ROBUST BEAMFORMING FOR OFDM MODULATED TWO-WAY MIMO RELAY NETWORK

A Thesis

by

JIANWEI ZHOU

Submitted to the Office of Graduate Studies of
Texas A&M University
in partial fulfillment of the requirements for the degree of

MASTER OF SCIENCE

May 2012

Major Subject: Electrical Engineering

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Approved by:

Chair of Committee,	Shuguang Cui
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ABSTRACT

Robust Beamforming for OFDM Modulated
Two-Way MIMO Relay Network. (May 2012)

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This thesis studies a two-way relay network (TWRN), which consists of two single antenna source nodes and a multi-antenna relay node. The source nodes exchange information via the assistance of the relay node in the middle. The relay scheme in this TWRN is amplify-and-forward (AF) based analog network coding (ANC). A robust beamforming matrix optimization algorithm is presented here with the objective to minimize the transmit power at the relay node under given signal to interference and noise ratio (SINR) requirements of source nodes. This problem is first formulated as a non-convex optimization problem, and it is next relaxed to a semi-definite programming (SDP) problem by utilizing the S-procedure and rank-one relaxation. This robust beamforming optimization algorithm is further validated in a MATLAB-based orthogonal frequency-division multiplexing (OFDM) MIMO two-way relay simulation system. To better investigate the performance of this beamforming algorithm in practical systems, synchronization issues such as standard timing offset (STO) and carrier frequency offset (CFO) are considered in simulation. The transmission channel is modeled as a frequency selective fading channel, and the source nodes utilize training symbols to perform minimum mean-square error (MMSE) channel estimation. BER curves under perfect and imperfect synchronization are presented to show the performance of TWRN with ANC. It is shown that the outage probability of robust beamforming algorithm is tightly related to the SINR requirements at the source nodes, and the outage probability increases significantly when the SINR requirements are high.

To my family

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Two years in Aggieland flies by much too fast, and I am fortunate to meet many nice friends here. All the members from our research group are great. The first group dinner made me feel so welcomed when I first moved from Hawaii to here. I enjoyed every hotpot or dumping dinner with them. Especially, I would like to thank Meng Zeng and Chuan Huang to take the effort and time to hold and organize these group dinners. The food they cook is the best I ever had in College Station. I also had a great time with my basketball fellows. Playing basketball with them help me to relax from intensive study. I would not forget the moments that I watched NBA and played video games with them.

Last but not the least, I would like to express my deepest love to my parents, my sister, my little nephew for their selfless support and encouragement to me. Although I can only spend very short amount of time with my family every year, they mean a lot to me.

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CHAPTER I

INTRODUCTION

In wireless communication, signal suffers from server attenuation due to various fading effects. To mitigate such issues, multiple antennas can be used to provide diversity and increase the reliability. In particular, multiple-input and multiple-output (MIMO) system can be implemented in a number of different ways to obtain either diversity gain or multiplexing gain [1]. However, for some small wireless devices such as mobile phone and PDA, it may not be practical to have the device equipped with multiple antennas due to size and power constraints. Besides MIMO techniques, relaying has also received much attention because of its capability to improve transmission coverage, network capacity, and system reliability [2]. In addition, relay nodes can be equipped with multiple antennas to take advantage of MIMO gains. Two kinds of relay schemes are commonly used in relay networks, namely amplify-and-forward (AF) and decode-and-forward (DF). Both schemes can be deployed to achieve cooperative diversity in wireless communication [3].

Meanwhile, systems with two-way transmission over relay nodes are well known as two-way relay networks (TWRNs), and recently TWRN has been well studied in our community [4] [5] [6] [7] [8]. A simplest TWRN consists of two source nodes and one relay node. Traditionally, four time slots are required to complete one round of information exchange between two source nodes. It was later found that network coding can be applied in TWRN to enhance network throughput by allowing signal mixture at the relay node. In particular, a simplified version of physical-layer network coding called analog network coding (ANC) [9] can be applied to reduce to the number

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The thesis follows the style of *IEEE Journal on Selected Areas in Communications*.

of transmission time slots from four to two. In the first time slot, source nodes S1 and S2 send signals to the relay node R simultaneously. In the second time slot, R multiplies the superimposed received signal by a beamforming matrix and broadcasts the resulting signal back to S1 and S2. Since ANC is an AF based network coding scheme, no decoding is necessary at the relay node. The above scheme requires the source nodes to have perfect channel state information (CSI) over all transmission channels such that the self-interference can be subtracted out at respective source nodes. This assumption is generally not true in practice, where the quality of channel estimation is a key factor to determine the overall performance of TWRN. In this thesis, we consider a robust beamforming design problem under channel uncertainty in an OFDM modulated TWRN with ANC. We aim to minimize the relay transmission power satisfying the signal to interference and noise ratio (SINR) requirements of source nodes. In order to investigate the performance of the robust beamforming optimization algorithm in a practical communication system, we test the algorithm in a MATLAB-based two-way relay network simulation system. In particular, source nodes perform MMSE channel estimation to obtain CSI, and the channel estimation error is treated as the channel uncertainty in our robust beamforming optimization problem. In addition, we know OFDM is sensitive to synchronization errors caused by symbol timing offset (STO) or carrier frequency offset (CFO), which lead to inter-symbol interference (ISI) and inter-carrier interference (ICI). In our simulation, cyclic prefix (CP) based STO detection scheme and CFO estimation scheme are employed to reduce the effect of synchronization offsets on the performance of TWRN.

The rest of the paper is organized as follows. Chapter II introduces the system model of the OFDM modulated TWRN with ANC and presents the robust beamforming optimization problem. Chapter III discusses the transformation of the original non-convex problem to a convex problem via the help of S-procedure and rank-one relaxation. Two methods are presented to reconstruct a rank-one beamforming matrix from the rank-one relaxed optimization problem. Chapter IV presents the simulation results. Chapter V concludes the thesis.

CHAPTER II

SYSTEM MODEL

In this thesis, we consider a robust beamforming design problem for a two-way relay network similar to that in [10]. The system consists of two source nodes S1 and S2 and relay node R as shown in Fig. 1. The relay node R has M ($M \geq 2$) antennas

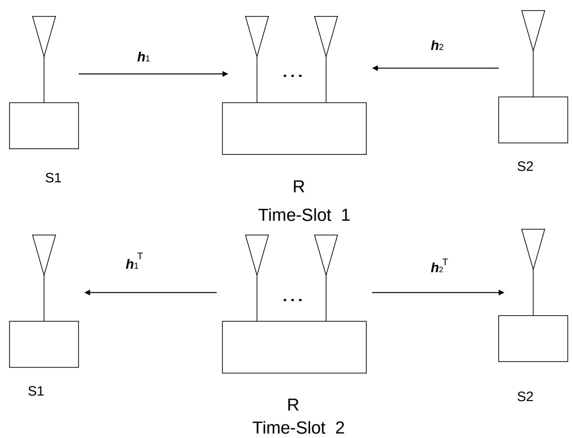

Figure 1: System Model

and source nodes S1 and S2 each has single antenna. In particular, we consider the transmit signals $s_1(n), s_2(n)$ from S1 and S2 as OFDM modulated with N subcarriers, where $n = 1, 2, ..., N$ denotes the subcarrier index. Let $\boldsymbol{h}_i(n) \in \mathbb{C}^{M\times 1}, i = 1, 2,$ denote the channel vector from source node i to relay node R at the nth subcarrier, which is assumed to be a flat channel due to OFDM. The exchange of one round of information between S1 and S2 in TWRN is completed in two time slots. In the first time slot, S1 and S2 transmit simultaneously to R. The received signal vector over the nth subcarrier at R is

$$\boldsymbol{y}_R(n) = \boldsymbol{h}_1(n)\sqrt{p_1}s_1(n) + \boldsymbol{h}_2(n)\sqrt{p_2}s_2(n) + \boldsymbol{z}_R(n), \tag{2.1}$$

where p_1 and p_2 denote the transmit power of S1 and S2, and z_R is circularly symmetric complex Gaussian (CSCG) noise with zero mean and covariance matrix $\sigma_R^2\boldsymbol{I}$. In the second time slot, R multiplies the received signal by a beamforming matrix $\boldsymbol{A}(n) \in \mathbb{C}^{M\times M}$ and broadcasts the resulting nth subcarrier signal back to S1 and S2. The transmit signal from R is

$$\boldsymbol{x}_R(n) = \boldsymbol{A}(n)\boldsymbol{y}_R(n). \tag{2.2}$$

Channel reciprocity is assumed, and the received signal at S1 and S2 are

$$y_1(n) = \boldsymbol{h}_1^T(n)\boldsymbol{A}(n)\boldsymbol{h}_1(n)\sqrt{p_1}s_1(n)+\boldsymbol{h}_1^T(n)\boldsymbol{A}(n)\boldsymbol{h}_2(n)\sqrt{p_2}s_2(n)+\boldsymbol{h}_1^T(n)\boldsymbol{A}(n)\boldsymbol{z}_R(n)+z_1(n), \tag{2.3}$$

$$y_2(n) = \boldsymbol{h}_2^T(n)\boldsymbol{A}(n)\boldsymbol{h}_2(n)\sqrt{p_2}s_2(n)+\boldsymbol{h}_2^T(n)\boldsymbol{A}(n)\boldsymbol{h}_1(n)\sqrt{p_1}s_1(n)+\boldsymbol{h}_2^T(n)\boldsymbol{A}(n)\boldsymbol{z}_R(n)+z_2(n). \tag{2.4}$$

For notational convenience, we drop the index n here, and the rest of the analysis applies to each OFDM subcarrier signal.

