

INVENTORIES AND CAPACITY UTILIZATION
IN GENERAL EQUILIBRIUM

A Dissertation

by

DANILO R. TRUPKIN

Submitted to the Office of Graduate Studies of
Texas A&M University
in partial fulfillment of the requirements for the degree of

DOCTOR OF PHILOSOPHY

December 2008

Major Subject: Economics

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ABSTRACT

Inventories and Capacity Utilization
in General Equilibrium. (December 2008)

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Chair of Advisory Committee: Dr. Leonardo Auernheimer

The primary goal of this dissertation is to gain a better understanding, in the context of a dynamic stochastic general equilibrium framework, of the role of inventories and capacity utilization (of both capital and labor) and, in particular, the relationship among them. These are variables which have long been recognized as playing an important role in the business cycle. An analysis of the association between inventories and capital utilization seems natural, for physical capital could be seen as a stock ultimately destined to be transformed into an inventory of finished goods. In the same way, inventories could be seen as a stock of physical capital already transformed into finished goods. Introducing variable rates of utilization of capacity, then both can be seen as providing a short-run adjustment “buffer stock” mechanism.

The analysis of the relationship between those variables is centered on the effects of two possible shocks: preference (demand) shocks and technology shocks. Impulse-response experiments show that inventories and the rate of capital utilization are mostly complements, while inventories and the rate of labor utilization are mostly substitutes. Moreover, low-persistence shocks emphasize the role of inventories as being a “shock absorber”, whereas high-persistence shocks emphasize the role of inventories as being a complement to consumption. Consistent with the stylized facts in the literature, simulation results show that inventory holdings are pro-cyclical, while

the inventory-to-sales ratio is counter-cyclical.

Two additional “themes” are explored. The first has to do with the treatment of uncertainty and the consequences of using, as it is done in most of the literature, a first-order approximation. By approximating the decision rules to a second order, we observe that higher exogenous uncertainty enhances the importance of the precautionary motive to holding inventories. The second additional theme is a more general framework for the analysis of capital utilization. We find that the two most common ways of modeling capital utilization can fit in a more general specification that incorporates spending on capital maintenance. Though the aforementioned results do not vary qualitatively after that concept is introduced, quantitative answers do.

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CHAPTER I

INTRODUCTION

A. Motivation

Previous studies have shown that inventory fluctuations play an important role in explaining the business cycle. Although inventory investment has averaged roughly one-half of 1 percent of U.S.' GDP over the post-World War II period, changes in inventory investment have averaged more than one-third the changes in quarterly GDP (see Fitzgerald [19]). Moreover, Blinder and Maccini [9] show that the drop in inventory investment accounted for 87% of the drop in total output during the average postwar recession period in the U.S. In words of Blinder, “business cycles are, to a surprisingly large degree, inventory cycles.” (Blinder [8], p. 8).

The same way, capacity - or factor - utilization, being a central component of the economy's supply side, is often considered as an indicator of the state of real activity. For instance, variable factor utilization is thought to account for much of the variation in the Solow residual (40-60 percent according to Basu and Kimball [2]), and thus provide an important insight into characterizing the business cycle.

In this work, I study the equilibrium conditions that characterize the relationship between capacity utilization (i.e., the intensity of use of both the stock of capital and the labor force) and inventory investment. Likewise, I am interested in answering the question of whether these variables respond symmetrically to shocks that may differ in their rates of persistence.

The subject associated with understanding the relationship between capacity utilization and inventory investment relies on the fact that inventories are in some

This dissertation follows the style of *Journal of Economic Theory*.

way an alternative form of capital, and capital, as well, is somehow a form of inventory. Once the possibility of variable rates of utilization of both capital and labor is also considered, then both such rates of utilization and inventories can be conceived as providing a short-run adjustment “buffer stock” mechanism. For example, consider a machine that produces good A . The more intensively this machine is used (together with labor), the more A we obtain to either consume or store as inventories. However, there is a trade-off between the benefit of raising the quantity of goods produced and the cost of an accelerated rate at which physical capital depreciates. Roughly speaking, it is this trade-off that I seek to characterize under a dynamic, general equilibrium framework, and for which, as far as I am aware, there is no literature that emphatically faces it.

By also picturing the role of inventories as a form of capital, Christiano [14] considered them as an investment component that is in fact a residual during the business cycle. For instance, consider a shock to the economy, say a bad productivity shock. Although there would be a contractive effect on investment, inventories are those which would especially help the agent in smoothing consumption. In a broader sense, inventories play the role of “buffering consumption from unexpected disturbances in production, and buffering production from unexpected disturbances in consumption.” (Christiano [14], p. 248.) This attribute of inventories is indeed what distinguishes them from other types of capital, all these types involving certain degrees of pre-commitment and time to build (not only in the sense of Kydland and Prescott [30], but rather in a more general interpretation).

In this work, I develop a dynamic, stochastic general equilibrium (DSGE) model that departs from standard models in that it introduces endogenous capital depreciation, variable use of the labor force, and inventory holdings.

Endogenous capital depreciation captures the idea that the depreciation rate

is a variable subject to choice by the user of the capital good. As such, this rate enters into the model as a function of the intensity of use of capital (see, e.g., the works of Greenwood, Hercowitz, and Huffman [21] on variable capital utilization in a real business cycle model, and of Rumbos and Auernheimer [37] on variable capital depreciation in an otherwise standard neoclassical growth model).

Variable labor utilization is introduced into the model by allowing the agent to modify the working time during the production process. Within the macroeconomic literature on inventories, for instance, Galeotti, Maccini, and Schiantarelli [20] also distinguish between an employment decision (extensive margin), and an hours-per-worker decision (intensive margin). I will briefly go back to this reference when I describe the related literature.

Finally, the demand for the zero-return assets is motivated here by the idea that a larger stock of inventories allows consumers either to match their tastes more effectively or to economize on shopping costs. Technically, this is done by introducing inventories in the utility function, as is done by Kahn, McConnell and Perez-Quiros [26]. This approach is different to the widely-used approach of introducing inventories as inputs in the production function (see, e.g., Kydland and Prescott [30]). Although the focus of this work is on the analysis of final good inventories, one can still apply this argument to the analysis of intermediate good inventories. For instance, one can think of producers with larger inventories of intermediate goods as ultimately increasing consumers' possibilities - and utility - by expanding the different alternatives.

Following the “factor-hoarding” literature (e.g., Burnside, Eichenbaum and Rebelo [11] and Burnside and Eichenbaum [10]), I will assume that the demand for productive factors is decided before the state of nature is known, while the remaining variables such as capacity utilization, consumption and investment both in capital and inventories, are decided later with full information.

About the uncertainty involved in this economy, the model will feature preference and technology shocks. These shocks are important for the analysis as they give rise to another motive for the agent to hold inventories. This motivation is linked to the “stock-out avoidance” motive - see, e.g., Kahn [25] - in that the agent shields herself from the uncertainty associated to the disentanglement between the commitment of factor inputs and the realization of the shocks.¹

Results based on impulse-response analysis show that low-persistence shocks to preferences that relatively raises utility on both consumption and inventories make the agent reduce transitorily inventory holdings. On the other hand, these shocks lead to an increase in the intensity of use of both capital and labor. This is an appealing result one expects to hold if inventories are interpreted as a buffer of consumption. In contrast, both low-persistence technology shocks and high-persistence shocks - either to preferences or technology - make inventories complement both rates of utilization. In other words, the latter emphasize mostly the role of inventories as being a complement to consumption instead of emphasizing their role as a “shock absorber”.

Based on the approximated decision rules arising from the benchmark model, the computation of the series’ first- and second-order moments brought results that are consistent with the U.S. evidence. The volatilities of consumption, inventory holdings, and of services from capital and labor are lower than the volatility of output, while the volatility of investment is higher. Furthermore, consistent with the facts documented in the business-cycle literature of inventories (see, e.g., Ramey and West [35] and Iacoviello et al. [24]) the inventory-to-sales ratio is countercyclical, while inventory investment is procyclical.

Two additional issues are investigated in this work. The first issue has to do

¹See also Khan and Thomas [27] for a thorough discussion on the pros and cons of stockout-avoidance type-of models.

with the treatment of uncertainty and the consequences of using, as it is done in most of the literature, a first-order approximation to the decision rules. By approximating to a second order, one observes, as expected, that higher exogenous uncertainty enhances the importance of the precautionary motive to holding inventories. The second additional issue is related to a more general framework for the analysis of capital utilization. Christiano, Eichenbaum, and Evans [15] proposed an alternative way of modeling variable capital utilization. Instead of assuming an endogenous rate of depreciation, they simply impose a cost in terms of output whenever the stock of capital is used more intensively. By introducing this alternative specification, results do not change qualitatively, but quantitatively. Furthermore, by introducing capital maintenance into the model presented in Chapter II, it is shown that both this model and Christiano et al.'s [15] specification become special cases of a more general formulation. While the qualitative features remain unaltered, quantitative results are affected.

B. Literature Review

I will begin this section with a brief review of the macroeconomic literature on inventories. Next, I will continue with a brief review of the macroeconomic literature on variable capacity utilization, or, as is also known, the factor-hoarding literature. The combination of both lines of research was indeed reviewed in the motivation. Fitzgerald [19], p. 12, summarizes the macroeconomic literature on inventories as follows: “This literature provides a good example of how theory and data interact in the ongoing process of research.” Once the review of this collection of works concludes, we will be in a position to understand why that is the case.

For this research, the main question has been whether inventory investment was

in fact a key determinant in the amplification and propagation of shocks. Motivated by this inquiry, in a classic paper Metzler [33] developed a model where exogenous, uncorrelated shocks, together with inventory investment, could generate the business cycle fluctuations of output observed in the data. Later, Holt, Modigliani, Muth, and Simon [23] introduced, in a simple linear-quadratic model of optimal inventory behavior, the “production smoothing” motive for holding inventories. They assumed convex costs of production and firms facing variable and uncertain demand for goods that are storable. Thus, according to this approach, a profit-maximizing firm will have incentives to hold inventories to smooth the time path of production in order to reduce average costs. As perhaps their mostly recognized role, inventories acted as a buffer stock in production. Notice that the production-smoothing notion implies two testable implications: the variance of sales exceeds the variance of production, and inventories and output are negatively correlated. The idea is simple. Since the cost of production is convex, firms will generate a surplus when sales are low, and use this surplus when sales are high. Providing microeconomic foundations to the business-cycle analysis of inventory behavior, this was considered the standard model until the early 1980s.

Works by Blinder [6], [7] and Blanchard [4], among others, put in evidence the weaknesses of the standard model in replicating some of the stylized facts on inventory cycles. These facts are: (i) at the aggregate level, output was more variable than sales, and (ii) inventory investment was procyclical. In order to explain these contradictions, the new class of models basically added shocks to the firm’s production costs. In this line, suppose the extreme case in which sales are constant. Assuming, as an extreme case, that sales are constant, then it is easy to see that since production is supposed to follow costs shocks, it will clearly be more volatile than sales, and also positively correlated with inventories.

At the same time, though focused on a different motivation, Kydland and Prescott [30] developed a model where inventories enter the production function, and uncertainty arises from productivity shocks. They found that both cyclical fluctuations in inventories and their correlation with output were consistent with the data.

On the other hand, Kahn [25] provided a theoretical model where firms followed a target on inventories, introducing this way a *stock-out avoidance* motive for inventory investment. Under the assumption that production takes time, firms demand inventories so as to be ready for unexpected increases in sales. The paper rationalizes this motive by introducing both a non-negativity constraint on inventories and serially-correlated demand shocks. These two assumptions allow to obtain the final goal of explaining the stylized fact that the variance of production exceeds that of sales.

More recently, Bils and Kahn [3] developed a model in which sales are simply an increasing function of inventory holdings. As such, final good inventories facilitate sales where a speculative, stockout-avoidance motive dominates the rationale for holding inventories. They show that both the markup of price over marginal cost and expected changes in marginal costs are the main determinants of the inventory cycle. In line with the evidence, their model generates the result that inventories vary in proportion to expected sales.

Other approaches simply assumed that the marginal cost of production was decreasing. This way, one eliminates the *production-smoothing* result. For instance, Ramey [34] found evidence of non-convex marginal costs in a large number of manufacturing industries.

Most of the literature on inventories has concentrated in modifying the *production-smoothing* model in order to replicate the evidence. However, there is an alternative theory of inventory behavior provided by the so-called (S,s) approach, which focuses

on the timing of deliveries rather than the timing of production (e.g., Caballero and Engel [12], Fisher and Hornstein [18], and the recent contributions by Khan and Thomas [27], [28]). Briefly, this theory states that the firm's choice about inventories is one in which it optimally chooses some minimum level, s , below which it does not let inventories fall. When inventory stocks reach that level, the firm orders a new lot, so that the stocks rise to the optimally chosen level, S . The assumption of a fixed cost of acquiring goods, in addition to the marginal cost, leads to this (S,s) behavior.

Recently, Galeotti et al. [20] developed a stockout-avoidance type-of model integrating inventory and labor decisions, distinguishing between an extensive and an intensive margin decision. Their model is closely related to this work, in particular, regarding the time structure of information and decisions. In Galeotti et al. [20] the firm decides on inventory investment and employment (the extensive margin) before sales and technology shocks are known. Later on, the firm adjusts the intensity of use of labor (the intensive margin). The main differences of this work with respect to Galeotti et al. [20] lies on that they use a partial equilibrium approach, and capital accumulation is absent. In contrast, this dissertation intends to integrate inventory investment with both endogenous capital utilization and variable labor intensity. Iacoviello et al. [24] is another recent approach closely related to this work. In a DSGE model, the firm decides on both inventory investment and capital utilization, while inventories provide utility to the consumer. Main differences between their specification and this work include the time structure of decisions and information, the absence of variable use of labor in their model, and the fact that they contemplate intermediate and final good inventories, while this work is focused on final good inventories.

Finally, it is worthwhile noting that, according to the empirical work by Wen [39], the stock-out avoidance approach à la Kahn [25] seems to have better potential than other theories for explaining inventory fluctuations. Distinguishing between low and

high-frequency time series, he found that this approach performs well at different cyclical frequencies and, most importantly, that demand shocks are the main source of the business cycle.

I will turn now to briefly review the literature related to variable capacity utilization and the business cycle. Although the idea of exploring variable capacity utilization with endogenous depreciation is not new - one could go back, for instance, to Keynes' notions of "user cost" - Calvo [13] and Greenwood et al. [21] are standard references in this literature. In particular, the latter introduce variable capacity utilization in a standard, real business cycle model, where shocks to investment generate business fluctuations, and where capital depreciation is a function of its intensity of use. In their model, as in this one, variable capacity utilization allows predicting the Keynesian type result of less than "full capacity equilibrium". Nevertheless, Greenwood et al. [21] only consider variable capital utilization, leaving aside variable labor intensity.

Burnside and Eichenbaum [10] studied the capacity-utilization issue introducing variable rates of utilization on both capital and labor. They analyze the role of these rates in propagating shocks over the business cycle in a model where capacity utilization is treated as a form of factor-hoarding. To model variable capacity utilization, they assume that the production function depends on effective capital services - the capital-utilization rate times the stock of capital - and on effective hours of work - labor effort times total hours of work. As in Greenwood et al. [21], they assume that the rate at which capital depreciates is a function of the capital-utilization rate.

More recently, Rumbos and Auernheimer [37] introduce variable capital utilization into a modified Ramsey-type model by modeling the notion of pure user cost. They find that the introduction of a variable utilization rate yields a slower rate of convergence toward the steady state, inducing more persistence in the transitional

dynamics.

Nowadays, variable capacity utilization, and especially variable capital utilization, are being widely used in standard, DSGE models in many areas of macroeconomics. For instance, Christiano et al. [15] introduce capital utilization in a study of nominal rigidities and monetary policy. Not surprisingly, one of the key features of their model that serve to account for the observed inertia in inflation and persistence in output is variable capital utilization. An interesting feature of their model economy is that it departs from the standard specification which assumes endogenous capital depreciation. They assume constant depreciation in a model where varying the capital-utilization rate implies a cost that is borne through less resources in consumption goods. In Chapter IV, I will go over this particular issue in order to compare how different the conclusions are if one took that approach.

C. Scope

The body of this dissertation contains five chapters. Chapter II introduces the general equilibrium model which later on is referred to as the benchmark specification. In that chapter, I describe the environment, discuss the assumptions of the model, and provide the optimality conditions for the benchmark setting.

Chapter III proceeds with the solution to the model, and shows the main results. I also discuss the specific functional forms and the baseline parameter values assumed for solving the model. An analysis of the impulse-response functions and the second-order-moment properties generated by the model is provided. Moreover, it shows the effects of inventories and capacity utilization, the relationship between them, and the source of shocks behind these results. Finally, I discuss the relevance of considering a stochastic model, and especially the effect of uncertainty on inventories and capacity

utilization.

Chapter IV provides an analysis of the robustness of the model results introduced in the preceding chapter. An alternative specification on preferences, one which can be approximated by a special case of the benchmark utility function, is discussed. Regarding the way of modeling variable capital utilization, an alternative setting in the literature is Christiano et al. [15]. I introduce the latter's assumptions into the benchmark model in order to compare the main results. Further, a more general treatment to specifying capital utilization is provided, where the benchmark and the Christiano et al. [15] specifications result as special cases. In Chapter V I present the conclusions of the dissertation.

CHAPTER II

A GENERAL-EQUILIBRIUM MODEL

Consider a standard, dynamic stochastic general equilibrium model, modified to incorporate inventory holdings and variable capacity utilization of both capital and labor. Inventory holdings are introduced into the model by assuming that they provide utility services as a proxy for decreasing shopping time and increasing variety. Being an asset, they also shield the agent from the uncertainty arising on the timing gap between production decisions and consumption and investment decisions. Variable capacity utilization is introduced into the model by allowing the agent to choose the rates at which capital and labor are used during the production process. Capital utilization will involve an endogenous capital depreciation rate, while labor intensity will involve an additional disutility term to the agent arising from adjustments on work time.

A. The Environment

The model economy is populated by a continuum of identical agents with unit mass. There is one good that may take the form of consumption, capital, and inventories.¹ Output is produced according to a production function $Y_t = F(e_t N_t, s_t K_t; \omega_t)$; where N_t and K_t denote the levels of labor and capital at time t , e_t and s_t denote the rates of labor and capital utilization at time t , and ω_t represents the exogenous, stochastic

¹Inventories are final-good inventories in this one-sector model. Other papers analyze both final and intermediate-good inventories, highlighting the importance of separating them. The main reason for studying both categories appears from the fact that, as the data show, intermediate-good inventories are relevant both as a fraction of total inventories (they are roughly more than one-third of total inventory holdings) and in terms of their relative volatility (they roughly represent almost one-third the variability of total inventories). For evidence on this issue, see, e.g., Blinder and Maccini [9].

time- t level of technology. As stated by this production function, what matters for producing output is the total amount of effective capital, $s_t K_t$, i.e., capital services per unit of time, and effective time of work, $e_t N_t$, i.e., labor services per unit of time. I will assume that markets are competitive in this economy. Given this assumption, the presence of firms will no play any relevant role, thus for now I assume that there are no firms. In Appendix C, I show that the same results arise in a model with competitive firms.

As is standard in the capital utilization literature, the model assumes that using capital more intensively raises the rate at which it depreciates (see, e.g., Calvo [13], Greenwood et al. [21], and Rumbos and Auernheimer [37] on the commonly also known endogenous capital depreciation literature). For that matter, I will assume that the time- t depreciation rate of capital, δ_t , is an increasing and convex function of s_t , given by $\delta_t = \delta(s_t)$. Therefore, the stock of capital evolves according to:

$$K_{t+1} = K_t(1 - \delta_t) + I_t, \quad (2.1)$$

where I_t denotes time- t gross investment in physical capital.

At any date t , the resource constraint of the representative agent is given by:

$$C_t + I_t[1 + h(I_t)] + Q_{t+1} - Q_t \leq Y_t, \quad (2.2)$$

where C_t denotes time- t households consumption, Q_t denotes inventory holdings at time t , and $h(I_t)$ is an increasing function that captures adjustment costs on investment, such that total investment cost, $I_t h(I_t)$, is convex.²

The typical agent in this economy maximizes her expected lifetime utility as

²I assume that inventories do not depreciate. Other studies introduce a fixed depreciation rate in order to capture inventory holding costs. In any case, this simplification is not central for the questions I intend to address.

given by

$$E_0 \sum_{t=0}^{\infty} \beta^t u(C_t, l_t, e_t, Q_t; z_t), \quad (2.3)$$

where $0 < \beta < 1$ is the discount factor, $l_t \equiv 1 - e_t N_t$ stands for leisure at time t - out of a normalized time endowment of 1 -, and z_t represents an exogenous, stochastic preference shock.

The reason why the third argument, e_t , and the fourth argument, Q_t , are listed in u plays an important role, so it warrants some discussion. On the third element, the rate of labor intensity, it is clear that it affects the agent's utility through its effect on leisure, since the time- t leisure is defined as $1 - e_t N_t$. However, by introducing a separate argument for e_t one may clearly understand the effects of allowing the agent to independently choose the intensity at which the leisure time is used. This will become clear below, once the assumptions made on the information structure are described. Moreover, the time- t instantaneous utility function further includes the stock of inventories at time t . Its motivation was briefly mentioned in the introduction, and it will become clearer later when studying its effects. The way they enter in u is consistent with others settings in the inventory literature (e.g., Iacoviello et al. [24], and Kahn et al. [26]).

B. Time Structure of Decisions and Information

At the beginning of date t , before the state of nature is known, the agent must decide on the time she will allocate to work at time t , N_t . The stock of capital available for production at time t , K_t , is given by the investment decision at the previous date. Thus, at the moment the shocks are finally revealed, the stocks of factors of production available for producing output are given. These assumptions, and in particular the assumption related to the labor decision, indicate that the model is,

somehow, a *stock-out avoidance* kind-of model. Remember that, in those models, the time structure of decisions and information plays an important role. That is, the fact that the stocks of factors of production are given at the moment the demand and the state of technology are revealed, gives rise to another motive for holding inventories - in addition to that of saving shopping time, which is that of protecting the agent from uncertainty.

