LOSS MODELING FOR PRICING CATASTROPHIC BONDS

A Thesis

by

JYOTIRMOY SIRCAR

Submitted to the Office of Graduate Studies of
Texas A&M University
in partial fulfillment of the requirements for the degree of

MASTER OF SCIENCE

August 2008

Major Subject: Civil Engineering

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Approved by:

Chair of Committee,   John B. Mander
Committee Members, Ivan Damnjanovic
                                 Kerry Back
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ABSTRACT

Loss Modeling for Pricing Catastrophic Bonds.

(August 2008)

Jyotirmoy Sircar, B.Eng., Osmania University, Hyderabad

Chair of Advisory Committee: Dr. John B. Mander

It is important to be able to quantify potential seismic damage to structures and communicate risk in a comprehendible way to all stakeholders. The risks involved with damage to constructed facilities due to catastrophic disasters can be hedged using financial instruments such as Catastrophic (CAT) bonds. This work uses the loss ratio (L_r), which is the ratio of the repair cost to the total replacement cost, to represent structural and non-structural damage caused by earthquakes.

A loss estimation framework is presented that directly relates seismic hazard to seismic response to damage and hence to losses. A key feature of the loss estimation approach is the determination of losses without the need for fragility curves. A Performance-Based Earthquake Engineering (PBEE) approach towards assessing the seismic vulnerability of structures relating an intensity measure (IM) to its associated engineering demand parameter (EDP) is used to define the demand model. An empirically calibrated tripartite loss model in the form of a power curve with upper and lower cut-offs is developed and used in conjunction with the previously defined demand model in order to estimate loss ratios. The loss model is calibrated and validated for

different types of bridges and buildings. Loss ratios for various damage states take into account epistemic uncertainty as well as an effect for price surge following a major hazardous event. The loss model is then transformed to provide a composite seismic hazard-loss relationship which is used to estimate financial losses from expected structural losses.

The seismic hazard-loss model is then used to assess the expected spread, that is the interest rate deviation above the risk-free (prime) rate in order to price two types of CAT bonds: indemnity CAT bonds and parametric CAT bonds. It is concluded that CAT bonds has the ability to play a major role in hedging financial risk associated with damage to a civil engineering facility as a result of a catastrophe. However, it is seen that a potential investor seeks a high degree of confidence when investing in CAT bonds as there is huge uncertainty surrounding the probability of occurrence of an event.

DEDICATION

DEDICATED TO MY FAMILY AND FRIENDS

ACKNOWLEDGEMENTS

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1. INTRODUCTION

1.1. Importance of the Research

It is important to be able to quantify potential seismic damage to structures and communicate risk in a comprehendible way to all stakeholders. The risks involved with damage to constructed facility due to catastrophic disasters can be hedged using financial instruments such as Catastrophic (CAT) bonds. This work uses the Loss Ratio (L_r), which is the ratio of the repair cost to the total replacement cost, to represent structural and non-structural damage caused by earthquakes. A loss estimation framework is first presented that directly relates seismic hazard to seismic response to damage and hence to losses. A key feature of the loss estimation approach is the determination of losses without the need for fragility curves. Relationships between intensity measures and engineering demand parameters are used to define the demand model. An empirically calibrated loss model in the form of a power curve with upper and lower cut-offs is used in conjunction with the demand model in order to estimate loss ratios. Loss ratios for various damage states take into account epistemic uncertainty as well as an effect for price surge following a major hazardous event.

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This thesis follows the style of *Journal of Structural Engineering*.

The loss model is calibrated and validated for different types of bridges and buildings. The loss model is then transformed to provide a composite seismic hazard-loss relationship and used to facilitate a transition between understanding expected structural loss and financial loss estimation. The seismic hazard-loss model is then used to assess the expected spread, that is the interest rate deviation above the risk-free (prime) rate in order to price two types of CAT bonds: indemnity CAT bonds and parametric CAT bonds. Conclusions are drawn as to the classes of constructed facilities that are most suited to the potential application of such financial instruments.

1.2.   Research Motivations

Earthquakes and other natural hazards have the capability of seriously damaging constructed facilities inflicting loss to life and limb. It is therefore necessary for all stakeholders to clearly identify and mitigate the risks to the greatest possible extent. Financial losses resulting from seismically damaged structures can be estimated using a four step approach that can be subdivided into four distinct tasks: (i) hazard analysis; (ii) structural analysis; (iii) damage analysis; and (iv) loss analysis. Loss ratio (L_r), which is the ratio of the repair cost to the total replacement cost, can be considered to be an effective parameter to represent costs associated with structural and non-structural damage caused by uncertain events like earthquakes. When these are integrated and averaged over all possible seismic scenarios, the Expected Annual Loss (EAL) can be computed. EAL can be used as a parameter by the insurance industry to indemnify a

possible portfolio of civil engineering facilities like buildings or bridges. Furthermore, EAL can be used as a basis for deriving the terms and conditions of hazard-related risk mitigating tools like catastrophic bonds commonly known as *CAT* bonds. Unlike insurance, CAT bonds are considered to provide a good hedging mechanism against natural disasters since their occurrence of being rare and unexpected is generally uncorrelated with financial market parameters (Doherty 1997). However, no fixed loss estimation methodology is implemented to design special-purpose vehicles like CAT bonds. Therefore there is a need to provide a straight-forward and robust methodology to facilitate a smooth transition between expected structural losses estimation and financial loss estimation. The presence of a robust loss estimation framework can ensure efficient and effective pricing of hazard-related financial risk-hedging tools and reduce the inherent uncertainty involved with these tools.

1.3. Problem Statement

In the event of the destruction caused by a major hazard, reconstruction can be efficiently carried out for damaged structures through the rapid infusion of funds. Catastrophic bonds, commonly termed CAT bonds are a new and useful financial instrument for this purpose. CAT bonds help mitigate and distribute the financial losses and risks caused by natural hazards, as they provide the desired level of flexibility. Insurers collect premium from owners of facilities and may then sell bonds at a stipulated value to investors. Since CAT bonds are considered to be high-risk high-return

bonds, investors may then see them as an attractive alternative investment option as bonds are listed with the Securities Exchange Commission; the given rating clarifies investor risk. It is therefore necessary to provide a robust loss-estimation framework which can then be used to price parameters such as spread or deviation from the risk-free rate and risk premium for CAT bonds. The framework should be capable of making a smooth transformation from estimating losses due to seismic damage to estimating CAT bond prices and account for the presence of both structural and non-structural components.

1.4.   Research Objectives

The overall objectives of this research are:

a) To develop a simplified loss estimation procedure that directly relates hazard to response and hence to losses. A key feature of the loss estimation approach is the determination of losses without the need for fragility curves. Simple relationships between previously calibrated intensity measures and engineering demand parameters are used to define the demand model. An empirically calibrated loss model in the form of a power curve with upper and lower cut-offs is used in conjunction with the demand model in order to estimate loss ratios. Loss ratios for each of the damage states take into account epistemic uncertainty as well as an effect for price surge following a major hazardous event.

b) To calibrate and validate the loss model for bridges designed as per Caltrans, Japan, New Zealand, and Damage Avoidance Design (DAD) philosophies, as well as Welded Steel Moment Frame and Concrete Frame Buildings.

c) To investigate the impact and contribution of structural and non-structural components to the total losses of civil engineering facilities.

d) To modify and implement the simplified loss estimation procedure appropriately in order to price and value CAT bonds.

e) To explore various trigger mechanisms and term structures of CAT bonds based on the loss-estimation procedure and propose valid schemes of operation.

2. LITERATURE REVIEW

Earthquakes and other natural hazards have the capability to inflict losses to life, limb and property. Since time immemorial, it has been man's endeavor to mitigate losses caused due to so called natural catastrophic events. The nuclear engineering industry was the the first to implement the use of risk-based hazard mitigation strategy through the use of probabilistic seismic hazard analysis and fragility curves. These helped in assessing the seismic vulnerability of structures and facilitated design for satisfying specified performance objectives (Kennedy et al. 1980; Kennedy and Ravindra 1984; Kennedy 1999). Initial risk and safety management practices for nuclear power plants were based on satisfying certain safety functions; redundancy and diversity in safety functions were necessary attributes. A whole set of so called hazards were created and included in design of the nuclear plants. The concept of credible events was devised and it was believed that if the nuclear plants had the capability to withstand all large credible events, then the plant would be capable of withstanding any credible event (Garrick and Christie 2002).

The first step towards realizing a performance based design objective for structures involves determining the risk at the site of an engineering project. Methods for evaluating the seismic risk at project sites were developed in order to express a ground motion parameter in terms of return period of the event (Cornell 1968). This amounted to developing what is commonly referred to as the seismic hazard curves (Kennedy

1999). Similarly years of studies led to relationships being developed as part of the process of quantifying seismic demands on structures in terms of ground motion parameters (Shome and Cornell 1999). This was part of the second step of the performance based design paradigm.

The aforementioned steps are considered to be the domain of seismologists, geologists, and structural civil engineers. However in lieu of increasing awareness amongst the general public about hazards and its impacts, there arose a necessity to relate demands to capacities and hence to probable losses. This led to the genesis of methods for earthquake loss estimation. In 1972, National Oceanic and Atmospheric Agency (NOAA) carried out studies for San Francisco area (Algermissen et al. 1972). The Federal Emergency Management Agency (FEMA) presented the National Academy of Sciences report titled Estimating Losses from Future Earthquakes in 1989. This report laid the ground work for carrying out loss estimation method development and studies. FEMA partnered with NIBS to develop a standard nationally applicable loss estimation framework for estimating losses on a regional basis. Thereafter, National Institute of Building Sciences (NIBS) carried out thirty major earthquake loss studies as part of the report submitted to (FEMA-249) in 1994 (Whitman et al. 1997). NIBS constituted an eight member committee of technical experts and an eighteen member committee to represent user interests in earthquake community. These two committees proposed a set of components and objectives for the loss estimation methodology. Simultaneously, NIBS contracted a venture between Risk Management Solutions (RMS) and the

California Universities of Research in Earthquake Engineering (CUREe) in order to develop standard earthquake loss estimation methodologies (Kircher et al. 2006).

Recent loss estimation frameworks incorporated in HAZUS were based on seismic fragility curve analysis. Fragility curves were developed as part of studies to quantify the seismic vulnerability of highway bridges (Mander and Basoz 1999) and for welded steel moment frame buildings (Kircher 2003). The main aim of these methods was to estimate losses for given structures based on discrete damage states. The damage states were calibrated based on both experimental data and analytical studies. Earthquake loss estimation was related closely to damage being incurred as a result of demand exceeding capacity. A different approach for assessing the vulnerability of buildings to seismic events was to express performance in terms of the assembly based vulnerability framework. The method had the capability to account for a building's seismic exposure as well as the impact on structural and nonstructural components (Porter et al. 2001).

The final step in the four step performance based design involved quantifying losses in terms of annual frequencies of events. Recent studies have quantified financial seismic vulnerability of structures (Dhakal and Mander 2006; Mander et al. 2007; Solberg et al. 2008) in terms of dollars per million dollar of asset for a given exposure time. This approach of expressing losses is considered apt since it expresses losses in terms of commonly used and observed quantities and can help link the engineering community with the financial community.

Losses from natural hazards caused inflation-adjusted catastrophe losses of about $98 billion between 1989 and 1998 (Bantwal and Kunreuther 2000). These losses were mostly due to hurricane Andrew and the Northridge earthquake. The main aim was therefore to transfer the risks arising out of the hazards itself ; since studies revealed that the insurance industry did not have the capability to withstand the effects of a large catastrophe (Cummins et al. 2002) and therefore special purpose vehicles (SPV) functioning as risk hedging schemes needed to be developed. The most prominent type of risk-linked security is the catastrophic risk (CAT) bond, which is a fully-collateralized SPV that defaults when a defined catastrophe occurs. These types of securities are deemed useful since they can utilize the capital markets to their benefit. Though the market for hazard and therefore risk-linked securities such as CAT bonds is small in comparison with non-life reinsurance market, it has increased and evolved over the last few years (Cummins 2008).

Due to its continuous evolution, the market for CAT bonds has not been standardized. One major drawback of CAT bonds has been an apparent high spread (ratio of premium to expected losses). However due to a stabilization of the CAT bond market coupled with an acceptance amongst the capital market has led to spread values reducing to about 2.3 in the first quarter of 2007 from 6.5 reported earlier (Cummins 2008). Though there are numerous studies carried out on pricing CAT bonds and its inherent risk (quantified through risk), none of it uses a specific loss-estimation procedure. In fact loss estimation methodologies are often bypassed and the focus remains on calibrating and designing other financial parameters.

Thus there is a need to provide a simple yet robust loss estimation framework that allows going from hazard to demand to response to losses and hence to perceived market risk. The main aim is to link a performance based design paradigm with tools for hedging probable losses (and hence risks) in order to ensure that no stakeholder loses money. More importantly, the loss estimation framework should contribute to the ultimate cause of reducing death, downtime, and destruction.

3. THEORETICAL DIRECT LOSS MODEL FOR SEISMICALLY DAMAGED STRUCTURES

3.1. Section Summary

Loss ratio which is the ratio of the repair cost to the total replacement cost, is an effective parameter to represent structural and non-structural damage caused by earthquakes. A probabilistic loss estimation framework is first presented that directly relates hazard to response and hence to losses. A key feature of the loss estimation approach is the determination of losses without need for customary demand-side fragility curves. Relationships between intensity measures and engineering demand parameters are used to define the demand model. An empirically calibrated loss model in the form of a power curve with upper and lower cut-offs is used in conjunction with the demand model in order to estimate loss ratios. Loss ratios for each of the damage states take into account epistemic uncertainty as well as an effect for price surge following a major hazardous event. The loss model is calibrated and validated for bridges designed to prevailing Caltrans, Japan, and New Zealand standards. The loss model is then transformed to provide a composite seismic hazard-loss relationship which is used to estimate expected annual loss for structures. The closed form four-step loss estimation method is applied to the bridges designed for ductility. Results of these ductile designs are compared to a bridge detailed to an emerging Damage Avoidance Design philosophy.

3.2.   Background

Financial losses resulting from seismically damaged structures can be estimated using a four-step approach that can be subdivided into four distinct tasks: (i) hazard analysis; (ii) structural analysis; (iii) damage and hence repair-cost analysis; and (iv) loss estimation. It is possible to use probabilistic risk assessment methodology to estimate expected annual financial loss (Dhakal and Mander 2006; Mander et al. 2007; Solberg et al. 2008) for structures using a combination of fragility curves with loss functions (Dhakal and Mander 2006; Mander et al. 2007; Solberg et al. 2008). The hazard analysis requires evaluation of seismic hazard at the constructed facility site, and generating intensity measures (IM) representative to the varying local hazard levels. The structural analysis involves predicting the response of the facility to increasing levels of ground shaking in terms of engineering demand parameters (EDP). The damage analysis uses EDPs to determine damage measures to the facility components from which repair costs can be estimated. The loss analysis involves determination of direct financial losses to the structure and its contents. Indirect losses such as downtime and death for a given level of shaking can also be determined in a similar fashion. These measures of performance are referred to as decision variables, since they can be used to inform stakeholder decisions about future performance. Each relationship, from location, seismic demand versus capacity, and capacity versus loss involves uncertainty and must be treated probabilistically.

Increasingly methods are being developed to quantify seismic damage in a way that becomes comprehendible to all. It is possible to assess Loss Ratios (L_r) for various

seismic scenarios. When these are integrated and averaged over all possible scenarios, the Expected Annual Loss (EAL) can be computed. Both L_r and EAL are reasonable parameters for stakeholders to comprehend as they are analogous to everyday occurrences such as fixing (or replacing) a car after a crash and the ongoing insurance cost of owning a car.

There is a need to develop a method that relates L_r to an EDP through a simple relationship in order to rapidly determine scenario losses as well as overall EAL. Previous work (Kircher et al. 1997, 2003; Mander and Basoz 1999) used demand-side fragility curves to estimate probable damage for a given (demand-side) IM. More recently, this has been extended by Dhakal and Mander (2006), Mander et al. (2007) and Solberg et al. (2008). Their work required the evaluation of a quadruple integral which inevitably led to lengthy numerical computations.

The objective of this work is to develop a direct closed form loss estimation framework that relates hazard to response to damage and hence to losses without the need of the classic demand-side fragility curves or the evaluation of convolution integrals. The framework for the proposed closed form loss estimation procedure is derived using data from existing work and also from reasonable replacement cost estimates. The approach is validated through a number of bridge cases following both ductile design and emerging Damage Avoidance Design (DAD) philosophies and seismic vulnerability of financial losses to these bridges is estimated.

3.3.   Theoretical Direct Loss Estimation Framework

A four-step approach involved in estimating financial losses to seismically damaged structures is shown in Figure 3-1. The main aim of using a direct four-step procedure for computing losses is to relate estimated losses in terms of well-known seismic demand and structural capacity parameters. From Figure 3-1, it should also be evident that the four steps from (a) to (d) can be visually inter-related through the use of log-log graphs. The four graphs and their inter-relationships via power equations (that plot as linear lines in log-log scale) are explained in what follows.

In order to estimate losses, it is imperative to provide a clear relationship between IM and an annual frequency (f_a), referred to herein as the seismic hazard-recurrence relationship. Figure 3-1(a) presents a graphical representation of the relationship between hazard recurrence rate and the intensity measure (IM). As seen, a straight line can be fitted through two points on a log-log plot and this represents a suitable approximation of the hazard-recurrence relationship. A previously used well-known relationship (Kennedy 1999; Cornell et al. 2002; Solberg et al. 2008) is given by

$$f_a(IM) = k_0 (IM)^{-k} \tag{3.1}$$

where k_0 and k are best-fit empirical constants.

Figure 3-1(b) presents, in log-log space, a straight line relationship IM and EDP. Using the same notation proposed by Cornell at al. (2002), the hazard intensity response in terms of drift θ (an EDP) is given by

$$D = a S_a{}^b \tag{3.2}$$
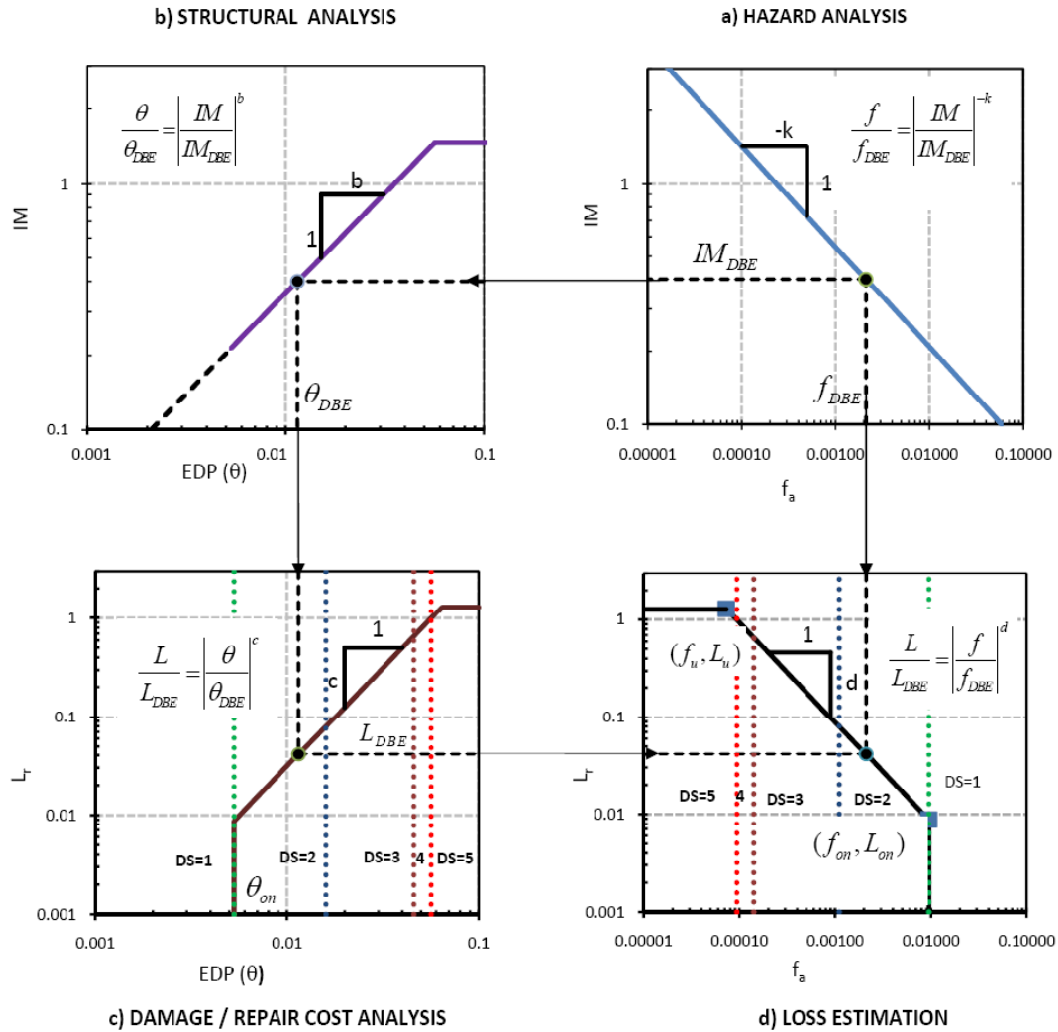
Figure 3-1 Summary of the four step approach used to estimate EAL. (a) Two points on the hazard recurrence curve are used to compute the IM (hazard analysis). (b) The IM's derived from (a) are used to compute interstory drifts using the hazard-drift curve (structural analysis). (c) The drifts obtained from (b) are used to compute loss ratios using the calibrated loss model (damage analysis). (d) Return periods for loss ratios are calculated from the hazard-loss curve (loss analysis). The area under the hazard recurrence-loss curve will give the EAL for the structure

where S_a = spectral acceleration; $D = \theta$ = drift (D is the notation used by Cornell et al. 2002) and a and b are constants.

The basis of this research is to take these fundamental previously proposed equations and to similarly extend them to incorporate losses.

For convenience, (3.1) can be recast as

$$\frac{f}{f_{DBE}} = \left| \frac{S_a}{S_{a_{DBE}}} \right|^{-k} \tag{3.3}$$

where f_a = annual frequency, f_{DBE} and $S_{a_{DBE}}$ are the annual frequency and spectral acceleration demand (an intensity measure IM) for design basis earthquake (DBE), typically taken as 10 percent in 50 years or $f_{DBE} = 1/475$.

