ACID PLACEMENT IN ACID JETTING TREATMENTS IN LONG HORIZONTAL WELLS

A Thesis

by

HARI SASONGKO

Submitted to the Office of Graduate Studies of Texas A&M University in partial fulfillment of the requirements for the degree of

MASTER OF SCIENCE

May 2012

Major Subject: Petroleum Engineering

Acid Placement in Acid Jetting Treatments in Long Horizontal Wells

Copyright 2012 Hari Sasongko

ACID PLACEMENT IN ACID JETTING TREATMENTS

IN LONG HORIZONTAL WELLS

A Thesis

by

HARI SASONGKO

Submitted to the Office of Graduate Studies of Texas A&M University in partial fulfillment of the requirements for the degree of

MASTER OF SCIENCE

Approved by:

Chair of Committee, A. Daniel Hill Committee Members, Ding Zhu Yuefeng Sun Head of Department, Stephen A. Holditch

May 2012

Major Subject: Petroleum Engineering

ABSTRACT

Acid Placement in Acid Jetting Treatments in Long Horizontal Wells.

(May 2012)

Hari Sasongko, B. Tech, Bandung Institute of Technology, Indonesia Chair of Advisory Committee: Dr. A. Daniel. Hill

In the Middle East, extended reach horizontal wells (on the order of 25,000 feet of horizontal displacement) are commonly acid stimulated by jetting acid out of drill pipe. The acid is jetted onto the face of the openhole wellbore as the drill pipe is withdrawn from the well. The jetting action helps to remove the drilling fluid filter cake and promote the acid to penetrate into the formation and form wormholes to stimulate the well. However, with very long sections of wellbore open to flow, the acid placement and subsequent wormhole distribution and penetration depths are uncertain.

This study has modeled the acid jetting process using a comprehensive model of acid placement and wormhole propagation in a horizontal well. It is presumed that the acid jetting tool removes the drilling mud filter cake, so that no filter cake exists between the end of the drill pipe and the toe of the well. Correspondingly, the model also assumes that there is an intact, low-permeability filter cake on the borehole wall between the end of the drill pipe and the heel of the well. The drill pipe is modeled as being withdrawn from the well during the acid jetting treatment, as is done in practice. The acidizing simulator predicts the distribution of acid and the depths of wormholes formed as functions of time and position during the acid jetting treatment. The model shows that the acid jetting process as typically applied in these wells preferentially stimulates the toe region of the horizontal well. Comparisons of the simulation predictions with published data for acid jetting treatments in such wells showed good general agreement. Based on the simulation study, this study presents recommendations for improved acid jetting treatment procedures to improve the distribution of acid injected into the formation.

DEDICATION

This thesis is dedicated to:

My mother, Siti Umaya

My father, Matrap

My special dedication is given to my wife, PDA, for her patience and support.

ACKNOWLEDGEMENTS

I would never have been able to finish my thesis without the guidance of my committee members, great help from my colleagues and support from my family and wife. I am heartily thankful to my supervisor, Dr. Hill and co-supervisor Dr. Zhu, whose encouragement, guidance and support from the initial (first time I came to PE Department) to the final level enabled me to develop an understanding of the subject. My greatest appreciation also goes to Dr. Sun for his help and support as my committee member.

I would like to thank to the Middle East Carbonate Stimulation Project (MECS) that has provided a financial support to the department to fund this study.

I also would like to thank my colleagues, especially at 712 office room, who as good friends, were always willing to help and giving the best suggestions. Many special thanks to Manabu and Hau which my research would not have been possible without their help.

Finally, I would like to thank to my wife, Putri D. Ayuningtyas, she always there cheering me up and stood by me through the good times and bad.

NOMENCLATURE

- a_{ix} = parameter in inflow equation, bbl/min-psi
- A_i = coefficients in solution matrix
- b_{jx} = parameter in inflow equation, bbl/min
- B_i = coefficients in solution matrix
- C_i = coefficients in solution matrix
- d = internal wellbore diameter, ft
- f_f = fanning friction factor, dimensionless
- k = permeability of reservoir rock, mD
- k_d = permeability of damaged region, mD
- 1 = length of reservoir segment, ft
- p_D = dimensionless pressure
- p_i = initial reservoir pressure, psi
- $p_w = pressure$ at any point in the wellbore, psi
- PV_{bt} = pore volume for breakthrough, dimensionless
- q_R = reservoir inflow rate per unit length of wellbore, bbl/min/ft
- $q_w =$ wellbore flow rate at any point, bbl/min
- $r_d =$ radius of damaged zone, ft
- $r_w =$ wellbore radius, ft
- r_{wh} = radius of wormhole length, ft
- s = skin factor, dimensionless

t = time, minutes

- t_D = dimensionless time
- V_i = interstitial velocity, cm/min
- $V_{i\text{-opt}}$ = interstitial velocity, cm/min

$$V = volume, ft^3$$

- x = position of any point in wellbore
- $\mu =$ fluid viscosity, cp
- $\rho = fluid density, lbm/ft^3$
- ξ = ellipsoidal coordinate dimension
- ϕ = porosity, fraction

TABLE OF CONTENTS

ABSTRACT		iii
DEDICATION		v
ACKNOWLEDGI	EMENTS	vi
NOMENCLATUR	E	vii
TABLE OF CONT	ENTS	ix
LIST OF FIGURE	S	xi
LIST OF TABLES	5	xiii
CHAPTER		
I INTR	ODUCTION	1
1. 1. 1.	 2 Literature Review	1 2 2 3 6
II ACID	JETTING MODEL DEVELOPMENT	7
2. 2.	 Model Formulation	7 10 11 12 13 14 15 15
	2.2.5 Skin Model	18

III	ACID J	ETTING SIMULATOR	20
	3.1 3.2	Problem Solution Acid Jetting Simulator	20 22
IV	RESUL	T AND DISCUSSION	25
	4.1 4.2	Case Study Running Result and Discussion	25 26
V	SUMM	ARY AND CONCLUSIONS	33
	5.1 5.2	Summary Recommendations	33 33
REFERENCES		34	
APPENDIX		36	
VITA			39

Page

LIST OF FIGURES

FIGUR	E	Page
1.1	Typical of the well completion in Al Shaheen field (Brink et al.2010)	3
1.2	Acid jetting assembly (Ritchie et al. 2008)	5
2.1	One segment in openhole section represents one cycle of the acid jetting treatment process.	8
2.2	For one treatment cycle, the acid jetting injection point is assumed stationary at the tip of each segment	8
2.3	At the second cycle, acid removes the filter cake in the current segment and creates wormholes in the previous segments	9
2.4	Wellbore flow schematic	11
2.5	Fluid interface movement inside wellbore	13
2.6	Typical of coreflood test results and numerical data fit (Buijse and Glasbergen 2005)	17
2.7	Schematic of near wellbore zone in openhole completion	18
3.1	The workflow of acid jetting treatment model at each step	22
3.2	The acid jetting simulator interface program	23
4.1	Wormholes length along openhole horizontal section at each treatment	27
4.2	Wormhole length at the toe segment as function of time	28
4.3	Wormhole length at the toe segment as function of cumulative injected acid	29
4.4	Interstitial velocity at the toe segment as function of time	30
4.5	Wormholes length along openhole horizontal section $PV_{bt-opt} = 0.147$ and $V_i = 0.228$ cm/min	31

FIGURE		
4.6 Caliper logs run after acid jetting treatments in Al Shaheen (Ritchie et al., 2010)	32	

LIST OF TABLES

TABLE		Page
4.1	Data input	26

CHAPTER I

INTRODUCTION

1.1 Research Background

Horizontal drilling is a technique which has been enthusiastically adopted by the oil industry in the past 20 years. The technology of horizontal wells in oil exploration, development, and production operations has grown rapidly and now, the drilling technology is able to deliver ultra-long horizontal well designs which allow the well to have a greater contact area with the hydrocarbon zone and the higher rates or production.