After the shocks are revealed, the agent chooses the rates of both capital utilization and labor intensity, s_t and e_t , in order to adjust the services of capital and labor to the new information. Finally, the agent decides how much to consume, C_t , and how much to invest for the next period both in physical capital, K_{t+1} , and in inventory holdings, Q_{t+1} . See the timeline in Figure 1.³

C. A Discussion on the Rate of Labor Intensity

Having described the information structure, it remains to explain the appearance of e_t as a separate element in the utility function. The argument goes as follows. Once the agent has already decided on the time devoted to work at t , N_t , the best she can do in response to a shock is to vary this pre-committed working time by choosing the appropriate rate of labor intensity. However, one should expect that, as the agent moves away from the previously optimal time to work, it does not come for free. In other words, it makes sense that the agent faces a cost in terms of disutility of changing the plans already made. This cost is captured here by identifying a separate argument that is additional to the “linear” effect of e_t on leisure, as measured by l_t . *Ex-ante*, whether the agent increases e_t or N_t will be the same in terms of disutility arising from effective work. *Ex-post*, changing e_t will not be the same as changing N_t

³Figures and tables are shown in Appendix E.

for it will involve an extra-cost in terms of disutility - despite the fact that N_t cannot be changed after the shock indeed.

Consider, as an example, the difference between extensive and intensive-margin decisions related to the demand for labor by the firm (see, e.g., Galeotti et al. [20] for a thorough discussion on the relation between inventories and employment decisions). The extensive margin refers to the firm's demand in terms of the number of employees - translated to this model as the amount of labor that the agent decides before the shocks are realized, N_t . The intensive margin refers to the firm's decision on the number of hours the workers will be effectively working, $e_t N_t$, which can differ from that previously established in labor contracts. That is translated to this work as the decision on the effective labor, which ultimately depends on the rate of labor utilization.

Both in theory and in practice, it is well known that a decision on the intensive margin that implies a change in the number of hours established in labor contracts, say extra hours or lay-offs, in general involves an extra cost for the firm. Extra hours are paid more than the average hourly wage rate, while lay-off hours, even though not productive, still involve some costs for the firm. The implications of a specific market structure that can support these considerations and, more in general, can support the representative-agent model described in this section will be carefully discussed in Appendix C.

One way of rationalizing the last argument under a representative agent problem implies thinking on adjustment costs in the form of disutility. The motivation can briefly be stated as follows. Consider an individual who has already planned her day regarding the time allocated to both work and leisure. Further, suppose this individual made plans already for the time immediately after work. Suppose now that all of a sudden she receives an assignment at the very last minute that requires

her to stay one more hour. What happens is that this individual has to make changes in her plans, which ultimately imply an extra cost in addition to the cost of working the extra hour. This extra cost on changing plans would not exist if the individual had originally planned on staying one more hour at work. The same occurs if this individual receives the news that she can leave the workplace one hour earlier. It is clear that this person is happier now because of this extra time for leisure, but one should expect this person would have been happier if she could originally make plans for that extra hour of leisure. It is this extra cost what led me to introduce the rate of labor utilization as a separate argument in utility, which I believe will also ease the exposition later on.

D. Dimensions of the Rates of Utilization

I have defined time- t labor services and capital services as $e_t N_t$ and $s_t K_t$ respectively. For ulterior use, let me now set $L_t \equiv e_t N_t$ and $S_t \equiv s_t K_t$. These are the flows of factor services per unit of time that are used to produce output with the technology given by $F(L_t, S_t)$ - I abstract for now from the level of technology, ω_t .

Variables e_t and s_t represent the intensities of use of labor and capital. In the case of the former, it can simply be interpreted as an index of the period- t utilization rate of the stock of labor, which in turns is measured here in fractions of a unit of time. In the case of the latter, s_t can be interpreted in two ways: (i) it can be understood as measuring the speed at which the stock of capital is operated per unit of time, with a given amount of labor services, or (ii) it can be understood as measuring the fraction of a unit of time at which the stock of capital is operated, given a capital-labor services ratio. To put this notion formally, consider the following argument based on Calvo [13] and Burnside and Eichenbaum [10]. Define $\bar{eN}(K, sK/eN)$ as the flow

of labor services required to operate capital at the maximum speed, when $S \equiv K$, with the capital-labor services ratio equal to sK/eN . This ratio can be interpreted as the amount of capital at the disposal of the typical individual during her shift. This implies that

$$\frac{K}{\overline{eN}(K, sK/eN)} = \frac{sK}{eN}; \quad (2.4)$$

and then, $s\overline{eN} = eN$. Finally, notice that the production function being homogeneous of degree one - which is the case, as it will be assumed later - implies that

$$F(s\overline{eN}, sK) = sF(\overline{eN}, K). \quad (2.5)$$

The expression $F(\overline{eN}, K)$ can be interpreted as the “full-capacity” output when the factor services ratio equals sK/eN and total supply of capital services is K . This reasoning is then intending to show that the rate of capital utilization, s , can be conceived as the share of the maximum output obtained when “machines” are operated a fraction s of a unit of time, per unit of time, for a given factor services ratio - e.g., $s = 0.2$ implies that capital is being used 20% of time per date, given a factor services ratio.

On the complementarity between the rates of labor and capital utilization, consider also the following reasoning. Assume a firm that uses different combinations of services from computers and workers to produce a unit of output under a given technology. Further, take the set up discussed so far, where capital and labor, i.e., computers and workers, can be utilized at different rates. Now, one might raise the question of whether, say, an increase in the intensity of use of labor would automatically imply an increase in the intensity of use of capital, and viceversa. After all, if we say that the workers are the ones who use the computers, any rise in the intensity of use of labor, suppose an extra-hour, would immediately imply a rise in the intensity

of use of computers - the extra-hour - other things the same.

A corollary of the argument made above would be that the two rates of capital utilization should be clearly related in a one-to-one basis. However, even when the argument is correctly placed, there is a mistake in the conclusion one is tempted to assert. In fact, we do not need to specify additional, technical assumptions on the relationship between these two variables. Again, consider the extra-hour arising from an increase in the rate of labor utilization. In some way, it does not necessarily mean that computers will also be used one more hour, but simply that individuals will work more for an hour, perhaps with the same or less computer services than before, on average. After all, these workers might be willing to change the technology towards a labor-intensive production process. Of course, if the production function is of a constant-return-to-scale technology, then the two rates will be positively related but, once more, it does not mean that they should have a one-to-one relationship imposed by assumption.

E. On the Properties of the Utility Function

A utility function $u(C_t, l_t, e_t, Q_t; z_t)$ that satisfies the aforementioned properties of e_t must satisfy the following assumptions: $u_3(\cdot) \geq 0$ for $e_t \leq 1$, and $u_{33}(\cdot) < 0$ for all e_t . In fact, these assumptions imply the existence of a cost, or a “punishment”, in terms of utility to any deviation of e_t from its *ex-ante* optimal choice \bar{e} . As shown below, it will be the case that $\bar{e} = 1$. In particular, the property $u_3(e_t = 1, \cdot) = 0$ comes from the fact that at this point, $e_t N_t = N_t$, which means that there is no deviation from the decision on leisure already planned, i.e., there are no extra costs in terms of disutility. Only at those points where $e_t \neq 1$, the agent can do better by getting closer to the optimal, *ex-ante* time allocation. For a very simple illustration

of a utility function that satisfies these properties, see Figure 2.

With regards to inventory holdings, I assume that $u_4 > 0$ and $u_{44} < 0$, for all Q_t . That is, the agent receives utility services that are proportional to inventory holdings, and, as it is commonly assumed for the consumption good as well, the marginal utility on inventories is decreasing. These assumptions are consistent with those considered in Iacoviello et al. [24] and Kahn et al. [26].

F. The Optimization Problem

Given the assumptions made so far, the competitive equilibrium corresponds to the solution for the aggregate of all representative individuals of this economy. I will discuss now the problem of the typical agent as was described above. The competitive-equilibrium problem with firms is discussed in Appendix C.

To posit this problem in as a stylized way, consider the following notation. Define as $\Gamma_{t-1} \equiv (\omega_{t-1}, z_{t-1})$ the set of previous date's realizations of the exogenous shocks. Then, the time- t set of state variables for the optimization problem will be given by $\Phi_t \equiv \{K_t, Q_t, \Gamma_{t-1}\}$. Among the time- t choice variables is N_t (decided before the shocks are revealed) and the set $\Omega_t \equiv \{C_t, e_t, s_t, K_{t+1}, Q_{t+1}\}$, which includes the variables to decide after the new information is received.

In order to present the problem in a way that reflects explicitly the informational constraints, I set up the following dynamic programming problem.⁴

$$V(\Phi_t) = \max_{N_t} E \{ \max_{\Omega_t} E [u(C_t, l_t, e_t, Q_t; z_t) + \beta V(\Phi_{t+1}) \mid \Gamma_t] \mid \Gamma_{t-1} \} \quad (2.6)$$

subject to (2.1) and (2.2), $K_{t+1} \geq 0$, $Q_{t+1} \geq 0$, and given K_0 and Q_0 .

⁴The way in which I describe the information structure and the timing of decisions is close to the approach followed by Christiano [14].

The solution to this problem is characterized by the following five optimality conditions - in addition to (2.1), (2.2) given with equality, and the transversality requirements.⁵

$$E \{ e_t \times [u_1(C_t, l_t, e_t, Q_t)F_1(e_t N_t, s_t K_t) - u_2(C_t, l_t, e_t, Q_t)] \mid \Gamma_{t-1} \} = 0 \quad (2.7)$$

$$u_1(C_t, l_t, e_t, Q_t)F_1(e_t N_t, s_t K_t) - u_2(C_t, l_t, e_t, Q_t) = -\frac{u_3(C_t, l_t, e_t, Q_t)}{N_t} \quad (2.8)$$

$$F_2(e_t N_t, s_t K_t) = \delta'(s_t)p_{k,t} \quad (2.9)$$

$$u_1(C_t, l_t, e_t, Q_t)p_{k,t} = \beta E \left\{ \left[\begin{array}{c} u_1(C_{t+1}, l_{t+1}, e_{t+1}, Q_{t+1}) \times \\ s_{t+1} F_2(e_{t+1} N_{t+1}, s_{t+1} K_{t+1}) + \\ (1 - \delta_{t+1}) p_{k,t+1} \end{array} \right] \mid \Gamma_t \right\} \quad (2.10)$$

$$u_1(C_t, l_t, e_t, Q_t) = \beta E \left[\begin{array}{c} u_1(C_{t+1}, l_{t+1}, e_{t+1}, Q_{t+1}) + \\ u_4(C_{t+1}, l_{t+1}, e_{t+1}, Q_{t+1}) \mid \Gamma_t \end{array} \right] \quad (2.11)$$

where $p_{k,t} = 1 + h(I_t) + I_t h'(I_t)$ can be interpreted as the relative price of a unit of capital good in terms of consumption good. That relative price, which even in steady state will be larger than one, arises from the existence of a cost of transforming a unit of consumption into a unit of capital.

Optimality condition (2.7) sets the expected marginal product of labor equal to the expected marginal utility of leisure, measured in terms of consumption. This equation is expressed in terms of expectations because, remember, the agent decides labor before the information is known. The second equation (2.8) refers to the choice of the rate of labor intensity, and it sets the difference between the marginal product of labor and the marginal utility of leisure equal to the marginal disutility of the rate

⁵To simplify notation, I write both the utility function and the production function without their respective states of preferences and technology, z_t and ω_t .

of labor intensity. Notice that if what was planned in (2.7) is indeed optimal when the shocks are realized, then because of the way I defined $u_3(\cdot)$, it must be the case that $e_t = 1$. This is the same as saying that the right-hand side of (2.8) is equal to zero, i.e. $u_3(\cdot) = 0$, and thus this condition reduces to (2.7) *ex-post*. Intuitively, this means that there is no shock that could make the agent deviate optimally from what she had already planned in regards to the allocation of time.

Equation (2.9) is standard in the literature of variable capital utilization. It characterizes efficient use of capital by stating that the stock of capital should be utilized at the rate s_t which sets the marginal benefit of capital services equal to the marginal user's cost. The latter is made up of two elements. The component $\delta'(s_t)$ is the marginal cost in terms of a higher depreciation from using capital more intensively, whereas the factor $p_{k,t} = 1 + h(I_t) + I_t h'(I_t)$ represents the current replacement cost of old in terms of new capital good.

Equation (2.10), although a bit “scary”, is a standard optimality condition on investment - an Euler equation. The left-hand side is the loss in current utility from an extra unit invested in physical capital. The right-hand side represents the discounted expected future utility obtained from that unit. The first term inside brackets, $s_{t+1} F_2(e_{t+1} N_{t+1}, s_{t+1} K_{t+1})$, is the marginal benefit in terms of production of a unit of capital in the next period - adjusted for its rate of utilization -, while the second term, $(1 - \delta_{t+1}) p_{k,t+1}$, represents the future value of this unit after depreciation, both margins in terms of consumption.

Finally, equation (2.11) characterizes the optimal decision of holding inventories. It sets the loss in current utility from an extra unit stored as inventory today equal to the discounted expected future utility obtained from that unit. The second term inside brackets, $u_4(C_{t+1}, L_{t+1}, e_{t+1}, Q_{t+1})$, represents the future utility received from an extra unit of inventory accumulated today, while the first term, $u_1(C_{t+1}, L_{t+1}, e_{t+1}, Q_{t+1})$,

represents its future value in terms of consumption, which is indeed the same as the value of an extra unit of consumption good.

CHAPTER III

SOLVING THE MODEL: THE MAIN RESULTS

In this chapter, I proceed with the solution to the model, and show the main results. First, I discuss the specific functional forms and the baseline parameter values assumed for solving the model. Second, I introduce the analysis of the impulse-response functions and the second-order-moment properties generated by the model. Later on, I present the effects of inventories and capacity utilization, the relationship between them, and the source of shocks behind these results. Finally, I discuss the relevance of considering a stochastic model, and, especially, the effect of uncertainty on inventories and capacity utilization.

A. Functional Forms and Parameter Values

The specific production technology is assumed to be

$$F(e_t N_t, s_t K_t; \omega_t) = \omega_t (e_t N_t)^{1-\alpha} (s_t K_t)^\alpha, \quad (3.1)$$

where $0 < \alpha < 1$ is the share of income allocated to effective capital.

The state of technology is assumed to evolve according to the following stochastic process

$$\ln \omega_t = \rho_\omega \ln \omega_{t-1} + \epsilon_{\omega t}, \quad (3.2)$$

where $\epsilon_{\omega t}$ is a serially uncorrelated process with zero mean and standard deviation σ_ω , and $0 < \rho_\omega < 1$ is the first-order autoregressive coefficient.

The utility function is specified by

$$u(C_t, l_t, e_t, Q_t; z_t) = z_t X_t + \eta l_t - \frac{\phi}{2} (e_t - 1)^2, \quad (3.3)$$

where X_t is the natural log of a CES bundle of consumption and inventory goods, defined as $X_t \equiv \ln[\theta C_t^{1-\gamma} + (1-\theta)Q_t^{1-\gamma}]^{\frac{1}{1-\gamma}}$, and $\eta > 0$, $\phi > 0$, $0 < \theta < 1$, $\gamma > 1$.

The first term in (3.3), $z_t X_t$, captures the notion that larger inventory holdings, Q_t , raise the marginal utility of any given purchase of consumption, C_t , either by reducing transaction costs or by better matching the consumer's tastes. The parameter γ is the inverse of an elasticity of substitution, which will rule the degree to which consumption and inventories are related. The shock z_t will work as a preference shifter that affects the relative taste between goods (consumption and inventories) and leisure at time t .

The second term in (3.3) shows that utility is linear in leisure, as in Hansen [22] and Rogerson [36], which implies this utility function is consistent with any degree of intertemporal substitutability of leisure at the individual level. One of several interpretations of this property is that the average time the individual devotes to work will be constant, and all fluctuations in labor services will result from indivisible labor decisions, i.e., either full time work or no work at all. In Appendix C, I discuss in detail the implications of working with this type of preferences on leisure.

The last term in the utility function represents exactly the idea discussed above of introducing an additional effect of variable labor intensity on the agent's preferences. The interesting feature of introducing that term this way in the utility function is that one obtains a relatively simple utility function with similar properties as those found by Burnside, Eichenbaum and Rebelo [11] and Burnside and Eichenbaum [10] in their studies of labor and capital hoarding.

I assume that the preference shock behaves according to the following stochastic process

$$\ln z_t = \rho_z \ln z_{t-1} + \epsilon_{zt}, \quad (3.4)$$

where ϵ_{zt} is a serially uncorrelated process with zero mean and standard deviation σ_z , and $0 < \rho_z < 1$.

I assume that total adjustment costs on investment, $I_t h(I_t)$, are quadratic. In particular, consider $h(I_t) = bI_t$, with $b > 0$.

Assume that the rate of capital depreciation is given by $\delta(s_t) = \delta_0 + s_t^v$, with $\delta_0 > 0$, and $v > 1$. The first term represents a fixed component of the depreciation rate, while the second term refers to the endogenous part of capital depreciation, the one which depends on its use. Notice that, as v goes to infinity, the second term approaches to zero, and the endogenous depreciation rate approaches to the standard, constant rate.

Before proceeding with the parameterization of the model economy, I define the length of a date to be a quarter, which seems appropriate since most of the literature have documented the parameters of my interest in terms of U.S. quarterly data. The baseline values for the model's tastes and technology parameters were chosen in the following manner. The value of the discount factor β is set at 0.988 to match an average annual real interest rate close to 4%. Moreover, as is standard in the literature - and this work is not the exception - this value is determined so that the model's average quarterly capital-to-output ratio, K/Y , is approximately that found in the data. The preference parameter η is chosen so that in equilibrium the agent spends on average approximately one-third of the time working. The value that results following this criterion is $\eta = 2.5$. The capital share parameter α is set to approach the average capital's share in the U.S. national income accounts, roughly being 0.36 - for details, see Appendix A.

The constant fraction of the depreciation rate, δ_0 , is arbitrarily set at 0.01, following Rumbos and Auernheimer [37]. Indeed, the cyclical properties of equilibria in the model do not depend on the value of this parameter. The parameter θ , represent-

ing the weight of the consumption good in the CES bundle of (3.3), is set to obtain an inventory-to-sales (I-S) ratio roughly in line with the average in the U.S. In this sense, I set $\theta = 0.98$, which approximately corresponds to an average I-S ratio of 0.8.¹

About the preference relation between consumption and inventories, the model requires $\gamma > 1$ for inventories to complement consumption. Although Kahn et al. [26] state that inventories are procyclical for sufficiently large values of γ (they set $\gamma = 10$), this model's simulations allow inventories to be procyclical at relatively low values. I will arbitrarily assume a benchmark value of $\gamma = 5$, but the main results are not particularly sensitive to the choice of this parameter. In Chapter IV and Appendix B, I discuss the possibility that γ approaches to 1. In this case, the utility function becomes a separable function in the logs of consumption and inventories. The benefits of analyzing this special case are: First - and the most obvious, that the utility function becomes simpler, and with it so does the study of the questions I intend to address; second, it allows me to identify more easily the kind of preference shocks I am interested in working with.

Regarding the second advantage discussed above, I must point out that in the benchmark utility function I have expressed the preference shock as a multiplicative shock that affects the whole CES bundle of goods, X_t . This way, it is not very clear that an expansionary preference shock could resemble a demand shock as a shock pulling consumption. The reason is that here a preference shock would shift tastes away from leisure but toward *both* consumption and inventory holdings.²In this sense, one may want to isolate a preference shock toward consumption goods and, indeed,

¹Sales here are simply defined as consumption plus capital investment, which is equivalent to the difference between output and inventory investment.

²Below I discuss in detail this kind of shocks, which are often called "labor supply" shocks. For now, notice that in essence this shock shifts the labor supply by altering the marginal rate of substitution between consumption and leisure.

away from inventories and leisure.

On the one hand, if one were to work with a utility function that is separable in consumption and inventories, that problem could be handled in a rather easy way by simply identifying the “target” of a variety of shocks. This variety of shocks may include shocks individually targeting consumption, inventories, and even the labor supply.³

On the other hand, the main cost in applying this particular, separable utility function is that one loses the property of inventories as being a complement to consumption. In any case, I will compare the results of both functional forms below when providing robustness and sensitivity analyses in depth.

The parameter related to the magnitude of adjustment costs on labor, ϕ , was arbitrarily chosen to be 0.5, although the results are not particularly sensitive to this choice. With respect to the adjustment costs on capital, parameter b was set at 1. This parameter was calibrated so that the average replacement cost of capital, $\bar{p}_k = 1 + 2b\bar{I}$, lies around a reasonable value - the baseline steady-state price of capital was 1.2 (see, e.g., Blanchard, Rhee, and Summers [5] for estimates in the U.S.)

The parameter indicating the degree of endogeneity of capital depreciation, v , together with the standard deviations of the innovations, σ_z and σ_ω , are set so that the cyclical volatilities of the model’s series approach U.S. cyclical volatilities. I assume baseline values $v = 2$, $\sigma_z = 1$, and $\sigma_\omega = 0.07$.⁴ Finally, the autoregressive parameters

³Of course, an alternative could be to add in the benchmark utility function a shock that affects only the consumption component of the CES bundle. However, as I will show and explain below, that would bring counterfactual and even counterintuitive results.

⁴The magnitudes of these baseline standard deviations are consistent with the business cycle literature - see, e.g., Arias et al. [1]. Standard deviations express percent deviations of the innovations from their steady state values, which are assumed to be one.

of the two shocks' stochastic processes, ρ_z and ρ_ω , are set equal to 0.95, as is standard in the literature.

It is worth noting that most of the parameters for which there are neither counterparts to validate from previous works nor a common way to calibrate, especially v , σ_z , and ρ_z , will be subject to robustness and sensitivity analyses in Chapter IV.

B. Model Solution

At this point, I make use of the specific functional forms (3.1) and (3.3), the stochastic processes (3.2) and (3.4), and the parameter values introduced in the preceding section. By plugging these specifications into the economy's resource constraint (2.2), the law of motion for capital (2.1), and the optimality conditions (2.7)-(2.11), one obtains a system of nine equations which completely describe the equilibrium behavior of the model's variables.

