Differential cross section for n-p elastic scattering in the angular region $50° < \theta^* < 180°$ at 459 MeV

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The differential cross section for n-p elastic scattering at 459 MeV in the c.m. angular region $50° < \theta^* < 180°$ has been measured with high statistical precision and good relative accuracy. The uncertainty in the absolute normalization (based on the simultaneously measured yield of deuterons from the $np \rightarrow d\pi^0$ reaction) was initially estimated to be $\sim 7\%$. The results agree well with back-angle data obtained independently at LAMPF but less well with results from Saclay and the Princeton-Pennsylvania Accelerator and, except for a normalization difference of 10%, are fairly well represented by a phase-shift fit. The pole-extrapolation method of Chew was used to extract the pion-nucleon coupling constant f^2 from the back-angle portion of the data. The value obtained, $f^2=0.069$, is somewhat smaller than the values 0.0735–0.0790 obtained from analyses of pion-nucleon scattering, tending to confirm the need for an upward renormalization of the angular distribution by $\sim 10\%$.

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I. INTRODUCTION

Knowledge of the nucleon-nucleon (N-N) interaction in the intermediate energy region (up to 800 MeV) has been greatly improved in the past two decades. In fact, the variety and quality of p-p scattering data are sufficient to lead to an unambiguous and fairly accurate determination of the $I=1$ phase-shift solutions throughout the region. Since the data for n-p scattering are fewer in number and generally less accurate, the determination of the $I=0$ partial waves is far less certain, especially at the higher energies, even on the assumption that all but the lowest $I=1$ partial waves are the same for n-p as for p-p scattering. The need for data of higher accuracy persists, especially since disagreements remain between the results obtained by different groups. Earlier papers published by the present group added significantly to the body of data. These papers presented measurements of the n-p differential cross section $d\sigma/d\Omega^*$ for wide angular regions at 647 MeV [1] and at 802 MeV [2], which resolved the considerable disagreements between earlier measurements. The results being reported here were obtained with the same apparatus and methods as were described in the earlier reports [1,2], but are for a lower energy.

Compilations of the existing N-N data maintained by Arndt *et al.* [3] and by Bystricky and Lehar [4] list four previous measurements of $d\sigma/d\Omega^*$ for n-p elastic scattering in the neighborhood of 460 MeV: results from the Princeton-Pennsylvania Accelerator (PPA) at 466 MeV for the c.m. angular region $56° < \theta^* < 180°$ [5], data from Saclay at 457 MeV for the region $152° < \theta^* < 180°$ [6], and results from LAMPF (the Clinton P. Anderson Meson Physics Facility) at 451 and 473 MeV for the region $118° < \theta^* < 180°$ [7]. While these data sets are in fair agreement at extreme back angles ($\theta^* \sim 180°$), they rapidly diverge as θ^* decreases; near 160°, for example, the Saclay values are $\sim 40\%$ higher than the PPA values, and the LAMPF values fall between these two extremes. The cross section values presented in the present paper are most nearly in agreement with the results of Ref. [7].

II. EXPERIMENTAL METHOD

Since the apparatus and techniques used in the present experiment were the same as those used in the previous experiments at 647 and 802 MeV [1,2], only the briefest discussion will be given here. The neutron beam was obtained by passing the LAMPF proton beam through a liquid deuterium (LD_2) target, and magnetically deflecting it into a heavily shielded beam dump. Neutrons produced by the $^2H(p,n)$ reaction in the target were tightly collimated at 0° forming a nearly monoenergetic beam, consisting of a rather narrow and intense peak of charge-exchange (CE) neutrons at about the proton beam energy and a broad spectrum of lower-energy neutrons (much less intense) coming from pion-production and three-body-breakup processes. Only the neutrons in the CE peak were used in the experiment. A target of liquid-hydrogen (LH_2) of thickness 13.2 cm was placed in the path of this beam, and individual charged particles

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produced by n-p reactions in the target were detected with a magnetic spectrometer containing two scintillator planes and four multiwire proportional chambers (MWPCs). Scintillator S_1 at the front of the spectrometer, and S_2 at the back of it, provided timing information used primarily for particle identification. Two of the MWPCs (W_1 and W_2) were placed in front of the magnet, and two (W_3 and W_4) in back of it. Each MWPC provided horizontal (x) and vertical (y) coordinates for a point on the path of the particle, thus overdetermining its trajectory. The laboratory scattering angle was determined from the "hit" positions in W_1 and W_2, and the angle of magnetic deflection ($\sim 22°$) was given by the hit positions in all four chambers, leading to momentum determination of accuracy $\sim \pm 0.35\%$, which corresponds to an uncertainty in neutron beam energy of $\sim \pm 3$ MeV. The spectrometer could be rotated about a vertical axis centered on the target. Its angular acceptance was $\sim 4°$, and its nominal angle setting was changed from 0° to 60° in increments of $\sim 4°$. The uncertainties in laboratory proton-recoil angle arose from multiple scattering in the LH_2 target and scintillator S_1 ($\pm 0.13°$ at $\sim 0°$ to $\pm 0.60°$ at $\sim 60°$), from multiple scattering and geometrical resolution effects within the spectrometer ($\pm 0.09°$ at 0° to $\pm 0.30°$ at $\sim 60°$), and from uncertainties in the spectrometer position and the MWPC alignment (0.10°). Combined in quadrature these give overall uncertainties in laboratory angles ranging from $\pm 0.19°$ at $\sim 0°$ to $\pm 0.68°$ at $\sim 60°$. The corresponding errors in θ^* range from $\pm 0.42°$ at 180° to $\pm 1.28°$ at $\sim 60°$.

Protons were distinguished from deuterons and pions of the same momentum by the difference in their flight time through the spectrometer and by the difference in the pulse height they produced in S_2. The latter constraint was important because accidental coincidences corrupted a fraction of the flight times. At each spectrometer angle, data were taken with the LH_2 target both filled and emptied, so as to determine the background coming from scattering in the target cell walls. This background rate was 5%–11% of the total rate. The fraction of time spent determining it varied from $\sim 20\%$ where it was low to $\sim 35\%$ where it was higher. The number of events accepted for full-target runs varied from $\sim 474\,000$ and $\sim 410\,000$ at spectrometer settings of 0° and 4°, respectively, through a minimum of $\sim 32\,000$ at 36°, and up to another maximum of $\sim 70\,000$ at 60°.

The criterion for acceptance of an event was a coincidence between S_1, S_2, and signals from at least three of the x and three of the y wire planes. All data for accepted events were sent to a computer, which wrote them onto magnetic tape for off-line analysis, but also processed a fraction of the events on-line, generating histograms and two-dimensional plots for on-line display. In the subsequent off-line analysis, the momentum of the particle was determined for each event from the MWPC coordinate data and a map of the magnetic field of the spectrometer. The procedure began with the coordinates of the incident path and an estimate of the momentum (provided by the angle of magnetic deflection). The horizontal deflection of the particle as it passed through the spectrometer was then calculated by numerical integration, yielding calculated coordinates of the emergent particle, which were compared with the observed coordinates. A χ^2 minimization process was then used to adjust the incident coordinates and momentum for optimum agreement between the calculated and measured coordinate values. More detailed discussion of this procedure can be found in Ref. [1].

