DISTURBANCE MODEL IDENTIFICATION AND MODEL FREE SYNTHESIS OF CONTROLLERS FOR MULTIVARIABLE SYSTEMS

A Thesis

by

KIRAN SOMASHEKAR SAJJANSHETTY

Submitted to the Office of Graduate Studies of
Texas A&M University
in partial fulfillment of the requirements for the degree of

MASTER OF SCIENCE

August 2012

Major Subject: Electrical Engineering

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ABSTRACT

Disturbance Model Identification and Model Free Synthesis of Controllers for Multivariable Systems. (August 2012)

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In this work, two different problems are addressed. In the first part, the problem of synthesizing a set of stabilizing controllers for unknown multivariable systems using direct data is analyzed. This is a model free approach to control design and uses only the frequency domain data of the system. It is a perfect complement to modern and post modern methods that begin the control design with a system model. A three step method, involving sequential design, search for stability boundaries and stability check is proposed. It is shown through examples that a complete set of stabilizing controllers of the chosen form can be obtained for the class of linear stable multivariable systems. The complexity of the proposed method is invariant with respect to the order of the system and increases with the increase in the number of input channels of the given multivariable system. The second part of the work deals with the problem of identification of model uncertainties and the effect of unwanted exogenous inputs acting on a discrete time multivariable system using its output information. A disturbance model is introduced which accounts for the system model uncertainties and the effect of unwanted exogenous inputs acting on the system. The frequency content of the exogenous signals is assumed to be known. A linear dynamical model of the disturbance is assumed with an input that has the same frequency content as that of the exogenous input signal. The extended model of the system is then subjected to Kalman filtering and the disturbance states estimates are used to obtain a least squares estimate of the disturbance model parameters. The proposed approach is applied to a linear multivariable system perturbed by an exogenous signal of known frequency content and the results obtained depict the efficacy of the proposed approach.

DEDICATION

To my mother

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CHAPTER I

INTRODUCTION AND OVERVIEW

1.1 Introduction

The recent advent in the direct use of experimental data for the synthesis of system controllers has initiated a novel paradigm of control design. This line of research puts aside the dogma that analysis of control must start with a model. Stability, robustness and optimality for a system can be proved contingent upon the assumption that the model of the system is within the conjectured tolerances. Ergo, theories which use mathematical models to build control laws give insufficient attention to the implications of possible future observations, which may be at odds with assumptions with which the model of the system is built. The Achilles heel of modern control theory has been this habit of "proof by assumption". Models may be useful in control design since they help formulate theories about suitable controller structures by extracting simple patterns from complex data. Nonetheless, there is a need to recognize that models may be misleading, since they can result in conclusions which cannot be drawn from the data alone. A classic example of this is seen in [1], wherein it is shown that an identified model of a high order system is non-stabilizable by a three-term controller and that the original data indicates that it is indeed stabilizable. Models are essentially a collection of our *a priori*

This thesis follows the style of *IEEE Transactions on Automatic Control*.

knowledge and prejudices. A careful examination of the control-relevant information in the data is required so as to be cognizant of situations in which our modeling assumptions might be inaccurate. This motivates the first problem that is being addressed in this work.

This work focuses on the use of data to synthesize sets of stabilizing controllers for a Multi-Input Multi-Output (MIMO) or a multivariable system. Synthesis of sets of stabilizing controllers is important, since performance and specification problems can be solved on this set during the design stage. Additionally, it is important in applications that use switching control which should ideally be carried out on a set of stable controllers. A survey of the results on modern free control till date [2-7] is given in Chapter II and it is seen that most of these techniques result in a single optimal controller or are restricted to Single Input Single Output (SISO) systems. It is reiterated that this work focuses on the synthesis of sets of stabilizing controllers for MIMO systems, thus emphasizing its novelty.

Given the input-output frequency domain data of an unknown linear multivariable system, a three step procedure to synthesize a stabilizing set of controllers is proposed. The structure of the controller is fixed, hence the form of the closed-loop characteristic equation of the system with the controller in place is known. However, the controller parameters do not appear linearly in this equation, the contrary of which is required during the second stage of the proposed approach. Hence, the controllers that appear at each input of the system are designed sequentially. At every stage of the sequential design, a stabilizing set of controllers is obtained using Neimark's D-Decomposition (search for stability regions) method. This is followed by the stage involving stability check, based on the Bode equivalent of Nyquist criterion. The three step procedure involving sequential design, search for stability regions and stability check gives a set of stabilizing controllers for the given multivariable system. Various sets of such controllers can be obtained by changing the structure of the controller assumed in the first place.

The second problem that is being addressed in this work is the estimation of disturbance acting on a discrete-time multivariable system. The disturbance includes model uncertainties and undesired exogenous inputs acting on the system, certain characteristics of which are unknown but play a major role in the system's dynamic behavior. Disturbance in the sense of system uncertainties, undesired exogenous input signals and noisy measurements can obscure the development of a viable control law for a system. Hence, it is important to identify a model for the disturbance from the actual system's partial state measurements. Exogenous signals considered in this work are sinusoidal signals whose frequencies are known, but the amplitudes and phases are unknown. Sinusoidal exogenous signals are encountered in a number of systems such as shape control of flexible membranes [8], active noise control [9, 10] and hard disk drives [11]. The disturbance representing the model uncertainties and sinusoidal exogenous signals is modeled as a dynamic system with a set of sinusoidal signals (with frequencies at which the exogenous signal perturbs the given system) as inputs. This disturbance term is added to the assumed model of the multivariable system to get a dynamically equivalent extended system. This kind of approach to modeling the disturbance is seen in [12], however, the parameters that appear in the dynamic model of the disturbance are not estimated (identified) adaptively from the actual system measurements. It is a method devised in the context of direct controller design and the input to the disturbance dynamic system is assumed to be a white noise process. An assumed initial model of the disturbance term is used throughout to tune the controller to obtain the required design specifications and the process noise covariance matrix is updated adaptively online. In this work, a methodology to estimate the disturbance parameters when its model is assumed to be a linear dynamical system with a set of sinusoidal signals as input is provided.

A linear time invariant or time varying dynamic model of the disturbance term is estimated by augmenting the disturbance term to the already known discrete time system model as,

$$\begin{aligned} x_m(k+1) &= F_m(x_m(k)) + D_m(k) \\ D_m(k+1) &= A_{Dm}(k)D_m(k) + B_{Dm}(k)u(k) \\ y_m(k) &= Cx_m(k) \end{aligned} \tag{1.1}$$

where x_m denotes the state of the model, $F_m(.)$ and C are the known part of the system dynamics and D_m denotes the disturbance term which is modeled as a linear dynamical system whose parameters. $A_{Dm}(k)$ and $B_{Dm}(k)$ are unknown and are to be estimated from the actual system's noisy output measurements, where k represents sample time. $u(k)$ denotes a mixture of sinusoidal signals which acts as an input to the disturbance model and the frequencies present in this mixture are assumed to be known. y_m denotes the output of the model. In the proposed approach, this extended system is subjected to Kalman filtering to obtain the disturbance state estimates in the presence of process and measurement noise of known statistics. A least squares estimate of $A_{Dm}(k)$ and $B_{Dm}(k)$ are obtained using these disturbance state estimates. The novelty of the solution lies in the way in which the disturbance model is estimated adaptively in order to obtain an identified model, which can further be used in the construction of a control law for the system. This can be regarded as the first step in the indirect method of control synthesis.

1.2 Overview

This thesis is divided into four chapters. After a brief introduction to the two problems in Chapter I, Chapter II gives a comprehensive treatment to the problem of model free control of multivariable systems, presenting a brief history of the various classical and modern control strategies till date. Some recent results on modern free control of SISO systems are reviewed and their applicability to MIMO systems is discussed. A novel three step method to synthesize sets of controllers for MIMO systems using direct data is proposed. The efficacy of the method is validated using two examples of stable linear multivariable systems. Figures depicting the stabilizing set of controllers for these examples are shown. Analysis and interpretation of the results obtained and directions for future research is entailed at the end of Chapter II. Chapter III addresses the issue of estimation of disturbance through partial state measurements of a system in a stochastic environment. Previous results to some of the related problems are revisited. A simulation result wherein certain characteristics of the disturbance are known is presented. Chapter IV concludes this thesis with a summary of the proposed techniques reiterating their novel aspects, their sphere of applicability and directions for future research.

CHAPTER II

MODEL FREE CONTROL OF MULTIVARIABLE SYSTEMS

2.1 Introduction

The problem of designing sets of stabilizing controllers for a system requires precise knowledge of the system. As described in the beginning of Chapter I, traditionally, if a system is unknown, it is subjected to the identification process that uses measurements of the system in order to obtain a system model. The identified model is then used for the synthesis of controllers. In many fields of science and engineering, one needs to deal with complex systems, where, approximations and simplifications made during the process of modeling may result in unreliable models, which lose the ability to capture the behavior of the actual system. A fatal consequence of such simplifications occurs when the designed controller stabilizes the model but not the actual system.

Model free approach to the design of controllers involves the synthesis of these controllers using either input-output data or frequency domain data. It is a significant improvement over classical control loop shaping approaches since complete sets of stabilizing controllers can be obtained and these sets can further be inspected to obtain controllers with the required performance specifications.

The earliest notion of model free control was seen in the work by Nyquist in 1932 [13], who introduced a means of predicting the stability of a closed loop system based on frequency measurements made on the open loop system. The Nyquist criterion was later enhanced by Bode [14] and several others by introducing a graphical design approach to reshape the open loop frequency response by a simple cascaded compensator to achieve the prescribed closed loop stability margins. The model based approach to control design was introduced by Kalman in 1960 [15, 16] which involved the use of state space models, state feedback control and quadratic optimization, guaranteeing stability and optimality. H_∞ theory [17] is also a model based approach to control design where the controller order is invariably high and typically of the order of the generalized plant whose state space model is obtained by the process of identification using the input-output data. In this work, the design of sets of fixed and lower order stabilizing controllers is carried out in contrast to the design approaches where in the controller order is unconstrained, since high order controllers are rarely implemented in practice.

Early approaches to model free design were made by researchers in the fuzzy logic and neural network community, but the fact that the stability and performance guarantees cannot be given, is a disadvantage. Several independent ideas regarding data driven control were also proposed in the context of optimal control. Some of the work done in this direction is seen in [2-5], where a mathematical model (a state equation or a difference equation) of the optimal controller is derived directly from the observed data. These works focused on algorithms to find the controller, however data driven control based on a dynamical system theory approach in data space is seen in [6]. Here, a method is proposed to design single optimal controller for a discrete time SISO system whose MacMillan degree and relative degree is assumed to be known. A recent result on model free design approach is also seen in [7] which uses input-output time series data. It provides stability conditions for a closed loop system in the framework of Lyapunov's second method. This method is also restricted to SISO systems and the order of the system in some sense is presumed to be known.

