K^+/π^+ enhancement in heavy-ion collisions

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Using the expanding fireball model, we calculate the K^+/π^+ ratio from heavy-ion collisions at 14.5 GeV/nucleon. A substantially larger ratio than that from the proton-nucleus collisions at the same energy is obtained if the contribution from the pion-pion interactions is included. This is supported by the preliminary data.

I. INTRODUCTION

Kaon production from heavy-ion collisions is a topic of great interest. For heavy-ion collisions at ≈ 1 GeV/nucleon, it has been shown that the kaon production probability is sensitive to the nuclear equation of state at high densities.[1] At ultrarelativistic energy collisions available at the future heavy-ion collider, it has been argued that a kaon might carry the signature for the quark-gluon plasma.[2]. Recently, there are preliminary data from the Alternating Gradient Synchrotron (AGS) at Brookhaven National Laboratory on kaon production from the reaction of Si on Au at an incident energy of 14.5 GeV/nucleon.[3] The expected ratio of K^+/π^+ is about 5% from the proton-nucleus collisions at the same energy. The measured ratio is about 20%. The factor of 4 enhancement has stimulated the speculation that a quark-gluon plasma might have been formed in the collision. However, measurements of nuclear stopping power at this energy indicates that the projectile is stopped by the target with the resulting energy density less than that for the formation of the quark-gluon plasma. From the analysis of the transverse energy distribution in such reaction, it has been shown that the data are consistent with the assumption that a fast-moving hadronic fireball has been formed from the projectile and part of the target.[4] It is thus unlikely that the enhanced production of kaons is due to the formation of the quark-gluon plasma. To find other explanations to this large value of K^+/π^+ ratio is therefore of great interest.

The first possibility one can think of is the associated production $\pi N \rightarrow \Lambda K$ from the interaction between the secondary pions and the nucleons. Indeed, in a recent study of Λ production from $\bar{p}$ annihilation in nuclei, we have found that the associated production from the secondary pions contributes almost equally to Λ production as that from the direct annihilation.[5] To take into account the contribution from the associated production to kaon production, we need to consider the interaction of pion with both the spectator and the participant nucleons in the target. As pions are produced most likely outside the target in high-energy collisions due to the time dilation of the production process in a fast moving frame, the contribution from the interaction of pions with the spectator nucleons is expected to be unimportant. The interaction of the pion with the participant nucleons will not lead to an appreciable contribution to kaon production either as the threshold energy for this process is much higher than the average energy of the pion in the frame of the participants. However, the number of created pions is very large, and the contribution from the pion-pion interaction might be important. In this paper, we shall demonstrate this important contribution to the K^+/π^+ ratio in high-energy heavy-ion collisions in an expanding fireball model. Similar conclusion have been reported previously by Kapusta and Mekjian[6] for a hadronic matter that is composed of only pions.

This paper is organized as follows. In Sec. II the hydrochemical model is introduced to describe the fireball expansion. The rates for the relevant reactions are calculated in Sec. III. Section IV gives the results of the calculations. Discussions and conclusions are given in Sec. V. In the Appendix the details of solving the hydrochemical model are described.

II. HYDRODYNAMICAL MODEL

We assume that a hadronic fireball is formed in the collision of two heavy ions at high energies and that kaons are produced from the nucleon-nucleon, pion-nucleon, and the pion-pion interactions during the expansion of the fireball. To model the fireball expansion, we use the hydrochemical model of Biro *et al.*,[7] in which the fireball is assumed to be in thermal but not chemical equilibrium, and the thermal energy in the fireball is converted into the collective flow energy via the relativistic hydrodynamical equations,

$$\partial_i T^{i0} = 0 \ , \tag{1a}$$

$$u_k \partial_i T^{ik} = 0 \ , \tag{1b}$$

$$\partial_i (\rho_a u^i) = \Psi_a \ . \tag{1c}$$

In the above, T^{ik} is the energy-momentum tensor given by

$$T^{ik} = (e+p) u^i u^k - p g^{ik} \ , \tag{2}$$

where e and p are the energy and pressure densities, g^{ik} is the standard metric tensor, and u^i is the four velocity defined by $u^i = \gamma(1, \mathbf{u})$, with $\mathbf{u}$ the local velocity and γ the associated Lorentz factor. The quantity ρ_a denotes the density of particle type a while the source term Ψ_a takes into account the various reactions that either change particle a into other kinds of particles or convert other particles into particle a. Taking the spatial average of these equations with the assumption of a uniform density distribution, we have

$$\frac{1}{V}\frac{d}{dt}[V(e\langle\gamma^2\rangle+p\langle\gamma^2u^2\rangle)]=0 , \tag{3a}$$

$$\frac{1}{V}\frac{d}{dt}(V\rho_a\langle\gamma\rangle)=\Psi_a , \tag{3b}$$

$$\frac{1}{V}\frac{d}{dt}(Vs\langle\gamma\rangle)=-\frac{1}{T}\sum_a\mu_a\Psi_a . \tag{3c}$$

