DURABLE GOODS, PRICE INDEXES, AND MONETARY POLICY

A Dissertation

by

KYOUNG SOO HAN

Submitted to the Office of Graduate Studies of
Texas A&M University
in partial fulfillment of the requirements for the degree of

DOCTOR OF PHILOSOPHY

August 2008

Major Subject: Economics

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ABSTRACT

Durable Goods, Price Indexes, and Monetary Policy. (August 2008)

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The dissertation studies the relationship among durable goods, price indexes and monetary policy in two sticky-price models with durable goods. One is a one-sector model with only durable goods and the other is a two-sector model with durable and non-durable goods.

In the models with durable goods, the COLI (Cost of Living Index) and the PPI (Producer Price Index) identical to the CPI (Consumer Price Index) measured by the acquisitions approach are distinguished, and the COLI/PPI ratio plays an important rule in monetary policy transmission. The welfare function based on the household utility can be represented by a quadratic function of the quasi-differenced durables-stock gaps and the PPI inflation rates. In the one-sector model, the optimal policy maximizing welfare is to keep the (acquisition) price and the output gap at a constant rate which does not depend on the durability of consumption goods. In the two-sector model with sticky prices, the central bank has only one policy instrument, so it cannot cope with distortions in both sectors. Simulation results show that the PPI is an adequate price index for monetary policy and that a policy of targeting core inflation constructed by putting more weight on prices in the sector producing more durable goods is near optimal.

In memory of my father

TABLE OF CONTENTS

LIST OF TABLES

CHAPTER I

INTRODUCTION

One of the most frequently used assumptions in economics, especially in macroeconomics, is that consumption goods are perishable, non-storable. Using the assumption of perishable goods, macroeconomic researchers have focused mainly on non-durables and service sectors but have paid little attention to the durables sector. This tradition seems to be justifiable since the share of the durable goods sector in the gross domestic production (GDP) is quite smaller than those of the non-durables and service sectors.[1]

However, consumer durables matter in economy and deserve more attention from economic researchers. First of all, in the economy with durables, the relative importance of some sector cannot be properly measured by its fraction of the GDP. Since the consumer derives utility from the stock of durables rather than the flow of the durables, the consumer's preference on durable goods cannot be well represented by the flow variables like the level of production in a specific period. The relative importance of the durable goods sector implied by utility functions is usually greater than when evaluated by the share of its production in the GDP. Moreover, the many empirical results, including Erceg and Levin (2006), and Monacelli (2008), show that the consumption of durable goods is more sensitive to interest rates and explains a large part of business

This dissertation follows the style and format of the Journal of International Economics.

[1] The share of the durable goods sector in the U.S. GDP is around 12.5% in Erceg and Levin (2006).

cycles. Thus, business cycle models, only with consumer non-durables, have limitations to explain the observed economic fluctuations.

Recently some studies have paid attention to the role of durable goods in macroeconomic models. Barsky et al. (2007) study the role of price stickiness in a model with durable and non-durable goods, and argue that sticky price models cannot explain the fact that the consumption of durables responds procyclically during the periods of economic expansion without the price stickiness of durable goods. Campbell and Hercowitz (2005), and Monacelli (2008) examine the role of collateralized debt in a business cycle model and a New Keynesian model with durable goods, respectively. Erceg and Levin (2006) compare the performance of alternative monetary policies in a sticky price model with durable and non-durable goods, and show that a policy targeting an appropriately weighted average of aggregate wage and price inflation performs better than a policy stabilizing final goods price inflation.

Even though these studies have contributed in extending and deepening our understanding about the role of durable goods in business cycles and monetary policy, they have overlooked alternative approaches[2] to measure price indexes in an economy with durable goods. They have, then, neglected the importance of the relation between price indexes and monetary policy. All of the above mentioned studies follow the acquisitions approach to price indexes in that the price index is a weighted average of prices of both durable and non-durable goods; the weights used are the share of each

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[2] According to Diewert (2003), there are the acquisitions, rental equivalence and user cost approaches to treat consumer durables in a consumer price index (CPI). Assuming complete rental markets for durable goods, the rental equivalence approach is equivalent to the user cost approach.

sector in the nominal GDP. Despite of its simplicity, the acquisitions approach is not clearly based on the consumer theory that leads to a cost of living index (COLI). The COLI in a general equilibrium model can be derived from the user cost approach to price indexes. The latter defines the CPI as a weighted average of the price of the non-durable good and the user cost of the durable good.[3]

The main purpose of this dissertation is to examine the relationship among durable goods, price indexes and monetary policy in a sticky price model. Recently many central banks have adopted a policy of inflation targeting. An inflation-targeting central bank announces its inflation target and has the responsibility to keep the growth rate of a specific price index around the target. However, the choice of which price index shall the central bank stabilize is not apparent, since there are many alternative measures to pick from in an economy with durable goods. This dissertation investigates which price index is more appropriate for the undertaking of monetary policy.

In addition, this dissertation exploits the implications of introducing durable goods in a sticky-price model as in Calvo (1983) and Yun (1996). I focus on two ways to introduce durable goods into macroeconomic models. One simple way is to formulate a one-sector model where only one durable consumption good exists; the durable goods can be thought of as a composite good of a variety of durable and non-durable goods. The second way is to build a two-sector model where one sector produces durable goods and the other sector yields non-durable goods. While using a one-sector model is more tractable for examining the aggregate effects of monetary shocks, it provides limited

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[3] The user cost of durable consumption goods is defined as the cost of using the services of the durable good during the period under consideration.

insight on explaining the alternative choices for undertaking monetary policy when central banks face a tradeoff between the durables sector and the non-durables sector. Since the consumption of durable goods responds more sensitively to monetary policy shocks, it is often observed that loose (tight) monetary policy during a recession (boom) causes a significant increase (decrease) in the consumption of durable goods while other sectors are still in recession (boom).

Chapter II examines the implications for optimal monetary policy of introducing consumer durables into a general equilibrium model with sticky prices. It generalizes a standard one-sector sticky-price model by eliminating the restriction that consumption goods are perishable. In the model with durable goods, the COLI and the PPI are naturally distinguished and the COLI/PPI ratio plays an important role in monetary policy transmission.[4] The social welfare can be expressed as a quadratic function of the quasi-differenced durables-stock gap and the PPI inflation rate. Although the introduction of durable goods changes the dynamics of the log-linear equilibrium and the form of the social welfare function, the policy that maximizes social welfare is to keep the price and the output gap at a constant rate which does not depend on the durability of consumption goods. Without cost-push shocks, a policy completely stabilizing inflation is also optimal.

Chapter III studies the implications of consumption goods with heterogeneous durability for the monetary policy that maximizes the social welfare. It formulates a two-sector sticky-price model where two sectors produce consumption goods with

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[4] The CPI measured by the acquisitions approach and the PPI are identical in the dissertation.

heterogeneous durability. Compared to two-sector models with only non-durable goods, the COLI/PPI ratio and the relative user cost play important roles in monetary policy transmission. The social welfare can be represented by a quadratic function of a weighted average of the sectoral quasi-differenced durables-stock gaps and the sectoral PPI inflation rates. The relative weight on the sectoral durables-stock gaps is a decreasing function of its depreciation rate. This implies that the central bank should implement monetary policy focusing more on the sector producing more durable goods. In this two-sector model, monetary policy cannot stabilize both sectors simultaneously since the central bank has only one policy instrument. The chapter shows that it is optimal to stabilize core inflation constructed by giving more weight on prices in the sector producing more durable goods. In addition, the numerical results show that the central bank should target to stabilize an adequately weighted average of the sectoral PPI inflation rates rather than the sectoral COLI inflation rates.

The rest of the dissertation is structured as the following. Chapter II exploits the implications of optimal monetary policy by introducing consumer durables into a one-sector sticky-price model. Chapter III studies a two-sector general equilibrium model with sticky price. Summary and concluding remarks are in Chapter IV.

CHAPTER II

IMPLICATIONS OF DURABLE GOODS

FOR OPTIMAL MONETARY POLICY

The objective of this chapter is to examine the implications for optimal monetary policy of introducing consumer durables into a general equilibrium model with sticky prices. Section 1 generalizes a standard one-sector New Keynesian model by eliminating the assumption that consumption goods are only perishable. Thus consumption goods are durable and accumulated, and households derive utility from services provided by the stock of durable goods. In the model, the COLI and the PPI are distinguished and the COLI/PPI ratio plays an important role.

In section 2, the log-linear approximation to the equilibrium is presented. Assuming price stickiness as in Calvo (1983), I derive a variant of the New-Keynesian Phillips Curve, in which inflation depends on the real interest rate gap as well as the output gap.

Section 3 shows that taking a second-order approximation to the discounted sum of period utilities, the central bank's period-loss can be expressed as a quadratic function of the durables-stock gap and the inflation rate measured by the acquisitions approach.

Even though the introduction of durable goods changes the dynamics of log-linear equilibrium and the form of the period-loss function, the optimal policy is to keep the price/output gap ratio constant as in the standard model.

Finally, section 4 evaluates the performance of several targeting rules and instrument rules. The results show that the central bank should target to stabilize the inflation rates measured by the acquisitions approach instead of the COLI inflation rates and that inflation targeting rules yield better outcomes rules stabilizing the output gaps in terms of the social welfare.

1. THE BASELINE MODEL

There are a large number of identical and infinitely-lived households. The representative household derives utility from services provided by the stock of a single final durable good while he derives disutility from supplying labor. The final durable

good is a composite of differentiated intermediate goods. Intermediate goods are perishable, and the production of intermediate goods requires only labor as an input. Stylistically, this set-up can be production of a durable good, housing, produced by labor (and perhaps a free material product).

Money can be introduced via a money-in-utility function approach. However, since the central bank will determine the interest rate via a monetary policy rule, the central bank will always supply enough money to meet money demand at any nominal interest rate. Thus, if utility is additively separable in real money balances,[5] and if the utility of real balances is small enough, the effect of money balances can be ignored. Thus the model can be considered cashless.

1.1. The Representative Household

The objective of the representative household is to maximize a lifetime utility function given by

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[u(K_t) - \int_0^1 v(H_t(i)) di \right] \tag{2.1}$$

Here E is the expectation operator, $0 < \beta < 1$ is the discount factor, K_t is the stock of the final durable good, and $H_t(i)$ is the quantity of labor of type i supplied to intermediate producer i in period t.

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[5] If utility is separable in money balances, the money balances have no effect for the rest of the model.

In each period, the household purchases D_t units of the final durable good at price P_t. The stock of the final durable good evolves according to the following law of accumulation

$$K_t = (1-\delta)K_{t-1} + D_t \qquad (2.2)$$

where $0 < \delta \leq 1$ denotes the depreciation rate of the final good. In the limiting case of $\delta = 1$, the model corresponds to the traditional one with a non-durable consumption good.

The household's budget constraint in period t is

$$B_t \leq R_{t-1}B_{t-1} - P_tD_t + \int_0^1 W_t(i)H_t(i)di + \int_0^1 \Pi_t(i)di - T_t \qquad (2.3)$$

Here B_t is the nominal value of (net) bond holdings (whether privately issued or claims to the government) which is the numeraire in the economy, R_t is the nominal interest rate paid on a riskless bond held at the end of period t, $W_t(i)$ is the nominal wage paid to labor of type i, $\Pi_t(i)$ is the nominal profit from sales of intermediate good i, and T_t represents net lump-sum tax collections by the government.

Combining with (2.2), the budget constraint (2.3) can be rewritten as

$$B_t \leq R_{t-1}B_{t-1} - P_t(K_t - (1-\delta)K_{t-1}) + \int_0^1 W_t(i)H_t(i)di + \int_0^1 \Pi_t(i)di - T_t \qquad (2.4)$$

Solving the household's utility maximization problem gives the Euler equation and the labor supply equation:

$$\frac{u_k(K_{t+1})}{u_k(K_t)} = \frac{1}{\beta R_t}\frac{Q_{t+1}}{Q_t} \qquad (2.5)$$

$$\frac{v_h(H_t(i))}{u_k(K_t)} = \frac{W_t(i)}{Q_t} \tag{2.6}$$

for $i \in [0,1]$, where u_k is the marginal utility of the durables-stock, v_h is the marginal disutility of working, and the user cost of consumer durable is defined as

$$Q_t \equiv P_t - E_t \frac{(1-\delta)P_{t+1}}{R_t} \tag{2.7}$$

where the second term in the RHS represents the discounted resale value of consumer durables. Equations (2.5) and (2.6) are similar to the first-order conditions for the household's maximization problem in a standard consumer problem *except* the user cost takes the place of the durable good price, which implies the user cost approach is appropriate when we have durable goods in the CPI. The user cost can be interpreted as the (true) cost of living index (COLI) [6].

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[6] Hereafter the consumer price index measured by the user cost approach is called the 'COLI'.

1.2. Firms and Price Setting

1.2.1. Final Good Producer

The final durable good producer is assumed to behave competitively. The final good firm uses a continuum of intermediate goods $Y_t(i)$ to produce output Y_t, according to the following constant elasticity substitution (CES) technology

$$Y_t \equiv \left[\int_0^1 Y_t(i)^{\frac{\theta-1}{\theta}} di \right]^{\frac{\theta}{\theta-1}}, \tag{2.8}$$

where $\theta > 1$ the elasticity of substitution across differentiated intermediate goods.