In general, it is not possible to solve this type of models analytically, given the strong non-linearity of the system's equations, additionally to its relatively large dimension. Therefore, I use a pure perturbation approach as in Schmitt-Grohé and Uribe [38] to obtain an approximate solution for the agent's decision rules. Our model could be summarized by a collection of equilibrium conditions that take the following general form:

$$\begin{aligned}
 E_t\{f(y_{t+1}, y_t, y_{t-1}, \Gamma_t)\} &= 0, \\
 \Gamma_t &= \rho\Gamma_{t-1} + \epsilon_t, \\
 E(\epsilon_t) &= 0, \\
 E(\epsilon_t\epsilon_t') &= \Sigma_\epsilon,
 \end{aligned} \tag{3.5}$$

where,

y : vector of the endogenous variables;

Γ : vector of the exogenous, stochastic shocks: ω , and z .

ϵ : vector of innovations corresponding to each shock.

A solution to this problem is a set of equations that relate variables in the current period with past state variables and current shocks, where the original system is satisfied. The resulting policy function can be written as

$$y_t = g(y_{t-1}, \Gamma_t). \quad (3.6)$$

A first-order linearization of the function g around the non-stochastic steady state \bar{y} , yields⁵

$$y_t = \bar{y} + g_y \hat{y} + g_\Gamma \hat{\Gamma}, \quad (3.7)$$

where $\hat{y} = y_{t-1} - \bar{y}$, $\hat{\Gamma} = \Gamma_t - \bar{\Gamma}$, $g_y = \frac{\partial g}{\partial y_{t-1}}$, and $g_\Gamma = \frac{\partial g}{\partial \Gamma_t}$. For details on the properties of the function g , and how to obtain it, see, e.g., Collard and Juillard [16] or Schmitt-Grohé and Uribe [38].

In the three sub-sections that follow, I will study the central question that motivates this work, that is the relationship between inventories and the intensity of use of the factors of production. For this purpose, I will first study the impulse-response functions that the model generates. In particular, these are the result of iterating the policy function (3.7) starting from an initial value given by the steady state. Second, I will analyze the properties of the first and second-order moments arising from the approximated policy function. Third, I will discuss how uncertainty affects the

⁵In the section related to “Does Uncertainty Matter?” I will discuss the properties of approximating the policy function up to a second order. For now, I will focus on the main results of this model, say impulse-response functions and first- and second-order moments, those that are not especially sensitive to the order of approximation. In particular, stochastic steady-state values and welfare comparisons are relevant subjects for this matter - see, e.g., Schmitt-Grohé and Uribe [38].

results of our model, and, in particular, how important it is to consider a stochastic specification as opposed to a simpler deterministic form. For the steady-state values and other details derived from the model, see Appendix A.

1. Impulse-Response Analysis

In this section, I describe the results obtained by studying the impulse-response functions brought by the model. The goal here is to characterize in detail the equilibrium properties of this economy. I define a *shock* as the unanticipated, one-date increase of one percent in the *i.i.d.* component of an exogenous, stochastic process. As the approximate solution to the model is defined, one can perform impulse-response analysis on a particular shock, suppose ϵ_ω , by assigning $\epsilon_\omega = 0.01$ at $t = 1$, while setting $\epsilon_\omega = 0$ for $t \geq 2$, and $\epsilon_z = 0$ for all t . The analysis that follows will show impulse-response functions for inventory holdings, the rates of labor and capital utilization, and labor and capital services, with the series expressed as percent deviations from steady state. I distinguish between high-persistence technological and preference shocks, with $\rho = 0.95$, and relatively low-persistence shocks, in particular $\rho = 0.7$.

a. The Benchmark Model

An expansionary, highly-persistent technology shock leads to an increase in both the stock of inventories and the rates of capacity utilization (see Figure 3).⁶ Inventory holdings rise for the agent accumulates goods that are produced more efficiently during the time when productivity is higher. Notice that the rate of labor utilization, e_t , rises at the first date, but declines immediately toward its steady state value, $\bar{e} = 1$. What is happening is that the *extensive-margin* decision taken by the agent

⁶By rates of capacity utilization I mean the rates of utilization of both capital and labor. Remember that capacity in this work refers to both factors of production.

regarding the allocation of time at t was already made before the shock is known. Later on, a positive “supply” shock makes the agent increase the intensity of use of labor (the *intensive margin*) even though it implies an extra cost (similar to the increased unit-cost from an extra-hour in the case of the firm). This only occurs for one date since by the next date the agent reoptimizes so that total amount of labor supplied (the *extensive margin*) is adjusted without costs.

After a positive, high-persistence preference shock toward consumption and inventories, the rates of capacity utilization increase - similarly as in the technology-shock case, while the response of inventories is negative only for the very short run (see Figure 4). Now the agent prefers to consume more in the present than in the future, thus in the very short run, when the labor force cannot be adjusted extensively - just intensively though with a cost, the inventory stock falls because *part of it is just consumed*. Later, when the labor force has increased, inventory holdings will also increase at higher rates because they must recover their target level - remember they are complementary to consumption.

A digression is in order at this point. Given the stochastic specification of this model, preference shocks - or loosely speaking, demand shocks - affect the marginal utilities of both consumption and inventories through the CES bundle in u . Then, one observes two effects here. On the one hand, since inventories act as a buffer for consumption, they could fall at impact as is shown in Figure 4 for the baseline parameters. On the other hand, since both marginal utilities go up, both inventories and consumption could indeed increase at impact due to the optimal agent’s response. The latter may dominate the first effect under the following conditions.

Consider relatively low costs of adjustment on either the intensive margin of labor or investment (parameters ϕ and b respectively). Let me set for a moment, and without loss of generality, $\phi = b = 0.2$, instead of the baseline parameters $\phi = 0.5$ and

$b = 1$. Figure 5 shows how the behavior of inventories and capacity utilization change. The first, and perhaps obvious, is that the rate of labor intensity goes strongly up, now that its costs are lower. The second, remarkable finding is that inventories increase after a demand shock indeed, a result that may appear counterintuitive. What is happening, though, is not at odds with the real world. Suppose an economy with either low adjustment costs on both labor contracts and investment plans, or very low adjustment costs on one of them. Now, suppose that all of a sudden individuals prefer more goods amid a decrease in the preference for leisure. In a state where labor and capital are already given, and in an economy where those adjustment costs are high, a demand shock would lead to the immediate depletion of some of the inventory holdings. That is, agents choose neither to increase production nor to deplete part of their capital because of the relatively high costs. On the other hand, an economy with lower costs will be tended to increase the intensity of use of labor, decrease part of their capital, and still raise some of their inventory holdings, as is shown in Figure 5.

Notice that this particular behavior of inventories is obtained without modifying the baseline inverse-elasticity of substitution between consumption and inventories (recall, $\gamma = 5$). In fact, if one raises this parameter, i.e., raises the complementarity between the two goods, the previous result not only remains but also is emphasized. In other words, for higher values of γ , inventory holdings may largely increase at impact, as discussed earlier. However, the origin of this result is quite different. This means that the agent values more inventories as a complement to consumption, thus even a demand shock may lead to the desire for more inventories. Roughly speaking, in this case the role of complementarity from inventories is governing the results against their role as a buffer stock for consumption.

To sum up this digression, notice that through a simple way, with a general equi-

librium model that would be standard except for the introduction of inventories and capacity utilization, one obtains results that are consistent with Kahn's [25] stockout-avoidance theory. Recall that his contribution was essentially to generate the stylized fact of procyclicality of inventories, in a partial-equilibrium model, without the need of non-standard assumptions as, for instance, the increasing-returns-to-scale assumption. Kahn [25] generates inventories' procyclicality by considering either a positive, serially-correlated uncertain demand, or firms having the possibility of backlogging excess demand, where they target a certain non-zero level of inventories. In fact, one also obtains this inventory-target in a general-equilibrium model as the one studied here, without the need of demand shocks being highly persistent. Sensitivity exercises showed that under conditions as the ones described above - low adjustment costs on both investment and labor and, to a lesser extent, somewhat high complementarity between consumption and inventories - the effect of rising inventories at impact, and their procyclical behavior, is also observed at very low-persistence preference shocks - say $\rho \approx 0.2$.

After an expansionary, low-persistence impulse to the state of technology, both inventories and the rates of labor and capital utilization respond relatively stronger compared to the high-persistence shock (although the rate of use of capital is slightly stronger, see Figure 6). The explanation is simple: the agent is prone to produce more goods, both to consume and to store as inventories, when the shock is relatively more transitory. This way, she will use a much higher rate of the capacity in the short run. This stronger effect of the shock's lower persistence does not occur with respect to the rate of capital utilization, mostly, because the agent uses more the rate of labor utilization to adjust production after very short-run impulses. Notice that the rate of labor intensity does not have persistence indeed, and the cost of changing it carries on for only one period. On the other hand, the cost of changing the intensity of use

of capital carries on for a longer duration, for it affects capital depreciation, and thus the speed of capital accumulation.⁷

An interesting result is shown in Figure 7. In response to a low-persistence, expansionary shock to the marginal utility of both consumption and inventories, inventory holdings sharply decrease at impact and stay below their steady state for several periods. A very short-run impulse to preferences leads to an abrupt rise in the demand that is not completely matched by the supply. The result is a fall in the stock of inventories with respect to its target level, even though the rates of capacity utilization still increase.

This property of inventories is what distinguishes them from the choice related to the use of factors of production. Whether inventory holdings decrease at impact depends on the nature of the shock - remember the special case where in fact they increase in response to a preference shock. And whether they stay below the steady state until convergence or simply fall during the very short run depends on persistence - consider the contrast between figures 4, 5 and 7. On the other hand, notice that the intensity of use of capacity responds simply symmetrically to each shock, for any rate of persistence. Moreover, inventories and physical capital respond in a different way to the shocks considered above. In a standard, real business cycle model an expansionary, preference shock (either persistent or non-persistent) leads to an immediate fall in investment, with the stock of capital decreasing in the short run. Again, the question is why the stock of inventories, an alternative form of capital as interpreted here, may not fall as a response to a persistent shock to preferences? See, for instance, Figure 5. And, again, as was mentioned above the answer lies on

⁷In Chapter IV, I especially discuss the persistence property of the two rates of factor utilization. In there, I discuss the non-persistence property of labor intensity, and the relatively high-persistence property of capital utilization.

the role of inventories in this model, for they are not only stored as an alternative to capital, but also as a complement to consumption.

Figures 8-9 go over high-persistence, impulse-response dynamics of total labor services and total capital services, that is, $e_t N_t$ and $s_t K_t$. Figure 8 shows that the amount of services from labor rises immediately after every shock, whether it is a technology or a preference shock. This is due, first, to an increase in the rate of labor utilization e_t , and later on, to the accommodation of the labor extensive margin, N_t .

On the other hand, the dynamics of capital services show different patterns. For both shocks, there is an immediate jump of these services because at the beginning the stock of capital is used more intensively. A high-persistence technology shock generates that these services keep higher for a long time - in fact, more than 8 years in the simulations - because the later fall in the intensity of use of capital is compensated by a rise in investment to build-up new capital (see Figure 8). In short, it is the replacement of the old capital what leads to a high and persistent level of capital services. On the contrary, after an expansionary shock to preferences, the level of capital services jumps too, yet it goes back more drastically to its steady state. This occurs because investment sharply falls on impact, causing the stock of physical capital to drop further together with its use (see Figure 9).

Figures 10-11 show the effects of shocks with a low persistence rate. After a technological impulse, the amounts of capital and labor services jump again on impact, but they decay strongly almost reaching their steady-state values in less than a year (see Figure 10). In general, in extreme cases where the shocks are not persistent, the rates of factor use will strongly increase at impact, but the levels of these factors will not rise later. The result makes sense, for the agent is optimizing by using more intensively her productive capacity today, and is not willing to pay more either for capital or employment being used tomorrow.

Less dramatic behaviors are seen in Figure 11 with respect to the effect of a preference shock on both factor services. This is also an interesting result which is related to Figure 7. In response to a demand shock, the agent can increase the use of the capacity, and in fact the rates of factor utilization rise. Although the shock is of a low-persistence rate - what would imply dramatic changes of capacity utilization, what indeed happens is that responses are in fact weaker than those to high-persistence shocks. The main reason is, once more, the presence of inventories, for the agent has the possibility to deplete them in order to raise consumption in the very short run. This way, inventories serve as a substitute to capital and labor when a demand shock is relatively less persistent. This would suggest they are negatively correlated in high-frequency data. This is indeed the relationship I am searching along this study. In order to get to that, I employ more formal methods in the next section where, among other statistics, second moments are obtained. For now, consider how would capacity and inventories respond in models where only one at a time is present.

b. The Effects of Inventories, Capital Utilization, and Labor Utilization

Figures 12 and 13 (12 showing high-persistence preference shocks, 13 showing low-persistence preference shocks) show both the impulse-response functions from the benchmark model and those from models where either inventories or capacity utilization are absent. The primary goal here is to show that inventories are a substitute to the factors of production when a preference shock hits the economy. To show this, we should observe that the absence of variable capacity utilization pushes inventories to respond more strongly to a demand shock. On the other hand, without the possibility of accumulating inventories both rates of use of capacity should also respond stronger to this type of shocks.

Figure 12 first shows the two series of inventories in response to a high-persistence

demand shock. It is clear that if capacity utilization is not allowed to vary in the model, then inventories' response is larger. That is, they fall farther from the steady state in response to an expansionary preference shock. The same occurs with labor utilization when there are no inventories, but somehow different is the response of the intensity of use of capital. About the latter, we note that the impact effect is slightly stronger in absence of inventories. However, after the first date the presence of inventories makes the rate of use of capital rise even higher. The reason is simply found in the fact that inventories have to be replenished so that the agent follows her inventory-target. Thus, the presence of these zero-return assets causes that capital be used even more intensively in the short run. Why does not it happen with labor utilization? Again, because labor utilization here lacks of persistence itself, which leads the agent to adjust it only for one period. And, at the first date, what happens is that the presence of inventories allows the agent to use them instead of increasing the rate of labor use, even though the stocks have to be replenish later on.

Similar results are observed when the preference shocks are less persistent, even though the differences between the benchmark model and the corresponding alternative specifications are larger. Consider Figure 13. The response of inventories to a 1% rise in the preference parameter almost reaches -0.4% when capacity utilization is fixed. But most importantly notice that the absence of variable capacity utilization raises the persistence of inventories. This, indeed, is an important result, in particular for analyses on monetary policy and the inflation-persistence issue. Furthermore, it becomes relevant for it implies that most studies on inventories that do not consider variable rates of utilization may overestimate their intrinsic rates of persistence.

The rate of labor utilization also responds stronger when inventories are absent, but the rate of capital use once again only responds more strongly at impact. Notice that the persistence of the last variable is almost the same in both cases. The conclu-

sion is that the rate of capital use is not as substitute to inventories as it is the rate of labor utilization. But again, this is happening because the former affects a state variable as the stock of capital. And as such, it becomes more relevant in the long run when inventories indeed need to be back to their steady state level.

Consider now the effect of technology shocks where either variable capacity utilization or inventories are absent from the specification. This is shown in figures 14 (high-persistence shocks) and 15 (low-persistence shocks). As one may expect, the response of inventories is lower when variable capacity utilization is absent from the specification. This occurs when shocks are either of low or of high persistence. The reason is simple. Inventories cannot be raised as much as the agent would desire if capacity utilization is not flexible in the very short run. In fact, in response to an expansionary, high-persistence technology shock inventories fall in the short run. One must have in mind that a persistent increase in productivity raises the opportunity cost of having inventories, which in the absence of a flexible rate of use of the factors of production leads to their transitory depletion. The same happens with both rates of utilization.

2. The Association Among Variables

In this subsection, I study the relationship among the variables in the model, and their volatility. These concepts are summarized by the correlation between any pairs of variables, and their relative standard deviations. Briefly, they are derived as follows.

First, take expectations to the policy function described in its general form by (3.6), $y_t = \bar{y} + g_y \hat{y} + g_\Gamma \hat{\Gamma}$, to obtain $E\{y_t\} = \bar{y}$.⁸

Then, take the second moment of the vector of endogenous variables $\{y_t\}$, in

⁸Once again, consider a first-order approximation to the policy function.

order to obtain the covariance matrix $\Sigma_y = E\{y_t - \bar{y}\}\{y_t - \bar{y}\}'$. This matrix is found through an expression of the form $\Sigma_y = g_y \Sigma_g g_y' + g_\Gamma \Sigma_\Gamma g_\Gamma'$. The covariance matrix is used, finally, to find the correlations among the model's variables and the relative standard deviations.

a. The Benchmark Model

Tables I-II show results on second moments of the model's variables based on the covariance matrix that arises from the approximated policy function. The computation of these moments included removing a trend by the Hodrik-Prescott filter with $\lambda = 1,600$, which seems appropriate since the model parameterization is made by accounting for quarterly facts as reference.

Table I shows relative standard deviations, i.e., the standard deviation of a variable relative to its mean, for the benchmark model, for a model without inventories but with variable capacity utilization, and finally for a model without variable capacity but with inventories - where I also distinguished between the absence of variable labor and the absence of variable capital. The goal here is to have an idea of how the variables behave under the benchmark specification, and compare these results with those results arising from models in which the variables of interest are absent.

First, notice that the volatilities of the variables in the benchmark model behave as reported in the U.S. business cycle data. In particular, the variability of consumption, labor services, capital services, and inventory holdings are lower than the volatility of output, whereas that of investment in physical capital is larger. Also, consistent with the evidence the stock of capital has low volatility, being much lower than those of the series just mentioned above. Perhaps a weak replication of this model is related to the volatility of the inventory-to-sales ratio. As a share of output volatility, the volatility of this ratio is higher than the one documented by previous

works (0.66 in the benchmark model, and 0.75 in the model without capacity utilization, versus 0.5 according to the evidence documented in Iacoviello et al. [24]). This is a fact that remains to be explained, yet it may ultimately depend on the measurement of the inventory series, that is, Iacoviello et al. [24] include intermediate and final-good inventories while this work treats inventories only as final goods.

Second, consider for a moment what we should expect about the series' variability in each of the specifications considered in Table I. This is more of a hypothetical exercise, a series of intuitive speculations on the model as is specified. Remember that inventories fulfil two roles here: they act as a buffer stock for consumption, but at the same time provide utility services that complement the latter. At this point, suppose we take away the possibility that the representative agent holds inventories, and consider first a demand shock. Recall that this would lead the agent to lower inventories at impact in order to raise consumption. But later on, inventory holdings would have to reach again their target level for they are a complement to consumption. Therefore, what if there are no inventories in the economy? In this case, only investment would act to buffer consumption. Consider now a technology shock. If there are no inventories, it happens that the rise in productivity and output would go only to consumption, instead of going to both consumption and inventories. As a conclusion, the presence of inventories would tend to increase the variability of output, but to smooth consumption and investment. This is what we can basically see in Table I by comparing the benchmark model with the *no-inventories* model.

How about assuming away variable capacity utilization? Suppose an economy where agents can store goods as inventories, but cannot change the services from labor and capital once the states of preferences and technology are revealed. A result we can verify from Table I is the difference in the volatilities of investment between the two specifications. As one would expect, variable capacity utilization leads to a rise in the

volatility of investment. I should mention that the contribution of technology shocks to the variability of investment is above 95%. But remember that technology shocks lead also to higher use of capacity. This way, a shock that moves the system away from its steady state, generates more investment variability when there is a chance to vary the use of capacity. In fact, not only investment but most variables' standard deviations go up in the benchmark model. Furthermore, notice that variable capital utilization leads to changes in the capital depreciation rate, which in turn generates more variation on investment, thus providing of another source of volatility.

Finally, if we look at the last two columns of Table I we see that in fact the rate of capital utilization has the leading role in terms of rising the volatilities in the benchmark model. There is an interesting result though with respect to the rate of labor utilization, and is the one related to inventories. Note that for both the inventory-to-sales (I-S) ratio and inventory holdings the *absence* of variable labor use leads to a rise in their volatilities. What is happening is that the rate of labor utilization has no persistence in the model. As such, any expansionary shock will in general lead to a rise in this rate and to a fall in inventories, depending on the nature of the shock. I will show later that these two variables will be negatively related indeed. Remember from the previous section that they act as substitutes in the benchmark model, which in general implies that the absence of one of them will cause the other variable's volatility to increase.

Table II shows the correlation matrix emerging from the benchmark model. The first noteworthy result is that, consistent with the evidence, output is negatively correlated with the I-S ratio (-0.72), while positively correlated with inventory investment (0.63). As it was mentioned in the introduction, the countercyclicality of the I-S ratio and the procyclicality of inventory investment have been largely documented in the empirical inventory literature. Perhaps the shortcoming here is that the countercyclical-

cality of the I-S ratio is higher than in the evidence, which documents a correlation close to -0.4.

As was mentioned above, inventories are negatively correlated with the intensity of use of labor. Notice, though, that the former are positively correlated with labor services. Once again, the lack of persistence of the intensive margin, added to the procyclicality of the extensive margin, is generating this result. On the other hand, both the rate of capital utilization and total capital services are positively correlated with inventories. Finally, the correlations arising in this model among capital, labor, consumption, and investment do not differ much with respect to those arising in standard business-cycle models.

b. The Relationship Between Capacity Utilization and Inventories, and the Source of Shocks

I study now the association between inventories and capacity utilization by source of uncertainty and degree of persistence. Table III shows the correlations among the three main variables distinguished by the two shocks and two extreme rates of persistence: $\rho = 0$ and $\rho = 0.95$.

As we observe from the table, low-persistence preference shocks make the correlation between inventories and the two rates of capacity utilization be negative. They simply act as substitute goods, which is something I already mentioned above. That is, non-persistent demand shocks lead inventories and utilization respond in an opposite way. But notice that as the shock's persistence goes up, this association becomes positive for the rate of capital utilization. Once again, as the duration of the preference shock is higher, capacity moves together with inventories as the latter are not allowed to fall farther from their target. Notice that this does not happen with the rate of labor utilization – its correlation with inventories is -0.62. Once

again, it is the implication that, in response to an expansionary shock for instance, the agent raises this rate only for one date, which indeed is the date when inventories fall whatever the shock's persistence. At the end, they will be negatively correlated at every persistence. However, given the relatively higher persistence of the services from labor - not its rate of use, but its total services, they will be positively correlated with inventories under demand uncertainty, as will also be capital services.

