159
0.61	$\frac{3}{2}^-$	0.890	0.375	0.246	0.078
0.79	$\frac{5}{2}^-$	0.366	0.901	−0.168	0.160
0.95	$\frac{7}{2}^-$			0.999	0.050
1.23	$\frac{3}{2}^-$	−0.407	0.910	0.082	0.014
1.39	$\frac{5}{2}^-$	0.638	−0.407	−0.633	0.164
1.41	$\frac{9}{2}^-$				1.00
1.55	$\frac{5}{2}^-$	0.613	−0.141	0.756	0.181
1.65	$\frac{1}{2}^-$	0.214		0.977	0.022
1.66	$\frac{3}{2}^-$	−0.183	−0.166	0.959	−0.139
2.10	$\frac{7}{2}^-$			−0.050	0.999
2.49	$\frac{3}{2}^-$	−0.090	−0.066	0.114	0.987
2.49	$\frac{5}{2}^-$	−0.287	−0.055	−0.007	0.956
2.51	$\frac{1}{2}^-$	−0.160		0.013	0.987

quire excitation by a two-step process (unlikely for this reasonably strong state), or considerable 0^+ ground-state configuration must also be present. Another possibility, also requiring a two-step excitation, is that this is the $\frac{7}{2}^-$ state predicted at essentially this energy, whose configuration is primarily $2p_{3/2}$ coupled to the 2^+ core, although it is unlikely such a configuration would be excited strongly in this reaction. This "extra" $l=1$ state of moderate strength persists throughout the Nb isotopes.[7]

Next, two $\frac{5}{2}^-$ states are predicted at 1.39 MeV ($C^2S=2.2$) and 1.55 MeV ($C^2S=2.0$), respectively, with $l=3$ states observed at 1.413 MeV ($C^2S=2.3$) and 1.573 MeV ($C^2S=1.0$) showing good agreement.

Then two $l=1$ states are predicted at 1.65 MeV ($C^2S=0.06$) and 1.66 MeV ($C^2S=0.11$) with $J=\frac{1}{2}$ and $\frac{3}{2}$, respectively. There is an experimentally observed $l=1$ level at 1.75 MeV ($C^2S=0.27$). There is some evidence from the experiment[7] that this state may be a doublet. Then there is a predicted $\frac{7}{2}^-$ level at 2.10 MeV, which may correspond to the experimentally observed 1.97-MeV weak state. This would primarily require a $1f_{5/2}$ hole coupled to the 2^+ first excited state in ^{100}Mo and can be excited by two-step processes. The ^{99}Nb states observed in β decay are shown in column c. As explained earlier, these states are assumed to have positive parity, and theoretical predictions for them were made assuming coupling of the $1g_{9/2}$, $2d_{5/2}$, $3s_{1/2}$, and $2d_{3/2}$ orbitals arbitrarily taken at 0.0, 2.0, 2.5, and 3.0 MeV, respectively, to the core. In column d we have shown the low-lying predicted levels, using the same parameters as for the negative-parity states. The correct number of levels is predicted, and the spins of the low-lying levels are consistent with the observed β-decay branches. The wave functions obtained in the present calculation are shown in Tables I and II. Using the mean value of the transitional probabilities $B(E2,\ 0^+\rightarrow 2^+)$[10,11] for ^{100}Mo and the parameters χ_1 and χ_2, the calculated absolute value of the spectroscopic quadrupole moment of the first 2^+ state in ^{100}Mo is found to be 0.49 b. This value lies between the predictions using the collective vibrational model and the rotational model. The former model gives $Q_{2^+}=0.0$ b and the latter model gives $\lvert Q_{2^+}\rvert = 0.68$ b. This result indicates that ^{100}Mo is probably a transitional nucleus in agreement with the speculations of Taketani *et al.*,[12] who suggest that the ^{98,100,102}Mo isotopes form a series of transitional nuclei leading to a new deformed region around $A\approx 104$.

TABLE II. The wave functions for the positive-parity states of ^{99}Nb are listed (see caption for Table I).

E_x (MeV)	I^π	$\lvert 0^+j\rangle$	$\lvert 2^+s_{1/2}\rangle$	$\lvert 2^+d_{3/2}\rangle$	$\lvert 2^+d_{5/2}\rangle$	$\lvert 2^+g_{9/2}\rangle$
0.0	$\frac{9}{2}^+$	0.972			−0.132	−0.196
0.30	$\frac{11}{2}^+$					1.00
0.84	$\frac{9}{2}^+$	0.169			−0.191	0.967
1.34	$\frac{7}{2}^+$			−0.012	−0.202	0.979
1.42	$\frac{5}{2}^+$	0.694	−0.152	−0.066	−0.144	−0.685

The known levels of ^{99}Nb are fitted quite well by the simple particle-core coupling model, and spectroscopic factors are also reproduced. Using the reduced transition probability for the $E2$ transition from the first excited 2^+ state to the ground state in ^{100}Mo, the reduced transition probabilities can be predicted for ^{99}Nb, but unfortunately no measurements are available.

ACKNOWLEDGMENTS

The authors thank Dr. P. Hoffmann-Pinther, Dr. T. Kishimoto, and Dr. R. L. Kozub for discussions on this work. One of us (P. K. B.) also thanks the Director, Dr. T. T. Sugihara, and the faculty and staff of the Cyclotron Institute for financial assistance and excellent working conditions.

[†] Work supported in part by the National Science Foundation and the Robert A. Welch Foundation.

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