Solving the final good producer's profit-maximization problem, taking the final good price P_t and intermediate goods prices $P_t(i)$ as given, yields demand functions for the typical intermediate good i:

$$Y_t(i) = \left(P_t(i) / P_t \right)^{-\theta} Y_t, \tag{2.9}$$

The aggregate price consistent with the final good producer earning zero profits is given by

$$P_t = \left[\int_0^1 P_t(i)^{1-\theta} di \right]^{\frac{1}{1-\theta}} \tag{2.10}$$

The aggregate price of intermediate goods is called the PPI, which here is identical to a CPI measured by the acquisitions approach.[7]

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[7] While the COLI and the PPI is not distinguished in a standard model, the COLI and the PPI need to be distinguished in this model with durables.

1.2.2. Intermediate Good Producers

Intermediate goods producers behave as monopolistic competitors. Each of the differentiated intermediate goods uses a specialized labor input in its production. The technology for producing intermediate goods type i is

$$Y_t(i) = A_t H_t(i) \tag{2.11}$$

Note that by assumption the time-varying exogenous technology factor $A_t > 0$ is identical across intermediate suppliers.

The total cost of supplying a quantity $Y_t(i)$ of good i is then given by

$$W_t(i) Y_t(i) / A_t \tag{2.12}$$

Differentiating this with respect to $Y_t(i)$ and combining the labor supply equation (2.6), the nominal marginal cost (NMC) of supplying good i is equal to

$$NMC_t(i) = \frac{v_h(Y_t(i) / A_t)}{u_k(K_t)} \frac{Q_t}{A_t} \tag{2.13}$$

The real marginal cost of providing good i is then defined as

$$MC_t(i) \equiv \frac{MC_t(i)}{P_t} = \frac{v_h(Y_t(i) / A_t)}{u_k(K_t)} \frac{Q_t}{A_t P_t} \tag{2.14}$$

In a standard model, the COLI/PPI ratio has no effect on the real marginal cost since the COLI is identical to the PPI. In contrast, in this model with durables, the COLI and the PPI are distinguished, and thus the real wages, W_t / Q_t and W_t / P_t evaluated by the household and producers, respectively, will generally have different values. An increase in the COLI/PPI ratio raises the real marginal cost through increases in the real wage since it generates excess demand in labor markets by lowering the real wage as

evaluated by the household or by increasing the real wage as evaluated by the intermediate good suppliers (or both). In addition, log-linearization shows that the COLI/PPI ratio is an increasing function of the real interest rates measured by the acquisitions approach.

Real effects of monetary policy to the model are introduced by assuming prices of the intermediate goods are sticky in a form given by Calvo (1983) and Yun (1996). Each supplier can choose its new price with probability $0 < 1 - \gamma < 1$ in a given period, while it keeps its old price fixed with probability γ.

Taking the price index and the demand function as given, a monopolistically competitive supplier that changes its price in period t chooses its new price $P_t(i)$ to maximize the expected present value of its profit

$$E_t \sum_{k=0}^{\infty} \gamma^k \lambda_{t,t+k} \left[(1+\tau)P_t(i) - MC_{t,t+k}(i) \right] Y_{t,t+k}(i) \tag{2.15}$$

where $\lambda_{t,t+k} = \beta^k (u_k(K_{t+k})/Q_{t+k})/(u_k(K_t)/Q_t)$, τ is a subsidy to intermediate good producers, $Y_{t,t+k}(i) = Y_{t+k} \left(P_t(i)/P_{t+k} \right)^{-\theta}$, and $NMC_{t,t+k}(i) = \dfrac{v_h(Y_{t,t+k}(i)/A_{t+k})}{u_k(K_{t+k})} \dfrac{Q_{t+k}}{A_{t+k}}$.

The first order condition for the optimal choice of P_t^* is given by

$$E_t \sum_{k=0}^{\infty} \gamma^k \lambda_{t,t+k} \left[(1+\tau)P_t^* - \mu NMC_{t+k}^* \right] Y_{t,t+k}^* = 0 \tag{2.16}$$

where $\mu \equiv \theta/(\theta-1)$ denotes the markup, $NMC^*_{t+k} = \dfrac{v_h(Y^*_{t,t+k}/A_{t+k})}{u_k(K_{t+k})} \dfrac{Q_{t+k}}{A_{t+k}}$, and

$Y^*_{t,t+k} = Y_{t+k}\left(P^*_t/P_{t+k}\right)^{-\theta}$. This implies that firms set their price equal to a discounted sum of expected nominal marginal cost.

It is notable that under the flexible-price regime, i.e., $\gamma = 0$, an intermediate good supplier will set its price equal to a proportional markup over nominal marginal cost:

$$P_t = \frac{\mu}{1+\tau} MC_t \tag{2.17}$$

As a fraction γ of prices are fixed, the price index evolves according to

$$P_t = \left[(1-\gamma)P_t^{*1-\theta} + \gamma P_{t-1}^{1-\theta}\right]^{\frac{1}{1-\theta}} \tag{2.18}$$

1.3. Fiscal and Monetary Policy

The fiscal authority chooses a subsidy rate τ that maximizes the utility of the domestic household in a zero inflation steady state. Meanwhile, the central bank determines the short-term nominal interest rate by a time-invariant commitment rule. The monetary authority (the government and the central bank) supplies bonds in amounts enough to meet bond demand at the target nominal-interest rate.

The budget constraint of the monetary authority in period t is given by

$$B_t - R_{t-1}B_{t-1} + T_t = \tau \int_0^1 P_t(i)Y_t(i)di \tag{2.19}$$

1.4. Market Clearing Conditions

The market clearing conditions (labor, intermediate goods, final good, and bond markets) in the model economy are given by

$$H_t = H_t^d = H_t^s$$

$$Y_t(i) = Y_t^d(i) = Y_t^s(i)$$

$$D_t = Y_t$$

$$B_t = B_t^d = B_t^s$$

Here the superscripts d and s implies demand and supply, respectively.

Walras' law holds in the model economy. Suppose labor and intermediate goods markets are in equilibrium. Then combining the household's budget constraint and the profit functions of intermediate suppliers yields the bond demand function in the form[8]

$$B_t^d = R_{t-1}B_{t-1} - T_t + \tau \int_0^1 P_t(i)Y_t(i)di + \int_0^1 P_t(i)Y_t(i)di - P_tD_t \qquad (2.20)$$

Rearranging the government budget constraint, one has the bond supply function

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[8] The profit function of intermediate supplier of type i is given by

$$\Pi_t(i) = (1+\tau)P_t(i)Y_t(i) - W_t(i)H_t(i)$$

given by

$$B_t^s = R_{t-1}B_{t-1} - T_t + \tau \int_0^1 P_t(i)Y_t(i)di \qquad (2.21)$$

Combining the bond market clearing condition ($B_t^d = B_t^s$), (2.20) and (2.21)

gives

$$\int_0^1 P_t(i)Y_t(i)di = P_tD_t \qquad (2.22)$$

From the zero profit condition of the final good producer, (2.22) can be rewritten

as

$$Y_t = D_t \qquad (2.23)$$

Equation (2.23) is the final good market clearing condition. Thus Walras' law holds in

the model.

2. EQUILIBRIUM

For a real variable X_t , we define $\bar{x}_t = \log(X_t^n / X)$, $x_t = \log(X_t / X)$, and

$\hat{x}_t = \log(X_t / X_t^n)$, where X and X_t^n are the steady state value and the natural level of

X_t , respectively.

2.1. Steady-State Equilibrium

If there is no shock, the model has a unique stationary equilibrium (with no inflation) described by

$$R = 1/\beta \tag{2.24}$$

$$D = Y \tag{2.25}$$

$$H = Y \tag{2.26}$$

$$Y = \delta K \tag{2.27}$$

$$\frac{v_h}{u_k} = (1 - \Phi)\frac{1}{q} \tag{2.28}$$

where $1 - \Phi \equiv (1 + \tau)/\mu$, and $q \equiv Q/P = 1 - \beta(1 - \delta)$.

2.2. Flexible-Price Equilibrium

When prices are flexible, monetary policy is neutral, and thus real variables are affected only by real disturbances (called productivity shocks in our model). In flexible-price equilibrium, quantities and prices of each differentiated good are equal:

$$Y_t(i) = Y_t, \text{ and } P_t(i) = P_t$$

The flexible price equilibrium can be represented by the following linearized system:

$$\sigma^{-1}\overline{k}_t + \omega\overline{y}_t + \frac{1-q}{q}\overline{rr}_t^A - (1+\omega)a_t = 0 \qquad (2.29)$$

$$\overline{k}_t = E_t\overline{k}_{t+1} - \sigma\overline{rr}_t^C \qquad (2.30)$$

$$\overline{y}_t = \frac{1}{\delta}\left(\overline{k}_t - (1-\delta)\overline{k}_{t-1}\right) \qquad (2.31)$$

$$\overline{rr}_t^C = \frac{1}{q}\left(\overline{rr}_t^A - (1-q)E_t\overline{rr}_{t+1}^A\right) \qquad (2.32)$$

Here $\sigma \equiv -u_k/(Ku_{kk})$ is the intertemporal elasticity of substitution of private expenditure, and $\omega \equiv Hv_{hh}/v_h$ is the elasticity of marginal disutility of work. I denote with $\overline{rr}_t^A$ and $\overline{rr}_t^C$ the natural rates of interest measured by the acquisitions approach and by the user cost approach, respectively.[9] (2.29) is the price-setting equation under the flexible-price regime, (2.30) is the Euler equation, (2.31) is the law of accumulation for durable goods, and (2.32) is the relationship between real interest rates by the acquisitions approach and the user cost approach. If the final good is not storable (e.g., non-durable goods or services) so that $\delta = q = 1$, then (2.29) is reduced to $\overline{k}_t = \overline{y}_t = \frac{1+\omega}{\sigma^{-1}+\omega}a_t$, which implies the natural rate of consumption of non-durables

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[9] In a model with durables, $\overline{rr}_t^C$ is corresponding to what Woodford (2000) has called *Wicksellian real interest rate*.

depends on only current technology shocks. In contrast, the natural rate of durables depends on both the present and past technology shocks.

Assuming the technology shocks $a_{j,t}$ are stationary, the natural rate of the durable good stock evolves according to the following process (derived in appendix A)

$$\eta_1 \bar{k}_t - \eta_2 \left(E_t \bar{k}_{t+1} + \bar{k}_{t-1} \right) = (1+\omega)\left(a_t - (1-q)E_t a_{t+1} \right) \tag{2.33}$$

or, equivalently,

$$\frac{\eta_2}{\upsilon} \left\{ \left(\bar{k}_t - \upsilon \bar{k}_{t-1} \right) + \beta \upsilon \left(E_t \bar{k}_{t+1} - \upsilon \bar{k}_t \right) \right\} = (1+\omega)\left(a_t - (1-q)E_t a_{t+1} \right) \tag{2.34}$$

where $\eta_1 = \left[q\sigma^{-1} + \left(1 + \beta(1-\delta)^2 \right)(\omega/\delta) \right]$, $\eta_2 = (1-\delta)(\omega/\delta)$, and $0 < \upsilon < 1 - \delta$ is the smaller root of the quadratic equation

$$\eta_1 \upsilon = \eta_2 (\beta \upsilon^2 + 1)$$

For example, suppose a_t follows an AR(1) process of the form

$$a_t = \rho_a a_{t-1} + \varepsilon_{a,t} \tag{2.35}$$

Here the error term $\varepsilon_{a,t}$ is identically and independently distributed. Then the natural rate of the durables-stock evolves according to the following AR(2) process

$$\bar{k}_t = \upsilon \bar{k}_{t-1} + \bar{a}_t \tag{2.36}$$

where $\bar{a}_t = (1+\omega)\zeta^{-1} a_t$, and $\zeta = \dfrac{\eta_1 - \beta \eta_2 (\upsilon + \rho)}{1 - \beta\rho(1-\delta)}$.

2.3. Sticky-Price Equilibrium

The log-linearization of the Euler equation and of the law of accumulation of durable goods around the zero-inflation steady-state, respectively, are given by

$$k_t = E_t k_{t+1} - \sigma\left(r_t - E_t \pi_{t+1}^C\right) \tag{2.37}$$

$$y_t = \frac{1}{\delta}\left(k_t - (1-\delta)k_{t-1}\right) \tag{2.38}$$

Here $\pi_t^C \equiv \log\left(Q_t / Q_{t-1}\right)$ is the COLI inflation rate, and the real interest rate measured by the user cost can be rewritten as

$$rr_t^C \equiv r_t - E_t \pi_{t+1}^C = \frac{1}{q} E_t\left(rr_t^A - (1-q)rr_{t+1}^A\right) \tag{2.39}$$

where $rr_t^A \equiv r_t - E_t \pi_{t+1}^A$ is the real interest rate measured by the acquisitions approach, and $\pi_t^A \equiv \log\left(P_t / P_{t-1}\right)$ is the PPI inflation rate.