Notice that the correlation between the two capacity variables is positive when uncertainty is governed by non-persistent preference shocks. Yet, it becomes almost negligible under the presence of highly persistent preference shocks. Once more, the lack of persistence of the rate of labor utilization leads through this result, for the services from these two variables are still positively correlated - though not shown in Table III.

When uncertainty is led by technology shocks only, the correlation between inventories and capacity is clearly positive if these are non-persistent. Explanations are the same as those given above. Recall that an expansionary technology shock makes these two concepts complement each other. That is, when productivity is higher the agent takes advantage of it and produces more of the three goods available in this economy: consumption, inventories and capital. When these shocks are more persistent, though, only the rate of capital utilization keeps its positive correlation with inventories. About the correlation between the two rates of utilization, their positive and strong association when shocks are non-persistent contrasts with their weaker positive correlation when shocks are highly-persistent. Simply, the lack of persistence of labor utilization explains this result.

Table IV shows the relationship between inventories and the rates of capacity utilization from alternative specifications. The purpose of this exercise is to isolate two variables at a time in order to have their association when the third variable of interest

is not in the the agent's choice set. For example, consider the correlation between inventories and the rate of capital use when preference shocks are non-persistent: this is of -0.60. It shows that in absence of variable labor utilization, inventories and capital utilization are negatively associated when uncertainty comes from these shocks only. That rate was in fact -0.84 if labor was allowed to vary. This means that the substitution between inventories and the variable rate of capital use is lower when labor utilization becomes fixed. The result basically shows that the two rates of capacity are in fact highly complementary to each other, and, when both available, they make inventories raise their substitution role in response to demand shocks. This also happens in regards with the correlation between inventories and the rate of labor utilization. A remarkable result emerges when one observes the correlation between the two rates of capacity utilization when inventories are not among the agent's choices. When demand shocks are non-persistent, this association almost reaches the 100% level, whereas in the benchmark model it was only 0.47. This pattern is also present under high-persistence preference shocks. Once again, this confirms the idea of inventories being a substitute for capacity utilization. When they are introduced into the model, they help lower the response of capacity utilization when a preference shock hits the economy. About technology shocks, we observe that the absence of a third variable does not affect much the results. This is not surprising, for that the effect of technology shocks is much clear and simpler than that from preference shocks.

3. Does Uncertainty Matter?

As we studied in the preceding sections, the solution to the benchmark model is found by applying a first-order approximation to the system of necessary conditions. Even though in general, and for this model in particular, behavioral results are not partic-

ularly sensitive to higher-order approximations, welfare comparisons can indeed be misleading this way - see, for example, Kim and Kim [29] and Schmitt-Grohé and Uribe [38]. An undesirable feature of the standard log-linearization is that it does not break the certainty equivalence property. The problem is that second- and higher-order terms of the equilibrium system are omitted, and thus it becomes impossible to obtain the effect of volatility in the model. On the contrary, a second-order approximation to the policy function allows for second-order terms to arise, causing volatility to affect the model's solution.

A corollary of this statement is that by linearizing our stochastic, benchmark specification up to a first order, in essence it becomes deterministic. For instance, the steady state found in order to linearize the system of necessary conditions was indeed a non-stochastic steady state, and the resulting impulse-response functions were basically algebraic forward iterations of the model's decision rules.

When the policy function is approximated up to a second order through the application, for instance, of the perturbation method developed by Schmitt-Grohé and Uribe [38], one can have a better description of what is happening in our model in terms of uncertainty. In principle, it will serve to find a stochastic steady state for the economy, i.e., the state to which all variables converge when one accounts for the effect of uncertainty. In general, as the variance of the exogenous shocks rise, the stochastic-steady state values get farther from the non-stochastic steady state. In particular, a second order-approximation will help us find the effect of increasing uncertainty on our variables' stochastic steady states, and especially on inventories and capacity utilization.

By linearizing up to a second order, impulse response functions will become the result of performing Monte Carlo simulations on our benchmark model. The technique implies pulling shocks from their distributions and evaluate how they affect

the system, repeating this experiment a large number of times so as to sketch an average response. On the other hand, stochastic steady-state values are the result of taking expectations to the second-order-approximated decision rules, which in turn bring about the non-stochastic steady states corrected by the effects of second-order moments, i.e., the auto-covariances - see, e.g., Schmitt-Grohé and Uribe [38].⁹

Figure 16 shows the stochastic steady-state values arising from different combinations of standard deviations on both preference and technology shocks.

We notice that the steady-state inventory-to-output ratio rises with higher volatility either from preference or technology shocks. We should expect this to happen since one of the motives to holding inventories comes from uncertainty, i.e., recall the stock-out avoidance motive leads the agent to store goods in order to protect herself from different realizations of the shocks. Thus, the stochastic steady-state inventory-to-output ratio is about 84% in an economy with no uncertainty, while it exceeds 100% when both shocks have standard deviations of 10%. Notice that technology shocks have a relatively higher impact on this ratio, for a technology volatility of 10% causes this ratio to increase to about 98% if there is no any volatility on preferences. On the other hand, the ratio does not exceed 90% when the standard deviation of preference shocks reaches 10% and there is no variance on technology shocks. This confirms the idea that inventories are partly held by a stock-out avoidance motive: an increase in the volatility of the preference shock - a demand shock, essentially - causes the agent to wish to hold a higher stock of inventories, in average, to protect herself against expected larger demand shocks.

The stochastic steady-state inventory-to-consumption ratio behaves similarly, indeed, as the inventory-to-output ratio. It is worth noting that, although not shown,

⁹These exercises were performed by employing Dynare v3, a skillful set of codes used to solve and estimate DSGE models.

the stochastic steady-state consumption-to-output ratio declines with increases in volatility. Yet, these reductions are weaker compared to the changes on the inventory ratios (they slightly fall from 75.6% when no any volatility to 74.6% when both are 10%). In this case, if one individualizes the changes on the standard deviations, once again technology shocks are clearly more important. This makes sense in that a larger size of technology shocks - supply shocks, essentially - leads the agent to allocate output and time not only onto higher amounts of capital and labor but also to improve the inventory stock. The agent may be engaging in precautionary saving that affects average consumption. For instance, think on an expected, large negative supply shock. The effect on the agent's decisions is simple: she will surely choose to hold a higher amount of inventories, once more to shield herself later. It is the potentially high cost of not having enough resources to consume after a bad realization what leads to these results. At the end, the average stocks of both capital and inventories would rise with respect to the average flows of output and consumption when uncertainty goes up.

The stochastic steady-state rates of both capital and labor utilization slightly fall as volatility goes up. The former falls from 14.88% when no volatility to 14.21% when both reach 10%. The latter falls even less, from 1.000 without variance to 0.997 if standard deviations are 10%. Although small, explanations for these falls could be found in that, as we noticed above, the increase in the average levels of capital and labor goes in opposite direction with respect to their rates of utilization. A higher level of uncertainty definitely leads to increases in average precautionary stocks leaving their average rates of utilization either unchanged or with small drops. Though not shown, the average gross investment-to-output ratio also falls slightly with the increase in uncertainty, in spite of the average stock of capital being higher. This is happening because the average depreciation rate falls when volatility grows,

which indeed simply comes from utilizing capital at lower intensity.

A conclusion one might be tempted to assert after reading the last paragraphs is simply the following. For otherwise identical economies, those with high variance are most likely to have lower averages in the consumption-to-output and the investment-to-output ratios, while higher averages in their inventory-to-output, and capital-to-output ratios.

As I mentioned above, a second-order approximation to the equilibrium system allows us to capture the effects of uncertainty on welfare. I now proceed to derive an approximation to the welfare function, one which becomes related, among other things, to uncertainty. To this matter, I will follow Collard and Dellas' [16] procedure to comparing welfare results. First, the welfare function is computed taking a second-order approximation to the utility function. Second, I compute first and second order theoretical moments for its arguments: consumption, leisure, inventories, and the rate of labor intensity. There is a difference here with respect to Collard and Dellas [16], since they compute these moments from a multitude of many-period simulations of the model's series. Results of course should not be different the other way, simply because the theoretical moments are the average values to which the simulation-based results are supposed to converge. Finally, these moments are then entered into the approximated utility function so as to compute the unconditional expected welfare.

A second-order Taylor approximation to the utility function (3.3) brought about the following result:

$$\begin{aligned}
 u \approx & \bar{u} + z_0(z_t - \bar{z}) + c_0(C_t - \bar{C}) + q_0(Q_t - \bar{Q}) + n_0(n_t - \bar{n}) \\
 & + e_0(e_t - \bar{e}) + \frac{1}{2} [c_1(C_t - \bar{C})^2 + q_1(Q_t - \bar{Q})^2 + e_1(e_t - \bar{e})^2 + co], \quad (3.8)
 \end{aligned}$$

where,

$$\begin{aligned}
z_0 &= \ln \bar{X}^{1/(1-\gamma)}; & c_0 &= \left(\frac{\bar{z}}{\bar{X}}\right) \theta \bar{C}^{-\gamma}; & q_0 &= \left(\frac{\bar{z}}{\bar{X}}\right) (1-\theta) \bar{Q}^{-\gamma}; & n_0 &= -\eta \bar{e} \\
e_0 &= -\eta \bar{n} - \phi(\bar{e} - 1); & c_1 &= \left(-\frac{1}{\bar{X}^2}\right) [\bar{X} \gamma \theta \bar{C}^{-\gamma-1} + \theta^2 \bar{C}^{-2\gamma} (1-\gamma)] \\
q_1 &= \left(-\frac{1}{\bar{X}^2}\right) [\bar{X} \gamma (1-\theta) \bar{Q}^{-\gamma-1} + (1-\gamma)(1-\theta)^2 \bar{Q}^{-2\gamma}]; & e_1 &= -\phi \\
co &= \theta \bar{C}^{-\gamma} \bar{X}^{-1} (z_t - \bar{z})(C_t - \bar{C}) + 2(1-\theta) \bar{Q}^{-\gamma} \bar{X}^{-1} (z_t - \bar{z})(Q_t - \bar{Q}) \\
&\quad - 2\eta(e_t - \bar{e})(n_t - \bar{n}) - 2\bar{z}\theta \bar{C}^{-\gamma} \bar{X}^{-2} (1-\theta) \bar{Q}^{-\gamma} (1-\gamma)(C_t - \bar{C})(Q_t - \bar{Q}),
\end{aligned}$$

and an over-bar indicates that the variable or function is evaluated at the stochastic steady state.

Taking the expectation to the approximated expression for the utility function in the right hand side (RHS) of (3.8) - denoted by u^* herein - one obtains the following:

$$E(u^*) = \bar{u} + \frac{1}{2} [c_1 \sigma_c^2 + q_1 \sigma_q^2 + e_1 \sigma_e^2 + 2cov], \quad (3.9)$$

where I use the facts that $E(x_t - \bar{x}) = 0$, and $E(x_t - \bar{x})^2 = \sigma_x^2$; while $E(x_t - \bar{x})(y_t - \bar{y}) = \sigma_{xy}$ is used inside the term cov .

Notice that coefficients c_1 , q_1 , and e_1 are all negative, yet we do not know the sign of the whole term comprised by covariances. For special conditions in which the last term becomes negligible, our result would imply that the higher the variables' auto-covariances, the lower the (approximated) unconditional expected welfare. A question emerges here, though, because the value of utility at the stochastic steady state, \bar{u} , changes with the shocks' standard deviations. In general, higher variances of the shocks imply higher variances of the endogenous variables. At the same time, they imply different averages for these variables, i.e., stochastic steady states, which in turn affect \bar{u} . Therefore, in principle we cannot say that increasing the model's exogenous variance implies lowering welfare. Indeed, from the benchmark model we

notice that increases only in preference-shocks' volatility lower welfare. Contrary to what the theory predicts, rises in the technology-shocks' volatility go the other way around. This counterintuitive result can be explained by the approximation error that causes taking the unconditional expectation of welfare rather than its conditional expectation, which would be more appropriate. A solution to this problem is, though, beyond this work, and subject to further investigation.

Finally, notice that were not for the second-order linearization of the utility function, we would only had $E(u^*) = \bar{u}$. Assuming the latter as true, and taking welfare comparisons based on it, would be nothing but misinterpreting the stochastic setting as if it was deterministic. There is a vast recent literature - see, e.g., Kim and Kim [29] and the references therein - discussing another inconsistency, which is that of comparing welfare by approximating the utility function up to a second order while approximating the policy function to a first order.

The lesson one can take from the previous discussion is that second-order approximations enhance our understanding of the behavior of capacity utilization, and most importantly of inventories. The latter are essentially closely related to the concept of uncertainty. We have seen that higher uncertainty implies, *ceteris paribus*, a higher inventory stock, but lower rates of capacity utilization, in average. The question of whether higher uncertainty leads to lower welfare, though, is still open. It is clear for preference shocks, yet it is not clear for technology shocks.

CHAPTER IV

ROBUSTNESS ANALYSIS

This chapter discusses the robustness of the results derived in the previous chapter. First, I assume an alternative specification for the utility function so that it becomes separable in consumption and inventories. This form of setting preferences can be approximated, indeed, as a special case of the benchmark model when the elasticity-of-substitution parameter, γ , approaches to one - see the proof in Appendix B. Second, I compare the benchmark model with an alternative way of modeling variable capital utilization introduced by Christiano et al. [15]. Finally, I find a more general set up where benchmark and Christiano et al. [15] become special cases. In order to obtain this general treatment on capital utilization, the model introduces a new variable into the analysis: capital maintenance.

A. The Separable Utility Function

In Section A of Chapter III, I briefly mentioned the alternative of assuming a utility function that was separable in consumption and inventories. In this case, one can distinguish between preference shocks that shift the labor supply - and thus affects the demand for both consumption and inventory holdings - from those preference shocks that shift the demand for consumption goods only.

What are the salient results now under this specification? In order to compare the implications of the benchmark model with those of a model with separable utility, consider first an analysis of the impulse-response functions with labor supply shocks.

A one-percent, high-persistence technology shock leads to a large, but slow response of inventory holdings - staying almost 2% above the steady state after 10 quarters, while both rates of capacity utilization respond similarly to the benchmark

model - see Figure 17. This is an interesting result for one may guess that changing the specification of the utility function will mostly affect the responses to preference shocks and just slightly those to technology shocks. What is happening is that, when consumption and inventories are complement, a technology shock causes both to increase at impact. When inventories and consumption are not complement in the utility function, the rise in inventory holdings is stronger, while that in consumption is weaker. In other words, there is a substitution of inventories for consumption. On this matter, low-persistence technology shocks lead to similar patterns - see Figure 19.

Regarding the preference shocks, the separable utility function allows one to distinguish between a shock that only hits the consumption good - call this a consumption shock - and a shock that hits both consumption and inventories - call this the labor-supply shock as the one seen for the general case. For any rate of persistence, both labor supply and consumption shocks in the separable-utility case lead to a decrease in inventory holdings - see Figures 18 and 20 for the case of labor-supply shocks. They will stay below the steady state not only at impact but also along the convergence through it. Recall that the response of inventories in the more general, benchmark case depended on the parameters assumed, in particular with respect to the adjustment costs on labor and investment. In the benchmark case, I showed that they fall at impact but then they grow even higher to stay above the steady state along the convergence path.

When the utility function is separable there are no conditions under which inventory holdings rise through higher values than the steady state in response to preference shocks - whether they are consumption or labor-supply shocks, of high or low persistence. The reason is simple. Inventories do not complement consumption, and as such they mostly fulfil the role of being a buffer stock. An expansionary labor-supply shock

raises the preference toward both consumption and inventories, and against leisure. However, the desirability of raising consumption implies that the agent takes on part of inventories. In the separable-utility case there is a substitution between consumption and inventories even though the demand shock goes toward both variables. The channel through which this occurs is the increase of the opportunity cost of a unit of inventory in terms of consumption. In fact, consumption rises because of the increase in the intra-temporal, marginal rate of substitution between consumption and leisure, which discourages leisure in favor of consumption - see equation (2.8). On the other hand, by equation (2.11), the increase in the marginal utility of consumption generates a rise in the RHS. This, in turn, leads to an increase in both the marginal utility of consumption tomorrow and the marginal utility of inventories left for tomorrow. Summarizing, the last two implications forces inventories to fall. But now, and differently to what one observes in the general case, they are not complement with consumption. Thus, they keep staying below the steady state until convergence without the replenishment needed before.

Of course, a consumption shock is more obvious. This shock directly hits on consumption against inventories and leisure, and inventory holdings fall not only at impact but keep falling for a few more dates.

Regarding the dynamics of capital and labor services, it does not differ largely under the two specifications. An expansionary preference shock leads to a higher response in the benchmark specification because of the larger need to replenish inventories. In this sense, inventories act more as substitutes for the intensity of capacity utilization under the separable-utility specification. That is, they help lower the response of capacity after a demand shock.

Consider now the differences we find between the second moments from the benchmark model and those from the separable-utility model following the discussion

of section III.B.2. Table V shows relative standard deviations from the separable-utility model on its baseline specification and specifications either without inventories or without capacity utilization. Notice that the main differences arising with respect to the benchmark model are related to the series of inventories and the I-S ratio (see also Table I). As one may expect, these two variables become more volatile in the separable-utility model. This happens basically because both preference and technology shocks generate greater responses in inventories. As shown above, a preference shock causes inventory holdings to have a larger, negative response compared to the general model, as they serve more to buffering consumption. Further, remember that a technology shock causes inventories to respond stronger in the separable-utility model.

Table VI shows the correlation matrix that corresponds to this setting. The countercyclicality of the inventory-to-sales ratio is quite close to the general specification (-0.68 versus -0.72), and inventory investment is still positively correlated with output (0.62). On the one hand, inventories are negatively correlated with the rate of labor utilization, but now they are almost not correlated with labor services. On the other hand, inventory investment behaves similarly in both specifications. It is negatively correlated with labor utilization, but positively correlated with the services from labor. This is another implication of the lack of persistence arising in the response of labor intensity.

A major implication of the separable-utility model is that inventories are almost not correlated with consumption, contrary to the general model where they are positively correlated (-0.05 versus 0.56). Of course, this comes from the assumption made in the benchmark setting that both concepts complement each other in the utility function. Also compared to the general model, the separable-utility specification leads to a lower correlation between inventories and both the rate of capital

utilization and the services from capital. Inventories are positively correlated with both variables in the benchmark (0.51 and 0.67 respectively), while not correlated with capital utilization, and weakly, positively correlated with capital services in the separable-utility set up (-0.02 and 0.24). Nevertheless, notice that inventory investment behaves somewhat similarly under the two specifications. By looking closely at the impulse-response functions of the inventory stock, both from preference and technology shocks, one can extract that, although their levels behave quite differently, changes behave likewise in these two specifications.

B. On the Rate of Capital Utilization

1. A Comparison with an Alternative Specification in the Literature

Our benchmark economic environment was built on the basis that variable capital utilization goes together with the assumption of endogenous depreciation. That is, the stock of capital is used to produce goods at different rates of intensity according to optimality conditions, and this intensity of use affects the depreciation rate. Consider equation (2.9) in Chapter II. For a given level of capital, raising the flow of capital services by one unit basically implies the following trade-off. There is a benefit arising from the marginal product of capital services (LHS), while there is a cost in higher depreciation of the current stock of capital, valued in terms of consumption goods (RHS).

Although most of the literature on variable capacity utilization generally assumes the specification of endogeneizing a capital depreciation rate, there are alternative specifications to treating the cost of varying the use of capital. A recent standard example is the set up by Christiano et al. [15] (CEE specification herein), mentioned briefly in the literature review. In that paper, variable capital utilization plays an

important role in explaining the observed inertia in inflation and persistence in output. However, their theoretical model assumes a constant depreciation rate, independent of the rate of capital utilization. Instead, they model the extra-cost of using capital more intensively with respect to the steady-state rate by an increasing, convex function of s_t , denoted herein by $a(s_t)$.

Thus, for a given stock of capital, varying capital services by one unit does not raise the depreciation rate, but involves a cost to the agent in terms of resources of $a'(s_t)K_t$. In CEE, the functional form for $a(s_t)$ is set to require that the rate of capital use, s_t , be 1 in steady state. Further, they assume that $a(1) = 0$. These two assumptions imply, first, that the steady-state stock of capital will equal the flow of capital services and, second, that the additional costs that depend on the rate of capital utilization, $a(s_t)K_t$, disappear from the resource constraint in steady state. Therefore, they are somehow modeling capital utilization in a similar fashion as labor utilization is modeled here. That is, recall that the labor extensive margin is assumed to be decided *before* the state of nature is known, while the labor intensive margin is set *after* the state of nature is realized. Also, remember that the costs of increasing both margins are the same in terms of disutility, except for an additional cost that only involves the intensive margin. This extra-cost in terms of disutility was assumed to have the form $\frac{\phi}{2}(e_t - 1)^2$, which implies a steady-state rate $e = 1$, and extra-costs - not in terms of resources as in CEE but in terms of welfare - arising from deviations when adjusting labor decisions *ex-post*.

The similarity between CEE's way of modeling capital utilization and this work's way of modeling labor utilization is evident. There is an interesting difference in their results that warrants a brief discussion though. In Chapter II we noticed that the way labor utilization is modeled leads to responses in the rate of labor intensity that are not persistent, even for highly persistent shocks. And remember, this result is simple

to understand. Because of the additional adjustment cost on the intensive margin, any persistent shock - either to preferences or technology - makes the agent change only temporarily the rate of labor utilization, while labor is given by pre-shock decisions. In absence of further shocks, the agent will only adjust the extensive margin by the second period, simply for it is cheaper in terms of disutility.

