MEASUREMENT ENHANCEMENT FOR STATE ESTIMATION

A Dissertation

by

JIAN CHEN

Submitted to the Office of Graduate Studies of
Texas A&M University
in partial fulfillment of the requirements for the degree of

DOCTOR OF PHILOSOPHY

May 2008

Major Subject: Electrical Engineering

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ABSTRACT

Measurement Enhancement for State Estimation. (May 2008)

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Chair of Advisory Committee: Dr. Ali Abur

After the deregulation of the power industry, power systems are required to be operated efficiently and economically in today's strongly competitive environment. In order to achieve these objectives, it is crucial for power system control centers to accurately monitor the system operating state. State estimation is an essential tool in an energy management system (EMS). It is responsible for providing an accurate and correct estimate for the system operating state based on the available measurements in the power system. A robust state estimation should have the capability of keeping the system observable during different contingencies, as well as detecting and identifying the gross errors in measurement set and network topology. However, this capability relies directly on the system network configuration and measurement locations. In other words, a reliable and redundant measurement system is the primary condition for a robust state estimation.

This dissertation is focused on the possible benefits to state estimation of using synchronized phasor measurements to improve the measurement system. The benefits are investigated with respect to the measurement redundancy, bad data and topology error processing functions in state estimation. This dissertation studies how to utilize the phasor measurements in the traditional state estimation. The optimal placement of measurement to realize the maximum benefit is also considered and practical algorithms are designed. It is shown that strategic placement of a few phasor measurement units (PMU) in the system can significantly increase measurement redundancy, which in turn can improve the capability of state estimation to detect and identify bad data, even during loss of measurements. Meanwhile, strategic placement of traditional and phasor measurements can also improve the state estimation's topology error detection and identification capability, as well as its robustness against branch outages. The proposed procedures and algorithms are illustrated and demonstrated with different sizes of test systems. And numerical simulations verify the gained benefits of state estimation in bad data processing and topology error processing.

To My Wife and Parents

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CHAPTER I

INTRODUCTION

1.1 Motivation

After the deregulation of the power industry, power systems are required to be operated efficiently and economically in a strongly competitive environment. In order to achieve these objectives it is crucial to accurately monitor the state of the power system as the operating conditions change during the daily operation. State Estimation, which determines the optimal estimate for the system state based on the available system measurements, has become an essential tool in modern control centers. The measurements are commonly provided by the remote terminal units (RTU) at the substations and include real/reactive power flows, power injections, and magnitudes of bus voltages and branch currents. Today, state estimators are widely used in almost every power system control center.

Performance of the state estimator relies heavily on its measurement system. When a new state estimator is put into service or an existing state estimator is upgraded, the measurement system needs to be well designed to ensure that the power system not only is observable, but also remains observable during all major contingencies. The problem of determining the best locations of measurements for state estimation is referred as the optimal measurement placement problem. This problem has been widely

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This dissertation follows the style of *IEEE Transactions on Power Systems.*

studied in the past and the results were documented in [1-17]. While the majority of these studies are concerned about the observability problem, some of them also consider the state estimation robustness against loss of measurements and outage of branches, which may happen during some contingencies. On the other hand, a reliable and redundant measurement system is essential in order to enable proper bad data and/or information processing.

In the recent years, synchronized phasor measurements have been introduced into power systems at selected substations in the system. Phasor measurement units (PMU) are devices that provide positive sequence phasor voltages and currents based on the measured voltage and current signals at substations. These signals are time synchronized by the help of global positioning system (GPS) satellites. As the numbers of PMUs increase in power systems, phasor measurements will play a dominant role in improving the performance of state estimators.

The idea of using synchronized phasor measurements for state estimation in not a new concept. In the pioneering work in PMU development and utilization done by Phadke et al. [18,19], it is argued that the state estimation problem can be solved by exclusive use of phasor measurements, if PMUs are installed at each bus. Later on, this requirement is relaxed in [20,21] based on the fact that each PMU can measure not only the bus voltage but also the currents along all the lines incident to the bus. This will also lead to a linear real-time state estimator, as opposed to the non-linear traditional state estimator which uses conventional measurements.

While the idea of using only phasor measurements appears very attractive due to its advantages in state estimation solution, it may not yet be practical since it requires a large number of PMUs to be installed in strategic system buses in order to accomplish this goal. Hence, a good comprise would be to incrementally improve the current traditional state estimators by introducing a limited number of phasor measurements. It has been shown that when phasor measurements are added to traditional measurement sets, accuracy of the state estimation can be improved [18,19,22]. Furthermore, it is recognized that PMUs can also be used to improve network observability [23].

This dissertation studies potential benefits of adding phasor measurements to existing measurement sets. The benefits are investigated with respect to the measurement redundancy, bad data and topology error processing functions. Optimal placement of phasor measurements in order to maximize these benefits is considered and practical engineering solutions are developed.

1.2 Objective

This dissertation is mainly focused on the possible benefits to state estimation of introducing phasor measurements, with respect to measurement redundancy, bad data processing and topology error processing. As state estimation constitutes the core of the on-line system security analysis, it acts like a filter between the raw data/information received from the system and all application functions that rely on the current state of the system. Therefore, the state estimator is required to have the capability to detect and identify gross errors in the measurement set and network topology. These objectives are accomplished by implementing proper bad data and topology error processing functions. However, bad data and topology error processing capability is closely related to the measurement redundancy problem. Even for an observable measurement system, bad data appearing in some measurements or topology errors associated with some branches may not be detected due to the deficiencies of the measurement system. In this dissertation, as a supplement of traditional measurements, the voltage and current phasor measurements from PMUs are incorporated into the commonly-used WLS state estimation algorithm. While the bad data and topology error processing capability is limited by the measurement system consisting of traditional measurements, adding a few extra PMUs can drastically improve the bad data and topology error processing capability,. Strategic PMU placement algorithms are also developed for this purpose. The developed PMU placement procedures can identify existing deficiencies in the measurement system and determine an optimal placement of PMUs to improve these deficiencies. The algorithm is designed in such a way that it can also be extended to incorporate traditional measurements, as well as to improve redundancy based on desired levels of reliability.

1.3 Contribution of the Dissertation

This dissertation shows that strategic placement of few PMUs in the system can significantly increase measurement redundancy, which in turn can improve the capability of the state estimator to detect and identify bad data, even during loss of measurements. Meanwhile, strategic placement of traditional and phasor measurements can also improve the state estimation's topology error detection and identification capability, as well as its robustness against branch outages. This dissertation explores how to utilize these phasor measurements to improve bad data processing and topology error processing capability in state estimation. The main contributions of the dissertation are listed below:

- Illustration of how phasor measurements can be used to improve measurement redundancy and bad data detection and identification capability.
- Development of a new algorithm that is designed for optimal placement of both traditional and phasor measurements, to improve the measurement redundancy of a given system to a desirable level. This allows design of measurement systems with different degrees of vulnerability against loss of measurements and bad data.
- Illustration of how phasor measurements are used to improve topology error detection and identification capability. Phasor measurements are shown to be capable of improving topology error processing capability for cases where this can not be done by the traditional measurements.
- Development of a new algorithm that is designed to obtain the optimal placement of measurements to improve topology error detection and identification. This placement also improves the robustness of state estimation against branch outages.

1.4 Outline of the Dissertation

The dissertation includes five chapters. Chapter I introduces the motivation, objectives, and contributions of the completed work. Chapter II describes the traditional state estimation problem—its definition, formulation, and its function in bad data processing and topology error processing. Furthermore, the new measurements with PMUs are introduced. Incorporation of phasor measurements in state estimation formulation is reviewed and discussed. A new formulation of state estimation with both traditional measurements and phasor measurements is described. Chapter III analyzes benefits of phasor measurements for bad data processing. It is shown that with a few PMUs, bad data detection and identification capability of a given system can be drastically improved. The critical measurements or critical pairs of measurements in the original system, in which the bad data is undetectable or unidentifiable, can be transformed into redundant measurements. An optimal placement algorithm that accomplishes this in an efficient manner is also developed and described in this chapter. Chapter IV analyzes benefits of phasor measurements for topology error processing. It is shown that phasor measurements can improve the system's topology error processing capability up to a desired level, so that any single branch topology error can be detected by state estimation using measurement residual analysis. The measurement system can also be further reinforced in order to not only detect but also identify topology errors. Description of the developed placement algorithm is given, and case studies carried out on different size test system are presented in this chapter. Following a summary of the contributions of the completed work, Chapter V discusses potential avenues for future research.

CHAPTER II

STATE ESTIMATION

In this chapter, the traditional state estimation problem is introduced, such as its definition, formulation and important functions. Before the main study of this dissertation is given, it is appropriate to provide a review for these primary problems and state of act in the area of state estimation. The review covers the models and assumptions in state estimation, the commonly used Weighted Least Squares (WLS) method to solve the state estimation problem, Chi-squares test and largest normalized residual test for bad data processing, as well as a geometric interpretation of the measurement residuals for topology error processing. The chapter will also review phasor measurements and their previous utilization in state estimation. A specific algorithm is provided to utilize the phasor measurements in traditional Weighted Least Square (WLS) method.

2.1 State Estimation Problem

Power system state estimation constitutes the core of the on-line power system monitoring, analysis and control functions. In modern power system, the control center receives the system-wide device information and measurement data through the Supervisory Control and Data Acquisition (SCADA) system. However, the information and measurement data provided by SCADA may not always be accurate and reliable due to errors in the measurements, telemetry failures, communication noise, etc. On the other hand, the collected measurements may not allow direct extraction of the corresponding a real-time AC operation state of the system. These concerns bring the development of state estimation [24,25].

State estimation acts like a filter between the raw measurements received from the system and all the application functions that require the most reliable data base for the current system operation state. State estimation use the measurement data from SCADA system, the status information about the circuit breakers (CB), switches and transformer taps, as well as the parameters of transmission lines, transformers, shunts capacitors/reactors and other devices, to estimate the state of the power system. Nowadays, state estimation has become one of the essential energy management system (EMS) functions. It is responsible for maintaining a reliable and accurate real-time data base, which will in turn be used by all other EMS functions.

State estimation typically includes the follow functions [27-29]:

- Topology processor: Gathers the status information about the CBs and switches in the system, and configures the bus-branch model of the system.
- Observablility analysis: Determines the available measurements in the system, and checks if these measurements are enough to obtain the state estimation solution for the entire power system. If not, identifies the unobservable branches and the observable islands in the power system.
- State estimation solution: Finds out the optimal estimated solution for the state of entire power system, using the gathered measurement data and devices information. The state of power system is usually obtained by solving a nonlinear

optimization problem, and given out in the form of complex bus voltages (magnitudes and angles) for all buses. Therefore, other variables, such as line flows, loads, and generator outputs can be calculated based on the estimated solution.

- Bad data processing: Detects existence of gross errors in the measurement data. If there is any bad measurement data, it should be identified and eliminated. However, it requires enough redundancy in the measurement system.
- Parameter and topology error processing: Detects parameter error in the network parameters, such as transmission line parameters, transformer tap parameters, as well as shunt capacitor/reactor parameters. Estimates the correct values if there is any erroneous parameter. Detects topology error in the network configuration. Identifies the topology error if there is enough measurement redundancy.

2.2 State Estimation Formulation

2.2.1 Models and Assumptions

State estimation problem generally only uses the single phase positive sequence circuit for modeling the power system. Power system is assumed to operate in the steady state under balanced conditions, which implies all bus loads and branch power flows will be three phase and balanced, all transmission lines are fully transposed, and all other devices are symmetrical in the three phases.

State estimation collects the measurement data from a various types of measurements installed in the power system. However, the most commonly used measurements include the following types:

- Line power flow measurements: Provide the real and reactive power flow along the transmission lines or transformers.
- Bus power injection measurements: Provide the real and reactive power injected at the buses.
- Voltage magnitude measurements: Provide the voltage magnitudes of the buses.

Furthermore, in some cases, especially for state estimation of distribution systems, the line current magnitude measurements may be taken into consideration, which provide the current flow magnitudes (Amps) along the transmission lines or transformers. The line current magnitude measurements are not discussed in this dissertation.

With the introduction of PMUs into state estimation, there will be two new types of measurements:

- Voltage phasor measurements: These are the phase angles and magnitudes of voltage phasors at system buses.
- Current phasor measurements: These are the phase angles and magnitudes of current phasors along transmission lines or transformers.

The utilization of these two types of phasor measurements is discussed in the later part of this chapter.

All types of measurements can be expressed in terms of the system state as below:

$$z=\begin{bmatrix} z_1 \\ z_2 \\ \vdots \\ z_m \end{bmatrix}=\begin{bmatrix} h_1(x_1,x_2,\cdots x_n) \\ h_2(x_1,x_2,\cdots x_n) \\ \vdots \\ h_m(x_1,x_2,\cdots x_n) \end{bmatrix}+\begin{bmatrix} e_1 \\ e_2 \\ \vdots \\ e_m \end{bmatrix}=h(x)+e \qquad (2.1)$$

where,

z is the vector of measurement, and z_i is the measured value of measurement *i*;

$h^T=\left[h_1(x), \quad h_2(x), \quad \cdots, \quad h_m(x)\right]$ and $h_i(x)$ is the nonlinear function relating measurement i to the state vector x;

$x^T=\left[x_1 \quad x_2 \quad \cdots \quad x_n\right]$ is the system state vector, including the voltage magnitudes and phase angles of all the buses excluding the reference bus phase angle;

$e^T=\left[e_1 \quad e_2 \quad \cdots \quad e_m\right]$ is the vector representing measurement errors, and e_i is measurement error of measurement *i*.

Regarding the general statistical properties of the measurement errors, the following assumptions are made:

- The measurement error e_i is assumed to have a normal distribution with zero mean and known standard deviation σ_i, i.e. $E(e_i)=0$;
- The measurement errors are assumed to be independent, i.e. $E[e_i e_j]=0$.

Hence, the covariance matrix of the measurement errors *R* is diagonal

$$R=Cov(e)=E[e\cdot e^T]=diag\left\{\sigma_1^2, \quad \sigma_2^2, \quad \cdots, \quad \sigma_m^2\right\}$$

The standard deviation σ_i of measurement *i* is set to reflect the expected accuracy of the corresponding meter used.

2.2.2 WLS State Estimation Algorithm

Weight Least Square (WLS) method is commonly used to solve the state estimation problem, which is formulated as the following optimization problem:

$$\begin{aligned} &Minimize \quad \sum_{i=1}^{m} W_{ii} r_i^2 \\ &subject\ to \quad z_i = h_i(x) + r_i \quad i = 1, \ldots, m \end{aligned} \tag{2.2}$$

where,

m is the number of measurements;

n is the number of system states;

$z^T = [z_1, \quad z_2, \quad \cdots, \quad z_m]$ is the vector of measurement;

$h^T = [h_1(x), \quad h_2(x), \quad \cdots, \quad h_m(x)]$ is the nonlinear function vector;

$x^T = [x_1 \quad x_2 \quad \cdots \quad x_n]$ is the system state vector.

