The Impact of Product Contamination in a Multi-Stage Food Supply Chain

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THE IMPACT OF PRODUCT CONTAMINATION IN A MULTI-STAGE FOOD SUPPLY CHAIN

A Dissertation

by

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Submitted to the Office of Graduate Studies of Texas A&M University in partial fulfillment of the requirements for the degree of

DOCTOR OF PHILOSOPHY

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May 2012

Major Subject: Industrial Engineering
ABSTRACT

The Impact of Product Contamination in a Multi-Stage Food Supply Chain. (May 2012)

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Food product contamination leading to a food borne illness is real and has potentially devastating impact on supply chain operations and cost. However, it is not well understood from the quantitative perspective. This research seeks to fill this gap by providing a generic model of a multi-stage food supply chain consisting of a supplier/grower, processing center and retailer(s) and analyzing the impact of food product contamination in this model. The supplier corresponds to the farm/grower of the raw material such as fruits and vegetables, the processing center processes the raw material into a final food product and the retailer corresponds to the supermarkets and grocery stores selling the food product to a customer. A situation where a contamination occurs at the supplier or processing center potentially resulting in a food borne illness to the customer is considered. The contamination is discovered through periodic sampling tests conducted by the grower, processing center or through the outbreak of a food borne illness. The supply chain is modeled utilizing a G/G/1 queuing system at the processing center and an order-up to policy at the retailer(s).
This research develops and compares multi-stage supply chain models with varying number of retailers. The negative dependence of contamination on the origin and mode of detection of the contamination is quantified. The differences in individual food product attributes which can impact the cost of contamination are analyzed. The impact of supply chain structure and properties and detection policies on the severity of potential contamination cases is studied. The most cost effective sampling strategies which companies can adopt in the event of product contamination are derived. The payoff from the implementation of a quality control process which can eradicate contamination is evaluated. A numerical study of the impact of a real-world contamination event on a tomato and lettuce supply chain is also conducted.

Finally, a traceability system capable of tracking and tracing back products in the event of a food product recall is incorporated in the supply chain model. The value of traceability for different supply chain scenarios is assessed through the implementation of an ARENA based simulation model.
ACKNOWLEDGEMENTS

I would like to thank my committee chair, Dr. Gary Gaukler for his constant guidance, support, understanding and patience. Most importantly it has been an immense learning experience working with him during the course of my research for which I am very grateful. I would also like to thank my committee members, Dr. Guy Curry, Dr. Victoria Salin and Dr. Natarajan Gautam for their valuable advice and time.

I would like to thank the institution of Texas A&M University, the Industrial Engineering and Systems Department and its staff and faculty for giving me the opportunity to pursue my Ph.D. My special thanks to Judy Meeks, colleagues and friends at Texas A&M University for their support and making my stay here a great experience. Finally, thanks to Vinay and my family for their endless support, patience and encouragement.
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CHAPTER I
INTRODUCTION

1.1. Introduction

The Centers for Disease Control and Prevention (CDC) estimate that every year about 76 million people contract a food borne illness, about 325,000 require hospitalization and about 5000 die in the United States. A testimony presented to the House of Representatives in 2007 by the United States Government Accountability Office (GAO) designates oversight of food safety as a high-risk area needing urgent attention as it impacts public health, citizen's rights, and economic growth and could result in significant injury or loss of life, reduced economic efficiency and effectiveness (Shames 2007).

The United States Department of Agriculture's (USDA) Economic Research Service (ERS) estimates that the annual economic cost of the five major food borne illnesses is approximately $6.9 billion (ERS 2001), (Golan, Roberts et al. 2004). Food borne illnesses can have widespread impact, for example a meat recall can affect the entire meat industry including downstream industries such as fast food restaurants and upstream suppliers such as ranchers (Golan, Krissoff et.al. 2004).

Over the past decade the size and number of food products recalled in the United States has increased significantly. There has also been an increase in recall cases.

This dissertation follows the style of INFORMS Transactions on Education.
classified as class I recalls i.e. for foods that pose the greatest risk of illness or death. For instance the size of meat and poultry recalls has increased from nearly 6 million pounds in 1988 to about 36 million pounds in 2003 (Dyckman and Lansburgh 2004). During 1993-96, the number of meat and poultry class I recalls averaged about 24 per year and amounted to 1.5 million pounds annually. During 1997-2000, class I recalls averaged 41 per year and reached 24 million pounds annually. The number of food recall cases has also increased over the past decade, with more than 500 cases reported in the year 2002. Additionally in the year 2003 most cases were categorized as class I recalls (Ollinger and Ballenger 2003). Food product recalls and the outbreaks associated with them highlight a number of deficiencies of the present food safety systems, and underscore the need for greater oversight, more effective industry practices, and stronger safeguards (Sundlof 2009).

1.2. **Food Product Contamination and Recall**

Several high-profile food product recall cases covering a wide variety of products and local as well as global food supply chains have occurred in the recent past. The consequences and the fallout these cases have had on businesses, economy and public health has been severe.

**Peanut product recalls, 2008:** In December 2008 the Food and Drug Administration (FDA) along with the Centers for Disease Control and Prevention (CDC), the Food Safety and Inspection Service (FSIS) and public health officials in various states started investigating the multi-state outbreak of human infections caused
by Salmonella Typhimurium, which had been found in peanut products produced by the Peanut Corporation of America (PCA). As of February 2009 as a part of the ongoing investigation and recall process CDC reported that 600 persons from 44 states had been infected with the outbreak strain of Salmonella Typhimurium apart from 1 person in Canada, and that the infection had contributed to 8 deaths (Sundlof 2009).

An investigation of an unopened container of King Nut peanut butter produced by PCA showed that it contained the same strain of Salmonella Typhimurium that was associated with illnesses linked to the outbreak. A recall of peanut butter and peanut paste produced by PCA was initiated. This recall was subsequently expanded to include all peanuts and peanut products, including all peanuts, granulated peanuts, peanut meal, peanut butter and peanut paste (Sundlof 2009).

The investigations revealed that PCA distributed potentially contaminated products to more than 300 consignee firms, many of whom further distributed these contaminated products for consumption as peanut butter or as ingredients in several different products, such as cookies, crackers, cereal, candy and ice cream. Many of the companies that received peanut and peanut products from PCA in turn recalled those products, thus increasing the scope of recall exponentially. The FDA released a list of nearly 1800 entries in 17 categories of products recalled by about 200 companies as part of a consumer information campaign (Sundlof 2009).

**Milk product recalls, 2008:** One of the most serious recall cases in recent times occurred due to the contamination of milk products through melamine in China. At the beginning of September 2008 it was reported that some brands of infant milk formula
were contaminated with melamine. The contamination with melamine happened in the primary production stage where it was intentionally added to raw milk at milk collection centers for at least 9 months (World Health Organization Expert Review Meeting 2008). Almost 300,000 babies were taken ill due to consumption of the contaminated milk powder, the death toll rose to 6 infants and more than 860 were hospitalized with 154 classified as severe cases by the end of the year 2008 (Branigan 2008).

Chinese inspectors on conducting investigations found the chemical melamine in several batches of infant milk powder produced by 22 companies nationwide. The authorities ordered a halt to the sale of the contaminated products which included among others well-known Chinese brands such as Sanlu, Mengniu, Yili and Yashili. (Xuequan 2008). Chinese officials seized and recalled more than 10,000 tons of baby formula. They seized 2,176 tons of milk powder in the warehouse of Sanlu Group, producer of the milk powder and recalled 8,218 tons that had been on the market (Yuxia 2008).

Several other countries also reported finding melamine in milk containing products, dairy and non-dairy products manufactured in China such as liquid milk, frozen yogurt dessert, biscuits, candies and in coffee drinks (World Health Organization Expert Review Meeting 2008). The repercussions of the contamination event were felt far and wide across the globe. The European Union announced a ban on imports of baby food containing Chinese milk; Australia, New Zealand, Japan, Singapore and India joined the rapidly growing list of countries pulling out contaminated Chinese food products from store shelves (Saputra et.al. 2008). The United States also issued a ban on Chinese food imports; such a broad ban by the Food and Drug Administration on goods
from an entire country rather than from a particular manufacturer was unusual and reflected the level of concern over how widespread the problem was. Importers to the United States had to certify that food products were free of diary or melamine failing which the goods were stopped at the border (Macartney 2008). The melamine scandal had devastating impact on China’s dairy industry, leading to numerous questions about the safety of food products. It was reported that Chinese milk exports had dropped by 92 percent since September 2008, when news of the contaminated milk had first emerged (Jacobs 2008).

So far the legal outcomes of the milk product contamination and recall case have also been severe with two men sentenced to death and the chairwoman of a dairy conglomerate in China receiving a life sentence for their roles in the scandal. A liability lawsuit seeking more than $5.2 million in compensation was also filed in Beijing against a group of dairy companies by the families of 213 children who died or fell ill from consuming tainted milk (McDonald 2009).

Jack in the Box recall, 1993: In 1993 there was an unusual increase in the number of children with bloody diarrhea in the state of Washington. This alerted state health officials to the possibility of a food borne illness outbreak. Health officials on investigation identified E. coli O157:H7-contaminated hamburgers from Jack in the Box restaurants as the cause of the outbreak. Totally, 73 Jack in the Box restaurants in the states of Washington, Idaho, California, and Nevada were found to be involved in the outbreak and recall. Seven hundred people became ill and four children died as a result of this food contamination. Epidemiologists at the Centers for Disease Control and
Prevention concluded that the outbreak was due to contamination in meat processing and cooking (Bell et al. 1994).

Jack in the Box reported losses of approximately $160 million from reduced sales and other costs (Roberts et al. 1997). These costs were due to the company's recall of all hamburger meat from their restaurants and legal costs.

**Spinach recall, 2006:** The 2006 E.coli outbreak in California due to the contamination of spinach resulted in a detrimental impact on public health and the local economy. The FDA reported 205 illnesses and 3 deaths due to the outbreak. Industry representatives estimated losses to range from $37 million to $74 million (Shames 2007).

**Meat, poultry recall, 1999:** Samples from a meat and poultry processing plant in Arkansas of the Thorn Apple Valley Inc Company based in Michigan tested positive for Listeria in January 1999. The plant operations were shut down and a recall of products worth £30 million took place. As a result the Thorn Company faced $ 5.1 million in losses of production and sales and reported $ 184 million as debt to creditors. Finally the company filed for bankruptcy and was bought by Iowa beef producers inc. (Skees et al. 2001).

1.3. **Traceability in the Food Supply Chain**

In the last decade there has been a growing interest in traceability systems as tools to enable the tracking of food products through the supply chain in the event of a food product recall. Golan, Krissoff et al. (2004) in their USDA Economic report define
traceability systems as recordkeeping systems designed to track the flow of product or product attributes through the production process or supply chain.

Traceability systems can help companies isolate the source and extent of these food safety or quality-control problems and companies have an incentive to invest in traceability systems because they help minimize the production and distribution of unsafe or poor quality products, which in turn minimizes the potential for bad publicity, liability and recalls (Golan, Krissoff et al. 2004) (Smith et al. 2005). Traceability can also reduce anonymity and facilitate the allocation of liability in a supply chain (Patel and Desai 2011), (Buhr 2003). The U.S. private sector has voluntarily adopted a diversified and significant capacity to trace food without government regulation, motivated by the desire to improve product recalls (Golan, Krissoff et al., 2004).

Companies are thus seeking to improve supply chain efficiency, create product differentiation and increase food safety and quality control, through the use of these traceability systems to keep track of product flow along the supply chain. The use of these systems has also been linked to lower cost distribution networks, lower recall costs and higher sales (Golan, Krissoff et al. 2004). Overall, companies can benefit monetarily by implementing these systems. Therefore these economic benefits are driving the study and development of traceability systems across the U.S. food supply chain (Golan, Krissoff et al. 2004).
1.4. Motivation

There has been a considerable increase in the number, severity and spread of food product contamination and recalls in the past few decades. Factors such as consumption patterns, demographics and the structure of the modern food supply chain are some of the reasons impacting the increase in food product contamination and recalls.

The change in national demographics and consumption patterns also underscore the need for a better understanding of food product contamination and recalls. The risk of severe or life-threatening symptoms from food borne illnesses is higher for older adults, young children, pregnant women and immune compromised individuals. A shift in U.S. demographics means that that more of the U.S. population is, and increasingly will be, susceptible to food borne illnesses.

Additionally a change in consumption patterns is taking place with people increasingly eating raw foods or foods with minimal processing increasing the risk of food borne illnesses, as these kinds of foods are often associated with food contamination. For example, according to the USDA leafy greens such as the spinach is one the food groups most likely to be associated with a food borne illness. The average consumption of spinach has risen by 180 percent from 1992 to 2005 (Shames 2008).

The globalization of Multinational Corporations (MNCs) due to economic forces has resulted in reorganization of production structures in the food supply chain to locations across the globe (Sideri 1997). MNCs are taking advantage of regions with different capital and labor markets. The pressures of cost reduction in the food supply
chain partly also due to the heavy competition in the retail grocery arena has given rise to what is now considered the mainstream food supply chain (Roth et al. 2008). This has resulted in labor-intensive production being relocated to cheaper labor economies to cut costs (Buckley 2009). Thus, the imported proportion of U.S. food consumption has witnessed a rapid increase. The FDA estimates that 80 percent of seafood and 20 percent of produce is imported from abroad (Roth et al. 2008).

The guarantee of lowest price is the order winner for products which are ordered through this selection process. Hence, suppliers tend to take short cuts in quality or product safety in an attempt to provide the lowest cost (Lyles et al. 2008). There is no specification or emphasis on product safety which makes products thus sourced more vulnerable to risk of contamination. However, recent reports of contamination in imported foods have raised serious concerns about quality risks of global sourcing. These incidents have raised questions about business and supply chain management practices which have resulted in contaminated foods reaching the end consumer (Roth et al. 2008). As more large-scale labor markets compete for international trade, the incentives to cut corners will increase and the overlooking of contaminated products may rapidly become a more common occurrence (Klarevas 2008).

Thus, modern day global food supply chains tend to be associated with greater vulnerability and risks of product contamination due to lack of accountability, lower visibility and quality failures. They are also prone to lower responsiveness due to longer lead times (Roth et al., 2008). Additionally the sheer length of the supply chain and
slower movement of inventory and events makes it difficult for supply chain partners to monitor and control the actions of up and downstream partners (Lyles et al., 2008).

For example if we consider the recent China-melamine-milk product recall scandal the World Health Organization attributed many of the problems to the rapid pace of development of food and agricultural production in the past few decades. However, they also say that the agencies in charge of food safety and quality control are not developing at the same pace (Schlein 2008). It should however be emphasized that these problems are universal to most global supply chains in today's world and are not just unique to Chinese supply chains (Lyles et al. 2008).

Even though food product contamination is real and has potentially devastating effects on companies and supply chains, it is still not very well understood from a quantitative perspective. This work seeks to fill that gap by providing a generic model of a food supply chain, and by quantifying the cost of contamination in that model and analyzing factors which impact the magnitude of the contamination event.

Companies build traceability systems to improve supply chain efficiency and build lower-cost distribution systems, but simply tracing a product in the supply chain does not improve supply management unless the traceability system is paired with an supply chain inventory control system (Golan, Krissoff et al. 2004) (Smith et al. 2005). In this work we consider the tracking and tracing of a product in the supply chain model built to evaluate the value of traceability in the event of a contamination.
1.5. **Scope of Research**

This research work attempts to quantitatively model the food contamination event in its entirety from the supply chain perspective. The cost of product contamination is quantified for a generic food supply chain. In particular, the amount of product that is affected by a contamination event and the time it takes from when the contamination is introduced until the contamination event is detected are quantified. This is done by considering the different origins and modes of detection of contamination. Various product properties and supply chain attributes which can impact the magnitude of a food contamination event are also analyzed. Further, the impact of incorporating product traceability in the event of a food contamination is also studied. In particular, this research work seeks to answer the following major questions:

1. How does the negative impact of a contamination event depend on the origin and the mode of discovery of the contamination? This is an important consideration; because quantifiable answers to this question may help companies better understand the risk profile of different stages of the food supply chain, and to be able to compare the relative merit of preventive quality control.

2. How do differences in individual product properties, supply chain structure and attributes impact the severity of contamination? Generalizable answers to this question may help companies understand the risks that certain product groups or supply chain configurations (e.g., long lead time global sourcing) pose.

3. What is the value of tracking and tracing products in the event of a food product contamination and recall? Quantifying the value of traceability in a given
supply chain scenario may help companies in evaluating the economic benefits of implementing traceability systems.

Due to the high risk of economic losses associated with a food product recall, companies when faced with a food product contamination scenario may have a tendency to delay a product recall. However, delaying a recall may also lead to higher costs when there is large scale food contamination leading to a food borne illness. Therefore a recall decision has to be made at the “optimal time” to minimize losses from the perspective of the companies. This decision will depend on the magnitude of the food contamination event and factors such as whether the event is local or global (Skees et al. 2001).

This research work by quantifying the scope of the contamination event can help companies make more informed decisions in the case of a contamination event. This will give an overall view and aid in getting a better picture of the impact of a food.

1.6. Literature Review

The literature in the areas of food contamination and recall deal with issues ranging from the medical costs, market impact, shareholder wealth and consumer reactions to implementation of safety systems and traceability of a food supply chain.

To aid policy making the USDA's Economic Research Service (ERS) uses previous case history data to estimate the costs for a number of food borne illnesses (ERS Food Borne Illness Calculator, 2009). For example, the ERS estimates that the annual economic cost of Salmonellosis is $2,544,394,334 (Golan 2003). These estimates are only for related medical costs which is just one of the indirect costs of a food safety
event and are based on data collected (Golan 2003). In our research we quantitatively approximate the overall costs of the event which include several other direct and indirect costs and not just the medical costs.

