

ESSAYS ON A MONOPOLIST'S PRODUCT CHOICE AND ITS EFFECT ON  
SOCIAL WELFARE

A Dissertation

by

SUNG ICK CHO

Submitted to the Office of Graduate Studies of  
Texas A&M University  
in partial fulfillment of the requirements for the degree of

DOCTOR OF PHILOSOPHY

August 2012

Major Subject: Economics

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## ABSTRACT

Essays on a Monopolist's Product Choice and its Effect on Social Welfare. (August 2012)

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This dissertation builds on earlier works by analyzing the provision of product quality by a monopolist and comparing that to a social planner. This paper extends the analysis of this problem to the discrete quality setting. Earlier works focused on a continuum of qualities and found no quality distortion for the highest qualities, but downward quality distortion for lower qualities.

The results in the discrete setting differ in that there can be an upward distortion of qualities provided by the monopolist for the highest qualities. The key to this distortion is that the monopolist focuses on the profit that can be extracted from the group of consumers that value quality the most. When there are neither too many nor only a few of these consumers relative to other market segments, it can lead the monopolist to bias its quality provision to extract more value from these consumers. This effect distorts quality at the high-end as compared to the social planner. This upward distortion of quality is found in the real world. In Texas, 30.6% of cable service providers offers an upward distorted service for higher taste consumers.

Besides the quality issue, I also examine how consumer distributions affect on price, profit, and social welfare. Under the various hypothetical consumer distributions, I simulate the above values, and I observe the effect of distribution changes.

When I apply this tool to the real data from Texas cable service industry, I can simulate the consumer type distribution in each franchise, and I can construct the demand curve. Finding consumer type distribution is the key for the demand estimation in this structure.

To my Parents, Wonhee, and So-Yeon

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## CHAPTER I

## INTRODUCTION

In this dissertation, I analyze a monopolist's product quality choice. We can find many firms, including a monopolist, which produce several products of different qualities. For instances, Microsoft produces 6 different editions of Windows 7 and Intel lists three different versions for 2nd Generation Intel® Core™ i7 Extreme Processor. Obviously, both Microsoft and Intel might choose a quality, as well as a price, for each edition or version strategically, considering market responses.

In fact, we have a long research history for the multi-product monopolist, in both sides of theoretical and empirical literature. Since two seminal works of Mussa and Rosen (1978) and Maskin and Riley (1984), numerous have investigated properties of products' qualities, prices, and their market shares. However, only a few papers consider monopolist's quality choice explicitly. Many works, including Mussa and Rosen (1978) and Maskin and Riley (1984), do not need to care about the endogenous quality problem, because they assume a continuous quality provision by the monopolist, so all possible qualities are provided after all. Other works, adopting a finite provision of products, just ignore the problem; they assume exogenously given quality levels. In this dissertation, I try to solve the endogenous quality selections by the monopolist.

When introducing various qualities of products, it is natural to consider different tastes of consumers on qualities. Mussa and Rosen (1978) introduces heterogenous consumers focusing on their responses on quality improvement. Some consumers were

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This dissertation follows the style of *Econometrica*.

supposed gladly to pay a big additional amount for the improvement of product quality, while others were reluctant to pay a sizable money. Targeting these consumers, the monopolist, which Mussa and Rosen (1978) considered, could get a bigger profit if it could offer a products catalog with various quality and price combinations. Technically, we have two different ways to introduce consumer heterogeneity. The first one is to consider only finite number of different types and the second one is a continuum of types.

Except Crawford and Shum (2007), every work adopting a continuum of consumer types does not consider a case that the monopolist is restricted to a finite number of products. In the real world, it is very difficult to find a firm which offers a perfect customization to its customers. One possible reason can be a fixed cost to introduce a new production line. Then, infinite addition of production lines will not be profitable. Besides, some psychology works argue that consumers are usually overwhelmed by tremendous choices. The limited set of offered products can be demanded ironically by consumers. Here, I consider the finite products by the monopolist having the continuous consumer types.

This dissertation's finite product differentiation model with the continuous consumer types can endogenize both the monopolist's quality choices for its products and the market share for each product. As I explained before, the continuous product differentiation with the continuous consumer types cannot endogenize the product quality. When we take the finite product differentiation with the finite consumer types, the model must take exogenous market shares. It is because the probability for each consumer type is given as primitive, and the given probability results in the market share for a product that aims at the consumer type. That is, this dissertation models the situation in which each product's market share is determined by its quality, and the monopolist chooses its product quality considering the product's expected market

share.

In the remaining part of the first chapter, I present an extensive literature review. The review categorizes the existing works into three groups, using the cardinality of provided products and consumer taste types. This taxonomy is quite useful to understand which variables are endogenized or interested in the literature.

In the second chapter, I will construct the endogenous product differentiation problem by both a monopolist and a social planner, in the finite products with the continuous consumer types environment.

First, the chapter arranges optimality conditions for the monopolist. I reproduce the results of Itoh (1983) regarding the optimal pricing scheme and Crawford and Shum (2007) about the optimal quality selection rule. In the quality selection rule, the average marginal benefit of consumers is greater than the increment of marginal cost when improving a quality. It looks similar to the monopolist's optimal pricing scheme obtained in the only quantity relevant setting.

I show that the famous hazard rate assumption<sup>1</sup> plays an important role in the uniqueness of monopolist's solution. It was a crucial assumption for the monotone quality and price schedule, in the works of the continuous product provision with the continuous consumer types.

Second, I reveal optimality conditions for a social planner. The optimality conditions, with zero fixed cost, require the social planner to charge a tariff at the marginal cost, simply equating the tariff to get a product to the marginal cost to produce it. For the quality and the market segmentation, the social planner should equate the average marginal benefit of consumers and the increment of marginal cost when improving a quality.

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<sup>1</sup>The derivative of an inverse hazard rate should be less than 1.

After completing the monopolist's and the social planner's optimality conditions, I develop numerical computation simulation procedures, which find exact optimal (quality and price) values from the optimality conditions. The simulation method and outcomes obtained from the various simulations are discussed in the third chapter.

There, I find possibility of the upward quality distortion for a product aiming at a consumer with the highest taste, comparing the monopolist's optimal choices and the social best outcomes. This is a quite new discovery, which cannot be obtained in any canonical models of both the finite products with the finite consumer types and the continuous products with the continuous consumer types. Only few authors, such as Meng and Tian (2008), reported the possibility, under the restricted setting, such as an industry with network externality. More comparisons between the monopolist's choice and the social planner's choice will be performed in the third chapter.

Besides the comparison, I can find relationships between the optimal price, quality, profit, and social welfare and the outsider parameters, like number of products and parameters for consumer type distribution. That is, I can observe how the optimal price, quality, profit, and social welfare change as the number of products increase. The one possible task is to show that the finite/continuous model approaches to the continuous/continuous model as the number of products increases. I can also show how the product's price, quality, profit and social welfare change as consumer distribution becomes more dispersed or more skewed. The detailed discussions about the distributional changes, like an effect of a more dispersed distribution on social welfare, are presented in the third chapter.

In the chapter IV, I will apply the model developed in the second and third chapters to empirical analysis. There, I simulate the parameters for consumer type distribution, using a real data in the cable industry. And I examine how demographic variables affect the consumer distribution. In the setting of this dissertation, the con-

sumer type distribution contains all information about consumer demand. Thus, we can induce the demand curve for an interested product using the previously simulated values, whenever we can fix consumer demographics and other product's characteristics. I also study the impact of the change of market environment on the demand curve. When a demographic variable changes, the change of the variable alters the consumer distribution. From this new consumer distribution, we can derive a new demand curve. We can examine how the demand changes, shifts, or rotates.

More importantly, I find this possibility of upward quality distortion is quite prevailing in the cable industry. In Texas, 30.6% of franchises shows the upward quality distortion. Especially in Amarillo, the estimated amount of welfare loss, induced by the upward distortion, is up to \$1,048,705 monthly. I expect this discovery will trigger a large volume of research for the upward quality distortion and its welfare loss.

The cable industry is a perfect example for the model developed in this dissertation. The cable service market is monopolized in each city by local authorities, and the cable service providers offer two or three different tiers. Thus, I will examine real data obtained from the cable industry, to support the results obtained in the second and third chapters, and to estimate the demand function.

I conclude this dissertation in the fifth chapter.

In the Appendix A, I thoroughly review the cable industry. The all proofs for propositions, lemmas, and corollaries, appeared in the Chapters II and III, are presented in the Appendix B. In the Appendix C, the uniqueness condition for the social planner's problem is discussed in detail. The explanation for data variables is given in the Appendix D, and the tables and figures are presented in the Appendix E.



## A. Literature Review

To clarify on where this dissertation stands, I start from classifying the existing literatures. The first criterion is whether firms choose price-quality combinations or price-quantity combinations. Where quality is a choice variable, a consumer usually needs and buys only one unit of product. Contrasting the differentiated qualities problem, the authors call the quantity-choosing model a nonlinear pricing model. While Mussa and Rosen (1978) investigated the differentiated qualities problem, many other papers, including the seminal paper Maskin and Riley (1984), have researched the nonlinear pricing problem. In many cases, the quantity variable in the nonlinear pricing model is directly interchangeable with the quality variable in the quality-choosing framework. Almost all results from the nonlinear pricing model can be reconfirmed in the quality-choosing model.

The pattern of competition is the second and more meaningful criterion. Early research focused on the monopolist's problem. Many literatures, which have followed Mussa and Rosen (1978) and Maskin and Riley (1984), have solved and analyzed the monopolist's behaviors and implications. However, the recent interest of researchers has moved to the competitive environment, especially to oligopoly.

In fact, introducing competition is quite difficult in this stream of models. Since price is one of the control variables of the firm, the direct extension to two, three or many firms may collapse down to the Bertrand competition. To avoid this collapse to the marginal cost pricing, some authors have assumed that the qualities are pre-determined. Firms can compete only with prices, at their own quality levels. Then, firms choose price, considering the incentive compatibility constraints. Even though this (pre-determined quality) restriction seems too expedient, it is quite useful, especially in the empirical works. (See Bresnahan (1981), Bresnahan (1987), Shepard

(1991), and Balan and Deltas (2010).) Others have developed a two-stage model; at the first stage, firms choose quality levels for their products, and at the second stage, they compete with prices. (See Champsaur and Rochet (1989) and Lahmandi-Ayed (2000).) Others have introduced different entry timing for firms, usually in the form of incumbent and entrant. (See Bonnisseau and Lahmandi-Ayed (2006).) Choi and Shin (1992), Donnenfeld and Weber (1992), and Donnenfeld and Weber (1995) modeled both two-stage quality-price choice and different entry timing of firms. More interestingly, the others have introduced a multi-dimensional consumer heterogeneity model. Unlike the canonical model of Mussa and Rosen (1978), where consumers differ only in the willingness to pay for quality improvement, consumers can be different in their location from stores, their brand loyalty for firms, and etc. In the model, firms may recover their market power for some of consumer segments. (See Ivaldi and Martimort (1994), Stole (1995), Rochet and Stole (1997), Rochet and Stole (2002), and Miravete and Röller (2004).)

The dimension of product characteristics is the third criterion . While Mussa and Rosen (1978) developed one-dimensional (quality) product characteristic, many descendants have introduced products with multiple characteristics. (See McAfee and McMillan (1988) and Armstrong (1996).) In this dissertation, I concentrate on only the canonical one-dimensional quality model.

In summary, when we are going to classifying the literatures, the first question is whether the paper adopts the quantity-choosing framework or the quality-choosing. The second question should be whether the paper works in the monopolistic market or the oligopolistic market. The last criterion is about the cardinality of product characteristics and consumer heterogeneity.

From now on, I will extensively review literatures. In there, I categorize papers into three groups; a finite product differentiation model with finite consumer types,

a continuous product differentiation model with continuous consumer types, and a finite product differentiation model with continuous consumer types, as I mentioned before. And in the each category, I ask the above three criteria to find the paper's exact position in the literature.

At first, consider the finite product differentiation model with finite consumer types. Donnenfeld and White (1988) is one of the earliest paper, which adopted the Mussa and Rosen framework and modified it to the finite product and finite type setting. They explicitly solved two-qualities and two-types model and showed various comparative statics results. We can find the extensive discussion for the general  $n$ -quality and  $n$ -type model in the chapter 6 of Wilson (1993). Besanko, Donnenfeld, and White (1988) added various constraints to the finite/finite model, to explain how regulations affect results. Salant (1989) extended the model to dynamic situation. In the paper, a product provides utility for a long time after purchasing. He applied this framework to the famous durable good monopoly problem. The model can be also applied to network externality problem. Kim and Kim (1996) added an interaction term of offered products in social welfare function, and so they can explain spill-over effect. Csorba (2008) introduced the network externality term in the utility function. Meng and Tian (2008) modified the individual rationality condition to incorporate the network externality, and analyzed the direction of quality distortion.

We can also find some empirical applications of this finite/finite model. Goolsbee and Petrin (2004) investigated the impact of the direct broadcast satellites on the cable television industry. In doing so, they divided consumers into several (finite) number of groups according to their income levels. Moreover, it is well known that the cable providers offer several (finite) number of different services. In addition, they assumed that the quality levels for these services were exogenously fixed. Unlike Goolsbee and Petrin (2004), the following papers have taken the firms which choose

the quality levels. Ghose and Sundararajan (2005) estimated the degree of quality degradation using the different editions of softwares. Hernandez and Wiggins (2008) and Dai, Liu, and Serfes (2010) applied the finite/finite framework to explain the effect of market concentration on the prices in the airline industry. While all other papers investigated the monopolist's problem, Goolsbee and Petrin (2004), Hernandez and Wiggins (2008), and Dai, Liu, and Serfes (2010) dealt the competitive environment. To do so, the works employed two-dimensional consumer heterogeneity.

A second literature focuses on continuous product differentiation and continuous consumer types. This continuous/continuous model includes the pioneering works of Mussa and Rosen (1978) and Maskin and Riley (1984). Besanko, Donnenfeld, and White (1987) further investigated quality distortion and also evaluated the effect of government regulation. Bousquet and Ivaldi (1997) analyzed network externality, directly included it in the utility function, and they estimated the demand side parameters using residential telephone data in France. Miravete (2002) also estimated the demand side parameters under the two-part tariff convention using U.S. data. Basaluzzo and Miravete (2007) introduced two-dimensional consumer heterogeneity in the monopolistic industry. They investigated the pricing scheme using local cellular telephone data in U.S.

Besides the above works for the monopolist, we also have many applications to the competitive environment. The applications include Champsaur and Rochet (1989), Ivaldi and Martimort (1994), Stole (1995), Rochet and Stole (1997), Rochet and Stole (2002), and Miravete and Röller (2004). They all adopted the continuous/continuous environment. While other papers mainly devoted to the theoretical works, Miravete and Röller (2004) estimated the nonlinear pricing scheme under the U.S. cellular phone competition.

Contrary to the finite/finite model, the continuous/continuous model doesn't

have the endogenous quality issue. In this model, the decision maker chooses a price schedule, which is a function to assign price to each quality level. She doesn't need to sort out quality levels. The only quality-related problem is to assign which quality level to which consumer type.

At third, I consider the finite product differentiation model with continuous consumer types. Since the decision maker should choose only finite number of qualities over the full support of qualities, it becomes an important topic which qualities should be offered. One easy way to deal with the quality selection problem is taking it exogenously given. Itoh (1983) constructed the monopolist's problem, solved an optimal pricing rule and found some comparative statics results including relationship with the number of products. However, he could not solve an optimal quality conditions, since he assumed the exogenously given qualities, so he didn't need to do. Gabszewicz, Shaked, Sutton, and Thisse (1986) also assumed the exogenously given qualities. In addition, they assumed that consumer types (income, rather than willingness to pay for quality improvement) are distributed uniformly. Upon these assumptions, they could show the market segmentation explicitly, and relate the finite product case to the infinite product case, as increasing the number of products to infinity. This was the first work to connect the finite/continuous model with the continuous/continuous model.

Verboven (2002b) tried to estimate the monopolist's market power empirically. In his reduced form estimations in a spreadsheet software market and a car engine market, he used comparative statics results obtained from the exogenously given quality model. Xia and Sexton (2010) extends this monopoly problem to the dynamic environment, to deal with durable goods. They also stuck to the exogenously given quality levels. We can find the network externality application in the finite/continuous model, too. Jing (2007) inserted the network externality into the utility function. He

compared the optimal prices with and without network externality. While doing so, he assumed the pre-determined quality level and the uniform consumer type distribution. In the competitive environment, the exogenously given quality assumption prevails, too. Bresnahan (1981), Bresnahan (1987), Shepard (1991), and Balan and Deltas (2010) commonly assumed the exogenously given quality level. Especially, first three papers proceeded to empirical works in the automobile market and the retail gasoline market.

Related with the empirical works, the framework from Berry, Levinsohn, and Pakes (1995), the famous paper in the empirical field, has been widely applied in our context, too. Basically, their work is constructed on the exogenously given product characteristics. Suppose that we have only one observed product characteristic, and further it is a quality level. Then, we can interpret the random coefficient for the characteristic as a vertical taste type, and the idiosyncratic error as a horizontal consumer type. This connection is already pointed out by Rochet and Stole (2002).

Here, I document quality-related (or quantity-related) works using the BLP method. We have several number of papers, which assumed finite exogenous qualities, and adopted the vertical consumer heterogeneity (quality taste) as the form of a random coefficient and the horizontal consumer heterogeneity (brand loyalty) as the form of an error term. Verboven (2002a) adopted two discrete, and exogenously given, qualities; a gasoline engine and a diesel engine. Firms chose one of them. Consumers were different in their annual mileage, which interacts with the engine choice. Leslie (2004) examined the pricing practice of Broadway theaters. They provided several different classes of seats. Mortimer (2007) also adopted similar strategy in the VHS/DVD movie market. Her quality choice was selling or renting. Lustig (2010) applied the framework to the privatized medicare system. In his work, the generosity of coverage by an insurance plan can be interpreted as the quality in our

framework. He considered two or three insurance plans with different qualities. McManus (2007) estimated the nonlinear pricing model. For his quantity variable, he took the specialty coffee sizes. Cohen (2008) is another empirical example for the nonlinear pricing model. He listed the several package sizes for paper towels.

Our last concern is the endogenous quality choice in the finite/continuous model. Crawford and Shum (2007) is the only paper, which dealt with the endogenous quality issue for the monopolist's problem. They imitated the framework from Itoh (1983), but they designed the endogenous qualities and found the condition for optimal qualities. They could gauge the degree of quality distortion in the monopolized cable market, using their theoretical results from the finite/continuous model. Here, it is worth noting that their central results are based upon variation in shares and that similar predictions could be obtained from a discrete/discrete model.

We have some papers, which investigated the endogenous quality issue in the competitive environment. Choi and Shin (1992) is one of the earliest work to deal with the endogenous quality problem explicitly. They constructed the competitive structure, based on the sequential entry of firms and the sequential choice of quality and price. To get an explicit solution, they assumed that the consumer types are uniformly distributed and the marginal cost is zero. Under these restrictive assumptions, they can solve the optimal quality levels for the firms. Donnenfeld and Weber (1992) and Donnenfeld and Weber (1995) took the same environment with Choi and Shin (1992). The only difference was that they considered three-firm competition.

Lahmandi-Ayed (2000) extended the above works to the general  $n$  firms competition. He adopted simultaneous entry of firms, instead of sequential entry, but still kept the sequential choice of quality and price. He introduced the strictly positive marginal cost, but the change of the marginal cost is restricted within the narrow range. The one common characteristic for the above four papers is that each firm

produces just one product. Of course, the competing firms should consider the incentive compatibility constraint, furnished by other firms, but the consideration pattern is not same with a multi-product monopolist. Bonnisseau and Lahmandi-Ayed (2006) introduced multi-product competing firms. The basic setup was similar to Choi and Shin (1992). There were an incumbent firm and an entering firm. The consumer types were uniformly distributed, and the marginal cost was zero. However, it is allowed for the incumbent to produce multiple products. Chu (2010) is another paper with the multi-product competing firms. He followed the incumbent-entrant framework, but didn't adopt the price competition after establishing quality level. The incumbent offered a quality and price combination, and then the entrant suggested a new combination. Instead of quality-price sequence, he introduced BLP-like horizontal consumer heterogeneity as a source of competition. The vertical consumer types, inserted in the form of a random coefficient, is assumed to follow the Weibull distribution. He estimated the impact of a direct broadcast satellite on the monopolized cable market.



## CHAPTER II

ENDOGENOUS AND FINITE PRODUCT DIFFERENTIATION BY A  
MONOPOLIST

## A. Introduction

It is very common for a firm to offer differentiated products in various grades. Examples include airlines, Broadway plays, grades of gasoline, and meal size in fast food restaurants. These products are close substitutes each other, but differ in quality.

This differentiation of products arises, because we have consumers with differing tastes for quality. Mussa and Rosen (1978), a monumental and actual beginning work of the product differentiation, introduces heterogeneous consumers focusing on their responses on quality improvement, as I described in the first chapter.

For the continuously distributed heterogeneous consumers, Mussa and Rosen (1978) and other literature find a monopolist's continuous product schedule, which maps all possible quality levels to price levels. Each consumer can find a uniquely appealing product. Although finding the continuous product schedule is theoretically interesting, firms in the real economy seldom provide the function, whose domain is every possible quality level. Usually, firms compose a menu with finite number of entries. In the continuous product differentiation model, the monopolist focuses only on optimal pricing scheme. Because it will provide products in every quality level, it does not need to solve for optimal quality level for each product in its catalog. In the finite product differentiation model, however, finding optimal quality levels and constructing optimal market share for each product become its task, in addition to finding optimal price levels.

In almost all the literature, especially Itoh (1983), discussing finite product and

continuous types, authors assume that quality levels were exogenously given to avoid the quality choice problem. This assumption makes the problem very tractable. Only small number of papers adopt the endogenously determined quality level.

In the real world, many firms can adjust their product qualities. Windows Media Center was not included in Windows Vista Home Basic edition, but it was later included in Windows 7 Home Basic edition. Obviously, Microsoft policies dictate which features would be included in each edition. This research challenges the endogenous quality problem in the environment with finite products and continuous types. Only a few papers, such as Choi and Shin (1992), Crawford and Shum (2007), and Chu (2010), precede this work.

Although the huge volume of relevant literature has discovered numerous valuable results, the above mentioned topic, the endogenous quality problem in the finite/continuous environment, still remains mostly unexplored. Specifically in the monopolist's problem, only Crawford and Shum (2007) considers endogenous qualities. For the problem, we need to establish an optimal pricing rule, an optimal quality selection, and market segmentation. Itoh (1983) establishes the monopolist's optimal pricing rule and the market segmentation with exogenously given quality levels, and Crawford and Shum (2007) endogenizes the quality selection rule.

However, we have no comparative statics results, except the work of Itoh (1983) between monopolist's profit and the number of products. In fact, no work examines the effect of consumer type distribution changes in the finite/continuous setting. In the real economy, firms may offer different product catalogs, in geographically separated markets. They may change their menus when the economy goes into a boom or a bust. If we know how the distribution changes affect the optimal choices by the monopolist, we can explain more about firms' business practices.

Furthermore, we know nothing about a social planner's problem in the setting

of finite/continuous environment. The comparison, between the social planner's first best choice and the firms' optimal choice constrained by the consumer's choice, is the first step for social welfare analysis. Both in the finite/finite and the continuous/continuous model, every social welfare research includes the analysis of the social planner's problem. However, we cannot find any preceding analysis for the social planner's problem in the finite/continuous model. Existing literature compares the firms' choice obtained from the finite/continuous setting with the social planner's choice obtained from the continuous/continuous setting.

This chapter constructs the endogenous product differentiation problem by both a monopolist and a social planner, in the finite/continuous model. In the problem, quality, as well as price, is also a control variable of a decision-maker.

At first, this work arranges the optimality conditions for the monopolist. I reproduce the results of Itoh (1983) regarding the optimal pricing scheme and Crawford and Shum (2007) about the optimal quality selection rule. At second, I reveal the optimality conditions for a social planner. The social planner should equate the average marginal benefit of consumers and the increment of marginal cost when improving a quality. While I find the optimality conditions, I also examine the conditions to guarantee the existence and uniqueness of solutions for the maximization problems. I describe the conditions in detail, in the sections C and D. In the continuing section, E, I present various relationships between the monopolist's and social planner's optimal choices, profit, and social welfare and the number of products. Conclusions are drawn in the section F.

## B. Model

The model adopted here is derived from Mussa and Rosen (1978). In this seminal paper, they analyze a vertically differentiated product market with one-dimensional heterogeneous consumers. In the market model, every product produced by a monopolist can be associated with a one-dimensional quality index. An individual consumer differs from another in that she has a different sensitivity to increase of quality. Consumers can be ordered by their sizes of the sensitivity.

First, I introduce a product and its producer, a monopolist. The products produced by the monopolist are same except for the quality,  $q$ . The monopolist can produce products at various quality levels. In this work, the monopolist can produce only finite numbers of different products, which differs from Mussa and Rosen (1978), but is similar to Itoh (1983). When the monopolist produce  $n$  different quality products, I list the products as  $q_1, q_2, \dots, q_n$ . The monopolist has a marginal cost function,  $C(q)$ , to produce a product of a quality  $q$ . This marginal cost is constant, regardless of the quantity of products, for any given quality level. That is, the total cost to produce  $n$  unit of products, whose quality level is  $q$ , is  $nC(q)$ . The monopolist can set price on each its product. These prices are designated as  $p_1, p_2, \dots, p_n$ .<sup>1</sup> Naturally,  $p_k$  is the price for the product  $q_k$ . For the notational convenience, I denote  $\mathbf{q} \equiv (q_1, q_2, \dots, q_n) \in \mathbb{R}^n$  and  $\mathbf{p} \equiv (p_1, p_2, \dots, p_n) \in \mathbb{R}^n$ .

For the marginal cost function, I should introduce an assumption, which will be maintained throughout the whole dissertation. All things in the Assumption 1 are standard in the literature.

**Assumption 1.** *The marginal cost function  $C$  is twice differentiable, strictly increas-*

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<sup>1</sup>When we consider a social planner's problem,  $p$  can be considered as a tariff. A consumer, to whom a social planner assigns  $q_k$  and who accepts this assignment, should pay a tariff,  $p_k$ .

ing and strictly convex. That is,  $C' > 0$  and  $C'' > 0$ . In addition,  $C(0) = 0$  and  $\lim_{q \rightarrow 0} C'(q) = 0$ .

Second, I discuss the continuum of consumer types. Some consumers highly value quality improvement, while others obtain only small increase in their utility from a quality improvement. I index the consumers as  $\theta$ . The consumer taste type  $\theta$  is distributed in  $[0, \bar{\theta}]$ , following a distribution function  $F$  and its density function  $f$ . Now, I can present a utility function of a consumer of type  $\theta$  when she consumes a product  $q$  paying a price  $p$ ;

$$(2.1) \quad U(q, p; \theta) = \theta q - p.^2$$

### 1. Market Segmentation and Demand of Product

Before diving into a main problem, I investigate a consumer segment for each product. The utility function (2.1) shows separability of  $q$  and  $p$  and linearity in  $\theta$ . These characteristics guarantee that consumers would be divided into several pieces according to their types.

To check a consumer's choice pattern, suppose that  $n$  products of (already chosen) different qualities are selling in the market, assuming  $q_1 < q_2 < \dots < q_n$ . In order that every provided products are meaningful,  $p_1 < p_2 < \dots < p_n$ . If  $p_1 > p_2$ , nobody would like to buy the product  $q_1$ . Under this setup, we can divide a market (or consumers) into  $n + 1$  segments. Consumers in the same segment buy a same product or all buy nothing. In addition, I assume that a consumer, who is indiffer-

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<sup>2</sup>We can easily generalize this utility form to  $U(q, p; \theta) = V(q; \theta) - p$ . In fact, we can recover almost all properties, which discovered in this dissertation, in this generalized form, when  $V$  is concave in  $q$ . However, this generalization makes our problem tremendously and unnecessarily complex. We can find all essential results from the simple linear form.

ent between two different products, selects a higher quality one. This is a standard tie-breaker.

Tentatively, assume that for all  $k \in \{1, 2, \dots, n-1\}$ ,

$$(2.2) \quad (p_{k+1} - p_k)(q_k - q_{k-1}) > (p_k - p_{k-1})(q_{k+1} - q_k),$$

where  $q_0 = 0$  and  $p_0 = 0$ . Technically, buying nothing will be replaced by buying  $q_0$  with its price  $p_0$ . In the whole dissertation,  $(q_0, p_0)$  will be considered as buying nothing. The equation (2.2) is just for that all  $n$  products are sold in the market. Let  $\theta_k \equiv (p_k - p_{k-1})/(q_k - q_{k-1})$ , then the above equation guarantees that for all  $k \in \{1, 2, \dots, n-1\}$ ,  $\theta_k < \theta_{k+1}$ .

**Proposition II.1. (Market Segmentation)** *Assume that the equation (2.2) holds. Then, for all  $k \in \{0, 1, 2, \dots, n\}$ , a consumer of type  $\theta$ , who belongs to  $[\theta_k, \theta_{k+1})$ , will buy  $q_k$ , where  $\theta_0 = 0$  and  $\theta_{n+1} = \bar{\theta}$ .<sup>3</sup>*

The Proposition II.1 shows that  $n$  cutoffs divide the market into  $n+1$  segments. The  $n+1$  exclusive segments compose the whole support for consumer types. A consumer, who belongs to the upper segment, consumes a higher quality product paying a higher price.

This market segmentation is the first step to calculate a market demand for each product. We know all consumers in  $[\theta_k, \theta_{k+1})$  want to buy a product  $q_k$  with a price  $p_k$ . If we know an exact consumer type distribution and a whole market population, we can calculate the number of consumers will buy a product  $q_k$ . Here, I add an assumption for a consumer distribution.

**Assumption 2.** *The distribution function  $F$  is twice differentiable and strictly increasing.*

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<sup>3</sup>The last cutoff,  $\theta_{n+1}$ , is considered  $\bar{\theta}$ , whenever a problem chooses  $n$  products.

Mussa and Rosen (1978) and some other works admit a upward jump of  $F$ . In this research, I will not allow the mass point, which may make the problem extremely messy. The Assumption 2 will be maintained throughout the whole dissertation.

Now, we can calculate the demand for each product. We know that consumers in the segment  $[\theta_k, \theta_{k+1})$  will buy the product  $q_k$ , and other segments will not buy the product  $q_k$ . Thus, for all  $k \in \{0, 1, \dots, n\}$ , the demand for the product  $q_k$ ,

$$D(q_k) = M \int_{\theta_k}^{\theta_{k+1}} dF(\theta),$$

where  $M$  is a whole market population and  $D(q_0)$  implies the volume who buy nothing.

Note that cutoff levels play crucial role to calculate a demand for each product. For notational convenience in the rest of this dissertation, I denote the set of every cutoffs as  $\boldsymbol{\theta} \equiv (\theta_1, \theta_2, \dots, \theta_n)$ . Additionally, I normalize  $M = 1$ . In fact, the size of  $M$  does not play a special role except scale change.

### C. The Monopolist's Problem

It is time to take a monopolist's problem. The monopolist maximizes its total (or aggregate) profit earned from every segmented markets.

$$(P.M) \quad \max_{\mathbf{q}, \mathbf{p}} \sum_{k=1}^n D(q_k)(p_k - C(q_k))$$

$$\text{such that for all } k, \theta_k q_k - p_k = \theta_k q_{k-1} - p_{k-1}.$$

Before proceeding to solve the problem directly, I'll examine the property of prices. From the constraints, we know that for all  $i$ ,

$$p_i - p_{i-1} = \theta_i(q_i - q_{i-1}).$$

Then,  $p_k = \sum_{j=1}^k \theta_j (q_j - q_{j-1})$ . Now, we can rewrite the problem (P.M) as follows.

$$(P.M') \quad \max_{\mathbf{q}, \boldsymbol{\theta}} \sum_{k=1}^n D(q_k) \left( \sum_{j=1}^k \theta_j (q_j - q_{j-1}) - C(q_k) \right).$$

In this problem, there is no  $p_j$ s, and a control variable  $\mathbf{p}$  is replaced by  $\boldsymbol{\theta}$ . Surprisingly,  $D(q_j)$  is not a function of  $q_j$  any more, because  $\theta_j$  and  $\theta_{j+1}$  are free variables now. When  $q_j$ s are given as  $q_1 < q_2 < \dots < q_n$ , we can establish a one-to-one relationship between  $\mathbf{p}$  and  $\boldsymbol{\theta}$ . Then, the problem (P.M) and the problem (P.M') are equivalent.

Let a combination  $(\mathbf{q}^m, \boldsymbol{\theta}^m)$  be a maximizer for the monopolist's problem (P.M'). Using the constraints of (P.M),  $\mathbf{p}^m$  can be obtained. The first result is existence of the maximizer in the monopolist's problem (P.M'), that is (P.M).

**Proposition II.2. (Existence of a Maximizer in the Monopolist's Problem)**

*Under the Assumptions 1 and 2, the maximizer  $(\mathbf{q}^m, \boldsymbol{\theta}^m)$  for the problem (P.M') exists.*

Before presenting the next proposition, I should introduce another assumption about distribution of consumers. This assumption is quite general for continuous distributions with a truncated interval. Under this assumption, any type in the support can be realized.

**Assumption 3.** *For all  $\theta \in [0, \bar{\theta}]$ ,  $f(\theta) > 0$ .*

The next proposition suggests conditions the maximizer for the problem (P.M') should satisfy. In this proposition and its proof, I implicitly assume that  $q_1^m < q_2^m < \dots < q_n^m$ . In fact, the implicit assumption is true. It will be proved in the section E.



**Proposition II.3. (Monopolist's Problem)** *Under the Assumptions 1, 2 and 3, for all  $k$ ,  $q^m$ ,  $p^m$  and  $\theta^m$  satisfy*

$$p_k^m = \sum_{i=1}^k \theta_i^m (q_i^m - q_{i-1}^m),$$

$$\left[ \theta_k^m - \frac{1 - F(\theta_{k+1}^m)}{F(\theta_{k+1}^m) - F(\theta_k^m)} (\theta_{k+1}^m - \theta_k^m) \right] = C'(q_k^m), \text{ and}$$

$$\theta_k^m - \frac{1 - F(\theta_k^m)}{f(\theta_k^m)} = \frac{C(q_k^m) - C(q_{k-1}^m)}{q_k^m - q_{k-1}^m}.$$

The Proposition II.3 is not a brand new result. The first and the third equations in Proposition II.3 appeared in Itoh (1983), which is the first work adopting the finite/continuous framework. Itoh (1983), however, assumes the exogenously given quality levels. The second equation is from Crawford and Shum (2007).

The first equation explains the monopolist would not take a marginal cost pricing. Moreover, the first and the third equations show that the price is greater than the marginal cost.

$$\begin{aligned} p_k^m &= \sum_{i=1}^k \theta_i^m (q_i^m - q_{i-1}^m) = \sum_{i=1}^k \left( \frac{1 - F(\theta_i^m)}{f(\theta_i^m)} + \frac{C(q_i^m) - C(q_{i-1}^m)}{q_i^m - q_{i-1}^m} \right) (q_i^m - q_{i-1}^m) \\ &= \sum_{i=1}^k \left( \frac{1 - F(\theta_i^m)}{f(\theta_i^m)} (q_i^m - q_{i-1}^m) \right) + C(q_k^m) > C(q_k^m). \end{aligned}$$

In the second equation, the increment of marginal cost does not match with the average marginal benefit of consumers. In fact, the increment is much smaller than the average marginal benefit, since

$$C'(q_k^m) = \left[ \theta_k^m - \frac{1 - F(\theta_{k+1}^m)}{F(\theta_{k+1}^m) - F(\theta_k^m)} (\theta_{k+1}^m - \theta_k^m) \right] < \theta_k^m < \mathbb{E}(\theta | \theta \in [\theta_k^m, \theta_{k+1}^m]).$$

In the third equation, we have an inverse hazard rate term, which will not appear in the social planner's optimality condition. Assuming that the monopolist takes a marginal cost pricing, the market is segmented, as dictated by the third equation

without a term,  $-(1-F(\theta_k^m))/f(\theta_k^m)$ . We can conjecture that the monopolist's market segmentation will shift upward, comparing to the case of marginal cost pricing.

The next proposition states uniqueness of the monopolist's maximizer. For this proposition, I add an assumption that is widely used in the literature, including Maskin and Riley (1984) and Itoh (1983).

**Assumption 4.** *Let  $H(\theta) \equiv (1 - F(\theta))/f(\theta)$ . Then,  $H'(\theta) < 1$ .*

In the continuous/continuous model, Assumption 4 ensures that the monopolist monotonically assigns qualities on consumer types. Here, it ensures uniqueness of the maximizer.

**Proposition II.4. (Uniqueness of a Maximizer in the Monopolist's Problem)** *Under the Assumptions 1, 2, 3 and 4, the maximizer  $(\mathbf{q}^m, \boldsymbol{\theta}^m)$  for the problem (P.M') is unique.*

Finding this set of sufficient conditions for the uniqueness for the monopolist's optimal solution is a quite substantial contribution. When we try to find a solution by computer simulations, we should guarantee uniqueness of the solution, in order that we should assure that the answer, a computer gives us, is what we are looking for. Moreover, the most important condition, the Assumption 4, is very familiar with us, and it is known to be a quite weak one.

#### D. Social Planner's Problem

In this section, I explain the first best outcome of a benevolent social planner, which is never explored before. I consider a market having  $n$  meaningful products, like in the last section. Since the benevolent social planner wants to maximize aggregated

social welfare, the problem is as follows;

$$(P.S) \quad \max_{\mathbf{p}, \mathbf{q}} \sum_{k=1}^n \int_{\theta_k}^{\theta_{k+1}} [sq_k - C(q_k)] f(s) ds$$

such that for all  $\theta \in [\theta_k, \theta_{k+1})$ ,  $k$ ,  $\theta_k q_k - p_k = \theta_k q_{k-1} - p_{k-1}$ .

We know that  $sq_k - p_k$  implies net utility (or consumer surplus), which a consumer of type  $s$  enjoys, and  $p_k - C(q_k)$  is profit margin of a monopolist selling one unit of  $q_k$ . Thus, the sum,  $(sq_k - p_k) + (p_k - C(q_k))$  becomes social welfare when the monopolist sells one unit of  $q_k$  to a consumer of type  $s$ . Note that  $p_k$  is a tariff, rather than a price.

The first step to solve the problem (P.S) is showing that the social planner exercises a marginal cost pricing (in fact, a marginal cost tariff) and makes no profit in supply side, that is for all  $k$ ,  $p_k = C(q_k)$ . It is easy to think that any tariff scheme will not affect the social planner's problem to maximize total surplus, because a tariff just divides a fixed total surplus into a consumer's share and a monopolist's share. It is not true. In this model, the prices (or the tariffs) determine market segments also, together with the qualities. A consumer may consume a different product under a different pricing scheme, even under the exact same set of product's qualities. Thus, the social welfare may depend on the tariff scheme. The Lemma II.1 shows that the maximum social welfare is achieved, under the marginal cost tariff scheme.

**Lemma II.1. (Marginal Cost Tariff in the Social Planner's Problem)** *Under the Assumption 1, the problem (P.S) is maximized when  $p_k = C(q_k)$ .*

Due to the Lemma II.1, we can replace  $\mathbf{p}$  with  $C(q_k)$ , in the problem (P.S). It makes the social planner's problem very tractable, since we do not need to consider tariffs.

Before solving the problem (P.S), I prove existence of a maximizer. Intuitively the existence is obvious, because consumer's utility increases linearly and producer's cost increases convexly. Thus, infinitely increasing the quality level cannot be the social planner's solution. There might be a proper quality level to maximize the social welfare.

Let  $(\mathbf{q}^*, \mathbf{p}^*)$  be a maximizer for the social planner's problem (P.S). Moreover, we can derive  $\boldsymbol{\theta}^*$  from  $(\mathbf{q}^*, \mathbf{p}^*)$  and constraints in the problem (P.S).