AF based ANC requires the source nodes to have perfect CSI of all channels so as to cancel out the self-interference $\boldsymbol{h}_1^T\boldsymbol{A}\boldsymbol{h}_1\sqrt{p_1}s_1$ from y_1 and $\boldsymbol{h}_2^T\boldsymbol{A}\boldsymbol{h}_2\sqrt{p_2}s_2$ from y_2. Perfect CSI is generally not practical, and a more realistic approach is to model the channels as $\boldsymbol{h}_1 = \hat{\boldsymbol{h}}_1 + \Delta\boldsymbol{h}_1$ and $\boldsymbol{h}_2 = \hat{\boldsymbol{h}}_2 + \Delta\boldsymbol{h}_2$, where $\hat{\boldsymbol{h}}_1$ and $\hat{\boldsymbol{h}}_2$ denote the estimated channels. In addition, the estimation errors are bounded as $\|\Delta\boldsymbol{h}_1\| \leq \epsilon_1$, $\|\Delta\boldsymbol{h}_2\| \leq \epsilon_2$. Replacing $\boldsymbol{h}$ by $\hat{\boldsymbol{h}}$ in the ANC self-interference cancelation, the resulting signal is

$$\tilde{y}_1 \approx \underbrace{(\hat{\boldsymbol{h}}_1^T \boldsymbol{A}\Delta\boldsymbol{h}_1 + \Delta\boldsymbol{h}_1^T \boldsymbol{A}\hat{\boldsymbol{h}}_1)\sqrt{p_1}s_1}_{\text{remaining self-interference}} + \underbrace{(\hat{\boldsymbol{h}}_1^T \boldsymbol{A}\hat{\boldsymbol{h}}_2 + \hat{\boldsymbol{h}}_1^T \boldsymbol{A}\Delta\boldsymbol{h}_2 + \Delta\boldsymbol{h}_1^T \boldsymbol{A}\hat{\boldsymbol{h}}_2)\sqrt{p_2}s_2}_{\text{desired signal}} + \underbrace{(\hat{\boldsymbol{h}}_1^T \boldsymbol{A} + \Delta\boldsymbol{h}_1^T \boldsymbol{A})\boldsymbol{z}_R + z_1}_{\text{noise}}, \tag{2.5}$$

$$\tilde{y}_2 \approx \underbrace{(\hat{\boldsymbol{h}}_2^T \boldsymbol{A}\Delta\boldsymbol{h}_2 + \Delta\boldsymbol{h}_2^T \boldsymbol{A}\hat{\boldsymbol{h}}_2)\sqrt{p_2}s_2}_{\text{remaining self-interference}} + \underbrace{(\hat{\boldsymbol{h}}_2^T \boldsymbol{A}\hat{\boldsymbol{h}}_1 + \hat{\boldsymbol{h}}_2^T \boldsymbol{A}\Delta\boldsymbol{h}_1 + \Delta\boldsymbol{h}_2^T \boldsymbol{A}\hat{\boldsymbol{h}}_1)\sqrt{p_1}s_1}_{\text{desired signal}} + \underbrace{(\hat{\boldsymbol{h}}_2^T \boldsymbol{A} + \Delta\boldsymbol{h}_2^T \boldsymbol{A})\boldsymbol{z}_R + z_2}_{\text{noise}}. \tag{2.6}$$

We would like to minimize the transmit power as a function of $\boldsymbol{A}$ given the SINR requirements at S1 and S2. The transmit power at R is

$$\begin{aligned} p_R(\boldsymbol{A}) &= \|\boldsymbol{A}\boldsymbol{h}_1\|^2 p_1 s_1^2 + \|\boldsymbol{A}\boldsymbol{h}_2\|^2 p_2 s_2^2 + \mathrm{tr}(\boldsymbol{A}^H\boldsymbol{A}) z_R^2, \\ &= (\hat{\boldsymbol{h}}_1 + \Delta\boldsymbol{h}_1)^H \boldsymbol{A}^H \boldsymbol{A}(\hat{\boldsymbol{h}}_1 + \Delta\boldsymbol{h}_1) p_1 s_1^2 \\ &\quad + (\hat{\boldsymbol{h}}_2 + \Delta\boldsymbol{h}_2)^H \boldsymbol{A}^H \boldsymbol{A}(\hat{\boldsymbol{h}}_2 + \Delta\boldsymbol{h}_2 p_2 s_2^2 + \mathrm{tr}(\boldsymbol{A}\boldsymbol{A}^H) z_R^2 \\ &= \hat{\boldsymbol{h}}_1^H \boldsymbol{A}^H \boldsymbol{A}\hat{\boldsymbol{h}}_1 p_1 s_1^2 + \hat{\boldsymbol{h}}_2^H \boldsymbol{A}^H \boldsymbol{A}\hat{\boldsymbol{h}}_2 p_2 s_2^2 + \mathrm{tr}(\boldsymbol{A}^H\boldsymbol{A}) z_R^2 \\ &\quad + 2\Re(\hat{\boldsymbol{h}}_1^H \boldsymbol{A}^H \boldsymbol{A}\Delta\boldsymbol{h}_1) p_1 s_1^2 + 2\Re(\hat{\boldsymbol{h}}_2^H \boldsymbol{A}^H \boldsymbol{A}\Delta\boldsymbol{h}_2) p_2 s_2^2 \\ &\quad + \Delta\boldsymbol{h}_1^H \boldsymbol{A}^H \boldsymbol{A}\Delta\boldsymbol{h}_1 p_1 s_1^2 + \Delta\boldsymbol{h}_2^H \boldsymbol{A}^H \boldsymbol{A}\Delta\boldsymbol{h}_2 p_2 s_2^2 \end{aligned} \tag{2.7}$$

The SINRs at S1 and S2 are

$$\text{SINR}_1 = \frac{|\hat{\boldsymbol{h}}_1^T \boldsymbol{A}\hat{\boldsymbol{h}}_2 + \hat{\boldsymbol{h}}_1^T \boldsymbol{A}\Delta\boldsymbol{h}_2 + \Delta\boldsymbol{h}_1^T \boldsymbol{A}\hat{\boldsymbol{h}}_2|^2 p_2 s_2^2}{|\hat{\boldsymbol{h}}_1^T \boldsymbol{A}\Delta\boldsymbol{h}_1 + \Delta\boldsymbol{h}_1^T \boldsymbol{A}\hat{\boldsymbol{h}}_1|^2 p_1 s_1^2 + \|(\hat{\boldsymbol{h}}_1^T + \Delta\boldsymbol{h}_1^T)\boldsymbol{A}\|^2 z_R^2 + z_1^2}, \tag{2.8}$$

$$\text{SINR}_2 = \frac{|\hat{\boldsymbol{h}}_2^T \boldsymbol{A}\hat{\boldsymbol{h}}_1 + \hat{\boldsymbol{h}}_2^T \boldsymbol{A}\Delta\boldsymbol{h}_1 + \Delta\boldsymbol{h}_2^T \boldsymbol{A}\hat{\boldsymbol{h}}_1|^2 p_1 s_1^2}{|\hat{\boldsymbol{h}}_2^T \boldsymbol{A}\Delta\boldsymbol{h}_2 + \Delta\boldsymbol{h}_2^T \boldsymbol{A}\hat{\boldsymbol{h}}_2|^2 p_2 s_2^2 + \|(\hat{\boldsymbol{h}}_2^T + \Delta\boldsymbol{h}_2^T)\boldsymbol{A}\|^2 z_R^2 + z_2^2}. \tag{2.9}$$

Let γ_1,γ_2 be the SINR requirements at S1 and S2 respectively; the robust optimization problem can be formulated as follows

$$\begin{aligned} \min_{\boldsymbol{A}} \quad & \max_{\|\Delta\boldsymbol{h}_1\|\le\epsilon_1, \|\Delta\boldsymbol{h}_2\|\le\epsilon_2} p_R \\ \text{s.t.} \quad & \min_{\|\Delta\boldsymbol{h}_1\|\le\epsilon_1, \|\Delta\boldsymbol{h}_2\|\le\epsilon_2} \text{SINR}_1 \ge \gamma_1 \\ & \min_{\|\Delta\boldsymbol{h}_1\|\le\epsilon_1, \|\Delta\boldsymbol{h}_2\|\le\epsilon_2} \text{SINR}_2 \ge \gamma_2. \end{aligned}$$

This problem can be equivalently reformulated as

$$\mathcal{Q}_1: \quad \min_{\boldsymbol{A}, t} \ t \tag{2.10}$$

$$\text{s.t.} \quad \min_{\|\Delta\boldsymbol{h}_1\|\le\epsilon_1, \|\Delta\boldsymbol{h}_2\|\le\epsilon_2} \text{SINR}_1 \ge \gamma_1 \tag{2.11}$$

$$\min_{\|\Delta\boldsymbol{h}_1\|\le\epsilon_1, \|\Delta\boldsymbol{h}_2\|\le\epsilon_2} \text{SINR}_2 \ge \gamma_2 \tag{2.12}$$

$$\max_{\|\Delta\boldsymbol{h}_1\|\le\epsilon_1, \|\Delta\boldsymbol{h}_2\|\le\epsilon_2} p_R \le t. \tag{2.13}$$

Definition 1. *Given the SINR requirements γ_1 and γ_2, the channel realizations $\boldsymbol{h}_1$ and $\boldsymbol{h}_2$, we define the outage as the event that $SINR_1 < \gamma_1$ or $SINR_2 < \gamma_2$.*

CHAPTER III

SDP FORMULATION

In $\mathcal{Q}_1$, $\Delta \boldsymbol{h}_1$ and $\Delta \boldsymbol{h}_2$ are continuous, which means that the number of constraints in $\mathcal{Q}_1$ is infinite. Therefore, $\mathcal{Q}_1$ needs to be transformed into a solvable problem with finite constraints. In fact, $\mathcal{Q}_1$ can be relaxed as a semidefinite programming (SDP) [11] problem by applying the S-procedure and rank-one relaxation. The resulting SDP problem is convex, and it can be solved efficiently using various interior point methods.