Combining (2.30) and (2.37) with definitions of real interest rates, one obtains

$$\hat{k}_t = E_t \hat{k}_{t+1} - \sigma\left(\frac{1}{q}(rr_t^A - \overline{rr}_t^A) - \frac{1-q}{q}E_t(rr_{t+1}^A - \overline{rr}_{t+1}^A)\right) \tag{2.40}$$

Log-linearizing to (2.15) and (2.18) yields a variant of the New Keynesian

Phillips Curve given by (derived in appendix A)

$$\pi_t^A = \beta E_t \pi_{t+1}^A + \varphi \left(\sigma^{-1} \hat{k}_t + \omega \hat{y}_t + \frac{1-q}{q}(rr_t^A - \overline{rr}_t^A) \right) \qquad (2.41)$$

where $\varphi \equiv \dfrac{(1-\gamma)(1-\gamma\beta)}{\gamma(1+\omega\theta)}$. The supply curve shows inflation in the economy with

durables depends positively on real interest rates and log-deviation of the durables-stock

from its natural level as well as positively on the output gap. Current and future real

interest rates have a positive effect on PPI inflation through the COLI/PPI ratio. Under

the assumption of perishable good, i.e., $\delta = 1$, (2.41) reduces to a standard New-

Keynesian Phillips curve

$$\pi_t^A = \beta E_t \pi_{t+1}^A + \varphi(\sigma^{-1} + \omega)\hat{y}_t$$

Using (2.31), (2.38), and (2.40) to substitute for the output gap and interest rate

terms in (2.41), the supply equation can be rewritten equivalently in the form

$$\pi_t^A = \beta E_t \pi_{t+1}^A + (1-q)\left(E_t \pi_{t+1}^A - \beta E_t \pi_{t+2}^A \right) + \chi_k^S \left[(\hat{k}_t - \upsilon \hat{k}_{t-1}) - \beta\upsilon(\hat{k}_{t+1} - \upsilon \hat{k}_t) \right] \quad (2.42)$$

where $\chi_k^S \equiv \varphi\eta_2 / \upsilon$, and υ and η_2 are defined in (2.34). Equations (2.40), (2.42), and

the monetary policy rule choosing r_t determine the equilibrium path of $\hat{k}_t$, π_t^A, and r_t.

3. OPTIMAL MONETARY POLICY

3.1. The Objective of Monetary Policy

The optimal commitment monetary policy rule will maximize the representative household's expected lifetime utility given by

$$W_0 \equiv U_0 - V_0, \tag{2.43}$$

where $U_0 \equiv E_0 \sum_{t=0}^{\infty} \beta^t u(K_t)$, and $V_0 \equiv E_0 \sum_{t=0}^{\infty} \beta^t \int_0^1 v(H_t(i)) di$.

Following Rotemberg and Woodford (1998), I take a second-order Taylor approximation of the welfare function (2.43) around the steady state values with zero inflation. A second-order approximation of the discounted sum of utility around the steady state values is given by (derived in appendix A)

$$W_0 = -\frac{\delta}{q}\frac{\theta}{\varphi} K u_k \sum_{t=0}^{\infty} \beta^t L_t + t.i.p + O\left(\|a\|^3\right) \tag{2.44}$$

Here the term *t.i.p.* denotes terms that are independent of monetary policy, $O\left(\|a\|^3\right)$ denotes the terms that are of third or higher order in the deviations of the various variables from their steady-state values, and the period-loss function is given by

$$L_t = \chi_k^L \left(\hat{k}_t - \upsilon \hat{k}_{t-1}\right)^2 + \left(\pi_t^A\right)^2. \tag{2.45}$$

where $\chi_k^L \equiv \frac{\varphi}{\theta}\frac{\eta_2}{\delta\upsilon}$ is the relative weight on the durables-stock gap variability. Equation

(2.45) reduced to the period-loss function $\frac{\varphi(\sigma^{-1}+\omega)}{\theta}\hat{y}_t^2 + \left(\pi_t^A\right)^2$ in a standard model

when $\delta = 1$. Comparing with a standard model, $\hat{k}_t - \upsilon \hat{k}_{t-1}$ takes the place of the output gap in the period-loss function. Taking into account that $0 < \upsilon < 1 - \delta$, it implies that once the durables-stock deviates from its natural level, it should be adjusted gradually to its natural level. In the extreme case of $\upsilon = 0$, only the current durables-stock matters so that the current level of durables-stock should be always stabilized around its natural level, ignoring inflation variability. If $\upsilon = 1 - \delta$, it is optimal that the central bank stabilizes current output around its natural level as in a standard model.

In addition, it is notable that comparing with habit formation models as in Amato and Laubach (2001), and Giannoni and Woodford (2003), introduction of durable goods has an opposite effect on the period-loss function. Suppose the level of output in the previous period is greater than its natural level. Then it is desirable that the current level of output should also be higher than its natural level in a model with habit formation since the period-loss can be written as

$$L_t^h = \chi_y^h \left(\hat{y}_t - \upsilon^h \hat{y}_{t-1} \right)^2 + \left(\pi_t^A \right)^2$$

where $0 < \upsilon^h < 1$. However, if the previous output gap is greater than zero, the current output should be lower than its natural level in a model with durables because the period-loss can be rewritten as

$$L_t = \chi_k^L \left(\delta \hat{y}_t + \delta \left((1-\delta) - \upsilon \right) \sum_{k=0}^{\infty} (1-\delta)^k \, \hat{y}_{t-k-1} \right)^2 + \left(\pi_t^A \right)^2$$

Note that $0 < (1-\delta) - \upsilon < 1$.

3.2. Optimal Monetary Policy

An optimal monetary policy seeks to minimize the discounted sum of losses (2.45) subject to the sticky-price equilibrium given by equations (2.40) and (2.41). The Lagrangian for this problem is given by

$$E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \begin{array}{l} \chi_k^L \left(\hat{k}_t - \upsilon \hat{k}_{t-1} \right)^2 + \left(\pi_t^A \right)^2 + 2\phi_{1,t} \left[\hat{k}_t - E_t \hat{k}_{t+1} + \sigma \left(rr_t^C - \overline{rr}_t^C \right) \right] \\ +2\phi_{2,t} \begin{bmatrix} \pi_t^A - \beta E_t \pi_{t+1}^A - (1-q) \left(E_t \pi_{t+1}^A - \beta E_t \pi_{t+2}^A \right) \\ -\chi_k^S \left((\hat{k}_t - \upsilon \hat{k}_{t-1}) - \beta \upsilon (E_t \hat{k}_{t+1} - \upsilon \hat{k}_t) \right) \end{bmatrix} \end{array} \right\} \tag{2.46}$$

First order necessary conditions with respect to rr_t^C, $\hat{k}_t - \upsilon \hat{k}_{t-1}$, and π_t^A are given by

$$\phi_{1,t} = 0 \tag{2.47}$$

$$\chi_w \left(\hat{k}_t - \upsilon \hat{k}_{t-1} \right) = \chi_\pi \left(\phi_{2,t} - \upsilon \phi_{2,t-1} \right) \tag{2.48}$$

$$\pi_t^A + \phi_{2,t} - (2-\delta)\phi_{2,t-1} + (1-\delta)\phi_{2,t-2} = 0 \tag{2.49}$$

for each $t \geq 0$. Following Woodford (1999), the optimal policy under a timeless perspective is that the central bank implements conditions (2.48) and (2.49) for all periods. Hence, the system of the economy under an optimal policy can be represented by equations (2.42), (2.48), and (2.49) given initial conditions, $\hat{k}_{-1} = 0$ and $\phi_{2,-2} = \phi_{2,-1} = 0$.

Combining (2.48) and (2.49), PPI inflation and the output gap under the optimal policy satisfy

$$(\pi_t^A - \upsilon \pi_{t-1}^A) + \frac{1}{\theta}(1-L)(\hat{y}_t - \upsilon \hat{y}_{t-1}) = 0 \tag{2.50}$$

where *L* is the lag operator.

Note that if there is no uncertainty which generates monetary policy to deviate from the optimal rule, equation (2.50) can reduce to the optimal policy rule in a standard model given by

$$\pi_t^A + \theta^{-1}(1-L)\hat{y}_t = 0 \qquad (2.51)$$

which implies that the optimal policy maximizing the social welfare is to keep the (acquisition) price and the output gap at a constant rate that does not depend on the durability of goods.

Cost-push shocks make the central bank face the conflicting goals of stabilizing both inflation and the output gap simultaneously. Here we have no cost-push shocks, so the strict inflation targeting is also an optimal monetary policy. In the next section, cost-push shocks are introduced into the supply equation (2.41) as in Clarida, Gali, and Gertler (1999) in order to compare a set of targeting rules and instrument rules.

4. NUMERICAL RESULTS

In this section, I compare several targeting rules and instrument rules with the optimal rule derived in the previous section. Section 4.1 calibrates the structural parameters and the shock processes in the model used for numerical analysis. Section 4.2

explains the results of numerical analysis. I use the AIM algorithm to solve rational

expectation models in this section. [10]

4.1. Calibration

The calibrated values are summarized in Table 1. Based on the empirical results

of Rotemberg and Woodford (1999), I calibrate structural parameters and the shock

processes of the model to simulate data at a quarterly frequency.

The discount factor β is set to the conventional value of 0.99 which implies the

annual real interest rate of 4%. I set γ =0.66, so an average lifetime of price contracts is

three quarters. An average markup in goods markets is 15%. Thus θ is set to 7.88.

Following Amato and Laubach (2001), I set ω=0.60 which is consistent with a Frisch

elasticity of 5 and a Cobb-Douglas production technology with a coefficient on labor of

0.75. I assume the period utility is a log function, so σ is set to 1. The depreciation rate

of durable goods is set to 0.40 which implies the annual depreciation rate of 87%,

Table 1
Calibrated parameter values in the baseline model

β	0.99	δ	0.40
γ	0.66	ρ_a	0.900
θ	7.88	sd(a)	0.036
σ	1.00	ρ_μ	0.900
ω	0.60	sd(μ)	0.036

[10] The AIM is Anderson and Moore (1985) implementation of the Blanchard and Kahn (1980) method, modified to take advantage of sparse matrix functions.

corresponding to the share of the non-durables sector in the U.S GDP. Finally, technology and cost-push shocks are assumed to independently follow AR(1) processes with the coefficients (ρ_a and ρ_μ) of 0.90 and the conditional standard deviations (sd(a) and sd(μ)) of 0.036.

4.2. Results

Table 2 reports welfare losses (the last column) and unconditional variances (columns 1-5) of the PPI and COLI inflation rates, the deviations of output and durables-stock from their steady-state levels and the nominal interest rates under the optimal policy and various alternative rules. The welfare losses are measured by

$$E_0[L] = (1-\beta)E_0\sum_{t=0}^{\infty}\beta^t L_t \qquad (2.52)$$

where L_t is the period-loss function defined in (2.45).

The first row shows the performance of the optimal monetary policy given by (2.50). The optimal rule minimizes the unconditional variance of nominal interest rate as well as the welfare loss.

The second and third rows in table 2 show the performance of targeting rules to stabilize PPI inflation and the output gap, respectively. It is evident that the inflation targeting rule approximates closely to the optimal rule while the rule stabilizing output gap results in poor outcomes. This is opposite from the results in Erceg and Levin (2006) in which prices are sticky by the assumption of fixed-duration (Taylor-type) contracts. As Erceg and Levin insist, the weights on relative price and wage dispersion are much

Table 2
Results on loss and unconditional variances with alternative rules in the baseline model

	$V_{\pi A}$	$V_{\pi C}$	V_y	V_k	V_r	$E_0[L]$
Optimal Rule	0.0023	0.0173	2.1100	1.8873	0.0139	0.0139
Inflation Targeting	0.0000	0.0163	3.6444	2.8003	0.0175	0.0188
Output Gap Targeting	0.5741	0.5742	0.0090	0.0069	0.4651	0.5451
Taylor Rule w/π_A	0.4906	0.4996	0.6334	0.4981	0.4357	0.4692
Taylor Rule w/π_C	0.4879	0.5526	0.9740	0.5635	0.4412	0.4675

smaller under fixed-duration contracts than in random-duration (Calvo-type) contracts. Hence, the bad performance of the aggregate inflation targeting rule in Erceg and Levin may be explained by the relatively low weight on variability of the output gap in the welfare function, not by the introduction of durable goods.

Also the table 2 implies the central bank should target to stabilize the PPI inflation rates rather than the COLI inflation rates. First of all, it often happens that the COLI inflation targeting rule makes the model unstable. Hence, a policy to stabilize COLI inflation may add additional instability to the economy. Second, the fourth and the fifth rows show that instrument rules responding the PPI inflation rates (row 4) and the COLI inflation rates (row 5), respectively, have similar performance. Despite these results, the central bank may want to target PPI inflation because the COLI cannot be computed accurately due to uncertainty regarding the depreciation rate for durable goods.[11]

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[11] Orphanides (1998) insists that it results in poor performance to determine the stance of monetary policy based on the variables that the central bank cannot measure with certainty.