W is the weight matrix, which is defined as the inverse of the covariance matrix of the measurement errors R :

$$W = R^{-1} = diag\left\{\frac{1}{\sigma_1^2}, \quad \frac{1}{\sigma_2^2}, \quad \cdots, \quad \frac{1}{\sigma_m^2}\right\}$$

The optimization problem in Equation (2.2) can be solved when the first-order optimality conditions are satisfied:

$$g(x) = \frac{\partial J(x)}{\partial x} = -H^T(x) R^{-1} [z - h(x)] = 0 \tag{2.3}$$

where $H(x)$ is called Jacobian matrix, and $H(x) = \dfrac{\partial h(x)}{\partial x}$ (2.4)

Equation (2.3) is a nonlinear equation, which can be further solved using an iterative solution scheme known as the Gauss-Newton method as shown below:

$$x^{k+1} = x^k - \left[G(x^k)\right]^{-1} \cdot g(x^k) \tag{2.5}$$

where,

k is the iteration index;

x^k is the solution vector at the *k*th iteration;

$$g(x^k) = -H^T(x^k) \cdot R^{-1} \cdot (z - h(x)^k) \tag{2.6}$$

$$G(x^k) = \frac{\partial g(x^k)}{\partial x} = H^T(x^k) \cdot R^{-1} \cdot H(x^k) \tag{2.7}$$

G(x) is called the gain matrix. It is sparse, positive definite and symmetric if the system is fully observable. At the kth iteration, it is decomposed into its triangular factors, and the following linear equation is solved using forward/back substitutions:

$$\left[G(x^k)\right]\Delta x^{k+1} = H^T(x^k)R^{-1}\left[z - h(x^k)\right] \tag{2.8}$$

where $\Delta x^{k+1} = x^{k+1} - x^k$

2.2.3 Bad Data Processing

One of the essential functions of state estimation is bad data processing function. State estimation is required to detect, identify and correct or eliminate the gross errors in the measurement data, in order to obtain an unbiased result. Hence, state estimation has to be equipped with some advanced features for bad data detection and identification [30,31,32].

Treatment of bad data depends on the method of state estimation used in the implementation. With the commonly used WLS method, detection and identification of bad data are done after the estimation solution by analyzing the measurement residuals.

In this dissertation, Chi-squares (χ^2) test will be used to process the measurement residuals to detect bad data in the measurement set. Once bad data are detected, the Largest Normalized Residual ($r^N_{\max}$) test will be used to identify bad data. These two tests will be described next.

2.2.4 Chi-squares Test

It can be shown that sum of squares of independent random variables will have a Chi-squares distribution, if each variable is distributed according to the Standard Normal distribution. Therefore, based on the given formulation of WLS estimation method, the objective function *J(x)* is expected to have a distribution which can be approximated as a Chi-squares distribution with at most *(m-n)* degrees of freedom, where *m* is the total number of measurements and *n* is the number of state variables.

Using the statistical properties of the objective function, the following steps can be defined as the Chi-squares χ^2 -test for bad data detection:

- Solve the WLS estimation problem and compute the objective function as defined by Equation (2.2):

  $$J(\hat{x}) = \sum_{i=1}^{m} \frac{(z_i - h_i(\hat{x}))^2}{\sigma_i^2}$$

  where $\hat{x}$ is the estimated state vector of dimension *n*.

- Check the detection confidence value $\chi^2_{(m-n),p}$ for the Chi-squares distribution with probability p (e.g. 95%) and (m-n) degrees of freedom. The probability p is defined as $p = \Pr(J(\hat{x}) \le \chi^2_{(m-n),p})$.
- Test if $J(\hat{x}) \ge \chi^2_{(m-n),p}$. If yes, then bad data will be suspected, else no bad data will be assumed to exist.

2.2.5 Largest Normalized Residual Test

Consider the linearized measurement equation, which is used at each iteration during the numerical solution of the WLS estimation problem:

$$\Delta z = H\Delta x + e \tag{2.9}$$

Applying the optimization criterion, the following expression can be derived for the optimal state update:

$$\Delta\hat{x} = (H^T R^{-1} H)^{-1} H^T R^{-1} \Delta z = G^{-1} H^T R^{-1} \Delta z \tag{2.10}$$

The calculated measurement updates based on the estimated state updates will be given by:

$$\Delta\hat{z} = H\Delta\hat{x} = HG^{-1}H^T R^{-1}\Delta z = K\Delta z \tag{2.11}$$

where $K = HG^{-1}H^T R^{-1}$ and is called the hat matrix. Furthermore, it can be proved that the matrix K has the following property: $K \cdot H = H$

Thus, the expression of measurement residuals can be derived as the follows:

$$
\begin{aligned}
r &= \Delta z - \Delta \hat{z} \\
&= (I-K)\Delta z \\
&= (I-K)(H\Delta x + e) \\
&= (I-K)e \\
&= Se
\end{aligned}
\tag{2.12}
$$

where $S = I - K$ and is called the sensitive matrix, which has the following property: $S \cdot R \cdot S^T = S \cdot R$. It represents the sensitivity of measurement residuals to the measurement errors.

Based on the assumption that the measurement errors have normal distributions, the statistical properties of measurement residual are derived as:

$$
\begin{aligned}
E(r) = E(S \cdot e) = S \cdot E(e) = 0 \\
Cov(r) = \Omega = E\left[rr^T\right] = S \cdot E\left[ee^T\right] \cdot S^T = SRS = SR
\end{aligned}
\tag{2.13}
$$

where Ω is the covariance matrix of measurement residuals.

Hence, the normalized value of the residual for ith measurement can be calculated as:

$$
r_i^N = \frac{|r_i|}{\sqrt{\Omega_{ii}}} = \frac{|r_i|}{\sqrt{R_{ii} S_{ii}}}
\tag{2.14}
$$

and the normalized residual vector r^N have a Standard Normalized Distribution, i.e.

$r_i^N \sim N(0,1)$

It can be derived that, with enough measurement redundancy, the largest normalized residual should correspond to the measurement with bad data. The Largest Normalized Residual ($r_{\max}^N$) Test uses this property to identify and subsequently eliminate bad data, which involves the following steps:

- Solve the WLS estimation problem and calculate the measurement residuals:

  $$r_i = z_i - h_i(\hat{x}) \quad i = 1, \cdots m$$

- Calculate the normalized residuals of the measurements:

  $$r_i^N = \frac{|r_i|}{\sqrt{\Omega_{ii}}} = \frac{|r_i|}{\sqrt{R_{ii} S_{ii}}} \quad i = 1, \cdots m$$

- Find the largest value r_k^N in the normalized residual corresponding to *k*th measurement;
- If $r_k^N > c$, the *k*th measurement is identified as bad data. Otherwise, no bad data will be suspected. Here, c is the chosen identification threshold (e.g. 3.0).
- Eliminate the *k*th measurement, and repeat the state estimation.

2.2.6 Topology Error in State Estimation

As introduced at the beginning of this chapter, state estimation problem is formulated based on a branch-to-bus electrical network model provided by the topology processor. The topology processor analyzes the status of all circuit breakers (CB) and switching devices to configure the bus-branch model of the power system. However, in some rare cases, the obtained status of certain CBs may be incorrect. When this happens, the topology processor generates wrong bus-branch model, which leads to a topology error.

Topology errors can be generally classified in two types:

- Branch status errors: This type of errors involves the status of network branches, which represent the transmission lines or transformers. For example, an inclusion error takes places when a disconnected element is assumed to be in service. And an exclusion error happens when an energized element is assumed to be out of service.

- Substation configuration errors: This type of errors affects the CBs which link different bus sections within the substation. A split error happens when an electric bus is erroneously modeled as two buses, while a merging error occurs when two actually separated buses is modeled as one bus. This type of errors generally can be detected as a multiple branch status error, but its identification need more detailed bus-section-switch model.

Topology errors will lead the state estimation to a significantly biased result or serious convergence problem. It is necessary for state estimation to develop effective mechanisms to detect and identify topology errors. With the commonly used WLS method, the topology error detection and identification can be realized by analyzing the measurement residuals after the estimation [33,34], which is introduced in the following section.

2.2.7 Residual Analysis for Topology Error Detection and Identification

The topology errors involve wrong network configuration in the generated bus-branch model, which leads to the incorrect nonlinear function *h(x)*. The effect of the topology errors then shows up in the Jacobian matrix *H*. This effect can be modeled in the following manner [34]:

$$H_t = H_e + E \tag{2.15}$$

where,

H_t is the true Jacobian matrix,

H_e is the incorrect Jacobian due to topology errors,

E is the Jacobian matrix error.

The true equation for the state estimation should be:

$\Delta z = H_t \Delta x + e$

But the following equation will be used erroneously instead:

$\Delta z = H_e \Delta x + e$

Measurement residuals will then have the following statistical properties due to the topology error:

$$\begin{aligned} r &= \Delta z - H_e \hat{x} = (I - K_e)(Ex + e) \\ E(r) &= (I - K_e)Ex \\ \text{cov}(r) &= (I - K_e)R \end{aligned} \tag{2.16}$$

where $K_e = H_e (H_e^T R^{-1} H_e)^{-1} H_e^T R^{-1}$, which is the hat matrix with topology errors.

Let Δf be the vector of branch flow errors, which represents the errors in the branch flows due to transmission line topology errors or other topology errors. Let M be the measurement-to-branch incidence matrix. The measurement bias Ex in Equation (2.15) can be expressed as:

$$Ex = M \Delta f \tag{2.17}$$

and the measurement residuals can be given by:

$$r = (I - K_e) M \Delta f \tag{2.18}$$

Therefore, given enough measurement redundancy, the existence of topology errors will affect measurement residuals. This implies that topology errors can be detected by checking the objective function *J(x)* and applying the Chi-squares (χ^2) test, or by checking the normalized residuals of measurements, assuming that analog bad data in measurements have already been identified and eliminated.

Let us consider the linear relationship between the measurement residuals and branch flow errors:

$$r = T\Delta f \tag{2.19}$$

where $T = (I - K_e)M$. When a single topology error exists in the ith branch, there will be a change in the corresponding branch flow $\Delta f_i = \alpha$ and $\Delta f_k = 0$ for $k \neq i$, where α is the scalar corresponding to the type of topology error. Thus, the measurement residual vector *r* will be collinear with the vector T_i, representing the ith column of matrix *T*.

A geometric interpretation of the measurement residuals can be used to identify single branch topology errors [33] applying the following procedure:

- Solve the WLS estimation problem and calculate the measurement residuals vector:

  $$r = z - h(\hat{x})$$

- Calculate the sensitive matrix of *T* for measurement residual r respect to branch flow errors Δf :

  $$T = (I - K_e)M$$

- Test the collinearity between the measurement residuals vector and the columns of the sensitive matrix of T, using their dot product:

$$\cos\theta_i = \frac{T_i^T r}{\|T_i\|\|r\|} \quad i = 1, \cdots n$$

where n is the number of branches in the system.

- If $\cos\theta_i \cong 1.0$, and other $\cos\theta_k < 1$ for $k \neq i$, a single branch topology error is suspected in the ith branch.

Note that, both detection and identification of topology errors based on the analysis of measurement residuals will require high enough measurement redundancy in the system. Moreover, in some cases, the capability of detection and identification is limited by the network configuration.

2.3 Synchronized Phasor Measurements

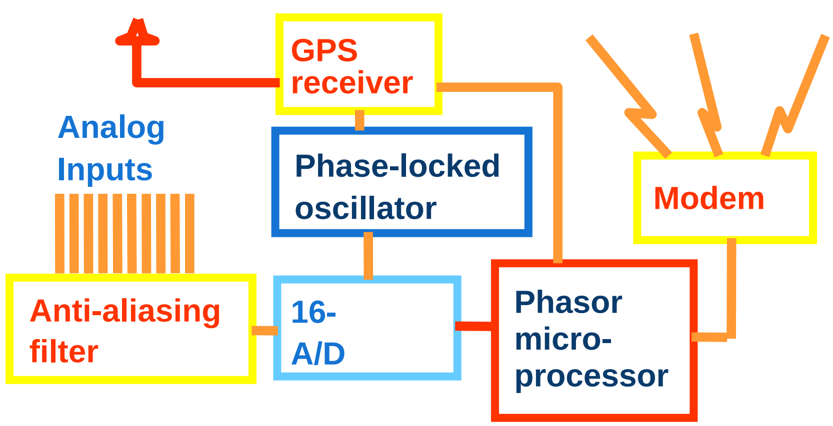

Figure 1 Typical Blocks of PMU

Phasor measurement units (PMUs) use the synchronization signals received from the GPS satellite system. By measuring the magnitude and phase angles of currents and voltages, multiple PMUs will provide coordinated system-wide measurements [35,36].

Figure 1 shows a typical synchronized phasor measurement unit configuration. The analog input signals are obtained from the secondary sides of the voltage and current transformers. The analog input signals are filtered by anti-aliasing filter to avoid aliasing errors. Then the signals will be sampled by the A/D converter. The sampling clock is phase-locked to the GPS time signal. The GPS receivers can provide uniform time stamps for PMUs at different locations. The phasor microprocessor calculates the values of phasor. The calculated phasors and other information are transmitted to appropriate remote locations over the modems or other communication tools.

In recent years, PMUs are becoming more common in the power systems due to their versatile utilization. PMUs have made significant improvements in the control and protection functions [37-39]. The wide-spread placement of PMUs also provides an opportunity to improve state estimation. Their benefits to the state estimation function have been studied and results of the work were reported in [18-20,40,41].

2.4 State Estimation with Phasor Measurements

PMUs can directly provide two types of measurements, namely bus voltage phasors and branch current phasors. A PMU placed at a given bus can provide voltage phasor at the bus and current phasors on several or all lines incident to that bus, as shown in Figure 2. Depending on the type of PMUs used, the number of channels used for measuring voltage and current phasors will vary.

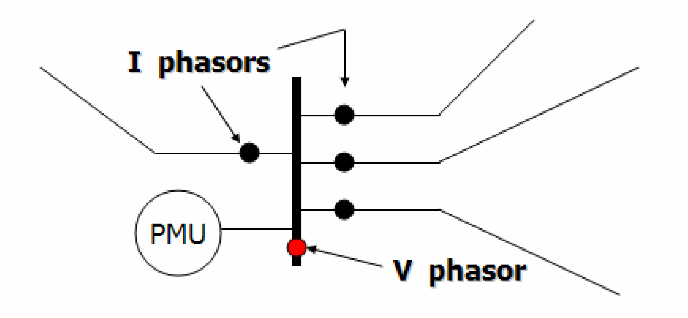

Figure 2 Phasor Measurement Provided by PMU

So far, there have been two optional methods which proposed to utilize the phasor measurements in the state estimation. These will be reviewed next.