Recently, Keel and Bhattacharyya [1] proposed a method to obtain sets of stabilizing controllers for systems without analytical models. Design of sets of stabilizing controllers is important because performance and specification problems can be solved on this set during design. It is also important in switching control which should ideally be done on a stable set of controllers. In [1], an example is shown, which indicates that an identified model of a high order system is non-Proportional Integral Derivative (PID) stabilizable, whereas the original data used to synthesize the controllers directly is PID stabilizable. This marks the importance of designing controllers directly based on data and not on models. A Signature based method was used to obtain sets of stabilizing controllers for SISO systems without making any assumptions on the order or the relative degree of the system. The controllers were designed using the frequency domain data and were of fixed order unlike methods where their order can be unconstrained. This technique is explained in the segment that follows.

2.1.1 Signature based Method - Model Free Synthesis of Controllers for SISO Systems

Some mathematical preliminaries are given in the beginning to aid in the better understanding of this method. Consider a real rational function,

$$P(s) = \frac{Y(s)}{U(s)} \tag{2.1}$$

where *Y(s)* and *U(s)* are polynomials of degrees *m* and *n*, respectively. *Y(s)* and *U(s)* have real coefficients and have no zeros on the *jω* axis. Let the number of open Right Half Plane (RHP) and open Left Half Plane (LHP) zeros and poles of *P(s)* be denoted as z_P^+, p_P^+ z_P^-, p_P^-. As *ω* runs from *0* to +∞ the net change of phase of *P(jω)* is,

$$\Delta_0^\infty \angle P(j\omega) = \frac{\pi}{2}[z_P^- - z_P^+ - (p_P^- - p_P^+)] = \frac{\pi}{2}\sigma(P) \tag{2.2}$$

where *σ(P)* called the Hurwitz Signature of *P(s)* is defined as in Equation (2.3).

$$\sigma(P) := z_P^- - z_P^+ - (p_P^- - p_P^+) = -(n-m) - 2(z^+ - p^+) \tag{2.3}$$

P(jω) can be written as,

$$P(j\omega) = P_r(\omega) + jP_i(\omega) \tag{2.4}$$

where $P_r(\omega)$ and $P_i(\omega)$ are the real and imaginary parts of $P(j\omega)$. They have real coefficients and have no real poles for $\omega \in (-\infty, +\infty)$ since $P(s)$ has no imaginary axis poles. The left hand side of Equation (2.2), can be calculated by developing formulas in terms of $P_r(\omega)$ and $P_i(\omega)$. $\omega_0 = 0$ is a zero of $P_i(\omega)$ since $P(s)$ is real. Let,

$$0 = \omega_0 < \omega_1 < \omega_2 < \ldots\ldots < \omega_{l-1} \tag{2.5}$$

represent zeros of $P_i(\omega) = 0$ which are of odd multiplicities, real, finite and non-negative. Signature of $P(s)$ can be written as shown in Equation (2.6) and Equation (2.7). For n-m even,

$$\sigma(P) = \left(sgn[P_i(\omega_0)] + 2\sum_{j=1}^{l-1}(-1)^j sgn[P_r(\omega_j)] + (-1)^l sgn[P_r(\omega_l)]\right)(-1)^{l-1} sgn[P_i(\infty^-)] \tag{2.6}$$

and n-m odd,

$$\sigma(P) = \left(sgn[P_i(\omega_0)] + 2\sum_{j=1}^{l-1}(-1)^j sgn[P_r(\omega_j)]\right)(-1)^{l-1} sgn[P_i(\infty^-) \tag{2.7}$$

where,

$$sgn(a) = \begin{cases} +1 \ \ if \ \ a > 0 \\ 0 \ \ if \ \ a = 0 \\ -1 \ \ if \ \ a < 0 \ . \end{cases} \tag{2.8}$$

The signature formulas are derived based on phase unwrapping. These concepts are used to get the stabilizing set of controllers given the frequency response measurements of a SISO system. Conditions on the closed loop characteristic equation in terms of signature which places all the poles of the closed loop system in the LHP gives the stabilizing set of controllers. Let *P(s)* denote a rational transfer function of a system, written as follows,

$$P(s) = \frac{M(s)}{N(s)} \tag{2.9}$$

where *M(s)* and *N(s)* denote the numerator of degree *m* and denominator of degree *n* of *P(s).* The relative degree *n-m* of the system *P* can be found from the high frequency slope of the Bode magnitude plot of *P(jω)*. If the system is stable, then $p^+ = 0$, hence z^+ can be found from the frequency data since $\sigma(P)$ in Equation (2.2) can be calculated from the phase plot of *P(jω)*.

If the system *P* is unstable, the parameters in the Equation (2.2) can be still be found, given that there exists a controller that can stabilize this system and is known, hence providing the knowledge of the closed loop frequency response.

Given the frequency measurements of *P*, a stabilizing set of PID controllers of the form,

$$C(s) = \frac{K_i + K_p s + K_d s^2}{s(1 + sT)} \tag{2.10}$$

where *T* is a fixed small positive value and K_p, K_i, K_d represent the proportional, integral and derivative parameters of the controller that are designed using the closed loop characteristic equation as shown in Equation (2.11).

$$\begin{aligned} R(s) &= s(1 + sT) + (K_i + K_p s + K_d s^2)P(s) \\ \bar{R}(s) &= R(s)P(-s) \end{aligned} \tag{2.11}$$

For closed loop stability, the signature of $R(s)$ and thus $\bar{R}(s)$ is required to satisfy Equation (2.12).

$$\begin{aligned} \sigma(R(s)) &= n + 2 - (p^- - p^+) \\ \sigma(\bar{R}(s)) &= n - m + 2z^+ + 2 \end{aligned} \tag{2.12}$$

$\bar{R}(j\omega)$ can be written as a quantity shown in Equation (2.13).

$$\begin{aligned}
\bar{R}(j\omega) &= j\omega(1+j\omega T)P(-j\omega)+(K_i+j\omega K_p-\omega^2 K_d)P(j\omega)P(-j\omega)\\
\bar{R}(j\omega) &= (K_i-K_d\omega^2)|P(j\omega)|^2-\omega^2 TP_r(\omega)+\omega P_i(\omega)+\\
&\qquad j\omega(K_p|P(j\omega)|^2+P_r(j\omega)+\omega^2 TP_i(\omega))\\
\bar{R}(j\omega) &= \bar{R}_r(\omega,Ki,Kd)+j\omega\bar{R}_i(\omega,K_p)
\end{aligned} \tag{2.13}$$

Setting the imaginary part $\bar{R}_i(\omega,K_p)$ to zero as shown in Equation (2.14), suitable values of K_p are selected.

$$\begin{aligned}
&\bar{R}_i(\omega,K_p)=0\\
&K_p=-\frac{P_r(\omega)+\omega TP_i(\omega)}{|P(j\omega)|^2}\\
&\quad=-\frac{\cos\phi(\omega)+\omega T\sin\phi(\omega)}{|P(j\omega)|}\\
&\quad=:g(\omega)
\end{aligned} \tag{2.14}$$

The function $g(\omega)$ is plotted and $K_p = K_p^*$ is selected such that the number of points at which it intersects $g(\omega)$ is equal to the number of frequency points that yield the required signature.

Let $\omega_1<\omega_2<\omega_3.......<\omega_{l-1}$ denote this set of frequencies which are distinct and of odd multiplicities. Determine strings of integers,

$$I=[i_0,i_1,i_2,.....i_l] \tag{2.15}$$

with $i_t \in \{+1,-1\}$ such that, when *n-m* is odd,

$$[i_0 - 2i_1 + 2i_2 + ... + (-1)^{l-1} 2i_{l-1} + (-1)^l i_l](-1)^{l-1} j = \sigma(\overline{R}). \tag{2.16}$$

When *n-m* is even,

$$[i_0 - 2i_1 + 2i_2 + ... + (-1)^{l-1} 2i_{l-1}](-1)^{l-1} j = \sigma(\overline{R}) \tag{2.17}$$

where $j = sgn[\overline{R}_i(\infty^-, K_p^*)]$.

For every fixed $K_p = K_p^*$, the (K_i, K_d) corresponding to closed loop stability are given by,

$$\overline{R}_r(\omega_t, K_i, K_d) i_t > 0 \tag{2.18}$$

where i_ts are strings satisfying Equation (2.16) and Equation (2.17) and ω_ts are the solutions of Equation (2.14).

Thus, sets of stabilizing controllers for SISO systems without any analytical models can be obtained using Signature based method. A direct extension of this method to MIMO systems may not be possible, since the controller parameters do not appear linearly in the closed loop characteristic equation. They appear in a multilinear or a nonlinear fashion and the same parameters appear in most of the coefficients of this equation. To get around this problem, a new solution is proposed and is explained in the next segment.

2.2 Model Free Method for Multivariable Systems

The model free methods that have been proposed till now have been restricted to SISO systems. In this work, a new design approach has been proposed which provides a method of model free controller synthesis for MIMO systems. The frequency domain data of the open loop system as in [1] is used for the controller synthesis. Some of the salient features of this method constitute the following,

- Sequential design of controllers
- Search for stability boundaries (Root invariant regions)
- Stability test

2.2.1 Sequential Design of Controllers

Designing controllers for multivariable systems directly from frequency domain data poses some problems. The controller parameters do not occur linearly in the closed loop characteristic equation as in the case of SISO systems. Instead, they occur as multilinear or nonlinear terms. Analyzing them using Routh Hurwitz, Neimark's D-Decomposition and Signature based methods is difficult, even when the model of the systems is known.

The proposed approach gets rid of this problem by inculcating a sequential design of controllers. The concept of sequential design becomes clear using an example as shown in Fig.1, which is a Two Input Two Output (TITO).

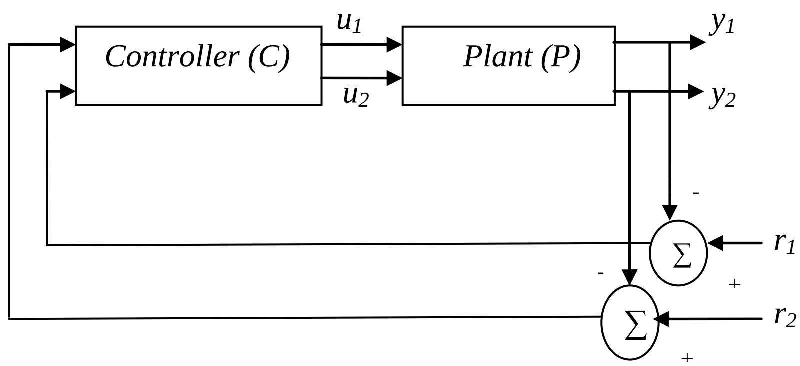

Fig.1. Two Input Two Output (TITO) System.