CHAPTER III

DURABLE GOODS, PRICE INDEXES,

AND MONETARY POLICY

Since the consumption of durable goods responds sensitively to monetary policy shocks, it is often observed that tight (loose) monetary policy in boom (recession) causes severe decrease (increase) in demand for consumer durables. However, one-sector price-sticky models, which are prevalent in monetary policy analysis, have a limited ability to deal with durable goods or to take account of the fact that the effects of monetary policy on real variables depend on the durability of consumption goods.

This chapter examines implications of consumption goods with different durability for the social welfare maximizing optimal monetary policy. In section 1, it formulates a two-sector general equilibrium model. In the model economy, households obtain utility provided by two types of consumer durables. To introduce the effects of monetary policy on real variables, I assume price-stickiness as in Calvo (1983).

Section 2 explains the equilibrium of the model. In the economy with durable goods, the COLI and the PPI are naturally distinguished.[12] The COLI/PPI ratio is proportional to the real interest rate measured by the acquisitions approach, which plays an important role in the equilibrium dynamics by affecting the real marginal cost.

In section 3, it defines the social welfare by taking a second-order approximation to the discounted sum of utilities. Also it shows that an optimal policy stabilization of each sector cannot be implemented because of insufficient policy instruments and that the second best is to stabilize a properly weighted average of the sectoral inflation rates as in Benigno (2004).

Finally, section 4 evaluates the performance of a set of targeting rules and instrument rules. Also looks for an optimal weight of the durables sector in the economy with both durable and non-durable goods. Results show that the central bank should target to stabilize PPI inflation instead of COLI inflation by focusing more on the durables sector.

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[12] Huang and Liu (2005) distinguish the COLI and the PPI by assuming final consumption goods are produced through two stages of processing.

1. A TWO-SECTOR MODEL

In this economy, there are a large number of identical and infinitely lived households. The representative household derives utility from services provided by two durable goods while he obtains disutility from supplying labors to firms producing intermediate goods. The production of intermediate goods requires labor as the only input. The final durable goods are a composite of differentiated intermediate goods.

1.1. Representative Household

The representative household seeks to maximize a discounted sum of utilities of the form

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[u(K_t) - \int_0^1 v(H_t(i)) di \right] \tag{3.1}$$

Here E is the expectation operator, $0 < \beta < 1$ is the discount factor, $H_t(i)$ is the quantity of labor of type i supplied to intermediate firm i in period t, and the index K_t is a Cobb-Douglas aggregator

$$K_t \equiv \frac{1}{\alpha_1^{\alpha_1} \alpha_2^{\alpha_2}} \left(K_{1,t} \right)^{\alpha_1} \left(K_{2,t} \right)^{\alpha_2} \tag{3.2}$$

where $\alpha_1 + \alpha_2 = 1$, and $K_{j,t}$ for $j = \{1, 2\}$ is the stock of the final durable good produced in sector j which evolves according to the following law of accumulation

$$K_{j,t} = (1 - \delta_j) K_{j,t-1} + D_{j,t} \tag{3.3}$$

for $j = \{1,2\}$, where $D_{j,t}$ is the amount purchased of the final durables produced in sector j, and $0 \le \delta_j < 1$ is the depreciation rate of the final durable good in sector j.

The budget constraint of the representative household in period t is

$$B_t \le R_{t-1}B_{t-1} + \int_0^1 W_t(i)H_t(i)di + \int_0^1 \Pi_t(i)di - P_{1,t}D_{1,t} - P_{2,t}D_{2,t} - T_t \qquad (3.4)$$

Here B_t is the nominal value of bonds, R_t is the nominal interest rate paid on a riskless bond held at the end of period t, $P_{j,t}$ is the price of the final durable good in sector j, $H_t(i)$ is the quantity of labor of type i supplied to intermediate good producer i, $W_t(i)$ is the nominal wage of labor of type i, $\Pi_t(i)$ is the nominal profit from sales of intermediate good i, and T_t represents nominal net lump-sum tax collections by the government.

Combining with (3.3), the budget constraint (3.4) can be rewritten as

$$B_t \le R_{t-1}B_{t-1} - \sum_{j=1}^{2} P_{j,t}(K_{j,t} - (1-\delta_j)K_{j,t-1}) + \int_0^1 W_t(i)H_t(i)di + \int_0^1 \Pi_t(i)di - T_t \qquad (3.5)$$

Solving the household's utility maximization problem gives the Euler equation, the relative demand equation, and the labor supply equation:

$$\frac{u_k(K_t)}{u_k(K_{t+1})} = \beta R_t E_t \frac{Q_t}{Q_{t+1}} \qquad (3.6)$$

$$\frac{\alpha_1}{\alpha_2}\frac{K_{2,t}}{K_{1,t}} = Q_{R,t}\left(\equiv \frac{Q_{1,t}}{Q_{2,t}}\right) \qquad (3.7)$$

$$\frac{v_h(H_t(i))}{u_k(K_t)} = \frac{W_t(i)}{Q_t} \qquad (3.8)$$

for $i \in [0,1]$, where u_k is the marginal utility of a composite of two final durables stocks, v_h is the marginal disutility of working, and the user cost of durable good consumption in sector j $Q_{j,t}$ is defined as

$$Q_{j,t} \equiv P_{j,t} - E_t \left[(1-\delta_j) P_{j,t+1} / R_t \right] \tag{3.9}$$

The cost of living index (COLI) Q_t is defined as [13]

$$Q_t \equiv Q_{1,t}^{\alpha_1} Q_{2,t}^{\alpha_2} \tag{3.10}$$

According to Cobb-Douglas preferences, the demand functions for the stocks of the two durables are given by

$$K_{j,t} = \alpha_j (Q_t / Q_{j,t}) K_t \tag{3.11}$$

1.2. Firms and Price Setting

1.2.1. Final Good Producers

I assume that final durables producers behave competitively. The technology for producing the final goods in sector j from intermediate goods is given by

$$Y_{j,t} \equiv \left[(1/n_j)^{\frac{1}{\theta}} \int_{N_j} Y_t(i)^{\frac{\theta-1}{\theta}} \, di \right]^{\frac{\theta}{\theta-1}} \tag{3.12}$$

for $j = \{1,2\}$, where the intervals of differentiated goods belong to the two sectors are, respectively, $N_1 \equiv [0, n_1]$ and $N_2 \equiv (n_1, 1]$, and $\theta > 1$ is the elasticity of substitution across

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[13] Q_t is also called the CPI measured by the user cost approach in price index theory.

goods produced within a sector. In the aggregator (3.12), n_j is the relative size of each sector and satisfies the identity $n_1 + n_2 = 1$.

Solving the firm's profit-maximization problem yields demand functions for the typical intermediate good i in sector j:

$$Y_t(i) = \frac{1}{n_j} \left(P_t(i) / P_{j,t} \right)^{-\theta} Y_{j,t} \qquad (3.13)$$

The sectoral price index consistent with the final good producer in each sector earning zero profits respectively, is given by

$$P_{j,t} = \left[\frac{1}{n_j} \int_{N_j} P_t(i)^{1-\theta} \, di \right]^{\frac{1}{1-\theta}} \qquad (3.14)$$

The producer price index (PPI) is defined as[14]

$$P_t \equiv P_{1,t}^{n_1} P_{2,t}^{n_2} \qquad (3.15)$$

1.2.2. Intermediate Good Producers

Intermediate goods producers behave as monopolistic competitors. Each of the differentiated intermediate goods uses a specialized labor input in its production. The technology for producing intermediate goods type $i \in N_j$ is

$$Y_t(i) = A_{j,t} H_t(i) \qquad (3.16)$$

Here I assume a time-varying exogenous technology factor $A_{j,t}$ is identical across suppliers within sector j.

The total cost of supplying a quantity $Y_t(i)$ of good i in sector j is given by

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[14] It is identical to the CPI measured by the acquisitions approach.

$$W_t(i)Y_t(i) / A_{j,t} \tag{3.17}$$

Differentiating this with respect to $Y_t(i)$ and combining (3.8) yields the nominal

marginal cost of supplying good i in sector j given by

$$NMC_{j,t}(i) = \frac{v_h(Y_t(i) / A_{j,t})}{u_k(K_t)} \frac{Q_t}{A_{j,t}} \tag{3.18}$$

The real marginal cost of providing good i in sector j is defined as

$$MC_{j,t}(i) \equiv \frac{NMC_{j,t}(i)}{P_t} = \frac{v_h(Y_t(i) / A_{j,t})}{u_k(K_t)} \frac{Q_t}{A_t P_t} \tag{3.19}$$

for $i \in N_j$. Since the COLI/PPI ratio affects the excess demand in labor markets, the real

marginal cost in sector j is an increasing function of the COLI/PPI ratio which depends

positively on real interest rates measure by the acquisitions approach.

To introduce real effects of monetary policy to the model, I assume prices of the

intermediate goods are sticky as in Calvo (1983). Each supplier in sector j can choose its

new price with the probability $0 < 1 - \gamma_j < 1$ in a given period, while it keeps its old price

fixed with the probability γ_j.

Given the price index and the demand function, a monopolistically competitive

supplier i in sector j that changes its price in period t chooses its new price $P_t(i)$ to

maximize the expected present value of its profit

$$E_t \sum_{k=0}^{\infty} \gamma_j^k \lambda_{t,t+k} \left[(1+\tau_j)P_t(i)Y_{t,t+k}(i) - MC_{j,t,t+k}(i) \right] Y_{t,t+k}(i) \tag{3.20}$$

where $\lambda_{t,t+k} = \beta^k (u_k(K_{t+k})/Q_{t+k})/(u_k(K_t)/Q_t)$, τ_j is a subsidy to intermediate good

producers in sector j, $Y_{j,t,t+k}(i) = n_j^{-1}\left(P_t(i)/P_{j,t+k}\right)^{-\theta} Y_{j,t+k}$ for $i \in N_j$, and

$$NMC_{j,t,t+k}(i) = \frac{v_h(Y_{j,t,t+k}(i)/A_{j,t+k})}{u_k(K_{t+k})} \frac{Q_{t+k}}{A_{j,t+k}} \text{ for } i \in N_j.$$

The optimal choice of $P_{j,t}^*$ satisfies the first order condition given by

$$E_t \sum_{k=0}^{\infty} \gamma_j^k \lambda_{t,t+k} \left[(1+\tau_j)P_{j,t}^* - \mu MC_{j,t,t+k}^*\right] Y_{j,t,t+k}^* = 0 \qquad (3.21)$$

where $\mu \equiv \theta/(\theta-1)$ denotes the markup, $MC_{j,t,t+k}^* = \dfrac{v_h(Y_{j,t,t+k}^*/A_{j,t+k})}{u_k(K_{t+k})} \dfrac{Q_{t+k}}{A_{j,t+k}}$, and

$Y_{j,t,t+k}^* = n_j^{-1}\left(P_{j,t}^*/P_{j,t+k}\right)^{-\theta} Y_{j,t+k}$.

Note that if prices are flexible, i.e., $\gamma_j = 0$, a intermediate firm sets its price equal

to a markup proportional to its nominal marginal cost:

$$P_{j,t} = \frac{\mu}{1+\tau_j} MC_{j,t} \qquad (3.22)$$

As a fraction γ_j of prices are fixed, the price index in sector j evolves according to

$$P_{j,t} = \left[(1-\gamma_j)P_{j,t}^{*\,1-\theta} + \gamma_j P_{j,t-1}^{1-\theta} \right]^{\frac{1}{1-\theta}} \tag{3.23}$$

Combining and log-linearizing to equations (3.21) and (3.23) yields the supply equation in sector j.

1.3. Fiscal and Monetary Policy

The government chooses subsidy rates τ_j that maximize the utility of the domestic household in a zero inflation steady state while the central bank determines the short-term nominal interest rate by a time-invariant commitment rule. The monetary authority (the government and the central bank) supplies bonds in amounts enough to meet bond demand at the target nominal-interest rate. The budget constraint of the monetary authority in period t is given by

$$B_t - R_{t-1}B_{t-1} + T_t = \tau_1 \int_0^{n_1} P_t(i)Y_t(i)di + \tau_2 \int_{n_2}^1 P_t(i)Y_t(i)di \tag{3.24}$$

2. EQUILIBRIUM

For a variable X_t, I define $x_t = \log\left(\dfrac{X_t}{X}\right)$, $\bar{x}_t = \log\left(\dfrac{X_t^n}{X}\right)$, and $\hat{x}_t = \log\left(\dfrac{X_t}{X_t^n}\right)$,

where X and X_t^n denote its steady state value and natural level, respectively.

2.1. Steady-State Equilibrium

If there is no shock, the model has a unique stationary equilibrium (with no inflation) described by

$$\beta R = 1 \tag{3.25}$$

$$Q^R \equiv \frac{Q_1}{Q_2} = \frac{\alpha_1}{\alpha_2}\frac{K_2}{K_1} \tag{3.26}$$

$$K \equiv \frac{1}{\alpha_1^{\alpha_1}\alpha_2^{\alpha_2}}\left(K_1\right)^{\alpha_1}\left(K_2\right)^{\alpha_2} \tag{3.27}$$

$$\delta_j K_j = D_j \tag{3.28}$$

$$\frac{v_h(Y(i)/A_j)}{u_{k_j}(K)}\frac{1}{A_j} = (1-\Phi_j)\frac{1}{q_j} \tag{3.29}$$

$$Y_j = D_j \tag{3.30}$$

$$n_j Y(i) = Y_j, \text{ for all } i \in N_j \tag{3.31}$$

$$H(i) = Y(i)/A_j, \text{ for all } i \in N_j \tag{3.32}$$

$$B = 0 \tag{3.33}$$

$$P_1/P_2 = A_2/A_1 \tag{3.34}$$

where $q_j \equiv Q_j/P_j = 1-\beta(1-\delta_j)$, and $1-\Phi_j \equiv (1+\tau_j)/\mu$. Combining with zero inflation, the Euler and the relative demand equations yield (3.25) and (3.26). From the

definition of the Cobb-Douglas index and the law of accumulation, one has (3.27) and

(3.28). The price setting equation under the flexible price regime (3.22) gives (3.29).