There have been several papers in literature which study consumer reactions, changes in consumer demand, consumer perceptions and sales losses due to a food contamination event. Jonge et al. (2007) apply structural equation modeling to understand the determinants influencing consumer perceptions of food safety incidents. The results indicate that some of these determinants are consumer trust in operators in the food supply chain, safety perceptions about product groups and personal recall experiences (Jonge 2007).

Marsh et al. (2004) empirically test the impact of meat product recalls on consumer demand in the USA by estimating an absolute price version of the Rotterdam demand model. Their findings indicate that FSIS's meat recall events have a significant impact on demand. They also find that any positive effects on substitutes for a recalled product were offset by a more general negative effect on meat demand as a whole. There was also a shift from meat to consumption of other products. In this research the authors are concerned about shift in consumption patterns, change in consumer demands and impact of information flow on consumer decisions in the face of a food contamination event.

Piggott and Marsh (2004) theoretically model consumer response to food safety information. Their paper develops an economic and empirical model to study whether food safety information about beef, pork and poultry has impacted meat consumption in
the U.S. over the last several decades. They find pre-committed levels of meat consumption which are impacted by time trends, seasonal factors and food safety information. Food safety information is found to be detrimental to demand and there are spillover or substitution effects from one meat to another (Piggott and Marsh 2004).

Pennings et al. (2002) study consumer reactions to a food contamination crisis. They show that by decoupling consumer risk response behavior into separate components of risk perception and risk attitude, a more robust prediction of consumer reactions is possible. Using an expected utility model and average sum score on risk-attitude scale they study the reactions of German, Dutch and American consumers to BSE (mad cow disease). They conclude that marketers’ response to a food contamination event should be based on whether the consumer behavior is driven by risk perceptions or risk attitudes (Pennings et al. 2002).

Thomsen et al. (2006) use an empirical model to measure sales losses for frankfurter brands which experienced a recall and the impact of the recall on other brands in the same category. They conclude from their model that recalls have adverse impact on sales of brands and the decline in sales is significant enough to warrant an incentive to minimize potential contamination events. They also state that brand equity provides a sign to the customers about food safety and can insulate firms from industry wide recall problems (Thomsen et al. 2006).

In the papers cited above the authors analyze consumer perceptions, behavior and lost sales in an attempt to arrive at a better understanding of a food contamination event by. In contrast, this research quantifies the impact of a food contamination event from a
supply chain perspective. The fallout of the event on the entire costs and product inventory is quantitatively approximated. The lost sales due to a food product recall is accounted for as a component of the direct costs of the food contamination event but the others costs involved are also considered.

There are other papers which analyze the fallout of a food contamination event on shareholders, stock markets and company valuations. Thomsen and McKenzie (2001) examine the reductions in valuations of food companies due to food contamination incidents. Their research seeks to quantify the impact of a recall on shareholder wealth (Thomsen and McKenzie 2001).

Salin (2000) uses real option valuation techniques to analyze how an agribusiness investor's decision will be influenced by the risk of a food safety incident. The author states that concerns about food safety affect investors’ perceptions as returns for a firm can be expected to fall drastically in the event of a product recall (Salin 2000). Hooker and Salin (1999) use stock market reactions to four recent recalls which vary by product type, scope and severity to study the impact of food recalls. A partial event analysis technique is used to demonstrate the impact of these recalls. The results provide insight about the value that financial markets place on food safety. The authors for these papers as in other cases do not study the various other direct or indirect costs of a recall. They mainly focus on the impact of recalls on company valuations and shareholder losses, which can be considered to be one of the indirect costs of a recall.

To summarize the related work cited above, most papers deal with certain indirect costs or fallouts of a food contamination event mainly from a consumer
perspective. In this research work however the product contamination and recall event is quantitatively modeled in its entirety from the supply chain perspective. The different origins and modes of detection of contamination, the product inventory affected and the direct as well as indirect costs of a food recall event are taken into account. Also various product and supply chain attributes which can impact the magnitude of a food contamination event are analyzed.

Akkerman et al. (2010) review quantitative operations management approaches to food distribution management, and relate this to challenges faced by the industry. One of the focus areas of this paper is food safety and they review literature in this area survey the research contribution and identify challenges for future research (Akkerman et al. 2010).

Tromp et al. (2010) model the transmission of salmonella through the broiler production chain. They utilize data collected from a Dutch broiler supply chain. They intend the model to be a tool for policymakers and industry to determine appropriate intervention strategies (Tromp et al. 2010).

Van Asselt et al. (2010) describe a method to filter the most important critical factors related to food safety risks in production chains. Their methodology comprises a comparison between a traditional and a new product from the same food chain using expert judgment, group discussion and individual ranking (Van Asselt et al. 2010).

In this dissertation work a generic food supply chain is modeled using inventory control and queuing principles to determine the impact of product and supply chain
properties on the scale of a contamination event. This model is subsequently applied to a recent real-time product contamination event.

Various systems and standards have also been developed over the past decades to identify, manage and reduce food safety risks. The best-known are the hazard analysis critical control point (HACCP) system, the ISO 22000 standard (ISO 2005) and the British BRC standards (British Retail Consortium 2004) (Akkerman et al. 2010). There are papers which discuss the use of specific software based procedures for the implementation of the HACCP system (Tuominen et al. 2003), (Van Gerwen et al. 1997). For example, Bertolini et al. (2007) utilize a fault tree analysis for implementation of the HACCP system.

Charlier and Valceschini (2008) study the economic incentives that the private sector has in the implementation of traceability in food chains in which specific sanitary risks exist (Charlier and Valceschini 2008). Based on case study illustrations of European meat and poultry firms, Buhr (2003) use case studies of European meat and poultry firms to analyze the role of traceability in a firm’s ability to limit the depth and size of product recall (Buhr 2003). Desai and Patel (2011) utilize case study data to model the key factors influencing traceability for the E.coli contamination of beef (Desai and Patel 2011).

Pouliot and Sumner show that exogenous increases in food traceability can create incentives for farms and marketing firms to supply safer food by increasing liability costs (Pouliot and Sumner 2008).
There are papers available in literature which discuss the use of RFIDs (Radio Frequency Identification), sensor networks and software agents utilized in the implementation of traceability systems for specific case studies (Abad et al. 2007), (Ngai et al. 2008) (Hannus et al. 2003), (Mousavi et al. 2005), (Thompson et al. 2005) and (Jedermann et al. 2006). The exact traceability system implemented in a particular food supply chain will depend on several factors including the product type and the electronic agreements in place between enterprises (Fritz and Schiefer 2009).

In this work, the tracking and tracing information from a traceability system is integrated with the supply chain model built to assess the value of traceability for different scenarios.

1.7. Summary

This chapter includes the introduction to the research, descriptions of recent food product contamination and recall cases, traceability systems, motivation and scope of the research. It also presents a summary of literature in the area of food product contamination, recalls and traceability systems in order to highlight the contribution this research.

It is stated that this research seeks to quantify the negative impact of a food contamination event from the supply chain perspective. A generic multi-stage supply chain model consisting of a grower, processing center and retailers is considered. This research intends to quantify the negative dependence of a contamination event on the origin and mode of detection on the contamination. It also aims to analyze the impact of product properties, supply chain structure and properties on the event. Further a
A numerical study based on the model is conducted. A traceability system is incorporated into the model to study the value of tracking and tracing products in a supply chain. The next chapter presents a detailed description of the supply chain model and origin and mode of detection of the contamination event. The various factors which impact the behavior of the model are then analyzed mathematically.
2.1. Model Overview

In a general food supply chain, growers of fruits and vegetables can market their produce through shippers, sell it directly to customers at farmer's markets and roadside stands, or sell it to processors. Direct sales to customers are small, accounting for only 2 percent of final fresh produce consumption in 1997 (Golan, Krissoff et.al. 2004). On the other hand, processing is an important part of the produce industry. In 2002, 86 percent of vegetables and fruits produced in the United States by weight went into processing (Golan, Krissoff et.al. 2004). A supply chain somewhat similar to this is depicted for processed food products by Kumar and Budin (2006). Figure 1 shows the basic multi-stage supply chain configuration.

1. Fresh fruits and vegetable, e.g., tomatoes, lettuce: The supplier corresponds to the farm or grower, the processing center processes the produce into a product such as bagged salad, canned spinach, or repackaged fruit for resale. The retailer is the supermarket which sells these processed or repackaged products to customers.

2. Canned produce, e.g., peanut butter: In this case the supplier is the grower of produce, the processing center is the plant processing and conserving the produce as well as performing the canning operation. The retailer is the grocery store or supermarket selling peanut butter.
Figure 1: Multi-stage food supply chain with multiple retailers

3. Meats: Most of the meat such as beef that Americans consume originates from cattle born and raised on one of the country's 800,000 cow-calf farms (Golan, Krissoff et.al. 2004). Cattle ready for slaughter are transported to slaughter or processing plants where beef carcasses are cut and packaged into “boxed beef” or “case-ready” retail cuts and delivered to grocery stores (Golan, Krissoff et.al. 2004). In this case, the supplier corresponds to the cow-calf farms; the processing center is the slaughter plant, and the retailer a grocery store or supermarket which sells the final product to the consumer.

In our model we consider a situation where a contamination has already occurred in the food supply chain which leads to a food borne illness. The detection of contaminated products and the subsequent initiation of a product recall can occur through one of the following modes (Teratanavat, Salin and Hooker 2005), (Dyckman and Lansburgh 2004):
1. Through periodic product sampling or testing conducted by companies or government agencies such as the Food and Drug Administration (FDA), Food Safety and Inspection Service (FSIS) or the firms itself.

2. Through the outbreak of food borne illness incidents as discovered by government agencies or reported by customers.

Teratanavat, Salin and Hooker (2005) state that the probability of discovering the recall problem through testing by the FSIS is 44.6%, the probability that it is discovered by the firms is 23.7%, the probability that the food borne illness incidents are discovered by government agencies is 17% and the probability that they are discovered by customers is 14.7% respectively.

2.2. Customer

The customer purchases the final product from one of the retailers, consumes it and, in the event of a product contamination having occurred, may contract a food borne illness. The time at which a customer purchases a product will be determined by the customer demand at that retailer and the service level offered by the retailer.

It is assumed that the customer consumes the product prior to the end of its shelf life and the time of consumption is uniformly distributed between the time of purchase and the end of shelf life of the product. The distribution of the time of consumption of a product will depend on the product type as fresh products such as tomatoes will be consumed sooner as compared to canned produce or frozen meats which can last for a longer time.
The shelf life of a product is denoted by \( H \) and is counted from the time the finished product enters the finished goods inventory (FGI) at the processing center. This is because the shelf life of a food product is commonly defined as the time period between the manufacture and the retail purchase of the product during which the product is of satisfactory quality (Kilcast and Subramaniam 2000).

The food borne illness will manifest itself after a specific incubation period. The incubation period is the interval between ingestion of a food contaminated with enough pathogens to cause illness and the appearance of the initial symptom of the illness (Investigation Operations Manual 2008 FDA). Further, it is assumed that there is an exogenous probability \( p \) that a customer who consumes the contaminated food product shows the illness symptoms. The probability of a customer consuming a product showing the symptoms in actuality will depend on the product type, customer demographics etc.

It is assumed that there is no time lag between a customer showing the illness symptoms and reports of the illness. Though realistically there will be a time lag between a customer experiencing and reporting an illness it is ignored as it is difficult to quantify mathematically. It is also supposed that every illness due to contamination is reported. In reality this may not be so as some illnesses may go unreported.

### 2.3. Retailers

Consider a supply chain with a total number of \( r \) retailers. It is assumed that the customer demand per time period at each of the retailers is independent of each other,
stationary over time and can be expressed as a normal random variable $D_i$, with known mean $\mu_i$ and standard deviation $\sigma[D_i]$ with $i = 1, \cdots, r$. The total demand over the multiple retailers is $D = \sum_{i=1}^{r} D_i$.

For mathematical tractability of the model, it is assumed that there is full backordering at the retailers. An order-up to policy is in place at each retailer; that is, the retailer sets a target inventory level, and every review period (e.g., every day) the retailer orders such that his inventory level is brought up to that target level. It is also assumed that the probability of encountering no demand at all during the review period is zero, and hence the retailer places an order every review period. The coefficient of variation of demand is assumed to be such that the probability of non-positive demand is negligible.

The lead time for receiving a shipment from the processing center to each retailer is assumed to be $l_i$ time periods for $i = 1, \cdots, r$. The lead time demand $D_{LT_i}$ also follows a normal distribution with mean $\mu_{LT_i}$ and standard deviation $\sigma[D_{LT_i}]$. Let the target service level for the retailer be $\beta_{\tau_i}$, then the retailer's order-up to level $R_{\tau_i}$, is given by $F(R_{\tau_i}) = \beta_{\tau_i}$, where $F$ represents the normal cumulative density function (cdf) of the lead time demand $D_{LT_i}$. In this control system, an order will be shipped from the processing center to each retailer every period. Hence if the lead time is $l$ periods from the processing center to the retailer, then there will always be $l_i$ orders outstanding in any period after an order is placed.
2.4. Processing Center

The processing center needs to satisfy the combined demand from all retailers $D = \sum_{i=1}^{r} D_i$, with standard deviation $\sigma[D] = \sum_{i=1}^{r} \sigma[D_i]$. Since it is assumed that there is backordering at the retailers, the processing center will see a demand stream that is equivalent to the aggregation of the retailer demand streams, and every period an order will be shipped from the processing center to each retailer.

The processing center is assumed to have finite capacity. This finite capacity is modeled by formulating a $G/G/1$ queuing system (Allen 1978), (Buzacott and Shantikumar 1993). Following our macro view of the basic supply chain, this single server does not represent a particular machine, but rather the abstraction of all the processing steps that happen at the processing center. A pictorial representation of the process is given in Figure 2.

Raw material arrives from the supplier to the processing center in batches of fixed size $k$. The inter-arrival time of batches to the processing center is denoted by the random variable $A_B$. Batches wait in the queue until service on individual items in the batch commences. In processing, the batches are broken apart and the individual items (e.g., single fruit or cartons of fruit) are served by a single server. The service time of an individual item in the batch is denoted by the random variable $S$. Both $A_B$ and $S$ are assumed to be known characteristics of the queue; that is, it is assumed that their distributions as well as the distributional parameters are known.

It is also assumed that each item in the batches of raw material arriving from the supplier is processed into $m$ number of final products, reaching the finished goods
inventory (FGI). For example, cartons of fruits may be processed to produce m cans of fruit. The processing center has the service level $\beta_p$.

Figure 2: Macro queuing model of the processing center

Such a batch processing system is similar to that examined in Curry and Deuermeyer (2002). Using the known customer demand distribution $D$ at the retailer and the supplier's fixed batch size $k$, the batch inter arrival time $A_B$ (which corresponds to the time between shipments from the supplier) is optimally set so as to replenish the processing center's FGI at a rate equal to the customer demand rate, as the inflows must
equal the outflows in steady state. The inter departure time between individual items is \( I \) and the replenishment of the FGI takes place at a rate \( R \). In steady state condition by the conservation principle the same number of units must depart the processing center as those that enter the processing center.

\[
E[D] = E[R] = \frac{1}{E[I]} = \frac{km}{E[A_B]} 
\]  

(2.1)

Therefore the expected batch inter arrival time can be set to be

\[
E[A_B] = \frac{km}{E[D]} = \frac{km}{E[R]} = kmE[I] 
\]  

(2.2)

In order to properly describe the FGI inventory system at the processing center, the optimal safety stock \( ss_{\text{total}} \) that the processing center will carry needs to be computed. The appropriate stocking level of finished goods inventory at the processing center, taking into account risk pooling due to demand variability and the variability of FGI replenishment through the queuing process.

**Lemma 2.4.1:** The safety stock at the processing center is

\[
ss_{\text{total}}[D_{LT}, R] = z \left( \sqrt{\sum_{i=1}^{r} \sigma[D_{LT_i}]^2 + \sigma[R]^2} \right), \text{where } z \text{ the standardized value corresponds to the service level at the processing center } \beta_p \text{.}
\]

**Proof:** The need for safety stock at the processing center is determined by two factors:

(1) the variability in the customer demand at the retailers and (2) the variability in the replenishment of the FGI at the processing center. The safety stock for (1) is dependent on the variability of the orders over the lead time placed by each retailer \( i \), and the safety stock for (2) is given by the variability of the replenishment rate \( R \) over the queue service time at the processing center. The safety stock for portion (1) is:
The variance of the FGI replenishment rate \( R \) has to be calculated in terms of the model parameters to estimate the safety stock in Lemma 2.4.1. To this end, the standard analysis method for \( G/G/1 \) queues with FCFS service is used to develop relationships between successive jobs through the system at the processing center. Then under suitable conditions when stationary state exists, taking the limit as the number of jobs goes to infinity yields relationships that are valid in steady state (Curry and Deuermeyer 2002).