Similarly,(3.2) can be recast as

$$\frac{\theta}{\theta_{DBE}} = \left| \frac{S_a}{S_{a_{DBE}}} \right|^{b} \tag{3.4}$$

Combining (3.3) and (3.4) gives

$$\frac{\theta}{\theta_{DBE}} = \left| \frac{S_a}{S_{a_{DBE}}} \right|^{b} = \left| \frac{f}{f_{DBE}} \right|^{\frac{-b}{k}} \tag{3.5}$$

where θ is the column (or interstory) drift on the structure for the considered event; θ_{DBE} is the interstory drift on the structure for the design basis event; and b is an exponent that is the slope of the line on the log-log plot shown in Figure 3-1(b).

At this stage, it is postulated that losses follow a similar power law form. This will be subsequently validated and calibrated later in this paper. Figure 3-1(c) represents

empirically calibrated results that relate structural (financial) losses and structural drifts which are given by

$$\frac{L}{L_{DBE}} = \left| \frac{\theta}{\theta_{DBE}} \right|^{c} \tag{3.6}$$

where θ_{DBE} = the drift induced by DBE and c = an empirically calibrated constant equal to the slope (in log-log space) of the line shown in Figure 3-1(c).

Combining (3.5) with (3.6) gives the interconnection between the four graphs in Figure 3-1:

$$\frac{L}{L_{DBE}} = \left| \frac{\theta}{\theta_{DBE}} \right|^{c} = \left| \frac{S_a}{S_{a\,DBE}} \right|^{bc} = \left| \frac{f}{f_{DBE}} \right|^{\frac{bc}{-k}} \tag{3.7}$$

Thus, by following the arrows in Figure 3-1, loss can be directly related to annual frequency. Simplifying (3.7) gives

$$\frac{L}{L_{DBE}} = \left| \frac{f}{f_{DBE}} \right|^{d} \tag{3.8}$$

where the exponents are inter-related by

$$d = \frac{bc}{-k} \tag{3.9}$$

In summary, as shown in Figure 3-1 and (3.7), the four-step loss model directly estimates losses due to inter-relationships between (a) hazard, (b) structural response, (c) damage and (d) loss.

3.4.   Derivation of Direct Loss Model

Although several methods of seismic risk assessment have been developed, most methods use *fragility curves* in order to predict probable damage for a given IM (Kircher et al. 1997; Mander and Basoz 1999; Kircher 2003). Such curves shall herein be referred to as *Demand-Side Fragility Curves*. Alternatively *Capacity-Side Fragility Curves* can be derived independently of conducting dynamic analysis. Such curves are derived directly from damage limit states with respect to the structure's pushover displacement capacity (and stability) such as column (or interstory) drift. Classic demand-side fragility curves could then be derived because of the explicit connection between ground shaking intensity and structural resistance as given by (3.4) and (3.5). Due to the multiplicative nature of damage spread, like their demand-side counterparts, capacity-side fragilities can be represented by a lognormal probability distribution. Thus only two parameters are needed to construct *Capacity-Side Fragility Curves*: (i) the median (the 50th percentile) EDP; and (ii) the lognormal standard deviation β_{RC} often referred to as the dispersion factor. This hypothesis is used to calibrate the loss model proposed in (3.6) and depicted in Figure 3-1(c).

Figure 3-2(a) presents an illustrative set of capacity-side fragility curves expressed in terms of column drift (an EDP). The levels of damage for the spaces between the curves are the same as the damage states used in HAZUS (Kircher et al. 1997; Mander and Basoz 1999; Kircher 2003) that is: (1) none; (2) slight; (3) moderate; (4) heavy; and (5) complete. In the context of this research, these damage states may be

thought of as (1) pre-yield damage; (2) tolerable (non-repairable) damage; (3) repairable damage; (4) irrepairable damage; and (5) collapse or toppling necessitating complete rebuild.

Associated with the occurrence of each damage state will be losses. Some slight and all moderate levels of damage necessitate repairs, while heavy and complete damage may require partial or total reconstruction. Such damage therefore leads to *financial losses* that can be expressed in terms of loss ratios. A loss ratio (L_r) is also defined as the repair cost ratio which is the reinstatement cost to the cost of a new facility built under normal conditions. Figure 3-2(b) presents a set of typical L_r for structures. As with any construction and contracting enterprise, there is a wide variety of bid prices and uncertainty as to the final costs. Following a disaster, owners and engineers may not always have the luxury of accepting the lowest bids. Moreover, contractors tend to inflate their bids to cover penal (over-time) rates due to the extraordinarily heavy work load demands that follow a disaster. Accordingly, losses on average may be assumed to be inflated 30% for price surge. As this class of variability cannot be easily modeled, but only estimated, it is defined as *epistemic uncertainty*. For each damage state shown in Figure 3-2(b), there are three bars. The central bar represents the median (50th percentile) L_r while the lower and upper bars represent one lognormal standard deviation either side of the median bars, that is the 16th and 84th percentiles, respectively.

When combining the probability for being within a damage state for a given EDP together with the associated loss ratio for each damage state, the loss ratio for that EDP

is found. The total probable financial loss due to earthquakes of a given probability is the sum of the corresponding values for the damage states. This can be written as

$$L_r\left[EDP\right]=\sum_{i=2}^{5}P_i[EDP]L_{ri} \tag{3.10}$$
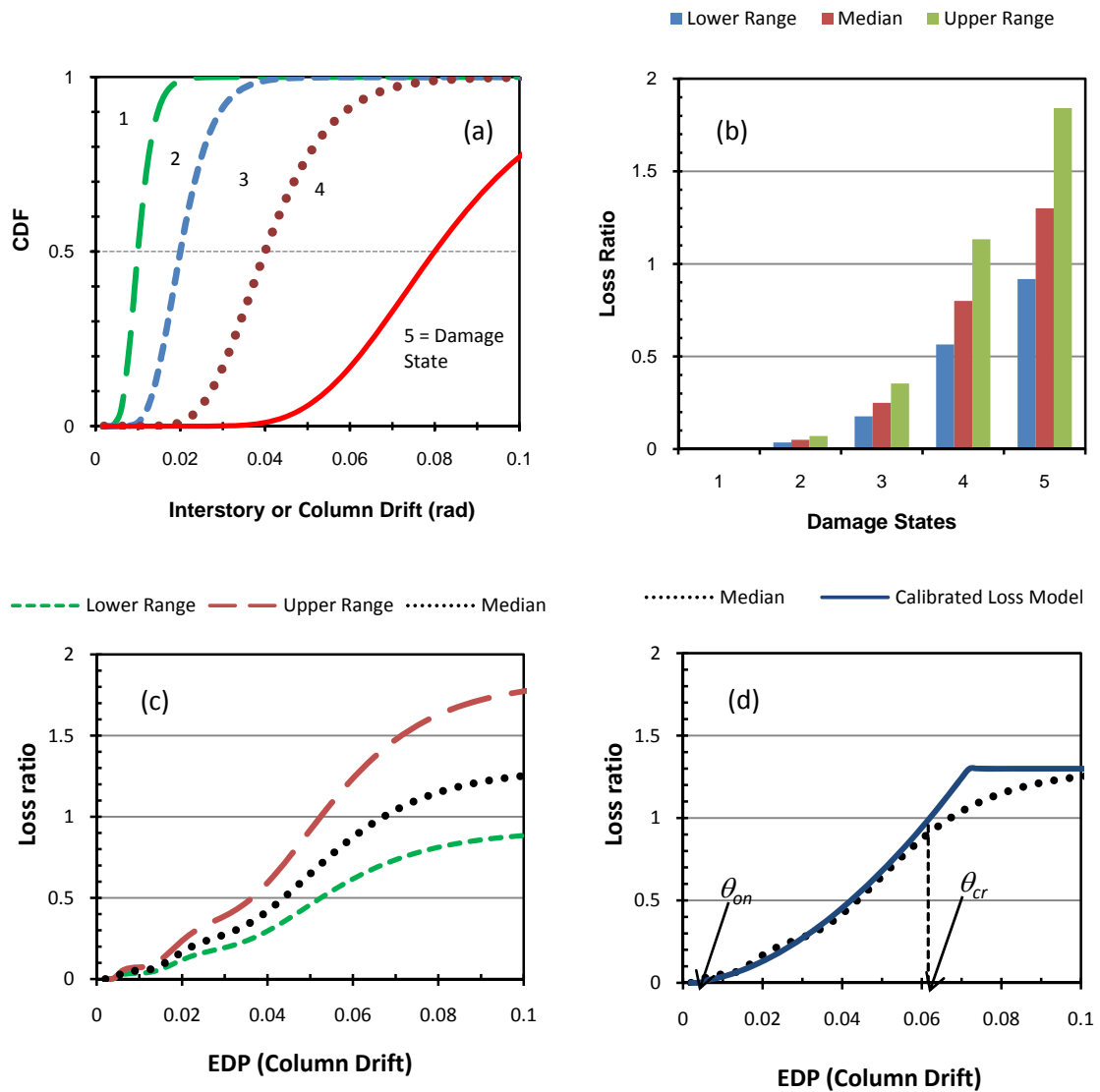
Figure 3-2 Procedure for deriving loss ratio: a) establish capacity-side fragility curves; b) estimate loss ratios for given damage states c) combine damage with losses across all damage states to give composite loss ratio; and d) loss model parameterization

where $P_i[EDP]$ and L_{ri} are the respective probability and loss ratios for the ith damage state.

Figure 3-2(c) illustrates the application of (3.10) and shows three resulting curves representing 16th, 50th, and 84th percentile losses respectively. In this graph, $\beta_{RC} = 0.3$ for aleatory randomness in capacity (the fragility), and $\beta_{UL} = 0.35$ for epistemic uncertainty in the estimated losses.

3.5.   Proposed Loss Model

It is herein proposed to use a two-parameter power curve, with upper and lower cutoffs to represent a loss ratio as a function of structural or column drift. The empirical model takes the form as shown in Figure 3-2(c) and is expressed as:

$$\frac{L}{L_c} = \left| \frac{\theta}{\theta_c} \right|^c \text{ and; } L_{on} \leq L \leq L_u = 1.3 \tag{3.11}$$

in which L = loss ratio; L_c = unit cost (normally taken as $L_c = 1$ for comparative studies); c = an empirically calibrated power; θ = column (or interstory) lateral drift (the EDP) ; θ_c = the critical drift defined as $\theta_c = f \theta_{DS5}$ where θ_{DS5} = drift value for complete damage (toppling or collapse); and f = adjustment factor for low damage structures. In general $f = 1$, but f may take other values for certain special structural types such as those with dampers or those employing DAD.

In (3.11) there is the restriction that $L_u \leq 1.3$ (to allow for price surge), and L_{on} = onset of damage (when $L < L_{on}$, $L=0$) which from (3.11) is given by

$$\frac{L_{on}}{L_c} = \left| \frac{\theta_{on}}{\theta_c} \right|^c \tag{3.12}$$

where θ_{on} = onset of damage (normally taken as $\theta_{on} = \theta_{DS2}$ where θ_{DS2} = drift value for Damage State 2).

From (3.11) it is evident that there are several parameters that need calibrating for the loss model, specifically $\theta_c, \theta_{on}, f, c$. These parameters are chosen to give a weighted least squares best-fit solution to a full analysis resulting from the implementation of (3.11).

3.6.   Loss Model Calibration Results

Following the study of Solberg et al. (2008), the empirical loss model was applied to bridge columns designed in accordance with the prevailing specifications of Caltrans, Japan, and New Zealand. Additionally, the empirical loss model was also calibrated for a bridge detailed using an emerging DAD philosophy. Best fit results for the model are listed in Table 3-1.

With the exception of the DAD case where $f = 1.15$, it is evident that the principal controlling parameter is $\tilde{\theta}_{DS5}$ because $f = 1.0$ (and thus $\tilde{\theta}_{cr} = \tilde{\theta}_{DS5}$) for the ductile pier. It is therefore essential that this parameter is reliably estimated. Although there are modest changes in the value of the exponent c, that parameter is fairly consistent for bridges.

Figure 3-3 presents four sets of results for bridges designed as per Caltrans, Japan, and New Zealand specifications as well as a bridge designed as per DAD philosophy. The results are the upper, central, and lower smooth curves which were derived using a least squares fit. The difference between the various specifications lies in the drifts at which onset of damage and complete damage occurs. This difference can be attributed to the varying levels of detailing followed by the specifications.

Table 3-1 Model Calibration Results for Different Bridge Types

Parameters	Caltrans	Japan	NZ	DAD	Comments
IM (DS2)	0.2000	0.2000	0.1750	0.57	*
IM (DS3)	0.7000	0.6000	0.5625	-	*
IM (DS4)	1.2875	1.1875	0.8375	-	*
IM (DS5)	1.3475	1.4000	0.9750	1.16	*
θ (DS2)	0.0053	0.0053	0.0062	0.03	*
θ (DS3)	0.0190	0.0160	0.0230	-	*
θ (DS4)	0.0510	0.0460	0.0440	-	*
θ (DS5)	0.0616	0.0566	0.0564	0.10	*
L_r (DS2)	0.05	0.05	0.05	0.1	~
L_r (DS3)	0.25	0.25	0.25	-	~
L_r (DS4)	0.80	0.80	0.80	-	~
L_r (DS5)	1.30	1.30	1.30	1.30	~

* Adopted from Mander et al. (2007)

~ Adopted from Mander et al. (2007) and Solberg et al. (2008) and includes 30 percent allowance for price surge

CALTRANS

JAPAN

NEW ZEALAND

DAD
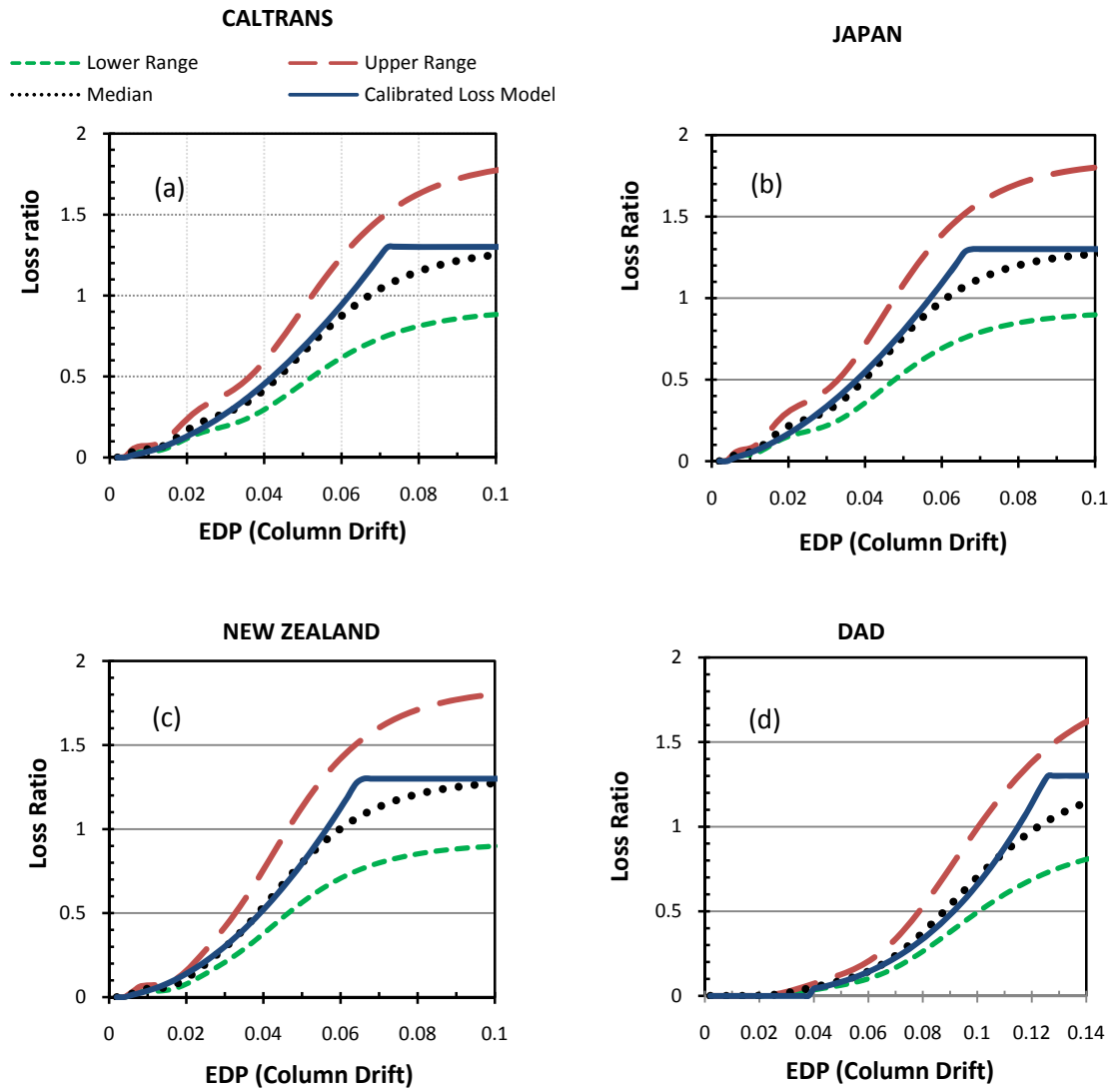
Figure 3-3 Loss model calibration for different bridge types

3.7.   Computing Annual Losses

Annual losses (*AL*) may be estimated by simply integrating the area beneath the curve in Figure 3-1(d) when that curve is plotted to natural scales. Thus in integral form, *AL* may be found by computing the following

$$AL = \int_0^{f_{on}} L_r \, df = L_u f_u + \frac{L_{DBE}}{f_{on} f_{DBE}^{\,d}} \int_{f_u}^{f_{on}} f^d \, df \tag{3.13}$$

This integral has the solution

$$AL = \frac{L_{on} f_{on} + d L_u f_u}{1 + d} \tag{3.14}$$

where in terms of the original structural parameters L_u and L_{on} are respectively defined by (3.11) and (3.12) while f_{on} and f_u are defined by

$$f_{on} = f_{DBE} \left| \frac{\theta_{DBE}}{\theta_{on}} \right|^{\frac{k}{b}} \tag{3.15}$$

and

$$f_u = f_{DBE} \left(L_u \right)^{\frac{1}{d}} \left| \frac{\theta_{DBE}}{\theta_c} \right|^{\frac{k}{b}} \tag{3.16}$$

At this stage it should be emphasized that all of the foregoing equations are probabilistic – that is implicit in the principal parameters are various forms of variability. This will now be dealt with in what follows.

3.8.   The Handling of Variabilities in Computation of Annualized Losses

It is necessary to incorporate the effects of various variabilities in the parameters in each of the four steps involved in estimating financial losses for seismically damaged structures. Variabilities consist of both uncertainty and randomness involved in estimating the demand over time produced by ground motion and the capacity of the structure to resist those demands (Cornell et al. 2002). Herein uncertainty and randomness are epistemic and aleatoric variabilities, respectively. Similarly financial loss estimates in (3.7) incorporate both aleatoric and epistemic uncertainties.

In order to estimate losses in each of the three parts to the capped loss model given by (3.11) and (3.12), it is essential to transform the median parameters to other fractiles, including the mean values of the parameters. This can be achieved by utilizing parameters quantifying the kind and degree of uncertainty in each of the parameters concerned. Due to the multiplicative (power) nature of the loss model, a lognormal distribution shall herein be assumed as being an appropriate representation of variability. Thus in general, if β represents the lognormal standard deviation (dispersion) in computing a variable y then assuming a lognormal distribution the relationship between the mean $\bar{y}$ and the median $\tilde{y}$ is given by

$$\bar{y} = \tilde{y}\exp\left(\frac{1}{2}\beta^2\right) \tag{3.17}$$

Similarly for other fractiles, say $x\%$ non-exceedance probabilities

$$y_{x\%} = \tilde{y}\exp(K_x\beta) \tag{3.18}$$

where K_x represents the standardized Gaussian random variable with a mean zero and standard deviation one.

For lognormal distributions, if β_{RC} and β_{RD} denote the aleotoric randomness in structural capacity and demand, and β_{UL} represents the epistemic uncertainty in loss estimation, then using the approach outlined by Kennedy et al. (1980), the total dispersion in each of the parameters involved in computing losses can be estimated.

Figure 3-4 presents the effect of the source (Figures 3-4(b) and (c)) and the consequent propagation of variability (Figures 3-4(c) and (d)) resulting in the overall variability in the loss model. Firstly, it will be noted that this model at this stage does not account for uncertainty in the seismic hazard (Figure 3-4(a)) – it is assumed to be crisp. However, it is well known (Shome and Cornell 1999; Vamvatsikos and Cornell 2002) that given a crisp input in terms of an input measure, randomness in structural response results due to nonlinear behavior and the general variability of ground motion input (demand) signals; this dispersion is defined as β_{RD} and is shown in Figure 3-4(b). As mentioned previously, there is also randomness in the structural capacity, β_{RC}. This affects the onset of damage and also when total damage collapse/toppling occurs and is shown in Figure 3-4(c).