Theorem 1 (S-procedure)**.** *Given Hermitian matrices $\boldsymbol{A}_j \in \mathbb{C}^{n\times n}$, vectors $\boldsymbol{b}_j \in \mathbb{C}^n$ and numbers $c_j \in \mathbb{R}$ for $j = 0, 1, 2$, define the functions $f_j(\boldsymbol{x}) = \boldsymbol{x}^H \boldsymbol{A}_j \boldsymbol{x} + 2\Re(\boldsymbol{b}_j^H \boldsymbol{x}) + c_j$ for $\boldsymbol{x} \in \mathbb{C}^n$. The following two conditions are equivalent.*

1. *$f_0(\boldsymbol{x}) \geq 0$ for every $\boldsymbol{x} \in \mathbb{C}^n$ such that $f_1(\boldsymbol{x}) \geq 0$ and $f_2(\boldsymbol{x}) \geq 0$;*
2. *There exist $\lambda_1, \lambda_2 \geq 0$ such that $\begin{pmatrix} \boldsymbol{A}_0 & \boldsymbol{b}_0 \\ \boldsymbol{b}_0^H & c_0 \end{pmatrix} \succeq \lambda_1 \begin{pmatrix} \boldsymbol{A}_1 & \boldsymbol{b}_1 \\ \boldsymbol{b}_1^H & c_1 \end{pmatrix} + \lambda_2 \begin{pmatrix} \boldsymbol{A}_2 & \boldsymbol{b}_2 \\ \boldsymbol{b}_2^H & c_2 \end{pmatrix}$.*

Some details about the S-procedure can be found in [12] and applications can be found in [13]. Here we need to transform the constraints of problem $\mathcal{Q}_1$ into the a quadratic form in terms of $\Delta \boldsymbol{h}$. According to Theorem 1, we can transform the constraints into linear matrix inequalities (LMI). Let us simplify the above constraints. First, we look at the SINR constraints. Note that

$$\boldsymbol{g}^T \boldsymbol{A} \boldsymbol{h} = \left[(\boldsymbol{h} \otimes \boldsymbol{1}_{M\times 1}) \odot (\boldsymbol{1}_{M\times 1} \otimes \boldsymbol{g})\right]^T \operatorname{vec}(\boldsymbol{A}),$$

where $\otimes$ denotes Kronecker product, $\odot$ denotes elements-wise product, and $\boldsymbol{1}_{M\times 1}$ is the all-one column vector. Let s_1 be the numerator of $SINR_1$ given in (2.8), then we have

$$\begin{aligned} s_1 &= \Big| \Big[(\hat{\boldsymbol{h}}_2 \otimes \mathbf{1}_{M\times 1}) \odot (\mathbf{1}_{M\times 1} \otimes \hat{\boldsymbol{h}}_1) \Big]^T \mathrm{vec}(\boldsymbol{A}) \\ &\quad + \Big[(\Delta \boldsymbol{h}_2 \otimes \mathbf{1}_{M\times 1}) \odot (\mathbf{1}_{M\times 1} \otimes \hat{\boldsymbol{h}}_1) \Big]^T \mathrm{vec}(\boldsymbol{A}) \\ &\quad + \Big[(\hat{\boldsymbol{h}}_2 \otimes \mathbf{1}_{M\times 1}) \odot (\mathbf{1}_{M\times 1} \otimes \Delta \boldsymbol{h}_1) \Big]^T \mathrm{vec}(\boldsymbol{A}) \Big|^2 p_2 s_2^2 \\ &= |\big(\tilde{\boldsymbol{h}}_2 \odot \breve{\boldsymbol{h}}_1 + \Delta\tilde{\boldsymbol{h}}_2 \odot \breve{\boldsymbol{h}}_1 + \tilde{\boldsymbol{h}}_2 \odot \Delta\breve{\boldsymbol{h}}_1\big)^T \boldsymbol{a}|^2 p_2 s_2^2, \end{aligned}$$

where $\boldsymbol{a} = \mathrm{vec}(\boldsymbol{A})$, $\tilde{\boldsymbol{h}}_i \triangleq \hat{\boldsymbol{h}}_i \otimes \mathbf{1}_{M\times 1}$ and $\breve{\boldsymbol{h}}_i \triangleq \mathbf{1}_{M\times 1} \otimes \hat{\boldsymbol{h}}_i$. Let $\bar{\boldsymbol{A}} \triangleq \boldsymbol{a}\boldsymbol{a}^H$, we rewrite s_1 as

$$\begin{aligned} s_1 &= \Big[\big(\tilde{\boldsymbol{h}}_2 \odot \breve{\boldsymbol{h}}_1\big)^T + \big(\Delta\tilde{\boldsymbol{h}}_2 \odot \breve{\boldsymbol{h}}_1\big)^T + \big(\tilde{\boldsymbol{h}}_2 \odot \Delta\breve{\boldsymbol{h}}_1\big)^T \Big] \bar{\boldsymbol{A}} p_2 s_2^2 \\ &\quad * \Big[\big(\tilde{\boldsymbol{h}}_2 \odot \breve{\boldsymbol{h}}_1\big)^* + \big(\Delta\tilde{\boldsymbol{h}}_2 \odot \breve{\boldsymbol{h}}_1\big)^* + \big(\tilde{\boldsymbol{h}}_2 \odot \Delta\breve{\boldsymbol{h}}_1\big)^* \Big] \\ &= c_1 + [\big(\tilde{\boldsymbol{h}}_2 \odot \breve{\boldsymbol{h}}_1\big)^T \bar{\boldsymbol{A}} p_2 s_2^2 \big(\Delta\tilde{\boldsymbol{h}}_2 \odot \breve{\boldsymbol{h}}_1\big)^* + \big(\Delta\tilde{\boldsymbol{h}}_2 \odot \breve{\boldsymbol{h}}_1\big)^T \bar{\boldsymbol{A}} p_2 s_2^2 \big(\tilde{\boldsymbol{h}}_2 \odot \breve{\boldsymbol{h}}_1\big)^*] \\ &\quad + [\big(\tilde{\boldsymbol{h}}_2 \odot \breve{\boldsymbol{h}}_1\big)^T \bar{\boldsymbol{A}} p_2 s_2^2 \big(\tilde{\boldsymbol{h}}_2 \odot \Delta\breve{\boldsymbol{h}}_1\big)^* + \big(\tilde{\boldsymbol{h}}_2 \odot \Delta\breve{\boldsymbol{h}}_1\big)^T \bar{\boldsymbol{A}} p_2 s_2^2 \big(\tilde{\boldsymbol{h}}_2 \odot \breve{\boldsymbol{h}}_1\big)^*] \\ &\quad + [\big(\Delta\tilde{\boldsymbol{h}}_2 \odot \breve{\boldsymbol{h}}_1\big)^T \bar{\boldsymbol{A}} p_2 s_2^2 \big(\tilde{\boldsymbol{h}}_2 \odot \Delta\breve{\boldsymbol{h}}_1\big)^* + \big(\tilde{\boldsymbol{h}}_2 \odot \Delta\breve{\boldsymbol{h}}_1\big)^T \bar{\boldsymbol{A}} p_2 s_2^2 \big(\Delta\tilde{\boldsymbol{h}}_2 \odot \breve{\boldsymbol{h}}_1\big)^*] \\ &\quad + [\big(\Delta\tilde{\boldsymbol{h}}_2 \odot \breve{\boldsymbol{h}}_1\big)^T \bar{\boldsymbol{A}} p_2 s_2^2 \big(\Delta\tilde{\boldsymbol{h}}_2 \odot \breve{\boldsymbol{h}}_1\big)^* + \big(\tilde{\boldsymbol{h}}_2 \odot \Delta\breve{\boldsymbol{h}}_1\big)^T \bar{\boldsymbol{A}} p_2 s_2^2 \big(\tilde{\boldsymbol{h}}_2 \odot \Delta\breve{\boldsymbol{h}}_1\big)^*], \end{aligned}$$

where $c_1 = \big(\tilde{\boldsymbol{h}}_2 \odot \breve{\boldsymbol{h}}_1\big)^T \bar{\boldsymbol{A}} p_2 s_2^2 \big(\tilde{\boldsymbol{h}}_2 \odot \breve{\boldsymbol{h}}_1\big)^*$. Let $\boldsymbol{h} = \mathrm{vec}(\boldsymbol{h}_1, \boldsymbol{h}_2)$, $\boldsymbol{G}_1 = [\boldsymbol{I}_M, \boldsymbol{O}_M]$, $\boldsymbol{G}_2 = [\boldsymbol{O}_M, \boldsymbol{I}_M]$, $\boldsymbol{D}_R = \boldsymbol{I}_M \otimes \mathbf{1}_{M\times 1}$, and $\boldsymbol{D}_L = \mathbf{1}_M \otimes \boldsymbol{I}_{M\times 1}$, then

$$\begin{aligned} \Delta\tilde{\boldsymbol{h}}_1 = \Delta\boldsymbol{h}_1 \otimes \mathbf{1}_{M\times 1} &= [\boldsymbol{I}_M \otimes \mathbf{1}_{M\times 1}]\Delta\boldsymbol{h}_1 &= \boldsymbol{D}_R \boldsymbol{G}_1 \Delta\boldsymbol{h}, \\ \Delta\breve{\boldsymbol{h}}_1 = \mathbf{1}_{M\times 1} \otimes \Delta\boldsymbol{h}_1 &= [\mathbf{1}_{M\times 1} \otimes \boldsymbol{I}_M]\Delta\boldsymbol{h}_1 &= \boldsymbol{D}_L \boldsymbol{G}_1 \Delta\boldsymbol{h}, \\ \Delta\tilde{\boldsymbol{h}}_2 = \Delta\boldsymbol{h}_2 \otimes \mathbf{1}_{M\times 1} &= [\boldsymbol{I}_M \otimes \mathbf{1}_{M\times 1}]\Delta\boldsymbol{h}_2 &= \boldsymbol{D}_R \boldsymbol{G}_2 \Delta\boldsymbol{h}, \\ \Delta\breve{\boldsymbol{h}}_2 = \mathbf{1}_{M\times 1} \otimes \Delta\boldsymbol{h}_2 &= [\mathbf{1}_{M\times 1} \otimes \boldsymbol{I}_M]\Delta\boldsymbol{h}_2 &= \boldsymbol{D}_L \boldsymbol{G}_2 \Delta\boldsymbol{h}. \end{aligned}$$