Newly drilled horizontal wells normally require acidizing to remove drilling mud damage before being put on production. For the ultra-long horizontal well, the efficient placement of conventional acids is critical. The difficulty of applying acid (HCl) in long horizontal wells to uniformly remove drilling damage has been identified by several authors (Brink et al. 2010; Hoch et al. 2010; Ritchie et al. 2010). In the long horizontal well completion type, due to the limitation of Coiled Tubing (CT), acid diversion, or foam technology application, Controlled Acid Jetting (CAJ) and Acid Jetting methods are considered as the effective solution to stimulate the well. This study is proposed to predict the acid placement model of the acid jetting stimulation treatment in the long horizontal wells.

This thesis follows the style of SPE Journal.

1.2 Literature Review

In this section, the basic idea of horizontal well technology and stimulation treatments required to improved well performance will be reviewed. Many papers and journals are available reporting the successfulness of horizontal well technology.

1.2.1 Long Horizontal Well Technology

Many fields in the world are geologically difficult to be explored using conventional technique which sometimes oil is found in thin layers but covers a large area, and developing using classic vertical completions will require a vast number of wells and if it is in offshore, it will require large number of platforms. For this case, horizontal technology is used to reduce number of wells and improve field economic performance. Completion methods and well intervention concepts are more complicated that require more attention when long horizontal well type is chosen.

One of the good example for a difficult and marginal field that require unconventional well completion strategy is the giant Al Shaheen field in Qatar peninsula (Thomasen et al. 2005). The field comprises a series of thin, stacked reservoirs, dominated by tight carbonates. Their appraisal study showed discourage results and concluded that development was not economically feasible if vertical wells were used. The two horizontal wells were drilled in 1993/1994 which both achieved world record horizontal lengths and put on test production and indicated the viability of horizontal well technology. The successful development of Al Shaheen is achieved by using 20,000+ ft horizontal section well. Up to now, about fourteen well records world longest horizontal section. Typical of the horizontal well completion configuration in AL Shaheen field is shown by next figure (**Fig. 1**). The completion concept in following figure is also showing the combination between Control Acid Jetting and Acid Jetting system.

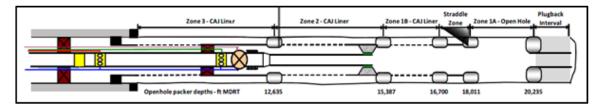


Fig. 1.1 – Typical of the well completion in Al Shaheen field (Brink et al. 2010)

1.2.2 Acid Jetting Stimulation Treatment

Several papers have been published showing that well completions and intervention methods including future well stimulation techniques are an important component in field development concept (Brink et al. 2010; Hoch et al. 2010; Ritchie et al. 2010). Those parameters will affect reservoir management scenario which also directly influences field economic performance.

In some typical laterally extensive low permeability chalk fields, such as Al Shaheen and Dan/Halfdan oil field (Hansen and Nederveen 2002; Lechner et al. 2009; Thomasen et al. 2005), matrix acidizing is considered as the most optimum stimulation treatment in a long horizontal well completion type. Their study clearly showed that low

permeability and formation damage problems during the drilling process require an acidizing stimulation treatment prior to production.

However, stimulating an ultra-long horizontal well is a challenging task. The length of the openhole horizontal section and pressure drop along wellbore prevent the acid placement reach and stimulate the toe section. Commonly used diversion agent, foams, or in-situ-viscosified acids do not work in such ultra-long horizontal. Coiled tubing intervention is not an option due the limitation of length of the well.

The solution that has been implemented is the acid jetting stimulation treatment where drillpipe is a workstring along with a dedicated jetting nozzle for a bottomhole assembly. The string is run to total depth (TD) through a rotating head mounted above the blowout preventer allowing the string to be pulled from the well, under pressure, while pumping acid. As the string is retracted, one stand at time, the acid is pumped (up to 20 bbl/min) where the combination of rate and pulling speed will determine the effectiveness of acid jetting treatment. The nozzle assembly is shown in **Fig. 2**.



Fig. 1.2 – Acid jetting assembly (Ritchie et al. 2008)

As drill pipe is pulled along the wellbore, acid is injected onto the face of the openhole and the jetting effect is expected to remove the filter cake by mechanical action. At the wellbore section where filter cake is no longer exists, if the wellbore pressure is higher than reservoir pressure, some amount of acid will flow into the formation, creating wormholes and enhancing formation conductivity. The process is repeated for each stand numerous times from the toe to the heel section.

1.3 Research Objective and Approach

The objective of the work is to study the acid placement and wormhole propagation model in acid jetting treatments in long horizontal wells. The model will consider the frictional pressure drop along wellbore and calculate wormhole propagation and estimate skin performance based on the amount of acid injected into formation. To achieve the research objective, this study is following several steps:

- Developing a fortran based software that capable of calculating wellbore flow model, an interface tracking movement, wormhole length, and skin evolution estimation during treatment.
- Developing a user friendly VBA Excel based software as an input and output interface.
- 3. Running the simulation predictions result using published field data for acid jetting treatments and presents recommendations for improved acid jetting treatment procedures to increase the effectiveness of the injected acid distribution into the formation.

CHAPTER II

ACID JETTING MODEL DEVELOPMENT*

2.1 Acid Jetting Model Concept

As noted in the previous chapter, the expected acid jetting stimulation effect comes from the jetting action of nozzle that will remove filter cake and the acid that will dissolve carbonate material and create wormhole. Technically, acid is pumped while drill pipe is retracted each stand (2 or 3 joints of pipes) at a time, then acid pumping is stopped to allow drill pipe sections to be disconnected and laid down. The time from the startoff injection until the pipes are disconnected is one cycle in the acid jetting treatment process.

The following steps were taken to model the acid jetting process:

- The openhole horizontal section is divided into a series of segments and each segment represents one cycle of the jetting treatment (Fig. 2.1).
- The process of retraction of one stand of drill pipe, which equals one cycle of the jetting treatment, is modeled as a single continuous injection process at the start point of every segment (**Fig. 2.2**). The duration and amount of acid injection in every segment depends on the drill pipe pulling speed and the length of one segment.

^{*}Part of data reported in this chapter is reprinted with permission from "Simulation of Acid Jetting Treatments in Long Horizontal Wells" by H. Sasongko, 2011. *SPE European Formation Damage Conference*, Copyright [2011] by Society of Petroleum Engineers.

Filter cake is represented by a thin low permeability layer on the borehole wall and it is assumed that after one cycle of the acid jetting treatment process, the filter cake is completely removed along the treated segment. This corresponds to the assumption that the filter cake is removed by the mechanical jetting action.

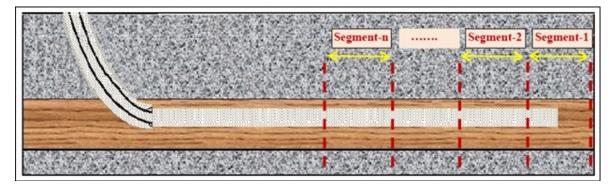


Fig. 2.1 – One segment in openhole section represents one cycle of the acid jetting treatment process

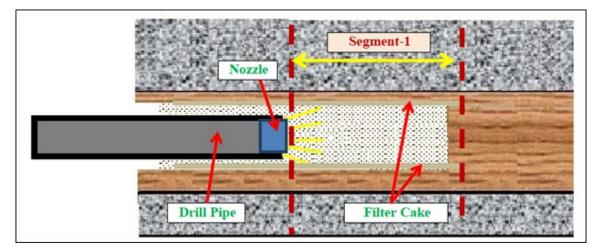


Fig. 2.2 – For one treatment cycle, the acid jetting injection point is assumed stationary at the tip of each segment

The injected acid amount is controlled by the duration of injection in each cycle, and the pumping rate and the drilling string pulling speed are matched to the field operation conditions.