In the above, V and T are the volume and the temperature of the fireball, respectively. The quantity s denotes the entropy density in the fireball. We take the local radial velocity (as in Ref. 7), i.e.,

$$\gamma\mathbf{u}=\mathbf{r}\frac{\dot{R}(t)}{R} , \tag{4}$$

where $\mathbf{r}$ is the radial vector and R is the radius of the fireball. Equation (4) leads to $\langle\gamma\rangle\approx1+33\dot{R}^2/160$, $\langle\gamma^2\rangle=1+0.6\dot{R}^2$, and $\langle\gamma^2u^2\rangle=0.6\dot{R}^2$. The chemical potential for particle type a is denoted by μ_a. At any instant of time during the expansion of the fireball, the thermodynamical quantities such as energy, pressure, etc., are determined from standard equilibrium thermodynamics. In the Appendix we describe explicitly how these equations are solved.

In our calculations we include only the following kinds of particles: nucleon, delta, lambda, pion, kaon, and antikaon. To take into account the change of the particle numbers, we consider the following reactions:

$$NN\leftrightarrow N\Delta , \tag{5a}$$

$$\pi N\leftrightarrow\Delta , \tag{5b}$$

$$\pi N\rightarrow\Lambda K , \tag{5c}$$

$$\pi\Delta\rightarrow\Lambda K , \tag{5d}$$

$$\pi\pi\leftrightarrow K\bar{K} , \tag{5e}$$

In Eqs. (5c) and (5d) we have neglected the inverse of the associated production as we expect a very small Λ density which leads thus to a negligible effect on the change of the number of the kaon and the lambda. For the same reason, we do not include other hyperons. The source functions are then given by

$$\begin{aligned}\Psi_N=&-\langle\sigma_{\pi N}^{\Delta}v_{\pi N}\rangle\rho_\pi\rho_N+\Gamma_\Delta\rho_\Delta-\tfrac{1}{2}\langle\sigma_{NN}^{N\Delta}v_{NN}\rangle\rho_N\rho_N\\&+\langle\sigma_{N\Delta}^{NN}v_{N\Delta}\rangle\rho_N\rho_\Delta-\langle\sigma_{\pi N}^{\Lambda K}v_{\pi N}\rangle\rho_\pi\rho_N ,\end{aligned} \tag{6a}$$

$$\begin{aligned}\Psi_\pi=&-\langle\sigma_{\pi N}^{\Delta}v_{\pi N}\rangle\rho_\pi\rho_N+\Gamma_\Delta\rho_\Delta-\langle\sigma_{\pi N}^{\Lambda K}v_{\pi N}\rangle\rho_\pi\rho_N\\&-\langle\sigma_{\pi\Delta}^{\Lambda K}v_{\pi\Delta}\rangle\rho_\pi\rho_\Delta-\langle\sigma_{\pi\pi}^{K\bar{K}}v_{\pi\pi}\rangle\rho_\pi\rho_\pi\\&+2\langle\sigma_{K\bar{K}}^{\pi\pi}v_{K\bar{K}}\rangle\rho_K\rho_{\bar{K}} ,\end{aligned} \tag{6b}$$

$$\begin{aligned}\Psi_\Delta=&\langle\sigma_{\pi N}^{\Delta}v_{\pi N}\rangle\rho_\pi\rho_N-\Gamma_\Delta\rho_\Delta+\tfrac{1}{2}\langle\sigma_{NN}^{N\Delta}v_{NN}\rangle\rho_N\rho_N\\&-\langle\sigma_{N\Delta}^{NN}v_{N\Delta}\rangle\rho_N\rho_\Delta-\langle\sigma_{\pi\Delta}^{\Lambda K}v_{\pi\Delta}\rangle\rho_\pi\rho_\Delta ,\end{aligned} \tag{6c}$$