Let $\mathrm{diag}(\boldsymbol{x})$ be the diagonal matrix with its entries being the elements of $\boldsymbol{x}$. Now s_1

can be written as

$$
\begin{aligned}
s_1 &= c_1 + 2\Re\Big(\big[(\tilde{\boldsymbol{h}}_2 \odot \breve{\boldsymbol{h}}_1)^T \bar{\boldsymbol{A}} p_2 s_2^2 \mathrm{diag}(\breve{\boldsymbol{h}}_1^*) \boldsymbol{D}_R \boldsymbol{G}_2 \\
&\quad + \big(\tilde{\boldsymbol{h}}_2 \odot \breve{\boldsymbol{h}}_1\big)^T \bar{\boldsymbol{A}} p_2 s_2^2 \mathrm{diag}(\tilde{\boldsymbol{h}}_2^*) \boldsymbol{D}_L \boldsymbol{G}_1 \big] \Delta \boldsymbol{h}^* \Big) \\
&\quad + \Delta \boldsymbol{h}^T [\boldsymbol{G}_2^T \boldsymbol{D}_R^T \mathrm{diag}(\breve{\boldsymbol{h}}_1) \bar{\boldsymbol{A}} p_2 s_2^2 \mathrm{diag}(\tilde{\boldsymbol{h}}_2^*) \boldsymbol{D}_L \boldsymbol{G}_1 \\
&\quad + \boldsymbol{G}_1^T \boldsymbol{D}_L^T \mathrm{diag}(\tilde{\boldsymbol{h}}_2) \bar{\boldsymbol{A}} p_2 s_2^2 \mathrm{diag}(\breve{\boldsymbol{h}}_1^*) \boldsymbol{D}_R \boldsymbol{G}_2 \\
&\quad + \boldsymbol{G}_2^T \boldsymbol{D}_R^T \mathrm{diag}(\breve{\boldsymbol{h}}_1) \bar{\boldsymbol{A}} p_2 s_2^2 \mathrm{diag}(\breve{\boldsymbol{h}}_1^*) \boldsymbol{D}_R \boldsymbol{G}_2 \\
&\quad + \boldsymbol{G}_1^T \boldsymbol{D}_L^T \mathrm{diag}(\tilde{\boldsymbol{h}}_2) \bar{\boldsymbol{A}} p_2 s_2^2 \mathrm{diag}(\tilde{\boldsymbol{h}}_2^*) \boldsymbol{D}_L \boldsymbol{G}_1] \Delta \boldsymbol{h}^* \\
&= \Delta \boldsymbol{h}^T \boldsymbol{Q_1} \Delta \boldsymbol{h}^* + 2\Re(\boldsymbol{q}_1^H \Delta \boldsymbol{h}^*) + c_1,
\end{aligned}
$$

where

$$
\begin{aligned}
\boldsymbol{Q}_1 &= \boldsymbol{G}_2^T \boldsymbol{D}_R^T \mathrm{diag}(\breve{\boldsymbol{h}}_1) \bar{\boldsymbol{A}} p_2 s_2^2 \mathrm{diag}(\tilde{\boldsymbol{h}}_2^*) \boldsymbol{D}_L \boldsymbol{G}_1 + \boldsymbol{G}_1^T \boldsymbol{D}_L^T \mathrm{diag}(\tilde{\boldsymbol{h}}_2) \bar{\boldsymbol{A}} p_2 s_2^2 \mathrm{diag}(\breve{\boldsymbol{h}}_1^*) \boldsymbol{D}_R \boldsymbol{G}_2 \\
&+ \boldsymbol{G}_2^T \boldsymbol{D}_R^T \mathrm{diag}(\breve{\boldsymbol{h}}_1) \bar{\boldsymbol{A}} p_2 s_2^2 \mathrm{diag}(\breve{\boldsymbol{h}}_1^*) \boldsymbol{D}_R \boldsymbol{G}_2 + \boldsymbol{G}_1^T \boldsymbol{D}_L^T \mathrm{diag}(\tilde{\boldsymbol{h}}_2) \bar{\boldsymbol{A}} p_2 s_2^2 \mathrm{diag}(\tilde{\boldsymbol{h}}_2^*) \boldsymbol{D}_L \boldsymbol{G}_1, \\
\boldsymbol{q}_1^H &= (\tilde{\boldsymbol{h}}_2 \odot \breve{\boldsymbol{h}}_1)^T \bar{\boldsymbol{A}} p_2 s_2^2 \mathrm{diag}(\breve{\boldsymbol{h}}_1^*) \boldsymbol{D}_R \boldsymbol{G}_2 + \big(\tilde{\boldsymbol{h}}_2 \odot \breve{\boldsymbol{h}}_1\big)^T \bar{\boldsymbol{A}} p_2 s_2^2 \mathrm{diag}(\tilde{\boldsymbol{h}}_2^*) \boldsymbol{D}_L \boldsymbol{G}_1, \\
c_1 &= \big(\tilde{\boldsymbol{h}}_2 \odot \breve{\boldsymbol{h}}_1\big)^T \bar{\boldsymbol{A}} p_2 s_2^2 \big(\tilde{\boldsymbol{h}}_2 \odot \breve{\boldsymbol{h}}_1\big)^*.
\end{aligned}
$$

The denominator t_1 is given as

$$
\begin{aligned}
t_1 &= \big(\Delta\tilde{\boldsymbol{h}}_1 \odot \breve{\boldsymbol{h}}_1\big)^T \bar{\boldsymbol{A}} p_1 s_1^2 \big(\Delta\tilde{\boldsymbol{h}}_1 \odot \breve{\boldsymbol{h}}_1\big)^* + \big(\tilde{\boldsymbol{h}}_1 \odot \Delta\breve{\boldsymbol{h}}_1\big)^T \bar{\boldsymbol{A}} p_1 s_1^2 \big(\tilde{\boldsymbol{h}}_1 \odot \Delta\breve{\boldsymbol{h}}_1\big)^* \\
&\quad + \big(\tilde{\boldsymbol{h}}_1 \odot \Delta\breve{\boldsymbol{h}}_1\big)^T \bar{\boldsymbol{A}} p_1 s_1^2 \big(\Delta\tilde{\boldsymbol{h}}_1 \odot \breve{\boldsymbol{h}}_1\big)^* + \big(\Delta\tilde{\boldsymbol{h}}_1 \odot \breve{\boldsymbol{h}}_1\big)^T \bar{\boldsymbol{A}} p_1 s_1^2 \big(\tilde{\boldsymbol{h}}_1 \odot \Delta\breve{\boldsymbol{h}}_1\big)^* \\
&\quad + \big(\boldsymbol{1}_{M\times 1} \otimes \boldsymbol{h}_1 + \boldsymbol{1}_{M\times 1} \otimes \Delta\boldsymbol{h}_1\big)^T \boldsymbol{E} \odot \bar{\boldsymbol{A}} \sigma_R^2 \big(\boldsymbol{I} \otimes \boldsymbol{h}_1 + \boldsymbol{I} \otimes \Delta\boldsymbol{h}_1\big)^* + z_1^2 \\
&= \big(\Delta\tilde{\boldsymbol{h}}_1 \odot \breve{\boldsymbol{h}}_1\big)^T \bar{\boldsymbol{A}} p_1 s_1^2 \big(\Delta\tilde{\boldsymbol{h}}_1 \odot \breve{\boldsymbol{h}}_1\big)^* + \big(\tilde{\boldsymbol{h}}_1 \odot \Delta\breve{\boldsymbol{h}}_1\big)^T \bar{\boldsymbol{A}} p_1 s_1^2 \big(\tilde{\boldsymbol{h}}_1 \odot \Delta\breve{\boldsymbol{h}}_1\big)^* \\
&\quad + \big(\tilde{\boldsymbol{h}}_1 \odot \Delta\breve{\boldsymbol{h}}_1\big)^T \bar{\boldsymbol{A}} p_1 s_1^2 \big(\Delta\tilde{\boldsymbol{h}}_1 \odot \breve{\boldsymbol{h}}_1\big)^* + \big(\Delta\tilde{\boldsymbol{h}}_1 \odot \breve{\boldsymbol{h}}_1\big)^T \bar{\boldsymbol{A}} p_1 s_1^2 \big(\tilde{\boldsymbol{h}}_1 \odot \Delta\breve{\boldsymbol{h}}_1\big)^* \\
&\quad + \breve{\boldsymbol{h}}_1^T \boldsymbol{E} \odot \bar{\boldsymbol{A}} z_R^2 \breve{\boldsymbol{h}}_1^* + 2\Re(\Delta\breve{\boldsymbol{h}}_1^T \boldsymbol{E} \odot \bar{\boldsymbol{A}} z_R^2 \breve{\boldsymbol{h}}_1^*) + \Delta\breve{\boldsymbol{h}}_1 \boldsymbol{E} \odot \bar{\boldsymbol{A}} z_R^2 \Delta\breve{\boldsymbol{h}}_1^* + z_1^2,
\end{aligned}
$$

where $\boldsymbol{E} = \boldsymbol{I}_M \otimes (\mathbf{1}_{M\times 1}\mathbf{1}_{M\times 1}^T)$. Let $c_2 = \breve{\boldsymbol{h}}_1^T \boldsymbol{E} \odot \bar{\boldsymbol{A}} z_R^2 \breve{\boldsymbol{h}}_1^* + z_1^2$, then

$$\begin{aligned} t_1 &= c_2 + 2\Re(\breve{\boldsymbol{h}}_1^T \boldsymbol{E} \odot \bar{\boldsymbol{A}} z_R^2 \boldsymbol{D}_L \boldsymbol{G}_1 \Delta \boldsymbol{h}^*) + \Delta \boldsymbol{h}^T [\boldsymbol{G}_1^T \boldsymbol{D}_R^T \mathrm{diag}(\breve{\boldsymbol{h}}_1) \bar{\boldsymbol{A}} p_1 \mathrm{diag}(\breve{\boldsymbol{h}}_1^*) \boldsymbol{D}_R \boldsymbol{G}_1 \\ &\quad + \boldsymbol{G}_1^T \boldsymbol{D}_L^T \mathrm{diag}(\tilde{\boldsymbol{h}}_1) \bar{\boldsymbol{A}} p_1 \mathrm{diag}(\tilde{\boldsymbol{h}}_1^*) \boldsymbol{D}_L \boldsymbol{G}_1 + \boldsymbol{G}_1^T \boldsymbol{D}_L^T \mathrm{diag}(\tilde{\boldsymbol{h}}_1) \bar{\boldsymbol{A}} p_1 \mathrm{diag}(\breve{\boldsymbol{h}}_1^*) \boldsymbol{D}_R \boldsymbol{G}_1 \\ &\quad + \boldsymbol{G}_1^T \boldsymbol{D}_R^T \mathrm{diag}(\breve{\boldsymbol{h}}_1) \bar{\boldsymbol{A}} p_1 \mathrm{diag}(\tilde{\boldsymbol{h}}_1^*) \boldsymbol{D}_L \boldsymbol{G}_1 + \boldsymbol{G}_1^T \boldsymbol{D}_L^T \boldsymbol{E} \odot \bar{\boldsymbol{A}} z_R^2 \boldsymbol{D}_L \boldsymbol{G}_1] \Delta \boldsymbol{h}^* \\ &= \Delta \boldsymbol{h}^T \boldsymbol{Q}_2 \Delta \boldsymbol{h}^* + 2\Re(\boldsymbol{q}_2^H \Delta \boldsymbol{h}^*) + c_2, \end{aligned}$$