2.4.1 Linear State Estimation with Only Phasor Measurements

The idea of using phasor measurements in state estimation is first presented in the pioneering work of Phadke et al. Initially it was proposed that every bus ought to be monitored by a PMU which would result in a simplified linear state estimation formulation. This requirement is further relaxed due to the fact that each PMU can measure not only the bus voltage phasor but also the current phasors along all lines incident to the bus.

However, in order to guarantee the observability of entire power system, it still needs enough PMUs are implemented at proper buses. Hence, although this type of state estimation has significant advantages comparing to traditional state estimation, its implementation in the power systems requires much more investment.

2.4.2 Hybrid State Estimation with Both Traditional and Phasor Measurements

Given the impracticality of placing many PMUs to support the linear state estimation with only phasor measurements, an intermediate solution is to use phasor measurements as additional inputs to the traditional state estimation. Some work has been done to incorporate the synchronized phasor measurements into the state estimation along with traditional measurement [42].

In this dissertation, a specific model is used to implement both the voltage and line current phasor measurements into traditional WLS state estimation. In this model, the voltage phasor measurements are used in the polar coordinates denoted as the angle θ_i and magnitude V_i for the voltage phasor at the certain bus *i*, which directly corresponds to the state variables θ_i and V_i. Therefore, there is a linear relation between the voltage phasor measurements and state variables.

However, the model of line current phasor measurement is nonlinear and more complicated. The line current phasor are written in rectangular coordinates, in terms of their real $I_{ij,(r)}$ and imaginary $I_{ij,(i)}$ parts for the current phasor in the branch from bus *i* to bus j. Consider the two-port $\pi -$model of a network branch show in Figure 3.

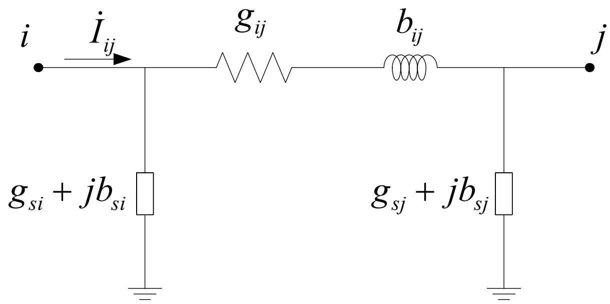

Figure 3 π-Model of a Network Branch

where,

$g_{ij} + jb_{ij}$ is the admittance of the series branch connecting buses i and j;

$g_{si} + jb_{si}$ is the admittance of the shunt branch connected at bus i.

The real and imaginary part of the current phasor along the branch from bus *i* to bus *j* can be expressed as the following formulations, which also represent the nonlinear measurement functions $h_I(x)$ relating current phasor measurements to the state variables:

$$I_{ij,(r)} = (V_i \cos\theta_i - V_j \cos\theta_j)g_{ij} - (V_i \sin\theta_i - V_j \sin\theta_j)b_{ij} + V_i \cos\theta_i g_{sh} - V_i \sin\theta_i b_{sh}$$
$$I_{ij,(i)} = (V_i \cos\theta_i - V_j \cos\theta_j)b_{ij} + (V_i \sin\theta_i - V_j \sin\theta_j)g_{ij} + V_i \cos\theta_i b_{sh} + V_i \sin\theta_i g_{sh}$$

(2.20)

Their corresponding elements in the Jacobian matrix H can also be obtained using Equation (2.4):

$$\frac{\partial I_{ij,(r)}}{\partial \theta_i} = -V_i \sin\theta_i g_{ij} - V_i \cos\theta_i b_{ij} - V_i \sin\theta_i g_{sh} - V_i \cos\theta_i b_{sh}$$
$$\frac{\partial I_{ij,(r)}}{\partial \theta_j} = V_j \sin\theta_j g_{ij} + V_j \cos\theta_j b_{ij}$$
$$\frac{\partial I_{ij,(r)}}{\partial V_i} = \cos\theta_i g_{ij} - \sin\theta_i b_{ij} + \cos\theta_i g_{sh} - \sin\theta_i b_{sh}$$
$$\frac{\partial I_{ij,(r)}}{\partial V_j} = -\cos\theta_j g_{ij} + \sin\theta_j b_{ij}$$

$$\frac{\partial I_{ij,(i)}}{\partial \theta_i} = -V_i \sin\theta_i b_{ij} + V_i \cos\theta_i g_{ij} - V_i \sin\theta_i b_{sh} + V_i \cos\theta_i g_{sh}$$
$$\frac{\partial I_{ij,(i)}}{\partial \theta_j} = V_j \sin\theta_j b_{ij} - V_j \cos\theta_j g_{ij}$$
$$\frac{\partial I_{ij,(i)}}{\partial V_i} = \cos\theta_i b_{ij} + \sin\theta_i g_{ij} + \cos\theta_i b_{sh} + \sin\theta_i g_{sh}$$
$$\frac{\partial I_{ij,(i)}}{\partial V_j} = -\cos\theta_i b_{ij} - \sin\theta_j g_{ij}$$

(2.21)

Using this model, both the bus voltage phasor and the line current phasor measurements can be easily incorporated into the traditional WLS state estimation problem shown in Equation (2.2). The solution algorithm will also remain the same as described in Section 2.2.

2.5 Summary

In this chapter, the traditional state estimation problem is briefly reviewed. Among its various functions, bad data and topology error processing are described in detail. The commonly used methods to detect and identify bad data as well as topology error are also reviewed. It is specifically noted that all of these bad data and topology error processing methods require high measurement redundancy.

The description of operation and properties of PMUs are also introduced in this chapter. PMUs have recently been populating power systems because of their wide applications in power system control and protection. The benefits of PMUs are also extended to the functions of state estimation. It is argued that state estimation based on only phasor measurements may require a large amount of PMUs and therefore may not be economically viable in the immediate future. A compromising alternative is to utilize the phasor measurements from PMUs to improve traditional state estimation. A specific model is introduced so that both voltage phasor and line current phasor measurements can be incorporated into the traditional WLS estimation method.

In the next chapter, one important benefit of PMUs to the state estimation, improving bad data detection and identification capability, will be discussed. The strategically placed PMUs will be used to improve traditional state estimation and its benefits to bad data processing will be shown.

CHAPTER III

OPTIMAL MEASURMENT PLACEMENT TO IMPROVE BAD DATA PROCESSING

In this chapter, PMUs are introduced into traditional state estimation to improve the bad data processing capability in state estimation. Bad data processing is an essential function to detect and identify the errors in measurement set, which is commonly integrated in the state estimation. Bad data processing capability is closely related to the measurement system, while bad data appearing in critical measurements can not be detected. In this chapter, it will be shown that by adding few extra PMUs at strategic locations, the bad data detection and identification capability of a given system can be drastically improved. A specific algorithm to obtain the optimal placement of extra PMUs or traditional measurements is also presented and illustrated with a simple example. Cases studies are carried out with different sizes of test systems, and simulation results are presented to demonstrate the gained benefits. Some studies and results have been presented in the previous paper [43].

3.1 Introduction

Bad data processing is an important function which is commonly integrated the state estimation. It is required for the state estimation to have the capability to detect, identify and correct the gross errors in the measurement set. Depending on the state estimation method used, bad data processing may be carried out as a part of the state estimation or as a post-estimation procedure. However, no matter what type of state estimation method employed, the bad data processing capability depends closely on the measurement configuration and redundancy.

In a given observable power system, measurements can be classified as either critical or redundant measurements. While a redundant measurement can be removed from the measurement system without observability problem, the removal of any critical measurement will cause the rest system unobservable. The critical measurements in the power system also lead to bad data detection problem. When a bad data takes place in the redundant measurement, it can detected by analyzing the objective function or measurement residuals. However, errors in the critical measurement cannot be detected. Therefore, a well-designed measurement system should not contain any critical measurement so that bad data processing can be accomplished.

Critical measurements in a given power system can be identified, either by the topological methods or numerical methods, such as those presented in [44] or [45]. The critical measurements can be improved to redundant measurements by adding a few measurements at the proper locations, as the result of increased measurement redundancy.

Although it is possible and feasible to improve measurement redundancy by adding traditional measurements, adding PMUs will potentially be a better alternative. As a new type of advanced measurement, a PMU placed at a given bus can provide multiple synchronized phasor measurements to the state estimation, which include the bus voltage phasor measurement and the current phasors on several or all lines incident to that bus. And using the model provided in Section 2.4, it is simple to incorporate these voltage and current phasor measurements into the WLS state estimation along with traditional measurements. In this chapter, it is shown that, given a power system which is fully observable with existing measurements, adding few PMUs can convert all existing critical measurements in the power system to redundant measurements. As a result of this improvement, it will make any bad data appearing in the measurement set detectable. An optimal PMU placement algorithm is developed for this purpose and presented in this chapter.

Besides bad data detection, another problem regarding bad data processing is the dad data identification, which also related to the measurement configuration and requires even higher redundancy. Two redundant measurements are defined as a critical pair, if their simultaneous removal from the measurement set will make the system unobservable. A single bad data in either measurement of a critical pair is detectable, but not identifiable. Hence, the placement of measurements to enable bad data identification is further discussed in this chapter. It is shown that the measurement redundancy can be further improved to a desirable level so that any bad data in the measurement set is identifiable.

It should be noted that the system is assumed to be already observable before further improving measurement redundancy. If the system is not observable, traditional measurements or PMUs can be added to improve the measurement system and make it fully observable, using the approaches provided in [21] or [23].

3.2 Linear Measurement Model with PMUs

A simplified DC approximation model for the measurement equations is often useful for analyzing the various problems related only to the measurement configuration. For a given network, the DC approximation model is obtained by assuming that all the bus voltage magnitude are already known and set to 1.0 per unit. Furthermore, all the branch series resistances and shunt elements are neglected. It leads the real power flow from bus *i* and bus *j* to the following simplified formulation:

$$P_{ij} = \frac{\sin\theta_{ij}}{x_{ij}} \tag{3.1}$$

And the real power injection at bus *i* can be expressed as the sum of the power flows along all branches incident to this bus:

$$P_i = \sum_{j \in \mathrm{N}_i} \frac{\sin\theta_{ij}}{x_{ij}} \tag{3.2}$$

where

x_{ij} is the reactance of branch *i*-*j*,

θ_{ij} is the phase angle difference between bus *i* and bus *j*,

N_i is the set bus numbers that are directly connect to bus *i*.

It should be noted that both the system observability and critical measurements problem are not only independent to the operating state of system, but also independent to the branch parameter. Therefore, all the reactance in the system branches can be assumed equal to 1.0 per unit. Using first order Taylor expansion around $\theta_{ij} = 0$ for

Equations (3.1) and (3.2), the relations between real power measurements and bus voltage phase angles can be expressed as linear functions:

$$P_{ij} = \theta_i - \theta_j + e \tag{3.3}$$

$$P_i = \sum_{j \in \mathrm{N}_i} \theta_i - \theta_j + e \tag{3.4}$$

As introduced in previous chapter, a PMU can measure both the voltage phasor of its own bus and current phasor along with the incident branches. It is obvious the voltage phasor measurement at bus i has the following linear function:

$$\theta_{(z),i} = \theta_i + e \tag{3.5}$$

where $\theta_{(z),i}$ is the angle part of voltage phasor measurement at bus *i*. Based on the above assumption about system operating state and network parameter, the real part of branch current can be simplified from Equation (2.20) to the following equation:

$$I_{ij,(r)} = \sin\theta_i - \sin\theta_j \tag{3.6}$$

Since the bus voltage phase angles in power system are relatively small, and the analysis result of measurements configuration is independent to the operating state of the system, Equation (3.6) can be further approximated to:

$$I_{ij,(r)} = \theta_i - \theta_j \tag{3.7}$$

Therefore, for a given network, the $P - \theta$ linear model for the real power and phasor measurement to the bus phase angles can be expressed in the following form:

$$z = H\theta + e \tag{3.8}$$

where,

z is the real power and phasor measurement vector, which contains real power flow, real power injection measurements, angle part of voltage phasor measurements, and real part of current phasor measurements;

θ is the bus phase angle vector;

H is the measurement Jacobian matrix for the real power and phasor measurements versus bus voltage angles;

e is the error vector corresponding to the real power and phasor measurements.

Note that the real and reactive power measurements, angle part and magnitude part of voltage phasor measurements, as well as real part and imaginary part of current phasor measurements are always in pairs in the measurement set. Hence, the analysis results based on $P-\theta$ linear model can be extended to the nonlinear full model without loss of generality.

3.3 Formulation of PMUs Placement Problem

In this section, a proposed procedure for PMUs placement in order to covert all critical measurements into redundant ones will be described. And a small tutorial example is given to illustrate the procedure in detail. The benefits of having this new measurement configuration are twofold: 1) the observability of system will no longer be vulnerable to the loss of any single measurement; and 2) any single bad data, no matter where it happens, can be detected.

The procedure is formulated as a three-step solution, including the following steps:

1) Identify the existed critical measurements in the original system;
2) Find candidate PMUs that an transform each critical measurement into a redundant one;
3) Choose the optimal set of PMUs among the candidates with minimum cost.

3.3.1 Identification of Critical Measurements

Based on its definition, a critical measurement is the measurement whose removal from the measurement set will result in an unobservable system. A power system will be observable only if the measurement Jacobian matrix H is of full rank. Hence, critical measurements in a given system can be identified by checking the algebraic dependency in the Jacobian matrix.

Consider an observable power system with n buses and m measurements. Using the linear $P-\theta$ measurement model, there will be $(n-1)$ state variables which correspond to all bus voltage angles except the reference bus. Therefore, the measurement Jacobian matrix H will be a $m\times(n-1)$ matrix with a column rank of $(n-1)$. Then, a set of $(n-1)$ measurements can be chosen out from the available m measurements in the system, so that the system can keep observable with only these $(n-1)$ measurements. These $(n-1)$ measurements are named as essential measurements, and other $(m-n+1)$ measurements are named as rest measurements. It should be noted that such a set of essential measurement may not be unique. However, all critical measurements in the system must be included in the set of essential measurement. And the $(m-n+1)$ rest measurements must be redundant (non-critical) measurements.

A numerical approach to identify the critical measurements in the power system by analyzing the Jacobian matrix is outlined as the following steps:

Step 1) Decompose the Jacobian matrix H into its lower trapezoidal L and upper triangular factors U by applying the Peters-Wilkinson [46] decomposition method:

$$\widetilde{H} = P \cdot H = \begin{bmatrix} L_b \\ M_R \end{bmatrix} \cdot U = L \cdot U \tag{3.9}$$

where,

$\widetilde{H}$ is the permuted matrix derived from H by suitably exchanging rows, which is equivalent to reordering the measurements.