$$\begin{aligned}\Psi_K\approx&\langle\sigma_{\pi N}^{\Lambda K}v_{\pi N}\rangle\rho_\pi\rho_N+\langle\sigma_{\pi\Delta}^{\Lambda K}v_{\pi\Delta}\rangle\rho_\pi\rho_\Delta+\tfrac{1}{2}\langle\sigma_{\pi\pi}^{K\bar{K}}v_{\pi\pi}\rangle\rho_\pi\rho_\pi\\&-\langle\sigma_{K\bar{K}}^{\pi\pi}v_{K\bar{K}}\rangle\rho_K\rho_{\bar{K}} ,\end{aligned} \tag{6d}$$

$$\Psi_\Lambda\approx\langle\sigma_{\pi N}^{\Lambda K}v_{\pi N}\rangle\rho_\pi\rho_N+\langle\sigma_{\pi\Delta}^{\Lambda K}v_{\pi\Delta}\rangle\rho_\pi\rho_\Delta , \tag{6e}$$

$$\Psi_{\bar{K}}=\tfrac{1}{2}\langle\sigma_{\pi\pi}^{K\bar{K}}v_{\pi\pi}\rangle\rho_\pi\rho_\pi-\langle\sigma_{K\bar{K}}^{\pi\pi}v_{K\bar{K}}\rangle\rho_K\rho_{\bar{K}} . \tag{6f}$$

In the above, Γ_Δ is the width of the Δ resonance and is taken to be 112 MeV. The average of the product of the cross section σ_{12}^{34} and the relative velocity v_{12} is defined by

$$\langle\sigma_{12}^{34}v_{12}\rangle=\int d^3\mathbf{k}_1\int d^3\mathbf{k}_2 f_1(\mathbf{k}_1)f_2(\mathbf{k}_2)\sigma_{12}^{34}v_{12} , \tag{7}$$

where f's are the normalized momentum distribution of the particles. In Eqs. (6b) and (6f), we only take into consideration kaon annihilation into two pions. The annihilation into more pions is neglected here because of our poor knowledge of the kaon annihilation cross section. As we shall show later, the cross section for kaon annihilation into two pions as determined by the detailed balance from the inverse cross section accounts for most of the annihilation cross section (as estimated in Ref. 6).

III. REACTION RATES

To evaluate Eq. (7) which is related to the reaction rate for a process, we make use of the assumption that the fireball is in thermal equilibrium. Then the normalized momentum distribution of the particles has the following form:

$$f(\mathbf{k})=[4\pi m^2K_2(m/T)]^{-1}\exp[-(k^2+m^2)^{1/2}/T] , \tag{8}$$

with K_2 the modified Bessel function. The relative momentum is given by

$$v_{12}=[(k_1\cdot k_2)^2-m_1^2m_2^2]^{1/2}/E_1E_2 , \tag{9}$$

where E_1 and E_2 are the energies of the two particles. Carrying out five of the six integrations in Eq. (7), we obtain

$$\langle\sigma_{12}^{34}v_{12}\rangle=\frac{T^4}{4(1+\delta_{m_1,m_2})m_1^2m_2^2K_2(m_1/T)K_2(m_2/T)}\int_{z_0}^{\infty}dz\,[z^2-(m_1/T+m_2/T)^2][z^2-(m_1/T-m_2/T)^2]K_1(z)\sigma_{12}^{34} , \tag{10}$$

where $z_0=\max(m_1+m_2, m_3+m_4)/T$, and K_1 is again the modified Bessel function. We have used the convention $\hbar=c=1$. For the cross sections $\sigma_{\pi N}^{\Delta}$, $\sigma_{NN}^{N\Delta}$, and $\sigma_{\pi N}^{\Lambda K}$, we use those from Cugnon *et al.*,[8] i.e.,

$$\sigma_{\pi N}^{\Delta}=\frac{12.62}{1+4[(s^{1/2}-1.232)/\Gamma_\Delta]^2}\text{fm}^2 , \tag{11a}$$

$$\sigma_{NN}^{N\Delta}=\frac{2(s^{1/2}-2.015)^2}{0.015+(s^{1/2}-2.015)^2}\text{fm}^2 , \tag{11b}$$

$$\sigma_{\pi N}^{\Lambda K}=\begin{cases}\dfrac{0.045(s^{1/2}-1.61)}{0.091}\text{fm}^2, & 1.7\geq s^{1/2}\geq 1.61,\\[2mm] \dfrac{0.0045}{s^{1/2}-1.6}\text{fm}^2, & s^{1/2}\geq 1.7 .\end{cases} \tag{11c}$$