Internally, the system shown above looks as in Fig.2. There are two controllers, one at each input of the system. In sequential design, at first, only one of the controllers, say C_2 is switched on and the other controller, C_1 is turned off. The closed loop characteristic equation is analyzed and is used to obtain a set of stabilizing C_2. Now, the controller C_1 is turned on and with every stabilizing C_2 in place, sets of stabilizing C_1 are obtained. This corresponds to one C_2 and a set of C_1 that stabilizes the entire system. This is repeated for every C_2 obtained in the first place to get a stabilizing set S_{C1}. This process is repeated with C_2 turned off and C_1 turned on to get set of stabilizing C_1. Then for every C_1 obtained, sets of stabilizing C_2 are obtained to get a set S_{C2}. The union of S_{C1}

and S_{C2} gives the stabilizing set. The exact method used to obtain these stabilizing sets S_{C1} and S_{C2} will be discussed in the segments that follow. This process can be extended to systems with any number of inputs and outputs and is not restricted to TITO systems or square systems, where the number of inputs and outputs are equal.

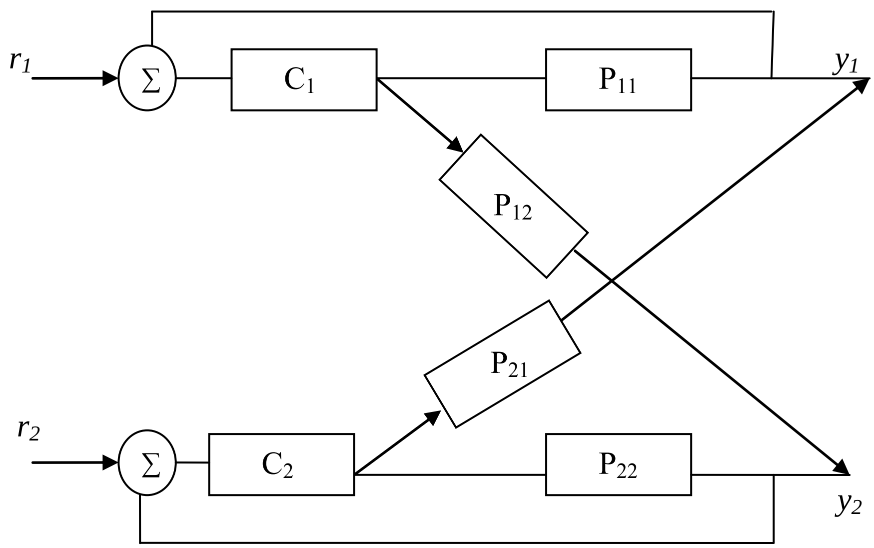

Fig.2. Internal Architecture of a TITO System.

A natural question that arises at this point is whether or not the Signature based method can be used in conjunction with the sequential design approach for MIMO systems, because sequential design approach results in a characteristic equation with linear controller parameters. In the case of Signature based method applied to SISO systems, the signature can be easily specified in terms of the relative degree of the system which can be calculated from the Bode magnitude plot of the system. However, in the case of multivariable systems, it is difficult to specify this signature condition at every stage of the sequential design. Although, the coefficients of the characteristic polynomial are linear in only one of the controller parameters, the calculation of signature of each of the subsystems (the loops corresponding to the controllers which are turned on) from the Bode magnitude plot of the multivariable system is not only difficult, but infact, not possible.

2.2.2 Search for Stability Boundaries

Stability boundaries represent the collection of all the points in the controller parameter space for which the corresponding characteristic equation of the closed loop system has at least one root on the imaginary axis. These boundaries define a partition of the parameter space in several regions, each region having a constant number of unstable roots for all the parameters inside the region.

The D-decomposition method suggested by Neimark [18, 19] in the 40s has been used to obtain these stability boundaries. This method has been used previously to obtain sets of stabilizing controllers for SISO systems when the model of the system is known. To the best knowledge of the author, it has not been used to obtain sets of stabilizing controllers for MIMO systems especially when their model is unknown. In this work, a technique is presented to obtain these stabilizing controllers for MIMO systems when their model is unknown and it is possible to do so because of the sequential design approach that is being followed.

Consider a characteristic polynomial $\pi(s,k)$, depending on a vector parameter *k* [20], as shown in Equation (2.19). The boundary of a stability domain (in the space *k*) is given by,

$$\pi(j\omega,k)=0, \quad -\infty<\omega<\infty. \tag{2.19}$$

If $k \in R$, then we obtain two equations (real and imaginary parts of Equation (2.19)), each containing the variable *k,* which define the stability boundary and on solving these equations for *k,* we get various *k,* which divide the real line into several intervals and each of these points (*k)*, define the boundary of the stability domain. Similarly, if $k \in R^2$, then we get two equations in two variables which define the parametric curve, *k(ω)*, for $-\infty < \omega < \infty$, specifying the boundary of the stability domain. Moreover, the curve *k(ω)* divides the plane into root invariant regions (regions with a fixed number of stable and unstable roots of *p(s,k)*). This is the basic idea of D-decomposition approach. This idea can be traced back to Vishnegradsky [21] who reduced a cubic polynomial to the form,

$$\pi(s,k)=s^3+k_1s^2+k_2s+1 \tag{2.20}$$

treating the coefficients k_1, k_2 as parameters. Then Equation (2.19) yields $k_1\omega^2 = 1$, $\omega(k_2 - \omega^2) = 0$. A hyperbola $k_1k_2 = 1$, which defines the stability boundary is obtained on eliminating ω. The stability domain is the set $k_1k_2 > 1$. When the model of the system is unknown, it is not possible to construct the characteristic polynomial of the system. In order to find the stability boundaries in this case, the characteristic equation whose zeroes denote the poles of the system is used (note the difference between characteristic polynomial and characteristic equation). It is convenient to use the characteristic equation as it contains the open loop system transfer function and since the frequency data is known, the characteristic equation can be written in terms of the data that is known. For example, consider a SISO system with the following transfer function,

$$P(s) = \frac{A(s)}{B(s)}. \tag{2.21}$$

Let $C(s)$ denote a controller which is to be designed so as to stabilize $P(s)$. The controller can be of any finite order, that is, the controller parameter space is finite dimensional. The closed loop dynamics of this system is characterized by,

$$1 + P(s)C(s) = 0. \tag{2.22}$$

The stability crossing boundaries T is the set of all points in the controller parameter space, for which Equation (2.22) has imaginary roots. In other words, these are the controller parameters for which the zeroes of the closed loop characteristic equation cross the imaginary axis from LHP to RHP or from RHP to LHP. The set of all points denoting the controller parameters between these crossings give root invariant regions, thus dividing the space of the parameters into stabilizing and non-stabilizing regions. Hence we need to find those controller parameters which satisfy,

$$1 + P(j\omega)C(j\omega) = 0. \tag{2.23}$$

Since $P(j\omega)$ is known, it is possible to find the root invariant regions for the model free case. In case of SISO systems, the controller parameters occur linearly in the characteristic equation/polynomial as in Equation (2.20) given that the controller chosen is linear. Hence, the root invariant regions can be easily found. But, in case of MIMO systems, the controller parameters occur in a nonlinear fashion. However, since we are using a sequential design approach, this problem is taken care of. Consider a MIMO system $P(s)$, which denotes a transfer function matrix whose row size is equal to the number of outputs to the system denoted as m and column size is equal to the number of inputs denoted as r. Let $C(s)$ denote the controller matrix whose row size is equal to r and column size is equal to m. The characteristic equation of this system is given by,

$$det(I + P(s)C(s)) = 0. \tag{2.24}$$

In general, the characteristic equation can be written as follows,

$$det(I + P(s)C(s)) = 1 + M(s) = 0 \tag{2.25}$$

where $M(s)$ is in terms of the transfer function of the subsystems, constituting the MIMO system and the controller transfer function. At every stage of the sequential design, $M(s)$ consists of coefficients which comprise of parameters of a single controller. Since each of the controllers is linear, these coefficients are also linear in those parameters. The search for stability boundaries can thus be done at every stage of the sequential design approach. Once these stability boundaries are obtained, the space of controller parameters is investigated for stable regions. The stability test is discussed in the next segment.

2.2.3 Stability Test

Having found the stability boundaries, it is required to check which of these regions contain a stabilizing set of controller parameters. Since each of these regions is root invariant, it is enough if the stability condition is checked at only one point in each of these regions. If one point is stabilizing, then the entire region forms a stabilizing set. Hence, there is a need to check if the term $1 + M(s)$ is stable for a point in each of the root invariant regions. The quantity $1 + M(s)$ can be written in terms of the frequency response measurements of the subsystems of a multivariable system and the controller whose parameter lie in the root invariant regions. A powerful test such as the Nyquist criterion can be used to determine the stability. It is a well known fact that the Nyquist criterion [13] provides a powerful test for closed-loop stability in terms of open-loop measured data. Let $M(j\omega)$ denote the frequency response measurement of a quantity which is in terms of the frequency response measurements of the subsystems of the given multivariable system and the controller whose stabilizability needs to be checked. Let ω_i, $i = 0, 1, 2, \ldots, k + 1$ with $\omega_0 = 0$ and $\omega_{k+1} = \infty$ denote the frequencies where the Nyquist plot of $M(s)$ cuts the negative real axis of the complex plane. In other words, these frequencies are the solutions to the following,

$$\angle M(j\omega) = n\pi, n = \pm 1, \pm 3, \pm 5, \ldots\ldots \qquad (2.26)$$

Define the set,

$$\Omega = \{\omega_0, \omega_1, \omega_2, \ldots\ldots, \omega_k, \omega_{k+1}\} \qquad (2.27)$$

where, $0 =: \omega_0 < \omega_1 < \omega_2 < \ldots\ldots < \omega_k < \omega_{k+1} := \infty$ and ω_0 and ω_{k+1} are included only if they satisfy the angle condition in the Equation (2.26). Introduce the corresponding sequence of integers,

$$\{i_0, i_1, i_2, \ldots\ldots, i_k, i_{k+1}\} \qquad (2.28)$$

where $i_t = 0$ if $|M(j\omega)| < 1$ and otherwise,

$$i_t = \begin{cases} +1 \text{ if } \frac{d}{d\omega}\angle M(j\omega)|_{\omega_t} > 0 \\ 0 \text{ if } \frac{d}{d\omega}\angle M(j\omega)|_{\omega_t} = 0 \\ -1 \text{ if } \frac{d}{d\omega}\angle M(j\omega)|_{\omega_t} < 0. \end{cases} \qquad (2.29)$$

Under the assumption that $M(s)$ has no imaginary axis poles, the number of counterclockwise encirclements of $-1+j0$ by the Nyquist plot of $M(s)$ is given by Equation (2.30).

$$i(M) := i_0 + \sum_{t=1}^{k} 2i_t + i_\infty \qquad (2.30)$$

For stability,

$$i(M) = p^+. \qquad (2.31)$$

Since only stable linear multivariable systems are being considered in this work, $p^+ = 0$ at every stage of the sequential design. Hence the controllers for which Equation (2.31) is satisfied are considered to be stabilizing. Expressions similar to Equation (2.29) can be derived when $M(s)$ has poles on the imaginary axis as shown in [22]. This primarily constitutes the stability check part of the proposed approach.