One might presume that this effect will also be present in CEE with respect to the rate of capital utilization. That is, suppose any shock, of any rate of persistence. At first, the agent would immediately respond by optimally adjusting the rate of capital utilization, given a certain level of capital. In absence of further shocks, the agent will take the decision between keeping the capital utilization rate outside the steady state and adjusting the stock of capital. The first involves an additional cost in terms of resources, given by $a(s_t)K_t$, while the second implies postponing consumption. Similar to the effect on labor utilization observed in Chapter III, one may think that the agent will choose to change only the stock of capital, depending on the adjustment costs to investment in some way. But the conclusion is that, even for null adjustment costs to investment and zero-persistence shocks, in most situations the agent will still choose a different rate of capital utilization from that of the steady state, even for several periods after the shock. In other words, as opposed to the effect of no-persistence on the rate of labor utilization studied so far, the rate of capital utilization in a CEE specification does have persistence indeed.

Why is this happening? To answer this question one has to look at the nature of investment decisions, which are basically inter-temporal. This is quite different from labor decisions, which are essentially intra-temporal. As I show below, specifying variable capital utilization à la CEE in this work does also generate a rate of capital utilization that is persistent, even in responses to shocks that are not persistent at all. In fact, either by looking at the first-order autocorrelation coefficient, or by looking

at the behavior of the impulse-response function - e.g., finding the half-life of the series, defined as the number of periods it remains above 0.5% after a unit-shock -, one cannot observe any differences between CEE and benchmark regarding the rate of capital utilization - see Table VII and Figures 21 and 22. If there is any difference at all, notice from Table VII that CEE displays higher autocorrelation coefficients, which provides some idea that it is quite persistent - though I do not test any of these hypotheses here.

An alternative specification to the benchmark model in line with CEE would imply the following changes. The law of motion for capital, equation (2.1), would now be

$$K_{t+1} = K_t[1 - \delta] + I_t, \quad (4.1)$$

where δ is the constant capital depreciation rate. The resource constraint (2.2) would now be

$$C_t + I_t[1 + h(I_t)] + Q_{t+1} - Q_t + a(s_t)K_t \leq Y_t, \quad (4.2)$$

where $a(s_t)$ is defined as mentioned above.

In the new specification, the first order-necessary condition (2.9) changes to the following equation

$$F_2(e_t N_t, s_t K_t) = a'(s_t), \quad (4.3)$$

whereas (2.10) becomes

$$u_1(C_t, l_t, e_t, Q_t) p_{k,t} = \beta E \left\{ \begin{array}{l} u_1(C_{t+1}, l_{t+1}, e_{t+1}, Q_{t+1}) \times \\ \left[\begin{array}{l} s_{t+1} F_2(e_{t+1} N_{t+1}, s_{t+1} K_{t+1}) + \\ (1 - \delta) p_{k,t+1} - a(s_{t+1}) \end{array} \right] \Big| \Gamma_t \end{array} \right\} \quad (4.4)$$

Notice that equation (4.3) only differs from (2.9) on the right hand side. Now the decision includes the marginal cost, in units of consumption, of increasing the

intensity of use of capital: $a'(s_t)$. In a different way, the decision before included the marginal cost in terms of capital deterioration, $\delta'(s_t)$, transformed to units of consumption good by $p_{k,t}$. On the other hand, Euler equation (4.4) introduces now the term $a(s_{t+1})$ on the RHS. It shows that the discounted, expected future marginal benefit of increasing physical capital is affected by an additional cost that depends on its future intensity of use. Of course, if the stock of capital is being used at its steady state, then no extra-costs will be accounted.

In order to compare the results from the benchmark model with those from the CEE model, let us assume that $a(s_t) = 0.5v(s_t^2 - 1)$. This functional form satisfies the requirements stated above, while v is a positive parameter that serves to normalize s_t to be 1 in steady state, as in CEE. Even though the benchmark setting does not normalize s_t this way, I scale CEE steady-state values so that we depart from the same position in order to compare them consistently.¹

Impulse-response analysis performed on the alternative, CEE specification shows that in general it does not differ from the benchmark specification. The only slight difference appears from the investment response to high-persistence preference shocks, say $\rho_z > 0.9$. Consider an expansionary shock. Then, the CEE's investment impulse-response function will be at all points below the one from benchmark - see Figure 23. The difference comes from the fact that in the benchmark model the cost of increasing s_t is "paid" with higher capital depreciation, i.e., with capital goods, whereas in the CEE-version it is paid with consumption goods taken from the resource constraint. When the cost is charged on capital depreciation, there is a need to replenish the stock of capital, thus investment does not decrease but in fact for highly-enough persistent, demand shocks it could rise. On the other hand, when the cost is paid over the agent's

¹See the notes about the dimensions of the rates of labor and capital utilization in Chapter II.

resources she will be relatively more willing to lower investment in order to smooth consumption.

What happens in the end is reflected by equations (2.9) and (4.3). In both, the LHS could be interpreted as the demand for capital services for a given level of K_t , while the RHS could be interpreted as the supply of capital services. The latter, indeed, is a function only of s_t in (4.3), but a function both of the marginal cost of capital depreciation and the price of capital in (2.9). Notice that, for given N_t and K_t , an expansionary preference shock leads to a rightward shift of the demand for capital services arising from an increase in the intensity of use of labor, e_t . This, in turn, causes a movement along the supply curve that finally leads to an increase in the rate of capital utilization.

This mechanism is at work in both specifications. Yet, the difference relies on the fact that the benchmark's supply curve includes a factor comprised by the price of capital, which solely depends on investment. Remember that in general investment falls in response to a preference shock, for the agent is prone to consume more rather than saving. It turns out, then, that the effect of a preference shock on the intensity of use of capital is muffled by a fall in investment, and viceversa; that is, the effect of a preference shock on investment is muffled by an increase in the intensity of use of capital. In addition, one shall notice that the higher the parameter v is in the benchmark specification (i.e., the closer we approach to the standard, fixed-capital-utilization model) the lower the response of s_t to a preference shock, since the capital-services supply schedule becomes steeper. Yet, at the same time, the magnitude of the response (decrease) of investment to a preference shock increases.

Consider now Table VIII, where I compare the volatilities in the CEE and in the benchmark settings. Notice that there are no important differences. The volatility of investment is slightly lower in CEE (2.9 versus 3.3) while those related both to the

rate of capital intensity and to capital services are slightly higher (1.1 versus 0.8 in benchmark). As mentioned above, movements in investment help muffling movements in the rate of use of capital more in benchmark than in CEE. As a result, investment becomes more volatile in the former, contrarily to what happens with the rate of use of capital. Thus, and because not only capital utilization but also labor utilization is more volatile, the standard deviation of output becomes larger in CEE. Finally, I compare the correlations arising from the benchmark case with those arising from the CEE specification - see Tables II and IX. Essentially, they are roughly similar. As one would expect from earlier comments, the main differences emerge from investment.

2. A More General Treatment for Modeling Capital Utilization

McGrattan and Schmitz [32] present evidence that suggests that incorporating expenditures on maintenance of physical capital into macroeconomic models will change the quantitative answers on some questions. Using a survey with Canadian data, they show that the activity of maintaining and repairing equipment and structures is generally large relative to investment and a substitute for investment to some extent. In this sense, it should also be evident the relationship between the notions of capital maintenance and the intensity of use of capital. Clearly, both affect the dynamics of the stock of physical capital. Yet, there is no much theoretical literature on this issue.²

In this section, I treat this particular issue, not only because of its relevance per-se, but also because it will help us find a general specification that will include our benchmark's and the CEE's way of modeling variable capital utilization as special cases.

²Exceptions are Collard and Kollintzas [17] and Licandro and Puch [31], among a few others.

In order to pursue this goal, let us start with the following assumption. Consider a general, implicit function

$$\Lambda(\delta_t, s_t, m_t) = 0, \quad (4.5)$$

where the variable m_t stands for the rate of capital maintenance at time t , which can be interpreted as units of maintenance services per unit of capital. Furthermore, assume that Λ is separable in its arguments, with $\Lambda_\delta > 0$, $\Lambda_s < 0$, and $\Lambda_m > 0$.

Now, consider the following case. Suppose the maintenance rate of capital is a constant, \bar{m} . Then, our previous function becomes $\Lambda(\delta_t, s_t, \bar{m}) = 0$, which can be re-expressed as $\delta_t = \delta(s_t, \bar{m})$. Given the assumptions stated above, one has $\delta_s > 0$, which can be associated with the function I assume in the benchmark model to specify the cost, for a given \bar{m} , of using capital more intensively. This suggests that our benchmark specification could be considered as a special case where higher use of capital raises its depreciation for a given, fixed rate of capital maintenance. Indeed, one could take this as an alternative interpretation. That is, in a more general setting one should say that increasing the intensity of use of capital not necessarily raises its depreciation as long as maintenance spending increases in a way that δ stays constant. Put it this way: It is as if I was assuming so far that there is no any way of keeping capital depreciating constantly if its use is permanently raised. But notice then that this was not completely accurate for one should have considered a way to “escape” from an increasing depreciation. This argument is then discussed in depth below.

Now, let us consider another case. Suppose the capital depreciation rate is a constant, $\bar{\delta}$. Then, the general function becomes $\Lambda(\bar{\delta}, s_t, m_t) = 0$, which can be re-expressed as $m_t = m(s_t, \bar{\delta})$, with $m_s > 0$. Furthermore, consider a change of variable such that m_t is now re-named as a_t . Notice that this variable a_t can be associated with the CEE’s cost function of capital utilization, i.e., the function $a(s_t)$ considered in

the preceding section. This suggests that the CEE's way of modeling variable capital utilization could be considered as a special case where maintenance and intensity of use of capital are complementary variables, at a given rate of capital depreciation, $\bar{\delta}$. This may be taken as another interpretation of how CEE works. Remember that $a(s_t)$ was an increasing, convex function. Thus, if we identify $a(s_t)$ with $m(s_t)$, it means that increasing the use of capital will imply a higher spending in maintenance, so that we can "buy" the idea of a constant depreciation rate. It is as if increasing the use of capital necessarily implies higher spending on maintenance so as to keep machines depreciating at a constant rate $\bar{\delta}$. As we observe, this would be a richer way to view CEE's setting, as we have a specification that is more general. Of course, neither the benchmark, nor the CEE settings interpret the more general idea of a treatment for variable capital utilization as the one given by the function Λ .

Consider, finally, how the general model works. I now revisit the problem (2.6), though assuming that the agent also chooses capital maintenance, m_t . I re-express the resource constraint by

$$C_t + I_t[1 + h(I_t)] + Q_{t+1} - Q_t + d(m_t)K_t \leq Y_t. \quad (4.6)$$

The law of motion of capital is still given by (2.1), where there is an additional constraint given by (4.5) The function $d(m_t)$ can be considered a general cost function of maintenance, where a special case is a linear specification $d(m_t) = m_t$. This way, one can derive the CEE setting by assuming $d(m_t) = m_t$, and $\Lambda(\bar{\delta}, s_t, m_t) = 0$, so that $m_t = m(s_t, \bar{\delta})$. Notice that the law of motion would become $K_{t+1} = K_t(1 - \bar{\delta}) + I_t$, as is specified by CEE.

In the same line of reasoning, the benchmark specification would be attained by assuming $m_t = \bar{m}$ at all t . Then, by substituting $\delta_t = \delta(s_t, \bar{m})$ into (2.1), and by assuming $d(\bar{m}) = 0$, one obtains the benchmark case.

A simple way of solving the general problem would be to re-express $\Lambda(\delta_t, s_t, m_t) = 0$ by a function $\delta_t = \delta(s_t, m_t)$. This is a reasonable step for that capital depreciation becomes then a function both of its intensity of use and its maintenance rate. An increase in the rate of capital utilization will rise the depreciation rate, as before. On the other hand, it is consistent to assume that an increase in maintenance will lower this rate.

It also seems appropriate to assume that the marginal depreciation of both intensity and maintenance is increasing in their respective arguments. However, it does not seem appropriate, at first, to assume that the marginal efficiency of maintenance is either increasing or decreasing with respect to the rate of capital utilization. In other words, there are no any technical presumptions indicating that δ_{sm} is either positive or negative.

Thus, I assume $\delta_s(s_t, m_t) > 0$, $\delta_{ss}(s_t, m_t) > 0$, $\delta_m(s_t, m_t) < 0$, $\delta_{mm}(s_t, m_t) > 0$, and $\delta_{sm}(s_t, m_t) = 0$. The last assumption is simply made for the sake of convenience.

Finally, it remains to consider how the spendings on maintenance are *paid* in this model. Let us assume a maintenance, unit-cost function, $d(m_t)$, that is increasing and convex on the units of maintenance services, and where at the steady state, $d(\bar{m}) = 0$.

To sum up, our problem becomes one of solving (2.6), subject to (4.6) and

$$K_{t+1} = K_t[1 - \delta(s_t, m_t)] + I_t. \quad (4.7)$$

After solving for the first-order, necessary conditions, the same system of equations (2.7)-(2.11) results, except that now (2.10) becomes

$$u_1(C_t, l_t, e_t, Q_t)p_{k,t} = \beta E \left\{ \begin{array}{l} u_1(C_{t+1}, l_{t+1}, e_{t+1}, Q_{t+1}) \times \\ \left[\begin{array}{l} s_{t+1}F_2(e_{t+1}N_{t+1}, s_{t+1}K_{t+1}) + \\ [1 - \delta(s_{t+1}, m_{t+1})]p_{k,t+1} - d(m_{t+1}) \end{array} \right] \Big| \Gamma_t \end{array} \right\} \quad (4.8)$$

In addition, we obtain the following equilibrium condition with respect to the maintenance variable,

$$d'(m_t) = -\delta_m(m_t)p_{k,t}, \quad (4.9)$$

where $\delta_m(m_t)$ results from the fact that the depreciation rate is separable in its two arguments.

Equilibrium condition (4.9) could be interpreted as the equality between the marginal cost of increasing maintenance services (LHS) and the marginal benefit caused by lowering the rate of capital depreciation (RHS) - the latter valued in terms of the replacement cost of a unit of capital. Condition (4.8) is the new Euler equation. The RHS includes the term $-d(m_{t+1})$ to show that the marginal benefit of increasing the capital stock has to be adjusted by the higher cost in maintenance services. Recall, though, that $d(\bar{m}) = 0$, so that at the steady state this additional term vanishes.

Note that condition (4.9) is a function of two variables only: m_t , and I_t - remember that $p_{k,t} = 1 + h(I_t) + h'(I_t)$. Therefore, I could write $m_t = m(I_t)$, and re-express the system of equilibrium conditions by replacing m_t for an expression on I_t . This way, investment will have an additional feature itself that will directly affect the capital depreciation rate. In fact, given the assumptions made both on $d(m_t)$ and $\delta_m(m_t)$, (4.9) implies that $m(I_t)$ will be a monotonically increasing function of I_t . One interpretation of this equilibrium result would be that increases in the capital replacement cost - directly associated with increases in investment at equilibrium -, lead to higher maintenance spendings, simply because the agent will not want her capital stock to depreciate further.

From the previous analysis it follows that, for a given rate of capital utilization, an increase in investment would generate, in equilibrium, a fall in the depreciation

rate.³ A final result that follows from condition (4.9) is that the steady-state rate of capital utilization will indeed depend (negatively) on the price of capital, as in the CEE specification. Remember that in the benchmark model the steady-state rate of capital utilization depended only on parameters, in particular β , δ_0 , and v . Now, it will also depend on steady-state investment, and the parameters related both to the adjustment costs on investment and the depreciation rate.

In order to solve the problem described so far, assume the rate of depreciation takes the form $\delta(s_t, m_t) = 0.5\bar{\delta}(s_t^{\nu_s} + m_t^{\nu_m})$, while the maintenance unit-cost function takes the form $d(m_t) = 0.5m_0(m_t^2 - 1)$. Parameters $\nu_s > 0$ and $\nu_m < 0$ are set so that variables s and m result 1 in steady state; m_0 is set to a sufficiently low value so that main results are not affected by its choice; while $\bar{\delta}$ is set to approximate the steady-state depreciation rate that resulted so far in both the benchmark and the CEE settings. Notice that the choice of these functional forms are still special cases of the general functions given above for Λ and d . In this sense, it is important to show under which conditions this model reduces to the benchmark and CEE specifications, as well as to the problem introduced in this section. This is briefly shown in Appendix D.

Consider now the results from the general model that includes both maintenance and endogenous depreciation. What do we learn by adding capital maintenance to the benchmark model, in addition to the theoretical finding of obtaining a general formulation for the treatment of variable capital utilization? Table X shows the volatilities found in the three specifications analyzed so far - the benchmark, CEE, and the general case, though I will focus on what the general adds with respect to

³We should not conclude, though, that investment and depreciation are negatively correlated, for the correlation matrix of this general specification shows they are positively correlated indeed - see Table XI. We will return to this result below after describing the functional forms and the parameter values on this specification.

the benchmark setting. As I mentioned above, investment becomes sensitive after the introduction of capital maintenance - recall that these two variables are perfectly correlated indeed. As such, the relative standard deviation of investment slightly falls from 3.27 to 2.96 in the general form. The introduction of maintenance spending slightly smoothes the response of investment, for it allows capital depreciate at lower rates. In this sense, the volatility of the rate of depreciation becomes much lower in the general specification (0.59 versus 1.08). In fact, notice that most of the volatilities decrease with the introduction of maintenance, being relatively important the drop in the relative standard deviation of capital intensity (from 0.78 to 0.53).

In Table XI, we can see the relationship among the model's variables from the general case. We observe now that the positive correlation between investment and the rate of capital depreciation becomes lower (0.70 versus 0.82). Remember that increases in investment imply proportional increases in maintenance, which in turn lower the rate of depreciation. But, still, investment and depreciation are positively correlated. What is happening is that both preference and technology shocks (expansionary, for instance) lead to increases in the intensity of use of capital, and thus in the depreciation rate. Investment will also rise in response to these shocks.⁴ At the end, this variable will be positively correlated with the depreciation rate. In other words, the positive effect of capital utilization on depreciation overcomes the negative effect of maintenance and investment. Notice that this result is consistent with the procyclicality of the depreciation rate pointed out in the capacity-utilization literature - see, for instance, Greenwood et al. [21].

⁴Recall that, for low-persistence preference shocks, investment is supposed to fall instead. However, the high-persistence rates of shocks used in the simulations, added to the positive effect of technology shocks on investment, in general make investment rise. Also, remember that the contribution of preference shocks to the variations in investment are rather small.

CHAPTER V

CONCLUSIONS

The primary goal of this dissertation is to gain a better understanding, in the context of a general equilibrium Walrasian stochastic framework, of the role of inventories and capacity utilization (of both capital and labor) and, in particular, the relationship among those variables. These are variables which have long been recognized as playing an important role, and having rather well defined associations both among each other and with other indicators in the business cycle. An analysis of the association between inventories and capacity utilization seems natural, since in some conceptual sense physical capital can be seen as a stock ultimately destined to be transformed into an inventory of finished goods. Likewise, inventories could be seen as a stock of physical capital already transformed into finished goods. Furthermore, once we introduce the possibility of variable rates of utilization of the capital stock, then both such rate of utilization and inventories can be seen as providing a short-run adjustment “buffer stock” mechanism. A variable rate of utilization of labor is also introduced, in a manner that is very much symmetric to the rate of capital utilization –the symmetry not being perfect, since changes in the rate of capital utilization change a state variable (the capital stock) while the same is not true of changes in the rate of labor utilization.

In Chapter II we discussed how our model differs from those in the existing literature. One methodological difference, which does not change the ultimate results, is that since firms are competitive, then they can be abstracted from, so that the analysis can be conducted in terms of the behavior of “agents” or “households”, performing all activities (production, investment and consumption). This is also explained in detail in Chapter II. Being this the case, one should see some of the assumptions as “mimicking” what we would observe for firms. One example is the

assumption of utility depending not only on leisure but also on “last minute” changes in labor (leisure): for the firms, this is the usual case of changes in the number of work hours. Another is the modeling of changes in preferences, which corresponds to firms’ changes in demand. Finally, the existence of inventories in the steady state is motivated by their presence in the utility function –mimicking, to some extent, the association between the levels of final goods inventories and sales at the level of the firm.

Introducing both variable capital and labor utilization, and inventories in a dynamic, stochastic general equilibrium model generates some limitations if one is interested in obtaining an analytical solution. The relatively large number of variables and equilibrium conditions, in addition to the presence of non-linear expressions, allows only to find approximate solutions. The analysis of the relationship between inventories and capacity utilization is centered on the effects of two possible shocks: preference (demand) shocks and technology shocks. The first is specified via shocks in a parameter of the utility function; the second, via shocks in a parameter of the production function. This allows not only to investigate the nature of those relationships, but also a discussion of the extent to which the results conform to the associations observed in the data, or the “stylized facts”. In these impulse-response experiments, a “shock” (it is more formally defined in Chapter III), means an increase, for only one date, of 1% of the random term. In each case, it will be useful to distinguish between “high” and “low” persistence of the shock.¹ The results, on which we elaborate in more detail in Chapter III, can be summarized as follows.

¹Remember that the stochastic structure of shocks is of the form $\ln \varsigma_t = \rho_\varsigma \ln \varsigma_{t-1} + \epsilon_{\varsigma t}$, where ϵ is the random component and ρ is the measurement of the shock’s persistence. We should also keep in mind the important difference between the persistence of a shock (measured by the coefficient ρ), and whatever is the “intrinsic”, internal persistence of the model.

Consider, first, the effects of a positive technology shock. For a relatively “high” persistence of the shock, inventories behave as being complement to capital utilization (i.e., they are positively associated), but as substitute to the rate of labor utilization –the latter resulting from the lack of persistence of the rate of labor utilization. Still, inventories are positively associated with (i.e., behave as complement to) labor services. For relatively “low” persistence of the shock, inventories act as a complement to (i.e., are positively associated with) both capital and labor utilization rates.

Second, consider the effects of a preference shock –more specifically, a positive shock, increasing the utility of both consumption and inventories. Here, a high-persistent shock, as in the case of a productivity shock, shows inventories as being a complement to capital utilization, but a substitute to the rate of labor utilization. Still, and as in the case of the technology shock, inventories are associated positively with (i.e., as a complement to) to labor services. For shocks of relatively low persistence, inventories behave as a substitute for both rates of capacity utilization (capital and labor).

