Structure of ^{91}Mo via the (*p*, *d*) and (*d*, *t*) Reactions*

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The (p,d) and (d,t) reactions on ^{92}Mo have been studied at bombarding energies of 38.6 and 40.6 MeV, respectively. The (p,d) reaction Q value was measured to be -10.446 ± 0.015 MeV. Excitation energies and angular distributions were measured for levels up to 3.5 MeV excitation. The distorted-wave Born-approximation spectroscopic factors indicate that essentially all of the $1g_{9/2}$ and $2p_{1/2}$ strength and 60–65% of the $2p_{3/2}$ and $1f_{5/2}$ strength were observed. The results are generally in good agreement with the unified-model calculations of Kitching.

I. INTRODUCTION

The properties of $N=49$ nuclei have been the subject of recent theoretical[1,2] and experimental[3-5] investigations. Kitching[1] has recently calculated properties of levels in ^{91}Mo up to about 3 MeV excitation using a unified-model description in which single quasiparticle states are coupled to anharmonic-core vibrations. The structure of ^{91}Mo below 2 MeV excitation has been studied with (d, t)[3] and $(^3\mathrm{He}, \alpha)$[4] reactions at bombarding energies of 23 and 18 MeV, respectively. In the present work, nuclear structure information is obtained for levels up to 3.5 MeV excitation using the (p, d) and (d, t) reactions at 40-MeV bombarding energy. The results are compared with previous experimental work and the theoretical predictions of Ref. 1. A study of isobaric-analog states excited in the (p, d) and (d, t) reactions on Mo isotopes has already been reported.[6]

II. EXPERIMENTAL PROCEDURE

The experimental configuration and data-analysis procedure was similar to that reported previously.[7] Data were taken with 38.6-MeV protons and 40.6-MeV deuterons incident on an isotopically enriched ^{92}Mo target whose weighed thickness was 1.10 mg/cm^2. Two counter telescopes spaced 5° apart were used simultaneously to reduce data acquisition time, and selected data points were checked by measurement with both systems. Each telescope consisted of a silicon surface-barrier detector of 1.0-mm thickness for ΔE, a 3-mm Si(Li) detector for E, and a third detector in anticoincidence with ΔE and E to eliminate pulses due to elastically scattered particles. The over-all resolution obtained was about 50 keV full width at half maximum. A peak-fitting computer code was used to extract yields for partially resolved levels. Spectra for the (p, d) and (d, t) reactions on ^{92}Mo are shown in Figs. 1 and 2, respectively.

Energy calibrations were obtained from neutron-pickup reactions on ^{60}Ni and the ^{12}C and ^{16}O contaminants present in the target. Absolute cross-section normalizations were obtained by integrating the beam current and checked by measuring proton elastic scattering cross sections at forward angles. Excellent agreement with optical-model predictions ($\lesssim 5\%$) was obtained.

III. DISTORTED-WAVE BORN-APPROXIMATION CALCULATIONS

Distorted-wave Born-approximation (DWBA) calculations, including finite-range and nonlocal (FRNL) corrections, were performed using the computer code DWUCK[8] and optical-model parameters from the literature.[9-11] These parameters and the FRNL parameters are listed in Table I. The same deuteron parameters were used for both the (p, d) and (d, t) reactions. Form factors were calculated for potentials of radius $1.15A^{1/3}$ fm, diffuseness 0.65 fm, and $\lambda_{so}=25$, with binding energies equal to the experimental separation energies.

The calculated and experimental cross sections are related by

$$\left(\frac{d\sigma}{d\Omega}\right)_{\text{exp}} = \frac{NC^2S}{2J+1}\left(\frac{d\sigma}{d\Omega}\right)_{\text{DW}},$$

where J is the transferred angular momentum, N is the normalization constant determined from the internal structure of the projectiles, and C^2S is the spectroscopic factor. We have used $N=2.54$ in the present work for both the (p, d)[12] and (d, t)[13] reactions.