2.3 Results

In this segment, results for the model free method applied to a second and a third order system are shown. Both the examples correspond to stable linear systems, hence $p^+ = 0$.

2.3.1 Example 1: Second Order System

Consider the following Two Input Two Output (TITO) second order stable multivariable system,

$$P(s) = \begin{bmatrix} \frac{1}{s+2} & \frac{1}{s+2} \\ \frac{1}{s+4} & \frac{1}{s+4} \end{bmatrix}. \tag{2.32}$$

A set of stabilizing controllers of the form,

$$C(s) = \begin{bmatrix} K_1 & 0 \\ 0 & K_2 \end{bmatrix} \tag{2.33}$$

where K_1 and K_2 represent gains is found using the proposed technique.

2.3.1.1 Theoretical Construction

The above system is first analyzed theoretically to get a stabilizing set of controllers of the chosen form as shown below,

$$P(s)=D_p(s)^{-1}N_p(s),\ C(s)=N_c(s)D_c(s)^{-1}, \tag{2.34}$$

$$D_p(s)=\begin{bmatrix} s+2 & 0 \\ 0 & s+4 \end{bmatrix},\ N_p(s)=\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}, \tag{2.35}$$

$$D_c(s)=\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},\ N_c(s)=\begin{bmatrix} K_1 & 0 \\ 0 & K_2 \end{bmatrix}. \tag{2.36}$$

The characteristic polynomial of the closed loop system is,

$$\begin{aligned} \pi(s,K_1,K_2) &= det(D_pD_c+N_pN_c) \\ &= det\left(\begin{bmatrix} s+2 & 0 \\ 0 & s+4 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}+\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} K_1 & 0 \\ 0 & K_2 \end{bmatrix}\right) \\ &= s^2+s(K_1+K_2+6)+2K_2+4K_1+8\ . \end{aligned} \tag{2.37}$$

Using Routh-Hurwitz criterion, the set of stabilizing controllers satisfy the following inequalities,

$$\begin{aligned} &K_1+K_2+6>0 \\ &2K_2+4K_1+8>0\,. \end{aligned} \tag{2.38}$$

The above inequalities are plotted using Maple and the stabilizing region obtained is shown in Fig.3.

2.3.1.2 Using Model Free Method

The closed loop characteristic equation of the system can be written in terms of the entries of the transfer function matrix of the given multivariable system, as shown in Equations (2.39-2.41). The zeroes of this characteristic equation correspond to the poles of the closed loop system.

$$\begin{aligned}\pi(s,K_1,K_2) &= det(I+PC)\\ &= 1+P_{11}C_1+P_{22}C_2+(P_{11}P_{12}-P_{21}P_{22})C_1C_2\\ &= 1+P_{11_r}K_1+P_{22r}K_2+G_rK_1K_2+j(P_{11_i}K_1+P_{22i}K_2+G_iK_1K_2)\end{aligned} \tag{2.39}$$

where,

$$\begin{aligned}G_r &= P_{11r}P_{22r}-P_{11i}P_{22i}-P_{21r}P_{12r}+P_{21i}P_{12i}\\ G_i &= P_{11i}P_{22r}+P_{11r}P_{22i}-P_{21i}P_{12r}-P_{21r}P_{12i}\end{aligned} \tag{2.40}$$

$$\begin{aligned}\pi(j\omega) = 1 &+ P_{11_r}(\omega)K_1+P_{22r}(\omega)K_2+G_r(\omega)K_1K_2+j(P_{11_i}(\omega)K_1\\ &+P_{22i}(\omega)K_2+G_i(\omega)K_1K_2)\end{aligned} \tag{2.41}$$

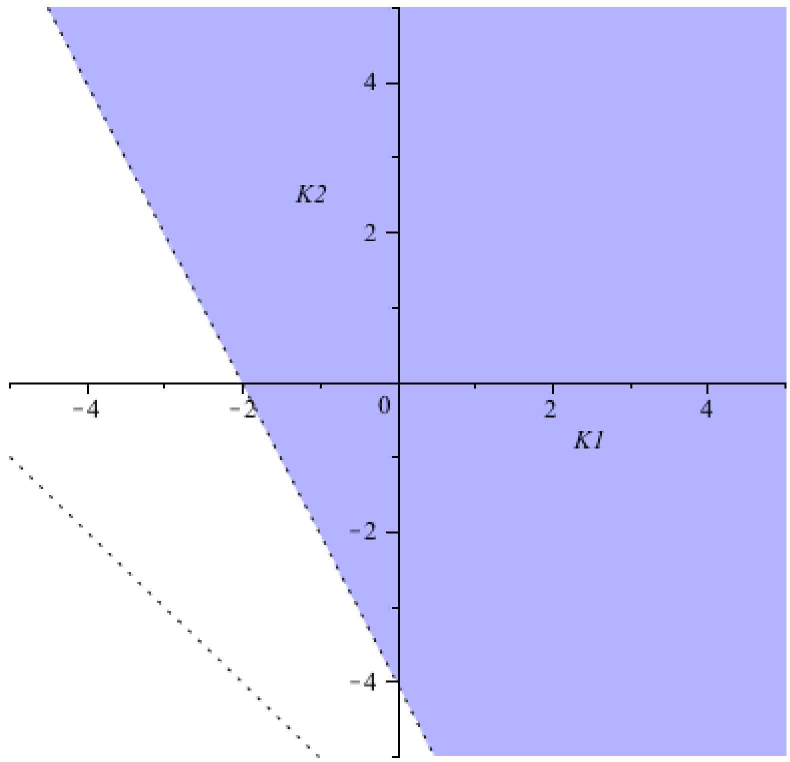

Fig.3. Theoretical Stabilizing Region (shaded area) for Example 1.

Now, given the open loop Bode frequency response of the multivariable second order system as shown in Fig.4, Equation (2.41) is analyzed through the proposed three stages of the control design. The number of inputs to the system is two; hence the sequential design stage undergoes two recursions. During each of the recursive stages, the characteristic equation passes through the other two steps, that is, the search for stability regions and stability check.

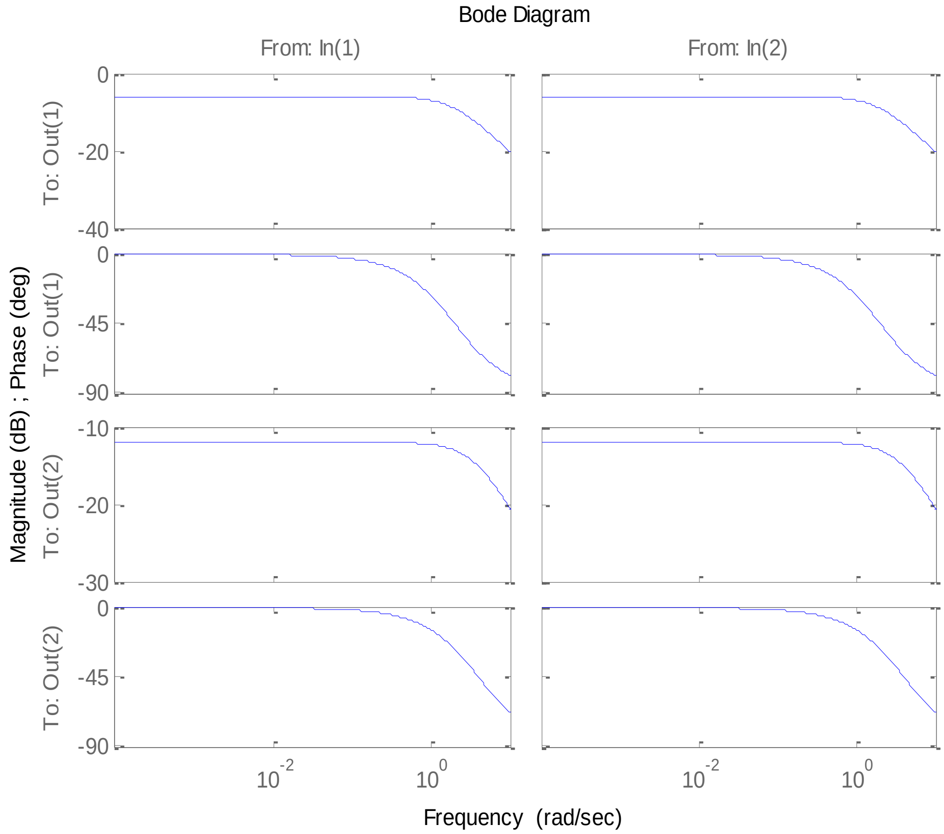

Fig.4. Open Loop Bode Frequency Response of Example 1.

The stabilizing region obtained for Example 1 using the proposed approach is shown in Fig.5.