**Proposition II.5. (Existence of a Maximizer in the Social Planner's Problem)** *Under the Assumptions 1 and 2, the maximizer  $\mathbf{q}^*$  for the problem (P.S) exists.*

With this proposition, we find the maximizer. The next proposition shows conditions that the maximizer for the problem (P.S) satisfies. In the proposition and its proof, I implicitly assume that for all  $k$ ,  $q_k^* < q_{k+1}^*$ , like in the monopolist's problem. In fact, it is also true. This strict monotonicity will be explained in the next section.

**Proposition II.6. (Social Planner's Problem)** *Under the Assumptions 1 and 2, for all  $k$ ,  $\mathbf{q}^*$ ,  $\mathbf{p}^*$  and  $\boldsymbol{\theta}^*$  satisfy*

$$\begin{aligned} p_k^* &= C(q_k^*), \\ \mathbb{E}(\theta | \theta \in [\theta_k^*, \theta_{k+1}^*]) &= \int_{\theta_k^*}^{\theta_{k+1}^*} s \frac{f(s)}{F(\theta_{k+1}^*) - F(\theta_k^*)} ds = C'(q_k^*), \text{ and} \\ \theta_k^* &= \frac{p_k^* - p_{k-1}^*}{q_k^* - q_{k-1}^*} = \frac{C(q_k^*) - C(q_{k-1}^*)}{q_k^* - q_{k-1}^*}. \end{aligned}$$

The first equation explains the marginal cost tariff scheme is a social best tariff schedule, and the third equation shows the market segmentation, induced by the Proposition II.1 and the marginal cost pricing, is social best.

The right hand side of the second equation in the Proposition II.6 is an increment of marginal cost of the product provider (e.g. monopolist). When the product

provider improves quality of a product  $q_k$  by one unit, the marginal cost to produce  $q_k$  increases by  $C'(q_k)$ . If the quality for a product  $q_k$  is improved by one unit, utility of a consumer of type  $\theta$  increases by  $\theta$ . The left hand side  $\mathbb{E}(\theta|\theta \in [\theta_k^*, \theta_{k+1}^*])$  is an average taste of consumers who buy the product  $q_k$ . Thus, the Proposition II.6 argues that social welfare is maximized when the average marginal benefit in demand side is equal to the increment of marginal cost in supply side, in each segment.

We can compare the results from the Proposition II.6 with the results from the Proposition II.3. Tentatively, assume that  $\theta_k^* = \theta_k^m$ . Then,

$$C'(q_k^m) = \left[ \theta_k^m - \frac{1 - F(\theta_{k+1}^m)}{F(\theta_{k+1}^m) - F(\theta_k^m)} (\theta_{k+1}^m - \theta_k^m) \right] < \mathbb{E}(\theta|\theta \in [\theta_k^*, \theta_{k+1}^*]) = C'(q_k^*).$$

If the market is segmented identically in both cases, then  $q_k^m < q_k^*$ . The monopolist seems to distort the quality of the product  $q_k$  downward. However,  $\theta_k^*$  does not need to be equal to  $\theta_k^m$ . For the direction of quality distortion, we need more careful investigation.

The third equations in the both problems direct the market segmentation. The only difference is the inverse hazard rate term in the monopolist's optimal condition. Tentatively, assume that  $\mathbf{q}^* = \mathbf{q}^m$ . Then,  $\theta_k^m > \theta_k^*$ . We can guess that the market will be more finely segmented across higher taste types. However,  $\mathbf{q}^*$  also does not need to be equal to  $\mathbf{q}^m$ . The pattern of market segmentation is not simple. The next chapter, using numerical computation works, serves thorough comparisons between the monopolist's choice and the social planner's choice.

We have one more salient topic in the social planner's problem: uniqueness of the maximizer. In fact, the uniqueness property is very important, when we find the maximizer numerically. Finding conditions for the uniqueness is, however, intensely technical and mathematical. I arrange the uniqueness topic in the appendix.

### E. Comparative Statics Results with Respect to the Number of Products

In this section, I examine comparative statics results: how the change of number of products affects the monopolist's (the social planner's) optimal qualities, prices (tariffs), and profit (social welfare).

The next proposition is about a relationship between the maximized profit of the monopolist and the number of offered products. To present the proposition, I define a profit function. Let

$$\Pi^m(\mathbf{q}, \boldsymbol{\theta}) \equiv \sum_{k=1}^n D(q_k) \left( \sum_{j=1}^k \theta_j (q_j - q_{j-1}) - C(q_k) \right).$$

**Proposition II.7. (Total Profit and  $n$ )** *Under the Assumptions 1, 2 and 3, defining  $\mathcal{V}(n) \equiv \Pi^m((q_1^m, \dots, q_n^m), (\theta_1^m, \dots, \theta_n^m))$ ,  $\mathcal{V}(n)$  is strictly increasing in  $n$ .*

Note that the Proposition II.7 is a rephrasing of the proposition 2 in Itoh (1983). Under the exogenously given quality levels, Itoh (1983) obtains the above comparative statics result.

We have a similar result in the social planner's problem. The next proposition shows that the total social welfare is strictly increasing in the number of provided products. To present the proposition, I define a social welfare function. Let

$$SW^{SP}(\mathbf{q}) \equiv \sum_{k=1}^n \int_{\theta_k}^{\theta_{k+1}} [sq_k - C(q_k)] f(s) ds.$$

Note that for all  $k$ ,  $\theta_k$  is determined by  $\theta_k = (C(q_k) - C(q_{k-1})) / (q_k - q_{k-1})$ .

**Proposition II.8. (Social Welfare and  $n$ )** *Under the Assumptions 1 and 2, defining  $V(n) \equiv SW^{SP}((q_1^*, q_2^*, \dots, q_n^*))$ ,  $V(n)$  is strictly increasing in  $n$ .*

This proposition implies that the expansion of the consumer choice set always increases total social welfare. I have not considered the possibility of a cost to intro-

duce an additional production line; a fixed cost. Without any additional cost to add the line, the increase of social welfare in the number of products is quite predictable. If we introduce the fixed cost to add a new production line, we cannot guarantee the increase. It may be possible to find an optimal number of products to maximize the total social welfare.

It is a good time to more discuss a fixed cost. Until now, I have ignored cost to add a new production line. Without the fixed cost, the monopolist (or the social planner) has an incentive to add production lines repeatedly, as argued by the Propositions II.7 and II.8. With the fixed cost, the monopolist (or the social planner) may have an optimal number of products to maximize its profit (or her social welfare). Suppose that a profit function is concave in the number of products and the fixed cost is constant. If the monopolist already produces sufficient number of products, the additional profit will be smaller than the additional cost to add a new line. At that point, the monopolist does not differentiate more. This logic can be applied to the social planner, too.

Now, it is time to explain the strict monotonicity of product's qualities (for all  $k$ ,  $q_k^* < q_{k+1}^*$ , not  $q_k^* \leq q_{k+1}^*$ ), which I implicitly assume while proving the Proposition II.6. According to Proposition II.8, the social welfare strictly increases whenever we add a new product. Suppose that  $q_k^* = q_{k+1}^*$ . Since  $p_k^* = C(q_k^*) = C(q_{k+1}^*) = p_{k+1}^*$ , two products are exactly same. It means we lose one possible product that can strictly increase the social welfare. Thus,  $(q_1^*, q_2^*, \dots, q_n^*)$  cannot be a maximizer if there is  $q_k^*$  such that  $q_k^* = q_{k+1}^*$ . The similar logic can be applied to the monopolist, too.

From now on, I examine what happens to the monopolist's problem when the number of offered products increase up to infinity. Intuitively, we can expect our finite/continuous model should approach the continuous/continuous model.

One of the most important outcomes in the continuous/continuous model is

a discordance between the monopolist's profit maximizing product catalog and the social planner's best product lineup. It is very well known that the monopolist offers a lower quality product to each consumer than the quality offered by the social planner to the same consumer. The literature calls this phenomenon a *downward distortion*. One more interesting discovery is that the monopolist does not distort a quality for its highest product. In other words, a consumer with a highest taste is treated with a same quality product from the monopolist and the social planner. The literature calls this phenomenon *no distortion on top*.

In the finite/continuous model, neither phenomena hold. In fact, we can find an upward-distorted product, aiming at a consumer-group of highest tastes. The possibility of upward distortion deserves to be examined in detail and I will cover it in the next chapter.

In this section, I prove that the monopolist's quality choices recover both a downward distortion and no distortion on top when the number of products goes to infinite, which is what the Proposition II.9 states. Before presenting the proposition, I should introduce new notations. Remember that the number of products is  $n$ . I denote  $q^*(\theta)$  as a quality of product which a consumer of type  $\theta$  will get at the social best and denote  $q^m(\theta)$  as a quality of product which the same consumer will get in the monopoly. Then, if  $\theta \in [\theta_k^*, \theta_{k+1}^*)$  and  $\theta \in [\theta_j^m, \theta_{j+1}^m)$ ,  $q^*(\theta) = q_k^*$  and  $q^m(\theta) = q_j^m$ . Next, I define  $\bar{q}$  as an efficient quality level for a consumer of a highest taste. At the social planner's optimal, the increment of marginal cost is equal to the marginal benefit of a consumer. Thus,  $C'(\bar{q}) = \bar{\theta}$ .

**Proposition II.9. (No Distortion on Top and Downward Distortion When  $n$  Goes to Infinity)** *As  $n$  goes to  $\infty$ , both  $q_n^*$  and  $q_n^m$  approach  $\bar{q}$ . In addition, as  $n$  goes to  $\infty$ , for all  $\theta \in [0, \bar{\theta}]$ ,  $q^*(\theta) \geq q^m(\theta)$ .*



Remember that  $q_n^*$  and  $q_n^m$  are the highest qualities provided by the social planner and the monopolist respectively. Generally,  $q_n^* \neq q_n^m$  in the finite/continuous model. However, the Proposition II.9 tells us that  $q_n^m$  goes to  $\bar{q}$  and  $q_n^*$ , as  $n$  goes to  $\infty$ . We can recover no distortion on top when  $n = \infty$ . Even though the downward distortion does not hold always when  $n$  is finite, the Proposition II.9 argues that we can recover  $q^*(\theta) \geq q^m(\theta)$  when  $n = \infty$ . To sum up, the Proposition II.9 states that the finite/continuous model converges to the continuous/continuous model, at least in the two above famous phenomena.

## F. Conclusion

In this chapter II, I analyze the finite product differentiation model with continuous consumer types. The model is the only possible mixture to endogenize both the product quality and the market share, even though only a few theoretical works adopt this model. Therefore, the thorough study for the finite/continuous model is necessary, if we want to research the monopolist's real optimal behaviors, and evaluate the social welfare impact of the behaviors.

The model is applied to both the monopolist's problem and the social planner's problem. I reconfirm the optimal market segmentation of Itoh (1983) in the endogenous quality setting. Remember that Itoh (1983) assumes the exogenously given quality levels. I resolve the optimal quality selection rule of Crawford and Shum (2007), in the explicit way. Remember that Crawford and Shum (2007) just analogizes the result in the continuous/continuous model, into the finite/continuous setting.

And then, I try to make a comparative static analysis; how the change of number of offered products affect the maximized profit, the maximized social welfare. The

Propositions II.7 and II.8 show that the maximized profit and the maximized social welfare strictly increases, as the number of products increases. At last, I show the two famous phenomena (no distortion on top and downward distortion), discovered in the continuous/continuous literature, are reobtained, when the number of products increases to infinity, that is, the finite/continuous model approaches to the continuous/continuous model. At least, about two phenomena, the finite/continuous model is a generalization of the continuous/continuous model.

## CHAPTER III

### COMPARATIVE STATICS ANALYSIS USING NUMERICAL SIMULATIONS

#### A. Introduction

Even though we find optimality conditions for the monopolist and the social planner from the chapter II, it is very difficult to add further theoretical results using the conditions. Unless we furnish some more restrictions for the marginal cost function and the consumer type distribution, the additional theoretical analysis has a big difficulty, since we have no reduced form solutions.

Here, I suggest the alternative way: depending on the numerical analysis. In the section B, I present the procedures for the numerical simulations. Using the procedures and the optimality conditions, obtained from the chapter II, I can find the exact value for the optimal quality, price, profit, and social welfare, for each possible combination of marginal cost function and consumer type distribution. Most of all, I find the possibility of upward quality distortion, which has been never reported in the previous researches without any further market frictions.

#### B. Computational Method to Find the Maximizers

In this section, I suggest computational methods to find the maximizers for the social planner's problem and the monopolist's problem. The Procedure 1 is for the social planner's maximizer and The Procedure 2 is for the monopolist's maximizer.

##### **Procedure 1. (Procedure for the Social Planer's Problem)**

1. Take an arbitrary  $\boldsymbol{\theta} = (\theta_1, \dots, \theta_n)$  such that  $\theta_1 < \theta_2 < \dots < \theta_n$ .

2. For given  $\theta_0 = 0$  and  $\theta_2$ , find  $\theta'_1$  and  $q'_1$  such that

$$C'(q'_1) = \mathbb{E}(\theta | \theta \in [\theta'_1, \theta_2]) \quad \text{and} \quad \theta'_1 = \frac{C(q'_1)}{q'_1}.$$

3. For  $k = 2, \dots, n$ , find  $\theta'_k$  and  $q'_k$  such that

$$\begin{aligned} C'(q'_k) &= \mathbb{E}(\theta | \theta \in [\theta'_k, \theta_{k+1}]), \quad \text{where } \theta_{n+1} = \bar{\theta}, \\ C'(q'_{k-1}) &= \mathbb{E}(\theta | \theta \in [\theta_{k-1}, \theta'_k]) \quad \text{and} \\ \theta'_k &= \frac{C(q'_k) - C(q'_{k-1})}{q'_k - q'_{k-1}}. \end{aligned}$$

4. Formulate  $\boldsymbol{\theta}'$  from the steps 2 and 3.

5. Repeat the steps 2, 3 and 4 until  $\boldsymbol{\theta}$  converges.

**Procedure 2. (Procedure for the Monopolist's Problem)**

1. Take an arbitrary  $\boldsymbol{\theta} = (\theta_1, \dots, \theta_n)$  such that  $\theta_1 < \theta_2 < \dots < \theta_n$ .

2. Calculate  $\mathbf{q}' = (q'_1, \dots, q'_n)$  using  $\boldsymbol{\theta}$  and the equation

$$C'(q'_k) = \theta_k - \frac{1 - F(\theta_{k+1})}{F(\theta_{k+1}) - F(\theta_k)} (\theta_{k+1} - \theta_k), \quad \text{where } \theta_{n+1} = \bar{\theta}.$$

3. Calculate  $\boldsymbol{\theta}' = (\theta'_1, \dots, \theta'_n)$  using  $\mathbf{q}'$  and the equation

$$\theta'_k - H(\theta'_k) = \frac{C(q'_k) - C(q'_{k-1})}{q'_k - q'_{k-1}} \quad \text{where } H(\theta'_k) = \frac{1 - F(\theta'_k)}{f(\theta'_k)} \quad \text{and } q_0 = 0.$$

4. Repeat the steps 2 and 3 until  $\boldsymbol{\theta}$  and  $\mathbf{q}$  converge.

The above procedures depend on the optimality conditions, obtained from the Propositions II.3 and II.6. Thus, every converging values should satisfy the optimality conditions. If the maximizers for the social planner's problem and for the monopolist's problem are respectively unique, the converging values, obtained from the Procedures 1 and 2, will be true maximizers. I already showed that the Assumption 4 was a sufficient condition for the uniqueness for the monopolist's optimal. In the appendix,

I thoroughly examine conditions for uniqueness for the social planner's optimal. The Condition 1, which will appear in the appendix, is a sufficient condition to guarantee the uniqueness of the social planner's maximizer.

Fortunately, both the Assumption 4 and the Condition 1 are not too restrictive. Many previous works show that various families of distributions satisfy Assumption 4. In the appendix, I show the Condition 1 is not too strong, by simulating various consumer distributions and marginal cost functions. Almost all ordinary families of distributions and cost functions satisfy the Condition 1. Moreover, note that the failures of the Condition 1 and the Assumption 4 do not imply that the maximizers are not unique.

### C. Effect of Number of Products

In the chapter II, I developed a theory for the finite product differentiation model with continuous consumer types. From now on, I will present both theoretical and numerical results, especially focusing on relationships between the monopolist's and social planner's optimal choices and the number of products,  $n$ .

Itoh (1983) shows that the monopolist's profit increases as  $n$  increases, when every product quality is exogenously given. Remember that I reproduced the same result in Proposition II.7, when the qualities are endogenously chosen. This dissertation solves the social planner's problem in the finite/continuous setting, for the first time. Thus, Proposition II.8, which states that the social welfare increases as  $n$  increases, is the first statement about the relationship.

Except Itoh (1983), we have no meaningful preceding work to investigate an effect of  $n$  on the profit/social welfare. It is because we cannot find closed form solutions from the optimality conditions in Propositions II.3 and II.6. Since we do not have

closed form solutions for the optimal quality, price, and market share, we do not have well-organized profit and social welfare functions. This situation makes it difficult to find interesting outcomes by changing  $n$ . This work adopts the numerical simulations to investigate the relationship between the profit/social welfare and  $n$ .

The first task is to confirm the results of Proposition II.9, which argues that the finite/continuous model recovers no distortion on top and downward distortion, as the number of products increases up to infinity. The Figure 1<sup>1</sup> vividly shows how we recover no distortion on top as  $n$  increases. For the simulation, I assume that consumer tastes are distributed over  $[0, 5]$ , by a truncated normal distribution with a mean, 2.5 and a standard deviation, 1. A marginal cost function is assumed quadratic;  $C(q) = (1/2)q^2$ . In fact, we have similar graphs, although we change means and standard deviations of distributions.

The second and more interesting work is to check the increasing pattern of the profit and the social welfare, according to the number of products. In the previous chapter, I already check that the maximized profit and the social welfare is strictly increasing, as the number of products increases. Check the Propositions II.7 and II.8. Here, I study how and how fast they are increasing. I show that the increment is diminishing and the increasing speed is very fast. We can achieve significant levels of the social welfare and the profit with only a small number of products.

Although it is quite natural to expect that the maximized profit or the maximized social welfare is concave in  $n$ ,<sup>2</sup> we cannot show the concavity mathematically. Instead,

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<sup>1</sup>In the Figure 1, I just connect discrete quality levels at midpoints of consumer intervals, for *provided quality lines*. It is only for easy comparison.

<sup>2</sup>The monopolist would like to take best positions, when it is forced to choose only a few different products. Thus, profit generated by newly introduced product may be smaller than profit generated previously introduced product. The same logic can be applied to the social planner.

the Simulation Result III.1 states that the maximized profit (or the social welfare) is concave in  $n$  generally.

**Simulation Result III.1. (Diminishing Increment of Profit or Social Welfare in  $n$ )** *Generally, the increment of the maximized profit (or the social welfare) by the monopolist (or the social planner) is diminishing in  $n$ .*

For the Simulation Result III.1, I tried various distributions: left-centered, right-centered, concentrated, dispersed, extremely skewed, twin-peaked, and thin-centered distributions. The Figures 2 and 3 show that the maximized profit and the maximized social welfare are concave in  $n$ , in almost all imaginable consumer distributions.

Next, I examine the increasing speed of the profit or the social welfare, when  $n$  increases.

**Simulation Result III.2. (Increasing Speed of Profit or Social Welfare in  $n$ )** *The profits and social welfare generally achieved by only a few products represents more than 95 percent of the profits and social welfare generated by an infinite number of products.*

The Table 1 shows that the maximized profit (or social welfare) increases very rapidly even when  $n$  is small. When  $n = 3$ , the simulated values explain over 96% of the value achieved when  $n = 100$ . The value obtained when  $n = 100$  is virtually equal to the maximized value obtained in the continuous/continuous model. If  $n$  increases up to 10, we can achieve over 99.5% of the value.

The monopolist (or social planner) in the real world may find an optimal number of production lines, in the case that it has a constant fixed cost to add one more production line. The Simulation Result III.1 tells us that the increment of profit (or social welfare) becomes smaller and smaller as  $n$  increases. It means that it cannot make enough profit to cover the cost to add a new production line, when  $n$  is enough

big. That is, it should stop adding a new production line. Moreover, we can expect that the moment will come very soon, due to the Simulation Result III.2.

#### D. Effect of Consumer Distribution Change

It is obvious that our model will predict different optimization results under different consumer distributions. The next direct question is how distribution change affects optimization results. The comparative statics analysis, usually using the implicit function theorem or the envelope theorem, reveals how the change of parameters defining distribution alters the optimized values. In this work, mathematical complexity hinders the usual comparative statics analysis. The numerical simulations come again.

Suppose that we know exact relationships between distribution and the monopolist's choice. We can describe many real world phenomena. Consider a boom and a bust in economy. A consumer distribution in a boom may be different with a distribution in a bust. One possible conjecture is that a distribution will have a much thicker right-side (higher tastes) tail in a boom. Knowing the relationship enables prediction of the monopolist's behaviors in a boom or a bust. Another example is comparison over societies or industries. Some societies may highly value diversity, while others can be populated with similar taste consumers. Some industries can be trend-sensitive, while others may have a steady consumption pattern. We can interpret these varieties as differences of consumer distributions.

I begin this section with a taxonomy of distributions.



## 1. Classification of Distributions

Here, I use two criteria to classify consumer distributions. The first one is a *center*. A left-centered distribution implies that a high proportion of consumers has low tastes. Considering a truncated normal distribution, I have two free selectable parameters: a mean and a standard deviation, when the support is fixed. The mean determines whether a distribution is left- or right-centered. As the mean changes from small to large, the distribution goes from severely left-centered to lightly left-centered, to symmetric, to lightly right-centered and to severely right-centered.

I have one more criterion, a *concentration*. I divide distributions into concentrated distributions and dispersed distributions. The degree of concentration is closely related to a standard deviation of distribution. If we have a large standard deviation, the distribution has a high peak and thin tails on both sides. I call this type of distributions a concentrated distribution. If we have a small standard deviation, the consumer types are widely dispersed over the support. I call this type of distributions a dispersed distribution.

We can cross over these two criteria, like a left-centered concentrated distribution. The Figure 4 shows examples for each class of distribution, when the support is  $[0, 5]$ .

## 2. Distortion on Top

In this subsection, I investigate the property of no distortion on top in detail. Almost all existing works have reported that a quality distortion does not happen on a consumer of a highest taste. Remember  $\bar{q}$  defined in the Chapter II, where  $\bar{q}$  is the socially efficient quality level for the consumer of the taste  $\bar{\theta}$ . Both in the discrete/discrete and the continuous/continuous model, the monopolist sells  $\bar{q}$  to the consumer of the type  $\bar{\theta}$ .

In our finite/continuous model, neither the monopolist nor the social planner provides  $\bar{q}$  to the consumer of the type  $\bar{\theta}$ . They are both serving consumer interval, including  $\bar{\theta}$  at the end of interval, not only to  $\bar{\theta}$ . Since they should consider other consumers besides  $\bar{\theta}$ , the product quality may be distorted downward from  $\bar{q}$ . Thus, for more meaningful interpretation, we need to check whether the monopolist's choice is equal to the social best for the highest group of consumers, rather than compare to  $\bar{q}$ .

**Simulation Result III.3. (Distortion on Top)** *Generally,  $q_n^* \neq q_n^m$ . That is, the highest quality provided by the monopolist may be distorted from the social best selection.*

As we saw in the previous section, the distortion on top does not happen in the case that  $n = \infty$ . When we have a uniform consumer distribution and a quadratic marginal cost function, we can analytically verify there is no distortion on top. Unfortunately, these are just special examples. In usual environments, quality for a highest interval tends to be distorted either downward or upward. The Figure 1 shows the instances of downward distortion on top.

From the Propositions II.3 and II.6, in order that  $q_n^*$  is equal to  $q_n^m$ ,

$$(3.2) \quad C'(q_n^m) = \theta_n^m = \mathbb{E}(\theta | \theta \in [\theta_n^*, \bar{\theta}]) = C'(q_n^*).$$

The equation (3.2) implies that  $\theta_n^m$  should be equal to a conditional mean over the interval  $[\theta_n^*, \bar{\theta}]$ . It usually does not happen. If  $\theta_n^m$  is greater than the conditional mean,  $q_n^m > q_n^*$ , the highest quality product can be distorted upward.

The Table 2 reports simulation results using six different truncated normal distributions. Each distribution has a right-side tail of different thickness and length. There, the monopolist and the social planner choose only three different products. In

all distributions,  $q_n^m < q_n^*$ , confirming the Simulation Result III.3.

Next, I define a *degree of quality distortion on top* as a ratio of qualities provided to a highest consumer group by the monopolist and by the social planner. That is,  $q_n^m/q_n^*$ . If the degree of quality distortion on top is greater than 1, it means upward distortion. Note that as the ratio recedes from 1 either upward or downward, the quality is heavily distorted.

The first task is to find a relationship between the degree of quality distortion and a center of distribution.

**Simulation Result III.4. (Degree of Quality Distortion on Top and Center of Distribution)**

- *Generally, the degree of quality distortion on top is larger when the distribution is left-centered, rather than right-centered.*
- *In the concentrated distribution, the degree of quality distortion on top becomes larger as the center of distribution goes to both extremes.*
- *In the dispersed distribution, the degree of quality distortion on top becomes smaller as the center of distribution goes to right.*

The Simulation Result III.4 is nothing but a summary for the Figure 5. The key element to explain the Simulation Result III.4 is for whom the monopolist cares. In a left-centered distribution, its main concern may be consumers of low tastes. Since the low quality product market is large, the monopolist wants to reduce consumers who will buy nothing. Thus,  $\theta_1^m$  will be quite small. In a right-centered distribution, it cares for consumers of high tastes. It does not need to have a very small  $\theta_1^m$ . Let  $\theta_{1,l}^m$  be  $\theta_1^m$  in the left-centered distribution and  $\theta_{1,r}^m$  be in the right-centered distribution. Obviously,  $\theta_{1,l}^m < \theta_{1,r}^m$ . The product quality increases as the consumer type increases.

Since  $\bar{\theta} - \theta_{1,l}^m > \bar{\theta} - \theta_{1,r}^m$ , the quality can travel a longer span in the left-centered distribution, and so it may arrive at a higher level.

The above scenario can be applied to the second and third argument in the Simulation Result III.4, except the right-centered concentrated distribution. In the case, consumer population of highest tastes is very large. Now, the right-side population is extremely important for the monopolist's profit. To fully exploit their tastes, the monopolist may offer a high quality product with a high price.

Next, I relate the degree of quality distortion with the degree of concentration.

**Simulation Result III.5. (Degree of Quality Distortion on Top and Degree of Concentration)**

- *The degree of quality distortion on top approaches 1 as the distribution becomes more dispersed (flatter).*
- *If the distribution is not too dispersed, the degree of quality distortion on top becomes smaller as the distribution becomes more concentrated.*
- *In the left-centered distribution and under the proper degree of concentration, the degree of quality distortion on top may be bigger than 1 (upward distortion).*

The first argument is related to a uniform distribution property. As  $\sigma$  goes to infinity, a distribution becomes a uniform distribution. As previously mentioned,  $q_n^* = q_n^m = \bar{q}$  when adopting the uniform distribution. Then, the degree of quality distortion on top becomes 1. For the second argument, we should consider a proportion of consumers of high tastes. The more concentrated the distribution is, the more thin the right-side tail. Then, the consumers of high tastes are ignorable by the monopolist. The highest quality product will be more distorted downward. The third argument is observed in the Figures 5 and 6. This upward distortion will be explained in detail in the next subsection.

### 3. Quality Flipping

In the previous subsection, we observed the possibility of upward distortion. I investigate the possibility in more detail.

To find a upward-distorted product, I make simulations adopting truncated normal distributions with a mean, 1 over a support,  $[0, 5]$ . The monopolist and the social planner choose three different products. The Table 3 shows upward distortion when  $2.03 \leq \sigma \leq 20$ , where  $\sigma$  is a standard deviation of distribution.

**Simulation Result III.6. (Upward Distortion)** *In some environments of distributions,  $q_k^m$  can be greater than  $q_k^*$ . That is, some consumers can enjoy a higher quality product in the monopoly, rather than the social best selection.*

Naturally, I should investigate a relationship between direction of quality distortion and a standard deviation of distribution. Since our distribution is left-centered, we have a thin right-side tail. The standard deviation determines how thin the right-side tail is. As the standard deviation grows, in other words, the distribution becomes more dispersed, we have a thicker right-side tail. See the Figure 7.

We can find an interesting association related to the thickness of tail. With a very thin tail of high tastes, the social planner cares about them only in proportion to their volume. The monopolist, however, almost ignores these consumers of high tastes, because their population is too small to yield a big profit. In this stage, the highest quality product is distorted downward. As the volume of consumers of high tastes increases, the social planner's concern for these consumers increases in proportion to their volume again. The monopolist, however, has a different incentive. The consumers of high tastes, who still comprise a small population but who can pay a large amount, abruptly become very attractive to the monopolist. In this phase, the monopolist reflects these consumers' tastes very significantly. Sometimes

the monopolist offers a product even distorted upward to fully exploit the great willingness to pay of these consumers. Consider the situation with enough number of consumers of high tastes. The increment of consumers of high tastes is not attractive any more. The product quality aiming at the consumers is already enough high. The monopolist's enthusiasm to produce an extreme edition is damped, while the social planner always keep her pace, in proportion to the consumers' volume.

**Simulation Result III.7. (Quality Flipping)** *Suppose that the distribution is left-centered. When the distribution has a thin tail,  $q_n^m$  is downward distorted. As the tail of the distribution become thicker,  $q_n^m$  approaches, and then passes over  $q_n^*$ . That is, the quality can be distorted upward. After that,  $q_n^m$  approaches  $q_n^*$  downwardly, when the distribution becomes uniform. The qualities provided by the monopolist and by the social planner can be flipped.*

The Table 3 and the Figure 8 vividly show the quality flipping phenomenon.

#### 4. Excluded Consumers from Consumption

Even in a social best product selection, some consumers will be excluded from consumption. Some consumers, who are located in a left side of support, have extremely low tastes. Since they extremely undervalue the worth of quality increase, even the benevolent social planner may want to exclude these consumers. She has to choose only a finite number of products.

In the monopolist's problem, the consumer of  $\theta_1^m$  has a highest taste among consumers who will not buy nothing. Thus, the proportion of excluded consumers is  $F(\theta_1^m)$  in the monopoly. Similarly,  $F(\theta_1^*)$  is the proportion of consumers excluded by the social planner. Before presenting the simulation results, I define a *ratio of excluded consumers* as  $F(\theta_1^*)/F(\theta_1^m)$ .

### Simulation Result III.8. (Patterns of Excluded Consumers)

- *The monopolist excludes more consumers from consumption than the social planner.*
- *Both in the monopolist's and the social planner's optimal, the proportion of excluded consumers decreases as the distribution evolves from left-centered to right-centered.*
- *The ratio of excluded consumers increases as the distribution evolves from left-centered to right-centered.*
- *The proportion of excluded consumers decreases as the distribution becomes more concentrated, if it is not too left-centered.*
- *The ratio of excluded consumers increases as the distribution becomes more concentrated.*

See the Figure 9. The panels in the first column show how density changes from the left-centered to the right-centered as the mean increases. From the panels in the second column, we can confirm the monopolist excludes more consumers than the social planner. The solid lines, which represents the proportion of excluded consumers at the monopoly optimal, are over the dotted lines, which represents the proportion of excluded consumers at the social best. The downward slopes of both lines explains that the proportion of excluded consumers decreases as the mean increases. The panels in the third column argue that the ratio of excluded consumers increases as the mean increases.

The Figure 10 completes the remaining parts of the Simulation Result III.9. The first three panels show the results from left-centered density, symmetric density, and right-centered density. We can check that the solid lines are above the dotted lines

again. In addition, the Figure 10 explains that the proportion of excluded consumers decreases as the standard deviation decreases; that is, the distribution becomes more concentrated. The ratio of excluded consumers increases as the distribution becomes more concentrated.

## 5. Information Rent

The existing literature has documented that information rent, which each consumer can take, increases as the consumer's type increases, in the monopoly market. That is, as  $\theta$  increases, the (net) utility,  $U(q, p; \theta) = \theta q - p$  increases. This result can be reconfirmed in our finite/continuous environment.

Let  $q(\theta)$  be a quality and  $p(\theta)$  be a price of a product that is purchased by the consumer of type  $\theta$ .

**Proposition III.1. (Increasing Information Rent in Type)** *If  $\theta < \theta'$ ,*

$$U(q(\theta), p(\theta); \theta) \leq U(q(\theta'), p(\theta'); \theta'),$$

*where the equality holds only when  $q(\theta) = q(\theta') = 0$ .*

I will now examine how the information rent is affected by distribution change.

**Simulation Result III.9. (Information Rent and Distribution Change)**

- *The information rent becomes smaller as the distribution changes from left-centered to right-centered.*
- *The information rent becomes smaller as the left-centered distribution changes from concentrated to dispersed.*
- *The information rent becomes smaller as the right-centered distribution changes from dispersed to concentrated.*



The above simulation results are supported by the Figures 11 and 12. This time, the monopolist chooses five different product qualities, rather than three products, which can show the information rents curve more smoothly.

The Figure 11 shows that the left-centered distributions have higher information rent curves than the right-centered distributions. The Figure 12 argues that the degree of concentration works in opposition, depending on whether the distribution is left- or right-centered. When the distribution is left-centered or symmetric, the concentrated distributions have higher information rent curves. When the distribution is right-centered, the dispersed distributions have higher information rent curves.

The key factor for the Simulation Result III.9 is the proportion of consumers of low tastes. As the proportion of consumers of low tastes increases, the information rent curve shift upward. The reason comes from the pricing scheme. We know that  $p_k^m = p_{k-1}^m + \theta_k^m(q_k^m - q_{k-1}^m)$ . Suppose that we have a thick volume of low tastes. Since the monopolist wants to fully exploit these low tastes, it offers a cheap price to the consumers. This cheaply offered price affects on the all prices of higher quality products. All prices of higher quality products should be offered at cheaper levels. Thus, if the monopolist really cares for the consumers of low tastes, it should endure a reduced ability to exploit consumers of higher tastes.

## 6. Profit and Social Welfare

Last, I report relationships between profit/social welfare and distribution change.

### **Simulation Result III.10. (Profit, Consumer Surplus and Social Welfare)**

- *Both profit and consumer surplus in the monopoly increase, when the proportion of consumers of high tastes increases.*
- *Both social welfare in the monopoly and at the social best increase, when the*

*proportion of consumers of high tastes increases.*

The Simulation Result III.10 is obvious. Consumers of higher tastes can get higher utilities, if ignoring prices. Thus, they have a higher willingness to pay. The social planner can achieve a higher social welfare, and the monopolist can obtain a higher profit. We know that consumers of higher tastes can enjoy higher information rents. Thus, we will get a higher total consumer surplus.

See the Figures 13 and 14. In the left-centered or the symmetric distributions, the more dispersed distribution implies a larger proportion of consumers of high tastes. Meanwhile, the more dispersed distribution implies a smaller proportion in the right-centered distribution. These simulation results strongly support the Simulation Result III.10.

I calculate total surplus in the monopoly. Total surplus is just a sum of the monopolist's profit and total consumer surplus; that is, social welfare induced by the monopolist. Now, I define an *efficiency score* achieved by the monopolist as follows;

$$\text{efficiency score} = \frac{\text{social welfare induced by the monopolist}}{\text{social welfare at the social best}}.$$

Since the social welfare is maximized by the social planner, the efficiency score cannot be strictly greater than 1.

**Simulation Result III.11. (Efficiency Score)** *The efficiency score increases as the distribution becomes more concentrated.*

If the consumers are very homogenous; that is, almost all consumer tastes are very similar, the monopolist and the social planner will provide a similar product catalog. Since everybody wants similar products, both the monopolist and the social planner should provide the similar ones. In this case, just obeying the consumers' demand is a best strategy for both decision-makers.

When the tastes of consumers are widely dispersed, the monopolist can utilize consumer heterogeneity. The monopolist offers a distorted product catalog from the social planner's list. This distortion decreases the efficiency score. See the Figure 15. When the distribution is extremely concentrated, the ratio is close to 1. As the distribution becomes more dispersed, the score declines rapidly.

## E. Conclusion

Using the computational procedures introduced in this chapter and the optimality conditions obtained from the chapter II, I could find the exact values for optimal qualities, prices, maximized profit and social welfare. Then, I could compare the monopolist's optimal with the social planner's optimal. The comparison is the first step for welfare analysis, which can induce policy implications.

From the comparison, I showed the possibility of upward quality distortion, which the existing literature has not previously reported. This discovery of the upward quality distortion, especially on the highest quality product, can stimulate research for an extreme product version.

This chapter also analyzed how the number of products affects on the monopolist's and the social planner's decisions. The profit/social welfare strictly increases as the number of products increases. Moreover, the increment of increase diminishes rapidly.

The chapter III also researched how distribution change affects the monopolist's and the social planner's choices. When a consumer distribution is dispersed, or left-centered, or both, the quality is less distorted and the amount of excluded consumers is large. When we have a large volume of consumers of low tastes, the quality is less distorted and information rents, which are enjoyed by consumers, increase. When we

have a proper level of thick right-side tail, the possibility of upward distortion arises. When the distribution is concentrated, the monopolist can achieve a higher efficiency level compared with the social best.

The major contribution of this chapter is that this is the first work for the social welfare analysis in the finite/continuous setting. I expect this work can trigger numerous following welfare related research, adopting the finite/continuous setting. Remember that the finite/continuous setting is the only model, which can endogenize both the quality and the market share.

## CHAPTER IV

ESTIMATIONS OF CONSUMER DISTRIBUTION AND MARKET DEMAND IN  
THE CABLE INDUSTRY

## A. Introduction

Until now, I have developed the endogenous quality choice model for a monopolist and a social planner. In the chapter II, I constructed the maximization problems for both decision makers, and found optimality conditions for the problems. In the chapter III, I calculated the market prices, qualities, profit, and social welfare, in a hypothetical market environment. In addition, I examined how the above market results change, as the underlying market environments change. By the numerical simulation, I could try and check all imaginable market environments.

Now, it is time to apply the previously obtained results to the real world. In this chapter IV, I examine the cable service industry.<sup>1</sup> The cable service industry perfectly fits the model, which has been developed in the previous chapters. First, the cable service market is monopolized by an exclusive contract with a local authority, in each franchise. Second, the cable service providers offer two or three service tiers with differing numbers of channels, which can be interpreted as qualities. Thus, the cable service provider is a monopolist offering two or three products of differentiated qualities. Adopting theoretical and numerical methods from the previous chapters, I can analyze the cable industry in the more rigid framework.

At first, I construct a consumer type distribution by simulating parameters using real data. The section B explains the data set, which includes a monthly fee for each service tier in each franchise. In addition, the set has the number of subscribers

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<sup>1</sup>In the appendix, I thoroughly review the cable service industry.

for each tier. This chapter assumes that the values result from a profit maximizing strategy by the cable service provider (in fact, the monopolist). In the chapter III, I could simulate the profit maximizing prices, qualities, and market shares, when I know an underlying consumer type distribution. In this chapter, I go the reverse direction, simulating a consumer type distribution from optimized (or assumed to be optimized) values. Thus, the main job in the section B is to recover a consumer type distribution in each franchise.

After recovering the consumer type distribution, I can simulate optimal values for the social planner: optimal quality and optimal tariff levels for each tier, and the number of subscribers assigned for each tier. Now, I can check the most important discovery, obtained in the chapter III: the upward quality distortion. I find the upward quality distortion is a very prevailing phenomenon. In Texas, 64 franchises among 209 franchises displays the upward quality distortion. It explains 30.6% of all relevant franchises. Considering almost all previous literature have ignored the possibility of upward quality distortion, it is a quite shocking result.

In the section D, I examine how demographic variables affect the consumer type distribution. Here, I assume that the consumer type distribution responds to underlying demographic factors, like population, female population, Hispanic population, city size, household's income level, age distribution, income distribution, number of households, and etc. I estimate how much each demographic variable affects mean, standard deviation, and upper bound for the consumer type distribution. Then, I can predict how the shape of distribution will change, according to the change of a demographic variable. In fact, the section D shows that the increase of population makes the consumer distribution more right-centered and more concentrated.