where

$$\begin{aligned} \boldsymbol{Q}_2 &= \boldsymbol{G}_1^T \boldsymbol{D}_R^T \mathrm{diag}(\breve{\boldsymbol{h}}_1) \bar{\boldsymbol{A}} p_1 \mathrm{diag}(\breve{\boldsymbol{h}}_1^*) \boldsymbol{D}_R \boldsymbol{G}_1 + \boldsymbol{G}_1^T \boldsymbol{D}_L^T \mathrm{diag}(\tilde{\boldsymbol{h}}_1) \bar{\boldsymbol{A}} p_1 \mathrm{diag}(\tilde{\boldsymbol{h}}_1^*) \boldsymbol{D}_L \boldsymbol{G}_1 \\ &\quad + \boldsymbol{G}_1^T \boldsymbol{D}_L^T \mathrm{diag}(\tilde{\boldsymbol{h}}_1) \bar{\boldsymbol{A}} p_1 \mathrm{diag}(\breve{\boldsymbol{h}}_1^*) \boldsymbol{D}_R \boldsymbol{G}_1 + \boldsymbol{G}_1^T \boldsymbol{D}_R^T \mathrm{diag}(\breve{\boldsymbol{h}}_1) \bar{\boldsymbol{A}} p_1 \mathrm{diag}(\tilde{\boldsymbol{h}}_1^*) \boldsymbol{D}_L \boldsymbol{G}_1 \\ &\quad + \boldsymbol{G}_1^T \boldsymbol{D}_L^T \boldsymbol{E} \odot \bar{\boldsymbol{A}} z_R^2 \boldsymbol{D}_L \boldsymbol{G}_1, \\ \boldsymbol{q}_2^H &= \breve{\boldsymbol{h}}_1^T \boldsymbol{E} \odot \bar{\boldsymbol{A}} z_R^2 \boldsymbol{D}_L \boldsymbol{G}_1, \\ c_2 &= \breve{\boldsymbol{h}}_1^T \boldsymbol{E} \odot \bar{\boldsymbol{A}} z_R^2 \breve{\boldsymbol{h}}_1^* + z_1^2. \end{aligned}$$

The constraint (2.11) is equivalent to

$$\begin{aligned} & \Delta \boldsymbol{h}^T (\boldsymbol{Q}_1 - \gamma_1 \boldsymbol{Q}_2) \Delta \boldsymbol{h}^* \\ & + 2\Re\big((\boldsymbol{q}_1 - \gamma_1 \boldsymbol{q}_2)^H \Delta \boldsymbol{h}^*)\big) + c_1 - \gamma_1 c_2 \geq 0 \\ \text{s.t.} \quad & \Delta \boldsymbol{h}^T \boldsymbol{G}_1^H \boldsymbol{G}_1 \Delta \boldsymbol{h}^* \leq \epsilon_1^2 \\ & \Delta \boldsymbol{h}^T \boldsymbol{G}_2^H \boldsymbol{G}_2 \Delta \boldsymbol{h}^* \leq \epsilon_2^2. \end{aligned}$$

According to the S-procedure, we have

$$\begin{pmatrix} \boldsymbol{Q}_1 - \gamma_1 \boldsymbol{Q}_2 + \lambda_1 \boldsymbol{G}_1^H \boldsymbol{G}_1 + \lambda_2 \boldsymbol{G}_2^H \boldsymbol{G}_2 & \boldsymbol{q}_1 - \gamma_1 \boldsymbol{q}_2 \\ \boldsymbol{q}_1^H - \gamma_1 \boldsymbol{q}_2^H & c_1 - \gamma_1 c_2 - \lambda_1 \epsilon_1^2 - \lambda_2 \epsilon_2^2 \end{pmatrix} \succeq 0$$

Similarly, for SINR2, we have

$$\begin{aligned}
s_2 &= c_3 + 2\Re\Big(\big[(\tilde{\boldsymbol{h}}_1 \odot \breve{\boldsymbol{h}}_2)^T \bar{\boldsymbol{A}} p_1 s_1^2 \mathrm{diag}(\breve{\boldsymbol{h}}_2^*)\boldsymbol{D}_R \boldsymbol{G}_1 \\
&\quad + \big(\tilde{\boldsymbol{h}}_1 \odot \breve{\boldsymbol{h}}_2\big)^T \bar{\boldsymbol{A}} p_1 s_1^2 \mathrm{diag}(\tilde{\boldsymbol{h}}_1^*)\boldsymbol{D}_L \boldsymbol{G}_2 \big] \Delta \boldsymbol{h}^* \Big) \\
&\quad + \Delta \boldsymbol{h}^T [\boldsymbol{G}_1^T \boldsymbol{D}_R^T \mathrm{diag}(\breve{\boldsymbol{h}}_2) \bar{\boldsymbol{A}} p_1 s_1^2 \mathrm{diag}(\tilde{\boldsymbol{h}}_1^*) \boldsymbol{D}_L \boldsymbol{G}_2 \\
&\quad + \boldsymbol{G}_2^T \boldsymbol{D}_L^T \mathrm{diag}(\tilde{\boldsymbol{h}}_1) \bar{\boldsymbol{A}} p_1 s_1^2 \mathrm{diag}(\breve{\boldsymbol{h}}_2^*) \boldsymbol{D}_R \boldsymbol{G}_1 \\
&\quad + \boldsymbol{G}_1^T \boldsymbol{D}_R^T \mathrm{diag}(\breve{\boldsymbol{h}}_2) \bar{\boldsymbol{A}} p_1 s_1^2 \mathrm{diag}(\breve{\boldsymbol{h}}_2^*) \boldsymbol{D}_R \boldsymbol{G}_1 \\
&\quad + \boldsymbol{G}_2^T \boldsymbol{D}_L^T \mathrm{diag}(\tilde{\boldsymbol{h}}_1) \bar{\boldsymbol{A}} p_1 s_1^2 \mathrm{diag}(\tilde{\boldsymbol{h}}_1^*) \boldsymbol{D}_L \boldsymbol{G}_2] \Delta \boldsymbol{h}^* \\
&= \Delta \boldsymbol{h}^T \boldsymbol{Q_3} \Delta \boldsymbol{h}^* + 2\Re(\boldsymbol{q}_3^H \Delta \boldsymbol{h}^*) + c_3,
\end{aligned}$$

where

$$\begin{aligned}
\boldsymbol{Q}_3 &= \boldsymbol{G}_1^T \boldsymbol{D}_R^T \mathrm{diag}(\breve{\boldsymbol{h}}_2) \bar{\boldsymbol{A}} p_1 s_1^2 \mathrm{diag}(\tilde{\boldsymbol{h}}_1^*) \boldsymbol{D}_L \boldsymbol{G}_2 + \boldsymbol{G}_2^T \boldsymbol{D}_L^T \mathrm{diag}(\tilde{\boldsymbol{h}}_1) \bar{\boldsymbol{A}} p_1 s_1^2 \mathrm{diag}(\breve{\boldsymbol{h}}_2^*) \boldsymbol{D}_R \boldsymbol{G}_1 \\
&+ \boldsymbol{G}_1^T \boldsymbol{D}_R^T \mathrm{diag}(\breve{\boldsymbol{h}}_2) \bar{\boldsymbol{A}} p_1 s_1^2 \mathrm{diag}(\breve{\boldsymbol{h}}_2^*) \boldsymbol{D}_R \boldsymbol{G}_1 + \boldsymbol{G}_2^T \boldsymbol{D}_L^T \mathrm{diag}(\tilde{\boldsymbol{h}}_1) \bar{\boldsymbol{A}} p_1 s_1^2 \mathrm{diag}(\tilde{\boldsymbol{h}}_1^*) \boldsymbol{D}_L \boldsymbol{G}_2, \\
\boldsymbol{q}_3^H &= (\tilde{\boldsymbol{h}}_1 \odot \breve{\boldsymbol{h}}_2)^T \bar{\boldsymbol{A}} p_1 s_1^2 \mathrm{diag}(\breve{\boldsymbol{h}}_2^*) \boldsymbol{D}_R \boldsymbol{G}_1 + \big(\tilde{\boldsymbol{h}}_1 \odot \breve{\boldsymbol{h}}_2\big)^T \bar{\boldsymbol{A}} p_1 s_1^2 \mathrm{diag}(\tilde{\boldsymbol{h}}_1^*) \boldsymbol{D}_L \boldsymbol{G}_2, \\
c_3 &= \big(\tilde{\boldsymbol{h}}_1 \odot \breve{\boldsymbol{h}}_2\big)^T \bar{\boldsymbol{A}} p_1 s_1^2 \big(\tilde{\boldsymbol{h}}_1 \odot \breve{\boldsymbol{h}}_2\big)^*.
\end{aligned}$$

