ESSAYS IN APPLIED MACROECONOMICS: ASYMMETRIC PRICE ADJUSTMENT, EXCHANGE RATE AND TREATMENT EFFECT

A Dissertation

by

JINGPING GU

Submitted to the Office of Graduate Studies of
Texas A&M University
in partial fulfillment of the requirements for the degree of

DOCTOR OF PHILOSOPHY

August 2008

Major Subject: Economics

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Approved by:

Co-Chairs of Committee,	Dennis W. Jansen
	Qi Li
Committee Members,	David A. Bessler
	Li Gan
Head of Department,	Larry Oliver

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ABSTRACT

Essays in Applied Macroeconomics: Asymmetric Price Adjustment, Exchange Rate and Treatment Effect. (August 2008)

Jingping Gu, B.A., Renmin University of China; M.A., Peking University

Co-Chairs of Advisory Committee: Dr. Dennis W. Jansen
Dr. Qi Li

This dissertation consists of three essays. Chapter II examines the possible asymmetric response of gasoline prices to crude oil price changes using an error correction model with GARCH errors. Recent papers have looked at this issue. Some of these papers estimate a form of error correction model, but none of them accounts for autoregressive heteroskedasticity in estimation and testing for asymmetry and none of them takes the response of crude oil price into consideration. We find that time-varying volatility of gasoline price disturbances is an important feature of the data, and when we allow for asymmetric GARCH errors and investigate the system wide impulse response function, we find evidence of asymmetric adjustment to crude oil price changes in weekly retail gasoline prices

Chapter III discusses the relationship between fiscal deficit and exchange rate. Economic theory predicts that fiscal deficits can significantly affect real exchange rate movements, but existing empirical evidence reports only a weak impact of fiscal deficits on exchange rates. Based on US dollar-based real exchange rates in G5 countries and a flexible varying coefficient model, we show that the previously documented weak

relationship between fiscal deficits and exchange rates may be the result of additive specifications, and that the relationship is stronger if we allow fiscal deficits to impact real exchange rates *non-additively* as well as nonlinearly. We find that the speed of exchange rate adjustment toward equilibrium depends on the state of the fiscal deficit; a fiscal contraction in the US can lead to less persistence in the deviation of exchange rates from fundamentals, and faster mean reversion to the equilibrium.

Chapter IV proposes a kernel method to deal with the nonparametric regression model with only discrete covariates as regressors. This new approach is based on recently developed least squares cross-validation kernel smoothing method. It can not only automatically smooth the irrelevant variables out of the nonparametric regression model, but also avoid the problem of loss of efficiency related to the traditional nonparametric frequency-based method and the problem of misspecification based on parametric model.

To my parents and my brother

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CHAPTER I

INTRODUCTION

Chapter II examines the possible asymmetric response of gasoline prices to positive and negative shocks of crude oil price. This topic has been investigated by many previous researchers. Bacon (1991) looked at the U.K.'s gasoline market and described the asymmetric adjustment as a "rockets and feathers" phenomenon. Prices rise like a rocket and fall like a feather. In Bacon's work he found that the asymmetry was in the response of gasoline prices to crude oil price changes.

Recent papers have looked at this issue, including Borenstein, Cameron and Gilbert (1997), who find asymmetry in the response of retail gasoline prices to crude oil price changes, and Bachmeier and Griffin (2003), who find no evidence of asymmetry in both the response of wholesale and retail gasoline prices to crude oil price changes based on two-step Engle-Granger estimation procedure. These papers both estimate a form of error correction model, but neither accounts for autoregressive heteroskedasticity in estimation and testing for asymmetry and neither takes the response of crude oil price into consideration.

Chapter II is based on bivariate VAR model and estimates both the retail gasoline price impulse response function (IRF) and the crude oil price IRF in a system context. For expository purposes we call these system-wide IRFs to distinguish them from the

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This dissertation follows the style of Journal of Monetary Economics.

IRF presented in previous paper. Two broad classes of models that allow potential asymmetric responses are estimated in this part: the traditional error correction model followed the previous literature on this topic and standard threshold model specification.

In addition, various specifications of heteroskedasticity are also considered in this chapter. In particular, we allow different asymmetric models for conditional volatility -- Exponential GARCH, GJR-GARCH, and logistic smooth transition GARCH -- to check whether the asymmetry conclusion is sensitive to the different choice of the conditional volatility model.

We find that time-varying volatility of gasoline price disturbances is an important feature of the data, and when we allow for asymmetric GARCH errors and investigate the system wide impulse response function, we find evidence of asymmetric adjustment to crude oil price changes in weekly retail gasoline prices.

Moreover, we suggest a signal extraction model based on consumer search which takes explicit account of both crude oil price volatility and retail gasoline price volatility. The model predicts positive correlation between crude oil price volatility and degree of asymmetry and negative correlation between retail gasoline price volatility and asymmetry. The empirical result supports the model's prediction. This explanation also reconciles the contrary theoretical conclusion in BCG (1997) and empirical evidence in Peltzman (2000) and Radchenko (2005).

Chapter III investigates the relationship between fiscal deficit and real exchange rate determination. In previous literature, the effect of monetary policy on exchange rates has been extensively explored, but not many serious attentions has been paid to empirically exploring the role of fiscal policy in foreign exchange markets.

Economic theory predicts that fiscal deficits can significantly affect real exchange rate movements, but existing empirical evidence reports only a weak impact of fiscal deficits on exchange rates. The early empirical work typically views fiscal policy as a separate tool from monetary policy and consider its stand-alone effects on exchange rates. Recently, more and more theoretical works begin to underscored the potentially complex interaction between monetary and fiscal policies, which means if one allows for fiscal deficits to affect exchange rates as a state variable, non-additively and nonlinearly, fiscal policy variables play an important (and statistically significant) role in explaining the real exchange rate dynamic adjustments.

In this chapter, based on US dollar-based real exchange rates in G5 countries and a flexible varying coefficient model, we show that the previously documented weak relationship between fiscal deficits and exchange rates may be the result of additive specifications, and that the relationship is stronger if we allow fiscal deficits to impact real exchange rates non-additively as well as nonlinearly.

We find that the speed of exchange rate adjustment toward equilibrium depends on the state of the fiscal deficit, whether it is in contraction or expansion. We find that a fiscal contraction in the US can lead to less persistence in the deviation of exchange rates from fundamentals, and faster mean reversion to the equilibrium.

This research contributes to the literature in two important aspects. First, it fills the gap in the literature by allowing for a fiscal policy variable (the changes in fiscal deficits) to act as a conditioning variable in modeling exchange rate movements. Second, it employs recently developed flexible varying coefficient in econometric analysis. This model appears to be particularly suitable for modeling fiscal deficits as a conditioning variable. The idea that fiscal deficits may effectively work as a binding constraint on monetary policy can hardly be modeled by a parametric (linear or nonlinear) model. From this perspective, the method improves on the popular parametric nonlinear smooth transition models used in the real exchange rate mean reversion literature. In addition, the findings in this chapter lend support to the argument that there are stronger monetary policy impacts on exchange rates in the absence of fiscal deficits.

Chapter IV proposes a kernel method to deal with the nonparametric regression model with only discrete covariates as regressors.

The traditional parametric method suffers from misspecification. The traditional nonparametric way to deal with the discrete covariates is frequency-based method which splits the data into small “cells”. When the number of cells exceeds the sample size, this approach is infeasible. Furthermore, the tests on whether there exists significant treatment effects or not based on this frequency approach may lead to a loss of power.

New approach proposed in this chapter is based on recently developed least squares cross-validation kernel smoothing method. It can not only automatically smooth the irrelevant variables out of the nonparametric regression model, but also avoid the problem of loss of efficiency related to the traditional nonparametric frequency based method and the problem of misspecification.

CHAPTER II

EVIDENCE ON ASYMMETRIC GASOLINE PRICE RESPONSES FROM ERROR CORRECTION MODELS WITH GARCH ERRORS

2.1 Introduction

Many observers claim that gasoline prices rise quickly but decline slowly. This issue of asymmetry in the response of gasoline prices to positive and negative shocks is one that has been investigated by previous researchers. Bacon (1991) looked at the U.K.'s gasoline market and described the asymmetric adjustment as a "rockets and feathers" phenomenon. Prices rise like a rocket and fall like a feather. In Bacon's work he found that the asymmetry was in the response of gasoline prices to crude oil price changes. He claimed that when crude oil prices increased, gasoline price responded very quickly, but when crude oil prices decreased, gasoline prices were much slower to respond.

Borenstein, Cameron and Gilbert (1997) (hereafter BCG) looked at the issue of asymmetry with an error correction model (or ECM). They estimated an ECM for weekly and semimonthly retail gasoline prices, with crude oil prices as an explanatory variable. They report strong evidence that gasoline prices respond asymmetrically to crude oil price changes, supporting the earlier findings by Bacon (1991).

Following up on BCG, Bachmeier and Griffin (2003) (hereafter BG) also use an ECM but with the two-step Engle-Granger estimation procedure instead of the less traditional approach used by BCG. BG investigates both daily spot (wholesale) gasoline prices and retail gasoline prices to see their response to crude oil price changes. BG report no evidence of asymmetry in the response of spot gasoline prices to crude oil price changes, and no evidence of asymmetry in the response of retail gasoline prices to crude oil price change if the two-step Engle-Granger estimation procedure is used for the ECM. BG claims BCG's asymmetry evidence may rest on their nonstandard estimation approach.

These papers provide differing conclusions regardng the retail gasoline price response to crude oil prices, based on different econometric model specifications and somewhat different data. We look again at the possible asymmetry of retail gasoline prices response, in order to consider a broader array of econometric models including system-wide impulse response functions and various specifications of heteroskedasticity. It is well known that assumptions of homoskedasticity, when inappropriate, can cause misleading inference, and we find that prior tests for asymmetry are strongly influenced by the treatment of possible heteroskedasticity.

Our paper investigates the system-wide retail gasoline price response instead of the univariate error correction model used in most previous research. The previous literature generally estimates a dynamic model for the retail gasoline price response and considers the impact of a crude oil price shock without allowing for crude oil prices to exhibit a dynamic response to the shock itself nor to changes in the retail price. This paper begins from the bivariate VAR model and estimates both the retail gasoline price impulse response function (IRF) and the crude oil price IRF in a system context. For expository purposes we call these system-wide IRFs to distinguish them from the IRF presented in BCG's paper.

Our approach is to estimate two broad classes of models that allow potential asymmetric responses. We focus on error correction models that are similar to those estimated by both BCG and BG. These models explicitly allow for differential dynamic responses to shocks. More specifically, the coefficients on explanatory variables including own-lags are allowed to vary depending on whether crude oil prices have increased or decreased. We will estimate versions of these models that follow the BCG and BG framework, and also versions that are more in accord with standard threshold model specifications, where there is a single threshold variable affecting all right hand side coefficients.[1] In all cases our estimation takes account of the presence of autoregressive conditional heteroskedasticity. In particular, we allow different asymmetric models for conditional volatility -- Exponential GARCH, GJR-GARCH, and

[1] Such threshold models are common in the literature. Bradley and Jansen (2000) is just one example.

logistic smooth transition GARCH -- to check whether the asymmetry conclusion is sensitive to the different choice of the conditional volatility model.

When considering these different models we focus on testing for asymmetric responses as indicated by different coefficients on positive and negative changes in crude oil prices. We attempt to characterize the nature of the asymmetry, including the speed of adjustment to a disturbance (rockets versus feathers), the size of adjustment to a disturbance, the permanence of shocks, and the impact of the initial state.

Overall, we find evidence of asymmetry in the weekly retail gasoline prices response on the crude oil prices in both classes of models. This asymmetry is both in the speed of adjustment to shocks and in the magnitude of adjustment. Rising crude oil prices tend to cause somewhat more rapid responses of gasoline prices than falling crude oil prices. Over comparable horizons, rising crude oil prices tends to cause a somewhat greater magnitude response of gasoline prices than falling crude oil prices. In all models we find it is important to account for ARCH errors.[2]

[2] In addition to the debate over the empirical facts on asymmetry, there is a debate over theoretical explanations, and a number of explanations have been offered. Market concentration is a usual suspect. However, it is fair to say that a consensus is lacking. Peltzman (2000) and Brown and Yucel (2000) note that it is difficult to find a model based on market concentration that will explain asymmetric downstream price adjustment to an upstream price change, although Borenstein and Shepard (2002) point to tacit collusion and margins increasing when expected future demand increases. Johnson (2002) claims asymmetry is due to consumer search costs, and provides evidence supporting the search explanation over the oligopolistic behavior explanation from consumer gasoline and diesel markets. However, Radchenko (2005) shows that asymmetry declines with volatility, and argues that this supports oligopolistic coordination explanations over search-based explanations.

2.2 Model Specification

We investigate two sets of models. The first is an asymmetric error correction model that was popular in the previous literature. This model is often specified as a single-equation ECM, and for our purposes the key feature is that it is estimated assuming homoskedastic disturbances. We will compare results for this model with a second set of models that are asymmetric error correction models with a version of heteroskedastic disturbances.

2.2.1 Models with Homoskedastic Disturbances

Our starting point is the basic mark-up model. Obviously crude oil is a vital input into gasoline production, and crude oil prices are one of the most important cost factors affecting gasoline prices. In fact, the cost of crude oil accounts for roughly half of the retail gasoline price. Therefore it is natural to begin our analysis with the following mark-up model, and one that has been is widely used in previous research.

$$LPG_t = \alpha + \beta \cdot LPC_t \tag{2.1}$$

Here LPG is the log price of retail gasoline and LPC is the log price of crude oil. Both the log price of gasoline and the log price of crude oil appear to be I(1) processes, non-stationary in levels but stationary in differences. Gasoline prices and crude oil prices also appear to be cointegrated. When we test for cointegration between the log spot gasoline price and the log spot crude oil price, and separately, between the log retail gasoline price and the log spot crude oil price, we cannot reject cointegration.

Cointegration between LPG and LPC is consistent with using an ECM, the approach we take here and the approach used earlier by both BCG and BG. We use a two-step estimation strategy to estimate the ECM.[3]

Symmetric Error Correction Model

Our starting point is a standard, symmetric ECM. This model specifies that changes in the log price of gasoline is a function of lagged changes in the log price of gasoline, current and lagged changes in the log price of crude oil, and the lagged error correction term. This specification assumes that crude oil prices are weakly exogenous to gasoline prices.

$$\Delta LPG_t = \alpha + \sum_{i=0}^{m} \beta_i \Delta LPC_{t-i} + \sum_{j=1}^{n} \gamma_j \Delta LPG_{t-j} + \lambda(LPG_{t-1} - \varphi LPC_{t-1}) + \varepsilon_t \qquad (2.2)$$

We use this model as a point of departure, in order to compare to a model allowing asymmetry.

Asymmetric Error Correction Model

An asymmetric ECM can be specified in a variety of ways, but the model frequently used by researchers in this area, including BCG and BG, specify different coefficients for positive and negative lagged changes in gasoline prices and for positive and negative

[3] We report the cointegration test results in Appendix Tables A1.

current and lagged changes in crude oil prices. In what follows the superscript '+' refers to a positive change and the superscript '-' refers to a negative change. A common asymmetric ECM specification is:

$$\begin{aligned} \Delta LPG_t &= \alpha + \sum_{i=0}^{m} (\beta_i^{+} \Delta LPC_{t-i}^{+} + \beta_i^{-} \Delta LPC_{t-i}^{-}) + \sum_{j=1}^{n} (\gamma_j^{+} \Delta LPG_{t-j}^{+} + \gamma_j^{-} \Delta LPG_{t-j}^{-}) \\ &+ \lambda (LPG_{t-1} - \varphi LPC_{t-1}) + \varepsilon_t \end{aligned} \tag{2.3}$$

Here $\Delta LPC^{+} = \Delta LPC$ if $\Delta LPC > 0$, and $\Delta LPC^{+} = 0$ if $\Delta LPC < 0$. Similarly, $\Delta LPC^{-} = \Delta LPC$ if $\Delta LPC < 0$, and $\Delta LPC^{-} = 0$ if $\Delta LPC > 0$. ΔLPG^{+} and ΔLPG^{-} are defined analogously.

As an alternative to equation (2.3), we can specify an asymmetric ECM within a standard threshold framework where the model switches between regimes depending on an indicator variable that switches value between zero and one depending on the relationship of the (single) threshold variable to a threshold value. In comparison, equation (2.3) has multiple threshold variables or states, depending on the sign of current and lagged values of ΔLPC and ΔLPG.

In our threshold asymmetric ECM, the threshold variable will be a lagged valued of LPG (or LPC), and the threshold level will be set at zero. We can write this threshold asymmetric ECM as:

$$\begin{aligned} \Delta LPG_t &= \alpha + \sum_{i=0}^{m} (\beta_i^{+} \Delta LPC_{t-i} \cdot I(Z_{t-d}) + \beta_i^{-} \Delta LPC_{t-i}) + \\ &\sum_{j=1}^{n} (\gamma_j^{+} \Delta LPG_{t-j} \cdot I(Z_{t-d}) + \gamma_j^{-} \Delta LPG_{t-j}) + \lambda (LPG_{t-1} - \varphi LPC_{t-1}) + \varepsilon_t \end{aligned} \tag{2.4}$$

Here $I(Z_{t-d})$ is the indicator variable, with $I(Z_{t-d}) = 1$ if $Z_{t-d} > 0$ and $I(Z_{t-d}) = 0$ if $Z_{t-d} < 0$. The variable Z is the threshold variable, either $Z = \Delta LPG$ or $Z = \Delta LPC$. The subscript d is called the delay parameter, and for our purposes $d=0, 1,$ or 2.

2.2.2 Models with GARCH Errors

This section describes our preferred specifications. It contains two points of departure from BCG and BG. First, in those papers the symmetry or asymmetry discussion is based on results from a univariate ECM or threshold model, which necessarily ignores system-wide dynamics and feedback between retail prices and crude oil prices. In constrast, we specify a bivariate vector ECM (VECM) and calculate IRFs for the system we estimate. Second, previous work assumes homoskedastic disturbances, but we will test for and model versions of GARCH.

A Bivariate VECM Model

We consider a bivariate VECM model of the form,

$$\begin{pmatrix} \Delta LPG \\ \Delta LPC \end{pmatrix}_t = A(L)\begin{pmatrix} \Delta LPG \\ \Delta LPC \end{pmatrix}_{t-1} + BZ_{t-1} + \begin{pmatrix} \varepsilon_G \\ \varepsilon_C \end{pmatrix}_t \tag{2.5}$$

Here $A(L) = \begin{pmatrix} A_{11}(L) & A_{12}(L) \\ A_{21}(L) & A_{22}(L) \end{pmatrix}$, where L is the lag operator, Z is the error correction term, and $B = (b_1, b_2)'$ is a 2x1 vector. The variance covariance matrix of $\begin{pmatrix} \varepsilon_G \\ \varepsilon_C \end{pmatrix}_t$ is given by:

$$\mathrm{var}\begin{pmatrix}\varepsilon_G \\ \varepsilon_C\end{pmatrix}_t = \begin{pmatrix}\sigma_{GG,t} & \sigma_{GC,t} \\ \sigma_{GC,t} & \sigma_{CC,t}\end{pmatrix} \tag{2.6}$$

We will test for autoregressive conditional heteroskedasticity in the disturbances in (2.5). In results to be reported below, we find ARCH in the retail gasoline price innovations, but not in the crude oil price innovations. We also find evidence that crude oil prices are weakly exogenous, that the error correction term does not enter the crude oil price equation, so $b_2 = 0$.

These empirical findings suggest a particular Cholesky decomposition and a particular specification of the ARCH structure of our model innovations. In particular, we model the crude oil price residual ε_C as exogenous, so ε_C has a contemporaneous effect on ε_G but ε_G does not have a contemporaneous effect on ε_C. We can write this more explicitly as:

$$\varepsilon_{Gt} = \pi_1 \varepsilon_{Ct} + \eta_t , \tag{2.7}$$

where we suppose π_1 is constant and η_t is that part of the innovation in the gasoline price equation that is uniquely determined by the gasoline market. Equation (2.7) specifies the contemporaneous correlation between ε_C and ε_G in a way consistent with our Cholesky decomposition. Note that we can substitute equation (2.7) into the bivariate VECM model (2.5) to get,

$$\begin{pmatrix}\Delta LPG \\ \Delta LPC\end{pmatrix}_t = \begin{pmatrix}A_{11}(L) & A_{12}(L) \\ A_{21}(L) & A_{22}(L)\end{pmatrix}\begin{pmatrix}\Delta LPG \\ \Delta LPC\end{pmatrix}_{t-1} + \begin{pmatrix}b_1 \\ b_2\end{pmatrix} Z_{t-1} + \begin{pmatrix}\pi_1 \varepsilon_{Ct} + \eta_t \\ \varepsilon_{Ct}\end{pmatrix} \tag{2.8}$$

From equation (8) we can derive our crude oil price equation as:

$$\Delta LPC_t = A_{21}(L) \cdot \Delta LPG_{t-1} + A_{22}(L) \cdot \Delta LPC_{t-1} + b_2 \cdot Z_{t-1} + \varepsilon_{Ct} \tag{2.9}$$

where we will eventually argue that $Var(\varepsilon_{Ct}) = \sigma_{cc}^2$, a constant.

We can also rewrite equation (2.9) to derive the crude oil price residual ε_C and substitute this into the gasoline price equation in the bivariate VECM model (2.8). Reorganizing, we obtain the following retail gasoline price equation:

$$\begin{aligned} \Delta LPG_t &= (A_{11}(L) - \pi_1 A_{21}(L)) \cdot \Delta LPG_{t-1} + \pi_1 \cdot \Delta LPC_t \\ &+ (A_{12}(L) - \pi_1 A_{22}(L)) \cdot \Delta LPC_{t-1} + b_1 \cdot Z_{t-1} + \eta_t \end{aligned} \tag{2.10}$$

Here contemporaneous crude oil prices enter as an explanatory variable, and the error term η_t is that part of the innovation in the retail gasoline price that is uniquely determined in the gasoline market. We will assume that this is the term embodying the GARCH effects.

We estimate equation (2.10) with its implicit decomposition of the gasoline price disturbance or innovation term. This equation for gasoline prices is also the form used in much of the previous research, so it allows easy comparison of our results with the previous work.

In estimating GARCH models, we specify an equation for the conditional mean and an equation for the conditional variance. For the conditional mean we look at two versions, an asymmetric ECM and an asymmetric threshold ECM. These are essentially the models reported in equations (2.3) and (2.4) above, with the explicit assumption on the disturbance term.

Asymmetric ECM:

$$\Delta LPG_t = \alpha + \sum_{i=0}^{m} (\beta_i^+ \Delta LPC_{t-i}^+ + \beta_i^- \Delta LPC_{t-i}^-) + \sum_{j=1}^{n} (\gamma_j^+ \Delta LPG_{t-j}^+ + \gamma_j^- \Delta LPG_{t-j}^-)$$
$$+ \lambda^+ (LPG_{t-1} - \varphi LPC_{t-1}) + \lambda^- (LPG_{t-1} - \varphi LPC_{t-1}) + \eta_t \tag{2.11}$$

Asymmetric Threshold ECM:

$$\Delta LPG_t = \alpha + \sum_{i=0}^{m} (\beta_i^+ \Delta LPC_{t-i} \cdot I(Z_{t-d}) + \beta_i^- \Delta LPC_{t-i}) + \sum_{j=1}^{n} (\gamma_j^+ \Delta LPG_{t-j} \cdot I(Z_{t-d})$$
$$+ \gamma_j^- \Delta LPG_{t-j}) + \lambda^+ (LPG_{t-1} - \varphi LPC_{t-1}) + \lambda^- (LPG_{t-1} - \varphi LPC_{t-1}) + \eta_t \tag{2.12}$$

For each specification, we consider the impact of autoregressive conditional heteroskedasticity (ARCH) on our estimation results. The standard LM test for ARCH indicates strong evidence for ARCH in residuals from both our asymmetric ECM models. We provide estimates of our asymmetric ECMs specified with asymmetric generalized autoregressive conditional heteroskedasticity (GARCH) errors. This means, in terms of equation (2.10), that the gasoline price residual η_t follows a GARCH process.

We extend the above asymmetric ECMs to incorporate different asymmetric GARCH specifications. In particular, we consider models with Exponential GARCH errors, GJR-GARCH errors, and Logistic Smooth Transition GARCH errors.

Asymmetric ECMs with Exponential GARCH Errors

The EGARCH model was proposed by Nelson (1991), and can capture possible asymmetries in the response volatility response.[4] Thus we specify the conditional mean as in the asymmetric ECM in equation (2.11) above or in the asymmetric threshold ECM in equation (2.12) above. For either model we specify an EGARCH model of the conditional variance as:

$$\log(\sigma_t^2) = \omega + \sum_{j=1}^{q} \eta_j \log(\sigma_{t-j}^2) + \sum_{i=1}^{p} \xi_i \left| \frac{\varepsilon_{t-i}}{\sigma_{t-i}} \right| + \sum_{k=1}^{r} \phi_k \frac{\varepsilon_{t-k}}{\sigma_{t-k}} \qquad (2.13)$$

This specification allows asymmetry in the response of the conditional variance to shocks. For instance, large positive shocks will have a greater impact on the variance than negative shocks when φ_K is positive.

Asymmetric ECMs with GJR - GARCH Errors

Another popular asymmetric conditional variance specification is introduced by Glosten, Jagannathan, and Runkle (1993), so called GJR- GARCH. The conditional variance equation will be

$$\sigma_t^2 = \omega + \sum_{j=1}^{q} \eta_j \sigma_{t-j}^2 + \sum_{i=1}^{p} \xi_i \varepsilon_{t-i}^2 + \sum_{k=1}^{r} \phi_k \varepsilon_{t-k}^2 \cdot f_k \qquad (2.14)$$

Here $f_k = 1$ when $\varepsilon_{t-k} > 0$; $f_k = 0$ when $\varepsilon_{t-k} \le 0$.

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[4] We also considered EGARCH-in-mean specifications, allowing the asymmetric volatility to directly impact the response of gasoline prices, but we found little evidence that time-varying volatility impacts the conditional mean.

Asymmetric ECMs with LST - GARCH Errors

A final more generalized model also investigate the asymmetric response of conditional variance is Logistic Smooth Transition GARCH model (LST-GARCH) which has been introduced in Gonzalez and Rivera (1998) and Hagerud (1996). The conditional variance equation of LST-GARCH(p,q) has the following form.

$$\sigma_t^2 = \omega + \sum_{j=1}^{q} \eta_j \sigma_{t-j}^2 + \sum_{i=1}^{p} (1 - F(\varepsilon_{t-i})) \cdot \xi_i \varepsilon_{t-i}^2 + \sum_{i=1}^{p} F(\varepsilon_{t-i}) \cdot \phi_i \varepsilon_{t-i}^2 \tag{2.15}$$

Here F(.) is logistic transition function,

$$F(\varepsilon_{t-i}) = \frac{1}{1 + \exp(-\lambda \varepsilon_{t-i})}, \ \lambda > 0 \tag{2.16}$$

If lambda is quite large, the LST-GARCH specification converges to the GJR-GARCH model. That is, the LST-GARCH is actually a more general form of GJR-GARCH model.[5]

2.3 Estimation Methods and Diagnostic Tests

We estimate the ECM models under homoskedasticity and also under various GARCH specifications. Estimation is by quasi-maximum likelihood. We assume the residual ε_t has a Student's t distribution, to better capture the excess kurtosis in the data. We use the Optmum procedure in GAUSS 6. When we estimate the asymmetric models, we take the GARCH estimation results as starting values. The different types of

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[5] The Appendix has a graph of the logistic smooth transition function for different values of λ. It makes clear that the bigger is lambda, the faster the transition between regimes.

asymmetric GARCH models are estimated to provide evidence on how sensitive the results are to the specific model of heteroskedasticity.

