

## Reply to “Comment on ‘Quantum search protocol for an atomic array’”

 Marlan O. Scully<sup>1,2</sup> and M. Suhail Zubairy<sup>1,3</sup>
<sup>1</sup>*Department of Physics and Institute for Quantum Studies, Texas A&M University, College Station, Texas 77843-4242*
<sup>2</sup>*Max-Planck-Institut für Quantenoptik, D-85748 Garching, Germany*
<sup>3</sup>*Department of Electronics, Quaid-i-Azam University, Islamabad, Pakistan*

(Received 21 June 2002; published 7 November 2002)

Our quantum search protocol indicates that entanglement is not required for  $\sqrt{N}$  speedup. We also reemphasize the quantum error correction mechanism in our scheme.

DOI: 10.1103/PhysRevA.66.056301

PACS number(s): 03.67.Lx, 42.50.Gy, 42.50.Ar

In the preceding Comment [1], the author discusses our recent proposal [2] based on a quantum search protocol utilizing an atomic array and states: “the optical approach of Ref. [2] could be accomplished with strictly classical means, requires exponential resources, and therefore does not represent quantum computation.”

This comment simply misses the main thrust and key point(s) of Ref. [2], the abstract of which states “In the present paper we show how modern quantum optics may provide a simple and practicable quantum search procedure, which may also yield insights into quantum search algorithms in general.”

A key point of Ref. [2] was to show that entanglement is not required to get the  $\sqrt{N}$  speedup, and the present comment does not dispute that by stating that “. . . the optical method of Ref. [2] presents a savings in temporal resources [ $O(\sqrt{N})$  versus  $O(N)$  queries].”

The observation about the requirements on hardware is well known. As noted by Lloyd [3] (a paper referred to in Ref. [2]) and others, entanglement reduces the amount of hardware from  $N$  to  $\log_2 N$ . As stated above, Ref. [2] shows in a simple scheme, how  $\sqrt{N}$  trials (instead of  $N$ ) are required for the search. This was also the main point of Grover’s original paper [4]. Grover’s algorithm can be implemented with or without entanglement, the advantage of entanglement is of course in terms of hardware. Reference [2] brings out in a simple practical scheme that the  $\sqrt{N}$  factor in data searches is more fundamental than Grover’s algorithm. A similar point was made in the experimental paper by Bucksbaum and collaborators [5]. In a separate paper [6], we have presented a possible implementation of Grover’s algorithm *with entanglement* using cavity QED methods. Unlike the scheme proposed in Ref. [2], Grover’s algorithm leads to the probabilistic outcome of search results even in an ideal system. The only exception is the  $N=4$  case as discussed in Ref. [6].

It is also well known that a purely classical scheme can also yield a  $\sqrt{N}$  speed up. Grover has recently given a simple pendulum system which demonstrates this nicely [7].

However there is more, which is strictly quantum. All such purely classical schemes are limited by the ability to resolve spectral lines and normal-mode frequencies. However Lorentzian tails are notoriously long. Thus the “needle” atom in our scheme [2] can be found by applying  $\sqrt{N}$  pulses, however there will always be error counts due to accidental excitation of “straw” atoms. Hopefully the number of error

counts per atom is small. Nevertheless if the number of atoms is large enough, these errors will be compounded into a large error count.

As discussed in Ref. [2], it is possible to eliminate the error counts by applying a sequence of  $2\pi$  pulses (Fig. 1) which cycle atoms in the “straw” level  $|s\rangle$  to the auxiliary level  $|h\rangle$  and back to  $|s\rangle$ , resulting in the net sign change of the ground state (Fig. 2). Here we elaborate the main idea and discuss the conditions when the error counts can be rendered negligible via our quantum error correction.

Consider the quantum system which can make an *unwanted* weak transition from state  $|s\rangle$  to  $|a\rangle$  as a result of an error signal of strength  $G$ . The Hamiltonian in the interaction picture is

$$H_{error}(t) = \hbar G |a\rangle\langle s| e^{i\delta t} + \text{H.c.}, \quad (1)$$

where  $\delta$  is the detuning, i.e., the perturbation need not be resonant with the transition  $|s\rangle \leftrightarrow |a\rangle$ . The transition probability is

$$P_{sa} = |G|^2 \frac{\sin^2(\delta t/2)}{(\delta/2)^2}. \quad (2)$$

In order to show that this unwanted transition can be suppressed via a sequence of short  $2\pi$  pulses on the  $|s\rangle \leftrightarrow |h\rangle$  transition, we divide the total time interval into  $\sqrt{N}$  short intervals  $\tau$  (see Fig. 1). The system evolves under  $G$  from an