In the section E, I construct the demand curve. Since I know a consumer distribution, I can make a demand curve for a given service tier, varying price levels.

Moreover, I also study the impact of the change of market environment on the demand curve. When the population increases, the discussion of the section D can describe how the change of population alters the consumer type distribution. Then, I can construct the new demand curve from the new consumer distribution.

The comparison between the original demand curve and the new demand curve displays shift, rotation, or both. Since the increase of population affects the consumer distribution, in a few different routes, such as mean, standard deviation, and upper bound, we can observe the rotation of the demand curve, as well as the shift of the demand curve.

## B. Data Description and Simulating Consumer Distributions

### 1. Data Description

In this dissertation, I gather the data for cable service providers, operating in Texas. Each cable service provider operates in the franchise, where the provider contracts with the local authority. In the most cases, we can find one provider in one franchise. Rarely, we can encounter multiple cable service providers in one franchise. Fortunately, all these rare cases happens in the large cities, like Houston, and the biggest provider has a dominant market position, in the point of market share. In fact, the other providers in the large cities have ignorable market shares. In Houston, there are Comcast and its small competitors. In this dissertation, I adopt only the monopolist or the largest firm in the city. Thus, in my data set, the unit of observation becomes a franchise (or a cable service provider, equivalently). Each franchise is considered as an independent market, since consumers have no choice except their exclusive service provider.

In Texas, we had, have, or had have 740 cable service providers. Among these

740 providers, only 426 providers are operating, now. Among these 426 providers, only 275 providers offer multiple services, usually two or three services among a basic service, an expanded basic service, and a digital basic service. In this chapter, I only consider the franchises with multiple service tiers.<sup>2</sup> I cannot match demographic variables in 16 franchises, and I remove 11 more providers, which are not No.1 service providers in their franchises. Finally, I have 248 provider-franchise observations, in which monopolists offer multiple service tiers.

The Table 4 summarizes the collected data on cable service providers. The data comes from WarrenCommunicationsNews (2012). For the more detailed definition and the description for each variable, confer the Appendix.

The Table 4 strongly implies that each provider plays a monopolist role in its franchise. Among all providers, 47% of them are affiliated to a big MSOs, like Comcast or Suddenlink. More than half are independent providers, or are affiliated to small MSOs, only operating in a few different franchises. In fact, even MSO-affiliated firms operate quite independently, because local authorities furnish different contractual conditions, including ownership requirement.

For each service tier, we can check that the number of channels and the monthly fees have significant differences across the cable service providers. The differences prevail even in the cable service providers affiliated to the same MSO. In fact, the number of channels widely distributes from 6 to 36 for the basic service, and from 22 to 91 for the expanded basic service, even only considering 76 cable service providers affiliated to Suddenlink. These wide differences suggest each cable service provider operates independently.

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<sup>2</sup>I assume that the number of offered tiers is exogenously given to the provider. Although the problem of how many services they offer is very important, I rule out the problem in this chapter. It is consistent with the previous chapters.



It is rational to think that this diversification comes from the provider's profit maximizing strategy. That is, the providers certainly respond to market environments, including a consumer type distribution, and so they generate quite different choices in the different markets, whatever their affiliations are.

This big diversification also motivates empirical works. It suggests the change of market environments can make a big difference on the provider's choices. From the data work, we can analyze how the market environment affects the consumer type distribution and the market demand, and how the consumer type distribution affects the provider's products choice.

The Table 4 also includes other information. The miles of plant, which a provider install, distributes from 7 to 29791. Comparing to the median, the mean is much bigger. It implies that we have a few big firms and many small operators. About the installation fees, we don't have enough differences, regardless of service tiers. The providers seem to want to promote upper class services, when a new customer visit them.

The Table 5 shows the demographic data, collected from United States Census Bureau, City-Data.com, and Google map. Our franchises are approximately 130 miles from the closest large city in average. The average population is 46089, and the median is 5563. Thus, we have a few franchises with large populations, and many franchises with small populations. It agrees with our observation in the Table 4 about the size distribution of the cable service providers.

For the Hispanic population, we have a significant fluctuation. In some franchises, we have a large portion of Hispanic population, while we have the small number of Hispanic people in the other franchises. The large fluctuation of Hispanic population gives a hint that the Hispanic population can be one of the reasons for a large fluctuation of product choices of firms. For income level or poverty ratio, we can also

find a large fluctuation. This diversification of market environments also stimulates empirical works. It is natural to conjecture that these big differences in the market environments induce the big differences in the offered products across markets.

## 2. Construction of the Consumer Distribution Simulating Parameters

In this subsection, I simulate five parameters, which represent operating cable service providers and market environments, in each franchise. For the cable service provider, I simulate a coefficient for marginal cost function with respect to a quality,  $a$ . Like previous chapters, the marginal cost is assumed as constant with respect to a quantity. The constant marginal cost, however, is increasing in the quadratic form, as the quality is improved. That is,  $C(q) = aq^2$ , where  $q$  is a quality and  $C(q)$  is a constant marginal cost when producing a product of  $q$  quality.

The remaining parameters are for the market environments. Especially, I simulate three parameters for a consumer type distribution. In each franchise (equivalently, in each market), a consumer type distribution is assumed to follow a truncated normal distribution. To define a truncated normal distribution, we need a four parameters: a mean, a standard deviation, a lower bound, and an upper bound. I assume the lower bound is zero, since nobody would like to value quality improvement negatively. The other three parameters are recovered by simulations, using the real world data.

Here, I introduce one more parameter, a total number of potential subscribers, which is not a direct parameter to explain a truncated normal distribution. The number implies a market size in each franchise. In fact, I have a data for total number of households, but it is not exactly same with the number of potential subscribers. Besides households, there are demands for cable services, like restaurants, sports bars, and hotels. In rare cases, some households subscribe two or three cable services. In the case that a cable service provider serves neighboring areas, we don't know

how many households there are. Thus, the parameter, the total number of potential subscribers, is also simulated, as a parameter to describe a franchise.

In the franchise with two products, I have the prices for the basic service and expanded basic service, and the number of subscribers for both service tiers. In addition, I have the number of channels each service tier carries. I use this number of channels for a proxy of product quality, in two different ways. First, I take the number of channels for the quality for the product. Second, I take the square root of the number of channels for the quality. The second method reflects the fact that one additional channel is very valuable when we have only a few channels, but the additional enjoyment is not much when we already have many channels. I will try both ways. However, coming results without any special descriptions, result from the second method.

Now, I possess the prices, qualities, and market shares. Assuming the monopolist (the provider) offers a profit maximizing combination of products, I look for the most probable truncated normal distribution and the number of potential households, which can match the real data. The simulation is performed as follows; I set up a hypothetical consumer type distribution, simulate the profit maximizing values for the monopolist, compare the simulated values with the data, renew the hypothetical consumer type distribution, and repeat the above steps until I find the closest simulated values with the data.

The Table 6 summarizes the simulation results. The lower table shows the results when I take the square root of the number of channels for a quality, while the upper table shows the results when I take the number of channels for a quality. From now on, I explain the lower table. After simulations, I am compelled to abandon some observations (franchises), because the predicted (profit maximizing) values obtained

from the simulation quite differ from the data.<sup>3</sup> I abandon 39 observations, which the upper bound is over 12. I find that the size of upper bound is closely related with the differences between the predicted values and the real data.

The simulation results, presented at the Table 6, are quite stable. For the coefficient of marginal cost function, almost all simulated values stay around the mean. We have only 10 providers, whose coefficients are outside of 2 standard deviations. The simulated values for the total number of potential subscribers reflect our demographic observations very well. The fact that the mean is much bigger than the median, implies that we have a few big markets and many small markets.

In the average franchise, we have 63155 potential subscribers, and the consumer types distribute over  $[0, 5.107607]$ , following the truncated normal distribution with  $-1.53716$  mean and  $7.060615$  standard deviation. That is, the density function is monotonically decreasing over the relevant support. However, the most distributions has a peak in the relevant support. Note that the median of the mean is  $1.162921$ , which is greater than  $0$ , a designated lower bound. Considering that the mean of the upper bound is  $5.107607$ , and the median is  $4.401373$ , the most of distributions are left-centered. Considering that the mean of the standard deviation is  $7.060615$ , and the median is  $1.498823$ , the most of distributions are not dispersed.

### C. Upward Quality Distortion and the Effect on Social Welfare

In the previous chapter, the possibility of upward quality distortion is one of the main discoveries. That is, the monopolist can offer a higher quality product than the social planner offers. This phenomenon is never reported in the canonical model

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<sup>3</sup>In the case of the number of channels for the quality, I obtain relatively better fits between the predicted values and the data. However, the square root is still more appealing, in the sense of the proxy for the quality.

of product differentiation. Here, I verify that the possibility really arises in the real world. Among the meaningful 209 franchises, which is explained in the last section, I find 64 franchises, where the upward distortion happens. Approximately, 30.6% of franchises show the upward distortion. In the real world, we can argue that the upward distortion is a quite prevailing phenomenon. This is very surprising, considering the existing literature have overlooked the possibility.

In fact, the most of big cities, like Houston, San Antonio, Austin, and Fort Worth, shows a downward quality distortion, the existing literature suggested. However, the examples for the upward distortion also include meaningful franchises, such as Amarillo, San Angelo, and Conroe. In Amarillo, the welfare loss induced by the upward quality distortion reaches \$1,048,705 per month. Moreover, the welfare loss explains almost 50% of possible social welfare at the social best. The Table 7 presents some franchises which are worthy of note.

Since I recover the consumer distribution from the simulations starting from the real data, there are an inevitable amount of difference between the predicted (profit maximizing) values and the real data. It is very difficult to discriminate whether the upward distortion comes from the profit maximization process or from the simulation error.<sup>4</sup> Thus, I also consider the possibility of upward quality distortion, comparing the predicted values from the simulated distribution and the data. The result is more stunning. Among 209 franchises, 188 franchises display the upward quality distortion, which explains approximately 90% of franchises. The examples include the most of all large cities, such as San Antonio, Austin, and Fort Worth. Houston, however, still shows the downward quality distortion. When I consider the number of channels for the quality, and hires the predicted values for comparison, 197 franchises (79.4%)

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<sup>4</sup>Here, I call the difference between the predicted values and the data the simulation error.

shows the upward quality distortion.

This prevailing phenomenon of the upward quality distortion gives us a new perspective on the policy making. The existing regulation, represented by Minimum Quality Standard, concentrates on the downward distortion, trying to guarantee cable viewers a minimum service level. Now, we should review the situation that the cable viewers subscribe even unwanted channels, by the service provider's profit maximizing behaviors.

Before proceeding to the next section, I also report the welfare loss induced by the downward quality distortion. As I mentioned earlier, the major portion of franchises (60.4%) experiences the downward quality distortion. Especially, San Antonio displays a huge welfare loss: \$30,861,898 per month. Kerrville has \$824,777 welfare loss per month, Tyler has \$795,152 per month, and Corrigan \$570,918 per month. To be fair, the results show that the welfare loss is a more serious problem in the downward distortion case, comparing to the upward distortion case. However, the welfare loss induced by the upward distortion also deserves careful consideration. The phenomenon is enough prevailing and the loss is enough severe.

#### D. Estimation of Consumer Distribution

Each franchise has a unique market environment. I collect the demographic data to explain each franchise's environment. It is rational to consider that the demographic variables are closely related to the consumer type distribution. A household with a high income may have a higher taste for the quality improvement. Of course, it may not have. Neighbors can affect to the taste. Suppose that every neighbors value the quality improvement highly. Then, the adjacent may also value highly. The household with many children may show a higher taste for adding TV channels.

The Table 8 shows the estimation results how the demographic variables affect the consumer distribution. About the total number of potential subscribers (*t. subs.*), the city area size (*ar\_land*) and the population (*pop\_2009*) are important. In the bigger city and in the more crowded city, we have a more potential subscribers. Obviously, it is very intuitive.

For the driving distances to a nearest big city, we have a more interesting interpretation. As the distance between a franchise and a big city (Metropolitan Statistical Area over 500,000 population) grows, the number of potential subscribers diminishes. It is easy to explain; we are going to a rural area. However, as the distance between a franchise and a metropolis (Metropolitan Statistical Area over 5,000,000 population<sup>5</sup>) decreases, we have less potential subscribers, surprisingly. A possible explanation is the existence of “reluctant” consumers. There are some households, which voluntarily don’t want to watch TV. In the metropolitan area, we can find more single-person households, which may have a smaller taste for watching TV. Possibly, there are more people with unique tastes, like game mania, orthodox book readers, and etc. These kind of people likely hate watching TV. In the metropolitan area, we can easily find substitutes for watching TV, rather than a mid-size city or a rural town.

The size of a household is negatively related to the number of potential subscribers, even though it is not significant. Controlling the race, income, and city size, a big family values other activities, rather than watching TV.

The mean of consumer distribution determines whether the distribution is left-centered or right-centered, together with the upper bound. The hypothesis of “reluctant” consumers is re-discovered here. In the metropolitan area, the consumer distribution becomes left-centered. We have more people who undervalues the qual-

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<sup>5</sup>In Texas, we have only two such Metropolitan Statistical Areas, Dallas-Fort Worth-Arlington and Houston-Sugar Land-Baytown.

ity improvement. In the mid and large size city over 500,000 population, however, the mean increases. The urban citizens, who may be easily affected by neighbors, has the higher tastes for the quality improvement.

The income distribution is identified as the very important factor for the mean of the consumer distribution. The more the low income households are, the smaller the mean is. In addition, the more the high income households are, the bigger the mean is. The household with a high income may display the high taste for adding TV channels.

Controlling all the other factors, Hispanic families don't like to spend too much time watching TV. The bigger portion of female population makes the left-centered consumer distribution, even though insignificantly. We can imagine a society of many girl-talk clubs.

Now, consider the upper bound. We can develop some more neighborhood stories here. The increase of population shifts the upper bound to the right, but the city size shifts it to the left. In the more spacious areas, people would like to find other enjoyment, besides watching TV. Moreover, other enjoyment, like outdoor activities, may involve their neighbors.

The income distribution plays a similar role with the case of the mean. The low income households decreases the upper bound, and the high income households increase the upper bound. However, the trigger income levels, which start to increase the parameters, are different. The upper bound becomes bigger at the lower income level, and the mean responds later.

The standard deviation of the consumer distribution defines how dispersed the distribution is. The bigger the standard deviation is, the more dispersed the distribution is. In the large city, we have more concentrated distribution; that is another example for the neighborhood effect. In the case of large portion of Hispanic house-



holds, the distribution becomes more dispersed. Possibly, Hispanic families have different tastes with the other races. When we have a higher proportion of high income households, the distribution becomes more concentrated. The high income households may have a stable and substantial lower bound of tastes, comparing to the low income households.

The Tables 9 and 10 examines which and how demographic factors affect the cutoff levels, inducing a given probability, and the probabilities, obtained from a given cutoff level. The city size and the population play some roles when determining the cutoff levels or the probabilities. Generally, the factors affect the mean or the upper bound of the distribution, and then the distributional change affects the cutoff levels and the probabilities.

In conclusion, some demographic variables can really change the consumer distribution. In the process, we can find some stories, like a neighborhood effect and “reluctant” consumers.

#### E. Estimation of Market Demand

In the section B, I suggest the method to make a consumer type distribution, using the real world data. In the section D, I find how the demographic variables affect the parameters for the consumer distribution.

Here, we have a data set for a cable service provider in Dallas. In Dallas, Time Warner Cable offers a basic service (24 channels, \$11.35), an expanded basic service (71 channels, \$44.5), and a digital basic service (125 channels, \$54.45). Then, we can construct the consumer type distribution for Dallas, and simulate the number of potential subscribers of Dallas.

Using the above menu, offered by Time Warner Cable, we can calculate the

quantity demanded for each service tier. Fixing every other values and only varying the price for the expanded basic service, I derive the demand curve for the expanded basic service. The first panel in the figure 16 shows the current demand curve for the expanded basic service.

In this time, we can make an experiment, changing other variables. First, suppose that the population in Dallas increases by 100,000. The increase changes the mean, the standard deviation, the upper bound, and the total number of potential subscribers. Thus, we can derive a brand new demand curve, using the new consumer distribution. The second panel displays the original demand curve and the new one for the expanded basic service. The third panel shows a new demand curve, when the median household income in Dallas increases by \$10,000. The last panel includes a new demand curve, when the ratio of households with income over \$150,000 increases by 1%.

In fact, Dallas is not a good example to show the shift of rotation or both for demand curve, since the market is too big to capture the change of demographic variables. Here, I present another example. Georgetown, a prosperous suburban city of Austin, is a mid-size city with population of 50885. There, Suddenlink offers a basic service (14 channels, \$5.99), an expanded basic service (65 channels, \$18.48), and a digital basic service (152 channels, \$52.34).

The impact of demographic variable changes appears more vividly, in the Figure 17. The panels show the increase of population by 10,000, the increase of median household income by 10,000, and the decrease of the proportion of higher income household by 1%.

## F. Conclusion

The most important discovery in this chapter IV is confirming the possibility of upward quality distortion. I show that the upward distortion is quite prevailing in the cable industry. In Texas, 64 franchises among 209 franchises displays the upward quality distortion. In some franchises, we can find the upward quality distortion even in the low-end product. Considering the limited volume of previous research, this discovery is really surprising. I expect this dissertation will be a trigger to introduce the serious research for the upward quality distortion and its welfare loss.

Another contribution of this chapter is the construction of the consumer type distribution and the demand curve for a given interested product. The construction of the consumer distribution is performed by the simulation using the real data of the cable industry. The section B includes the detailed methodology. In the section E, I construct the demand curve. Since I already simulate the consumer type distribution, I can also simulate each product's market share, varying price levels and fixing other variables.

In the section D, I examine how demographic variables, like population, income, city size, race, and gender, affect the consumer distribution. It is performed while I relate the above demographic variables to the parameters for the consumer type distribution.

Naturally, the next topic is how the demographic variables affect the market demand. The effect of a demographic variable's change on the consumer distribution can be traced by the estimation results, obtained from the section D. Then, the effect of a distribution change on the demand curve can be obtained by the simulation, suggested in the section E. The change of demographic variables can induce the shift or rotation or both of the demand.

## CHAPTER V

### CONCLUSION

This dissertation adopts the finite product differentiation model with continuous consumer types. The model is applied to both the monopolist's problem and the social planner's problem. I reobtain the optimality conditions for the monopolist, originally obtained by Itoh (1983) and Crawford and Shum (2007). I reconfirm the optimal market segmentation of Itoh (1983), even in the endogenous quality setting, and resolve the optimal quality selection rule of Crawford and Shum (2007), in the more direct way. Moreover, I also solve the optimality conditions for the social planner, for the first time. In the chapter III, I introduce the numerical procedures. Using these computational procedures and the optimality conditions obtained from the chapter II, I can find the exact values for optimal qualities, prices, maximized profit and social welfare.

Next, I compare the monopolist's optimal with the social planner's optimal. The comparison is the first step for welfare analysis, which can induce policy implications. Some part of comparison is performed analytically, while other part is performed numerically. From the comparison, I show the possibility of upward quality distortion, which the existing literature has not previously reported. This discovery of the upward quality distortion, especially on the highest quality product, can stimulate research for an extreme product version. Although we can easily find the flagship product with an extremely high quality, like an Intel's deca-core Xeon CPU and a premium booth in Yankee Stadium, we have no precedent research for this kind of extreme editions.

I confirm this possibility of upward quality distortion, in the chapter IV. In fact, it is quite prevailing in the cable industry. In Texas, 64 franchises among 209 franchises displays the upward quality distortion. In some franchises, we can find the

upward quality distortion even in the low-end product. Considering the limited volume of previous research, this discovery is really surprising. I expect this dissertation will be a trigger to introduce the serious research for the upward quality distortion and its welfare loss.

This dissertation also analyzes how the number of products affects on the monopolist's and the social planner's decisions. The profit/social welfare strictly increases as the number of products increases. Moreover, the increment of increase diminishes rapidly. The chapter III also researches how distribution change affects the monopolist's and the social planner's choices. When a consumer distribution is dispersed, or left-centered, or both, the quality is less distorted and the amount of excluded consumers is large. When we have a large volume of consumers of low tastes, the quality is less distorted and information rents, which are enjoyed by consumers, increase. When we have a proper level of thick right-side tail, the possibility of upward distortion arises. When the distribution is concentrated, the monopolist can achieve a higher efficiency level compared with the social best.

In the chapter IV, I construct the consumer type distribution by simulating parameters using a real data in the cable industry. I examine how demographic variables, like population, income, city size, race, and gender, affect the consumer distribution. It is performed while I relate the above demographic variables to the parameters for the consumer type distribution.

When we have a consumer distribution, we can make a demand curve for a given service tier, varying price levels. I construct the demand curve in the chapter IV. Moreover, I also study the impact of the change of market environment on the demand curve. When the population increases, the change of population alters the consumer distribution. Using this new consumer distribution, we can make a new demand curve. In the Economics textbook level, we can expect that the change of

outside parameter induces the shift of demand curve. Here, the story is much complex. The increase of population affect the consumer distribution, in a few different routes: mean, standard deviation, and upper bound. In fact, it affect the total number of potential subscribers, which is another determinant for the demand curve. We can observe the rotation of the demand curve, as well as the shift of the demand curve.

This dissertation can be a first stepping stone between theoretical framework for endogenous quality choice and empirical model using finite product differentiation. Essentially, empirical models take a discrete product space, since data is collected in the discrete space. Moreover, numerous followers of Berry, Levinsohn, and Pakes (1995) usually have assumed that firm's quality level is exogenously given and only price is endogenously chosen considering its expected market share. In many cases, firms choose quality considering the expected market share, too. In addition, the market share of a product depends on the product quality, as well as its price. This work can help empirical researchers simulate the endogenously chosen quality, together with the endogenously chosen price and the endogenously determined market share.

The immediate future works, therefore, are empirical applications of this work, even though I already start the work here. Crawford and Shum (2007) is a valuable and unique previous empirical analysis with the endogenous qualities in the monopoly, like this work. Other future investigations would be extending this dissertation's framework into competitive environment. Although we already have several preceding papers of finite products and endogenous qualities, almost all works adopt quite restrictive environments, like a uniform consumer distribution and a zero marginal cost. Chu (2010) begins the study from the empirical side but it makes a theoretical improvement also. He adopts even more flexible consumer distribution and estimates both quality and market share, even though he needs some restrictions. He exploits

consumers' brand loyalty and incumbent-entrant structure. However, in empirical application or extension to the competitive environments or both, many tasks and topics remain to be developed.

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## APPENDIX A

### OVERVIEW OF THE CABLE INDUSTRY

In this appendix, I survey the cable industry. At first, I describe the chronological history of the industry. While describing, I focus on a structural change of the industry, usually induced by technological innovations or governmental interventions. And then I describe the technology of the industry. In the third subsection, I listed essential governmental interventions. The remaining subsections are devoted to describing big players of the industry: MSOs and media conglomerates.

The cable industry has had a relatively short life. Parsons (2008) argues that the industry is very young, compared with other industries' process of development. Although the industry has grown only a short time, its influence over mass culture, especially America's mass culture, is enormous. Thus, many authors have been feeling it is a good time to comb through the industry's history. See Mullen (2008), Parsons (2008), and Webb (1983) for examples. Both Parsons (2008) and Mullen (2008) employ a chronological method, but Mullen (2008) pays more attention to the roles of people around the industry, while Parsons (2008) emphasizes the institutional effects like regulation. Webb (1983) analyzes causes and consequences of the industry using economic logic.

#### A Brief History of the Industry

##### An Eve of the Birth of the Cable Industry

In the 19th century, radio was already used in the form of "wireless telegraph." In early times, the "wireless" use of radio was usually applied to ships in the ocean. It didn't, however, take a long before recognizing there was no technical difficulty

to apply the radio technology to multiple anonymous recipients, instead of a single known recipient like a ship. In the 1920s, the interest of industry using the radio technology moved from telegraphing to broadcasting radio. In 1926, NBC, the first radio station, was founded, and in 1928, CBS followed. The U.S. government, which observed that the radio technology could become the essential part of mass media, introduced the Radio Act of 1927 and created the FRC. Ever since, the government have put various regulations on the industry.

The era of radio rapidly had evolved to the era of television. The government needed a new act, the Communication Act of 1934, and the FRC was replaced with the FCC. In the 1930s, the major radio stations sensed the potential power of televisions. In fact, “Hollywood” was enjoying their heyday in these times, and many persons regarded television as a potential mini-theater at home. Since the late 1930s, numerous potential television broadcast stations were waiting to get their licenses from the FCC. But the enemy of the entrepreneurs was not the FCC, but World War II. The spectrums had to be assigned to military uses and the broadcasting licenses were limited at the minimum level. After the war, numerous stations, over 100, were licensed and began to broadcast. The limited spectrums were exhausted in a very short time, and there was much confusion in radio uses. After all, the FCC declared all additional licensing should be frozen. While freezing additional stations, there were efforts to exploit existing signals from already licensed stations. These efforts triggered the cable industry.

#### An Early Stage: a CATV Business and Beginning of Government Regulations

John Walson, an appliance dealer, made a cable system for the first time in 1948. Mahanoy City in Pennsylvania is surrounded by mountains and so this city couldn't get signals from Philadelphia. Walson had difficulties to sell televisions in this city

and he decided to set a big antenna on Tuscarora Mountain. The households in Mahanoy city were connected to this antenna, and became the first cable subscribers. Walson offered free cable service for buying a television set, and this service became the foundation of Service Electric Television, which is one of the biggest MSOs today.

On the other hand, we have three more plausible stories for the first cable service. Leroy E. Parsons of Astoria, Oregon installed an antenna on the top of a hotel near his home just to secure a clean screen. But right after that, he found the business opportunity from this antenna. It happened in late 1948. James Y. Davidson of Tuckerman, Arkansas did a similar thing to Parsons. The difference was that he made a 100-foot big antenna to receive signals from faraway Memphis. After the demonstration of a football game: University of Tennessee versus University of Mississippi, he was recognized as the center of cable business in this region. In Lansford, Pennsylvania, Robert J. Tarlton began Panther Valley TV in 1950. Though he started a little later than his competitors for the “first,” the Panther Valley TV established its own business model faster than others after merging with Jerrold Electronics, which supplied cable equipments including a CATV-specific coaxial cable. The Panther Valley TV charged an installation fee and then a monthly subscription fee, which is very similar to a cable service charge today.

We call this kind of service a CATV; a community antenna television. In this stage, the function of the cable industry was just to retransmit signals from broadcasting stations operating in big cities. For more information about a CATV and early pioneers, see Lockman and Sarvey (2005) and Robichaux (2002). Both two books are wholly devoted to CATV pioneers.

During the 1950s, the FCC could not determine its position. The FCC, basically, was not for this industry. The CATV used wired cable like telephone, but they transmitted the signal from broadcasting stations. In the viewpoint of the FCC,



it was neither telephone-like nor broadcast-like. Of course, it was both telephone-like and broadcast-like. Virtually, the standpoint of the FCC was no intervention through all the 1950s. The CATV industry rapidly expanded their businesses without regulations.

The philosophy of the FCC was changed dramatically in the 1960s after the CATV industry employed microwaves. Local small broadcasting stations began to strongly complain that CATV threatened their survival. Through the microwaves, the CATV industry could supply high quality and capital-intensive programs produced by urban-area stations to local subscribers in rural areas. It stole numerous audiences from local small broadcasting stations. The FCC decided to stand on the local stations' side although Congress and Court still didn't determine their positions. Until this time, Congress could not pass any bill about the CATV business and courts gave some mixed judgements. The FCC required all CATV companies should deliver their local stations' programs (a must-carry rule) and strictly limited the adoption of microwaves.

In the late of 1960s, the government and some governmental bodies including the FCC determined their position at last. The FCC considered the CATV as a top prospect to break down the strong three-network oligopoly in the broadcasting industry. At this time, the U.S. broadcasting market was ruled by ABC, CBS, and NBC. The regulations of the FCC aimed at invigorating local broadcasting stations. Thus, the CATV systems, which were medium to large size, were asked to produce and deliver their own programs (a local origination rule).

Since this local broadcasting, a recent form of cable television began to replace the CATV system. That is, the cable industry added the local broadcasting service on just a retransmitting service. The FCC's intention got a small success, but a big failure at large. In some communities, cable service providers constructed active

relationship with local schools and local groups. They cooperated to produce regional programs, which was exactly FCC hoped. But in most part, this regulation induced a huge financial crisis among the local CATV companies.

In 1972, the FCC modified their regulations. Apparently, it seemed to strengthen the regulations. They added the mandatory delivering the programs of assigned local stations and they obligated contracts with local governments in CATV company's service area. But the real face was different; the FCC loosened the requirement to be under the regulation. Now, the companies only in big cities fell under the regulation. In fact, the FCC wanted to supply a remedy for the financial crisis of the CATV companies. The obligation of municipal contracts was, in the point of fact, helpful to companies, because it limited a franchising fee of municipality. In 1974, people began to think that a CATV business could make a profit.

In spite of non-cooperative attitude of FCC in the 1960s, CATV industry still continued to expand. There appeared a few prototypes of a future MSO. And prototypes of a pay cable channel also entered the stage. After a few experiments in the 1950s, some pay channels were formed. The Gridtronics, owned by Warner Communication, started the service. It eventually had grown into The Movie Channel. Another pay channel, the Z Channel, was a foundation for Showtime. In 1972, Sterling Manhattan Cable, owned by the Time, Inc., launched a pay channel, named by Home Box Office.

#### An Era of Satellites: a Rise of the Cable Networks

In the early 1970s, the cable industry was congested. Pay televisions using cables were prohibited, companies in a big city were severely regulated by the FCC, and the plan to utilize satellites was pending by endless debates among related government bodies. Moreover, the conflicts about the copyright of broadcasting programs were

still in courts. Nobody could squeeze a growth plan out of the industry. In 1975, accurately after 1972, the agonies of the industry suddenly melted down with the satellite debut in the cable industry. Since that, the satellites became the essential part of the industry growth.

In the 1960s, pioneering satellite experiments were being tried. Numerous CATV companies had much interest on that, since AT&T land lines asked expensive charges. But the “discussion” between the FCC and other governmental bodies had continued until 1972. That is, for the CATV companies, the satellites were just a pie in the sky.

In the 1970s, the FCC eliminated almost all possible legal obstacles in the industry. In 1974, the regulation to produce local programs was abolished. In 1976 and 1980, the FCC allowed all kinds of importation of programs from any broadcast stations, step by step. Except for the municipal contracts, almost all obligations were disappeared.<sup>1</sup> Most of all, the cable company could use their longing satellites in 1975. It seemed that the industry could grow easily and rapidly, with satellites together. But it was not easy like that. The rental fee for the satellites<sup>2</sup> and the cost of dishes and converters were not cheap enough. In 1979, the FCC repealed the specific technological requirements on a satellite and a receiving dish and converter. And then, the cable providers could employ the “cheap” technologies according to their profit level.

The first forerunners employing a satellite technology were pay-cable stations, es-

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<sup>1</sup>We should note one thing here: Although the FCC removed the local program obligations, the local programs could hold their positions. It was because of maintaining municipal contracts obligations. Local governments and civil activists asked for the cable providers to produce the local programs. As the prospect of the industry became positive, the competition for the service contract became fierce. Thus local governments had strong bargaining power.

<sup>2</sup>Of course, the rental fee for a satellite was reverted to a cable broadcasting network, not a cable service provider.

pecially HBO. But the other broadcasting stations began to adopt the technology, not before long the HBO's employment. In these times, the local broadcasting stations, like New York's WOR and Chicago's WGN, grew the nation-wide super-stations by delivering their signals using the satellites. In the late 1970s, the other cable-based networks started to make revenue. The networks like USA, C-SPAN, ESPN, and many Spanish networks began the nation-wide services through satellites.

During the 1980s, people began to think that the cable industry would be a cash cow. Using the satellite technology, newly arriving networks produced numerous attractive programs. In 1980, there were only 12 networks, but in 1992, the number of operating networks were over 60. In the deregulation mood of the Reagan administration, the Cable Act of 1984 really promoted astonishing growth of the industry. The main objective of the Act was a compromise between the cable industry and the municipalities. But the Act, the first industry-specific Act, maintained almost all existing traditions. The large MSOs, which will be explained in following sections in detail, got a chance to leap one more time. Usually, a *laissez faire* policy helps big players. These big MSOs became bigger and bigger, and they moved into the area of broadcasting networks. In conclusion, it made a big development of cable-specific broadcasting. Helped by MSO's financing, many networks could begin their adventurous businesses.

In those days, some thought that the narrow-tasted channels also would be profitable in the multi-channel age. They launched the so-called "culture" networks. ABC launched ABC-ARTS. CBS had CBS-Cable and NBC had the Entertainment Channel. In addition, there was Bravo<sup>3</sup>. These "culture" networks couldn't last long. ABC-ARTS and the Entertainment Channel merged into A&E, and A&E added more

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<sup>3</sup>At that time, this network broadcasted non-Hollywood movies, usually international art films.

popular programs. Bravo also added some popular films in their program lineup.

In 1980, Time, Inc. set up Cinemax, as a complement of HBO. In 1984, Lifetime was founded aiming for housewife TV viewers. In 1985, John Hendricks launched Discovery Channel. And the Discovery networks added The Learning Channel, Travel Channel, Animal Planet, Discovery Health Channel and Discovery Home and Leisure.

There was another big success story. The owner of TBS,<sup>4</sup> Ted Turner, launched CNN in 1980. In early years, it underwent difficult days to attract viewers. CNN, however, aggressively secured international news sources, and it gave CNN a big success, especially, through the Gulf war. Turner added TNT, Cartoon Network, and TCM in turn.

In the 1990s, there appeared networks with more restricted audience targets. Comedy Central and E! Entertainment Television aimed their own special audiences. The first considered politically progressive young people. The second aimed young and easy-going persons. But the most outstanding success story went to MTV. MTV made a big success not only on the broadcasting scene, but also the whole cultural scene. MTV could make a new MTV generation and it was a cultural phenomenon, jumping over TV's limitation. Another phenomenon worthy of noting is advent of home shopping channels. In the 1980s, several home shopping channels made successes. Before launching of home shopping channels, people thought that the mutual interaction would play a big role in success of these channels. But at that time, they just used phone call interactions. After that, the interest on the mutual interaction technology moved to an internet, not a cable. Recently, the home shopping channels are facing with a big challenge of an internet-based business.

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<sup>4</sup>At first, it was a local broadcasting station in Atlanta area. By delivering the signal using the satellite, it grew a nation-wide super-station.

## Convergence of Communication related Industries

In the 1980s, the industry expanded astonishingly. The Cable Act of 1984 removed almost all obstacles in the industry. It was obvious that big players would play in the region of cable service providing. The first one was TCI. The TCI already had 2 million subscribers in 1981. During the 1980s, the TCI acquired many small cable systems and rival systems. In the early 1980s, the TCI made a hub and spokes-like big system near Pittsburgh. The economic benefit of this system stimulated other MSOs. In conclusion, almost all metropolitan areas were equipped by this hub and spokes system. To do this, the MSOs merged, acquired, and exchanged numerous local systems. The most notable story was about “swaps” between Milwaukee for Time Warner and Chicago for TCI in 1998.

By 1990, Comcast also obtained a seat for a big MSO. Comcast started in 1963, and by the numerous merger and acquisition, it grew a big MSO. Comcast expanded one more time when adding MediaOne’s system after merger and acquisition with AT&T. And then, the breakup of TCI gave Comcast a chance to the largest one in the industry. As of 2008, Comcast secured 24.2 million subscribers.

Warner-Amex and ATC were another big MSOs in the 1980s. And in the 1990s, ATC had become the big part of a giant Time Warner. After a huge size M&A deal of Time Warner in 1989, Time Warner obtained NewChannel in 1995. It had 13.1 million subscribers in 2008. The third positioned, Cox Communications had 5.3 million. On the other hand, Service Electric, which we reviewed in the Section 1.2, was fourteenth positioned and had 29 thousand subscribers.

In these times, that is, in the 1990s and the 2000s, the most outstanding phenomenon was a convergence of industry and technology. Telephone, internet, and cable business in addition to broadcasting industry were rapidly coming together under

one roof. Today, almost all cable MSOs offer the cable-internet-telephone package. Moreover, the cable MSOs have a substantial share for broadcasting networks. In 1996 Time Warner merged with Turner Broadcasting, and in 2001 it acquired AOL. Here are some notable mergers: in 1993 Walt Disney Co. acquired ABC, in 1997 Microsoft acquired 11.5% share of Comcast, and in 1999 CBS and Viacom merged.

In 1992, Congress passed the Cable Consumer Protection and Competition Act, mainly led by Senator Albert Gore. Unlike the Act of 1984, this one strengthened some regulations. Especially, this Act limited the total number of subscribers in one MSO. But this Act quickly replaced by the Communication Act of 1996. In 1996, people already expected the industry convergence, so they needed the new bill to manage relevant industries together. The Act of 1996 gave a chance to a telephone companies. Until 1996, the telephone companies were more regulated than others. Moreover, by the new integrated Act, some new technologies, like the Ku-band satellite broadcasting system, entered the cable industry as competitors. As the cable MSOs have had new chances for a business, they have had new competitors, who were not before.

## A Basic Technological Explanation on the Industry

### A Cable Technology

In this section, we examine technical aspects of the cable industry. The essence of the cable industry is to take broadcasting signals and to deliver them to end-node consumers, households. At first, we describe the receiving and re-distributing process. Of course, we will deal other functions relevant with the cable industry.

The first and most important function in the cable industry is receiving broadcasting signals and re-distributing. In early years, broadcasting stations started to

emit signals, but the signals couldn't go very far. Moreover, they were affected by geography. So people constructed a big antenna on a high hill to collect the signals and distributed them through cables connected with the big antenna. The signals collected by the big antenna were sent to a *headend* at first. The *headend* can amplify and process the signals.

When the broadcasting stations employed microwaves, the function of *headend* had also expanded to receive microwaves. In the form of microwaves, the signals could travel longer distances. And the new *headend* could change the spectrum of microwaves and emit the changed. That is, the signal from a broadcasting station could travel longer and longer distances. It suggested a possibility of a big broadcasting station. In the 1970s, we could use satellites to distribute broadcasting signals. A *headend*, of course, evolved to accept the satellite technologies.

A *headend* has other functions than receiving and re-distributing. A *headend* employs various playback technologies, so it can adjust broadcasting schedules and sequences. Sometimes it can insert a locally prepared program between regular broadcasting programs. Many *headends* are connected to facilities to make own programs. In recent years, a *headend* can process internet services and cable telephone services. Now, almost all cable service providers offer a bundle consisting with a cable service, a telephone service, and an internet service.

From the *headend*, the signals go to a subscriber's home. To connect the *headend* to the subscriber's television, we use a cable. There are some different kinds of cables; a feeder is a thick cable from the *headend* and a drop connects a feeder with a subscriber's home. The signal moves through these cables from the *headend* to a subscriber's home. In this process, there happen amplifications a few times.

Once the signal arrives at the subscriber's home, we need a converter box. This converter box tunes the arrived signals by VHF dials or alternative tuning device.



Today almost all televisions include the converter box, so many people can use the cable services just from wall. The converter box, however, is still used for new services; digital channels, interactive services, and digital video recording. Sometimes the converter box is used for cable-related services; a high-speed internet and a cable telephone.

In recent days, the major part of cable services is no longer re-distributing the air-signals from the broadcasting stations. After the communication satellites became popular, there have arrived numerous cable-based program services or networks. Using the communication satellites, they emit signals which a household cannot receive by a usual antenna. Only cable service providers can receive these and re-distribute into a subscriber's home by cables. The familiar channels, like CNN, ESPN, and MTV, are cable-based networks. We cannot receive these channels' programs only using a regular antenna, while we are able to catch the programs from ABC, CBS, NBC, and Fox. There are many networks using air-signals,<sup>5</sup> but most air-signal networks are affiliated with ABC, CBS, NBC, Fox and PBS.<sup>6</sup> The technical part is almost same in these different stations, only except whether using air-signals or not.