The losses can only be estimated; as mentioned this is part of the contracting enterprise, such epistemic uncertainty (also shown in Figure 3-4(c)) is given by β_{UL}.
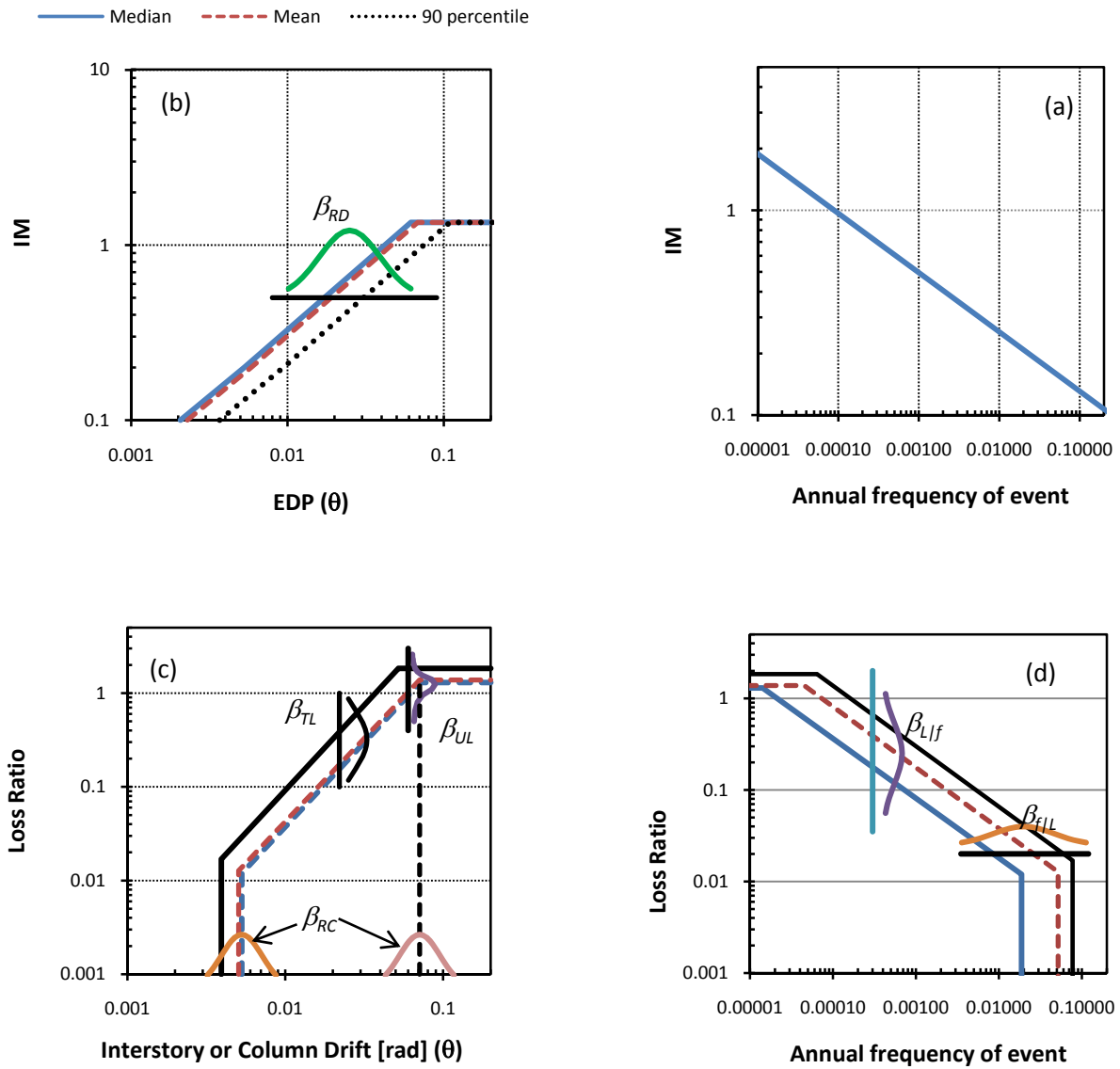
Figure 3-4 Summary of the four step approach after incorporating aleotoric and epistemic uncertainties: (a) the seismic-hazard relation is assumed to be crisp hence only one line is shown; (b) the seismic response primarily involves β_{RD} - the randomness in demand. (c) the loss model includes randomness in structural capacity, β_{RC} as well as epistemic uncertainty in the loss model, β_{UL}. (d) shows the hazard-loss curve with the area under the outer dotted line representing a 90% non-exceedance probability of loss

From (3.11), it is evident that the total loss β_{TL} in L is affected by variabilities in L_r, θ and θ_c which are accounted for by β_{UL}, β_{RD} and β_{RC} respectively. Thus according to Kennedy et al. (1980), β_{TL} is given by

$$\beta_{TL} = \sqrt{\beta_{UL}^2 + c^2 \beta_{RS}^2} \tag{3.19}$$

where the parameter β_{RS} represents the total variability associated with the structure and is given by

$$\beta_{RS} = \sqrt{\beta_{RD}^2 + \beta_{RC}^2} \tag{3.20}$$

These losses and their associated variabilities, shown in Figure 3-4(c) are transformed into the total hazard loss model in Figure 3-4(d).

However, it is now important to change the perspective of the variabilities from drift (in Figure 3-4(c)) to annual frequency in Figure 3-4(d). Thus the variability of f_{on} for a given drift in (3.15) is given by

$$\beta_{f_{on}|\theta} = \frac{k}{b} \beta_{RS} = \frac{k}{b} \sqrt{\beta_{RD}^2 + \beta_{RC}^2} \tag{3.21}$$

For variability in annual frequency given losses it follows from (3.8) and (3.19)

$$\beta_{f|L} = \frac{k}{b} \sqrt{\frac{\beta_{UL}^2}{c^2} + \beta_{RS}^2} \tag{3.22}$$

Note that β_{UL} remains unchanged in capping the losses. This means that both losses at onset of damage and ultimate collapse have β_{UL} associated with them.

Using the foregoing dispersion factors, $\beta_{UL}, \beta_{f|L}, \beta_{f_{on}|\theta}$, in conjunction with the

median coordinates $(\tilde{f}_{on}, \tilde{L}_{on})$ and $(\tilde{f}_u, \tilde{L}_u)$, it is possible to calculate a mean loss curve

utilizing (3.17) when applying (3.14) as follows

$$\overline{EAL} = \frac{\overline{L}_{on}\overline{f}_{on} + d\overline{L}_u\overline{f}_u}{1+d} \tag{3.23}$$

where $(\overline{f}_{on}, \overline{L}_{on})$ and $(\overline{f}_u, \overline{L}_u)$ are the mean values of the primary loss curve coordinates

shown in Figure 3-4(c).

3.9.   Case Studies: Ductile Versus Damage Avoidance Bridge Piers

The closed form direct loss model was implemented to three bridge piers designed for

same loading, material, and geological characteristics as per prevailing Caltrans, Japan,

and New Zealand specifications as well as Damage Avoidance Design (DAD). Details of

the prototype bridge piers have been used from previously related work (Mander et al.

2007; Solberg et al. 2008). Figure 3-5 presents the DAD version of the prototype

structural concrete (partially prestressed) bridge pier taken from a notionally long bridge

which is considered for carrying out the illustrative analysis. The bridge has a

longitudinal span of 40m, transverse deck width of 10m, and pier height of 7m. The

superstructure is assumed to have a seismic weight of 7000 kN. In this study,

$IM_{DBE} = 0.4g$ i.e. the intensity measure for the DBE with 10% probability in 50 years

(i.e. return period of 475 years) is 0.4g. The response IM's for the various levels of

damage for the bridges is listed in Table 3-1 along with the corresponding drifts. This data was adopted from the previous work of Solberg et al. (2008).

The DAD bridge pier employs armored connection details proposed by Mander and Cheng 1997. Damage is precluded due to a combination of rocking action along with post-tensioning tendons and dampers to provide stiffness and supplemental energy dissipation. The DAD pier has a steel plate at the pier-to-pile cap connection to allow rocking to occur without significant damage to the surrounding concrete (Solberg et al. 2008).

Results of the direct financial loss analysis for the different bridge designs described above are summarized in Table 3-2 which presents median estimates of the parameters required to completely define the direct loss model. Graphically presented in a similar fashion to Figure 3-1, Figure 3-6 presents results using the *median* parameters for the four different bridge design and detailing cases.

The first half of Table 3-2 lists the *median* estimates for the parameters along with the associated EAL while the second half of the table presents the dispersions required to compute parameters required for computing values for the mean estimates of annualized loss. Given that the EAL now incorporates a 30% allowance for price surge, the results are now comparable to those previously computed numerically by using the more lengthy quadruple integral approach in Mander et al. (2007) and Solberg et al. (2008).

The effect of total dispersion in the model can be seen in Figure 3-7 where the median and 84th (median plus one lognormal standard deviation) are plotted. It is evident

that the combined dispersion in losses, capacity of the structure and seismic demand on the structure, significantly affect the potential loss outcome for a given annual frequency. Also plotted in Figure 3-7 is the mean (expected value of the losses), given an annual frequency. It is this curve that is used to compute EAL with those results shown in figure.
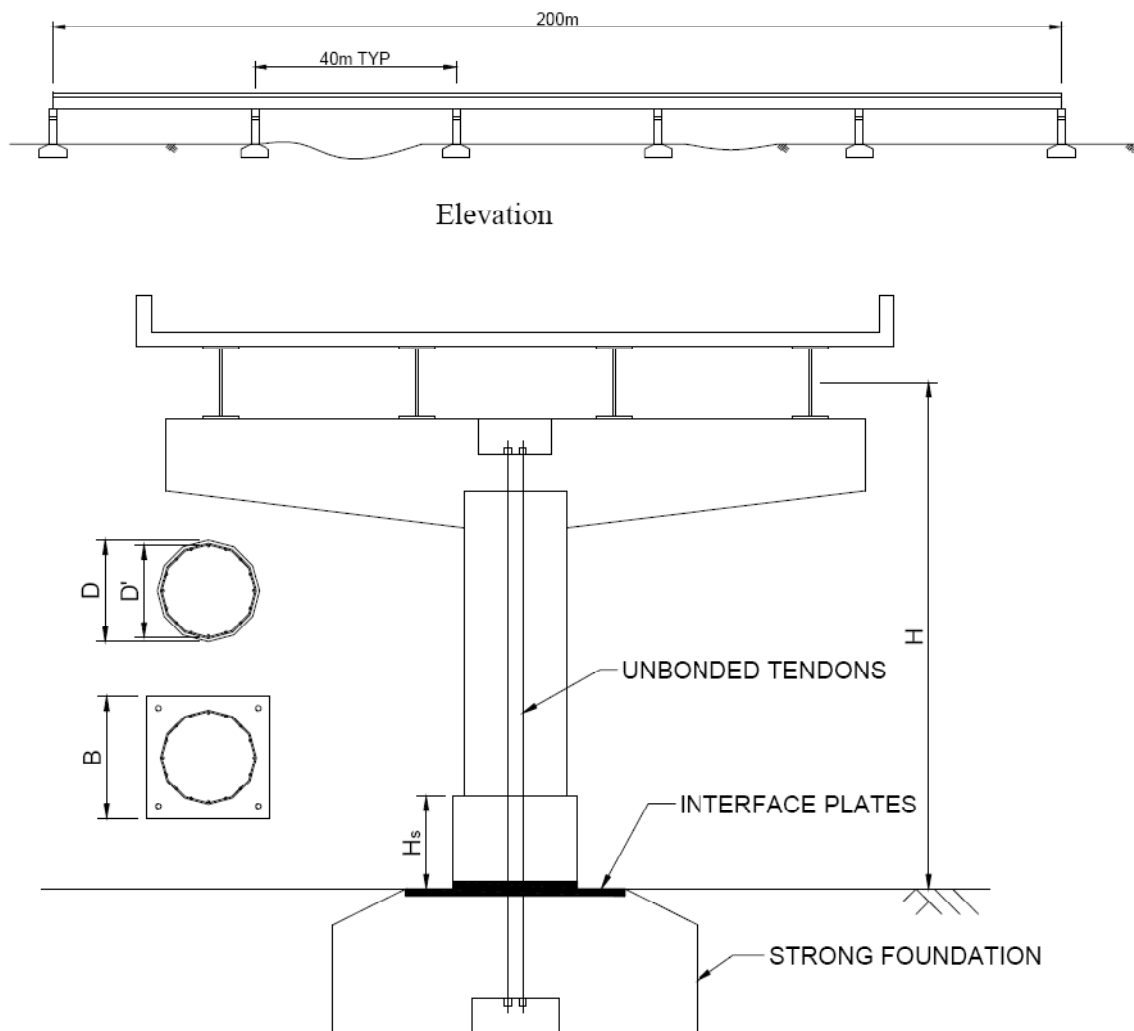
**Figure 3-5 Prototype bridge and design details of the DAD and ductile piers
(adapted from Mander et al. 2007; Solberg et al. 2008)**

Table 3-2 Summary of Parameters Used in Four Step Direct Loss Estimation Procedure

	Parameters	Caltrans	Japan	NZ	DAD	Remarks
a)	Hazard IM_{DBE}	0.4	0.4	0.4	0.4	Assigned
	f_{DBE}	0.0021	0.0021	0.0021	0.0021	from
	k	3.45	2.4	3	3	Solberg et al. (2008)
b)	$\tilde{\theta}_{DBE}$	0.0117	0.0115	0.0163	0.0165	Calibrated from
	b	1.25	1.23	1.27	1.69	IDA
c)	$\tilde{\theta}_{on}$	0.0053	0.0053	0.0062	0.03	Yield drift
	θ_{cr}	0.0616	0.0566	0.0564	0.115	Calibrated
	f	1	1	1	1.15	Calibrated
	c	1.8	1.7	1.9	3	Calibrated
	$d = \dfrac{bc}{-k}$	-0.6522	-0.8713	-0.8043	-1.6900	(3.9)
	$\tilde{L}_{DBE}$	0.050	0.066	0.095	0.003	(3.6)
d)	$\tilde{L}_{u}$	1.3	1.3	1.3	1.3	Assigned
	$\tilde{L}_{on}$	0.012	0.018	0.015	0.018	(3.12)
	$\tilde{f}_{u}$	0.0000142	0.0000686	0.0000809	0.0000574	(3.16)
	$\tilde{f}_{on}$	0.0187282	0.0095442	0.0206514	0.0007284	(3.15)
	β_{RD}	0.42	0.4	0.43	0.42	Solberg 2008
	β_{RC}	0.30	0.30	0.30	0.30	Assigned
	β_{UL}	0.35	0.35	0.35	0.35	Assigned
	$\beta_{L_{on}|\theta}$	0.35	0.35	0.35	0.35	Assigned
	$\beta_{f_{on}|\theta}$	1.425	0.976	1.239	0.916	(3.21)
	$\beta_{f|L}$	1.522	1.055	1.313	0.939	(3.22)
	$\bar{L}_{u}$	1.38	1.38	1.38	1.38	(3.17)
	$\bar{L}_{on}$	0.013	0.019	0.016	0.019	"
	$\bar{f}_{u}$	0.0000450	0.0001196	0.0001916	0.0000893	"
	$\bar{f}_{on}$	0.051661	0.015361	0.044467	0.001108	"
	EAL	*$1,771*	*$1,118*	*$2,553*	*$272*	(3.23)
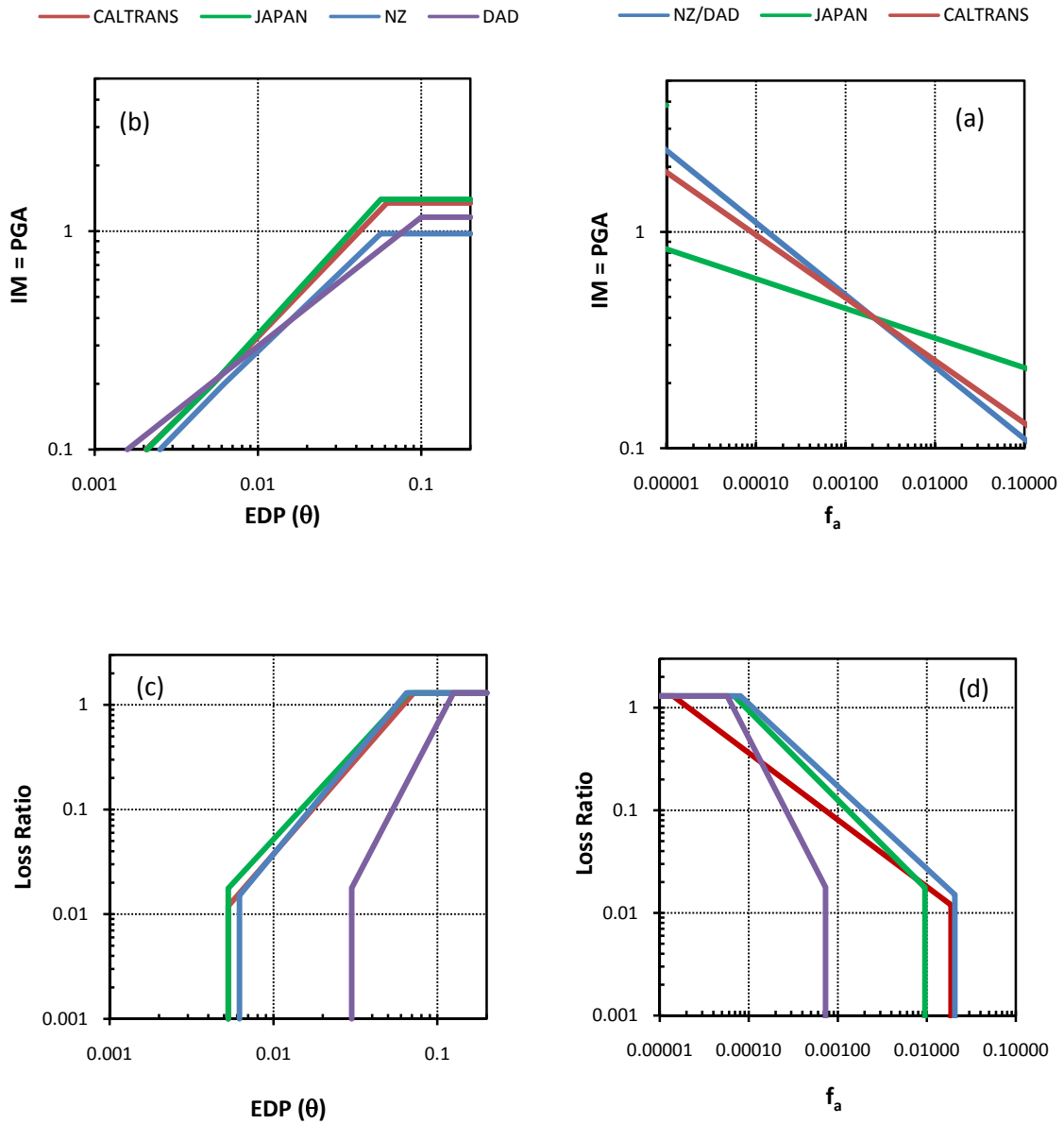
Figure 3-6 Four step loss estimation results for different bridge types showing results for the median values
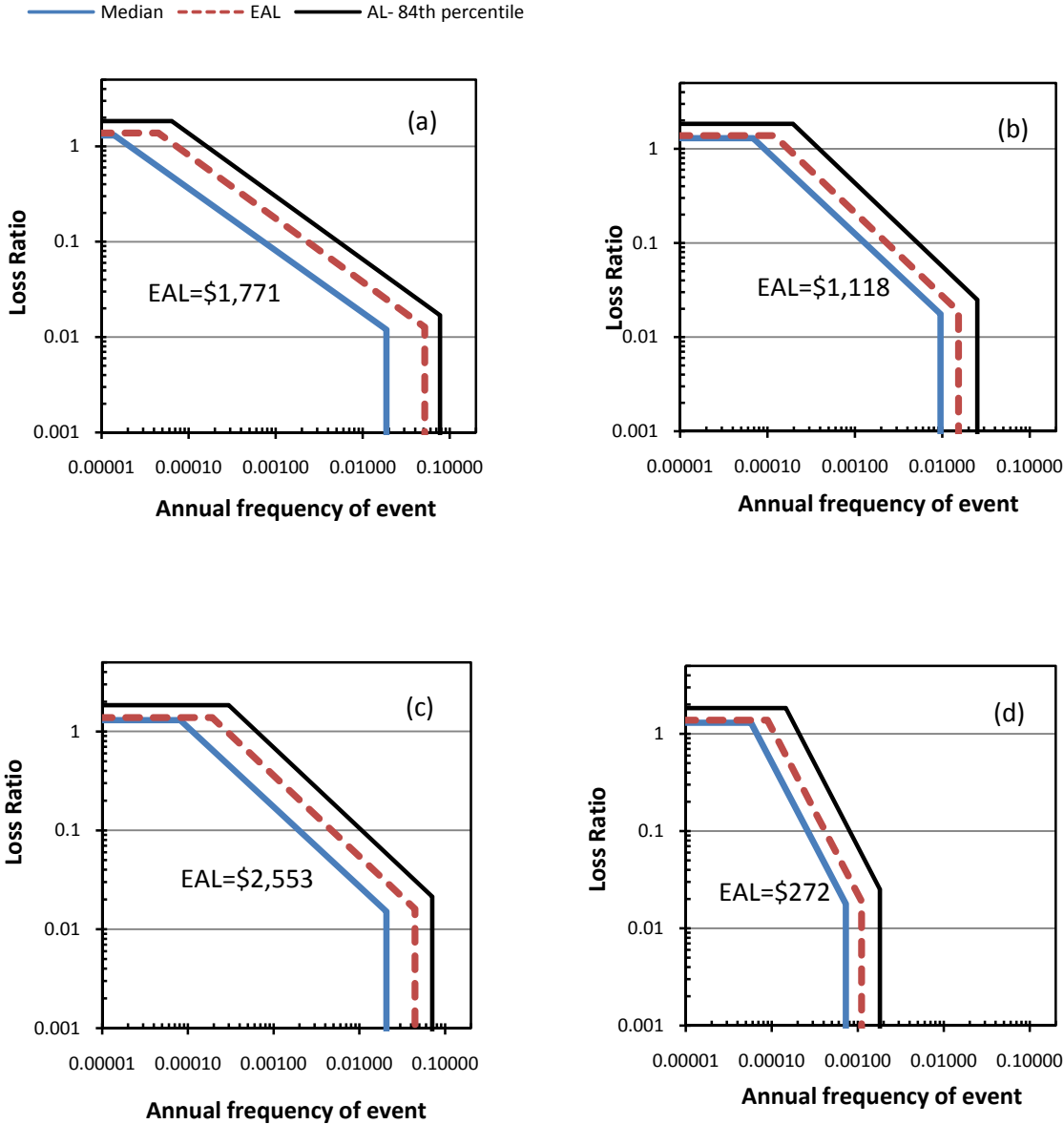
Figure 3-7 Seismic loss-hazard graph showing median, mean, and 84th percentile (median plus one standard deviation) loss curves for (a) Caltrans; (b) Japan; and (c) New Zealand ductile piers; and (d) the DAD bridge pier

3.10. Swing Analysis: Parameter Sensitivity Study

A swing analysis was carried out to investigate the sensitivity of the various parameters impact on EAL. Figure 3-8 presents the results of the swing analysis in the form of a tornado diagram for the New Zealand bridge. Each parameter shown in Figure 3-8 was varied by $\pm 10\%$ and EAL was recalculated. The variation of the result compared to the standard mean value was calculated and ranked.
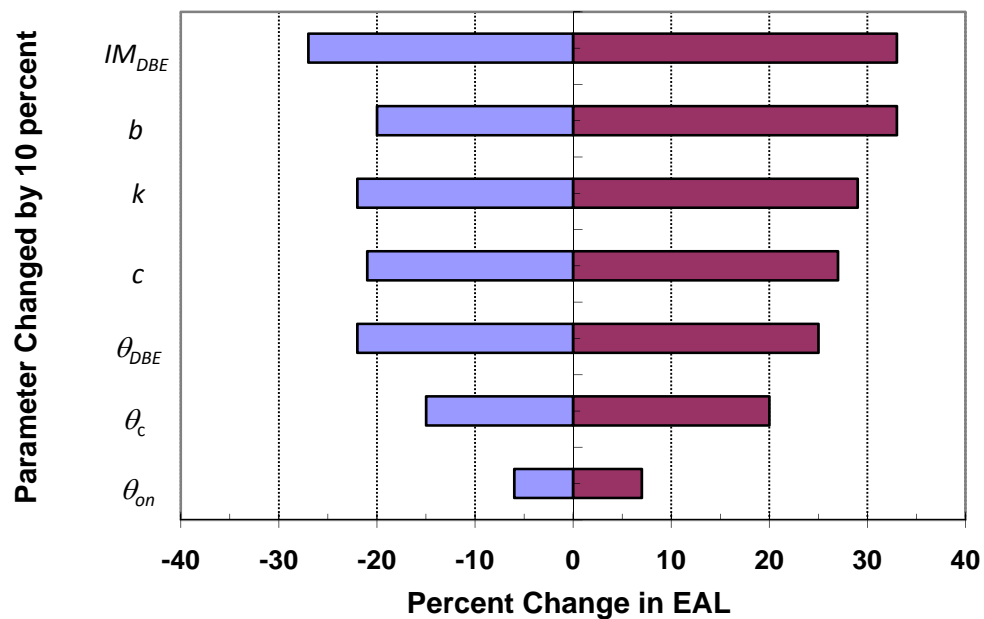
Figure 3-8 Tornado diagram representing results of swing analysis performed to determine sensitivity of the most important parameters affecting EAL for NZ bridge pier

Six parameters showed changes markedly higher than the 10% variation, namely: IM_{DBE}, b, k, c, θ_{DBE}, and θ_c. These parameters can be grouped to represent seismic hazard demand (IM_{DBE}, k), structural response demand (b, θ_{DBE}) and structural damage capacity (c, θ_c); clearly it is of paramount importance to have dependable local hazard and specific structural models for an accurate estimation of the expected losses as seen by the importance of these parameters. Other parameters, the foremost being θ_{on} produce variations smaller than the 10% perturbation. Although still important in producing a dependable estimate for EAL, slight errors in their estimation will not produce a distorted view of EAL.