IV. RESULTS AND DISCUSSION

A summary of experimental results is given in Table II. Our excitation energies for low-lying levels are in excellent agreement with those mea-

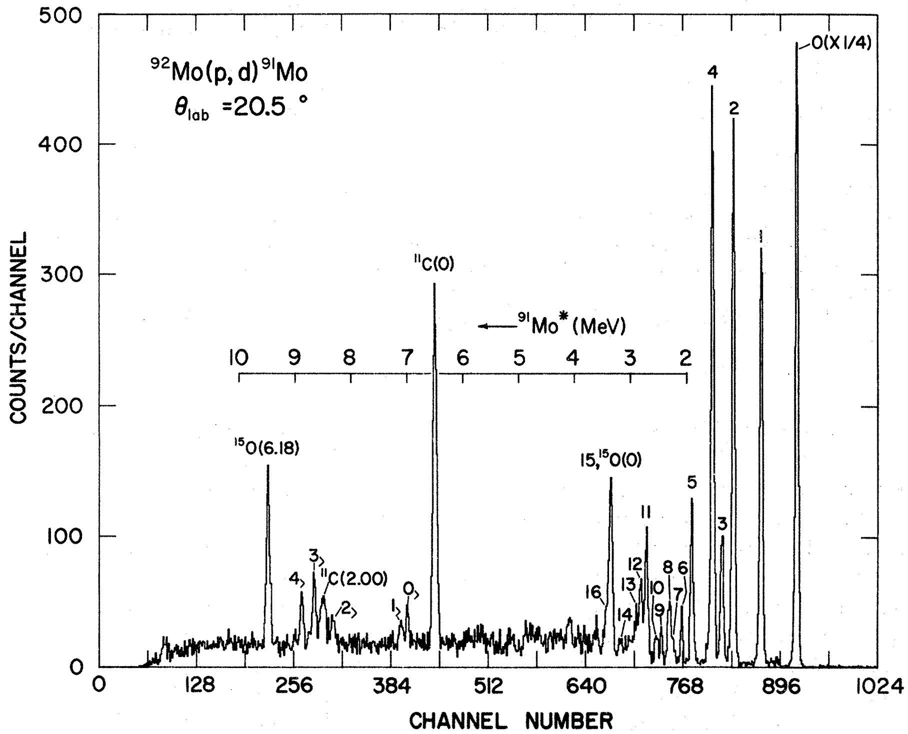

FIG. 1. Spectrum for the ^{92}Mo$(p,d)^{91}$Mo reaction at $\theta_{lab} = 20.5°$. An approximate ^{91}Mo excitation-energy scale is shown. Peak numbers having the subscript > are isobaric-analog states.

sured by Bassani and Picard,[4] who studied the ^{92}Mo(^{3}He, α)^{91}Mo reaction. In addition, we have measured the Q value for the ^{92}Mo$(p, d)^{91}$Mo reaction to be -10.446 ± 0.015 MeV. This corresponds to a mass excess of -82.209 MeV for ^{91}Mo, in agreement with the value of -82.188 ± 0.029 MeV derived by Wapstra and Gove.[14]

Angular distributions and DWBA predictions for

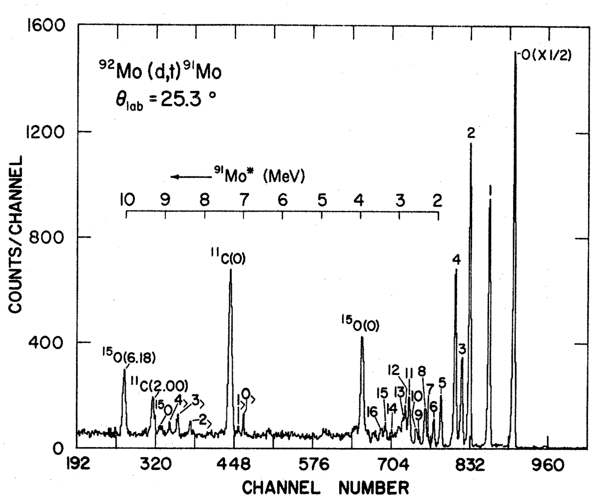

FIG. 2. Spectrum for the ^{92}Mo$(d,t)^{91}$Mo reaction at $\theta_{lab} = 25.3°$. See also Fig. 1 caption.