#### Competing Technologies with Cable

A MMDS uses microwave relays. The receiver collects signals in the form of microwaves, and it re-broadcasts in the form of air-signals in the limited area. Since it needs air-signals, the number of channels should be limited, usually around 10. In the middle of 1980s, that is, in the first arrival of multi-channel age, the MMDS

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<sup>5</sup>The name of local air-signal networks begins with K(west of the Mississippi) or W(east of the Mississippi). This name is a license granted by FCC.

<sup>6</sup>There are a few independent local air-signal networks. They are usually "homes" for local Major League Baseball teams or "broadcasting platform" for religious bodies.

had disappeared. But its extremely low cost and the wireless form can be another alternative someday.

A SMATV is serviced only for hotels, hospitals, and so on. This is a small size CATV industry. A hotel installs a dish for SMATV service on its rooftop and entire hotel rooms are connected to the dish by cables. This system doesn't need expensive utility polls, but it was not profitable enough to overcome the competition with rich cable operators. It is, however, still attractive in remote areas.

The DBS, which usually we call a dish, are the strongest rival for the cable industry. Since the middle of 1970s, they have expanded their business and now they have a substantial proportion in the multichannel television services industry.

The first "dish"-like technology was DTH satellite service using the C-band signals. This service requires 10-12-foot earth station; a big dish. When it was introduced first, people really welcomed it. Especially in rural areas, it was quite popular. At that time, it asked just one-time installation fee. But after the invention of scrambling technology, the operators charged a monthly fee. It removed the attractiveness of the service. Except in a few rural areas, this technology has quickly disappeared. And then, the DBS using the Ku band signals came. It requires 18-20-inch dishes to install anywhere around a house. Although there were eight service providers at early days, we have only two major companies, DirecTV and Echostar (DISH Network). The weakness of DBS is the limited number of service providers, for the number of orbital slots for static satellites is limited.

### The History of Regulations on the Industry

A tone of governmental regulations always depends on the political viewpoint of a party in power. In the Republican administration, the tone was quite generous. The government would remove regulations and would like to promote private enterprise

atmosphere. On the other hand, the Democratic administration stressed strict regulations and consumer protection. They pursued competition in a controlled industry.

**The Communication Act of 1934** This act, which was following the Radio Act of 1927, formulated the FCC. Since after this act, the FCC has regulated all relevant industries. At that time, the relevant industries were classified into two categories: a mass communication and an intercommunication. The mass communication included radio and television broadcasting. The intercommunication included telephone and telegraph. Especially, they called the intercommunication as a common carrier.

The characteristics of the mass communication were described by emission of air-signal to unknown mass and one-way communication. On the other hand, the intercommunication could specify receiving subjects and could get the response from the receivers. In the mass communication, the business should use the spectrum, which was allocated by the FCC, and so the FCC had justification for regulations on these businesses. In the case for the intercommunication, the businesses needed some geographical monopolistic power to secure local network facilities. Thus, the FCC guaranteed some degree of market power and then regulated their businesses, especially the FCC did rate regulation.

In the case for the cable television business, the situation was much more complicated. The cable industry didn't employ air signal and it could specify the receiving subject. Moreover, they needed some local network facilities. But it didn't admit intercommunication, although it was technologically possible. And the main contents of this business was ones of the mass communication. That is, the cable television business was neither a mass communication nor intercommunication. At the early years of the industry, the FCC didn't consider the industry as a common carrier, so it could avoid a rate regulation.

**Belknap Dispute (1951-1954)** In 1951, J. E. Belknap and Associates claimed that it would employ microwave as a common carrier. They argued that they would sell broadcasting signals to CATV companies using microwave. So microwave could be a common carrier in this case. But at that time, the only consumer CATV company was J. E. Belknap and Associates itself. Thus, the main issue was whether a CATV business was a common carrier or not, since the final and conclusive service was a CATV, not a microwave business. Since the Communication Act of 1934, the FCC still hadn't decide their position about CATV businesses. In this reason, J. E. Belknap and Associates reclaimed the same argument after dividing company into a microwave part and a CATV part. Although it got a permission in 1954, it couldn't begin its business.

While this claim proceeded, some issues, which would become major controversial things during a few following decades, rose on the surface. The first one is copyright. The contents of CATV businesses were from broadcasting companies. Although the broadcasting stations had benefits from re-distributing of their signal, the problem on property right for their creations still remained unsolved.

The second one was the concern of the FCC to protect small local broadcasting stations. The import of popular programs made by big urban broadcasting stations, using microwaves and CATV, might do harm to local stations. Since after this claim, the FCC had maintained their propensity for diversity and equal opportunity of broadcasting in rural area until the Reagan administration would appear.

**Cox Report (1958) and 1959 Report and Order of the FCC** In 1958, the Senate Committee issued the staff report, authored by Cox, about the debate on the cable industry. Until this report, the FCC had held their position, in which they

were reluctant to intervene the industry operation. But the report suggested that the FCC should regulate the industry as a new media. Especially, the report pointed that the industry could be harmful to a local small broadcasters, and so the FCC should protect these broadcasters.

The 1959 Report and Order of the FCC was a response on the report. Although the FCC explained the CATV wasn't harmful much to a local broadcasters, the FCC recommended that CATA operators should obtain retransmission consent from broadcasters whose signal CATV operators would want to carry.

**S.2653 (1959)** At the late of 1950s, the first industry specific bill was prepared. The bill, S.2653, required that the FCC made mandatory licensing procedure for CATV business. The CATV operators should carry local broadcasting signals and they were prevented to duplicate long distance signals or other cable channels' programs. The most outstanding figure of this bill was the fact that the CATV industry would be regulated by a mass communication, not a common carrier. After a big exhaustive debates, the bill was defeated and died in committee.

**Carter Mountain Decision (1962)** The FCC denied to license a microwave application of Carter Mountain Transmission Corp. The FCC judged that microwave import might be harmful to local broadcasters and the FCC decided to protect them. This decision became a new momentum for the expansion of regulatory authority. Note that the FCC still had considered the industry as a common carrier. That is, the FCC took the first step to regulate the industry, but the FCC didn't take the whole viewpoint of S.2653.

**The Seiden Report (1964)** As of 1964, there was still no official Congressional

regulation on the industry. The FCC hired Dr. Seiden to investigate the effect of the CATV industry on local broadcasters, especially UHF television stations. The report showed CATV to be less of a threat to the broadcast stations, and so Dr. Seiden preferred to support UHF stations, rather than regulate CATV.

**1965 and 1966 Report and Order of the FCC** Still with no Congressional support, the FCC set up their regulatory position at last. The FCC acknowledged that they needed a new category to regulate the industry, neither a mass communication nor an intercommunication. And the FCC dealt with the industry aiming the public interest. Their actions were close to Cox Report and S.2653, rather than the Seiden Report. From this time, the FCC began to intervene the industry effectively for the first time.

**The Rostow Report (1968)** In 1968, a task force, leaded by Eugene Rostow, was formed by the White House of Johnson administration. They recommended the FCC to relax regulations on the industry. It was because they decided that the cable industry could be a good alternative to collapse three network oligopoly in television broadcasting industry. After the report, people had recognized the industry's new role as an alternative for the mass communication industry.

**Two Court Decisions in 1968** In the case of *United States vs. Southwestern Cable*, the Supreme Court admitted the jurisdiction of the FCC in the industry. This became the first legal support for the regulations of the FCC. In another case of *United Artists Television vs. Fortnightly Corp.*, the Supreme Court removed the copyright liability from the CATV industry. The court recognized the CATV as one of viewer's tools to catch broadcasting signals. Thus, the CATV company was free of copyright

liability, like an antenna company.

**1972 Report and Order of the FCC** This was the first extensive regulations for the industry. Under this rule, the cable systems were required to have at least 20 channels and two-way capacity. If the system had more than 3,500 subscribers, it should offer one access channel. And the system, which operated in the top 100 markets, had to provide at least three public, government, and educational channels.

About the importation of signals, cable systems were granted the right to import distant signals. The number of signals the system could import depended on the market size. Top market systems could import 3 additional independent signals. But systems in markets below 100 could carry only one independent, which is called “anti-leapfrogging.” Of course, the local broadcasters’ signals should be carried by the systems.

The Certificate of Compliance was added to the regulation set for the industry. Every cable operators should take this Certificate, and this was related with municipalities. The FCC gave municipalities some degree of control power on the cable operators, instead of capping franchise fees at 3%. The operators should take the Certificate from bargaining with municipalities.

As a whole, the 1972 rule was constructed by the viewpoint of liberalism in the Nixon administration. Indeed, the Certificate played a role to protect cable systems from local fickle politicians. And the new importation policy was quite favorable to the operators. At that time, the NCTA evaluated this rule as the first step to end the industry freeze.

**Deregulating Steps During 1974-1980** In 1974, the FCC abandoned the rule to

originate programming in the top market systems.<sup>7</sup> In 1976, the FCC removed any restriction on the importation of signals. Then in 1980, the FCC removed the all remaining “anti-leapfrogging” rule.

**The Copyright Act of 1976** Contrary to the Court decision of 1968, the act guaranteed the property rights of broadcasters on programming. But the act got rid of all exhaustive conflicts between broadcasters and cable operators, employing a “compulsory license” system. Every cable operators should pay some royalties to the Copyright Royalty Tribunal and it re-distributed that money to broadcasters.

**The Cable Communications Policy Act of 1984** The main purpose of this Act was a compromise between the interests of the cable industry and those of the municipalities in which cable systems wished to do business. At that time, cable franchising process was quite corrupted and so this compromise was very important. Apparently, this Act handed a win to municipalities, giving some regulatory powers and rights to secure leased access channels. The Act, however, was based on the deregulation mind of the Reagan administration. While establishing franchising process gave some regulation power to cities, it freed cable operators from unreasonable bargaining process with municipalities. Additionally, consolidating regulations implied cable operators were free as long as they were under the regulation. Thanks to this favorable Act, the cable industry could grow very fast, especially a large MSO could.

**Must-Carry Decision in 1985** In 1980, Ted Turner petitioned the FCC for elimination of the must-carry rules, arguing that they violated the First Amendment rights

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<sup>7</sup>By the 1969 rule, the systems, which had more than 3,500 subscribers, should offer their own local programming.



of the Constitutional law. But the FCC didn't respond to the petition. Meanwhile, Quincy Cable TV, Inc. in Quincy, Washington, decided not to carry three local Spokane broadcast signals, choosing to import three stations from Seattle instead. The FCC ordered carriage of local signals and charged a fine. At last, both cases went to the court. In 1985, the District of Columbia Circuit Court ruled the FCC's must-carry rules were an unconstitutional infringement of the cable operator's First Amendment rights.

**Abandon of Price Control (1986)** On December 29, 1986, the FCC abandoned the control for cable subscriber rates. In the first six months of deregulation, the average cost of basic service increased between 10.6% and 14.6%, depending on the estimates. We should note that an industry with the possibility of a natural monopoly are usually controlled by a rate regulation.

**The Cable Consumer Protection and Competition Act of 1992** The era of a *laissez faire* arisen from the Cable Act of 1984 had gone in 1992. The 1992 Cable Act was led by Senator Albert Gore, and he recognized US consumers as victims of a greedy and out-of-control cable industry. This new Act proclaimed a new mood of the Clinton administration.

This new Act was the most harsh regulation for the industry. The FCC could do a rate regulation and even a tier regulation. As a matter of fact, the FCC ordered a freeze on all cable rate increases and then began a systematic rate rollback in 1993. Must-carry was reinstated. The service guideline was introduced in detail. The regulatory power of municipality was enhanced.

**The Telecommunication Act of 1996** Since 1934, the communication industry

had changed dramatically, and it invented numerous new technologies. Thus, it was inevitable to compose a new Act, the Telecommunication Act. This act mainly concerned about the telecommunication industry focusing on the telephone industry, not the cable industry. But the Act had significantly influenced over the cable industry.

The Clinton administration pursued to increase competition, and so they removed barriers between relevant industries. Right after the Act, the telephone companies began to enter the cable industry. Moreover, the cable operators could possess cable networks.

Additionally, this Act relaxed the rate regulation. The MSOs with less than 50,000 subscribers were freed immediately.

#### The Rise and Fall of MSOs

Through the 1970s, TelePrompTer dominated the cable TV business.<sup>8</sup> And in the 1980s, TCI became a No.1 market sharer. After falling of TCI, the domination had passed to Time Warner and Comcast, in turn. As of 2008, Comcast secured the largest subscribers, and Time Warner and Cox followed Comcast. The Table 11 shows the ranking of MSOs in 2008.<sup>9</sup>

**TelePrompTer Corporation** Irving B. Kahn was one of the first leaders of a recent style MSO. In 1959, Kahn's TelePrompTer began to acquire CATV systems. By the mid-1960s, TelePrompTer had grown to one of the biggest in U.S. with 14 systems and 70,000 subscribers. TelePrompTer began to lose its leading role in the industry since 1971, when Kahn was convicted of bribery during the Johnstown, Pennsylvania

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<sup>8</sup>The substantial amount of descriptions in this section was from explanation of Wikipedia.

<sup>9</sup>Confer the NCTA website.

franchise renewal process. During all 1970s, TelePrompTer had been in difficulty, and at last it was sold to Westinghouse's Group W Cable Subdivision in 1981.

**Cox Communications, Inc.** In 1962, Cox Enterprises dived into the cable business, purchasing a number of cable systems in Pennsylvania, California, Oregon and Washington. In 1999 Cox acquired the cable television assets of Media General in Virginia. In 2000 Cox Communications acquired Multimedia Cablevision with assets in Kansas, Oklahoma and North Carolina. On November 1, 2005, Cox sold all of its Texas, Missouri, Mississippi and North Carolina properties, as well as some systems in Arkansas, California, Louisiana and Oklahoma to Cebridge Communications. The sale closed in 2006 and those systems were transitioned by their new owner from Cox to Suddenlink Communications, a new brand of Cebridge Communications.

**Tele-Communication, Inc.** TCI was a cable television provider for much of its history controlled by John Malone. The company came into being in 1968, following the merger of Western Microwave, Inc. and Community Television, Inc.

In 1956, Bob Magness decided to raise some money for a CATV system in Memphis, Texas. Two years later, he built 6 systems with some partners, serving a total of 12,000 homes. In 1968, the companies moved to Denver and became Tele-Communications Inc. TCI went public in 1970. At the time, it was the 10th largest cable company in the United States. By 1972, with 100,000 subscribers, Magness needed someone with more business knowledge to run the operation. He decided to hire John Malone, president of Jerrold Electronics, a division of General Instrument. By 1981, Malone had made TCI the largest cable company in the United States. His major business skill was merging and acquiring small to mid-size other companies. Between 1972 and 1988, TCI completed 482 acquisition or sale deals.

After a failed merger attempt with Bell Atlantic in 1994, it was purchased in 1999 by AT&T. On June 24, 1998, AT&T, the nation's largest provider of telephone service, announced a plan to buy TCI, for \$32 billion in stock and \$16 billion in assumed debt. This marked the first major merger between phone and cable since deregulation after the Reagan administration. AT&T completed its acquisition March 9, 1999, and TCI became AT&T Broadband and Internet Services, the company's largest unit. TCI's cable television assets were later acquired by Charter Communications and the Comcast Corporation.

**American Television and Communications** Bill Daniels, commonly known as the "Father of Cable Television," constructed a CATV system in New Mexico in the 1950s and employed a microwave relay for the first time. He founded Daniels and Associates, a cable brokerage firm in 1958. In 1968, Daniels and Associates gathered a number of systems and created a new cable company, ATC. Right after its birth, it was the third largest cable company in the country with more than 100,000 subscribers. In 1973, ATC bought the systems from Time, Inc., but the whole ATC including the part from Time, Inc. was purchased back by Time, Inc. in 1978.

**Comcast Corporation** American Cable Systems was founded in 1963 by Ralph J. Roberts, and in 1969 it was incorporated in Pennsylvania under the name Comcast Corporation. Comcast bought 25% of Group W Cable in 1986, doubling its size. Two years later, it purchased a 50% share in Storer Communications, Inc. Comcast became the third largest cable operator in 1994 following its purchase of Maclean-Hunter's American division.

In June 1997, Microsoft bought 11% of Comcast for \$1 billion. This was another evidence that Microsoft had much interest in a cable industry, together with Paul

Allen's Charter Communications. A substantial amount of Wall Street investment followed Bill Gates.

In 2001, Comcast announced it would acquire the assets of the largest cable television operator at the time, AT&T Broadband (AT&T's spin off cable TV service) for \$44.5 billion. In 2002, Comcast acquired all assets of AT&T Broadband, thus making Comcast the largest cable television company in the United States with over 22 million subscribers. In 2005, Comcast joined Adelphia purchase with Time Warner Cable.<sup>10</sup>

**Adelphia Communications Corporation** John Rigas opened his first cable system in 1953. In 1973, he put all his businesses together, and create Adelphia. He and his family made this company to the nation's sixth largest MSO, with more than 5 million subscribers. But the excessive investment to telephony business brought a huge financial crisis. As a result, it bankrupted in 2002, and its assets were officially acquired by Time Warner and Comcast on July 31, 2006.

**Cablevision Systems Corporation** Cablevision Systems Corporation was founded in 1973 under leading of Charles Dolan. As of 2008, it was the 5th largest cable provider in the US, with most customers residing in New York, New Jersey, Connecticut, and parts of Pennsylvania.

**Group W Cable** The Westinghouse Broadcasting Company, also known as Group W, was the broadcasting division of Westinghouse Electric Corporation, founded in 1886. It had operated in radio and television broadcasting since the 1920s. In 1981,

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<sup>10</sup>See the subsections for Adelphia and Time Warner Cable.

Group W purchased cable TV system operator TelePrompTer, which it renamed Group W Cable the following year. Group W would leave the cable TV system business in 1986. Their assets, covering 2.1 million subscribers and valuing \$2.1 billion, went to ATC, Comcast, Daniels and Associates, Century Communications, and TCI.

**Time Warner Cable, Inc.** Time Life Broadcasting entered the CATV field in 1965. Its most historically significant investment came to a company called Sterling Communications. In fall of 1964, Sterling applied for a franchise to wire New York. The heavy amount of investment for Manhattan, however, was a big burden for Sterling. After a big failure in Manhattan, Time Inc. abandoned its CATV business, selling assets to ATC in 1973. But in 1978 Time Inc. reentered system operation, purchasing the whole ATC back.

In 1974, Warner Communications entered the cable television industry by forming Warner Cable in Ohio and Virginia. In 1977, Warner Cable's Columbus, Ohio unit introduced the QUBE, the world's first interactive television programming system. Despite its technological innovation and vision, the creation of the QUBE and its relative financial failure meant that Warner Communications needed outside capital to expand beyond Columbus, Ohio. In December 1979, Warner Communications and American Express each contributed \$75 million to form a joint venture with two divisions: Warner Amex Cable Company and Warner Amex Satellite Entertainment Company.

Time Warner Cable was formed in 1989 through the merger of Time Inc.'s cable television company, and Warner Cable, a division of Warner Communications. It also includes the remnants of the defunct QUBE interactive TV service. It became the nation's largest cable provider after TCI, with 5 million subscribers.

On July 31, 2006, Time Warner Cable and Comcast completed a deal to purchase

Adelphia's assets for \$17 billion. Time Warner Cable gained 3.3 million of Adelphia's subscribers, a 29% increase, while Comcast gained almost 1.7 million subscribers. Adelphia stockholders received 16% of Time Warner Cable.

In addition to Adelphia's coverage being divided up, Time Warner Cable and Comcast also agreed to exchange some of their own subscribers in order to consolidate key regions. An example of this was the Los Angeles market, which was mostly covered by Comcast and Adelphia (and some areas of the region already served by TWC), went under Time Warner Cable. In Philadelphia, previously was split between Time Warner and Comcast, the majority of cable subscribers went to Comcast. Time Warner subscribers in Philadelphia were swapped with Comcast in early 2007. Similarly, the Houston area, which was under Time Warner, was swapped to Comcast, while the Dallas metro area was changed to Time Warner. In the Twin Cities, Minneapolis was Time Warner and Saint Paul was Comcast. That whole market is now Comcast.

**Charter Communications, Inc.** This company was founded in Delaware in 1993. In 1998, Paul Allen, co-founder of Microsoft with Bill Gates, bought its controlling share, and the new Charter gathered numerous systems during a few following years. By 2001, Charter was the fourth largest cable company with more than 6 million subscribers. But the rise of Charter couldn't continue, suffering a lawsuit for former executives' financial report fraud.

On March 28, 2009, Charter Communications filed for a Chapter 11 bankruptcy. For assets of Charter, Comcast and Time Warner Cable would be interested in. Especially, Time Warner has some incentives in California, while Comcast has a positive attitude for assets of New England.

## The Media Conglomerates

The media conglomerates own lots of subsidiary companies in the television related fields: air signal broadcasting, cable networks, film and music recording makers as contents, cable operators, DBS operators, spin-off magazines and online business, and etc. In this section, we briefly describes the company structure of major media conglomerates. To investigate the structure, we usually consulted with each companies' internet website.

**Comcast Corporation** The national No.1 cable operator, Comcast, is the owner of Comcast Spectator (which owns a NBA team, Philadelphia 76ers) and several cable networks including Comcast SportsNet, E! Entertainment Television, Style Network, G4, the Golf Channel, and Versus.

**E. W. Scripps Company** This was founded by Edward W. Scripps in 1878. It started as a newspaper company and added numerous newspapers, and television and radio stations, mostly associated with ABC. For the cable networks, it owns HGTV, DIY Network, Food Network, and etc.

**General Electric** GE of Thomas Edison also has its subsidiary company in the entertaining field: NBC Universal. GE owns 80% share of NBC Universal and the remaining 20% is owned by Vivendi, a French media group.

In a movie industry, NBC Universal owns Universal Studios. It also has 3 parks and resorts in Los Angeles, Orlando, and Tokyo. The major part of the company lies in television networks. It owns NBC, CNBC, Bravo, Syfy, Telemundo, USA Network, and the Weather Channel. Together with Microsoft, it launched MSNBC in 1996.



On the other hand, NBC Universal has some shares of A&E<sup>11</sup>, the History Channel, the Biography Channel, National Geographic, and TiVo.

**Liberty Media Corporation** This company, which is controlled by John Malone, has 48% interest of DirecTV, a DBS operator. A major league baseball team, Atlanta Braves and a film maker, Overture Films, are also its property. For the cable networks, it owns some regional franchises of FSN and QVC. On the other hand, it has some interests for Discovery Communications, which is controlled by its founder, John Hendricks. Discovery Communications owns Discovery Channel, TLC, Animal Planet, Discovery Health Channel, Science Channel, Military Channel, and etc.

**News Cooperation** Rupert Murdoch has lots of newspapers and book publishing companies in Oceania. In US, News Corporation owns a series of Fox companies. It has film makers: 20th Century Fox, Fox Searchlight Pictures, Fox Studios, and Blue Sky Studios. For the air signal broadcasting, it has Fox Broadcasting Company. In the cable networks, it includes Fox Movie Channel, Fox News Channel, Fox College Sports, Fox Sports Net, Fuel TV, FX, National Geographic, and Speed. It has newspapers like the Wall Street Journal, Dow Jones, and New York Post. And Its most prominent book publisher is HarperCollins Publishers. It also owns an internet blogging site, MySpace.

**Time Warner Inc.** Time Warner Inc. consists of 5 subgroups: AOL LLC, Time Inc., HBO, Turner Broadcasting System, and Warner Bros. Entertainment.

Time Inc. has numerous magazines including Entertainment Weekly, Fortune,

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<sup>11</sup>Recently, A&E Television Networks, which is a joint venture of Hearst, Disney, and NBC, acquired Lifetime Entertainment Services. It was on August 27, 2009.

Health, In Style, Money, People, Sports Illustrated, and Time. HBO has two brands: HBO and Cinemax. Turner Broadcasting System plays in the area of cable networks. It includes Adult Swim, Boomerang, Cartoon Network, CNN, HLN, TBS, TCM, truTV, Turner Classic Movies, and Turner Network Television. The last branch is Warner Bros. Entertainment. It has Warner Bros. Pictures, Warner Bros. Television, and DC Comics. The CW television network is a joint venture of CBS Corporation and Warner Bros. Entertainment. Note that Warner Music Group is not owned by Time Warner Inc. any more. It is sold in 2004.

Importantly note that Time Warner Cable, the national No. 2 cable operator, was separated from Time Warner Inc. in March, 2009. Time Warner Inc. disposed its 84% share of Time Warner Cable, receiving one time money dividend.

**Viacom/CBS** Viacom/CBS was split in 2005: Viacom and CBS corporation. But both companies are still controlled by Sumner Redstone of National Amusements. So we consider both firms together.

Viacom owns cable networks including BET, CMT, Comedy Central, MTV, Nickelodeon, Nick at Nite, Noggin, Spike, the N, TV Land, and VH1. And it owns film maker, Paramount Pictures. CBS corporation consists of numerous CBS subsidiaries (including television networks, film makers, and radio stations), Simon & Schuster (a book publisher), Showtime, and the CW television network, a joint venture with Warner Bros.

**Vivendi** The historic French media group, Vivendi, currently owns Canal+ Group (French television networks) and Universal Music Group. It has controlling stakes in Activision Blizzard (video games), Maroc Telecom (telecommunication operator in Morocco) and SFR (telecommunication operator in France). Last, it has 20% share of

NBC Universal.

**Walt Disney Group** Walt Disney Group has many subsidiary companies in many entertaining fields. The first field is a motion pictures. The Walt Disney Studio Entertainment include Walt Disney Pictures which includes Walt Disney Animation Studios, Pixar Animation Studios and DisneyToon Studios, Touchstone Pictures, Hollywood Pictures, and Miramax Films. To distribute their movies, Disney Group owns Walt Disney Studios Home Entertainment. About music business, Disney Music Group distributes their music properties under Walt Disney Records, Hollywood Records, and Lyric Street Records. On the other hand, Disney Theatrical Productions which includes Disney Live Family Entertainment and Disney on Ice produces substantial amount of theatrical performances.

Disney's Parks and Resorts owns 11 theme parks in 5 resort locations: Disneyland Resort (Anaheim, California), Walt Disney World Resort (Lake Buena Vista, Florida), Tokyo Disney Resort (Urayasu, Chiba), Disneyland Resort Paris (Marne La Valle, France), and Hong Kong Disneyland (Penny's Bay, Lantau Island).

Disney Media Networks comprise a vast array of broadcast, cable, radio, publishing and Internet businesses. The Disney-ABC Television Group includes the ABC Television Network (including ABC Daytime, ABC Entertainment and ABC News divisions), the Disney Channels, ABC Family, and SOAPnet. And the company holds equity interest in Lifetime Entertainment Services and A&E Television Networks.

Last, Walt Disney Group owns an 80% share of the ESPN network. The remaining share of the ESPN network goes to Hearst Corporation, which has numerous newspapers and magazines.

## APPENDIX B

## PROOFS

Proof for the Section B in the Chapter II

*Proof. (Proposition II.1: Market Segmentation)* We want to show that for all  $k \in \{0, 1, 2, \dots, n\}$  and  $j \neq k$ ,  $\theta q_k - p_k \geq \theta q_j - p_j$ , and  $\theta q_k - p_k > \theta q_j - p_j$  when  $k < j$ . Suppose that  $k > j$ . Then,

$$\begin{aligned}
& \theta(q_k - q_j) - (p_k - p_j) \\
&= \theta\{(q_k - q_{k-1}) + (q_{k-1} - q_{k-2}) + \dots + (q_{j+1} - q_j)\} \\
&\quad - \{(p_k - p_{k-1}) + (p_{k-1} - p_{k-2}) + \dots + (p_{j+1} - p_j)\} \\
&= \{\theta(q_k - q_{k-1}) - (p_k - p_{k-1})\} + \{\theta(q_{k-1} - q_{k-2}) - (p_{k-1} - p_{k-2})\} \\
&\quad + \dots + \{\theta(q_{j+1} - q_j) - (p_{j+1} - p_j)\} \\
&\geq \{\theta_k(q_k - q_{k-1}) - (p_k - p_{k-1})\} + \{\theta_{k-1}(q_{k-1} - q_{k-2}) - (p_{k-1} - p_{k-2})\} \\
&\quad + \dots + \{\theta_{j+1}(q_{j+1} - q_j) - (p_{j+1} - p_j)\} = 0.
\end{aligned}$$

Now, suppose that  $k < j$ . Then,

$$\begin{aligned}
& \theta(q_k - q_j) - (p_k - p_j) = -[\theta(q_j - q_k) - (p_j - p_k)] \\
&= -[\theta\{(q_j - q_{j-1}) + (q_{j-1} - q_{j-2}) + \dots + (q_{k+1} - q_k)\} \\
&\quad - \{(p_j - p_{j-1}) + (p_{j-1} - p_{j-2}) + \dots + (p_{k+1} - p_k)\}] \\
&> -[\{\theta_j(q_j - q_{j-1}) - (p_j - p_{j-1})\} + \{\theta_{j-1}(q_{j-1} - q_{j-2}) - (p_{j-1} - p_{j-2})\} \\
&\quad + \dots + \{\theta_{k+1}(q_{k+1} - q_k) - (p_{k+1} - p_k)\}] = 0.
\end{aligned}$$

□

Proofs for the Section C in the Chapter II

In this subsection, I prove the existence, uniqueness and the first order conditions for the monopolist's solution.

*Proof. (Proposition II.2: Existence of a Maximizer in the Monopolist's Problem)* Our objective function,  $\Pi : \mathbb{R}^{n \times 2} \Rightarrow \mathbb{R}$  is defined by

$$\Pi(\mathbf{q}, \boldsymbol{\theta}) = \sum_{k=1}^n \left\{ \left( \int_{\theta_k}^{\theta_{k+1}} f(\theta) d\theta \right) \left( \sum_{j=1}^k \theta_j (q_j - q_{j-1}) - C(q_k) \right) \right\}.$$

This function's inputs are  $\mathbf{q}$  and  $\boldsymbol{\theta}$  and the output is the monopolist's profit. We can easily establish the compactness of domain and the continuity of  $\Pi$  in  $\mathbf{q}$  and  $\boldsymbol{\theta}$ .  $\square$

*Proof. (Proposition II.3: Monopolist's Problem)* For the first equation, remind of the constraint in (P.M);  $p_i - p_{i-1} = \theta_i (q_i - q_{i-1})$ . Summing up from 1 to  $k$ ,  $p_k = \sum_{i=1}^k \theta_i (q_i - q_{i-1})$ .

Let  $O_j \equiv D(q_j) \left( \sum_{i=1}^j \theta_i (q_i - q_{i-1}) - C(q_j) \right)$ . Then, our objective function becomes  $\sum_{j=1}^n O_j$ . For  $k$  such that  $1 < k < n$ ,

$$\begin{aligned} \frac{\partial \sum_{j=1}^n O_j}{\partial q_k} &= \frac{\partial O_k}{\partial q_k} + \frac{\partial O_{k+1}}{\partial q_k} + \dots + \frac{\partial O_n}{\partial q_k}, \text{ and} \\ \frac{\partial \sum_{j=1}^n O_j}{\partial \theta_k} &= \frac{\partial O_{k-1}}{\partial \theta_k} + \frac{\partial O_k}{\partial \theta_k} + \frac{\partial O_{k+1}}{\partial \theta_k} + \dots + \frac{\partial O_n}{\partial \theta_k}. \end{aligned}$$

Differentiating the objective function with respect to  $q_k$ ,

$$\begin{aligned} \frac{\partial \sum_{j=1}^n O_j}{\partial q_k} &= D(q_k) [\theta_k - C'(q_k)] + \sum_{j=k+1}^n D(q_j) (\theta_k - \theta_{k+1}) \\ &= \left[ D(q_k) \theta_k - \sum_{j=k+1}^n D(q_j) (\theta_{k+1} - \theta_k) \right] - D(q_k) C'(q_k) \\ &= [\{F(\theta_{k+1}) - F(\theta_k)\} \theta_k - \{1 - F(\theta_{k+1})\} (\theta_{k+1} - \theta_k)] \\ &\quad - \{F(\theta_{k+1}) - F(\theta_k)\} C'(q_k). \end{aligned}$$

Differentiating the objective function with respect to  $\theta_k$ ,

$$\begin{aligned} \frac{\partial \sum_{j=1}^n \mathbf{O}_j}{\partial \theta_k} &= f(\theta_k) \left( \sum_{j=1}^{k-1} \theta_j (q_j - q_{j-1}) - C(q_{k-1}) \right) \\ &\quad - f(\theta_k) \left( \sum_{j=1}^k \theta_j (q_j - q_{j-1}) - C(q_k) \right) + \sum_{j=k}^n D(q_j) (q_k - q_{k-1}) \\ &= -f(\theta_k) [\theta_k \{q_k - q_{k-1}\} - (C(q_k) - C(q_{k-1}))] + (1 - F(\theta_k)) (q_k - q_{k-1}). \end{aligned}$$

Now, I consider the case that  $k = 1$ . In fact,  $\partial(\sum_{j=1}^n \mathbf{O}_j)/\partial q_1$  has a same result with the previous case. Differentiating the objective function with respect to  $\theta_1$ ,

$$\begin{aligned} \frac{\partial \sum_{j=1}^n \mathbf{O}_j}{\partial \theta_1} &= -f(\theta_1) [\theta_1 (q_1 - q_0) - C(q_1)] + \sum_{j=1}^n D(q_j) (q_1 - q_0) \\ &= -f(\theta_1) [\theta_1 (q_1 - q_0) - \{C(q_1) - C(q_0)\}] + (1 - F(\theta_1)) (q_1 - q_0). \end{aligned}$$

Last, I consider the case that  $k = n$ .

$$\begin{aligned} \frac{\partial \sum_{j=1}^n \mathbf{O}_j}{\partial q_n} &= \frac{\partial \mathbf{O}_n}{\partial q_n} = D(q_n) [\theta_n - C'(q_n)] = (1 - F(\theta_n)) [\theta_n - C'(q_n)] \\ &= [\{F(\bar{\theta}) - F(\theta_n)\} \theta_n - \{1 - F(\bar{\theta})\} (\bar{\theta} - \theta_n)] - \{F(\bar{\theta}) - F(\theta_n)\} C'(q_n). \end{aligned}$$

Differentiating the objective function with respect to  $\theta_1$  and  $\theta_n$ , we get a consistent result with the case that  $1 < k < n$ . The first order conditions imply the second and the third equations in this proposition. Note that the second order conditions for the maximizers hold by Assumption 1.  $\square$

*Proof. (Proposition II.4: Uniqueness of a Maximizer in the Monopolist's Problem)* Let both  $(\mathbf{q}, \boldsymbol{\theta})$  and  $(\mathbf{q}', \boldsymbol{\theta}')$  be the maximizer for the problem (P.M') such that  $(\mathbf{q}, \boldsymbol{\theta}) \neq (\mathbf{q}', \boldsymbol{\theta}')$ . Before diving into the proof, I define  $L(\theta)$  such that  $L(\theta) \equiv \theta - H(\theta)$ . By Assumption 4,  $L'(\theta) > 0$ .

First, suppose that  $\mathbf{q} = \mathbf{q}'$ . Then, for all  $k$  and  $k - 1$ ,  $(C(q_k) - C(q_{k-1})) / (q_k - q_{k-1}) = (C(q'_k) - C(q'_{k-1})) / (q'_k - q'_{k-1})$ . By Proposition II.3,  $L(\theta_k) = L(\theta'_k)$ . Since  $L$

is strictly increasing,  $\boldsymbol{\theta} = \boldsymbol{\theta}'$ . Thus,  $\mathbf{q} \neq \mathbf{q}'$ .

Second, suppose that  $\mathbf{q} \neq \mathbf{q}'$ . Then, there exists  $k$  such that  $q_k \neq q'_k$  and for all  $j < k$ ,  $q_j = q'_j$ .<sup>12</sup> Without loss of generality, we can assume that  $q_k < q'_k$ . From Proposition II.3, we know that

$$(B.1) \quad L(\theta_k) = \frac{C(q_k) - C(q_{k-1})}{q_k - q_{k-1}} \text{ and } L(\theta'_k) = \frac{C(q'_k) - C(q'_{k-1})}{q'_k - q'_{k-1}}.$$

Since  $C$  is strictly convex,  $L$  is strictly increasing,  $q_{k-1} = q'_{k-1}$  and  $q_k < q'_k$ , the equation (B.1) implies that  $\theta_k < \theta'_k$ .

Meanwhile, we have one more equation from Proposition II.3;

$$(B.2a) \quad \theta_{k-1} - \frac{1 - F(\theta_k)}{F(\theta_k) - F(\theta_{k-1})}(\theta_k - \theta_{k-1}) = C'(q_{k-1}) \text{ and}$$

$$(B.2b) \quad \theta'_{k-1} - \frac{1 - F(\theta'_k)}{F(\theta'_k) - F(\theta'_{k-1})}(\theta'_k - \theta'_{k-1}) = C'(q'_{k-1}).$$

Since  $q_{k-1} = q'_{k-1}$  and  $\theta_{k-1} = \theta'_{k-1}$ ,

$$(B.3) \quad \frac{1 - F(\theta_k)}{F(\theta_k) - F(\theta_{k-1})}(\theta_k - \theta_{k-1}) = \frac{1 - F(\theta'_k)}{F(\theta'_k) - F(\theta'_{k-1})}(\theta'_k - \theta'_{k-1}).$$

Let  $K(b|a) \equiv \{(1 - F(b))/(F(b) - F(a))\}(b - a)$ , given  $a$ . Then,

$$(B.4) \quad \begin{aligned} \frac{\partial K}{\partial b} &= \frac{-f(b)}{F(b) - F(a)}(b - a) - \frac{(1 - F(b))f(b)}{(F(b) - F(a))^2}(b - a) + \frac{1 - F(b)}{F(b) - F(a)} \\ &= \frac{f(b)}{F(b) - F(a)} \left\{ -(b - a) - \frac{1 - F(b)}{F(b) - F(a)}(b - a) + \frac{1 - F(b)}{f(b)} \right\} \\ &= \frac{f(b)}{F(b) - F(a)} \{(a - K(b|a)) - (b - H(b))\}. \end{aligned}$$

---

<sup>12</sup>I assume that  $q_0 = q'_0 = 0$  as always.

Then, by the equation (B.1) and (B.2),

$$\begin{aligned} \frac{\partial K(\theta|\theta_{k-1})}{\partial \theta} \Big|_{\theta=\theta_k} &= \frac{f(\theta_k)}{F(\theta_k) - F(\theta_{k-1})} \{(\theta_{k-1} - K(\theta_k|\theta_{k-1})) - (\theta_k - H(\theta_k))\} \\ &= \frac{f(\theta_k)}{F(\theta_k) - F(\theta_{k-1})} \left\{ C'(q_{k-1}) - \frac{C(q_k) - C(q_{k-1})}{q_k - q_{k-1}} \right\} < 0. \end{aligned}$$

Similarly,  $\{\partial K(\theta|\theta_{k-1})/\partial \theta\}|_{\theta=\theta'_k} < 0$ .

Now, consider the function  $K(\theta|\theta_{k-1})$  on the interval  $[\theta_k, \theta'_k]$ . We know that  $K(\theta|\theta_{k-1})$  is continuous in  $\theta$  on  $[\theta_k, \theta'_k]$  and  $K(\theta_k|\theta_{k-1}) = K(\theta'_k|\theta_{k-1})$  by the equation (B.3). Since both  $\{\partial K(\theta|\theta_{k-1})/\partial \theta\}|_{\theta=\theta_k} < 0$  and  $\{\partial K(\theta|\theta_{k-1})/\partial \theta\}|_{\theta=\theta'_k} < 0$ , we have at least 2 different points in  $(\theta_k, \theta'_k)$ , satisfying  $\partial K(\theta|\theta_{k-1})/\partial \theta = 0$ . The Figure 18 will help us understand the above logic.