P is the permutation matrix;

L is the lower trapezoidal matrix;

U is the upper triangular matrix;

L_b is the $(n-1)\times(n-1)$ lower triangular sub-matrix, whose rows corresponds to the essential measurements;

M_R is the $(m-n+1)\times(n-1)$ lower rectangular sub-matrix, whose rows corresponds to the rest redundant measurements.

Step 2) Both the matrix of L and U are of full rank for an observable system. Hence, the rank of the Jacobian matrix H can is exactly the rank of transformed factor L', which is given by:

$$L' = L \cdot L_b^{-1} = \begin{bmatrix} L_b \cdot L_b^{-1} \\ M_R \cdot L_b^{-1} \end{bmatrix} = \begin{bmatrix} I_{(n-1)} \\ K_R \end{bmatrix} \tag{3.10}$$

where,

$I_{(n-1)}$ is the identity matrix of dimension $(n-1)$;

K_R is the lower rectangular sub-matrix in the transformer factor L'.

Note that since L_b is of full rank, and its inverse is multiplied from the right, as shown in Equation (3.10), the row identities will be well preserved in the transformed factor matrix L'. Hence, each row of L' still corresponds to the certain measurement, respectively. If one column of K_R is null, it will be indicated that the corresponding essential measurement is linear independent to others measurements. Therefore, if a column of K_R contains all zero elements, then the measurement corresponding to the row index will be identified as critical.

The procedure can be illustrated using a simple example. Consider the small five-bus power system and its measurement configuration shown in Figure 4.

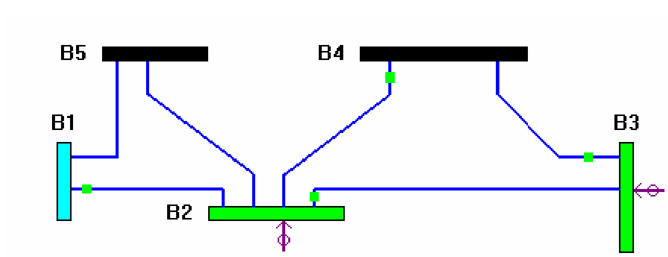

Figure 4 Five-bus Test System

The critical measurements in the system can be easily identified by applying the above procedure. Calculating the transformed factor matrix of L' for the example, the result is shown as follows:

$$\begin{bmatrix} I_{(n-1)} \\ K_R \end{bmatrix} = \begin{array}{c} P_2 \\ P_3 \\ P_{24} \\ P_{12} \\ P_{34} \\ P_{23} \end{array} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0.5 & 0.5 & 0 \\ 0 & -0.5 & 0.5 & 0 \end{bmatrix} \begin{array}{l} \leftarrow \ \textit{critical} \\ \\ \\ \leftarrow \ \textit{critical} \\ \\ \\ \end{array}$$

Checking the transformed lower rectangular sub-matrix K_R, there are two columns with all zero elements. Therefore, two measurements are identified as critical, which are the power injection measurement at bus 2 and the power flow measurement at the branch from bus 1 to bus 2, denoted as P_2 and P_{12}, respectively.

3.3.2 Identifying the Candidate PMUs for Eliminating Critical Measurements

Once the critical measurements are identified, a set of candidate PMUs is selected for each critical measurement. The effects of candidate PMUs are studied if their installations will transform the corresponding critical measurements into redundant ones. The effects can be revealed by checking the linear dependency in the transformed factor matrix L' after assuming their installations.

With all candidate PMUs installed in the system, the measurement Jacobian matrix of H can be partitioned into two sub-matrices.

$$H = \begin{bmatrix} H_{used} \\ H_{pmu} \end{bmatrix}$$

where,

H_{used} is the sub-matrix whose rows correspond to the existing measurements in the system;

H_{pmu} is the sub-matrix whose rows correspond to the phasor measurements associated with candidate PMUs.

Repeating the procedure in the above section, now the transformed factor L' is given by:

$$L' = L \cdot L_b^{-1} = \begin{bmatrix} L_b \cdot L_b^{-1} \\ M_R \cdot L_b^{-1} \\ M_{pmu} \cdot L_b^{-1} \end{bmatrix} = \begin{bmatrix} I_{(n-1)} \\ K_R \\ K_{pmu} \end{bmatrix} \tag{3.11}$$

where K_{pmu} is the lower rectangular sub-matrix corresponding to the phasor measurements associated with candidate PMUs.

The effects of those measurements can be obtained easily by simply tracing the columns for the critical measurements in the transformed matrix L'. For a certain row corresponding to a new measurement, those non-zero elements in the columns of original critical measurements indicates that these critical measurements can be improved by introducing the new measurement.

Considering the five-bus example given above, let us assume that there is a candidate PMU installed at bus 1 only. Including the measurements associated with this PMU, L' will take the following form:

$$L' = \begin{bmatrix} I_4 \\ K_R \\ K_{pmu} \end{bmatrix} = \begin{array}{c} P_2 \\ P_3 \\ P_{24} \\ P_{12} \\ P_{34} \\ P_{23} \\ \theta_1 \\ I_{12} \\ I_{15} \end{array} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0.5 & 0.5 & 0 \\ 0 & -0.5 & 0.5 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0.5 & 1.5 & 2 \end{bmatrix} \begin{array}{l} \\ \\ \\ \\ \\ \\ \left.\begin{array}{l} \\ \\ \\ \end{array}\right\} \text{PMU at bus 1} \end{array}$$

As shown above, a PMU placed at bus 1 is assumed to provide three phasor measurements, namely, the voltage phase angle measurement θ_1 and the current phasor I_{12} and I_{15}. By checking the existence of non-zero elements in the sub-matrix of K_{pmu}, it shows that both the critical measurements P_2 and P_{12} can be improved to redundant measurements by introducing the new phasor measurements of the PMU at bus 1.

3.3.3 PMU Placement Problem

The final step involves the optimal selection of the PMUs from the list of candidates, which can improve all critical measurements in the system with minimum cost.

An incidence matrix B that relates PMUs to their associated phasor measurements is formed. The element of B is defined as the follows:

$$B(i,j) = \begin{cases} 1 & \text{if PMU at bus } i \text{ provides the measurement } j \\ 0 & \text{otherwise} \end{cases}$$

For the five-bus system in Figure 4, assume that there are five candidate PMUs corresponding to all five buses in the system. And also assume each candidate PMU has one voltage phasor measurement for its own bus and several current phasor measurements for its incident branches, yielding the incidence matrix of B shown in the following equation:

$$B = \begin{array}{c} \\ PMU_1 \\ PMU_2 \\ PMU_3 \\ PMU_4 \\ PMU_5 \end{array} \begin{array}{c} \begin{array}{ccccccccccccccccc} \theta_1 & I_{12} & I_{15} & \theta_2 & I_{21} & I_{23} & I_{24} & I_{25} & \theta_3 & I_{32} & I_{34} & \theta_4 & I_{42} & I_{43} & \theta_5 & I_{51} & I_{52} \end{array} \\ \begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \end{bmatrix} \end{array}$$

Next, let us define another matrix R. It is formed using the binary form of the columns of K_{pmu} in Equation (3.11) which correspond to the critical measurements. It is defined as

$$R(i,j) = \begin{cases} 1 & if\ K_{pmu}(i,j) \neq 0\ and\ measurement\ "j"\ is\ critical \\ 0 & otherwise \end{cases}$$

Note that the binary matrix R provides a compact representation of these critical measurements (columns) that will be improved to redundant ones by given phasor measurements with candidate PMUs (rows).

For the same five-bus system which has only two critical measurements (corresponding to column 1 and column 4 in K_{pmu}), the matrix R will be obtained as follows:

$$K_{pmu} = \begin{array}{c} \theta_1 \\ I_{12} \\ I_{15} \\ \theta_2 \\ I_{21} \\ I_{23} \\ I_{24} \\ I_{25} \\ \theta_3 \\ I_{32} \\ I_{34} \\ \theta_4 \\ I_{42} \\ I_{43} \\ \theta_5 \\ I_{51} \\ I_{52} \end{array} \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0.5 & 1.5 & 2 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 0.5 & 1.5 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0.5 & 0.5 & 0 \\ 0 & 0.5 & 0.5 & 1 \\ 0 & 0.5 & 0.5 & 0 \\ 0 & 0.5 & 0.5 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0.5 & 0.5 & 0 \\ 1 & 0.5 & 1.5 & 2 \\ 1 & 0.5 & 1.5 & 2 \\ 1 & 0.5 & 1.5 & 1 \end{bmatrix} \Rightarrow R = \begin{bmatrix} 0 & 0 \\ 0 & 1 \\ 1 & 1 \\ 0 & 1 \\ 0 & 1 \\ 1 & 1 \\ 0 & 0 \\ 0 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \\ 1 & 1 \\ 1 & 1 \\ 1 & 1 \end{bmatrix}$$

Since each PMU supplies more than one phasor measurements (typically a voltage phasor measurement and several current phasor measurements), the incidence matrix B will be used to represent the incidence relation between phasor measurements and candidate PMUs. Then, the following matrix product will yield the relationship between candidate PMUs and critical measurements:

$$F = R^T \cdot B^T \tag{3.12}$$

Calculating F for the above example system, yielding the following result:

$$F = R^T \cdot B^T = \begin{bmatrix} 1 & 1 & 0 & 0 & 3 \\ 2 & 3 & 1 & 1 & 3 \end{bmatrix}$$

Note that each row of F corresponds to a critical measurement, and each column of F corresponds to a candidate PMU. The following constraint will ensure the requirement of PMU placement that for every critical measurement, there will be at least on PMU to make the critical measurement redundant:

$$F \cdot X \geq \hat{1} \tag{3.13}$$

where,

$\hat{1}$ is a vector, whose entries are all equal to 1;

X is a binary (0/1) vector, whose entries is defined as

$$X(i) = \begin{cases} 1 & \textit{if the candidate PMU i is chosen} \\ 0 & \textit{otherwise} \end{cases}$$

The installation of selected PMUs will guarantee network observability in case of loss any of these critical measurements in the original system. In other words, all previously critical measurements now are transformed into redundant measurements.

An optimization problem is then formulated, whose objective is to select a set of PMUs with minimum installation cost so that all critical measurements are transformed into redundant ones. Due to the types of variables involved, it can be formed as an integer programming problem, which is stated as the following equation:

$$\begin{aligned} &\textit{Minimize} \quad \sum_{i}^{n} c_i \cdot x_i \\ &\textit{subject to} \quad F \cdot X \geq 1 \end{aligned} \tag{3.14}$$

where,

n is the number of candidate PMUs in the system;

c_i is the cost of installing candidate PMU i.

The solution vector X of the optimization problem given in Equation (3.14) will provide the strategic placement of PMUs, which eliminates all critical measurements with minimum cost.

Solution of this problem for the above five-bus example yields that any one of candidate PMUs installed at the bus 1, bus 2 or bus 5 can achieve the objective. Only one PMU at any of these three buses can transform both critical measurements into redundant ones, so the PMU with minimum installation cost should be chosen.

3.3.4 Algorithm

An optimal PMU placement algorithm is developed based on the procedure given in the previous discussion. This algorithm includes the following steps:

Step 1) Build the measurement Jacobian matrix H based on system configuration, which includes both the original measurements in the system and the phasor measurements with the candidate PMUs. Also build the incidence matrix B.

Step 2) Factorized H and use back-substitution to obtain transformed factor matrix L' as Equation (3.11);

Step 3) Identify the critical measurements corresponding to the null columns in the sub-matrix K_R, and form the compact binary matrix R from the sub-matrix K_{pmu};

Step 4) Solve the integer programming problem given by Equation (3.14). The optimal locations for the PMUs will be given by the non-zero entries in the integer solution vector X.

3.4 Optimal Placement for Mixed Measurements

The above described algorithm can be easily revised so that not only PMUs but also traditional measurements are considered as candidate measurements to improve the critical measurements. The required revisions to the algorithm are given in detail below.

Since now the candidate measurement set includes two types of measurement, namely, phasor measurements with PMUs and traditional measurements, thus the corresponding measurement Jacobian H_{cand} also includes two sub-matrices as follows:

$$H_{cand} = \begin{bmatrix} H_{pmu} \\ H_{trad} \end{bmatrix}$$

where H_{trad} is the sub-matrix in the measurement Jacobian matrix, which rows correspond to the candidate traditional measurements.

Therefore, now the measurement Jacobian matrix *H* has the following form, with assuming that all candidate measurements are installed:

$$H = \begin{bmatrix} H_{used} \\ H_{pmu} \\ H_{trad} \end{bmatrix}$$

where H_{used} , H_{pmu} and H_{trad} correspond to the existing, candidate phasor measurements, and candidate traditional measurements, respectively.

Applying the same decompose method provided in above sections, Jacobian matrix H can be decomposed into the lower trapezoidal and upper triangular factors:

$$\widetilde{H} = P \cdot H = L \cdot U = \begin{bmatrix} L_b \\ M_R \\ M_{pmu} \\ M_{trad} \end{bmatrix} \cdot U \tag{3.15}$$

where, as in Equation (3.11), L and U represent the lower and upper factors. In the matrix of L, the rows of L_b and M_R correspond to the essential and rest redundant measurements, which already exist in the system. The rows of M_{pmu} and M_{trad} correspond to the candidate phasor measurement and traditional measurements, respectively. It should be noted that, since the sub-matrix H_{used} is of full rank with the original observable system, the rows exchanging during the decomposition only affects the rows in H_{used}.

Appling the same transformation as done in Equation (3.12), the sub-matrices K_R, K_{pmu} and K_{trad} will be formed in the similar procedure:

$$L' = L \cdot L_b^{-1} = \begin{bmatrix} L_b \cdot L_b^{-1} \\ M_R \cdot L_b^{-1} \\ M_{pmu} \cdot L_b^{-1} \\ M_{trad} \cdot L_b^{-1} \end{bmatrix} = \begin{bmatrix} I_{(n-1)} \\ K_R \\ K_{pmu} \\ K_{trad} \end{bmatrix} \tag{3.16}$$

Note that now the critical measurements can still be identified as done in above sections, by checking the null columns in K_R.

The objective is still to introduce the non-zero elements into the certain columns of L' by placing PMUs or traditional measurements at strategic locations. Hence, the constraint matrix F will be revised as:

$$F = \begin{bmatrix} F_{pmu} & F_{trad} \end{bmatrix} \tag{3.17}$$

where,

$F_{pmu} = R_{pmu}^{T} \cdot B^{T}$

$F_{trad} = R_{trad}^{T}$

R_{pmu} and R_{trad} is the binary form of the columns K_{pmu} and K_{trad} that correspond to the critical measurements;

B is the same PMU and phasor measurement incidence matrix as defined in previous section.