3.11. Discussion

It is evident from Figure 3-6 that the loss model is able to distinguish the different levels of damage across the broad spectrum of frequent to very rare earthquakes – that is from the onset of damage to collapse, respectively. For moderately frequent $(f_a \sim 0.005)$ events, the ductile bridges of the three countries have similar response levels $(\theta = 0.01)$ resulting in similar levels of loss $(0.02 < L_r < 0.04)$. But outside this range the loss outcomes differ somewhat partly because of different structural strength and ductility capabilities, but mostly because of different seismic-hazard frequency relations as depicted in Figure 3-6(a). These different attributes are all integrated in the evaluation of the expected annual losses. Interestingly, the Japanese bridge fares the best, largely

because of its higher strength. EAL estimates for the Caltrans and Japanese bridges are $1,800 and $1,100 per $million of asset value, the former resulting from a slightly higher ductility and less onerous seismicity for very rare events. Compared to the Caltrans and Japan counterparts, the New Zealand ductile bridge pier has both lower strength and ductility leading to an EAL = $2,500 per $million of asset value.

The DAD bridge was deliberately designed to have similar response attributes for the New Zealand ductile structure. But due to the major changes in detailing via armoring of the critical connections, damage while not eliminated entirely, is avoided for a much wider band of earthquakes. Whereas, the New Zealand ductile design one could expect to see damage for earthquakes where $f_a < 0.02$ (return period > 48 years), this reduces to $f_a < 0.00073$ (return period > 1370 years) for the DAD bridge. In turn this results in 90% reduction in EAL ($270 vs. $2500 per $Million of asset value). Interestingly, the earthquake frequency at toppling is similar for these two bridges.

3.12. Closure

The work conducted as part of the research can be summarized into the following:

1. A four-step closed-form loss estimation methodology that relates hazard to response and hence to losses without the need for classic demand-side fragility curves was proposed and validated. The closed-form solution is formulated in terms of well understood seismic hazard and structural design and capacity parameters.

2. Structural response can be related to losses through a parameterized empirical loss model in the form of a tripartite power curve. The principal part of that model conforms to a simple power curve relationship relating the L_r to EDP for that structural system. Based on information gathered from existing literature and other previous studies, the loss model was calibrated for a number of bridge types. It was seen that $1.7 < c < 1.9$ for ductile piers and $c = 3$ for DAD piers. The parameter that most affects loss is $\tilde{\theta}_{DS5}$, that is the drift at the onset of "complete damage" or toppling/collapse. This parameter is largely dependent on the degree of ductility inherent in the structural system used..

3. When accounting for all variabilities in terms of randomness and uncertainty, the resulting hazard-loss model can be integrated across all possible earthquakes to derive the expected annual loss, EAL. To obtain a sense for the upper bound on loss, it is straight forward to formulate losses for other fractiles, such as the 90th percentile non-exceedance probability.

4. DEVELOPMENT AND IMPLEMENTATION OF PARAMETRIC LOSS MODELING FOR SEISMICALLY DAMAGED BUILDINGS

4.1. Section Summary

A method is proposed to implement capacity-side fragility curves in conjunction with estimated damage state dependent loss ratios in order to derive a financial loss model. The stochastic model is expressed in the form of loss ratio which is a function of commonly accepted structural capacity parameter such as interstory or column drift. Loss ratios for each of the damage states incorporating epistemic uncertainty and price surge effects following a major hazardous event are assigned in order to derive upper and lower bound estimates of total expected loss for a given structural drift. An empirical loss model in the form of a power curve with upper and lower cut-offs is proposed and calibrated for various structural frame types as well as drift-sensitive non-structural components. The calibrated loss models are used in conjunction with hazard rate-drift demand relationships for different types of steel buildings in order to compute the expected annual seismic loss. Illustrative examples highlighting the difference between brittle (pre-Northridge/low-code) and ductile (post-Northridge/high-code) as well as the impact of non-structural components to seismic loss are presented.

4.2.   Background

Performance Based Earthquake Engineering (PBEE) involves a broad four-step analysis process which can be divided into the following: (i) seismic hazard assessment; (ii) analysis of demand on structure; (iii) comparison of estimated demand against predicted capacity of structure and quantification of the extent of expected damage given demand and capacity; and (iv) estimation of losses based on incurred damage. Owners can set up desired performance limits for their facilities and instruct engineers to carry out the design based on the target specifications. Hazard is directly related to demand, response, and hence to losses, and therefore renders PBEE to be an iterative process (Dhakal and Mander 2006). Previous hazards have shown that financial losses are very dependent on damage to buildings (Kircher 2003). However it is necessary to incorporate all forms of losses arising from things such as structural and non-structural damage, downtime, injuries, and even death. This is in contrast to traditional codes which primarily aim to ensure life-safety and prevent collapse (Liu et al. 2004) and therefore implicitly accounts for only structural damage.

Earthquake loss estimation methods developed as part of the FEMA/NIBS program involves use of capacity curves based on engineering parameters and fragility curves based on probability of damage to buildings. Losses are usually based on damage states that contribute heavily to that loss type and can be computed as per the estimates provided for structural system as well as nonstructural drift and acceleration drift sensitive components (Kircher 2003). Default values of repair and replacement costs for the above have been presented previously (Kircher et al. 2006). It is possible to assess

loss ratios or the ratio of repair cost to replacement costs for different seismic scenarios. The four step process suggested by PBEE involves estimating financial losses as a function of hazard intensity, demand, and response. Each level of hazard expressed in terms of probabilities of exceedance in a given exposure period has a corresponding damage condition. It is possible to quantify these damage conditions through the use of acceptable parameters such as inter-storey drift ratio. Therefore it is possible to assess future seismic losses and design the structure to meet specified performance objectives (Liu et al. 2003). It is possible to assess *Loss Ratios* (L_r) for various seismic scenarios given the extent of damage. When these are integrated and averaged over all possible scenarios, the Expected Annual Loss (*EAL*) can be computed. Both L_r and *EAL* are acceptable parameters for stakeholders to comprehend as it considers the return period and intensity of the event (Goulet et al. 2007). Previous work expressed the financial seismic vulnerability of structures in terms of loss ratios and EAL has been conducted for bridges (Dhakal and Mander 2006; Mander et al. 2007; Solberg et al. 2008) and precast concrete buildings with hollow-core floor systems (Dhakal et al. 2006).

There is a need to develop a parametric loss model that relates L_r to structural capacity parameters and hence provide a transition for estimating financial losses. The objective of this work is to develop an analytical parametric loss model by using data from existing work and also from reasonable replacement cost estimates. The analytical approach is validated through a number of widely known building cases designed to both ductile and brittle designs as well as non-structural components. The parametric model is then implemented to commonly designed building structures in order to study the

seismic financial vulnerability of structural and non-structural components as well as a combination of both.

4.3.   Theoretical Basis of Parametric Loss Model

The basis of the theoretical approach used in this work is presented in Figure 4-1. Although several methods of seismic risk assessment have been developed, most methods use *fragility curves* in order to predict probable damage for a given intensity measure (*IM*). Such curves shall herein be referred to as *Demand-Side Fragility Curves* (Kircher 2003; Kircher et al. 2006). Because of the explicit connection between *IM* and engineering demand parameter (*EDP)* derived through incremental dynamic analysis (*IDA*), it is possible to express fragility curves of earthquake damage in terms of an EDP (such as drift). These curves express the probability of damage due to earthquakes as a function of an EDP such as column or interstory drift. On the other hand, *capacity-side fragility curves* can be derived using structural capacity parameters which are generally chosen to be the same as those used to define demand. This is done in order to compare and contrast predicted demand with expected capacity. Due to the multiplicative nature of damage spread, like their demand-side counterparts, capacity-side fragilities conform to a lognormal probability distribution. Thus only two parameters are needed to construct *capacity-side fragility curves*: (i) the median (the 50th percentile) *EDP*; and (ii) the lognormal standard deviation β, often referred to as the dispersion factor. Figure 4-1(a) presents a set of damage dependent fragility curves expressed in terms of structural
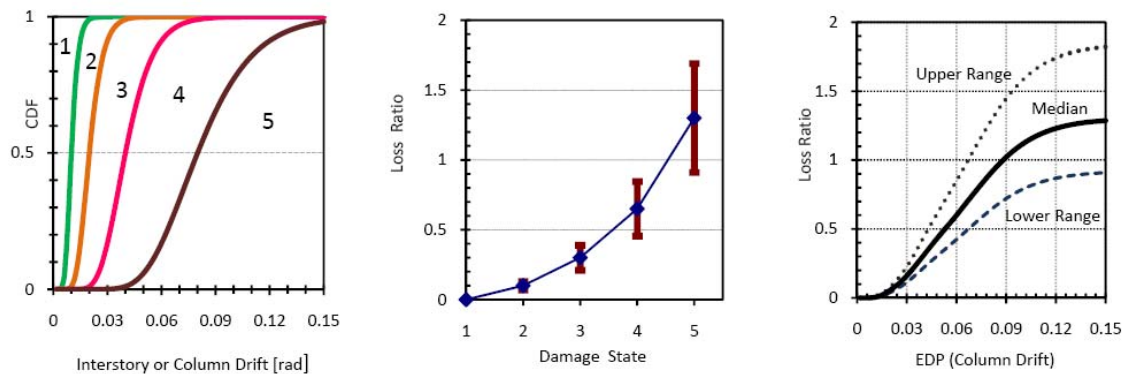
Figure 4-1 Derivation of capacity-side loss model: a) capacity-side fragility curves for the 5 damage states; b) loss ratio estimates for the given damage states including the effect of price surge; and c) integration of losses for damage states for given EDP showing variability to give composite loss ratio

capacity parameters such as inter-story or column drift. The levels of damage for the spaces between the curves are the same as used in HAZUS (Kircher 2003; Kircher et al. 2006) that is: (1) none; (2) slight; (3) moderate; (4) heavy; and (5) complete. These are however defined for ductile structures. Depending on the circumstances these can be related to structural performance (Immediate Occupancy, Life-Safety, and Collapse Prevention) or remedial measures to reinstate full operational service.

In general, *DS1* represents pre-yield response and therefore no damage occurs. The intermediate damage states namely *DS2*, *DS3*, and *DS4* can be defined for various damage magnitudes. The boundary for DS2 and DS3 is defined as being the limit wherein the structure would be unusable until repairs are made. Similarly the boundary for DS3 and DS4 would occur when the structure is deemed irreparable i.e. components

need to be rebuilt or the structure must be replaced. DS5 represents full collapse or toppling of the structure.

Associated with the occurrence of each damage state will be financial losses that can be expressed in terms of loss ratios. A loss ratio (Lr) is also defined as the repair cost ratio which is the reinstatement cost to the cost of a new facility built under normal conditions. Figure 4-1(b) illustrates a set of possible Lr for structures. It is evident that there is a degree of underlying uncertainty with the estimated values. This uncertainty arises from the widespread nature of losses from damage together with the uncertain competitive environment at the time the disaster occurs. Accordingly, losses are inflated 30% for price surge. As this class of variability cannot be easily modeled, but only estimated, it is defined as epistemic uncertainty. For each damage state shown in Figure 4-1(b), there are three possible values. The central value represents the median (50th percentile) L_r while the lower and upper values represent one lognormal standard deviation either side of the median values, that is the 16th and 84th percentiles, respectively.

When combining the probability for being within a damage state for a given capacity value together with the associated loss ratio for each damage state, the loss ratio for that capacity value is found. The total probable financial loss due to earthquakes of a given probability is the sum of the corresponding values for the damage states. Figure 4-1(c) illustrates the application of the aforementioned and shows the three resulting curves. In this graph, $\beta_{RC} = 0.3$ for aleatory randomness in capacity (the fragility), and

$\beta_{UL} = 0.35$ for epistemic uncertainty in the estimated losses. Thus the lower, central, and upper curves represent 16, 50 (median), and 84 percentile L_r, respectively.

As the number of parameters, the extent of data and the amount of computation necessary to represent the resulting curves in Figure 4-1(c) is not trivial, it is desirable to have a simpler model that makes seismic performance-based analysis and design more tractable. In fact with a suitable loss model, the entire step of damage analysis (Step iii mentioned above) can be bypassed and one can go directly from the *IDA* (Step ii, Structural Analysis) to loss (Step iv).

4.4. Methodology

Step 1: Assign Damage Limit States

It is possible to predict the expected drift (or any relevant EDP) for an earthquake with a certain level of intensity if the 16th, 50th, and 84th percentile values of drift are known. Default values of damage and loss parameters for typical 3-story, 9-story, and 20 story WSMF buildings designed to pre and post Northridge standards and reinforced concrete buildings designed to varying levels of code design (high, moderate, and low) are used from previous work (Kircher 2003) and the Advanced Engineering Building Module of HAZUS. Fragility curves are generated using the principle of two parameter lognormal distribution approach. Drifts are incremented at suitable steps. Fragility curves are generated for various damage states for the various cases. The data points corresponding to the fragility curve are used for subsequently estimating losses.

Step 2: Assign Loss Ratios

Loss ratios or loss functions are needed to quantify the degree of loss for a specified damage state. The data points corresponding to the fragility curves for a specific damage state imply the probability of being in or exceeding that damage state. Multiplying the same with loss ratio for a given damage state will provide an estimate of expected losses for that damage state. This is due to temporary increase in prices of material and labor following an earthquake. To accommodate uncertainty in predicting L_r, losses are inflated 30% for price surge. For each damage state shown in Figure 4-1(b), there are three points (indicated as diamond-shaped). The central point represents the median (50th percentile) L_r while the lower and upper bars represent one lognormal standard deviation either side of the median bars, that is the 16th and 84th percentiles, respectively.

Step 3: Calculate Total Losses

Since there are 3 values of loss ratios pertaining to every damage state, there will be 3 different total losses for a particular damage state corresponding to the 16th, 50th, and 84th percentile loss functions. Loss ratios are arrived at by calculating the probability of being in that damage state multiplied by the loss ratio of that damage state. The total probable financial loss due to earthquakes of a given probability is the sum of the corresponding values for the damage states. More formally this can be written as

$$L_r\left[EDP\right] = \sum_{i=2}^{5} P_i[EDP]L_{ri} \tag{4.1}$$

where $P_i[EDP]$ and L_{ri} are the respective probability and loss ratios for the ith damage state. It is clearly evident that as the probability of being in higher damage states are

multiplied by their higher loss ratios, the higher damage states contributes greater to the total losses. This is despite the fact that there is not a great likelihood of damage caused due to earthquakes falling into the higher damage states. In the graphs that follow, three curves are plotted. These lower, middle, and upper curves correspond to the 16th, 50th (median), and 84th percentile for assumed epistemic uncertainty given by a dispersion factor of $\beta_{UL} = 0.35$.

Step 4: Calibrate the Empirical Loss Model

It is therefore proposed to use a simple power curve, with upper and lower cutoffs to represent a loss ratio as a function of structural or column drift, in the form:

$$\frac{L}{L_c} = \left| \frac{\theta}{\theta_c} \right|^c \text{ and; } L_{on} < L < L_u = 1.3 \tag{4.2}$$

in which L = loss ratio; L_c = unit cost (normally taken as L_c =1 for comparative studies c = an empirically calibrated power; θ = column (or interstory) lateral drift (the EDP) ; θ_c = the critical drift defined as $\theta_c = f\theta_{DS5}$ where θ_{DS5} = drift value for complete damage (toppling or collapse); and f = adjustment factor for low damage structures. In general f =1, but f may take other values for certain special structural types such as those with dampers or those employing DAD.

In (4.2) there is the restriction that $L_u \leq 1.3$ (to allow for price surge), and $L_{on} =$ onset of damage (when $L < L_{on}$, $L=0$) which from (4.2) is given by

$$\frac{L_{on}}{L_c} = \left| \frac{\theta_{on}}{\theta_c} \right|^c \tag{4.3}$$

where θ_{on} = onset of damage (normally taken as $\theta_{on} = \theta_{DS2}$ where θ_{DS2} = drift value for

Damage State 2).

From (4.2) it is evident that there are several parameters that need calibrating for

the loss model specifically $f, c, \theta_{cr}, \theta_{on}$. These parameters were chosen to give a least

squares best-fit solution.

4.5.   Loss Model Calibration Results

The empirical loss model was applied to several concrete and steel building types as well

as non-structural components. The results from the least squares calibration for welded

steel moment frame buildings of different heights designed as per both pre-Northridge

(brittle) and post-Northridge (ductile) in various locations are given in Table 4-1. A so-

called pre-Northridge connection condition implies that buildings are provided with

connections with construction flaws typical of buildings constructed prior to the 1994

Northridge earthquake. However these connections have not been damaged by

earthquake ground shaking. Likewise a so-called post-Northridge connection implies

that the building has new or retrofitted beam column connections as per SAC Steel

Project reference guidelines (Wen and Song 2003). It can be clearly observed that the

loss model indicates higher losses for the pre-Northridge type connections and lower

losses for post-Northridge type connections. This can be attributed to the overall higher

ductility capability inherent in the post-Northridge connections as a result of improved

detailing and construction practice.

The results from the empirical calibration for concrete buildings designed to low-code, moderate-code, and high-code and drift-sensitive non-structural components are presented in Table 4-2. Low-code, moderate-code, and high-code represent increasing levels of ductility in design specifications.

On examining the results for both steel and concrete building frames, it is evident that the principal controlling parameter is $\tilde{\theta}_{DS5}$ (because in general $f=1$ for most designs). Clearly it is essential that this parameter be reliably assessed for accurate loss estimates. There are modest changes in the value of the exponent c, and it tends to increase slightly with the height of steel buildings. This reflects the fact that at low drift levels, damage is similar, but it is the P-delta effects and the greater propensity for toppling at large drifts for tall steel structures that causes this interplay between the parameters θ_{cr} and c.

The results of loss model calibration for drift-sensitive non-structural components indicate that these are more susceptible to damage for small drifts and hence ground motions.

Figure 4-2 presents graphs for loss model calibration of six different structural types. The first four graphs present the results for WSMF buildings designed as per pre- and post-Northridge specifications in various locations (Los Angeles, Seattle, and Boston), while the last two graphs give results for low-rise concrete frames detailed to low and high (code) levels of ductility. The figure clearly shows that the expected losses are lesser for buildings designed to higher levels of ductility. Thus the parameter that

principally affects the degree of loss is $\tilde{\theta}_{DS5}$, the onset of collapse. This is related to the

goodness of the ductile detailing present in the structure.