When one cycle of acid jetting is done, the injection point is moved to the tip of the next segment and the same process repeated while in the previous segments, filter cake is no longer exists. As a new acid jetting cycle process begins, filter cake on the face of the borehole is removed by mechanical action, and at the same time, some amount of acid will go to the previous segments and flow into the formation creating wormhole. The amount of acid that goes into the formation and the length of wormhole propagation depend on the pressure inside the wellbore, reservoir properties, and the presence of damage around the wellbore. A depiction of this process can be seen in **Fig. 2.3** below.

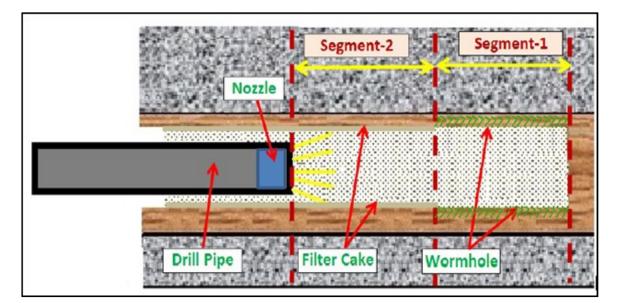


Fig. 2.3 – At the second cycle, acid removes the filter cake in the current segment and creates wormholes in the previous segments

Based on the concept shown in **Fig. 2.3**, as the nozzle is pulled away to segment-3, wormholes also start to grow in the segment-2, and wormhole length in segment-1 keeps growing while segment-2 is treated. This process is repeated for all segments of the well.

2.2 Model Formulation

In this section, an acidizing model for each cycle in acid jetting treatment will be presented. The acid placement model includes an interface tracking function to trace the movement of interfaces of different fluids in the wellbore, a wellbore flow model to calculate pressure drop and material balance inside the wellbore, and a transient reservoir outflow model. This work is developed based on previous work in Mishra et al. (2007) and Nozaki et al. (2010).

In a field practice, acid flows inside drillpipe through nozzle, removes filter cake and will give a chance to some amount of acid flow into the formation creating wormhole. To simulate each cycle process in acid jetting treatment, several models have been developed. The wellbore model will handle the pressure drop and material balance inside wellbore, the interface tracking model will track the interfaces movement between different fluids in the wellbore, the transient reservoir flow model, the wormhole model to predict the wormhole growth, and the skin model to estimate skin evolution during treatment.

2.2.1 Wellbore Model

The wellbore model will calculate the material balance inside wellbore and pressure drop along wellbore. It is assume that the fluid used during acid jetting treatment is single phase and incompressible fluid. The schematic of the wellbore model is shown in **Fig. 2.4** below.

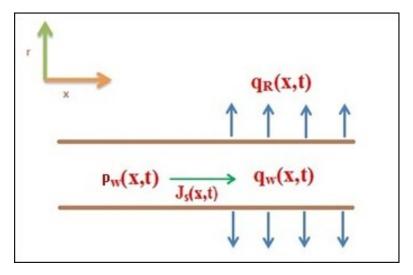


Fig. 2.4 – Wellbore flow schematic

Wellbore pressure is represented by p_w , q_w represents the flow rate inside wellbore and q_R is the specific reservoir outflow per unit length. The flow rate changes along wellbore is mainly caused by the amount of fluid that flows into reservoir. By using material balance concept, this process can be written as,

$$\frac{\partial q_w(x,t)}{\partial x} = -q_R(x,t) \tag{2.1}$$

Equation 2.1 implies that the flow rate decrease per unit length inside wellbore, should be equal to the specific reservoir outflow, q_R (bbl/ft).

Pressure drop due to friction factor along wellbore is obtained by assuming a steady state flow in a horizontal section and the fluid has a constant density by using Fanning equation (Economides et al. 1994),

$$\frac{\partial p_w}{\partial x} = -1.525 f_f \frac{\rho(q_w(x,t))^2}{d^5}$$
(2.2)

where p_w is in psi, x is in ft, ρ is in lb_m/ft^3 , d is in inches, and q_w is in bpm.

Equation 2.2 will estimate a pressure drop in a horizontal section during the acid injection process.

2.2.2 Reservoir Flow Model

The reservoir flow is calculated by a transient inflow equation with variable injection rate (Lee et al. 2003),

$$-\frac{2\pi kl}{\mu}(p_i - p_w) = \sum_{j=1}^n \Delta q_R^j \left[p_D(t_{D_n} - t_{D_{j-1}}) + q^n s^n \right]$$
(2.3)

The Equation 2.3 assumes a transient condition which the wellbore rate and reservoir inflow at any location are not constant with time during acidizing process, where,

$$\Delta q_R^j = q_R^j - q_R^{j-1} \tag{2.4}$$

$$p_D \approx \frac{1}{2}(lnt_D + 0.80907)$$
 (2.5)

$$t_D = \frac{4.395 \times 10^{-6} kt}{\emptyset \mu c_t r_w^2} \tag{2.6}$$

If Eq. 2.3 is divided by 1 (length of segment), and re-arranging, the transient injection rate per unit length of wellbore at time t^n can be written as,

$$q_R^n = -aJ_x(p_R - p_w^n) - b_{Jx}$$
(2.7)

where,

$$a_{Jx} = \frac{4.91816x10^{-6}k}{\mu[p_D(t_{D_n} - t_{D_{n-1}}) + s^n]}$$
(2.8)

$$b_{J\chi} = \frac{\sum_{j=1}^{n-1} \Delta q_R^j \left[p_D(t_{D_R} - t_{D_{j-1}} \right] - q_R^{n-1} p_D(t_{D_R} - t_{D_{n-1}})}{p_D(t_{D_R} - t_{D_{n-1}}) + s^n}$$
(2.9)

The Eq. 2.8 is using oilfield units of bpm/ft for injection rate, mD for permeability unit, and c_p for viscosity.

2.2.3 Fluid Interface Tracking Model

The concept of fluid interface tracking model is built based on a model that was developed by Eckerfield et al. (1998). The acid displacement model uses a discretized solution approach which is integrated with the reservoir flow, wormhole, and skin model. **Fig. 2.5** shows the schematic of the interface between injected acid and wellbore fluid.

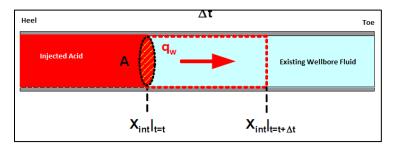


Fig. 2.5 – Fluid interface movement inside wellbore

By assuming that the interface movement is a piston-like displacement, the velocity of an interface located at x_{int} is expressed,

$$\frac{dx_{int}}{dt} = \frac{q_w}{A}\Big|_{x=x_{int}}$$
(2.10)

In discrete form the location of interface at time $(t+\Delta t)$ can be written as,

$$x_{int}|_{t=t+\Delta t} = x_{int}|_{t=t} + \frac{q_w}{A}\Big|_{x=x_{int}} \Delta t$$
(2.11)

where A is the cross-sectional area of flow in the pipe and the Eq. 2.11 is solved by discretizing the wellbore into small segments and assuming constant q_w over each segment.

After the fluid interface and the outflow rates in the wellbore are estimated then the growth of wormhole during the injection time is determined. Length of wormhole is calculated by integrating growth in every small time step.

2.2.4 Wormhole Model

In carbonates formation, the purpose of acidizing treatment is to dissolve the matrix, forming new channels (wormholes) that bypass the damaged areas and lead to the wellbore improving the flow condition. These wormholes are formed when the matrix of the porous and permeable rock is dissolved by reactive fluids like HCl.

Wormholes formation in carbonates has been widely discussed where the structure of wormholes depends on many factors such as flow geometry, injection rate, reaction kinetics, and mass transfer rate. In this work, two models of the wormholing process will be implemented.