These findings warrant some further considerations. Notice, first, that inventories and the rate of capital utilization are complements in three of the four cases described above, while inventories and the rate of utilization of labor are substitutes in three of them. It is worth remembering, though, that both rates of utilization complement each other in response to any shocks of any persistence. One can infer that both rates of utilization associate differently with inventories because, as “shock absorbers”, both inventories and the rate of labor utilization result in less of an effect carried on to subsequent periods than changes in the rate of capital utilization and the depreciation rate. In other words, differences in the internal persistence of inventories and the rates of capacity utilization are central for understanding these results.

Second, and related to the first comment, notice that low-persistence shocks

generate more clear results, in the sense that inventories complement both rates of utilization in response to technology shocks (as one would expect), while they substitute both rates in response to preference shocks (as one would also expect). High-persistence shocks, on the other hand, emphasize mostly the role of inventories as being a complement to consumption instead of emphasizing their role as a “shock absorber”.²

By considering the effects of the two shocks simultaneously, and assuming persistence rates and standard deviations as those reported in the literature, one finds that inventories are associated positively with capital utilization, but negatively with labor utilization. Furthermore, inventory holdings are pro-cyclical, while the inventory-to-sales ratio is counter-cyclical, being both findings consistent with the stylized facts in the literature.

Related to the main line of the work, two additional “themes” have been analyzed. The first, discussed in Chapter III, has to do with the treatment of uncertainty and the consequences of using, as it is done in most of the literature, a first-order approximation. As indicated before, the complexity of the model makes it impossible to derive a closed solution. Being the model stochastic, it also implies that some features are affected by the way in which the solution is approximated. In general, a first-order approximation to the model brings satisfactory results if one is interested in characterizing the variables’ dynamics. By approximating the decision rules to a second order, Schmitt-Grohé and Uribe [38] have shown that this dynamics is not affected by the shocks’ volatility. However, second- and higher-order approximations do become relevant when studying stochastic steady-state values and making welfare

²In chapter IV we analyzed the case of a utility function that is separable in consumption and inventories. Under this specification, preference shocks generate a negative association between inventories and both rates of utilization that is independent of the persistence of the shocks.

comparisons. In this way, the stochastic steady-state behavior of both inventories and capacity utilization was analyzed by using a second-order approximation. Interestingly, as uncertainty rises, the average of inventories increases relative to consumption and output, whereas the averages of both rates of utilization fall. As one would expect, higher exogenous uncertainty enhances the importance of the supposedly most relevant motive to holding inventories: the precautionary motive. Welfare comparisons were also performed by approximating to a second order both the decision rules and the utility function. Results here are mixed. One expects that as the shocks' standard deviations are raised, expected welfare falls. It turns out that this only happens if we raise the volatility of preference shocks, for if we raise the volatility of technology shocks then welfare would increase. This remains to be explained on the grounds of more accurate approximation procedures. However, this issue is left for further research.

The second additional theme, discussed in chapter IV, is a more general framework for the analysis of capital utilization. Christiano et al. [15] proposed an alternative way of modeling variable capital utilization. Instead of higher rates of capital utilization increasing the rate of depreciation, simply imposes a cost in terms of output. By introducing this alternative specification into our model, we showed that the results do not change qualitatively, but that the main differences are quantitative. Furthermore, by introducing capital maintenance into the benchmark model, one finds that both Christiano et al. [15] and the benchmark specification become special cases of a more general formulation. A natural way of introducing this concept is to assume that maintenance, as capital utilization, affects depreciation. Once again, while the qualitative features of our model remain unaltered, this additional subject becomes relevant quantitatively.

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APPENDIX A

MODELING DETAILS

System of necessary conditions: Applying the functional forms

$$E \left\{ e_t \left[z_t (\theta C_t^{1-\gamma} + (1-\theta)Q_t^{1-\gamma})^{-1} \theta C_t^{-\gamma} (1-\alpha) \left(\frac{Y_t}{e_t n_t} \right) - \eta \right] \mid \Gamma_{t-1} \right\} = 0 \quad (\text{A.1})$$

$$N_t \left[z_t (\theta C_t^{1-\gamma} + (1-\theta)Q_t^{1-\gamma})^{-1} \theta C_t^{-\gamma} (1-\alpha) \left(\frac{Y_t}{e_t n_t} \right) - \eta \right] = \phi(e_t - 1) \quad (\text{A.2})$$

$$\alpha \left(\frac{Y_t}{s_t K_t} \right) = v s_t^{v-1} (1 + 2bI_t) \quad (\text{A.3})$$

$$\begin{aligned} & z_t (\theta C_t^{1-\gamma} + (1-\theta)Q_t^{1-\gamma})^{-1} \theta C_t^{-\gamma} (1 + 2bI_t) \\ = & \beta E \left\{ \begin{array}{l} z_{t+1} (\theta C_{t+1}^{1-\gamma} + (1-\theta)Q_{t+1}^{1-\gamma})^{-1} \theta C_{t+1}^{-\gamma} \times \\ \left[\begin{array}{l} s_{t+1} \alpha \left(\frac{Y_{t+1}}{s_{t+1} K_{t+1}} \right) + \\ [1 - (\delta_0 + s_{t+1}^v)] (1 + 2bI_{t+1}) \end{array} \right] \mid \Gamma_t \end{array} \right\} \quad (\text{A.4}) \end{aligned}$$

$$\begin{aligned} & z_{t+1} (\theta C_{t+1}^{1-\gamma} + (1-\theta)Q_{t+1}^{1-\gamma})^{-1} \theta C_{t+1}^{-\gamma} \\ = & \beta E \{ [\theta C_{t+1}^{-\gamma} + (1-\theta)Q_{t+1}^{-\gamma}] \mid \Gamma_t \} \quad (\text{A.5}) \end{aligned}$$

$$C_t + I_t(1 + bI_t) + Q_{t+1} - Q_t = Y_t \quad (\text{A.6})$$

$$K_{t+1} = K_t[1 - (\delta_0 + s_t^v)] + I_t \quad (\text{A.7})$$

Steady-State Expressions:

$$s = \left[1 - \frac{\beta(1 - \delta_0)}{\beta(v - 1)} \right]^{1/v} \quad (\text{A.8})$$

$$\delta = \delta_0 + s^v \quad (\text{A.9})$$

$$AB[D(1 + 2b\delta K)]^{\frac{\alpha}{\alpha-1}} - K(sD - \delta) - K^2b\delta(2sD - \delta) = 0 \quad (\text{A.10})$$

$$N = sK [D(1 + 2b\delta K)]^{\frac{1}{1-\alpha}} \quad (\text{A.11})$$

$$Q = B \left(\frac{sK}{N} \right)^\alpha \quad (\text{A.12})$$

$$C = AQ \quad (\text{A.13})$$

$$I = \delta K \quad (\text{A.14})$$

where,

$$A \equiv \left[\frac{(1 - \beta)\theta}{(1 - \theta)\beta} \right]^{1/\gamma} \quad (\text{A.15})$$

$$B \equiv \frac{(1 - \alpha)\beta(1 - \theta)}{[\theta A^{1-\gamma} + (1 - \theta)]\eta(1 - \beta)} \quad (\text{A.16})$$

$$D \equiv \frac{vs^{v-1}}{\alpha} \quad (\text{A.17})$$

Baseline Parameters:

β	δ_0	ν	α	γ	θ	η	ϕ	b	σ_z	σ_ω	ρ_z	ρ_ω
.988	.01	2	.36	5	.98	2.5	.5	1	.01	.007	.95	.95

Steady-State Values:

Capital	K	2.41
Consumption	C	0.26
Depreciation Rate	δ	0.03
Inventories	Q	0.29
Investment	I	0.08
Labor	N	0.33
Output	Y	0.34
Rate Capital Utilization	s	0.15
Rate Labor Utilization	e	1.00

APPENDIX B

OBTAINING THE SEPARABLE UTILITY FUNCTION FROM THE BENCHMARK. THE LINK BETWEEN CONSUMPTION AND INVENTORIES

By expression (3.3) in Chapter II, it is assumed a utility function of the form

$$u(C_t, 1 - e_t N_t, e_t, Q_t; z_t) = z_t X_t + \eta(1 - e_t N_t) - \frac{\phi}{2}(e_t - 1)^2, \quad (\text{B.1})$$

where $X_t \equiv \ln[\theta C_t^{1-\gamma} + (1 - \theta)Q_t^{1-\gamma}]^{\frac{1}{1-\gamma}}$.

Then, the marginal utility of consumption is given by

$$u_c(\cdot) = \frac{z_t \theta}{[\theta C_t^{1-\gamma} + (1 - \theta)Q_t^{1-\gamma}]C_t^\gamma}, \quad (\text{B.2})$$

the marginal utility of inventories results

$$u_q(\cdot) = \frac{z_t(1 - \theta)}{[\theta C_t^{1-\gamma} + (1 - \theta)Q_t^{1-\gamma}]Q_t^\gamma}, \quad (\text{B.3})$$

and the cross derivative results

$$u_{cq}(\cdot) = -\frac{z_t(1 - \gamma)\theta(1 - \theta)}{[\theta C_t^{1-\gamma} + (1 - \theta)Q_t^{1-\gamma}]^2(C_t Q_t)^\gamma}. \quad (\text{B.4})$$

Consider the following notes.

1. For inventory goods to complement consumption goods, it is sufficient to assume $\gamma > 1$. This way, $u_{cq}(\cdot) > 0$ under the assumptions stated above.
2. Notice that as $\gamma \rightarrow 1$, the three derivatives (B.2)-(B.4) converge to the following expressions:

$$u_c(\cdot) \approx \frac{z_t \theta}{C_t}, \quad (\text{B.5})$$

$$u_q(\cdot) \approx \frac{z_t(1-\theta)}{Q_t}, \quad (\text{B.6})$$

$$u_{cq}(\cdot) \approx 0. \quad (\text{B.7})$$

More formally, apply L'hopital's rule to the expression X_t to show the following:

$$\begin{aligned} \lim_{\gamma \rightarrow 1} X_t &= \lim_{\gamma \rightarrow 1} \left\{ \frac{\ln[\theta C_t^{1-\gamma} + (1-\theta)Q_t^{1-\gamma}]}{1-\gamma} \right\} \\ &= \lim_{\gamma \rightarrow 1} \left\{ \frac{\theta C_t^{1-\gamma}(\ln C_t) + (1-\theta)Q_t^{1-\gamma}(\ln Q_t)}{\theta C_t^{1-\gamma} + (1-\theta)Q_t^{1-\gamma}} \right\} \\ &= \theta \ln C_t + (1-\theta) \ln Q_t. \end{aligned}$$

Thus, as $\gamma \rightarrow 1$, one can rewrite the utility function (3.3) in terms of a utility function that is separable in consumption and inventories:

$$u(\cdot) = z_t[\theta \ln C_t + (1-\theta) \ln Q_t] + \eta(1 - e_t N_t) - \frac{\phi}{2}(e_t - 1)^2. \quad (\text{B.8})$$

The most clear advantage one obtains by considering this case is that it allows us to highlight the “true” role of inventories of being a buffer stock for consumption. A consequence of this assumption is that the response of inventory holdings to a positive demand shock - a rise in z_t -, for instance, will be negative whatsoever the degree of persistence of the shock. In other words, the consumer will be more prone to use inventories as a response to an increase in demand, instead of accumulating them to be used as a complement for consumption.

APPENDIX C

FIRMS AND THE COMPETITIVE EQUILIBRIUM

There are a number of market specifications that can support the allocation arising from the representative-agent problem. In order to keep things simple, assume that the typical household maximizes her lifetime utility by choosing contingency plans for consumption, inventory holdings, capital investment - and, with it, the rate of capital utilization -, and the supply of labor services. On the other hand, the firm maximizes profits by choosing at every period an expected demand for capital services, S_t^d - in equilibrium, equal to $s_t K_t$ -, and an expected demand for labor services, L_t^d - in equilibrium, equal to $e_t N_t$.

I will now describe a competitive-equilibrium specification that is consistent with Hansen's [22] and Rogerson's [36] indivisible-labor models.¹ I assume that, before the state of nature is known, there is a market for state-contingent labor services where households choose their expected supply of labor - the extensive margin - and an associated state-contingent rate of labor utilization - the intensive margin. The difference with Hansen-Rogerson model is that, in this economy, at the beginning of every period t firms and labor suppliers face a state-contingent wage schedule that specifies the wage as a function of the information set $\Gamma_t \equiv (\omega_t, z_t)$. This means that the equilibrium wage depends upon the realization of the technology and the preference shocks. Similarly, before the state of nature is realized, households - who own the capital stock - and firms trade state-contingent rental contracts on capital which specify the quantity traded and the rental rate as functions of the information

¹This specification was also employed by Burnside, Eichenbaum, and Rebelo [11] and Burnside and Eichenbaum [10] in developing the factor-hoarding models.

set $\Gamma_t \equiv (\omega_t, z_t)$. With this specification, the *ex-post* rental rate depends upon the realized rate of capital utilization through its dependence on the shocks to technology and preferences.

Households

For simplicity, I state the household's problem the same way as in Chapter II.² The typical household solves the following problem,

$$V(\Phi_t) = \max_{N_t} E \{ \max_{\Omega_t} E [u(C_t, l_t, e_t, Q_t; z_t) + \beta V(\Phi_{t+1}) \mid \Gamma_t] \mid \Gamma_{t-1} \} \quad (\text{C.1})$$

subject to (2.1) - the law of motion of capital -, (3.3) - the specific utility function -, the shocks processes, and her budget constraint given by,

$$C_t + I_t[1 + h(I_t)] + Q_{t+1} - Q_t \leq w_t e_t N_t + r_t s_t K_t, \quad (\text{C.2})$$

where, w_t and r_t are the wage and rental rate on labor and capital services, both given to the household.

The first order conditions are

$$E [u_1(C_t, l_t, e_t, Q_t)w_t - u_2(C_t, l_t, e_t, Q_t) \mid \Gamma_{t-1}] = 0 \quad (\text{C.3})$$

$$u_1(C_t, l_t, e_t, Q_t)w_t - u_2(C_t, l_t, e_t, Q_t) = -\frac{u_3(C_t, l_t, e_t, Q_t)}{N_t} \quad (\text{C.4})$$

$$r_t = \delta'(s_t)p_{k,t} \quad (\text{C.5})$$

²To be strictly consistent with Hansen [22] and Rogerson [36], one should write the household's work-leisure decision in terms of the probability of working. However, one can see below that factor prices and quantities are, at the end, determined the same way as in those models. This occurs because I reduce the infinite-agent-with-unit-mass problem to a single-agent problem with linear utility in leisure as the one described in Rogerson [36]. This simplification is made for the sake of describing a simple labor market with standard supply and demand schedules.

$$u_1(C_t, l_t, e_t, Q_t)p_{k,t} = \beta E \left\{ \begin{array}{l} u_1(C_{t+1}, l_{t+1}, e_{t+1}, Q_{t+1}) \times \\ [s_{t+1}r_{t+1} + (1 - \delta_{t+1})p_{k,t+1}] \mid \Gamma_t \end{array} \right\} \quad (\text{C.6})$$

$$u_1(C_t, l_t, e_t, Q_t) = \beta E \left[\begin{array}{l} u_1(C_{t+1}, l_{t+1}, e_{t+1}, Q_{t+1}) + \\ u_4(C_{t+1}, l_{t+1}, e_{t+1}, Q_{t+1}) \mid \Gamma_t \end{array} \right] \quad (\text{C.7})$$

The first equation is the necessary condition on the labor supply as the extensive margin, where the wage rate is written inside the expectation operator to indicate that contracts are written conditional on the realization of the shocks. The second expression refers to the decision on the intensity of use of labor, where the wage rate is now denoted *ex-post*. Notice that what is going to ultimately define the amount of labor services supplied by the household will be the decision related to the intensity of use of labor, at the *ex-post* wage, w_t . The third condition is related to the rate of capital utilization, which, as in the case of the labor supply, is going to define the supply of capital services by the household, given the *ex-post* rental rate, r_t . Conditions four and five are related to the decisions on investment both in capital and inventories.

Firms

The firm's problem can be stated as follows. At every date t , the firm maximizes expected profits by

$$\max_{(L_t, S_t)} E_t [F(L_t, S_t; \omega_t) - w_t L_t + r_t S_t \mid \Gamma_{t-1}]$$

subject to the Cobb-Douglas production technology given by $F(L_t, S_t; \omega_t) = \omega_t L_t^{1-\alpha} S_t^\alpha$ - a modified version of (3.1) indicating that the firm chooses to rent expected labor and capital services according to households' supply, at the expected factor prices w_t

and r_t .³

The first order conditions are

$$E_t [F_1(L_t^d, S_t^d) - w_t \mid \Gamma_{t-1}] = 0 \quad (\text{C.8})$$

$$E_t [F_2(L_t^d, S_t^d) - r_t \mid \Gamma_{t-1}] = 0 \quad (\text{C.9})$$

That is, the expected marginal products of labor services and capital services must equal the *ex-ante* wage and rental rates.

Ex-post, the firm's best response is to choose labor and capital services according to

$$F_1(L_t^d, S_t^d) - w_t = 0 \quad (\text{C.10})$$

$$F_2(L_t^d, S_t^d) - r_t = 0 \quad (\text{C.11})$$

Equilibrium

Define the competitive equilibrium as a sequence $\{C_t, Q_t, K_t, N_t, s_t, e_t, w_t, r_t\}_{t=0}^{\infty}$ so that households and firms solve their optimization problems, and the markets for factors of production and goods clear.

Condition (C.3) together with (C.8), and condition (C.6) together with (C.9) determine the trade of state-contingent contracts on labor and capital services at the expected factor prices $E_t\{w_t \mid \Gamma_{t-1}\}$ and $E_t\{r_t \mid \Gamma_{t-1}\}$. Once the states of technology and preferences are realized, equilibria in the markets for labor and capital services need that $L_t^d = e_t N_t$ and $S_t^d = s_t K_t$. This implies that the equilibrium wage and rental rate must satisfy conditions (C.4)-(C.5) and (C.10)-(C.11). This will be held indeed for contracts are state-contingent, which means that the firm commits to pay

³Notice that the expected output price was normalized to 1, so that the relative price of capital to output, p_k , is as specified in Chapter II. Remember that in the steady state, $p_k > 1$, for the steady state rate of transformation from output to capital is positive.

wage and rental rate according to the realized marginal products of labor and capital services. Otherwise, it would not be an equilibrium to the firm.

Naturally, the two equations describing the equilibrium quantities of labor and capital services will be the same as those found in the representative-agent problem of Chapter II, namely, conditions (2.8) and (2.9). Finally, equilibrium in the output market is obtained from the market clearing condition:

$$C_t + I_t[1 + h(I_t)] + Q_{t+1} - Q_t = y_t. \quad (\text{C.12})$$

To sum up, the market-specification could be described through the following steps:

1. At the beginning of each period, households and firms trade contingent contracts on labor and capital services, according to expected prices and quantities.
2. These contracts specify, first, the amounts of labor and capital, N_t and K_t , supplied with certainty by the household.
3. Given these stocks of labor and capital, the decision on the services is a decision on the rates of use of these productive factors, and I assume these are decided by the household.
4. Contingent contracts imply a commitment on behalf of the firm to pay wage and rental rate at the end of the period, according to the services received.
5. Equilibrium prices and quantities of these services ultimately depend on the firm's maximizing behavior. For it is the firm who specify, *ex-ante*, a state-contingent plan where, for any bundle of factor services (L_t, S_t) , it is willing to pay $w_t(\omega_t, z_t)L_t + r_t(\omega_t, z_t)S_t$.

6. After the vector of states of technology and preferences (ω_t, z_t) is realized, there is no trade between firm and household. The household optimally chooses the rates of labor and capital utilization according to the plan traded with the firm. Finally, equilibrium quantities of factor services and prices result.

APPENDIX D

THE BENCHMARK AND CEE AS SPECIAL CASES

Consider the following specification for the function Λ :

$$\Lambda(\delta_t, s_t, m_t) = \Lambda_0 + \Lambda_1 \delta_t^{v_1} + \Lambda_2 s_t^{v_2} + \Lambda_3 m_t^{v_3} = 0, \quad (\text{D.1})$$

and the following specification for d :

$$d(m_t) = d_0 + d_1 m_t^{v_4}. \quad (\text{D.2})$$

Then, it is straightforward to note that our benchmark case requires

$$\Lambda_3 = d_0 = d_1 = 0; \Lambda_0 = -\delta_0; \Lambda_1 = \Lambda_2 = v_1 = 1, \quad (\text{D.3})$$

while the CEE case requires

$$\Lambda_0 = 0; \Lambda_1 = v_1 = 1; \Lambda_2 = \bar{\delta}; v_2 = 2; \Lambda_3 = \frac{-2\bar{\delta}}{v}; \quad (\text{D.4})$$

$$d_0 = 0; d_1 = v_4 = 1; \quad (\text{D.5})$$

where v , as in the CEE specification, is set so that $\bar{s} = 1$.

Finally, the general case treated in Chapter IV, where maintenance becomes a control variable and depreciation is endogenous, requires

$$\Lambda_0 = 0; \Lambda_1 = v_1 = 1; \Lambda_2 = \Lambda_3 = 0.5\bar{\delta}; v_2 = v_s; v_3 = v_m; \quad (\text{D.6})$$

$$d_0 = -0.5; d_1 = 0.5; v_4 = 2. \quad (\text{D.7})$$

APPENDIX E

FIGURES AND TABLES

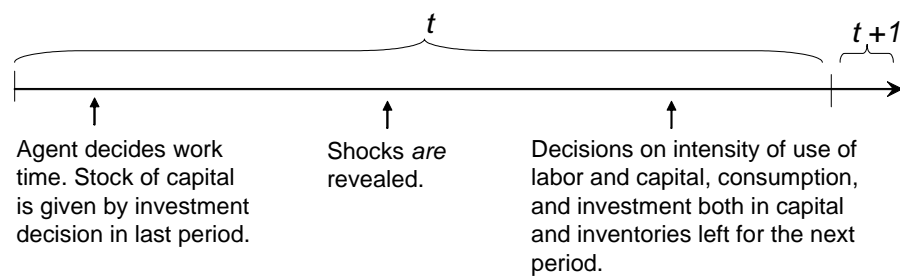


Fig. 1. Timeline of Decisions and Information.