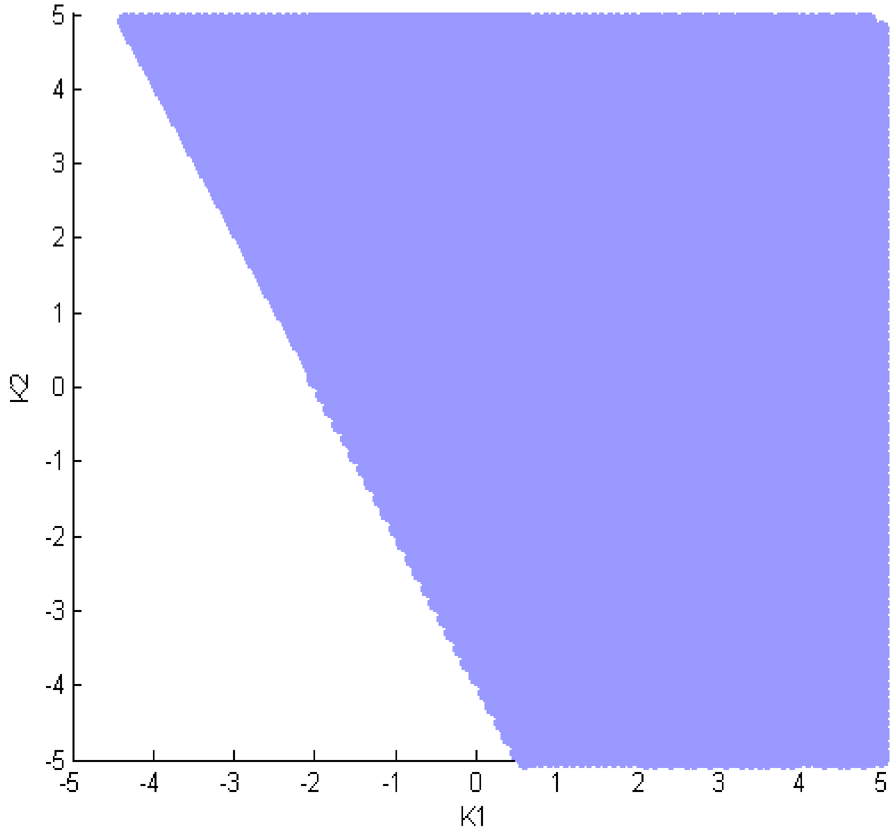

Fig.5. Stabilizing Region Using Proposed Approach for Example 1.

2.3.2 Example 2: Third Order System

Consider a TITO third order stable multivariable system as shown in Equation (2.42). Even in this case, a set of stabilizing controllers of the form (2.43), where both the controllers are gains is found.

$$P(s)=\begin{bmatrix}\frac{1}{s+2} & \frac{1}{s+3}\\ \frac{1}{s+1} & \frac{1}{s+1}\end{bmatrix} \tag{2.42}$$

$$C(s)=\begin{bmatrix}K_1 & 0\\ 0 & K_2\end{bmatrix} \tag{2.43}$$

2.3.2.1 Theoretical Construction

The system in Equation (2.42) is analyzed theoretically to get a stabilizing set of controllers of the chosen form as shown below,

$$P(s)=D_p(s)^{-1}N_p(s),\ C(s)=N_c(s)D_c(s)^{-1}, \tag{2.44}$$

$$D_p(s)=\begin{bmatrix}(s+2)(s+3) & 0\\ 0 & s+1\end{bmatrix},\ N_p(s)=\begin{bmatrix}s+3 & s+2\\ 1 & 1\end{bmatrix}, \tag{2.45}$$

$$D_c(s)=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix},\ N_c(s)=\begin{bmatrix}K_1 & 0\\ 0 & K_2\end{bmatrix}. \tag{2.46}$$

The characteristic polynomial of the closed loop system is,

$$\begin{aligned}p(s,K_1,K_2) &= det(D_pD_c+N_pN_c)\\ &= s^3+s^2(K_1+K_2+6)+s(11+5K_2+4K_1)+6+6K_2+3K_1+K_1K_2.\end{aligned} \tag{2.47}$$

Using Routh-Hurwitz criterion, the following set of inequalities, if satisfied, give a set of stabilizing controllers,

$$\begin{aligned} &K_1 + K_2 + 6 > 0 \\ &4K_1^2 + 5K_2^2 + 8K_1K_2 + 38K_1 + 47K_2 + 72 > 0 \qquad (2.48) \\ &K_1K_2 + 3K_1 + 6K_2 + 6 > 0 \,. \end{aligned}$$

The stabilizing region obtained using Equation (2.48) through Maple is shown in Fig.6.

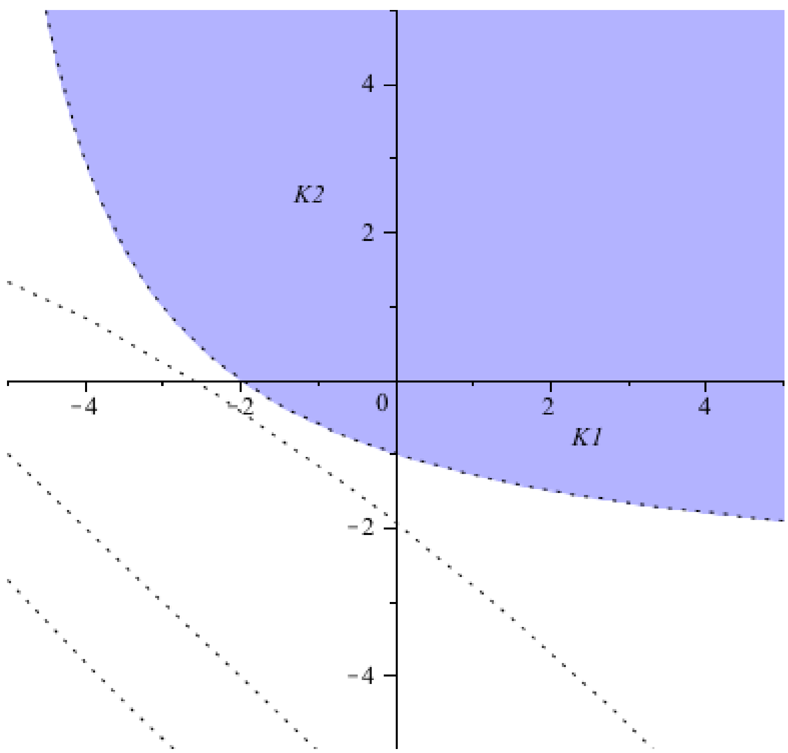

Fig.6. Theoretical Stabilizing Region (shaded area) for Example 2.

2.3.2.2 Using Model Free Method

The Bode frequency response of the multivariable third order system given in Equation (2.42) is shown in Fig.7.

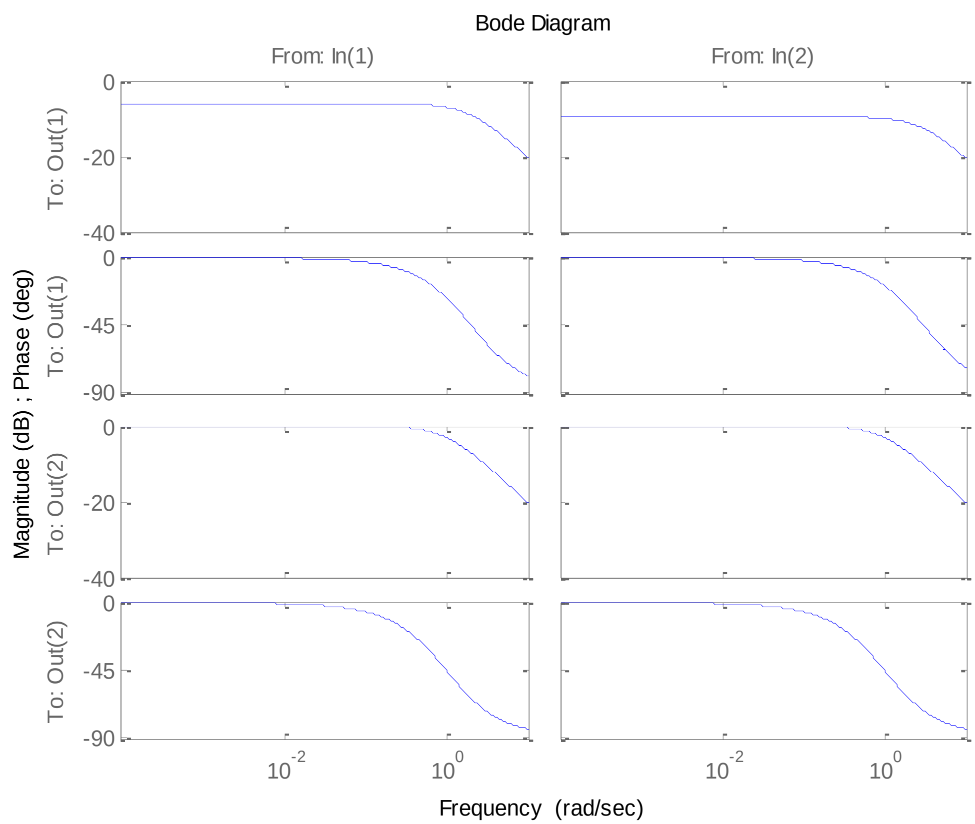

Fig.7. Open Loop Bode Frequency Response of Example 2.

The closed loop characteristic equation in terms of the frequency response of the open loop system, as shown in Equation (2.41), is analyzed through the three stages of the control design to get the stabilizing region shown in Fig.8.

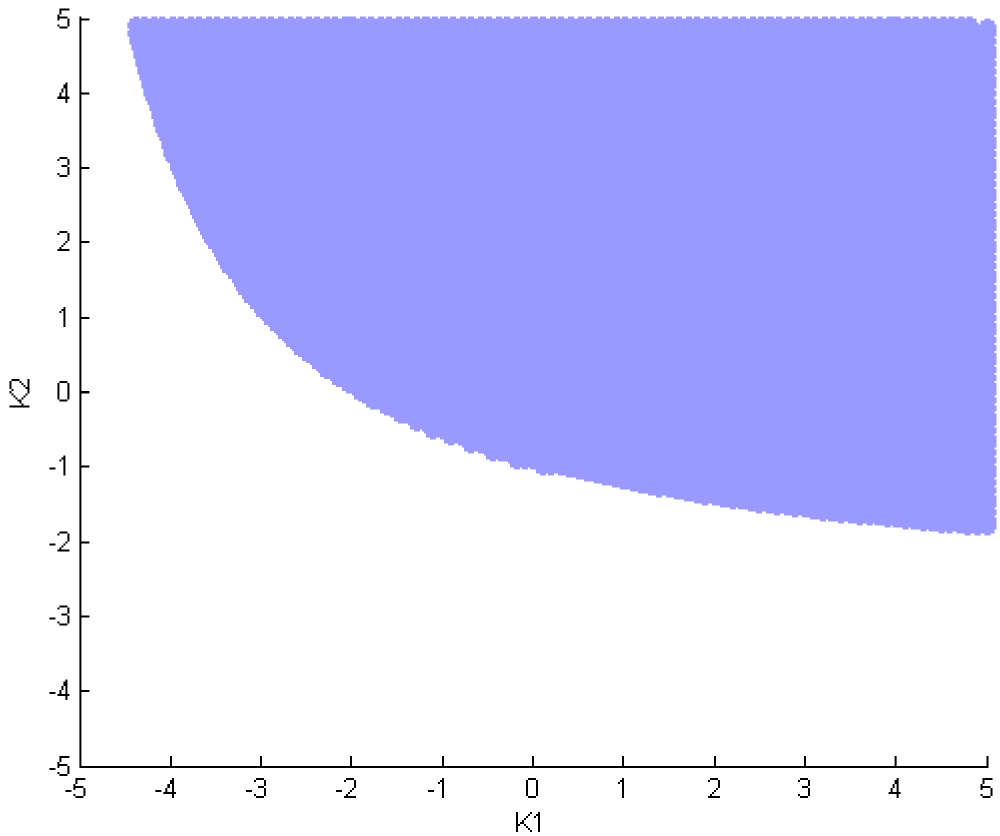

Fig.8. Stabilizing Region Using Proposed Approach for Example 2.