Let  $\Theta = \{\theta \in [\theta_k, \theta'_k] | \partial K(\theta|\theta_{k-1})/\partial \theta = 0\}$ , and let  $\theta_1 \equiv \min \Theta$  and  $\theta_2 \equiv \max \Theta$ . Obviously,  $\theta_1 < \theta_2$ . Since  $\{\partial K(\theta|\theta_{k-1})/\partial \theta\}|_{\theta=\theta_k} < 0$  and  $\{\partial K(\theta|\theta_{k-1})/\partial \theta\}|_{\theta=\theta'_k} < 0$ ,

$$(B.5) \quad K(\theta_1|\theta_{k-1}) < K(\theta_k|\theta_{k-1}) = K(\theta'_k|\theta_{k-1}) < K(\theta_2|\theta_{k-1}).$$

By Assumption 3 and the equation (B.4), we know that  $\theta_{k-1} - K(\theta|\theta_{k-1}) = \theta - H(\theta)$  if  $\partial K(\theta|\theta_{k-1})/\partial \theta = 0$ . Since  $\theta_1, \theta_2 \in \Theta$ ,  $\theta_{k-1} - K(\theta_1|\theta_{k-1}) = \theta_1 - H(\theta_1) = L(\theta_1)$  and  $\theta_{k-1} - K(\theta_2|\theta_{k-1}) = \theta_2 - H(\theta_2) = L(\theta_2)$ . Since  $L'(\theta) > 0$  and  $\theta_1 < \theta_2$ ,  $\theta_{k-1} - K(\theta_1|\theta_{k-1}) < \theta_{k-1} - K(\theta_2|\theta_{k-1})$ . Thus,  $K(\theta_1|\theta_{k-1}) > K(\theta_2|\theta_{k-1})$ , which contradicts to the equation (B.5).  $\square$

## Proofs for the Section D in the Chapter II

In this subsection, I prove the existence, uniqueness and the first order conditions for the social planner's solution.

*Proof. (Lemma II.1: Marginal Cost Tariff in the Social Planner's Problem)*

To solve the problem (P.S), the social planner should find the optimal  $\mathbf{q}$ ,  $\mathbf{p}$ , and  $\boldsymbol{\theta}$ .



We can assume that the social planner's tariff scheme is  $p_k = P_k(\mathbf{q}, \boldsymbol{\theta})$ . Then, we can rewrite the problem (P.S) as

$$(P1') \quad \max_{\mathbf{q}, P_1, \dots, P_n} \sum_{k=1}^n \int_{\theta_k}^{\theta_{k+1}} [sq_k - C(q_k)] f(s) ds$$

such that for all  $k$ ,  $\theta_k q_k - p_k = \theta_k q_{k-1} - p_{k-1}$  and  $p_k = P_k(\mathbf{q}, \boldsymbol{\theta})$ .

In the problem (P1'), the social planner should find the optimal set of qualities and the optimal tariff scheme.

Let  $\mathbf{q}^*$  be a maximizer when for all  $k$ ,  $P_k(\mathbf{q}, \boldsymbol{\theta}) = C(q_k)$ . Let  $\tilde{\mathbf{q}}$  be a maximizer when for all  $k$ ,  $P_k(\mathbf{q}, \boldsymbol{\theta}) = \tilde{C}_k(\mathbf{q}, \boldsymbol{\theta})$  and for some  $k$ ,  $\tilde{C}_k(\mathbf{q}, \boldsymbol{\theta}) \neq C(q_k)$ . Using  $\tilde{\mathbf{q}}$ ,  $\tilde{C}_k(\mathbf{q}, \boldsymbol{\theta})$  and the cutoff constraint, we can find the set of  $\tilde{\boldsymbol{\theta}}$ .

Now, consider a combination  $(\tilde{\mathbf{q}}, \tilde{\boldsymbol{\theta}}, (C(\tilde{q}_1), C(\tilde{q}_2), \dots, C(\tilde{q}_n)))$  and a combination  $(\tilde{\mathbf{q}}, \tilde{\boldsymbol{\theta}}, (\tilde{C}_1(\tilde{\mathbf{q}}, \tilde{\boldsymbol{\theta}}), \tilde{C}_2(\tilde{\mathbf{q}}, \tilde{\boldsymbol{\theta}}), \dots, \tilde{C}_n(\tilde{\mathbf{q}}, \tilde{\boldsymbol{\theta}})))$ . Take an arbitrary interval,  $[\tilde{\theta}_k, \tilde{\theta}_{k+1}]$ . The social welfare in the interval is

$$\begin{aligned} & \int_{\tilde{\theta}_k}^{\tilde{\theta}_{k+1}} \left( \theta \tilde{q}_k - \tilde{C}_k(\tilde{\mathbf{q}}, \tilde{\boldsymbol{\theta}}) \right) d\theta + \left( \tilde{C}_k(\tilde{\mathbf{q}}, \tilde{\boldsymbol{\theta}}) - C(\tilde{q}_k) \right) \left( F(\tilde{\theta}_{k+1}) - F(\tilde{\theta}_k) \right) \\ &= \int_{\tilde{\theta}_k}^{\tilde{\theta}_{k+1}} \theta \tilde{q}_k d\theta - C(\tilde{q}_k) \left( F(\tilde{\theta}_{k+1}) - F(\tilde{\theta}_k) \right) = \int_{\tilde{\theta}_k}^{\tilde{\theta}_{k+1}} (\theta \tilde{q}_k - C(\tilde{q}_k)) d\theta. \end{aligned}$$

The above equation shows that the combination  $(\tilde{\mathbf{q}}, \tilde{\boldsymbol{\theta}}, (C(\tilde{q}_1), C(\tilde{q}_2), \dots, C(\tilde{q}_n)))$  and the combination  $(\tilde{\mathbf{q}}, \tilde{\boldsymbol{\theta}}, (\tilde{C}_1(\tilde{\mathbf{q}}, \tilde{\boldsymbol{\theta}}), \tilde{C}_2(\tilde{\mathbf{q}}, \tilde{\boldsymbol{\theta}}), \dots, \tilde{C}_n(\tilde{\mathbf{q}}, \tilde{\boldsymbol{\theta}})))$  give us the same social welfare when  $P_k = \tilde{C}_k(\mathbf{q}, \boldsymbol{\theta})$ .

That is, the marginal cost tariff scheme guarantees at least the same welfare level with the other possible tariff scheme, when we adopt the other scheme's optimal qualities and cutoffs combination. We know the combination  $(\mathbf{q}^*, \boldsymbol{\theta}^*)$  is the optimal combination, under the tariff scheme  $P_k(\mathbf{q}, \boldsymbol{\theta}) = C(q_k)$ . Thus, the combination  $(\mathbf{q}^*, \boldsymbol{\theta}^*)$  gives us a greater social welfare with the combination  $(\tilde{\mathbf{q}}, \tilde{\boldsymbol{\theta}})$ .  $\square$

*Proof.* (**Proposition II.5: Existence of a Maximizer in the Social Planner's Problem**) At the beginning, I clarify the objective function. This function's input is  $\mathbf{q}$  and its output is the value of total social surplus. That is, the function  $SW : \mathbb{R}^n \Rightarrow \mathbb{R}$  is defined by

$$(B.6) \quad SW(\mathbf{q}) = \sum_{k=1}^n \int_{\theta_k}^{\theta_{k+1}} [sq_k - C(q_k)] f(s) ds,$$

where  $\theta_k q_k - C(q_k) = \theta_k q_{k-1} - C(q_{k-1})$ , for all  $k$ .

Without loss of generality, we can assume that  $q_1 \leq q_2 \leq \dots \leq q_n$ . We define  $\bar{q}$  such that  $C'(\bar{q}) = \bar{\theta}$ . Then,  $q_n \leq \bar{q}$ . Any  $q$  over than  $\bar{q}$  cannot be an element of the maximizer for (P.S), because

$$\frac{C(q) - C(\bar{q})}{q - \bar{q}} > C'(\bar{q}) = \bar{\theta} \geq \theta \Rightarrow \theta q - C(q) < \theta \bar{q} - C(\bar{q}),$$

for all  $q > \bar{q}$  and  $\theta \in [0, \bar{\theta}]$ . Therefore,  $0 \leq q_1 \leq q_2 \leq \dots \leq q_n \leq \bar{q}$ . The domain of  $SW$ , the relevant subset of  $\mathbb{R}^n$ , is compact.

Now, we need to show that  $SW$  is a continuous function of  $\mathbf{q}$ . Since  $SW$  is determined by  $\mathbf{q}$  and  $\boldsymbol{\theta}$ , which is determined by  $\mathbf{q}$ , we have to show that *i*)  $SW$  is a continuous function of  $\mathbf{q}$  when  $\boldsymbol{\theta}$  is fixed, *ii*)  $\boldsymbol{\theta}$  is a continuous function of  $\mathbf{q}$  and *iii*)  $SW$  is a continuous function of  $\boldsymbol{\theta}$ .

The first step is obvious, since  $C$  is continuous.

For the next two steps, let

$$(B.7) \quad \theta_k(q_{k-1}, q_k) = \begin{cases} (C(q_k) - C(q_{k-1})) / (q_k - q_{k-1}) & \text{when } q_k > q_{k-1} \\ C'(q_{k-1}) = C'(q_k) & \text{when } q_k = q_{k-1}. \end{cases}$$

This definition is exactly same with  $\theta_k$  in the problem (P.S) except when  $q_k = q_{k-1}$ . In the problem (P.S),  $\theta_k$  is not defined when  $q_k = q_{k-1}$ . Note that  $\theta_k$  is simply a slope

between  $(q_k, C(q_k))$  and  $(q_{k+1}, C(q_{k+1}))$ , and

$$\lim_{q_k \rightarrow q_{k-1}} \frac{C(q_k) - C(q_{k-1})}{q_k - q_{k-1}} = C'(q_{k-1}) = C'(q_k) = \lim_{q_{k-1} \rightarrow q_k} \frac{C(q_k) - C(q_{k-1})}{q_k - q_{k-1}}.$$

In the problem (P.S), consumers who are in  $[\theta_{k-1}, \theta_{k+1})$  will buy the product  $q_{k-1}$  or the exactly same one  $q_k$ . In our new definition, consumers who are in  $[\theta_{k-1}, C'(q_{k-1}))$  will buy the product  $q_{k-1}$  and consumers who are in  $[C'(q_{k-1}), \theta_{k+1})$  will buy the product  $q_k$ . Thus, our newly extended definition in the equation (B.7) gives the same social welfare with the equation (B.6).

The definition in the equation (B.7) guarantees the continuity of  $\theta$  in  $\mathbf{q}$ . The second step is done. Since  $F$  has no mass point,  $SW$  is a continuous function of  $\theta$ . The third is also done.

Since the relevant domain is compact and the objective function is continuous in  $\mathbf{q}$ , there exists a maximizer by the Weierstrass theorem.  $\square$

*Proof. (Proposition II.6: Social Planner's Problem)* The first and third equations come from the Proposition II.1 and the Lemma II.1 directly.

Let  $O_j \equiv \int_{\theta_j}^{\theta_{j+1}} [sq_j - C(q_j)] f(s) ds$ . Then, our objective function is  $\sum_{j=1}^n O_j$ . For  $k$  such that  $1 < k < n$ ,

$$\frac{\partial \sum_{j=1}^n O_j}{\partial q_k} = \frac{\partial O_{k-1}}{\partial q_k} + \frac{\partial O_k}{\partial q_k} + \frac{\partial O_{k+1}}{\partial q_k}.$$

Letting  $\theta_k(q_k, q_{k-1}) \equiv (C(q_k) - C(q_{k-1})) / (q_k - q_{k-1})$  and using the Leibniz integral rule,<sup>13</sup>

$$\frac{\partial O_{k-1}}{\partial q_k} = \frac{\partial \theta_k(q_k, q_{k-1})}{\partial q_k} [\theta_k q_{k-1} - C(q_{k-1})] f(\theta_k)$$

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<sup>13</sup>The Leibniz rule is  $\frac{d}{d\alpha} \int_{a(\alpha)}^{b(\alpha)} f(x, \alpha) dx = \frac{db(\alpha)}{d\alpha} f(b(\alpha), \alpha) - \frac{da(\alpha)}{d\alpha} f(a(\alpha), \alpha) + \int_{a(\alpha)}^{b(\alpha)} \frac{\partial}{\partial \alpha} f(x, \alpha) dx$ .

$$\begin{aligned}
& - 0 \left( \because \frac{\partial \theta_{k-1}(q_{k-1}, q_{k-2})}{\partial q_k} = 0 \right) + 0 \left( \because \frac{\partial}{\partial q_k} [sq_{k-1} - C(q_{k-1})]f(s) = 0 \right), \\
\frac{\partial O_k}{\partial q_k} &= \frac{\partial \theta_{k+1}(q_{k+1}, q_k)}{\partial q_k} [\theta_{k+1}q_k - C(q_k)] f(\theta_{k+1}) \\
& \quad - \frac{\partial \theta_k(q_k, q_{k-1})}{\partial q_k} [\theta_k q_k - C(q_k)] f(\theta_k) + \int_{\theta_k}^{\theta_{k+1}} [s - C'(q_k)]f(s)ds, \text{ and} \\
\frac{\partial O_{k+1}}{\partial q_k} &= -\frac{\partial \theta_{k+1}(q_{k+1}, q_k)}{\partial q_k} [\theta_{k+1}q_{k+1} - C(q_{k+1})] f(\theta_{k+1}) \\
& \quad + 0 \left( \because \frac{\partial \theta_{k+2}(q_{k+2}, q_{k+1})}{\partial q_k} = 0 \right) + 0 \left( \because \frac{\partial}{\partial q_k} [sq_{k+1} - C(q_{k+1})]f(s) = 0 \right).
\end{aligned}$$

Since  $\theta_j q_j - C(q_j) = \theta_j q_{j-1} - C(q_{j-1})$ ,  $\partial(\sum_{j=1}^n O_j)/\partial q_k = \int_{\theta_k}^{\theta_{k+1}} [s - C'(q_k)]f(s)ds$ .

Now, I consider the case that  $k = 1$ .

$$\begin{aligned}
\frac{\partial \sum_{j=1}^n O_j}{\partial q_1} &= \frac{\partial O_1}{\partial q_1} + \frac{\partial O_2}{\partial q_1} \\
&= \frac{\partial \theta_2(q_2, q_1)}{\partial q_1} [\theta_2 q_1 - C(q_1)]f(\theta_2) - \frac{\partial \theta_1(q_1, q_0)}{\partial q_1} [\theta_1 q_1 - C(q_1)]f(\theta_1) \\
& \quad + \int_{\theta_1}^{\theta_2} [s - C'(q_1)]f(s)ds + 0 \left( \because \frac{\partial \theta_3(q_3, q_2)}{\partial q_1} = 0 \right) \\
& \quad - \frac{\partial \theta_2(q_2, q_1)}{\partial q_1} [\theta_2 q_2 - C(q_2)]f(\theta_2) + 0 \left( \because \frac{\partial}{\partial q_1} [sq_2 - C(q_2)]f(s) = 0 \right) \\
&= \int_{\theta_1}^{\theta_2} [s - C'(q_1)]f(s)ds (\because \theta_1 q_1 - C(q_1) = 0).
\end{aligned}$$

Last, I consider the case that  $k = n$ .

$$\begin{aligned}
\frac{\partial \sum_{j=1}^n O_j}{\partial q_n} &= \frac{\partial O_{n-1}}{\partial q_n} + \frac{\partial O_n}{\partial q_n} \\
&= \frac{\partial \theta_n(q_n, q_{n-1})}{\partial q_n} [\theta_n q_{n-1} - C(q_{n-1})]f(\theta_n) - 0 \left( \because \frac{\partial \theta_{n-1}(q_{n-1}, q_{n-2})}{\partial q_n} = 0 \right) \\
& \quad + 0 \left( \because \frac{\partial}{\partial q_n} [sq_{n-1} - C(q_{n-1})]f(s) = 0 \right) + 0 \left( \because \frac{\partial \bar{\theta}}{\partial q_n} = 0 \right) \\
& \quad - \frac{\partial \theta_n(q_n, q_{n-1})}{\partial q_n} [\theta_n q_n - C(q_n)]f(\theta_n) + \int_{\theta_n}^{\bar{\theta}} [s - C'(q_n)]f(s)ds \\
&= \int_{\theta_n}^{\bar{\theta}} [s - C'(q_n)]f(s)ds.
\end{aligned}$$

Since  $\theta_{n+1} = \bar{\theta}$ , the first order conditions imply that for all  $k$ ,

$$\begin{aligned} \int_{\theta_k}^{\theta_{k+1}} [s - C'(q_k)] f(s) ds &= 0 \\ \Leftrightarrow \int_{\theta_k}^{\theta_{k+1}} s \frac{f(s)}{F(\theta_{k+1}) - F(\theta_k)} ds &= C'(q_k) \left( \int_{\theta_k}^{\theta_{k+1}} \frac{f(s)}{F(\theta_{k+1}) - F(\theta_k)} ds \right) = C'(q_k). \end{aligned}$$

We can easily show that the second order conditions for the maximizers hold.

The convexity assumption for  $C$  in the Assumption 1 crucially plays.  $\square$

### Proofs for the Section E in the Chapter II

*Proof. (Proposition II.7: Total Profit and  $n$ )* Let  $\mathbf{q}^m = (q_1^m, q_2^m, \dots, q_n^m)$  and  $\boldsymbol{\theta}^m = (\theta_1^m, \theta_2^m, \dots, \theta_n^m)$  be a maximizer when we have  $n$  different goods and let  $\mathbf{q}^{m'} = (q_1^{m'}, q_2^{m'}, \dots, q_{n+1}^{m'})$  and  $\boldsymbol{\theta}^{m'} = (\theta_1^{m'}, \theta_2^{m'}, \dots, \theta_{n+1}^{m'})$  be a maximizer when we can choose  $n + 1$  products. Note that  $p_k^m = \sum_{i=1}^k \theta_i^m (q_i^m - q_{i-1}^m)$  and  $p_k^{m'} = \sum_{i=1}^k \theta_i^{m'} (q_i^{m'} - q_{i-1}^{m'})$ . Assume that  $q_1^m < q_2^m < \dots < q_n^m$ , in order that every product is meaningful. Additionally, assume  $0 < \theta_1^m < \theta_2^m < \dots < \theta_n^m < \bar{\theta}$  so that all  $n$  products are meaningful in the market.

Take an arbitrary  $\tilde{\theta}$  such that  $\theta_n^m < \tilde{\theta} < \bar{\theta}$ , and take  $\tilde{q}$  such that  $\tilde{\theta} > (C(\tilde{q}) - C(q_n^m)) / (\tilde{q} - q_n^m)$  and  $q_n^m < \tilde{q} < \bar{q}$ . Since  $\tilde{\theta} > \theta_n^m = C'(q_n^m)$  by the Proposition II.3 and since  $\lim_{\tilde{q} \rightarrow q_n^m} \{(C(\tilde{q}) - C(q_n^m)) / (\tilde{q} - q_n^m)\} = C'(q_n^m)$ , we can always find a proper  $\tilde{q}$  slightly over than  $q_n^m$ . Last, I define  $\tilde{p} = p_n^m + \tilde{\theta}(\tilde{q} - q_n^m)$ , which exactly coincides with the pricing formula.

Now, I introduce a new product combination that is added  $\tilde{\theta}$ ,  $(q_1^m, q_2^m, \dots, q_n^m, \tilde{q})$ ,  $(\theta_1^m, \theta_2^m, \dots, \theta_n^m, \tilde{\theta})$  and  $(p_1^m, p_2^m, \dots, p_n^m, \tilde{p})$ . We can easily confirm that for  $k$  such that  $1 \leq k < n$ , a consumer in the segment  $[\theta_k^m, \theta_{k+1}^m)$  buys the product  $q_k^m$  at the price  $p_k^m$ . That is, a consumer in  $[0, \theta_n^m)$  consumes the exactly same product at the same price with the original profit maximizing offer of the monopolist when the number of

product is  $n$ . By the definition of  $\tilde{p}$  and the linear structure of utility, a consumer in  $[\theta_n^m, \tilde{\theta}]$  would like to buy  $q_n^m$ . Meanwhile, a consumer in  $[\tilde{\theta}, \bar{\theta}]$  would like to buy  $\tilde{q}$ . The monopolist's new profit in the segment  $[\tilde{\theta}, \bar{\theta}]$  is  $(1 - F(\tilde{\theta}))(\tilde{p} - C(\tilde{q}))$ . We can check

$$(B.8) \quad (\tilde{p} - C(\tilde{q})) - (p_n^m - C(q_n^m)) = \tilde{\theta}(\tilde{q} - q_n^m) - (C(\tilde{q}) - C(q_n^m)) > 0,$$

by the formulation of  $(\tilde{q}, \tilde{\theta}, \tilde{p})$ . Then,

$$\begin{aligned} & \Pi^m \left( (q_1^m, \dots, q_n^m, \tilde{q}), (\theta_1^m, \dots, \theta_n^m, \tilde{\theta}) \right) \\ &= \Pi^m(\mathbf{q}^m, \boldsymbol{\theta}^m) - (1 - F(\theta_n^m))(p_n^m - C(q_n^m)) \\ & \quad + (F(\tilde{\theta}) - F(\theta_n^m))(p_n^m - C(q_n^m)) + (1 - F(\tilde{\theta}))(\tilde{p} - C(\tilde{q})) \\ &= \Pi^m(\mathbf{q}^m, \boldsymbol{\theta}^m) + (1 - F(\tilde{\theta}))((\tilde{p} - C(\tilde{q})) - (p_n^m - C(q_n^m))). \end{aligned}$$

By the above equation (B.8),

$$\Pi^m \left( (q_1^m, \dots, q_n^m, \tilde{q}), (\theta_1^m, \dots, \theta_n^m, \tilde{\theta}) \right) > \Pi^m(\mathbf{q}^m, \boldsymbol{\theta}^m).$$

Since  $\mathbf{q}^{m'}$  and  $\boldsymbol{\theta}^{m'}$  is a maximizer when the monopolist can choose  $n+1$  products,

$$\Pi^m(\mathbf{q}^{m'}, \boldsymbol{\theta}^{m'}) \geq \Pi^m \left( (q_1^m, \dots, q_n^m, \tilde{q}), (\theta_1^m, \dots, \theta_n^m, \tilde{\theta}) \right).$$

Therefore,  $\Pi^m(\mathbf{q}^{m'}, \boldsymbol{\theta}^{m'}) > \Pi^m(\mathbf{q}^m, \boldsymbol{\theta}^m)$ , which implies that  $\mathcal{V}(n)$  is strictly increasing in  $n$ .  $\square$

*Proof. (Proposition II.8: Social Welfare and  $n$ )* Let  $(q_1^*, q_2^*, \dots, q_n^*)$  be a maximizer when we have  $n$  different goods and let  $(q_1^{**}, q_2^{**}, \dots, q_{n+1}^{**})$  be a maximizer when we can choose  $n+1$  products.

Without loss of generality, we can assume that  $q_1^* \leq q_2^* \leq \dots \leq q_n^*$ . And I assume that all  $n$  products are meaningful, that is, each product has consumers who buy it.

Since  $p_k^* = C(q_k^*)$ ,  $q_1^* < q_2^* < \dots < q_n^*$ , in order that we have  $n$  different products.

Then, for all  $k$ , since  $C$  is strictly convex,

$$\theta_k^* = \frac{C(q_k^*) - C(q_{k-1}^*)}{q_k^* - q_{k-1}^*} < C'(q_k^*) < \frac{C(q_{k+1}^*) - C(q_k^*)}{q_{k+1}^* - q_k^*} = \theta_{k+1}^*.$$

From the Proposition II.6, we know that  $q_k^*$  satisfies  $\mathbb{E}(\theta|\theta \in [\theta_k^*, \theta_{k+1}^*]) = C'(q_k^*)$ .

Since  $C'' > 0$ ,  $q_k^* = C'^{-1}(\mathbb{E}(\theta|\theta \in [\theta_k^*, \theta_{k+1}^*]))$ , which is a strictly increasing function.

Now, take an arbitrary  $\theta^{ins}$  such that  $\theta_k^* < \theta^{ins} < \theta_{k+1}^*$  and denote

$$\mathbb{E}(\theta_1^{ins}) \equiv \int_{\theta_k^*}^{\theta^{ins}} s \frac{f(s)}{F(\theta^{ins}) - F(\theta_k^*)} ds \text{ and } \mathbb{E}(\theta_2^{ins}) \equiv \int_{\theta^{ins}}^{\theta_{k+1}^*} s \frac{f(s)}{F(\theta_{k+1}^*) - F(\theta^{ins})} ds.$$

Take  $q_1^{ins}$  and  $q_2^{ins}$  to satisfy  $\mathbb{E}(\theta_1^{ins}) = C'(q_1^{ins})$  and  $\mathbb{E}(\theta_2^{ins}) = C'(q_2^{ins})$ .

Since  $\mathbb{E}(\theta_1^{ins}) < \mathbb{E}(\theta|\theta \in [\theta_k^*, \theta_{k+1}^*]) < \mathbb{E}(\theta_2^{ins})$  and  $C'^{-1}$  is strictly increasing,  $q_1^{ins} < q_k^* < q_2^{ins}$ . By the maximizing conditions,

$$\begin{aligned} & \int_{\theta_k^*}^{\theta_{k+1}^*} [sq_k^* - C(q_k^*)]f(s)ds \\ &= \int_{\theta_k^*}^{\theta^{ins}} [sq_k^* - C(q_k^*)]f(s)ds + \int_{\theta^{ins}}^{\theta_{k+1}^*} [sq_k^* - C(q_k^*)]f(s)ds \\ &< \int_{\theta_k^*}^{\theta^{ins}} [sq_1^{ins} - C(q_1^{ins})]f(s)ds + \int_{\theta^{ins}}^{\theta_{k+1}^*} [sq_2^{ins} - C(q_2^{ins})]f(s)ds. \end{aligned}$$

Keeping other  $q_j^*$ s and only replacing  $q_k^*$  with  $q_1^{ins}$  and  $q_2^{ins}$ , we get the same social welfare from other intervals, and the higher social welfare from the  $k$ th interval (now divided by two intervals). That is,  $SW^{SP}((q_1^*, \dots, q_{k-1}^*, q_1^{ins}, q_2^{ins}, q_{k+1}^*, \dots, q_n^*)) > SW^{SP}((q_1^*, \dots, q_k^*, \dots, q_n^*))$ . Since  $(q_1^{**}, q_2^{**}, \dots, q_{n+1}^{**})$  is a maximizer when we can choose  $n+1$  products,

$$SW^{SP}((q_1^{**}, \dots, q_k^{**}, \dots, q_{n+1}^{**})) \geq SW^{SP}((q_1^*, \dots, q_{k-1}^*, q_1^{ins}, q_2^{ins}, q_{k+1}^*, \dots, q_n^*)).$$

Therefore,  $SW^{SP}((q_1^{**}, \dots, q_k^{**}, \dots, q_{n+1}^{**})) > SW^{SP}((q_1^*, \dots, q_k^*, \dots, q_n^*))$ , which im-

plies that  $V(n)$  is strictly increasing in  $n$ .  $\square$

From now on, I show that two famous continuous product phenomena, the no distortion on top and the downward distortion properties, can be recovered in our finite model when  $n$  goes to infinite.

*Proof. (Proposition II.9: No Distortion on Top and Downward Distortion when  $n$  goes to Infinity)* Suppose that  $n = \infty$  and  $\theta_n^*$  (or  $\theta_n^m$ )  $< \bar{\theta}$ . This cannot be an optimal. If we insert a higher quality product aiming at a consumer with a taste between  $\theta_n^*$  (or  $\theta_n^m$ ) and  $\bar{\theta}$ , we can increase social welfare (or profit).<sup>14</sup> Therefore,  $\theta_n^*$  (or  $\theta_n^m$ )  $\rightarrow \bar{\theta}$  as  $n \rightarrow \infty$ . Then,  $q_n^*$  (or  $q_n^m$ ) goes to  $\bar{q}$ , which explains the no distortion on top.

We know that  $\theta_{k+1}^* \rightarrow \theta_k^*$  and  $\theta_{k-1}^* \rightarrow \theta_k^*$ , as  $n \rightarrow \infty$ . Then,

$$(B.9) \quad \lim_{n \rightarrow \infty} C'(q_k^*) = \lim_{n \rightarrow \infty} \mathbb{E}(\theta | \theta \in [\theta_k^*, \theta_{k+1}^*]) = \lim_{n \rightarrow \infty} \mathbb{E}(\theta | \theta \in [\theta_{k-1}^*, \theta_k^*]) = \theta_k^*.$$

Similarly, for all  $k > 1$ ,  $\theta_{k+1}^m \rightarrow \theta_k^m$ , as  $n \rightarrow \infty$ . We know that

$$\lim_{n \rightarrow \infty} \frac{F(\theta_{k+1}^m) - F(\theta_k^m)}{\theta_{k+1}^m - \theta_k^m} = f(\theta_k^m),$$

and so

$$\lim_{n \rightarrow \infty} \left( \frac{1 - F(\theta_{k+1}^m)}{F(\theta_{k+1}^m) - F(\theta_k^m)} (\theta_{k+1}^m - \theta_k^m) \right) = \frac{1 - F(\theta_k^m)}{f(\theta_k^m)} = H(\theta_k^m).$$

Then,

$$(B.10) \quad \lim_{n \rightarrow \infty} C'(q_k^m) = \theta_k^m - \frac{1 - F(\theta_k^m)}{f(\theta_k^m)} = \theta_k^m - H(\theta_k^m).$$

Let  $Q_1(\theta) \equiv \theta$  and  $Q_2(\theta) \equiv \theta - H(\theta)$ . By the definition of  $q^*(\theta)$  and  $q^m(\theta)$  and the equations (B.9) and (B.10),  $Q_1(\theta) = \lim_{n \rightarrow \infty} C'(q^*(\theta))$  and  $Q_2(\theta) = \lim_{n \rightarrow \infty} C'(q^m(\theta))$ .

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<sup>14</sup>Check the proofs for the Propositions II.8 and II.7.



Since for all  $\theta$ ,  $H(\theta) \geq 0$ ,  $Q_1(\theta) - Q_2(\theta) = H(\theta) \geq 0$ . Thus,  $Q_1(\theta) \geq Q_2(\theta) \Leftrightarrow \lim_{n \rightarrow \infty} C'(q^*(\theta)) \geq \lim_{n \rightarrow \infty} C'(q^m(\theta))$ . Since  $C'$  is strictly increasing,  $q^*(\theta) \geq q^m(\theta)$ , when  $n = \infty$ . It shows the downward distortion.  $\square$

### Proof for the Section D in the Chapter III

*Proof. (Proposition III.1: Increasing Information Rent in Type)* Without loss of generality, we can assume that  $\theta \in [\theta_j^m, \theta_{j+1}^m)$ . For the first case, suppose that  $\theta' \in [\theta_j^m, \theta_{j+1}^m)$ . Then,  $q(\theta) = q(\theta') = q_j^m$  and  $p(\theta) = p(\theta') = p_j^m$ .

$$U(q(\theta'), p(\theta'); \theta') - U(q(\theta), p(\theta); \theta) = (\theta' q_j^m - p_j^m) - (\theta q_j^m - p_j^m) = (\theta' - \theta) q_j^m \geq 0,$$

where the equality holds only when  $q_j^m = q_0^m = 0$ .

For the second case, suppose that  $\theta' \in [\theta_k^m, \theta_{k+1}^m)$ , where  $k > j$ . Then,

$$\begin{aligned} U(q(\theta'), p(\theta'); \theta') - U(q(\theta), p(\theta); \theta) &= (\theta' q_k^m - p_k^m) - (\theta q_j^m - p_j^m) \\ &\geq (\theta_k^m q_k^m - p_k^m) - (\theta q_j^m - p_j^m) = (\theta_k^m q_{k-1}^m - p_{k-1}^m) - (\theta q_j^m - p_j^m) \\ &> (\theta_{k-1}^m q_{k-1}^m - p_{k-1}^m) - (\theta q_j^m - p_j^m) = \cdots = (\theta_{j+1}^m q_j^m - p_j^m) - (\theta q_j^m - p_j^m) > 0. \end{aligned}$$

$\square$

## APPENDIX C

## UNIQUENESS OF THE SOCIAL PLANNER'S MAXIMIZER

In this appendix, I will introduce a condition to guarantee the uniqueness of maximizer for the social planner's problem. From the Proposition II.6, we know the conditions that the solution for the problem (P.S) should satisfies. The Proposition II.5 argues that the solution exists. We, however, still do not know whether the solution is unique or not. The uniqueness may be a crucial factor if we want to find the solution computationally. If we have multiple solutions, the answer obtained by the numerical computation is only one of multiple solutions. We cannot find any information about other solutions. If we have only one solution, the answer a computer finds is "the" solution.

To describe the condition which guarantees uniqueness of maximizer, we need to newly define some concepts, for the interpretational and notational convenience. I make two notations related with conditional expectation. For given  $\theta_l$  and  $\theta_u$  such that  $0 \leq \theta_l \leq \theta \leq \theta_u \leq \bar{\theta}$ ,

$$\begin{aligned}\mathcal{E}_l(\theta|\theta_l) &\equiv \mathbb{E}(s|s \in [\theta_l, \theta]) = \int_{\theta_l}^{\theta} s \frac{f(s)}{F(\theta) - F(\theta_l)} ds \text{ and} \\ \mathcal{E}_u(\theta|\theta_u) &\equiv \mathbb{E}(s|s \in [\theta, \theta_u]) = \int_{\theta}^{\theta_u} s \frac{f(s)}{F(\theta_u) - F(\theta)} ds.\end{aligned}$$

Denote the product served to consumers in  $[\theta_l, \theta]$  as  $q_l$  and the product served in  $[\theta, \theta_u]$  as  $q_u$ . From the Proposition II.6, we know  $C'(q_l) = \mathcal{E}_l(\theta|\theta_l)$  and  $C'(q_u) = \mathcal{E}_u(\theta|\theta_u)$ . Since  $C''(q) > 0$ , we can define

$$q_l(\theta|\theta_l) \equiv C'^{-1}(\mathcal{E}_l(\theta|\theta_l)) \text{ and } q_u(\theta|\theta_u) \equiv C'^{-1}(\mathcal{E}_u(\theta|\theta_u)).$$

Now, I introduce a condition which are crucial for uniqueness of maximizer.

**Condition 1.** For arbitrarily given  $\theta_l$  and  $\theta_u$  such that  $0 \leq \theta_l < \theta_u \leq \bar{\theta}$  and for all  $\theta$  such that  $\theta_l \leq \theta \leq \theta_u$ ,

$$\frac{\theta - \mathcal{E}_l(\theta|\theta_l)}{C''(q_l(\theta|\theta_l))} \frac{(\theta - \mathcal{E}_l(\theta|\theta_l))f(\theta) + (\mathcal{E}_l(\theta|\theta_l) - \theta_l)f(\theta_l)}{F(\theta) - F(\theta_l)} + \frac{(\mathcal{E}_u(\theta|\theta_u) - \theta)^2}{C''(q_u(\theta|\theta_u))} \frac{f(\theta)}{F(\theta_u) - F(\theta)}$$

$$< q_u(\theta|\theta_u) - q_l(\theta|\theta_l)$$

and

$$\frac{(\theta - \mathcal{E}_l(\theta|\theta_l))^2}{C''(q_l(\theta|\theta_l))} \frac{f(\theta)}{F(\theta) - F(\theta_l)} + \frac{\mathcal{E}_u(\theta|\theta_u) - \theta}{C''(q_u(\theta|\theta_u))} \frac{(\mathcal{E}_u(\theta|\theta_u) - \theta)f(\theta) + (\theta_u - \mathcal{E}_u(\theta|\theta_u))f(\theta_u)}{F(\theta_u) - F(\theta)}$$

$$< q_u(\theta|\theta_u) - q_l(\theta|\theta_l).$$

The above Condition 1 looks very complicated and messy. In fact, it is very difficult to find economic or mathematical interpretations at a glance. I will discuss and suggest some interpretations about the Condition 1, after presenting proposition for uniqueness.

The next proposition states that under the general assumptions, the the maximizer for the problem (P.S) is unique if the Condition 1 holds.

**Proposition B. 1. (Uniqueness of Maximizer in the Social Planner's Problem)** *Under the Assumptions 1, 2 and 3, the maximizer  $\mathbf{q}^*$  for the problem (P.S) is unique, if the Condition 1 holds.*

Except the Condition 1, the conditions to guarantee the uniqueness for the maximizer are considerably mild. Obviously, our next task is to understand and explain what Condition 1 implies technically or economically.

### Interpretation of Condition 1

From the Proposition II.6, we know

$$\theta_k^* = \frac{C(q_k^*) - C(q_{k-1}^*)}{q_k^* - q_{k-1}^*}.$$

In fact, we can easily find  $\mathbf{q}^*$  once we establish  $\boldsymbol{\theta}^*$ . Thus, we can rewrite the above equation as follows;

$$(C.1) \quad \theta_k^* = \frac{C(q_k^*(\boldsymbol{\theta}^*)) - C(q_{k-1}^*(\boldsymbol{\theta}^*))}{q_k^*(\boldsymbol{\theta}^*) - q_{k-1}^*(\boldsymbol{\theta}^*)}.$$

Let

$$M_k(\boldsymbol{\theta}) \equiv \theta_k - \frac{C(q_k(\boldsymbol{\theta})) - C(q_{k-1}(\boldsymbol{\theta}))}{q_k(\boldsymbol{\theta}) - q_{k-1}(\boldsymbol{\theta})}.$$

Using this definition, we can reveal the hidden meaning of the Condition 1. In fact, the Condition 1, which looks very complicated, only requires that  $\partial M_k(\boldsymbol{\theta})/\partial \theta_k > 0$  whenever  $M_k(\boldsymbol{\theta}) = 0$ . Since the above equation (C.1) enforces  $M_k(\boldsymbol{\theta}^*) = 0$ , the Condition 1 can guarantee the uniqueness of  $\boldsymbol{\theta}$  satisfying the equation (C.1).

From now on, I examine the economic property of the Condition 1. In the Conditions 1 and 2, which will appear when we prove the Proposition 1, each term includes the density function  $f$ , the marginal cost function  $C$  and their crosses. Thus, we have two approaches to explain the condition. The first approach starts from the consumer distribution and the second one is built on the cost structure.

To begin with, I investigate the upper and lower bounds of both sides in the Condition 1. Since  $\theta_l \leq \mathcal{E}_l(\theta|\theta_l) \leq \theta \leq \mathcal{E}_u(\theta|\theta_u) \leq \theta_u$ , the left hand side of the condition is not negative. That is, the lower bound is 0. On the other hand, the right hand side of the Condition 1 is upper bounded, since  $q_u(\theta|\theta_u) - q_l(\theta|\theta_l) < q_u(\theta_u|\theta_u) - q_l(\theta_l|\theta_l)$ . Then, both left hand sides of the Condition 1 should be also upper bounded.

Now, I consider properties related with the hazard rate. By the Assumption 3, we know that  $\lim_{\theta \rightarrow \theta_l} \{f(\theta)/(F(\theta) - F(\theta_l))\} = \infty$ ,  $\lim_{\theta \rightarrow \theta_l} \{f(\theta_l)/(F(\theta) - F(\theta_l))\} = \infty$ ,  $\lim_{\theta \rightarrow \theta_u} \{f(\theta)/(F(\theta_u) - F(\theta))\} = \infty$ , and  $\lim_{\theta \rightarrow \theta_u} \{f(\theta_u)/(F(\theta_u) - F(\theta))\} = \infty$ . And we know  $\lim_{\theta \rightarrow \theta_l} (\theta - \mathcal{E}_l(\theta|\theta_l)) = 0$ ,  $\lim_{\theta \rightarrow \theta_l} (\mathcal{E}_l(\theta|\theta_l) - \theta_l) = 0$ ,  $\lim_{\theta \rightarrow \theta_u} (\mathcal{E}_u(\theta|\theta_u) - \theta) = 0$ , and  $\lim_{\theta \rightarrow \theta_u} (\theta_u - \mathcal{E}_u(\theta|\theta_u)) = 0$ . In order that the left hand sides are upper bounded, following all four

$$\frac{(\theta - \mathcal{E}_l(\theta|\theta_l))f(\theta)}{F(\theta) - F(\theta_l)}, \frac{(\mathcal{E}_l(\theta|\theta_l) - \theta_l)f(\theta_l)}{F(\theta) - F(\theta_l)}, \frac{(\mathcal{E}_u(\theta|\theta_u) - \theta)f(\theta)}{F(\theta_u) - F(\theta)}, \text{ and } \frac{(\theta_u - \mathcal{E}_u(\theta|\theta_u))f(\theta_u)}{F(\theta_u) - F(\theta)}$$

are upper bounded. Now, denote the bound as  $M$ .