The optimal selection of PMUs and traditional measurements can still be formulated as Equation (3.14), using the revised constraint matrix F , shown as the following form:

$$\begin{aligned} &\mathit{Minimize} \quad \sum_{i}^{n} c_i \cdot x_i \\ &\mathit{subject\ to} \quad F \cdot X \geq 1 \end{aligned} \tag{3.18}$$

However, where,

n is total number of candidate PMUs and candidate traditional measurements;

c_i is the installation cost of a candidate PMU or a candidate traditional measurement;

X is still the binary vector, whose entry x_i correspond a PMU or a traditional measurement. If the entry equals to 1, the corresponding PMU or traditional measurement is chosen, otherwise not chosen.

3.5 Improving Measurement Redundancy for Bad Data Identification

After the improvement of measurement system with the procedure given in above Section 3.3 or Section 3.4, the bad data detection capability of state estimation is ensured. However, the bad data identification capability is still limited by the existence of critical pairs of measurement in the system. Hence, the measurement redundancy in the system may be required to further increase to a desired level in order to ensure the bad data identification capability.

Such an objective also can be achieved by install extra PMUs or traditional measurements at the strategic locations in the system. The optimal placement of PMUs and traditional measurements can also be formed as an integer programming problem, however, through a more complicate procedure.

3.5.1 Identification of Critical Pairs of Measurement

The critical pairs of measurement can also be identified by examining the linear dependency in the Jacobian matrix H. Assuming the measurement system has been improved using the method provided in Section 3.3 and 3.4, therefore, there is no critical measurement in the system.

After the decomposition of Jacobian matrix H as Equation (3.10), the measurements in the system are classified into two categories: essential measurements and rest redundant measurements. It is easily concluded that the critical pairs of measurement must be formed by two essential measurements, or an essential measurement and a rest measurement, since the simultaneous removal two rest measurements will not affect the observability. Hence, there are also two expressions in the transformed lower rectangular factor K_R, respectively:

Type I: A critical pair with an essential measurement and a rest measurement corresponds to a certain column of K_R, which only has one non-zero element. And the non-zero element corresponds to the rest measurement, and the column corresponds to the essential measurement.

Type II: A critical pair of two essential measurements corresponds to two certain columns of K_R, which are collinear with each other.

Therefore, the critical pairs of measurements can be identified by checking the sub-matrix of K_R, based on the above two properties. And the optimal placement problem of PMUs or traditional measurements also can be formulated based on those properties.

3.5.2 PMU and Traditional Measurement Placement Problem

The procedure and algorithm to obtain the optimal placement of PMUs and traditional measurements can be obtained by modifying the given procedure and algorithm in Section 3.3 and Section 3.4.

Assuming all possible candidate measurements are installed, the measurement Jacobian matrix H has the following form:

$$H = \begin{bmatrix} H_{used} \\ H_{pmu} \\ H_{trad} \end{bmatrix}$$

Applying the decomposition as Equation (3.16) and linear transform as Equation (3.17), the transformed lower factor L' is given by:

$$L' = L \cdot L_b^{-1} = \begin{bmatrix} L_b \cdot L_b^{-1} \\ M_R \cdot L_b^{-1} \\ M_{pmu} \cdot L_b^{-1} \\ M_{trad} \cdot L_b^{-1} \end{bmatrix} = \begin{bmatrix} I_{(n-1)} \\ K_R \\ K_{pmu} \\ K_{trad} \end{bmatrix}$$

Define the new matrix W, which corresponds to the collinear relationship between the columns in the sub-matrix K_R. Since the two collinear columns in K_R correspond to a critical pair of measurements, a column of matrix W is then defined to represent to this critical pair: For the k th critical pair corresponding to two collinear columns *i* and *j*, find their first non-zero elements $K_R(m,i)$ and $K_R(m,j)$ in same row *m*. Therefore, the elements in the k th column of matrix W is defined as

$$W(l,k) = \begin{cases} 1/K_R(m,i) & l = i \\ -1/K_R(m,j) & l = j \\ 0 & otherwise \end{cases}$$

Note that the product of $K_R \cdot W$ must be a null matrix, where each row correspond to a rest measurement, and each column correspond to a critical pairs with two essential measurements. And the non-zero element in the product of $K_{pmu} \cdot W$ or $K_{trad} \cdot W$ represents the improvement to the corresponding critical pair with installation of new PMUs or traditional measurements, respectively.

Another type of critical pairs with an essential measurement and a rest measurement correspond to the columns in K_R with only one non-zero element. If there is another non-zero element in the corresponding columns of K_{pmu} and K_{trad}, it means that the installation of this PMU or traditional measurement can improve the critical pairs.

Therefore, the following revised optimization formulation will yield a solution that will help the system to approach the desired level for bad data identification:

$$\begin{aligned} &Minimize \quad \sum_{i}^{n} c_i \cdot x_i \\ &subject\ to \quad F \cdot X \geq 1 \end{aligned}$$

where the constraint matrix F has the following definition:

$$F = [F_{pmu} \ F_{trad}]$$

However, the sub-matrices of F_{pmu} and F_{trad} have more complicated definitions, which respectively are defined as:

$$F_{pmu} = \begin{bmatrix} R_{pmu} & K_{pmu} \cdot W \end{bmatrix}^T \cdot B^T$$

$$F_{trad} = \begin{bmatrix} R_{trad} & K_{trad} \cdot W \end{bmatrix}^T$$

where R_{pmu} and R_{trad} are the binary forms of the columns K_{pmu} and K_{trad} that correspond to these critical pairs with an essential measurement and a rest measurement.

3.6 Simulation Results

In order to simulate the proposed method and evaluate its performance, a program is developed. Two systems with different sizes, IEEE 57-bus and 118-bus test system are used for the simulations. The results are presented in four parts. Section 3.6.1 and Section 3.6.2 show the placement procedure applied to these two systems. Section

3.6.3 illustrated the benefits of such placement for the IEEE 118-bus system, where a previously undetected of bad data becomes detectable as a result of PMU placement. In Section 3.6.4, simulation results about improving measurement redundancy to bad data identification and using both PMUs and traditional candidate measurements are presented. The integer programming problem is solved using TOMLAB Optimization Toolbox [47].

All simulation cases use the following assumptions:

1) There are no special bus limitations for the placement of PMUs. It means that every bus is assumed to be a candidate location for PMU installation.
2) Installation costs of all PMUs are the same. Due to lack of any information about installation cost of PMUs, all PMUs are given the same cost in the optimization.
3) Current phasor along all branches incident to a bus will be measured by the PMU at that bus.

These assumptions are only made for convenience in carrying out the simulations. However, the proposed method also can work well without these assumptions.

3.6.1 IEEE 57-bus System

Since using the decoupled linear $P-\theta$ model, only the real power injection and line flow measurements are considered here. The measurements system is already designed to make the entire system observable. The simulated IEEE 57-bus system network and measurement configuration are shown in Figure 5. The system has a total of

33 real power flow measurements and 32 injection measurements. Hence, the number of state variables is ($n = 57 - 1 = 56$), and the number of measurements is $m = 65$.

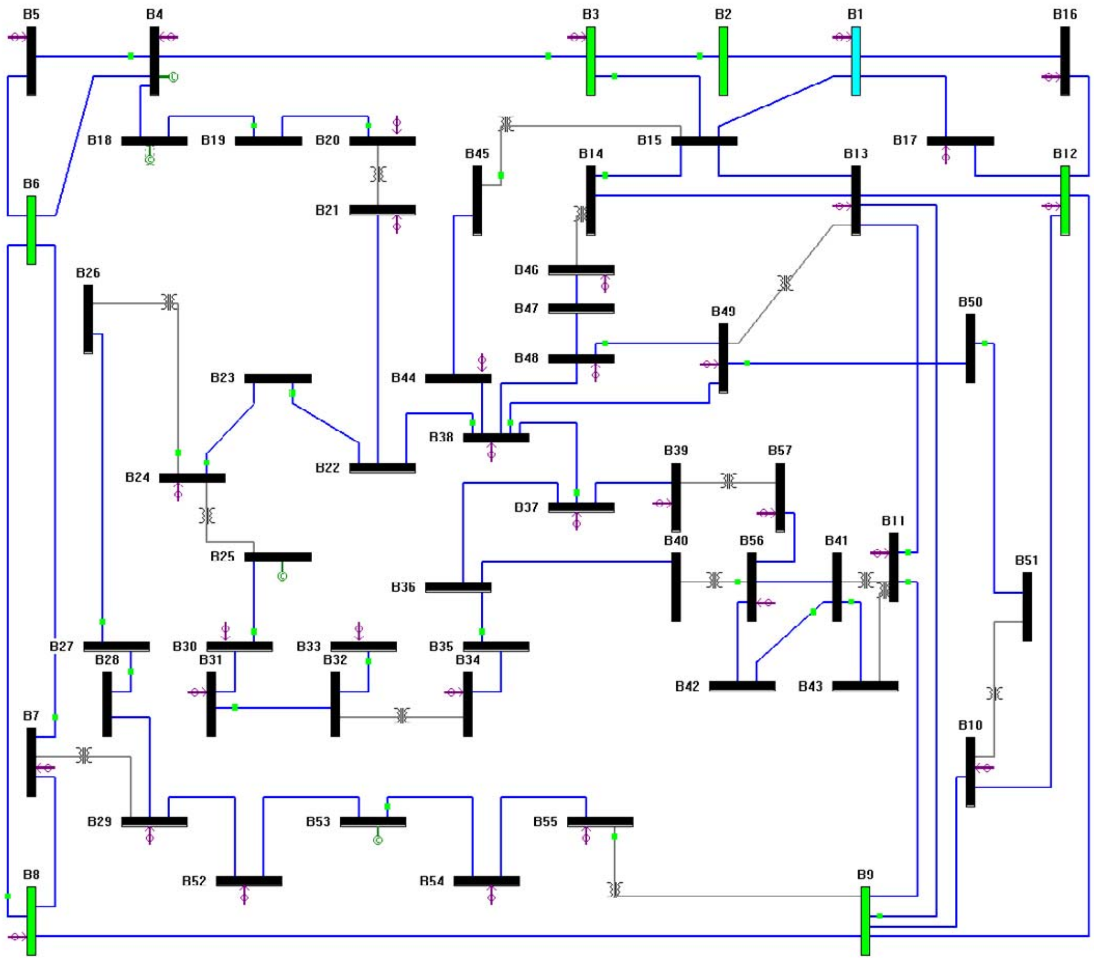

Figure 5 Network Diagram and Measurement Configuration for IEEE 57-bus System

The measurement configuration contains 13 critical measurements, which are listed in Table 1.

Table 1 Critical Measurements in IEEE-57 bus System

Critical measurement	Measurement type	Measurement location
1	power flow	Bus 41 to Bus 43
2	power flow	Bus 36 to Bus 35
3	power flow	Bus 42 to Bus 41
4	power flow	Bus 40 to Bus56
5	Injection	Bus 11
6	Injection	Bus 24
7	Injection	Bus 39
8	Injection	Bus 37
9	Injection	Bus 46
10	Injection	Bus 48
11	Injection	Bus 57
12	Injection	Bus 56
13	Injection	Bus 34

Using the method presented in previous sections, the optimal placement for PMUs is obtained as the result of integer programming problem. The optimal solution yields phasor measurement units at bus 34 and bus 46. As a result of installing these two PMUs, no critical measurements will exist in this system.

3.6.2 IEEE 118-bus System

In this case, IEEE 118-bus system is chosen for the simulation. A measurement set is chosen to make the system fully observable. A total of 39 injection measurements and 114 power flow measurements are chosen, yielding a total of ($n = 118 - 1 = 117$)

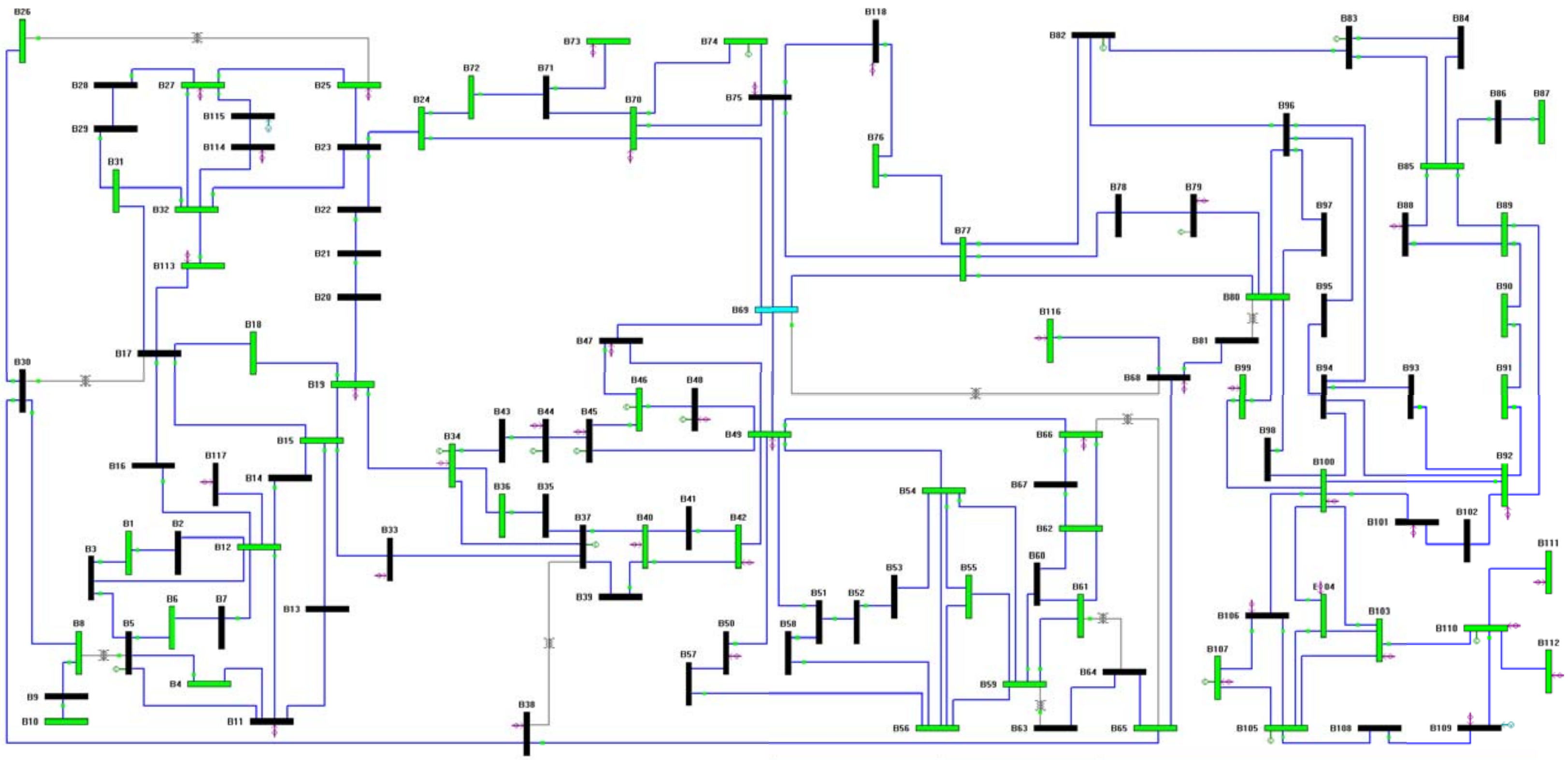

Figure 6 Network Diagram and Measurement Configuration for

IEEE 118-bus System

state variables and $m = 153$ measurements, the system network and measurement configuration is shown in Figure 6.