the (d, t) and (p, d) reactions on ^{92}Mo are shown in Figs. 3–7. The data are fitted reasonably well for both reactions, although the curves are slightly out of phase with the data for the (d, t) reaction in some cases. A pronounced J dependence, which arises largely from the deuteron spin-orbit potential, is predicted for the (p, d) reaction. However, no evidence for such an effect is observable in the data, and all the $l_n = 1$ and $l_n = 4$ data are best fitted by the $J = l_n + \frac{1}{2}$ calculations for $\theta_{c.m.} \lesssim 40°$ (see, e.g., the distributions for the 1.904- and 2.085-MeV levels in Fig. 4). The spectroscopic factors from the (p, d) and (d, t) reactions are in good agreement for $l_n = 1$ and $l_n = 3$ levels (Table II), while the (p, d) values are 20–50% lower for $l_n = 2$ and $l_n = 4$ transitions. Some of this is probably due to the relatively poor fits to the $l_n = 2$ (p, d) data in the vicinity of the forward maximum (Figs. 3–5), which gives rise to a normalization ambiguity. For $l_n = 4$ transitions, the momentum matching is not favorable for the (p, d) reaction at the bombarding energies and Q values involved here. The (p, d) DWBA transition amplitude is peaked at a considerably smaller radius than for the (d, t) reaction, so the calculated (p, d) cross section is more likely to contain unrealistic contributions from the nuclear interior.

The ground, 0.652-, 1.155-, and 1.533-MeV states of ^{91}Mo appear to contain most of the hole

TABLE I. Optical-model and FRNL parameters used in DWBA calculations (MeV fm units).

Particle	V	r_0	a	W	$4W_D$	r_I	a_I	V_{so}	r_{so}	β^2 [a]	R [b]
p [c]	47.4	1.170	0.750	5.84	12.56	1.320	0.570	6.20	1.010	0.85	
d [d]	100.8	1.099	0.835		53.64	1.344	0.747	6.53	1.099	0.54	0.695
t [e]	151.1	1.240	0.685	24.06		1.432	0.870			0.25	0.845
n		1.150	0.65					$\lambda_{so}=25$		0.85	

[a] Nonlocal parameter used in DWUCK.
[b] Finite-range parameter for (p, d) and (d, t) reactions, respectively, used in DWUCK.
[c] Proton parameters were obtained from the equations of Ref. 9:

$V=54-0.32E+0.4ZA^{-1/3}+24(N-Z)/A$,
$W=0.22E-2.7$,
$W_D=11.8-0.25E+12(N-Z)/A$,
$a_I=0.51+0.7(N-Z)/A$, with r_0, a, r_I, V_{so}, r_{so}, and $a_{so}=a$ fixed.

[d] See Ref. 10.
[e] See Ref. 11.

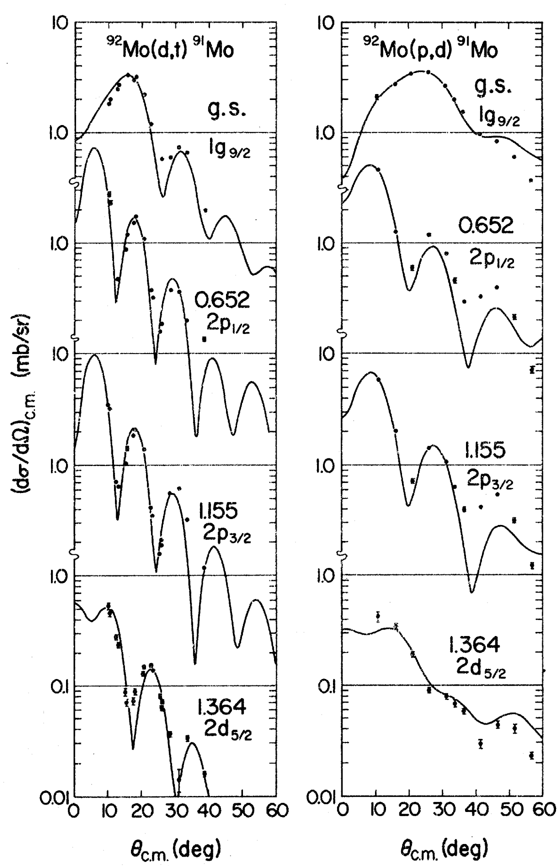

FIG. 3. Angular distributions for (d,t) and (p,d) reactions to the ground, 0.652-, 1.155-, and 1.364-MeV levels of ^{91}Mo. The curves are DWBA calculations for the nlj indicated.