2.2.4.1 Volumetric Model

The volumetric wormhole model that had been implemented (Economides et. al, 1994) is assumed that a constant fraction of rock volume is dissolved in the region penetrated by wormholes. For radial flow, the formula for volumetric model is,

$$r_{\rm wh} = \sqrt{r_{\rm w}^2 + \frac{\bar{\nu}}{\pi \emptyset P V_{\rm bt}}}$$
(2.12)

where r_{wh} is the radius of the region penetrated by wormholes and \overline{V} is the volume of acid injected per unit length of formation. PV_{bt} is the parameter that represents the number of pore volumes of acid required to propagate wormholes through a certain distance which highly depends on the rock mineralogy. The PV_{bt} is usually obtained from laboratory experiment using a linear core.

2.2.4.2 Buijse Semi-empirical Model

The wormhole model is using a semi-empirical model presented by Buijse and Glasbergen (2005). They developed an improved empirical model which the growth rate of the wormhole front is modeled as function of acid velocity in the pores (interstitial velocity). In their study, several parameters such as temperature, acid concentration, permeability, and mineralogy are not modeled explicitly but are incorporated in the model in two constans, W_{eff} and W_B . These two values are obtained from the results of a core flow experiments in the laboratory.

The interstitial velocity, V_i, is the average acid velocity in the pores, defined by,

$$V_{i} = \frac{q}{2\pi r l \phi}$$
(2.13)

with the q is the volumetric acid injection rate, r is the distance of the wormhole tip from the center of the wellbore.

In the model, the growth rate of the wormhole front, V_{wh} , is expressed as a function of the interstitial fluid velocity (V_i),

$$V_{wh} = W_{eff} \cdot V_i^{\frac{2}{3}} \cdot B(V_i)$$
(2.14)

The B-function in above equation describes the compact dissolution regime at low values of V_i . The expression for $B(V_i)$ is formulated as,

$$B = \left(1 - exp(-W_B \cdot V_i^2)\right)^2$$
(2.15)

The expression in Eq. 2.15 shows that at high value of V_i , the B-function is equal to 1 and has no effect on V_{wh} in Eq. 2.14. When the V_i is less than the optimum value, $V_{i\text{-opt}}$, the B-function has a value less than 1, and has an inhibiting effect on the wormhole growth rate V_{wh} . The constant W_B is the wormhole B-factor which directly related to the optimum injection rate.

The pore volumes to break through is expressed by PV_{bt},

$$PV_{bt} = \frac{V_i}{V_{wh}} = \frac{V_i^{1/3}}{W_{eff} B(V_i)}$$
(2.16)

In the Eq. 2.16, the constants of W_{eff} and W_B are determined experimentally by fitting Eq. 2.15 and Eq. 2.16 to the results of core flow test. A simpler method to calculate the constants W_{eff} and W_B are using the following equations,

$$W_{eff} = \frac{V_{i-opt}^{1/3}}{PV_{bt-opt}}$$
(2.17)

$$W_B = \frac{4}{V_{i-opt}^2} \tag{2.18}$$

Both equations give acceptable values for W_{eff} and W_B although the accuracy is less compared to a direct fit of the curve.

The example of the results of data fitting to estimates the value of W_{eff} and W_B can be seen as **Fig. 2.6** below which the fitting procedure is recommended to be done numerically.

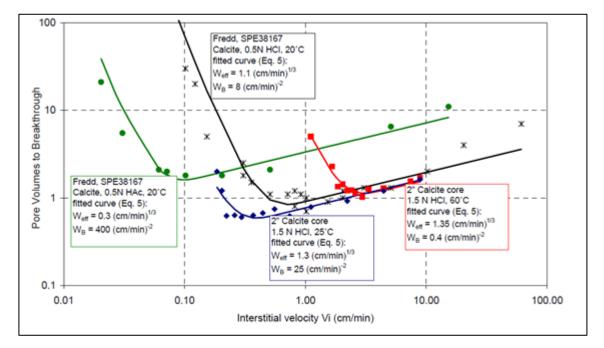


Fig. 2.6 – Typical of coreflood test results and numerical data fit (Buijse and Glasbergen 2005)

The experimentally core flow test is usually used a linear core geometry which the interstitial fluid velocity, V_i , is independent of the position in the core (Eq. 2.13). In this case, the wormhole velocity, V_{wh} , depends only on the injection rate not on the position of the wormhole front inside core (Eq. 2.14). For radial geometry, however, V_i is a function of the distance R from wellbore and can be express as:

$$V_i(R) = \frac{q_R}{2\pi R h \phi} \tag{2.19}$$

In Eq. 2.19, the wormhole growth will depend on the injection rate and the position of the wormhole front in the formation where the V_{wh} decreases with the increasing of the wormhole penetration.

2.2.5 Skin Model

The skin factor model in this acid placement model is the skin for an openhole completion based on Hawkin's formula (Hawkins 1956). The flow inside the wormhole is assumed steady state flow and is considered as infinite conductivity channels (Brownscombe and Collins 1950). **Fig. 2.7** shows a damaged region of r_d , and a stimulated region of r_{wh} around an openhole completion with the wellbore radius of r_w .

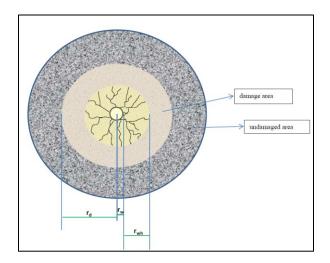


Fig. 2.7 – Schematic of near-wellbore zone in an openhole completion

The original permeability around wellbore can be altered due to fines migration from drilling and completion fluid. In openhole completions, the wellbore have direct contact with the formation and damage skin can cause significant pressure drop and reduce the well productivity performance.

For the case of $r_{wh} < r_d$, local skin factor can be derived by applying Hawkin's formula (Economides et al. 1994),

$$s(x) = \frac{k}{k_d(x)} \ln\left(\frac{r_d(x)}{r_{wh}(x)}\right) - \ln\left(\frac{r_d(x)}{r_w}\right)$$
(2.20)

For the case of $r_{wh} > r_d$,

$$s(x) = -\ln\left(\frac{r_{wh}(x)}{r_w}\right)$$
(2.21)

where r_{wh} is radius of region penetrated by wormholes at that particular point and is calculated from the wormhole model.

During acid injection process the wormholes are created and propagate into the formation. The propagation of wormholes into the formation lowers the skin and enhances the productivity of the well. The evolution of skin factors for each type of completions are discussed in separate chapter.

CHAPTER III

ACID JETTING SIMULATOR*

3.1 Problem Solution

This section briefly describes the numerical solution of acid jetting placement in long horizontal wells. All models in the stimulation process including wellbore flow model, skin calculation, fluid interface tracking, reservoir inflow, wormhole growth, and skin evolution were incorporated into a numerical simulator and solved in a discretized manner in time and space.

The initial and boundary conditions to solve the equations in acid jetting model systems,

$$q_w(x,0) = 0$$
 (3.1)

$$p_w(x,0) = p_R \tag{3.2}$$

$$q_w(x,t) \qquad = 0; x \ge L \tag{3.3}$$

The first initial boundary implies that at the initial condition, wellbore flow rate at any point is zero. This condition is achieved by assuming that the initial pressure in wellbore is equal to the initial average reservoir pressure (Eq. 3.2). The other assumption (Eq. 3.3) assumes that there is no lateral flow in the wellbore beyond the toe of the well (x > L).

^{*}Part of data reported in this chapter is reprinted with permission from "Simulation of Acid Jetting Treatments in Long Horizontal Wells" by H. Sasongko, 2011. *SPE European Formation Damage Conference*, Copyright [2011] by Society of Petroleum Engineers.