From market clearing conditions, I have (3.30)-(3.33). When $\tau_1 = \tau_2 = \tau$, equation (3.34)

holds.

Combining (3.28), (3.26), and $(P_1 Y_1)/(P_2 Y_2) = n_1/n_2$, one has

$$\frac{\alpha_1}{\alpha_2} = \frac{\delta_2}{\delta_1} \frac{q_1}{q_2} \frac{n_1}{n_2} > \frac{n_1}{n_2} \text{ iff } \delta_1 < \delta_2 \tag{3.35}$$

Suppose that the sector 1 produces durable goods while the sector 2 yields non-durable

goods, i.e., $0 < \delta_1 < \delta_2 = 1$. Then $\alpha_1 \left(= (Q_1 K_1)/(QK) \right) > n_1 \left(= (P_1 Y_1)/(PK) \right)$ holds by

(3.35), that is, the relative importance of the durables sector (sector 1) in the CPI (by the

user cost approach) α_1 is always greater than in the PPI (or the CPI by the acquisitions

approach) n_1 as Diewart (2003) insists.

2.2. Log-linearized System

2.2.1. Real Interest Rates and Relative Prices

In the model with durable goods, real interest rates change across economic

agents and commodities and over time. From the definitions of price indexes and real

interest rates, log-linear approximations of the real interest rates and of the relative

prices are given by

$$rr_{j,t}^A \equiv r_t - E_t \pi_{j,t+1}^A, \tag{3.36}$$

$$rr_t^A \equiv r_t - E_t \pi_{t+1}^A = n_1 rr_{1,t}^A + n_2 rr_{2,t}^A, \tag{3.37}$$

$$rr_{j,t}^{C} \equiv r_t - E_t\pi_{j,t+1}^{C} = \frac{1}{q_j}\left(rr_{j,t}^{A} - (1-q_j)E_t rr_{j,t+1}^{A}\right) \tag{3.38}$$

$$rr_t^{C} \equiv r_t - E_t\pi_{t+1}^{C} = \alpha_1 rr_{1,t}^{C} + \alpha_2 rr_{2,t}^{C} \tag{3.39}$$

$$q_t^{R} \equiv \log(Q_1/Q_2) = \frac{1-q_1}{q_1}rr_{1,t}^{A} - \frac{1-q_2}{q_2}rr_{2,t}^{A} + p_t^{R} \tag{3.40}$$

$$p_t^{R} \equiv \log(P_1/P_2) = E_t p_{t+1}^{R} + rr_{1,t}^{A} - rr_{2,t}^{A} \tag{3.41}$$

for $j = 1,2$, where the PPI and COLI inflation rates in sector j are defined, respectively,

as $\pi_{j,t}^{A} \equiv \log(P_{j,t}/P_{j,t-1})$ and $\pi_{j,t}^{C} \equiv \log(Q_{j,t}/Q_{j,t-1})$. The PPI and COLI inflation in the

economy are defined as $\pi_t^{A} = n_1\pi_{1,t}^{A} + n_2\pi_{2,t}^{A}$ and $\pi_t^{C} = \alpha_1\pi_{1,t}^{C} + \alpha_2\pi_{2,t}^{C}$, respectively. In

(3.36) and (3.38), $rr_{j,t}^{A}$ and $rr_{j,t}^{C}$ are the real interest rates in sector j measured by the

acquisitions and the user cost approaches, respectively. In (3.37) and (3.39), rr_t^{A} and

rr_t^{C} are the real interest rates in the economy measured by the acquisitions and the user

cost approaches, respectively. In (3.40) and (3.41), q_t^{R} and p_t^{R} are the relative prices in

the COLI and the PPI, respectively.

2.2.2. Flexible-Price Equilibrium

When prices are fully flexible, monetary policy is neutral, and thus real variables

are affected only by real disturbances (called productivity shocks in this paper). In

flexible-price equilibrium, quantities and prices of differentiated goods are identical

within sector j:

$$n_j Y_t(i) = Y_{j,t}, \text{ and } P_t(i) = P_{j,t}$$

Combining the price setting equation (3.22), market-clearing conditions, and the Euler equation, one gets the log-linearized equilibrium in the case of flexible prices as a linear function of the current and past productivity shocks.

$$\sigma^{-1}\overline{k}_t + \omega\overline{y}_{j,t} + \overline{q}_t - \overline{p}_{j,t} - (1+\omega)a_{j,t} = 0 \tag{3.42}$$

$$\overline{k}_t = E_t\overline{k}_{t+1} - \sigma\overline{rr}_t^{C} \tag{3.43}$$

$$\overline{k}_{2,t} - \overline{k}_{1,t} = \overline{q}_t^{R} \tag{3.44}$$

$$\overline{k}_t = \alpha_1\overline{k}_{1,t} + \alpha_2\overline{k}_{2,t} \tag{3.45}$$

$$\overline{y}_{j,t} = \frac{1}{\delta_j}\left(\overline{k}_{j,t} - (1-\delta)\overline{k}_{j,t-1}\right) \tag{3.46}$$

Here $\sigma \equiv -u_k/(Ku_{kk}) > 0$ is the intertemporal elasticity of substitution of aggregates of durable good stocks, $\omega \equiv Yv_{yy}/v_y > 0$ represents the elasticity of the marginal disutility of work, and

$$\overline{q}_t - \overline{p}_{j,t} = \begin{cases} \dfrac{1-q_1}{q_1}\overline{rr}_{1,t}^{A} - \alpha_2\overline{q}_t^{R} & \text{when } j = 1 \\[2ex] \dfrac{1-q_1}{q_1}\overline{rr}_{1,t}^{A} + \alpha_1\overline{q}_t^{R} & \text{when } j = 2 \end{cases}.$$

From the definition of $\overline{rr}_t^{C}$ and equations (3.44)-(3.46), the Euler equation (3.43) can be rewritten as

$$\overline{k}_{j,t} - E_t\overline{k}_{j,t+1} = \eta_{3j}\left(\overline{q}_t^{R} - E_t\overline{q}_{t+1}^{R}\right) - (\sigma/q_j)E_t\left(\overline{rr}_{j,t}^{A} - (1-q_j)E_t\overline{rr}_{j,t+1}^{A}\right) \tag{3.47}$$

where

$$\eta_{3j} = \begin{cases} \alpha_2 q_1(1-\sigma^{-1}) & \text{when } j = 1 \\ -\alpha_1 q_2(1-\sigma^{-1}) & \text{when } j = 2 \end{cases}$$

Using (3.46) to eliminate $\bar{y}_{j,t}$ in (3.42), one obtains

$$(\sigma^{-1} + \omega/\delta_j)\bar{k}_{j,t} - (1-\delta_j)(\omega/\delta_j)\bar{k}_{j,t} + \frac{1-q_j}{q_j}\overline{rr}_{j,t}^A = \eta_{3j}\bar{q}_t^R + (1+\omega)a_{j,t} \qquad (3.48)$$

Using (3.48) to substitute for the interest rate terms in (3.47), one gets

$$\eta_{1j}\bar{k}_{j,t} - \eta_{2j}\left(\beta E_t\bar{k}_{j,t+1} + \bar{k}_{j,t-1}\right) = \eta_{3j}\bar{q}_t^R - (1+\omega)(a_{j,t} - (1-q_j)E_t a_{j,t+1}) \qquad (3.49)$$

where $\eta_{1j} = q_j\sigma^{-1} + (1+(1-q_j)(1-\delta_j)(\omega/\delta_j))$, $\eta_{2j} = (1-\delta_j)(\omega/\delta_j)$ for $j \in \{1,2\}$, and

Finally $\bar{k}_{j,t}$ and $\bar{q}_t^R$ under the flexible-price regime are determined From (3.44) and (3.49).

2.2.3. Sticky-Price Equilibrium

In this section, I discuss the log-linear approximation of the equilibrium under the sticky-price regime. The sticky-price equilibrium can be represented by the linearization system which consists of two brocks: the demand and the supply blocks.

First, I begin to explain the demand block. Log-linear approximations of the Euler equation, the relative demand equation, the definition of the durables-stock index, and the law of accumulation, respectively, are given by

$$k_t = E_t k_{t+1} - \sigma rr_t^C \qquad (3.50)$$

$$k_{2,t} - k_{1,t} = q_t^R \qquad (3.51)$$

$$k_t = \alpha_1 k_{1,t} + \alpha_2 k_{2,t} \qquad (3.52)$$

$$y_{j,t} = d_{j,t} = \frac{1}{\delta_j}\left(k_{j,t} - (1-\delta)k_{j,t-1}\right) \qquad (3.53)$$

Log-linearizing to (3.21) and (3.23) yields the supply equation in sector j (derived in appendix B) given by

$$\pi_{j,t}^A = \beta E_t \pi_{j,t+1}^A + \varphi_j \left(\sigma^{-1} \hat{k}_t + \omega \hat{y}_{j,t} + \hat{q}_t - \hat{p}_{j,t} \right) \tag{3.54}$$

for $j=1,2$, where $\varphi_j = \dfrac{(1-\gamma_j)(1-\gamma_j \beta)}{\gamma_j (1+\omega\theta)}$, and $\hat{q}_t - \hat{p}_{j,t} = \begin{cases} \dfrac{1-q_1}{q_1} \widehat{rr}_{1,t}^A - \alpha_2 \hat{q}_t^R & \text{when } j = 1 \\[3mm] \dfrac{1-q_2}{q_2} \widehat{rr}_{2,t}^A + \alpha_1 \hat{q}_t^R & \text{when } j = 2 \end{cases}$.

Comparing to a standard two-sector model, the supply equation (3.54) has several distinct characteristics. First, the current inflation rate in sector j depends positively on both the past and the present output gaps because the household utility is a function of the durables-stock index. Second, in sector j, the real interest rates have positive effect on the PPI inflation by changing the COLI/PPI ratio. Finally, the inflation rates in sector j depend also on the relative user cost q_t^R. Under the assumption of perishable goods, i.e., $\delta_j = 1$ for $j=1,2$, (3.54) reduces to the supply equation as in Aoki (2001) and Benigno (2004) where two sectors produce only non-durable goods.

It is easy to show that the system collapses into IS curves and supply equations that determine $\hat{k}_{j,t}$ and $\pi_{j,t}^A$ conditional on the path of nominal interest rates r_t :

$$\hat{k}_{j,t} = E_t\hat{k}_{j,t+1} + \eta_{3j}\left(\hat{q}_t^R - E_t\hat{q}_{t+1}^R\right) - (\sigma/q_j)E_t\left(\widehat{rr}_{j,t}^A - (1-q_j)\widehat{rr}_{j,t+1}^A\right) \tag{3.55}$$

$$\pi_{j,t}^A = \beta E_t\pi_{j,t+1}^A + \varphi_j\left(\begin{array}{c}(\sigma^{-1}+(\omega/\delta_j))\hat{k}_{j,t} - (1-\delta_j)(\omega/\delta_j)\hat{k}_{j,t-1} \\ +\dfrac{1-q_j}{q_j}\widehat{rr}_{j,t}^A - \eta_{3j}\hat{q}_t^R\end{array}\right) \tag{3.56}$$

where $\eta_{3,j}$ is defined in (3.47).

Using (3.56) to substitute for real interest rate terms in (3.55), the supply equations can be rewritten equivalently in the form

$$\begin{aligned}
\pi_{j,t}^A &= \beta E_t\pi_{j,t+1}^A + (1-q_j)\left(E_t\pi_{j,t+1}^A - \beta E_t\pi_{j,t+2}^A\right) - \eta_{3j}\left(\hat{q}_t^R - E_t\hat{q}_{t+1}^R\right) \\
&\quad + \chi_{k_j}^S\left[(\hat{k}_{j,t} - \upsilon_j\hat{k}_{j,t-1}) - \beta\upsilon_j(\hat{k}_{j,t+1} - \upsilon_j\hat{k}_{j,t})\right]
\end{aligned} \tag{3.57}$$

where $\chi_{k_j}^S \equiv \varphi_j\eta_{2,j}/\upsilon_j$, and υ_j and $\eta_{2,j}$ is defined in (3.49), and $0 < \upsilon_j < 1 - \delta_j$ is the smaller root of the quadratic equation

$$\eta_{1j}\upsilon_j = \eta_{2j}(\beta\upsilon_j^2 + 1)$$

3. OPTIMAL MONETARY POLICY

3.1. The Objective of Monetary Policy

In this section, I look for an optimal commitment policy rule which maximizes the representative household's expected life time utility given by

$$W_0 \equiv U_0 - V_0 \tag{3.58}$$

where $U_0 \equiv E_0 \sum_{t=0}^{\infty} \beta^t u(K_t)$, and $V_0 \equiv E_0 \sum_{t=0}^{\infty} \beta^t \int_0^1 v(H_t(i)) di$.