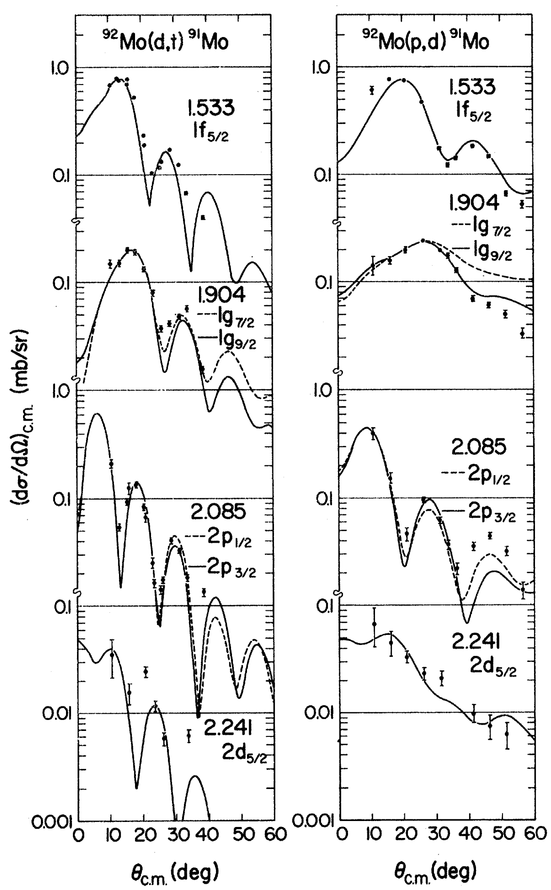

FIG. 4. Angular distributions for (d,t) and (p,d) reactions to the 1.533-, 1.904-, 2.085-, and 2.241-MeV levels of ^{91}Mo. The curves are DWBA calculations for the nlj indicated.

TABLE II. Summary of results from neutron-pickup reactions on ^{92}Mo.

Peak number [a]	^{91}Mo* (MeV ± keV)	l_n	J^{π} [b]	C^2S_n (p, d) [c]	(d, t) [c]	(d, t) [d]	$(^3$He, $\alpha)$ [e]
0	0	4	$\frac{9}{2}^+$	7.05	8.74	8.7	9.1
1	0.652 ± 5	1	$\frac{1}{2}^-$	1.50	1.65	1.7	2.7
2	1.155 ± 5	1	$\frac{3}{2}^-$	1.91	1.97	2.1	3.0
3	1.364 ± 10	2	$(\frac{5}{2})^+$	0.10	0.19	0.2	
4	1.533 ± 10	3	$\frac{5}{2}^-$	2.43	2.60	2.4	3.0
5	1.904 ± 15	4	$\frac{7}{2}^+$	0.87	1.51		
			$\frac{9}{2}^{+*}$	0.58	0.79		(0.9–1.3)
6	2.085 ± 15	1	$\frac{1}{2}^-$	0.15	0.18		
			$\frac{3}{2}^-$	0.14	0.15		
7	2.241 ± 20	(2)	$(\frac{3}{2}^+)$	0.023	(0.024)		
			$(\frac{5}{2}^+)$	0.019	(0.018)		
8	2.299 ± 15	1, 4	$(\frac{1}{2}, \frac{3}{2})^-$	(0.085)	(0.085)		
			$(\frac{9}{2}^+)$	(0.10)	(0.10)		
8	2.340 ± 15						
9	2.462 ± 25	3	$(\frac{5}{2})^-$	0.13	0.15		
10	2.547 ± 25	2	$\frac{3}{2}^+$	0.029	0.047		
			$\frac{5}{2}^+$	0.024	0.035		
11	2.727 ± 20	3	$(\frac{5}{2})^-$	0.60	0.65		
12	2.824 ± 20	4	$\frac{7}{2}^+$	0.54	1.11		
			$\frac{9}{2}^{+*}$	0.36	0.57		
13	2.898 ± 20	1	$\frac{1}{2}^-$	0.10	0.12		
			$\frac{3}{2}^-$	0.096	0.096		
14	3.188 ± 25	1	$\frac{1}{2}^-$	0.049	0.049		
			$\frac{3}{2}^-$	0.048	0.044		
15	3.330 ± 25	1,3	$(\frac{1}{2}, \frac{3}{2})^-$	(0.062)	(0.059)		
			$(\frac{5}{2})^-$	(0.16)	(0.15)		
16	3.455 ± 20	1	$\frac{1}{2}^-$	0.13	0.15		
			$\frac{3}{2}^-$	0.12	0.12		
$0_>$	6.99 ± 30	4	$\frac{9}{2}^+$	0.28	0.25		
$1_>$	7.12 ± 30	1	$\frac{1}{2}^-$	0.18	0.20		
$2_>$	8.34 ± 30	1	$(\frac{3}{2}^-)$	0.15	0.14		
$3_>$	8.66 ± 30	1	$(\frac{3}{2}^-)$	0.33	0.25		
$4_>$	8.87 ± 30	3	$(\frac{5}{2}^-)$	0.41	0.64		