2.4 Concluding Remarks

The complexity of the proposed approach increases with the increase in the number of inputs to the system, since the number of stages in the sequential design is directly proportional to the number of inputs. However, for a given number of inputs, increase in the order of the system does not increase the complexity. For example, the TITO systems given by Equations (2.32) and (2.42) are analyzed using the same form of the characteristic equation given by Equation (2.41), irrespective of their order. This can be regarded as one of the advantages of the proposed approach. The examples shown indicate that the stabilizing region obtained using the proposed approach recovers the entire set of stabilizing controllers of the chosen form. However, it is required to provide a formal proof indicating that the entire set is always recovered, or to provide a counterexample that falsifies the former statement. Different sets of stabilizing controllers can be obtained for a given system by changing the form of the controller chosen in the first place. The examples shown in the previous segment are restricted to stable linear multivariable systems of finite order. The proposed approach can also be extended to unstable linear multivariable systems if the number of unstable poles p^+ of the system is known. Further research is required to handle a larger class of systems which includes systems that are described using linear Partial Differential Equations (PDEs), nonlinear systems and discrete time systems.

CHAPTER III

DISTURBANCE MODELING

3.1 Introduction

Understanding the effect of disturbance on a system's output is a ubiquitous issue. Disturbance of various kinds is seen in almost all kinds of systems ranging from complex systems, such as aircrafts, spacecrafts, ecological systems whether abiotic, such as fires in forests and wave action in rocky intertidal zone, or biotic, such as disease and predation, to simple systems like hard disk drives. Disturbance results from a combination of phenomena that cannot be measured individually.

In applications wherein it is desired to devise control actions for systems, the synthesis of the control actions with unknown disturbance inputs poses serious problems. Consequently, it is necessary to determine or estimate certain characteristics of disturbance from system outputs which could ease the construction of a feasible control law for that system. This is an indirect method of synthesizing a controller. Direct methods involve tuning the controller parameters to meet the specified performance requirements, while simultaneously rejecting the unknown disturbance input.

Methods to synthesize controllers for the purpose of regulation in SISO systems considering sinusoidal disturbance inputs with unknown amplitudes, frequencies and phase has been reported in the literature using both direct [9, 23] and indirect methods [24-28]. Some of them have also been extended to MIMO systems, of which a direct method is seen in [29]. In [30], an indirect method of regulation with an adaptive internal model is presented to track sinusoidal reference signals with unknown amplitudes and frequencies for linear MIMO systems. An adaptive observer is presented to estimate the amplitudes and the frequencies of the sinusoidal signals where only the number of sinusoids in the disturbance is assumed to be known. A direct method of adaptive regulation in the presence of unknown sinusoidal disturbance is also discussed in [31], wherein a regulator design approach is proposed. The desired regulator is designed within a set of *Q*-parameterized stabilizing controllers. A properly constructed set of such controllers is considered to introduce triangular decoupling in part of the closed-loop system dynamics. The decoupling allows for significant simplification in the design of the adaptive regulator and the analysis of the properties of the resulting adaptive closed-loop system. However, there exists a constraint wherein the set of controllers that can decouple the closed loop dynamics of the given system may be empty. An indirect method for regulation of nonlinear MIMO systems where only the number of frequencies in the disturbance is assumed to be known was proposed in [32].

The concept of Disturbance Accommodating Control (DAC) was proposed in the early 1970s [33], which is a direct method of controller synthesis in the presence of disturbance which is quantified to include both the system model parameter uncertainties and external unknown disturbance/noise inputs. The main objective of DAC is to make necessary corrections to the nominal control input to accommodate for external disturbances and system uncertainties [33-36]. The disturbance accommodating observer approach has shown to be extremely effective for disturbance attenuation [37-39]. However, the performance of the observer can significantly vary for different types of exogenous disturbances, which is due to observer gain sensitivity. An extension of the observer based DAC is seen in [12], which uses a robust control approach wherein both the system states and the disturbance term are estimated using a Kalman filter from the measurements of the system. The disturbance term is modeled as a linear dynamical system with white noise process as an input to this system. The states estimated using Kalman filtering are used to develop a nominal control law while the estimated disturbance term is used to make necessary corrections to the nominal control input to minimize the effect of system uncertainties and the external disturbance. The process noise covariance is updated adaptively online.

In this work, a method to estimate disturbance, which includes system model uncertainties and exogenous input signals whose frequencies are known, is proposed. Disturbance is modeled as a linear dynamic system with a set of sinusoidal signal inputs at frequencies with which the exogenous signals (external disturbance) is assumed to perturb the given discrete time system. The idea of modeling the disturbance term as a dynamic system is already seen in [12], but in this work, this model is updated adaptively online using Recursive Least Squares (RLS) technique. The way the disturbance model is estimated/identified is explained at length in the next segment.

3.2 Disturbance Model Identification

Consider an n^{th} order, *m* output and *r* input system of the following form,

$$\begin{aligned} x(k+1) &= F(x(k)) + B\xi(k) + w(k) \\ y(k) &= Cx(k) + v(k) \end{aligned} \tag{3.1}$$

where *x* denotes the state of the system, *F(x)* is assumed to be linear in *x*, either time invariant, *F(x(k)) = Ax(k)* or time varying, *F(x(k))= A(k)x(k)*. The true state matrix (*A*) is assumed to be unknown. ξ is the external disturbance representing the exogenous input to the given system and the frequencies at which it perturbs the system is assumed to be known. *y* represents the output of the system and *C* is the output matrix that is assumed to be known. *w* represents the process noise and *v* is the measurement noise, the statistics of which are assumed to be known. The external disturbance dynamics is,

$$\xi(k+1) = L_1(x(k),\xi(k)) + u(k). \tag{3.2}$$

The assumed (known) system model is,

$$\begin{aligned} x_m(k+1) &= F_m(x_m(k)) \\ y_m(k) &= Cx_m(k) \end{aligned} . \tag{3.3}$$

The external disturbance and the model uncertainties can be lumped into a disturbance term D as follows,

$$D(k) = \Delta F(x(k)) + B\xi(k) \tag{3.4}$$

where $\Delta F(x(k)) = F(x(k)) - F_m(x_m(k))$. The true model of the system can be written in terms of the known model as,

$$\begin{aligned} x(k+1) &= F_m(x(k)) + D(k) + w(k) \\ y(k) &= Cx(k) + v(k). \end{aligned} \tag{3.5}$$

Equation (3.5) can also be written in terms of the disturbance, which is modeled as a dynamical system to get an extended system as follows,

$$\begin{aligned} x(k+1) &= F_m(x(k)) + D(k) + w(k) \\ D(k+1) &= \Delta F(x(k+1)) + B\xi(k+1) = \Delta F(x(k+1)) + B(L_1(x(k),\xi(k)) + u(k)) \\ &= L_2(x,D) + Bu(k) \\ y(k) &= Cx(k) + v(k). \end{aligned} \tag{3.6}$$

The extended system, assuming $F_m(x(k)) = A_m x(k)$ can be written as,

$$\begin{bmatrix} x(k+1) \\ D(k+1) \end{bmatrix} = \begin{bmatrix} A_m & I_{(n\times n)} \\ L_{2x} & L_{2D} \end{bmatrix} \begin{bmatrix} x(k) \\ D(k) \end{bmatrix} + \begin{bmatrix} 0 \\ B \end{bmatrix} u(k) + \begin{bmatrix} w(k) \\ 0_{(n\times 1)} \end{bmatrix} \tag{3.7}$$

$$\begin{aligned} z(k+1) &= J_1 z(k) + J_2 u(k) + W(k) \\ y(k) &= [C \;\; 0_{(m\times n)}]\, z(k) + v(k) = H z(k) + v(k). \end{aligned} \tag{3.8}$$

The disturbance term is modeled as,

$$D_m(k+1) = A_{Dm} D(k) + B_{Dm} u(k). \tag{3.9}$$

The known model of the system can now be written as,

$$\begin{bmatrix} x_m(k+1) \\ D_m(k+1) \end{bmatrix} = \begin{bmatrix} A_m & I_{(n\times n)} \\ 0_{(n\times n)} & A_{Dm} \end{bmatrix} \begin{bmatrix} x(k) \\ D(k) \end{bmatrix} + \begin{bmatrix} 0_{(n\times r)} \\ B_{Dm} \end{bmatrix} u(k) \tag{3.10}$$

$$\begin{aligned} z_m(k+1) &= J_{1m} z_m(k) + J_{2m} u(k) \\ y_m(k) &= [C \;\; 0_{(m\times n)}]\, z_m(k) = H z_m(k) \end{aligned} \tag{3.11}$$

where,

$$z_m(k+1) = \begin{bmatrix} x_m(k+1) \\ D_m(k+1) \end{bmatrix},\; J_{1m} = \begin{bmatrix} A_m & I_{(n\times n)} \\ 0_{(n\times n)} & A_{Dm} \end{bmatrix},\; J_{2m} = \begin{bmatrix} 0_{(n\times r)} \\ B_{Dm} \end{bmatrix},\; H = [C \;\; 0_{(m\times n)}].$$

The disturbance term in Equation (3.9) is adaptively updated using Kalman filter and Recursive Least Squares (RLS) technique.

The discrete time update and measurement update equations for a Kalman filter starting with some initial values of J_{1m} and J_{2m} are given in Equation (3.12) and (3.13),

$$\begin{aligned} \hat{z}_m^-(k+1) &= J_{1m}\hat{z}_m(k) + J_{2m}u(k) \\ P^-(k+1) &= J_{1m}P(k)J_{1m}^T + Q \end{aligned} \tag{3.12}$$

$$\begin{aligned} K(k+1) &= P^-(k+1)H^T(HP^-(k+1)H^T + R)^{-1} \\ \hat{z}_m(k+1) &= \hat{z}_m^-(k+1) + K(k)(y(k+1) - H\hat{z}_m^-(k+1)) \\ P(k+1) &= (I - K(k)H)P^-(k+1) \end{aligned} \tag{3.13}$$

where P^- denotes the *a priori* state estimate error covariance and P denotes the *a posteriori* state estimate error covariance. $\hat{z}_m^-$ denotes the *a priori* extended state estimate and $\hat{z}_m$ denotes the *a posteriori* extended state estimate. K denotes the Kalman filter gain or the blending factor, Q is the process noise covariance and R is the measurement noise covariance matrix.