The neutron beam flux was monitored at the collimator exit by a pair of counter telescopes placed symmetrically at 25° to the left and right of the beam axis, which detected charged particles recoiling from a polyethylene disk of thickness 2.54 cm placed in the beam. The ratio of counts in the left and right telescopes could be used as an indicator of the stability of the beam profile. Typically, the statistical uncertainty in this measurement was $\sim \pm 0.2\%$ or better, and occasionally as small as 0.1%. Seldom did this ratio for a given run differ by more than 0.3% from its overall average for the whole experiment. The absolute calibration of the monitor was achieved by measurement, in the MWPC spectrometer, of the yield of deuterons from the $np \rightarrow d\pi^0$ reaction; by isospin arguments the total cross section σ_n for this reaction is expected to be one-half of the total cross section σ_p for the $pp \rightarrow d\pi^+$ reaction at the same total energy in the c.m. system, and this latter cross section was presumed to be reasonably well known ($\sim 5\%$ accuracy [8]). At the time of the analysis, the overall uncertainty in this method of normalization was estimated to be $\sim 7\%$ [9] (although there now is reason to believe that this error may have been an underestimate). On a plot of momentum *vs* laboratory scattering angle, the deuterons from the $np \rightarrow d\pi^0$ reaction fall on a well-defined locus, different from that of the protons from $np \rightarrow pn$ scattering. The deuteron data were analyzed separately, in order to determine their relative angular distribution in the c.m. system, which was assumed to have the functional form

$$\frac{d\sigma}{d\Omega^*} \propto A + \cos^2\theta^* + B\cos^4\theta^* \ .$$

The values of A and B were determined by a least-squares fit to the data and were found to be $A = 0.223$ and $B = -0.092$ [10]. The final-state particles from the $np \rightarrow d\pi^0$ reaction at the energy of this experiment (459 MeV) have the same total c.m. energy as those from the $pp \rightarrow d\pi^+$ reaction at 462 MeV. At that energy the total cross section for the latter reaction was taken to be $\sigma_p = 1.66$ mb. Thus the value assumed for the $np \rightarrow d\pi^0$ total cross section in calibration of the neutron flux monitor was $\sigma_n = 0.83$ mb. This will be discussed further in Sec. III C.

The incident neutron spectrum was reconstructed from the observed recoil proton spectrum by use of the known n-p kinematics. Only those events which fell within a narrow window containing the quasifree CE peak of the reconstructed neutron spectrum were used in the n-p cross-section determination. As the spectrometer was moved to larger angles, the CE peak of the reconstructed spectrum was broadened and shifted by plural and multiple scattering and energy-loss effects in the LH_2 target and the spectrometer, and window placement became less certain. Correction for this was provided by a Monte

Carlo calculation, which is described in Ref. [1]. The corrections were small at small spectrometer angle settings, but became as large as 5.5% at some of the larger angles. The fractional accuracy of the corrections is estimated to be 10%. Small corrections were also made for absorption of both protons and deuterons in the target and spectrometer. For deuterons these varied smoothly from 2.2% at deuteron energy 295 MeV to 2.4% at 250 MeV. For protons the corrections were 0.1% or less.

III. RESULTS AND DISCUSSION

The final center-of-mass differential cross-section values are presented in Table I, in the form $d\sigma/d\Omega^*$ as well as in the alternative form $d\sigma/du$ (where $-u$ is the square of the four-momentum transfer to the recoil proton), along with the corresponding values of u and the center of mass angle θ^*. The data obtained with the spectrometer set at 0° spanned the c.m. angular range $176° < \theta^* < 184°$. Since no significant asymmetry about 180° was seen, the points were pooled and are shown in the 176°–180° range. The errors listed are statistical only. The corrections discussed in the preceding section have uncertainties which are much smaller than these statistical errors. The uncertainty in the $\sim 2\%$ deuteron absorption correction contributes negligibly to the 7% normalization uncertainty assigned to the entire angular distribution. This normalization is based on the assumption that the total cross section $\sigma_n(459)$ for the $np \rightarrow d\pi^0$ reaction at 459 MeV is 0.83 mb. If a better value of $\sigma_n(459)$ is obtained in the future, the present cross section values should be renormalized by the factor $\sigma_n/0.83$ mb. As noted earlier, the uncertainties in θ^* range from $\pm 0.42°$ at 180° to $\pm 1.28°$ at $\sim 60°$. The corresponding errors in $d\sigma/d\Omega^*$ are negligible at 180° and ~ 0.07 mb/sr at 60°.

A. Comparison with other experiments

The results of the present experiment are compared with other n-p differential cross-section measurements available in the energy region 450–475 MeV in Fig. 1. The full angular distribution is shown in Fig. 1(a), and compared there with the results from PPA at 466 MeV [5], which span the same angular region. Although there is a qualitative similarity of the two data sets, the differences are very real, especially in the region of the cross-section minimum near 100°. The data reported here also are not in good agreement with the results from Saclay at 457 MeV [6] [Fig. 1(b)]. Actually, the agreement was better before the second renormalization [6] of the Saclay data. The agreement with the LAMPF measurements at 451 and 473 MeV [7] [Fig. 1(b)] is rather good over most of the angular region. This is not very surprising since most of the methods and apparatus, and in particular the method of normalization, were the same in Ref. [7] as in the present experiment. The main difference is that the measurements reported here were made with a monoenergetic neutron beam while the beam of Ref. [7] had a continuum of neutron energies ("white spectrum") which were binned according to time of flight. The reason for the disagreement at extreme back angles is not understood.

Since the n-p differential cross section varies only slowly with energy, and there exist accurate measurements with good absolute normalization from TRIUMF at 418 and 493 MeV [11], a comparison with those data seems worthwhile, even though they are outside the database of phase-shift fit C450 (see next paragraph). This comparison is made in Fig. 2. The clutter has been reduced in this plot by averaging neighboring points in pairs between 130° and 170°, and in groups of four below 130°. Similarly, the TRIUMF data have been averaged (where appropriate) to give a point spacing of $\sim 2°$. An upward renormalization of the present data by $\sim 7\%$ would put the two experiments more or less in agreement.