Table 4-1 Loss Model Parameters for Welded Steel Moment Frame Buildings (Kircher 2003)

	Pre Northridge	Post Northridge					
Interstory Drifts	All heights All locations	3-Story LA	9-Story LA	3-Story Seattle	9-Story Seattle	All heights Boston	Non-Structural Components
θ (DS2)	0.01	0.01	0.01	0.01	0.01	0.01	0.004
θ (DS3)	0.015	0.02	0.02	0.0175	0.0175	0.015	0.008
θ (DS4)	0.025	0.04	0.04	0.03	0.03	0.025	0.025
θ (DS5)	0.04	0.1	0.08	0.08	0.06	0.04	0.05
Loss Ratios							
L_r (DS2)	0.08	0.005	0.005	0.005	0.005	0.005	0.05
L_r (DS3)	0.2	0.1	0.1	0.1	0.1	0.1	0.2
L_r (DS4)	0.8	0.5	0.5	0.5	0.5	0.5	0.65
L_r (DS5)	1.3	1.3	1.3	1.3	1.3	1.3	1.3
θ_{on}	0.01	0.01	0.01	0.01	0.01	0.01	0.004
θ_c	0.04	0.10	0.08	0.08	0.06	0.04	0.05
f	1	1	1	1	1	1	1
c	1.8	1.6	2	1.7	1.9	2.2	1.2

Table 4-2 Loss Model Parameters for Concrete Frames (AEBM – HAZUS)

Parameters	Seismic Code Level of Structural Design					
	Low Rise Concrete Frame			Mid Rise Concrete Frame		
	Low Code	Moderate Code	High Code	Low Code	Moderate Code	High Code
Interstory Drifts						
$\theta\,(DS2)$	0.005	0.005	0.005	0.0033	0.0033	0.0033
$\theta\,(DS3)$	0.008	0.0087	0.01	0.0053	0.0058	0.0067
$\theta\,(DS4)$	0.02	0.0233	0.03	0.0133	0.0156	0.02
$\theta\,(DS5)$	0.05	0.06	0.08	0.0333	0.04	0.053
Loss Ratios						
$L_r\,(DS2)$	0.08	0.005	0.005	0.08	0.005	0.005
$L_r\,(DS3)$	0.2	0.1	0.1	0.2	0.1	0.1
$L_r\,(DS4)$	0.8	0.5	0.5	0.8	0.5	0.5
$L_r\,(DS5)$	1.3	1.3	1.3	1.3	1.3	1.3
θ_{on}	0.005	0.005	0.005	0.0033	0.0033	0.0033
θ_c	0.05	0.06	0.08	0.0333	0.04	0.053
f	1	1	1	1	1	1
c	1	1.3	1.4	1	1.3	1.4
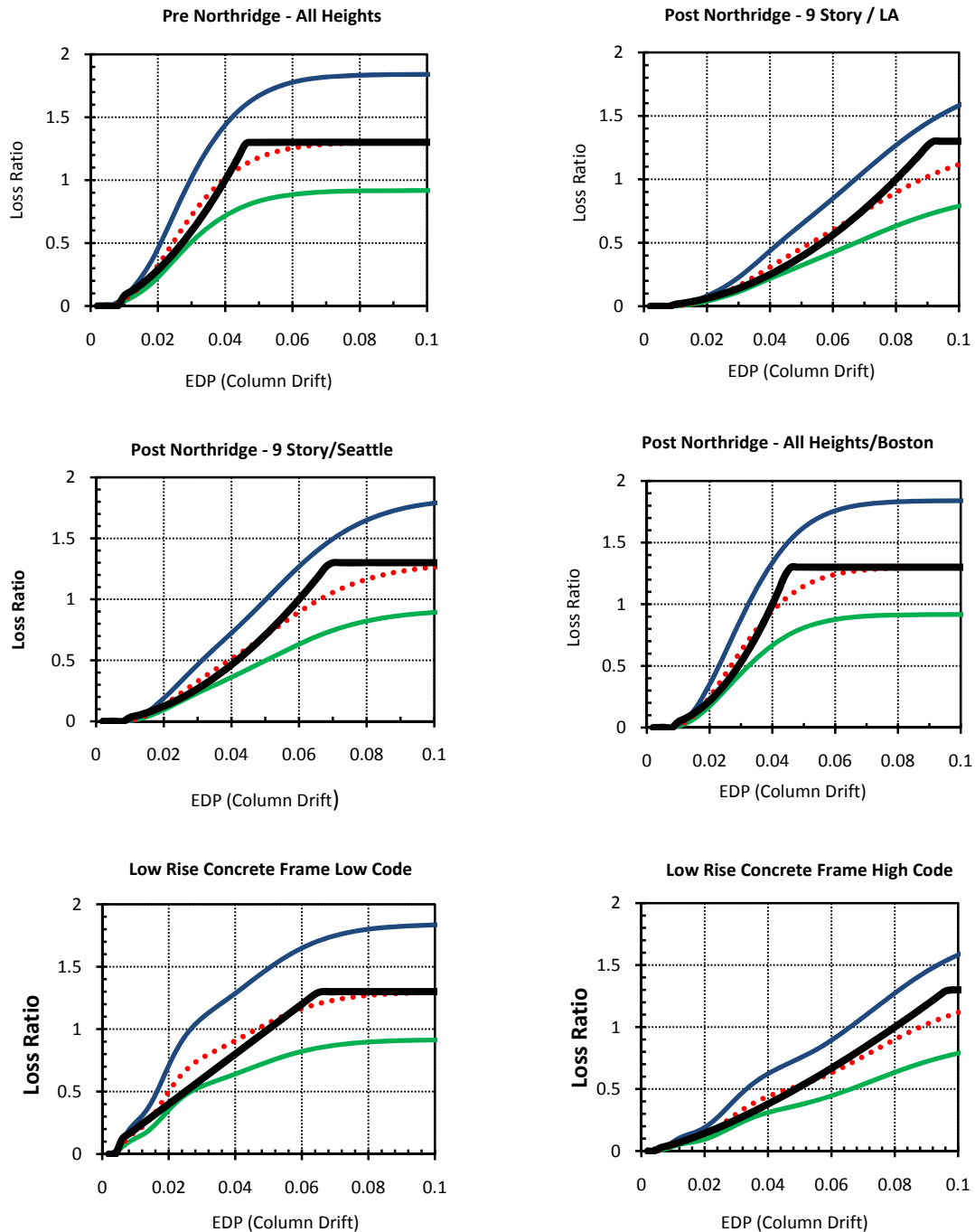
Figure 4-2 Sample of results for fitting the simplified power model to the detailed computed loss model for welded steel moment frame buildings and low rise concrete frames

Figure 4-3 presents two sets of results indicating the comparative capability of the empirically calibrated loss model. Figure 4-3(a) illustrates loss model results for WSMF of 3, 9, and 20 stories designed in Los Angeles as per post-Northridge guidelines. Figure 4-3(b) presents four sets of results: two curves correspond to WSMF buildings designed to pre-Northridge (brittle) and post-Northridge (ductile) specifications and two curves corresponds to low rise concrete frames designed to low-code (brittle) and high-code (ductile) specifications.
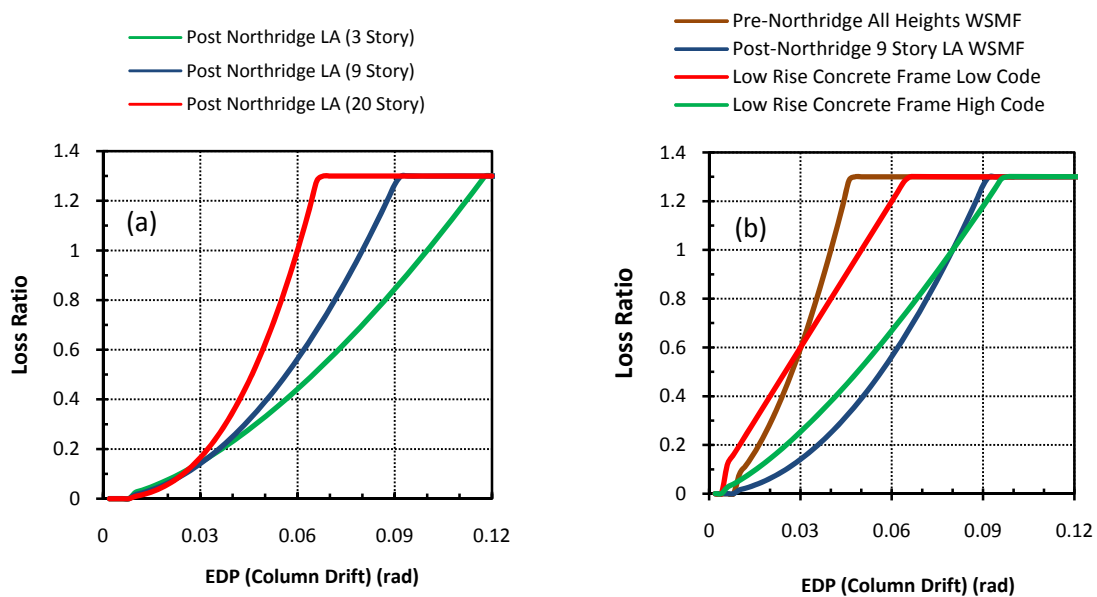
Figure 4-3 Comparison of loss models (a) effect of building height on the calibrated empirical loss model; (b) effect of ductility capability

As expected ductile buildings experience somewhat less damage for the same values of drifts. Moreover a ductile WSMF is much more robust when compared to a ductile low rise concrete frame. This can be attributed to the fact that well-detailed steel structures possess an inherently higher degree of ductility when compared to concrete structures. However it must be mentioned, that a well detailed concrete structure has the capability to provide good performance at high drifts and delay the onset of toppling or collapse.

4.6. Calibrating Composite Parametric Loss Model with Structural and Non-Structural Components

It is generally known that non-structural components make up about 80% of the total cost of a building. It is therefore useful to develop and calibrate the parametric loss model in such a way that it reflects the combined effect of both structural and non-structural components.

In general onset of damage will be governed by the component have a lower value of drift at onset; however the toppling or collapse is primarily governed by the structure alone. The drifts at other damage states can be weighted as per the chosen weighting ratio. However for a ductile building, the drift at onset of collapse may be weighed since it is possible that non-structural components reach onset of collapse earlier compared to structural components. The following conditions summarize the aforementioned hypothesis.

For brittle buildings (or buildings with structural components having earlier onset of collapse)

$$\theta_{DS2} = \min(\theta_{DS2_{structural}}, \theta_{DS2_{non-structural}}) \tag{4.4}$$

$$\theta_{DS5} = \theta_{DS5_{structural}} \tag{4.5}$$

For ductile buildings (or with buildings having non-structural components with earlier onset of damage

$$\theta_{DS2} = \min(\theta_{DS2_{structural}}, \theta_{DS2_{non-structural}}) \tag{4.6}$$

$$\theta_{DS5} = (1-w)\theta_{DS5_{structural}} + w\theta_{DS5_{non-structural}} \tag{4.7}$$

where w indicates the percentage of non-structural components in a facility.

It is also hypothesized that (4.1) can be modified appropriately to incorporate weighting factors for the loss ratios. The loss ratio for each damage state is assumed to be the sum of the loss ratio of each component for each damage state multiplied by their weighting factors. More formally this can be represented by the following equation

$$L_r[EDP] = w\left(\sum_{i=2}^{5} P_i[EDP]L_{ri}\right)_{non-structural} + (1-w)\left(\sum_{i=2}^{5} P_i[EDP]L_{ri}\right)_{structural} \tag{4.8}$$

The final parameter which needs calibrating is the power c. As mentioned before this can be obtained from a least-squares fit. However for ductile buildings it is possible to suggest the following

$$c = wc_{non-structural} + c_{structural} \tag{4.9}$$

Figure 4-4 presents results of the loss model after incorporating the 80:20 weighting factors for non-structural and structural components in each of the parameters

used to describe the loss model. As evident from the plots, the loss model has the capability to incorporate and reflect the losses sustained by the structure as a whole. This is useful in proceeding towards computation of annualized losses for the entire structure.
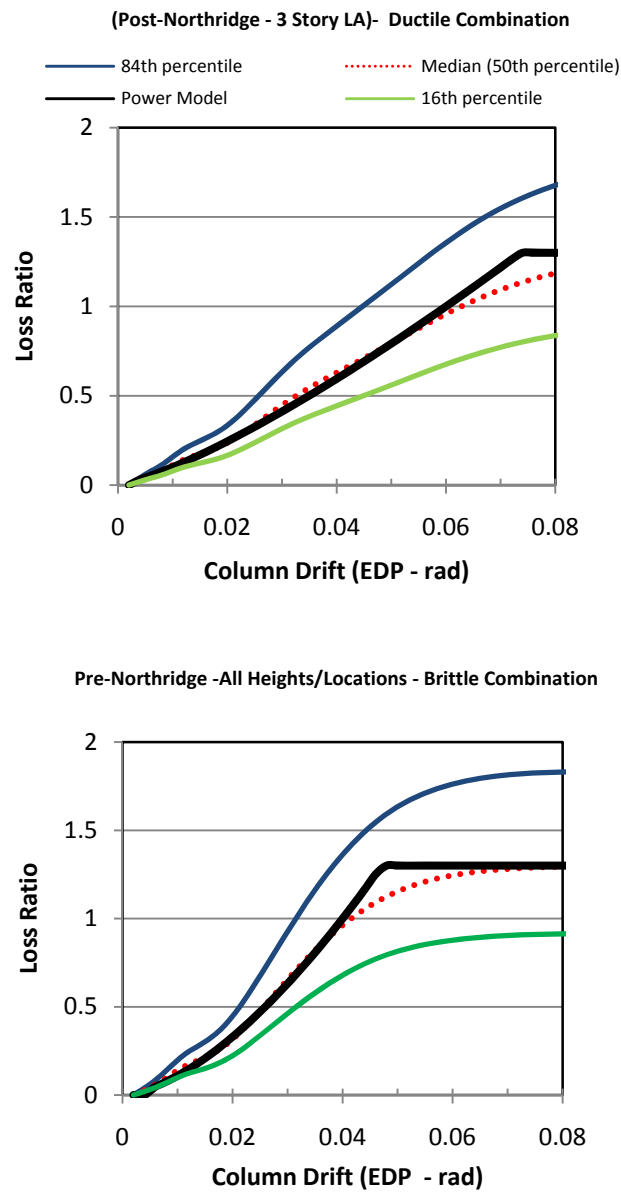
Figure 4-4 Results of directly combining losses weighted 80%:20% for non-structural: structural losses, respectively

4.7.   Implementation of the Direct Loss Model for the Estimation of Seismic Loss for Buildings

The main objective of developing, calibrating, and validating the parametric loss model is its implementation for estimating the expected losses in seismically damaged structures, specifically buildings. The loss model can also be used to check the expected losses for a given design of a new structure. The four-step PBEE loss estimation approach involves a progression from hazard to demand to response and hence to losses. However in the absence of an explicit relationship between seismic hazard recurrence rate and IM (either Peak Ground Acceleration (PGA) or Spectral Acceleration (S_a)), it is necessary to modify the conventional approach of computing losses and still account for all the underlying uncertainties. Often drift demands on structures are related to a level of ground shaking representative of a particular seismic hazard recurrence rate. Non-linear time history analysis of analytical models representing structural behavior of brittle and ductile WSMF with and without torsional effects subjected to SAC ground motions of 2% (*Maximum Considered Earthquake - MCE*), 10% (*Design Basis Earthquake - DBE*), and 50% (*Frequently Arriving Earthquake - FAE*) probability of exceedance in 50 years for Los Angeles have been reported previously. Results of structural performance were quantified in terms of maximum column drift ratio (MCDR); median responses as well as the dispersion of computed response were reported (Wen and Song 2003).

It is possible to plot the hazard recurrence curve expressing the relationship between hazard recurrence rate and intensity measure (*IM)* (Kennedy 1999; Cornell et al.

2002; Solberg et al. 2008) and the equation relating hazard intensity response in terms of

a common demand parameter (herein considered to be drift $\theta = D$) (Cornell at al. 2002)

on a log-log scale. It is possible to recast those equations as the following

$$\frac{f_a}{f_{DBE}} = \left| \frac{IM}{IM_{DBE}} \right|^{-k} \tag{4.10}$$

$$\frac{\theta}{\theta_{DBE}} = \left| \frac{IM}{IM_{DBE}} \right|^{b} \tag{4.11}$$

where f_a = annual frequency of occurrence; IM_{DBE} is shaking intensity for design basis

earthquake, θ is the interstory drift on the structure for the considered event, θ_{DBE} is the

interstory drift on the structure for the design basis event, and b is an exponent.

Since the analysis performed reports resulting relating the left hand side of (4.10)

and (4.11), it is necessary to transform the equations into a suitable format relating drift

demands to probabilities of occurrence. This is given by

$$\frac{\theta}{\theta_{DBE}} = \left| \frac{IM}{IM_{DBE}} \right|^{b} = \left| \frac{f_a}{f_{DBE}} \right|^{\frac{-b}{k}} = \left| \frac{f_a}{f_{DBE}} \right|^{a} \tag{4.12}$$

where the exponent a is given by

$$a = -\frac{b}{k} \tag{4.13}$$

Figure 4-5(a) illustrates the aforementioned equation wherein it is possible to plot

points in a straight line on log-log scale relating drift demand with probability of
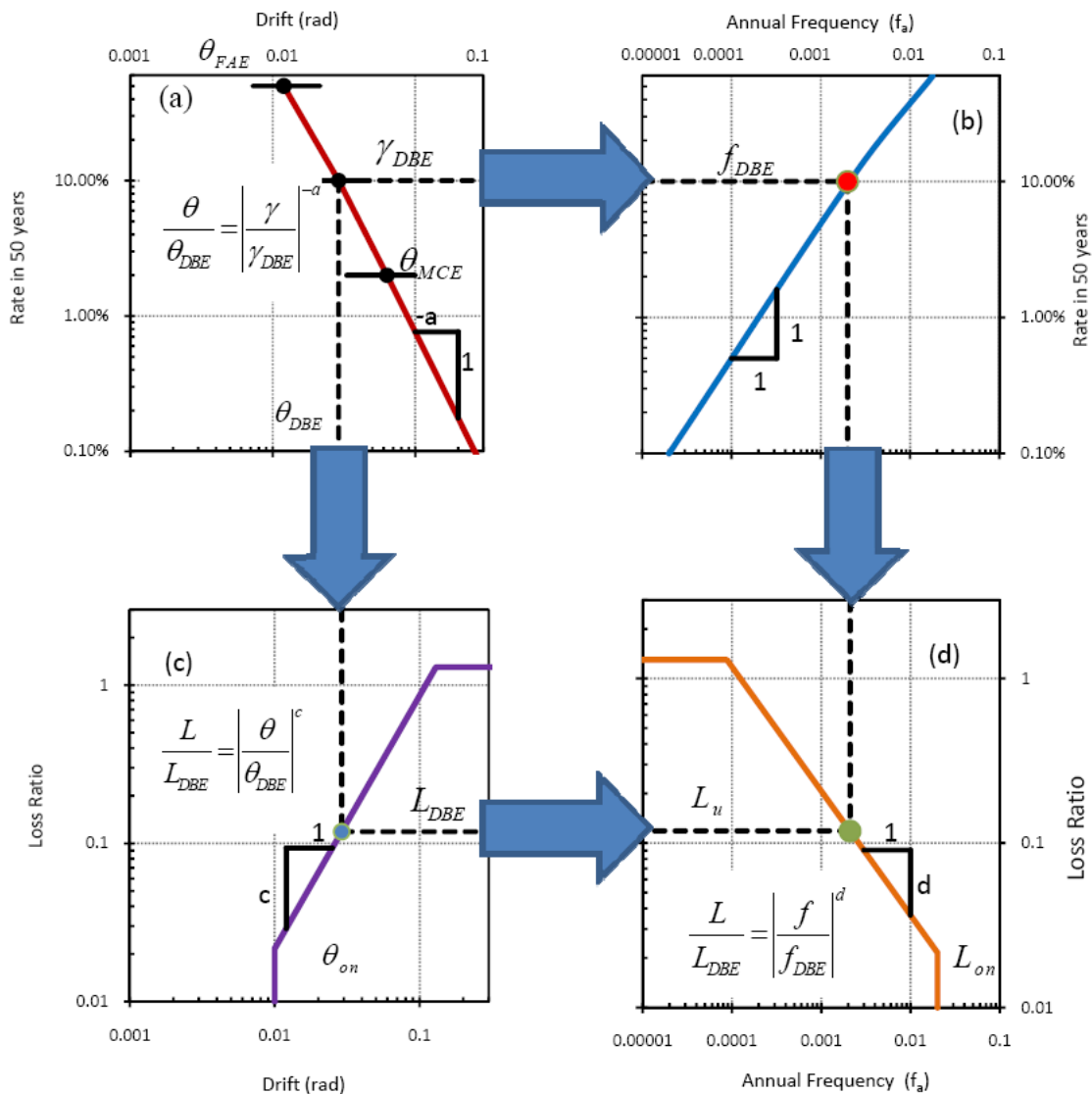
Figure 4-5 Adaptation of direct loss model to the cases where no incremental dynamic analysis results are available. (a) Drift demands (with underlying uncertainty) are imposed due to earthquakes with different occurrence rates in 50 years. (b) The rate is transformed into its equivalent annual frequency of occurrence. (c) The demand drifts are used to compute loss ratios using the calibrated loss model. (d) Hazard-loss curve (loss ratio vs. frequency)

exceedance in 50 years. It is also possible to obtain the exponent a by conducting a regression analysis between θ and f_a.

Recasting (4.2) as

$$\frac{L}{L_{DBE}} = \left|\frac{\theta}{\theta_{DBE}}\right|^c \qquad (4.14)$$

where

$$\frac{L_{DBE}}{L_c} = \left|\frac{\theta_{DBE}}{\theta_c}\right|^c \qquad (4.15)$$

where $L_c =$ unit loss.

It follows from (4.15) and (4.12) that a composite identity equation can be written in the form

$$\frac{L}{L_{DBE}} = \left|\frac{\theta}{\theta_{DBE}}\right|^c = \left|\frac{f}{f_{DBE}}\right|^d \qquad (4.16)$$

where the exponent d is given by

$$d = ac \qquad (4.17)$$

Figure 4-5(d) presents the relationship between losses and the annual frequency of the event. The plot is a culmination of the aforementioned procedure and can be used to estimate losses for scenario events that have a given annual frequency or compute expected annual losses (EAL) for structures.

In doing so it must be emphasized that the above equations only represent the median values. In fact to plot the median loss curve only two key sets of coordinates are

needed (f_{on}, L_{on}) and (f_u, L_u). These can be transformed from median values to mean values for EAL calculations. This approach was outlined in section 3.6 and is summarized here for sake of completeness.

4.8. Computing Annualized Losses

The mean estimate of the losses can be given by the following

$$\overline{EAL} = \frac{\overline{L}_{on}\overline{f}_{on} + d\overline{L}_u\overline{f}_u}{1+d} \tag{4.18}$$

Therefore the modified four-step approach for estimating financial losses for seismically damaged structures shown in Figure 4-4 needs to account for uncertainty and randomness. In general if β represents the lognormal standard deviation (dispersion) in computing a variable y then following the approach outlined in Kennedy et al. (1980), the mean $\overline{y}$ and the median $\tilde{y}$ can be related by the following

$$\overline{y} = \tilde{y}\exp\left(\frac{1}{2}\beta^2\right) \tag{4.19}$$

Similarly for other fractiles, say *x%* non-exceedance probabilities

$$y_{x\%} = \tilde{y}\exp(K_x\beta) \tag{4.20}$$

The mean estimate of f_{on} = the annual frequency of event at onset of damage is given by

$$\overline{f}_{on} = \tilde{f}_{on}\exp\left(\frac{1}{2}\beta_{f_{on}|\theta}^2\right) \tag{4.21}$$

where

$$\beta_{f_{on}|\theta} = \frac{k}{b}\sqrt{\beta_{RD}^2 + \beta_{RC}^2}$$

(4.22)

and

$$\tilde{f}_{on} = f_{DBE}\left|\frac{\tilde{\theta}_{DBE}}{\tilde{\theta}_{on}}\right|^{\frac{k}{b}}$$

(4.23)

The mean estimate of L_{on} = the loss at onset may be calculated as per the following

$$\bar{L}_{on} = \tilde{L}_{on}\exp\left(\frac{1}{2}\beta_{L_{on}|\theta}^2\right)$$

(4.24)

where

$$\beta_{L_{on}|\theta} = \beta_{UL}$$

(4.25)

and

$$\frac{\tilde{L}_{on}}{\tilde{L}_c} = \left|\frac{\tilde{\theta}_{on}}{\tilde{\theta}_c}\right|^c$$

(4.26)

The mean estimate of f_u = the annual frequency at collapse may be given by

$$\bar{f}_u = \tilde{f}_u\exp\left(\frac{1}{2}\beta_{f|L}^2\right)$$

(4.27)

where

$$\beta_{f|L} = \frac{k}{b}\sqrt{\frac{\beta_{UL}^2}{c^2} + \beta_{RC}^2 + \beta_{RD}^2}$$

(4.28)

and

$$\tilde{f}_u = f_{DBE}\left(\tilde{L}_u\right)^{\frac{1}{d}}\left|\frac{\tilde{\theta}_{DBE}}{\tilde{\theta}_c}\right|^{\frac{k}{b}}$$

(4.29)

The mean estimate of the ultimate loss $= L_u$ is given by the following

$$\overline{L}_u = \tilde{L}_u \exp\left(\frac{1}{2}\beta_{UL}^2\right) \tag{4.30}$$

where $\tilde{L}_u = 1.3$ (assigned to incorporate price surge) and β_{UL} is the uncertainty associated with the loss estimation.