Following Rotemberg and Woodford (1999), I take a second-order Taylor approximation of the welfare function (3.58) around the steady state values with zero inflation. Then an approximation of the discounted sum of utility takes the form (derived in appendix B)

$$W_0 = -\frac{1}{2} K u_k \sum_{t=0}^{\infty} \beta^t L_t + t.i.p + O\left(\|a\|^3\right) \tag{3.59}$$

Here the term *t.i.p.* denotes terms that are independent of monetary policy, $O\left(\|a\|^3\right)$ denotes the terms that are of third or higher order in the deviations of the various variables from their steady-state values, and the period-loss function is given by

$$L_t = \frac{\alpha_1 \delta_1}{q_1 n_1} \sum_{j=1}^{2} n_j \left[\chi_{k_j}^L \left(\hat{k}_{j,t} - \upsilon_j \hat{k}_{j,t-1} \right)^2 + \chi_{\pi_j}^L \left(\pi_{j,t}^A \right)^2 \right] + \alpha_1 \alpha_2 (1 - \sigma^{-1}) \left(\hat{q}_t^R \right)^2 \tag{3.60}$$

where $\chi_{k_j}^L = \eta_{2j} / (\delta_j \upsilon_j) = (1 - \delta_j)(\omega / \delta_j) / (\delta_j \upsilon_j)$, and $\chi_{\pi_j}^L = \theta / \varphi_j$. The period-loss function has several distinct characteristics. First of all, $\chi_{k_j}^L$ is an decreasing function of δ_j, which means that the central bank should put more weight on the sector producing

more durable goods. Second, $\chi^{L}_{\pi_j} = \theta / \varphi_j$ is an increasing function of γ_j, which implies

the central bank should focus more on the sector where prices are stickier, as Aoki

(2001) and Benigno (2004) argue. Finally, comparing to a standard model, $\hat{k}_{j,t} - \upsilon \hat{k}_{j,t-1}$

takes the place of the output gap in the period-loss function. Taking into account that

$0 < \upsilon_j < 1 - \delta_j$, it implies that once the durables-stock deviates from its natural level, it

should be adjusted gradually to its natural level. Hence, targeting rules that require

keeping the output and/or the durables stock at their natural levels may yield bad

outcomes in terms of social welfare, since such rules do not allow the durables stock to

be adjusted gradually when the durables stock gap is not equal to zero.

3.2. Optimal Monetary Policy

An optimal monetary policy seeks to minimize the discounted sum of losses

(3.60) subject to the sticky-price equilibrium. The Lagrangian for this problem is given

by

$$E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \begin{array}{c} \sum_{j=1}^{2} \dfrac{\alpha_j \delta_j}{q_j} \left[\dfrac{\eta_{2j}}{\delta_j \upsilon_j} \left(\hat{k}_{j,t} - \upsilon_j \hat{k}_{j,t-1} \right)^2 + \dfrac{\theta}{\varphi_j} \pi^A_{j,t} \right] \\ + \alpha_1 \alpha_2 (1 - \sigma^{-1}) \left(q^R_t - \bar{q}^R_t \right)^2 \\ + 2 \sum_{j=1}^{2} \phi_{j,t} \left[\begin{array}{c} \pi^A_{j,t} - \beta E_t \pi^A_{j,t+1} - (1 - q_j) \left(E_t \pi^A_{j,t+1} - \beta E_t \pi^A_{j,t+2} \right) \\ + \eta_{3j} \left(\hat{q}^R_t - E_t \hat{q}^R_{t+1} \right) \\ - \chi^S_{k_j} \left[(\hat{k}_{j,t} - \upsilon_j \hat{k}_{j,t-1}) - \beta \upsilon_j (\hat{k}_{j,t+1} - \upsilon_j \hat{k}_{j,t}) \right] \end{array} \right] \end{array} \right\} \qquad (3.61)$$

The first-order necessary conditions with respect to $\hat{k}_{j,t}$ and $\pi^A_{j,t}$ are given by

$$\frac{\alpha_1}{q_1}\frac{\eta_{21}}{\upsilon_1}\left[\left(\hat{k}_{1,t}-\upsilon_1\hat{k}_{1,t-1}\right)-\beta\upsilon_1\left(\hat{k}_{1,t+1}-\upsilon_1\hat{k}_{1,t}\right)\right]$$
$$-\alpha_1\alpha_2(1-\sigma^{-1})\hat{q}_t^R-\alpha_2 q_1(1-\sigma^{-1})\left(\phi_{1,t}-\beta^{-1}\phi_{1,t-1}\right) \tag{3.62}$$
$$-\chi_{k_j}^S\left[(\phi_{1,t}-\upsilon_1\phi_{1,t-1})-\beta\upsilon_1\left(\phi_{1,t+1}-\upsilon_1\phi_{1,t}\right)\right]=0$$

$$\frac{\alpha_2}{q_2}\frac{\eta_{22}}{\upsilon_2}\left[\left(\hat{k}_{2,t}-\upsilon_2\hat{k}_{2,t-1}\right)-\beta\upsilon_2\left(\hat{k}_{2,t+1}-\upsilon_2\hat{k}_{2,t}\right)\right]$$
$$+\alpha_1\alpha_2(1-\sigma^{-1})\hat{q}_t^R-\alpha_1 q_2(1-\sigma^{-1})\left(\phi_{2,t}-\beta^{-1}\phi_{2,t-1}\right) \tag{3.63}$$
$$-\chi_{k_j}^S\left[(\phi_{2,t}-\upsilon_2\phi_{2,t-1})-\beta\upsilon_2\left(\phi_{2,t+1}-\upsilon_2\phi_{2,t}\right)\right]=0$$

$$\frac{\alpha_j\delta_j}{q_j}\frac{\theta}{\varphi_j}\pi_{j,t}^A+\phi_{j,t}-(2-\delta)\phi_{j,t-1}+(1-\delta)\phi_{j,t-2}=0\text{, for }j=1,2. \tag{3.64}$$

For simplicity, I assume the utility is a logarithm function ($\sigma=1$) which is separable in terms of each good. Then combining (3.62), (3.63) and (3.64) yields

$$(\pi_{j,t}^A-\upsilon_j\pi_{j,t-1}^A)+\frac{1}{\theta}(1-L)(\hat{y}_{j,t}-\upsilon_j\hat{y}_{j,t-1})=0 \tag{3.65}$$

for $j=1,2$, for each $t\geq 0$.

The optimal policy in the economy is to have (3.65) hold in each sector. However, the optimal policy is not feasible. As Benigno (2004) notes, the central bank has only one policy instrument, so it cannot cope with distortions in both sectors. In the next section, I investigate the performance of a set of targeting rules and instrument rules in order to find whether it is optimal to give higher weight to the sector producing more durable goods.

4. NUMERICAL RESULTS

In this section, I compare the performance of several targeting rules and instrument rules. In section 4.1, I begin with the calibration of the structural parameters and the shock processes in the model used for numerical analysis. Section 4.2 then explains the results of numerical analysis. I use the AIM algorithm to solve rational expectation models in this section.

4.1. Calibration

The calibrated values are summarized in Table 3. Based on the empirical results of Rotemberg and Woodford (1999), and Erceg and Levin (2006), I calibrate structural parameters and the shock processes of the model to simulate data at a quarterly frequency.

The discount factor β is set to the conventional value of 0.99 which implies the annual real interest rate is 4%. I set $\gamma = 0.66$, so an average lifetime of price contracts is three quarters. An average markup in goods markets is 15%. Thus θ is set to 7.88.

Table 3
Calibrated parameter values in the two-sector model

β	0.99	n_1	0.125	ρ_{a1}	0.900
γ	0.66	α_1	0.166	$sd(a_1)$	0.036
θ	7.88	δ_1	0.025	ρ_{a2}	0.900
σ	1.00	δ_2	1.000	$sd(a_2)$	0.036
ω	0.60				

Following Amato and Laubach (2001), I set ω=0.60 which is consistent with a Frisch elasticity of 5 and a Cobb-Douglas production technology with a coefficient on labor of 0.75. I assume the period utility is a log function, so σ is set to 1. Following Erceg and Levin (2006), the depreciation rates, δ_1 and δ_2, are set to 0.025 and 1, respectively, which means that the annual depreciation rate of the final good in sector 1 is 10% and that the final good in sector 2 is non-durable. Finally, technology shocks in sector 1 and 2 are assumed to independently follow AR(1) processes with the coefficients (ρ_{a1} and ρ_{a1}) of 0.90 and the conditional standard deviations (sd(a_1) and sd(a_2)) of 0.036.

4.2. Results

This section compares outcomes of a set of instrument rules (rows 1-3) and targeting rules (rows 4-8) in terms of the welfare loss given by

$$E_0[L] = (1-\beta)E_0 \sum_{t=0}^{\infty} \beta^t L_t \qquad (3.66)$$

where L_t is the period-loss function defined in (3.60). Table 4 reports welfare losses (the last column) and unconditional variances (columns 4-7) of the sectoral inflation rates and output, and the nominal interest rates under various targeting rules and instrument rules.

The rows 1-3 in table 4 show outcomes of three targeting rules.[15] The first row is the performance of the optimal inflation targeting rule ($n\pi_{1,t}^A + (1-n)\pi_{2,t}^A = 0$) with the weight on the durables sector n chosen optimally. The second row indicates the results of

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[15] Svensson (1999) distinguishes targeting rules from instrument rules.

Table 4
Results on loss and unconditional variances with alternative rules in the two-sector model[a]

	κ_1	κ_2	n[b]	$V_{\pi 1}$	$V_{\pi 2}$	V_{y1}	V_{y2}	V_r	$E_0[L]$
Targeting Rules									
Optimal index			0.38	0.040	0.015	40.7	3.438	0.017	0.003
Inflation				0.145	0.003	144	4.787	0.040	0.006
Stock gap				1.132	0.633	1607	4.276	0.992	0.103
Instrument Rules[c]									
Inflation (z=A)	>100	0.0	0.41	0.031	0.014	42.6	3.502	0.015	0.003
Inflation (z=C)	1.5	0.0	0.15	0.379	0.099	1097	3.999	0.228	0.043
Stock gap	0.0	>100	0.15	0.057	0.023	52.1	3.881	0.022	0.004
Taylor Rule (z=A)	0.5	1.5		0.075	0.013	91.5	3.883	0.031	0.005
Taylor Rule (z=C)	0.5	1.5		0.077	0.013	91.3	3.929	0.033	0.005

[a]The welfare loss and unconditional variances are multiplied by 10^3.
[b]n is the relative weight on the durables sector chosen optimally.
[c]Instrument rules have the form $r_t = \kappa_1\left(n\pi_{1,t}^z + (1-n)\pi_{2,t}^z\right) + \kappa_2\left(ny_{1,t} + (1-n)y_{2,t}\right)$, for $z \in \{A,C\}$.

the PPI inflation targeting rule ($\pi_t^A = 0$). Since the PPI inflation targeting is nested in the optimal inflation targeting, it is natural that the latter is better than the former in terms of the social welfare. The third row shows that the durables-stock gap targeting ($\hat{k} = 0$) is the worst rule among a set of rules under consideration.

The table 4 also considers three types of instrument rules. First, the rows 4-5 show outcomes of instrument rules responding only to an optimally weighted average of sectoral inflation rates. The results indicate that central banks can considerably reduce the welfare loss by giving more weight on the durables sector and by adjusting its policy rate responding to the PPI inflation rates instead of the COLI inflation rates. The sixth row shows it yields bad outcomes to adjust the interest rate by the instrument rule depending only on the present durables-stock gap ($nk_{1,t} + (1-n)k_{2,t} = 0$), even if

coefficients, n and κ_2, are selected optimally. Finally, the seventh and eighth rows show the performance of Taylor rules responding to the current inflation rate and output gap with coefficients (κ_1 and κ_2) 1.5 and 0.5, respectively. The inflation rates in the seventh and eighth rows are defined as the PPI and the COLI inflation rates, respectively. The results show that the Taylor rule responding to PPI inflation is slightly better and that the performance of instrument rules can be improved by adding the inflation rates terms.

To summarize the results in table 4, first of all, it is optimal to target core inflation giving more weight to the sector producing more durable goods. As shown in the third column, the relative weight of the durables sector n chosen optimally is greater than 12.5%. The result provides justification for a policy targeting core inflation rather than aggregate inflation as in Aoki (2001), and offers another criterion for the construction of such an index, while Aoki suggests to measure core inflation base on relative price stickiness.

Second, it is evident that the inflation-targeting rule overwhelms the durables-stock gap targeting rule, which is an opposite result to Erceg and Levin (2006). Their results may be explained by the low (relative) weight on inflation variability in the period-loss function, not by the introduction of durable goods.