B. Phase-shift fit

A phase-shift prediction of the differential cross section, obtained with the SAID computer program of Arndt *et al.* [3] and labeled C450, is also shown in Fig. 1. The database for this phase-shift analysis (PSA) contains published and unpublished p-p, n-p, and p-n (quasifree) data in the 425–475 MeV energy region. The differential cross-section values included are those of the present experiment and the previously published LAMPF results [7], but not those from PPA [5] or Saclay [6]. Polarization or analyzing power data included are those from LAMPF [12,13], TRIUMF [14,15], and Chicago [16]. Also included are some spin-correlation data from TRIUMF [14] and LAMPF [17], a few polarization transfer data [15,18,19], and some measurements of the Wolfenstein D, R, and A parameters [16]. While they are limited in number, these "two-spin" measurements are of critical importance to the phase-shift analysis.

The fit was made with 18 free parameters, including partial-wave phase shifts up to $^3J_6$ and coupling parameters up to ϵ_6. The $I=1$ parameters were determined primarily by the p-p data and the $I=0$ parameters by the n-p data. The n-p data set included 583 points and the fitting program reconciled differences between the various data sets by allowing each to have a floating normalization. The χ^2 value for the fit to the renormalized points was 798, giving χ^2_ν (χ^2 per degree of freedom) of 1.41. The fact that this number is greater than 1.0 indicates the presence of unknown random errors other than those of counting statistics. The renormalization factor applied to the present data in the C450 fit was 1.10, somewhat larger than the 7% normalization uncertainty originally assumed for these data, primarily because of uncertainty in the $pp \rightarrow \pi^+ d$ cross section. Possible isospin noninvariance effects are presumed to be small but cannot be ruled out.

C. Charge-exchange region

It has long been known [20] that the shape of the peak in the backward angle (CE) region can be described rather well by the empirical double-exponential formula

$$\frac{d\sigma}{du} = \alpha_1 \exp(\beta_1 u) + \alpha_2 \exp(\beta_2 u) .$$

Least-squares fits to the backward-angle data of the ex-

TABLE I. Differential cross section for n-p elastic scattering at 459 MeV.

θ^* (deg)	$d\sigma/d\Omega^*$ (mb/sr)	$-u$ $(GeV/c)^2$	$d\sigma/du$ $[mb/(GeV/c)^2]$	θ^* (deg)	$d\sigma/d\Omega^*$ (mb/sr)	$-u$ $(GeV/c)^2$	$d\sigma/du$ $[mb/(GeV/c)^2]$	θ^* (deg)	$d\sigma/d\Omega^*$ (mb/sr)	$-u$ $(GeV/c)^2$	$d\sigma/du$ $[mb/(GeV/c)^2]$
179.65	9.601±0.281	0.0000	140.21±4.10	136.62	2.272±0.052	0.1176	33.15±0.75	91.35	1.208±0.036	0.4205	17.62±0.52
178.95	9.693±0.130	0.0001	141.42±1.90	135.94	2.059±0.050	0.1212	30.04±0.73	90.72	1.112±0.035	0.4253	16.22±0.51
178.24	9.485±0.100	0.0002	138.38±1.45	135.26	2.150±0.051	0.1248	31.36±0.74	89.10	1.208±0.045	0.4374	17.63±0.65
177.54	9.386±0.107	0.0004	136.94±1.56	134.57	2.096±0.049	0.1284	30.58±0.72	88.47	1.094±0.043	0.4422	15.96±0.62
176.84	9.355±0.095	0.0007	136.48±1.38	133.89	2.009±0.048	0.1321	29.31±0.70	87.84	1.190±0.045	0.4469	17.35±0.66
176.13	9.123±0.118	0.0010	133.09±1.72	130.00	1.645±0.054	0.1538	24.00±0.78	87.21	1.237±0.045	0.4516	18.05±0.66
175.43	8.935±0.120	0.0014	130.35±1.74	129.32	1.835±0.057	0.1577	26.77±0.83	86.58	1.250±0.046	0.4564	18.24±0.67
174.73	8.432±0.168	0.0018	123.02±2.45	128.65	1.826±0.057	0.1617	26.63±0.83	85.95	1.272±0.047	0.4611	18.56±0.68
174.56	8.037±0.153	0.0019	117.26±2.23	127.97	1.776±0.056	0.1657	25.91±0.81	85.32	1.279±0.047	0.4658	18.66±0.69
173.86	8.184±0.151	0.0025	119.41±2.21	127.63	1.655±0.055	0.1677	24.15±0.80	85.45	1.199±0.045	0.4648	17.49±0.66
173.16	7.843±0.148	0.0031	114.43±2.16	127.29	1.637±0.054	0.1697	23.86±0.79	84.82	1.253±0.046	0.4695	18.28±0.67
172.81	7.692±0.148	0.0034	112.22±2.17	126.96	1.704±0.054	0.1717	24.86±0.79	84.69	1.179±0.046	0.4705	17.21±0.67
172.45	7.595±0.147	0.0037	110.80±2.15	126.62	1.719±0.056	0.1738	25.08±0.81	84.19	1.265±0.048	0.4742	18.46±0.70
172.11	7.478±0.146	0.0041	109.11±2.13	126.28	1.495±0.052	0.1758	21.81±0.76	83.57	1.316±0.049	0.4789	19.20±0.71
171.75	7.419±0.148	0.0045	108.24±2.15	125.61	1.621±0.054	0.1799	23.65±0.78	82.94	1.292±0.048	0.4836	18.84±0.70
171.40	7.395±0.147	0.0048	107.89±2.14	124.94	1.524±0.052	0.1840	22.23±0.76	82.32	1.294±0.049	0.4882	18.88±0.71
170.70	6.902±0.140	0.0057	100.70±2.04	124.26	1.497±0.052	0.1882	21.85±0.76	81.69	1.338±0.049	0.4929	19.52±0.72
170.00	6.540±0.138	0.0065	95.41±2.01	123.59	1.591±0.054	0.1924	23.21±0.78	81.07	1.360±0.050	0.4975	19.85±0.73
169.30	6.528±0.138	0.0075	95.24±2.01	122.92	1.430±0.051	0.1966	20.86±0.74	79.75	1.378±0.038	0.5073	20.11±0.56
168.60	6.245±0.136	0.0085	91.11±1.99	122.25	1.350±0.049	0.2009	19.69±0.72	79.13	1.435±0.039	0.5119	20.93±0.57
167.89	5.891±0.131	0.0096	85.95±1.92	120.48	1.417±0.041	0.2122	20.68±0.60	78.51	1.522±0.040	0.5164	22.20±0.60
167.19	5.534±0.130	0.0107	80.74±1.90	119.81	1.409±0.041	0.2165	20.56±0.59	77.89	1.487±0.040	0.5210	21.69±0.59
167.06	5.608±0.110	0.0109	81.81±1.60	119.14	1.327±0.039	0.2209	19.36±0.57	77.27	1.459±0.041	0.5256	21.29±0.59
166.49	5.361±0.126	0.0119	78.22±1.84	118.48	1.355±0.040	0.2253	19.77±0.58	76.65	1.539±0.041	0.5301	22.46±0.60
166.36	5.286±0.108	0.0121	77.12±1.58	117.81	1.257±0.038	0.2297	18.33±0.56	76.03	1.520±0.041	0.5346	22.17±0.60
165.66	5.449±0.107	0.0134	79.50±1.56	117.14	1.319±0.040	0.2342	19.24±0.58	75.42	1.435±0.041	0.5391	20.94±0.60
164.96	4.933±0.106	0.0148	71.97±1.55	116.48	1.312±0.040	0.2386	19.14±0.58	74.80	1.549±0.042	0.5436	22.60±0.62
164.37	4.848±0.110	0.0159	70.73±1.60	115.82	1.269±0.039	0.2431	18.51±0.57	73.58	1.500±0.045	0.5524	21.89±0.66
164.26	5.289±0.107	0.0162	77.16±1.56	115.15	1.218±0.038	0.2476	17.77±0.56	72.96	1.607±0.046	0.5568	23.44±0.67
163.67	5.036±0.111	0.0174	73.47±1.62	114.49	1.186±0.039	0.2521	17.31±0.56	72.35	1.637±0.045	0.5612	23.88±0.66
163.56	4.601±0.103	0.0176	67.12±1.50	113.06	1.228±0.034	0.2620	17.92±0.50	71.74	1.667±0.046	0.5656	24.31±0.66
162.97	4.853±0.110	0.0189	70.80±1.60	112.40	1.127±0.033	0.2666	16.44±0.48	71.13	1.694±0.049	0.5700	24.72±0.71
162.86	4.636±0.102	0.0191	67.63±1.48	111.74	1.172±0.034	0.2712	17.10±0.49	70.75	1.598±0.046	0.5727	23.32±0.67