To understand the meaning of  $M$  better, I will add one more assumption for the density function. Let  $\underline{m} \equiv \min_{\theta \in [\theta_l, \theta_u]} f(\theta)$  and  $\overline{m} \equiv \max_{\theta \in [\theta_l, \theta_u]} f(\theta)$ , and assume that  $k\underline{m} \geq \overline{m}$ . This assumption means that fluctuation of distribution is limited. Clearly,  $k \geq 1$  and if  $k = 1$ , the distribution is uniform.<sup>15</sup>

Here, I examine only the first one. The other three inequalities have similar stories.

$$(C.2) \quad \frac{(\theta - \mathcal{E}_l(\theta|\theta_l))f(\theta)}{F(\theta) - F(\theta_l)} \leq M.$$

By the definition for  $\underline{m}$ ,  $\overline{m}$  and  $k$ ,

$$(C.3) \quad \begin{aligned} (\theta - \mathcal{E}_l(\theta|\theta_l)) \frac{f(\theta)}{F(\theta) - F(\theta_l)} &\leq \left\{ \theta - \frac{1}{2} \frac{\underline{m}}{\overline{m}} (\theta + \theta_l) \right\} \frac{\overline{m}}{\underline{m}(\theta - \theta_l)} \\ &\leq k \frac{\theta}{\theta - \theta_l} - \frac{1}{2} \frac{\theta + \theta_l}{\theta - \theta_l} \\ &= \frac{1}{2} \frac{1}{\theta - \theta_l} ((2k - 1)\theta - \theta_l). \end{aligned}$$

---

<sup>15</sup>Related with this assumption, we can establish some results like  $\underline{m} \leq f(\theta) \leq \overline{m}$ ,  $(1/2)\underline{m}(\theta^2 - \theta_l^2) \leq \int_{\theta_l}^{\theta} s f(s) ds \leq (1/2)\overline{m}(\theta^2 - \theta_l^2)$ , and  $\underline{m}(\theta - \theta_l) \leq F(\theta) - F(\theta_l) \leq \overline{m}(\theta - \theta_l)$ . These results will be utilized in the following discussion.

This equation gives us a very feeble hint for a relationship between  $M$  and  $k$ . For given  $M$ , if

$$(C.4) \quad k \leq \frac{1}{2} + \frac{\theta - \theta_l}{\theta} M + \frac{1}{2} \frac{\theta_l}{\theta},$$

the inequality (C.2) holds. Here, we should check that  $k > 1/2 + \{(\theta - \theta_l)/\theta\}M + (1/2)(\theta_l/\theta)$  does not imply the inequality (C.2) does not hold. The inequality (C.4) is just one possible sufficient condition to guarantee the inequality (C.2). We can conclude that if  $k$  is enough small, that is the density function is quite flat, our inequalities hold easily. Meanwhile, we can argue that the Condition 1 holds, if  $M$  is enough big. Note that if the equation (C.4), the strong sufficient condition, holds,  $M$  should be greater than  $1/2$ . For the reason, consult with the fact that  $k \geq 1$  and the equation (C.3).

Now, let us consider the cost structure. Applying the upper bound of each inequality, the following inequality

$$(C.5) \quad \left\{ \frac{\theta - \mathcal{E}_l(\theta|\theta_l)}{C''(q_l(\theta|\theta_l))} + \frac{\mathcal{E}_u(\theta|\theta_u) - \theta}{C''(q_u(\theta|\theta_u))} \right\} 2M \leq q_u(\theta|\theta_u) - q_l(\theta|\theta_l)$$

implies the Condition 1. Remember that the equation (C.5) is a very strong sufficient condition for the Condition 1. This strong sufficient condition introduced here is just for exploring a possible family of distributions and cost structures to agree with our concern.

Assume that for all  $q$  in the relevant support, that is  $[0, C'^{-1}(\bar{\theta})]$ , there exist  $\underline{c}$  and  $\bar{c}$  such that  $\underline{c} \leq C''(q) \leq \bar{c}$ . That is, the acceleration of  $C$  stays in a bounded area. In other words, the acceleration of  $C$  does not fluctuate highly. If  $C$  is quadratic,  $C''(q)$  is constant for all  $q$ , that is the acceleration of  $C$  does not change.

Now, consider the following inequality

$$(C.6) \quad \frac{C'(q_u(\theta|\theta_u)) - C'(q_l(\theta|\theta_l))}{q_u(\theta|\theta_u) - q_l(\theta|\theta_l)} \leq \frac{\underline{c}}{2M}.$$

Since  $C'(q_u(\theta|\theta_u)) = \mathcal{E}_u(\theta|\theta_u)$  and  $C'(q_l(\theta|\theta_l)) = \mathcal{E}_l(\theta|\theta_l)$ , the inequality (C.6) is equivalent with

$$(\mathcal{E}_u(\theta|\theta_u) - \mathcal{E}_l(\theta|\theta_l)) \frac{2M}{\underline{c}} \leq q_u(\theta|\theta_u) - q_l(\theta|\theta_l).$$

Since  $\underline{c} \leq C''(q_u(\theta|\theta_u))$  and  $\underline{c} \leq C''(q_l(\theta|\theta_l))$ ,

$$\frac{\theta - \mathcal{E}_l(\theta|\theta_l)}{C''(q_l(\theta|\theta_l))} + \frac{\mathcal{E}_u(\theta|\theta_u) - \theta}{C''(q_u(\theta|\theta_u))} \leq \frac{\theta - \mathcal{E}_l(\theta|\theta_l)}{\underline{c}} + \frac{\mathcal{E}_u(\theta|\theta_u) - \theta}{\underline{c}}.$$

Thus, the equation (C.6) implies the equation (C.5).

The left hand side in the inequality (C.6) is the slope of  $C'$  between  $q_l(\theta|\theta_l)$  and  $q_u(\theta|\theta_u)$ . When  $q_l(\theta|\theta_l)$  and  $q_u(\theta|\theta_u)$  are very close, it approaches to  $C''$ . Generally, it is greater than  $\underline{c}$ , which is the smallest value of  $C''$ . If  $(C'(q_u(\theta|\theta_u)) - C'(q_l(\theta|\theta_l)))/(q_u(\theta|\theta_u) - q_l(\theta|\theta_l)) > \underline{c}$ , we need that  $M$  is enough small for the inequality (C.6). If the left hand side is not quite different with  $\underline{c}$ , it is also very helpful for the inequality (C.6) to hold. Thus, we can conclude that if the acceleration of  $C$  is stable, that is  $C''$  is not very changeable, and  $M$  is not quite big, the equation (C.6) and the equation (C.5) hold. We know these equations are sufficient conditions to guarantee the Condition 1.

In fact, holding both two inequalities (C.4) and (C.6) simultaneously is too excessively sufficient for the Condition 1. Obviously, we know that if the inequalities (C.4) and (C.6) hold, then The Condition 1 also hold. But this is too strong. From the inequality (C.6), we know  $2M \leq 1$ . The equation (C.4) obstinately requires that  $M$  should be greater than  $1/2$ . That is, only when  $M = 1/2$ , and so  $k = 1$ , both the inequalities (C.4) and (C.6) can hold. In addition, both inequalities hold in equalities.

That is, the distribution should be uniform and the second derivative of cost function should be constant, like a quadratic cost function. We can conclude that at least we have a unique maximizer for the problem (P.S), if  $F$  is uniform and  $C$  is quadratic.

Fortunately, the Condition 1 is much less restrictive compared to the inequalities (C.4) and (C.6), which are sufficient conditions for the Condition 1. Assume that a cost function is quadratic, and assume that we have a truncated normal distribution in an interval  $[0, 2]$  with a mean 1. In this case, the Condition 1 holds for any standard deviation. We can confirm same thing in asymmetric truncated normal distributions with intervals  $[0, 3/2]$  and  $[0, 3]$ .

#### Proof for the Proposition B. 1

From now on, I prove the uniqueness. This proof needs a few lemmas and a long process until we reach the final destination. I present some lemmas and prove them, before proving the proposition. First of all, I introduce a condition, which is a weaker version for the Condition 1.

**Condition 2.** For arbitrarily given  $\theta_l$  and  $\theta_u$  such that  $0 \leq \theta_l < \theta_u \leq \bar{\theta}$ , and for all  $\theta$  such that  $\theta_l \leq \theta \leq \theta_u$ ,

$$\frac{(\theta - \mathcal{E}_l(\theta|\theta_l))^2}{C''(q_l(\theta|\theta_l))} \frac{f(\theta)}{F(\theta) - F(\theta_l)} + \frac{(\mathcal{E}_u(\theta|\theta_u) - \theta)^2}{C''(q_u(\theta|\theta_u))} \frac{f(\theta)}{F(\theta_u) - F(\theta)} < q_u(\theta|\theta_u) - q_l(\theta|\theta_l).$$

We can easily show that the Condition 1 implies the Condition 2. Check that both

$$\frac{(\mathcal{E}_l(\theta|\theta_l) - \theta_l)f(\theta_l)}{F(\theta) - F(\theta_l)} \geq 0 \text{ and } \frac{(\theta_u - \mathcal{E}_u(\theta|\theta_u))f(\theta_u)}{F(\theta_u) - F(\theta)} \geq 0.$$

Thus, if the Condition 1 holds, then the Condition 2 also holds.

The first lemma gives a hint for uniqueness when only two products, lower and higher, are provided.



**Lemma B. 1. (Clue for Uniqueness when  $n = 2$ )** *Suppose that the Assumptions 1 and 2 hold. Let  $N(\theta|\theta_l, \theta_u) \equiv (C(q_u(\theta|\theta_u)) - C(q_l(\theta|\theta_l)))/(q_u(\theta|\theta_u) - q_l(\theta|\theta_l))$ . For given  $\theta_l$  and  $\theta_u$  such that  $0 \leq \theta_l < \theta_u \leq \bar{\theta}$  and for  $\theta$  such that  $\theta_l \leq \theta \leq \theta_u$ ,  $\theta$ , such that  $N(\theta|\theta_l, \theta_u) = \theta$ , exists. Additionally,  $\theta$ , such that  $N(\theta|\theta_l, \theta_u) = \theta$ , is unique if the Condition 2 holds.*

*Proof. (Existence)* Define  $\mathcal{E}_l(\theta_l|\theta_l) \equiv \lim_{\theta \rightarrow \theta_l} \mathcal{E}_l(\theta|\theta_l)$  and  $\mathcal{E}_u(\theta_u|\theta_u) \equiv \lim_{\theta \rightarrow \theta_u} \mathcal{E}_u(\theta|\theta_u)$ .

Check that by the strict convexity of  $C$ ,

$$N(\theta_l|\theta_l, \theta_u) = \frac{C(q_u(\theta_l|\theta_u)) - C(q_l(\theta_l|\theta_l))}{q_u(\theta_l|\theta_u) - q_l(\theta_l|\theta_l)} > C'(q_l(\theta_l|\theta_l)) = \mathcal{E}_l(\theta_l|\theta_l) = \theta_l$$

and

$$N(\theta_u|\theta_l, \theta_u) = \frac{C(q_u(\theta_u|\theta_u)) - C(q_l(\theta_u|\theta_l))}{q_u(\theta_u|\theta_u) - q_l(\theta_u|\theta_l)} < C'(q_u(\theta_u|\theta_u)) = \mathcal{E}_u(\theta_u|\theta_u) = \theta_u.$$

Obviously,  $N(\theta|\theta_l, \theta_u)$  is continuous in  $\theta$  in  $[\theta_l, \theta_u]$ . Since  $N$  is continuous,  $N(\theta_l|\theta_l, \theta_u) > \theta_l$ , and  $N(\theta_u|\theta_l, \theta_u) < \theta_u$ ,  $\theta$ , satisfying  $N(\theta|\theta_l, \theta_u) = \theta$ , exists by the intermediate value theorem.

**(Uniqueness)** Now, we are moving to the proof for the uniqueness. The first step of proof is to establish the equivalence between the uniqueness of maximizer and the following tentative condition;

$$(C.7) \quad \text{for all } \theta \text{ such that } N(\theta|\theta_l, \theta_u) = \theta, N'(\theta|\theta_l, \theta_u) < 1.$$

( $\Rightarrow$ ) For this direction, I adopt a contraposition. Assume that there exists  $\theta$  satisfying both  $N(\theta|\theta_l, \theta_u) = \theta$  and  $N'(\theta|\theta_l, \theta_u) \geq 1$ . We denote this one as  $\hat{\theta}$ . Then, there is  $\theta$  such that  $\theta < \hat{\theta}$  and  $N(\theta|\theta_l, \theta_u) < \theta$  or  $\theta$  such that  $\theta > \hat{\theta}$  and  $N(\theta|\theta_l, \theta_u) > \theta$ . By the intermediate value theorem, we need other  $\theta$  satisfying  $N(\theta|\theta_l, \theta_u) = \theta$  than  $\hat{\theta}$ , at least one from  $[\theta_l, \hat{\theta})$  or  $(\hat{\theta}, \theta_u]$ . Therefore,  $\theta$  satisfying  $N(\theta|\theta_l, \theta_u) = \theta$  is not unique.

( $\Leftarrow$ ) Suppose that  $\theta$  satisfying  $N(\theta|\theta_l, \theta_u) = \theta$  is not unique. Then, we can find  $\theta_1$  and  $\theta_2$  which both satisfy that  $N(\theta|\theta_l, \theta_u) = \theta$  and  $N'(\theta|\theta_l, \theta_u) < 1$ . Without loss of generality, we can assume that  $\theta_1 < \theta_2$ . Then, there are  $\theta_3$  and  $\theta_4$  such that  $N(\theta_3|\theta_l, \theta_u) < \theta_3$ ,  $N(\theta_4|\theta_l, \theta_u) > \theta_4$ , and  $\theta_1 < \theta_3 < \theta_4 < \theta_2$ . Then, we need at least one  $\theta$  such that  $\theta_3 < \theta < \theta_4$ ,  $N(\theta|\theta_l, \theta_u) = \theta$ , and  $N'(\theta|\theta_l, \theta_u) > 1$ . It is a contradiction.

Now, we can derive  $N'(\theta|\theta_l, \theta_u)$ .

$$\begin{aligned} N'(\theta|\theta_l, \theta_u) &= \frac{C''(q_u(\theta|\theta_u))q'_u(\theta|\theta_u) - C''(q_l(\theta|\theta_l))q'_l(\theta|\theta_l)}{q_u(\theta|\theta_u) - q_l(\theta|\theta_l)} \\ &\quad - \frac{C'(q_u(\theta|\theta_u)) - C'(q_l(\theta|\theta_l))}{(q_u(\theta|\theta_u) - q_l(\theta|\theta_l))^2} (q'_u(\theta|\theta_u) - q'_l(\theta|\theta_l)) \\ &= \frac{1}{q_u(\theta|\theta_u) - q_l(\theta|\theta_l)} \left\{ \left( C''(q_u(\theta|\theta_u)) - \frac{C'(q_u(\theta|\theta_u)) - C'(q_l(\theta|\theta_l))}{q_u(\theta|\theta_u) - q_l(\theta|\theta_l)} \right) q'_u(\theta|\theta_u) \right\} \\ &\quad + \frac{1}{q_u(\theta|\theta_u) - q_l(\theta|\theta_l)} \left\{ \left( \frac{C'(q_u(\theta|\theta_u)) - C'(q_l(\theta|\theta_l))}{q_u(\theta|\theta_u) - q_l(\theta|\theta_l)} - C''(q_l(\theta|\theta_l)) \right) q'_l(\theta|\theta_l) \right\}. \end{aligned}$$

Since

$$q'_u(\theta|\theta_u) = \frac{1}{C''(q_u(\theta|\theta_u))} \frac{d\mathcal{E}_u(\theta|\theta_u)}{d\theta} = \frac{(\mathcal{E}_u(\theta|\theta_u) - \theta)}{C''(q_u(\theta|\theta_u))} \frac{f(\theta)}{F(\theta_u) - F(\theta)} \quad ^{16}$$

and

$$q'_l(\theta|\theta_l) = \frac{1}{C''(q_l(\theta|\theta_l))} \frac{d\mathcal{E}_l(\theta|\theta_l)}{d\theta} = \frac{(\theta - \mathcal{E}_l(\theta|\theta_l))}{C''(q_l(\theta|\theta_l))} \frac{f(\theta)}{F(\theta) - F(\theta_l)},$$

$$\begin{aligned} N'(\theta|\theta_l, \theta_u) &= \\ &= \frac{1}{q_u(\theta|\theta_u) - q_l(\theta|\theta_l)} \left\{ (\mathcal{E}_u(\theta|\theta_u) - N(\theta|\theta_l, \theta_u)) \frac{(\mathcal{E}_u(\theta|\theta_u) - \theta)}{C''(q_u(\theta|\theta_u))} \frac{f(\theta)}{F(\theta_u) - F(\theta)} \right\} \end{aligned}$$

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<sup>16</sup>Since  $\mathcal{E}_u = \int_{\theta}^{\theta_u} s [f(s)/(F(\theta_u) - F(\theta))] ds = [1/(F(\theta_u) - F(\theta))] \int_{\theta}^{\theta_u} s f(s) ds$ ,

$$\frac{d\mathcal{E}_u}{d\theta} = \frac{1}{F(\theta_u) - F(\theta)} (-\theta f(\theta)) + \frac{f(\theta)}{\{F(\theta_u) - F(\theta)\}^2} \int_{\theta}^{\theta_u} s f(s) ds = (\mathcal{E}_u - \theta) \frac{f(\theta)}{F(\theta_u) - F(\theta)}.$$

$$+\frac{1}{q_u(\theta|\theta_u) - q_l(\theta|\theta_l)} \left\{ (N(\theta|\theta_l, \theta_u) - \mathcal{E}_l(\theta|\theta_l)) \frac{(\theta - \mathcal{E}_l(\theta|\theta_l))}{C''(q_l(\theta|\theta_l))} \frac{f(\theta)}{F(\theta) - F(\theta_l)} \right\}.$$

The second step of proof is to establish that the Condition 2 implies the equation (C.7). In fact, it is obvious. Substituting  $\theta$  into  $N(\theta|\theta_l, \theta_u)$  and applying the Condition 2 complete the proof.  $\square$

The next lemma states the possibility of uniqueness when three products are provided: a lower, a middle and a higher.

**Lemma B. 2. (Clue for Uniqueness when  $n = 3$ )** *Suppose that the Assumptions 1, 2 and 3 hold. For given  $\theta_l$  and  $\theta_u$  such that  $0 \leq \theta_l < \theta_u \leq \bar{\theta}$  and for  $\theta_1$  and  $\theta_2$  such that  $\theta_l \leq \theta_1 < \theta_2 \leq \theta_u$ , let*

$$N(\theta_1|\theta_l, \theta_2) \equiv \frac{C(q_m(\theta_1|\theta_2)) - C(q_l(\theta_1|\theta_l))}{q_m(\theta_1|\theta_2) - q_l(\theta_1|\theta_l)}$$

and

$$N(\theta_2|\theta_1, \theta_u) \equiv \frac{C(q_u(\theta_2|\theta_1)) - C(q_m(\theta_2|\theta_1))}{q_u(\theta_2|\theta_u) - q_m(\theta_2|\theta_1)}.$$

Then,  $(\theta_1, \theta_2)$ , satisfying both  $N(\theta_1|\theta_l, \theta_2) = \theta_1$  and  $N(\theta_2|\theta_1, \theta_u) = \theta_2$  simultaneously, uniquely exists, if the Condition 1 holds.

*Proof. (Existence)* I define  $\theta_0$  so that it satisfies  $N(\theta_0|\theta_l, \theta_u) = \theta_0$ . By the Lemma B. 1,  $\theta_0$  is uniquely determined. In the same line, we know that  $\theta_1$  is uniquely determined by  $\theta_l$  and  $\theta_2$ , just substituting  $\theta_2$  into the position of  $\theta_u$  in the Lemma B. 1. Then, we can construct a function,  $\theta_1(\theta_2|\theta_l)$ . Similarly, we can make a function,  $\theta_2(\theta_1|\theta_u)$ . By the construction of the functions,  $\theta_1(\theta_2 = \theta_u|\theta_l) = \theta_2(\theta_1 = \theta_l|\theta_u) = \theta_0$ . Obviously,  $\theta_1(\theta_2 = \theta_l|\theta_l) = \theta_l$  and  $\theta_2(\theta_1 = \theta_u|\theta_u) = \theta_u$ . The above equations implies that the function  $\theta_1(\theta_2|\theta_l)$  starts from  $\theta_l$  and finishes at  $\theta_0$ , while  $\theta_2(\theta_1|\theta_u)$  travels from  $\theta_0$  and finishes at  $\theta_u$ .

Since  $N(\theta_1|\theta_l, \theta_2)$  is continuous in  $\theta_2$  as well as  $\theta_1$  and  $N(\theta_2|\theta_1, \theta_u)$  is continuous in  $\theta_1$  as well as  $\theta_2$ ,  $\theta_1(\theta_2|\theta_l)$  is continuous in  $\theta_2$  and  $\theta_2(\theta_1|\theta_u)$  is continuous in  $\theta_1$ . The above conditions and the continuity guarantee the existence of  $(\theta_1, \theta_2)$ , satisfying  $N(\theta_1|\theta_l, \theta_2) = \theta_1$  and  $N(\theta_2|\theta_1, \theta_u) = \theta_2$ . See the Figure 19. At the intersection of  $\theta_1(\theta_2|\theta_l)$  and  $\theta_2(\theta_1|\theta_u)$ ,  $N(\theta_1|\theta_l, \theta_2) = \theta_1$  and  $N(\theta_2|\theta_1, \theta_u) = \theta_2$  simultaneously.

**(Uniqueness)** The question about uniqueness is about the number of intersections of  $\theta_1(\theta_2|\theta_l)$  and  $\theta_2(\theta_1|\theta_u)$  in the Figure 19.

First of all, we check the sign of  $\partial N(\theta_1|\theta_l, \theta_2)/\partial\theta_2$  and  $\partial N(\theta_2|\theta_1, \theta_u)/\partial\theta_1$ .

$$\begin{aligned} \frac{\partial N(\theta_1|\theta_l, \theta_2)}{\partial\theta_2} &= \frac{\partial}{\partial\theta_2} \left\{ \frac{C(q_m(\theta_1|\theta_2)) - C(q_l(\theta_1|\theta_l))}{q_m(\theta_1|\theta_2) - q_l(\theta_1|\theta_l)} \right\} \\ &= \frac{1}{q_m(\theta_1|\theta_2) - q_l(\theta_1|\theta_l)} \left\{ C'(q_m(\theta_1|\theta_2)) - \frac{C(q_m(\theta_1|\theta_2)) - C(q_l(\theta_1|\theta_l))}{q_m(\theta_1|\theta_2) - q_l(\theta_1|\theta_l)} \right\} \frac{\partial q_m}{\partial\theta_2} \\ &= \frac{(\mathbb{E}(s|s \in [\theta_1, \theta_2]) - N(\theta_1|\theta_l, \theta_2)) (\theta_2 - \mathbb{E}(s|s \in [\theta_1, \theta_2]))}{q_m(\theta_1|\theta_2) - q_l(\theta_1|\theta_l)} \frac{f(\theta_2)}{C''(q_m(\theta_1|\theta_2)) (F(\theta_2) - F(\theta_1))} > 0, \end{aligned}$$

and

$$\begin{aligned} \frac{\partial N(\theta_2|\theta_1, \theta_u)}{\partial\theta_1} &= \frac{\partial}{\partial\theta_1} \left\{ \frac{C(q_u(\theta_2|\theta_u)) - C(q_m(\theta_2|\theta_1))}{q_u(\theta_2|\theta_u) - q_m(\theta_2|\theta_1)} \right\} \\ &= \frac{1}{q_u(\theta_2|\theta_u) - q_m(\theta_2|\theta_1)} \left\{ \frac{C(q_u(\theta_2|\theta_u)) - C(q_m(\theta_2|\theta_1))}{q_u(\theta_2|\theta_u) - q_m(\theta_2|\theta_1)} - C'(q_m(\theta_2|\theta_1)) \right\} \frac{\partial q_m}{\partial\theta_1} \\ &= \frac{(N(\theta_2|\theta_1, \theta_u) - \mathbb{E}(s|s \in [\theta_1, \theta_2])) (\mathbb{E}(s|s \in [\theta_1, \theta_2]) - \theta_1)}{q_u(\theta_2|\theta_u) - q_m(\theta_2|\theta_1)} \frac{f(\theta_1)}{C''(q_m(\theta_2|\theta_1)) (F(\theta_2) - F(\theta_1))} > 0. \end{aligned}$$

Now, I will concentrate on  $\theta_1(\theta_2|\theta_l)$ . From the above result, we can check  $\partial N(\theta_1|\theta_l, \theta_2)/\partial\theta_2 > 0$ . That is, if  $\theta_2 < \tilde{\theta}_2$ , for given  $\theta_1$ ,  $N(\theta_1|\theta_l, \theta_2) < N(\theta_1|\theta_l, \tilde{\theta}_2)$ . That is,  $N(\theta_1|\theta_l, \cdot)$  will shift upward as  $\theta_2$  increases. To understand how  $\theta_2$  affects on the determination of  $\theta_1$ , we adopt the analysis using linearization and limit. See the Figure 20. First, the increase from  $\theta_2$  to  $\tilde{\theta}_2$  changes  $N(\theta_1|\theta_l, \theta_2)$  to  $N(\theta_1|\theta_l, \tilde{\theta}_2)$ . Second, we can find a new  $\theta$  along  $N(\theta_1|\theta_l, \tilde{\theta}_2)$ . Denoting  $\theta_1 = \theta_1(\theta_2|\theta_l)$  and  $\tilde{\theta}_1 = \theta_1(\tilde{\theta}_2|\theta_l)$ ,

we know

$$(C.8) \quad N'(\theta_1|\theta_l, \tilde{\theta}_2)(\tilde{\theta}_1 - \theta_1) + (N(\theta_1|\theta_l, \tilde{\theta}_2) - N(\theta_1|\theta_l, \theta_2)) = \tilde{\theta}_1 - \theta_1.$$

The above equation (C.8) can be easily understood in the Figure 20;  $A + B$  should equal to  $\tilde{\theta}_1 - \theta_1$ . Rearranging both sides of the equation (C.8) and dividing by  $\tilde{\theta}_2 - \theta_2$ ,

$$\frac{N(\theta_1|\theta_l, \tilde{\theta}_2) - N(\theta_1|\theta_l, \theta_2)}{\tilde{\theta}_2 - \theta_2} = \left(1 - N'(\theta_1|\theta_l, \tilde{\theta}_2)\right) \frac{\tilde{\theta}_1 - \theta_1}{\tilde{\theta}_2 - \theta_2}.$$

Taking the limitation in both sides,

$$\begin{aligned} \lim_{\tilde{\theta}_2 \rightarrow \theta_2} \frac{N(\theta_1|\theta_l, \tilde{\theta}_2) - N(\theta_1|\theta_l, \theta_2)}{\tilde{\theta}_2 - \theta_2} &= \lim_{\tilde{\theta}_2 \rightarrow \theta_2} \left(1 - N'(\theta_1|\theta_l, \tilde{\theta}_2)\right) \frac{\tilde{\theta}_1 - \theta_1}{\tilde{\theta}_2 - \theta_2} \\ &\Leftrightarrow \frac{\partial N(\theta_1|\theta_l, \theta_2)}{\partial \theta_2} = (1 - N'(\theta_1|\theta_l, \theta_2)) \frac{d\theta_1}{d\theta_2}. \end{aligned}$$

Thus,  $d\theta_1/d\theta_2 = [1/(1 - N'(\theta_1|\theta_l, \theta_2))] [\partial N(\theta_1|\theta_l, \theta_2)/\partial \theta_2]$ . In the similar way, we can get

$$\frac{d\theta_2}{d\theta_1} = \frac{1}{(1 - N'(\theta_2|\theta_1, \theta_u))} \frac{\partial N(\theta_2|\theta_1, \theta_u)}{\partial \theta_1}.$$

When  $\theta_1 = N(\theta_1|\theta_l, \theta_2)$ ,

$$\begin{aligned} &N'(\theta_1|\theta_l, \theta_2) + \frac{\partial N(\theta_1|\theta_l, \theta_2)}{\partial \theta_2} \\ &= \frac{1}{q_m(\theta_1|\theta_2) - q_l(\theta_1|\theta_l)} \left\{ \frac{(\mathbb{E}(s|s \in [\theta_1, \theta_2]) - \theta_1)^2}{C''(q_m(\theta_1|\theta_2))} \frac{f(\theta_1)}{F(\theta_2) - F(\theta_1)} \right\} \\ &\quad + \frac{1}{q_m(\theta_1|\theta_2) - q_l(\theta_1|\theta_l)} \left\{ \frac{(\theta_1 - \mathcal{E}_l(\theta_1|\theta_l))^2}{C''(q_l(\theta_1|\theta_l))} \frac{f(\theta_1)}{F(\theta_1) - F(\theta_l)} \right\} \\ &\quad + \frac{(\mathbb{E}(s|s \in [\theta_1, \theta_2]) - \theta_1)}{q_m(\theta_1|\theta_2) - q_l(\theta_1|\theta_l)} \left\{ \frac{(\theta_2 - \mathbb{E}(s|s \in [\theta_1, \theta_2]))}{C''(q_m(\theta_1|\theta_2))} \frac{f(\theta_2)}{F(\theta_2) - F(\theta_1)} \right\} \\ &< 1 \text{ (by the Condition 1)}. \end{aligned}$$

When  $\theta_2 = N(\theta_2|\theta_1, \theta_u)$ , by the Condition 1,

$$N'(\theta_2|\theta_1, \theta_u) + \frac{\partial N(\theta_2|\theta_1, \theta_u)}{\partial \theta_1} < 1.$$

Rearranging these inequalities, we get

$$\frac{1}{(1 - N'(\theta_1|\theta_l, \theta_2))} \frac{\partial N(\theta_1|\theta_l, \theta_2)}{\partial \theta_2} < 1 \text{ and } \frac{1}{(1 - N'(\theta_2|\theta_1, \theta_u))} \frac{\partial N(\theta_2|\theta_1, \theta_u)}{\partial \theta_1} < 1.$$

Therefore, whenever both  $\theta_1 = N(\theta_1|\theta_l, \theta_2)$  and  $\theta_2 = N(\theta_2|\theta_1, \theta_u)$  hold,

$$\frac{d\theta_1}{d\theta_2} < 1 \text{ and } \frac{d\theta_2}{d\theta_1} < 1.$$

See the Figure 19 again. The slope for  $\theta_2(\cdot)$  is less than 1 and the slope for  $\theta_1^{-1}(\cdot)$  is greater than 1 whenever two functions intersect. Then, the intersection should be unique in the same logic appeared in the proof of Lemma B. 1.  $\square$

The next lemma extends two previous lemmas in the situation where  $n$  products are provided.

**Lemma B. 3. (Clue for Uniqueness for general  $n$ )** *Suppose that the Assumptions 1, 2 and 3 hold. For given  $\theta_l$  and  $\theta_u$  such that  $0 \leq \theta_l < \theta_u \leq \bar{\theta}$  and for  $\theta_1, \theta_2, \dots, \theta_n$  such that  $\theta_l = \theta_0 \leq \theta_1 < \dots < \theta_n \leq \theta_u = \theta_{n+1}$ , let*

$$N(\theta_k|\theta_{k-1}, \theta_{k+1}) \equiv \frac{C(q_k(\theta_k|\theta_{k+1})) - C(q_{k-1}(\theta_k|\theta_{k-1}))}{q_k(\theta_k|\theta_{k+1}) - q_{k-1}(\theta_k|\theta_{k-1})},$$

*for all  $k = 1, 2, \dots, n$ . Then,  $(\theta_1, \dots, \theta_n)$ , satisfying  $N(\theta_k|\theta_{k-1}, \theta_{k+1}) = \theta_k$  for all  $k$ , uniquely exists, if the Condition 2 holds.*

*Proof.* I will use mathematical induction. The Lemma B. 1 proves this lemma when  $n = 1$ . Suppose that  $(\theta_1, \dots, \theta_{k-1})$ , satisfying the conditions listed in the Lemma B. 3, uniquely exists when  $n = k - 1$ .

Now, take an arbitrary  $\theta$  so that  $\theta_l < \theta < \theta_u$ , and denote it as  $\theta_k$ . Then, we can find a unique combination  $(\theta_1, \dots, \theta_{k-1})$  on the interval  $[\theta_l, \theta_k]$ , by the induction hypothesis. Especially, we get a uniquely determined  $\theta_{k-1}$  by  $\theta_k$ . For  $\theta_k$  to satisfy the condition in the Lemma B. 3,  $\theta_k$  should be also determined by  $\theta_{k-1}$  and given

$\theta_u$ . Thanks to the Lemma B. 2, we know the combination  $(\theta_{k-1}, \theta)$ , satisfying our desirable conditions, is uniquely exists.  $\square$

Finally, I can prove the Proposition B. 1.

*Proof. (Proposition B. 1: Uniqueness of Maximizer in the Social Planner's Problem)* For this proof, I will change the problem (P.S) into a problem whose argument is  $\theta$ , rather than  $q$ . To do so, I need to establish an one to one relationship between  $\theta$  and  $q$ . We already know that  $\theta$  is uniquely determined if  $q$  is fixed.

Suppose that  $\theta$  is given. Since  $C$  is strictly convex,  $C'(q_1) > (C(q_1) - C(q_0))/(q_1 - q_0) = C(q_1)/q_1$ . Then,

$$\frac{d(C(q_1)/q_1)}{dq_1} = \frac{1}{q_1^2}(C'(q_1)q_1 - C(q_1)) > 0.$$

Because  $C(q_1)/q_1$  is strictly increasing in  $q_1$ ,  $q_1$ , satisfying  $\theta_1 = C(q_1)/q_1$ , is uniquely determined when  $\theta_1$  is given.

Now, assume that  $q_{k-1}$  is uniquely determined by given  $\theta$ . From the strict convexity of  $C$ , we know that  $C'(q_k) > (C(q_k) - C(q_{k-1}))/ (q_k - q_{k-1})$ . Then, for the uniquely determined  $q_{k-1}$ ,

$$\begin{aligned} & \frac{\partial ((C(q_k) - C(q_{k-1}))/ (q_k - q_{k-1}))}{\partial q_k} \\ &= \frac{1}{(q_k - q_{k-1})^2} \{C'(q_k)(q_k - q_{k-1}) - (C(q_k) - C(q_{k-1}))\} > 0. \end{aligned}$$

Thus,  $q_k$ , satisfying  $\theta_k = (C(q_k) - C(q_{k-1}))/ (q_k - q_{k-1})$ , is also uniquely determined by given  $\theta_k$ .

By the constraint in (P.S) and the Proposition II.6, we know that the maximizer  $q^* = (q_1^*, q_2^*, \dots, q_n^*)$  satisfies

$$(C.9) \quad \theta_k^* = \frac{C(q_k^*) - C(q_{k-1}^*)}{q_k^* - q_{k-1}^*} \text{ and } C'(q_k^*) = \mathbb{E}(\theta | \theta \in [\theta_k^*, \theta_{k+1}^*]).$$

Instead of  $\mathbf{q}^*$ , I will find  $\boldsymbol{\theta}^* = (\theta_1^*, \theta_2^*, \dots, \theta_n^*)$  satisfying the above two equations. The one to one relationship between  $\boldsymbol{\theta}$  and  $\mathbf{q}$ , established above, guarantees finding  $\boldsymbol{\theta}^*$  is equivalent with finding  $\mathbf{q}^*$ .

Now, I define the function  $S(\boldsymbol{\theta}) = (S_1(\boldsymbol{\theta}), S_2(\boldsymbol{\theta}), \dots, S_n(\boldsymbol{\theta})) : \mathbb{R}^n \Rightarrow \mathbb{R}^n$  as follows;

$$S_k(\boldsymbol{\theta}) = \frac{C(C'^{-1}(\mathbb{E}(\theta|\theta \in [\theta_k, \theta_{k+1}])) - C(C'^{-1}(\mathbb{E}(\theta|\theta \in [\theta_{k-1}, \theta_k])))}{C'^{-1}(\mathbb{E}(\theta|\theta \in [\theta_k, \theta_{k+1}])) - C'^{-1}(\mathbb{E}(\theta|\theta \in [\theta_{k-1}, \theta_k]))}.$$

At the maximizer,  $\theta_k^* = S_k(\boldsymbol{\theta}^*)$  for all  $k$ .

By the equation (C.9) and the Lemma B. 3,

$$S_k(\boldsymbol{\theta}) = \frac{C(q_k(\theta_k|\theta_{k+1})) - C(q_{k-1}(\theta_k|\theta_{k-1}))}{q_k(\theta_k|\theta_{k+1}) - q_{k-1}(\theta_k|\theta_{k-1})} = N(\theta_k|\theta_{k-1}, \theta_{k+1}).$$

Remind the Lemma B. 3 for  $N(\theta_k|\theta_{k-1}, \theta_{k+1})$ . Then, the Lemma B. 3 guarantees the unique  $(\theta_2^*, \dots, \theta_n^*)$  in the interval  $[\theta_1, \bar{\theta}]$  for given  $\theta_1$ . Note that a series of previous lemmas does not give us any clue for choice between  $q_0$  (buying nothing) and  $q_1$  (buying a minimum-quality good). Thus, we need to show  $\theta_1^*$  is also uniquely determined. In fact, it is obvious. Replacing  $q_l$  as  $q_0 = 0$ , the Lemmas B. 1 and 2 easily proves that  $\theta_1^*$  is uniquely determined.

Now, we have a unique  $\boldsymbol{\theta}^*$ . Then, the relationship established in this proof earlier guarantees the unique  $\mathbf{q}^*$ . □



## APPENDIX D

## DATA AND VARIABLE DEFINITIONS

The variables in this dissertation are divided in two categories. The first category is variables for the cable service provider in each franchise. Mainly, each franchise is city-level, and the cable service provider serves only in the franchise. Sometimes, the provider may serve neighboring areas of the main franchise. My unit of observation is a franchise. In most cases, each franchise has only one cable service provider. In the case that multiple providers are operating, I choose only the biggest one. This case only happens in the large cities, such as Houston, and the other provider's market shares are ignorable, comparing to the biggest's. I gathered the data set from *Television & Cable Factbook 2012*, published by Warren Communications News. Here, I list all variables and their explanation which are gathered from WarrenCommunicationsNews (2012).