**Lemma 2.4.2:** The variance of the FGI replenishment rate \( R \) is,

\[
\text{Var}[R] = \left( \frac{1}{\frac{1}{E[D]}} \right)^2 \left( \frac{E[A_B]^2 \cdot \text{Var}[A_B] \cdot \left(1 - \frac{kE[S]}{E[A_B]} \right)^2 + \left(1 - \frac{kE[S]}{E[A_B]} \right)^2 + 2(E[A_B] - kE[S])E[S]}{k \cdot m} \right) + E[S^2] - \left( \frac{1}{E[D]^2} \right).
\]

**Proof:** A standard approach in queuing literature called the Allen-Cullen approximation is used to develop the following relations (Allen 1978). The notation and development parallels Buzacott and Shanthikumar (1993) and the renewal approximation is used to estimate the second moment of 1 (Curry and Deuermeyer 2002).

\[
E[I^2] = \frac{E[M^2] + 2E[M]E[S]}{k \cdot m} + E[S^2] \tag{2.4}
\]

\[
E[M^2] = E[A_B]^2 (c^2[A_B](1 - u^2) + (1 - u)^2) \tag{2.5}
\]
\[ E[M] = E[A_B] - kE[S] \]  \hspace{1cm} (2.6)

\( M \) is the idle time of the machine/server and \( c^2[A_B] \) is the squared coefficient of variation of the batch arrival time,

\[ c^2[A_B] = \frac{\text{Var}[A_B]}{E[A_B]^2} \]  \hspace{1cm} (2.7)

The server steady state utilization factor \( u \) is given by,

\[ u = \frac{E[S_B]}{E[A_B]} \]  \hspace{1cm} (2.8)

where the batch service time \( S_B \) is the sum of \( k \) individual item service times,

\[ E[S_B] = kE[S] \]  \hspace{1cm} (2.9)

The variance of the item inter departure time can be calculated using the standard relationship for variance,

\[ \text{Var}[I] = E[I^2] - E[I]^2 \]  \hspace{1cm} (2.10)

By substitution from equations (2.1) and (2.5) to (2.11):

\[ \text{Var}[I] = \left( \frac{E[A_B]^2 \left( \frac{\text{Var}[A_B]}{E[A_B]^2} \left( 1 - \left( \frac{kE[S]}{E[A_B]} \right)^2 \right) + \left( 1 - \frac{kE[S]}{E[A_B]} \right)^2 \right) + 2(E[A_B] - kE[S])E[S]}{km} + E[S^2] \right) \\
- \left( \frac{1}{E[D]} \right)^2 \]  \hspace{1cm} (2.11)
The variance of the FGI replenishment rate $\sigma^2[R]$ using the Delta method (Taylor's series approximation) is:

$$\text{Var}[R] = \left( \frac{1}{E[I]^2} \right)^2 \text{Var}[I]$$  \hspace{1cm} (2.12)

From equations (2.1) and (2.12), $\text{Var}[R]$ in terms of the model parameters is:

$$\text{Var}[R] = \left( \frac{1}{E[D]^2} \right)^2 \left( \frac{E[A_B]^2 \left\{ \text{Var}[A_B] \left( 1 - \left( \frac{kE[S]}{E[A_B]} \right)^2 \right) + \left( 1 - \frac{kE[S]}{E[A_B]} \right)^2 \right\}}{\text{km}} \right) + \frac{2(E[A_B] - kE[S])E[S]}{\text{km}} + E[S^2] + E[S^2]$$

$$- \left( \frac{1}{E[D]^2} \right)^2$$  \hspace{1cm} (2.13)

2.5. Supplier

It is assumed that there is ample raw material and there is instantaneous production in place at the supplier. Separate inventory holding or storage facilities are not modeled at the supplier. Inventory holding at the supplier is neglected because there are usually few, if any, production steps at the supplier. Hence, the system at the supplier is modeled as a push system that sends raw material directly to the processing center in batches. The processing center acts as a pull system which receives batches of raw
material from the supplier based on the expected demand rate. For simplicity, it is assumed that the produce batches are assembled instantaneously at the supplier.

In line with the needs of the processing center, the supplier ships a batch of size \( k \) to the processing center every \( \frac{km}{E[D]} \) periods, where \( D = \sum_{i=1}^{r} D_i \). The lead time from the supplier to the processing center is denoted by \( L \).

2.6. Model Evaluation

This model considers the stage at which the contamination is introduced into the supply chain and the mode of detection of the contamination. The relative impact of contamination origination at the supplier/grower, versus contamination occurring at a processing center is contrasted. As described above the detection of contamination can take place through sampling of the product or through food borne illness reports. The sampling can be conducted at the supplier, processing center or retailer along the supply chain.

The following are the performance metrics of the model:

- \( \tau_j^i \), which represents the time to detection of a contamination event, originating at a supply chain node \( i \) and being discovered through mode \( j \);
- \( \Omega_j^i \), which represents the amount of contaminated stock in the case of a food contamination event originating at supply chain node \( i \) and being discovered through mode \( j \);
- \( \omega_j^i \), denoting the amount of contaminated stock sold;
• $y^1_i$, denoting the amount of contaminated stock that remains unsold (that is, in inventory) in the case of a food contamination event.

The origin of contamination $i$ is defined as $i = 1$, if origin of contamination is at the supplier, and $i = 2$, if origin of contamination is at the processing center. The mode of detection $j$ is defined as $j = 1$ if mode of detection of the contamination event is through sampling at the supplier, $j = 2$ if mode of detection of the contamination event is through sampling at the processing center, $j = 3$ if mode of detection of the contamination event is through sampling one of the retailers and $j = 4$ if mode of detection is through food borne illness incidents.

To perform the analysis, the time to detection, the amount of contaminated stock and the amount of sold and unsold stock are modeled for each mode of detection and for each origin of contamination.

The contamination is assumed to be ongoing until it is discovered. This means that after a contamination is introduced at a location; all subsequent products that go through that location are contaminated as well. A quality control process such as cleaning of equipment, water used being changed etc. may however lead to the elimination of the source of contamination.

In this model it is initially assumed that the entire system continues to operate unchanged, until the contamination is discovered and later on a quality control process which eradicates contamination is introduced. This helps in quantifying the complete impact of a contamination occurrence until its detection. Finally, for simplicity it is also assumed that sampling happens in zero time and is non-destructive by nature. The
sampling time will depend on the product and contamination type, however for simplicity it is assumed that sampling happens instantaneously. It is presumed that sampling reliably detects the contamination if a contamination is present.

2.7. **Impact of Origin and Mode of Detection of Contamination**

The impact of the origin of contamination and the mode of detection of contamination on the model and the associated costs to the supply chain in the event of a contamination are now analyzed. The timeline from the start of the contamination incident to the detection and initiation of the food recall event for each of the modes of detection of the food contamination event is modeled and the expected time to detection and the total contaminated stock is calculated.

2.7.1. **Origin at Supplier**

The supplier ships batches of raw material to the processing center. For the case of origin of contamination at the supplier without loss of generality, at time $t = 0$ the first batch of contaminated raw material is shipped from the supplier to the processing center. This batch reaches the processing center after the constant shipping lead time $L$. 
The cycle time of the item at the processing center $CT_p$ is the sum of the queue time, the individual service time and the process time delay within a batch. The process time delay arises because on an average each item within a batch waits for the item(s) in front of it to be processed. The average wait time for a finished product in the finished goods inventory $t_{FGI}$ before being shipped to the retailer is determined by the safety stock at the processing center and the average demand fulfilled by the processing center. The position of a product entering the FGI is assumed to be uniformly distributed and due to flow conservation the amount of product entering the FGI is equal to the amount exiting the FGI. Similarly, the average time a product spends on a retailer shelf $t_{H1}$ is determined by the safety stock at the retailer and the average demand fulfilled by the retailer.
The time line is chosen such that after the onset of contamination, when the first contaminated batch is assembled and shipped from the supplier time is set to \( t = 0 \) and henceforth it is referred to as the start time (Figure 3).

### 2.7.2. Origin at Processing Center

The processing center processes raw material received from the supplier into finished products. For origin of contamination at the processing center, it is assumed that the contamination is introduced at the end of processing of a particular batch, which is after the last product in that particular batch finishes processing.

![Figure 4: Origin of contamination at processing center](image)

Without loss of generality, at time \( t = 0 \) processing of the first batch begins after the introduction of contamination. The expected service time for each product is \( E[S] \).
The contamination is assumed to last for the entire service duration time. This assumption is valid as the service time duration is much lower compared to the other time periods in our model such as the lead time \( l_i \) and incubation time \( T_{ln} \). The time line is chosen such that, when the processing of the first batch begins after the introduction of contamination, the time is set to \( t = 0 \) and is referred to as the start time (Figure 4).

### 2.7.3. Contamination Detection through Sampling

The sampling strategy is a policy decision that is defined by the frequency with which a sample is taken. In real implementations, sampling strategies are derived from industry standards such as ISO 22000. There is literature available on the implementation of food safety systems as discussed in our initial review (Akkerman et al. 2010), (Tuominen et al. 2003), (Van Gerwen et al. 1997) and (Bertolini et al. 2007). The selected sampling strategy will also depend on the product type and the type of contamination which may occur that in turn will influence the severity of the food borne illness caused. However, we do not consider specific food safety management systems so as to be able to adapt it to different product types and food borne illnesses caused.

In our model, we describe the frequency of sampling by the parameter \( N \), which is defined as the number of batches between successive samples. Thus, a high value of \( N \) corresponds to infrequent sampling and a low value corresponds to frequent sampling. Sampling may be done at any of the nodes in the supply chain.

It is assumed that a sampling strategy has been established such that a single item belonging to one batch in every \( N \) batches is sampled at the supplier. At the processing center, two sampling regimen are of interest, depending on whether we want to hedge
against potential contamination that originates either at the supplier, or at the processing
center itself. If we want to detect contamination that originates at a supplier via
sampling, we sample once every Nkm items arrive, finish processing and leave the FGI
for the case of origin of contamination at the supplier. If we want to detect
contamination that originates at the processing center, we sample one in every Nkm
finished items. At the retailer, one in every Nkm items is sampled prior to being sold.
The sampling will therefore be conducted in the following manner:

1. At the supplier the time of sampling is assumed to be uniformly
distributed between the start time of the contamination until the time the $N^{th}$
contaminated batch is assembled and shipped.

2. At the processing center the time of sampling is assumed to be uniformly
distributed between the time the first contaminated batch arrives until the time the
$(Nkm)^{th}$ product leaves the FGI for origin at the supplier. For origin at the processing
center the time of sampling is assumed to be uniformly distributed between the time the
first contaminated product enters the FGI until the time $(Nkm)^{th}$ product leaves the
FGI.

3. At any of $r$ retailers the time of sampling is assumed to be uniformly
distributed between the time the first contaminated product arrives until the time the
$(Nkm)^{th}$ product is sold.

The results of the subsequent analysis are summarized in the following
Proposition:
Proposition 2.7.3.1:

1. If the contamination origin is at the supplier and contamination detection is through sampling, then the expected times to detection will be in the order: \( E[\tau_3^1] > E[\tau_3^2] > E[\tau_1^1] \) and the expected amounts of contaminated stock will be in the order: \( E[\Omega_3^1] > E[\Omega_3^2] > E[\Omega_1^1] \).

2. If the contamination origin is at the processing center and contamination detection is through sampling, then the expected times to detection will be in the order: \( E[\tau_3^2] > E[\tau_3^3] \), and the expected amounts of contaminated stock will be in the order: \( E[\Omega_3^2] > E[\Omega_3^3] \).

3. The above results are true for any sampling strategy that is, any choice of \( N \), that a company may adopt.

Proof: For origin of contamination at the supplier and,

1. Sampling at supplier

The time of sampling is assumed to be uniformly distributed between the time at which the first contaminated batch is assembled and shipped to the time at which the \( N^{th} \) contaminated batch is assembled and shipped. Therefore, the sampling time \( T_{F1S} \) will be a random variable uniformly distributed over the interval \( \left[ 0, \frac{(N-1)km}{E[D]} \right] \). Since the \( N^{th} \) batch will be shipped after time \( \frac{(N-1)km}{E[D]} \) from the start of contamination. The expected amount of contaminated stock produced in this case will be:

\[
E[\Omega_1^1] = E[D]E[T_{F1S}] \tag{2.14}
\]

2. Sampling at processing center
The time of sampling is assumed to be uniformly distributed between the time the first contaminated batch arrives until the time the \((N_{km})\)th product leaves the FGI. In this case the sampling \(T_{F2S}\) time can be assumed to be a random variable uniformly distributed over the interval \([L, L + \frac{(N-1)km}{E[D]} + CT_P + t_{FGI}]\), where \(CT_P\) is the cycle time of the item at the processing center and \(L\) is the lead time from the supplier to the processing center.

\[
CT_P = \frac{u}{u - 1} E[S]\left(\frac{c^2(A_B) + c^2(S)}{2}\right) + \frac{k + 1}{2} E[S] \tag{2.15}
\]

\[
c^2[A_B] = \frac{\text{Var}[A_B]}{E[A_B]^2} \text{ from equation (2.7).}
\]

\[
t_{FGI} = \frac{ss_{\text{total}}[DLT_i R]}{E[D]} + \frac{E[D]}{2} \tag{2.16}
\]

The expected amount of contaminated stock will be:

\[
E[\Omega_{\frac{1}{2}}] = E[D]E[T_{F2S}] \tag{2.17}
\]

3. Sampling at the retailer

At each retailer the time of sampling is assumed to be uniformly distributed between the time the first contaminated product arrives until the time the \((N_{km})\)th product is sold. The first contaminated product will reach the retailer after time \(L + \frac{(N-1)km}{E[D]} + CT_P + t_{FGI} + l_i\), the shelf time is \(t_{H_i} = \frac{ss_{\text{total}}[DLT_i R]}{E[D_i]}\), it will be purchased by the customer after time \(\frac{1}{E[D_i]}\) if the retailer service level \(\beta_{R_i} = 1\), however if \(\beta_{R_i} < 1\) then there will be an additional delay of \(\frac{1 - \beta_{R_i}}{E[D_i]}\) before the first product is purchased. Therefore the
(Nkm)\textsuperscript{th} product will be purchased by the customer after time \( \frac{Nkm}{E[D_1]} + \frac{Nkm(1-\beta_1)}{E[D_1]} = \frac{Nkm(2-\beta_1)}{E[D_1]} \). The sampling time at retailer \( i, T_{F3Si} \) will be a random variable uniformly distributed over the interval \( [L + CT_p + t_{FGI} + 1 + \frac{(N-1)km}{E[D]} + CT_p + t_{FGI} + 1 + t_{H_i} + \frac{Nkm(2-\beta_1)}{E[D_1]}] \). The time to detection of contamination through sampling at \( r \) retailers will be the random variable, which will be the minimum among the sampling times at each retailer \( Y = \min[T_{F3S1}, T_{F3S2}, \ldots, T_{F3Sr}] \). If \( F_{T_{F3S}}[T_{F3S}] \) is the distribution of \( T_{F3S} \) the corresponding cdf of \( Y \) will be,

\[
F_Y(y) = P(Y \leq y) = (\min[T_{F3S1}, T_{F3S2}, \ldots, T_{F3Sr}] \leq y) \\
= 1 - P(\min[T_{F3S1}, T_{F3S2}, \ldots, T_{F3Sr}] > y) \\
= 1 - P(T_{F3S1} > y, T_{F3S2} > y, \ldots, T_{F3Sr} > y) = 1 - [P(T_{F3S} > y)]^r \\
= 1 - [1 - [P(T_{F3S} \leq y)]^r] \\
= 1 - [1 - F_{T_{F3S}}[T_{F3S}]|^r]
\] (2.18)

The expected time to detection of contamination through sampling at \( r \) retailers will be \( E[Y] \). The expected amount of contaminated stock will be:

\[
E[\Omega_3^1] = E[Y](E[D_1] + E[D_2] + \ldots + E[D_r]) = E[Y]E[D]
\] (2.19)

The expected time to detection for each of the three cases above is:

\[
E[\tau^1] = E[T_{F1S}] = \frac{(N-1)km}{2E[D]} \] (2.20)
\[
E[\tau_2] = E[T_{F2S}] = \frac{2L + CT_P + \frac{(N-1)km}{E[D]} + t_{FGI}}{2}
\]  
(2.21)

\[
E[\tau_3] = E[T_{F3S_i}]
\]

\[
= \frac{2L + 2CT_P + \frac{(N-1)km}{E[D]} + 2t_{FGI} + 2l_i + t_{H_i} + \frac{Nkm(2-\beta_{h_i})}{E[D_i]}}{2}
\]  
(2.22)

for \(i = 1, \cdots, r\). Since \(N > 0\), \(L > 0\), \(t_{FGI} > 0\), \(E[S] > 0\), \(k > 0\), \(l_i > 0\), \(H > 0\), \(E[D_i] > 0\), \(E[D] > 0\), \(CT_P > 0\), \(m > 0\), \(H > 0\) therefore,

\[
E[\tau_3] > E[\tau_2] > E[\tau_1]
\]  
(2.23)

\[
E[\Omega_3] > E[\Omega_2] > E[\Omega_1]
\]  
(2.24)

Also the variances of the time to detection are:

\[
\text{Var}[\tau_1] = \text{Var}[T_{P1S}], \text{Var}[\tau_2] = \text{Var}[T_{F2S}], \text{Var}[\tau_3] = \text{Var}[T_{F3S_i}]
\]  
(2.25)

For origin of contamination at the processing center,

1. **Sampling at the processing center**

The time of sampling is assumed to be uniformly distributed between the time the first batch after the onset of contamination starts processing until the time the \((Nkm)^{th}\) product leaves the FGI. The sampling time \(T_{F2P}\) is a random variable, assumed to be uniformly distributed over the interval \([0, NkE[S] + t_{FGI}]\). The amount of stock to be recalled or to be investigated further will be

\[
E[\Omega_2] = E[D]E[T_{F2P}]
\]  
(2.26)