[a] See Figs. 1 and 2.
[b] Where appropriate, preferred spins are denoted by an asterisk (*). For J assignments of peaks 0 through 5 see also Refs. 3 and 4.
[c] Present work. Values for $T_>$ levels were obtained using a form factor radius of $1.23A^{1/3}$ fm in the DWBA calculations (see Ref. 6).
[d] See Ref. 3.
[e] See Ref. 4.

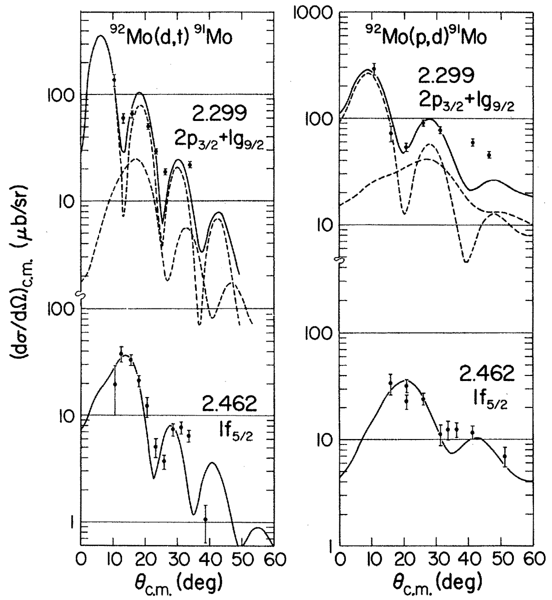

FIG. 5. Angular distributions for (d,t) and (p,d) reactions to the 2.299- and 2.462-MeV levels of ^{91}Mo. The curves are DWBA calculations for the nlj indicated.