Let $c_4 = \breve{\boldsymbol{h}}_2^T \boldsymbol{E} \odot \bar{\boldsymbol{A}} z_R^2 \breve{\boldsymbol{h}}_2^*$, then

$$\begin{aligned}
t_2 &= c_4 + 2\Re(\breve{\boldsymbol{h}}_2^T \boldsymbol{E} \odot \bar{\boldsymbol{A}} z_R^2 \boldsymbol{D}_L \boldsymbol{G}_2 \Delta \boldsymbol{h}^*) + \Delta \boldsymbol{h}^T [\boldsymbol{G}_2^T \boldsymbol{D}_R^T \mathrm{diag}(\breve{\boldsymbol{h}}_2) \bar{\boldsymbol{A}} p_2 \mathrm{diag}(\breve{\boldsymbol{h}}_2^*) \boldsymbol{D}_R \boldsymbol{G}_2 \\
&\quad + \boldsymbol{G}_2^T \boldsymbol{D}_L^T \mathrm{diag}(\tilde{\boldsymbol{h}}_2) \bar{\boldsymbol{A}} p_2 \mathrm{diag}(\tilde{\boldsymbol{h}}_2^*) \boldsymbol{D}_L \boldsymbol{G}_2 + \boldsymbol{G}_2^T \boldsymbol{D}_L^T \mathrm{diag}(\tilde{\boldsymbol{h}}_2) \bar{\boldsymbol{A}} p_2 \mathrm{diag}(\breve{\boldsymbol{h}}_2^*) \boldsymbol{D}_R \boldsymbol{G}_2 \\
&\quad + \boldsymbol{G}_2^T \boldsymbol{D}_R^T \mathrm{diag}(\breve{\boldsymbol{h}}_2) \bar{\boldsymbol{A}} p_2 \mathrm{diag}(\tilde{\boldsymbol{h}}_2^*) \boldsymbol{D}_L \boldsymbol{G}_2 + \boldsymbol{G}_2^T \boldsymbol{D}_L^T \bar{\boldsymbol{A}} z_R^2 \boldsymbol{D}_L \boldsymbol{G}_2] \Delta \boldsymbol{h}^* \\
&= \Delta \boldsymbol{h}^T \boldsymbol{Q}_4 \Delta \boldsymbol{h}^* + 2\Re(\boldsymbol{q}_4^H \Delta \boldsymbol{h}^*) + c_4,
\end{aligned}$$

where

$$\begin{aligned}
\boldsymbol{Q}_4 &= \boldsymbol{G}_2^T\boldsymbol{D}_R^T\mathrm{diag}(\breve{\boldsymbol{h}}_2)\bar{\boldsymbol{A}}p_2\mathrm{diag}(\breve{\boldsymbol{h}}_2^*)\boldsymbol{D}_R\boldsymbol{G}_2 + \boldsymbol{G}_2^T\boldsymbol{D}_L^T\mathrm{diag}(\tilde{\boldsymbol{h}}_2)\bar{\boldsymbol{A}}p_2\mathrm{diag}(\tilde{\boldsymbol{h}}_2^*)\boldsymbol{D}_L\boldsymbol{G}_2 \\
&\quad + \boldsymbol{G}_2^T\boldsymbol{D}_L^T\mathrm{diag}(\tilde{\boldsymbol{h}}_2)\bar{\boldsymbol{A}}p_2\mathrm{diag}(\breve{\boldsymbol{h}}_2^*)\boldsymbol{D}_R\boldsymbol{G}_2 + \boldsymbol{G}_2^T\boldsymbol{D}_R^T\mathrm{diag}(\breve{\boldsymbol{h}}_2)\bar{\boldsymbol{A}}p_2\mathrm{diag}(\tilde{\boldsymbol{h}}_2^*)\boldsymbol{D}_L\boldsymbol{G}_2 \\
&\quad + \boldsymbol{G}_2^T\boldsymbol{D}_L^T\boldsymbol{E}\odot\bar{\boldsymbol{A}}z_R^2\boldsymbol{D}_L\boldsymbol{G}_2, \\
\boldsymbol{q}_4^H &= \breve{\boldsymbol{h}}_2^T\boldsymbol{E}\odot\bar{\boldsymbol{A}}z_R^2\boldsymbol{D}_L\boldsymbol{G}_2, \\
c_4 &= \breve{\boldsymbol{h}}_2^T\boldsymbol{E}\odot\bar{\boldsymbol{A}}z_R^2\breve{\boldsymbol{h}}_2^* + z_2^2.
\end{aligned}$$

Next, the constraint (2.12) is equivalent to

$$\begin{pmatrix} \boldsymbol{Q}_3 - \gamma_2\boldsymbol{Q}_4 + \lambda_3\boldsymbol{G}_1^H\boldsymbol{G}_1 + \lambda_4\boldsymbol{G}_2^H\boldsymbol{G}_2 & \boldsymbol{q}_3 - \gamma_2\boldsymbol{q}_4 \\ \boldsymbol{q}_3^H - \gamma_2\boldsymbol{q}_4^H & c_3 - \gamma_2 c_4 - \lambda_3\epsilon_1^2 - \lambda_4\epsilon_2^2 \end{pmatrix} \succeq 0$$

Now we address the p_R. Let $\boldsymbol{K}$ be the commutation matrix such that $\mathrm{vec}(\boldsymbol{A^T}) = \boldsymbol{K}\mathrm{vec}(\boldsymbol{A})$, then

$$\begin{aligned}
p_R &= \hat{\boldsymbol{h}}_1^H\boldsymbol{A}^H\boldsymbol{A}p_1s_1^2\hat{\boldsymbol{h}}_1 + \hat{\boldsymbol{h}}_2^H\boldsymbol{A}^H\boldsymbol{A}p_2s_2^2\hat{\boldsymbol{h}}_2 + \mathrm{tr}(\boldsymbol{A}^H\boldsymbol{A}) \\
&\quad + 2\mathrm{Re}(\hat{\boldsymbol{h}}_1^H\boldsymbol{A}^H\boldsymbol{A}p_1s_1^2\Delta\boldsymbol{h}_1) + 2\mathrm{Re}(\hat{\boldsymbol{h}}_2^H\boldsymbol{A}^H\boldsymbol{A}p_2s_2^2\Delta\boldsymbol{h}_2) \\
&\quad \Delta\boldsymbol{h}_1^H\boldsymbol{A}^H\boldsymbol{A}p_1s_1^2\Delta\boldsymbol{h}_1 + \Delta\boldsymbol{h}_2^H\boldsymbol{A}^H\boldsymbol{A}p_2s_2^2\Delta\boldsymbol{h}_2 \\
&= (\boldsymbol{1}_{M\times 1}\otimes\hat{\boldsymbol{h}}_1)^T\boldsymbol{E}\odot[\boldsymbol{K}\bar{\boldsymbol{A}}\boldsymbol{K}^Tp_1s_1^2](\boldsymbol{1}_{M\times 1}\otimes\hat{\boldsymbol{h}}_1)^* \\
&\quad + (\boldsymbol{1}_{M\times 1}\otimes\hat{\boldsymbol{h}}_2)^T\boldsymbol{E}\odot[\boldsymbol{K}\bar{\boldsymbol{A}}\boldsymbol{K}^Tp_2s_2^2](\boldsymbol{1}_{M\times 1}\otimes\hat{\boldsymbol{h}}_2)^* + \mathrm{tr}(\bar{\boldsymbol{A}}) \\
&\quad + 2\Re((\boldsymbol{1}_{M\times 1}\otimes\boldsymbol{h}_1^*)^T\boldsymbol{E}\odot[\boldsymbol{K}\bar{\boldsymbol{A}}\boldsymbol{K}^Tp_1s_1^2]\boldsymbol{D}_L\boldsymbol{G}_1\Delta\boldsymbol{h}) \\
&\quad + 2\Re((\boldsymbol{1}_{M\times 1}\otimes\boldsymbol{h}_2^*)^T\boldsymbol{E}\odot[\boldsymbol{K}\bar{\boldsymbol{A}}\boldsymbol{K}^Tp_2s_2^2]\boldsymbol{D}_L\boldsymbol{G}_2\Delta\boldsymbol{h}) \\
&\quad + \Delta\boldsymbol{h}^H\boldsymbol{G}_1^T\boldsymbol{D}_L^T\boldsymbol{E}\odot[\boldsymbol{K}\bar{\boldsymbol{A}}\boldsymbol{K}^Tp_1s_1^2]\boldsymbol{D}_L\boldsymbol{G}_1\Delta\boldsymbol{h} \\
&\quad + \Delta\boldsymbol{h}^H\boldsymbol{G}_2^T\boldsymbol{D}_L^T\boldsymbol{E}\odot[\boldsymbol{K}\bar{\boldsymbol{A}}\boldsymbol{K}^Tp_2s_2^2]\boldsymbol{D}_L\boldsymbol{G}_2\Delta\boldsymbol{h} \\
&= c_0 + 2\Re(\boldsymbol{q}_0^H\Delta\boldsymbol{h}) + \Delta\boldsymbol{h}^H\boldsymbol{Q}_0\Delta\boldsymbol{h},
\end{aligned}$$

where

$$
\begin{aligned}
\boldsymbol{Q}_0 &= \boldsymbol{G}_1^T \boldsymbol{D}_L^T \boldsymbol{E} \odot [\boldsymbol{K} \bar{\boldsymbol{A}} \boldsymbol{K}^T p_1 s_1^2] \boldsymbol{D}_L \boldsymbol{G}_1 \\
&\quad + \boldsymbol{G}_2^T \boldsymbol{D}_L^T \boldsymbol{E} \odot [\boldsymbol{K} \bar{\boldsymbol{A}} \boldsymbol{K}^T p_2 s_2^2] \boldsymbol{D}_L \boldsymbol{G}_2, \\
\boldsymbol{q}_0^H &= \breve{\boldsymbol{h}}_1^H \boldsymbol{E} \odot [\boldsymbol{K} \bar{\boldsymbol{A}} \boldsymbol{K}^T p_1 s_1^2] \boldsymbol{D}_L \boldsymbol{G}_1 \\
&\quad + \breve{\boldsymbol{h}}_2^H \boldsymbol{E} \odot [\boldsymbol{K} \bar{\boldsymbol{A}} \boldsymbol{K}^T p_2 s_2^2] \boldsymbol{D}_L \boldsymbol{G}_2, \\
c_0 &= \breve{\boldsymbol{h}}_{\boldsymbol{1}}^T \boldsymbol{E} \odot [\boldsymbol{K} \bar{\boldsymbol{A}} \boldsymbol{K}^T p_1 s_1^2] \breve{\boldsymbol{h}}_1^* \\
&\quad + \breve{\boldsymbol{h}}_{\boldsymbol{2}}^T \boldsymbol{E} \odot [\boldsymbol{K} \bar{\boldsymbol{A}} \boldsymbol{K}^T p_2 s_2^2] \breve{\boldsymbol{h}}_1^* + \mathrm{tr}(\bar{\boldsymbol{A}}).
\end{aligned}
$$