Finally, the table 2 implies the central bank should target to stabilize the PPI inflation rather than the COLI inflation. It often happens that the COLI inflation targeting rule makes the model unstable, which implies monetary policy to stabilize the COLI inflation targeting may add additional instability to the economy. In addition, the instrument rules responding PPI inflation show better performance. Finally, note that in

practice determining the stance of monetary policy mechanically by rules responding to the COLI inflation rates may lead to worse outcomes because it is difficult to measure COLI inflation accurately. According to Orphanides (1998), implementing monetary policy based on variables that the central bank cannot measure with certainty may well result in poor performance.

CHAPTER IV

SUMMARY AND CONCLUSIONS

This dissertation studied the relationship among durable goods, price indexes and monetary policy in two sticky-price models. One is a one-sector model with only durable goods and the other is a two-sector model with both durable and non-durable goods.

The general equilibrium models with the durables have several characteristics. First, the COLI and the PPI (the CPI measured by the acquisitions approach) are distinguished in the model. The COLI consists of the user costs of durable goods which depend on positively on the real interest rate measured by the acquisitions approach while the PPI is made of prices purchasing durable goods. By the result, the supply equation depends positively on the COLI/PPI ratio proportional to the real interest rate, which explains why consumption of durable goods responds more sensitively to monetary policy shocks.

Second, the social welfare can be represented by a quadratic function of the quasi-differenced durables-stock gaps and the PPI inflation rates. It means that once the durables-stock deviates from its natural level, it should be recovered toward the natural level gradually. Hence, a policy stabilizing strictly the output gap or the durables-stock gap may not be optimal.

Third, even if the introduction of durable goods changes the dynamics of log-linear equilibrium and the form of the social welfare function, the optimal policy in the one-sector model is to keep the (acquisition) price and the output gap at a constant rate.

The rate does not depend on the durability of the good. Without cost-push shocks, a policy completely stabilizing inflation is also optimal.

Fourth, in the two-sector model with durable and non-durable goods, the relative importance of the durables sector to the non-durables sector in the social welfare depends positively on the durability of the durable goods. In addition, the central bank cannot stabilize the inflation rates of both sectors since it has only one policy instrument. Hence, a policy stabilizing a properly weighted average of the sectoral inflation rates giving more weight on the durables sector is optimal. This conclusion is consistent with Aoki who insists a policy targeting core inflation rather than aggregate inflation. However, the dissertation offers another criterion for the construction of core inflation.

Finally, the numerical simulations show a policy targeting core inflation constructed by the sectoral PPI inflation rates yields better performance than by the sectoral COLI inflation rates. This implies that a PPI is an adequate price index for monetary policy.

In conclusion, central banks face a tradeoff between the durables sectors and the non-durables sectors, which could comes from the fact that demands for heterogeneous goods have different sensitivities to monetary policy shocks depending positively on the durability of the good. It is desirable that monetary policy stabilizes core inflation constructed by the sectoral PPI inflation rates and by giving more weight to the sector producing more durable goods.

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APPENDIX A

A.1. Derivation of the Supply Equation (2.41)

This section derives equation (2.41) in chapter II. Combining the definitions of $\lambda_{t,t+k}$ and $MC^*_{t,t+k}(i)$, equation (2.16) can be rewritten as

$$E_t \sum_{k=0}^{\infty} (\gamma\beta)^k \left[(1+\tau)P^*_t u_k(K_{t+k})/Q_{t+k} - \mu v_h(Y^*_{t,t+k}/A_{t+k})/A_{t+k} \right] Y^*_{t,t+k} = 0 \qquad (A.1)$$

Combining (2.29), log-linear approximations to equation (A.1) yields

$$E_t \sum_{k=0}^{\infty} (\gamma\beta)^k \left[\begin{array}{l} (1+\omega\theta)\left(p^*_t - \sum_{s=1}^{k} \pi^A_{t+s} \right) + \\ \left(\dfrac{\delta\sigma^{-1}+\omega}{\delta}\hat{k}_t - \dfrac{(1-\delta)\omega}{\delta}\hat{k}_{t-1} + \dfrac{1-q}{q}(rr^A_t - \overline{rr}^A_t) \right) \end{array} \right] = 0 \qquad (A.2)$$

where $p^*_t \equiv \log\left(P^*_t / P_t \right)$.

Since p^*_t does not depend on k, equation (A.2) can be expressed as

$$\begin{aligned} p^*_t &= \gamma\beta\left(p^*_{t+1} + E_t\pi^A_{t+1} \right) \\ &+ \frac{1-\gamma\beta}{1+\omega\theta}\left(\frac{\delta\sigma^{-1}+\omega}{\delta}\hat{k}_t - \frac{(1-\delta)\omega}{\delta}\hat{k}_{t-1} + \frac{1-q}{q}(rr^A_t - \overline{rr}^A_t) \right) \end{aligned} \qquad (A.3)$$

In addition, log-linearization of (2.18) yields

$$p^*_t = \frac{\gamma}{1-\gamma}\pi^A_t \qquad (A.4)$$

Using (A.4) to substitute for p^*_t in (A.3), one obtains a New Keynesian Phillips Curve of the form

$$\pi_t^A = \beta E_t \pi_{t+1}^A + \varphi \left(\frac{\delta\sigma^{-1} + \omega}{\delta} \hat{k}_t - \frac{(1-\delta)\omega}{\delta} \hat{k}_{t-1} + \frac{1-q}{q}(rr_t^A - \overline{rr}_t^A) \right)$$

where $\varphi \equiv \dfrac{(1-\gamma)(1-\gamma\beta)}{\gamma(1+\omega\theta)}$.

A.2. Analytical Solution to Flexible-Price Equilibrium

In this section, I derive an analytical solution to flexible-price equilibrium given by equation (2.33).

Combining (2.29) and (2.31), one gets

$$(\sigma^{-1} + \frac{\omega}{\delta})\overline{k}_t - \frac{\omega}{\delta}(1-\delta)\overline{k}_{t-1} + \frac{1-q}{q}\overline{rr}_t^A - (1+\omega)a_t = 0 \tag{A.5}$$

Using (2.32) to substitute for $\overline{rr}_t^C$ in (2.30), one gets

$$\overline{k}_t = E_t\overline{k}_{t+1} - (\sigma/q)E_t\left[\overline{rr}_t^A - (1-q)E_t\overline{rr}_{t+1}^A\right] \tag{A.6}$$

Updating (A.5) one period and taking expectations in period t, one obtains

$$(\sigma^{-1} + \frac{\omega}{\delta})E_t\overline{k}_{t+1} - \frac{\omega}{\delta}(1-\delta)\overline{k}_t + \frac{1-q}{q}E_t\overline{rr}_{t+1}^A - (1+\omega)E_t a_t = 0 \tag{A.7}$$

Using (A.5) and (A.7) to substitute for interest terms in (A.6), one gets

$$\eta_1\overline{k}_t - \eta_2\left(E_t\overline{k}_{t+1} + \overline{k}_{t-1}\right) - (1+\omega)\left(a_t - (1-q)E_t a_{t+1}\right) = 0 \tag{A.8}$$

As the solution to (A.8) has a form $\overline{k}_t = \upsilon\overline{k}_{t-1} + \overline{a}_t$ where $\overline{a}_t \equiv (1+\omega)\zeta^{-1}a_t$, one obtains

$$\left(\eta_1\upsilon - \eta_2(\beta\upsilon^2 + 1)\right)\overline{k}_{t-1}$$
$$\eta_1\overline{a}_t - \beta\eta_2\left(\upsilon\overline{a}_t + E_t\overline{a}_{t+1}\right) - \zeta\left(\overline{a}_t - (1-q)E_t\overline{a}_{t+1}\right) = 0 \tag{A.9}$$

Since the equation (A.9) holds any values of $\hat{K}_{t-1}^n$ and $\bar{a}_t$, one obtains (2.33). In addition, a_t follows an AR(1) process defined in (2.35), and thus $E_t\bar{a}_{t+1} = \rho\bar{a}_t$, which gives (2.36).

A.3. The Representative Household's Welfare

This section provides details of the derivation of equations (2.44) and (2.45) in chapter II which are the second-order approximation to the representative household's welfare. The second-order Taylor series approximation of the utility of consumption is given by

$$u(K_t) = Ku_k\left(k_t + \frac{1}{2}(1-\sigma^{-1})k_t^2\right) + t.i.p + O\left(\|a\|^3\right). \tag{A.10}$$

Assuming $\Phi = 0$, the second-order approximation of the disutility function is given by

$$\int_0^1 v(Y_t(i)/A_t)di = \frac{\delta K}{q}u_k\left\{\begin{array}{l} y_t + \frac{1}{2}(1+\omega)y_t^2 - (1+\omega)a_t\,y_t \\ +\frac{1}{2}(\theta^{-1}+\omega)\,\mathrm{var}_i\,y_t(i) \end{array}\right\} + t.i.p + O\left(\|a\|^3\right), \tag{A.11}$$

where var_i denotes the variance of the distribution of values for the differentiated goods.

Combining (A.10) and (A.11) with (2.43), one gets

$$W_0 = E_0 K u_k \sum_{t=0}^{\infty} \beta^t \left(k_t + \frac{1}{2}(1-\sigma^{-1})k_t^2 \right)$$

$$-\frac{\delta K}{q} u_k E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \begin{array}{l} y_t + \frac{1}{2}(1+\omega)y_t^2 - (1+\omega)a_t\, y_t \\[2mm] + \frac{1}{2}(\theta^{-1}+\omega)\,\mathrm{var}_i\, y_t(i) \end{array} \right\} + t.i.p + O\left(\|a\|^3\right) \qquad \text{(A.12)}$$

Using the law of accumulation of durable goods and the market clearing condition $D_t = Y_t$, one gets

$$y_t = \frac{1}{\delta}\left(k_t - (1-\delta)k_{t-1}\right) - \frac{1-\delta}{2\delta^2}\left(k_t - k_{t-1}\right)^2 \qquad \text{(A.13)}$$

Combining (A.12) and (A.13), one has

$$W_0 = -\frac{1}{2}\frac{\delta K}{q} u_k \sum_{t=0}^{\infty} \beta^t \left\{ \begin{array}{l} \frac{1}{\delta}\left[\begin{array}{l} (\eta_1 k_t - \eta_2(\beta k_{t+1} + k_{t-1}))k_t \\[1mm] -2(1+\omega)\left(a_t - (1-q)a_{t+1}\right)k_t \end{array} \right] \\[4mm] +(\theta^{-1}+\omega)\,\mathrm{var}_i\, y_t(i) \end{array} \right\} + t.i.p + O\left(\|a\|^3\right) \quad \text{(A.14)}$$

where $\eta_1 = \left[q\sigma^{-1} + \left(1 + \beta(1-\delta)^2\right)(\omega/\delta) \right]$, $\eta_2 = (1-\delta)(\omega/\delta)$, and $0 < \upsilon < 1-\delta$ is the smaller root of quadratic equation

$$\eta_1 \upsilon = \eta_2(\beta\upsilon^2 + 1)$$

Note that I implicitly assume that the initial value of the stock of durables is equal to its steady state value and, thus, use the formula $\sum_{t=0}^{\infty} \beta^t C_{t-1} = \sum_{t=0}^{\infty} \beta^{t+1} C_t$ for any variable C.

Combining (A.8) to substitute for a_t in (A.14), the discounted welfare function can be rewritten as

$$W_0 = -\frac{1}{2}\frac{\delta K}{q}u_k\sum_{t=0}^{\infty}\beta^t\left\{\begin{array}{l}\frac{1}{\delta}\left[\begin{array}{l}\left(\eta_1 k_t - \eta_2(\beta k_{t+1}+k_{t-1})\right)k_t\\-2\left(\eta_1\overline{k}_t - \eta_2(\beta\overline{k}_{t+1}+\overline{k}_{t-1})\right)k_t\end{array}\right]\\+(\theta^{-1}+\omega)\,\mathrm{var}_i\,y_t(i)\end{array}\right\}+t.i.p+O\left(\|a\|^3\right)\quad\text{(A.15)}$$

Combining (2.34) with (A.15), one obtain the form

$$W_0 = -\frac{1}{2}\frac{\delta K}{q}u_k\sum_{t=0}^{\infty}\beta^t\left\{\begin{array}{l}\frac{1}{\delta}\left[\begin{array}{l}\frac{\eta_2}{\upsilon}\left((k_t-\upsilon k_{t-1})-\beta\upsilon(k_{t+1}-\upsilon k_t)\right)k_t\\-2\frac{\eta_2}{\upsilon}\left((\overline{k}_t-\upsilon\overline{k}_{t-1})-\beta\upsilon(\overline{k}_{t+1}-\upsilon\overline{k}_t)\right)k_t\end{array}\right]\\+(\theta^{-1}+\omega)\,\mathrm{var}_i\,y_t(i)\end{array}\right\}$$
$$+t.i.p+O\left(\|a\|^3\right)$$

(A.16)

where υ is defined in (2.34).