The last n entries of $\hat{z}_m$ correspond to the disturbance state estimates $\hat{D}_m$. These estimates are used to adaptively update the model given in Equation (3.9) as shown below,

$$\begin{aligned} D_m(k+1) &= A_{Dm}D(k) + B_{Dm}u(k) \\ D_m^T(k+1) &= \left[D_m^T(k) \;\; u^T(k)\right]\begin{bmatrix} A_{Dm}^T \\ B_{Dm}^T \end{bmatrix}. \end{aligned} \tag{3.14}$$

Let,

$$m_k = \hat{D}_m^T(k+1), \;\; n_k = \left[\hat{D}_m^T(k) \;\; u^T(k)\right], \; L = \begin{bmatrix} \hat{A}_{Dm}^T \\ \hat{B}_{Dm}^T \end{bmatrix} \tag{3.15}$$

$$\begin{bmatrix} m_1 \\ m_2 \\ : \\ : \\ m_k \end{bmatrix} = \begin{bmatrix} n_1 \\ n_2 \\ : \\ : \\ n_k \end{bmatrix} L \;\; \Rightarrow M = NL \;\Rightarrow\; L = (N^T N)^{-1} N^T M \tag{3.16}$$

where,

$$M = \begin{bmatrix} m_1 \\ m_2 \\ : \\ : \\ m_k \end{bmatrix} \; and \;\; N = \begin{bmatrix} n_1 \\ n_2 \\ : \\ : \\ n_k \end{bmatrix} \tag{3.17}$$

$$\begin{aligned} N^T N &= n_1^T n_1 + n_2^T n_2 + \dots\dots\dots\dots + n_k^T n_k \\ N^T M &= n_1^T m_1 + n_2^T m_2 + \dots\dots\dots\dots + n_k^T m_k \end{aligned} \tag{3.18}$$

$$L(k+1) = (\lambda N^T N + n_{k+1}^T n_{k+1})^{-1} (\lambda N^T M + n_{k+1}^T m_{k+1}) \tag{3.19}$$

where λ denotes the forgetting factor and $0 \le \lambda \le 1$.

If $\lambda = 1$, then all the previous disturbance state estimates and the input are used to get an estimate of the disturbance model parameters. If $\lambda = 0$, only the present values are used to get the disturbance model parameter estimates. For any other value of $0 < \lambda < 1$, the present estimates and inputs are given more importance than all the previous values. An initial batch processing step followed by a recursive procedure given in the Equation (3.19) is used to obtain the disturbance model. The efficacy of this approach is shown using an example in the next segment.

3.3 Results

In this section, an example is presented, wherein the disturbance model is assumed to be linear time invariant. Results for various values of forgetting factor λ are presented. The parameters of a third order, TITO discrete time system whose output information is available are given as follows,

$$F_m(x(k)) = A_m\, x(k)\,,\ L_2(x,D) = A_D D\,, \tag{3.20}$$

$$A_m = \begin{bmatrix} -0.89 & 0 & 0 \\ 0 & 0.5 & 0 \\ -0.2 & -0.1 & -0.3 \end{bmatrix},\ A_D = \begin{bmatrix} 0 & 0.1 & 1 \\ 1 & 0 & 0.1 \\ -0.2 & -0.3 & -0.8 \end{bmatrix}, \tag{3.21}$$

$$B = \begin{bmatrix} 0.5 & 0 \\ 0.2 & 0.1 \\ 0 & 0.8 \end{bmatrix},\ C = \begin{bmatrix} 0.1 & 0.1 & 0.2 \\ 0 & 0.5 & 0 \end{bmatrix}. \tag{3.22}$$

The process and measurement noise are assumed to be normally distributed, their statistics is assumed to be known and the two inputs u are assumed to be a mixture of sinusoidal signals. The initial state is assumed to be unknown and the estimation error covariance matrix is selected such that the error from the disturbance states is assumed to be more than that of the system states.

The initial values for A_{Dm} and B_{Dm} are chosen as follows,

$$A_{Dm} = \begin{bmatrix} 0.1 & 0 & 0 \\ 0 & 0.1 & 0 \\ 0 & 0 & 0.1 \end{bmatrix}, \; B_{Dm} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \\ 1 & 1 \end{bmatrix} \tag{3.23}$$

The outputs of the extended model with the disturbance term updated recursively in comparison with the actual system outputs with a forgetting factor $\lambda = 1$ are shown in Fig.9 and.Fig.10. As can be seen in these two figures, the disturbance model outputs exactly track the system outputs. This is a cross-validation step and the figures indicate satisfactory performance in terms of output tracking. The convergence of the disturbance model parameters is shown in Fig.11 and.Fig.12.

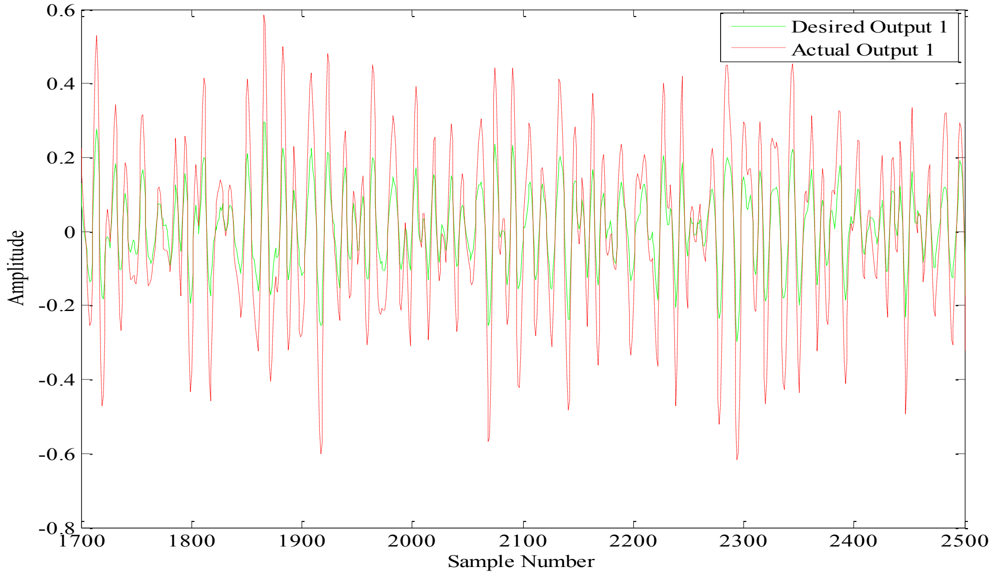

Fig.9. Output 1 of Disturbance Identified Model v/s System Output 1.

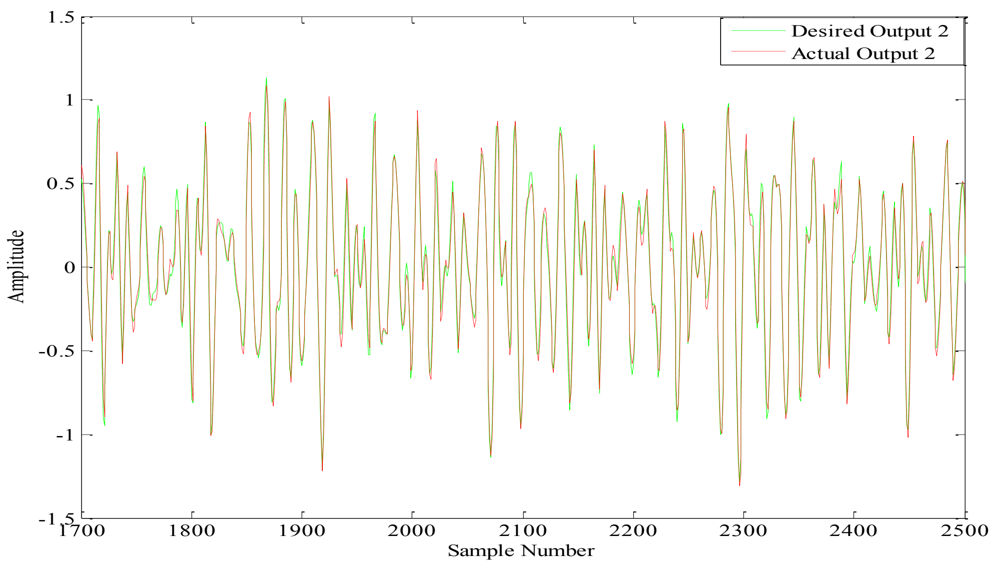

Fig.10. Output 2 of Disturbance Identified Model v/s System Output 2.

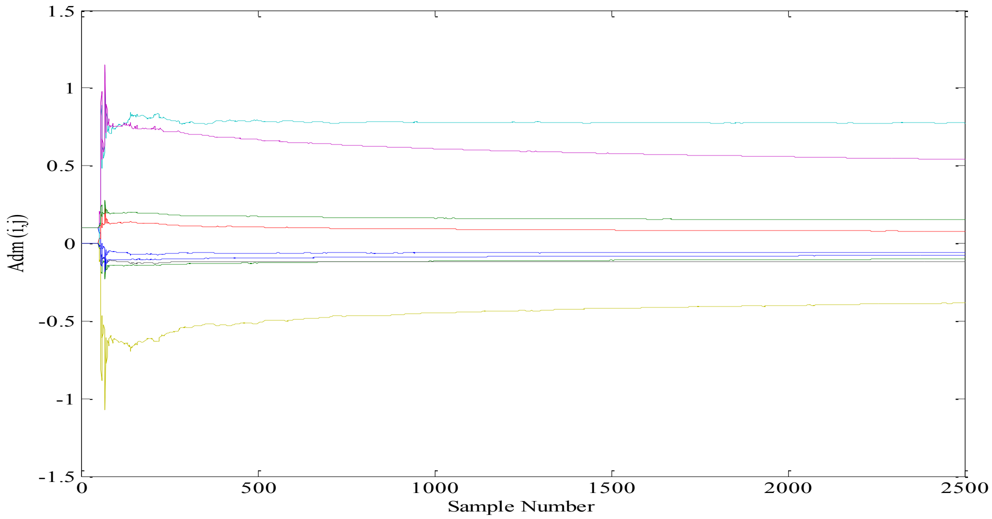

Fig.11. Convergence of the Entries in A_{Dm}.