A variety of diagnostic tests are used to check that the asymmetric GARCH-type models are correctly specified. First, we examine Lagrange multiplier (LM) tests for autoregressive conditional heteroskedasticity (ARCH) in the residuals (Engle 1982). Second, we calculate the Ljung and Box (1979) Q statistics for the level and the square of the residuals, to examine possible serial correlation in the residuals. Third, we examine the ability of the models to deal with potential biases and identify potential misspecification of the conditional variance. We calculated three tests proposed by Engle and Ng (1993) -- the Sign Bias Test, the Negative Size Bias Test and the Positive Size Bias Test. We also calculate the joint test.

For these tests we define the dummy variable S_{t-1}^{-} to be one when the residual ε_{t-1} is negative and equals zero otherwise. We define S_{t-1}^{+} to be one minus S_{t-1}^{-}. Finally, we define v_t^2 as the squared normalized residuals. The sign bias test, negative size bias test and positive bias test are based on the following regressions.

$$v_t^2 = a + b \cdot S_{t-1}^{-} + e_t \tag{2.17a}$$

$$v_t^2 = a + b \cdot S_{t-1}^{-}\varepsilon_{t-1} + e_t \tag{2.17b}$$

$$v_t^2 = a + b \cdot S_{t-1}^{+}\varepsilon_{t-1} + e_t \tag{2.17c}$$

The sign bias test is based on the significance of the slope coefficient on S_{t-1}^{-} in (2.17a). The negative size bias test is based on the significance of the slope coefficient on $S_{t-1}^{-}\varepsilon_{t-1}$ in (2.17b). The positive size bias test is based on the significance test of the slope coefficient on $S_{t-1}^{+}\varepsilon_{t-1}$ in (2.17c). Engle and Ng (1993) also propose a joint test based on regression

$$v_t^2 = a + b_1 \cdot S_{t-1}^{-} + b_2 \cdot S_{t-1}^{-}\varepsilon_{t-1} + b_3 \cdot S_{t-1}^{+}\varepsilon_{t-1} + e_t \tag{2.18}$$

The t-ratios for b_1, b_2 and b_3 corresponds to the sign bias, negative size and positive size bias tests, respectively. The joint test is the LM test calculated as $T \cdot R^2$. If the variance model is correctly specified, all tests should be insignificant, which would indicate that the variables realized in the past but not included in the variance model have no power to predict the standardized residuals.

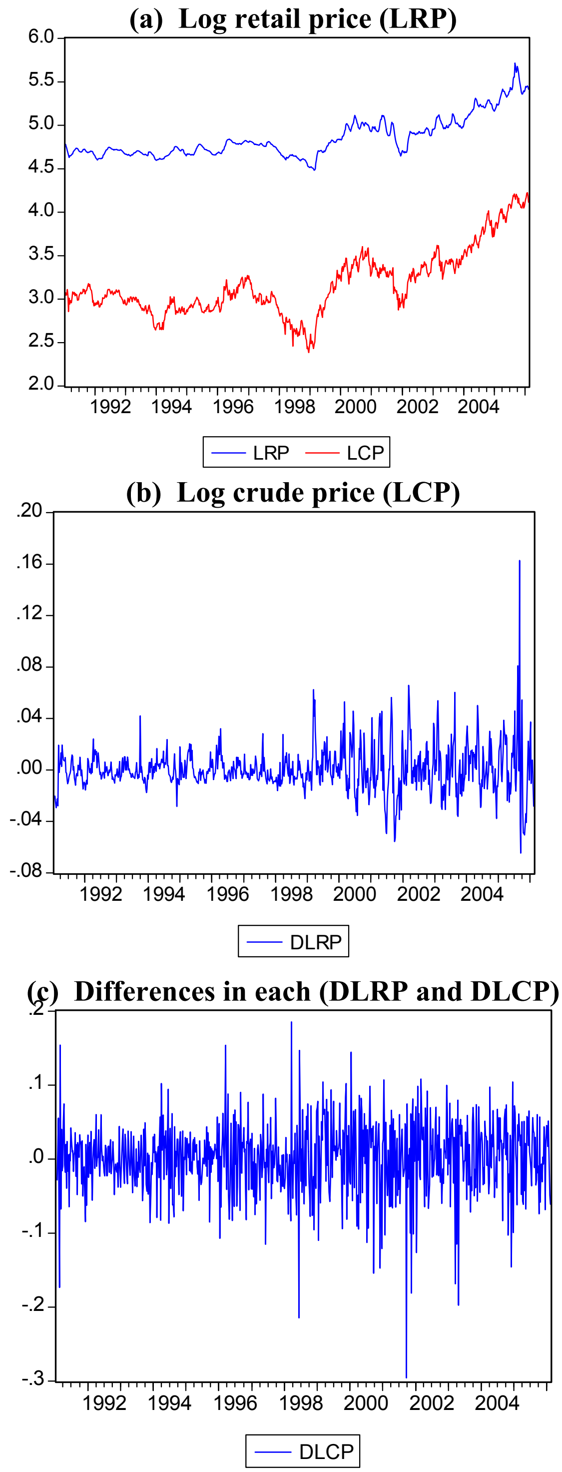


Figure 2.1: Retail Gasoline Price and Crude Oil Price (1/21/1991-2/13/2006)

2.4 Data

Our retail price series is available from the U.S. Department of Energy website[6] from 1990. We use the U.S. regular conventional retail gasoline prices and the West Texas Intermediate crude oil price. There are some holes in the data series so that we begin our sample from January 1991. Our data covers the period from January 21, 1991 to February 13, 2006, a total 787 observations. This data set is similar to the weekly data set used by BCG, but our series span a longer period. BCG's weekly data sets cover the period from March 1986 to November 1992.

Figure 2.1 (a) provides a graph of the log retail gasoline price (LRP) and the log crude oil price (LCP), weekly, from January 1991 through the middle of February 2006. Retail gasoline prices, after holding fairly steady from 1990 through 1997, exhibited some downward trend in 1998, followed by an upward trend to a new plateau of sorts by 2000, then increased volatility but an upward trend running through 2005. Crude oil prices show more apparent volatility, including a more pronounced decline during 1997 – 1998, a more dramatic upward movement in 1999, and a fairly steady upward trend from 2002 through 2005. For future reference, there seems to be a narrowing of the gap between these series after 2002.

Figure 2.1 (b) and (c) provides graphs of the log difference of gasoline prices (DLRP) and crude oil prices (DLCP). A noteworthy feature of both the gasoline price and the crude oil price date is the apparent increase in volatility. For the data

[6] http://www.eia.doe.gov/

on differences in gasoline prices, volatility seems to have increased after 1997. For the data on difference in crude oil prices, the volatility seems to have increased since 1996.

Table 2.1: Data Description, Retail Gasoline Price and Crude Oil Price (Sample Period 1/21/1991 – 2/19/2006, weekly)

	LRP	LCP	DLRP	DLCP
Mean	4.853	3.177	0.001	0.001
Standard Deviation	0.233	0.377	0.018	0.049
Skewness	1.188	0.820	1.365	-0.638
Kurtosis	3.919	3.361	13.610	5.884
Observations	787	787	786	786

Note: LRP is log retail price of gasoline; LCP is log crude oil price; DLRPis difference in LRP; DLCP is difference in LCP.

Table 2.1 summarizes some statistical features of the data. The standard deviation of crude oil prices, in levels and differences, is higher than the standard deviation of retail gasoline prices. The differenced data show exhibit fat tails, especially retail gasoline prices.

2.5 Estimation Results

We first examine the weekly data set for retail gasoline prices using the asymmetric ECM from the prior literature. A variety of diagnostic tests are conducted, to check for possible misspecification of the models. We then turn to our results for the asymmetric error correction models with GARCH errors. We then estimate various types of asymmetric GARCH error models. Finally, we present and analyze the impulse response functions of the ECM and threshold models.

Table 2.2 presents our coefficient estimates for the asymmetric ECM for weekly retail gasoline prices in column headed ECM. These are estimates for the homoskedastic model. Prior researchers have tested asymmetry with Wald tests of the hypothesis that the coefficients on the positive and negative price changes are equal. We report results of tests for asymmetry in Table 2.3. The column headed "ECM" reports tests for equality of coefficients on explanatory variables DLCP, DLCP(-1), DLRP(-1), and DLRP(-2). Note that equality of the coefficients on crude prices is not rejected, indicating no asymmetry in the adjustment of retail gasoline prices to crude oil price changes. Only the coefficients on DLRP(-1) are found to be statistically significantly different for positive and negative values of DLRP(-1).

Table 2.2: Models for Retail Gasoline Prices

	ECM		**ECM-EGARCH**	
	Coefficient	Std. Error	Coefficient	Std. Error
			Mean Equation	
C	0.003	0.001	-0.001	0.001
DLRP_P(-1)	0.234	0.044	0.355	0.043
DLRP_N(-1)	0.601	0.075	0.441	0.063
DLRP_P(-2)	0.069	0.044	0.124	0.044
DLRP_N(-2)	0.077	0.072	0.214	0.060
DLCP_P	0.076	0.020	0.055	0.011
DLCP_N	0.071	0.017	0.031	0.011
DLCP_P(-1)	0.140	0.020	0.111	0.011
DLCP_N(-1)	0.089	0.018	0.060	0.011
EC_P(-1)	-0.068	0.014	-0.032	0.011
EC_N(-1)	0.003	0.016	-0.015	0.008
			Variance Equation	
C			-0.585	0.145
$\lvert\varepsilon_{t-1}\rvert/(GARCH_{t-1})^{1/2}$			0.297	0.065
$(\varepsilon_{t-1})/(GARCH_{t-1})^{1/2}$			0.075	0.039
$Ln(GARCH_{t-1})$			0.961	0.013
T-DIST. DOF			5.039	0.767
R-squared	0.428		0.386	
S.E. of regression	0.013		0.014	
Log likelihood	2275.334		2531.968	

Table 2.3: Testing for Asymmetry

Variables		ECM	ECM-EGARCH
DLCP	F-stat.	0.025	1.695
	p-value	0.875	0.193
DLCP(-1)	F-stat.	2.680	8.638
	p-value	0.102	0.003
DLRP(-1)	F-stat.	14.091	1.004
	p-value	0.000	0.317
DLRP(-2)	F-stat.	0.008	1.178
	p-value	0.931	0.278

Note: Wald tests of equivalent coefficients on positive and negative changes in price variables)

However, we run a series of diagnostic tests on the ECM model. Results of the ARCH test are reported in Table 2.4. The first entry in Table 2.4, headed "Retail Gasoline Price Model: ECM" reports results of the ARCH test for our ECM model. Note that for lags one, two, or three, the test result is to strongly reject homoskedasticity.

Table 2.5 reports the Ljung-Box Q statistic for the residuals of the ECM model, both the levels and the squares of the residuals. We calculate the first 30 estimated residual autocorrelations. The test statistic Q(30) is 56.28 for the level of the residuals, and 100.80 for the squared residuals. The Chi-square critical value at the 5% significance level is 43.77. Thus for the traditional ECM we can strong evidence of serial correlation in both level and squared residuals.

Table 2.4: ARCH LM Tests

Retail Gasoline Price Model: ECM

Lag	1	3	5
F-statistics	19.519	21.955	15.683
Obs.*R-squared	19.092	61.030	71.745
p-value	0.000	0.000	0.000

Retail Gasoline Price Model: ECM-EGARCH

Lag	1	3	5
F-statistics	0.030	0.213	0.323
Obs.*R-squared	0.030	0.642	1.626
p-value	0.862	0.887	0.899

Crude Oil Price Model

Lag	1	3	5
F-statistics	3.262	1.046	1.250
Obs.*R-squared	3.257	3.143	6.249
p-value	0.071	0.371	0.284

Retail Gasoline Price Model: Threshold ECM

Lag	1	3	5
F-statistics	28.432	20.637	15.369
Obs.*R-squared	27.503	57.637	70.440
p-value	0.000	0.000	0.000

Retail Gasoline Price Model: Threshold ECM-EGARCH

Lag	1	3	5
F-statistics	0.160	0.207	0.368
Obs.*R-squared	0.160	0.623	1.349
p-value	0.690	0.892	0.930

Table 2.5: Ljung-Box Q Statistics
(Homoskedastic ECM versus ECM-EGARCH)

	Levels of the residuals				Squares of the residuals			
Lags	ECM		ECM-EGARCH		ECM		ECM-EGARCH	
	Q-Stat	Prob	Q-Stat	Prob	Q-Stat	Prob	Q-Stat	Prob
1	0.34	0.56	0.14	0.71	19.19	0.00	0.03	0.86
2	5.42	0.07	1.78	0.41	23.06	0.00	0.41	0.82
3	19.59	0.00	5.42	0.14	69.26	0.00	0.65	0.89
4	26.49	0.00	5.50	0.24	95.28	0.00	0.75	0.95
5	27.38	0.00	7.05	0.22	96.26	0.00	1.62	0.90
6	27.68	0.00	7.79	0.25	96.53	0.00	2.38	0.88
7	28.32	0.00	8.77	0.27	97.14	0.00	3.05	0.88
8	28.54	0.00	8.80	0.36	99.18	0.00	3.87	0.87
9	32.51	0.00	9.72	0.37	99.21	0.00	3.99	0.91
10	32.52	0.00	10.85	0.37	99.22	0.00	4.57	0.92
11	34.04	0.00	11.40	0.41	99.30	0.00	5.31	0.92
12	34.04	0.00	11.61	0.48	99.37	0.00	5.39	0.94
13	35.70	0.00	14.39	0.35	99.38	0.00	5.67	0.96
14	35.83	0.00	14.90	0.39	99.40	0.00	5.73	0.97
15	36.60	0.00	15.03	0.45	99.40	0.00	6.12	0.98
16	37.08	0.00	16.03	0.45	99.44	0.00	6.17	0.99
17	37.47	0.00	16.03	0.52	99.64	0.00	6.22	0.99
18	39.57	0.00	17.46	0.49	99.67	0.00	6.22	1.00
19	42.36	0.00	19.60	0.42	99.76	0.00	6.24	1.00
20	44.68	0.00	20.94	0.40	100.03	0.00	6.27	1.00
21	44.68	0.00	21.10	0.45	100.04	0.00	7.05	1.00
22	45.08	0.00	21.51	0.49	100.19	0.00	7.09	1.00
23	45.10	0.00	22.00	0.52	100.21	0.00	7.12	1.00
24	49.00	0.00	25.68	0.37	100.23	0.00	7.48	1.00
25	50.16	0.00	27.13	0.35	100.24	0.00	7.61	1.00
26	53.45	0.00	28.61	0.33	100.30	0.00	7.84	1.00
27	54.00	0.00	28.97	0.36	100.34	0.00	8.25	1.00
28	54.84	0.00	29.09	0.41	100.51	0.00	8.25	1.00
29	56.26	0.00	31.15	0.36	100.77	0.00	9.11	1.00
30	56.28	0.00	31.29	0.40	100.80	0.00	9.16	1.00

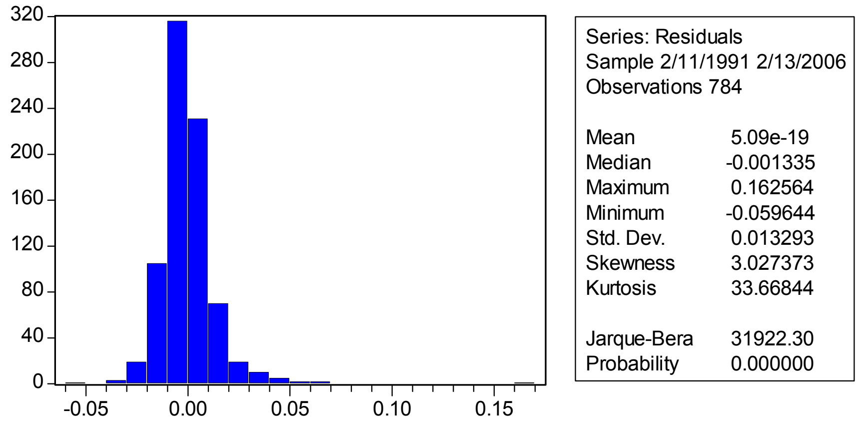


Figure 2.2: Residuals from the Traditional Homoskedastic ECM

These diagnostic tests suggest that the traditional ECM is misspecified, so we turn to models allowing heteroskedasticity. Estimation results are reported in Table 2.2 under the column headed ECM-EGARCH. For the ECM-EGARCH model, we report estimates for both the mean equation and the variance equation. We specify that the conditional distribution of disturbances is a t-distribution and estimate the degrees of freedom as five. We do this because the residuals from the homoskedastic ECM show strong kurtosis, as can be seen in Figure 2.2.

For the ECM with EGARCH errors we cannot reject the null of no remaining ARCH, as can be seen in Table 2.4 under the heading “Retail Gasoline Price Model: ECM-EGARCH.” The probability values are hugely greater than even ten percent for lags one, two, or three. Further, we report results of the Ljung-Box Q statistic in Table 2.5 under the headings “ECM-EGARCH.” We again calculate the first 30 estimated residual autocorrelations, and the statistic Q(30) equals 31.29 for the level of the residuals and 9.16 for the squared residuals. The Chi square critical value for the 5% significance level is 43.77. Thus for our ECM specification with EGARCH errors we find no serial correlation in the levels or squares of the residuals.

We report asymmetry tests in Table 2.3. Recall that when we conduct this test for the ECM with homoskedastic errors we can't reject symmetry for the coefficients on crude oil price changes, a finding that is in accord with BG's conclusion. Note, however, that when we conduct this test for our ECM-EGARCH model we strongly reject the null of symmetry of the crude oil price coefficients. The probability value for this hypothesis test is 0.003. Thus we find that allowing EGARCH errors is key for our asymmetry result, and that BG's conclusion to the contrary is due to specifying homoskedastic errors.

Table 2.6: Sign and Size Bias Tests

Models		Sign bias test	Negative size bias test	Positive size bias test	Joint test	
ECM-ARCH	coef.	-0.480	-15.544	-13.269	F stat.	1.504
	t stat.	-2.054	-1.117	-1.270	Chi square	4.512
	prob.	0.040	0.264	0.205	Prob.	0.212
ECM-GARCH	coef.	-0.386	-2.139	-5.435	F stat.	1.081
	t stat.	-1.617	-0.145	-0.517	Chi square	3.242
	prob.	0.106	0.885	0.605	Prob.	0.356
ECM-EGARCH	coef.	-0.324	-9.208	-0.207	F stat.	0.558
	t stat.	-1.176	-0.521	-0.017	Chi square	1.675
	prob.	0.240	0.603	0.986	Prob.	0.643

We also calculate the sign and size bias tests for our ECM-EGARCH model, and report the results in Table 2.6. We cannot reject the hypothesis that the residuals from our ECM-EGARCH model have not sign bias, negative size bias, or positive size bias at even the ten percent significance level. To compare this model with others, we also estimated an ECM-ARCH model and an ECM-GARCH model. We do not report the model estimates here, but the diagnostic tests in Table 2.6 indicate that the ECM-ARCH model exhibits sign bias, and the ECM-GARCH model marginally rejects sign bias at the ten percent significance level. The probability value is 0.106, providing some motivation for estimating asymmetric GARCH-type models.

Table 2.7 reports estimation results for two other asymmetric GARCH type models – the GJR-GARCH and logistic smooth transition GARCH (LST-GARCH). We also include the EGARCH model results from Table 2 for ease of comparison. Notice that the estimates of the mean equation of these three models are quite close; the asymmetry conclusion is not sensitive to the different asymmetric conditional variance specifications. In terms of the two new models, the parameter lambda in the LST-GARCH model is quite large, indicating a fast transition between the two regimes. In this case the LST-GARCH model converges to the GJR-GARCH model. Finally, in the different models of the conditional variance the terms governing asymmetry are statistically significant, and all coefficients estimates indicate the asymmetry is such that positive shocks have bigger effects than negative shocks.

Table 2.7: Models for Retail Gasoline Price: Asymmetric GARCH Type Models

	ECM-EGARCH			ECM-GJR-GARCH			ECM-LST-GARCH	
	Coefficient	Std. Error		Coefficient	Std. Error		Coefficient	Std. Error
	Mean Equation			Mean Equation			Mean Equation	
C	-0.001	0.001		0.000	0.001		0.000	0.001
DLRP_P(-1)	0.355	0.043		0.337	0.055		0.337	0.055
DLRP_N(-1)	0.441	0.063		0.458	0.061		0.458	0.061
DLRP_P(-2)	0.124	0.044		0.133	0.052		0.133	0.052
DLRP_N(-2)	0.214	0.060		0.188	0.059		0.188	0.059
DLCP_P	0.055	0.011		0.053	0.013		0.053	0.013
DLCP_N	0.031	0.011		0.034	0.010		0.034	0.010
DLCP_P(-1)	0.111	0.011		0.100	0.013		0.100	0.013
DLCP_N(-1)	0.060	0.011		0.068	0.011		0.068	0.011
EC_P(-1)	-0.032	0.011		-0.033	0.009		-0.033	0.009
EC_N(-1)	-0.015	0.008		-0.013	0.009		-0.013	0.009
	Variance Equation			Variance Equation			Variance Equation	
c	-0.585	0.145	c	0.000	0.000	c	0.000	0.000
$\lvert\varepsilon_{t-1}\rvert/(GARCH_{t-1})^{1/2}$	0.297	0.065	ε_{t-1}^{2}	0.391	0.140	$(1-F)\varepsilon_{t-1}^{2}$	0.391	0.140
$(\varepsilon_{t-1})/(GARCH_{t-1})^{1/2}$	0.075	0.039	$GARCH_{t-1}$	0.402	0.050	$GARCH_{t-1}$	0.402	0.050
$Ln(GARCH_{t-1})$	0.961	0.013	$f^{*}\varepsilon_{t-1}^{2}$	0.578	0.229	$F\varepsilon_{t-1}^{2}$	0.578	0.227
						lambda	41320	375059
T-DIST. DOF	5.039	0.767		3.817	0.661		3.817	0.640
Log likelihood	2531.968			2502.041			2502.041	

In order to better interpret their results, both BCG and BG provide a type of impulse response function. These trace out the response of gasoline prices to a temporary one-period shock to crude oil prices, where crude oil prices are treated as exogenous and future values of crude oil prices do not respond to the shock to crude oil prices. We present this impulse response function in Figure 2.3 for the homoskedastic ECM model and the ECM-EGARCH model. We graph the absolute value of the response to a negative and a positive shock to crude oil prices, in order to better see the possible asymmetric response to increases and decreases in crude oil prices. It is readily apparent that the homoskedastic ECM is quite symmetric, while asymmetry is stronger for the ECM/EGARCH model, at least in terms of the difference between the response to a positive and a negative shock to the price of crude. Thus the impact on gasoline prices of a positive shock to crude oil prices is greater in absolute value than the response of gasoline prices to a negative shock to crude oil prices. Moreover, the difference persists over time, and hints at a rockets and feathers hypothesis, since crude prices rise faster than they fall in response to a crude shock.

The impulse response function in Figure 2.3 illustrates the asymmetry in response to a crude oil shock. However, these are not standard impulse response functions in that they hold crude oil prices constant after the shock. We also present more standard impulse response functions that show the response of gasoline prices to crude oil prices shocks – and to gasoline price shocks – in a system context. These impulse response functions obviously require us to model crude oil prices as well as gasoline prices.

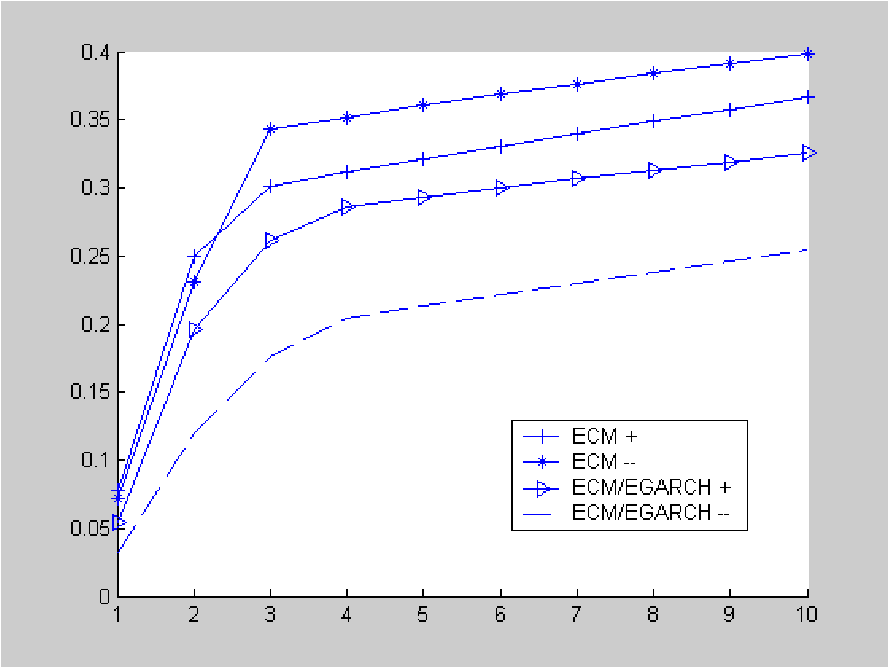


Figure 2.3: Impulse Response Function: BG and BCG Specifications

Table 2.8: Model for Crude Oil Price

Dependent Variable:	DLCP	
Variable	Coefficient	Std.Error
C	0.002	0.002
DLRP(-1)	0.256	0.124
DLRP(-2)	0.217	0.123
DLRP(-3)	-0.304	0.112
DLCP(-1)	-0.137	0.037
DLCP(-2)	-0.141	0.039
R-squared	0.037	
S.E. of regression	0.048	
Log likelihood	1266.06	

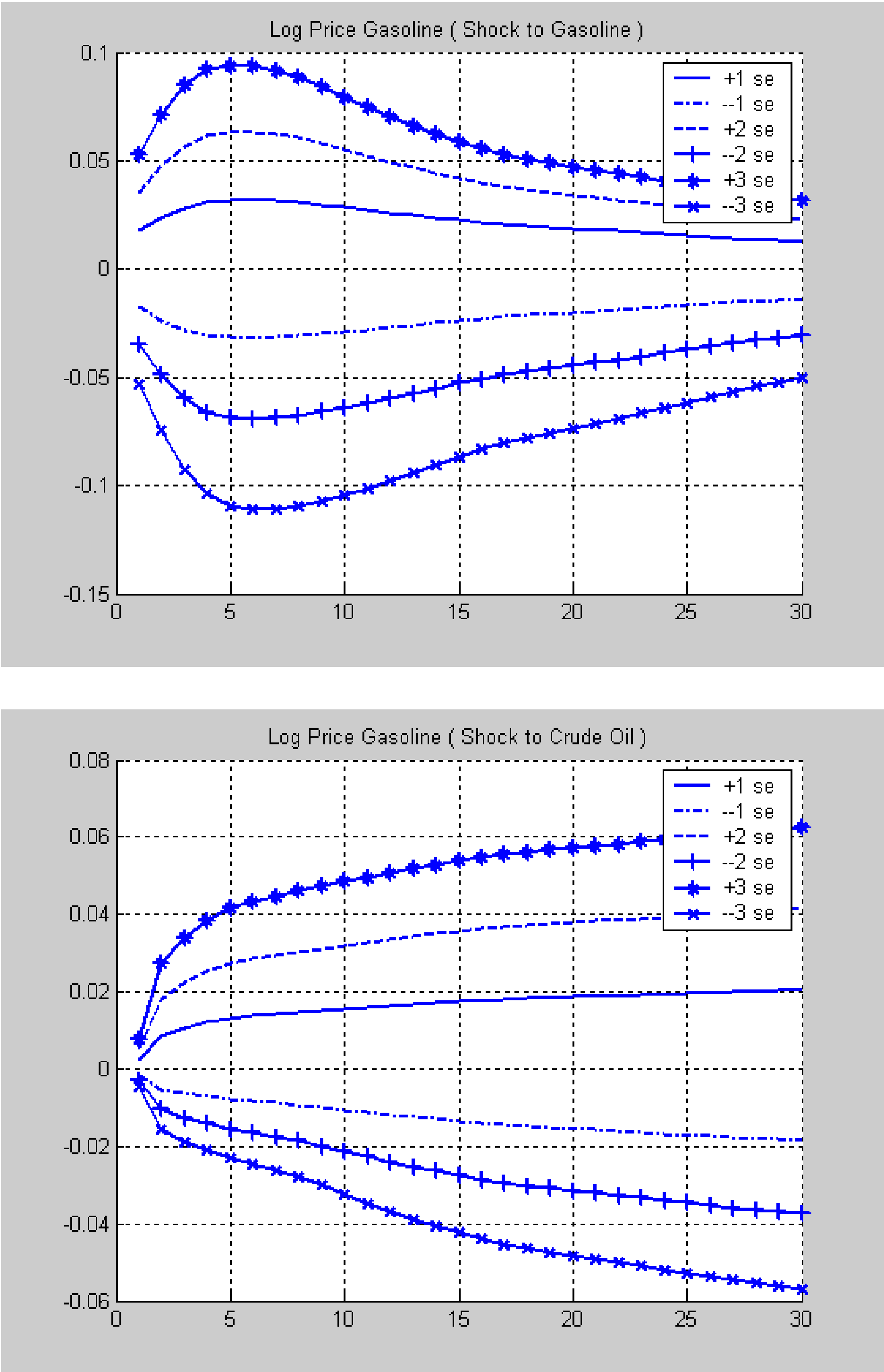


Figure 2.4: Impulse Response Functions: System-Wide ECM-EGARCH

Table 2.8 shows the estimation results for the crude oil price equation. We found the error correction term was not significant when we estimate the crude oil price model, indicating that crude oil prices are weakly exogenous. We conducted diagnostic ARCH LM tests, and report these results in Table 2.4 under the heading "Crude Oil Price Model." Note that we cannot reject the null hypothesis of no autoregressive conditional heteroskedasticity in the crude oil price model. These empirical results support our assumption that the disturbance term in the crude oil price equation is homoskedastic.