2. **Sampling at the retailer**

At the retailer the time of sampling is assumed to be uniformly distributed between the time the first contaminated product arrives until the time the \((Nkm)^{th}\) product is sold.
The first contaminated product will reach the retailer after time $E[S] + t_{FG1} + l_i$ and the $(N_{km})^{th}$ product will be sold after time $N_{k}E[S] + t_{FG1} + l_i + t_{H_i} + \frac{N_{km}(2-\beta_i)}{E[D_i]}$. The sampling time $T_{F2P_i}$ is assumed to be a random variable uniformly distributed over the interval $\left[E[S] + t_{FG1} + l_i, N_{k}E[S] + t_{FG1} + l_i + t_{H_i} + \frac{N_{km}(2-\beta_i)}{E[D_i]}\right]$. The time to detection of contamination through sampling at $r$ retailers will be the random variable, which will be the minimum among the sampling times at each retailer $Y = \min[T_{F3S_1}, T_{F3S_2}, \ldots, T_{F3S_r}]$. If $F_{T_{F3P}}[T_{F3P}]$ is the distribution of $T_{F3P}$, the corresponding cdf of $Y$ will be,

$$F_Y(y) = P(Y \leq y) = P(\min[T_{F3P_1}, T_{F3P_2}, \ldots, T_{F3P_r}] \leq y)$$

$$= 1 - P(\min[T_{F3P_1}, T_{F3P_2}, \ldots, T_{F3P_r}] > y)$$

$$= 1 - P(T_{F3P_1} > y, T_{F3P_2} > y, \ldots, T_{F3P_r} > y) = 1 - [P(T_{F3P} > y)]^r$$

$$= 1 - [1 - F_{T_{F3P}}[T_{F3P}]]^r$$

(2.27)

The expected time to detection of contamination through sampling at $r$ retailers will be $E[Y]$. The expected amount of contaminated stock will be:

$$E[\Omega_3^2] = E[Y](E[D_1] + E[D_2] + \ldots, + E[D_r]) = E[Y]E[D]$$

(2.28)

The expected time to detection for each of the three cases above is:

$$E[\tau_3^2] = E[T_{F2P}] = \frac{N_{k}E[S] + t_{FG1}}{2}$$

(2.29)
\[ E[\tau_2^2] = E[T_{F3S_1}] = \]
\[
\frac{(Nk+1)E[S]+2t_{FG1}+2l_1+t_{H1}+\frac{Nkm(2-\beta_{r_1})}{E[D_1]}}{2}
\]

for \( i = 1, \cdots, r \), since \( N > 0, L > 0, t_{SS} > 0, E[S] > 0, k > 0, l_1 > 0, \frac{Nkm(2-\beta_{r_1})}{E[D_1]} > 0, H > 0 \) therefore
\[ E[\tau_2^2] > E[\tau_2^2] \quad (2.31) \]
\[ E[\Omega_2^2] > E[\Omega_2^2] \quad (2.32) \]
Also the variances of the time to detection are
\[ \text{Var}[\tau_2^2] = \text{Var}[T_{F2P}], \text{Var}[\tau_2^2] = \text{Var}[T_{F3P_1}] \quad (2.33) \]

This proposition shows that regardless of how often one chooses to sample, detection always takes longest (and the amount of inventory contaminated is always largest) when sampling at any of the retailers. Detection is quickest (and the amount affected is smallest) when sampling at the supplier (processing center) for the case of origin at the supplier (processing center). The benefit of sampling is always greatest when sampling is done closest to the actual location where the contamination occurred.

Therefore, the location of sampling is more important than the sampling regimen a company or government agencies may adopt. This also implies that understanding where and with what probability contamination originates, should be of prime importance to the decision maker.
2.7.4. Contamination Detection through Food Borne Illnesses

It is in the company and supply chain’s interest to detect contamination before it leads to massive food borne illness outbreaks. Food recall investigations are initiated when a certain number of food borne illness incidents are reported and linked to a certain food product. It is assumed that a certain number $T, T \geq 1$, of illnesses have to be reported before a contamination can be discovered through food borne illness incidents.

Lemma 2.7.4.1: The expected time to detection, variance of the time to detection and the expected contaminated stock for origin of contamination at the supplier and mode of detection through food borne illnesses in a multiple retailer supply chain is,

$$E[\tau_4^1] = E[Y], Var[\tau_4^1] = \sigma^2[Y], E[\Omega_n^1] = E[D]E[Y], \text{ where } Y =$$

$$\text{Max} \left[ Z_1, Z_2, \cdots, Z_{[T/p]} \right] \text{ and the time of reporting of the food borne illness by the } n^{th} \text{ customer is the random variable } Z_n = C_i + T_i, C_i \text{ is the consumption time of the } n^{th} \text{ consumer, } n = 1 \cdots \left[ \frac{T}{p} \right].$$

$$C_T \sim \left[ \left( \frac{T}{p} \right)^{km} \cdot \frac{k+1}{E[D]} \cdot L + C T_p + l_i + t_{FGI} + t_{H_i} + \frac{\left( \frac{T}{p} \right)^{2-\beta_i}}{E[D]} + L + C T_p + H \right].$$

The expected time to detection, variance of the time to detection and the contaminated stock for origin of contamination at the processing center and mode of detection through food borne illnesses is

$$E[\tau_4^2] = E[Y], Var[\tau_4^2] = \sigma^2[Y], E[\Omega_n^2] = E[D]E[Y], \text{ where }$$

$$Y = \text{Max} \left[ Z_1, Z_2, \cdots, Z_{[T/p]} \right] \text{ and the time of reporting of the food borne illness by the } n^{th}$$
customer is the random variable \( Z_n = C_i + T_{in} \). \( C_i \) is the consumption time of the \( n^{th} \) consumer, \( n = 1, \ldots, \left\lfloor \frac{T}{p} \right\rfloor \).

\[
C_T \sim \left[ \left( \frac{T}{p} \right)^{\frac{T}{p}} E[S] + l_i + t_{FGI} + t_{Hi} + \frac{\left( \frac{T}{p} \right) (2 - \beta_{ri})}{E[D_i]} \right] \left( \frac{T}{p} \right)^{\frac{T}{p}} E[S] + H \right].
\]

for \( i = 1, \ldots, r, T \geq 1 \).

**Proof:** For origin of contamination at the supplier, for \( T \) customers to show the symptoms, expected number of customers required to purchase and consume the products is \( \left\lfloor \frac{T}{p} \right\rfloor \). The \( \left( \left\lfloor \frac{T}{p} \right\rfloor \right)^{th} \) product after the introduction of contamination will be sold after time \( \frac{\left( \frac{T}{p} \right) km}{E[D]} + L + CT_p + t_{FGI} + l_i + t_{Hi} + \frac{\left( \frac{T}{p} \right) (2 - \beta_{ri})}{E[D_i]} \). As before we assume \( X_T \) to be the time between the purchase of the \( \left( \left\lfloor \frac{T}{p} \right\rfloor \right)^{th} \) product by a customer and the end of its shelf life, where \( \frac{T}{p} \) gives the batch number to which \( \left( \left\lfloor \frac{T}{p} \right\rfloor \right)^{th} \) the product belongs. The time of consumption of the product by the \( \left( \left\lfloor \frac{T}{p} \right\rfloor \right)^{th} \) customer is a random variable \( C_\left\lfloor \frac{T}{p} \right\rfloor \) which is uniformly distributed between

\[
\left[ \frac{\left( \frac{T}{p} \right) km}{E[D]} + L + CT_p + t_{FGI} + l_i + t_{Hi} + \frac{\left( \frac{T}{p} \right) (2 - \beta_{ri})}{E[D_i]} + X_T \right].
\]

where \( X_T = H - \)
\( t_{FGI} + t_i + t_{Hi} + \frac{\left( T_0 / (2 - \beta r_i) \right)}{E[D_i]} \). Then the expected time at which the \( \left( \left\lceil \frac{T}{p} \right\rceil \right) \)th customer shows symptoms of disease after consumption of the contaminated product will be \( E \left[ C_{\left\lceil \frac{T}{p} \right\rceil} + T_{In} \right] \) and the variance will be \( \text{Var} \left[ C_{\left\lceil \frac{T}{p} \right\rceil} + T_{In} \right] \). The time of reporting of the food borne illness by the \( n \)th customer is the random variable \( Z_n = C_i + T_{In} \), \( C_n \) is the consumption time of the \( n \)th consumer. \( Z_n \) are \( \left\lceil \frac{T}{p} \right\rceil \) independent identically distributed random variables (iid) for \( n = 1, \ldots, \left\lceil \frac{T}{p} \right\rceil \). It is to be noted that the time of consumption may not be in the order of purchase of the product. Let the time at which the \( \left\lceil \frac{T}{p} \right\rceil \)th illness is reported be the random variable, which will be the maximum among the reporting times of each of the \( \left\lceil \frac{T}{p} \right\rceil \) customers \( Y = \text{Max} \left[ Z_1, Z_2, \ldots, Z_{\left\lceil \frac{T}{p} \right\rceil} \right] \). If \( F_Z(z) \) is the distribution of, then the corresponding cdf of \( Y \) will be \( F_Y(y) = P \left[ Z_1 \leq y, Z_2 \leq y, \ldots, Z_{\left\lceil \frac{T}{p} \right\rceil} \leq y \right] = \left[ F_Z(y) \right]^{\left\lceil \frac{T}{p} \right\rceil} \). The expected time at which \( T \) cases will be reported is \( E[Y] = E \left[ \text{Max} \left[ Z_1, Z_2, \ldots, Z_{\left\lceil \frac{T}{p} \right\rceil} \right] \right] \). The contaminated stock to be investigated or destroyed after the reporting of \( \left\lceil \frac{T}{p} \right\rceil \) cases of the disease will be \( E[\Omega_1^T] = E[D]E[Y] \). The expected time to detection \( E[\tau_{14}^T] = E[Y] \) and the variance of the time to detection \( \text{Var}[\tau_{14}^T] = \text{Var}[Y] \).
For origin of contamination at the processing center, the \( \left( \left[ \frac{T}{p} \right] \right) \) th product after the introduction of contamination will be sold after time \( \left[ \frac{T}{p} \right] \) E[S] + t\(_{FGL}\) + l\(_i\) + t\(_{H_i}\) + \( \left[ \frac{T}{p} \right] \) \( (2-\beta_{r_1}) \) \( \frac{E[D_1]}{E[D_1]} \). As before we assume \( X_T \) to be the time between the purchase of the \( \left( \left[ \frac{T}{p} \right] \right) \) th product by a customer and the end of its shelf life. The time of consumption of the product by the \( \left( \left[ \frac{T}{p} \right] \right) \) th customer is a random variable \( C_{\left[ \frac{T}{p} \right]} \) which is uniformly distributed between \( \left[ \frac{T}{p} \right] \) E[S] + t\(_{SS}\) + l\(_i\) + \( \frac{\left[ \frac{T}{p} \right] (2-\beta_{r_1})}{E[D_1]} \), \( \left[ \frac{T}{p} \right] \) E[S] + t\(_{FGL}\) + l\(_i\) + t\(_{H_i}\) + \( \frac{\left[ \frac{T}{p} \right] (2-\beta_{r_1})}{E[D_1]} \) + \( X_T \). where \( X_T = H - \left( t_{FGL} + l_i + t_{H_i} + \frac{\left[ \frac{T}{p} \right] (2-\beta_{r_1})}{E[D_1]} \right) \). Then the expected time at which the \( \left( \left[ \frac{T}{p} \right] \right) \) th customer shows symptoms of disease after consumption of the contaminated product will be \( E \left[ C_{\left[ \frac{T}{p} \right]} + T_{ln} \right] \) and the variance will be \( \text{Var} \left[ C_{\left[ \frac{T}{p} \right]} + T_{ln} \right] \). The time of reporting of the food borne illness by the \( n \) th customer is the random variable \( Z_n = C_i + T_{ln} \). \( C_n \) is the consumption time of the \( n \) th consumer. \( Z_n \) are \( \left[ \frac{T}{p} \right] \) independent identically distributed random variables (iid) for \( n = 1, \ldots, \left[ \frac{T}{p} \right] \). It is to be noted that the time of consumption may not be in the order of purchase of the product. Let the time at which the \( \left( \left[ \frac{T}{p} \right] \right) \) th illness is reported be the random variable, which will be the maximum among the reporting times of each of the \( \left[ \frac{T}{p} \right] \) customers \( Y = \text{Max} \left[ Z_1, Z_2, \ldots, Z_{\left[ \frac{T}{p} \right]} \right] \). If \( F_Z(z) \) is the distribution of, then the
corresponding cdf of $Y$ will be $F_Y(y) = P\left[Z_1 \leq y, Z_2 \leq y, \ldots, Z_{\left\lceil \frac{T}{p} \right\rceil} \leq y\right] = [F_Z(y)]^{\left\lceil \frac{T}{p} \right\rceil}.

The expected time at which $T$ cases will be reported is $E[Y] = E\left[\max\left[Z_1, Z_2, \ldots, Z_{\left\lceil \frac{T}{p} \right\rceil}\right]\right].$

The contaminated stock to be investigated or destroyed after the reporting of $\left\lceil \frac{T}{p} \right\rceil$ cases of the disease will be $E[\Omega_4] = E[D]E[Y].$ The expected time to detection $E[\tau_4^2] = E[Y]$ and the variance of the time to detection $\text{Var}[\tau_4^2] = \text{Var}[Y].$

The following proposition shows under what circumstances the expected time to detection through food borne illness will be greater than through sampling efforts. This allows the selection of an appropriate sampling strategy if we want to detect the food contamination through sampling and preempt detection through food borne illnesses.

Ideally, a company will want to sample less frequently (i.e., use a sampling strategy with a low $N$) to keep sampling costs at a minimum. Thus, we would like to find the least-frequent sampling strategy that can still - in expectation - provide contamination detection before a food borne illness incident is raised. Firstly, the following sampling strategy parameters are defined:

$$N_1 = 2 \left( \frac{\left\lceil \frac{T}{p} \right\rceil}{k} - 1 \right) + \frac{E[D]}{km} \left( 2L + 2CT_p + t_{FGI} + t_l + t_{H_1} + \frac{\left\lceil \frac{T}{p} \right\rceil (2 - \beta_{ri})}{E[D_i]} + H + 2T_{ln} \right) + 1$$

(2.34)
\[ N_2 = \left( \frac{|T|}{p} - \frac{1}{k} \right) + \frac{E[D]}{k m} \left( C T_p + l_i + t_{H_i} + \frac{|T|}{p} \left( 2 - \beta_{r_i} \right) \right) + H + 2T_{in} \]

\[ + 1 \] (2.35)

\[ N_3 = \frac{E[D_i]}{E[D_i] + E[D] (2 - \beta_{r_i})} \left( \frac{|T|}{p} - \frac{1}{k} \right) \]

\[ + \frac{E[D]}{k m} \left( \frac{|T|}{p} \left( 2 - \beta_{r_i} \right) \right) + H + 2T_{in} - t_{FGi} - l_i - t_{H_i} \]

\[ + 1 \] (2.36)

\[ N_4 = \frac{1}{k E[S]} \left( 2E[S] \left( \frac{|T|}{p} \right) + \frac{|T|}{p} \left( 2 - \beta_{r_i} \right) \right) + H + 2T_{in} + l_i \]

\[ + t_{H_i} \] (2.37)

\[ N_5 = \frac{E[D_i]}{k E[S] E[D_i] + km (2 - \beta_{r_i})} \left( 2E[S] \left( \frac{|T|}{p} \right) + \frac{|T|}{p} \left( 2 - \beta_{r_i} \right) \right) + H + 2T_{in} - t_{FGi} \]

\[ - l_i - t_{H_i} - E[S] \] (2.38)
Proposition 2.7.4.2:

1. If contamination origin is at the supplier, then when sampling takes place at location \(i\) (\(i = 1:\) supplier, \(i = 2:\) processing center, \(i = 3:\) retailer), a sampling strategy can preempt detection through a food borne illness illness for any \(N < N_i\).

2. If contamination origin is at the processing center, then when sampling takes place at location \(i\) (\(i = 4:\) processing center, \(i = 5:\) retailer), a sampling strategy can preempt detection through a food borne illness illness for any \(N < N_i\).

3. \(N_1 > N_2 > N_3\).

4. \(N_4 > N_5\).

Proof: From Lemma 2.7.4.1, for origin at the supplier, \(E[T_4^1] = E\left[\frac{C_{[T]}|p|}{k}\right] + E[T_{in}]\) for \([T]\) customers purchasing and consuming the product for \(T\) customers to report an illness.

\[
C_T \sim \left[\frac{\left(\frac{|T|}{k} - 1\right) km}{E[D]} + L + CT_p + l_i + t_{FGL} + t_{Hi} + \frac{|T|}{E[D]}(2 - \beta_i) + \frac{|T|}{E[D]}(2 - \beta_i)\right] + L + CT_p + H
\]

(2.39)
The expected time to detection through sampling at the different nodes is given in Proposition 2.7.3.1. The different expected times to detection can now be compared to obtain the condition under which the expected time to detection through sampling will be lesser than the expected time through food borne illnesses.