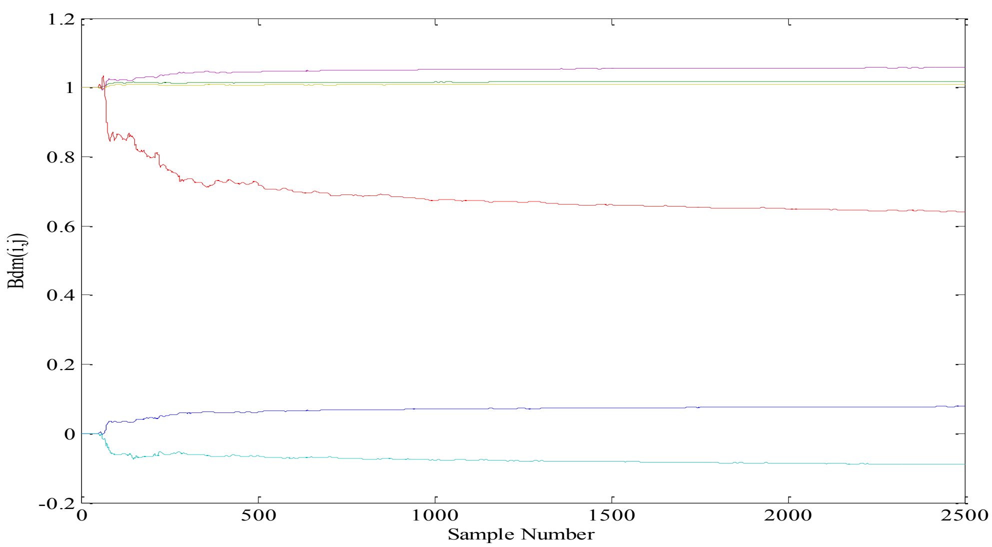

Fig.12. Convergence of the Entries in B_{Dm}.

The convergence of the disturbance model parameters for $\lambda = 0.95$ and $\lambda = 0.9$ is shown in Fig.13 and Fig.14. As seen in these figures, time taken for the parameters to converge increases with decrease in the forgetting factor.

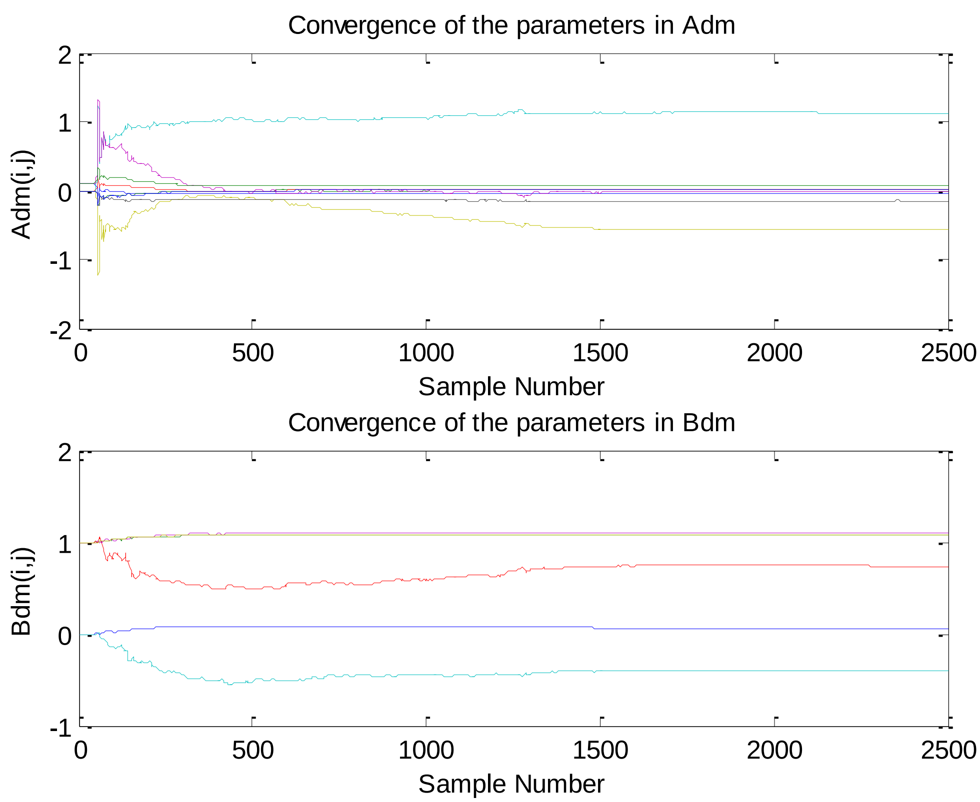

Fig.13. Convergence of Disturbance Model Parameters for λ = 0.95.

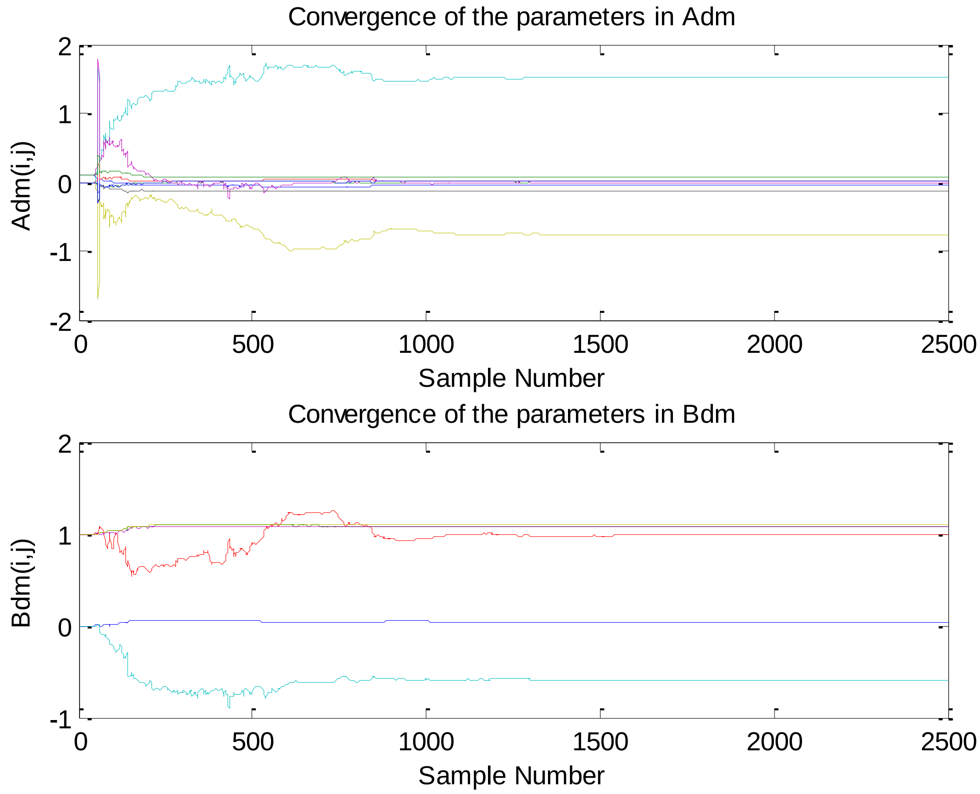

Fig.14. Convergence of Disturbance Model Parameters for $\lambda = 0.9$.

3.4 Concluding Remarks

A method to identify disturbance which includes the system model uncertainties and the exogenous inputs is proposed. The exogenous inputs are assumed to be a set of sinusoidal signals whose frequencies are known, but the amplitudes and phases are unknown. From the simulation results, it can be seen that the proposed approach performs well in the sense of the convergence of the identified disturbance model parameters. Cross validation indicates that the output of the system matches with that of the model with an identified disturbance term. Further research needs to be pursued for the case when no information about the exogenous inputs is known a priori. There is also a need to extend the proposed method to nonlinear systems and systems described using Partial Differential Equations (PDEs).

CHAPTER IV

CONCLUSIONS AND SUMMARY

This work is aimed at the investigation of two problems and the development of appropriate solutions for each of them. First, the problem of synthesis of stabilizing controllers for a multivariable system using the data alone, and the other, the problem of estimation of disturbance acting on a system using the knowledge of partial state measurements.

Data based synthesis of controllers is an important area of research since there is no assumption made about the system to be controlled. In other words, it eliminates the need to identify an unknown system before formulating control laws for that particular system. Most of the data based techniques proposed in the literature, till date, either concentrate on the synthesis of a single controller or are confined to SISO systems. In this work, a class of linear stable multivariable systems is considered. A three step procedure involving sequential design, search for root invariant regions and stability check is proposed that generates a set of stabilizing controllers given the frequency response measurements of the system. The form of the controller is chosen beforehand. Different sets of controllers can be obtained by changing the form of the controller chosen in the first place. Examples of TITO second and third order system are used to depict the efficacy of the proposed technique.

Disturbance is an undesired phenomenon that can inhibit a system's performance. Undesired exogenous signals that act as an input to a system can obscure the development of a control law for that system. In this work, the disturbance is assumed to include the model uncertainties of the system as well as the exogenous inputs acting on the system. It is assumed that there is access to only the outputs of the actual system and not to its states. Once the form of the disturbance is known, it becomes easier to construct actions that can cancel this disturbance while simultaneously achieving the required design specifications. As a result, a procedure is proposed to estimate the disturbance using system output measurements. Disturbance is modeled as a linear dynamical system with a set of sinusoidal signals acting as input to this system. The frequencies of this set of sinusoidal signals are assumed to be the same as the frequencies at which the exogenous signals perturb the given system. The disturbance is appended to the known model of the system to get a dynamically equivalent extended model. Kalman filter is used to obtain the disturbance state estimates which are then used to obtain a least squares estimate of the disturbance parameters. Simulation results for a third order, TITO discrete time system are shown which demonstrate good convergence properties and cross validation approves the veracity of the proposed disturbance modeling technique.

4.1 Future Research

The proposed model free method of synthesizing controllers for MIMO systems is applicable to linear systems. Specifically, the examples considered in this work are only stable MIMO systems. For unstable systems, the number of open loop unstable poles should be known to apply the proposed technique, or a method to find the number of unstable poles from the data needs to be devised. Further research needs to be pursued to include a more general class of systems like nonlinear systems and discrete time systems.

The proposed approach to disturbance modeling assumed that the frequencies at which the exogenous signals act on a given discrete time MIMO system are known *a priori*. This needs to be extended to the case where no such information is known beforehand. An elegant stability proof has to be given for the proposed estimation technique. There is also a need to extend the approach to a higher class of systems.

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