Figure 2.4 presents impulse response functions for our ECM-EGARCH model. For nonlinear models the state of the world at the time of the shock is important. We picked April 15, 1991 as the day of the shock, a day near the beginning of our sample and not subject to extreme events.

The top panel in Figure 2.4 illustrates the response of gasoline prices to a shock to gasoline prices. There is little evidence of asymmetry. Both a positive and a negative three standard error shock generates just over a five percent magnitude change in gasoline price for the first period. The impact of a positive shock increases close to ten percent at the fifth period. The impact of negative shock increases to over ten percent at the fifth period. The 'rocket and feather' effect does not exist in the gasoline response to a shock to gasoline prices. This type of gasoline price – to - gasoline price impulse response function was not investigated in BG and BCG.

The bottom panel in Figure 2.4 illustrates the response of gasoline prices to a shock to crude oil prices. Here, unlike the impulse response function in Figure 2.3, crude oil prices are themselves allowed to respond to the shock. Most important for our purposes, the impact of a crude oil price shock on gasoline prices shows a strong persistence for all shocks both positive and negative. Indeed there is a tendency for the positive shock to have a bigger impact on gasoline prices. The positive three standard error shock generate gasoline price increases of 4.2 percent, but the negative three standard error shock generate gasoline price decreases of 2.3 percent until period 5. For the positive shock gasoline prices take 11 periods to increase 5 percent, while for the negative shock gasoline prices takes 22 periods to decrease 5 percent.

Figure 2.5 presents confidence intervals for the impulse response functions. These are empirical confidence intervals generated by Monte Carlo simulation with 1,000 draws. In order to facilitate presentation we illustrate these confidence intervals for a two standard error impulse to gasoline prices (the top panel) and crude oil prices (the bottom panel). Note that, in the top panel, the confidence intervals easily overlap between positive and negative shocks. However, in the bottom panel the confidence intervals do not overlap, at least at short horizons. At the fifth period the confidence band for a positive shock ranges roughly over the interval .023 ~ .034, whereas for a negative shock the confidence band ranges roughly over the interval -.009 ~ -.022.

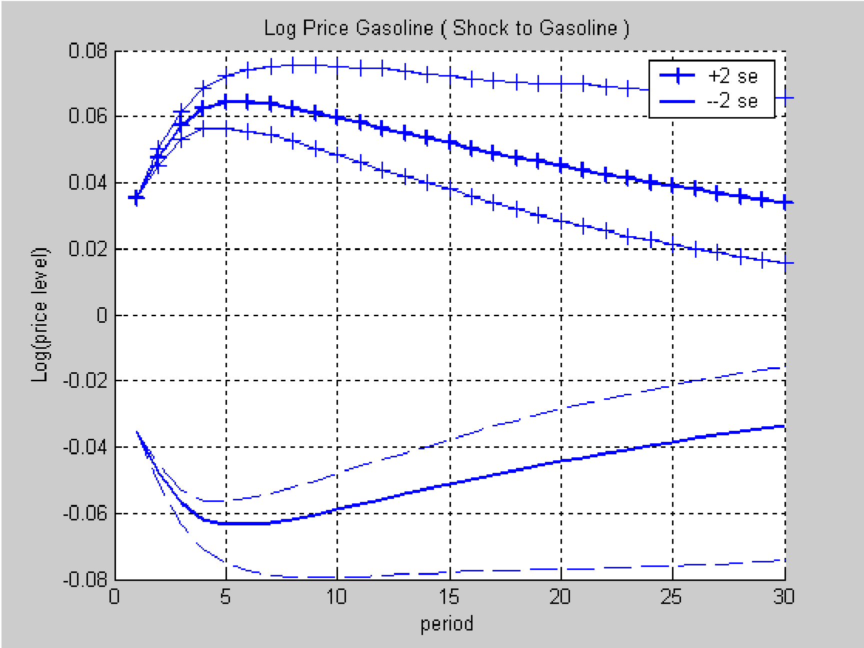
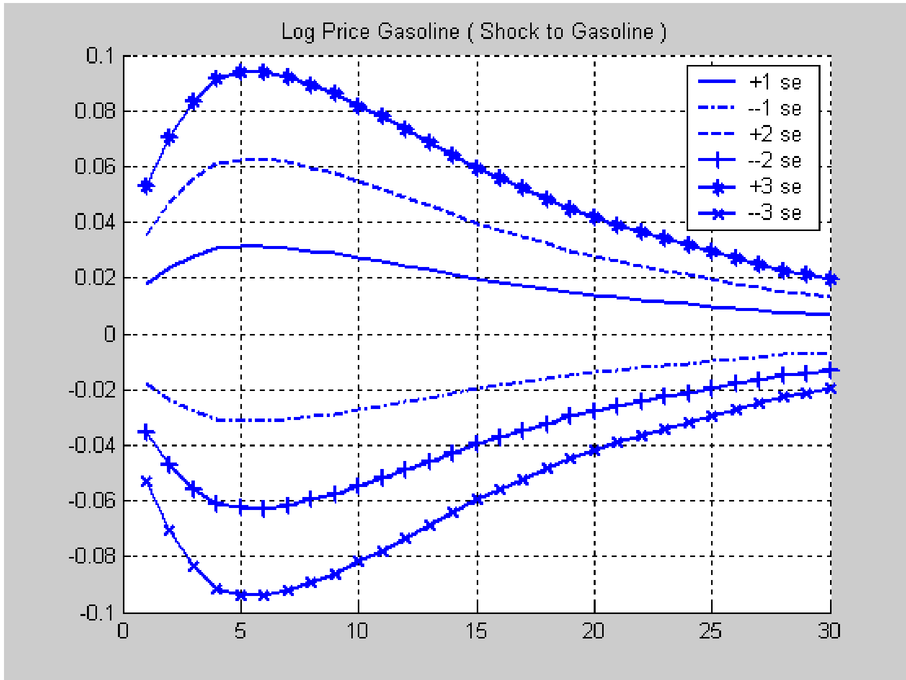


Figure 2.5: Impulse Response Functions: System-Wide ECM-EGARCH (90% Confidence Bands)

Table 2.9: Model for Retail Gasoline Price: Threshold Models

Threshold Variable	DLCP				DLCP(-1)				DLRP(-1)			
	Threshold_ECM		Threshold_Egarch		Threshold_ECM		Threshold_Egarch		Threshold_ECM		Threshold_Egarch	
	Coefficient	Std. Error	Coefficient	Std. Error	Coefficient	Std. Error	Coefficient	Std. Error	Coefficient	Std. Error	Coefficient	Std. Error
			Mean Equation				Mean Equation				Mean Equation	
DLRP(-1)	0.170	0.044	0.363	0.045	0.308	0.044	0.348	0.049	0.631	0.065	0.475	0.057
DLRPG(-1)*Z	0.378	0.064	0.055	0.064	0.070	0.065	0.063	0.062	-0.408	0.090	-0.128	0.084
DLRPG(-2)	0.271	0.050	0.192	0.046	-0.004	0.040	0.136	0.044	0.070	0.041	0.170	0.044
DLRPG(-2)*Z	-0.314	0.064	-0.046	0.062	0.196	0.064	0.077	0.060	-0.016	0.063	-0.036	0.062
DLPC	0.072	0.017	0.031	0.011	0.064	0.015	0.029	0.010	0.042	0.014	0.031	0.008
DLPC*Z	0.009	0.030	0.029	0.018	0.015	0.020	0.027	0.012	0.068	0.020	0.031	0.012
DLPC(-1)	0.113	0.017	0.067	0.010	0.088	0.018	0.060	0.011	0.105	0.013	0.076	0.007
DLPC(-1)*Z	-0.015	0.022	0.023	0.013	0.042	0.032	0.049	0.018	0.017	0.022	0.021	0.013
EC(-1)	-0.038	0.011	-0.022	0.007	-0.072	0.011	-0.022	0.007	-0.027	0.011	-0.019	0.006
EC(-1)*Z	-0.009	0.016	-0.001	0.009	0.050	0.016	-0.006	0.009	-0.015	0.016	-0.003	0.009
c	0.000	0.001	-0.001	0.000	0.000	0.001	-0.001	0.000	0.003	0.001	0.000	0.000
			Variance Equation				Variance Equation				Variance Equation	
c			-0.754	0.177			-0.435	0.123			-0.814	0.182
$\lvert\varepsilon_{t-1}\rvert/(GARCH_{t-1})^{1/2}$			0.364	0.073			0.254	0.055			0.356	0.073
$(\varepsilon_{t-1})/(GARCH_{t-1})^{1/2}$			0.079	0.043			0.068	0.034			0.092	0.045
$Ln(GARCH_{t-1})$			0.948	0.016			0.973	0.011			0.941	0.017
df			5.243	0.817			4.770	0.717			4.928	0.766
R-squared	0.434		0.378		0.427		0.381		0.431		0.388	
S.E. of regression	0.013		0.014		0.013		0.014		0.013		0.014	
Log likelihood	2279.47		2529.76		2274.49		2533.10		2276.87		2531.33	



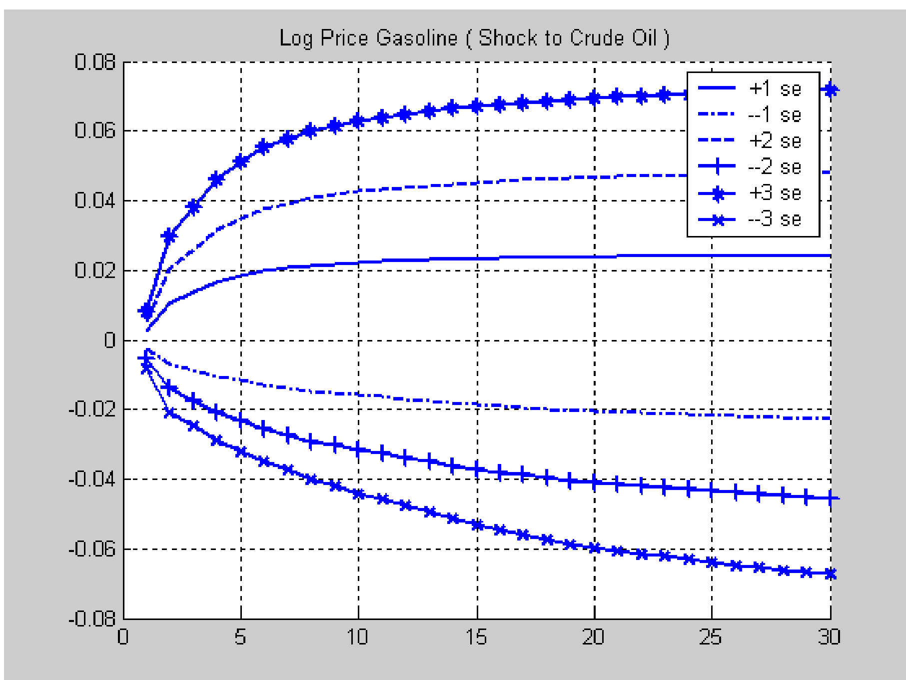


Figure 2.6: Impulse Response Functions: System-wide Threshold ECM-EGARCH

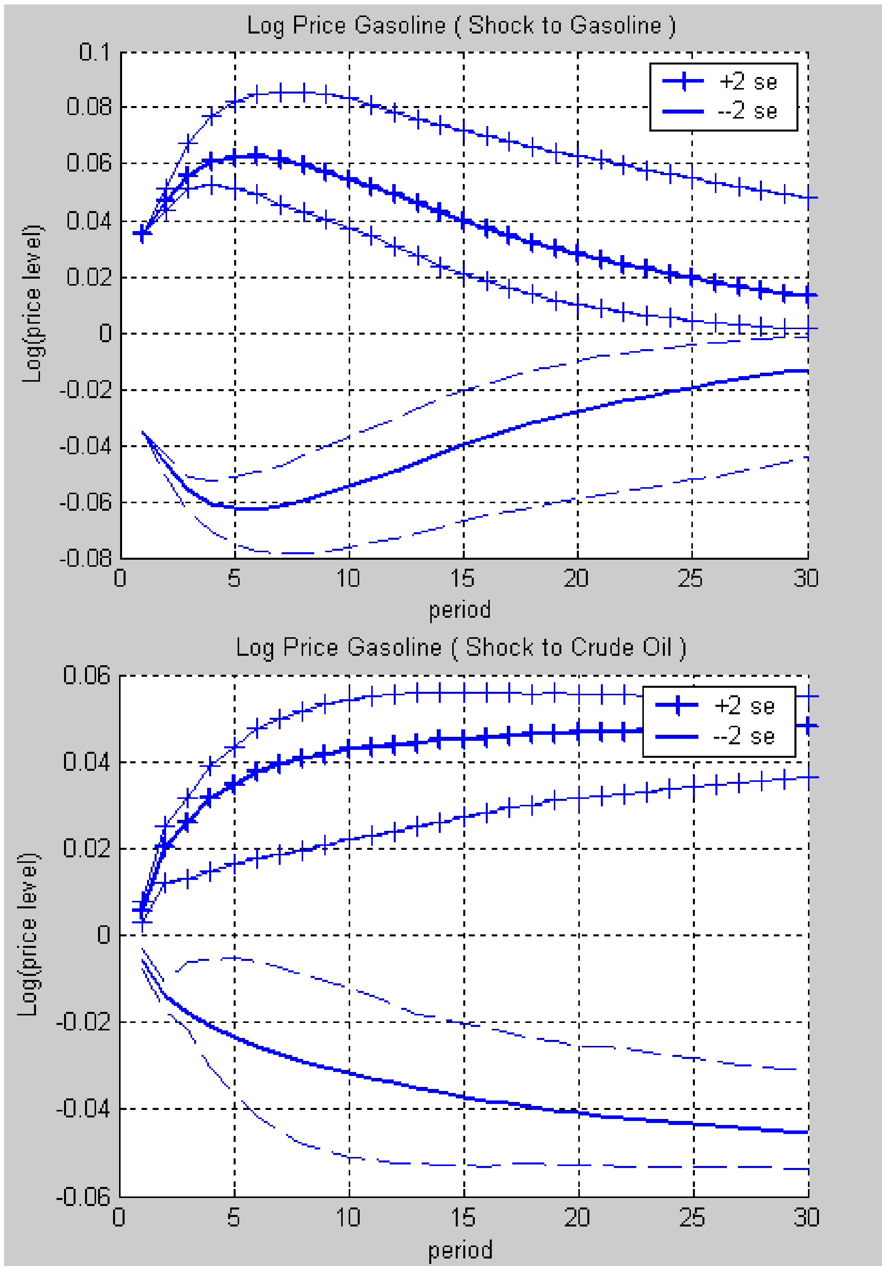


Figure 2.7: Impulse Response Functions: System-wide Threshold ECM-EGARCH (90% confidence bands)

We conclude from these impulse response functions that the system-wide impacts of an impulse to crude prices is asymmetric, with a tendency for gasoline prices to respond more (in magnitude) to positive shocks and a tendency for gasoline prices to exhibit more persistence in response to positive shocks.

We also estimate standard asymmetric threshold ECM models, with results reported in Table 2.9. We report six sets of results. We report results for three different choices of the threshold variable: DLCP, DLCP(-1), and DLRP(-1). For each choice of threshold variable, we report estimates of both a homoskedastic ECM model and an ECM-EGARCH model. There are several things to note in this table. First, the EGARCH models are preferred on statistical grounds to the homoskedastic models. The homoskedastic models all fail an ARCH test. The ECM-EGARCH models all have much higher likelihood values than their homoskedastic counterparts.[7] Second, the interaction terms involving the threshold variables are not always, nor even mostly, significant, though there is usually at least one significant interaction term in each equation, especially each EGARCH specification. Finally, the log likelihood values give us little reason to prefer one threshold variable over the other, although there is a slightly better fit when the threshold variable is the lag change in the gasoline price, DLRP(-1).

Figure 2.6 presents impulse response functions for the model with the threshold variable as the lag of the crude oil price changes[8], DLCP(-1). The top panel shows the response of wholesale gasoline prices to a shock in the gasoline price equation. There is

[7] ARCH test statistics are reported in Appendix Tables 4(1).
[8] We present the model with threshold variable DLCP(-1) to facilitate a forecasting exercise we describe later in the text. That model also fits in-sample slightly better than the alternatives.

little obvious asymmetry here. The bottom panel in Figure 2.6 illustrates the response of gasoline prices to a shock to crude oil prices. Here there is more evidence of asymmetry. Positive shocks have a larger impact, especially large positive shocks. The positive three-standard-error shock causes an increase in gasoline prices of 5.1% at period five, while the negative three-standard-error shock causes a decrease in gasoline prices of 3.1%. For the positive shock gasoline price takes 8 periods to increase 6%, while for the negative shock gasoline price takes 21 periods to decrease 6%. All crude shocks exhibit strong persistent effects on gasoline prices, with positive shocks exhibiting a somewhat greater magnitude of response than negative shocks. Thus the impulse response functions indicate a rockets and feathers type pattern in the response of gasoline prices to crude oil shocks similar to that documented by earlier researchers.

One caveat to interpretations of the impulse response functions in Figure 2.6 is the wide confidence bands we find and illustrate in Figure 2.7. The estimated impulse response functions for the threshold models are less precise, and hence the confidence bands overlap at all horizons.

2.6 Prediction Tests and Forecasts

An alternative method used in BG's paper to choose between the symmetric and asymmetric model is based on an out-of-sample forecasting comparison. BG find that their symmetric model gives better forecasts than their asymmetric model. We conduct a small out-of-sample forecast comparison based on our symmetric ECM and our asymmetric ECM-EGARCH models. We choose two forecasting horizons, one week ahead and four weeks ahead. Three models are examined here -- the symmetric model, the asymmetric ECM-EGARCH, and the asymmetric threshold ECM-EGARCH errors.

We consider a variety of forecasting periods: 50 weeks, 100 weeks, 150 weeks, and 200 weeks. Results are reported in Table 2.10. We find for most cases that, no matter the forecasting horizon, the ECM-EGARCH model beats the symmetric model. For the one-step ahead forecast over 100 weeks, the sum of squared prediction errors of the symmetric model is 0.075, while for the ECM-EGARCH model it is 0.072. The threshold ECM-EGARCH model also beats the symmetric model at the shorter horizon, but not at the longer horizon. Comparing the two asymmetric models, the threshold ECM-EGARCH tends to beat the ECM-EGARCH at the one-week horizon, but not at the four-week horizon.

Table 2.10: Forecast Comparison

		(1)	(2)	ratio	(3)	ratio
Horizon	Forecast Periods	Symmetric ECM	ECM-EGARCH	(2)/(1)	Threshold ECM- EGARCH	(3)/(1)
h=1	50	0.0656	0.0644	0.9819	0.0635	0.9687
	100	0.0747	0.0716	0.9587	0.0726	0.9719
	150	0.0864	0.0863	0.9985	0.0848	0.9810
	200	0.0958	0.0961	1.0031	0.0945	0.9859
h=4	50	0.0638	0.0622	0.9749	0.0645	1.0113
	100	0.0729	0.0718	0.9848	0.0739	1.0138
	150	0.0845	0.0840	0.9939	0.0866	1.0252
	200	0.0939	0.0939	0.9990	0.0965	1.0271

2.7 Further Discussion on Asymmetric Gasoline Price Response

In this paper, we estimate the system wide gasoline price and crude oil price response system and produce the impulse response function based on this system. More important, our analysis on system wide gasoline crude oil price response suggests a way to decompose the crude oil volatility and gasoline price volatility, and our empirical results support the idea that gasoline price volatility follows an EGARCH process. These empirical results and the characteristics of the data suggest that both crude oil price volatility and gasoline price volatility play an important role determining the asymmetric response of gasoline prices to crude oil prices.

Many theoretical papers have discussed reasons for the asymmetry of gasoline price response. Some popular explanations include oligopoly theory, search costs, inventory management, and the behavior of markup over the business cycle. BCG (1997) discuss several explanations for the apparent asymmetry. One is these is based on search theory, crude oil volatility, and a signal-extraction model. They argue that increased volatility of crude oil prices caused reduced consumer search in response to an increase in gasoline prices. This reduced search leads to a temporary increase in market power for retail gasoline dealers, and an asymmetry in the response of retail gasoline prices to an increase in crude prices.

Interestingly, Peltzman (2000) and Radchenko (2005) both report empirical evidence that the correlation between asymmetry and crude volatility is negative. This evidence is contrary to the argument outlined in BCG. We suggest a way to reconcile these discrepancies is to consider a signal extraction model that takes explicit account of both crude oil price volatility and retail gasoline price volatility. That is, crude oil price volatility may lead to retail price volatility, but retail prices may be volatile for reasons other than just crude oil price volatility. We present a simple signal extraction model in the appendix to illustrate our arguments, and show that our model suggests the following. First, the correlation between crude oil price volatility and asymmetry is indeed positive, as suggested by BCG. Second, the correlation between retail gasoline price volatility (other than that caused by crude oil price volatility) and asymmetry is negative. Because retail price volatility seems to have increased by more than crude oil price volatility in the second half of our sample, our model would suggest that asymmetry has declined for this reason. Our model might also reconcile Peltzman's empirical findings, because he looked at crude volatility and asymmetry, without controlling for retail price volatility.[9]

2.8 Conclusion

There is a debate in the literature concerning the existence and character of asymmetry in the response of gasoline prices to crude oil shocks. Borenstein, Cameron and Gilbert (1997) claim that retail gasoline prices respond asymmetrically to crude oil price changes, while Bachmeier and Griffin (2003) find no evidence of asymmetry in both retail and wholesale gasoline prices if the two-step Engle-Granger estimation procedure is used. We also get this result. That is, using the two-step Engle-Granger estimation procedure and assuming homoskedasticity, we can't reject the hypothesis of a symmetric response of gasoline prices to a crude oil price shock. However, we find that the homoskedastic ECM does not pass a battery of diagnostic tests.

[9] In the appendix we also graph the markup of retail gasoline prices over crude oil prices. Our signal extraction model predicts this would decrease with increased retail gasoline price volatility, and indeed this seems to be the case in the later part of our sample.

We estimate an ECM with EGARCH errors, as well as several other models allowing asymmetry in the conditional variance. This model does pass our diagnostic tests, and our analysis of impulse response functions indicates substantial asymmetry in both magnitude of response, speed of response, and persistence. Thus we conclude that asymmetry exists, and it requires estimation of an ECM with heteroskedastic errors in order to reliably detect this asymmetry. Our impulse response functions allow us to characterize this asymmetry, and for our models including the many-state model of BCG and a standard two-state threshold model, we find impulse response function patterns that are consistent with Bacon's rockets and feathers claim. That is, gasoline prices seem to rise more quickly in response to a positive crude oil price shock than they fall in response to an equivalent magnitude but negative crude oil price shock. Moreover, we find that gasoline prices respond in somewhat larger magnitude to a positive crude oil price shock, and the response to a positive shock is somewhat more persistent.

CHAPTER III

FISCAL DEFICITS AND EXCHANGE RATES

3.1 Introduction

The effect of monetary policy on exchange rates has been extensively explored (e.g., Eichenbaum and Evans, 1995; Almeida, Goodhart and Payne, 1998; Bonsar-Neal, Roley and Sellon, 2000; Andersen et al., 2003; Engel and West, 2005). Indeed, monetary fundamentals are typically considered to be among the most important economic factors driving exchange rate movements (as suggested, e.g., by the monetary model of exchange rate determination.) In fact, some argue that foreign exchange market reactions to macroeconomic news lies in market participants' anticipation of how the monetary authority will react to the news (Almeida, Goodhart, and Payne, 1998, p.406). Early theoretical works (e.g., Obstfeld, 1985; Frenkel and Razin, 1986; Devereux and Purvis, 1990) found that both monetary and fiscal policy could have substantial effects on floating exchange rates, although little serious attention has been paid to empirically exploring the role of fiscal policy in foreign exchange markets. A notable exception is Chinn (1997), perhaps the first comprehensive empirical work on the topic, who documents the difficulties in finding evidence of the role of fiscal policy on exchange rates based on various additive model specifications. Cheung, Chinn and Pascual (2005) also consider an additive regression model and find weak evidence that fiscal deficits may affect real exchange rate movements. Nevertheless, the early empirical work typically views fiscal policy as a separate tool from monetary policy and consider its stand-alone effects on exchange rates.

Recently, the literature has underscored the potentially complex interaction between monetary and fiscal policies (Sargent, 1999; Dixit and Lambertini, 2003; Linnemann and Schabert, 2003; Schabert, 2004). These recent works exploit the fact that monetary policy may not be independent of the fiscal decisions of governments, which can be binding constraints on the effectiveness of monetary policy. Moreover, the fiscal theory of price level (e.g., Leeper, 1991; Sims, 1994; Woodford, 1995; Canzoneri, Cumby, and Diba, 2001) emphasizes that fiscal policy alone can play a crucial role in affecting the price level and, potentially, both inflation and exchange rates.[10]

Economic theory predicts that changes in fiscal deficits should have an important impact on foreign exchange rate movements, but existing empirical work only finds weak evidence supporting the theory. This paper uses the G5 countries' real exchange rate data to show that if one only allows for fiscal policy variables to enter an exchange rate regression model additively – even if nonlinearly -- then fiscal policy variables are not significant in explaining real exchange rate movements. However, if one allows for fiscal deficits to affect exchange rates non-additively and nonlinearly, fiscal policy variables play an important (and statistically significant) role in explaining the real exchange rate dynamic adjustments.

[10] In particular, the fiscal theory of the price level argues that fiscal policy, rather than monetary policy, determines the general price level and inflation. Specifically, it views the government intertemporal budget constraint as an equilibrium condition where the price level adjusts to accommodate changes in fiscal conditions.

Our research contributes to the literature in two important aspects. First, it fills the gap in the literature by allowing for a fiscal policy variable (the changes in fiscal deficits) to act as a conditioning variable in modeling exchange rate movements. This implicitly addresses the role of fiscal policy via its interactions with monetary policy. Essentially, "monetary policy can be constrained by fiscal policy if fiscal deficits grow large enough to require monetization of government debt" (Sargent, 1999, p.1463). The existing literature mostly considers the fiscal policy variables as direct information variables, which assume the role of fiscal policy as a policy tool separate from monetary policy. That approach, since it does not allow for the possible non-additive and nonlinear interaction between fiscal and monetary policy variables, may fail to reveal the impact of fiscal policy variables on the exchange rate determination. Second, we employ recently developed flexible varying coefficient model (Cai, Fan, Yao, 2000; Ahmad, Leelahanon and Li, 2005) in our econometric analysis. This model appears to be particularly suitable for modeling fiscal deficits as a conditioning variable. The idea that fiscal deficits may effectively work as a binding constraint on monetary policy can hardly be modeled by a parametric (linear or nonlinear) model, as no theory has made an explicit suggestion about functional forms. In this regard, our method improves on the popular parametric nonlinear smooth transition models used in the real exchange rate mean reversion literature (e.g., Michael, Nobay, and Peel, 1997; Taylor and Peel, 2000).