The power constraint can be rewritten as

$$
\begin{pmatrix} -\boldsymbol{Q}_0 + \kappa_1 \boldsymbol{G}_1^H \boldsymbol{G}_1 + \kappa_2 \boldsymbol{G}_2^H \boldsymbol{G}_2 & -\boldsymbol{q}_0 \\ -\boldsymbol{q}_0^H & t - c_0 - \kappa_1 \epsilon_1^2 - \kappa_2 \epsilon_2^2 \end{pmatrix} \succeq 0.
$$

Finally, the original problem $\mathcal{Q}_1$ is formulated as

$$
\begin{aligned}
\mathcal{Q}_2 : \quad & \min_{\bar{\boldsymbol{A}}, t, \boldsymbol{\lambda}, \boldsymbol{\kappa}} \; t \\
\text{s.t.} \quad & \begin{pmatrix} \boldsymbol{Q}_1 - \gamma_1 \boldsymbol{Q}_2 + \lambda_1 \boldsymbol{G}_1^H \boldsymbol{G}_1 + \lambda_2 \boldsymbol{G}_2^H \boldsymbol{G}_2 & \boldsymbol{q}_1 - \gamma_1 \boldsymbol{q}_2 \\ \boldsymbol{q}_1^H - \gamma_1 \boldsymbol{q}_2^H & c_1 - \gamma_1 c_2 - \lambda_1 \epsilon_1^2 - \lambda_2 \epsilon_2^2 \end{pmatrix} \succeq 0 \\
& \begin{pmatrix} \boldsymbol{Q}_3 - \gamma_2 \boldsymbol{Q}_4 + \lambda_3 \boldsymbol{G}_1^H \boldsymbol{G}_1 + \lambda_4 \boldsymbol{G}_2^H \boldsymbol{G}_2 & \boldsymbol{q}_3 - \gamma_2 \boldsymbol{q}_4 \\ \boldsymbol{q}_3^H - \gamma_2 \boldsymbol{q}_4^H & c_3 - \gamma_2 c_4 - \lambda_3 \epsilon_1^2 - \lambda_4 \epsilon_2^2 \end{pmatrix} \succeq 0 \\
& \begin{pmatrix} -\boldsymbol{Q}_0 + \kappa_1 \boldsymbol{G}_1^H \boldsymbol{G}_1 + \kappa_2 \boldsymbol{G}_2^H \boldsymbol{G}_2 & -\boldsymbol{q}_0 \\ -\boldsymbol{q}_0^H & t - c_0 - \kappa_1 \epsilon_1^2 - \kappa_2 \epsilon_2^2 \end{pmatrix} \succeq 0 \\
& \mathrm{Rank}(\bar{\boldsymbol{A}}) = 1,
\end{aligned}
$$

where $\boldsymbol{\lambda} = (\lambda_1, \lambda_2, \lambda_3, \lambda_4)$, and $\boldsymbol{\kappa} = (\kappa_1, \kappa_2)$. Since the rank-one constraint is not convex, the problem $\mathcal{Q}_2$ is not a convex problem. However, if we ignore the rank-one constraint, this problem is relaxed into a SDP problem which is convex:

$$\begin{aligned}
\mathcal{Q}_3: \quad & \min_{\bar{\boldsymbol{A}},t,\boldsymbol{\lambda},\boldsymbol{\kappa}} \; t \\
\text{s.t.} \quad & \begin{pmatrix} \boldsymbol{Q}_1 - \gamma_1\boldsymbol{Q}_2 + \lambda_1\boldsymbol{G}_1^H\boldsymbol{G}_1 + \lambda_2\boldsymbol{G}_2^H\boldsymbol{G}_2 & \boldsymbol{q}_1 - \gamma_1\boldsymbol{q}_2 \\ \boldsymbol{q}_1^H - \gamma_1\boldsymbol{q}_2^H & c_1 - \gamma_1 c_2 - \lambda_1\epsilon_1^2 - \lambda_2\epsilon_2^2 \end{pmatrix} \succeq 0 \\
& \begin{pmatrix} \boldsymbol{Q}_3 - \gamma_2\boldsymbol{Q}_4 + \lambda_3\boldsymbol{G}_1^H\boldsymbol{G}_1 + \lambda_4\boldsymbol{G}_2^H\boldsymbol{G}_2 & \boldsymbol{q}_3 - \gamma_2\boldsymbol{q}_4 \\ \boldsymbol{q}_3^H - \gamma_2\boldsymbol{q}_4^H & c_3 - \gamma_2 c_4 - \lambda_3\epsilon_1^2 - \lambda_4\epsilon_2^2 \end{pmatrix} \succeq 0 \\
& \begin{pmatrix} -\boldsymbol{Q}_0 + \kappa_1\boldsymbol{G}_1^H\boldsymbol{G}_1 + \kappa_2\boldsymbol{G}_2^H\boldsymbol{G}_2 & -\boldsymbol{q}_0 \\ -\boldsymbol{q}_0^H & t - c_0 - \kappa_1\epsilon_1^2 - \kappa_2\epsilon_2^2 \end{pmatrix} \succeq 0.
\end{aligned}$$

Given the convexity of the SDP problem $\mathcal{Q}_3$, the optimal solution can be solved using various interior point methods. If the resulting solution $\bar{\boldsymbol{A}}^{\boldsymbol{*}}$ is of rank-one, then the optimal beamforming matrix at relay $\boldsymbol{A}^*$ can be easily obtained as $\boldsymbol{A}^* = \text{ivec}(\boldsymbol{a})$, where $\bar{\boldsymbol{A}}^{\boldsymbol{*}} = \boldsymbol{a}\boldsymbol{a}^H$ and ivec$(\cdot)$ is the inverse operation of vec$(\cdot)$.

An outage occurs if $\mathcal{Q}_3$ is infeasible or the resulting $\boldsymbol{A}^*$ is not rank one. If $\mathcal{Q}_3$ is not feasible, a hybrid approach is adopted to formulate the optimization problem with perfect CSI similar in [4]. If $\mathcal{Q}_3$ is feasible but $\boldsymbol{A}^*$ is not rank-one, it is impossible to reconstruct the exact optimal beamforming matrix. In this case, a rank-one reconstruction method is needed to obtain a feasible rank-one solution, although this solution is generally suboptimal compared with the exact optimal solution. Here, in addition to the principle eigenvector based method [10], we also include randomization based method [14] to increase the chance of obtaining a feasible rank-one solution.

A. Principle Eigenvector Based Rank-One Approximation

Since $\bar{\boldsymbol{A}}^*$ is Hermitian, $\bar{\boldsymbol{A}}^*$ can be decomposed into $\boldsymbol{U}\boldsymbol{\Sigma}\boldsymbol{U}^H$ by singular value decomposition (SVD). Let $\boldsymbol{A}^* = \text{ivec}(\boldsymbol{u}_1)$, where $\boldsymbol{u}_1$ is the column vector in $\boldsymbol{U}$ that corresponds to the largest eigenvalue. The resultant $\boldsymbol{A}^*$ is rank-one but not necessarily feasible with respect to the constraints in $\mathcal{Q}_1$. Therefore, every reconstructed rank-one matrix needs to be subject to a feasibility check. The principle eigenvector method is generally inferior to the randomization based method introduced next, but this method is computationally inexpensive, and it provides a fair performance when the SINR requirements are not stringent.

B. Randomization Based Rank-One Approximation

A set of candidate weight vectors $\{\boldsymbol{w}_l\}$ [15] can be generated from $\bar{\boldsymbol{A}}^*$. Let $\{\boldsymbol{w}_l\} = \boldsymbol{U}\boldsymbol{\Sigma}^{1/2}\boldsymbol{v}_l$, where $\boldsymbol{v}_l$ is a vector of circularly symmetric complex Gaussian random variables with zero mean and an identity covariance matrix. This ensures $E[\boldsymbol{w}_l\boldsymbol{w}_l^H] = \bar{\boldsymbol{A}}^*$. Each sample of $\text{ivec}(\boldsymbol{w}_l)$ needs to be checked against the feasibility conditions in $\mathcal{Q}_1$, and the best solution will be selected among all the feasible solutions if at least one solution exists.

Here, we define the outage probability in our robust beamforming algorithm as the probability that the hybrid approach fails to obtain a feasible solution or the rank-one reconstruction methods fail to reconstruct a feasible rank-one solution.

CHAPTER IV

SIMULATION RESULTS

In this chapter, we present MATLAB-based simulations to investigate the source node BERs in our TWRN and the outage probability of our robust beamforming algorithm. In this simulation system, the bit streams from source nodes S1 and S2 are modulated as QPSK signals. The number of subcarriers in this OFDM system is 64, and 16 subcarriers among the 64 subcarriers are used as virtual carriers (VCs) at both ends of the transmission band to mitigate the adjacent channel interference (ACI). Cyclic prefix of length 16 is inserted to each of OFDM symbol.

Each tap of $\boldsymbol{h_1}$ and $\boldsymbol{h_2}$ is modeled as Rayleigh fading, and the distances between the two source nodes to relay node are assumed to be the same, where $\boldsymbol{h}_1$ and $\boldsymbol{h}_2$ are estimated by sending pilot symbols from R to S1 and S2 and running MMSE channel estimations respectively. In addition to the desired signal, the source received signals also contain self-interference signals, amplified relay noise, and its own channel noise. As a result of multiple sources of noise and residue interference, the SINRs at the source nodes are usually very low, which could lead to high bit error rates when the source nodes decode the received signals.