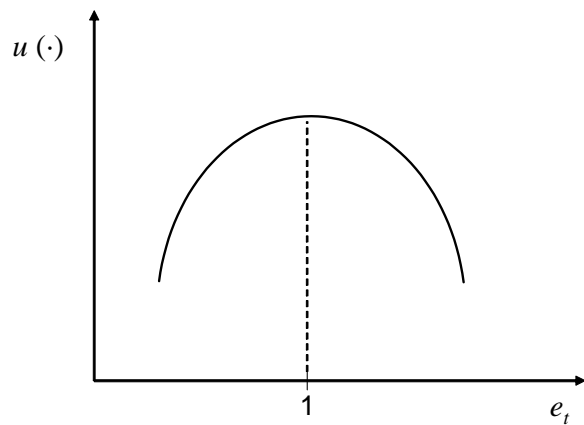


Fig. 2. An Illustration.

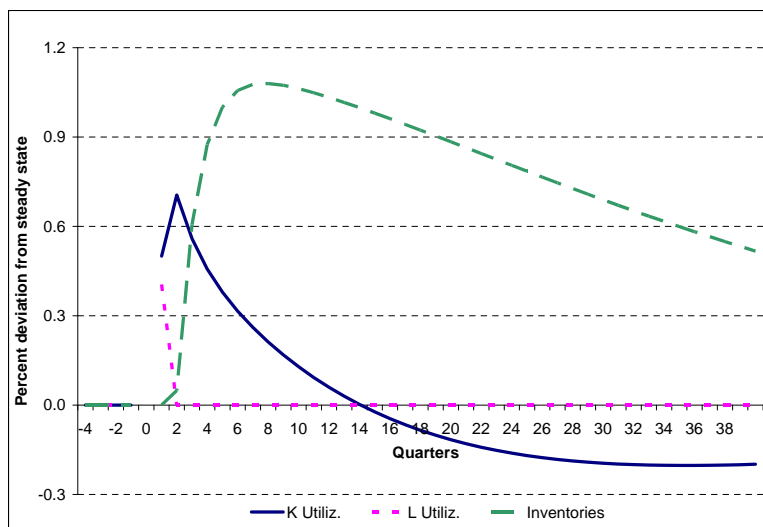


Fig. 3. Benchmark Specification. Inventories, Labor Utilization, Capital Utilization. High-Persistence Technology Shock.

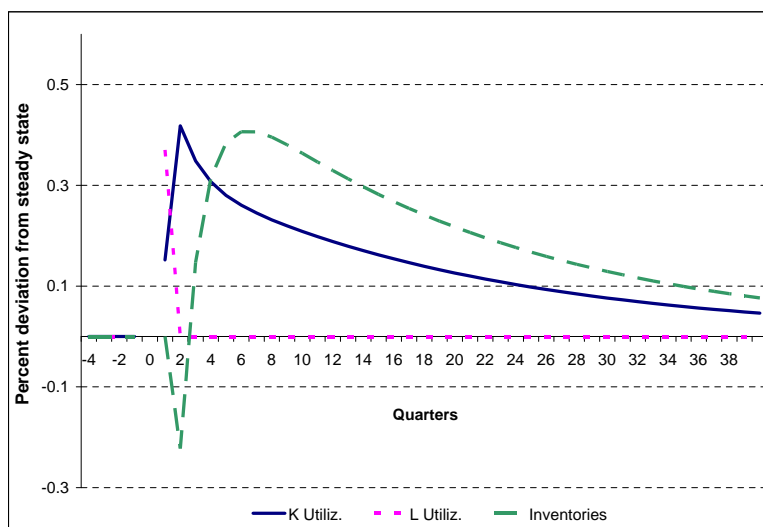


Fig. 4. Benchmark Specification. Inventories, Labor Utilization, Capital Utilization. High-Persistence Preference Shock.

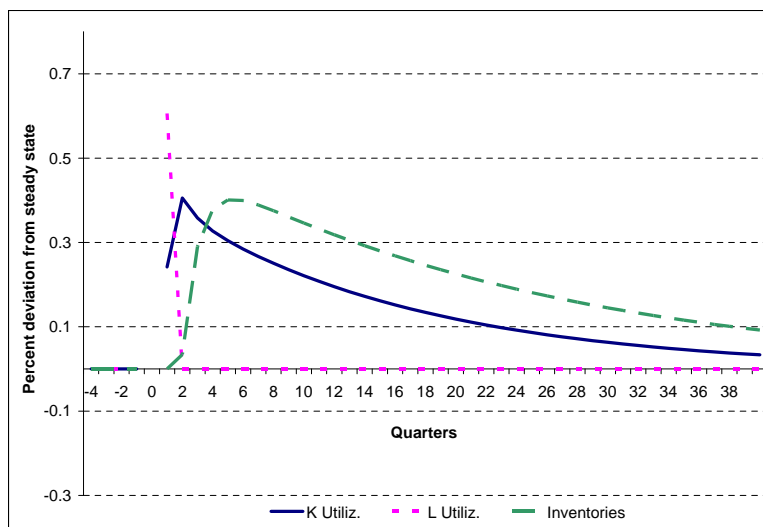


Fig. 5. Benchmark Specification. Inventories, Labor Utilization, Capital Utilization. High-Persistence Preference Shock, Low Adjustment Costs.

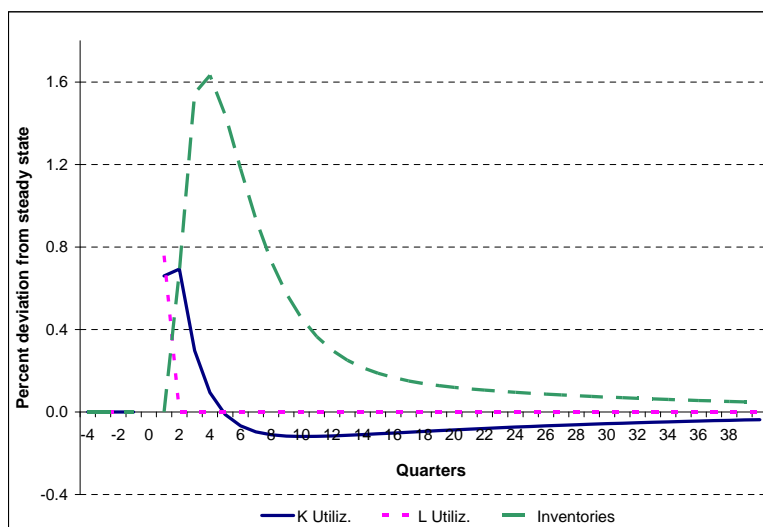


Fig. 6. Benchmark Specification. Inventories, Labor Utilization, Capital Utilization. Low-Persistence Technology Shock.

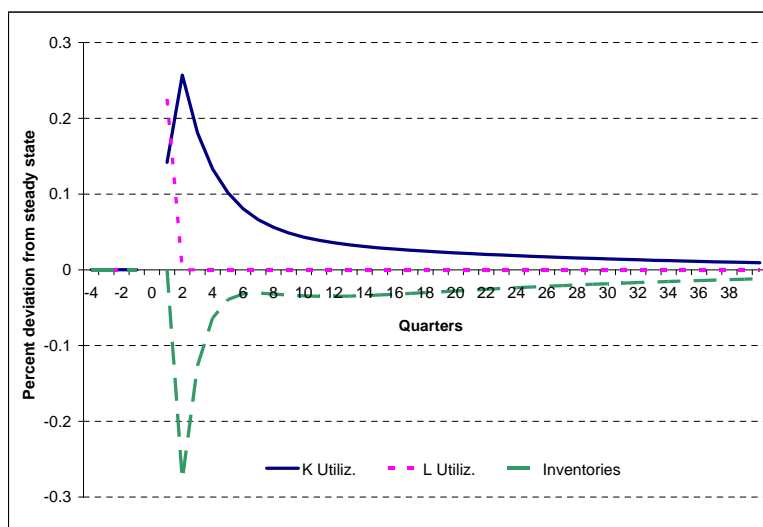


Fig. 7. Benchmark Specification. Inventories, Labor Utilization, Capital Utilization. Low-Persistence Preference Shock.

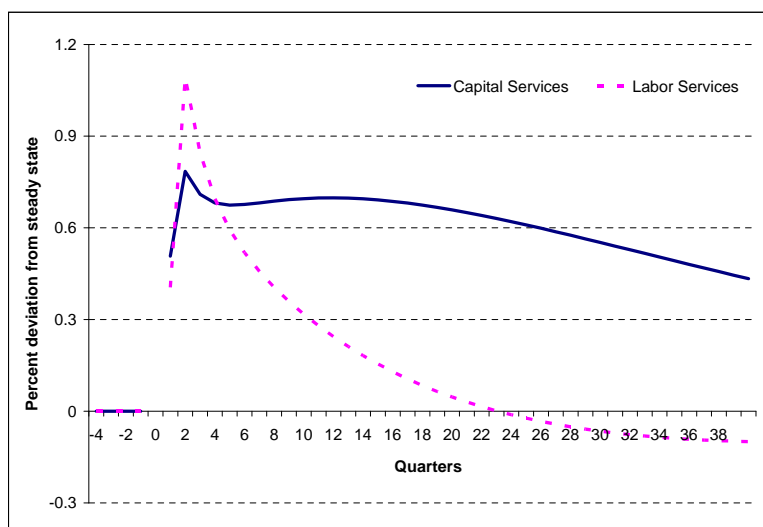


Fig. 8. Benchmark Specification. Labor and Capital Services. High-Persistence Technology Shock.

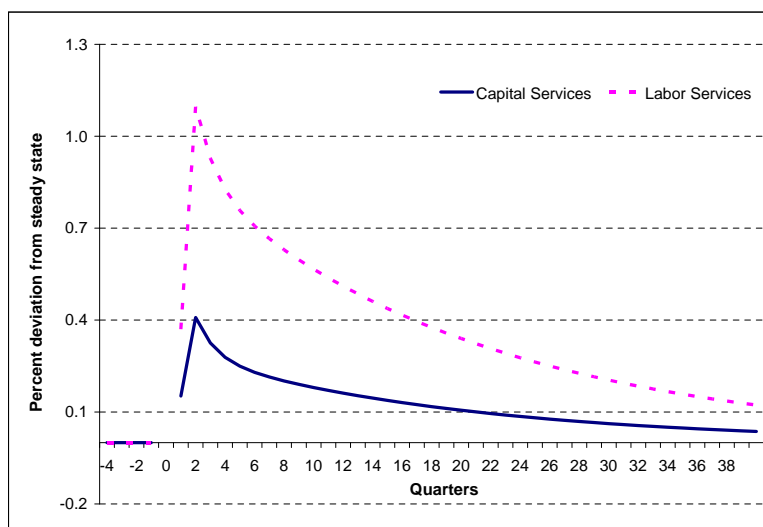


Fig. 9. Benchmark Specification. Labor and Capital Services. High-Persistence Preference Shock.

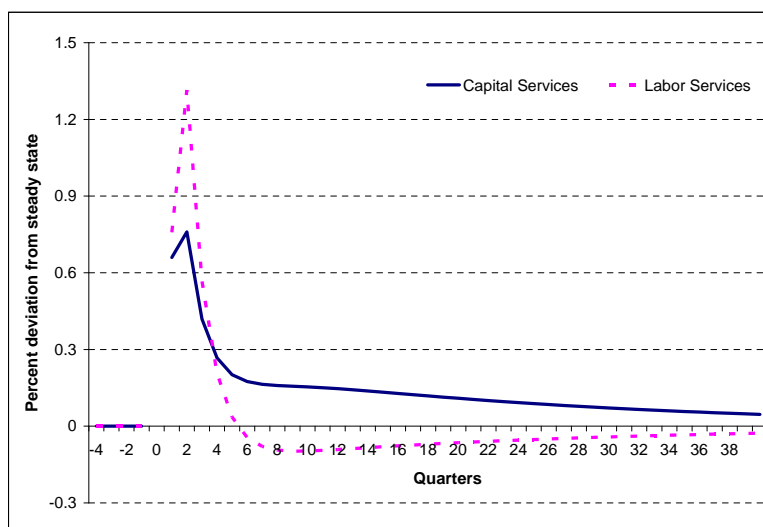


Fig. 10. Benchmark Specification. Labor and Capital Services. Low-Persistence Technology Shock.

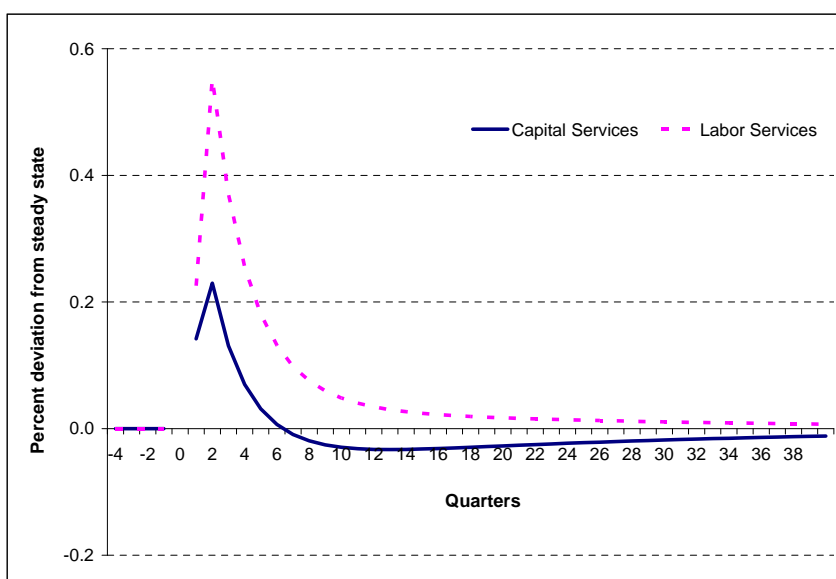


Fig. 11. Benchmark Specification. Labor and Capital Services. Low-Persistence Preference Shock.

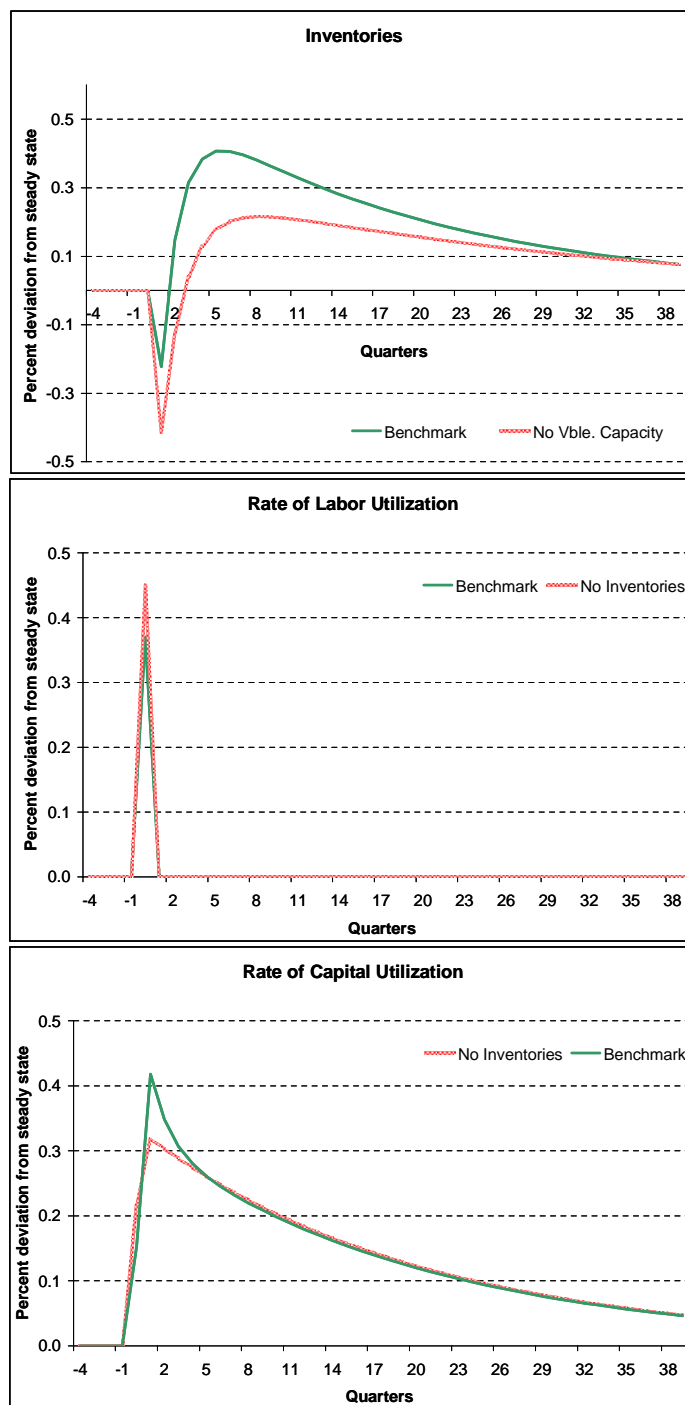


Fig. 12. Effects of Inventories and of Capacity Utilization. High-Persistence Preference Shock.

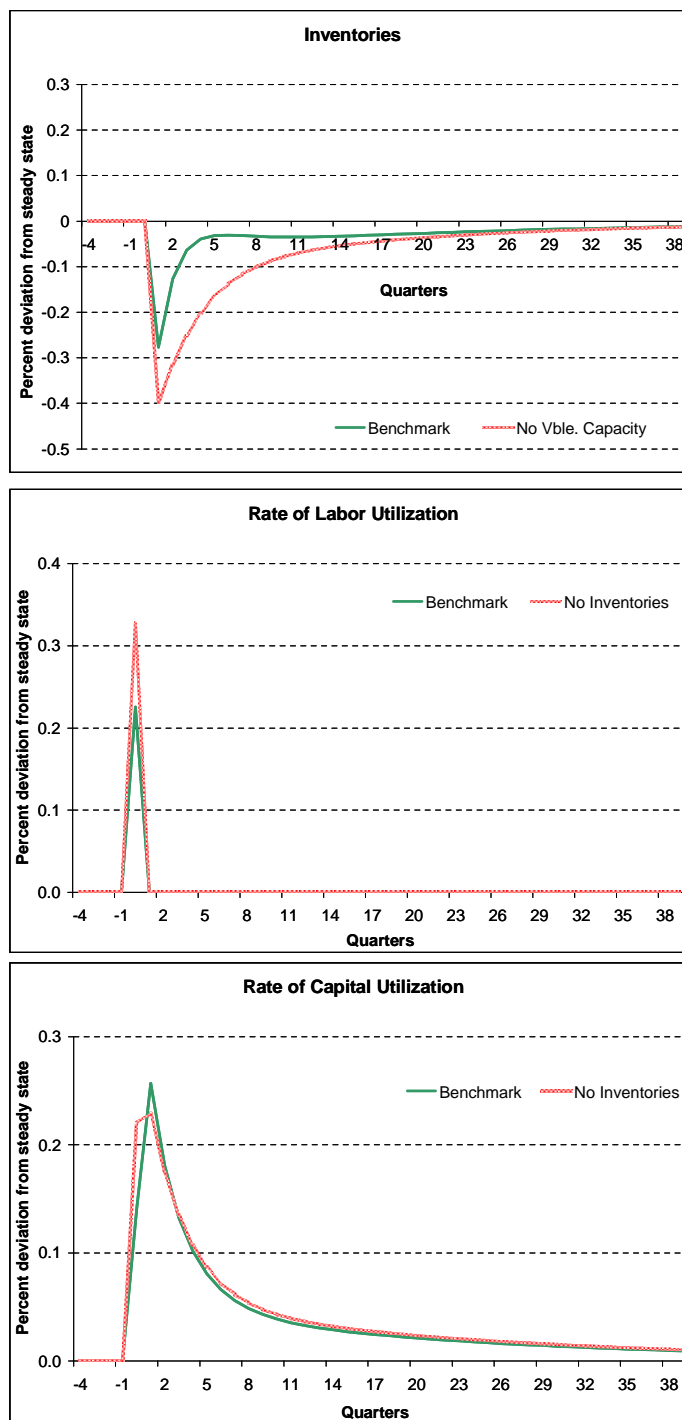


Fig. 13. Effects of Inventories and of Capacity Utilization. Low-Persistence Preference Shock.

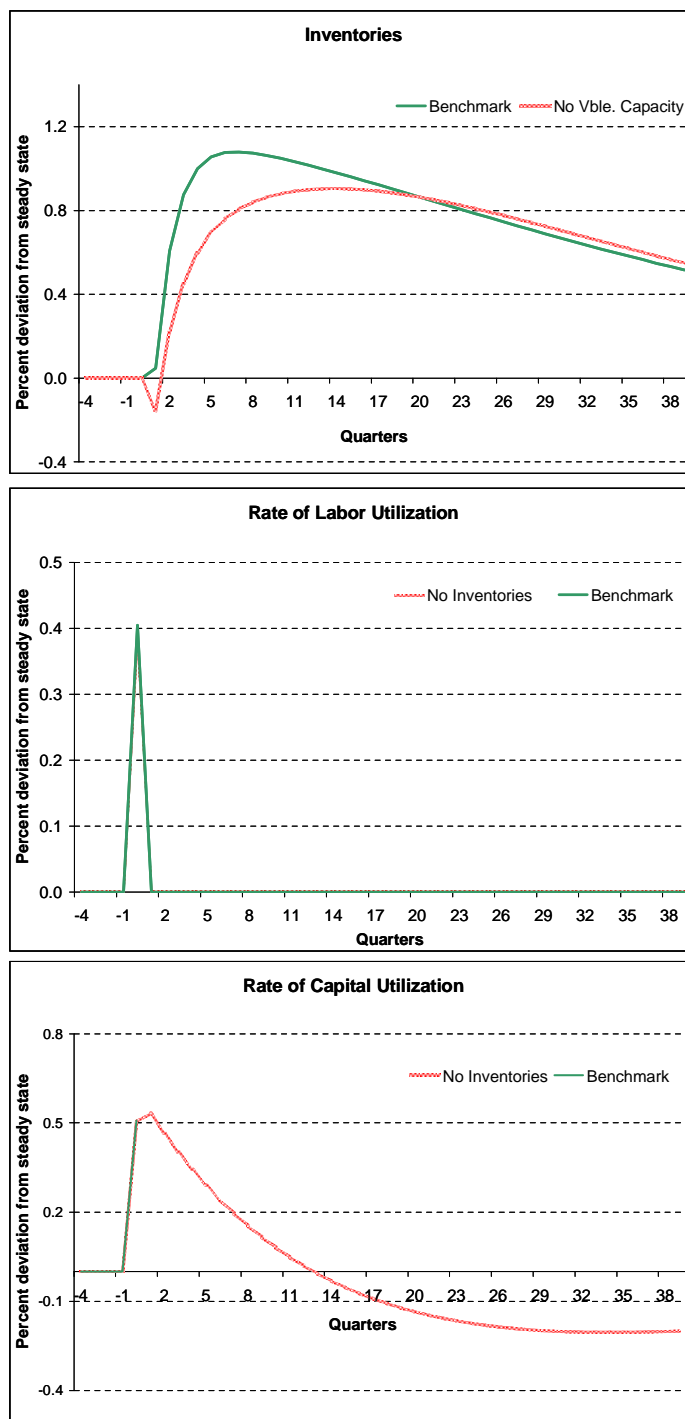


Fig. 14. Effects of Inventories and of Capacity Utilization. High-Persistence Technology Shock.