In the above, the center-of-mass energy $s^{1/2}$ is in units of GeV. The cross section $\sigma_{\pi\Delta}^{\Lambda K}$ is taken to be the same as $\sigma_{\pi N}^{\Lambda K}$ evaluated at the corresponding center-of-mass energy while the cross section $\sigma_{\pi\pi}^{K\bar{K}}$ is taken from Ref. 6, i.e.,

$$\sigma_{\pi\pi}^{K\bar{K}}\approx 0.34\text{ fm}^2 . \tag{12}$$

All the inverse cross sections are determined from the detailed balance relation. In the energy range we are considering, the kaon annihilation cross section obtained from the detailed balance has magnitude of about 10 mb that is comparable to that estimated in Ref. 6 based on quark counting and the antiproton annihilation cross section.

IV. RESULTS

We have considered specifically the reaction Si on Au at an incident energy of 14.5 GeV/nucleon and at zero impact parameter. For heavy-ion collisions at such high energy, it is probably reasonable to assume that the projectile sweeps through the target on a straight trajectory and moves forward together with the target nucleons that are in the volume traversed by the projectile. In this simplified geometrical picture, we can determine easily the number of participants and the total energy in the resulting compound system. Using a uniform density for both the projectile and the target, we obtain that the number of participants is about 102 and the total center-of-mass energy of the compound system is about 256 GeV. These numbers are consistent with that estimated from the empirical data.[9] Assuming equilibrium distributions for nucleons, deltas, and pions, and taking the initial number of K^+ to be 5% of that of the π^+, we can determine the temperature and the relative abundance of these particles if the initial density is specified. We first assume that the initial baryon density is twice that of the normal nuclear matter density. The results from solving the relativistic hydrodychemical equations are given below. In Fig. 1, we show the time evolution of the temperature and the radius of the fireball. Except for the first few fm/c, both quantities change essentially linearly with time. Although the initial temperature is very high with a value of about 225 MeV, the energy density is only about 800 MeV/fm^3 and is still smaller than the critical energy density for forming the quark-gluon plasma as estimated from lattice gauge calculations. Figure 2 shows the total number of nucleons, deltas, pions, and kaons as functions of time. We see that the kaon number approaches a constant value after about 4 fm/c when the temperature and density of the fireball begin to drop. We note that most of the kaons are created from the reaction $\pi\pi\rightarrow K\bar{K}$ as expected. The initial decrease of the pion number is due to the conversion of pions to kaons. In the later time when kaon number remains constant, both the pion and the nucleon numbers increase as a result of the decay of the deltas. We stop the hydrochemical calculation when the nucleon mean free path is comparable to the diameter of the fireball. This happens at about 12 fm/c after the fireball expansion. At freezeout, there are about 18 kaons, 142 pions, and 26 deltas. Eventually, the deltas will decay into nucleons and pions so the final pion number is about 168, which leads to a K^+/π^+ ratio of about 16% which is a factor of 3 enhancement over the initial value and is not too far off from the measured ratio of 20%. To see how the results depend on the initial condition of the fireball, we have also carried out calculations for different initial densities. We first show in Fig. 3 the initial temperature and the initial number of nucleons, deltas, and pions in the fireball as functions of the initial density. We see that the temperature does not increase very quickly with the density once the density is above the normal nuclear density. This is because of the increase of the deltas as the density is increased. In Fig. 4 we show the results for the K^+/π^+ ratio as a function of the initial density of the fireball. The solid curve is from

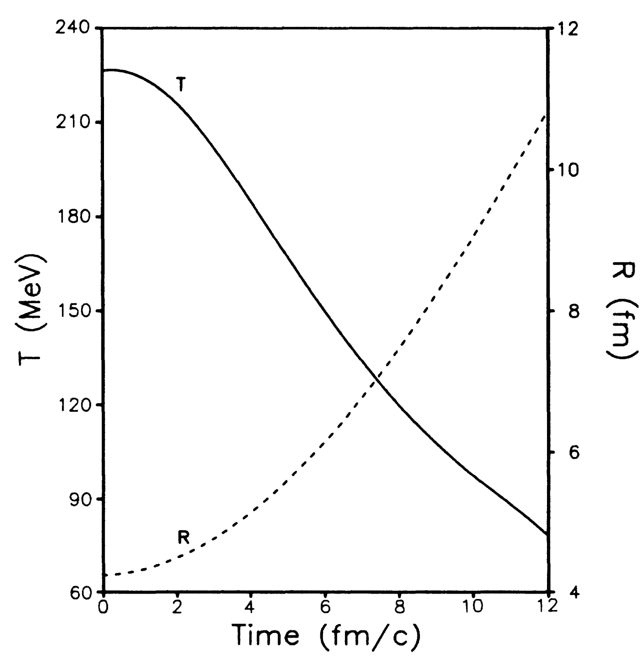

FIG. 1. Time evolution of the temperature T and the radius R of the fireball for the reaction Si on Au at 14.5 GeV/nucleon. The initial density ρ of the fireball is taken to be twice the normal nuclear matter density ρ_0.