The rest of this paper is organized as follows: Section 2 discusses the relationship between exchange rates and fiscal policy, Section 3 presents econometric methodology;

Section 4 describes the data and empirical results; and finally, Section 5 concludes the paper.

3.2 Real Exchange Rates, Fundamentals and Fiscal Policy

3.2.1 Exchange Rates and Fundamentals

The basic relationship between exchange rate and fundamentals is often based on the Purchasing Power Parity (PPP). The PPP assumes

$$E_t = \frac{P_t^*}{P_t} \tag{3.1}$$

where E_t is the nominal exchange rate measured by the units of foreign currency per unit of domestic currency. P_t is the price level of the home country, and P_t^* is the price level of the foreign country. Using lower case letters to represent the log level of the variables, PPP can also be represented as follows:

$$e_t = p_t^* - p_t \tag{3.2}$$

The deviation from the PPP or the relative price fundamental $p^* - p$ is defined as $z_t = e_t - (p_t^* - p_t)$, which is termed the real exchange rate, has been a subject of numerous earlier studies (e.g., Michael, Nobay, and Peel, 1997; Taylor and Peel, 2000; Taylor 2002).

Other studies have also used the monetary fundamental in exchange rate determination. In this case, the parity is usually expressed as

$$e_t = (m_t^* - m_t) - (y_t^* - y_t) \tag{3.3}$$

where m_t is the log of the money supply of the home county, y_t is the log of real income of the home country, and m_t^*, y_t^* represent the same variables of the foreign country. Hence, the deviation from the monetary fundamental can be defined as $z_t = e_t - [(m^* - m) - (y^* - y)]$, which is also used to double check the robustness of the main analysis based on the real exchange rate as determined by the price fundamentals.

3.2.2 Fiscal Policy and Real Exchange Rates

Compared with the literature on monetary policy and exchange rates, relatively few works emphasize the effect of fiscal policy. The earlier works (e.g., Obstfeld, 1985; Frenkel and Razin, 1986; Devereux and Purvis, 1990) typically focus on the potential effects of fiscal policy on real exchange rate as real shock or the national income account identity. Specifically, government expenditure may change the gap between national savings and investments. Hence, an expansionary fiscal policy through either an increase in government expenditure or a tax cut in the domestic country will decrease national savings, which reduces the supply for the domestic currency and could drive up the real exchange rate. Analogously, an expansionary fiscal policy abroad could reduce the foreign country's savings and drive up the interest rate of the foreign country in the setting of a small open economy. A higher interest rate thus will reduce the investment of the country, which increases the gap between national savings and investments and raises the supply of domestic currency. Then, the real exchange rate might fall. These works, however, emphasize the additively separable effect of fiscal policy on macroeconomic variables and exchange rates, and such additively separable effect is usually expressed as a linear regression model. As we mentioned earlier, additive models often fail to detect the important role that fiscal deficits (or surplus) can play in the real exchange rate dynamic adjustments.

The recent fiscal theory of price level tends to re-emphasize the importance of fiscal policy in the place of monetary policy in the determination of price level and exchange rates. In the traditional monetary economics, the price level cannot be determined uniquely in the equilibrium. More than one inflation path could exist with the equilibrium condition even though the nominal quantity of money is given. In the fiscal theory of price level (e.g., Leeper, 1991; Sims, 1994; Woodford, 1995; Canzoneri, Cumby, and Diba, 2001), the intertemporal government budget constraint just holds under the equilibrium condition. The equilibrium price level is determined by the government fiscal policy. Some researchers (e.g., Dupor, 2000; Daniel, 2001) extend the fiscal theory of price level to the open economy. After carefully tailoring the models, it can lead to the uniqueness of the price level and exchange rates. However, somewhat similar to the earlier works on fiscal policy as a real shock, these works also tend to focus more on additively separable effects of fiscal policy on exchange rates than on the effects of the interactions between monetary and fiscal policies.

Recently, many researchers (Sargent, 1999; Dixit and Lambertini, 2003; Linnemann and Schabert, 2003; Schabert, 2004) have underscored the potentially complex interactions between monetary and fiscal policies. Although specific implications from these models may be different, they all emphasize the role of fiscal policy as a conditioning information variable indicating the binding constraint of monetary policy actions. As clearly pointed out by Sargent (1999), the administrative independence of central banks does not by itself imply that monetary policy is independent of the fiscal decisions of governments. As an aspect of the "unpleasant monetarist arithmetic", if fiscal policy is managed to create a permanent stream of deficits, it follows arithmetically that monetary policy must supply a permanent stream of seigniorage sufficient to make up the budge shortfall. In this extreme case, monetary policy would not be capable of influencing the time path of domestic inflation and thus may not affect the exchange rate. Certainly, in the bilateral exchange rate, monetary policy and fiscal policy in both countries could matter, though one might dominate the other.

3.3 Econometric Methodology

Many earlier works (Michael, Nobay, and Peel, 1997; Taylor and Peel, 2000) model the nonlinear adjustment of the exchange rate deviation from the fundamentals using a smooth transition autoregressive (STAR) model:

$$z_t = \sum_{j=1}^{p} \alpha_j z_{t-j} + \sum_{j=1}^{p} \alpha_j^* z_{t-j} \phi[\theta, z_{t-d}] + u_t \tag{3.4}$$

where $\phi[\theta, z_{t-d}]$ is a smooth parametric transition function (e.g., exponential or logistic functions), which determines the degree of the mean reversion. Note that to simplify the notation, we use z_t to represent the demeaned exchange rate deviation from the fundamentals. The underlying premise is that there exists a certain nonlinear adjustment process pushing exchange rates returning to the fundamentals in the long run, and the adjustment speed depends on how far the exchange rate is away from the prediction of the fundamentals.

In this paper we choose a flexible functional form, the semiparametric varying coefficient model, which nests the STAR model as a special case and is given by:

$$z_t = X_t^{'}\beta(s_{t-d}, s_{t-d}^{*}) + u_t \tag{3.5}$$

where $X_t^{'} = (1, z_{t-1}, z_{t-2}, ..., z_{t-k})$. s_{t-d} and s_{t-d}^{*} are state variables, s_{t-d} is fiscal deficits of the home country, and s_{t-d}^{*} is fiscal deficits of the foreign country.

Note that equation (3.5) can be motivated by the recent literature on the interdependence of monetary and fiscal policy. Specifically, based on equation (3.2), we can see that when monetary policy in the domestic or the foreign country (or both) is adjusted in the earlier periods (at time t-j) to influence the future path of inflation (at time t), it certainly affects the future real exchange rate (at time t). Such influence of monetary policy in the future real exchange rate can be reflected as the dynamic adjustment of real exchange rate over time in equation (3.5). In this context, it should be obvious why fiscal deficits are used as the conditioning variables in the equation, as they implicitly reflect their role of potential constraints on the monetary policy in the domestic and foreign countries. It is also interesting to contrast the motivation here with that of many earlier works motivating nonlinear dynamic adjustment. The earlier works (e.g., Michael, Nobay, and Peel, 1997; Taylor and Peel, 2000) motivate the nonlinear dynamic adjustment based on commodity arbitrage, and existence of transaction costs may render arbitrage infeasible with small deviations. These works emphasize the market-driven mechanism in driving the adjustment. By contrast, our motivation focuses on the monetary policy impact on exchange rates via affecting inflation and the existence of fiscal deficits as the constraint on the monetary policy effectiveness.

We will also consider the following benchmark linear regression model:

$$z_t = \alpha + \sum_{j=1}^{k} z_{t-j}\beta_j + s'_{t-d}\gamma + s^*_{t-d}\gamma^* + u_t \qquad \text{t=1,…,n} \tag{3.6}$$

Similar to Jansen et al. (2007), we use the Generalized Likelihood Ratio (GLR) test suggested in Cai, Fan and Yao (2000) to conduct model specification tests. Specifically, we conduct model specification test between linear model and varying coefficient model. The linear model is the null hypothesis and the varying coefficient model is the alternative hypothesis. The GLR test is calculated from the sums of squared residual based on linear model and smooth coefficient model.

$$GLR = \frac{\sum_{t=1}^{n}\hat{u}_t^2 - \sum_{t=1}^{n}\tilde{u}_t^2}{\sum_{t=1}^{n}\tilde{u}_t^2}$$

where $\hat{u}_t$ is the residual from the null hypothesis linear model, and $\tilde{u}_t$ is the residual from the alternative smooth coefficient model. If the GLR statistics is big, we will reject the null hypothesis of linear model.

In Cai, Fan and Yao (2000), a nonparametric bootstrap process is proposed to evaluate the p-value of the test. Since the nonparametric estimation is always consistent under both null and alternative hypotheses, we bootstrap the centralized residuals from the nonparametric varying coefficient model instead of the linear model. Let u_t^* represent the centralized bootstrap errors based the residual of the vary coefficient model. u_t^* follows the ‘wild’ bootstrap distribution conditions (see Li and Wang (1998) for more details). Then we can bootstrap the p-value of GLR test according to the following steps:

Step 1: For t = 1; …, n, centralize the residuals based on smooth coefficient model, generate u_t^* according to the 'wild' bootstrap distribution conditions, and compute $z_t^* = X_t^{'}\hat{\beta}(s_{t-d}, s_{t-d}^*) + u_t^*$, where $X_t^{'} = (1, z_{t-1}, z_{t-2}, ..., z_{t-k})$ for $t = 1,...,n.$ based on bootstrap residual u_t^*.

Step 2: Obtain the least square estimator of using the bootstrap z_t^*

$$\hat{\beta}_0^* = (\sum_{t=1}^{n} X_t X_t')^{-1} \sum_{t=1}^{n} X_t z_t^*$$

Then compute the bootstrap OLS residual by $\hat{u}_{0,t}^* = z_t^* - X_t' \hat{\beta}_0^*$.

Step 3: Compute the kernel smoothing estimator of varying coefficient model $\hat{\beta}^*(s_{t-d}, s_{t-d}^*)$ based on the bootstrap z_t^*

$$\hat{\beta}^*(s_t, s_t^*) = (\sum_{j=1}^{n} X_j X_j' K_h(\frac{S_{j-d} - S_t}{h}))^{-1} \sum_{j=1}^{n} X_j z_j^* K_h(\frac{S_{j-d} - S_t}{h})$$

Here $S_t = (s_t, s_t^*)$ and $K_h(\bullet)$ is the product kernel. Then compute the bootstrap varying coefficient residual by $\hat{u}_t^* = z_t^* - X_t^{'}\hat{\beta}^*(s_{t-d}, s_{t-d}^*)$

Step 4: Compute the bootstrap GLR statistics by

$$GLR_n^* = \frac{\sum_{t=1}^{n} \hat{u}_{0,t}^{*2} - \sum_{t=1}^{n} \hat{u}_t^{*2}}{\sum_{t=1}^{n} \hat{u}_t^{*2}}$$

Step 5: Repeat steps 1- 4 1000 times, and obtain the empirical distribution of the GLR_n^* statistics. Find the critical value according to the bootstrap distribution of GLR_n^* statistics. We use $GLR_{n,\alpha}^*$ to represent the α percentile of the empirical distribution. If if $GLR_n > GLR_{n,\alpha}^*$, the null hypothesis is rejected at the significance level α.

3.4 Empirical Results

3.4.1 Data

In this paper, we consider US dollar-based exchange rate of G5 countries – U.K., Canada, Italy, France, and Japan. Because Germany experienced the reunification in 1990 that accompanied with substantial increase in fiscal deficits, there obviously exists a structure break in the German data. Furthermore, the monthly data for the German fiscal deficit, which may give us relatively more observations, is not available. Therefore, we decide not to include German here because we cannot conduct meaningful analysis using only the quarterly data (only about 60 observations either before or after the presumed break in 1990).

Most of the monthly and quarterly data for US, UK, Canada, Italy, France and Japan are obtained from Datastream. Specifically, Datastream is the source for the monthly exchange rate (E_t), consumer price index (P_t), real, seasonally adjusted gross domestic product (Y_t). M_4 is used for United Kingdom, while M_1 is used for all the other countries. The government deficit (FD_t) of United Kingdom is from the website of national statistics of U.K. The government deficit of Japan and part of the other Japan data are obtained from the OECD Main Economic Indicators. The government deficit data of all the other countries are from Datastream.

Also, the monthly data of Japan's government deficit is not available and the quarterly data of France is too short (less than 50 observations). Hence, we focus on bilateral US dollar-based exchange rates for the two sets of the four countries for both monthly and quarterly data. The monthly data covers the period from January 1975 to December 2006 for Canadian dollar, from May 1994 to December 2006 for French Franc, from January 1987 to December 2006 for Italian lira, from April 1984 to December 2006 for British pound. The quarterly data covers the period from 1975:1 to 2006:4 for Canadian dollar, from 1980q1 to 2006q4 for Japanese yen, from 1987q1 to 2006q4 from Italian lira, and from 1982q3 to 2006q4 from British Pound.

All the data is converted by taking logs. The fiscal deficit data is normalized by GDP. The state variables s_t and s_t^* here are the U.S. and the other country's change of the relative government deficit to GDP (change of the log difference between government deficit and GDP), i.e.,

$$s_t = (\frac{FD_t}{GDP_t}) - (\frac{FD_{t-1}}{GDP_{t-1}})$$

The US government deficit (normalized by GDP) is approximately from -2.5 to 2.5 percent (positive number represents government surplus, negative represents government deficit). Canadian deficits are lower than U.S., about -1.5 to +1.5 percent. France, Italy and U.K. are all higher than U.S. Italy is highest which is from -15 percent to over 40 percent. Figure 3.1 plots quarterly US fiscal deficits and real exchange rates.

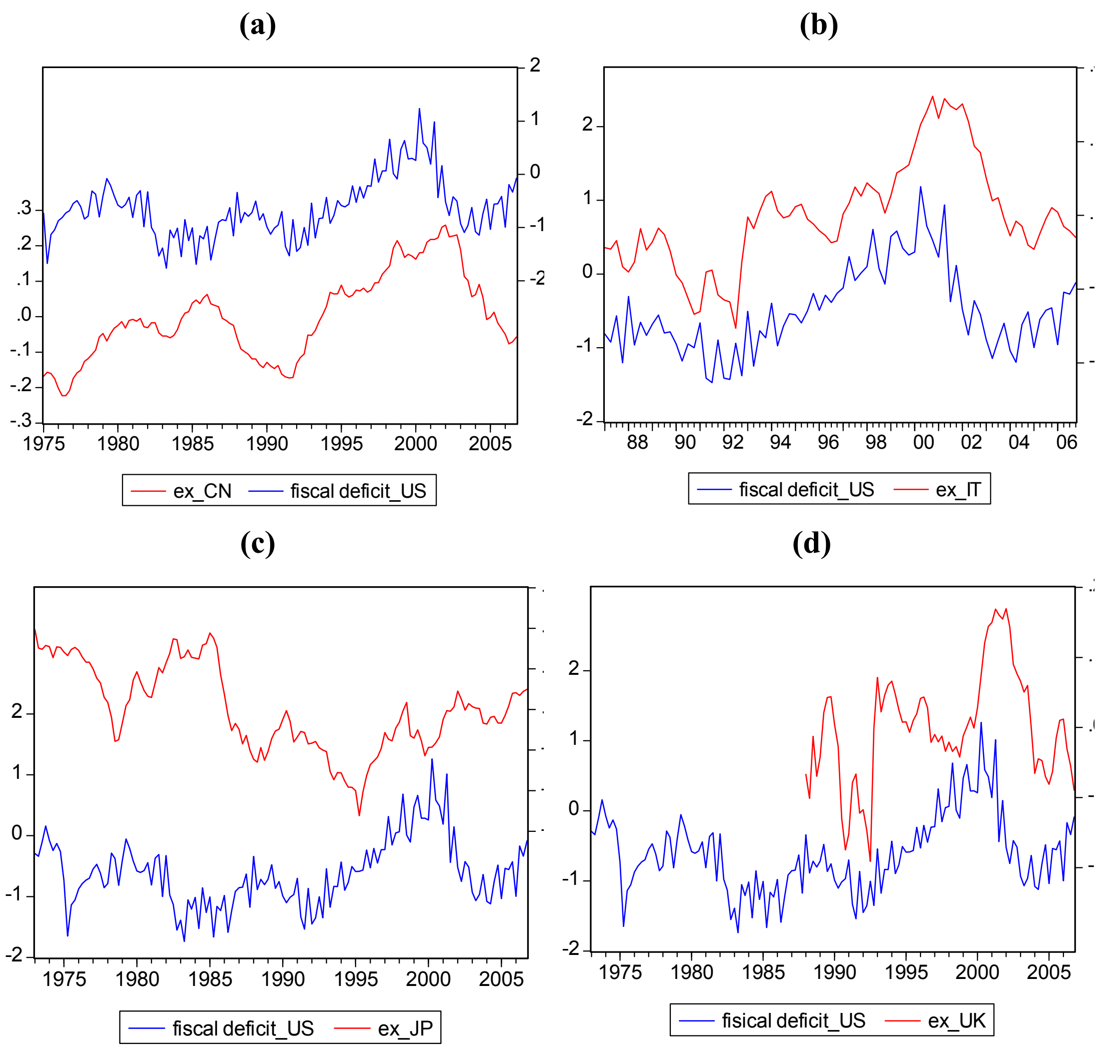


Figure 3.1: Real Exchange Rate and Fiscal Deficit of U.S. (Quarterly data)

3.4.2 Model Specification Tests

We consider five real exchange rates: Canadian dollar, Italy lira, French franc, British pound and Japanese yen. We estimate the model using the real exchange rate as

z_t as described by equation (5). To determine the lag, here we apply Hurvich, Simonoff and Tsai's (1998) nonparametric version of corrected Akaike information criterion (AICc) due to its impressive finite-sample properties. Specifically, we simultaneously choose the lag of the fiscal policy variable and the lags of the real exchange rate variable using the nonparametric kernel method. For monthly data, we allow the maximum lag value *d* up to 7 for the fiscal policy variable, as the impact of the fiscal policy variable could last for more than half a year, and chose the value of *d* (among *d=1,2,…,7*) that minimizes the corrected AIC criteria. We also allow the real exchange rate to have either a single lag or the combinations of two different lags, with the maximum lag of up to 3. The number of cases considered along this dimension is also 7. Hence, we totally consider 49 (=7x7) cases. We estimate all of the combinations using nonparametric kernel method and choose the one with the smallest AIC. For the quarterly data, we allow the maximum lag up to 5 for the fiscal policy variable and the same maximum lag for the real exchange rate. The results of lag length selection of various models are reported in Table 3.1. For example, for the Canadian exchange rate model with monthly data, we select *d=2, j=1* and *j=2*, which results in the model $z_t = \beta_0(s_{t-2}, s^*_{t-2}) + z_{t-1}\beta_1(s_{t-2}, s^*_{t-2}) + z_{t-2}\beta_2(s_{t-2}, s^*_{t-2}) + u_t$.

Given the selected lags, we test the null hypothesis of linear models against the alternative of more flexible varying coefficient models. In Table 3.2, the bootstrap critical values, GLR statistics and p-values are reported.

Table 3.1: Lag Length Determination of the Real Exchange Rate Models

Country	Differences of the Real Exchange Rates				Real Exchange Rates			
	Quarterly Data		Monthly Data		Quarterly Data		Monthly Data	
	j	*d*	*j*	*d*	*j*	*d*	*j*	*d*
Canada	1	1	1	3	1,2	2	1,2	2
Italy	1	3	1	4	1	3	1,2	1
France	-	-	1	4	-	-	1,2	3
Japan	1	3	-	-	1	1	-	-
UK	1	2	1	3	1	2	1	1

Note: table 3.1 reports selected lag values j and d in $z_t = \beta_0(s_{t-d}, s^*_{t-d}) + \sum_{j=1}^{k} z_{t-j}\beta_1(s_{t-d}, s^*_{t-d}) + u_t$ based on the corrected AIC criterion. For example, for the France exchange rate model with monthly data, we select $d=3$, $j=1$ and $j=2$, which results in the model $z_t = \beta_0(s_{t-3}, s^*_{t-3}) + z_{t-1}\beta_1(s_{t-3}, s^*_{t-3}) + z_{t-2}\beta_2(s_{t-3}, s^*_{t-3}) + u_t$.

Table 3.2: Testing the Null of a Linear Model (using Real Exchange Rates)

$$H_0 : z_t = \alpha + \sum_{j=1}^{k} z_{t-j}\beta_j + s'_{t-d}\gamma + s^*_{t-d}\gamma^* + u_t \text{ v.s. } H_1 : z_t = X'_t\beta(s_{t-d}, s^*_{t-d}) + u_t$$

(a) Test results using monthly data

country	Period (Monthly)	Obs.	Critical Value 1%	5%	10%	20%	GLR	Prob.
Canada	01/1975-12/2006	384	0.263	0.234	0.217	0.205	0.286	0.005
France	05/1994-12/2006	152	0.303	0.243	0.225	0.207	0.214	0.155
Italy	01/1987-12/2006	240	0.231	0.198	0.180	0.160	0.296	0.000
UK	04/1984-12/2006	273	0.276	0.242	0.225	0.205	0.230	0.090

(b) Test results using quarterly data

country	Period (Quarterly)	Obv.	Critical Value 1%	5%	10%	20%	GLR	Prob.
Canada	1975q1-2006q4	128	0.475	0.403	0.373	0.336	0.488	0.010
Italy	1987q1-2006q4	80	0.540	0.445	0.399	0.349	0.451	0.048
Japan	1973q1-2006q4	136	0.423	0.370	0.343	0.312	0.348	0.088
UK	1988q1-2006q4	76	0.541	0.473	0.416	0.377	0.500	0.038

In the case of monthly data, the linear regression model is rejected at the 1 percent level for Canada and Italy, and at the 10 percent level for U.K. The nonlinearity results are also strong for quarterly data. The GLR statistics show that we reject linear models at the 5 percent level for Canada, Italy and UK, and at the 10 percent for Japan. Therefore, we reject linear regression models in favor of the varying coefficient models. We will use the varying coefficient model in the econometric analysis in the remaining part of this paper. We would also like to mention that, when we estimate linear regression models, in almost all the cases considered, we find that the coefficients of government fiscal deficit variables are not significantly different from zero, which would lead one to reach an incorrect conclusion that the fiscal policy variables do not affect the exchange rates movements (based on misspecified linear regression models). However, from the GLR testing results, we know that fiscal policy variables significantly affect exchange rates in an non-additive and nonlinear way.

Table 3.3: GLR Tests on Insignificance of the Intercept Terms

$H_0 : \beta_0(s_{t-d}, s^*_{t-d}) = 0$ versus $H_1 : \beta_0(s_{t-d}, s^*_{t-d}) \neq 0$

(a) Test results using monthly data

country	Period (Monthly)	Obs.	Critical Value 1%	5%	10%	20%	GLR	Prob.
Canada	01/1975-12/2006	384	0.171	0.139	0.127	0.108	0.061	0.880
France	05/1994-12/2006	152	0.245	0.179	0.155	0.126	0.058	0.863
Italy	01/1987-12/2006	240	0.176	0.133	0.111	0.097	0.076	0.440
UK	04/1984-12/2006	273	0.254	0.219	0.203	0.175	0.085	0.893

(b) Test results using quarterly data

country	Period (Quarterly)	Obs.	Critical Value 1%	5%	10%	20%	GLR	Prob.
Canada	1975q1-2006q4	128	0.561	0.370	0.300	0.245	0.085	0.955
Italy	1987q1-2006q4	80	0.576	0.500	0.441	0.365	0.177	0.780
Japan	1973q1-2006q4	136	0.399	0.340	0.309	0.278	0.286	0.178
UK	1988q1-2006q4	76	0.442	0.334	0.293	0.243	0.093	0.948

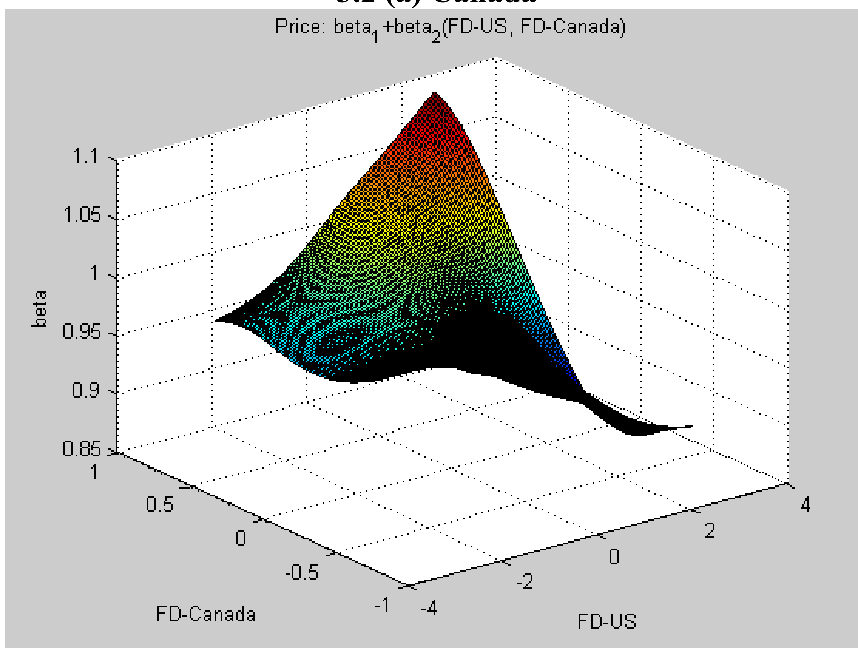
We further conduct separate tests for testing the null hypothesis that $\beta_0(s_{t-d}, s^*_{t-d})$ =0. Under this null hypothesis, the fiscal policy variables do not affect exchange rates in an additive separable fashion. The testing results are reported in Table 3.3. Table 3.3 shows that we cannot reject the null hypothesis of $\beta_0(s_{t-d}, s^*_{t-d})$ =0 for all the real exchange rates and for either monthly or quarterly data. These results suggest that the additively separable response of the real exchange rate to the fiscal deficits (i.e., $\beta_0(s_{t-d}, s^*_{t-d})$) is insignificant using either monthly or quarterly data. The results of Table 3.2 and Table 3.3 together suggest that the fiscal deficits serve as state variables which affect the exchange rates through their interactions with lagged values of real exchange rates (for example, z_t depends on $z_{t-j}\beta_1(s_{t-d}, s^*_{t-d})$ for some positive integer *j* and *d*).

3.4.3 Real Exchange Rate Adjustment

In order to explore the dynamics of the real exchange rate adjustment, we plot the dynamic response coefficient $\beta_1(s_{t-d}, s^*_{t-d})$ (or the sum of $\beta_1(s_{t-d}, s^*_{t-d})$ and $\beta_2(s_{t-d}, s^*_{t-d})$, in the case of multiple coefficients), which is a nonparametric function of domestic government deficit s_{t-d} and foreign government deficit s^*_{t-d}. We do not plot the estimated $\beta_0(s_{t-d}, s^*_{t-d})$, as it is found to be not significantly different from zero.