When the stimulation starts, the injection rate at heel (x = 0) is defined as Equation 3.4 below,

$$q_w(0,t) \qquad = q_i(t) \tag{3.4}$$

The work steps to solve the acid jetting model are:

- 1. Dividing the wellbore horizontal section into small segments.
- 2. Locate the acid injection point position.
- 3. Discretize the injection time into small time steps.
- 4. Apply the boundary and initial conditions.
- Calculate the initial parameter such as skin value for each segment and fluid interface positions.
- 6. Calculate the pressure drop and reservoir outflow value and estimate the acid movement inside wellbore.
- Calculate the volume of acid injected into each segment each time step based on pressure drop distribution and fluid interface position.
- 8. Based on the injected acid volume and by using wormhole model, the wormhole propagation is estimated for each segment.
- 9. Calculate the new skin based on the latest wormhole length in previous calculation.
- 10. Record the current parameters in current acid injection position for all segments in horizontal section as a new initial condition for a new acid injection point.

The workflow can be simplified and seen as Fig. 3.1 below.

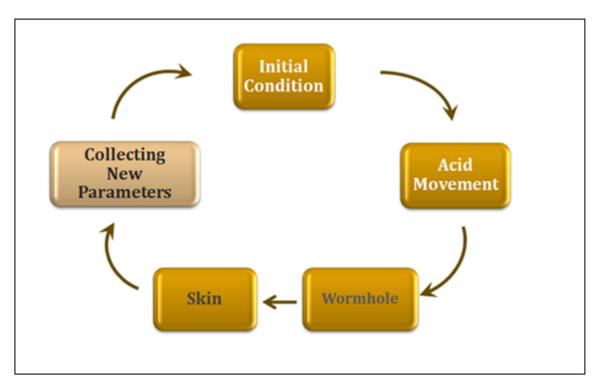


Fig. 3.1 – The workflow of acid jetting treatment model at each step

3.2 Acid Jetting Simulator

The acid jetting simulator is developed by implementing all the models and solutions in FORTRAN-90 language program. The software is equipped with the user friendly interface program to perform the tasks automatically and to reduce hassle involved while analyzing the result. The main simulator program is created as an *.exe file and it reads the data from a *.txt input file while the output result file is provided as an *.txt file. In this system, the interface is needed to help user creates a readable input file and extracts the result file.

The interface program is made using VBA, excel due to its simplicity and easy to use program, and can be seen as **Fig. 3.2** below.

	Well Acid Sti	mulat <u>io</u>	n		Bac	k	Next					
Start	Well Configuration	Flu	iid Propertie	s Tr	eatment Sch	nedule	Result	is				
Well Configurations and Reservoir Properties												
Wellbore OD	8.5	in										
Casing ID	8.1	in										
Pipe Roughness	0.0001											
Total Vertical Depth	3320	ft										
Measured Depth	17500	ft										
Wellbore Length	15000	ft										
Tubing Tail Location	0	ft										
nitial Pressure	1200	psi										
Fluid Viscosity	1	op										
Total Compressibility	5.00E-06	psi ⁻¹										
Rock Type	DOLOMITE -	1										
Rock Density	2.85	g/cm ³										
		grom										
	el 📕	Jarom										
Wormhole Mode	el E Buijs e-Glas bergen 🔹]										
Wormhole Mode Model Type	Buijs e-Glas bergen 🔸] grow										
Wormhole Mode Model Type Discretized Prop	Buijse-Glasbergen •]										
Wormhole Mode Model Type Discretized Prop Completion Type	Buijse-Glasbergen • perties Open Hole]										
Wormhole Mode Model Type Discretized Prop Completion Type Number of Segments	Buijs e-Glas bergen • perties Den Hole 20 •]										
Wormhole Mode Model Type Discretized Prop Completion Type Number of Segments	Buijse-Glasbergen • perties Open Hole]]] [ft						Input Initial				
Wormhole Mode Model Type Discretized Proj Completion Type Number of Segments Grid Length, dx	Buijs e-Glas bergen • perties Den Hole 20 •]						I Input Initial alue			ergen Wormhole fodel	
Wormhole Mode Model Type Discretized Prop Completion Type Number of Segments	Buijs e-Glas bergen • perties Den Hole 20 •]	Beginning Point	End Point	Anistropy Index	Porosity			Damage penetration			WH Length History inch
Wormhole Mode Model Type Discretized Prop Completion Type Number of Segments Grid Length, dx	Buijs e-Glas bergen • perties Cpen Hole 20 • 750]]]ft Number of		End Point x2(tt)		Porosity	Horizontal	Permeability Impairment		N	/lodel Interstitial	Lengt History
Wormhole Mode Model Type Discretized Prop Completion Type Number of Segments Grid Length, dx Segment Number # 1	Buijs e-Glas bergen • perties Open Hole 20 • 750 Status	ft Grids	Point x1(ft) 0	<mark>ж₂ (ft)</mark> 750	Index I _{ani} 1	φ 0.3	Horizontal Permeability k _H (md) 10	Permeability Impairment Ratio R _{KS} (k ₄ /k) 100	penetration L₄(ft) 0.0001	PVbt PV _{bt-set} 0.23	Acidel Interstitial Velocity V _{ireet} (cm/min) 0.41	Lengtl History inch 1.2E-0
Wormhole Mode Model Type Discretized Prop Completion Type Number of Segments Grid Length, dx Segment Number # 1 2	Buijs e-Glas bergen • perties Dpen Hole 20 • 750 Status open/close open open	ft Number of Grids # 1 1	Point ×1(ft) 0 750	<mark>×₂(ft)</mark> 750 1500	Index Iani 1 1	¢ 0.3 0.3	Horizontal Permeability k _H (md) 10 10	Alue Permeability Impairment Ratio R _{KS} (k ₄ /k) 100 100	penetration L₄(ft) 0.0001 0.0001	PVbt PVbt 0.23 0.23	Acidel Interstitial Velocity V _{iraet} (om/min) 0.41 0.41	Lengt History inch 1.2E-0 3.6084
Wormhole Mode Model Type Discretized Prop Completion Type Vumber of Segments Srid Length, dx Segment Number # 1 2 3	Buijs e-Glas bergen • perties Cpen Hole 20 • 750 Status open/close open open open	ft Number of Grids # 1 1 1 1	Point ×1(ft) 0 750 1500	<mark>ж₂ (ft)</mark> 750 1500 2250	Index Iani 1 1 1	φ 0.3 0.3 0.3	Horizontal Permeability k _H (md) 10 10 10	Permeability Impairment Ratio P _{KS} (k ₄ /k) 100 100 100	penetration L₄(t) 0.0001 0.0001 0.0001	PVbt PVbt 0.23 0.23 0.23 0.23	Acidel Interstitial Velocity Vireet (cm/min) 0.41 0.41 0.41	Lengt History inch 1.2E-0 3.6084 5.1810
Wormhole Mode Model Type Discretized Proj Completion Type Vumber of Segments Srid Length, dx Segment Number # 1 2 3 4	Buijs e-Glas bergen • perties Open Hole 20 • 750 Status open/close open open open open	tt Number of Grids # 1 1 1 1	Point x1(ft) 0 750 1500 2250	<mark>ж₂(ft)</mark> 750 1500 2250 3000	Index Iani 1 1 1 1 1 1	¢ 0.3 0.3 0.3 0.3	Horizontal Permeability k _H (md) 10 10 10 10	Permeability Impairment Ratio Price (k ₄ /k) 100 100 100 100 100	Penetration L ₄ (ft) 0.0001 0.0001 0.0001	PVbt PVbt 0.23 0.23 0.23 0.23	Acidel Interstitial Velocity V.;-est (cm/min) 0.41 0.41 0.41 0.41	Lengt History inch 1.2E-0 3.6084 5.1810 6.2836
Wormhole Mode Model Type Discretized Proy Completion Type Number of Segments Grid Length, dx Segment Number # 1 2 3 4 5	Buijs e-Glas bergen • perties Open Hole 20 • 750 Status open/close open open open open	 	Point ×1(ft) 0 750 1500 2250 3000	<mark>ж₂(ft)</mark> 750 1500 2250 3000 3750	Index I 1 1 1 1 1 1 1 1 1 1	φ 0.3 0.3 0.3 0.3 0.3 0.3	Horizontal Permeability k _H (md) 10 10 10 10 10 10	Permeability Impairment Ratio Price (k ₄ /k) 100 100 100 100 100 100 100 100 100	Penetration L₄(k) 0.0001 0.0001 0.0001 0.0001 0.0001	PVbt PVbt 0.23 0.23 0.23 0.23 0.23 0.23	100del Interstitial Velocity 0.41 0.41 0.41 0.41 0.41 0.41 0.41 0.41	Lengti History inch 1.2E-0 3.6084 5.1810 6.2836 7.1869
Wormhole Mode Model Type Discretized Proj Completion Type Number of Segments Grid Length, dx Segment Number # 1 2 3 4	Buijs e-Glas bergen • perties Open Hole 20 • 750 Status open/close open open open open	tt Number of Grids # 1 1 1 1	Point x1(ft) 0 750 1500 2250	<mark>ж₂(ft)</mark> 750 1500 2250 3000	Index Iani 1 1 1 1 1 1	¢ 0.3 0.3 0.3 0.3	Horizontal Permeability k _H (md) 10 10 10 10	Permeability Impairment Ratio Price (k ₄ /k) 100 100 100 100 100	Penetration L ₄ (ft) 0.0001 0.0001 0.0001	PVbt PVbt 0.23 0.23 0.23 0.23	Acidel Interstitial Velocity V.;-est (cm/min) 0.41 0.41 0.41 0.41	Lengtł History