This measurement set contains a total of 29 critical measurements, which are listed in Table 2.

Table 2 Critical Measurements in IEEE 118-bus System

Critical Measurement	Critical Measurement
Power flow 7-12	Power flow 83-84
Power flow 31-32	Power flow 86-87
Power flow 29-31	Power flow 85-86
Power flow 8-9	Power flow 96-97
Power flow 9-10	Power flow 54-55
Power flow 3-5	Power flow 54-59
Power flow 1-3	Power flow 63-59
Power flow 1-2	Power flow 59-60
Power flow 5-6	Power flow 59-61
Power flow 12-14	Power flow 64-61
Power flow 35-36	Power injection 114
Power flow 51-52	Power injection 117
Power flow 52-53	Power injection 34
Power flow 77-78	Power injection 79
Power flow 95-96	

The solution of the optimization problem yields a total of 13 PMUs to be placed at the following buses in order to transform all critical measurements into redundant ones: 7, 10, 12, 28, 32, 35, 52, 59, 61, 80, 83, 86, 94.

3.6.3 Bad Data Processing Capability

The same 118 bus system is used to illustrate bad data processing benefits gained as a result of PMU placement. In order to show these benefits, one of the critical measurements in the system is assumed to be corrupted by gross error. State estimation is executed and the results of bad data processing reveal no bad data. This is expected due to the criticality of the measurement. After applying the proposed PMU placement procedure, the gross error in the same measurement can be detected.

The original measurement set is populated with traditional measurements. These are 2 voltage, 39 pairs of injection and 114 pairs of power flow measurements. All measurements in the system are assumed to have the same standard deviation of $\sigma = 0.001$ per unit. Chi-squares test is used for bad data detection.

A single bad measurement is simulated for the real power flow measurement from bus 3 to bus 5. The true real power flow value of $P_{3-5} = \text{-}0.68103$ is replaced by a gross error as $P_{3-5} = \text{-}0.98103$. However, the state estimation fails to detect this bad data, yielding an objective function value of 0.0260. This value is far below the Chi-squares test threshold of 93.94 (corresponding to 95% confidence level and 73 degrees of freedom).

After introducing new PMUs using the proposed approach, 13 PMUs are installed at buses 7, 10, 12, 28, 32, 35, 52, 59, 61, 80, 83, 86 and 94. Repeating the state estimation with the updated measurement set yields an objective function value of 44443, which is clearly above the threshold of 228.58 (based on 95% confidence level and 195 degrees of freedom). Hence, the Chi-squares test will flag the presence of bad data. Furthermore, the largest normalized residual test is also performed and the gross error is identified, having the largest normalized residual of $r_i^N = 210.8$.

3.6.4 Redundancy Improvement with Mixed Measurements

As discussed in Section 3.4 and 3.5, the algorithm can be further modified to consider the optimal placement of mixed measurements and improve the system measurement redundancy to the bad data identification level. Therefore, IEEE 57-bus system is used to evaluate the performance of both two extended algorithms. The system network and original measurement configuration is same as shown in Figure 5. In addition to the PMUs, some injection measurements are also considered as candidate measurements in this particular simulation. It is assumed that any bus without an injection measurement is assumed as a candidate for injection measurement placement. The cost of a PMU installation is assumed to be twice the cost of place injection measurement.

Table 3 Optimal Candidates for IEEE 57-bus System with Both PMUs and Traditional Measurements

Redundancy level	PMU location	Injection measurement location
No critical measurement		Bus 35, 47
No critical pair	Bus 1	Bus 14, 19, 22, 27, 32, 36, 41, 45, 51, 53

The results of simulations are shown in Table 3. As the required redundancy level increase, so does the number of measurements to be placed. This is evident from the results of Table 3, where for the case of the IEEE 57-bus system, a total of 1 PMUs and 12 extra injection measurements are need when the system is improved to ensure any single bad data identification.

3.7 Conclusion

The essential objective of this chapter is to illustrate the benefits of adding a few PMUs for state estimation, even when the system is initially fully observable without these devices. In this chapter, it is shown that, PMUs will provide increased bad data detection and identification capability, which may come handy during contingencies and existence of bad data in low redundancy pockets of the system.

The problem of PMU placement is formulated and solved as an integer programming problem. The solution provides the minimum number of strategically located PMUs that will eliminate measurement criticality in the entire system. This implies that any bad data appearing on single measurement will be detectable. It is shown that depending on the measurement configuration and the system topology, this goal can be achieved by using only a few extra PMUs to transform several critical measurements into redundant ones in the system.

The placement problem is then extended to also incorporate traditional measurements as candidates for placement. Furthermore, it is also shown that an extended algorithm can be used to determine optimal measurement placement for a further desired level to enable bad data identification capability in the system. This allows design of measurement systems with different degrees of vulnerability against loss of measurements and/or bad data. Several simulation results are provided to illustrate the proposed placement procedure and its effectiveness in enhancing bad data processing.

CHAPTER IV

OPTIMAL MEASUREMENT PLACEMENT TO IMPROVE TOPOLOGY ERROR PROCESSING

In this chapter, PMUs are used to improve the given measurement system for the objective to ensure topology error processing capability in state estimation. Existing methods to detect and identify topology errors are dependent on the measurement configuration and network topology. Hence, the capability to effectively process topology errors is closely linked to proper measurement design. In particular, a topology error associated with a given branch may not be detectable with the existing measurement configuration. It is possible to efficiently improve the topology error processing capability for a given system by strategically placing few extra measurements. A systematic procedure is developed in order to accomplish this objective by using not only the traditional measurements but also the phasor measurements from PMUs. Case studies and numerical simulations are also provided in order to illustrate the proposed measurement placement strategy.

4.1 Introduction

The state estimation problem is formulated based on an electrical model provided by the Topology Processor. The topology processor processes the status information about the circuit breakers (CB) and switching devices in the system, and configures the bus-branch model of the system. This procedure is typically based on the assumption that network topology and parameters are perfectly known and correct. While true for most cases, this assumption may not hold for certain situations where the status of some circuit breakers may not be known or may even be wrong. In these rare cases, the bus-branch model generated for the state estimator is wrong, leading to a topology error.

Pioneering work for topology error detection and identification methods are based on normalized residual tests [34]. The relation between the capability to detect and identify topology errors and the existing measurement and network configuration is also systematically presented. It is shown that, in a given power system, some branches can be classified into sets of topology error undetectable and unidentifiable branches. The topology errors that occur in these branches can not be detected and identified, respectively. The conditions upon detectability and identifiability of topology errors are analyzed in detailed in [34] and [35]: A single branch error can not be detected if the following conditions happen: 1) it is an irrelevant branch (branch with no incident measurements), or 2) the removal of branch from the original network causes the rest of network unobservable (critical branch). A single branch error is unidentifiable if it happens in either one of critical pair of branches, which simultaneous removal of both branches causes the system unobservable.

In general, adding traditional extra measurements, which include power flow measurements and injection measurements, can improve the topology error processing capability of both detection and identification. However, not all branches can be made error detectable by using traditional measurements, since topology error processing capability also depends on the network configuration.

As discussed in previous chapters, the state estimation is expected to benefit from the PMUs rapid populating in today's power systems. It is easy to incorporate phasor measurements into the existing state estimator, as presented in Section 2.4, and it is proved that PMUs utilization in state estimator can improve the measurement redundancy, as discussed in Chapter III. Therefore, in this chapter, the benefits of phasor measurements to the state estimator are discussed regarding the topology error processing capability.

The objective of this study is to strategically place a mix set of few phasor and few traditional measurements in order to drastically enhance the topology error processing capability for a given system. In this chapter, an algorithm is also developed to find the optimal measurement placement to achieve this objective. This algorithm will be described and illustrated by examples. Furthermore, the improvements in topology error processing capability as a result of introducing new measurements will be verified by simulations.

4.2 Topology Error Detection and Identification

4.2.1 Residual Analysis of Topology Error

The effect of topology errors shows up in the measurements equations, which can be modeled in the Jacobian matrix:

$$H_t = H_e + E \tag{4.1}$$

where H_t is the true Jacobian matrix, H_e is the incorrect Jacobian due to topology errors, and E is the Jacobian matrix error.

With existing topology errors, the measurement residual vector is derived:

$$r = \Delta z - H_e \hat{x} = (I - K_e)(Ex + e) \tag{4.2}$$

where I is the identity matrix, and

$$K_e = H_e (H_e{}^t R^{-1} H_e)^{-1} H_e{}^T R^{-1} \tag{4.3}$$

Let Δf be the vector of branch flow errors, and M be the measurement-to-branch incidence matrix. Then the measurement bias vector Ex can be written as a linear combination of errors in network branch flows:

$$Ex = M\Delta f \tag{4.4}$$

and the measurement residual vector will be given by:

$$r = (I - K_e) M\Delta f = T\Delta f \tag{4.5}$$

It means a topology error in a certain branch will produce a residual vector that is collinear with the corresponding column of T. Based on their collinearity, a geometric interpretation of measurement residuals can identify the topology error.

However, a topology error in certain branch corresponding to a null column of T can not be detected, because now this specific topology error will not affect the measurement residuals. And a topology error in the one of two branches with collinear columns in T can not be identified, while the topology errors in either of branches will cause identical same measurement residuals.

4.2.2 Detectability and Identifiability of Branch Topology Error

Therefore, the detectability and Identifiability of branch topology error is directly related to the algebraic dependency among the columns of T. Due to the properties of matrix linear transformation, those dependencies can be further deduced from a simpler matrix. The method is described as follows [34]:

For an observable system, there must be a full rank Jacobian matrix *H*. And by reordering the measurements, the Jacobian matrix *H* can be written as

$$H = \begin{bmatrix} H_1 \\ H_2 \end{bmatrix}$$

where sub-matrix H_1 is square and of full rank.

Define two new matrices *R* and *G* as the following definitions:

$$\begin{aligned} F &= H_2 H_1^{-1} \\ G &= \begin{bmatrix} -F & I \end{bmatrix} \end{aligned} \tag{4.6}$$

It can be proved that

$$T = RG^T (GRG^T)^{-1} GM \tag{4.7}$$

Because the first two factors RG^T and $(GRG)^{-1}$ must also be of full rank for the observable systems, the algebraic dependencies among the columns of T are exactly equivalent to those among the columns of the matrix of *GM*. If M is also portioned into two blocks M_1 and M_2 as the same order of measurements in H, we can find the detectability and identifiability of branch topology errors, by only studying the dependencies in the following matrix:

$$\begin{aligned} GM &= M_2 - FM_1 \\ &= M_2 - H_2 H_1^{-1} M \end{aligned} \tag{4.8}$$

In other words, single topology error in the branch corresponding to a null column in the matrix of GM is not detectable. A topology error in either of two branches having collinear columns in GM can not be identified by measurement residual analysis.

4.3 Linear Measurement Model with PMUs

During the study of detectability of topology error, we still can use the simplified real power approximation model. Now all bus voltage magnitudes as 1.0 p.u., and the shunt elements and branch resistances are also neglected. It should be noted that these simplifying assumptions have in general no effect on the analysis of topology errors.

Therefore, the $P-\theta$ linear model for the real power and phasor measurement to the bus phase angles can be expressed as:

$$z = H\theta + e \tag{4.9}$$

where ,

z is the real power and phase angle measurement vector.

θ is the phase angle vector.

H is the decoupled real power-phase angle measurement Jacobian.

e is the measurement error vector.

Now let us consider a PMU installed at bus *i*, providing the bus voltage phase angle measurement θ_i as measurement k in the measurement vector. Note that, irrespective to any changes of network topology, the *k*-th row of *H* corresponding to this phasor measurement should always be given by:

$$H(k,j)=\begin{cases}1 & j=i\\ 0 & otherwise\end{cases}$$

This implies that, the rows corresponding to the phasor measurement in *E* should always be zero. Furthermore, considering the given form in Equation (4.4), the corresponding row in the measurement-to-branch incidence matrix *M*, must also be null.

4.4 Formulation of Measurement Placement Problem

A two-stage measurement placement strategy will be proposed in this section. For simple illustration of this procedure, it will be described using a tutorial example. The objective of this procedure is to improve the topology error processing capability up to the desirable level by adding both the phasor measurements and traditional measurements with minimal cost.

4.4.1 Measurement Placement to Enable Topology Error Detection

In the first stage, new measurements will be placed to make sure that any single branch topology error in the power system will be detectable, and the system will remain observable after the outage of any single branch.

This procedure is formulated as a three-step solution:

1) Identify the branches whose single branch topology error is undetectable;
2) Determine the effects of candidate measurements on each of existing branches;
3) Choose the optimal set of measurements among the candidate measurements.

4.4.1.1 Identification of Topology Error Undetectable Branch

As shown in Section 4.2, the topology error undetectable branches correspond to the null columns in the matrix *GM*. Using this property, a numerical method is simply developed to identify these branches.

Let us consider an observable system. Its essential measurements and rest measurements can be identified as the procedure described in Section 3.3, by applying the Peters-Wilkinson decomposition method to the Jacobian matrix *H*. And the measurements will be reordered during the decomposition, so that the Jacobian matrix *H* can be written in the following form:

$$H = \begin{bmatrix} H_1 \\ H_2 \end{bmatrix}$$

where H_1 and H_2 correspond to the essential and rest measurements respectively. Similar reordering for *M* will result:

$$M = \begin{bmatrix} M_1 \\ M_2 \end{bmatrix}$$

Then we can calculate the dependency matrix *GM* for this system, as Equation (4.8):

$$GM = M_2 - H_2 H_1^{-1} M_1$$

The null columns in matrix *GM* indicate the topology errors in corresponding branches have no effect on the measurement residuals. In other words, these branches are topology error undetectable. Note that each column of *GM* corresponds to a certain network branch.