4.9. Results of Annual Loss Analyses

The parametric direct loss model was applied to four types of steel moment resisting frames with both ductile and brittle connections. Details of the moment resisting frames have been used from previously related work (Wen and Song 2003), wherein drift demand estimates and their associated dispersions were provided for different types of frames subjected to different return rates of seismic motions. The frames studies included those with and without the impact of torsional motion. Table 4-3 lists median values of parameters used for defining all the four graphs of Figure 4-6 for both ductile and brittle steel welded moment frame buildings. Results for the annual loss calculations for EAL (mean annual loss) are also presented in Table 4-3. The first block of Table 4-3 lists median estimates of parameters for both brittle and ductile buildings based on data reported by Wen and Song (2003). The second block lists the relevant parameters that define the loss equations with the results plotted in Figure 4-5. The third block in Table 4-3 shows the coordinates of the junction points of the tripartite curves plotted in Figure 4-6. It is observed that brittle buildings can have EAL in the order of about $9,000 while

ductile buildings possess a possible EAL of some $3,000. Based on the median estimates, along with the variability data reported by Wen and Song (2003), the fourth block of table presents the calculated dispersion factors that are used to assess the modified (mean) coordinates listed in the fifth block of Table 4-3. Finally the EAL is computed from these listed values. It is clearly evident that brittle buildings are susceptible to higher degree of damage for more frequent (and correspondingly less intense) events. Though the onset of damage might be the same for both ductile and brittle buildings, damage accumulates more rapidly with more intense shaking for brittle buildings leading to collapse/toppling at more frequently occurring events. Figure 4-6(b) presents plots indicating EAL for brittle and ductile buildings of different structural configurations. The plots show that buildings without torsional motions are comparatively more robust than those prone to some torsional response. As expected, ductile buildings perform better and cause lesser damage (indicated by lower values of EAL). It can be clearly seen that the 3 story 1 bay brittle WSMF suffers the highest amount of losses amongst all the brittle buildings. This can be attributed to a combination of a lack of ductility and appropriate amount of structural redundancy required to preclude failure.

Table 4-3 Summary of Parameters Used in Four Step Simplified Loss Estimation Procedure for Three-Story Moment Frame with Ductile and Brittle Welded Connections

Building Type	Ductile Frames		Brittle Frames		Remarks (Equations)
	1 bay	3 bay	1 bay	3 bay	
θ_{FAE}	0.0144	0.0137	0.0147	0.0136	Wen and Song (2003)
θ_{DBE}	0.029	0.0249	0.0291	0.026	,,
θ_{MCE}	0.0662	0.0514	0.0684	0.0614	,,
f_{FAE}	0.01	0.01	0.01	0.01	,,
f_{DBE}	0.0021	0.0021	0.0021	0.0021	,,
f_{MCE}	0.0004	0.0004	0.0004	0.0004	,,
a	-0.47	-0.41	-0.48	-0.47	(4.13)
θ_{on}	0.01	0.01	0.01	0.01	(Kircher 2003)
f	1	1	1	1	Calibrated
θ_{cr}	0.10	0.10	0.04	0.04	(Kircher 2003)
c	1.6	1.6	1.8	1.8	Calibrated
d	-0.7557	-0.6542	-0.8604	-0.8424	(4.17)
$\tilde{L}_{DBE}$	0.1380	0.1081	0.5640	0.4605	(4.15)
$\tilde{L}_{u}$	1.3	1.3	1.3	1.3	Assigned
$\tilde{f}_{u}$	0.00010821	0.00004703	0.00079767	0.00061417	(4.29)
$\tilde{L}_{on}$	0.0251	0.0251	0.0825	0.0825	(4.26)
$\tilde{f}_{on}$	0.02005983	0.01960280	0.01966952	0.01621816	(4.23)
β_{RD}	0.43	0.39	0.42	0.4	Wen and Song (2003)
β_{RC}	0.30	0.30	0.30	0.30	Assigned
β_{UL}	0.35	0.35	0.35	0.35	Assigned
$\beta_{L_{on}|\theta}$	0.35	0.35	0.35	0.35	(4.25)
$\beta_{f_{on}|\theta}$	1.110	1.203	1.080	1.068	(4.22)
$\beta_{f|L}$	1.203	1.317	1.154	1.146	(4.28)
$\overline{L_{u}}$	1.38	1.38	1.38	1.38	(4.30)
$\overline{f}_{u}$	0.0002231	0.0001119	0.0015522	0.0011848	(4.27)
$\overline{L}_{on}$	0.027	0.027	0.088	0.088	(4.24)
$\overline{f}_{on}$	0.037147	0.040438	0.035235	0.028699	(4.21)
EAL	*$3,107*	*$2,830*	*$8,908*	*$7,213*	(4.18)
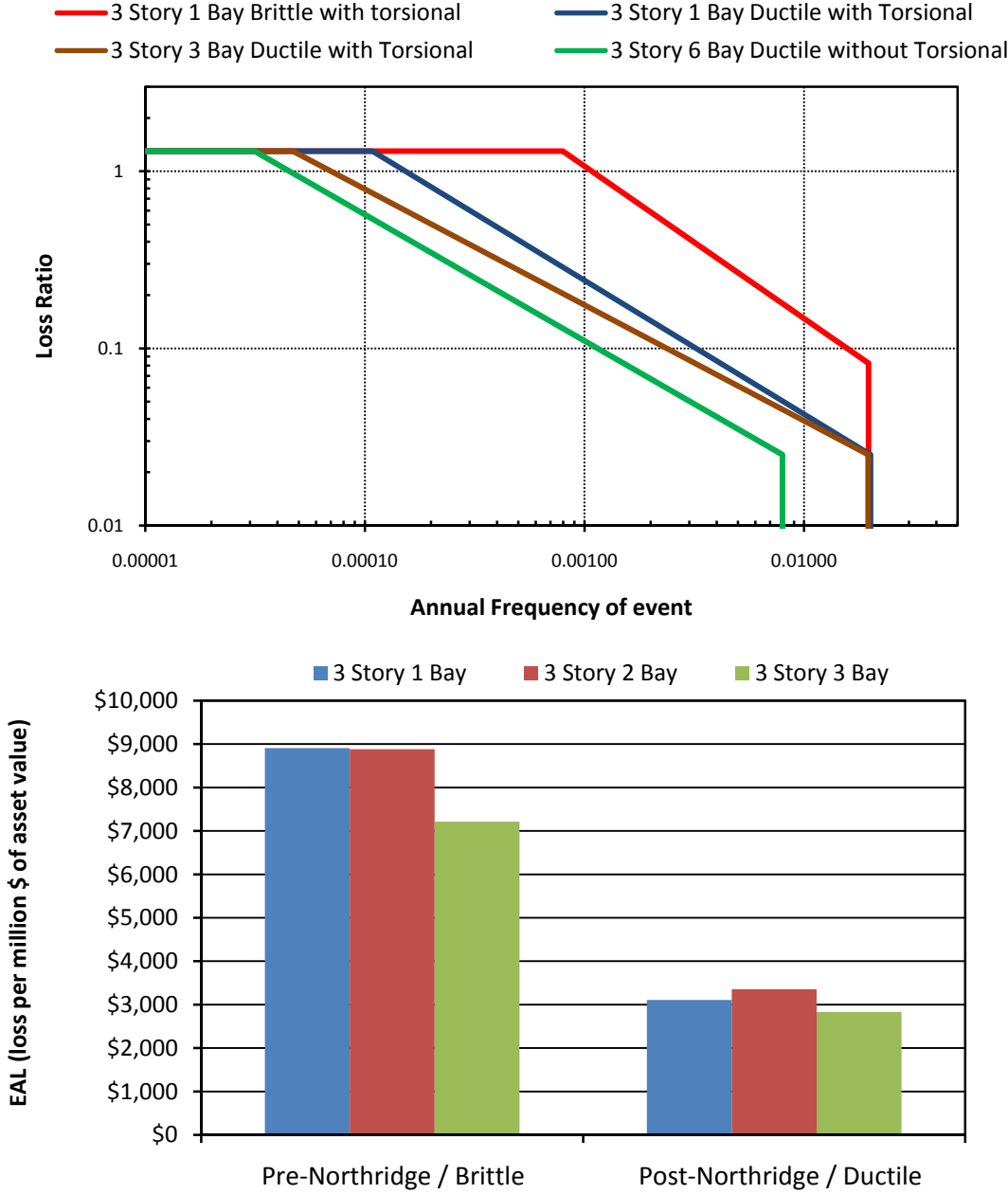
Figure 4-6 (a) Loss-frequency relations for different welded steel moment frame buildings; (b) results of EAL for the brittle and ductile steel welded moment frame buildings

In general, a relevant estimate of expected annual losses (EAL) resulting from earthquake damage, must include all possible components involved in the constructed facility. Therefore in order to provide realistic comprehensive estimates, the impact of non-structural components along with structural components has been investigated. Previous studies (Porter et al. 2001) presented seismic vulnerability of buildings and its contents using an assembly-based fragility approach. As mentioned before, non-structural elements are susceptible to earlier onset of damage as well as ultimate collapse. Since the cost of a commercial building facility is dominated primarily by non-structural components (in the order of 80% of the total cost), it is expected that annual losses for a structure incorporating the effect of both structural and non-structural losses would be somewhat higher when compared to a structure incorporating the impact of structural losses only. Figure 4-7 presents plots of EAL for brittle and ductile WSMF for structural, non-structural, and total losses. For computing total losses, a weighting factor of 80:20 indicating contribution of non-structural and structural components is assumed. This is a reasonable assumption for a mid-rise WSMF. Sensitivity analysis on the weighting factors indicates that the extent of total losses is fairly insensitive to the relative values of weighing factors. It is observed that although brittle WSMF buildings may suffer a higher extent of structural losses, the difference in the extent of non-structural losses is insignificant. Therefore, the total losses in brittle (EAL = $20,300 per $million) and ductile (EAL = $20,300 per $million) buildings are similar.
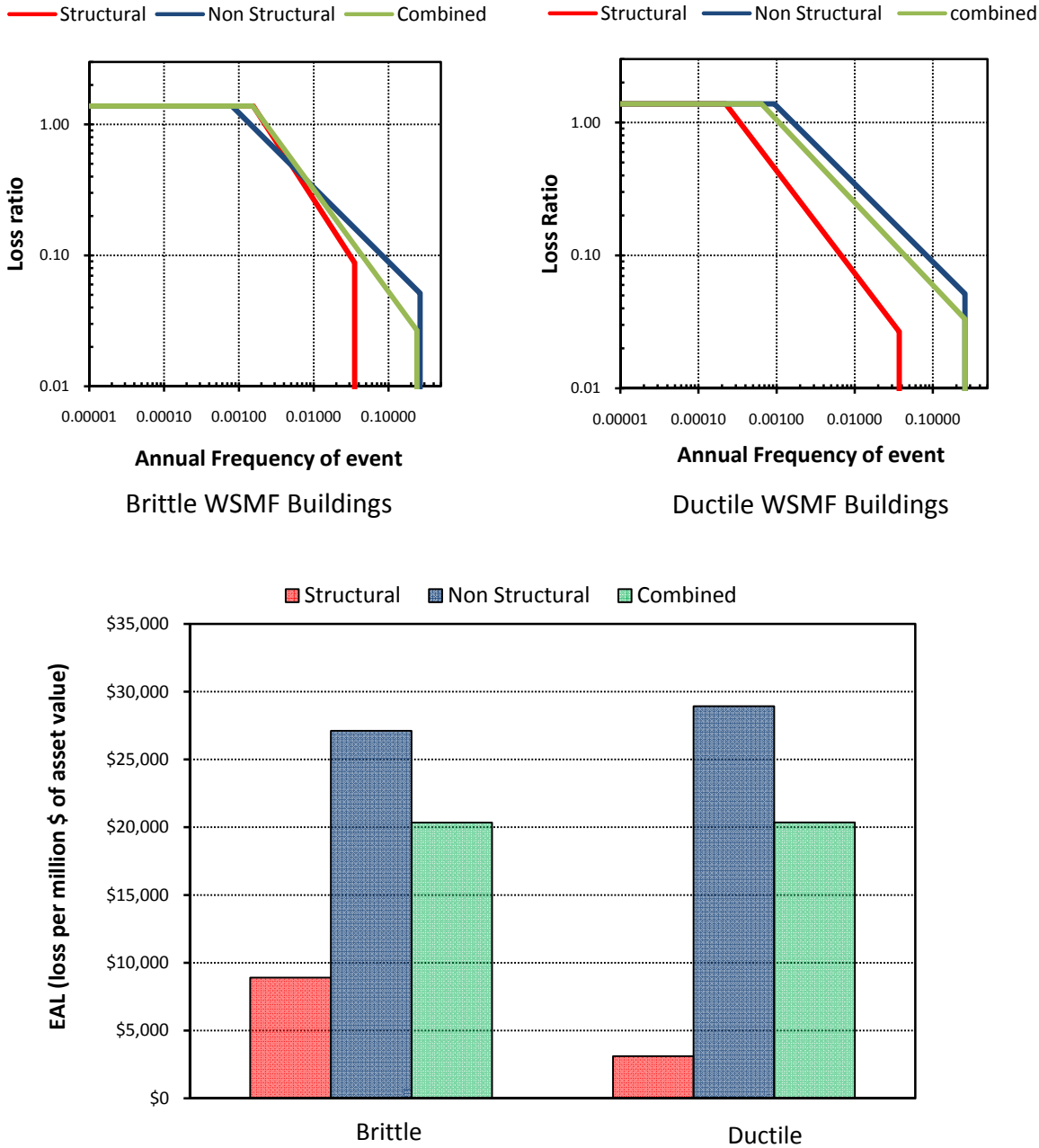
Figure 4-7 Significance of non-structural damage to EAL for brittle and ductile welded steel moment frame buildings

4.10. Discussion

As seen from Figures 4-2 to 4-4, it is evident that for low drift levels, there is reasonable agreement between the models but as the structure exceeds its median value for complete damage or collapse there is noticeable difference between the two models. This is of little consequence as the probabilities of occurrence for these excessively large drifts are correspondingly very small. For most structures the degree of loss depends largely on the parameter θ_{cr}, that is the critical drift normally equal to $\tilde{\theta}_{DS5}$ or the drift at which complete damage (and loss) or toppling occurs. Figure 4-5 and Figure 4-6 reveal that the structural component for estimated annual losses for brittle buildings is much more when compared to ductile buildings. The structural component of losses is about $9,000 for brittle buildings and $3,000 for ductile buildings. Structural component of losses differs only slightly with difference in structural configuration. The major difference in contribution of structural losses arises from the inherent ductility capability of the building which is a function of the goodness of the detailing and to a lesser extent the effect of torsional motions.

When non-structural components are included in the analysis the combined losses are surprisingly similar. Interestingly, the implication from this is that to reduce EAL, it is not so necessary to construct more ductile buildings, but rather stronger and stiffer buildings that delay the onset of non-structural damage. In an increasingly hazard-prone yet high-stakes environment, it is essential that engineering facilities be designed in a manner that minimizes direct financial losses arising from damage and downtime.

4.11. Closure

The work conducted as part of the research can be summarized into the following:

1. Capacity-side fragility curves can be used to derive a probabilistic seismic loss model. Only four parameters are needed to completely define the resulting capped power-curve loss model. Two parameters relate to structural performance and define the onset of damage (θ_{on}) and the onset of collapse (θ_c) or toppling. There is an assumed upper bound loss of $L_u = 1.3$, where $L_u \geq 1$ accounts for expected price surge following a catastrophic event. The fourth parameter c is the only empirically calibrated parameter. Values for the parameter have been calibrated for a variety of steel and concrete building types where c is more or less constant ($c = 1$). The model implicitly accommodates both aleatory and epistemic uncertainty.

2. A sensitivity study reveals that losses are most sensitive to a reliable estimate of θ_c, and relatively insensitive to the calibrated parameter c.

3. The loss model has been incorporated into an overall loss estimation framework and applied to welded steel moment frame buildings with both brittle (pre-Northridge) and ductile (post-Northridge) buildings. Results show that expected annual losses (EAL) to be in the order of $9,000 and $3,000 per $Million of structural value for the former and later respectively.

4. The loss model has also been applied to the non-structural components of buildings. Non-structural damage to frame structures tends to be insensitive to the building specific details. However, with EAL in the order of $28,000, this tends to overshadow structural losses. Total EAL (building and contents) is in the order of

$20,000 per $Million of total asset value for welded steel moment frame with both brittle and ductile connections, respectively. To mitigate such a high degree of loss, buildings need to be constructed stronger and stiffer.

5. It should be noted that the foregoing analysis concerns only the structure and considers neither business interruption, losses to amenities, nor does it consider the potential loss to life and limb. Incorporation of such losses is the subject of ongoing research.

5. APPLICATION OF LOSS MODELING FOR PRICING CATASTROPHIC BONDS

5.1. Section Summary

Natural hazards cause death, damage, downtime, and destruction. Associated with the hazards are long-term reconstruction and rehabilitation implications. Recently, hazard-linked securities such as catastrophic (CAT) bonds have found acceptance as a potential risk-mitigating measure. The benefits of investing in catastrophic bonds are two-fold. On one-hand, it insures facilities against potential loss of operation in the wake of a natural hazard; on the other hand they act as potentially safe yet enticing investment opportunities for the discerning investor, since the probability of a mega catastrophe is always slim. This section presents a cost-benefit based hypothesis for pricing the risk component of catastrophic bonds called spread. The direct loss model derived and illustrated in the previous sections is used to design a CAT bond after incorporating both the uncertainty in the occurrence of a loss-causing event that triggers the bond and the potential confidence seeked by an average investor. It is shown that most well-designed and detailed structures (including bridges and buildings) are inherently safe and provide a good degree of confidence to the investor. The loss model is used to compute and validate the high degree of confidence required by an investor for a given risk taken. It is also shown that spread ratio which is the ratio of spreads to estimated annual bond loss

can be considered to be a random variable which is distributed lognormally about the area of interest for most common insurance linked securities.

5.2. Background

An increasingly risk-prone yet risk-averse world has necessitated the need for concerted mitigation efforts to minimize death, damage, destruction, and downtime. Though life-safety remains the primary goal of the structural engineering community, it has become increasingly important to implement suitable methodologies that ensure an optimum allocation of resources and minimal amount of losses. In general, structural engineers can provide estimates of earthquake and other hazard related losses; however decision making is left in the hands of stakeholders. In order to bridge the gap between the civil engineering and the decision-making communities, there is a need to provide tools that are easily implementable in both the fields. Performance-based earthquake engineering (PBEE) can be considered to be a hazard-mitigation design paradigm wherein an effort is made to capture the overall performance of civil engineering facilities over all possible behavior modes when subjected to a various ranges of seismic actions (Bradley et al. 2007). This design methodology is increasingly seen as a comprehensive tool to satisfy specified performance objectives. However, there is always a residual probability of incurring damages beyond the predicted margins. In order to mitigate these possible risks, new measures called risk-linked securities are being employed. Catastrophic (CAT) bonds are popular risk-linked securities which yield returns based on either the occurrence of a natural catastrophe or upon the actual claims filed (Loubergé et al.

1999). CAT bonds are considered to be ideal tools to facilitate reconstruction and rehabilitation in the event of a major natural hazard. In general, the issuers of these risk-linked securities use the services of a specialized loss modeling agency in order to price the bonds. Since CAT bonds are increasingly being employed as a beneficial financial risk mitigating measure, there has been an overall attempt to standardize the characteristics and dispel some of the fears associated with investing in such schemes (Cummins 2008).

Figure 5-1 presents a representation of the possible combination of PBEE with a risk mitigating measure such as catastrophic bonds. The figure illustrates that it is possible to use a PBEE based loss estimation procedure in conjunction with appropriate financial instruments to provide an advantageous hazard mitigation strategy.

5.3.   Seismic Loss Estimation

Present state-of-the-art loss estimation approach suggested by Pacific Earthquake Engineering Research (Cornell and Krawinkler 2000; Deierlein 2004) entails a broad four-step procedure wherein a progression is made from hazard to demand to response to loss. Each of these steps can be carried out independently and then combined in order to provide results suited for various decision makers. Each of the steps involves both randomness and propagation and therefore has to be treated probabistically (Baker and Cornell 2008). More formally, the procedure can be represented mathematically in the an integral format (Deierlein 2004; Dhakal and Mander 2006; Mander et al. 2007)
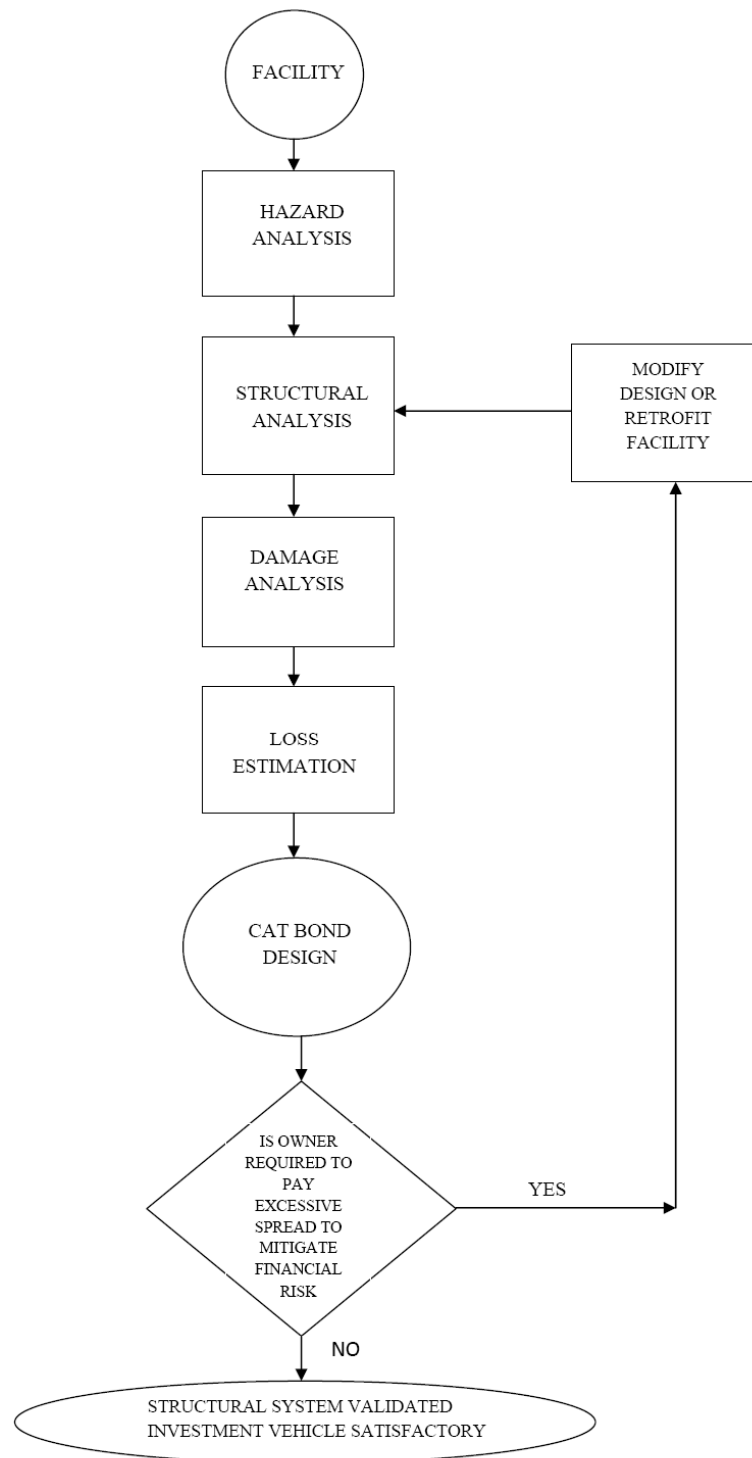
Figure 5-1 Hypothetical combination of Performance Based Earthquake Engineering with a risk-linked security such as CAT bonds

The variables involved in the integral include f_a = annual frequency of occurrence of an event; IM = intensity measure (usually characterized through spectral acceleration S_a or peak ground acceleration PGA); EDP = an engineering demand parameter such as interstory drift; dm= damage measure such as the maximum interstory drift not causing any damage; dv = an appropriate decision variable like loss ratio; and L_r = loss ratio defined as the ratio of repair cost to replacement cost.