Fig. 3.2 – The acid jetting simulator interface program

There are 5 input sections in the interface program with the first section is "Start" containing the program info and copy right statement. The well configuration data, reservoir properties and wormhole model option can be inputted in "Well Configuration" section, while the fluid injection properties is placed in "Fluid Properties" section. Drillpipe pulling speed and acid injection rate during treatment are defined in the

"Treatment Schedule" section with the wormhole performance, skin along wellbore and acid injection in horizontal section after treatment can be seen in "Result" section.

Following are the steps to run the acid jetting simulator program:

- 1. Put the acid jetting executable file and the interface program into the same folder.
- 2. Open the excel file, and press the "Start" button to start the program.
- 3. Input the well data in the well configuration section, fluid properties and treatment schedule then press "Run" to start the acid jetting simulation.
- The post treatment result is available in result section where the wormhole length, skin performance and acid injection volume along wellbore profile can be seen graphically.

CHAPTER IV

RESULT AND DISCUSSION*

4.1 Case Study

In this section, a case study is performed to see how acid is distributed and affected the well performance after the acid jetting stimulation treatment. All data in the example case are taken from published paper based on field practice. Acid is injected at target volume coverage of 0.5 bbl/ft with the pulling speed of drill pipe is about 50 ft/min.

For the wormhole model, the example case uses a value of 0.53 for optimum pore volume to breakthrough, PV_{bt-opt} , and value of 1.75 cm/min for optimum interstitial velocity, V_i -opt. These values were obtained from a laboratory experiment using 1" x 6" core with 15% HCl at 15 °F (Furui et al. 2010).

Average reservoir pressure data is not available in published papers, the best estimation value is then assumed in the example case including filter cake thickness and average rock permeability.

^{*}Part of data reported in this chapter is reprinted with permission from "Simulation of Acid Jetting Treatments in Long Horizontal Wells" by H. Sasongko, 2011. *SPE European Formation Damage Conference*, Copyright [2011] by Society of Petroleum Engineers.

The input data summaries for the example case are presented in Table 4.1 below.

V	Vell Data				
OH Diameter	8.6	inch			
TVD/MD	5,000/25,000	ft			
Openhole Section	20,000	ft			
Average Pr	2,500	psi			
Filter Cake Thickness	0.1	inch			
Segment Length	1000	ft			
Number of Segments	20				
Reservoir Data					
Wormhole Model	Buijse - Glasbergen				
Rock Type	Limestone				
Rock Porosity	31%				
Rock Permeability	5	mD			
Acid Data					
Injected Acid Amount	0.5	bbl/ft			
Pumping Rate	20	bpm			
Acid Viscosity	1	ср			

Table 4.1 – Data input

4.2 Running Result and Discussion

The distribution of wormhole along the wellbore at the end of each treatment cycle is shown in **Fig. 4.1** below. It is evident from this plot that wormholes longer than 2 inches long were created only in the last 5,000 feet of the wellbore towards the toe.

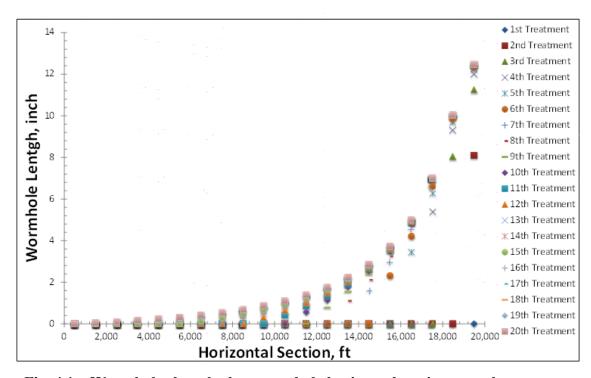


Fig. 4.1 – Wormholes length along openhole horizontal section at each treatment

The toe section of the well is the first to be stimulated, so it is expected that the toe section is the most stimulated zone. Less anticipated are the facts that most of the well (up to 75%) receives almost no matrix stimulation other than filter cake removal and that the wormholes in the toe section cease growing significantly during the treatment. These results are caused by the decreasing flux into the formation as more and more of the filter cake is removed by the acid jetting treatment, making the wormholing process is very slow.

Figures 4.2 - 4.4 clarify this result. Fig. 4.2 shows that the wormhole growth in the toe segment is negligible during the final 75% of the treatment. However, as indicated in Fig. 4.3, a plot of the toe segment wormhole length versus cumulative acid volume injected into that segment shows that the wormhole growth was very small

through this latter part of the treatment even though the segment continued to receive acid during the entire treatment.

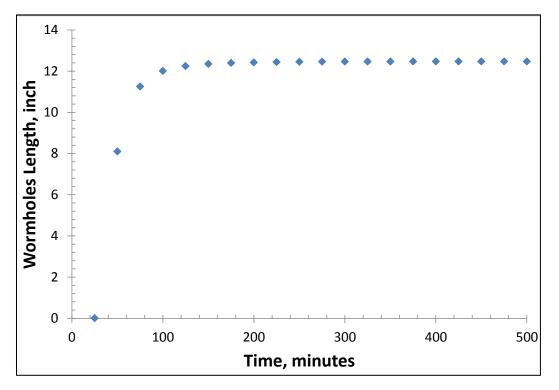


Fig. 4.2 – Wormhole length at the toe segment as function of time

The slow wormhole growth throughout most of the treatment is caused by the interstitial flux falling well below the optimal value of 1.75 ml/min. For velocities below the optimum, the amount of acid needed to propagate wormholes a given distance increases dramatically with a decrease in velocity. **Fig. 4.4** shows that the acid flux fell to less than 0.1 cm/min into the toe segment where the flux will always be the highest of anywhere along the well for an acid jetting treatment of a relatively homogeneous

formation. This reduced flux results in wormhole growth being orders of magnitude below the optimum.