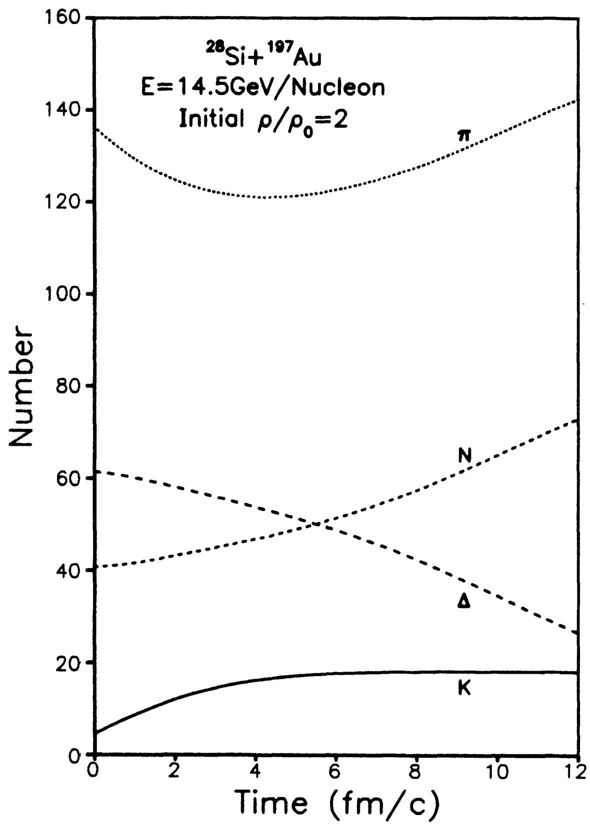

FIG. 2. Time evolution of the number of nucleons, deltas, pions, and kaons for the same reaction as in Fig. 1.

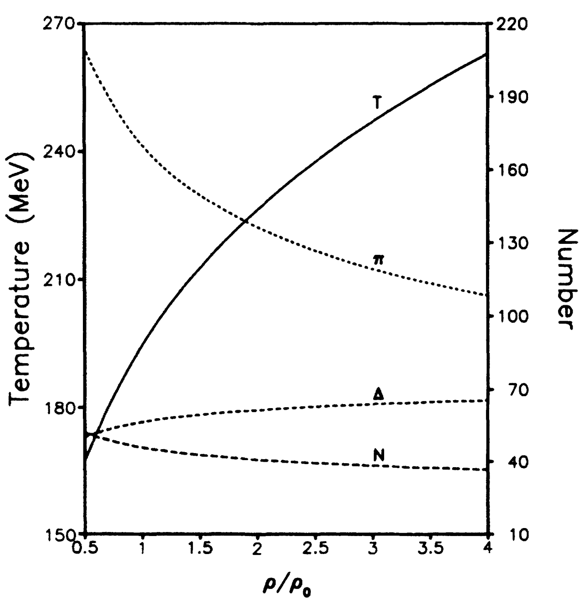

FIG. 3. The initial temperature and the number of nucleons, deltas, and pions in the fireball as functions of its initial density for the reaction in Fig. 1.

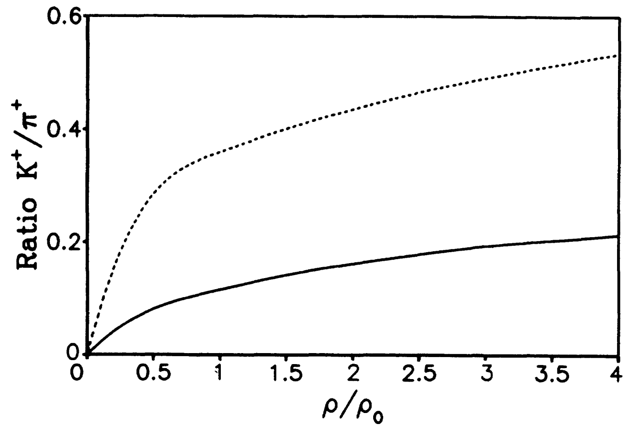

FIG. 4. The K^+/π^+ ratio as a function of the initial density. The solid curve is from the hydrochemical model, while the dashed curve is obtained by assuming that kaons are in chemical equilibrium in the initial state of the fireball.