TABLE I. (*Continued*).

θ^* (deg)	$d\sigma/d\Omega^*$ (mb/sr)	$-u$ $(\mathrm{GeV}/c)^2$	$d\sigma/du$ $[\mathrm{mb}/(\mathrm{GeV}/c)^2]$	θ^* (deg)	$d\sigma/d\Omega^*$ (mb/sr)	$-u$ $(\mathrm{GeV}/c)^2$	$d\sigma/du$ $[\mathrm{mb}/(\mathrm{GeV}/c)^2]$	θ^* (deg)	$d\sigma/d\Omega^*$ (mb/sr)	$-u$ $(\mathrm{GeV}/c)^2$	$d\sigma/du$ $[\mathrm{mb}/(\mathrm{GeV}/c)^2]$
162.27	4.525±0.107	0.0205	66.02±1.57	111.08	1.154±0.033	0.2758	16.83±0.49	70.52	1.690±0.048	0.5743	24.65±0.70
161.57	4.546±0.105	0.0221	66.33±1.52	110.42	1.166±0.034	0.2804	17.01±0.50	70.14	1.739±0.046	0.5770	25.37±0.68
160.87	4.588±0.104	0.0238	66.94±1.52	109.76	1.168±0.034	0.2850	17.04±0.49	69.53	1.749±0.049	0.5813	25.52±0.71
160.17	4.403±0.104	0.0255	64.23±1.52	109.10	1.138±0.034	0.2897	16.61±0.50	68.92	1.751±0.047	0.5856	25.55±0.69
159.47	4.394±0.102	0.0273	64.10±1.49	108.45	1.129±0.034	0.2944	16.47±0.49	68.31	1.857±0.049	0.5898	27.09±0.72
158.77	4.292±0.104	0.0292	62.61±1.51	107.79	1.140±0.033	0.2991	16.63±0.49	67.70	1.842±0.050	0.5941	26.88±0.74
158.07	4.132±0.104	0.0311	60.28±1.51	105.73	1.134±0.045	0.3139	16.54±0.66	67.09	1.890±0.050	0.5983	27.57±0.73
155.93	4.010±0.079	0.0375	58.50±1.16	105.08	1.157±0.045	0.3186	16.87±0.66	66.48	1.874±0.050	0.6025	27.34±0.73
155.23	3.846±0.078	0.0396	56.11±1.13	104.42	1.125±0.044	0.3234	16.42±0.65	65.10	1.931±0.050	0.6120	28.18±0.73
154.53	3.970±0.078	0.0418	57.92±1.13	103.77	1.177±0.045	0.3281	17.17±0.65	64.50	1.911±0.050	0.6161	27.88±0.73
153.84	3.885±0.078	0.0441	56.69±1.13	103.12	1.122±0.046	0.3329	16.37±0.67	63.89	2.091±0.054	0.6202	30.51±0.79
153.14	3.740±0.076	0.0465	54.56±1.11	102.47	1.002±0.043	0.3376	14.62±0.62	63.29	2.038±0.053	0.6242	29.74±0.77
152.45	3.599±0.075	0.0488	52.51±1.09	102.15	1.062±0.044	0.3400	15.49±0.64	62.69	2.084±0.053	0.6283	30.41±0.77
151.75	3.617±0.075	0.0513	52.77±1.10	101.83	1.083±0.045	0.3424	15.79±0.66	62.08	2.148±0.054	0.6323	31.33±0.78
151.06	3.398±0.074	0.0538	49.58±1.07	101.51	1.047±0.044	0.3448	15.28±0.64	61.96	2.148±0.055	0.6331	31.35±0.80
150.36	3.542±0.075	0.0563	51.68±1.09	101.18	1.095±0.045	0.3472	15.97±0.65	61.48	2.129±0.055	0.6363	31.06±0.80
149.67	3.554±0.075	0.0589	51.85±1.10	100.86	1.092±0.044	0.3495	15.93±0.64	61.36	2.129±0.055	0.6371	31.06±0.80
147.53	3.141±0.060	0.0673	45.83±0.88	100.21	1.082±0.045	0.3543	15.78±0.65	60.76	2.173±0.056	0.6410	31.70±0.82
146.83	3.089±0.059	0.0702	45.06±0.86	99.57	1.159±0.045	0.3591	16.90±0.65	60.16	2.151±0.057	0.6449	31.38±0.83
146.14	3.185±0.060	0.0730	46.47±0.87	98.92	1.110±0.045	0.3639	16.20±0.66	59.56	2.216±0.058	0.6488	32.33±0.84
145.45	2.940±0.058	0.0759	42.89±0.84	98.28	1.052±0.045	0.3687	15.34±0.66	58.96	2.186±0.058	0.6527	31.90±0.84
144.76	3.061±0.059	0.0789	44.65±0.85	97.63	1.096±0.045	0.3735	16.00±0.66	58.36	2.318±0.061	0.6565	33.82±0.88
144.07	2.854±0.057	0.0819	41.64±0.84	96.99	1.051±0.045	0.3782	15.34±0.65	57.77	2.253±0.059	0.6604	32.88±0.86
143.38	2.826±0.057	0.0850	41.23±0.83	95.82	1.176±0.034	0.3870	17.16±0.50	56.67	2.381±0.049	0.6673	34.74±0.71
142.69	2.826±0.057	0.0881	41.23±0.83	95.18	1.107±0.034	0.3918	16.15±0.49	56.08	2.382±0.048	0.6710	34.76±0.70
142.00	2.771±0.056	0.0913	40.43±0.81	94.54	1.081±0.034	0.3966	15.78±0.49	55.48	2.448±0.050	0.6747	35.72±0.72
139.36	2.431±0.053	0.1038	35.47±0.78	93.90	1.207±0.035	0.4013	17.61±0.51	54.89	2.537±0.050	0.6784	37.01±0.74
138.68	2.524±0.054	0.1072	36.82±0.78	93.26	1.154±0.035	0.4061	16.84±0.50	54.29	2.527±0.049	0.6820	36.87±0.72
137.99	2.418±0.053	0.1107	35.28±0.77	92.63	1.144±0.034	0.4109	16.70±0.50	53.70	2.596±0.052	0.6856	37.88±0.76
137.31	2.379±0.052	0.1141	34.71±0.76	91.99	1.113±0.034	0.4157	16.24±0.50	53.11	2.649±0.052	0.6892	37.18±0.76