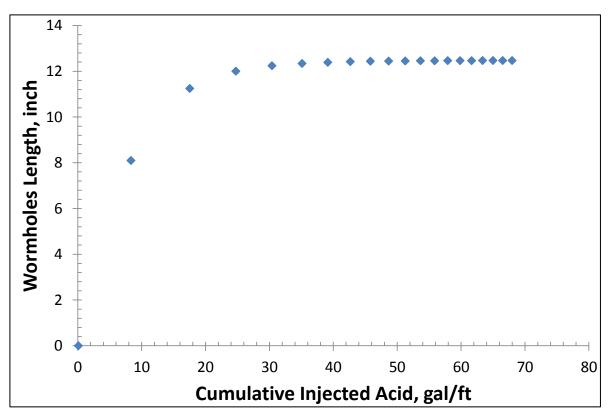


Fig. 4.3 – Wormhole length at the toe segment as function of cumulative injected acid

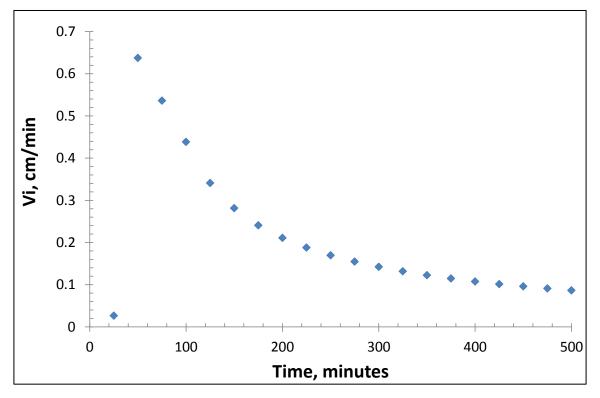


Fig. 4.4 – Interstitial velocity at the toe segment as function of time

For the sections of the well away from the toe, by the time these were exposed to the acid by the removal of the filter cake, the flux into the formation was too low to efficiently propagate wormholes. The stimulation of most of the well was only through filter cake removal by jetting, not by matrix stimulation. In fact, the well would have been treated just as effectively by replacing the acid with water for the last 75% of this treatment.

The results shown here are obviously very sensitive to the characteristics of the wormhole propagation process, as represented by the optimal interstitial flux and optimal pore volumes to breakthrough. If the values of these parameters are decreased, as suggested by the results of large core experiments and scale-up procedures (Furui et al., 2010), the results are dramatically improved. Using an optimal PV_{bt} of 0.147 and an optimal V_i of 0.228 cm/min as presented by Furui et al., for the same acid injection conditions, the wormhole distribution shown in **Fig. 4.5** is obtained. For this case, the entire well has achieved some matrix stimulation. The characteristics of the wormholing process are clearly critical information needed to predict the outcome of matrix stimulation of carbonates.

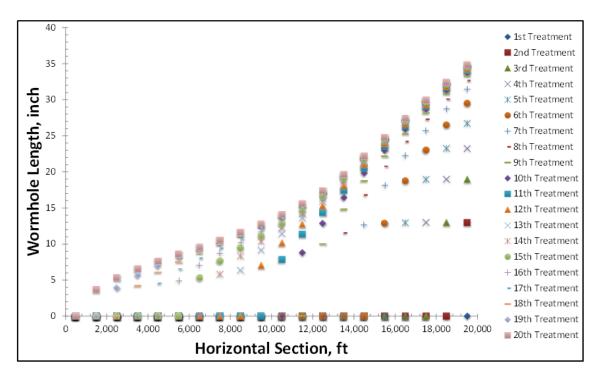


Fig. 4.5 – Wormholes length along openhole horizontal section $PV_{bt-opt} = 0.147$ and $V_i = 0.228$ cm/min

If the acid flux becomes too low to efficiently propagate wormholes in the formation, face dissolution of the wellbore, resulting in wellbore enlargement, will occur. Caliper logs run after acid jetting treatments in Qatar (Ritchie et al., 2010)

indicate that significant wellbore enlargement did occur as shown in **Fig. 4.6**. This problem could be mitigated by increasing injection rate if possible as the stimulated interval gets longer, or temporarily plugging off the section towards the toe before proceeding with more jetting.

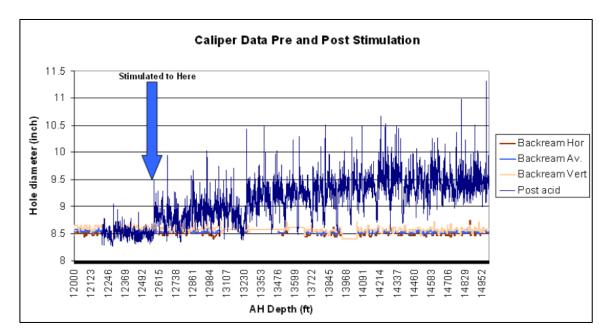


Fig. 4.6 – Caliper logs run after acid jetting treatments in Al Shaheen (Ritchie et al., 2010)

CHAPTER V

SUMMARY AND CONCLUSIONS

5.1 Summary

We have applied a horizontal well carbonate acidizing model to simulate acid jetting treatments that are performed in long openhole sections in carbonate reservoirs. The model shows that failure to maintain sufficient acid flux into the formation as longer and longer sections are exposed can result in very inefficient matrix stimulation. To maintain wormhole growth throughout such a treatment, the acid flux into the formation cannot drop too far below the optimal flux value.

5.2 Recommendations

These results show that acid jetting treatments could be improved by:

- 1. Increasing injection rate as more formation face is exposed by filter cake removal.
- 2. Temporarily plugging off the toe end of the well to increase the flux in the section towards the heel.
- 3. The study also showed that wormhole creation in the matrix in acid jetting of long horizontal wells is very sensitive to the wormholing behavior in the rock/acid system. Careful characterization of the optimal wormholing conditions for a particular acid system in a particular reservoir would greatly enhance the capability to design improved acid jetting treatments.

REFERENCES

- Brink, D.I., Ernst, S.D., Banerjee, S.N., Ekwue, W., Ritchie, B. et al. 2010. Improved Reservoir Management from Application of Intelligent Reservoir Completion Technology in an Extended-Reach Well in the Giant Al Shaheen Field, Offshore Qatar. Paper SPE 134934 presented at the SPE Annual Technical Conference and Exhibition held in Florence, Italy, 19 – 22 September 2010. http://dx.doi.org/10.2118/134934-MS.
- Brownscombe, E.R. and Collins, F. 1950. Pressure Distribution in Unsaturated Oil Reservoirs. *Trans. AIME*. **189**, 371-372. SPE-950371-G.
- Buijse, M.A. and Glasbergen, G. 2005. A Semi-Empirical Model to Calculate Wormhole Growth in Carbonate Acidizing. Paper SPE 96892 presented at the SPE Annual Technical Conference and Exhibition held in Dallas, Texas, USA, 9 – 12 October 2005. http://dx.doi.org/10.2118/96892-MS.
- Eckerfield, L.D., Zhu, D., Hill, A.D., Thomas, R.L., Robert, J.A. et al. 2000. Fluid Placement Model for Horizontal-Well Stimulation. Paper SPE 49103 presented at the SPE Annual Technical Conference and Exhibition held in New Orleans, Louisiana, USA, 27 – 30 September 1998. http://dx.doi.org/10.2118/49103-MS.
- Economides, M. J., Hill, A.D., and Ehlig-Economides, C. 1994. *Petroleum Production System*. Englewood Cliffs, New Jersey: Prentice Hall.
- Furui, K., Burton, R.C., Burkhead, D.W., Abdelmalek, N.A., Hill, A.D. et al. 2010. A Comprehensive Model of High-Rate Matrix Acid Stimulation for Long Horizontal Wells in Carbonate Reservoirs. Paper SPE 134265 presented at the SPE Annual Technical Conference and Exhibition held in Florence, Italy, September 19-22, 2010. http://dx.doi.org/10.2118/134265-MS.
- Hansen, J.H. and Nederveen, N. 2002. Controlled Acid Jet (Caj) Technique for Effective Single Operation Stimulation of 14,000+ Ft Long Reservoir Sections. Paper SPE 78318 presented at the European Petroleum Conference held in Aberdeen, United Kingdom, 29 – 31 October 2002. http://dx.doi.org/10.2118/78318-MS.
- Hawkins, M.F. 1956. A Note on the Skin Effect. J. Pet Tech. 8 (12):65-66.
- Hoch, E., Ohrt, H.B., Brink, D.I., Flikkema, J., and Norman, R. 2010. Pushing the Limits for Field Development Al Shaheen Field, Offshore Qatar. Paper SPE 138301 presented at the Abu Dhabi International Petroleum Exhibition and Conference held in Abu Dhabi, UAE, 1 – 4 November 2010. http://dx.doi.org/10.2118/138301-MS.