Figure 5-2 illustrates the four-step PBEE procedure through a direct four-step loss estimation process wherein hazard is directly mapped to response to damage and hence to losses. Note that all the figures are plotted on a log-log scale and can be inter-related through the composite equation given by

$$\frac{L}{L_{DBE}} = \left| \frac{\theta}{\theta_{DBE}} \right|^{c} = \left| \frac{S_a}{S_{a_{DBE}}} \right|^{bc} = \left| \frac{f}{f_{DBE}} \right|^{\frac{bc}{-k}} \tag{5.1}$$

in which L = loss ratio; θ= drift (an EDP); S_a = spectral acceleration; f_a = annual frequency; k, b, c are exponents (slopes of curves in log-log space shown in Figures 5-1 a, b, and c respectively); and the subscript DBE refers to the reference parameters for the design basis earthquake.

Figure 5-2(a) represents the hazard-intensity curve; it is assumed that the input for plot is crisp and therefore no uncertainty is considered. Figure 5-2(b) represents the hazard intensity-seismic demand plot. Figure 5-2(c) represents a parametric loss model used to assess damage to constructed facilities. In this figure, structural capacities (characterized in terms of interstory drifts) are related to possible damage states (quantified in terms of loss ratios) of the structure.
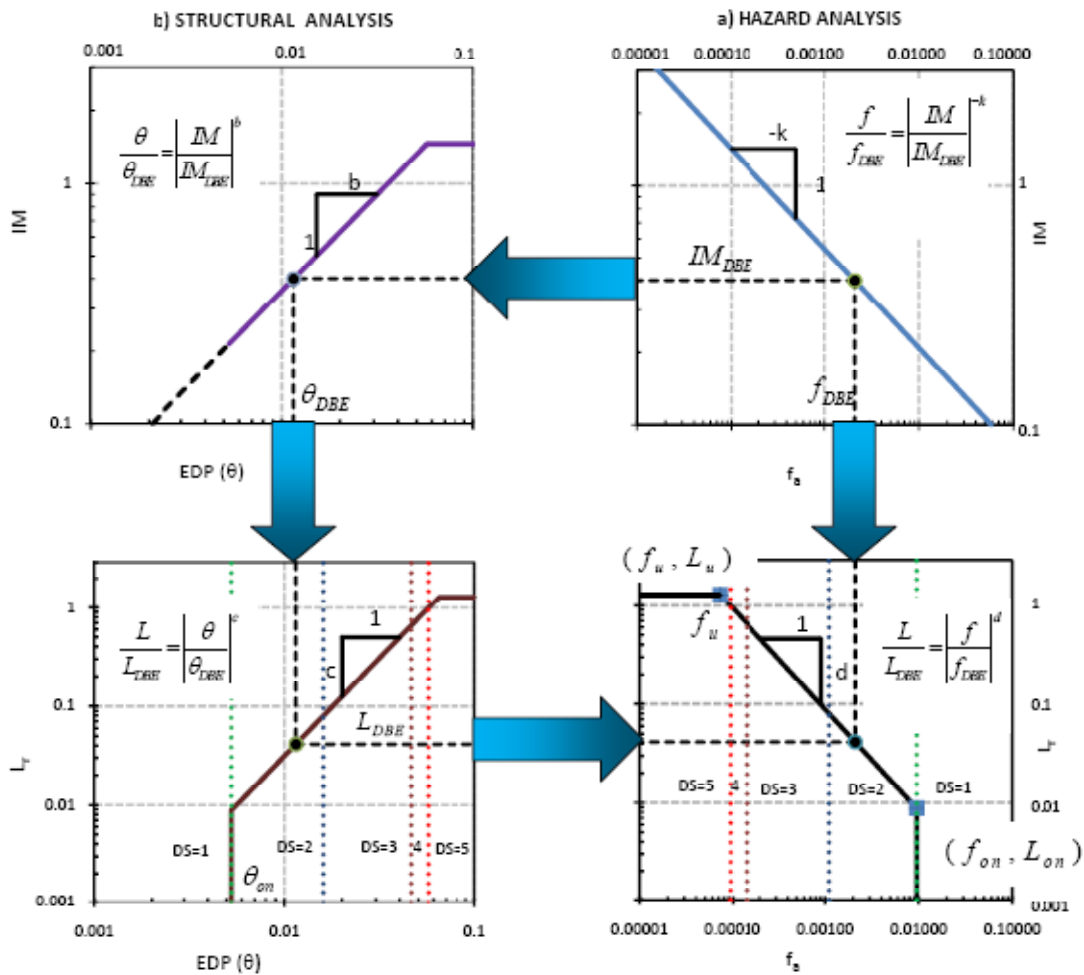
Figure 5-2 Four step loss estimation procedure outlined by PBEE. The arrows indicate the progression from hazard to response to damage and therefore to losses

The loss model is obtained using capacity-side fragility curves and discrete damage states suggested by HAZUS (Mander and Basoz 1999). Predicted demand is compared with the capacity obtained by conducting analysis on a non-linear model of the structure. As shown in sections 1 and 2, this can be expressed through a relationship relating losses with drifts and is given by

$$\frac{L_{DBE}}{L_c} = \left| \frac{\theta_{DBE}}{\theta_c} \right|^c \tag{5.2}$$

where L_c = unit loss, a random variable in itself that has a median value of $\tilde{L}_c = 1$.

All of the above are convoluted to obtain the loss-frequency curve shown in Figure 5-2(d) which illustrates the loss-frequency relation given by

$$\frac{L_a}{L_{DBE}} = \left| \frac{f_a}{f_{DBE}} \right|^d \tag{5.3}$$

where $d = \dfrac{bc}{-k}$.

Eq. (5.3) can be used to compute the annual frequency of the event for which a specific value of loss is exceeded. In other words, the annual frequency of the event can be considered to be equivalent to an annual probability of risk of loss to the investor.

5.4. Catastrophic Bonds

In general, bonds are debt instruments issued for raising capital by borrowing for a stipulated period. Bonds pay the principal along with an interest (a specified amount commonly termed coupon); based on the terms of the bond, coupons might be paid at

stipulated intervals throughout the period till maturity or otherwise. Catastrophic (CAT) bonds are one-to-five years single or multi-peril risk mitigating measures (Kunreuther 2001) or risk linked securities (Cummins 2008) whose coupon payments as well as principal payments depend on the occurrence of a specific catastrophe. Thus the bonds might forfeit payments if any of the specified risks is exceeded as per the terms and conditions.

Figure 5-3 illustrates the basic format of a CAT bond implementation. The entire operation involves four main parties i.e. the owner of the facility, the insurance or reinsurance agency, the collateral, and investors. The owner of the facility pays a premium to the insurance agency in order to obtain protection from natural catastrophes. The insurance agency has the primary objective of providing cover or protection to the owner and therefore designs the entire structure of the CAT bonds based on its own studies. A special purpose vehicle (SPV) is set up and it sells CAT bonds to the capital market (investors); the capital obtained is deposited in a collateral account wherein it receives interest. The SPV is designed to make stipulated regular payments to the investors in terms of coupons. Proceeds of the coupons are obtained from the premium charged from the owner as well as the interest obtained from the collateral. If a triggering event (catastrophe of a defined magnitude/losses exceeding a defined amount) occurs during the lifetime (within the maturity period) of the bond, then the investors stand the chance of forfeiting their entire principal as well as coupon payments, depending on the bond terms and conditions. The triggering conditions are clearly specified and are often based on modeling of expected losses due to a given hazard (as

shown in the previous section). Alternatively, the investors are paid their entire principal as well as the coupons if the specified event does not occur. In order to compensate investors for the possible loss of their principal, an additional amount over the prime risk-free rate (often London Interbank Offered Rate LIBOR) called *spread* is paid. The spread depends on the inherent vulnerability of the engineering facility to hazards as well as the possible risk-appetite of the investor. For example, a risk-averse investor will choose to invest in a bond which is designed to cover a potentially safe facility. The investor will be paid a lower spread component as he/she is confident about obtaining the principal back at the end of the bond period.

The two major types of CAT bonds include the parametric bond and the indemnity bond. A parametric CAT bond uses a physical parameter (such as magnitude of an earthquake) to determine whether default is triggered i.e. when losses of a catastrophe are to be covered by CAT bonds. It has been reported that parametric bonds are subject to basis risk (Cummins 2008). Basis risk is defined as the difference between the actual losses suffered by the insurance agency and the cumulative pool of losses that hinders it from receiving the amount in order to completely hedge risk. An indemnity CAT bond is triggered when the actual losses of the issuer above the set limit of losses. Indemnity bonds are subject to what is commonly known as moral hazard risk. This phenomenon occurs when the insurance agency pays out a certain cost for controlling loss that it is perceived to be greater than that required for debt forgiveness. It is believed that the insurer is at an advantage to pay the claims though losses might not have been to

that extent (Lee and Yu 2002). Moral hazard is caused due to inadequate loss control efforts by the insurer issuing CAT bonds (Doherty 1997).

The following section illustrates the application of the loss estimation method mentioned before in designing parametric and indemnity CAT bonds.
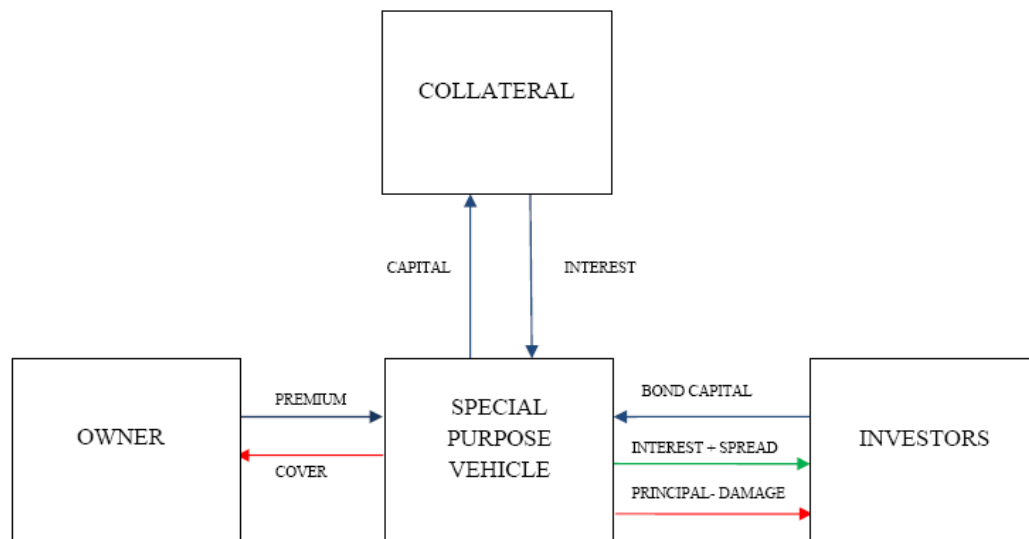
Figure 5-3 Cash flow of a typical CAT bond: red arrows indicate bond was triggered, green arrows indicate otherwise

5.5.   The Design of CAT Bond from Loss Estimates

Figure 5-2(d) illustrated the seismic loss-frequency curve for a prototype constructed facility on a log-log scale. It is possible to represent the constructed facility of both indemnity and parametric bonds on the same figure with the axes rotated (most relevant financial literature plot annual probabilities of occurrence of events against losses).

Figures 5-4(a) and 5-4(b) represents a possible design of an indemnity bond using the frequency-loss curve. Figures 5-4(c) and 5-4(d) illustrate a possible design of a parametric bond. Note that deductible indicates a proportion of the amount which is to be paid by the owner towards the coverage of losses. Insurance indicates a primary source of insuring agency (differing from a CAT bond SPV) which might be involved in absorbing a layer of losses. These two blocks are user specified and can be set up as per agreed terms. Figures 5-4(a) and 5-4(c) clearly represent the losses which are to be covered by CAT bonds. As observed from the figures, the two types of bonds differ in the nature of losses covered. While the shaded part of Figure 5-4(a) representing CAT bond losses depends on the losses of the issuer, the shaded part of Figure 5-4(c) depends on the exceedance of a parameter such as IM. Figures 5-4(b) and 5-4(d) are drawn based on the shaded areas of Figures 5-4(a) and 5-4(c) respectively and have the total losses normalized to unity. This makes it easy to compute expected annual bond loss *eabl* (losses to be covered by CAT bonds) based on the area of the frequency-loss diagram. For example, *eabl* for a parametric bond is simply equal to the magnitude of the maximum frequency of event (and hence the intensity) covered by the bond. More formally this can be expressed as the following

$$eabl = f_a \qquad (5.4)$$

For an indemnity bond, it amounts to calculating the area of the frequency loss curve after deducting the area (or losses) covered by deductible and insurance.
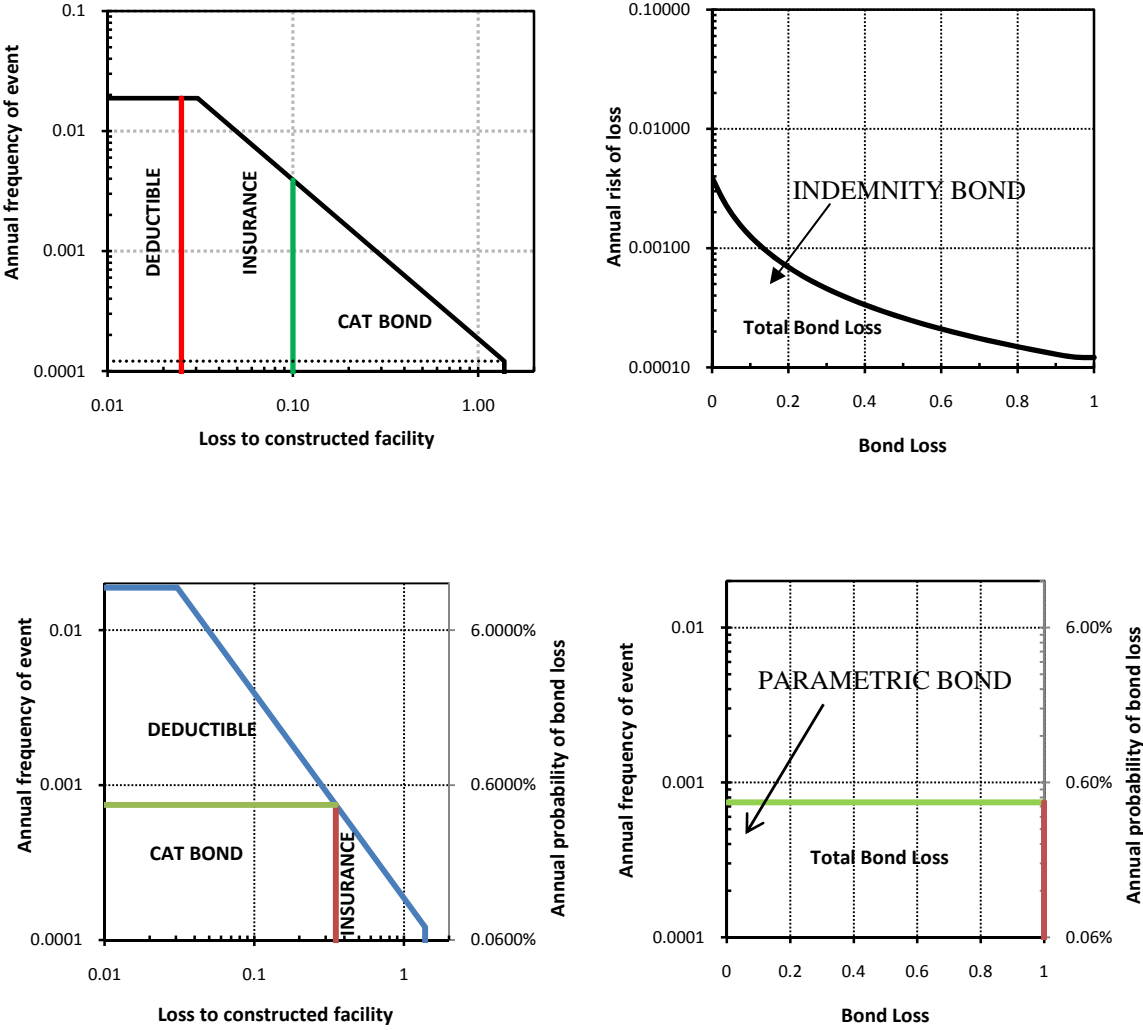
Figure 5-4 (a) Design of a typical indemnity CAT bond; (b) indemnity CAT bond loss; (c) partitioning of insurance coverage and the design of a typical parametric CAT bond; and (d) parametric CAT bond loss

5.6.   Computation of Spread Based on Losses

As mentioned previously, spread is an additional amount (quantified through an interest rate above risk free prime rate) paid in order to compensate investors for investing in a potential defaulting entity. Spread increases with an increase in the risk-appetite of the investor and is subject to the possible losses suffered by an engineering facility. If spread is denoted by the variable S, it can be represented by the following equation

$$S = r - i_p \qquad (5.5)$$

where r represents the rate offered by the CAT bond and i_p represents the risk-free prime rate.

Based on previous work (Kunreuther 2001) the following cost-benefit hypothesis can be used to determine spread. Let $p(L)$ represent the probability of occurrence of a loss (or the annual frequency of occurrence of a specified event), and B represent the amount of the bond used to cover the facility. In order to ensure that CAT bond investment is equivalent to normal investment, the following relationship can be suggested on implementing a simple cost-benefit ideology

$$(1 - p(L))rA - p(L)A = i_p A \qquad (5.6)$$

where 1- $p(L)$ represents the probability that no loss occurs. (5.6) can be further simplified to obtain the rate offered by the CAT bond. Dividing (5.6) through by A and substituting for

$$r = \frac{i_p + p(L)}{1 - p(L)} \qquad (5.7)$$

Without loss of generality, (5.7) can be further simplified using binomial approximation to give the following

$$r = (i_p + p(L))(1 + p(L)) \tag{5.8}$$

Implementing (5.5), spread can be given by the following equation

$$Spread = r - i_p = p(L) + i_p p(L) + p(L)^2 \tag{5.9}$$

This amounts to the following

$$Spread = p(L)(1 + i_p + p(L)) \tag{5.10}$$

As expected, spread is essentially proportional to the probability of loss - the greater the probability, the greater the spread. (5.10) indicates that a chance of higher probability of losses indicates a higher spread component which amounts to investment in a riskier structure. However it must be remembered that (5.10) is obtained without considering any variabilities in hazard, the structure or the losses depend on damage. Therefore (5.10) should be inflated by a suitable number indicating uncertainty in predicting the probability of loss (annual occurrence of event) as well as the risk-free prime rate. It is possible to model the movements of the risk-free rate using the well known Cox-Ingersoll-Ross (CIR) interest rate model or any suitable model. It is possible to define a term spread ratio indicating the ratio of spread to estimated annual bond loss *eabl*.

For a parametric bond, the estimated annual bond loss is equivalent to the probability of occurrence of loss (or frequency of loss causing event). Spread ratio can be denoted by the variable R_s and can be given by the following

$$R_s = \frac{Spread}{p(L)} = 1 + i_p + p(L) \tag{5.11}$$

Without loss of generality, it can be assumed that for a potential loss causing event, the probability of occurrence of the event will be equal to its frequency of occurrence. Thus

$$f_a = p(L) \qquad (5.12)$$

Therefore (5.12) can be recast as

$$R_s = \frac{Spread}{f_a} = 1 + i_p + p(L) \qquad (5.13)$$

To illustrate the above, consider a hypothetical example in order to maximize spread. For example, if an investor buys a CAT bond to cover a hypothetical structure designed to withstand an event with a frequency of occurrence = 0.02, and if the risk-free prime rate is 10%, then the spread ratio is evaluated to be equal to 1.12. But this spread ratio is considerably smaller than that offered by the risk-averse capital markets for CAT bonds, where observed spread ratios are generally greater than 3 (R_s >3). Practical values of spreads on CAT bonds for covering natural hazards such as earthquakes have varied between 2% and 8% (Source: Merrill Lynch and Brown Brothers Harriman. Mexico issued an earthquake linked CAT bond in 2006 which offered a spread of 2.30% (230 points above Spread). It is therefore seen that CAT bond offer higher spreads than what a cost-benefit hypothesis suggests. This can be attributed to the higher degree of uncertainty surrounding the risk aversion level of an investor which in turn is linked to the potential vulnerability of a structure to losses. It is hypothesized that higher spreads offered on CAT bonds are a result of high degree of uncertainty surrounding the occurrence of a catastrophic event such as an earthquake along with the high risk-averseness (quantified through a high degree of confidence or

probability of non-exceedance of losses) of the investor. This can be explained by the high degree of randomness and uncertainty on the loss-frequency relationship shown on Figure 5-2(d).