periment were attempted with sums of varying numbers of exponential terms, and χ^2_ν was found to be a minimum for the two-term fit. This fit is shown in Fig. 3, and the values obtained for the parameters α_1, β_1, α_2, and β_2 are presented in Table II, where they are compared with the parameters obtained by least-squares fitting of the backward-angle data from other experiments at nearby energies. The errors given are statistical errors of the fitting process and do not include systematic uncertainties. Considerable scatter is seen in the values obtained for some of these parameters. The comparison can be simplified by calculation of β, the $u \rightarrow 0$ limit of the logarithmic slope of $d\sigma/du$,

$$\lim_{u\rightarrow 0} \frac{d}{du} \ln \frac{d\sigma}{du} = \frac{\alpha_1\beta_1+\alpha_2\beta_2}{\alpha_1+\alpha_2} = \beta ,$$

which combines them into a single quantity, also shown in Table II. The quantity β is independent of normalization errors in each data set, and only characterizes the backward-angle shape of each angular distribution.

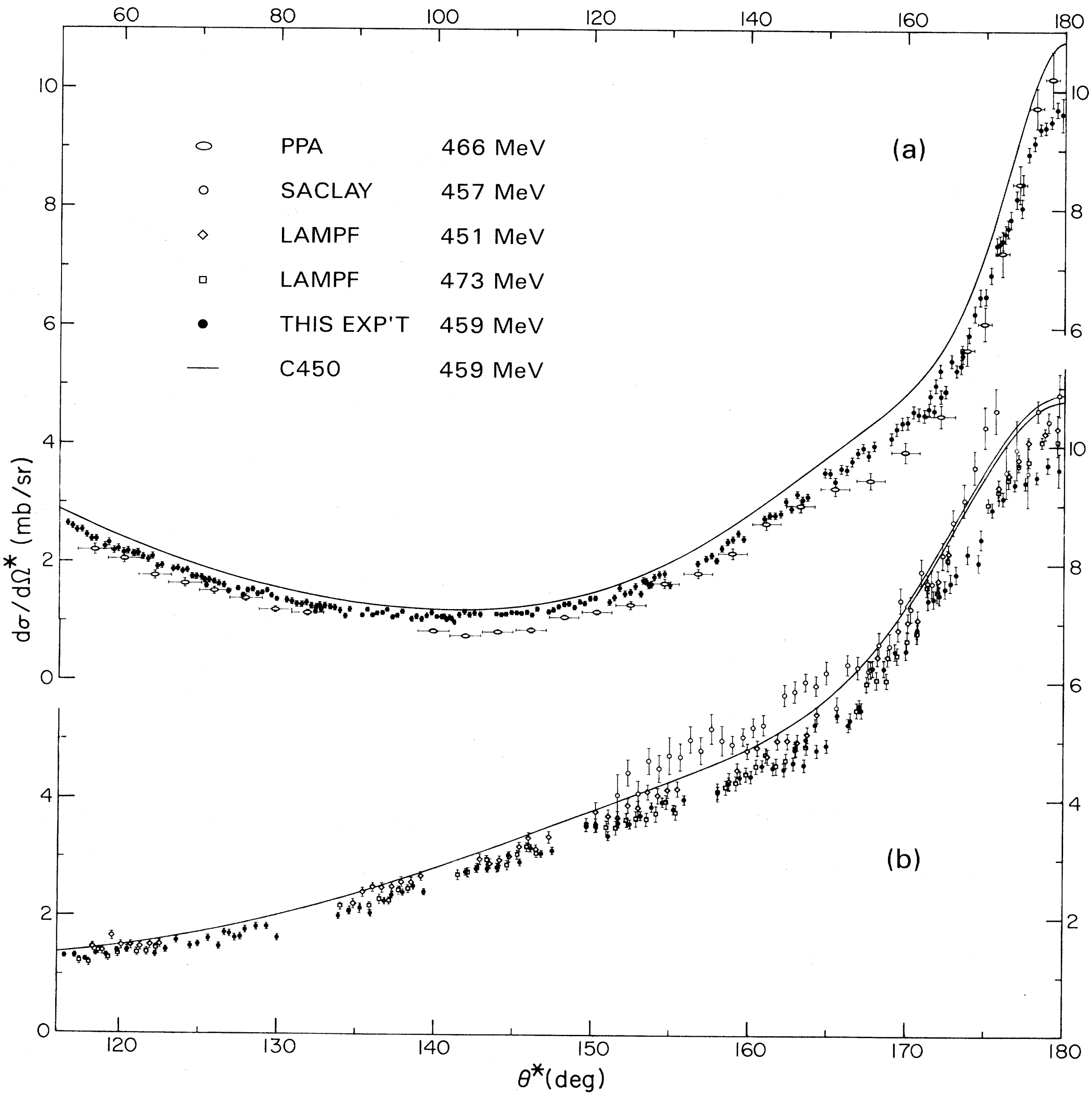

FIG. 1. Comparison of *n-p* differential cross section results of this experiment at 459 MeV with those of other experiments at nearby energies and with the single-energy phase-shift solution C450 given by SAID [3] (solid lines). (a) Comparison of the full angular distribution with data from PPA [5] at 466 MeV. (b) Comparison of the backward-angle part of the angular distribution (angle scale magnified) with data from Saclay [6] at 457 MeV and from LAMPF [7] at 451 and 473 MeV.