- Lechner, M., Ernst, S.D., Pitts, M.J., and Lopdrup, T.P. 2009. Case Study: Improved Reservoir Management from a Surface Controlled Two-Zone Open Hole Packer Completion in a Horizontal Well in Al Shaheen Field, Offshore Qatar. Paper SPE 13671 presented at the International Petroleum Technology Conference held in Doha, Qatar, 7 – 9 December 2009. http://dx.doi.org/10.2523/13671-MS.
- Lee, J., Rollins, J.B., and Spivey, J.P. 2003. *Pressure Transient Testing*, Vol. 9, Richardson, Texas: Textbook Series, SPE.
- Mishra, V., Zhu, D., Hill, A.D., and Furui, K. 2007. An Acid Placement Model for Long Horizontal Wells in Carbonate Reservoirs. Paper SPE 107780 presented at the European Formation Damage Conference held in Scheveningen, The Netherlands, 30 May – 1 June 2007. http://dx.doi.org/10.2118/107780-MS.
- Nozaki, M. and Hill, A.D. 2010. A Placement Model for Matrix Acidizing of Vertically Extensive, Heterogeneous Gas Reservoirs. *SPE Prod & Fac* **25** (3): 388-397. SPE-124881-PA. http://dx.doi.org/10.2118/124881-PA.
- Ritchie, B., Abbasy, I., Pitts, M.J., and White, B. 2008. Challenges in Completing Long Horizontal Wells Selectively. SPE Paper 116541 presented at the SPE Asia Pacific Oil and Gas Conference and Exhibition held in Perth, Australia, 20 – 22 October 2008. http://dx.doi.org/10.2118/116541-MS.
- Ritchie, B., Pitts, M.J., Sebastianus, V.D.V., and Aziz, A. I. 2010. Increasing the Reach and Selectivity of Completions in Extended Reach Horizontal Wells. SPE Paper 135670 presented at the IADC/SPE Asia Pacific Drilling Technology Conference and Exhibition held in Ho Chi Minh City, Vietnam, 1 – 3 November 2010. http://dx.doi.org/10.2118/135670-MS.
- Thomasen, J.B., Al-Emadi, I.A., Noman, R., Øgelund, N.P., and Damgaard, A. 2005. Realizing the Potential of Marginal Reservoirs: The Al Shaheen Field Offshore Qatar. SPE Paper 10854 presented at the International Petroleum Technology Conference held in Doha, Qatar, 21 – 23 November 2005. http://dx.doi.org/10.2523/10854-MS.

APPENDIX

The schematic of the wellbore model is shown in Fig. A-1.

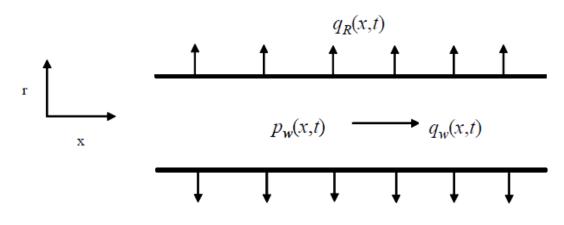


Fig. A-1 – Wellbore flow schematic (Mishra et al. 2007).

Since the flow rate changes along wellbore is caused by the fluid flowing into formation, using material balance concept, we have:

$$\frac{\partial q_w(x,t)}{\partial x} = -q_R(x,t) \tag{A-1}$$

and for steady state flow in horizontal pipe, the pressure drop is (oil field unit):

$$\frac{\partial p_w}{\partial x} = -1.525 f_f \frac{\rho(q_w(x,t))^2}{d^5} \tag{A-2}$$

The reservoir flow model is assumed as a transient flow:

$$-\frac{2\pi kl}{\mu}(p_i - p_w) = \sum_{j=1}^n \Delta q_R^j \left[p_D(t_{D_n} - t_{D_{j-1}}) + q^n s^n \right]$$
(A-3)

where,

$$\Delta q_R^j = q_R^j - q_R^{j-1} \tag{A-4}$$

$$p_D \approx \frac{1}{2}(lnt_D + 0.80907)$$
 (A-5)

$$t_D = \frac{4.395 x 10^{-6} kt}{\emptyset \mu c_t r_w^2} \tag{A-6}$$

If equation A-3 divided by l (length of segment), and re-arranging, we have the transient injection rate per unit length of wellbore at time t^n as;

$$q_R^n = -aJ_x(p_R - p_w^n) - b_{Jx}$$
(A-7)

where,

$$a_{Jx} = \frac{4.91816x10^{-6}k}{\mu[p_D(t_{D_n} - t_{D_{n-1}}) + s^n]}$$
(A-8)

$$b_{jx} = \frac{\sum_{j=1}^{n-1} \Delta q_R^j \left[p_D(t_{D_n} - t_{D_{j-1}} \right] - q_R^{n-1} p_D(t_{D_n} - t_{D_{n-1}})}{p_D(t_{D_n} - t_{D_{n-1}}) + s^n}$$
(A-9)

Solution approach

If the wellbore is segmented as in Fig. A-2 below, we can re-write equation A-7 as;

$$q_{w,i+1/2} - q_{w,i-\frac{1}{2}} = \Delta x_i [aJ_{x,i}(p_R - p_{w,i}) - b_{Jx,i}$$
(A-10)

for i = 1,2,3,4,5

$$p_{w,i+1} - p_{w,i} = -\frac{(\Delta x_{i+1} + \Delta x_i)}{2} \xi_i q_{w,i+1/2}$$
(A-11)

for i = 1,2,3,4,5 and ξ_i is a function of q_w .

diagonal matrix system as:

$$\begin{pmatrix} A_{1} & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & C_{1}(q) & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & A_{2} & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & C_{2}(q) & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & A_{3} & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & C_{3}(q) & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & C_{4}(q) & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & C_{4}(q) & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & A_{5} \end{pmatrix} \begin{pmatrix} p_{w,1} \\ q_{w,3/2} \\ p_{w,2} \\ q_{w,5/2} \\ p_{w,3} \\ q_{w,7/2} \\ p_{w,4} \\ q_{w,9/2} \\ p_{w,5} \end{pmatrix}$$

$$\begin{pmatrix} A_{1}P_{i} + B_{1} + Q_{w} \\ 0 \\ A_{2}i + B_{2} \\ 0 \\ A_{3}P_{i} + B_{3} \\ 0 \\ A_{5}P_{i} + B_{5} \end{pmatrix}$$

$$(A-12)$$

where,

$$A_i = \Delta x_i a_{Jx,i}$$

$$B_i = \Delta x_i b_{Jx,i}$$

$$C_i = (\Delta x_{i+1} + \Delta x_i)\xi_i/2$$

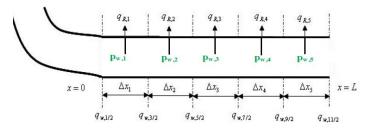


Fig. A-2 – Segmented wellbore diagram.

VITA

Name:	Hari Sasongko
Place of Birth:	Bandung, Indonesia
Permanent Address:	Pancur, RT 4 RW 1, Kabupaten Rembang Jawa Tengah Province Indonesia, 59262
Email Address:	sasongko_hari@yahoo.com
Education:	B.S., Petroleum Engineering, Bandung Institute of Tech, 2001 MS., Petroleum Engineering, Texas A&M University, 2012 College Station, Texas, USA