Model: $z_t = \beta_0(s_{t-d}, s^*_{t-d}) + z_{t-1}\beta_1(s_{t-d}, s^*_{t-d}) + z_{t-2}\beta_2(s_{t-d}, s^*_{t-d}) + u_t$

3.2 (a) Canada



3.2(b) Italy

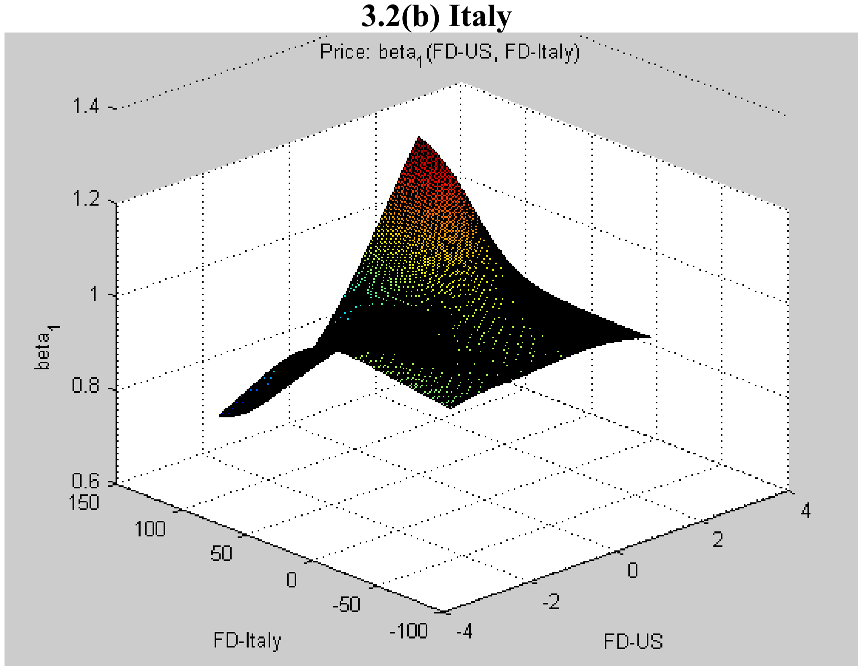
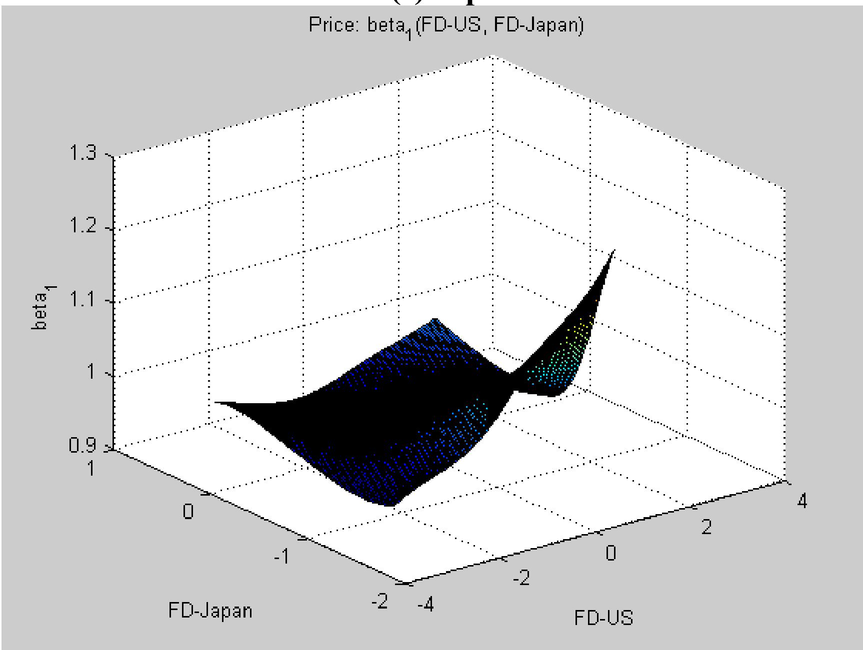


Figure 3.2: Exchange Rate Dynamic Adjustment Coefficients (Quarterly data)

3.2(c) Japan



3.2(d) UK

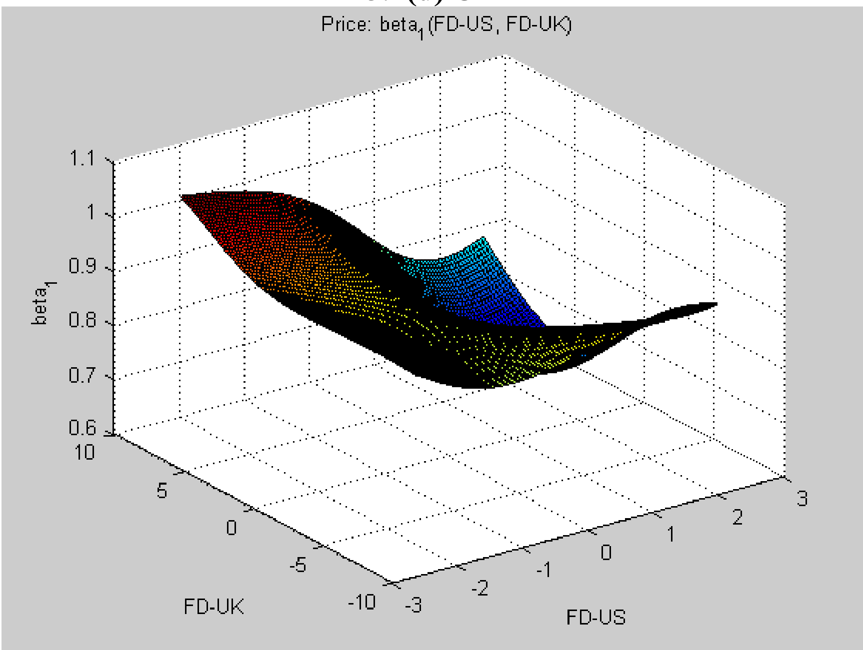
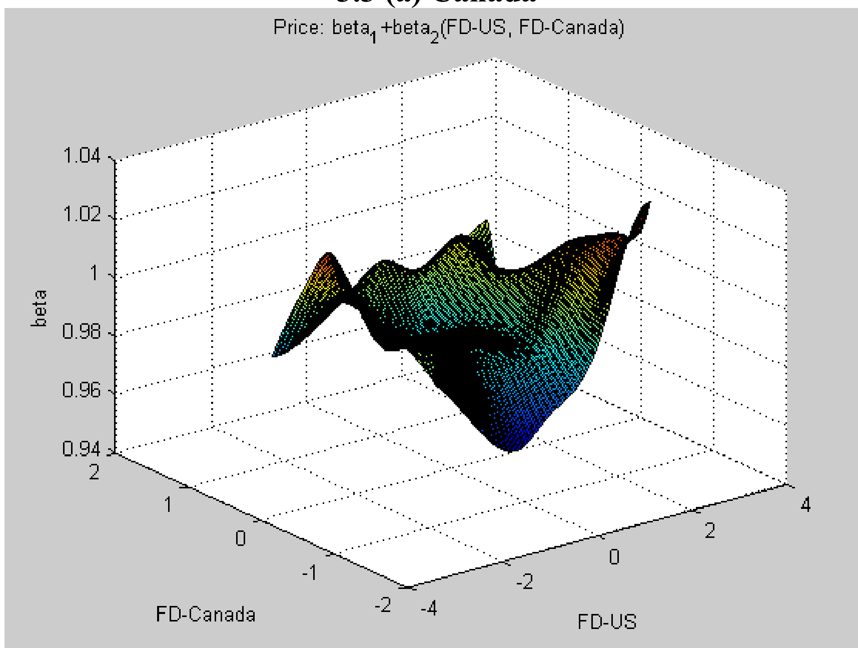


Figure 3.2. ***(Continued)***

Model: $z_t = \beta_0(s_{t-d}, s^*_{t-d}) + z_{t-1}\beta_1(s_{t-d}, s^*_{t-d}) + z_{t-2}\beta_2(s_{t-d}, s^*_{t-d}) + u_t$

3.3 (a) Canada



3.3 (b) Italy

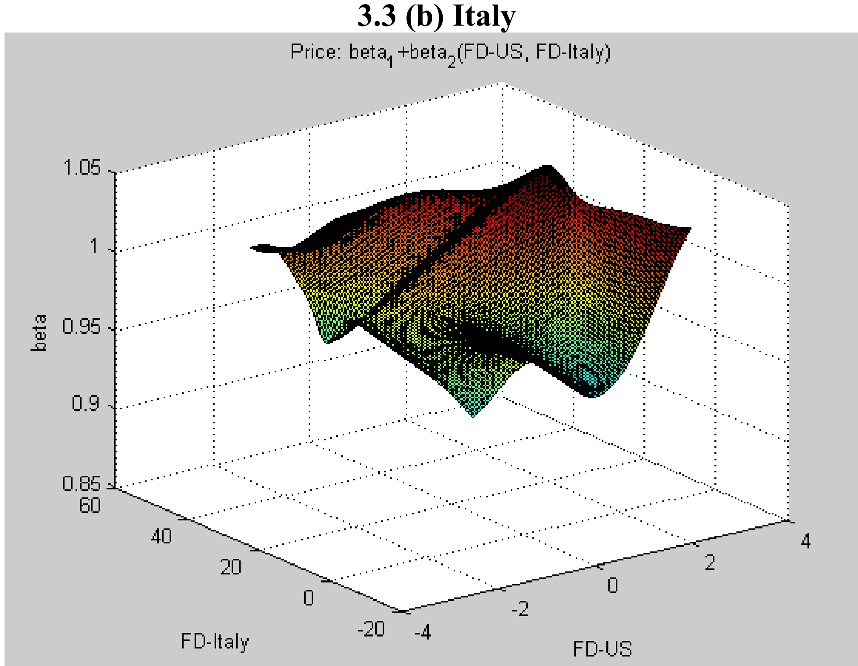
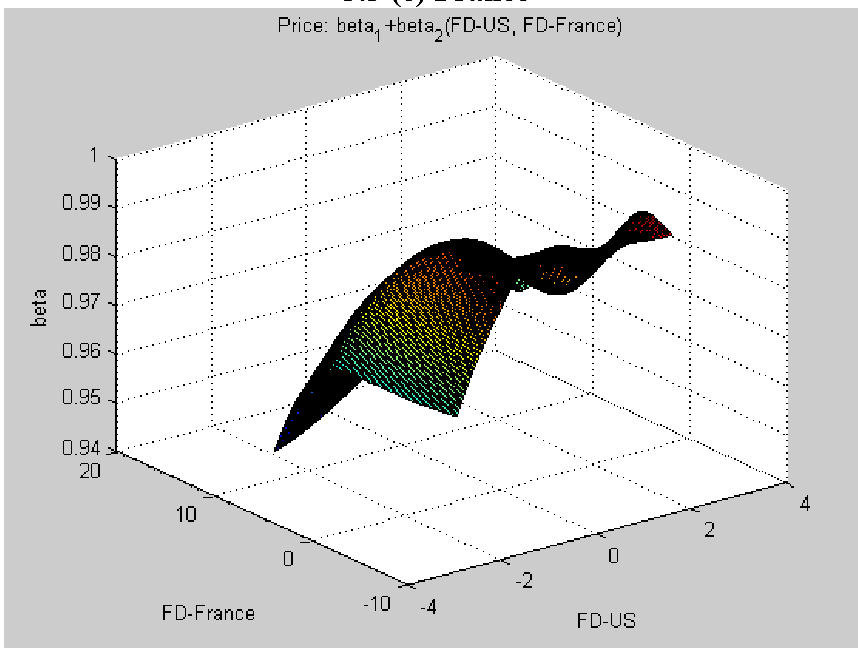


Figure 3.3: Exchange Rate Dynamic Adjustment Coefficients (Monthly data)

3.3 (c) France



3.3 (d) UK

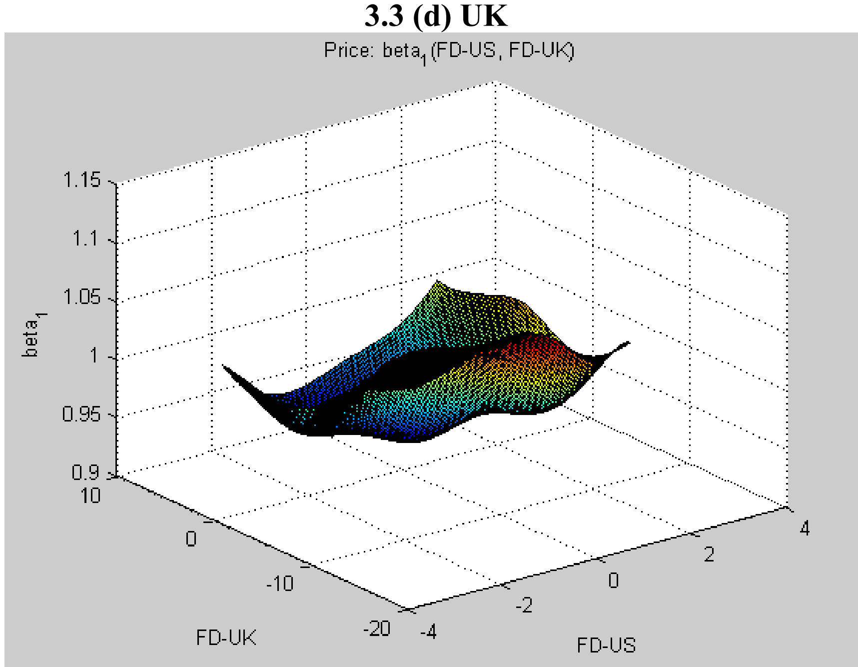


Figure 3.3. ***(Continued)***

Following the earlier works on mean reversion using the parametric nonlinear models, we first check the mean reverting property in the varying coefficient model. For example, as shown in Figures 3.2 and 3.3, the dynamic response coefficient (or the sum of these coefficients in the case of multiple lags) is less than one except some extreme points, indicating the global stability of the estimation. In other words, the figures indicate the mean reversion of the real exchange rates while the speed of such reversion tends to be low. The sum of the coefficients for the quarterly data is generally lower than that for the monthly data, indicating clearer evidence for mean reversion. Similar conclusions also apply to other cases to be considered.

In order to better understand the 2-dimensional exchange rate response coefficients curve $\beta_1(s_{t-d}, s^*_{t-d}) + \beta_2(s_{t-d}, s^*_{t-d})$ as shown in Figure 3.2, we fix one variable and look at the marginal effect of the other variable. When Canada experiences fiscal expansion in the earlier period (fix s^*_{t-d}<0), the speed of mean reversion is faster when the US experiences a fiscal contraction (i.e., s_{t-d}>0, a decrease in the US fiscal deficits) compared to a US fiscal expansion (s_{t-d}<0). However, when Canada experiences fiscal contraction in the earlier period (fix s^*_{t-d}>0), the speed of mean reversion becomes slower as the US moves from the fiscal expansion to the fiscal contraction. On the other hand, when the US experiences a fiscal expansion (fix s_{t-d}<0), the exchange rate response coefficients are not much affected, regardless of the fiscal expansion or contraction in Canada. This might imply that the US fiscal policy is more important than Canadian fiscal policy in determining the bilateral exchange rate. Finally, when US experiences a fiscal contraction (fix s_{t-d} >0), the speed of mean reversion becomes faster as Canada moves from fiscal expansion to contraction (i.e., from s^*_{t-d} <0 to s^*_{t-d} >0).

In the case of Italy, the lira exchange rate adjustment is also affected by interactions between the two countries' fiscal policy variables. When the US experiences a fiscal expansion in the earlier period (fix s_{t-d} <0), the speed of mean reversion is faster when Italy experiences a fiscal contraction (s^*_{t-d} >0), compared with a fiscal expansion in Italy. However, if the US experiences a fiscal contraction instead, the speed of mean reversion becomes slower when Italy moves from the fiscal expansion to the fiscal contraction. On the other hand, when Italy experiences fiscal expansion (fix s_{t-d} <0), the exchange rate response coefficients are not much affected, regardless of the fiscal expansion or contraction in the US.

The dynamic response coefficients are lower when Japan moves from fiscal expansion to fiscal contraction, regardless of fiscal contraction or expansion in the US. In this sense, the case of Japan is similar to that of Italy. In the case of UK, the most noticeable pattern is that when the UK experiences fiscal contraction, the British pound exchange rate dynamic response coefficient is lower (i.e., the speed of mean reversion is faster) when the US also experiences the fiscal contraction compared to the fiscal expansion. In summary, for Canadian and the UK real exchange rates, the US fiscal contraction is presumably the more dominant of the two countries involved in the bilateral exchange rate. When US fiscal contraction is cooperated with fiscal contraction or fiscal expansion in the other country, it can lead to a lower dynamic response coefficient and a faster speed of mean reversion of the bilateral real exchange rate. The reason why there is more pronounced nonlinear effect of Italian fiscal deficits than the US fiscal deficits on the lira exchange rate may lie in the fact that Italian fiscal deficits as percentage of GDP are far greater than that of the US and is the highest among G-7 countries. As for Japan, it is well known for its less exposure to external influence.

The results for the monthly data (Figure 3.3) largely confirm that the interactions of fiscal polices in the two countries affect the exchange rate response coefficients, although the nonlinear patterns may be somewhat different in some cases from those for quarterly data. It is likely that economic factors and the relative importance of the two countries in contributing these factors may weigh differently in driving real exchange rate adjustments at different frequencies. The nonlinear effect of fiscal deficits on the Canadian dollar dynamic response for the monthly data is not as clear as for the quarterly data. Italy still has a lower dynamic response coefficient when it experiences moderate but not large fiscal contraction in the earlier period when the US experiences fiscal expansion. Also, when Italy is in fiscal expansion, some moderate (but not large) fiscal contraction in the US will lead to a lower response coefficient. The French Franc has a lower dynamic response coefficient when France experiences fiscal contraction and particularly when the US also experiences fiscal expansion. Similarly, the British pound exchange rate dynamic response coefficient is lower and thus it has a faster speed of mean reversion when the UK experiences the fiscal contraction. Hence, the basic conclusion mostly remains that a country's fiscal contraction, if cooperated with certain fiscal policy in the other country, can lead to a lower dynamic response coefficient and a faster speed of mean reversion of the bilateral real exchange rate.

Table 3.4: Testing the Null of a Linear Model (using Deviations from the Monetary Fundamental)

$H_0 : z_{m,t} = \alpha + \sum_{j=1}^{k} z_{m,t-j}\beta_j + s_{t-d}^{'}\gamma + s_{t-d}^{*}\gamma^{*} + u_t$ versus $H_1 : z_{m,t} = X_t^{'}\beta(s_{t-d}, s_{t-d}^{*}) + u_t$

(a) Test results using monthly data

country	Period (Monthly)	Obs.	Critical Value 1%	5%	10%	20%	GLR	Prob.
Canada	01/1975-12/2006	384	0.268	0.238	0.228	0.214	0.324	0.000
France	05/1994-12/2006	152	0.322	0.260	0.235	0.213	0.658	0.000
Italy	01/1987-12/2006	240	0.211	0.186	0.174	0.158	0.472	0.000
UK	04/1984-12/2006	273	0.308	0.275	0.253	0.230	0.318	0.003

(b) Test results using quarterly data

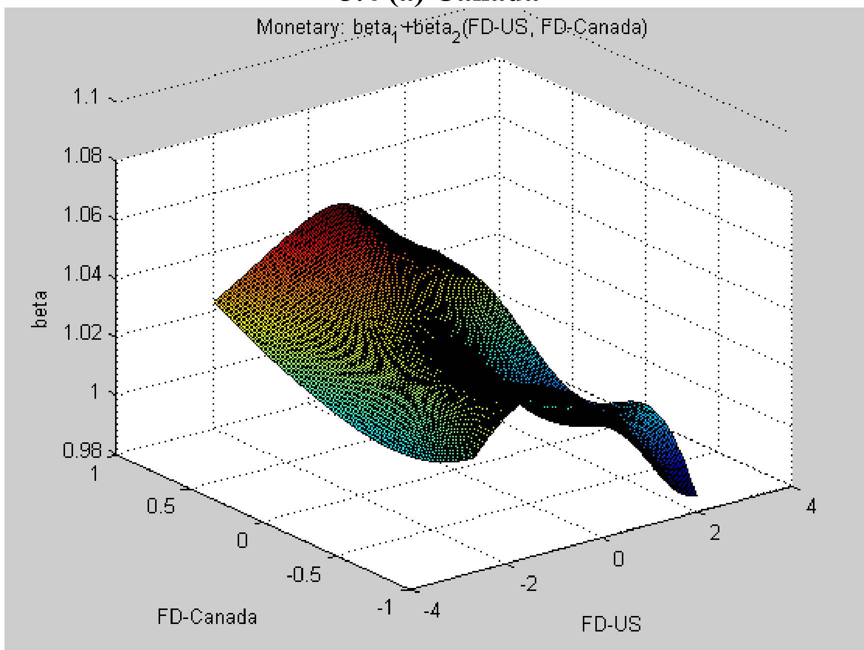
country	Period (Quarterly)	Obs.	Critical Value 1%	5%	10%	20%	GLR	Prob.
Canada	1975q1-2006q4	128	0.503	0.440	0.395	0.355	0.341	0.240
Italy	1987q1-2006q4	80	0.769	0.583	0.483	0.404	0.628	0.030
Japan	1980q1-2006q4	108	0.689	0.620	0.585	0.517	0.621	0.050
UK	1982q3-2006q4	98	0.424	0.362	0.333	0.298	0.358	0.053

3.4.4 Robustness Checks

As a robustness check, we replace the real exchange rate z_t by another variable which can be obtained the same way as z_t except that we replace the price fundamental with the monetary fundamental. We use $z_{m,t}$ to denote this new dependent variable and repeat the analysis using $z_{m,t}$. In Table 3.4, the GLR statistics indicate rejection of the linearity at the 1 percent level for all the four currencies with monthly data, and at the 10 percent level for all currencies except for Canada for the quarterly data. The dynamic response coefficient functions are reported in Figure 3.4 for quarterly data and Figure 3.5 for monthly data. Figure 3.4 is quite similar to Figure 3.2 based on the price fundamental and thus the basic inference remains the same. Perhaps, the only slight difference lies in the case of British Pound, where fiscal contraction in the US combined with fiscal expansion (rather than fiscal contraction) in the UK in the earlier period coincides with the lower response coefficient and faster speed of mean reversion. For the monthly data, similar to Figure 3.3(a), the fiscal policy effect on Canadian exchange rate (Figure 3.5(a)) does not have a clear pattern. For other three currencies (Italy, France and UK), when they experience fiscal contraction in the earlier period (s^*_{t-d}>0), the exchange rate response coefficient is generally lower, particularly when the US experience fiscal expansion or loosely increase in fiscal deficits (s_{t-d}<0). Overall, the basic conclusion remains the same that the role of fiscal expansion in US is likely to cause decreasing bilateral exchange rate response coefficients and increasing the speed of exchange rate mean reversion, except for Italy whose fiscal deficit (relative to GDP) is a magnitude larger than that of US.

$$z_{m,t} = \beta_0(s_{t-d}, s^*_{t-d}) + z_{m,t-1}\beta_1(s_{t-d}, s^*_{t-d}) + z_{m,t-2}\beta_2(s_{t-d}, s^*_{t-d}) + z_{m,t-3}\beta_3(s_{t-d}, s^*_{t-d}) + u_t$$

3.4 (a) Canada



3.4 (b) Italy

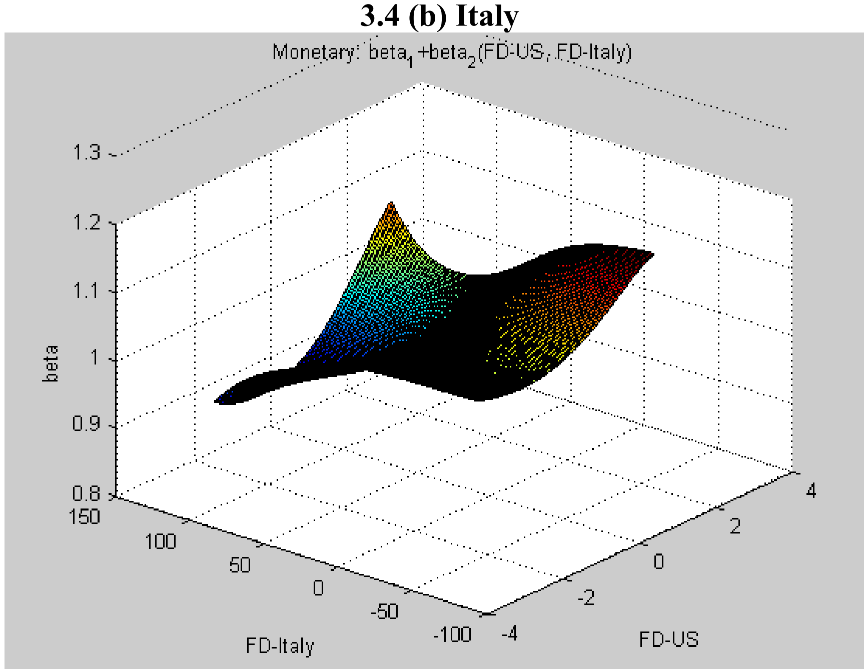
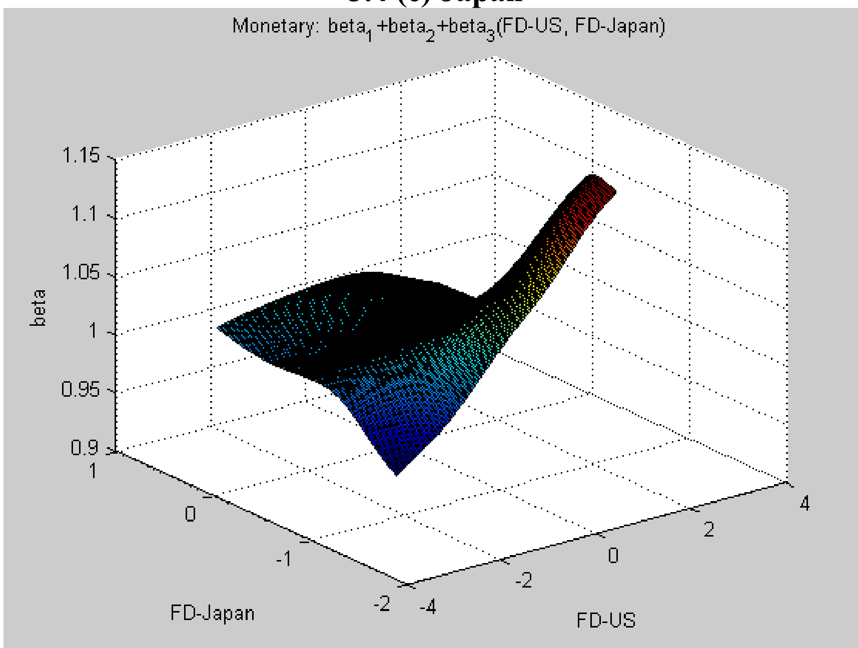


Figure 3.4: Exchange Rate Dynamic Adjustment Coefficients (Quarterly data, Monetary Fundamental)