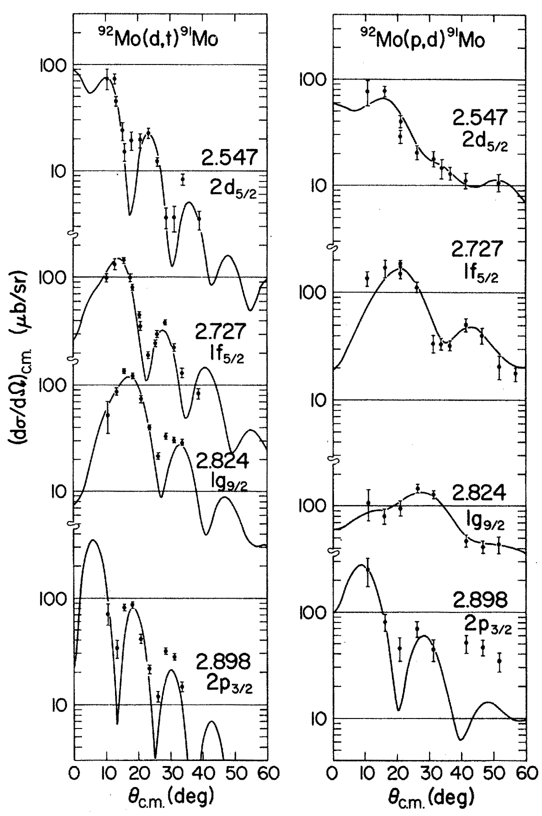

FIG. 6. Angular distributions for (d,t) and (p,d) reactions to the 2.547-, 2.727-, 2.824-, and 2.898-MeV levels of ^{91}Mo. The curves are DWBA calculations for the nlj indicated.

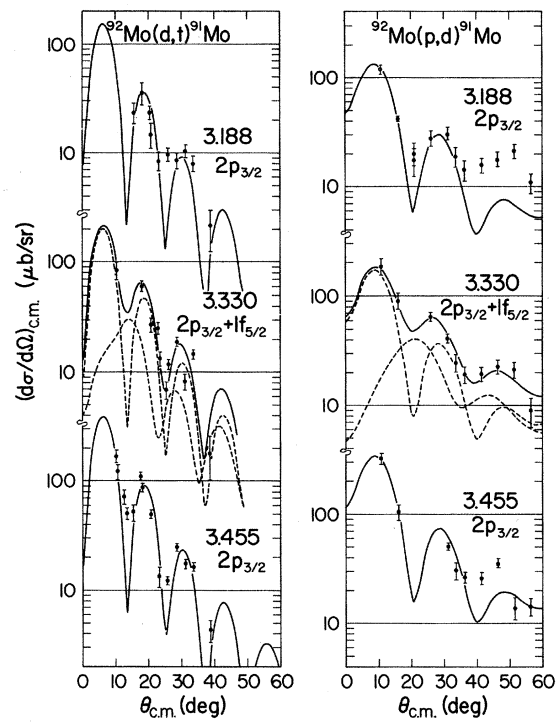

FIG. 7. Angular distributions for (d,t) and (p,d) reactions to the 3.188-, 3.330-, and 3.455-MeV levels of ^{91}Mo. The curves are DWBA calculations for the nlj indicated.

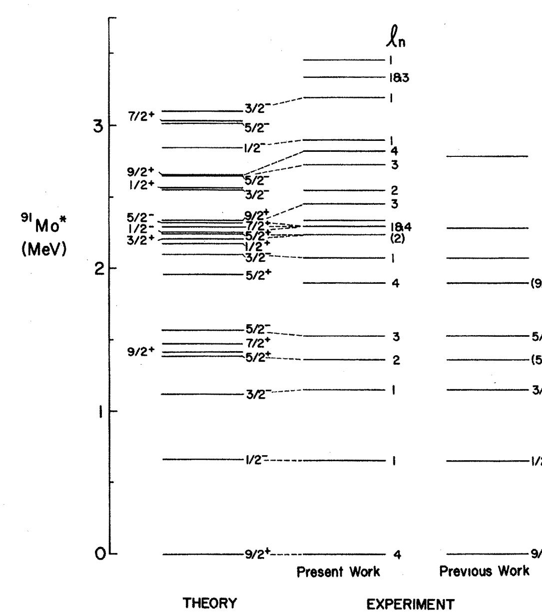

FIG. 8. Energy levels of ^{91}Mo. Theoretical calculations are those of Ref. 1 (levels having $J \geq \frac{11}{2}$ and $J^{\pi} = \frac{7}{2}^-$ and $\frac{9}{2}^-$ are not shown). The previous experimental work is that of Refs. 3 and 4.

strength from the $1g_{9/2}$, $2p_{1/2}$, $2p_{3/2}$, and $1f_{5/2}$ orbits, respectively, in agreement with previous work.[3,4] Our angular distributions for the 1.904-MeV level (Fig. 4) confirm the tentative $l_n = 4$ assignment for this state by Bassani and Picard.[4] Our spectroscopic factors for these levels are generally in good agreement with those of Ref. 3 (Table II).