*mrnk* the franchise's TV market ranking

*cpct* the provider's maximum channel capacity

*2wyc* whether the provider can offer 2-way capable service or not

*mplt* the provider's installed total miles of plant

*mplt\_c* the provider's installed miles of plant (coaxial)

*mplt\_f* the provider's installed miles of plant (fiber optic)

*VOD* whether the provider offer VOD service or not

*PPV* whether the provider offer pay per view service or not

*itnt* whether the provider offer internet service or not

*itnt\_s* the number of households subscribing internet service

*tlps* the number of households subscribing telephone service

*tlps\_mo* monthly fee for subscribing telephone service

*bs\_nc* number of channels, which basic service offers

*bs\_air* number of channels using off-air or microwave or translator, which basic service offers

*bs\_sat* number of channels using satellite, which basic service offers

*bs\_ins* installation fee for basic service

*bs\_mo* monthly fee for basic service

*bs\_s* the number of subscribers of basic service

*ebs\_nc* number of channels, which expanded basic service offers

*ebs\_ins* installation fee for expanded basic service

*ebs\_mo* monthly fee for expanded basic service

*ebs\_s* the number of subscribers of expanded basic service

*dbs\_nc* number of channels, which digital basic service offers

*dbs\_ins* installation fee for digital basic service

*dbs\_mo* monthly fee for digital basic service

*dbs\_s* the number of subscribers of digital basic service

I have one more service provider related variable, which is not obtained from WarrenCommunicationsNews (2012). It is about whether the provider is an MSO or not, and its ranking. The data comes from National Cable & Telecommunications Association.<sup>17</sup>

*d\_rk\_MSO* whether the provider is an MSO, which is ranked inside 25, or not

The second category of variables is about the demographic variables, which describes each franchise. The data has three different sources. The first and major

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<sup>17</sup>For the source, visit <http://www.ncta.com/Stats/TopMSOs.aspx>.

source is United States Census Bureau.<sup>18</sup> I find some additional variables, like city-level poverty ratio, in City-Data.com.<sup>19</sup> Last, for driving distances from the nearest large city, I consult with Google map service.<sup>20</sup> To define “large,” I use the Metropolitan Statistical Area, which is defined United States Census Bureau. These are demographic variables at franchise-level.

*dist\_5M* driving distance to the nearest MSA with population over 5M

*dist\_1M* driving distance to the nearest MSA with population over 1M

*dist\_0.5M* driving distance to the nearest MSA with population over 0.5M

*pop\_2009* the franchise’s population in 2009

*pop\_crt* the franchise’s population-increase rate from 2000

*rt\_pop\_fm* the franchise’s ratio of female population

*rt\_pop\_w* the franchise’s ratio of white population

*rt\_pop\_h* the franchise’s ratio of Hispanic population

*rt\_pop\_b* the franchise’s ratio of black population

*den\_pop* the franchise’s population density (per square mile)

*age* the franchise’s median resident age

*inc\_2009* the franchise’s median household income in 2009

*inc\_2000* the franchise’s median household income in 2000

*inc\_pc* the franchise’s income per capita

*rt\_pov* the franchise’s ratio of population in poverty in 2009

*rt\_inc\_x\_y* the franchise’s ratio of households whose income is greater than  $xk$  and less than  $yk$

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<sup>18</sup>For the source, visit <http://www.census.gov>.

<sup>19</sup>For the source, visit <http://www.city-data.com>.

<sup>20</sup>I obtain and construct the data, using the “Get directions” tap at [maps.google.com](http://maps.google.com).

*rt\_inc\_z\_* the franchise's ratio of households whose income is greater than  $zk$

*ar\_land* the franchise's land area (square mile)

*av\_h\_sz* the franchise's average household size (number of people)

*lv\_cst* the franchise's cost of living index in 2011 (US: 100)

All above variables are used in the estimation processes, in themselves or by some operations.

Unfortunately, WarrenCommunicationsNews (2012) does not offer the perfect set of data. I recover some missing variables for the cable service providers. I use the other related variables, whenever they are available. For example, when *bs\_s*, the number of subscribers of basic service, is missing, I use the variables, like *pop\_2009*, *rt\_pop\_h*, *av\_h\_sz*, *inc\_2009*, *rt\_inc\_0\_10*, *d\_rk\_mso*, *mplt*, *d\_tlps*, *bs\_nc*, *bs\_mo*, *ebs\_s*, and *dbb\_s*, to restore *bs\_s*.

## APPENDIX E

## TABLES AND FIGURES

Table 1. Increasing Speed of Profit (or Social Welfare)

| mean | st. dev. | $n = 1$ |        | $n = 2$ |        | $n = 3$ |        | $n = 10$ |        |
|------|----------|---------|--------|---------|--------|---------|--------|----------|--------|
|      |          | profit  | SW     | profit  | SW     | profit  | SW     | profit   | SW     |
| 1    | 1        | 0.8310  | 0.8454 | 0.9303  | 0.9369 | 0.9620  | 0.9657 | 0.9952   | 0.9957 |
| 2    | 1        | 0.8593  | 0.8861 | 0.9433  | 0.9557 | 0.9694  | 0.9764 | 0.9963   | 0.9972 |
| 2.5  | 1        | 0.8737  | 0.9067 | 0.9501  | 0.9649 | 0.9734  | 0.9816 | 0.9969   | 0.9979 |
| 3    | 1        | 0.8884  | 0.9266 | 0.9569  | 0.9735 | 0.9773  | 0.9864 | 0.9974   | 0.9985 |
| 4    | 1        | 0.9182  | 0.9600 | 0.9700  | 0.9866 | 0.9846  | 0.9933 | 0.9983   | 0.9993 |
| 2.5  | 0.5      | 0.9182  | 0.9623 | 0.9676  | 0.9862 | 0.9825  | 0.9928 | 0.9979   | 0.9991 |
| 2.5  | 1        | 0.8737  | 0.9067 | 0.9501  | 0.9649 | 0.9734  | 0.9816 | 0.9969   | 0.9979 |
| 2.5  | 2        | 0.8722  | 0.8873 | 0.9529  | 0.9590 | 0.9758  | 0.9790 | 0.9973   | 0.9977 |
| 2.5  | 5        | 0.8855  | 0.8882 | 0.9587  | 0.9597 | 0.9789  | 0.9795 | 0.9977   | 0.9977 |
| 2.5  | 10       | 0.8880  | 0.8887 | 0.9597  | 0.9600 | 0.9794  | 0.9796 | 0.9977   | 0.9978 |

The above st. dev. is the standard deviation for normal distribution before truncation.

I normalize the profit and the social welfare when  $n = 100$  as 1.

Table 2. Optimal Quality Level with Truncated Normal Consumer Types Distribution

| mean | $\bar{\theta}$ | $\sigma_1$ | $\sigma_2$ | $\theta_3^*$ | $\theta_3^m$ | $q_3^*$ | $q_3^m$ |
|------|----------------|------------|------------|--------------|--------------|---------|---------|
| 1    | 2              | 1.0000     | 1.7089     | 1.4047       | 1.6731       | 1.6819  | 1.6731  |
| 1    | 2              | 0.2000     | 1.0000     | 1.1238       | 1.1369       | 1.2459  | 1.1369  |
| 1    | 5              | 1.0000     | 0.9182     | 2.1501       | 2.5641       | 2.6459  | 2.5641  |
| 1    | 5              | 1.5870     | 1.0000     | 2.8857       | 3.5366       | 3.5658  | 3.5366  |
| 1    | 8              | 1.0000     | 0.9173     | 2.1513       | 2.5631       | 2.6476  | 2.5630  |
| 1    | 8              | 2.5685     | 1.0000     | 4.4136       | 5.4562       | 5.4944  | 5.4562  |

$\sigma_1$  and  $\sigma_2$  are the standard deviation before and after truncation, respectively.

$\theta^*$  and  $q^*$  are the choice by the social planners and  $\theta^m$  and  $q^m$  by the monopolist.

Fig. 1. Quality Provided to Each Taste Type

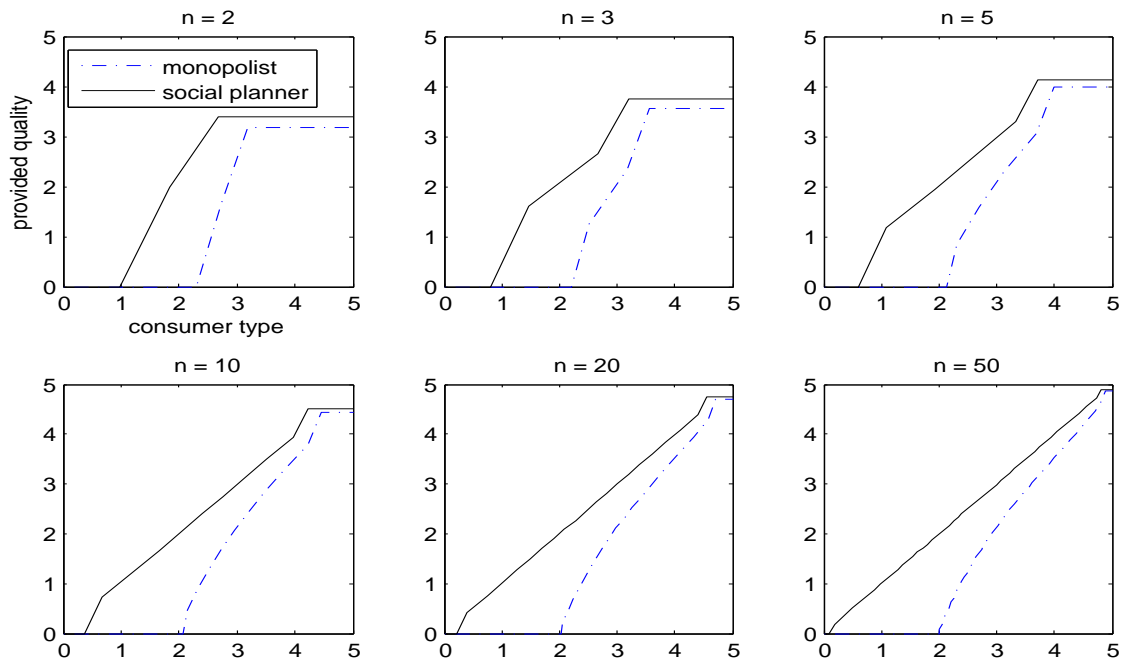


Table 3. Two-way Quality Distortion

| $\sigma$ | $\theta_3^*$ | $\theta_3^m$ | $q_3^*$ | $q_3^m$ | Social Welfare | Profit |
|----------|--------------|--------------|---------|---------|----------------|--------|
| 0.5      | 1.4291       | 1.6212       | 1.7064  | 1.6211  | 0.6232         | 0.2940 |
| 1        | 2.1501       | 2.5630       | 2.6459  | 2.5630  | 1.1041         | 0.4832 |
| 1.5      | 2.8054       | 3.4286       | 3.4690  | 3.4286  | 1.8143         | 0.7880 |
| 2        | 3.1453       | 3.8645       | 3.8652  | 3.8644  | 2.4661         | 1.1016 |
| 2.01     | 3.1497       | 3.8697       | 3.8701  | 3.8697  | 2.4772         | 1.1072 |
| 2.02     | 3.1540       | 3.8747       | 3.8749  | 3.8747  | 2.4882         | 1.1129 |
| 2.03     | 3.1583       | 3.8797       | 3.8796  | 3.8797  | 2.4991         | 1.1185 |
| 2.04     | 3.1625       | 3.8847       | 3.8842  | 3.8847  | 2.5099         | 1.1240 |
| 2.05     | 3.1668       | 3.8895       | 3.8888  | 3.8895  | 2.5207         | 1.1296 |
| 2.1      | 3.1866       | 3.9123       | 3.9105  | 3.9123  | 2.5733         | 1.1568 |
| 2.5      | 3.3039       | 4.0403       | 4.0343  | 4.0403  | 2.9257         | 1.3457 |
| 3        | 3.3879       | 4.1246       | 4.1182  | 4.1245  | 3.2300         | 1.5178 |
| 5        | 3.5066       | 4.2325       | 4.2292  | 4.2325  | 3.7490         | 1.8295 |
| 10       | 3.5554       | 4.2728       | 4.2720  | 4.2728  | 3.9958         | 1.9854 |
| 20       | 3.5674       | 4.2825       | 4.2823  | 4.2825  | 4.0600         | 2.0268 |
| 50       | 3.5708       | 4.2851       | 4.2852  | 4.2851  | 4.0782         | 2.0386 |
| 100      | 3.5712       | 4.2855       | 4.2856  | 4.2855  | 4.0808         | 2.0403 |
| uniform  | 3.5714       | 4.2857       | 4.2857  | 4.2857  | 4.0816         | 2.0408 |

$\sigma$  is the standard deviation before truncation.

$\theta^*$  and  $q^*$  are the choice by the social planners and  $\theta^m$  and  $q^m$  by the monopolist.

Table 4. Summary of Cable Data

| variable        | unit    | min  | max    | mean     | median | st. dev. |
|-----------------|---------|------|--------|----------|--------|----------|
| <i>mplt</i>     | mile    | 7    | 29791  | 477.0391 | 70     | 536.382  |
| <i>d_itnt</i>   | dummy   | 0    | 1      | 0.774194 | 1      | 0.419643 |
| <i>d_tlps</i>   | dummy   | 0    | 1      | 0.387097 | 0      | 0.503308 |
| <i>d_rk_MSO</i> | dummy   | 0    | 1      | 0.471774 | 0      | 0.504349 |
| <i>bs_ins</i>   | dollar  | 10   | 55     | 37.10039 | 38     | 9.967326 |
| <i>ebs_ins</i>  | dollar  | 10   | 68.34  | 37.70283 | 38     | 10.98089 |
| <i>db_s_ins</i> | dollar  | 10   | 68.34  | 38.15913 | 38     | 11.0266  |
| <i>bs_nc</i>    | channel | 6    | 72     | 22.34274 | 17     | 13.44649 |
| <i>bs_air</i>   | channel | 3    | 19     | 9.758065 | 9      | 3.854671 |
| <i>bs_mo</i>    | dollar  | 4.95 | 61.95  | 21.78021 | 19.95  | 11.56839 |
| <i>bs_s</i>     | people  | 64   | 790000 | 17651.94 | 1839   | 51168.8  |
| <i>ebs_nc</i>   | channel | 5    | 62     | 34.81463 | 38     | 13.43525 |
| <i>ebs_mo</i>   | dollar  | 4.57 | 49.99  | 25.74219 | 23.325 | 11.05807 |
| <i>ebs_s</i>    | people  | 43   | 526208 | 19813.39 | 1942   | 40791.92 |
| <i>db_s_nc</i>  | channel | 7    | 148    | 41.28796 | 32     | 31.82128 |
| <i>db_s_mo</i>  | dollar  | 3.99 | 54.99  | 17.85573 | 13.95  | 16.77435 |
| <i>db_s_s</i>   | people  | 9    | 480000 | 33966.13 | 5012   | 25093.67 |

st. dev. is the standard deviation.



Table 5. Summary of Demographic Data

| variable           | unit            | min   | max     | mean     | median  | st. dev. |
|--------------------|-----------------|-------|---------|----------|---------|----------|
| <i>dist_5M</i>     | mile            | 0     | 638     | 169.1653 | 134     | 109.4857 |
| <i>dist_1M</i>     | mile            | 0     | 552     | 138.2177 | 107     | 100.8309 |
| <i>dist_0.5M</i>   | mile            | 0     | 457     | 127.0444 | 103     | 88.27156 |
| <i>pop_2009</i>    | people          | 266   | 2257926 | 46088.65 | 5562.5  | 115728.6 |
| <i>pop_crt</i>     | percent         | -22.3 | 126.6   | 6.366524 | 3.7     | 12.52698 |
| <i>rt_pop_fm</i>   | percent         | 39.5  | 61.1    | 52.04556 | 52.2    | 2.155879 |
| <i>rt_pop_w</i>    | percent         | 3.9   | 97.5    | 56.97056 | 57.75   | 18.75092 |
| <i>rt_pop_h</i>    | percent         | 0.4   | 96.2    | 30.87218 | 24.85   | 19.29593 |
| <i>rt_pop_b</i>    | percent         | 0     | 51.9    | 9.549194 | 4.8     | 12.85585 |
| <i>den_pop</i>     | people/sq. mile | 49    | 6792    | 1301.484 | 1221    | 654.674  |
| <i>age</i>         | year            | 24.4  | 45.9    | 35.21895 | 34.95   | 3.388071 |
| <i>inc_2009</i>    | dollar          | 16161 | 139007  | 38825.43 | 37214.5 | 9526.69  |
| <i>inc_2000</i>    | dollar          | 15400 | 92778   | 31235.65 | 29493.5 | 7241.295 |
| <i>inc_pc</i>      | dollar          | 9225  | 64712   | 19219.22 | 18199.5 | 4059.255 |
| <i>rt_pov</i>      | percent         | 1.4   | 45      | 19.83105 | 19.45   | 8.739056 |
| <i>rt_inc_0_10</i> | percent         | 0     | 34.5    | 11.50806 | 10.9    | 6.134368 |
| <i>rt_inc_200_</i> | percent         | 0     | 38      | 1.59879  | 1.2     | 1.23711  |
| <i>ar_land</i>     | sq. mile        | 0.55  | 579.4   | 22.12012 | 4.925   | 48.94505 |
| <i>av_h_sz</i>     | people          | 2.2   | 3.7     | 2.613306 | 2.6     | 0.170542 |
| <i>lv_cst</i>      | —               | 76.1  | 103.4   | 82.38185 | 80.4    | 4.968363 |

st. dev. is the standard deviation.

Table 6. Summary of Simulated Parameters

Case 1: Taking  $\#$  as a quality, 248 observations.

| parameter                          | min        | max       | mean      | median    | st. dev.  |
|------------------------------------|------------|-----------|-----------|-----------|-----------|
| $m(\text{mean})$                   | -0.8720859 | 3.597811  | 1.745755  | 1.753276  | 0.6279741 |
| $\sigma(\text{st. dev.})$          | 0.0627664  | 10.11214  | 1.301668  | 1.202148  | 0.9449456 |
| $\bar{\theta}(\text{upper bound})$ | 0.1655705  | 5.285252  | 1.698061  | 1.222921  | 1.149143  |
| $M(\text{t. subs.})$               | 81.14688   | 1874962.5 | 50041.64  | 6841.152  | 199500.1  |
| $a$                                | 0.0004494  | 0.078143  | 0.0139462 | 0.0070602 | 0.01466   |

Case 2: Taking  $\sqrt{\#}$  as a quality, 209 meaningful observations.

| parameter                          | min      | max      | mean     | median   | st. dev. |
|------------------------------------|----------|----------|----------|----------|----------|
| $m(\text{mean})$                   | -197.845 | 47.50065 | -1.53716 | 1.162821 | 19.8733  |
| $\sigma(\text{st. dev.})$          | 0.633634 | 359.358  | 7.060615 | 1.498823 | 30.71782 |
| $\bar{\theta}(\text{upper bound})$ | 2.604025 | 11.80095 | 5.107607 | 4.401373 | 2.025908 |
| $M(\text{t. subs.})$               | 117.4732 | 2314081  | 63155.07 | 8919.308 | 254496.2 |
| $a$                                | 0.033127 | 0.884143 | 0.132846 | 0.074576 | 0.160553 |

 $\#$  is the number of channels.

st. dev. is the standard deviation.

t. subs. is the total number of potential subscribers.

Table 7. Welfare Loss in Notable Franchises Displaying Upward Quality Distortion

| franchise    | t. subs. | prof   | cs/m   | sw/m    | sw/sb   | wl/a    | wl/p |
|--------------|----------|--------|--------|---------|---------|---------|------|
| AMARILLO     | 234809   | 654817 | 404894 | 1059711 | 2108417 | 1048705 | 49.7 |
| SAN ANGELO   | 131319   | 296858 | 318643 | 615501  | 957492  | 341991  | 35.7 |
| CONROE       | 99208    | 263821 | 254159 | 517980  | 1098787 | 580806  | 52.9 |
| GEORGETOWN   | 78448    | 55490  | 489743 | 545233  | 574605  | 29372   | 5.1  |
| GREENVILLE   | 62250    | 103804 | 38784  | 142587  | 760347  | 617760  | 81.2 |
| HUNTSVILLE   | 49082    | 318740 | 355404 | 674144  | 911207  | 237063  | 26.0 |
| MINEOLA      | 35112    | 25060  | 6049   | 31109   | 322481  | 291372  | 90.4 |
| MT. PLEASANT | 24302    | 55920  | 21620  | 77541   | 234336  | 156795  | 66.9 |
| ANDREWS      | 21422    | 47475  | 68720  | 116194  | 212156  | 95962   | 45.2 |
| CRYSTAL CITY | 16840    | 157922 | 102828 | 260750  | 261992  | 1242    | 0.5  |
| PLEASANTON   | 13665    | 7851   | 19893  | 27745   | 121274  | 93529   | 77.1 |
| NAVASOTA     | 11695    | 5271   | 760    | 6031    | 84596   | 78565   | 92.9 |
| DALHART      | 10916    | 40915  | 45793  | 86708   | 113367  | 26659   | 23.5 |
| COLUMBUS     | 6540     | 3271   | 3439   | 6709    | 87386   | 80677   | 92.3 |
| GILMER       | 6355     | 6376   | 1130   | 7506    | 39836   | 32330   | 81.2 |
| WHITESBORO   | 5733     | 12060  | 15195  | 27256   | 33492   | 6236    | 18.6 |
| MT. VERNON   | 4393     | 10163  | 3940   | 14103   | 42173   | 28071   | 66.6 |
| SABINAL      | 3443     | 4316   | 3425   | 7741    | 28946   | 21205   | 73.3 |
| GANADO       | 2109     | 13786  | 6853   | 20639   | 44839   | 24200   | 54.0 |
| TUSCOLA      | 1199     | 1412   | 513    | 1926    | 6820    | 4895    | 71.8 |
| ROSCOE       | 1008     | 1801   | 528    | 2328    | 8726    | 6398    | 73.3 |
| TEXHOMA      | 902      | 4409   | 4228   | 8637    | 10640   | 2003    | 18.8 |
| COOPER       | 516      | 923    | 1021   | 1945    | 5157    | 3212    | 62.3 |

t. subs. is the total number of potential subscribers. (unit: people)

prof is the profit of the monopoly. (unit: dollar)

cs/m is the consumer surplus in the monopoly. (unit: dollar)

sw/m is the social welfare in the monopoly. (unit: dollar)

sw/sb is the social welfare at the social best. (unit: dollar)

wl/a is the amount of welfare loss induced by the upward distortion. (unit: dollar)

wl/p is the percent of welfare loss. (unit: percent)

Table 8. Estimated Coefficients for Consumer Type Distribution (Parameters)

|                      | mean                         | st. dev.                   | upper bound                 | t. subs.                    |
|----------------------|------------------------------|----------------------------|-----------------------------|-----------------------------|
| <i>ar_land</i>       | -0.061428<br>(0.0691736)     | 0.089675<br>(0.10513)      | -0.0177109**<br>(0.0074001) | 814.2932***<br>(263.8655)   |
| <i>dist_5m</i>       | 0.0401105*<br>(0.0240105)    | -0.0595175*<br>(0.0365456) | -0.0024297<br>(0.0026045)   | 194.9267**<br>(91.72563)    |
| <i>dist_05m</i>      | -0.0623385***<br>(0.0261046) | 0.0760575*<br>(0.0396737)  | 0.0046534*<br>(0.0028139)   | -250.7468**<br>(99.57697)   |
| <i>pop_2009</i>      | 0.000022<br>(0.0000211)      | -0.0000299<br>(0.0000321)  | 4.96e-06**<br>(2.26e-06)    | 0.9809839***<br>(0.0805236) |
| <i>den_pop</i>       | 0.0000791<br>(0.0024471)     | 0.0011224<br>(0.0037192)   | 0.000222<br>(0.0002652)     | 11.3361<br>(9.334743)       |
| <i>rt_pop_fm</i>     | -0.6564762<br>(0.6335078)    | 1.079083<br>(0.9628052)    | -0.0537857<br>(0.0686615)   | 1619.52<br>(2416.542)       |
| <i>rt_pop_h</i>      | -0.1955067*<br>(0.1175894)   | 0.3341066*<br>(0.1787123)  | -0.0019423<br>(0.0127642)   | 217.6518<br>(448.5496)      |
| <i>av_h_sz</i>       | 9.674475<br>(8.902405)       | -12.78075<br>(13.52988)    | -0.5232359<br>(0.9656676)   | -52702.32<br>(33958.6)      |
| <i>lv_cst</i>        | -0.1345113<br>(0.3791828)    | -0.0667913<br>(0.5762821)  | -0.0337358<br>(0.0410905)   | 688.1137<br>(1446.409)      |
| <i>ln_inc</i>        | -6.348325<br>(10.72374)      | 6.33373<br>(16.29795)      | -1.477376<br>(1.15925)      | -7837.052<br>(40906.18)     |
| <i>rt_inc_0_10</i>   | -0.5023324<br>(0.3612038)    | 0.8816102<br>(0.5489576)   | -0.0697276*<br>(0.038888)   | -16.12913<br>(1377.827)     |
| <i>rt_inc_75_100</i> | -0.0152204<br>(0.4610764)    | -0.3410856<br>(0.700744)   | 0.1023775**<br>(0.0495067)  | 1729.897<br>(1758.796)      |
| <i>rt_inc_100_</i>   | -1.220839***<br>(0.4690323)  | 2.285005***<br>(0.7128354) | 0.0002127<br>(0.0509159)    | -144.6192<br>(1789.144)     |
| <i>rt_inc_150_</i>   | 2.2565**<br>(1.050953)       | -3.287437**<br>(1.597238)  | 0.0041301<br>(0.1140859)    | -2050.069<br>(4008.903)     |
| <i>rt_inc_200_</i>   | -0.933054<br>(1.168443)      | 0.704138<br>(1.775799)     | -0.0106244<br>(0.1268381)   | 980.4974<br>(4457.074)      |
| $\bar{\theta}$       | -3.921158***<br>(0.6630877)  | 6.680463***<br>(1.007761)  | N/A                         | -1562.127<br>(2529.376)     |
| cons                 | 125.2684<br>(122.0835)       | -143.8579<br>(185.5426)    | 27.19189**<br>(13.10746)    | 58741.39<br>(465692.8)      |
| # of obs             | 209                          | 209                        | 209                         | 209                         |
| $R^2$                | 0.2220                       | 0.2479                     | 0.1132                      | 0.9310                      |

st. dev. is the standard deviation.

t. subs. is the total number of potential subscribers.

The numbers in the parentheses are the standard deviations.

\* means the estimate is significant at 90% level.

\*\* means the estimate is significant at 95% level.

\*\*\* means the estimate is significant at 99% level.

Table 9. Estimated Coefficients for Consumer Type Distribution (Cutoff Levels)

|                      | cutoff(40)                 | cutoff(50)                 | cutoff(75)                 | cutoff(90)                 | cutoff(95)                  | cutoff(98)                  |
|----------------------|----------------------------|----------------------------|----------------------------|----------------------------|-----------------------------|-----------------------------|
| <i>ar_land</i>       | -0.0062051*<br>(0.0036684) | -0.0070875*<br>(0.0041229) | -0.0096826*<br>(0.0054662) | -0.0119493*<br>(0.0062733) | -0.0132506**<br>(0.0065043) | -0.0147127**<br>(0.0066629) |
| <i>dist_5m</i>       | -0.0010207<br>(0.0012911)  | -0.0013201<br>(0.001451)   | -0.0020148<br>(0.0019238)  | -0.0023315<br>(0.0022079)  | -0.0023849<br>(0.0022892)   | -0.0023871<br>(0.002345)    |
| <i>dist_05m</i>      | 0.0006033<br>(0.0013949)   | 0.0010711<br>(0.0015677)   | 0.0023193<br>(0.0020785)   | 0.0031888<br>(0.0023854)   | 0.0035612<br>(0.0024733)    | 0.0038959<br>(0.0025336)    |
| <i>pop_2009</i>      | 2.47e-06**<br>(1.12e-06)   | 2.66e-06**<br>(1.26e-06)   | 3.19e-06*<br>(1.67e-06)    | 3.65e-06*<br>(1.92e-06)    | 3.94e-06**<br>(1.99e-06)    | 4.29e-06**<br>(2.04e-06)    |
| <i>den_pop</i>       | 0.0001201<br>(0.0001314)   | 0.000146<br>(0.0001477)    | 0.0001981<br>(0.0001959)   | 0.0002118<br>(0.0002248)   | 0.0002067<br>(0.0002331)    | 0.0001976<br>(0.0002388)    |
| <i>rt_pop_fm</i>     | -0.02832<br>(0.0340368)    | -0.0349469<br>(0.0382537)  | -0.0511756<br>(0.0507175)  | -0.0584545<br>(0.0582063)  | -0.058431<br>(0.0603494)    | -0.0563347<br>(0.0618211)   |
| <i>rt_pop_h</i>      | 0.0036145<br>(0.0063274)   | 0.0038458<br>(0.0071114)   | 0.0038877<br>(0.0094284)   | 0.0029601<br>(0.0108205)   | 0.0020158<br>(0.011219)     | 0.0007267<br>(0.0114925)    |
| <i>av_h_sz</i>       | -0.0444236<br>(0.4786997)  | -0.1617713<br>(0.5380075)  | -0.4589145<br>(0.7133003)  | -0.6224744<br>(0.8186243)  | -0.6542898<br>(0.8487659)   | -0.6364215<br>(0.8694631)   |
| <i>lw_cst</i>        | -0.017315<br>(0.0203694)   | -0.0190863<br>(0.022893)   | -0.0236407<br>(0.030352)   | -0.0259327<br>(0.0348336)  | -0.0265761<br>(0.0361162)   | -0.0275667<br>(0.0369969)   |
| <i>ln_inc</i>        | -0.664002<br>(0.5746622)   | -0.7723282<br>(0.6458591)  | -1.098729<br>(0.856292)    | -1.345654<br>(0.9827298)   | -1.438825<br>(1.018914)     | -1.486918<br>(1.04376)      |
| <i>rt_inc_0_10</i>   | -0.0131248<br>(0.0192775)  | -0.0163315<br>(0.0216659)  | -0.0265535<br>(0.028725)   | -0.0366553<br>(0.0329665)  | -0.0428154<br>(0.0341803)   | -0.0495845<br>(0.0350138)   |
| <i>rt_inc_75_100</i> | -0.0018222<br>(0.0245414)  | 0.0092358<br>(0.027582)    | 0.0430825<br>(0.0365687)   | 0.0703774*<br>(0.0419683)  | 0.0826093*<br>(0.0435136)   | 0.0923878**<br>(0.0445746)  |
| <i>rt_inc_100_</i>   | 0.0100366<br>(0.02524)     | 0.009513<br>(0.028367)     | 0.0070302<br>(0.0376095)   | 0.0038016<br>(0.0431628)   | 0.0017688<br>(0.0447521)    | -0.0002248<br>(0.0458434)   |
| <i>rt_inc_150_</i>   | 0.0546479<br>(0.0565545)   | 0.0570987<br>(0.0635613)   | 0.0619516<br>(0.0842707)   | 0.0598377<br>(0.0967139)   | 0.0541828<br>(0.1002749)    | 0.0436754<br>(0.1027201)    |
| <i>rt_inc_200_</i>   | -0.073196<br>(0.0628761)   | -0.0761099<br>(0.070666)   | -0.0778222<br>(0.0936903)  | -0.0683849<br>(0.1075244)  | -0.0580565<br>(0.1114834)   | -0.0440597<br>(0.1142019)   |
| cons                 | 11.47385*<br>(6.497615)    | 13.65469*<br>(7.302627)    | 19.79582**<br>(9.681959)   | 24.0788**<br>(11.11157)    | 25.6045**<br>(11.5207)      | 26.458**<br>(11.80163)      |
| # of obs             | 209                        | 209                        | 209                        | 209                        | 209                         | 209                         |
| R <sup>2</sup>       | 0.0816                     | 0.0723                     | 0.0666                     | 0.0763                     | 0.0855                      | 0.0968                      |

The cutoff( $p$ ) is the taste level such that  $F(\text{cutoff}(p)) = n \times 0.01$ .

The numbers in the parentheses are the standard deviations.

\* means the estimate is significant at 90% level.

\*\* means the estimate is significant at 95% level.

\*\* means the estimate is significant at 99% level.

Table 10. Estimated Coefficients for Consumer Type Distribution (Probability)

|                      | prob(0.5)                 | prob(1)                   | prob(2)                   | prob(3)                    | prob(4)                   | prob(5)                   |
|----------------------|---------------------------|---------------------------|---------------------------|----------------------------|---------------------------|---------------------------|
| <i>ar_land</i>       | 0.0001296<br>(0.000165)   | 0.0004229<br>(0.0003441)  | 0.0012358*<br>(0.0006663) | 0.0019291**<br>(0.0008031) | 0.0030163*<br>(0.001765)  | 0.0039632<br>(0.0047912)  |
| <i>dist_5m</i>       | 0.0000357<br>(0.0000581)  | 0.0000638<br>(0.0001211)  | 0.0001027<br>(0.0002345)  | 0.0001794<br>(0.0002941)   | 0.0002558<br>(0.0004484)  | 0.0008166<br>(0.0008168)  |
| <i>dist_05m</i>      | -9.87e-06<br>(0.0000627)  | -0.0000134<br>(0.0001308) | -0.0000354<br>(0.0002534) | -0.0000428<br>(0.0003128)  | -0.0001742<br>(0.0004526) | -0.0007946<br>(0.0007836) |
| <i>pop_2009</i>      | -5.18e-08<br>(5.05e-08)   | -1.55e-07<br>(1.05e-07)   | -4.39e-07**<br>(2.04e-07) | -6.81e-07***<br>(2.45e-07) | -9.09e-07*<br>(4.81e-07)  | -1.03e-06<br>(1.20e-06)   |
| <i>den_pop</i>       | -8.27e-06<br>(5.91e-06)   | -0.0000162<br>(0.0000123) | -0.0000264<br>(0.0000239) | -0.0000145<br>(0.0000289)  | -0.0000422<br>(0.0000455) | -0.0000504<br>(0.000071)  |
| <i>rt_pop_fm</i>     | 0.002221<br>(0.0015311)   | 0.004291<br>(0.0031928)   | 0.0069087<br>(0.0061821)  | 0.0076436<br>(0.0074656)   | 0.010589<br>(0.0104373)   | -0.0051362<br>(0.0168942) |
| <i>rt_pop_h</i>      | -0.0001312<br>(0.0002846) | -0.0001455<br>(0.0005935) | 0.0000574<br>(0.0011493)  | -0.000934<br>(0.0014336)   | -0.0022753<br>(0.0018436) | -0.0020757<br>(0.0031476) |
| <i>av_h_sz</i>       | -0.0066205<br>(0.0215333) | -0.0177804<br>(0.044904)  | -0.0406902<br>(0.0869462) | -0.006503<br>(0.1069106)   | 0.0513013<br>(0.1512391)  | -0.1105867<br>(0.2550867) |
| <i>lv_cst</i>        | 0.0001801<br>(0.0009163)  | 0.001041<br>(0.0019107)   | 0.0035614<br>(0.0036997)  | 0.0056418<br>(0.0044767)   | -0.0010323<br>(0.006531)  | -0.000459<br>(0.0122953)  |
| <i>ln_inc</i>        | -0.0022486<br>(0.0258499) | 0.0103596<br>(0.0539056)  | 0.0647001<br>(0.1043758)  | 0.1060982<br>(0.1263626)   | 0.31667**<br>(0.1573806)  | 0.3195182<br>(0.2754897)  |
| <i>rt_inc_0_10</i>   | 0.0001542<br>(0.0008672)  | 0.0008093<br>(0.0018083)  | 0.0026155<br>(0.0035014)  | 0.0035704<br>(0.0042565)   | 0.0027503<br>(0.0053529)  | -0.0077056<br>(0.0104966) |
| <i>rt_inc_75_100</i> | 0.0018734*<br>(0.0011039) | 0.0027312<br>(0.0023021)  | 0.0017632<br>(0.0044575)  | -0.0003045<br>(0.0053798)  | -0.0051271<br>(0.0067029) | 0.0045993<br>(0.0111709)  |
| <i>rt_inc_100_</i>   | 0.0000865<br>(0.0011354)  | 5.61e-07<br>(0.0023676)   | -0.0007157<br>(0.0045843) | -0.0012292<br>(0.0055383)  | -0.0062162<br>(0.0079187) | -0.0050359<br>(0.0125896) |
| <i>rt_inc_150_</i>   | -0.0023329<br>(0.002544)  | -0.0047189<br>(0.005305)  | -0.0091287<br>(0.010272)  | -0.0164212<br>(0.0125309)  | -0.010637<br>(0.0162417)  | -0.0206197<br>(0.0239152) |
| <i>rt_inc_200_</i>   | 0.0038712<br>(0.0028283)  | 0.0077388<br>(0.005898)   | 0.0144221<br>(0.0114202)  | 0.0215024<br>(0.013911)    | 0.0093025<br>(0.0251788)  | -0.0513939<br>(0.0537044) |
| cons                 | 0.0321673<br>(0.2922809)  | -0.1105275<br>(0.6095027) | -0.6714479<br>(1.180161)  | -1.16472<br>(1.42774)      | -2.903509<br>(1.81518)    | -1.682281<br>(3.200077)   |
| # of obs             | 209                       | 209                       | 209                       | 204                        | 129                       | 64                        |
| $R^2$                | 0.0756                    | 0.0682                    | 0.0693                    | 0.0931                     | 0.1522                    | 0.2359                    |

The prob( $\theta$ ) is the probability,  $F(\theta)$ .

The numbers in the parentheses are the standard deviations.

\* means the estimate is significant at 90% level.

\*\* means the estimate is significant at 95% level.

\*\*\* means the estimate is significant at 99% level.