Combining (A.16) and $\sum_{t=0}^{\infty}\beta^t\,\mathrm{var}_i\,\hat{y}_t(i)=\dfrac{\gamma\theta^2}{(1-\gamma)(1-\gamma\beta)}\sum_{t=0}^{\infty}\beta^t\left(\pi_t^A\right)^2$, the second-

order approximation to the household's welfare can be rewritten as[16]

$$W_0 = -\frac{\delta K}{q}\frac{\theta}{\varphi}u_k\sum_{t=0}^{\infty}\beta^t\left\{\frac{\varphi}{\theta}\frac{\eta_2}{\delta\upsilon}\left(\hat{k}_t-\upsilon\hat{k}_{t-1}\right)^2+\left(\pi_t^A\right)^2\right\}$$
$$+t.i.p+O\left(\|a\|^3\right)$$

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[16] See Woodford (2003, Chapter 6).

APPENDIX B

B.1. Derivation of the Supply Equation (3.54)

This section derives equation (3.54) in chapter III. Combining the definitions of $\lambda_{t,t+k}$ and $MC^*_{t,t+k}(i)$, equation (3.21) can be rewritten as

$$E_t \sum_{k=0}^{\infty} \left(\gamma_j \beta \right)^k \left[(1+\tau_j) P^*_{j,t} u_k(K_{t+k})/Q_{t+k} - \mu v_h (Y^*_{j,t,t+k}/A_{j,t+k})/A_{j,t+k} \right] Y^*_{j,t,t+k} = 0 \text{ (B.1)}$$

Combining (3.42), log-linear approximations to equation (B.1) yields

$$E_t \sum_{k=0}^{\infty} \left(\gamma \beta \right)^k \begin{bmatrix} (1+\omega\theta) \left(p^*_{j,t} - \sum_{s=1}^{k} \pi^A_{j,t+s} \right) + \\ \left(\sigma^{-1} \hat{k}_{t+k} + \omega \hat{y}_{j,t+k} + \hat{q}_{t+k} - \hat{p}_{j,t+k} \right) \end{bmatrix} = 0 \qquad \text{(B.2)}$$

where $p^*_{j,t} \equiv \log\left(P^*_{j,t}/P_t \right)$.

Since p^*_t does not depend on k, equation (B.2) can be expressed as

$$\begin{aligned} p^*_{j,t} = \gamma_j \beta \left(p^*_{j,t+1} + E_t \pi^A_{j,t+1} \right) \\ + \frac{1-\gamma_j \beta}{1+\omega\theta} \left(\sigma^{-1} \hat{k}_t + \omega \hat{y}_{j,t} + \hat{q}_t - \hat{p}_{j,t} \right) \end{aligned} \qquad \text{(B.3)}$$

In addition, log-linearization of (3.23) yields

$$p^*_{j,t} = \frac{\gamma_j}{1-\gamma_j} \pi^A_{j,t} \qquad \text{(B.4)}$$

Using (B.4) to substitute for $p^*_{j,t}$ in (B.3), one obtains the supply equation in sector j of the form

$$\pi_{j,t} = \beta E_t \pi_{j,t+1} + \varphi_j \left(\sigma^{-1} \hat{k}_t + \omega \hat{y}_{j,t} + \hat{q}_t - \hat{p}_{j,t} \right)$$

where $\varphi_j = \dfrac{(1-\gamma_j)(1-\gamma_j\beta)}{\gamma_j(1+\omega\theta)}$.

B.2. The Representative Household's Welfare

This section provides details of the derivation of equations (3.59) and (3.60) in chapter III which are the second-order approximation to the representative household's welfare. The second-order Taylor series approximation of the utility of consumption is given by

$$u(K_t) = Ku_k\left(k_t + \frac{1}{2}(1-\sigma^{-1})k_t^2\right) + t.i.p + O\left(\|a\|^3\right). \tag{B.5}$$

From the definition of k_t, (B.5) can be written as

$$u(K_t) = u_{1,t} + u_{2,t} - \frac{1}{2}\alpha_1\alpha_2(1-\sigma^{-1})Ku_k\left(k_{1,t} - k_{2,t}\right)^2 + t.i.p + O\left(\|a\|^3\right) \tag{B.6}$$

where $u_{j,t} = \alpha_j Ku_k\left(k_{j,t} + \frac{1}{2}(1-\sigma^{-1})k_{j,t}^2\right)$.

In the case of any good i in sector j, a second-order approximation of the disutility of labor can be written in the form

$$\begin{aligned} v(Y_t(i)/A_{j,t}) = {} & v + v_{y_j}\left(Y_t(i) - Y(i)\right) + \frac{1}{2}v_{y_jy_j}\left(Y_t(i) - Y(i)\right)^2 \\ & + v_{A_jy_j}\left(A_{j,t} - A_j\right)\left(Y_t(i) - Y(i)\right) + t.i.p. + O\left(\|a\|^3\right) \end{aligned} \tag{B.7}$$

where $v_{y_j} = \partial v/\partial Y(i)$, $v_{y_jy_j} = \partial^2 v/\partial Y(i)^2$, and $v_{A_jy_j} = \partial^2 v/\left(\partial A_j\partial Y(i)\right)$ for all $i \in N_j$.

Using the formula $X_t - X = X(x_t + x_t^2/2)$ and arranging the terms that are of third or higher order in the deviations of the various variables from their steady-state values, (B.7) is rewritten in the form

$$v(Y_t(i)/A_{j,t}) = v_{y_j} Y(i) \left(y_t(i) + \frac{1}{2} y_t(i)^2 \right) + \frac{1}{2} v_{y_j y_j} Y(i)^2 y_t(i)^2$$
$$+ v_{A_j y_j} A_j Y(i) a_{j,t} y_t(i) + t.i.p. + O\left(\|a\|^3\right) \tag{B.8}$$

Using the definition of ω and $v_{A_j y_j} = -(1+\omega)v_{y_j}/A_j$, one obtains

$$v(Y_t(i)/A_{j,t}) = v_{y_j} Y(i) \left(y_t(i) + \frac{1}{2}(1+\omega)\left(y_t(i)\right)^2 - (1+\omega)a_{j,t} y_t(i) \right)$$
$$+ t.i.p + O\left(\|a\|^3\right) \tag{B.9}$$

Integrating (B.9) over the goods i belonging to sector j, I have

$$v_{j,t} \equiv \int_{N_j} v(Y_t(i)/A_{j,t}) di$$
$$= n_j v_{y_j} Y(i) \left(\begin{array}{l} E_{i \in N_j} y_t(i) + \frac{1}{2}(1+\omega)\left[\left(E_{i \in N_j} y_t(i)\right)^2 + \mathrm{var}_{i \in N_j} y_t(i) \right] \\ -(1+\omega)a_{j,t} E_{i \in N_j} y_t(i) \end{array} \right) \tag{B.10}$$
$$+ t.i.p + O\left(\|a\|^3\right)$$

where $E_{i \in N_j}$ and $\mathrm{var}_{i \in N_j}$ denote, respectively, the mean and variance of the distribution of values for the differentiated goods i in sector j.

Combining $Y_j = n_j Y(i)$ and the Taylor series approximation to the CES production function, $y_{j,t} = E_{i \in N_j} y_t(i) + \frac{1}{2}(1-\theta^{-1}) \mathrm{var}_{i \in N_j} y_t(i) + \left(\|a\|^3\right)$, (B.10) can be rewritten as

$$v_{j,t} = v_{y_j} Y_j \left(\begin{array}{c} y_{j,t} + \dfrac{1}{2}(1+\omega)y_{j,t}^2 - (1+\omega)a_{j,t}y_{j,t} \\ + \dfrac{1}{2}(\theta^{-1}+\omega)\operatorname{var}_{i\in N_j} y_t(i) \end{array} \right) + t.i.p + O\left(\|a\|^3\right) \qquad (B.11)$$

Assuming $\Phi_j = 0$, and using $v_{y_j} Y_j = \dfrac{\alpha_j \delta_j}{q_j} Ku_k$, (B.11) can be rewritten as the

form

$$v_{j,t} = \dfrac{\alpha_j \delta_j}{q_j} Ku_k \left(\begin{array}{c} y_{j,t} + \dfrac{1}{2}(1+\omega)y_{j,t}^2 - (1+\omega)a_{j,t}y_{j,t} \\ + \dfrac{1}{2}(\theta^{-1}+\omega)\operatorname{var}_{i\in N_j} y_t(i) \end{array} \right) + t.i.p + O\left(\|a\|^3\right) \qquad (B.12)$$

Here $W_{j,0}$ is defined as

$$W_{j,0} \equiv \sum_{t=0}^{\infty} \beta^t \left(u_{j,t} - v_{j,t} \right) + t.i.p + O\left(\|a\|^3\right)$$

$$= \alpha_j Ku_k \sum_{t=0}^{\infty} \beta^t \left\{ \begin{array}{c} \left(k_{j,t} + \dfrac{1}{2}(1-\sigma^{-1})k_{j,t}^2 \right) \\ -\dfrac{\delta_j}{q_j} \left(\begin{array}{c} y_{j,t} + \dfrac{1}{2}(1+\omega)y_{j,t}^2 - (1+\omega)a_{j,t}y_{j,t} \\ + \dfrac{1}{2}(\theta^{-1}+\omega)\operatorname{var}_{i\in N_j} y_t(i) \end{array} \right) \end{array} \right\} \qquad (B.13)$$

$$+ t.i.p + O\left(\|a\|^3\right)$$

Combining (B.13) and (3.49), one has

$$W_{j,0} = -\dfrac{\alpha_j \delta_j}{2q_j} Ku_k \sum_{t=0}^{\infty} \beta^t \left\{ \begin{array}{c} \delta_j^{-1}\left(\eta_{1j}k_{j,t} - \eta_{2j}(\beta k_{j,t+1} + k_{j,t-1}) \right)k_{j,t} \\ -2\delta_j^{-1} \left(\begin{array}{c} \eta_{1j}\bar{k}_{1,t} - \eta_{2j}\left(\beta \bar{k}_{1,t+1} + \bar{k}_{1,t-1} \right) \\ -\eta_{3j}\bar{q}_t^R \end{array} \right) k_{j,t} \\ +(\theta^{-1}+\omega)\operatorname{var}_{i\in N_j} y_t(i) \end{array} \right\} \qquad (B.14)$$

$$+ t.i.p + O\left(\|a\|^3\right)$$

where η_{1j} and η_{2j} are defined in (3.49), and η_{3j} is defined in (3.47).

$$W_{j,0} = -Ku_k \frac{\alpha_j \delta_j}{2q_j} \sum_{t=0}^{\infty} \beta^t \left\{ \begin{array}{l} \dfrac{\eta_{2j}}{\delta_j \upsilon_j}\left(\hat{k}_{j,t} - \upsilon_j \hat{k}_{j,t-1}\right)^2 + \dfrac{2\eta_{3j}}{\delta_j}(1-\sigma^{-1})\bar{q}_t^R k_{j,t} \\ + (\theta^{-1} + \omega)\operatorname{var}_{i \in N_j} y_t(i) \end{array} \right\} \tag{B.15}$$

$$+t.i.p + O\left(\|a\|^3\right)$$

Combining (B.6) and (B.15), the social welfare function can be written as

$$W_0 = \sum_{t=1}^{2} W_{j,0} - \frac{1}{2}\alpha_1 \alpha_2 (1-\sigma^{-1}) Ku_k \sum_{t=0}^{\infty} \beta^t \left(k_{1,t} - k_{2,t}\right)^2$$

$$+t.i.p + O\left(\|a\|^3\right)$$

$$= -\frac{Ku_k}{2} \sum_{t=0}^{\infty} \beta^t \left\{ \begin{array}{l} \displaystyle\sum_{j=1}^{2} \dfrac{\alpha_j \delta_j}{q_j}\left[\dfrac{\eta_{2j}}{\delta_j \upsilon_j}\left(\hat{k}_{j,t} - \upsilon_j \hat{k}_{j,t-1}\right)^2 + (\theta^{-1} + \omega)\operatorname{var}_{i \in N_j} y_t(i)\right] \\ + \alpha_1 \alpha_2 (1-\sigma^{-1})\left(\hat{q}_t^R\right)^2 \end{array} \right\} \tag{B.16}$$

$$+t.i.p + O\left(\|a\|^3\right)$$

Finally, combining $\displaystyle\sum_{t=0}^{\infty} \beta^t \operatorname{var}_{i \in N_j} \hat{y}_t(i) = \frac{\gamma_j \theta^2}{(1-\gamma_j)(1-\gamma_j \beta)} \sum_{t=0}^{\infty} \beta^t \left(\pi_{j,t}^A\right)^2$ and (B.16),

the welfare function can be written as[17]

$$W_0 = -\frac{1}{2}Ku_k \sum_{t=0}^{\infty} \beta^t \left\{ \begin{array}{l} \displaystyle\sum_{j=1}^{2} \dfrac{\alpha_j \delta_j}{q_j}\left[\dfrac{\eta_{2j}}{\delta_j \upsilon_j}\left(\hat{k}_{j,t} - \upsilon_j \hat{k}_{j,t-1}\right)^2 + \dfrac{\theta}{\varphi_j}\left(\pi_{j,t}^A\right)^2\right] \\ + \alpha_1 \alpha_2 (1-\sigma^{-1})\left(\hat{q}_t^R\right)^2 \end{array} \right\}$$

$$+t.i.p + O\left(\|a\|^3\right)$$

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[17] See Woodford (2003, Chapter 6).

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