1. Sampling at supplier

\[ E[\tau_1] = \frac{(N-1)\text{km}}{2E[D]} + 2L + 2CT_p + l_i + t_{FGI} + \frac{[T]}{p} (2 - \beta_{r_i}) + H + 2T_{In} \]

\[ N < 2 \left(\frac{[T]}{p} - 1\right) + \frac{E[D]}{km} \left(2L + 2CT_p + t_{FGI} + l_i + t_{Hi} + \frac{[T]}{p} (2 - \beta_{r_i}) + H + 2T_{In}\right) + 1 \]

(2.41)

2. Sampling at the processing center

\[ E[\tau_2] = \frac{2L + CT_p + (N-1)\text{km}}{2E[D]} + t_{FGI} \]

\[ N_2 = \text{Max}(N), \text{ for } E[\tau_1] > E[\tau_2] \]

The condition which defines the sampling strategy will be:

\[ N < 2 \left(\frac{[T]}{p} - 1\right) + \frac{E[D]}{km} \left(CT_p + l_i + t_{Hi} + \frac{[T]}{p} (2 - \beta_{r_i}) + H + 2T_{In}\right) + 1 \]

(2.42)
3. Sampling at the retailer $E[\tau_3^3] = \frac{2L + 2CT_p + \frac{(N-1)km}{E[D]} + 2t_{FGI} + 2I + t_{HI} + \frac{Nkm(2-\beta_{r_1})}{E[D]}}{2}$.

let $N_3 = \text{Max}(N)$, for $E[\tau_4^4] > E[\tau_3^3]$

The condition which defines the sampling strategy will be:

$$N < \frac{E[D_i]}{E[D_i] + E[D](2 - \beta_{r_1})} \frac{T}{p} \frac{k}{k - 1}$$

$$+ \frac{E[D]}{km} \left( \frac{T}{p} \frac{k}{E[D_i]} + H + 2T_{In} - t_{FGI} - l_i - t_{Hi} \right) + 1 \quad (2.43)$$

Comparing $N_1, N_2$ as $L > 0, t_{ss} > 0$, therefore $N_1 > N_2$ especially for long lead times.

Comparing $N_2, N_3$, $2 \left( \frac{T}{p} \right) \frac{k}{k - 1} + \frac{E[D]}{km} \left( \frac{T}{p} \frac{(2 - \beta_{r_1})}{E[D_i]} + H + 2T_{In} - t_{FGI} - l_i - t_{Hi} \right) + 1 <$

$$2 \left( \frac{T}{p} \right) \frac{k}{k - 1} + \frac{E[D]}{km} \left( CT_p + l_i + t_{Hi} + \frac{T}{p} \frac{(2 - \beta_{r_1})}{E[D_i]} + H + 2T_{In} \right) + 1$$

as $t_{FGI}, CT_p > 0$

and $\frac{E[D_i]}{E[D_i] + E[D](2 - \beta_{r_1})} < 1$, therefore $N_3 < N_2$ and $N_1 > N_2 > N_3$.

For origin at the processing center, $E[\tau_4^4] = E \left[ C_{\frac{T}{p}} \right] + E[T_{In}]$ for $\frac{T}{p}$ customers purchasing and consuming the product for $T$ customers to report an illness.

$$C_T \sim \left[ \frac{T}{p} \right] E[S] + l_i + t_{FGI} + t_{HI} + \frac{T}{p} \frac{(2 - \beta_{r_1})}{E[D_i]} \frac{T}{p} E[S] + H \right] \quad (2.44)$$
\[ E[\tau^2_4] \]
\[
= \frac{2 \left( \frac{T}{l} \right) E[S] + l + t_{FG1} + t_{H_1} + \left( \frac{T}{l} \right) \left( 2 - \beta_{R_1} \right) \frac{E[D_i]}{E[D_i]} + H + 2T_{In}}{2} 
\]

(2.45)

Now the different expected times to detection can be compared to obtain the condition under which the expected time to detection through sampling will be lesser than the expected time through food borne illnesses.

1. Sampling at processing center
\[ E[\tau^2_4] = \frac{NkE[S] + t_{FG1}}{2}, \]
let \( N_4 = \text{Max}(N), \) for \( E[\tau^2_4] > E[\tau^2_1], \)

The condition which defines the sampling strategy will be:
\[
N < \frac{1}{kE[S]} \left( 2E[S] \left( \frac{T}{l} \right) + \left( \frac{T}{l} \right) \left( 2 - \beta_{R_1} \right) \frac{E[D_i]}{E[D_i]} + t_{H_1} + H + 2T_{In} + l_i \right) 
\]

(2.46)

2. Sampling at the retailer
\[ E[\tau^2_5] = \frac{\left( Nk + 1 \right) E[S] + 2t_{FG1} + 2l + t_{H_1} + \frac{Nkm(2 - \beta_{R_1})}{E[D_i]} }{2}, \]
let \( N_5 = \text{Max}(N), \) for \( E[\tau^2_5] > E[\tau^2_3], \)

the condition which defines the sampling strategy will be:
\[
N < \frac{E[D_i]}{kE[S]E[D_i] + km(2 - \beta_{R_1})} \left( 2E[S] \left( \frac{T}{l} \right) + \left( \frac{T}{l} \right) \left( 2 - \beta_{R_1} \right) \frac{E[D_i]}{E[D_i]} + H + 2T_{In} - t_{FG1} - l_i \right) - t_{H_1} - E[S] 
\]

(2.47)
Comparing $N_4, N_5$\

$$E[S] < \left(2E[S] \left[\frac{T}{T_p}\right] + \frac{T_p(2-\beta_{r_1})}{E[D_1]} + H + 2T_{in} - t_{FG1} - l_i - t_{Hi} - 2L + C T_p + \frac{(N-1)km}{E[D]} - NkE[S] > 1 \text{ (when sampling at processing center)} \right) \text{ as } t_{ss} > 0, E[S] > 0,$$

$$\frac{E[D_1]}{kE[S]E[D_1] + km(2-\beta_{r_1})} < \frac{E[D_1]}{kE[S]E[D_1]} = \frac{1}{KE[S]} \text{ as } km(2 - \beta_{r_1}) > 0 \therefore N_4 > N_5.$$

The preceding proposition gives the least frequent sampling strategy that a company can adopt to keep sampling costs at a minimum and still preempt detection by customer reports of food borne illnesses. This Proposition also shows that one can afford to sample less frequently when doing it at the supplier, especially with long lead times from the grower to the processing center. Similarly, one can sample less frequently at the processing center as compared to sampling at the retailer.

In the following Proposition the mode of detection of contamination is held constant. The performance metrics are then compared across the different origins of contamination to derive conditions under which the impact of contamination will be most significant.

**Proposition 2.7.4.3:** For a given mode of detection of contaminated stock, the expected time to detection and the expected amount of contaminated stock are most when the origin of contamination is at the supplier as opposed to the processing center when:

1. $2L + C T_p + \frac{(N-1)km}{E[D]} - NkE[S] > 1 \text{ (when sampling at processing center)}.$ For $N \gg 1$ the condition reduces to $2L + C T_p + \frac{N km (1-u)}{E[D]}$, where $u$ is the utilization factor.
2. \[ 2L + CT_p + \frac{(N-1)km}{E[D]} - (Nk + 1)E[S] > 1 \text{ (when sampling at retailer).} \]

For \( N \gg 1 \) the condition reduces to \( 2L + 2CT_p + \frac{Nkm}{E[D]} (1 - u) > 1 \).

3. \[ L + CT_p - \frac{km}{E[D]} + \left[ \frac{m}{E[D]} \right] (1 - u) > 1 \text{ (Food borne illness).} \]

**Proof:**

1. Sampling at processing center, \( E[\tau_1^1] = \frac{2L + CT_p + \frac{(N-1)km}{E[D]} + t_{FGI}}{2} \) and \( E[\tau_1^2] = \frac{\frac{Nkm}{E[D]} + t_{FGI}}{2} \), \( E[\tau_1^1] > E[\tau_1^2] \) if

\[
2L + CT_p + \frac{(N-1)km}{E[D]} - NkE[S] > 1 \tag{2.48}
\]

for \( N \gg 1 \) we assume \( N - 1 \cong N \) and \( u = \frac{E[S]E[D]}{m} \) the above condition reduces to

\[
2L + CT_p + \frac{Nkm(1-u)}{E[D]} > 1 \tag{2.49}
\]

2. Sampling at retailer,

\[
E[\tau_2^1] = \frac{2L + 2CT_p + \frac{(N-1)km}{E[D]} + 2t_{FGI} + 2l_1 + t_{H1} + \frac{Nkm(2-\beta_1)}{E[D]}}{2}
\]

and \( E[\tau_2^2] = \frac{\frac{(Nk+1)E[S] + 2t_{FGI} + t_{H1} + 2l_1 + \frac{Nkm(2-\beta_1)}{E[D]}}{2}, E[\tau_2^1] > E[\tau_2^2] \) if

\[
2L + CT_p + \frac{(N-1)km}{E[D]} - (Nk + 1)E[S] > 1 \tag{2.50}
\]

for \( N \gg 1 \) we assume \( N - 1 \cong N \) and \( u = \frac{E[S]E[D]}{m} \) the condition reduces to

\[
2L + 2CT_p + \frac{Nkm}{E[D]} (1 - u) > 1 \tag{2.51}
\]

3. Food borne illness,
This Proposition shows that for a realistic sampling strategy at the processing center or any of the retailers, where $N \gg 1$, the impact of contamination is highest when the origin is at the supplier as opposed to the processing center. This is because the conditions in the Proposition hold as the terms on the left hand side are positive, and for all practical purposes one can assume $L \geq 1$, in particular for global supply chains. A moderately frequent or a fairly infrequent sampling strategy with $N \gg 1$ can be assumed in most cases because too frequent sampling results in a prohibitively high cost of sampling.

2.8. Quality Control Process

Consider the implementation of a quality control process (such as equipment clean up, water change etc.) which is assumed to eradicate the contamination. This process can occur at the supplier or the processing center and negates the earlier assumption of an ongoing contamination. It is also assumed that the quality control process takes place in zero time once every $Q^*$ batches after the start of contamination.
Therefore on implementation of the quality control process the number of contaminated batches will be $Q^*$ (when a contamination occurs).

For the scenario where origin of contamination is at the supplier or the processing center, in the absence of a quality control process the contamination will be ongoing. The number of contaminated batches $Q$ for detection through a food borne illness can be calculated from Proposition 2.7.4.1. If there is a quality control process in place the number of contaminated batches will be $Q^*$. Now we analyze the following options:

1. $Q^* < Q$: In this case, either the contaminated stock is entirely in the supply chain (and has not been sold to customers), or some amount of product has reached the consumers. However, in this worst case, the amount that has reached consumers is less than the critical amount that would raise a food borne illness outbreak event. This situation is similar to the scenario of contamination detection through sampling in the absence of a quality control process discussed above.

2. $Q^* \geq Q$: The food borne illness outbreak event will take place prior to the quality control process being able to rectify the problem. In this case, the implementation of the quality control process appears redundant, if we assume that upon a food borne illness outbreak, processing and sale of the affected products will be halted anyway.

Under this assumption, it is only sensible to implement the quality control process if the process is such that $Q^* < Q$. We now evaluate the total budget that necessary for implementing such a process. Let $C_q$ be the total annual budget that has to
be allocated by the company for quality control so that they can meet the requirement $Q^* < Q$. The cost $C_q$ is now evaluated in terms of the known model parameters. Let $c_q$ be the cost per quality control process taking place and $E[D]$ be expected annual demand.

**Proposition 2.8.1:** The minimum cost per time period of a quality control process which will eradicate contamination prior to a food borne illness outbreak is $C_q > \frac{E[D]c_q}{kmQ}$.

**Proof:** The number of quality control processes taking place per year as decided by the total budget is $\frac{c_q}{c_q}$. The number of batches processed per time period will be $\frac{E[D]}{km}$. A quality control process will take place once every $Q^*$ batches where,

$$Q^* = \frac{E[D]}{km} \frac{c_q}{c_q}$$  \hspace{1cm} (2.53)$$

Since $Q^* < Q$, therefore

$$C_q > \frac{E[D]c_q}{kmQ}$$  \hspace{1cm} (2.54)$$

where $E[D]$ is the expected annual demand.

This proposition suggests that if a company cannot dedicate at least a budget of $C_q$ to a quality control regimen, then it should not employ quality control at all. This is of managerial interest, because it shows that the benefits from quality control (in terms of contamination health scare avoidance) are decidedly not linear in the effort and money spent on quality control; rather there is a threshold of quality control effort below which there is no impact at all.
2.9. **Cost of Contamination**

In the event of a product contamination, the cost of contamination will be higher depending on the number of products affected and the number of consumers of the product (Lippincott 2008). Together, these factors determine what is called the “direct cost” of the contamination event. This direct cost consists of the cost of destroying the unsold contaminated stock, recalling the sold contaminated stock, and the cost associated with lost sales during the recall period.

As opposed to the direct cost, the “indirect costs” of a food contamination event are the liability, medical costs and the costs of deteriorating brand value and negative reputation effects (Golan 2003), (Lippincott 2008), (Golan, Krissoff et.al. 2004), (Ollinger and Ballenger 2003). While these indirect costs (especially through litigation and reputation effects) may in some cases be higher than the direct costs, they are typically hard to quantify directly.

In this research, it is chosen to model the indirect costs as a consequence of the scale of the food contamination event. In essence, the larger the scale of the food contamination event as measured by the amount of product (sold or unsold) affected, and the time to detection of the contamination event, the greater are the resulting indirect costs likely to be. For example, if one considers only the medical costs, then the higher the number of contaminated products sold, the higher the number of customers reporting illnesses and filing medical claims is likely to be. Now a general expression for the total cost incurred due to the food contamination incident is derived. The total expected cost
of a contamination event $C_{\text{Total}}(i,j)$ is the sum total of the expected direct and expected indirect costs:

$$C_{\text{Total}}(i,j) = C_D(i,j) + C_I(i,j)$$

(2.55)

where $j = 1, \cdots, 4$ for $i = 1,2$ for different cases of origins and modes of detection of contamination. The direct costs incurred due to the food contamination event are the following:

1. The cost of destroying the contaminated safety stock if it is in the inventory or refunding the customer if it is sold.

2. The loss of sales during the time taken from opening to closure of the recall case under criterion agreed upon by the FSIS and the firms in the supply chain (Teratanavat et. al. 2005). The FSIS is the Food Safety and Inspection service of the United States Agricultural Department (USDA) and is responsible for the safety of food products. The time taken to close the recall case can be assumed to be a function of the contaminated stock (sold and unsold). This is because as the amount of contaminated stock produced increases the amount of sold or unsold stock will also increase. In turn, longer will be the timeframe required to conduct retrieval of the stock and close the recall case.

Let $\Omega^i_j$ denote the total contaminated stock, $\omega^i_j$ denote the amount of sold contaminated stock and $\gamma^i_j$ denote the amount of unsold contaminated stock. Then the expected direct cost will be $C_D(i,j)$:

$$C_D(i,j) = cE[\gamma^i_j] + rE[\omega^i_j] + rE[D]T_r(\omega^i_j, \gamma^i_j)$$

(2.56)
where \(E[D]\) is the expected demand during one period, \(T_r\) is the time taken to close the recall case, \(c\) is manufacturing cost per item and \(r\) is the retail cost per item. The time taken to close the recall case is assumed to be a function of the sold and unsold contaminated stock, since the greater the amount of contaminated stock, the longer will be the time required to track it.

The indirect costs are modeled by taking as proxies the amount of contaminated inventory. Then, the expected indirect cost will be a function of the total contaminated stock and amount of contaminated stock sold:

\[
C_i(i, j) = F\left( \Omega_i^j, \omega_i^j \right)
\]

such that

\[
\frac{dF}{d \Omega_i^j} \geq 0, \frac{dF}{d \omega_i^j} \geq 0.
\]

**Proposition 2.9.1:** The expected amount of sold and unsold contaminated stock at the expected time to detection of the food contamination event for the different origins and modes of detection of contamination is:

1. **Origin at supplier**

   \[
   E[\tau_i^j] = \begin{cases} 
   \left[ \frac{E[\tau_i^j] - \left( CT_P + L + t_{FGI} + t_{H_i} + l_i - \frac{k}{E[D]} \right)}{m \frac{E[D]}{E[D_i]} + \frac{(2 - \beta_r)}{E[D_i]}} \right], \\
   E[\tau_i^j] - \left( CT_P + L + t_{FGI} + + t_{H_i} + l_i - \frac{k}{E[D]} \right) > 0 \\
   0, \quad \text{otherwise}
   \end{cases}
   \]

   for \(j = 1 \cdots 4\).