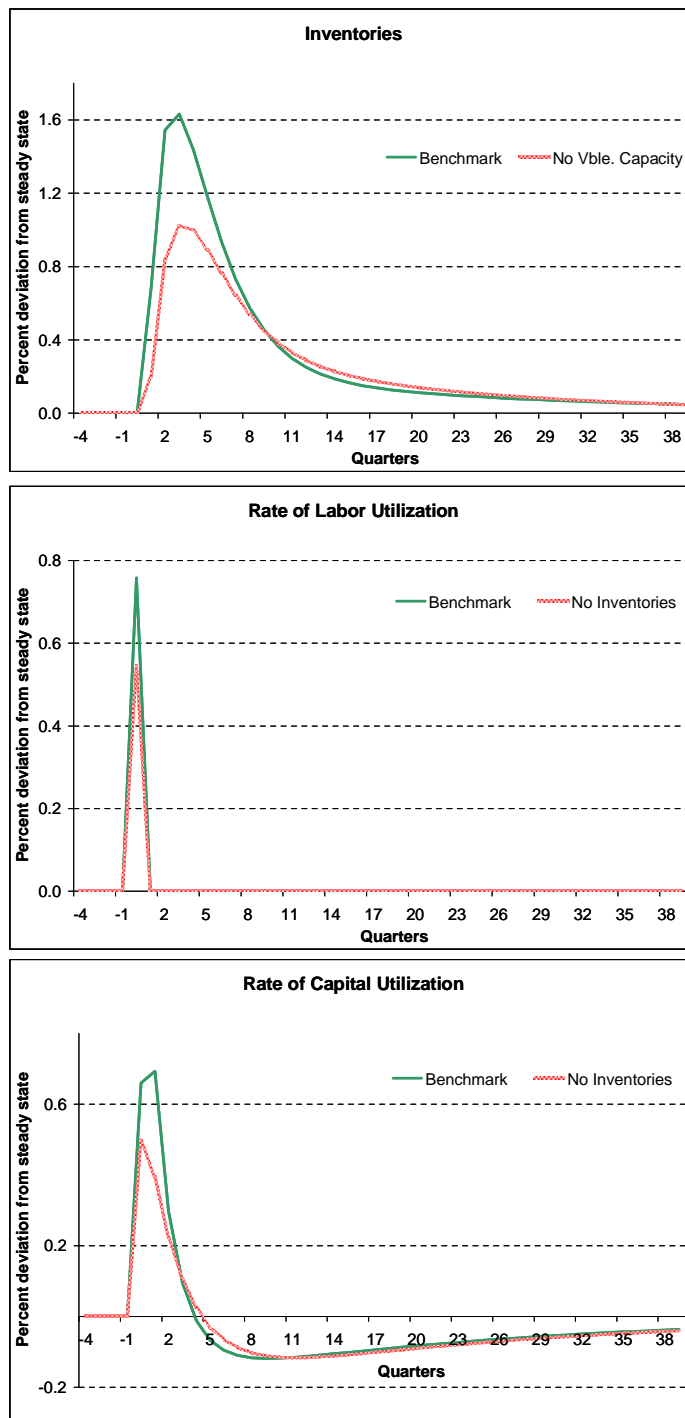


Fig. 15. Effects of Inventories and of Capacity Utilization. Low-Persistence Technology Shock.

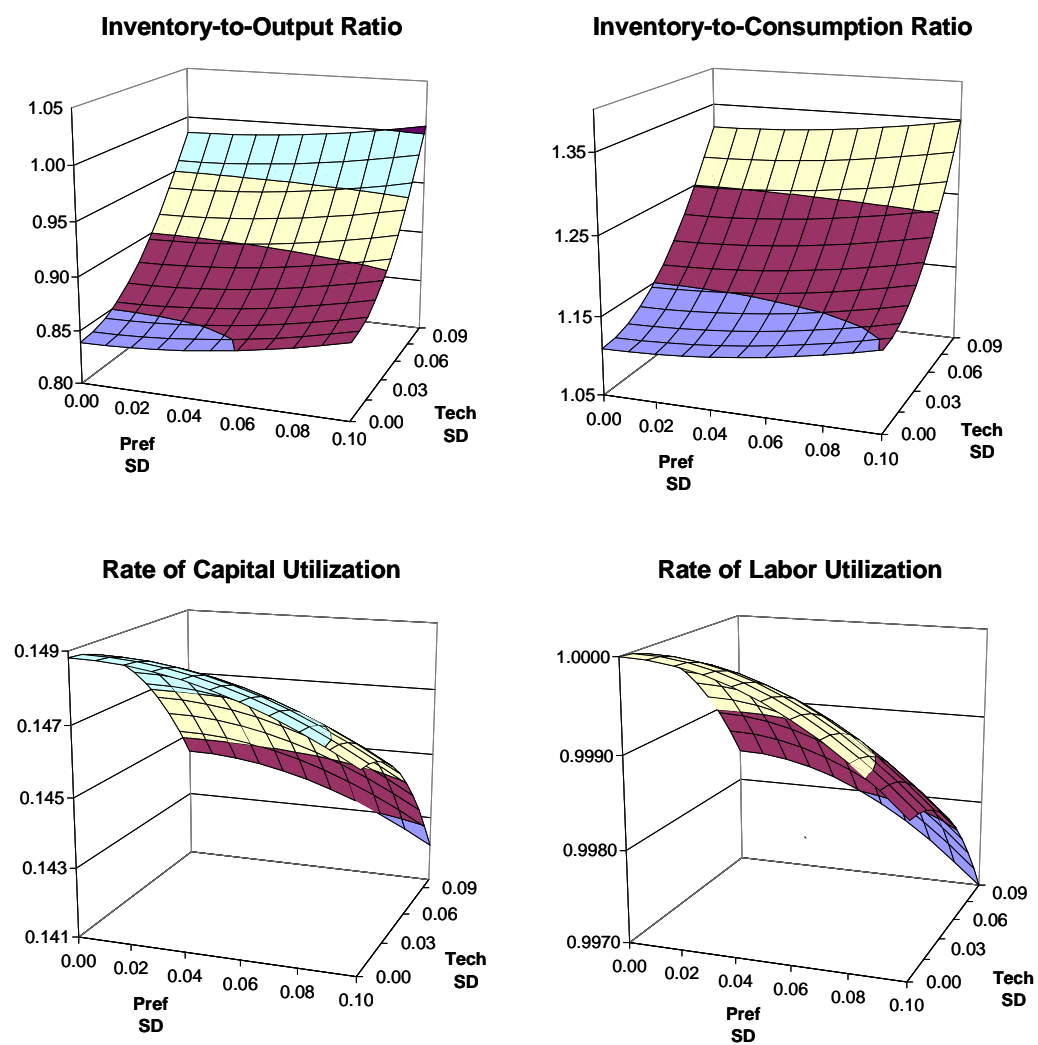


Fig. 16. Stochastic Steady State Values.

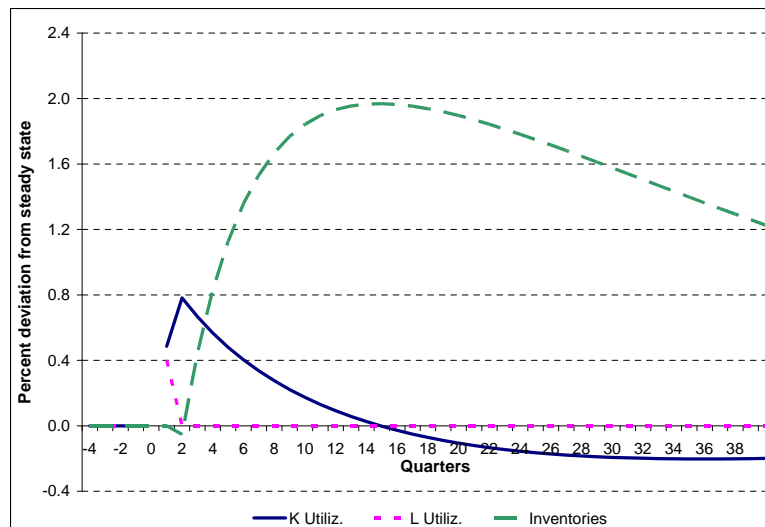


Fig. 17. Separable-Utility Specification. Inventories, and Labor and Capital Utilization. High-Persistence Technology Shock.

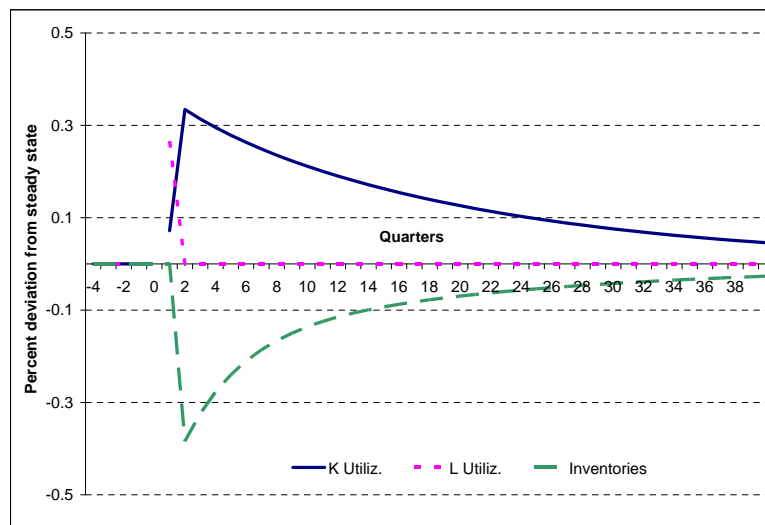


Fig. 18. Separable-Utility Specification. Inventories, and Labor and Capital Utilization. High-Persistence Preference Shock.

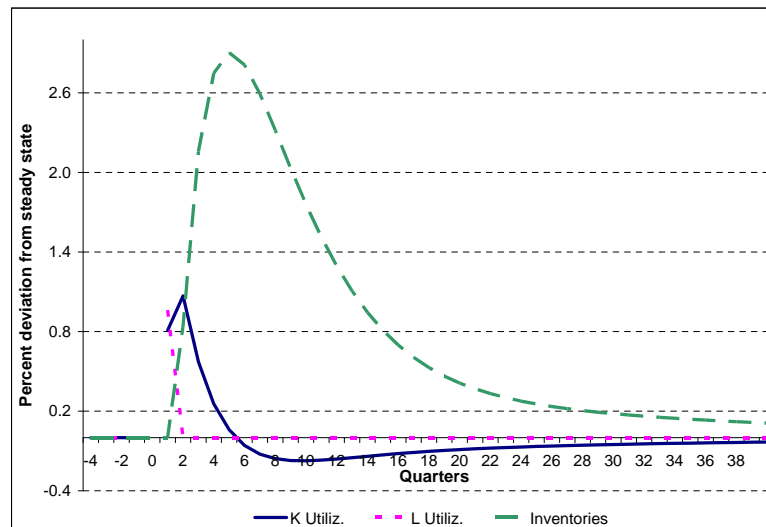


Fig. 19. Separable-Utility Specification. Inventories, and Labor and Capital Utilization. Low-Persistence Technology Shock.

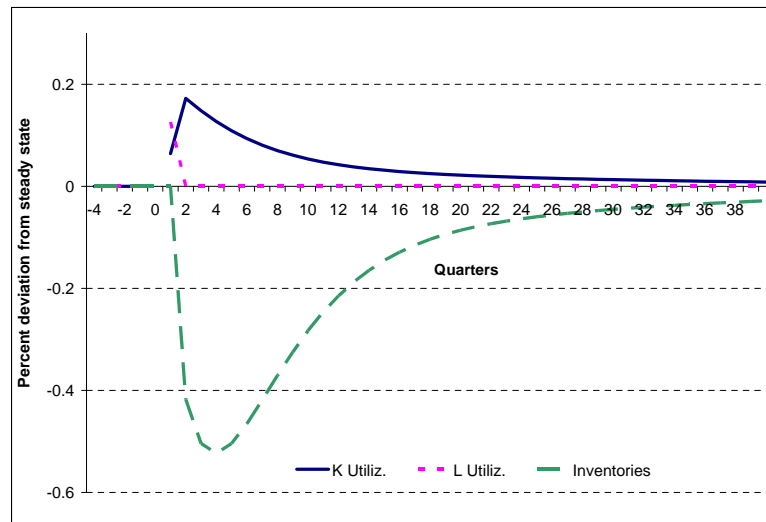


Fig. 20. Separable-Utility Specification. Inventories, and Labor and Capital Utilization. Low-Persistence Preference Shock.

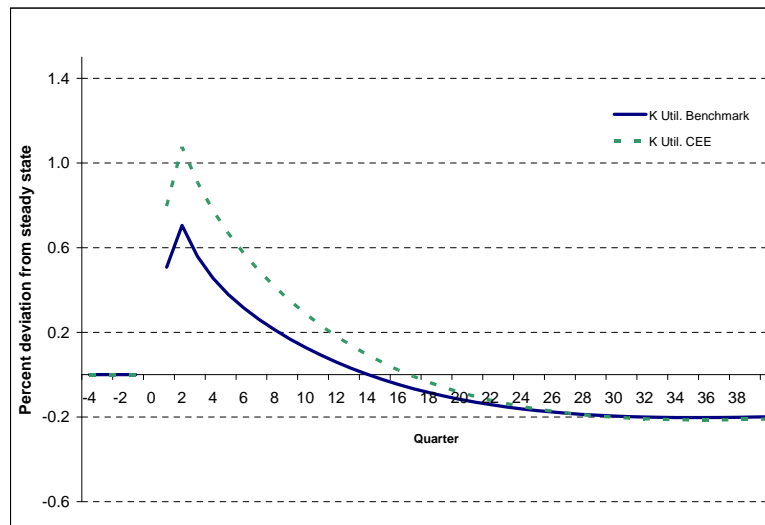


Fig. 21. Effect on Rate of Capital Intensity: CEE Vs. Benchmark. High-Persistence Technology Shock.

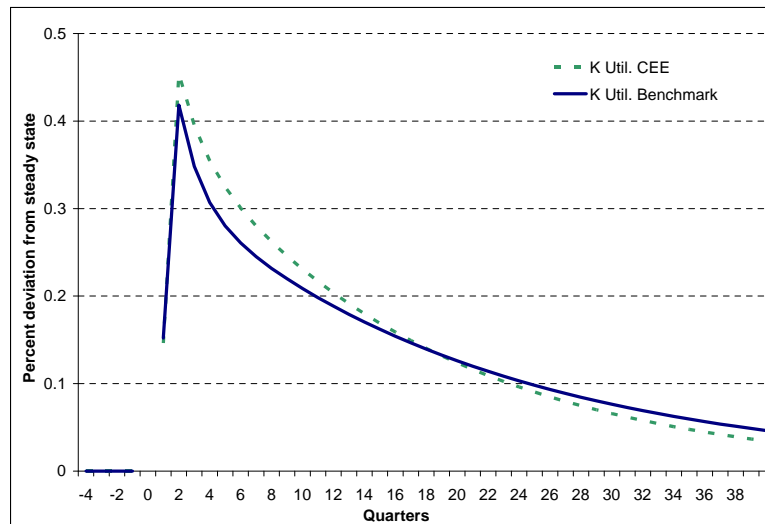


Fig. 22. Effect on Rate of Capital Intensity: CEE Vs. Benchmark. High-Persistence Preference Shock.

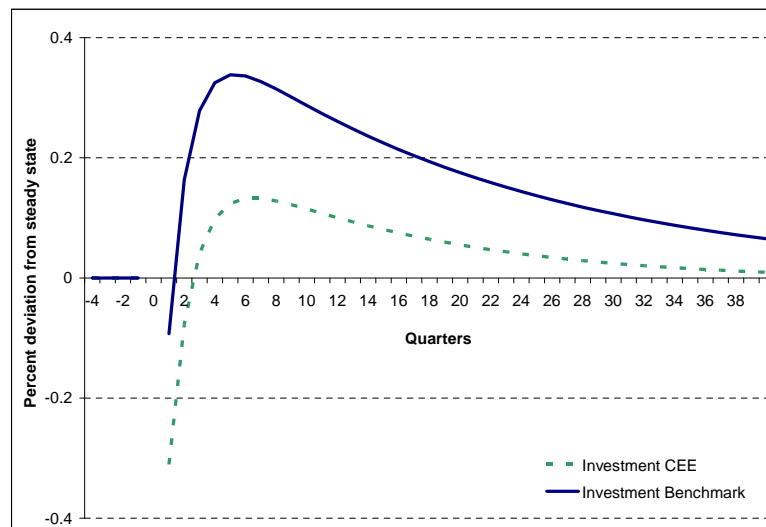


Fig. 23. Effect on Investment: CEE Vs. Benchmark. High-Persistence Preference Shock.

Table III. Correlation among Inventories, Capital Utilization, and Labor Utilization:
Benchmark Specification.

	Preference shock			Technology shock		
	Non-persistent Shock					
	Inventories	Rate Cap. Utiliz.	Rate Labor Utiliz.	Inventories	Rate Cap. Utiliz.	Rate Labor Utiliz.
Inventories	1.00	-0.84	-0.87	1.00	0.70	0.87
Rate Cap. Utiliz.		1.00	0.47		1.00	0.96
Rate Labor Utiliz.			1.00			1.00
	Persistent Shock					
Inventories	1.00	0.54	-0.62	1.00	0.49	-0.34
Rate Cap. Utiliz.		1.00	0.01		1.00	0.33
Rate Labor Utiliz.			1.00			1.00

Table IV. Correlation among Inventories, Capital Utilization, and Labor Utilization:
Isolating Effects.

	Preference shock			Technology shock		
	Non-persistent Shock					
	Inventories	Rate Cap. Utiliz.	Rate Labor Utiliz.	Inventories	Rate Cap. Utiliz.	Rate Labor Utiliz.
Inventories	1.00	-0.60	-0.82	1.00	0.70	0.82
Rate Cap. Utiliz.		1.00	0.99		1.00	0.99
Rate Labor Utiliz.			1.00			1.00
	Persistent Shock					
Inventories	1.00	0.46	-0.72	1.00	0.40	-0.36
Rate Cap. Utiliz.		1.00	0.19		1.00	0.41
Rate Labor Utiliz.			1.00			1.00

Note: Each correlation between two variables implies that the third one is absent from the specification.

Table V. Relative Standard Deviations: Sep.-Util. Specification.

	Baseline	No Inventories	No Variable Capacity Util.	No Variable Capital Util.	No Variable Labor Util.
Capital	0.3	0.3	0.3	0.3	0.3
Capital Services	0.8	0.7	-	-	0.9
Capital Utilization	0.8	0.7	-	-	0.8
Consumption	1.3	1.3	1.1	1.1	1.2
Depreciation	1.1	0.9			1.2
Invent.-Sales	2.1	-	2.3	2.1	2.3
Inventories	1.3	-	1.1	0.9	1.4
Investment	3.3	3.5	2.8	2.9	3.2
Labor	1.7	1.3	1.3	1.3	1.7
Labor Services	1.6	1.3	-	1.2	-
Labor Utilization	0.4	0.5	-	0.4	-
Output	2.0	1.7	1.5	1.5	2.1

Table VI. Correlation Matrix: Sep.-Util. Specification.

	Capital	Capital Serv.	Capital Util.	Cons.	Deprec	Invent.	Invent. Invest.	Invent.-Sales	Invest.	Labor	Labor Serv.	Labor Util.	Output
Capital	1.00	0.38	0.06	0.27	0.06	0.84	0.02	0.32	0.20	0.03	0.11	(0.13)	0.22
Capital Serv.		1.00	0.95	0.83	0.95	0.24	0.62	(0.59)	0.85	0.79	0.93	0.00	0.98
Capital Util.			1.00	0.80	1.00	(0.02)	0.65	(0.75)	0.88	0.85	0.94	0.09	0.98
Cons.				1.00	0.80	(0.05)	0.13	(0.78)	0.70	0.95	0.84	0.39	0.81
Deprec					1.00	(0.02)	0.65	(0.75)	0.88	0.85	0.94	0.09	0.98
Invent.						1.00	0.23	0.59	0.14	(0.27)	(0.04)	(0.41)	0.12
Invent. Invest.							1.00	(0.11)	0.46	0.16	0.61	(0.61)	0.62
Invent.-Sales								1.00	(0.65)	(0.93)	(0.70)	(0.59)	(0.68)
Invest.									1.00	0.77	0.69	0.36	0.92
Labor										1.00	0.83	0.50	0.84
Labor Serv.											1.00	(0.06)	0.91
Labor Util.												1.00	0.11
Output													1.00

Table VII. Rate of Capital Utilization: Coefficients of Autocorrelation.

Order	Benchmark	CEE
1	0.787	0.800
2	0.460	0.494
3	0.230	0.262
4	0.064	0.085
5	-0.057	-0.046

VITA

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