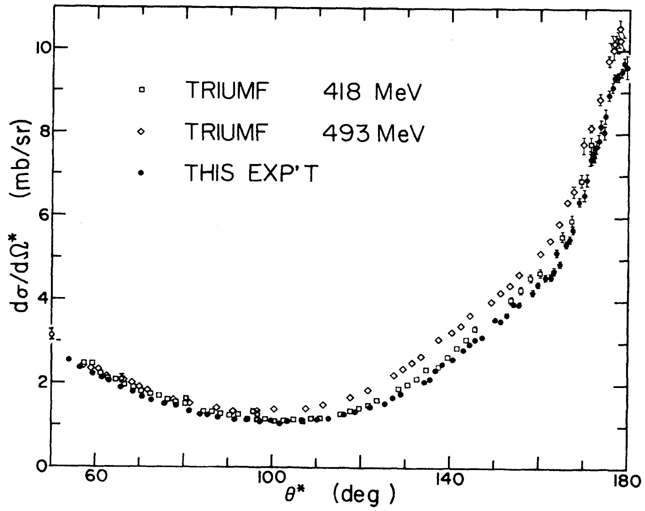

FIG. 2. Comparison of the results of this experiment with data from TRIUMF [11] at 418 and 493 MeV. The open circles are TRIUMF points obtained by detection of neutrons (81° and 96° at 418 MeV; 50°, 66°, and 97° at 493 MeV).

Better agreement is seen among the values obtained for β. Alternatively, the quantity $\alpha_1+\alpha_2$ (the $u\to 0$ limit of $d\sigma/du$) is independent of the shape but is directly related to the normalization of the backward-angle data; specifically, $d\sigma/d\Omega^*(180°)=k^2(\alpha_1+\alpha_2)/\pi$, where k is the nucleon c.m. momentum. These values are also shown in Table II. Included in the table are values obtained in an experiment done at SIN [21], very similar to the earlier LAMPF experiment [7] (the values of β must be read from a small figure in Ref. [21], but the values of $\alpha_1+\alpha_2$ are tabulated there). The value of $d\sigma/d\Omega^*(180°)$ given by the C450 phase-shift analysis solution is 10.82 mb. All of the tabulated values (except that for Ref. [6]) are lower than this by more than one standard deviation. In particular, the experiments normalized by means of the $np\to d\pi^0$ cross section (Refs. [7,21] and this experiment) are low by many standard deviations (those of [7] are 6% and 7% low, that of [21] is 12% low, and that from this experiment is 9% low). This is strong evidence that the $np\to d\pi^0$ cross section used in each case was too low.

A more meaningful test of the plausibility of the angular distribution at backward angles can be obtained by use of the pole-extrapolation method of Chew [22] in a

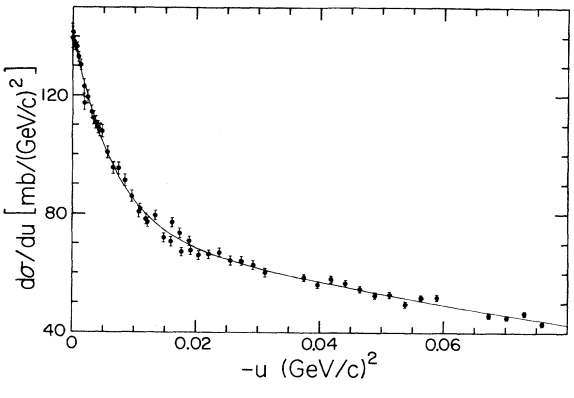

FIG. 3. Double-exponential fits to the large-angle ($\theta^*>145°$) data of this experiment, given by the parameters in Table II.

determination of the pion-nucleon coupling constant f^2. This is a test which is sensitive to the normalization as well as the shape and leads to a result of perhaps more physical significance. The method is based on the conjecture that there are poles in the real part of the N-N scattering amplitude caused by one-pion exchange at the unphysical values

$$\cos\theta^*=\pm(1+\mu^2/2k^2)$$

(where μ is the charged-pion rest mass) as well as branch points at

$$\cos\theta^*=\pm(1+4\mu^2/2k^2)\ ,\quad \pm(1+9\mu^2/2k^2)\dots\ ,$$

due to higher-order processes. The differential cross section can be written [23] in terms of the pion-nucleon coupling constant $g^2=(2m/\mu)^2f^2$ (where m is the neutron rest mass), the total energy $E^*=\sqrt{k^2+m^2}$ of the neutron in the c.m. system, and the quantity $x=\cos\theta^*+1+\mu^2/2k^2$, as

$$\frac{d\sigma}{d\Omega^*}=\frac{g^4}{4E^{*2}}\frac{(1+\cos\theta^*)^2}{x^2}+\frac{A}{x}+B\ ,$$

where the terms containing A and B represent higher-order processes and the remainder is the one-pion contribution. Although A and B are unknown functions of x, they are known to be finite at $x=0$. The experimental quantities x and $d\sigma/d\Omega^*$ are used to calculate values of a new variable $y(x)$,

TABLE II. Double exponential fits to the n-p charge-exchange scattering data in the 460 MeV energy region. The angular range is approximately $145°<\theta^*<180°$.

Exp't.	Ref.	T (MeV)	No. of points	χ^2_ν	α_1	β_1	α_2	β_2	$\frac{\alpha_1\beta_1+\alpha_2\beta_2}{\alpha_1+\alpha_2}$	$\frac{d\sigma}{d\Omega^*}(180°)$
PPA	5	466	12	0.35	62.0±3.4	5.18±0.84	85.8±4.7	152±19	90.7±12.7	10.29±0.40
Saclay	6	457	42	1.39	97.1±3.1	8.88±1.03	60.0±3.2	204±22	83.2±11.5	10.72±0.31
LAMPF	7	451	58	1.60	75.6±0.8	6.05±0.11	75.3±1.0	131±5	68.6±2.6	10.16±0.09
LAMPF	7	473	63	1.23	69.7±0.7	6.00±0.09	72.7±0.9	136±5	72.5±2.6	10.06±0.08
SIN	21	460							66.7±2.7	9.51±0.15
Present		459	54	1.48	75.4±1.2	7.08±0.30	66.4±1.1	154±6	75.9±3.1	9.73±0.11

$$x^2\frac{d\sigma}{d\Omega^*}=y(x)=\frac{g^4}{4E^{*2}}\left[x-\frac{\mu^2}{2k^2}\right]+Ax+Bx^2\ ,$$

which has the property that terms containing the unknown functions A and B vanish at $x=0$. Since $x=0$ is in the unphysical region, an extrapolation procedure must be used to obtain $y(0)$. The physical $y(x)$ values are least-squares fitted with the n-term polynomial $\sum_{i=0}^{n-1}a_i x^i$, and $y(0)$ is given by a_0, since all other terms vanish at $x=0$. From this it follows, after some substitutions and rearrangement, that

$$f^2=\sqrt{a_0(k^2+m^2)}(k/m)^2\ .$$

The data for three regions of backward angle were least-squares fitted with n-term polynomials, n being varied from 3 to ~ 10. Two criteria can be used to determine the optimum number of terms for the polynomial. One is that the value of χ^2_ν be a minimum. The second is a determination by means of the F test [24] of whether the addition of the nth term improved the fit significantly, i.e., calculation of the probability $P(n)$ that the addition of the nth term caused a significant reduction in χ^2_ν. If $P(n)$ remains near 1.0 as n increases, the addition of the nth term has resulted in significant improvement, but a sharp drop of $P(n)$ is an indication that addition of the nth term has not done so.