2. **Origin at processing center**
\[
E[\omega_j^2] = \begin{cases} 
\left[ E[\tau_j^2] - (t_{FGI} + l_i + t_{H_I}) \right] / E[S] + \frac{(2 - \beta_{r_I})}{E[D_I]} \left( m / E[D] + \frac{(2 - \beta_{r_I})}{E[D_I]} \right), & \text{if } \frac{E[\tau_j^2] - (t_{FGI} + l_i + t_{H_I})}{E[S] + \frac{(2 - \beta_{r_I})}{E[D_I]}} > 0 \\
0, & \text{otherwise}
\end{cases}
\]

for \( j = 2 \cdots 4 \),


**Proof:**

1. **Origin supplier**

The \( N^{th} \) contaminated product will be sold at time \( \frac{(N-1)k_m}{E[D]} + L + CT_P + t_{FGI} + t_{H_I} + l_i + \frac{N(2-\beta_{r_I})}{E[D_I]} \). At the time to detection \( E[\tau_j^2] \) the expected amount of contaminated stock will be

\[
E[\omega_j^1] = \begin{cases} 
\left[ E[\tau_j^1] - \left( CT_P + L + t_{FGI} + l_i + + t_{H_I} - \frac{k}{E[D]} \right) \right] / \left( m / E[D] + \frac{(2 - \beta_{r_I})}{E[D_I]} \right), & \text{if } \frac{E[\tau_j^1] - \left( CT_P + L + t_{FGI} + l_i + t_{H_I} - \frac{k}{E[D]} \right)}{m / E[D] + \frac{(2 - \beta_{r_I})}{E[D_I]}} > 0 \\
0, & \text{otherwise}
\end{cases}
\]

Otherwise \( E[\omega_j^1] = 0 \) for \( j = 1 \cdots 4 \) as the amount of stock cannot be negative

2. **Origin processing center**

The \( N^{th} \) contaminated product will be sold at time \( NE[S] + t_{FGII} + l_i + t_{H_I} + \frac{N(2-\beta_{r_I})}{E[D_I]} \). At the time to detection \( E[\tau_j^2] \) the expected amount of contaminated stock will be
\[ E[\omega_j^2] \]
\[
= \begin{cases} 
  \frac{E[\tau_j^2] - (t_{FG1} + l_i + t_{Hi})}{E[S]} + \frac{(2 - \beta_{r_j})}{E[D_i]}, & \text{if } \frac{E[\tau_j^2] - (t_{FG1} + l_i + t_{Hi})}{E[S]} + \frac{(2 - \beta_{r_j})}{E[D_i]} > 0 \\
  0, & \text{otherwise}
\end{cases} \tag{2.59}
\]

Otherwise \( E[\omega_j^2] = 0 \) for \( j = 2 \cdots 4 \) as the amount of stock cannot be negative. Since the total contaminated stock is the sum of the sold and unsold contaminated stock

\[ E[\omega_j^2] = E[\Omega_j^2] - E[\gamma_j] \tag{2.60} \]

### 2.10. Cost of Sampling

For the scenario where the mode of detection of contamination is through sampling the additional cost of sampling can also be considered. The cost of sampling will depend on the sampling strategy, increasing as the sampling is done more frequently and decreasing as the sample is done less frequently. The annual cost of sampling is defined as \( C_S(N) \). According to the present sampling strategy in the model, we sample once in every \( N \) batches or once in every \( Nkm \) items, where \( k \) and \( m \) are constant for a given product. The total cost of contamination in this case will be:

\[ C_{Total}(i,j) = C_D(i,j) + C_I(i,j) + C_S(N) \tag{2.61} \]

For example, in the case where we sample once in every \( N \) batches, if we consider the annual demand rate to be \( E[D] \), the total number of batches produced in a single year will be \( \frac{E[D]}{km} \). The number of samples per year will be \( \frac{E[D]}{Nkm} \). The a priori probability of contamination occurring during a single year is assumed to be \( q \). The
probability of no contamination during the year will therefore be \((1 - q)\). The unit cost of sampling is assumed to be \(c_s\). The annual cost of the number of unsuccessful samples will be

\[
C_s(N) = \frac{E[D]}{Nkm} (1 - q) c_s
\]  

(2.62)

2.11. Varying Number of Retailers

The impact of the supply chain structure, and in particular the number of retailers in the supply chain on the performance metrics in event of a product contamination is now investigated.

It is assumed that initially the overall product demand described by random variable \(D\) and standard deviation \(\sigma_D\) is aggregated at a single retailer. As the number of retailers increase demand is distributed among them such that, \(D = \sum_{i=1}^{r} D_i\) for \(i = 1, \cdots, r\), where \(D_i\) is the customer demand at each of the retailers. It is also assumed that the expected demand and standard deviation of demand are the same at each retailer since the aim is to compare the impact of contamination on the supply chain and we want to avoid bias towards any of the retailers. The processing center now needs to fulfill demand \(D\).

We are interested whether and how the time to detection and the amount of contaminated product vary when the number of retailers is varied. Note that if the number of retailers decreases, then each retailer needs to satisfy a larger portion of the
total demand, which stays constant. The following Proposition analyzes the variation in the safety stock as the number of retailers varies in the supply chain.

**Proposition 2.11.1:** The total safety stock and the time for the safety stock to be sold at the processing center increases as the number of retailers increase.

**Proof:** As stated in Lemma 2.4.1, the safety stock is determined by two factors (1) the variability in the customer demand at each retailer and (2) the variability in the replenishment rate of the FGI at the processing center.

It is first proved that the variability due to the replenishment rate the same as the number of retailers increase. Consider, two supply chains with \( j \) and \( k \) retailers respectively such that \( < k \). The coefficient of variation for the replenishment process \( CV_R \) in both the cases can be assumed to be equal, as the processing centers in both supply chains have identical queuing processes in place. The parameters for the supply chain with \( j \) retailers is denoted by the suffix \( j \) and the parameters for the case of \( k \) retailers is denoted by the suffix \( k \). The coefficient of variation for the two cases is:

\[
CV_{Rj} = \frac{\sigma[R]_j}{E[R]} \quad \frac{\sigma[R]_j}{E[D]} \quad (2.63)
\]

\[
CV_{Rk} = \frac{\sigma[R]_k}{E[R]} \quad \frac{\sigma[R]_k}{E[D]} \quad (2.64)
\]

as \( E[R] = E[D] \)

\[
E[R] = E[D] = E[\sum_{i=1}^{j} D_i] = \sum_{i=1}^{j} E[D_i] = \sum_{i=1}^{j} E[D_i] \quad (2.65)
\]

As the demand at each retailer \( D_i \) is independent of each other

\[
CV_{Rj} = \frac{\sigma[R]_k}{\sum_{i=1}^{k} E[D_i]} \quad CV_{R} = CV_{Rk} \quad (2.66)
\]
Hence \( \sigma[R]_j = \sigma[R]_k \) \hspace{1cm} (2.67)

Now the variability due to the demand process in the two cases is analyzed to arrive at a relationship between the safety stocks. Consider the stock at \( r \) retailers being consolidated at a single retailer. At the single retailer, \( E[D] = \sum_{i=1}^{r} E[D_i] \).

\[
\sigma[D_{LT}] = \sqrt{\sum_{i=1}^{r} \sigma^2[D_{LT_i}]} = \sqrt{r \sigma^2[D_{LT_i}]} = \sqrt{r} \sigma[D_{LT_i}] \hspace{1cm} (2.68)
\]

This is due to risk pooling. The safety stock at the processing center for the case of a single retailer from Lemma 2.4.1 will be:

\[
[ss_{total}[D_{LT}, R]]_{1} = z \left( \sum_{i=1}^{r} \sigma[D_{LT_i}]^2 + \sigma[R]_s^2 \right) \hspace{1cm} (2.69)
\]

For the supply chain with \( r \) retailers the safety stock at the processing center will be:

\[
[ss_{total}[D_{LT}, R]]_{r} = z \left( r^2 \sigma[D_{LT_i}]^2 + \sigma[R]_s^2 \right) \hspace{1cm} (2.70)
\]

Therefore as the numbers of retailers \( r \) increase and

\[
[ss_{total}[D_{LT}, R]]_{1} \leq [ss_{total}[D_{LT}, R]]_{r} \hspace{1cm} (2.71)
\]

Now the performance metrics can be compared as the numbers of the retailers in the supply chain vary.

**Proposition 2.11.2:**

1. When sampling at the supplier, the expected time to detection and the expected amount of contaminated stock remain the same as the number of retailers varies.
2. For all other modes of detection, the expected time to detection and the expected amount of contaminated stock increase as the number of retailers increase.

Proof:

1. The expected time to detection and the expected amount of contaminated stock will be the same for the supply chain as the number of retailers vary for detection through sampling at the supplier. The expected time to detection is

\[
E[\tau_1] = \frac{(N-1) km}{2E[D]}
\]

and the expected amount of contaminated stock is

\[
E[\Omega_1] = E[\tau_1]E[D]
\]

from Proposition 2.7.3.1.

2. The expected time to detection will be

\[
E[\tau_2] = \frac{2L + 2CT_p + \frac{(N-1) km}{E[D]} + t_{FGI}}{2}
\]

for detection through sampling at the processing center as the number of retailers varies.

The expected amount of contaminated stock will be

\[
E[\Omega_2] = E[\tau_2]E[D]
\]

The time for the FGI stock to be sold

\[
t_{FGI} = \frac{ss_{total} |D_{LT, r}| R + E[D]}{E[D]}
\]

increases as the number of retailers increase, therefore the expected time to detection and the expected amount of contaminated stock will also increase as the number of retailers increase, given that all other parameters are the same.

3. The expected time to detection will be

\[
E[\tau_3] = \frac{2L + 2CT_p + \frac{(N-1) km}{E[D]} + 2t_{FGI} + 2l_i + t_{H1} + \frac{N km(2-\beta r_i)}{E[D]}}{2}
\]

for \( i = 1, \ldots, r \) for detection through sampling at a retailer, and the expected amount of contaminated stock is

\[
\]
4. The expected time to detection will be

\[ E[\tau_4^i] = \frac{2 \left( \frac{T_i}{P_i} - 1 \right)^{km} + 2L + 2CT_p + l_i + t_{FGI} + t_{H_i}^2 E[D_i]}{2E[D_i] + H + 2T_{In}} \]

for \( i = 1, \cdots, r \) for detection through a food borne illness, and the expected amount of contaminated stock is \( E[\Omega_4^i] = E[\tau_4^i]E[D] \).

The components which contribute to the expected time to detection and are different for each of these cases are:

1) The time for the FGI to be sold which increases as the number of retailers increase and hence will contribute to an increase in the expected time to detection.

2) The time the product spends on the retailer shelf \( t_{H_i} = \frac{ssr_i + E[D_i]}{E[D]} \)

\[ \frac{z\sigma[D_{LT_i}]}{E[D_i]} + \frac{1}{2} = \frac{z\sigma[D_{LT_i}]\sqrt{r}}{E[D]}, \] where \( z \) is the standardized value corresponding to the retailer service level. As the number of retailers increase \( t_{H_i} \) also increases.

3) The time after which the product will be purchased by the customer once it reaches the retailer, this will be \( \frac{T_i}{P_i}(z-\beta_i) \) for the case of which also increases as the number of retailers increase as the amount of demand fulfilled by each retailer \( E[D_i] \) decreases.

Therefore, the expected time to detection and the expected amount of contaminated stock will also increase as the number of retailers’ increase, for detection through sampling at a retailer or through a food borne illness. The results of Proposition
2.11.1 are for origin at the supplier, similar results can also be proved for origin at the processing center.

Thus, the performance metrics remain the same irrespective of the number of retailers in the supply chain for detection through sampling at the supplier. The performance metrics are higher (that is, worsen) the more retailers the supply chain has, for all other modes of detection. This behavior is mainly due to risk pooling when demand is consolidated at a single location. In addition, the amount of safety stock and the time for the safety stock to be sold at the processing center also increases as the number of retailers increase leading to an increase in the performance metrics.

This finding implies that in a highly arborescent supply chain with many retailers, the time required to close a recall case will be longer, because the length of the recall period will depend on the total amount of contaminated stock as well as the location and spread of the stock. As the number of retailers increase the contaminated stock will be distributed or spread out at more number of retailers and its customers as opposed to the spread when there are fewer retailers. Therefore, an otherwise identical sampling strategy implemented at the processing center or a retailer will result in contamination being detected faster for a supply chain with fewer retailers, as compared to a supply chain with a larger number of retailers. Simplicity in supply chain structure benefits contamination detection in this case.
2.12. Comparison of Product Attributes and Supply Chain Properties

A sensitivity analysis is performed to study the impact of the input parameters on the output of the model, the total contaminated stock $\Omega_j$ and the time to detection $\tau_j$. The test is conducted for all the cases of origin of contamination and modes of detection of the contamination and the results were similar. The results below are for a specific case where origin is at the supplier and detection is through a food borne illness. The results for all cases can also be proved similarly and will result in the same conclusion.

Proposition 2.12.1: Regardless of the origin of the contamination and the mode of detection, the expected amount of contaminated stock $E[\Omega_j]$ and the expected time to detection $E[\tau_j]$ vary as follows:

1. An increase in the incubation time $T_{in}$ results in an increase in $E[\Omega_j]$ and $E[\tau_j]$.

2. An increase in the lead time from the supplier to the processing center $l$ results in an increase in $E[\Omega_j]$ and $E[\tau_j]$.

3. An increase in the lead time from the processing center to the retailer $L$ results in an increase in $E[\Omega_j]$ and $E[\tau_j]$.

4. An increase in the shelf life of the product $H$ results in an increase in $E[\Omega_j]$ and $E[\tau_j]$.

5. An increase in the number of batches after which the sampling is periodically conducted (sampling strategy) $N$ will result in an increase in $E[\Omega_j]$ and $E[\tau_j]$. 
6. An increase in the number of customers $T$ having to report a food borne illness before a food borne illness scare is recorded will result in increase in $E[\bar{\Omega}^1_i]$ and $E[\tau^1_i]$.

7. An increase in the probability $p$ of a customer contracting a food borne illness on consuming the contaminated product results in a decrease in $E[\bar{\Omega}^1_i]$ and $E[\tau^1_i]$

Proof:

1. \[
\frac{\partial \tau^1_i}{\partial T_{in}} = 1, \frac{\partial \bar{\Omega}^1_i}{\partial T_{in}} =
\]
E[D] \hspace{1cm} (2.74)

Which are positive as $E[D] > 0$, therefore as $T_{in}$ increases $\tau^1_i, \bar{\Omega}^1_i$ increase.

2. \[
\frac{\partial \tau^1_i}{\partial L} = 1, \frac{\partial \bar{\Omega}^1_i}{\partial L} =
\]
E[D] \hspace{1cm} (2.75)

Which are positive as $E[D] > 0$, therefore as $L$ increases $\tau^1_i, \bar{\Omega}^1_i$ increase.

3. \[
\frac{\partial \tau^1_i}{\partial l_i} = \frac{1}{2}, \frac{\partial \bar{\Omega}^1_i}{\partial l_i} =
\]
E[D] \hspace{1cm} (2.76)

Which are positive as $E[D] > 0$, therefore as $l_i$ increases $\tau^1_i, \bar{\Omega}^1_i$ increase.

4. \[
\frac{\partial \tau^1_i}{\partial H} = \frac{1}{2}, \frac{\partial \bar{\Omega}^1_i}{\partial H} =
\]
E[D] \hspace{1cm} (2.77)

Which are positive as $E[D] > 0$, therefore as $H$ increases $\tau^1_i, \bar{\Omega}^1_i$ increase.
5. \[
\frac{\partial \tau_4^1}{\partial T} = \frac{m}{pE[D]} - \frac{\beta_{R_i} - \tau_4^1}{2pE[D]} \frac{\partial \Omega_4^1}{\partial T} =
\]
\[
E[D]\left(\frac{m}{pE[D]} - \frac{\beta_{R_i} - \tau_4^1}{2pE[D]}\right) = (2.78)
\]
Which are positive as \(E[D] > 0, p > 0, m > 0, \beta_{R_i} \leq 1\) therefore as \(T\) increases \(\tau_4^1, \Omega_4^1\) increase.

6. \[
\frac{\partial \tau_4^1}{\partial N} = \frac{km}{2E[D]} \frac{\partial \Omega_4^1}{\partial N} =
\]
\[
E[D]\frac{km}{2E[D]} = (2.79)
\]
Which are positive as \(E[D] > 0, k > 0, m > 0\) therefore as \(N\) increases \(\tau_4^1, \Omega_4^1\) increase.

7. \[
\frac{\partial \tau_4^1}{\partial p} = \frac{\partial \Omega_4^1}{\partial p} =
\]
\[
E[D] = (2.80)
\]
Which are positive as \(E[D] > 0\), therefore as \(p\) increases \(\tau_4^1, \Omega_4^1\) increase.

The results of the sensitivity analysis as shown in Proposition 2.12.1 are recorded in Table 1. An increase in the output parameters on increasing the input parameters is denoted by "+" and a decrease in the output parameters on increasing the input parameters is denoted by "-".

Beyond these basic sensitivity results, it is also of interest to determine when sampling strategies are more effective than the strategy of waiting for customer reports of illnesses. To this end, the difference between expected time to detection for the mode of detection through food borne illnesses and detection through sampling is evaluated:
\[
\Delta = E[\tau_4^1] - E[\tau_i^1],\text{ where } j = 1, 2, 3 \text{ for } i = 1 \text{ and } j = 2, 3 \text{ for } i = 2.\text{ The higher this}
\]
difference is, the greater will be the benefit from sampling as compared to waiting for customer reports.

Tables 2 and 3 show the impact of product attributes on the relative benefit from a sampling approach. In these tables, higher values of the product attributes are denoted by the symbol ◆, and lower values are denoted by the symbol ◇. Cases in which no conclusion can be drawn about the values of these product attributes are denoted by “varies” in the tables.

### Table 1: Comparison of product attributes and detection policies

<table>
<thead>
<tr>
<th>Input parameters</th>
<th>Amount of contaminated stock</th>
<th>Time to detection</th>
</tr>
</thead>
<tbody>
<tr>
<td>Incubation time</td>
<td>+</td>
<td>+</td>
</tr>
<tr>
<td>Lead time from supplier to processing center</td>
<td>+</td>
<td>+</td>
</tr>
<tr>
<td>Lead time from processing center to retailer</td>
<td>+</td>
<td>+</td>
</tr>
<tr>
<td>Shelf life</td>
<td>+</td>
<td>+</td>
</tr>
<tr>
<td>Sampling strategy</td>
<td>+</td>
<td>+</td>
</tr>
<tr>
<td>Number of customers reporting a food borne illness</td>
<td>+</td>
<td>+</td>
</tr>
<tr>
<td>Probability of a customer contracting a food borne illness</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>
As Tables 2 and 3 shows, if a product causes a food borne illness with a higher expected incubation time, then the expected time to detection through food borne illnesses will also increase. This is because customers will take longer to show symptoms of the disease and hence will report the illness later as compared to products which cause illnesses with lower expected incubation times.

For example, fresh produce such as tomatoes and spinach cause salmonella related illnesses, which have a relatively shorter incubation time as compared to the incubation time for Campylobacter jejuni caused by pork products (Strohbehn and Beattie 1995).