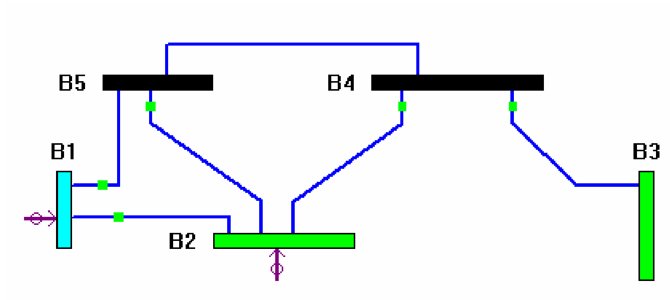

Figure 7 Five-bus Test System

The procedure is illustrated using a simple example. Let us consider the simple five-bus power system and its measurement configuration, as shown in Figure 7. Applying the above procedure, the topology error undetectable branches can be easily found. The *GM* matrix for this example is obtained as:

$$GM = \begin{array}{c} \begin{array}{cccccc} b_{1-2} & b_{1-5} & b_{2-5} & b_{2-4} & b_{3-4} & b_{4-5} \end{array} \\ \begin{bmatrix} 0.33 & -0.33 & 0.33 & 0 & 0 & 0 \\ -0.33 & 0.33 & -0.33 & 0 & 0 & 0 \\ 0.33 & -0.33 & 0.33 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

Note that there are three null columns in *GM*, corresponding to branches 4-5, 2-4 and 3-4. Among them, branch 4-5 is an irrelevant branch, while both branches 2-4 and 3-4 are critical branches, whose removal will lead to an unobservable network.

4.4.1.2 Identifying the Relevant Candidate Measurements

Now we assume that all candidate measurements are installed in the network, so the Jacobian matrix H will contain three sub-matrices as below:

$$H = \begin{bmatrix} H_1 \\ H_2 \\ H_c \end{bmatrix}$$

where the rows of the sub-matrix H_c correspond to the candidate measurements. Similar partitioning will apply to *M*:

$$M = \begin{bmatrix} M_1 \\ M_2 \\ M_c \end{bmatrix}$$

where the rows of the sub-matrix M_c correspond to the candidate measurements.

The new GM matrix will then be written as:

$$GM_{new} = \begin{bmatrix} GM \\ GM_c \end{bmatrix}$$

where

$$\begin{aligned} GM &= M_2 - H_2 H_1^{-1} M_1 \\ GM_c &= M_c - H_c H_1^{-1} M \end{aligned} \tag{4.10}$$

The non-zero elements in the rows of the sub-matrix GM_c, indicates that the branches are made error detectable by the corresponding candidate measurements.

As an example, consider a set of candidate measurements for the given five-bus test system in Figure 7. The candidate measurements include injection measurements at buses 3, 4 and 5, and flow measurement on branch 4-5, and phasor measurements at buses 2, 3, 4 and 5. Then this candidate measurements set yields a new *GM* as shown below:

$$
GM_{new} = \left[\begin{array}{c} GM \\ \hdashline GM_c \end{array}\right] =
\begin{array}{c|cccccc|}
 & b_{1-2} & b_{1-5} & b_{2-5} & b_{2-4} & b_{3-4} & b_{4-5} \\
 & 0.33 & -0.33 & 0.33 & 0 & 0 & 0 \\
 & -0.33 & 0.33 & -0.33 & 0 & 0 & 0 \\
 & 0.33 & -0.33 & 0.33 & 0 & 0 & 0 \\
\hdashline
P_3 & 0 & 0 & 0 & 0 & 0 & 0 \\
P_4 & 0.33 & -0.33 & -0.67 & 1 & 0 & -1 \\
P_5 & -0.33 & 0.33 & 0.67 & -1 & 0 & 1 \\
P_{4-5} & -0.33 & 0.33 & 0.67 & -1 & 0 & -1 \\
\theta_2 & 0.67 & 0.33 & -0.33 & 0 & 0 & 0 \\
\theta_3 & 0.67 & 0.33 & -0.33 & 1 & -1 & 0 \\
\theta_4 & 0.67 & 0.33 & -0.33 & 1 & 0 & 0 \\
\theta_5 & 0.33 & 0.67 & 0.33 & 0 & 0 & 0
\end{array}
$$

Note that adding a new injection measurement at bus 3 will have no effect for topology error processing capability. However, the installation of an injection at bus 4 will enable detection of previously undetectable topology errors on branches 4-5 and 2-4. Moreover, adding a phasor measurement at bus 3 will make previously critical branches 2-4 and 3-4, topology error detectable.

4.4.1.3 Optimal Placement of Measurements

The above example illustrates the effectiveness of placing new measurements on transforming previously undetectable branch errors into detectable ones. However, this process should also optimize the cost in order to accomplish the most optimal investment. Hence, the objective is set as making any single branch topology error detectable with minimal measurement installation cost. This problem can be formulated and solved using integer programming as shown below:

$$\min \quad \sum_{i}^{n} c_i x_i$$
$$\textit{subject to} \quad W_c^T \cdot X \geq \hat{1} \tag{4.11}$$

where,

n is the number of candidate measurements,

c_i is the cost of installing candidate measurement,

X is the binary vector, whose entry x_i indicates if the candidate measurement *i* should be chosen:

$$x_i = \begin{cases} 1 & \text{if } i\text{th candidate measurement is chosen} \\ 0 & \text{otherwise} \end{cases}$$

$\hat{1}$ is a vector, whose length is the number of topology error undetectable branches and all entries are equal to 1.

W_c is a binary matrix, which is formed using the binary form of the columns in GM_c that correspond to the topology error undetectable branches. It is defined as

$$W_c(i,j) = \begin{cases} 1 & \text{if } \mathrm{GM_c(i,j)} \neq 0 \text{ \& branch "j" "undetectable"} \\ 0 & \text{otherwise} \end{cases}$$

Note that the matrix W_c provides a compact representation of relations between topology error undetectable branches (columns) and given candidate measurements (rows).

The inequality constraint condition ensures that, for every topology error undetectable branch in the previous network, at least one non-zero elements exist in its corresponding column in the new matrix *GM*. It guarantees that, after improvement, there will be no topology error undetectable branch in the network. Thus, the solution vector X of the above optimization problem will provide the strategic placement of measurement.

For the example five-bus system, there are three topology undetectable branches (corresponding column 4, 5, 6 in GM_c), so its binary matrix W_c is obtained as (in transpose):

$$W_c^T = \begin{array}{c} b_{2-4} \\ b_{3-4} \\ b_{4-5} \end{array} \begin{bmatrix} 0 & 1 & 1 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Solution of integer programming problem for the example system yields a set of measurements including both voltage phasor measurement at bus 3 and power injection measurement at bus 4. Note that only voltage phasor measurement at bus 3 can eliminate the criticality of branch 3-4, while no traditional measurement can accomplish this due to the limitation imposed by the network configuration.

4.4.2 Measurement Placement to Enable Topology Error Identification

The above described procedure which is referred as the first-stage will ensure detection of any single branch topology error in the system, but it will not guarantee identification of these errors. In other words, there may still be some critical pairs of branches in the network, whose topology errors can not be identified by using residual analysis. While it would be nice to allow all branch errors to be identifiable, the cost may be prohibitively large. Instead, a less ambitious, yet practical objective which is to ensure topology error identification for certain set of important branches, is considered.

Accomplishing this objective necessitates a procedure that is similar to the one described in previous section:

1) Identify the existing critical pairs of branches;
2) Determine the candidate measurements that will transform those critical pairs of branches.
3) Choose the optimal set of measurements among the candidate measurements set.

It will be assumed that the measurement system is already optimized using the first-stage procedure, i.e. all network branches are already topology error detectable.

4.4.2.1 Identification of the Critical Pairs of Branches

The topology error unidentifiable branches correspond to the collinear columns in matrix *GM* and they can be identified by "normalizing" each column of *GM* as below:

$$gm'(i,j) = gm(i,j) \times \frac{1}{gm(k,j)} \tag{4.12}$$

where, $gm(k,j)$ is the first non-zero element in column j. Following this normalization, the collinear columns will be identical to each other. Searching for the identical columns in the normalized matrix GM', the critical pairs of branches can be identified.

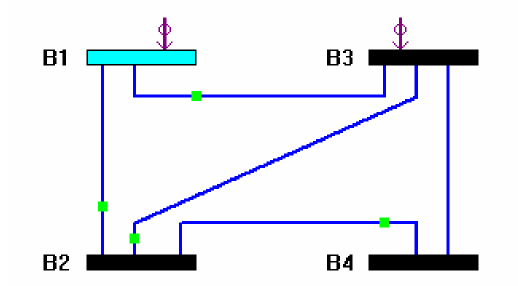

Figure 8 Four-bus Test System

Consider the four-bus example system given in Figure 8. After successful application of the first stage procedure, its measurement system contains no topology error undetectable branches. The matrix GM and its normalized form GM' will be given as below:

$$GM = \begin{array}{c} \begin{array}{ccccc} b_{1-2} & b_{2-3} & b_{1-3} & b_{2-4} & b_{3-4} \end{array} \\ \begin{bmatrix} 0.4 & 0.2 & -0.4 & 0.2 & -0.2 \\ 0.2 & 0.6 & -0.2 & -0.4 & 0.4 \\ -0.4 & -0.2 & 0.4 & -0.2 & 0.2 \end{bmatrix} \end{array}$$

$$GM' = \begin{array}{c} \begin{array}{ccccc} b_{1-2} & b_{2-3} & b_{1-3} & b_{2-4} & b_{3-4} \end{array} \\ \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 0.5 & 3 & 0.5 & -2 & -2 \\ -1 & -1 & -1 & -1 & -1 \end{bmatrix} \end{array}$$

By checking the columns in the matrix of GM', two critical pairs of branches are easily identified: the first branch pair containing branches 1-2 and 1-3 and the second critical pair composed of branches 2-4 and 3-4.

4.4.2.2 Identifying the Relevant Candidate Measurements

Let us define a new matrix U which will have as many columns as the number of critical branch pairs. For the kth critical pair of branches including branch i and branch j, the elements of column k will be given as:

$$u(l,k)=\begin{cases} 1/gm(m,i) & l=i \\ -1/gm(m,j) & l=j \\ 0 & otherwise \end{cases} \tag{4.13}$$

where, $gm(m,x)$ is the first non-zero element in column x of the matrix GM. The two non-zero elements corresponding to branches i and j must be in same row.

Define a new matrix P as the product:

$$P = GM_c \cdot U \tag{4.14}$$

where, a non-zero element $p(i,j)$ in the matrix P indicates that the ith candidate measurement will transform the jth critical pair of branches by making their branch errors identifiable.

Again, the above given four-bus example system will be used to illustrate this step. There are 2 critical pairs of branches, the candidate measurements set includes 3 phasor measurements at buses 2, 3 and 4, a flow measurement in branch 3-4, and 2 injections at buses 2 and 4. The matrix U and GM_c for this system will be given as:

$$U=\begin{bmatrix} 2.5 & 0 \\ 0 & 0 \\ 2.5 & 0 \\ 0 & 5 \\ 0 & 5 \end{bmatrix}$$

$$GM_c = \begin{array}{c} P_2 \\ P_4 \\ P_{3-4} \\ \theta_2 \\ \theta_3 \\ \theta_4 \end{array} \begin{bmatrix} -0.2 & 0.4 & 0.2 & -0.6 & 0.6 \\ 0.2 & -0.4 & -0.2 & 0.6 & -0.6 \\ -0.2 & 0.4 & 0.2 & -0.6 & 0.6 \\ 0.6 & -0.2 & 0.4 & -0.2 & 0.2 \\ 0.4 & 0.2 & 0.6 & 0.2 & -0.2 \\ 0.6 & -0.2 & 0.4 & 0.8 & 0.2 \end{bmatrix}$$

Substitute them into the product $P = GM_c \cdot U$:

$$P = \begin{array}{c} P_2 \\ P_4 \\ P_{3-4} \\ \theta_2 \\ \theta_3 \\ \theta_4 \end{array} \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 2.5 & 0 \\ 2.5 & 0 \\ 2.5 & 5 \end{bmatrix}$$

Note that the traditional measurements including two injections and a flow measurement can not affect the error identifiability of the critical pair, while the phasor measurement at bus 4 will transform both critical pairs and enable error identification when they have topology errors.

4.4.2.3 Optimal Placement of Measurements

This part is similar to the corresponding step of the first stage procedure. Integer programming formulation leads to the following problem:

$$\begin{aligned} &\min \quad \sum_i^n c_i x_i \\ &\textit{subject to } Q^T \cdot X \geq \hat{1} \end{aligned} \tag{4.15}$$

where Q is the reduced binary form of the matrix of P that correspond to the selected critical pairs of branches. Note that the selected critical pairs are those pairs including the important branches. It is defined as

$$q(i,j) = \begin{cases} 1 & \text{if } p(i,j) \neq 0 \text{ and "j" critical pair is selected} \\ 0 & \text{otherwise} \end{cases}$$

Other variables and matrices are the same as in Equation (4.11).

In the above simple example, it is trivial to see the optimal solution as the phasor measurement at bus 4. After the installation of this measurement, the topology error processing capability of the system is further improved so that any single branch topology error will not only be detectable but also identifiable.

4.4.3 Algorithm for Two-stage Optimal Placement

Having described the first and second stage procedures in previous two sections, the overall procedure which combines these two stages is summarized as below:

Stage I: Improving topology error detection

Step 1) Form the measurement Jacobian H and measurement-to-branch incidence matrix M based on the network and available measurement set. Form H_c and M_c based on the available candidate measurements including both traditional and phasor measurements.

Step 2) Partition H and M according to essential and rest redundant measurements:

$$H = \begin{bmatrix} H_1 \\ H_2 \end{bmatrix} \qquad M = \begin{bmatrix} M_1 \\ M_2 \end{bmatrix}$$

Step 3) Calculate the matrices GM and GM_c from Equation (4.10). Also calculate the binary form matrix W_c from GM_c.

Step 4) Solve the integer programming problem

$$\min \quad \sum_{i}^{n} c_i x_i$$
$$subject\ to\ W_c^{\ T} \cdot X \geq \hat{1}$$

to get the optimal placement of measurements.

Stage II: Improving topology error identification

Step 1) Based on the essential measurements obtained in Stage I, reorder the measurements so that H and M are partitioned as:

$$H = \begin{bmatrix} H_1 \\ H_2 \end{bmatrix} \qquad M = \begin{bmatrix} M_1 \\ M_2 \end{bmatrix}$$

Note that now the sub-matrices H_1 and M_1 are still the same, but the sub-matrices H_2 and H_2 include the extra installed measurements decided in Stage I.