The results of the fitting procedure for varying numbers of terms n are summarized in Table III. For each data set the optimum value of f^2 is underlined. The criterion used to determine the optimum value of n for each case was somewhat arbitrary; it was required that both $P(n+1)$ and $P(n+2)$ be less than 0.90. For these choices of optimum n it is seen that the value of χ^2_ν also is at or near a minimum. In the case of the 111 point data set, however, there are several reasons for believing that the choice of a seven-term rather than a nine-term polynomial might be better. The value of χ^2_ν reaches its first minimum with the seven-term fit, and does not change much with higher orders. Furthermore, the value obtained for f^2 is in better agreement with the values obtained with the other fits, and the error for f^2 is much smaller than with the nine-term fit. Incidentally, this error merely reflects a statistical error in the determination of a_0, and is much smaller than the uncertainties associated with the choice of the optimum number of terms, or the number of points included in the data set, and is comparable to the normalization uncertainty of the data. The fit to the 111-point data set is shown in Fig. 4. The fits for the less extended data sets give curves which are not discernibly different.

In a recent analysis of pion-nucleon elastic-scattering data [25], the value obtained for f^2 was 0.0735 ± 0.0015. (An earlier determination [26] gave 0.079 ± 0.001, and there has been some controversy over the matter [27].) This value of 0.0735, which will be labeled $f^2_{\pi N}$ for convenience in the following discussion, is $\sim 6\%$ higher than the value 0.069 seemingly indicated in Table III. Since $f^2\propto\sqrt{a_0}$ this is an indication that the a_0 value determined in the fitting process is $\sim 12\%$ too low, which in turn implies that the cross section values obtained in this experiment should be renormalized upward by 12%. This is fairly consistent with the finding in the phase-shift analysis of SAID (see Sec. III B) that an upward renormalization of the data by 10% is needed [28]. In fact, an upward renormalization of only 7% would bring the present determination of f^2 within the error of $f^2_{\pi N}$, and even with no renormalization, the upper error limit of the 83-point determination is about equal to the lower error limit of $f^2_{\pi N}$. (Note that to reach agreement with the higher value $f^2=0.079$ an upward renormalization of the data of this experiment by $\sim 30\%$ would be required, and such a large renormalization would put this experiment in sharp conflict with the TRIUMF measurements [11], at least for the back-angle region.)

This leads to the suspicion that the error in the total cross section for the $\sigma(pp\to d\pi^+)$ reaction in this energy region has been underestimated. Recent data [29–38] available for $\sigma(pp\to d\pi^+)$ are shown in Fig. 5. The data were taken from a compilation by Laptev and Strakovsky [39]. Most data obtained earlier than 1970 have relatively large error bars and are omitted in order to avoid cluttering the graph. The value assumed in the analysis of the present data is shown by the lower star. The $\pm 3\%$ uncertainty in neutron beam energy translates into a $\pm 6\%$ uncertainty in this value. It is more or less in agreement with the older data [40]. An upward renor-

TABLE III. Pion-nucleon coupling constant determination from data of this experiment.

	54 points $145°\le\theta^*\le 180°$			83 points $122°\le\theta^*\le 180°$			111 points $101°\le\theta^*\le 180°$		
n	χ^2_ν	$P(n)$	f^2	χ^2_ν	$P(n)$	f	χ^2_ν	$P(n)$	f^2
3	3.530	1.00	0.0447 ± 0.0010	17.20	1.00	0.0107 ± 0.0024	17.68	1.00	imaginary
4	1.503	1.00	$\underline{0.0648\pm0.0018}$	3.640	1.00	0.0489 ± 0.0009	10.33	1.00	0.0255 ± 0.0013
5	1.524	0.42	0.0615 ± 0.0054	1.772	1.00	0.0645 ± 0.0013	2.832	1.00	0.0532 ± 0.0010
6	1.517	0.73	0.0410 ± 0.0202	1.715	0.94	$\underline{0.0696\pm0.0023}$	1.534	1.00	0.0658 ± 0.0012
7	1.525	0.61	imaginary	1.680	0.89	0.0613 ± 0.0050	1.507	0.91	$\underline{0.0691\pm0.0020}$
8				1.701	0.24	0.0644 ± 0.0090	1.520	0.31	0.0703 ± 0.0032
9				1.673	0.86	0.0330 ± 0.0294	1.455	0.98	0.0576 ± 0.0063
10							1.468	0.20	0.0598 ± 0.0094
11							1.477	0.48	0.0498 ± 0.0181

TABLE IV. Pion-nucleon coupling constant determination from data of other experiments at nearby energies.

	Saclay (Ref. [6]) 457 MeV (42 points) $152^\circ \le \theta^* \le 180^\circ$			Saclay (Ref. [6]) 457 MeV (39 points)[a] $152^\circ \le \theta^* \le 178^\circ$			Saclay (Ref. [6]) 457 MeV (36 points)[b] $152^\circ \le \theta^* \le 176^\circ$			PPA (Ref. [5]) 466 MeV (22 points) $100^\circ \le \theta^* \le 180^\circ$		
n	χ^2_ν	$P(n)$	f^2	χ^2_ν	$P(n)$	f^2	χ^2_ν	$P(n)$	f^2	χ^2_ν	$P(n)$	f^2
3	1.831	1.00	0.0522±0.0025	1.880	1.00	0.0492±0.0035	2.00	1.00	imaginary	10.57	0.95	imaginary
4	1.427	1.00	0.0768±0.0053	1.464	1.00	0.0815±0.0067	1.351	1.00	0.0904±0.0068	8.137	0.98	0.0185±0.0072
5	1.439	0.58	0.0915±0.0145	1.414	0.86	0.1129±0.0160	1.066	1.00	0.1449±0.0145	1.315	1.00	0.0643±0.0034
6	1.479	0.09	0.0851±0.0489	1.434	0.52	0.1477±0.0376	0.849	0.99	0.2361±0.0282	0.925	0.99	0.0743±0.0045
7	1.304	0.98	imaginary	1.352	0.91	imaginary	0.877	0.13	0.2498±0.0908	0.706	0.97	0.0848±0.0062
8	0.994	1.00	imaginary	1.004	1.00	imaginary	0.805	0.93	imaginary	0.688	0.74	0.0770±0.0108
9	1.005	0.56	imaginary	0.974	0.83	imaginary	0.835	0.02	imaginary	0.701	0.60	0.0661±0.0206
10	1.013	0.61	0.4260±1.3590	1.005	0.19	imaginary	0.756	0.94	1.542±0.4879			
11							0.778	0.39	2.172±0.8506			
12							0.806	0.28	3.224±0.3714			