In addition to the 1.364-MeV level, which was assigned $(\frac{5}{2}^+)$ by Ref. 3, the levels at 2.241 and 2.547 MeV appear to have angular distributions corresponding to $l_n = 2$ transfers (Figs. 3, 4, and 6). This suggests some mixing with the $2d_{5/2}$ and/or $2d_{3/2}$ shells in the ^{92}Mo ground state, although none of these levels is strongly excited ($C^2S \lesssim 0.2$). [The (d, t) data for the 2.241-MeV level are not conclusive (Fig. 4), but the (p, d) distribution appears to be dominated by an $l_n = 2$ shape.] The levels at 2.085, 2.898, 3.188, and 3.455 MeV all have $l_n = 1$ angular distributions (Figs. 4, 6, and 7) and are assigned $(\frac{1}{2}, \frac{3}{2})^-$. The 2.462- and 2.727-MeV levels are excited by $l_n = 3$ transfers (Figs. 5 and 6) and are assigned $(\frac{5}{2})^-$ from shell-model considerations. An $l_n = 4$ level is observed at 2.824 MeV (Fig. 6) with a $\frac{9}{2}^+$ spectroscopic factor of about 0.5. A spin of $\frac{7}{2}$ for this level would imply a large $1g_{7/2}$ admixture in the ^{92}Mo ground state, which seems unlikely in view of the relatively small $[2d_{5/2}]^2$ components (Table II).

Particle groups 7 and 8 (Figs. 1 and 2) were separated by use of a peak-fitting code. Three Gaussian peaks were required to fit the data, indicating that group 8 contains two peaks at 2.299 and 2.340 MeV excitation. The data from both reactions for the 2.299-MeV component indicate some excitation by an $l_n = 1$ transfer (Fig. 5), and the (p, d) data are best fitted if an $l_n = 4$ component is also included. A similar combination is shown for the (d, t) data, although the fit is not significantly improved over that for a pure $l_n = 1$ transition in this case. No meaningful angular distributions were obtained for the weakly excited 2.340-MeV component. The angular distributions for the group at 3.330 MeV excitation in ^{91}Mo (Fig. 7) indicate that this group contains at least two levels. The data are fitted quite well by a combination of $l_n = 1$ and $l_n = 3$ DWBA curves. The levels observed at 6.99, 7.12, 8.34, 8.66, and 8.87 MeV excitation correspond to isobaric analogs of proton-hole states in ^{91}Nb, and are the subject of previous papers.[6]

TABLE III. Summed strengths and hole-state energy centroids.

	$\sum C^2 S_<$			$\langle E \rangle$ (MeV)	
nlj	(p, d)	(d, t)	Sum rule [a]	Expt.	Theory [b]
$1g_{9/2}$	8.09	10.20	9.7	0.31	
$2p_{1/2}$	1.69	1.86	1.8	0.87	0.93
$2p_{3/2}$	2.28	2.34	3.6	1.43	1.37
$1f_{5/2}$	3.32	3.55	5.4	1.87	1.85
$2d_{5/2}$	0.14	0.25	0		

[a] Determined from data of Refs. 6 and 7, and H. Ohnuma and J. L. Yntema, Phys. Rev. 176, 1416 (1968); 178, 1654 (1969).

[b] Quasiparticle energies of Ref. 1.

V. SUMMARY AND CONCLUSIONS

The level scheme of ^{91}Mo obtained from the present experiment is compared with previous experimental results[3,4] and the theoretical predictions of Kitching[1] in Fig. 8. (The levels of Ref. 1 having $J \geq \frac{11}{2}$ and $J^\pi = \frac{7}{2}^-$ or $\frac{9}{2}^-$ have been omitted.) Levels are predicted which correspond with most of our measurements to within 200 keV. A notable exception is the $l_n = 4$ level at 1.904 MeV, which is predicted to lie about 500 keV lower in energy.