the hydrochemical model calculations. We see that its value increases with the initial density and reaches to 20% at about four times the normal nuclear matter density. This is because a higher initial temperature leads to a smaller initial number of pions in the fireball as shown in Fig. 3. Also shown in this figure by the dashed curve is the K^+/π^+ value that one would have if kaons are assumed to be in chemical equilibrium at the initial fireball density and temperature. This value is substantially larger than the prediction of the hydrochemical model.

V. CONCLUSIONS

We have studied the K^+/π^+ ratio in high-energy heavy-ion collisions in the expanding fireball model. Due to the large number of pions in the fireball, the appreciable cross section for the reaction $\pi\pi\rightarrow K\bar{K}$, and its relatively low threshold energy, this process leads to a substantial enhancement of the K^+/π^+ ratio compared with that from the proton-nucleus reaction at the same energy. The calculated value for this ratio depends on the initial density of the fireball and increases as the initial density of the fireball increases. Since the projectile is stopped by the participant nucleons of the target even at AGS energies, we expect that the initial density should be relatively large. In this case, the observed ratio of K^+/π^+ from heavy-ion collisions at AGS can be largely explained by the reaction $\pi\pi\rightarrow K\bar{K}$ from the secondary pions created from the reaction.

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APPENDIX

In this Appendix we describe explicitly how we solve the relativistic hydrochemical equations as given by Eq.

(3). We rewrite it as a difference equation. In terms of the time step Δt, it can be expressed as

$$R^n = R^{n-1} + \dot{R}^{n-1}\Delta t \;, \quad \text{(A1a)}$$

$$\rho_a^n = \rho_a^{n-1} + \dot{\rho}_a^{n-1}\Delta t \;, \quad \text{(A1b)}$$

$$s^n = s^{n-1} + \dot{s}^{n-1}\Delta t \;, \quad \text{(A1c)}$$

where n denotes the nth time step. In the above, the time derivatives of the radius, density, and entropy are given, respectively, by

$$\dot{R}^{n-1} = \left[\frac{5}{3(e^{n-1}+p^{n-1})} \left[\frac{e^0 V^0}{V^{n-1}} - e^{n-1} \right] \right]^{1/2} , \quad \text{(A2a)}$$

$$\dot{\rho}_a^{n-1} = -\rho_a^{n-1} \frac{\dot{V}^{n-1}}{V^{n-1}} + \Psi_a^{n-1} \;, \quad \text{(A2b)}$$

$$\dot{s}^{n-1} = -s^{n-1} \frac{\dot{V}^{n-1}}{V^{n-1}} - \frac{1}{T^{n-1}} \sum_a \mu_a^{n-1} \Psi_a^{n-1} \;. \quad \text{(A2c)}$$

The volume at the nth time step is given by V^n and can be determined by the radius of the fireball. The rate of change of the volume is thus related to that of the radius. The initial energy density and volume of the fireball are denoted by e^0 and V^0, respectively. The source function Ψ_a^{n-1} are calculated according to Eq. (6). The temperature T^n, the energy density e^n, the pressure p^n, and the chemical potential μ_a^n at the nth time step are determined from the equilibrium thermodynamics, i.e.,

$$T^n = \frac{1}{s_n} \left[e^n + p^n - \sum_a \mu_a^n \rho^n \right] , \quad \text{(A3a)}$$

$$e^n = T^n \sum_a \rho_a^n \left[3 + \frac{m_a K_1(m_a/T^n)}{T^n K_2(m_a/T^n)} \right] , \quad \text{(A3b)}$$

$$p^n = T^n \sum_a \rho_a^n \;, \quad \text{(A3c)}$$

$$\mu_a^n = T^n \ln \left[\frac{2\pi^2 \rho_a^n}{\gamma_a m_a^2 T^n K_2(m_a/T^n)} \right] , \quad \text{(A3d)}$$

where m_a and γ_a are, respectively, the mass and the spin-isospin degeneracy factor of particle type a. Once the initial condition of the fireball is given, the evolution of the fireball can thus be determined by solving the above equations. The results in this paper are obtained with a time step of $\Delta t = 0.01$ fm/c and do not change significantly with smaller time steps.

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