	PPA (Ref. [5]) 466 MeV (18 points) $116^\circ \le \theta^* \le 180^\circ$			PPA (Ref. [5]) 466 MeV (12 points) $151^\circ \le \theta^* \le 180^\circ$			LAMPF (Ref. [7]) 451 MeV (40 points) $118^\circ \le \theta^* \le 180^\circ$			LAMPF (Ref. [7]) 473 MeV (42 points) $117^\circ \le \theta^* \le 180^\circ$		
n	χ^2_ν	$P(n)$	f^2	χ^2_ν	$P(n)$	f^2	χ^2_ν	$P(n)$	f^2	χ^2_ν	$P(n)$	f^2
3	10.82	0.98	imaginary	1.402	1.00	0.0530±0.0034	2.844	1.00	0.0424±0.0013	2.771	1.00	0.0449±0.0012
4	3.672	1.00	0.0494±0.0036	0.271	1.00	0.0723±0.0058	0.738	1.00	0.0659±0.0023	0.828	1.00	0.0649±0.0021
5	0.457	1.00	0.0757±0.0040	0.307	0.18	0.0740±0.0139	0.749	0.50	0.0692±0.0059	0.850	0.08	0.0654±0.0054
6	0.463	0.62	0.0791±0.0066	0.308	0.64	0.0905±0.0295	0.769	0.23	0.0726±0.0139	0.838	0.78	0.0786±0.0116
7	0.496	0.32	0.0760±0.0124				0.792	0.02	0.0718±0.0459	0.820	0.81	0.0210±0.1205

[a]Points for three largest angles omitted.
[b]Points for six largest angles omitted.

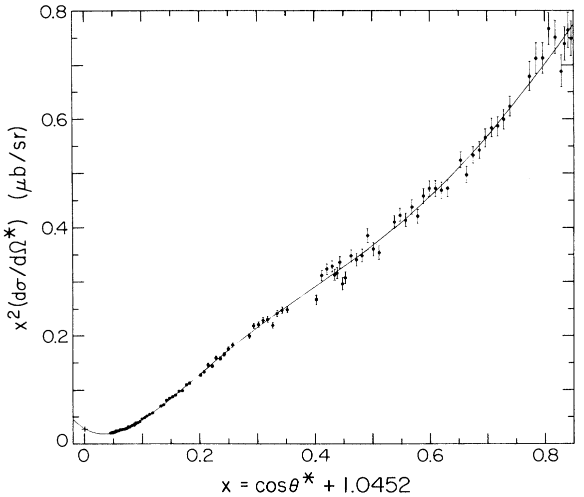

FIG. 4. Polynomial fits to the values of $x^2 d\sigma/d\Omega^*$ calculated from the data of this experiment. The solid line is a seven-term fit to the 111 points in the region 101°–180°. A six-term fit to the 83 points in the region 122°–180° is shown by a dotted line, which deviates only slightly from the solid line in the region $x \sim 0.5$.

malization of the point by 10% (shown by the upper star) would put it in better agreement with the more recent measurements of Giles [29]. This strengthens the argument for the ~10% renormalization suggested by both the phase-shift analysis and the coupling constant determination.

The pole-extrapolation procedure was also used to extract f^2 values from the data of the other experiments shown in Fig. 1, and the results are assembled in Table IV. The criteria used to determine the optimum number of terms is the same as was used in Table III, and once again the optimum values are indicated by underlining. The values of f^2 given by the Saclay data tend to be high, and are made higher by elimination of the points at extreme backward angles. The value given by the PPA data is too high if all of the points are used, but are consistent with the value 0.0735, within rather large error bars, if only the backward-angle data are used. The values given by the earlier LAMPF experiment are quite low if the same statistical criterion is used to determine the optimum number of terms. They are subject, of course, to the same normalization error as the data of the present experiment.

IV. CONCLUSIONS

The data presented here are the best available in this energy region. They are in reasonable agreement with an independent LAMPF experiment covering a smaller angular region [7], but not with data from PPA [5] and Saclay [6]. There does seem to be a problem with absolute normalization, however, which various evidence indicates

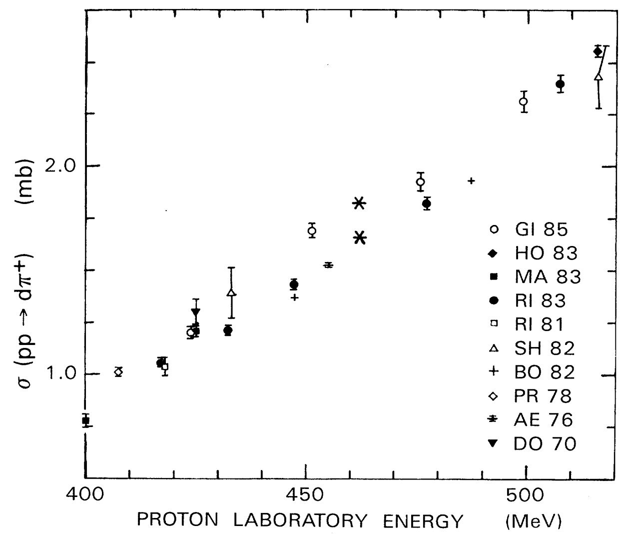

FIG. 5. Experimental values of the total cross section for the $pp \rightarrow d\pi^+$ reaction between 400 and 520 MeV. The references are as follows: GI 85–Giles [29]; HO 83–Hoftiezer *et al.* [30]; MA 83–Mathie *et al.* [31]; RI 83–Ritchie *et al.* [32]; RI 81–Ritchie *et al.* [33]; SH 82–Shimizu *et al.* [34]; BO 82–Boswell *et al.* [35]; PR 78–Preedom *et al.* [36]; AE 76–Aebischer *et al.* [37]; DO 70–Dolnick [38]. No errors were quoted on the points labeled BO 82. The lower star shows the value assumed in the analysis of the data of the present experiment, and the upper star a value 10% higher.

is about 10% low. Since this normalization was based on simultaneous detection of protons from n-p elastic scattering and deuterons from the $np \rightarrow d\pi^0$ reaction, it is simplest to assume that the cross section used for the latter reaction (0.83 mb) was too low by 10%. Since this cross section, by isospin conservation rules, should be one-half the cross section for the $pp \rightarrow d\pi^+$ reaction, the cross section for the latter reaction should have been 1.83 rather than the 1.66 mb assumed, provided that isospin conservation is valid.

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