Table 2: Impact of product attributes on sampling benefit, origin at supplier

<table>
<thead>
<tr>
<th>Product attributes</th>
<th>Supplier</th>
<th>Processing center</th>
<th>Retailer</th>
</tr>
</thead>
<tbody>
<tr>
<td>Expected Incubation time</td>
<td>◆</td>
<td>◆</td>
<td>◆</td>
</tr>
<tr>
<td>Shelf life</td>
<td>◆</td>
<td>◆</td>
<td>◆</td>
</tr>
<tr>
<td>Retailer service level</td>
<td>◇</td>
<td>◇</td>
<td>Varies</td>
</tr>
</tbody>
</table>

Similarly, an increase in shelf life of a product means that a customer may consume a product much later as compared to a product with shorter shelf life, as we assume that a customer consumes a product between the time he/she purchases it and discards it on the expiry of its shelf life. This in turn implies that if a food borne illness is
caused by product with a longer shelf life it will be reported later as compared to one with shorter shelf life. Therefore, ceteris paribus, products with a shorter shelf life, such as fresh produce, tend to have lower expected time to detection through food borne illnesses as compared to products with a longer shelf life such as milk powder, tea and cereal.

Further, in the case of contamination origin at the supplier and sampling at the supplier or processing center, products with lower service levels at the retailer reach the customer in expectation at a later time, because there is the chance of a stock out. Therefore, in case of a contamination event, customer reports of illness will also tend to take a longer time, as compared to products with higher service levels, which are purchased faster.

Table 3: Impact of product attributes on sampling benefit, origin at processing center

<table>
<thead>
<tr>
<th>Sampling at → Product attributes</th>
<th>Supplier</th>
<th>Retailer</th>
</tr>
</thead>
<tbody>
<tr>
<td>Expected Incubation time</td>
<td>◆</td>
<td>◆</td>
</tr>
<tr>
<td>Shelf life</td>
<td>◆</td>
<td>◆</td>
</tr>
<tr>
<td>Retailer service level</td>
<td>◊</td>
<td>Varies</td>
</tr>
<tr>
<td>Expected customer demand rate</td>
<td>◊</td>
<td>Varies</td>
</tr>
</tbody>
</table>
For the case of contamination origin at the processing center and sampling at the processing center, products with a lower expected demand rate will be bought less quickly by customers. Hence, any reports of food borne illness will tend to take longer, compared to products with higher demand rates.

As particular examples of food product types that may exhibit these product characteristics, one can contrast meat products (such as hams, processed meats, and roasts), with fresh vegetable products such as tomatoes.

Using the results from Tables 2 and 3, it can be concluded that meat products such as hams and roasts, which have higher values of incubation time, shelf life and typically lower values of retailer service levels, receive greater benefit from sampling.

Comparatively, products such as tomatoes which have relatively higher values of service levels, and lower shelf lives and incubation times, receive lower benefit from sampling. Processed pork products, for example, typically have a lower service level of approximately 87%-90% (Matsa 2009), a shelf life of a few months (Food Storage and Shelf life Recipetips.com) and can cause food borne illnesses due to the bacteria Campylobacter jejuni which has an average incubation time of 4 days (Strohbehn and Beattie 1995). Products such as tomatoes comparatively have a higher service level of about 96%, shelf life of 2.5 weeks and can cause the food borne illness Salmonellosis which has an average incubation time of 18 hours (Strohbehn and Beattie 1995).

Thus, these product attributes determine that for these meat products a given sampling strategy tends to be more beneficial than waiting for customer reports of food borne illness. If one wants a sampling strategy for products such as tomatoes to be as
successful as a given sampling strategy for hams and roasts, one needs to sample more frequently. Adoption of a strategy of more frequent sampling will, however, lead to a higher sampling cost. Conversely, one can sample less frequently in the case of products such as ham and roasts.

2.13. Summary

This chapter presents a detailed description of the multi-stage food supply chain model and its behavior in the event of a product contamination. The raw material (fruits, vegetables etc.) arrive in batches from the supplier to the processing center from where they are processed into the final product and added to the finished goods inventory. The final product is then sold at the retailers and reaches the customer. In the event of a product contamination occurring along the supply chain, the customer contracts a food borne illness. A G/G/1 queuing process at the processing center and an order up to inventory policy at the retailers is modeled and the safety stock at the processing center is evaluated.

The performance metrics of the model; the amount of contaminated stock (sold and unsold) and time to detection of the contamination event are established. The impact of the origin and mode of discovery contamination (sampling or a food borne illness) on the performance metrics is analyzed. The cost of food product contamination in terms of the performance metrics of the model is evaluated. The minimum annual budget required for a company to implement a quality control process which can eradicate contamination prior to the outbreak of a food borne illness is determined.
In this chapter the impact of product attributes, supply chain structure and properties on the performance metrics in the event of a contamination is also studied. Supply chains with varying number of retailers and different product groups (tomatoes vs. pork products) are compared and contrasted. A sensitivity analysis is also conducted to analyze the effect of input parameters of the model on the time to detection and the amount of contaminated stock.

In the next chapter, a numerical case study is conducted based on the mathematical model presented in this research. Two real world incidents of food product contamination in the tomato and lettuce supply chains leading to food borne illnesses are considered.
3.1. **Real World Food Product Contamination**

In this section two recent high-profile cases of food product contamination and recalls are examined and related numerical results are derived based on the mathematical model presented in Chapter II.

**1. Tomato recall, June 2008:** From mid April 2008 there were a total of 1251 persons infected with Salmonella Saintpaul identified in 43 states, the District of Columbia across the U.S. and Canada. At least 229 of these persons had been hospitalized and there were two outbreak-associated deaths. States reporting illnesses linked to the outbreak included Arizona, California, Colorado, Connecticut, Idaho, Illinois, Indiana, Kansas, New Mexico, Oklahoma, Oregon, Texas, Utah, Virginia, Washington, and Wisconsin (FDA Recalls and Alerts, 2008). Investigations revealed that jalapeno peppers were a major source of contamination and serrano peppers were also a source. Additionally tomatoes were also believed to be a source particularly early in the outbreak.

Jalapeno peppers were traced back to distributors in the United States that received produce grown and packed in Mexico. The outbreak strain was isolated from samples of jalapeno peppers collected in a US warehouse and a patient's home and from samples of Serrano peppers and water collected on a farm in Mexico (CDC 2008).
The FDA recommended that retailers, restaurateurs, and food service operators not offer for sale and service some varieties of tomatoes and also asked consumers to avoid eating fresh jalapeno peppers and food products made with jalapeno peppers during the investigation process (FDA Recalls and Alerts 2008).

The recall process had far-reaching economic impact. The U.S. tomato industry took a $100 million hit as restaurants temporarily dropped tomatoes from their menus, and farmers had to leave crops to rot in packing houses (Associated Press 2008). McDonald’s, Wal-Mart, Burger King, Kroger, Outback Steakhouse, Winn-Dixie and Taco Bell were among the companies that withdrew tomatoes. Restaurants including Taco Bell, KFC, Red Lobster, Olive Garden and Chipotle Mexican Grill Inc.; stopped serving most varieties of tomatoes in 15 states. Grocery stores such as Kroger, Trader Joe’s and Publix also pulled out some varieties of tomatoes from their stores in several states based on the FDA advisory (Johnson 2008).

A study by Flanders (2008) about the economic impact of the outbreak on the state of Georgia's tomato production states that decreased demand for Georgia tomatoes due to the tomato consumption warning resulted in diminished markets for all tomatoes. It estimates the total production values losses in the state at $13.9 million. The report states that the loss of production value had negative impact on the Georgia economy as decreased grower income led to reduced economic activity. It concludes that the decreased output impact totals $11.8 million, and combined with the decline in tomato sales, the total economic output decrease in the Georgia economy is about $25.7 million (Flanders 2008).
2. **Lettuce recall, 2010:** An ongoing case of recall started in May 2010 due to the contamination of lettuce resulting in an E.coli outbreak. To date, there have been 19 cases of E. coli infections in Michigan, Ohio, and New York. These illnesses include 12 individuals who have been hospitalized, and 3 with a potentially life threatening complications. Preliminary investigations trace back the contamination to a farm in Yuma, Arizona (FDA Recalls and Alerts 2010).

3.2. **Data Sources**

Based on the mathematical model, input data is generated for two cases similar to the ones cited above. Input data for the numerical study is taken from various public sources, including USDA, ERS data, U.S. Bureau of Labor Statistics, FDA etc. It is assumed that contamination occurs in a tomato and lettuce supply chain leading to food borne illnesses caused by salmonella and E.coli respectively.

The data for the commercial disappearance of tomatoes and lettuce annually is available for the years 1992-2006. For example, for the year of 2006 it is reported as 5,919,900,000 lbs (USDA/ ERS data 2010) for tomatoes. Commercial disappearance data can be used to proxy national demand of a food product (Blisard et al. 1999), (Outlaw et al. 1994). The data for the total number of food retailers which sell tomatoes and lettuce nationwide such as supermarkets, grocery stores, fruit and vegetable markets is also available through census data (US Census Bureau Economic census). The commercial disappearance data is averaged over the total number of food retailers and statistical data fitting is used to fit it to a normal distribution and obtain the expected
customer demand $E[D]$ and standard deviation of the demand $\sigma[D]$ for both tomatoes and lettuce in a year at a single, “average” retailer. It is also assumed that the expected demand and standard deviation are the same at $r$ retailers in the since we aim to compare the impact of contamination on the supply chain and want to avoid bias towards any of the $r$ retailers due to the amount of customer demand at each of them.

The data on retail stock outs are from the Commodity and Services Survey, which is used by the U.S. Bureau of Labor Statistics (BLS) to compute the consumer price index (CPI). This data is used to estimate the service levels for tomatoes and lettuce at the retailer (Matsa 2009). The stock out rate for tomatoes and lettuce is 3.6% and 1.8% approximately (Matsa 2009). In the numerical study therefore the retailer service level for tomatoes and lettuce is varied from 95% to 99%.

The shelf life of tomatoes $H$ depending on the variety varies from 1 to 5 weeks and the shelf life of lettuce is between 7 to 12 days (Food storage and shelf life, Recipetips.com).

The average range of the incubation period for common food borne illnesses $T_{in}$ is available from medical research data and is as listed in Table 4 (Strohbehn and Beattie 1995). The incubation time for Salmonellosis and E.coli is varied between these ranges from the data available.

The lead times $L$ and $l$ for tomatoes are assumed to be higher than the lead times for lettuce, as the tomatoes are imported to the USA from Mexico as compared to lettuce which is a locally grown product. It is also assumed that the maximum lead time of
either tomatoes or lettuce is less than or equal to either of their shelf lives. The lead times are then varied to obtain consistent results.

Table 4: Incubation time

<table>
<thead>
<tr>
<th>Food borne illness</th>
<th>Range of incubation time</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bacillus cereus</td>
<td>0.5-15 hrs</td>
</tr>
<tr>
<td>Campylobacter jejuni</td>
<td>1-7 days</td>
</tr>
<tr>
<td>Escherichia (E) coli</td>
<td>2-4 days</td>
</tr>
<tr>
<td>Salmonellosis</td>
<td>12-24 hrs</td>
</tr>
<tr>
<td>Clostridium botulinum</td>
<td>12-36 hrs</td>
</tr>
</tbody>
</table>

We vary the values of the individual item service times $E[S]$ for tomato and lettuce so as to hold the server utilization factor at the processing center from 50% to 98%. This is because tomatoes and lettuce are both products commonly purchased products with high demand, it can be assumed that the server stays moderately to highly busy at the processing center during the time period when this produce is in season.

The number of customers having to report a food borne illness before a food borne illness scare can be recorded, $T$ is assumed to vary from 1 to maximum of 1300. In the case of the 2008 tomato contamination, the total number of customers reporting an illness was 1251 (FDA Recalls and Alerts 2008), whereas in the recent case of lettuce the total number of cases are approximately equal to 19 to date (FDA Recalls and Alerts 2010). It can be assumed that the number of food borne illnesses which were reported
before a food borne illness scare could be recorded, is less than the total number of cases which were finally recorded.

The probability $p$ of a customer contracting a food borne illness on consuming a contaminated food product is such that $0 < p \leq 1$. The case of $p = 0$ is not considered as it would imply that the contamination cannot be discovered through a food borne illness.

The data from retailer specifications for baby lettuce and heirloom tomatoes at a particular grower/processing center (Earthbound Farm Organic) is used to evaluate the batch size $k$ (Earthbound farm organic food service products 2010). It is assumed that each batch consists of case(s). The case(s) in turn consists of the raw material which is processed into the final product. Lettuce is shipped in batches consisting of 45 cases. Tomatoes are shipped in batches consisting of 88 cases each (Earthbound Farm Organic Food service Products 2010).

The retailer specifications at Earthbound farm organic are used to evaluate the number of final products each item of raw material is processed into $m$. The batches arriving from the supplier consist of cases of raw material. Each case of lettuce is processed into 1 bag of lettuce weighing 3 lbs. Therefore, $m = 1$ for lettuce. Similarly, each case of tomatoes is processed into a bag of tomatoes weighing 10 lbs. Hence, $m = 1$ for tomatoes (Earthbound farm organic food service products 2010).

The standard deviation of the batch inter arrival time $\sigma_{A_B}$ and the service time $\sigma[S]$ are varied extensively. In all cases the variation is found to have no impact on our results.
The sampling strategy is a policy decision that is defined by the frequency with which a sample is taken. This frequency is described by the parameter $N$ which is defined as the number of batches between successive samples. Thus, a high value of $N$ corresponds to infrequent sampling and a low value corresponds to frequent sampling.

To further analyze the impact of the sampling strategy $N$ on the results we set the values of $p$, $\sigma[A_B]$ and the service time $\sigma[S]$ to fixed, reasonable values as cited in Table 5 and described above. The time to detection and the amount of contaminated stock are varied as functions of $N$ by setting $N = 1, \ldots, 500$. Based on the results it can be concluded that the results are consistent for a wide range of values of $N$ and follow similar trends.

Also a sensitivity analysis of, $\sigma[A_B]$ and $\sigma[S]$ is performed and there are no significant changes in the trends.

The number of retailers $r$ in the multiple retailer supply chain is varied from 1-100 and the corresponding results are analyzed.

Numerical results from the model are generated for the range of input parameter values described above. The results are found to be consistent and follow the same trends over the range of the input values. Table 5 gives the values of a specific set of input parameters used for tomato and lettuce, which is one among the given range of input parameter values.
Table 5: Input parameters for numerical study

<table>
<thead>
<tr>
<th>Input Parameters</th>
<th>Tomato</th>
<th>Lettuce</th>
</tr>
</thead>
<tbody>
<tr>
<td>Processing center service level $\beta_p$</td>
<td>0.9</td>
<td>0.95</td>
</tr>
<tr>
<td>Retailer service level $\beta_{r_l}$</td>
<td>0.95</td>
<td>0.95</td>
</tr>
<tr>
<td>Customer demand rate (lbs/day) $D$</td>
<td>45.83</td>
<td>57.01</td>
</tr>
<tr>
<td>Standard deviation of demand rate (lbs/day) $\sigma[D]$</td>
<td>6.12</td>
<td>7.82</td>
</tr>
<tr>
<td>Batch size $k$</td>
<td>88</td>
<td>45</td>
</tr>
<tr>
<td>Standard deviation of batch inter arrival time $\sigma[A_B]$</td>
<td>0.03</td>
<td>0.033</td>
</tr>
<tr>
<td>Item service time (days) $S$</td>
<td>0.0215</td>
<td>0.01752</td>
</tr>
<tr>
<td>Standard deviation of item service time (days) $\sigma[S]$</td>
<td>0.0015</td>
<td>0.0000365</td>
</tr>
<tr>
<td>Supplier to processing center lead time (days) $L$</td>
<td>2.55</td>
<td>0.405</td>
</tr>
<tr>
<td>Processing center to retailer lead time (days) $l$</td>
<td>1.46</td>
<td>0.365</td>
</tr>
<tr>
<td>Sampling strategy (number of batches) $N$</td>
<td>$1, \cdots, 500$</td>
<td>$1, \cdots, 500$</td>
</tr>
<tr>
<td>Shelf life (days) $H$</td>
<td>8</td>
<td>7.3</td>
</tr>
<tr>
<td>Number of people showing symptoms $T$</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>Incubation time (days) $T_{in}$</td>
<td>0.75</td>
<td>2.99</td>
</tr>
<tr>
<td>Probability of contracting illness $p$</td>
<td>0.65</td>
<td>0.65</td>
</tr>
<tr>
<td>Number of final products each item of raw material is processed into $m$</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Number of retailers $r$</td>
<td>$1, \cdots, 100$</td>
<td>$1, \cdots, 100$</td>
</tr>
</tbody>
</table>
3.3. Validation of Data Impact

The impact that each input parameter has on the output of the model is also studied by varying each individual input parameter numerically. For input parameters such as $E[D], L, l, T_{in}, k, m, \sigma[D], \beta$ which take on a fixed value as stated in the data description above and Table 5, the input parameters are varied by $\pm 50\%$ of the fixed base parameter value. For input parameters such as $S, \sigma[S], \sigma[A_B], T, p, N, r$ which follow a range of values we again vary the parameters by $\pm 50\%$ of their minimum and maximum values wherever viable. The corresponding change in the outputs the time to detection and the amount of contaminated stock is then noted. The impact of the input parameters on the output is classified as high or low in Table 6 depending on the relative change it causes to the output parameters.