Step 2) Recalculate the matrices GM and GM_c. Identify the existing critical pairs of branches in the system by checking the matrix GM.

Step 3) Form the matrix U for all critical pairs of branches based on its definition in Equation (4.12). Then, based on the selected critical pairs, obtain its reduced binary matrix Q.

Step 4) Solve the integer programming problem

$$\min \quad \sum_{i}^{n} c_i x_i$$
$$subject\ \ to\ \ Q^T \cdot X \geq \hat{1}$$

to get the optimal placement of measurements.

4.5 Simulation Results

A program is developed in order to simulate the proposed procedure and evaluate its performance. Two test systems with different sizes are used for the simulations. Section 4.5.1 and 4.5.2 show the placement procedure applied to these two systems. Section 4.5.3 illustrates the benefits of new measurement placement, where a previously undetected topology error becomes detectable and identifiable as a result of new measurement placement. Integer programming problem is solved using the TOMLAB Optimization Toolbox [47].

There are three assumptions in the simulations: (1) there is no upper limit for placing PMUs, so that every bus is treated as a candidate for PMU placement; (2) some injection measurements are also considered as candidate measurements, i.e. any bus without an injection measurement is a candidate for injection measurement placement; (3) the cost of a PMU installation is assumed to be twice the cost of installing an injection measurement. However, these assumptions are made mostly for convenience and the proposed method also can work well without these assumptions.

4.5.1 14-bus Test System

As shown in Figure 9, a modified version of IEEE 14-bus system is used to test the performance of the proposed method. Using the real power linear $P-\theta$ model, only the real power injection and branch flow measurement are considered. The test system has 18 branches and a total of 19 measurements, including 10 branch flow measurements and 9 injection measurements.

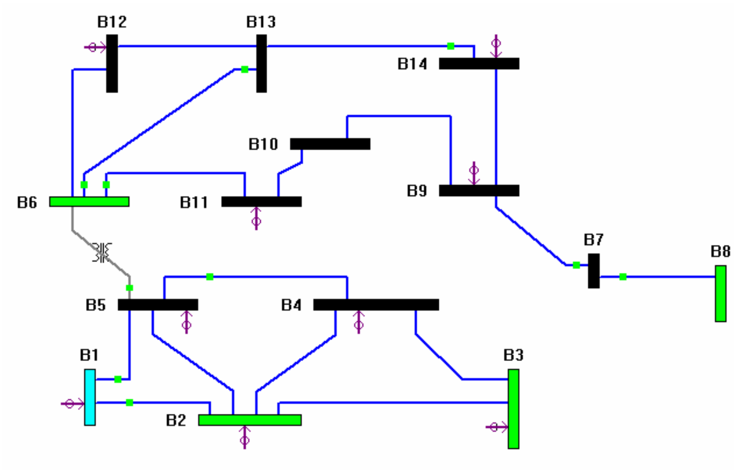

Figure 9 14-bus Test System

Using the method proposed in above sections, there are 5 branches identified as topology error undetectable, which are listed in Table 4.

Table 4 Topology Error Undetectable Branches in Test System

Branch	Locations
1	Bus 5 to Bus 6
2	Bus 7 to Bus 8
3	Bus 7 to Bus 9
4	Bus 6 to Bus 12
5	Bus 12 to Bus 13

By solving the integer programming problem as Equation (4.11), the first-stage optimal placement of candidate measurements is obtained. The optimal solution yields a selection including voltage phasor measurement at bus 8 and injection measurement at bus 13. As a result of adding these two measurements to original measurement set, any single branch topology error in the network can be detected by residual analysis.

By checking the improved system, it is indicated that there are 14 critical pairs of branches in the system, as listed in Table 5.

Table 5 Critical Pairs of Branches in 14-bus Test System

Critical pair	Branch	
1	Branch 2-4	Branch 2-3
2	Branch 2-4	Branch 3-4
3	Branch 2-4	Branch 4-5
4	Branch 2-3	Branch 3-4
5	Branch 2-3	Branch 4-5
6	Branch 3-4	Branch 4-5
7	Branch 5-6	Branch 7-8
8	Branch 5-6	Branch 7-9
9	Branch 7-8	Branch 7-9
10	Branch 9-10	Branch 10-11
11	Branch 9-10	Branch 6-11
12	Branch 9-14	Branch 13-14
13	Branch 10-11	Branch 6-11
14	Branch 6-12	Branch 12-13

Assume branch 5-6 is an important tie-line between two areas, and the objective of second-stage improvement is to enable the topology error identification capability in this branch. In other words, the second-stage should eliminate the critical pairs associated with branch 5-6. The solution of the second-stage optimal measurement placement yields only one phasor measurement at bus 9, so that branch topology error on branch 5-6 can be identified.

Considering the more ambitious objective, i.e. eliminating all critical pairs of branches in the system, the solution of the second-stage optimal measurement placement yields a total of 6 phasor measurements at buses 3, 7, 10, 11, 12 and 14.

4.5.2 IEEE 30-bus Test System

In this case, the IEEE 30-bus system is used to test the proposed method. A total of 19 injection measurements and 15 branch flow measurements are already installed in the network, making the system fully observable. The configuration of network and the locations of measurements are shown in Figure 10.

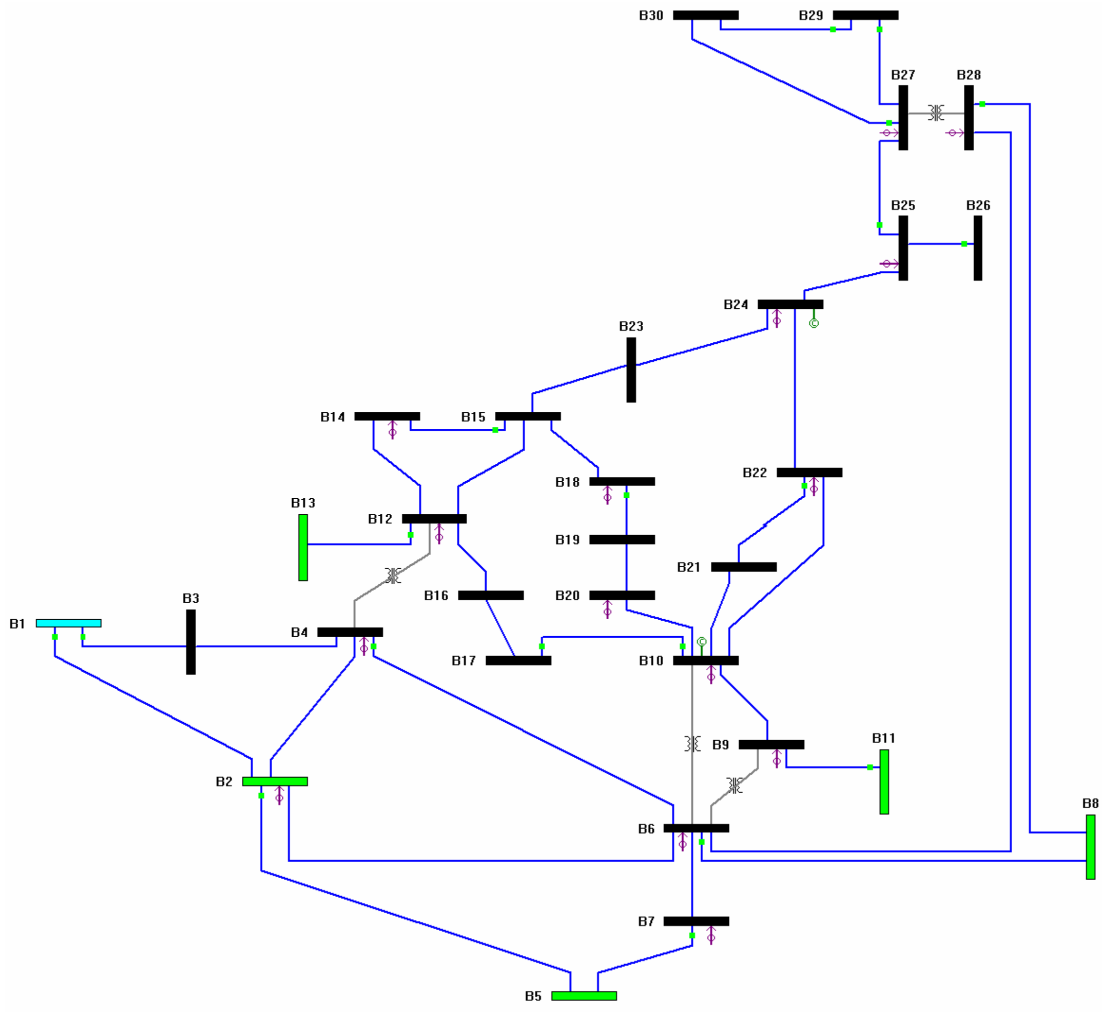

Figure 10 IEEE 30-bus Test System

Among the total 41 branches in the network, there are 9 branches which are topology error undetectable. These branches are listed in Table 6.

Table 6 Topology Error Undetectable Branches in IEEE 30-bus System

Branch	Locations
1	Bus 9 to Bus 11
2	Bus 12 to Bus 13
3	Bus 12 to Bus 16
4	Bus 15 to Bus 12
5	Bus 16 to Bus 17
6	Bus 17 to Bus 10
7	Bus 24 to Bus 23
8	Bus 25 to Bus 26
9	Bus 15 to Bus 23

The solution of the first-stage optimization problem yields a total of 2 injection measurements and 3 phasor measurements in order to eliminate these topology error undetectable branches. The 2 injection measurements should be placed at buses 15 and 16, while 3 phasor measurements should be placed at buses 11, 13 and 26.

After the installation of new measurements, there are still 29 critical pairs of branches in the system. Here, it is assumed that the branches 24-23 and 21-22 are two important branches, which must be topology error identifiable. The second-stage optimal measurement placement yields a total of two phasor measurements at buses 21 and 23, and one injection measurement at bus 19. These additional measurements will transform all critical pairs associated with these two important branches into topology error identifiable branches.

4.5.3 Topology Error Processing Capability

The IEEE 30-bus system is used to illustrate topology error processing benefits gained as a result of new measurement installation. In order to show these benefits, a single branch topology error is assumed to occur in one of the topology error undetectable branches.

First, the state estimation is executed and the result of topology error processing reveals no topology errors as expected. After applying the proposed placement procedure, the same topology error can be detected and even identified.

These are 2 voltage, 15 pairs of injection, and 19 pairs of power flow measurements in the original system. Hence, the number is state variables is 59 ($n=30\times2-1=59$), and the number of measurements is 70. All measurements in the system are assumed to have the same standards deviation of $\sigma=0.001$ p.u.. Chi-squares test is used for topology error detection, and the geometric interpretation of the measurement residuals is used to identify the topology error.

A branch topology error is simulated by disconnecting the branch 23-24, while the network model used by the state estimator erroneously assumed the branch to be in service. In this case, the state estimation fails to detect this error, yielding an objective function value of 0.0037. This value is far below the Chi-square test threshold of 19.6751 (corresponding to 95% confidence level and 11 degrees of freedom).

After introducing new measurements after the first-stage optimal measurement placement, 2 injection measurements are installed at buses 16 and 17, while 3 phasor measurements are installed at buses 11, 13 and 26. Repeating the state estimation with the updated measurement set yields an objective function value of 176.6125, which is clearly above the threshold of 32.6706 (corresponding to 95% confidence level and 21 degrees of freedom). Hence, the Chi-square test will flag the presence of topology error. Furthermore, the geometric interpretation of the measurement residuals is also used to identify the topology error. But the analysis of normal residual returns the possible topology error is in the branch 15-23 and branch 23-24, which form a critical pair of branches.

Hence, the second stage placement procedure is applied, yielding 1 injection and 2 phasor measurements which are installed in the system to enable the topology error identification capability on branches 22-21 and 24-23. The state estimation is executed again, and the real topology error branch 23-24 is then detected and identified correctly.

4.6 Conclusion

This chapter investigates the benefits to state estimation by strategically adding a few measurements, especially the phasor measurements to improve measurement system. It is shown that a few extra measurements can provide increased topology error detection and identification capability. It is also shown that, with the advantage of phasor measurements, it is feasible to eliminate all topology error vulnerable branches in the system by installing new extra measurements.

The optimal strategies can be implemented in order to determine locations and types of few new measurements that will significantly enhance topology error processing capability of a given system. The problem of measurement placement is formulated and solved as two-stage integer programming problem. Each stage will provide the strategic placement of measurements that will improve the topology error processing capability to the desirable level. Simulations on example systems are carried out to verify the effectiveness of the proposed measurement placement schemes in improving the capability of state estimators to detect and identify branch topology errors.

CHAPTER V

CONCLUSION

5.1 Summary

This dissertation is mainly focused on the benefits of introducing PMUs to traditional state estimation, regarding the measurement redundancy, bad data processing and topology error processing. In today's power system control center, state estimation constitutes the primary part of energy management system. It is required to provide the correct and accurate operating state of entire power system to the system operator and other analysis and control functions. Therefore, a robust state estimation should have the capability to detect and identify the gross errors in measurement set and network topology, as well to keep the system observable. However, this capability of state estimation is directly related to the system network configuration and measurement locations. For a given system with low measurement redundancy, critical measurements and branches exist as the deficiencies in the measurement system. It is necessary to enhance the measurement system, so as to ensure the robustness of state estimation against the loss of measurement or branch, and errors in measurement set or network topology. In this dissertation, a new type of measurement, synchronized phasor measurement, is introduced into traditional state estimation and its benefits to state estimation are studied with respect to measurement redundancy, bad data processing and topology error processing. The main achievements of this dissertation are listed as following:

- Illustration of how phasor measurements can be used to improve measurement redundancy and bad data detection and identification capability.
- Development of a new algorithm that is designed for optimal placement of both traditional and phasor measurements, to improve the measurement redundancy of a given system to a desirable level. This allows design of measurement systems with different degrees of vulnerability against loss of measurements and bad data.
- Illustration of how phasor measurements are used to improve topology error detection and identification capability. Phasor measurements are shown to be capable of improving topology error processing capability for cases where this can not be done by the traditional measurements.
- Development of a new algorithm that is designed to obtain the optimal placement of measurements to improve topology error detection and identification. This placement also improves the robustness of state estimation against branch outages.

5.2 Suggestions for Future Research

The study work in this dissertation can be an important basis for future research related to state estimation. In general, future research directions based on this dissertation can be summarized below:

- The benefits of phasor measurements to topology error processing can be studied with respect to different topology error processing methods.

- The possible benefits to state estimation of introducing phasor measurements can be studied with respect to the parameter error processing.
- For optimal measurement placement to enhance the measurement system, other possible contingencies, such as loss of RTU or outage of substation, may be considered in the future research.

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