3.4 (c) Japan



3.4 (d) UK

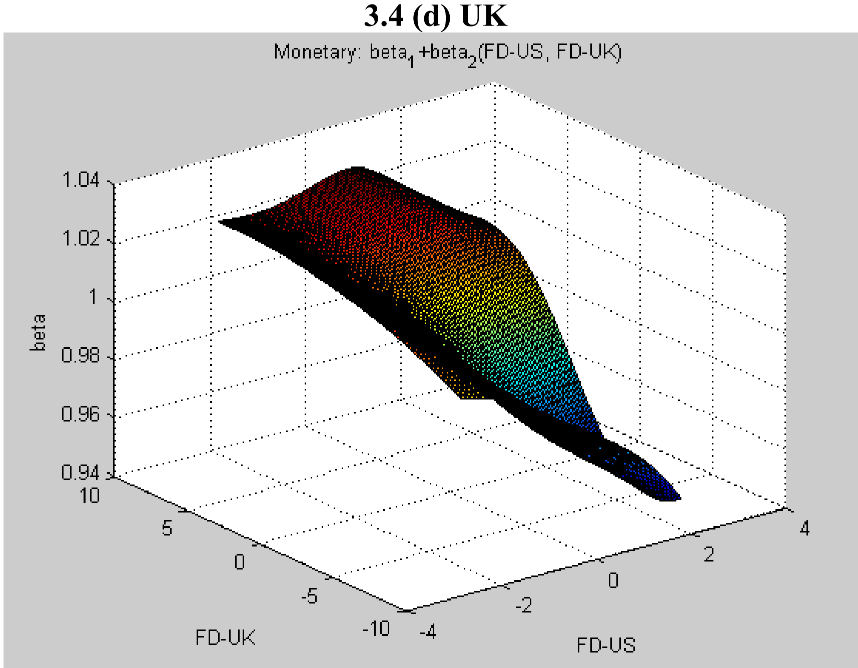
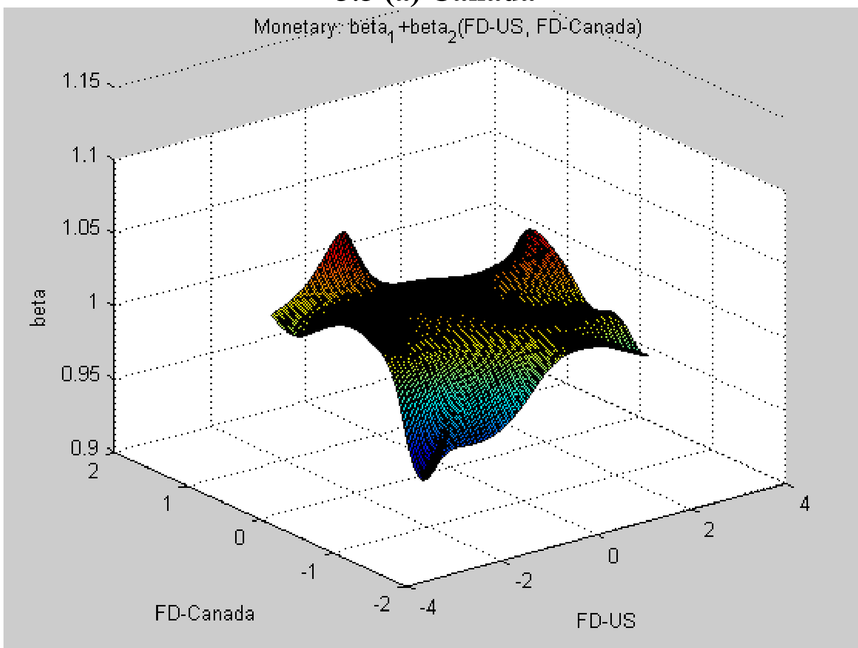


Figure 3.4. ***(Continued)***

$$z_{m,t} = \beta_0(s_{t-d}, s^*_{t-d}) + z_{m,t-1}\beta_1(s_{t-d}, s^*_{t-d}) + z_{m,t-2}\beta_2(s_{t-d}, s^*_{t-d}) + z_{m,t-3}\beta_3(s_{t-d}, s^*_{t-d}) + u_t$$

3.5 (a) Canada



3.5 (b) Italy

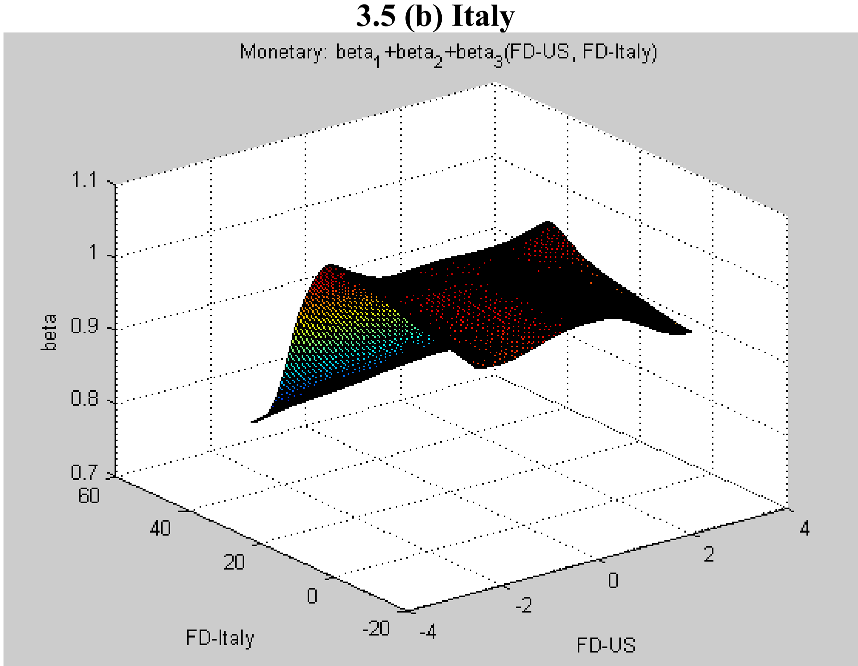
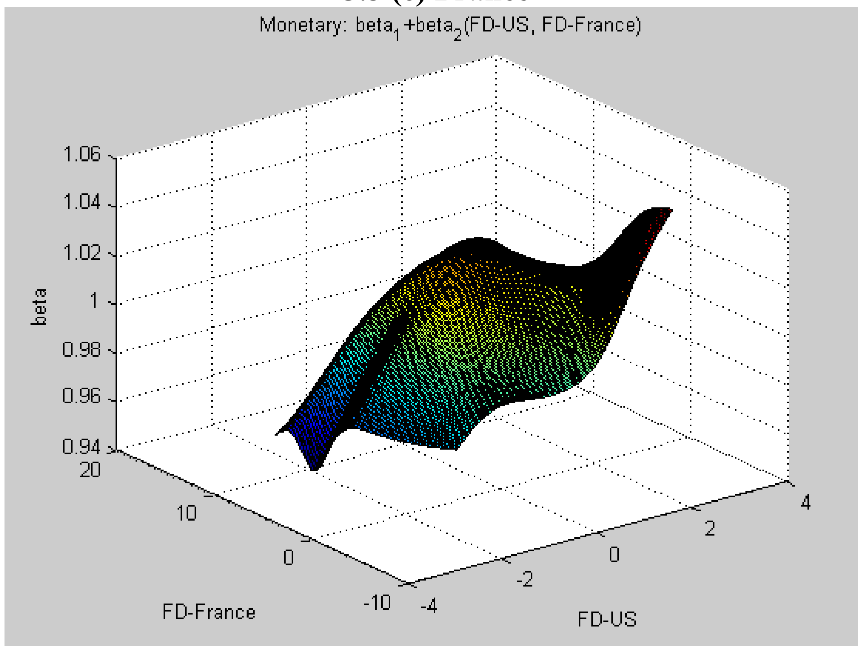


Figure 3.5: Exchange Rate Dynamic Adjustment Coefficients (Monthly data, Monetary Fundamental)

3.5 (c) France



3.5 (d) UK

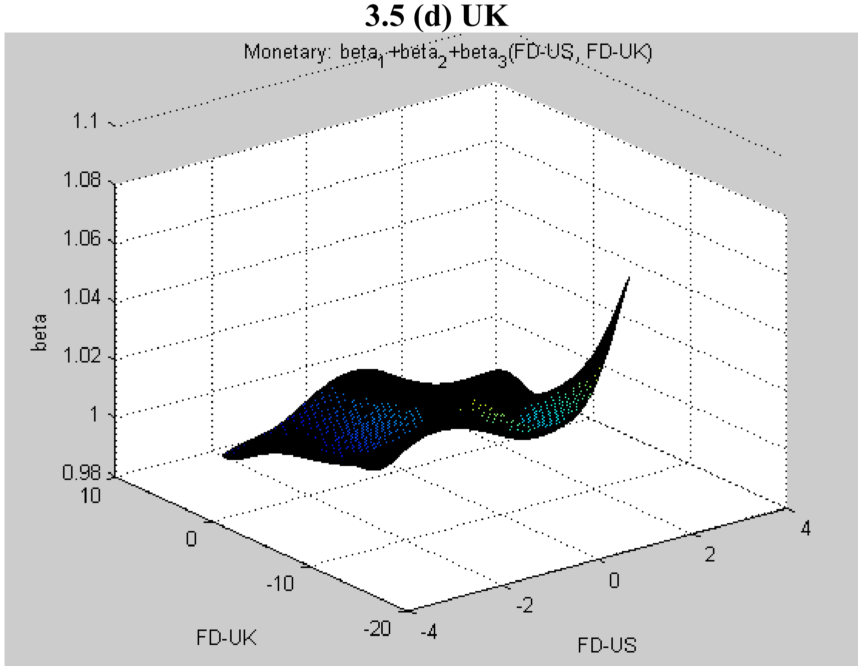
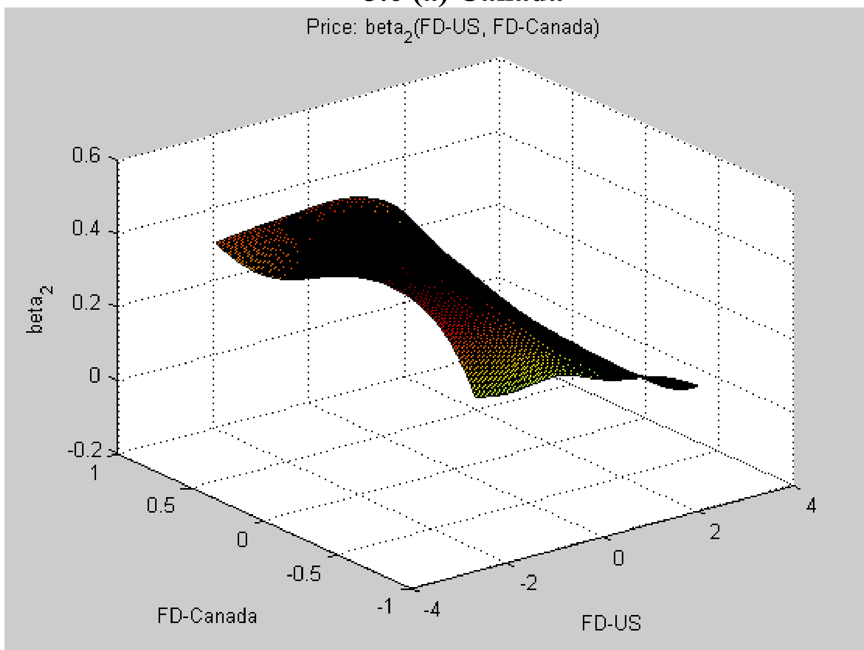


Figure 3.5. ***(Continued)***

$$\Delta z_t = \beta_1(s_{t-d}, s^*_{t-d}) + \Delta z_{t-i}\beta_2(s_{t-d}, s^*_{t-d}) + u_t$$

3.6 (a) Canada



3.6 (b) Italy

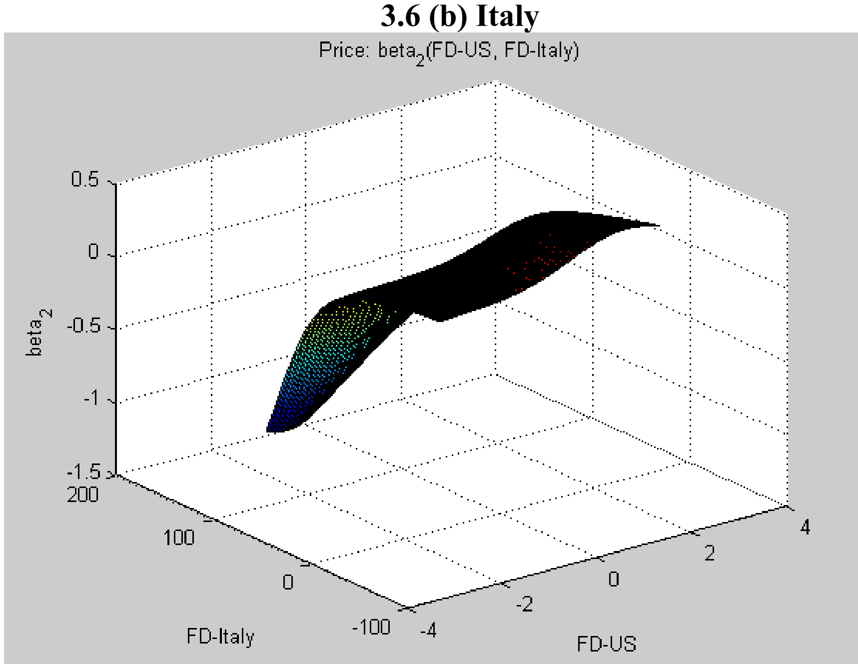
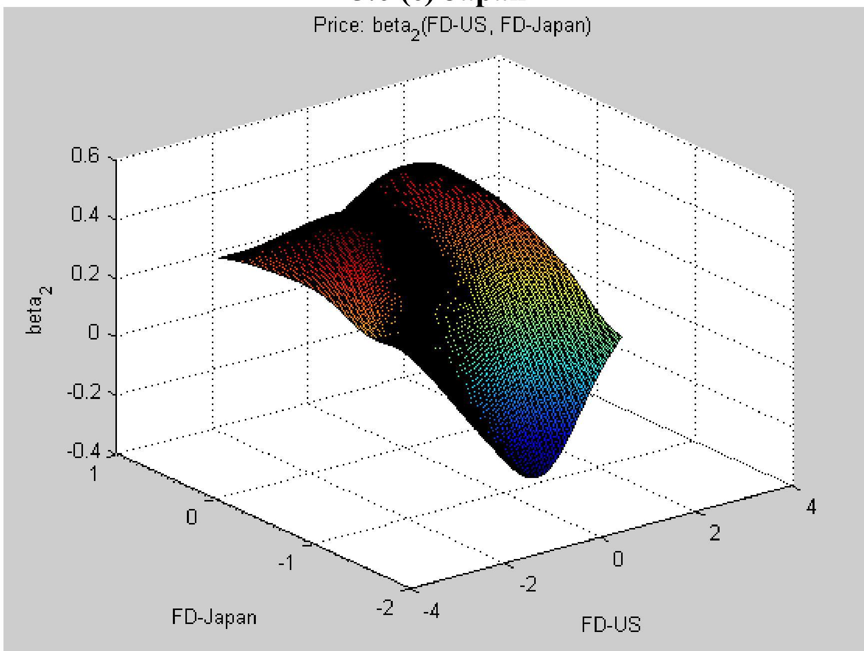


Figure 3.6: Exchange Rate Dynamic Adjustment Coefficients (Quarterly data, first differences)

3.6 (c) Japan



3.6 (d) U.K.

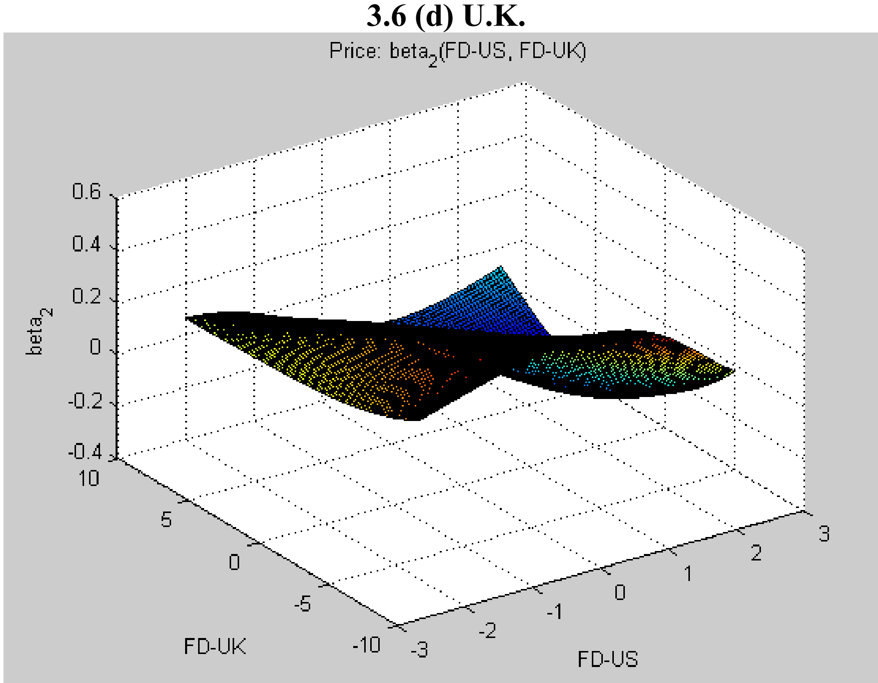


Figure 3.6. ***(Continued)***

Table 3.5: Testing the Null of a Linear Model (using Differences of Real Exchange Rates)

$$H_0 : \Delta z_t = \alpha + \sum_{j=1}^{k} \Delta z_{t-j}\beta_j + s'_{t-d}\gamma + s^*_{t-d}\gamma^* + u_t \text{ versus } H_1 : \Delta z_t = \Delta X'_t \beta(s_{t-d}, s^*_{t-d}) + u_t$$

where $\Delta X'_t = (1, \Delta z_{t-1}, ..., \Delta z_{t-k})$.

3.5(a) Test results using monthly data

country	Period (Monthly)	Obs.	Critical Value 1%	5%	10%	20%	GLR	Prob.
Canada	01/1975-12/2006	384	0.184	0.164	0.156	0.146	0.226	0.000
France	05/1994-12/2006	152	0.252	0.216	0.201	0.181	0.287	0.003
Italy	01/1987-12/2006	240	0.204	0.180	0.168	0.153	0.206	0.010
UK	04/1984-12/2006	273	0.399	0.315	0.258	0.220	0.301	0.060

3.5(b) Test results using quarterly data

country	Period (Quarterly)	Obs.	Critical Value 1%	5%	10%	20%	GLR	Prob.
Canada	1975q1-2006q4	128	0.343	0.297	0.273	0.243	0.454	0.000
Italy	1987q1-2006q4	80	0.542	0.444	0.394	0.338	0.642	0.000
Japan	1973q1-2006q4	136	0.407	0.371	0.339	0.304	0.451	0.003
UK	1988q1-2006q4	76	0.741	0.429	0.357	0.307	0.354	0.118

As additional robustness check, we also repeat the analysis using Δz_t, the first difference of the variable z_t, which is clearly stationary. By contrast, the variable z_t appears to be highly persistent. In Table 3.5, the GLR statistics indicate rejection of the linearity at the conventional significance level for all the four currencies for both monthly and quarterly data, with the exception that the linearity null cannot be rejected for UK at the 10% level for the quarterly data (with a p-value of 0.118). The estimated dynamic response coefficient functions for Δz_t using the quarterly data are plotted in

Figure 3.6. Specifically, when the US moves from fiscal expansion to fiscal contraction, the Canadian dollar exchange rate responds less to changes in the earlier period's deviation. Also, when Italy moves from fiscal expansion to fiscal contraction, the Italian lira exchange rate responds less to changes in the earlier period's deviation. The UK has the noticeably lower response coefficient when both the UK and the US have fiscal contractions. Japan, however, has the lower response coefficient when the US is close to the balanced budget while Japan is in the fiscal expansion. Overall, the general conclusion from Figure 3.6 is in line with the basic findings reported above.

3.5. Conclusions

We investigate dynamic adjustment of dollar-based bilateral real exchange rates of G5 countries (Canada, Italy, France, Japan and the United Kingdom). We find statistically significant evidence of nonlinear real exchange rate adjustments due to changes in the fiscal deficit for both monthly and quarterly data. A country's fiscal contraction, when appropriately cooperated with certain fiscal policy in the other country, can lead to a faster speed of mean reversion for bilateral real exchange rates. Our results lend support to the argument that monetary authorities can effectively influence exchange rates only when they are supported by contractionary fiscal policies. Noteworthy, assuming PPP holds, our findings are generally consistent with the evidence in Catao and Terrones (2005) that inflation is nonlinearly dependent on fiscal deficits based on a panel data spanning 107 countries and 1960-2001. Our findings are also broadly in line with Andersen et al. (2003) and Ehrmann and Fratzscher (2005), who report asymmetric response of exchange rate responses to macroeconomic fundamentals during different economic environments.

Finally, the results of this paper have some important implications for future research. It has been documented, mostly based on parametric additive regression models, that commonly used fundamental variables such as inflation, money supply, and interest rates have only weak impacts on the determination of exchange rates (mostly based on parametric additive linear models). Our study suggests that allowing for non-additive nonlinear effects of fiscal policy may provide a new avenue in examining the relationship between exchange rate and fiscal and monetary policy variables. It might also shed more light on other issues such as the documented stronger response of US monetary policy to inflation expectations in the post-1980 period (Boivin and Giannoni, 2006), during which fiscal deficits increased dramatically. In sum, this study, together with Jansen et al. (2007), strongly suggests the importance of modeling fiscal deficits as conditioning variables (in a non-additive nonlinear manner), rather than direct information variables (in an additive linear or nonlinear manner), in empirical analysis of monetary policy and the related issues.

CHAPTER IV

ESTIMATING AVERAGE TREATMENT EFFECTS WITH DISCRETE COVARIATES

4.1 Introduction

Many researchers work on how to estimate the average treatment effects. This technique has been widely applied in labor economics, education and macroeconomics. For example, Ichino and Winter-Ebmer (1998) estimate the local average treatment effect to access the long-run human capital loss during the World War II. Lechner (1999) estimates treatment effects to evaluate the job training program in East Germany after unification. In macroeconomics, the macroeconomics policy can also be viewed as "treatment" such as dollarization (Edwards and Magendzo (2003)), foreign exchange intervention (Galati, Melick and Micu (2005)), inflation targeting (Lin and Ye (2007)).

Previous parametric works include the method of instrument variable, propensity score matching method (Rosenbaum and Rubin, 1983) or covariates matching method (Rubin, 1980). The disadvantages of the parametric way are obvious. The misspecification of the parametric index model such as Logit or Probit model will impact the magnitude of the treatment effect significantly or even lead to the wrong sign of the estimation. So, recent years, more and more researchers begin to investigate the nonparametric way to deal with this topic.

Many nonparametric estimations deal with the case with continuous variables. In practice, sometimes we need to deal with the case with many discrete variables or only discrete variables in our econometric model. For example, in the economics of education area, many studies look at the impact of treatments such as the impact of charter schools attendance (or private school attendance, etc.) on student performance. The left hand side variable is a measure of student performance. The right hand side explanatory variables include indicators for type of school (0/1 charter/not charter); student mover (0/1 moved schools or not); indicators for various student characteristics (e.g. special education, gender, race or ethnicity, economically disadvantaged). In macroeconomics area, as the examples we just mentioned above, the left hand side could be measures of macroeconomic performance such as output, growth rate, inflation, etc. The right hand side explanatory variables could include policy treatment variable and other indicators for different countries characteristics (e.g. exchange rate regime dummy, openness dummy, regional dummy, trade agreement dummy, independence dummy, etc.).

The traditional nonparametric way to deal with the discrete covariates is frequency-based method which splits the data into small “cells”. The first disadvantage related to this approach is when the number of cells exceeds the sample size, this sample splitting is infeasible. Furthermore, the tests on whether there exists significant treatment effects or not based on this frequency approach will lead to a loss of power. This is another disadvantage related to the traditional method.

In this paper, we propose a kernel method to deal with the scenario with only discrete covariates. This new approach is based on recently developed least squares cross-validation kernel smoothing method. It can not only automatically smooth the irrelevant variables out of the nonparametric regression model, but also avoid the problem of loss of efficiency related to the traditional nonparametric frequency based method.

The structure of this chapter will be as follows. We construct the nonparametric estimator of average treatment effects in section 4.2. The asymptotic distribution of the proposed estimator is discussed here. In section 4.3, we report the Monte Carlo simulation results. From mean squared error based on new method and traditional frequency-based method, we can find the kernel method performs better than the traditional method. Main steps of proof are included in appendix.

4.2 Estimating Average Treatment Effects with Discrete Covariates

4.2.1 The Construction of the Estimator

The construction of the estimator follows the same steps as Li et al. (2007). We also follow the same notation as in Li et al. (2007).

We use a dummy variable t to denote an individual has been treated or not. If an individual i received a treatment, $t_i = 1$. If an individual i does not received a treatment, $t_i = 0$. So, $t_i \in \{0,1\}$, for i = 1,…, n. We use y_i to denote the outcome of each individual i. For each individual i, if this individual has been treated, the outcome will be $y_i(1)$, otherwise the outcome will be $y_i(0)$. Then, a general way to represent the outcome will be $y_i(t_i)$. For, $y_i(1)$ and $y_i(0)$, we couldn’t observe both of them at the same time. Then, we write

$$y_i = t_i y_i(1) + (1 - t_i) y_i(0). \tag{4.1}$$

We are interested in the average treatment effect which is defined as:

$$\tau = E[y_i(1) - y_i(0)]. \tag{4.2}$$

x_i represents a series of pre-treatment variables, say q variables. So, x_i is a vector with dimension $1 \times q$. As we mentioned above, for each individual i, if he is treated, we observe the outcome $y_i(1)$, otherwise we observe the outcome $y_i(0)$. we couldn't observe $y_i(1)$ and $y_i(0)$ at the same time. Without more assumptions, according to equation (4.2), we couldn't estimate the average treatment effect τ consistently.

One important assumption we need here is 'unconfoundedness condition' which has been discussed in Rosenbaum and Rubin (1983). The 'unconfoundedness condition' is stated as:

Assumption (A1) (Unfoundedness):

Conditional on x_i, the treatment indicator t_i is independent of the potential outcome.

Let $\tau(x)$ denote the average treatment effect conditional on x. This conditional treatment effect can be written as,

$$\tau(x) = E[y_i(1) - y_i(0) \mid x_i = x] \tag{4.3}$$

According to Assumption (A1), t_i is independent of the potential outcome, we can rewrite the equation (4.3) as

$$\tau(x) = E[y_i \mid t_i = 1, x_i = x] - E[y_i \mid t_i = 0, x_i = x]. \tag{4.4}$$

Now, if we can consistently estimate the two terms on the right hand side of equation (4.4), we can estimate the conditional treatment effect $\tau(x)$. For these two terms, $E[y_i \mid t_i = 1, x_i = x]$ and $E[y_i \mid t_i = 0, x_i = x]$, we can estimate both of them based on nonparametric methods. Then, based on Assumption (A1), we can easily calculate the average treatment effect by

$$\tau = E[\tau(x_i)]. \tag{4.5}$$

We use $g(x_i, t_i)$ to represent right hand term in equation (4.4), $E[y_i \mid x_i, t_i]$. So, we have,

$$y_i = g(x_i, t_i) + u_i \tag{4.6}$$

The conditional expectation of u_i is zero, $E[u_i \mid x_i, t_i] = 0$.

Let $g_0(x_i)$ denotes $g(x_i, t_i = 0)$, and $g_1(x_i)$ denotes $g(x_i, t_i = 1)$. We can transform the equation (4.6) as following,

$$y_i = g_0(x_i) + [g_1(x_i) - g_0(x_i)]t_i + u_i \tag{4.7}$$

$$= g_0(x_i) + \tau(x_i)t_i + u_i$$

where $\tau(x_i) = g_1(x_i) - g_0(x_i)$.

According to equation (4.7), we have,

$$\tau(x_i) = \frac{\operatorname{cov}(y_i, t_i \mid x_i)}{\operatorname{var}(t_i \mid x_i)} \tag{4.8}$$

Since t_i equals to either 0 or 1, so the conditional mean of t_i will be,

$$\mu(x_i) = \Pr(t_i = 1 \mid x_i) \equiv E[t_i \mid x_i] \tag{4.9}$$

Then, the average treatment effect can be written as,

$$\tau = E[\tau(x_i)] = E\left\{\frac{(t_i - \mu_i)y_i}{\mathrm{var}(t_i \mid x_i)}\right\}. \tag{4.10}$$

Now, we can use nonparametric technique to estimate the average treatment effects base on equation (4.10). Different from the case in Li et al. (2007), we'll focus on the case with discrete covariates only.

4.2.2 Nonparametric Estimator of Average Treatment Effects

Let x_s represent the s-th component of x. Suppose x_s is a discrete covariates with c_s different categories. We give different values to different categories. So, x_s takes the value from $\{0,1,...,c_s - 1\}(c_s \geq 2, s = 1,...,k)$.

For unordered regressors, we propose using the following kernel function which is based on Aitchison and Aitken's (1976)

$$l(x_{is}, x_s, \lambda_s) = \begin{cases} 1, & when \quad X_{is} = x_s \\ \lambda_s, otherwise. & \end{cases} \tag{4.11}$$

When $\lambda_s = 0$, the kernel function $l(x_{is}, x_s, \lambda_s)$ will change to the standard indicator function. When $\lambda_s = 1$, the kernel function $l(x_{is}, x_s, \lambda_s)$ will change to the uniform weight function. So, $\lambda_s \in [0,1]$ for all s=1,…,r.