Table 11. Number of Subscribers of Top 10 MSOs in 2008

| Rank | MSO                                  | Subscribers |
|------|--------------------------------------|-------------|
| 1    | Comcast Corporation                  | 24,182,000  |
| 2    | Time Warner Cable, Inc.              | 13,069,000  |
| 3    | Cox Communications, Inc.             | 5,328,304   |
| 4    | Charter Communications, Inc.         | 5,045,700   |
| 5    | Cablevision Systems Corporation      | 3,108,000   |
| 6    | Bright House Networks LLC            | 2,307,778   |
| 7    | Mediacom Communications Corporation  | 1,318,000   |
| 8    | Suddenlink Communications            | 1,268,674   |
| 9    | Insight Communications Company, Inc. | 707,600     |
| 10   | CableOne, Inc.                       | 669,469     |

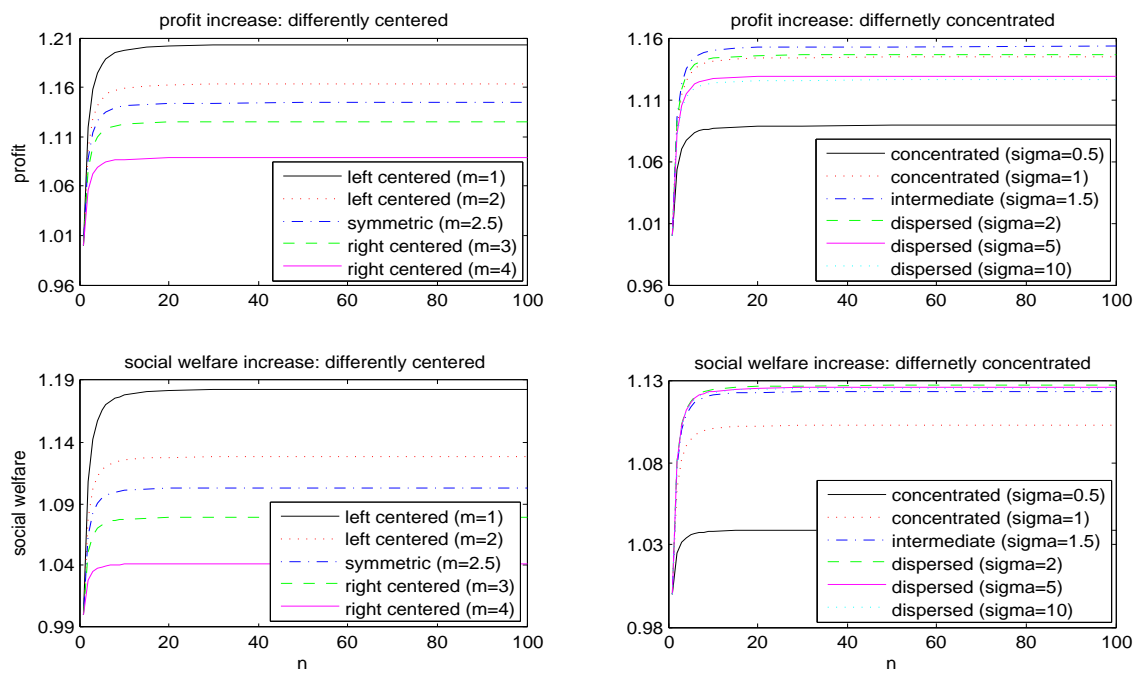
Fig. 2. Diminishing Increment of Profit or Social Welfare in  $n$ 



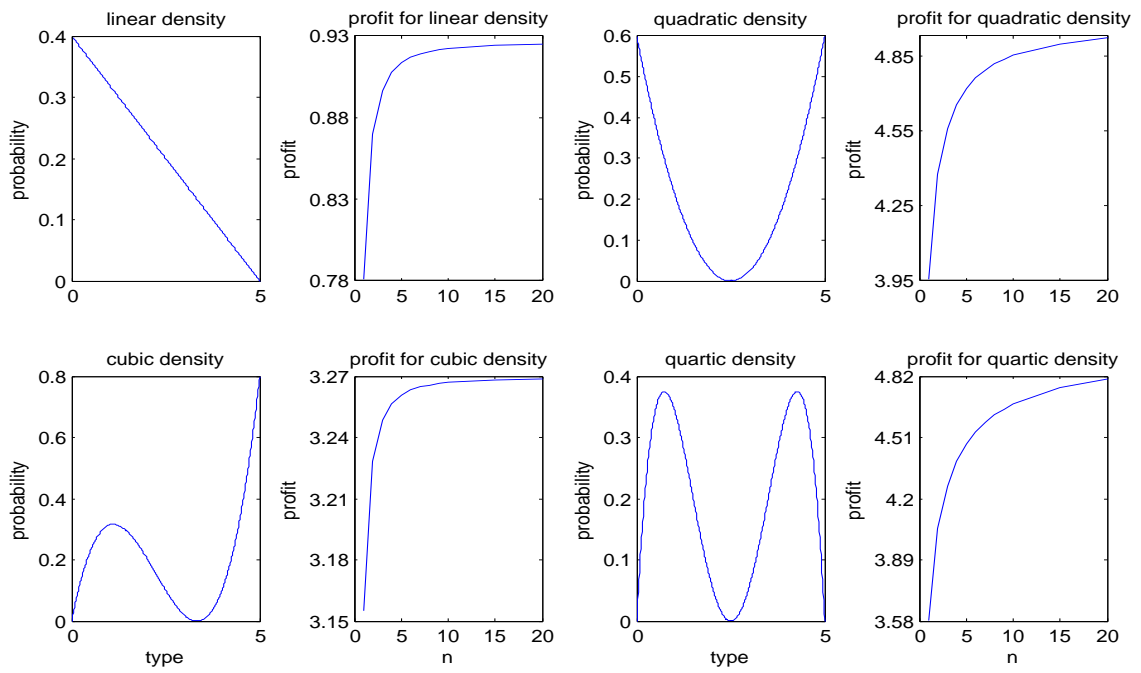
Fig. 3. Diminishing Increment of Profit in  $n$ 

Fig. 4. Classification of Consumer Distributions

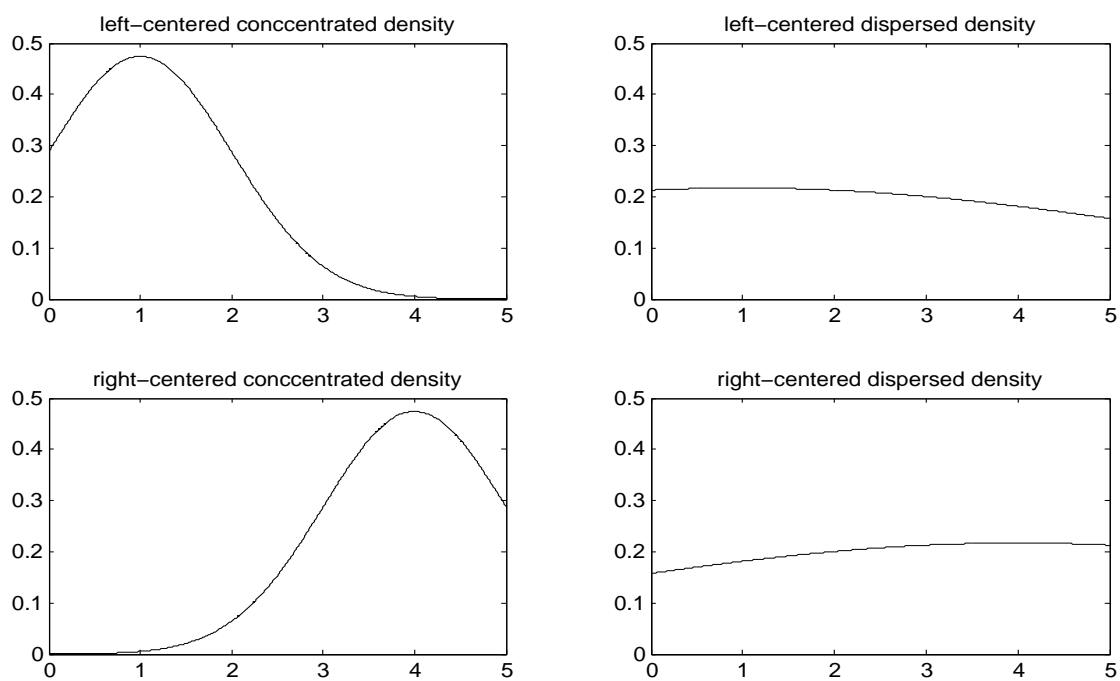


Fig. 5. Degree of Quality Distortion on Top and Center of Distribution

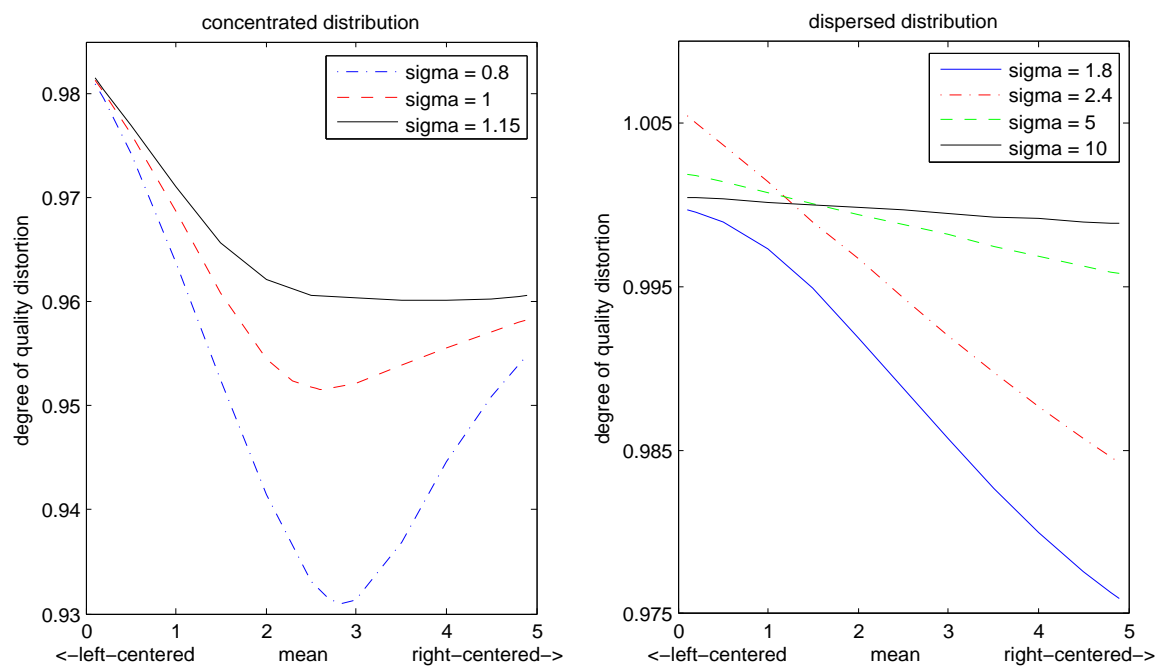


Fig. 6. Degree of Quality Distortion on Top and Degree of Concentration

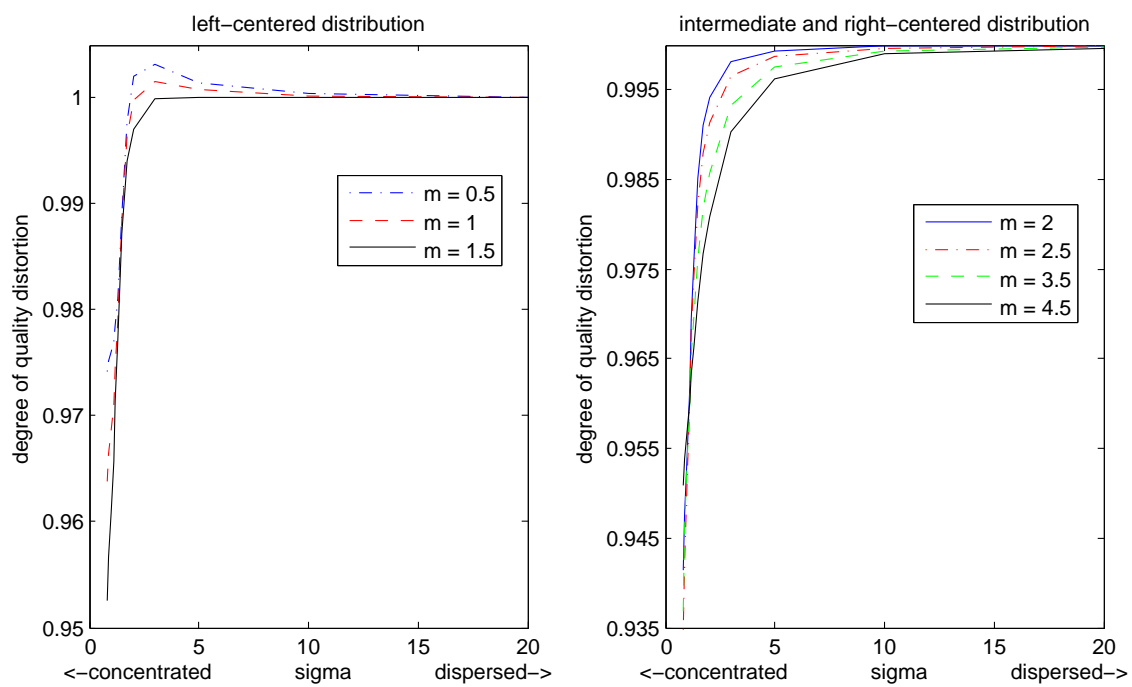


Fig. 7. Density of Types and Standard Deviation

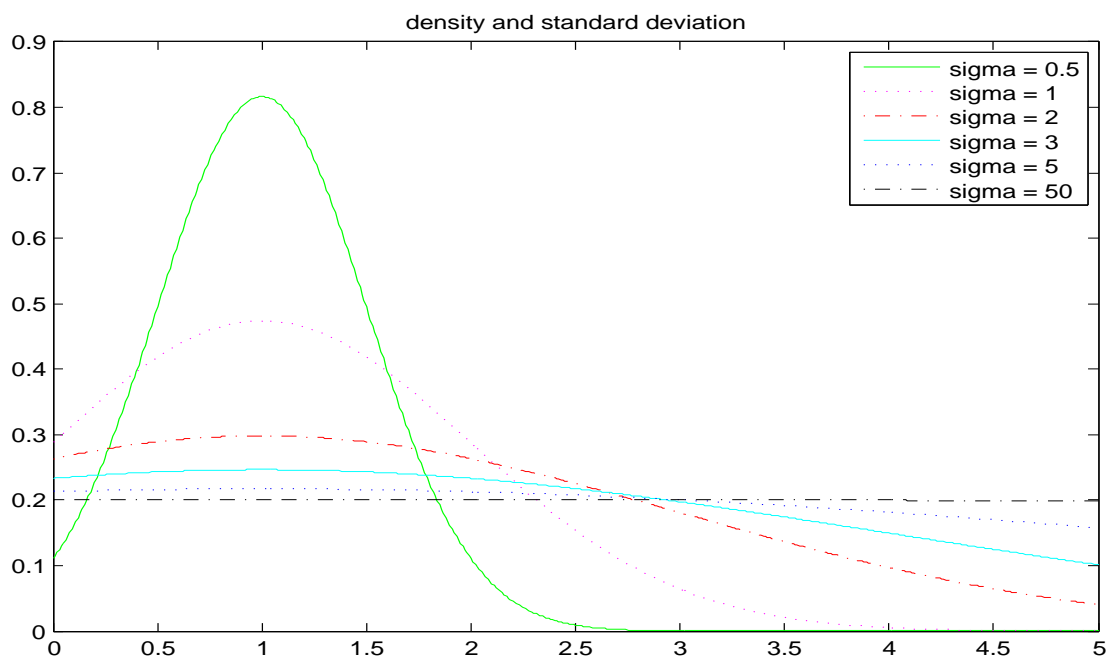


Fig. 8. Offered Quality and Thickness of Right Side Tail

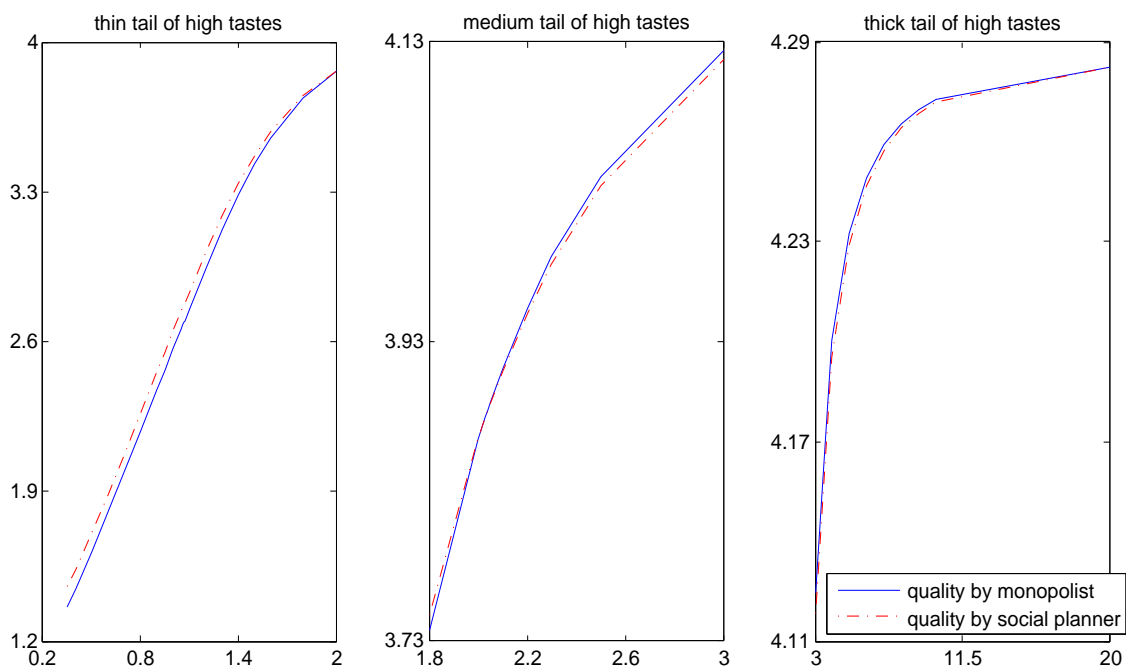


Fig. 9. Excluded Consumers: Differently Centered

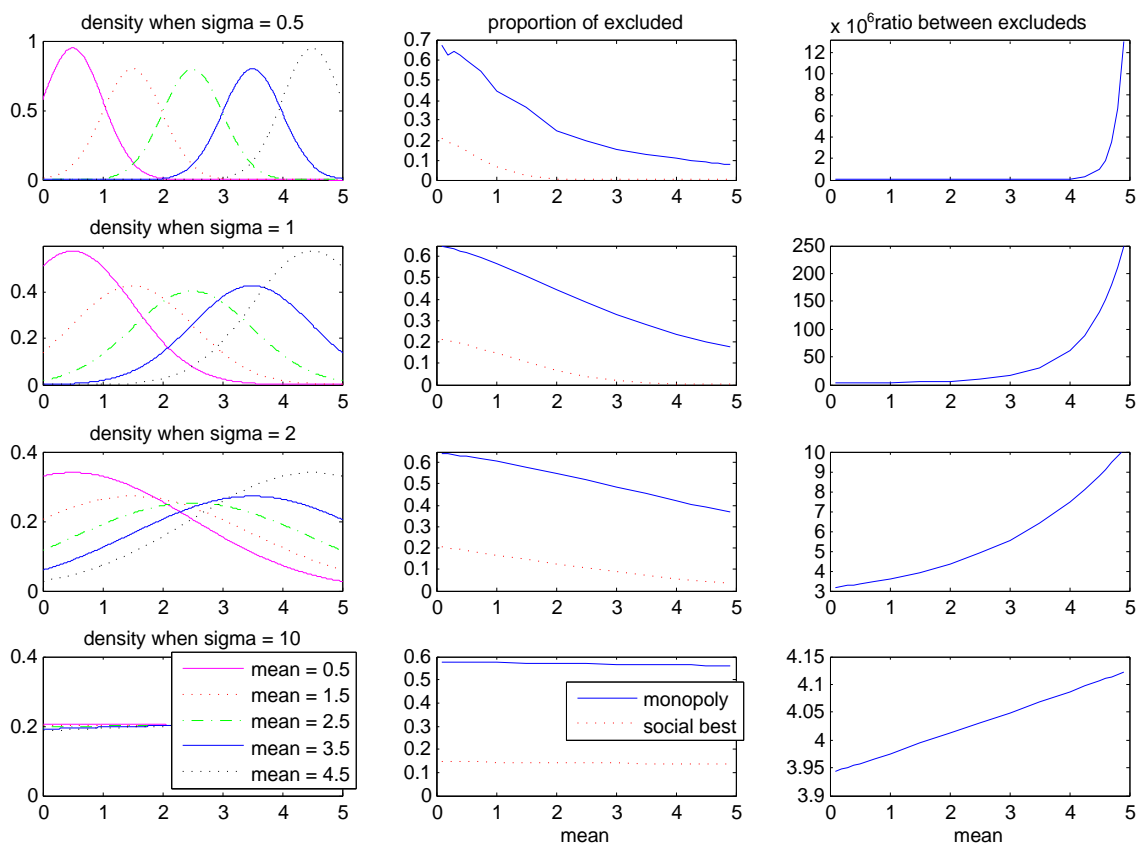


Fig. 10. Excluded Consumers: Differently Concentrated

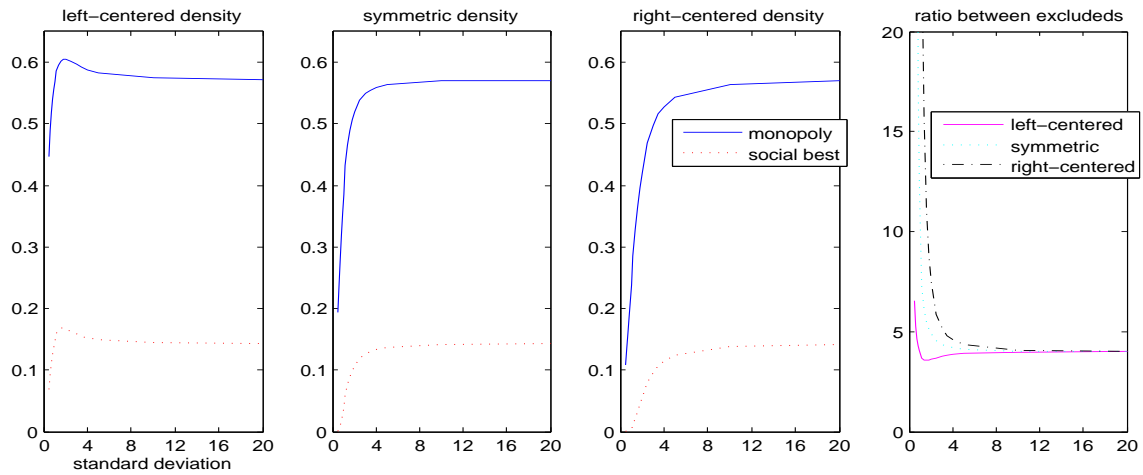


Fig. 11. Information Rents: Differently Centered

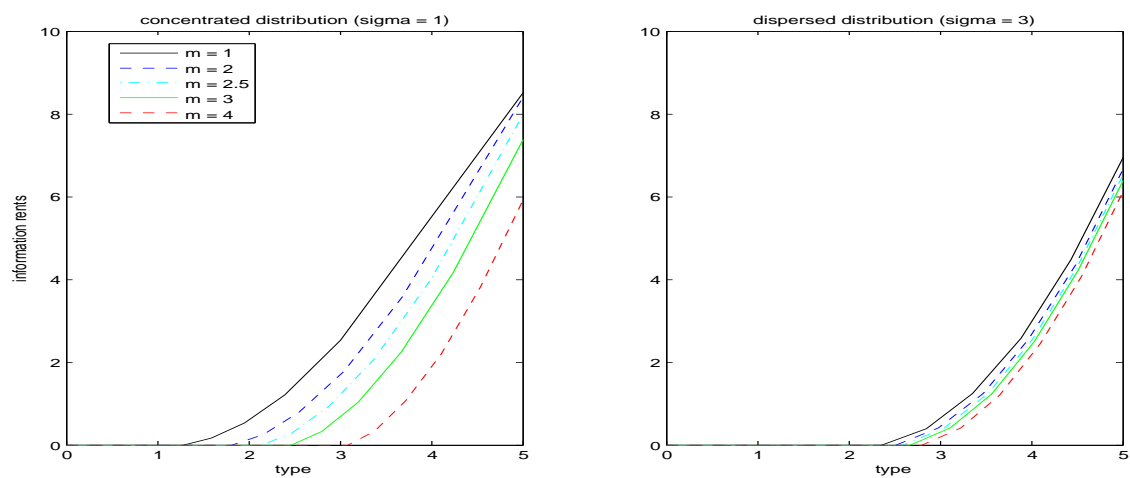




Fig. 12. Information Rents: Differently Concentrated

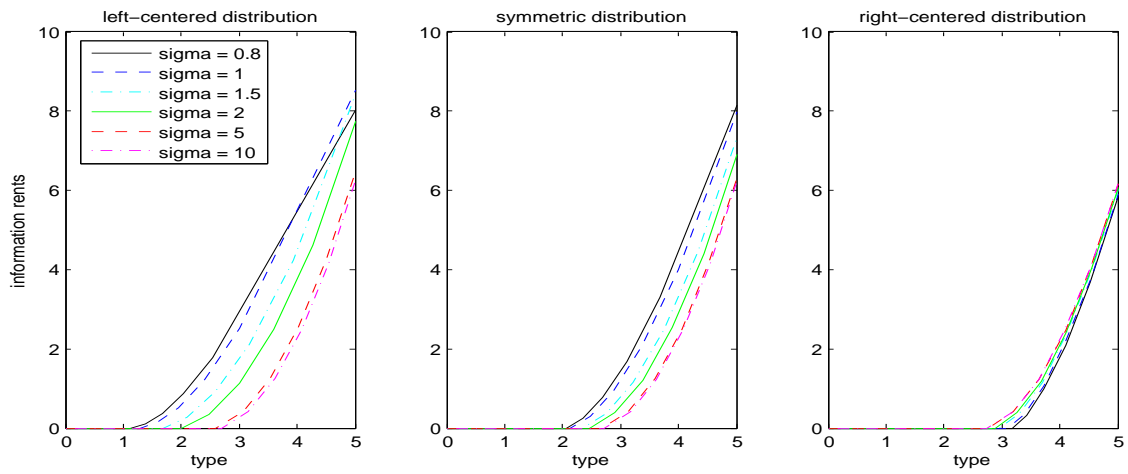


Fig. 13. Profit and Consumer Surplus in the Monopoly

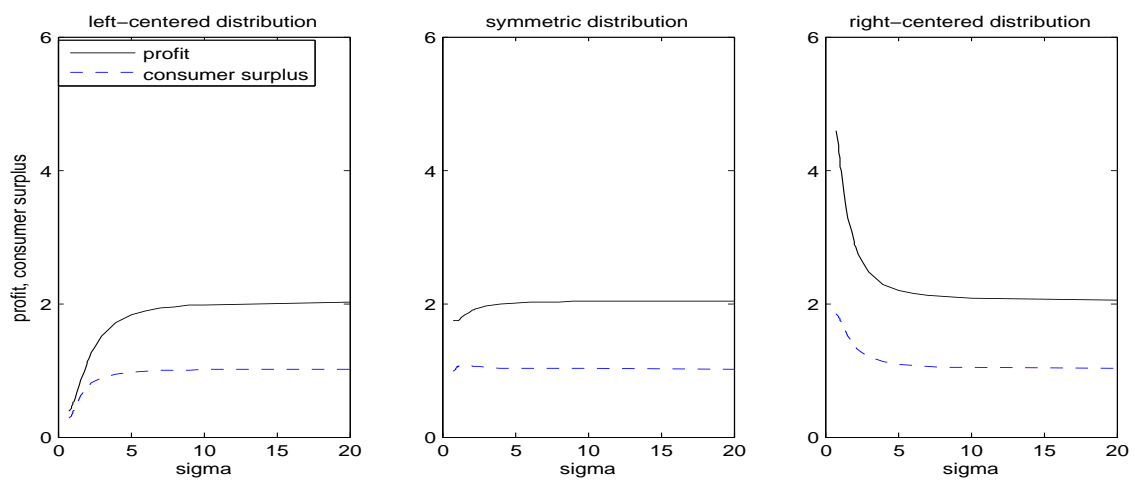


Fig. 14. Social Welfare in the Monopoly and Social Welfare at the Social Best

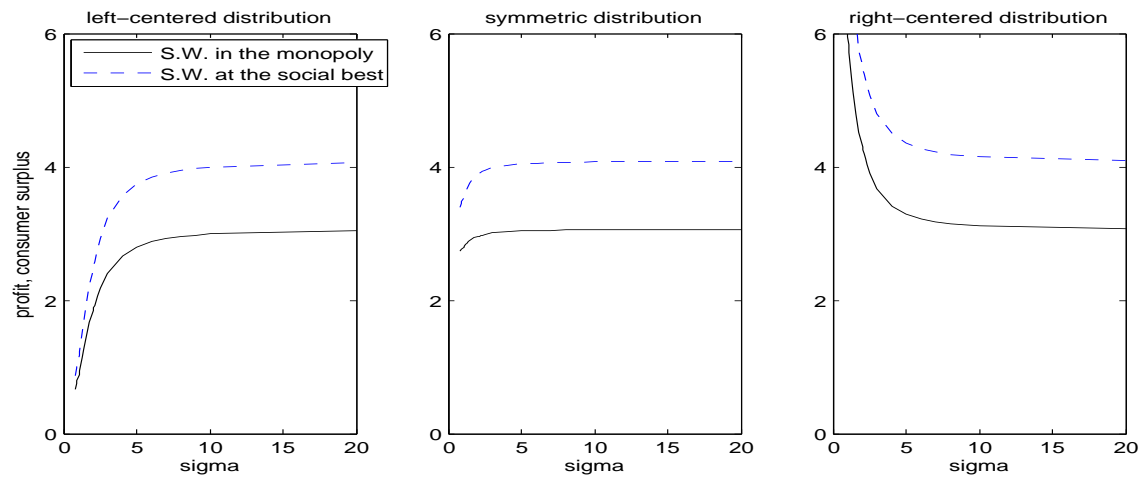


Fig. 15. The Efficiency Score Achieved by the Monopolist

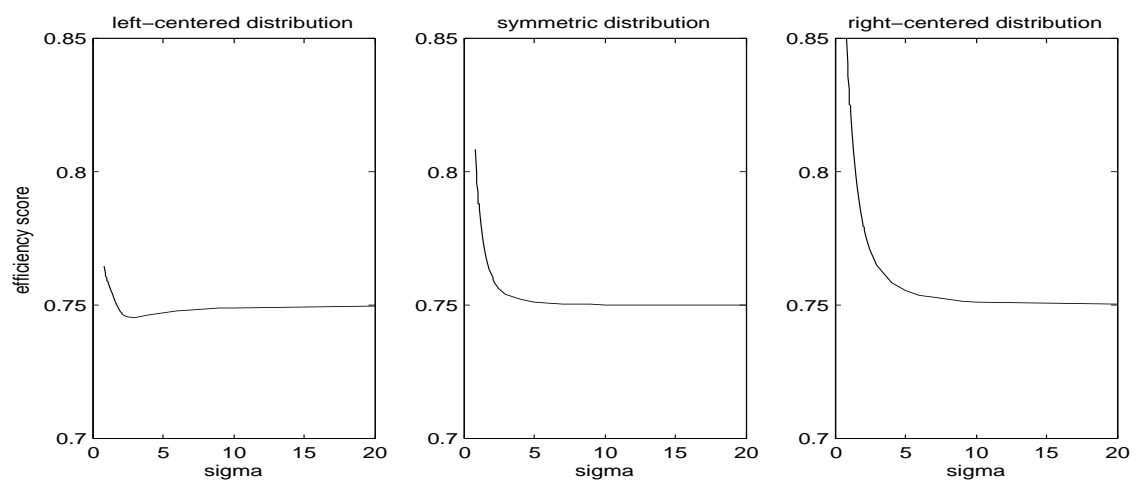


Fig. 16. The Shift or Rotation of Demand Curves in Dallas

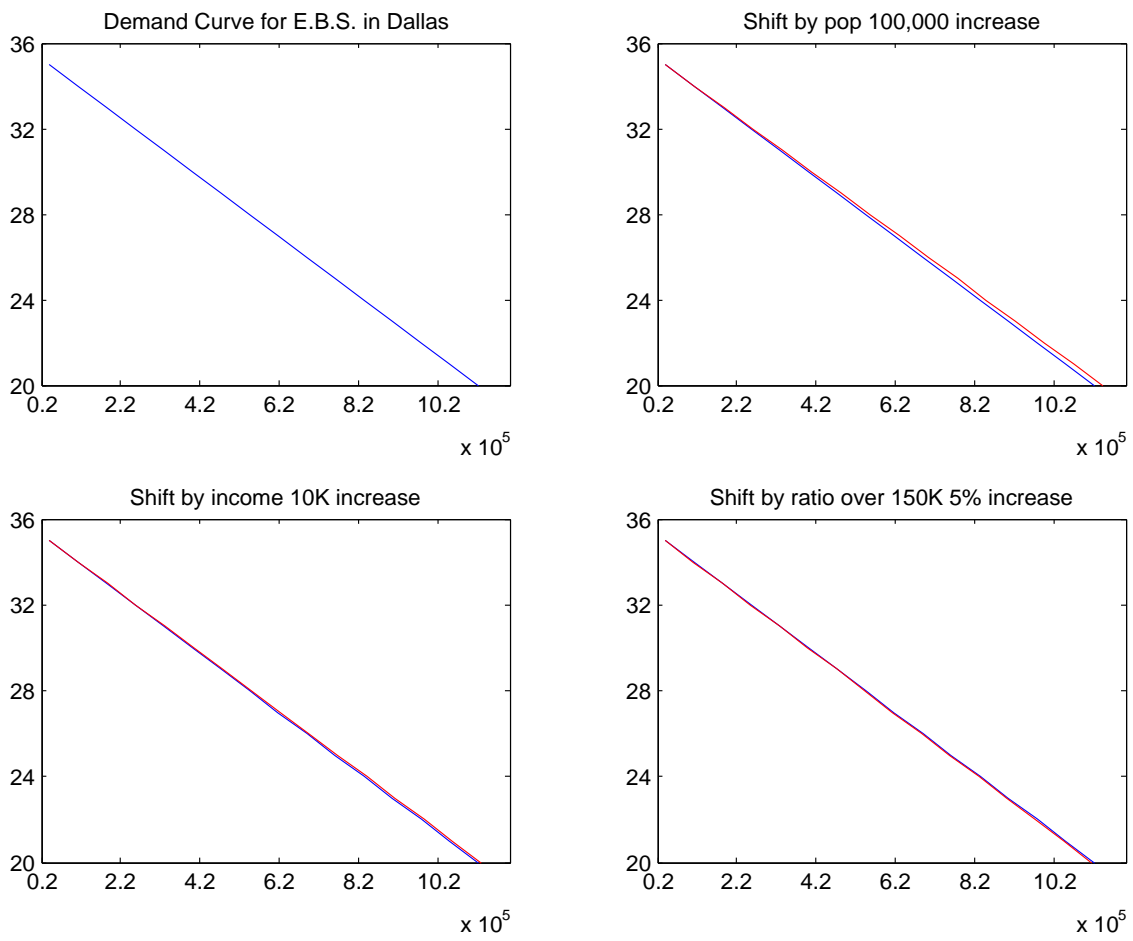


Fig. 17. The Shift or Rotation of Demand Curves in Georgetown

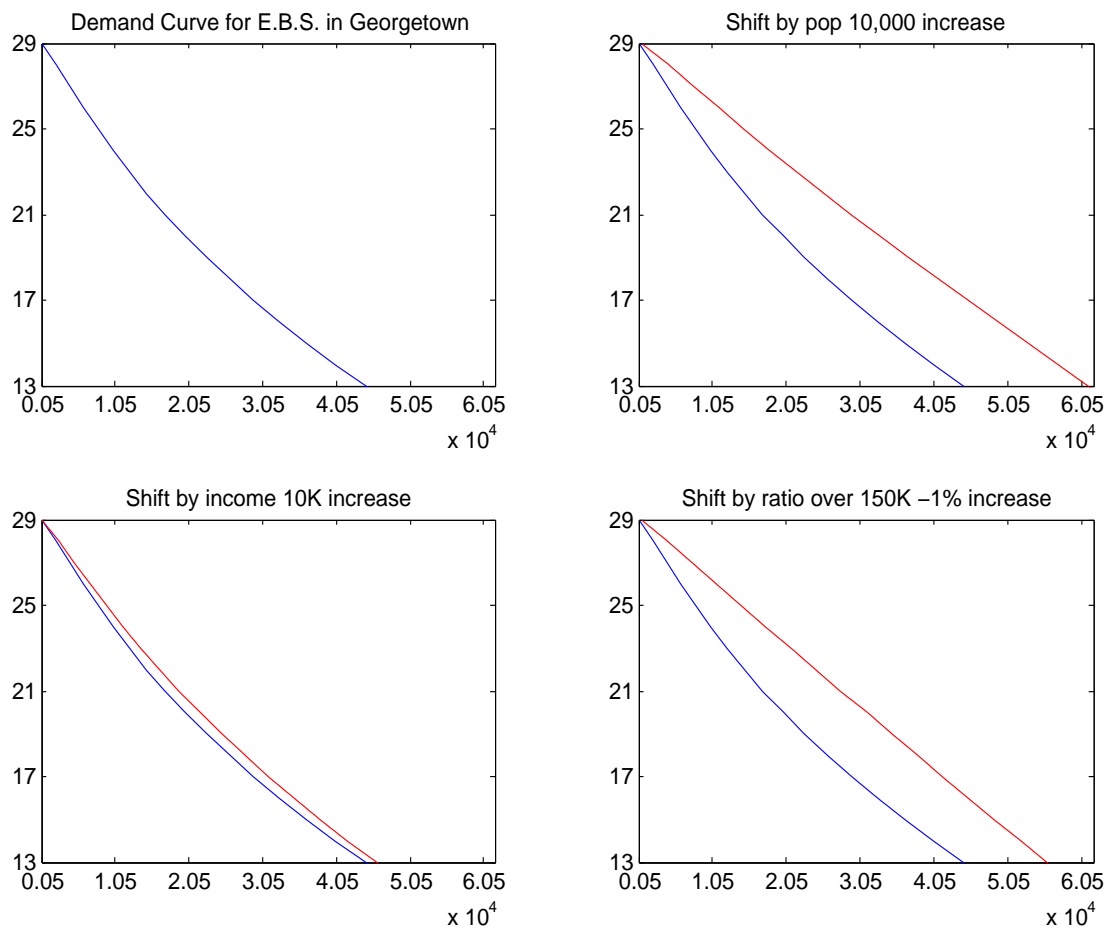


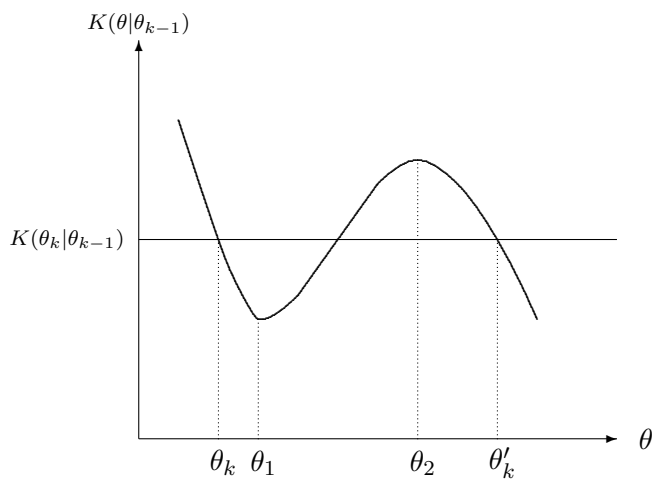
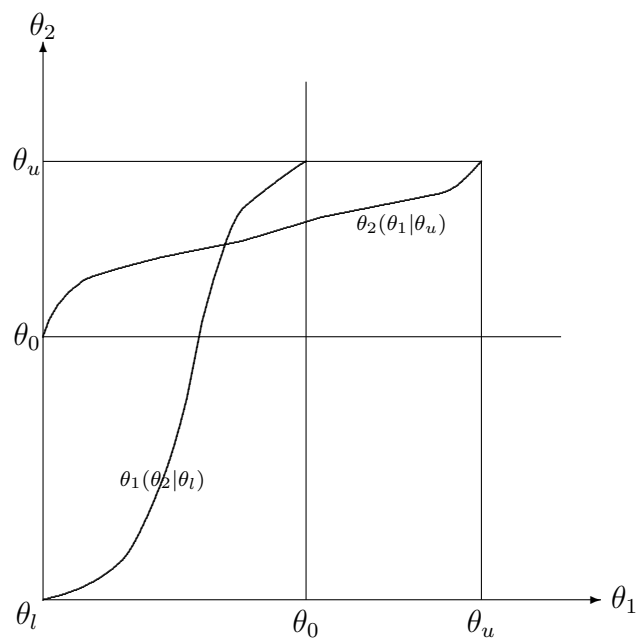
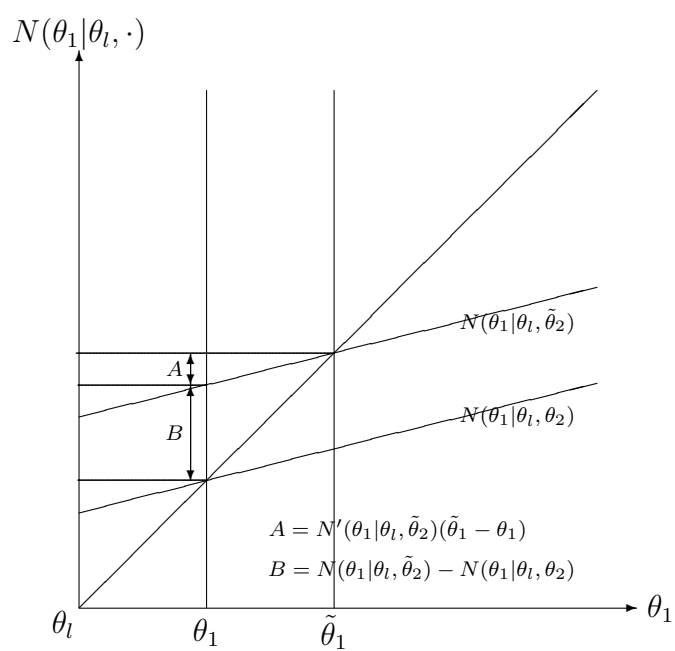
Fig. 18.  $\theta$ s Satisfying  $\partial K(\theta|\theta_{k-1})/\partial\theta = 0$ Fig. 19.  $\theta_1(\theta_2|\theta_l)$  and  $\theta_2(\theta_1|\theta_u)$ 

Fig. 20. Newly Determined  $\theta_1$  as  $\theta_2$  Increases

## VITA

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