The total hole strengths obtained in the present work are compared with sum-rule limits in Table III. In summing the spectroscopic factors, the $l_n = 1$ components at 0.652, 2.299, and 2.898 MeV excitation were assumed to have $J = \frac{1}{2}$ on the basis of theoretical predictions (Fig. 8), while all other $l_n = 1$ levels are included in the $2p_{3/2}$ sum. With the exception of the 2.340-MeV level, all other $T_<$ levels are included in the $2d_{5/2}$, $1f_{5/2}$, and $1g_{9/2}$ sums. It appears that essentially all of the strength from the $1g_{9/2}$ and $2p_{1/2}$ orbits has been observed, as well as 60–65% of the $2p_{3/2}$ and $1f_{5/2}$ strength. The hole-state energy centroids (i.e., the spectroscopic factor-weighted averages) from the present work are in excellent agreement with the quasiparticle energies of Ref. 1 (Table III). The experimental centroids for $2p_{3/2}$ and $1f_{5/2}$ might be slightly higher in energy, however, since not all the hole strength from these orbits was observed.

The total $l_n = 2$ strength of 0.2 would indicate that the $1g_{9/2}$ subshell is about 98% filled in ^{92}Mo, i.e., the probability for a $[g_{9/2}]^{10}$ configuration is about 0.9. It has been suggested[1] that the lowest-lying $\frac{5}{2}^+$ level in each $N = 49$ nucleus is basically of a collective nature and excited by a two-step process, since this level in ^{87}Sr is excited only weakly in both stripping and pickup reactions, with the main single-particle state unpopulated in the pickup reaction. However, the fact that the ground-state proton configurations of ^{86}Sr and ^{88}Sr are quite different[15] might also give rise to these single-neutron-transfer results for ^{87}Sr, even if only direct, one-step processes are involved. If a core excitation is involved in the transition to the 1.364-MeV level in ^{91}Mo, the $N = 50$ shell closure for ^{92}Mo is even more complete than our results suggest.

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Study of Hole-State Analogs in Mo Isotopes*

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A study of isobaric-analog states (IAS) excited in the (p,d) and (d,t) reactions on ^{92,94,96}Mo, the $(^3\text{He},\alpha)$ reaction on ^{92,96}Mo, and the $^{98}\text{Mo}(p,d)^{97}\text{Mo}$ reaction is reported. Coulomb energies and angular distributions were measured for analogs of all parent states observed in a $(d,{}^3\text{He})$ study on Mo targets, and some additional nuclear structure information was obtained. Distorted-wave Born-approximation calculations were performed, and the resulting spectroscopic factors reveal a pronounced decrease in $l=1$ hole strength with increasing mass number as well as fluctuations in the ratios $C^2S_n(p,d)/C^2S_n(d,t)$. The first effect is discussed in terms of mixing with the dense spectrum of $T_<$ levels in the IAS region.

I. INTRODUCTION

Isobaric-analog states (IAS) in the $A=90$ region have been studied extensively via proton resonances[1] and, more recently, in proton stripping reactions.[2-4] However, no extensive studies of $T_>$ hole states have been reported for this region, which probably reflects the fact that relatively high bombarding energies are required to obtain meaningful direct-reaction data for the IAS. In the present work, IAS excited in the (p, d) and (d, t) reactions on ^{92,94,96}Mo and the $^{98}\text{Mo}(p, d)^{97}\text{Mo}$ reaction were investigated. Differential cross-section measurements were also obtained at a few angles for the $(^3\text{He}, \alpha)$ reaction on ^{92,96}Mo. A comparison of spectroscopic strengths deduced from these reactions is presented, with further comparisons to the parent state results of Ohnuma and Yntema (OY)[5] who studied the $(d, {}^3\text{He})$ reaction on the molybdenum isotopes. A brief account of this work has been reported elsewhere.[6]

II. EXPERIMENTAL PROCEDURE

The experimental configuration and data analysis procedure was similar to that reported previously.[4] Data were taken with 38.6-MeV protons, 40.6-MeV deuterons, and 35.0-MeV helions incident on isotopically enriched Mo targets whose weighed thicknesses ranged from 0.554 to 1.10 mg/cm^2. Two