For ordered regressors, we propose using the following kernel function,

$$l(x_{is}, x_s, \lambda_s) = \begin{cases} 1, & if\ X_{is} = x_s, \\ \lambda_s^{|X_{i,t} - x_t|}, & if.\ X_{is} \neq x_s. \end{cases} \tag{4.12}$$

Same as the unordered case, when $\lambda_s = 0$, the kernel function $l(x_{is}, x_s, \lambda_s)$ will change to the standard indicator function. When $\lambda_s = 1$, the kernel function $l(x_{is}, x_s, \lambda_s)$ will change to the uniform weight function. So, $\lambda_s \in [0,1]$ for all s=1,…,r.

Based on equation (4.11) and (4.12), we can construct the following product kernel function,

$$L(x_i, x, \lambda) = \prod_{s=1}^{r} l(x_{is}, x_s, \lambda_s), \tag{4.13}$$

When $\lambda_s \neq 0$, we can see the summation of the kernel weight function does not equal to one. Since this kernel weight function shows up in both the numerator and denominator of the nonparametric estimator, we can multiply the kernel function in both numerator and denominator by same non-zero constant number at the same time. So, this summation does not equal to one will not affect the following nonparametric estimation of $\mu(x)$ in equation (4.15).

For any $x \in D$, where *D* is the range of *x*, we can obtain the probability function of $p(x)$ based on the following equation,

$$\hat{p}(x) = \frac{1}{n}\sum_{i=1}^{n} L(X_i, x, \lambda), \tag{4.14}$$

For conditional expectation of t_i, we can estimate $\mu(x) = E(t_i \mid x_i = x)$ based on the following equation (4.15)

$$\hat{\mu}(x) = \frac{n^{-1}\sum_{i=1}^{n} t_i L(x_i, x, \lambda)}{\hat{p}(x)} \tag{4.15}$$

Now we discuss some special cases. If $\lambda_s = 0$ for all s=1,…,r, the kernel function changes to the indicator function, the new estimator proposed here will change back to the frequency-based estimator with discrete covariates which has been discussed in previous literature. If $\lambda_s = 1$ for some *s*, the kernel function will change to the uniform weight function, then the new estimator propose in equation (4.15) will be unrelated to x_s. In this scenario, the regressor x_s is an "irrelevant" regressor and it will be automatically smoothed out from the nonparametric model.

Based on least squares cross-validation method with discrete covariates, we find the optimal $(\lambda_1, \lambda_2, ..., \lambda_s)$ to minimize the following target function

$$CV(\lambda) = \sum_{i=1}^{n} \left[t_i - \hat{t}_{-i}(x_i) \right]^2, \tag{4.16}$$

where $$\hat{t}_{-i}(x_i) = \frac{n^{-1} \sum_{j=1, j \neq i}^{n} t_j L(x_i, x_j, \lambda)}{\hat{p}_{-i}(x_i)} \tag{4.17}$$

$\hat{t}_{-i}(x_i)$ is the leave-one-out kernel estimator of $E(t_i \mid x_i)$, and

$$\hat{p}_{-i}(x_i) = \frac{1}{n} \sum_{j=1, j \neq i}^{n} t_j L(x_i, x_j, \lambda) \tag{4.18}$$

$\hat{p}_{-i}(x_i)$ is the leave-one-out estimator of $p(x_i)$.

Let $(\hat{\lambda}_1, \hat{\lambda}_2, ..., \hat{\lambda}_r)$ represent the optimal $(\lambda_1, \lambda_2, ..., \lambda_s)$ which minimize the equation (4.16)

We use x_{-s} to denote x without x_s, i.e. $x_{-s} = (x_1, ..., x_{s-1}, x_{s+1}, ... x_r)$, and use $D_{-s}(D_s)$ to denote the support of $x_{-s}(x_s)$.

We call $g(x)$ a *constant function* with respect to x_s if

$g(x_s, x_{-s}) = g(z_s, x_{-s})$ for all $x_s, z_s \in D_s$ and all $x_{-s} \in D_{-s}$.

The equation above means when x_s changes, $g(\bullet)$ does not change at all. That is x_s is an "irrelevant" variable and it should be removed from the econometric model. In this section, we want all the variables are relevant variables here. Thus, $g(x)$ is *not* a constant function with respect to x_s for all s=1,…, r. So, we need the following assumptions to get the results.

Assumption (A2)

(i). $\{x_i, y_i, t_i\}_{i=1}^n$ *are independent and identically distributed as* $\{x, y, t\}$. *Letting D denote the support of* x*, then* $\max_{x \in D}|g(x)| < \infty$.

(ii). $E[y_i^2 \mid x_i = x]$ *is bounded on* $x \in D$.

Assumption (A3) *The only values of* $(\lambda_1, \lambda_2, ..., \lambda_r)$ *that make*

$\sum_{x \in D} p(x)\{\sum_{z \in D} p(x)[t(x) - t(z)]L(x, z, \lambda)\}^2 = 0$ *is* $\lambda_s = 0$ *for all s=1,…, r.*

Assumption (A2).(i) is quite standard. Assumption (A2).(ii) implies that $t(x)$ is not a constant function for any component $x_s \in D_s$.

The following result is proved by Li et al (2006).

Lemma 2.1 *Under assumptions (A2).(i) and (A2).(ii), we have*

$\hat{\lambda}_s = O_p(n^{-1})$ *for s=1,…, r.*

Lemma 2.1 shows that the least squares cross-validation method selected smoothing parameters converge to zero at a fast rate of n^{-1}.

Define $\hat{t}_i \equiv \hat{E}(t_i \mid X_i)$ which denote the kernel estimate of $E(t_i \mid X_i)$. We have

$$\hat{t}_i = \frac{n^{-1}\sum_{j=1}^{n} t_j L_{\lambda, x_i, x_j}}{\hat{p}_i}, \tag{4.19}$$

where $\hat{p}_i = n^{-1}\sum_{j=1}^{n} L_{\lambda, x_i, x_j}$ is a kernel estimator of $p(X_i)$.

Based on the results above, we can have the following estimator of average treatment effect, τ, by

$$\hat{\tau} = \frac{1}{n}\sum_{i=1}^{n} \frac{(t_i - \hat{t}_i) y_i}{\hat{t}_i (1 - \hat{t}_i)}. \tag{4.20}$$

Select optimal $(\hat{\lambda}_1, \hat{\lambda}_2, \ldots., \hat{\lambda}_r)$ based on the least squares cross-validation method.

The asymptotic distribution of estimated average treatment effect, $\hat{\tau}$, is given by the following theorem.

Theorem 4.2.1 *Under assumptions (A1) to (A2), we have*

$\sqrt{n}(\hat{\tau} - \tau) \rightarrow N(0, V)$ *in distribution,*

where $V = \mathrm{var}\left(\dfrac{t_i - \mu_i}{\mu_i (1 - \mu_i)}\right) + \mathrm{var}(\tau(x_i))$.

The proof of Theorem 4.2.1 is given in the appendix.

4.3 Monte Carlo Simulations

In this section we conduct some simulations to investigate the finite sample property of the proposed test.

We generate data and design the Monte Carlo simulation according to the following steps:

First, the outcome of treatment effect is generated by

$$y_i = g_0(x_i) + \tau(x_i)\ t_i + \varepsilon_i$$

$$= \alpha_0 + \alpha_1 x_{i1} + \alpha_2 x_{i2} + \tau(x_i)\ t_i + \varepsilon_i \tag{4.21}$$

where x_1 is an irrelevant discrete variable. It takes four possible value {0, 1, 2, 3} with probability, $P[x_1 = 0] = 0.2$, $P[x_1 = 1] = 0.3$, $P[x_1 = 2] = 0.2$ and $P[x_1 = 3] = 0.3$. Let x_2 is a relevant discrete variable. It takes three possible value {0, 1, 2} with probability, $P[x_1 = 0] = 0.4$, $P[x_1 = 1] = 0.2$ and $P[x_1 = 2] = 0.4$.

Since x_1 is an irrelevant variable and x_2 is a relevant variable, we use $(\alpha_o, \alpha_1, \alpha_2, \tau)' = (1/2, 0, 1/2, \tau)$, and standard deviation of ε takes value, $\sigma_\varepsilon = 1/2$.

Second, the Probit model used for estimating the propensity score is give by

$$T_i = \beta_0 + \beta_1 x_{i1} + \beta_2 x_{i2} + \beta_3 (x_{i2})^2 + \eta_i, \tag{4.22}$$

$$t_i = \begin{cases} 1\ if\ \Phi(T_i) > 0.5 \\ 0 \quad otherwise \end{cases} \tag{4.23}$$

Where $\Phi(\bullet)$ is a standard normal cumulative distribution function.

Since x_1 is an irrelevant variable and x_2 is a relevant variable, we use $(\beta_o, \beta_1, \beta_2, \beta_3)' = (-1, 0, 1/2, 1/2)$, and standard deviation of η takes value, $\sigma_\eta = 1$.

Here τ could equal to different values. In this section, we choose τ equals to {1, 2, 3}. For each value of τ, we do simulation according to the following steps:

1. Draw $\{x_1, x_2\}$ randomly from {0, 1, 2, 3} and {0, 1, 2} respectively with probability mentioned above. Draw $\{\eta, \varepsilon\}$ randomly from Normal distribution with standard deviation $\sigma_\varepsilon = 1/2$ and $\sigma_\eta = 1$ respectively. Sample size is n.

2. Calculate y according to equation (4.21). Calculate t according to equation (4.22) and (4.23).

3. Calculate $\hat{t}_i$ based on equation (4.19); calculate $\hat{\tau}$ based on equation (4.20).

$$\hat{t}_i = \frac{n^{-1}\sum_{j=1}^{n} t_j L_{\lambda, x_i, x_j}}{\hat{p}_i}$$

$$\hat{\tau} = \frac{1}{n}\sum_{i=1}^{n} \frac{\left(t_i - \hat{t}_i\right) y_i}{\hat{t}_i\left(1 - \hat{t}_i\right)}$$

4. Compute $\hat{\tau}_{freq}$ with $\lambda = 0$.

5. Repeat the above step 1 to 4 1000 times and get 1000 $\hat{\tau}$ and $\hat{\tau}_{freq}$

6. Calculate mean squared error of $\hat{\tau}$ and $\hat{\tau}_{freq}$

$$MSE\_\hat{\tau} = \frac{1}{M}\sum_{j=1}^{M}\left(\hat{\tau}_j - \tau\right)^2$$

$$MSE\_\hat{\tau}_{freq} = \frac{1}{M}\sum_{j=1}^{M}\left(\hat{\tau}_{freq,j} - \tau\right)^2 .$$

For each λ, we use Brent method and Newton method for line search and maximize the CV function based on least squares cross-validation.

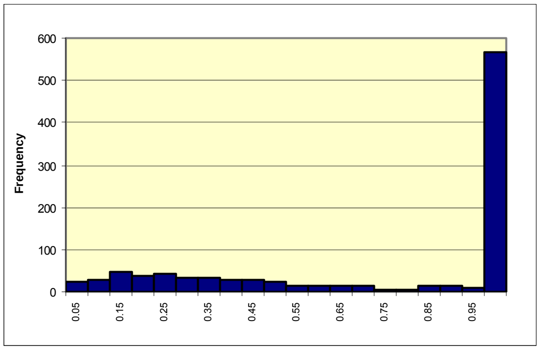
We do simulation based on different sample size n=100, 200, 400. The simulation results are reported in table 4.1. In this table, we compare the results based on nonparametric least squares cross-validation kernel method and traditional frequency based method. We can see the mean squared error based on kernel method is always smaller than the mean squared error based on frequency method. Mean squared error decreases when sample size n increases.

In figure 4.1, we report the histogram of λ_1 and λ_2 with sample size n=100, 200, 400. We know x_1 is the irrelevant variable, the bandwidth of x_1, λ_1**,** is close to 1, which means x_1 variable is smoothing out from the nonparametric regression model. x_2 is relevant variable, the bandwidth of x_2, λ_2**,** is very small and it decreases when sample size n increases. The simulation results are consistent with theoretical prediction.

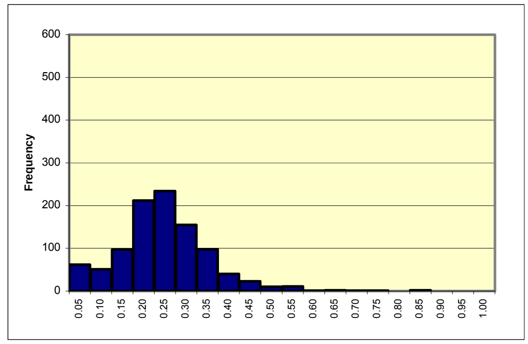
Table 4.1: MSE of Kernel and Frequency Estimators: Average Treatment Effects

	N=100		N=200		N=400	
τ	$MSE\_\hat{\tau}$	$MSE\_\hat{\tau}_{freq}$	$MSE\_\hat{\tau}$	$MSE\_\hat{\tau}_{freq}$	$MSE\_\hat{\tau}$	$MSE\_\hat{\tau}_{freq}$
1	0.1704	0.2119	0.0960	0.1575	0.0355	0.0809
2	0.1354	0.1634	0.0824	0.1470	0.0315	0.0804
3	0.1104	0.1356	0.0714	0.1400	0.0280	0.0801

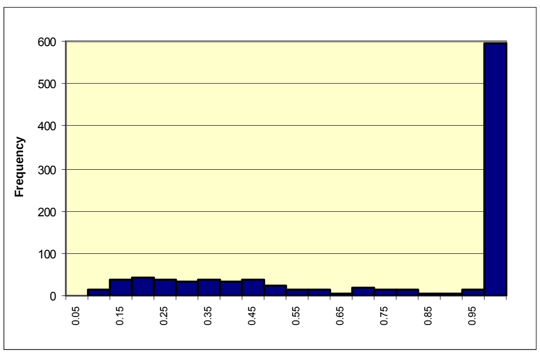
$\lambda_1, n = 100$



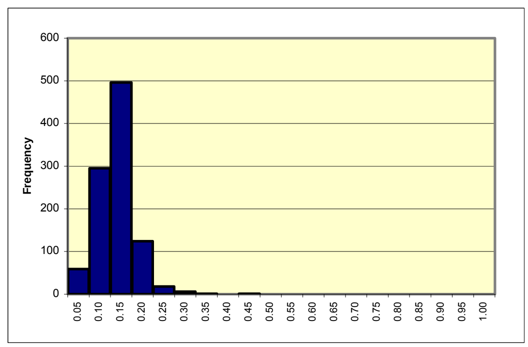
$\lambda_2, n = 100$



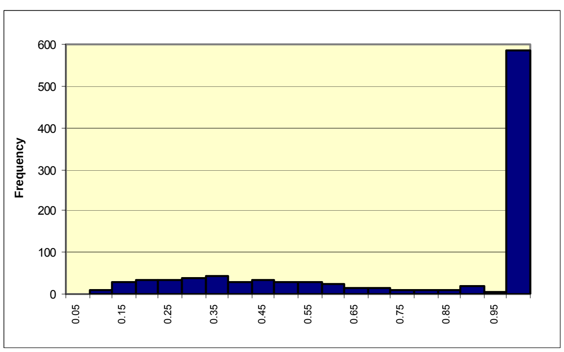
$\lambda_1, n = 200$



$\lambda_2, n = 200$



$\lambda_1, n = 400$



$\lambda_2, n = 400$

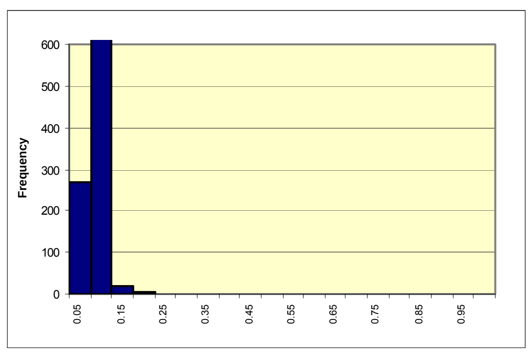


Figure 4.1: Histogram of λ_1 and λ_2

4.4 Conclusion

In chapter IV, a kernel method to deal with the nonparametric regression model with only discrete covariates is proposed.

The traditional parametric method suffers from misspecification. The traditional nonparametric way to deal with the discrete covariates is frequency-based method which splits the data into small “cells”. When the number of cells exceeds the sample size, this sample splitting approach is infeasible. Furthermore, the tests on whether there exists significant treatment effects or not based on this frequency approach may lead to a loss of power.

This new approach proposed in this chapter is based on recently developed least squares cross-validation kernel smoothing method. It can automatically detect the irrelevant variables and smooth them out of the nonparametric regression model. At the same time, it can also avoid the problem of loss of efficiency related to the traditional nonparametric frequency based method.

In this chapter, we construct the nonparametric estimator of average treatment effects based on kernel smoothing method and get the asymptotic distribution of the proposed estimator. According to the results of Monte Carlo simulation, mean squared errors based on new method are always smaller than the mean squared errors based on traditional frequency-based method. The kernel method performs better than the traditional method.

CHAPTER V

SUMMARY AND CONCLUSIONS

There is a debate in the literature concerning the existence and character of asymmetry in the response of gasoline prices to crude oil shocks. Borenstein, Cameron and Gilbert (1997) claim that retail gasoline prices respond asymmetrically to crude oil price changes, while Bachmeier and Griffin (2003) find no evidence of asymmetry in both retail and wholesale gasoline prices if the two-step Engle-Granger estimation procedure is used. We also get this result. That is, using the two-step Engle-Granger estimation procedure and assuming homoskedasticity, we can't reject the hypothesis of a symmetric response of gasoline prices to a crude oil price shock. However, we find that the homoskedastic ECM does not pass a battery of diagnostic tests.

We estimate an ECM with EGARCH errors, as well as several other models allowing asymmetry in the conditional variance. This model does pass our diagnostic tests, and our analysis of impulse response functions indicates substantial asymmetry in both magnitude of response, speed of response, and persistence. Thus we conclude that asymmetry exists, and it requires estimation of an ECM with heteroskedastic errors in order to reliably detect this asymmetry.

Moreover, we suggest a signal extraction model based on consumer search which takes explicit account of both crude oil price volatility and retail gasoline price volatility. The model predicts positive correlation between crude oil price volatility and degree of asymmetry and negative correlation between retail gasoline price volatility and asymmetry. The empirical result supports the model's prediction. This explanation also reconciles the contrary theoretical conclusion in BCG (1997) and empirical evidence in Peltzman (2000) and Radchenko (2005).

In chapter III, I investigate dynamic adjustment of dollar-based bilateral real exchange rates of G5 countries (Canada, Italy, France, Japan and the United Kingdom). We find statistically significant evidence of nonlinear real exchange rate adjustments due to changes in the fiscal deficit for both monthly and quarterly data. A country's fiscal contraction, when appropriately cooperated with certain fiscal policy in the other country, can lead to a faster speed of mean reversion for bilateral real exchange rates. Our results lend support to the argument that monetary authorities can effectively influence exchange rates only when they are supported by contractionary fiscal policies.

This research contributes to the literature in two important aspects. First, it fills the gap in the literature by allowing for a fiscal policy variable (the changes in fiscal deficits) to act as a conditioning variable in modeling exchange rate movements. Second, it employs recently developed flexible varying coefficient in econometric analysis. This model appears to be particularly suitable for modeling fiscal deficits as a conditioning variable. The idea that fiscal deficits may effectively work as a binding constraint on monetary policy can hardly be modeled by a parametric (linear or nonlinear) model. From this perspective, the method improves on the popular parametric nonlinear smooth transition models used in the real exchange rate mean reversion literature.

The results of this chapter have some important implications for future research. It has been documented, mostly based on parametric additive regression models, that commonly used fundamental variables such as inflation, money supply, and interest rates have only weak impacts on the determination of exchange rates (mostly based on parametric additive linear models).

In chapter IV, I propose a kernel method to deal with the nonparametric regression model with only discrete covariates. This new approach is based on recently developed least squares cross-validation kernel smoothing method. It can automatically detect the irrelevant variables and smooth them out of the nonparametric regression model. At the same time, it can also avoid the problem of loss of efficiency related to the traditional nonparametric frequency based method.

In this chapter, we construct the nonparametric estimator of average treatment effects based on kernel smoothing method and get the asymptotic distribution of the proposed estimator. According to the results of Monte Carlo simulation, mean squared errors based on new method are always smaller than the mean squared errors based on traditional frequency-based method. The kernel method performs better than the traditional method.

REFERENCES

Ahmad, I., Leelahanon, S., Li, Q., 2005. Efficient estimation of a semiparametric partially linear varying coefficient model. Annals of Statistics 33, 258-283.

Aitchison, J., Aitken, C.G.G., 1976. Multivariate binary discrimination by the kernel method. Biometrika, 63, 413-420.

Almeida, A., Goodhart, C., Payne, R., 1998. The effects of macroeconomic news on high frequency exchange rate behavior. Journal of Financial and Quantitative Analysis 33, 383-406.

Andersen, T., Bollerslev, T., Diebold, F.X., Vega, C., 2003. Micro effects of macro announcements: real-time price discovery in foreign exchange. American Economic Review 93, 38-62.

Bachmeier, L. J., Griffin, J.M., 2003. New evidence on asymmetric gasoline price responses. Review of Economics and Statistics 85(3), 772-776.

Bacon, R.W., 1991. Rockets and feathers: the asymmetric speed of adjustment of UK retail gasoline prices to cost changes. Energy Economics 13, 211–18.

Balke, N.S., Brown, S.P.A., Yucel, M.K.1998. Crude oil and gasoline prices: an asymmetric relationship? Federal Reserve Bank of Dallas Economic Review, First Quarter 1998, 2–11.

Boivin, J., Giannoni, M.P., 2006. Has monetary policy become more effective? Review of Economics and Statistics 88, 445-462.

Bonser-Neal, C., Roley, V.V., Sellon, G. H., 2000. The effect of monetary policy actions on exchange rates under interest-rate targeting. Journal of International Money and Finance, 19, 601-631.

Borenstein, S., 1991. Selling costs and switching costs: explaining retail gasoline margins. Rand Journal of Economics 22, 354–69.

Borenstein, S., Cameron, A.C., Gilbert, R, 1997. Do gasoline prices respond asymmetrically to crude oil prices? Quarterly Journal of Economics 112, 305–39.

Borenstein, S., Shepard, A, 2002. Sticky prices, inventory, and market power in wholesale gasoline markets. Rand Journal of Economics 33, 116-139.

Bradley, M.D., Jansen, D.W., 2000. Are business cycle dynamics the same across countries? Testing nonlinearity around the world, Studies in Nonlinear Dynamics and Econometrics 4(2), 51-71.

Cai, Z., Fan, J., Yao, Q., 2000. Functional-coefficient regression models for nonlinear time series. Journal of American Statistical Association 95, 941-956.

Canzoneri, M., Cumby, R., Diba, B., 2001. Is the price level determined by the needs of fiscal solvency? American Economic Review 91, 1221-123.

Catao, L. A.V., Terrones, M.E., 2005. Fiscal deficits and inflation. Journal of Monetary Economics 52, 529-554.

Cheung, Y.W., Chinn, M. D., Pascual, A.C., 2005. Empirical exchange rate models of the nineties: are any fit to survive? Journal of International Money and Finance 24, 1150-1175.

Chinn, M. D., 1997. Paper pushers or paper money? Empirical assessment of fiscal and monetary models of exchange rate determination. Journal of Policy Modeling 19, 1221-1238.

Daniel, B.C., 2001. The fiscal theory of the price level in an open economy. Journal of Monetary Economics 48, 293-308.

Devereux, M. B., Purvis, D. D., 1990. Fiscal policy and the real exchange rate. European Economic Review 34, 1201-1211.

Dixit, A.V., Lambertini, L., 2003. Interactions of commitment and discretion in monetary and fiscal policies, American Economic Review 93, 1522-1542.

Dupor, B., 2000. Exchange rates and the fiscal theory of the price level. Journal of Monetary Economics 45, 613-630.

Edwards, S., Magendzo, I., 2003. Dollarization and economic performance: what do we really know? International Journal of Finance and Economics 8, 351-363.

Ehrmann, M., Fratzscher, M., 2005. Exchange rates and fundamentals: new evidence from real-time data. Journal of International Money and Finance 24, 317-341.

Eichenbaum, M., Evans, C., 1995. Some empirical evidence on the effects of shocks to monetary policy on exchange rates. Quarterly Journal of Economics, 110, 975-1009.

Engel, C., West, K.D., 2005. Exchange rates and fundamentals. Journal of Political Economy 113, 485-517.

Engle, R.F., Granger, C.W.J, 1987. Co-integration and error correction: representation, estimation, and testing. Econometrica 55, 251-276.

Engle, R.F., NG,V.K., 1993. Measuring and testing the impact of news on volatility. The Journal of Finance 58(5), 1749-1778.

Frenkel, J.A., Razin A., 1986. Fiscal policies in the world economy. Journal of Political Economy 94, 564-594.

Galati, G., Melick, W., Micu, M., 2005. Foreign exchange market intervention and expectations: the yen/dollar exchange rate. Journal of International Money and Finance 24, 982-1011.

Glosten, L., Jagannathan, R., Runkle, D., 1993. Relationship between the expected value and the volatility of the nominal excess return on stocks. The Journal of Finance, 48, 1779–1801.

Gonzalez-Riviera, G., 1998. Smooth-transition GARCH models. Studies in Nonlinear Dynamics and Econometrics 3, 61–78.

Hagerud, G. E., 1996. A smooth transition GARCH model for assets returns. Stockholm Working Paper Series in Economics and Finance 162, Stockholm School of Economics.

Hurvich, C. M., Simonoff, J. S., Tsai, C. L., 1998. Smoothing parameter selection in nonparametric regression using an improved Akaike information criterion. Journal of the Royal Statistical Society (Series B) 60, 271-293.

Ichino, A., Winter-Ebmer, R., 1998. The long-run educational cost of World War II: an example of local average treatment effect. CEPR Publication DP1895.

Jansen, D.W., Li, Q., Wang, Z., Yang, J., 2007. Fiscal policy and asset markets: a semiparametric analysis. Journal of Econometrics, Forthcoming.

Johnson, R. N., 2002. Search costs, lags and prices at the pump. Review of Industrial Organization 20, 33-50.

Krugman, P., 1979. A model of balance of payments crises. Journal of Money, Credit, and Banking 11, 311-325.

Leeper, E. M., 1991. Equilibria under ‘active’ and ‘passive’ monetary and fiscal policies. Journal of Monetary Economics 27, 129-147.

Lechner, M., 1999. Earnings and employment effects of continuous off-the-job training in East Germany after unification. Journal of Business and Economic Statistics 17, 74-90.

Li, Q., Racine, J., Wooldridge, J.M., 2007. Efficient estimation of average treatment effects with mixed categorical and continuous data. Working paper.

Li, Q., Wang, S., 1998. A simple consistent bootstrap test for a parametric regression function. Journal of Econometrics 87, 145-165.

Lin, S., Ye, H., 2007. Does inflation targeting really make a difference? Evaluating the treatment effect of inflation targeting in seven industrial countries. Journal of Monetary Economics 54, 2521-2533.

Linnemann, L., Schabert, A., 2003. Fiscal policy in the new neoclassical synthesis. Journal of Money, Credit and Banking 35, 911-929.

Ljung, G. M., Box, G. E. P., 1978. On a measure of lack of fit in time series models. Biometrika, 65, 297–303.

Michael P., Nobay, A. R., Peel, D. A., 1997. Transactions costs and nonlinear adjustment in real exchange rates: an empirical investigation. Journal of Political Economy 105, 862-879.

Obstfeld, M., 1985. Floating exchange rates: experience and prospects. Brookings Papers on Economic Activity 2, 369-450.

Peltzman, S., 2000. Prices rise faster than they fall. Journal of Political Economy 108, 466-502.

Radchenko, S, 2005. Oil price volatility and the asymmetric response of gasoline prices to oil price increases and decreases. Energy Economics 27, 708-730.