5.7. Defining Spread Ratio as a Random Variable

Another relationship between expected loss and spread (Christofides 2004) is based upon the risk aversion level of the investor and is given by the following

$$Spread = (EAL)^{\frac{1}{\rho}} \qquad (5.14)$$

where EAL is the Expected Loss and $\rho \geq 1$, is a Risk Aversion Level (RAL). The values of ρ are found to be in the range 1.65 ± 0.15. For example, a parametric CAT bond designed to cover losses for design basis events $EAL = eabl = f_{DBE} = 0.002$ will offer a spread of 2.31% if a risk aversion factor of 1.65 (medium risk-aversion) is applied to (5.14).

Table 5-1 lists the annual losses and annual spreads on insurance linked securities (ILS) issued between 2001 and 2003. It is reported that ILS issued mainly over the last five years were priced at multiples of over 6 times their expected annual losses (Christofides 2004). However no underlying logic behind the pricing of these securities was provided and it was assumed that the values provided were accepted by the market.

However, it is postulated that spread ratio ($Spread / eabl$) can be considered to be a lognormal distribution around the area of interest. Figure 5-5 presents two graphs: one of them corresponds to the ILS data obtained from Lane Financial

(http://www.lanefinancialllc.com/pub/sec1/Trends_Review_2002_8-23-2002.pdf); the other graph corresponds to a fitted lognormal distribution of spread ratio with median value of 6 and a lognormal standard deviation of 0.38. As observed, the plot clearly indicates a fairly good correlation between the obtained data and the lognormal fit. However for very high and low spread ratios, the lognormally distributed data do not fit that well. However for practical values of spreads (<10%), the model gives pretty good results. Thus it can be said that spread ratio is a lognormally distributed variable with a median value of around 6 and a lognormal standard deviation of around 0.4.
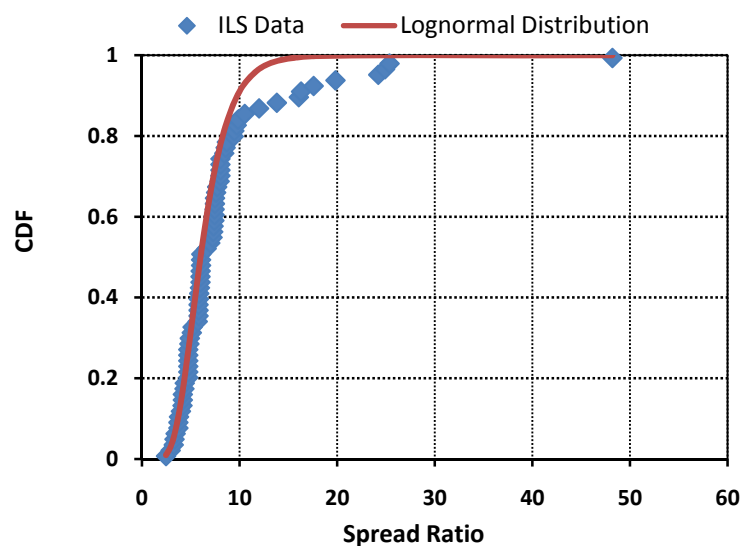
Figure 5-5 ILS data and fitted lognormal distribution for spread ratios. (http://www.lanefinancialllc.com/pub/sec1/Trends_Review_2002_8-23-2002.pdf)

Table 5-1 ILS Data from Lane (2000, 2001, 2002). http://www.lanefinancialllc.com

SPV	Annual EL	Annual Spread	SPV	Annual EL	Annual Spread
Atlas Re II Class A	0.05	2.41	Studio Re Ltd.	0.65	5.17
Atlas Re II Class B	0.9	6.84	Studio Re Ltd. Shares	1.71	8.11
Redwood Capital I	0.53	5.58	Fujiyama	2.06	7.1
Redwood Capital II	0.22	3.04	Mosaiic 2A	0.42	4.08
Residential Re 2001	0.68	5.06	Mosaiic 2B	2.84	8.36
St. Agatha Re	1.14	6.84	Halyard Re	0.63	4.56
Trinom Class A-1	1.11	8.11	Domestic Re	0.5	3.74
Trinom Class A-2 (Pre)	0.67	4.06	Concentric Re	0.42	3.14
Trinom Class A-2 (Post)	0.67	16.22	Juno Re	0.45	4.26
Redwood Capital I	0.72	7.1	Residential Re	0.44	3.71
Trinom (Pre)	3.11	10.14	Kelvin 1^{st}E	4.45	10.97
Trinom (Post)	3.11	18.25	Kelvin 2^{nd}E	0.3	4.82
Fujiyama	0.67	4.06	Golden Eagle B	0.17	2.99
Pioneer A Jun-02	1.28	6.08	Golden Eagle A	0.63	5.48
Pioneer A Dec-02	1.28	5.32	Namazu Re	0.75	4.56
Pioneer A Mar-03	1.28	5.58	Atlas Re A	0.11	2.74
Pioneer B Jun-02	1.27	5.07	Atlas Re B	0.23	3.75
Pioneer B Sep-02	1.27	5.32	Atlas Re C	3.24	14.19
Pioneer B Dec-02	1.27	5.32	Seismic Ltd	0.23	4.56
Pioneer B Mar-03	1.27	4.82	Alpha Wind FRN	0.63	4.62
Pioneer C Jun-02	1.28	6.08	Alpha Wind Prefs	1.46	7.1
Pioneer C Sep-02	1.28	6.08	Residential Re	0.54	4.16
Pioneer C Dec-02	1.28	6.08	NeHi	0.7	4.16
Pioneer C Mar-03	1.28	6.08	MedRe Class A	0.22	2.64
Pioneer D Jun-02	0.22	1.77	MedRe Class B	1.16	5.93
Pioneer D Sep-02	0.22	1.77	PRIME Hurricane	1.27	6.59
Pioneer D Dec-02	0.22	1.77	PRIME EQEW	1.33	7.6
Pioneer D Mar-03	0.22	1.77	Western Capital	0.55	5.17
Pioneer E Jun-02	1.29	4.31	Halyard Re	0.22	5.58
Pioneer E Dec-02	1.29	4.82	Golden Eagle 2001	0.75	5.58
Pioneer E Mar-03	1.29	4.82	SR Wind C1A-1	0.68	5.83
Pioneer F Jun-02	1.31	7.6	SR Wind C1A-2	0.76	5.32
Pioneer F Dec-02	1.31	7.6	NeHi	0.93	4.56
Pioneer F Mar-03	1.31	7.6	Golden Eagle 2001	1.18	7.1
Residential Re 2002	0.67	4.97	SR Wind C1 B-1	1.07	7.1
St. Agatha Re	1.14	6.84	SR Wind C1 B-2	1.13	6.59

5.8. The Need for Investor Conservatism and High Spread Ratios

Spread ratios significantly higher than (5.13) are needed to compensate for the uncertainty associated with the occurrence of a bond-default trigger mechanism. If this is set as a certain PGA for example, there will be a number of possible frequencies (annual probabilities) for which this may occur; f_a which has got an underlying uncertainty around its occurrence. Strictly speaking, this indicates that spread is a random variable. Eq. (5.11) holds good for an investment neutral with respect to uncertainty. Figure 5-6 illustrates the uncertainty associated with occurrence of an event given a trigger event.
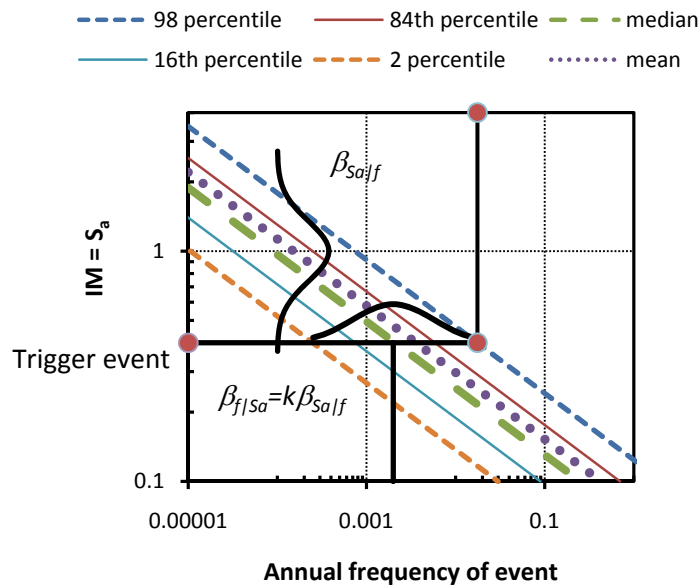

Figure 5-6 Illustration of uncertainty surrounding an annual frequency of occurrence given a trigger event intensity measure

As shown in the figure, it is possible to quantify the total uncertainty for annual occurrence of an event for a given trigger using the loss model mentioned before. Using (5.1) it is possible to relate the intensity measure with annual frequency as the follows

$$\left| \frac{IM}{IM_{DBE}} \right| = \left| \frac{f_a}{f_{DBE}} \right|^{\frac{1}{-k}} \tag{5.15}$$

Therefore the annual frequency of an event for a given IM can be given by

$$f_a = f_{DBE} \left| \frac{IM}{IM_{DBE}} \right|^{-k} \tag{5.16}$$

As evident from the plot, there is an uncertainty associated with prediction of an intensity measure for a given annual frequency. Based on the calibration done on the SAC suite of earthquakes for design basis events, it is possible to measure and quantify the uncertainty associated with the intensity measure. Prediction of IM for a given frequency is akin to predicting the hazard demand. If IM is considered to be a lognrormally distributed variable about the area of interest, then it is possible the quantify the dispersion in its value. Representing the lognormal standard deviation (dispersion) of the IM as $\beta_{S_a|f}$, it is possible to use this dispersion to compute $\beta_{f|S_a}$ -the dispersion of annual frequency of an event given an intensity measure, provided the slope k of the hazard curve is known. The relation between both the dispersions is given by the following

$$\beta_{f|S_a} = k\beta_{S_a|f} \tag{5.17}$$

Figure 5-6 illustrates the application of the above towards defining an associated investor conservatism which translates into higher spread ratios. As mentioned before,

the high values of spread ratios can be attributed to this very uncertainty surrounding a trigger event. However since this uncertainty is measurable, it is possible to compute non-exceedance probabilities for each event. .If $\overline{f}_a$ indicates the mean estimate of the annual frequency of an event, then the relationship between $\overline{f}_a$ and the median estimate of the annual frequency of event is given by

$$\overline{f}_a = \tilde{f}_a \exp\left(\frac{1}{2}\beta_{f|S_a}^{\ 2}\right)$$ (5.18)

Similarly, if $f_{a_{x\%}}$ indicates the annual frequency of an event for a given non-exceedance probability denoted by $x\%$, then the relationship between $f_{a_{x\%}}$ and the median estimate of the annual frequency of event is given by

$$f_{a_{x\%}} = \tilde{f}_a \exp\left(K_x \beta_{f|S_a}\right)$$ (5.19)

where K_x represents the standardized Gaussian random variable with a mean zero and standard deviation one.

Suppose now $\beta_{S_a|f} = 0.4$ as measured for a suite of earthquakes with a constant annual frequency, and k =3 (California). Thus from (5.17) $\beta_{f|S_a} = 1.2$. It follows from (5.18) that $\overline{f}/f = 2.05$ and from (5.19) $f_{98\%}/f = 5.4$. This ratio is related to the spread assigned to give a 98% non-exceedance probability of default. This explains the need for both high spread ratios and perceived investor conservatism.

Figure 5-6 presents plots of median and mean estimates of annual frequency of an event for a design basis event (return period of 475 years) along with plots of 2, 16, 84, and 98 percent non-exceedance probabilities. As evident from the plots, there is

considerable uncertainty surrounding an annual frequency of an event for a given intensity measure, which is the triggering mechanism for a parametric CAT bond. This translates into a higher spread for a given event (using (5.10). However it is possible to design a parametric CAT bond based on investor specified non-exceedance probabilities as will be illustrated in what follows.

5.9. Example Design Procedure of Parametric CAT Bond for Different Structures

In general, a parametric bond is triggered if the measurable physical parameter like a specified IM is exceeded. The inherent safety of an investment could be evaluated using the process outlined in Figure 5-7. As illustrated in Figure 5-7(a), the investor could choose a given non-exceedance probability (for the investment to be safe) in order to compute the particular annual frequency of event. Using the seismic hazard relationship described in (5.15), it is possible to compute the corresponding IM. As mentioned previously, it is possible to measure the dispersion in prediction of annual frequency of an event given an IM. As illustrated in Figure 5-7(b), the lines indicate the 2, 16th percentile, median (50th percentile), mean, 84th, and 98th percentile plots for frequency-IM plots after incorporating the mentioned uncertainty. This is important for the investor as it the possible spread around prediction of IM which could trigger the bond and cause a potential loss to investment.
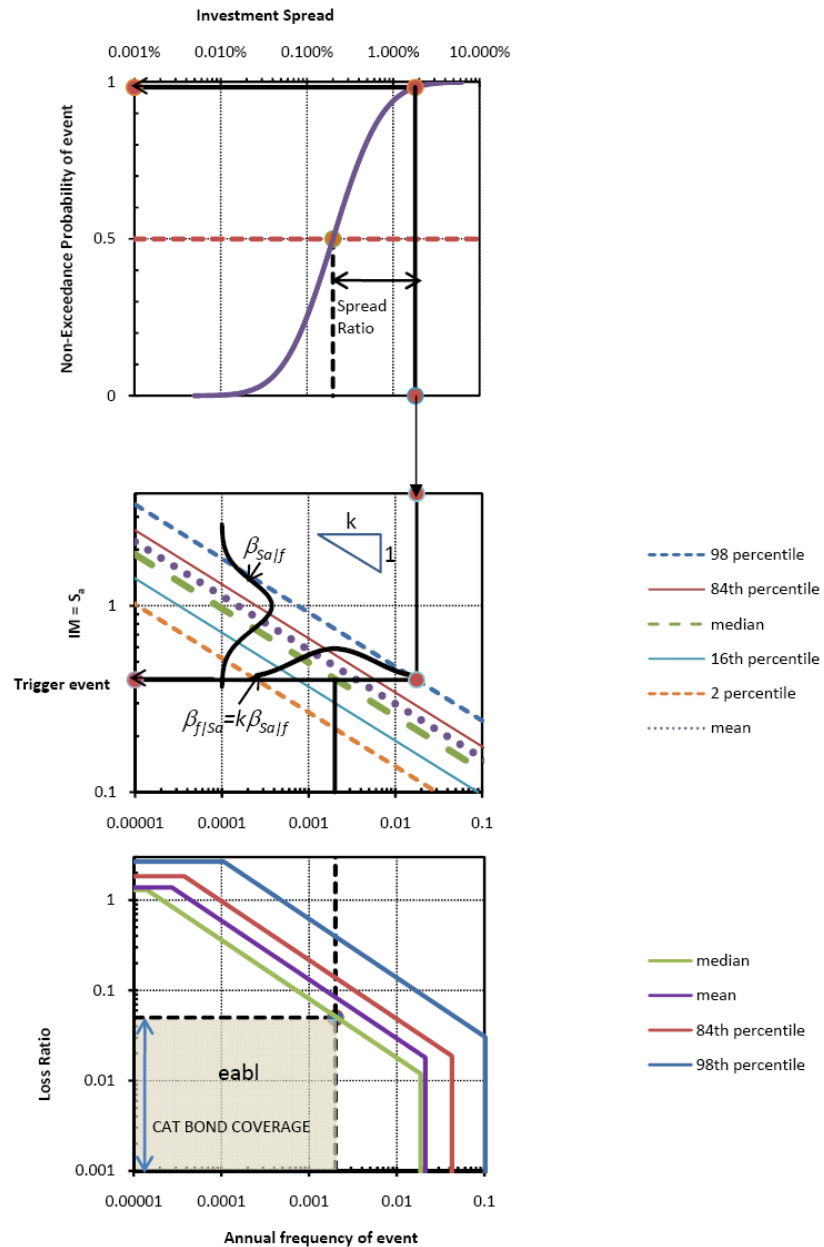
Figure 5-7 Description of process for evaluating a parametric CAT bond for the investor. (a) Investor can choose to know the annual frequency for a given event with a non-exceedance probability; (b) the annual frequency can be used to find out the corresponding IM (characterized in terms of spectral acceleration Sa) and evaluate the uncertainty for reaching the triggering IM (characterized by $\beta_{Sa|f}$); and (c) the annual frequency for the chosen event can be used to compute the total annual losses and the estimated annual bond losses

The owner, on the other hand, is only concerned with losses on his facility for an earthquake with a given return period. Thus the seismic loss-frequency curve shown in Figure 5-7(c) is an important tool for the owner as it helps assess the probability of losses for a given event. However, all the three plots shown have an implication for the broker (or the SPV in charge of covering the owner's losses and issuing the CAT bond). The SPV has to be immunized against possible basis risk arising due to difference in perceived losses suffered by the owner and it. The inherent uncertainty in predicting an IM given the annual frequency of event is thus a major source of basis risk and it can be removed by quantifying and computing it appropriately.

Conversely it is possible to evaluate expected confidence (non-exceedance probabilities) desired by an investor for a given event and spread ratio. As mentioned before, spread ratio is a random variable and it is possible to attach non-exceedance probabilities to it. Using (5.11), spread ratio for a given non-exceedance probability (as indicated in Figure 5-7(a)) can be given by

$$R_{s_{x\%}} = (1 + i_p + f_a) \exp(-\frac{1}{2}\beta_{f|S_a}^2 + K_x \beta_{f|S_a}) \tag{5.20}$$

Similarly it is possible to compute the confidence level (indicated by K_x) if the spread ratio, risk-free prime-rate, and $\beta_{Tf|L}$ are known. This can be expressed as the following

$$K_x = \frac{\ln\left(\frac{R_s}{(1 + i_p + f_a)}\right) + \frac{1}{2}\beta_{f|S_a}^2}{\beta_{f|S_a}} \tag{5.21}$$

The above approach is very useful for both the owner and investor perspective. It not only quantifies the uncertainty surrounding the occurrence of triggering event, but also helps assess possible losses and its uncertainties if the event was to occur. It is hypothesized that the above approach can be used fruitfully for parametric CAT bond design for both risk-averse and risk taking investors.

5.10. Discussion

As seen from the above studies, it is possible to make a transition from hazard loss estimation to design of suitable risk-investment measures. The main motive of issuing CAT bonds is to ensure inflow of funds for reconstruction purposes in the event of a loss causing hazard. These bonds are also considered good investments since their behavior is essentially uncorrelated with the market volatilities. However, the underlying risk to the investor lies in the form of a potential devastating event wherein he loses his investment. The investor therefore seeks a high degree of confidence on his investment, which can be attributed to the wide dispersion in predicting annual frequencies (or probabilities) of loss making events. As seen from Sections 3 and 4, there are a number of uncertainties involved in estimating losses; an aggregate of these uncertainties produces a wide range of possible scenarios causing the investor to be much more risk-averse.

It is also seen that defining a parametric bond through the loss model developed in Sections 3 and 4 is much more simpler because of the direct relationship between the

spread and annual frequency of a loss making event, quantified through spread ratio. Figure 5-4 indicates that computing the total bond loss entails estimating only the annual probability of loss making event, as the losses are normalized to unity.

5.11. Closure

The studies conducted as part of this work can be summarized into the following points:

1. It is possible to use a PBEE based direct loss estimation approach to make a transition from seismic loss estimation to designing risk mitigating measures such as CAT bonds. In the direct loss model, this would entail computing commonly understood capacity and design parameters and therefore predicting losses for a particular bond.

2. It is possible to partition the seismic loss hazard curve into various components of possible reinsurance and loss-mitigation. Specifically the seismic loss-hazard curve can be used to compute estimated annual bond losses (or losses on the investment) for both parametric and indemnity type bonds.

3. The high degree of confidence sought by an investor on a CAT bond investment can be explained using a combination of cost-benefit hypothesis as well as the uncertainty surrounding the loss-estimation approach. Therefore, it is possible to predict an expected level of user-confidence and design a CAT bond thereafter. It is also possible to define spread ratio or the ratio of spread to annual bond losses as a lognormally distributed variable for a parametric variable based on existing date.

4. Possible loss coverage of an asset for a specific bond can be estimated and therefore the suitability of issuing CAT bonds for insuring the facility can be predicted. In essence, the direct loss model facilitates evaluation of potential vulnerability of a facility and its associated suitability for implementation as a CAT bond insured facility.

6. CONCLUSIONS AND RECOMMENDATIONS FOR FUTURE WORK

The work done as part of this thesis can be summarized into the following points

1) A direct loss estimation procedure that relates hazard to response and hence to losses without the need for classic demand-side fragility curves was proposed and validated. The closed-form solution is formulated in terms of well understood seismic hazard and structural design and capacity parameters.

2) A parameterized empirical loss model in the form of a tripartite power curve was proposed. Only four parameters are needed to completely define the resulting capped power-curve loss model. The parameterized loss model was calibrated and validated for different types of bridges, concrete and steel frame buildings, as well as drift-sensitive non-structural components.

3) When accounting for all variabilities in terms of randomness and uncertainty, the resulting hazard-loss model can be integrated across all possible earthquakes to derive the expected annual loss, EAL. To obtain a sense for the upper bound on loss, it is straight forward to formulate losses for other fractiles, such as the 90th percentile non-exceedance probability.

4) Estimated annual losses were computed for bridges and buildings with and without the effect of non-structural components in order to examine the

vulnerability of structures and express them in terms of annualized dollar loss per million dollars of asset.

5) The loss model was applied towards pricing emerging risk-mitigating measure called catastrophic (CAT) bonds. An emphasis was laid on understanding the pricing of the risk component of these bonds and quantifying these through terms of confidence measures.

6) The annual probability of a loss causing event was quantified using a cost-benefit analysis through the spread ratio. A framework for computing annualized bond losses for the parametric bond was derived based on the spread ratio and the possible coverage of CAT bond.

7) Possible design schemes of parametric CAT bonds were explored based on existing spread ratio and an user-defined or required spread. The design schemes were illustrated through hypothetical examples on bridges and buildings.

The following areas might be considered suitable for future research

1) Calibrating the direct loss model for a variety of structures such as hydraulic systems and other commonly vulnerable systems.

2) Exploring the possibilities of applying the direct loss model in pricing other similar risk mitigating measures.

3) Exploring the implications of EAL towards pricing an indemnity CAT bond.

4) Carrying out a portfolio analysis using the approach outlined in the thesis in order to estimate the suitability of CAT bonds as a popular risk-mitigating measure.

5) Estimating a suitable price for the embedded call option in a CAT bond.

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VITA

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