In an OFDM system, the receivers also need to consider signal distortions due to synchronization offsets. In order to take the N-point FFT at the receivers, the receivers need to know the exact starting point of each OFDM symbol. In particular, symbol timing offset (STO) may cause inter-symbol interference (ISI) or inter-carrier interference (ICI) depending on the estimation of the starting point [16]. If the estimated starting point is before the exact starting point but after the end of the channel response to the previous OFDM symbol, a phase offset $e^{j2\pi k\delta/N}$ occurs, where k is the subcarrier index, δ is STO, and N is the FFT size. This phase offset can be compensated easily if δ is known. For the case where the estimated starting point is before the end of the channel response to the previous OFDM symbol or after the exact starting point, both ISI and ICI occur. Signal distortions that are caused by ISI or ICI are very difficult to compensate. Therefore, only the first case of STO is considered in this simulation system, and δ is an integer that is randomly generated from $[0, 8]$. Since CP is a replica of the ending part of the OFDM symbol, the similarities between CP and the corresponding data part of the OFDM symbol can be utilized for STO detection, where minimum mean square searching can be used to determine the STO [17]

$$\hat{\delta} = \underset{\delta}{\text{argmin}} \left\{ \sum_{i=\delta}^{N_g-1+\delta} (\mid y[n+i] \mid - \mid y^*[n+N+i] \mid)^2 \right\}. \tag{4.1}$$

The other type of major signal distortion due to synchronization comes from carrier frequency offset (CFO). When the receivers convert the passband signal from the carrier frequency to the baseband frequency, there will be unavoidable CFO due to the physical nature of phase lock loop. Let $\triangle f$ denote the subcarrier spacing in OFDM and $f_{\texttt{off}}$ be the CFO; we define $\varepsilon = f_{\texttt{off}} / \triangle f$ as the normalized CFO. Let ε be randomly generated from $[0, 0.1]$ in this simulation system. A CFO of ε causes a phase offset of $e^{j2\pi n\varepsilon/N}$, where n is the subcarrier index. Such a phase rotation in time domain corresponds to a frequency shift in the frequency domain. Therefore, the inter-carrier signal interferes with one another. The CFO can be found from the phase angle of the product of CP and its corresponding data part [17]

$$\hat{\varepsilon} = \frac{1}{2\pi} \arg \left\{ \sum_{i=-N_g}^{-1} (y^*[n]y[n+N]) \right\} \tag{4.2}$$

To show the effect of synchronization offsets on the performance of TWRN, we consider both perfect and imperfect synchronization in this simulation system. For the perfect synchronization case, we only consider the effect of channel estimation error. For the imperfect synchronization case, the effects of STO and CFO are compensated using the schemes mentioned above. In the simulations, the received signal noise and interference are decreased gradually by increasing the SNR of the system. As we can see from Fig. 2, synchronization offsets greatly increase the source node BERs, and the BERs of the system with imperfect synchronization decrease with a much slower rate over SNR compared with the BERs of the system with perfect synchronization. Therefore, it it critical for the source nodes to correctly estimate and detect the STO and CFO. In our future work, more sophisticated STO detection and CFO estimation schemes need to be studied to reduce the effect of synchronization offsets.

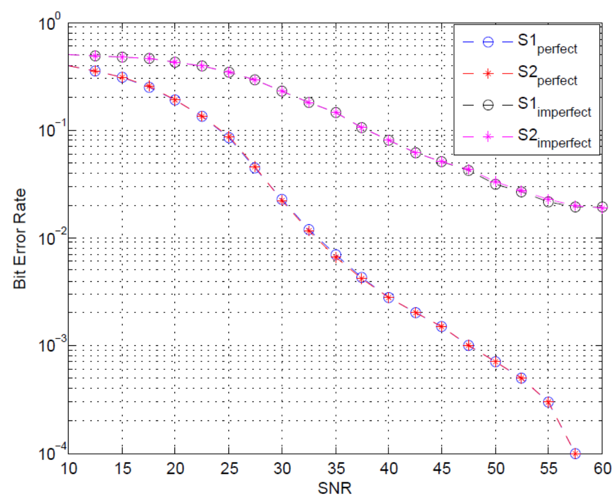

Figure 2: Source Nodes Bit Error Rates

Next, we show the relationship between the outage probability and the SINR requirements at the source nodes. We set the SNR at the relay node to be 40 dB, and we gradually increase the SINR requirements from 5 dB to 40 dB. As we see from Fig. 3, the outage probability increases as the SINR requirements increase. In particular, the outage probability begins to increase significantly when the SINR requirements reach 20 dB, and the outage probability becomes almost one when the

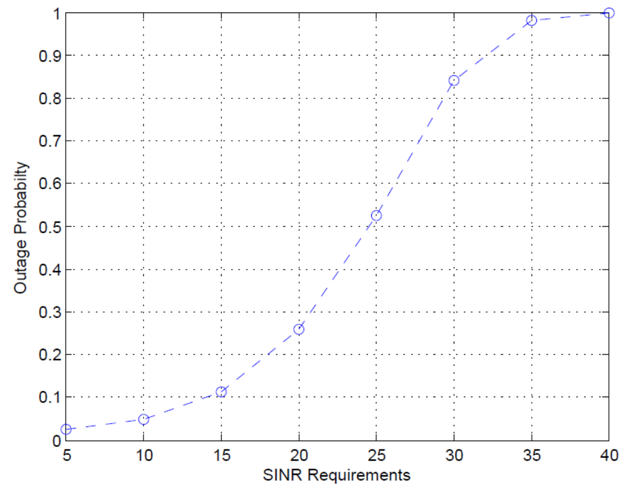

Figure 3: Robust Beamforming Outage Probability

SINR requirements reach 40 dB. This suggests that the achievable SINRs at the source nodes are generally much lower than the SNR at the relay node, and it is very difficult to generate a beamforming matrix satisfying the high SINR requirements at the source nodes.

CHAPTER V

CONCLUSION

In this thesis, we studied a robust relay beamforming optimization problem with channel uncertainty in the OFDM modulated TWRN with ANC. It is shown that this optimization problem can be relaxed to a convex optimization problem via the S-procedure and a rank-one relaxation. Rank-one reconstruction via the principal eigenvector method and randomization method are used to reconstruct a rank-one solution. In addition to the theoretical analysis, we validated the performance of TWRN and the robust beamforming outage probability through MATLAB simulations. Performance under perfect and imperfect synchronization is presented to show the overall performance of TWRN with ANC. As a result of multiple sources of noise and residue interference, the SINRs at the sources node are usually very low, which lead to high BERs at the source nodes. Furthermore, the source nodes BERs will be increased significantly if the effect of STO and CFO are not correctly compensated. We showed that the outage probability of our robust beamforming algorithm is tightly related to the SINR requirements, and the outage probability becomes very high if high SINRs are required at the source nodes.

REFERENCES

[1] D. Tse and P. Viswanath, *Fundamentals of Wireless Communication*, Cambridge University Press, Cambridge, 2005.

[2] T. Cover and A. E. Gamal, “Capacity theorems for the relay channel,” *IEEE Trans. Inf. Theory*, vol. 25, no. 5, pp. 572–584, 1979.

[3] A. Beck and Y. C. Eldar, “Cooperative diversity in wireless networks,” *IEEE Trans. Inf. Theory*, vol. 52, no. 9, pp. 3062–3080, 2004.

[4] R. Zhang, Y-C. Liang, C. Choy, and S. Cui, “Optimal beamforming for two-way multi-antenna relay channel with analogue network coding,” *IEEE J. Sel. Areas Commun.*, vol. 27, no. 5, pp. 699–712, Jun. 2009.

[5] D. Gunduz, A. Goldsmith, and H. V. Poor, “MIMO two-way relay channel: Diversity-multiplexing tradeoff analysis,” in *Asilomar Conference on Signal, Systems and Computers*, Pacific Grove, CA, Jun. 2008.

[6] N. Lee, H. Yang, and J. Chun, “Achievable sum-rate maximizing af relay beamforming scheme in two-way relay channels,” in *Proc. IEEE International Conference on Communications Workshop*, Beijing, May 2008.

[7] V. Havary-Nassab, S. Shahbazpanahi, and A. Grami, “Optimal distributed beamforming for two-way relay networks,” *IEEE Trans. Signal Process.*, vol. 58, no. 3, pp. 1238 – 1250, Mar. 2010.

[8] R. Vaze and R. W. Heath, “Optimal amplify and forward strategy for two-way relay channel with multiple relays,” in *Proc. IEEE Information Theory Workshop*, Volos, Greece, Jun. 2009.

[9] S. Zhang, S.-C Lewis, and P. P. Lam, "Physical-layer network coding," in *Proc. IEEE MobiComm*, 2006.

[10] A. Aziz, M. Zeng, J. Zhou, C. N. Georghiades, and S. Cui, "Robust beamforming with channel unvertanity for two-way relay networks," in *Proc. IEEE ICC- Signal Processing for Communications Symposium*, Ottawa, Canada, Jun. 2012.

[11] D. P. Palomar and Y. C. Eldar, *Convex Optimization in Signal Processing and Communications*, Cambridge University Press, Cambridge, U.K., 2009.

[12] A. Beck and Y. C. Eldar, "Strong duality in nonconvex quadratic optimization with two quadratic constraints," *SIAM J. Opt.*, vol. 17, no. 3, pp. 844–860, Oct. 2006.

[13] G. Zheng, K. Wong, A. Paulraj, and B. Ottersten, "Robust collaborative relay beamforming," *IEEE Trans. Signal Process.*, vol. 57, no. 8, pp. 3130–3143, Aug. 2009.

[14] Z.-Q. Luo and W. Yu, "An introduction to convex optimization for communications and signal processing," *IEEE J. Sel. Areas Commun.*, vol. 24, no. 8, pp. 1426–1438, Aug. 2006.

[15] N. D. Sidiropoulos, T. N. Davidson, and Z.-Q. Luo, "Transmit beamforming for physical-layer multicasting," *IEEE Trans. Signal Process.*, vol. 54, no. 6, pp. 2239 – 2252, Jun. 2006.

[16] L. Lu, T. Wang, S . C. Liew, and S. Zhang, "Implementation of physical-layer network coding," *Physical Communication*, Mar. 2012.

[17] Y. S. Cho, J. Kim, W. Y. Yang, and C. G. Kang, *MIMO-OFDM Wireless Communication with MATLAB*, IEEE Press: J. Wiley and Sons (Asia), Hoboken,

NJ, 2010.