Rosenbaum, P.R., Rubin, D.B., 1983. The central role of the propensity score in observational studies for causal effects. Biometrika 70, 41-55.

Rubin, D.B., 1980. Bias reduction using Mahalanobis-metric matching. Biometrics 36, 293-298.

Sargent, T.J., 1999. A primer on monetary and fiscal policy. Journal of Banking and Finance 23, 1463-1482.

Schabert, A., 2004. Interactions of monetary and fiscal policy via open market operations. Economic Journal, 114, C186-C206.

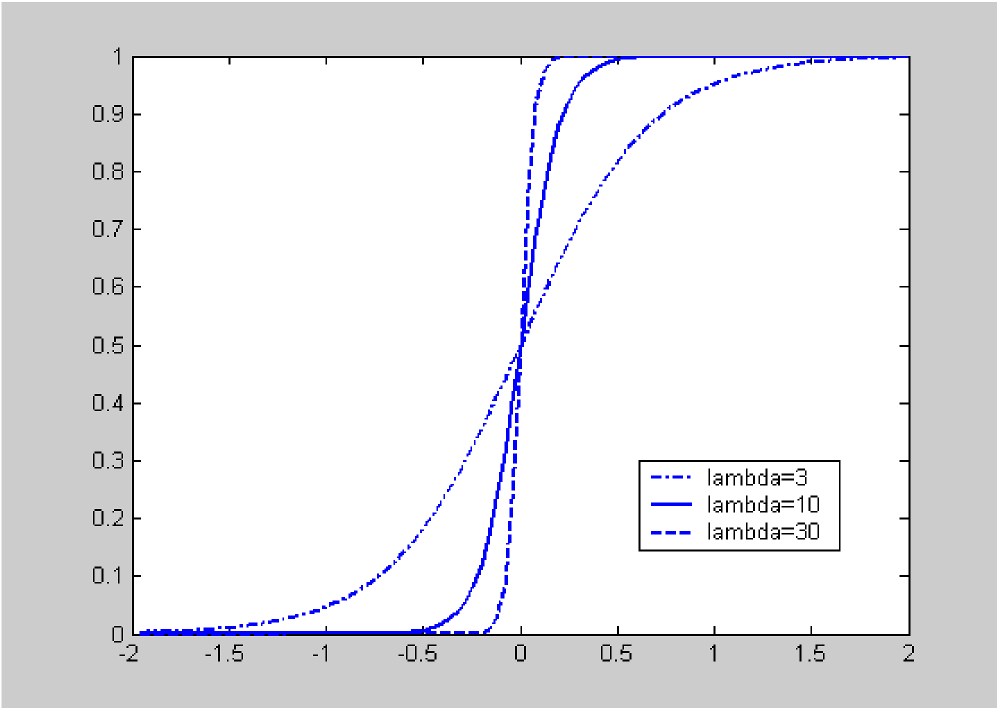
Sims, C. A., 1994. A simple model for the determination of the price level and the interaction of monetary and fiscal policy. Economic Theory 4, 381–399.

Taylor, A.M., 2002. A century of purchasing-power parity. Review of Economics and Statistics 84, 139-150.

Taylor, M. P., Peel, D. A., 2000. Nonlinear adjustment, long-run equilibrium and exchange rate fundamentals. Journal of International Money and Finance 19, 33-53.

Woodford, M., 1995. Price level determinacy without control of a monetary aggregate. Carnegie-Rochester Conference Series on Public Policy 43, 1-46.

APPENDIX A



Note:Horizontal axis has values of ε; vertical axis values of F(λ,ε)

Figure A1: Logistic Smooth Transition Function for Different Values of λ

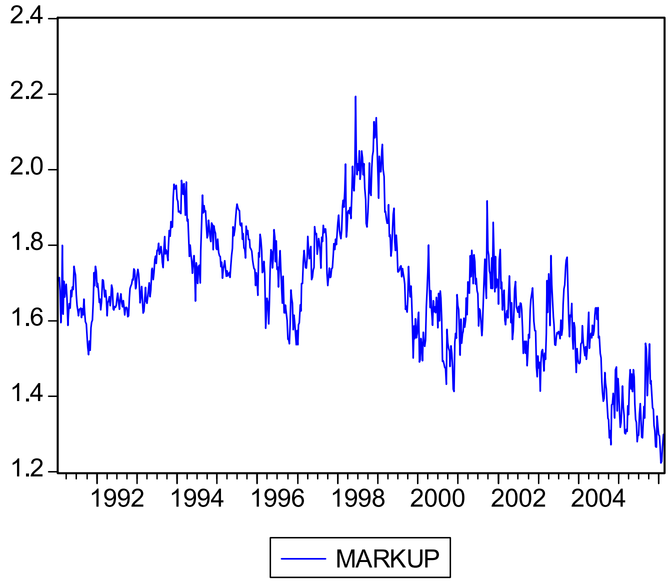


Figure A2: Mark-up Between Gasoline Price and Crude Oil Price

Table A1: Cointegration Analysis on Log of Retail Gasoline Price and Crude Oil Price
(Sample: weekly, 01/21/1991-02/13/2006)

Unrestricted Cointegration Rank Test (Trace)

Hypothesized No. of CE(s)	Eigenvalue	Trace Statistic	0.05 Critical Value	Prob.**
None *	0.062059	55.67441	25.87211	0.0000
At most 1	0.007101	5.573179	12.51798	0.5163

**MacKinnon-Haug-Michelis (1999) p-values

APPENDIX B

A signal-extraction model of consumer search in the retail gasoline market

Let p_c denote the crude oil price and $p_{g,i}$ the gasoline price of gas station *i*. Let $z_{g,i}$ be the specific factor related to gas station *i*. When a consumer shops at gas station *i*, the gasoline price of this station, $p_{g,i}$, is observable. Upon seeing this price, the consumer decides whether or not to search another gas station.

We model the price of gas at station i as consisting of a markup over the crude price and an idiosyncratic station – specific component:

$$p_{g,i} = \alpha p_c + z_{g,i}$$

where

$$p_c \sim N(\bar{p}_c, \sigma^2) \text{ and } z_{g,i} \sim N(0, \tau^2).$$

We assume that the crude price and the station-specific idiosyncratic shock are i.i.d. normal.

The consumer's search rule is as follows. When consumers observe that the gasoline price $p_{g,i}$ has changed, they attribute this price change either to a cost change -- a crude oil price shock -- or an idiosyncratic change at specific to gas station i, $z_{g,i}$. If a consumer thinks a gasoline price change is due to a crude oil shock, they consider this an aggregate shock that is the same for every gas station, so that the gasoline price would change proportionally to crude oil prices for every gas station. In such a case, consumers would not search in response to the observed change in gas prices. If, on the other hand, a consumer thinks a gasoline price change is due to the idiosyncratic shock specific to gas station i, $z_{g,i}$, he will choose to search the next gas station. In this case, the consumers decide whether they will keep searching or not, according to the expected relative change between gasoline price and crude oil price, $E(p_{g,i} - \alpha p_c)$.

In the above model, we have the following expressions for the variance of gasoline prices, and for the covariance between gasoline prices and crude oil prices:

$$\begin{aligned}\text{var}(p_{g,i}) &= E(p_{g,i} - E(p_{g,i}))^2 = E[\alpha p_c + z_{g,i} - \alpha \bar{p}_c]^2 \\ &= E\left[\alpha^2 (p_c - \bar{p}_c)^2 + z_{g,i}^2 + 2\alpha(p_c - \bar{p}_c) z_{g,i}\right] = \alpha^2\sigma^2 + \tau^2\end{aligned}$$

$$\begin{aligned}\text{cov}(p_{g,i}, p_c) &= E[(p_{g,i} - Ep_{g,i})(p_c - \bar{p}_c)] = E[(\alpha(p_c - \bar{p}_c) + z_{g,i})(p_c - \bar{p}_c)] \\ &= E[(\alpha(p_c - \bar{p}_c)^2] + E[z_{g,i}(p_c - \bar{p}_c)] = \alpha\sigma^2\end{aligned}$$

In this case, the mean and variance of $p_{g,i}$ and p_c will be:

$$\begin{pmatrix} p_{g,i} \\ p_c \end{pmatrix} \sim N\left(\begin{pmatrix} \alpha\bar{p}_c \\ \bar{p}_c \end{pmatrix}, \begin{pmatrix} \alpha^2\sigma^2 + \tau^2 & \alpha\sigma^2 \\ \alpha\sigma^2 & \sigma^2 \end{pmatrix}\right)$$

For the consumers, the gasoline price $p_{g,i}$ is observable when they reach the gas station i, but the crude oil price is unobservable. When the consumers observe the change of gasoline price, they will form an expectation of how much the change comes from the crude oil price shock and how much from the idiosyncratic shock to a particular station. From a standard information extraction procedure, we have,

$$E(p_c \mid p_{g,i}) = \bar{p}_c + \frac{\operatorname{cov}(p_c, p_{g,i})}{\operatorname{var}(p_{g,i})}(p_{g,i} - Ep_{g,i}) = \bar{p}_c + \frac{\alpha\sigma^2}{\alpha^2\sigma^2 + \tau^2}(p_{g,i} - \alpha\bar{p}_c)$$
$$= \frac{\alpha\sigma^2}{\alpha^2\sigma^2 + \tau^2} p_{g,i} + (1 - \frac{\alpha^2\sigma^2}{\alpha^2\sigma^2 + \tau^2})\bar{p}_c$$

In this case, the expected markup will be,

$$E(p_{g,i} - \alpha p_c) = p_{g,i} - \alpha E(p_c \mid p_{g,i}) = p_{g,i} - \frac{\alpha^2\sigma^2}{\alpha^2\sigma^2 + \tau^2} p_{g,i} - \alpha(1 - \frac{\alpha^2\sigma^2}{\alpha^2\sigma^2 + \tau^2})\bar{p}_c$$
$$= (1-\phi)p_{g,i} - \alpha(1-\phi)\bar{p}_c = (1-\phi)(p_{g,i} - \alpha\bar{p}_c)$$

where $\phi = \frac{\alpha^2\sigma^2}{\alpha^2\sigma^2 + \tau^2}$.

Note the importance of the variance terms. When σ^2 increase, ϕ increases, so $1-\phi$ decreases. The markup will decrease, ceteris paribus. Then consumers will tend not to search. When τ^2 increases, ϕ decreases, and $1-\phi$ increases. The markup will increase, ceteris paribus. In this case consumers will tend to keep searching. Further, when search behavior increases, the degree of asymmetry in response to crude shocks will decrease.

From our empirical work, both gasoline price and crude oil price volatility increased in recent years, and the gasoline price volatility increased more than the crude oil price. This is consistent with the above signal extraction model.

APPENDIX C

Proof of Theorem 4.2.1

From the definition of $L(x_j, x_i, \lambda) = \Pi_{s=1}^{r} L(x_{js}, x_{is}, \lambda_s)$, it is easy to see that

$$L(x_j, x_i, \lambda) = 1(x_j = x_i) + O_p\left(\sum_{s=1}^{r} \hat{\lambda}_s\right) = 1(x_j = x_i) + O_p\left(n^{-1}\right), \tag{C1}$$

where we have used $\hat{\lambda}_s = O_p\left(n^{-1}\right)$ by Theorem 4.2.1 of Li, Ouyang and Racine (2006).

Given that (18), we immediately have

$$\hat{t}_i = \tilde{t}_i + O_p\left(n^{-1}\right). \tag{C2}$$

Where $\tilde{t}_i = \sum_j t_j 1(x_j = x_i) / \sum_j 1(x_j = x_i)$ is the frequency-based estimator of $E(t_i \mid x_i)$.

In this appendix we will use the tilde notation to denote a frequency-based nonparametric estimator. For example, the frequency-based average treatment effects estimator is obtained from $\hat{\tau}$ by replacing $\hat{t}_i$ by $\tilde{t}_i$, i.e.,

$$\tilde{\tau} = \frac{1}{n} \sum_{t=1}^{n} \frac{(t_i - \tilde{t}_i) y_i}{\tilde{t}_i (1 - \tilde{t}_i)}. \tag{C3}$$

$$\tau = E\left[\frac{1}{n} \sum_{t=1}^{n} \frac{(t_i - \mu_i) y_i}{\mu_i (1 - \mu_i)}\right] = E\left(\tau_i\right). \tag{C4}$$

Recall that $\mu_i = \Pr\left(t_i = 1 \mid X_i\right) \equiv E\left(t_i \mid X_i\right)$.

Define $v_i = t_i - \mu_i \equiv t_i - E(t_i \mid X_i)$ so that $t_i = \mu_i + v_i$ and $E(v_i \mid X_i) = 0$. From

$\tilde{t}_i = \sum_j t_j 1(x_j = x_i) / \sum_j 1(x_j = x_i)$,

$\tilde{\mu}_i = \sum_j \mu_j 1(x_j = x_i) / \sum_j 1(x_j = x_i) = \sum_j (\mu_j + v_j) 1(x_j = x_i) / \sum_j 1(x_j = x_i)$,

We obtain,

$$\tilde{t}_i = \tilde{\mu}_i + \tilde{v}_i \tag{C5}$$

Where $\tilde{\mu}_i = \sum_j \mu_j 1(x_j = x_i) / \sum_j 1(x_j = x_i)$ and

$\tilde{v}_i = \sum_j v_j 1(x_j = x_i) / \sum_j 1(x_j = x_i)$. From $E(\tilde{v}_i \mid X_i) = 0$ it is easy to show that

$\tilde{v}_i = O_p(n^{-1})$.

Also note that $w_i = \mu_i(1-\mu_i)$, $\tilde{w}_i = \tilde{t}_i(1-\tilde{t}_i)$.

We further define some intermediate quantities:

$$\bar{\tau} \overset{def}{=} \frac{1}{n}\sum_{i=1}^{n} \frac{(t_i - \mu_i) y_i}{w_i} = \frac{1}{n}\sum_{i=1}^{n} \frac{v_i y_i}{w_i} \tag{C6}$$

$$\breve{\tau} \overset{def}{=} \frac{1}{n}\sum_{i=1}^{n} \frac{(t_i - \tilde{t}_i) y_i}{w_i} \tag{C7}$$

By adding and subtracting terms we obtain

$$\tilde{\tau} - \tau = (\tilde{\tau} - \breve{\tau}) + (\breve{\tau} - \bar{\tau}) + (\bar{\tau} - \tau). \tag{C8}$$

Substituting the following identity

$$\frac{1}{\tilde{w}_i} = \frac{1}{w_i} + \frac{w_i - \tilde{w}_i}{w_i^2} + \frac{(w_i - \tilde{w}_i)^2}{w_i^2 \tilde{w}_i} \tag{C9}$$

into $\tilde{\tau} - \bar{\tau}$, and note that $\tilde{w}_i - w_i = O_p(n^{-1})$, we obtain

$$(\tilde{\tau} - \breve{\tau}) = \frac{1}{n}\sum_{i=1}^{n}(t_i - \tilde{t}_i)y_i\left(\frac{1}{\tilde{w}_i} - \frac{1}{w_i}\right)$$

$$= \frac{1}{n}\sum_{i=1}^{n}(t_i - \tilde{t}_i)y_i\left(\frac{w_i - \tilde{w}_i}{w_i^2} + \frac{(w_i - \tilde{w}_i)^2}{w_i^2\tilde{w}_i}\right)$$

$$= \frac{1}{n}\sum_{i=1}^{n}(t_i - \tilde{t}_i)y_i\left(\frac{w_i - \tilde{w}_i}{w_i^2}\right) + O_p\left(n^{-1}\right) \tag{C10}$$

Note that $\mu_i - \tilde{\mu}_i = \left[\mu_i\tilde{p}_i - \tilde{\mu}_i\tilde{p}\right]/\tilde{p}_i$, and $\mu_i\tilde{p}_i - \tilde{\mu}_i\tilde{p} = n^{-1}\sum_{j=1}^{n}(\mu_i - \mu_j)1(x_j - x_i) \equiv 0$,

we obtain

$$\mu_i - \tilde{\mu} = 0. \tag{C11}$$

Using $\tilde{t}_i = \tilde{\mu}_i + \tilde{v}_i$, we have

$$w_i - \tilde{w}_i = \mu_i - \mu_i^2 - (\tilde{t}_i - \tilde{t}_i^{\,2})$$

$$= (\mu_i - \tilde{t}_i)\left[1 - (\mu_i + \tilde{t}_i)\right]$$

$$= (\mu_i - \tilde{\mu}_i - \tilde{v}_i)\left[1 - 2\mu_i + (\mu_i - \tilde{\mu}_i) - \tilde{v}_i\right] \tag{C12}$$

$$= -\tilde{v}_i\left[1 - 2\mu_i - \tilde{v}_i\right],$$

since $\mu_i - \tilde{\mu} = 0$ by (C11).

Combining (C10), (C11) and (C12), and note that $\tilde{v}_i = O_p\left(n^{-1}\right)$, we get

$$(\tilde{\tau} - \breve{\tau}) = \frac{1}{n}\sum_{i=1}^{n}(t_i - \tilde{t}_i)y_i\left(\frac{w_i - \tilde{w}_i}{w_i^2}\right) + O_p\left(n^{-1}\right)$$

$$= \frac{1}{n}\sum_{i=1}^{n}(\mu_i + v_i - \tilde{\mu}_i - \tilde{v}_i)y_i\frac{1}{w_i^2}\left(-\tilde{v}_i\right)\left[1 - 2\mu_i - \tilde{v}_i\right] + O_p\left(n^{-1}\right) \tag{C13}$$

$$= \frac{1}{n}\sum_{i=1}^{n}\frac{v_i y_i \tilde{v}_i (2\mu_i - 1)}{w_i^2}_{\quad i} + O_p\left(n^{-1}\right)$$

Using $\tilde{v}_i = n^{-1}\sum_{j=1}^{n} v_j 1(x_j = x_i) / \tilde{p}_i$, and $\tilde{p}_i = 1/p_i + O_p\left(n^{-1/2}\right)$, and put these into equation (C13), we get,

$$(\tilde{\tau} - \breve{\tau}) = \frac{1}{n^2}\sum_{i=1}^{n}\sum_{j=1}^{n}\frac{y_i v_j}{w_i^2 p_i}[v_i(2\mu_i - 1)]1\left(x_j = x_i\right) + O_p\left(n^{-1}\right)$$

$$= \frac{1}{n^2}\sum_{i=1}^{n}\sum_{j=1}^{n}\frac{y_i v_i^2}{w_i^2 p_i}[2\mu_i - 1]$$

$$+ \frac{1}{n^2}\sum_{i=1}^{n}\sum_{j\neq 1}^{n}\frac{y_i}{w_i^2 p_i} v_j 1\left(x_j = x_i\right)[v_i(2\mu_i - 1)] + O_p\left(n^{-1}\right) \qquad \text{(C14)}$$

$$= \frac{1}{n(n-1)}\sum_{i=1}^{n}\sum_{j\neq 1}^{n}\frac{y_i v_i v_j}{w_i^2 p_i}(2\mu_i - 1)1\left(x_j = x_i\right) + O_p\left(n^{-1}\right)$$

$$= \frac{2}{n(n-1)}\sum_{i=1}^{n}\sum_{j>i}^{n} H_{ij} + O_p\left(n^{-1}\right),$$

where

$$H_{ij} = H\left(z_i, z_j\right) = \frac{1}{2} v_i v_j\left[y_i\left(2\mu_i - 1\right)\frac{1}{w_i^2 p_i} + y_j\left(2\mu_j - 1\right)\frac{1}{w_j^2 p_j}\right]1\left(x_j = x_i\right) \qquad \text{(C15)}$$

with $z_i = \left(x_i, t_i, u_i\right)$.

Define $H_i = E\left(H_{ij} \mid z_i\right)$, then we have (since $E\left(v_j \mid x_j\right) = 0$)

$$H_i = 0 + \frac{1}{2} v_i E\left[v_j y_j\left(2\mu_j - 1\right)\frac{1}{w_j^2 p_j}1\left(x_j = x_i\right) \mid z_i\right]. \qquad \text{(C16)}$$

Replace $y_j = g_{0j} + \tau_j t_j + u_j = g_{0j} + \tau_j(\mu_j + v_j) + u_j$ in (33) and note that $E(v_j \mid x_j) = 0$, we obtain

$$H_i = \frac{1}{2} v_i E\left[v_j \left(g_{0j} + \tau_j(\mu_j + v_j) + u\right)(2\mu_j - 1)\frac{1}{w_j^2 p_j} 1(x_j = x_i) \mid z_i \right]$$

$$= \frac{1}{2} v_i \tau_i (2\mu_j - 1)\frac{1}{w_j^2 p_j} E\left[v_j^2 1(x_j = x_i) \mid z_i\right]$$

$$= \frac{1}{2} v_i \tau_i (2\mu_j - 1)\frac{1}{w_i},$$

where in the last equality we used

$$E\left[v_j^2 1(x_j = x_i) \mid z_i\right] = E\left\{E\left[(t_j - \mu_j)^2 \mid x_j\right] z_i\right\} = E\left[(\mu_j - \mu_j^2) 1(x_j = x_i) \mid z_i\right]$$

$$= \sum_{x_j \in D} p(x_j)(\mu_j - \mu_j^2) 1(x_j - x_i) = p(x_j)(\mu_j - \mu_j^2) \equiv p_i w_i . \qquad \text{(C17)}$$

Hence, by the U-statistic H-decomposition we have

$$(\tilde{\tau} - \breve{\tau}) = 0 + \frac{2}{n}\sum_{i=1}^{n} H_i + \frac{2}{n(n-1)}\sum_{i=1}^{n}\sum_{j>i}^{n}\left[H_{ij} - H_i - H_j + 0\right]$$

$$= \frac{1}{\sqrt{n}}\sum_{i=1}^{n} v_i \tau_i (2\mu_j - 1)\frac{1}{w_i} + O_p(n^{-1}).$$

Next, we consider J_{2n}. Using (C11) we have

$$(\tilde{\tau} - \breve{\tau}) = \frac{1}{n}\sum_{i=1}^{n}\frac{1}{w_i}\left[(t_i - \tilde{t}_i) y_i - v_i y_i\right]$$

$$= \frac{1}{n}\sum_{i=1}^{n}\frac{1}{w_i}\left[(\mu_i + v_i - \tilde{\mu}_i - \tilde{v}_i) - v_i\right] y_i$$

$$= -\frac{1}{n}\sum_{i=1}^{n}\frac{\tilde{v}_i y_i}{w_i} \tag{C18}$$

$$= -\frac{1}{n(n-1)}\sum_{i=1}^{n}\sum_{j\neq i}^{n}\frac{y_i v_j}{w_i\, p_i}1\left(x_j = x_i\right)$$

$$= -\frac{2}{n(n-1)}\sum_{i=1}^{n}\sum_{j>i}^{n}H_{2,ij}$$

where $H_{2,ij} = H_2\left(z_i, z_j\right) = \frac{1}{2}\left(\frac{y_j v_j}{w_i p_i} + \frac{y_j v_i}{w_j p_j}\right)1\left(x_i = x_j\right)$ with $z_i = \left(x_i, t_i, u_i\right)$.

From $E\left(v_i \mid x_i\right) = 0$ and $E\left(u_i \mid x_i, t_i\right) = 0$, it follows that $E\left(H_{2,ij}\right) = 0$ because $y_j = g_{0j} + \tau_j t_j + u_j = g_{0j} + \tau_i\left(\mu_j + v_j\right) + u_j$. Define $H_{2,i} = E\left(H_{2,ij} \mid z_i\right)$, then we have

$$H_{2,i} = E\left(H_{2,ij} \mid z_i\right)$$

$$= 0 + \frac{1}{2}v_i E\left[\frac{\left(g_{0j} + \tau_i\left(\mu_j + v_j\right) + u_j\right)}{w_j p_j}1\left(x_j = x_i\right) \mid z_i\right]$$

$$= \frac{v_i}{2}\sum_{x_j} p\left(x_j\right)\frac{\left(g_{0i} + \tau_i \mu_i\right)}{w_i p_i}1\left(x_j = x_i\right) + s.o. \tag{C19}$$

$$= \frac{1}{2}\frac{v_i\left(g_{0i} + \tau_i \mu_i\right)}{w_i}$$

Hence, by the U-statistic H-decomposition we have

$$(\breve{\tau} - \bar{\tau}) = -\left[0 + \frac{2}{n}\sum_{i=1}^{n}H_{2,i} + \frac{2}{n(n-1)}\sum_{i=1}^{n}\sum_{j>i}^{n}\left[H_{2,ij} - H_{2,i} - H_{2,j} + 0\right]\right] \tag{C20}$$

$$= -\frac{1}{n}\sum_{i=1}^{n}\frac{\left(g_{0i} + \tau_i \mu_i\right)v_i}{w_i} + O_p\left(n^{-1}\right).$$

Finally, we consider $\bar{\tau} - \tau$.

$$\bar{\tau} - \tau = \frac{1}{n}\sum_{i=1}^{n}\left(\frac{v_i y_i}{w_i} - \tau\right) \tag{C21}$$

$$= \frac{1}{n}\sum_{i=1}^{n}\left(\frac{v_i\left(g_{0i} + \tau_i t_i + u_i\right)}{w_i} - \tau_i\right) + \frac{1}{n}\sum_{i=1}^{n}\left(\tau_i - \tau\right).$$

$$\sqrt{n}\left(\tilde{\tau} - \tau\right) = \sqrt{n}\left[(\tilde{\tau} - \breve{\tau}) + (\breve{\tau} - \bar{\tau}) + (\bar{\tau} - \tau)\right]$$

$$= \frac{1}{\sqrt{n}}\sum_{i=1}^{n}\left(v_i\tau_i\left(2\mu_i - 1\right)\frac{1}{w_i} - \frac{v_i\left(g_{0i} + \tau_i\mu_i\right)}{w_i} + \frac{v_i\left(g_{0i} + \tau_i t_i + u_i\right)}{w_i} - \tau_i\right)$$

$$+ n^{-1/2}\sum_{i=1}^{n}\left(\tau_i - \tau\right) + O_p\left(n^{-1/2}\right) \tag{C22}$$

$$= n^{-1/2}\sum_{i=1}^{n}\left[\frac{v_i u_i}{w_i} + \left(\tau_i - \tau\right)\right] + O_p\left(n^{-1/2}\right),$$

where we used the fact that $w_i^{-1}\left[2v_i\tau_i\mu_i - v_i\tau_i + v_i^2\tau_i\right] - \tau_i = \frac{\tau_i}{w_i}\left[\left(\mu_i + v_i\right)^2 - \left(\mu_i + v_i\right)\right] = 0$

because $\left(\mu_i + v_i\right)^2 - \left(\mu_i + v_i\right) = t_i^2 - t_i = 0$ since $t_i^2 = t_i$, ($t_i \in \{0,1\}$).

By the Lindeberg central limit theorem, we know that

$$\sqrt{n}\left(\tilde{\tau} - \tau\right) \to N\left(0, V\right) \text{ in distribution,}$$

where $V = \mathrm{var}\left(\frac{v_i u_i}{w_i}\right) + \mathrm{var}\left(\tau_i\right)$ (since $\mathrm{cov}\left(\tau_i, u_i\right) = 0$). It can be easily shown that

$$\mathrm{var}\left(\frac{v_i u_i}{w_i}\right) = E\left[v_i^2 u_i^2 / w_i^2\right] = E\left\{\left(t_i - \mu_i\right)^2 \sigma^2\left(x_i, t_i\right) / \left[\mu_i^2\left(1 - \mu_i^2\right)\right]\right\} = E\left[\frac{\sigma^2\left(x_i, 1\right)}{\mu_i} + \frac{\sigma^2\left(x_i, 0\right)}{\mu_i}\right].$$

VITA

Name: Jingping Gu

Address: Department of Economics, Texas A&M Universtiy, 4228TAMU,
College Station, TX 77843-4228

Email Address: jgu@econmail.tamu.edu

Education: Ph.D., Economics, Texas A&M University, August 2008
M.A., Economics, Peking University, China, 2001
B.A., Economics, Renmin University of China, China, 1997

Fields of Specialization: Macro/Monetary Economics
Econometrics
International Economics