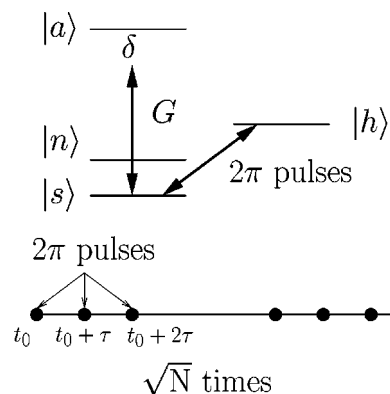


FIG. 1. Scheme for quantum error correction. The Hamiltonian (1) describes the interaction between the levels  $|s\rangle$  and  $|a\rangle$ . A sequence of ultrashort  $2\pi$  pulses at times  $t_0, t_0 + \tau, \dots$  between the levels  $|s\rangle$  and  $|h\rangle$  leads to error correction.

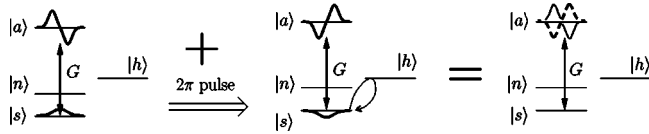


FIG. 2. Destructive interference between the  $|s\rangle \rightarrow |a\rangle$  transition is due to the phase change of state  $|s\rangle$  by  $\pi$  due to the  $2\pi$  pulse between  $|s\rangle$  and  $|h\rangle$ . The state  $|a\rangle$  does not undergo any phase change during the application of the  $2\pi$  pulse.

initial time  $t_0$  to  $t_0 + \tau$ . At  $t_0 + \tau$  we apply an ultrashort  $2\pi$  pulse on the transition  $|s\rangle \leftrightarrow |h\rangle$ . The system then evolves from  $t_0 + \tau$  to  $t_0 + 2\tau$  under  $G$  followed by a  $2\pi$  pulse and so on.

The transition probability at the end of  $\sqrt{N}$  such cycles will be [8]

$$\bar{P}_{sa} = \tan^2\left(\frac{\delta\tau}{2}\right) P_{sa}, \quad (3)$$

where  $P_{sa}$  is given by Eq. (2). Thus the application of a sequence of  $2\pi$  pulses on an auxiliary transition leads to the suppression of an unwanted transition between  $|s\rangle$  and  $|a\rangle$ , provided that the small interval and the detuning  $\delta$  are such that

$$\tan^2\left(\frac{\delta\tau}{2}\right) \ll 1. \quad (4)$$

The suppression arises from a destructive interference of the transition amplitude. This destructive interference is due to a phase change of the state  $|s\rangle$  (and not of  $|a\rangle$ ) by  $\pi$  due to the application of the  $2\pi$  pulse (Fig. 2). This also explains our choice of an auxiliary transition for the application of the  $2\pi$  pulse as we want to selectively produce a phase change, so that the interference can occur.

We also note that there is a related discussion of this result within the context of the quantum Zeno effect [9].

In summary, the use of an atomic array to demonstrate an  $\sqrt{N}$  speed up does not require entanglement. Furthermore, an essential feature of our search protocol is that we can protect against unwanted straw atom excitation via the  $2\pi$ -pulse quantum error correction. The connection with the Zeno effect and the  $2\pi$ -pulse quantum error correction is an interesting if somewhat controversial subject.

Coffey is correct in stating that there are insights to be gained by taking a classical model. However to miss the quantum aspects of our paper is to miss half of the fun!

The authors gratefully acknowledge the support from the Air Force Research Laboratory (Rome, New York), DARPA-QuIST, TAMU Telecommunication and Informatics Task Force (TITF) Initiative, the Office of Naval Research, and the Welch Foundation.

[1] M.W. Coffey, preceding paper, Phys. Rev. A **66**, 046301 (2002).  
 [2] M.O. Scully and M.S. Zubairy, Phys. Rev. A **64**, 022304 (2001).  
 [3] S. Lloyd, Phys. Rev. A **61**, 010301 (1999).  
 [4] L.K. Grover, Phys. Rev. Lett. **79**, 325 (1997).  
 [5] J. Ahn, T.C. Weinacht, and P.H. Bucksbaum, Science **287**, 463 (2000); P.L. Knight, *ibid.* **287**, 411 (2000).  
 [6] M.O. Scully and M.S. Zubairy, Proc. Natl. Acad. Sci. U.S.A.

**98**, 9490 (2001).  
 [7] L.K. Grover and A.M. Sengupta, Phys. Rev. A **65**, 032319 (2002).  
 [8] G.S. Agarwal, M.O. Scully, and H. Walther, Phys. Rev. Lett. **86**, 4271 (2001).  
 [9] See, for example, P. Facchi and S. Pascazio, in *Progress in Optics*, edited by E. Wolf (North-Holland, Amsterdam, 2001), Vol. 42, and references therein.