It can be seen from Table 6 that numerically a variation in the input parameters of customer demand rate, retailer service level and the number of final products each item of raw material is processed into have the highest impact on the outputs of the model. It can be noted that in our model the customer demand rate, service level and number of final products each item of raw material is processed into have been obtained from real time data.
Table 6: Impact of input parameters on the performance metrics

<table>
<thead>
<tr>
<th>Input parameters</th>
<th>Impact on performance metrics</th>
</tr>
</thead>
<tbody>
<tr>
<td>Retailer service level</td>
<td>High</td>
</tr>
<tr>
<td>Customer demand rate (lbs/yr)</td>
<td>High</td>
</tr>
<tr>
<td>Standard deviation of demand rate</td>
<td>Low</td>
</tr>
<tr>
<td>Batch size</td>
<td>Low</td>
</tr>
<tr>
<td>Standard deviation of batch inter arrival time</td>
<td>Low</td>
</tr>
<tr>
<td>Item service time (yrs)</td>
<td>Low</td>
</tr>
<tr>
<td>Standard deviation of item service time (yrs)</td>
<td>Low</td>
</tr>
<tr>
<td>Supplier to processing center lead time (yrs)</td>
<td>Low</td>
</tr>
<tr>
<td>Processing center to retailer lead time (yrs)</td>
<td>Low</td>
</tr>
<tr>
<td>Sampling strategy (number of batches)</td>
<td>Low</td>
</tr>
<tr>
<td>Shelf life (yrs)</td>
<td>Low</td>
</tr>
<tr>
<td>Number of people showing symptoms</td>
<td>Low</td>
</tr>
<tr>
<td>Incubation time (yrs)</td>
<td>Low</td>
</tr>
<tr>
<td>Probability of contracting illness</td>
<td>Low</td>
</tr>
<tr>
<td>Number of final products each item of raw material is processed into</td>
<td>High</td>
</tr>
<tr>
<td>Number of retailers</td>
<td>Low</td>
</tr>
</tbody>
</table>
3.4. Discussion of Numerical Results

For the case when contamination occurs at the supplier or the processing for both tomatoes and lettuce we observe the following from the numerical results as depicted by the graphs in Figures (5-12). The number of retailers is $r = 10$.

![Figure 5: Tomato: Time to detection (Origin Supplier)](image1)

![Figure 6: Tomato: Amount of contaminated stock (Origin Supplier)](image2)
A sampling strategy implemented closest to the node with the highest probability of contamination is most effective. In the case of origin at the supplier for both tomatoes and lettuce there is a difference of approximately one order of magnitude between the amount of contaminated stock for sampling at the supplier and sampling at the processing center. The corresponding difference between the amount of contaminated stock for sampling at the supplier and sampling at the retailer is almost to the order of two magnitudes. For origin at the processing center, the difference between the amount
of contaminated stock for sampling at the processing center and sampling at the retailer is to the order of one magnitude. This implies that the location of sampling has a very high impact on the performance metrics and, in turn, the severity of the contamination event.

![Figure 9: Lettuce: Time to detection (Origin Supplier)](image)

It can also be seen that for both tomatoes and lettuce sampling at the processing center detects contamination close to the time of detection through a food borne illness. This implies that in most cases it may be barely worth it to sample at the processing center.

It is also observed that sampling at the retailer can only detect the contamination prior to a food borne illness event, if sampling is done at least every \( N = 2 \) batches (or every 1760 lbs) for tomato. Similarly, sampling at the retailer can only detect the contamination prior to a food borne illness event, if sampling is done at least every
N = 3 batches (or every 405 lbs) for lettuce. This result holds irrespective of the origin of the contamination. This implies that for a moderate to less frequent sampling strategy, sampling at the retailer is redundant because detection through a food borne illness occurs much before a successful sampling. These results can be seen from the graphs in
Figures 5-12 where the time to detection and the amount of contaminated stock are plotted as functions of the sampling strategy $N$ for the case of origin at the supplier and processing center for both tomato and lettuce.

![Graph](image)

**Figure 12**: Lettuce: Amount of contaminated stock (Origin Processing Center)

In addition, we can observe that even sampling at the processing center may not be useful for some sampling strategies. In some cases, it may be redundant or barely worth sampling at the processing center. From the graphs in Figures 5-12 it can be seen a sampling strategy at the processing center is only successful if $N < 28$ (sampling every 24640 lbs of tomatoes). If a company cannot afford to sample at this frequency, then it should likely not sample at all, because any sampling strategy that samples less frequently will on average not yield any benefit.

Similarly for lettuce the sampling at the processing center detects contamination sooner than a food borne illness resulting in lesser contaminated stock when $N < 60$. 
batches (or 8100 lbs of lettuce) for both origin at supplier and the processing center. Therefore if a company cannot sample at a frequency \( N < 60 \) then it should not invest in any other sampling strategy.

This result is particularly significant as in the U.S. the legal rule in the event of food contamination is “strict liability” where the seller of the product to the customer involved in more stages of handling the food is held responsible. The seller cannot claim lack of prior knowledge of the contamination or that the incident was not foreseeable (Sumner, Pouliot 2008). Specifically, in the model in this research the processing center will be responsible even in the case where origin of contamination is at the supplier. Therefore the above result where by using a particular sampling strategy the processing center can preempt detection through a food borne illness becomes even more crucial.

On the other hand, when sampling at the supplier, it is possible to obtain positive benefits from a less-frequent sampling strategy: in our example, a sampling strategy of \( N < 32 \) (or every 28160 lbs) results in detection of contamination prior to a food borne illness event for tomato and a sampling strategy of \( N < 73 \) (or every 9855 lbs) results in detection of contamination prior to a food borne illness event for lettuce. This observation again underlines the importance of the choice of location for establishing a useful sampling strategy.

The Figures 13-20 depict the variation in the expected time to detection and the expected amount of contaminated stock decrease as the number of retailers varies for various modes of detection for origin at the supplier and processing center for tomato and lettuce. It can be seen that for detection through sampling at the supplier the
performance metrics remain the same, while for all modes of detection the performance metrics increase as the number of retailers increase for both tomato and lettuce. The sampling strategy is N = 5 for tomatoes and N = 20 for lettuce.

Figure 13: Tomato: Time to detection (Origin Supplier) N=5

Figure 14: Tomato: Amount of contaminated stock (Origin Supplier) N=5
Figure 15: Tomato: Time to detection (Origin Processing Center) N=5

Figure 16: Tomato: Amount of contaminated stock (Origin Processing Center) N=5
Figure 17: Lettuce: Time to detection (Origin Supplier) N=20

Figure 18: Lettuce: Amount of contaminated stock (Origin Supplier) N=20
3.5. **Summary**

In this chapter a numerical case study based on the mathematical model presented in this research is conducted. Two real world contamination events which
occur in a tomato and lettuce supply chain and lead to food borne illnesses and product recall are considered.

The evaluation of the data from diverse public sources is described and the impact of data on the performance metrics is assessed. The performance metrics are evaluated for supply chains with increasing number of retailers and implementing various sampling strategies for both tomato and lettuce.

In the next the concept of product traceability in the supply chain is introduced and discussed in context of the model presented in this research.
CHAPTER IV

TRACEABILITY OF FOOD PRODUCTS IN THE SUPPLY CHAIN

4.1. Traceability Systems

A traceability system enables the tracking and trace back of food products in the supply chain in the event of a product contamination and recall. In order to initiate an effective recall, a traceability system must be able to accurately trace backward and forward in the supply chain in order to find all products affected by the contamination (Jansen-Vullers et al. 2003). Thus, a traceability system may allow for faster and more precise recall of contaminated food products from the supply chain possibly mitigating costs for companies and health risks for consumers.

Traceability systems can comprise of technology such as RFID (Radio Frequency Identification) electronic product tags. RFID is a contactless interrogation method for identification of objects (Gaukler and Seifert 2007). An RFID system essentially consists of three parts: the RFID tag itself, the RFID reader device, and a backend IT system (Gaukler and Seifert 2007). RFID technology can be utilized to track and trace the food product’s origin and when combined with sensor technology can allow for the monitoring of a product’s location within the supply chain (Fritz and Schiefer 2009). For example, RFID tags maybe attached to individual batches or boxes of cereal. RFID tagging can in turn be combined with information systems networks, internet communication and software agents to fully realize the range of tracking and tracing functionalities (Fritz and Schiefer 2009).
There are papers available in literature which discuss in detail the technology utilized in the implementation of RFID based traceability systems in food supply chains (Abad et al. 2007), (Ngai et al. 2008) (Hannus et al. 2003), (Mousavi et al. 2005), (Thompson et al. 2005) and (Jedermann et al. 2006). The exact traceability system implemented in a particular food supply chain will depend on the product type and also on the electronic agreements in place between enterprises (Fritz and Schiefer 2009).

In this research however, we are interested in evaluating the value of implementing any such RFID based traceability system in the supply chain. Quantifying the value of traceability in a given supply chain scenario may help companies in evaluating the necessity and economic benefits of implementing traceability systems.

To this end, we simulate a supply chain model equipped with traceability capabilities in ARENA. We then formulate a performance metric to help assess the impact of the supply chain structure, properties and the levels of traceability implemented on the value of traceability.

4.2. Validation of the Simulation Model

At first we simulate the supply chain model presented in this research in ARENA and validate it by comparing the simulation results with the numerical case study results in Chapter III.

As described in Section 2.1, consider a supply chain consisting of a supplier, processing center and retailer(s). The supplier ships raw material in batches to the processing centers at time, the batches are broken down at the processing centers on
arrival and wait in a queue for processing. On completion of processing the finished 
products are added to the finished goods inventory at the processing center. The 
retailer(s) places orders every period on checking the customer demand. The processing 
center ships finished products to the retailer(s) every period. The customer demand 
follows a normal distribution with known mean and standard deviation.

It is assumed that an ongoing contamination originates at the supplier at time \( t = 0 \) and the mode of detection of the contamination is through the outbreak of a food borne 
illness. As discussed in Section 2.7.4, food recall investigations are initiated when a 
certain number of food borne illness incidents \( T, \; T \geq 1 \), are reported and linked to a 
certain food product.

The supplier start shipping contaminated batches at the start of the simulation 
and the entire subsequent operation of this supply chain model is simulated in ARENA 
(see Appendix) for both the tomato and lettuce case studies described in Chapter III. The 
input data of the model is similar to that utilized in the numerical study and given in 
Table 5, Chapter III. The model is terminated when the specified \( T \) food borne illnesses 
are reported. The simulation length will correspond to the time to detection of the 
contamination. The total amount of contaminated stock (rounded off to the nearest 
decimal) at the time of detection which is in the form of raw material and finished 
products in the supply chain is one of the outputs of the model.

The outputs from simulation for 50 replications are now compared to the results 
of the numerical case study in Chapter III in order to validate the simulation model in
ARENA. The results from the numerical study and the simulation for $T = 100, 500$ and $1000$, for a single retailer $r = 1$. are presented in Tables 7 and 8.

It can be seen that time to detection of the contamination and the amount of contaminated stock from the numerical study and the simulation coincide reasonably well for different values of $T$. Therefore we can consider the simulation of the supply chain model to be valid.

<table>
<thead>
<tr>
<th>Tomato</th>
<th>Numerical study</th>
<th>Simulation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of food borne illness outbreaks (T)</td>
<td>Time to detection (days)</td>
<td>Amount of contaminated stock (lbs)</td>
</tr>
<tr>
<td>100</td>
<td>12.92</td>
<td>593</td>
</tr>
<tr>
<td>500</td>
<td>33.1</td>
<td>1531</td>
</tr>
<tr>
<td>1000</td>
<td>59</td>
<td>2705</td>
</tr>
</tbody>
</table>

### 4.3. Supply Chain Model with Traceability

The supply chain described above is now extended so as to consist of four suppliers $S_1, S_2, S_3, S_4$, two processing centers $P_1, P_2$ and a cluster of retailer(s) $R$ as depicted in Figure 21. It is assumed that all suppliers supply to both the processing centers and both the processing centers supply to all the retailer(s). This assumption is
made so as to avoid bias towards any node in the supply chain due to its mode of operation.

Table 8: Comparison of numerical study and simulation results (Lettuce)

<table>
<thead>
<tr>
<th>Lettuce</th>
<th>Numerical study</th>
<th>Simulation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of food</td>
<td>Time to</td>
<td>Time to</td>
</tr>
<tr>
<td>borne illness</td>
<td>detection</td>
<td>detection</td>
</tr>
<tr>
<td>outbreaks (T)</td>
<td>(days)</td>
<td>(days)</td>
</tr>
<tr>
<td></td>
<td>Amount of</td>
<td>Amount of</td>
</tr>
<tr>
<td></td>
<td>contaminated</td>
<td>contaminated</td>
</tr>
<tr>
<td></td>
<td>stock (lbs)</td>
<td>stock (lbs)</td>
</tr>
<tr>
<td>100</td>
<td>15.65</td>
<td>15.94</td>
</tr>
<tr>
<td></td>
<td>893</td>
<td>945</td>
</tr>
<tr>
<td>500</td>
<td>32.11</td>
<td>30.17</td>
</tr>
<tr>
<td></td>
<td>1831</td>
<td>1685</td>
</tr>
<tr>
<td>1000</td>
<td>44.4</td>
<td>38.2</td>
</tr>
<tr>
<td></td>
<td>2534</td>
<td>2235</td>
</tr>
</tbody>
</table>

It is presumed that the supply chain is equipped with a traceability system capable of imparting information about product origin and location in real time. This traceability information is captured in the simulation by recording as outputs the origin and location of stock at each stage of the supply chain such as the raw material in transit to the processing center from the supplier, batches in queue at the processing center, finished products at the FGI etc.

It is also assumed that an ongoing contamination occurs at one of the suppliers leading to a food borne illness outbreak and recall since we aim to evaluate the value of a traceability system in such a scenario. This scenario will not be possible if there is a
quality control process which eliminates the contamination at the supplier or processing center, as discussed in Section 2.8. We however ignore the implementation of such a quality control process for the time being as this will lead to the contaminated products dispersing further downstream along the supply chain and reaching the customer resulting in a food borne illness outbreak. This will give us a bigger picture of the value of traceability in the supply chain.

Henceforth, this model will be referred to as the “base case” and further modifications will be made to the base case to create different scenarios for comparison. By varying the supply chain structure, properties and also the information generating capabilities of the traceability system we aim to assess whether some scenarios benefit more from a traceability system as compared to others.
4.4. Model Evaluation

A study conducted among meat and poultry companies implementing a traceability system states that the value of traceability for the participants depends on the amount of contaminated stock and the subsequent contamination costs (Buhr 2003). The cost of contamination in turn is a function of the total amount of contaminated stock as discussed in Section 2.9. Therefore the performance metric of the simulation model is the amount of contaminated stock distributed at various stages of the supply chain in the form of raw material and finished products. It is assumed that this performance metric in turn estimates the value of traceability.

The supply chain simulated in ARENA corresponds to the tomato case study described in Chapter III. A Salmonella contamination occurs at a tomato farm leading to a food borne illness outbreak and recall. Analogous to this in the simulation model it is assumed that a salmonella contamination occurs at the supplier \( S_1 \) leading to a food borne illness outbreak. Table 9 gives the input parameters of the simulation base model, these parameters have been evaluated as in the numerical study case in Chapter III.

Initially for the base case, it is assumed that both processing centers fulfill in equal parts of the demand at the retailer(s), i.e. 50% of the total demand at the retailers. The four suppliers also fulfill in equal parts the demand to the processing centers, i.e. 25% of the total demand fulfilled by the processing centers. All other attributes of the suppliers, processing centers and retailer(s) such as the lead time from the supplier to the processing center, batch size etc. are also assumed to be same. These assumptions are
made so as to build an equitable base model against which other models with varying attributes can be compared.

<table>
<thead>
<tr>
<th>Input Parameters</th>
<th>Tomato</th>
</tr>
</thead>
<tbody>
<tr>
<td>Batch size</td>
<td>10</td>
</tr>
<tr>
<td>Total customer demand (lbs/day)</td>
<td>200</td>
</tr>
<tr>
<td>Standard deviation of demand (lbs/day)</td>
<td>5</td>
</tr>
<tr>
<td>Items processed into</td>
<td>1</td>
</tr>
<tr>
<td>Incubation time (days)</td>
<td>0.74</td>
</tr>
<tr>
<td>Shelf life (days)</td>
<td>7</td>
</tr>
<tr>
<td>Lead time: Suppliers to Processing centers (days)</td>
<td>2</td>
</tr>
<tr>
<td>Lead time: Processing centers to Retailer(s) (days)</td>
<td>1</td>
</tr>
<tr>
<td>Service time (days)</td>
<td>0.007</td>
</tr>
<tr>
<td>Standard deviation of service time (days)</td>
<td>0.0005</td>
</tr>
<tr>
<td>Number of customers contracting illness</td>
<td>200</td>
</tr>
<tr>
<td>Probability of contracting illness</td>
<td>0.65</td>
</tr>
<tr>
<td>Demand fulfilled by $P_1P_2$ (lbs/day)</td>
<td>100</td>
</tr>
<tr>
<td>Demand fulfilled by $S_1S_2S_3S_4$ (lbs/day)</td>
<td>25</td>
</tr>
</tbody>
</table>
At the start of the simulation the suppliers start shipping batches of the raw material to the processing centers; this includes the contaminated raw material shipped from supplier \( S_1 \) to the processing centers \( P_1, P_2 \). The amount and location of the contaminated stock originating from the supplier \( S_1 \) at each stage of the supply chain is the output of the simulation, for example the amount of raw material shipped from supplier \( S_1 \) to the processing centers \( P_1, P_2 \), the number of finished products processed by \( P_1, P_2 \) etc. It should be noted that the total contaminated stock can be either be in the form of raw material or finished products in the supply chain.

The simulation is terminated when the number of customers contracting a food borne illness, \( T = 200 \). At the termination, the length of the simulation will estimate the time to detection of the contamination through a food borne illness outbreak. Table 10 gives the outputs of the simulation for the base case for 50 replications. In Table 11 we list the sample mean, half-width of a 95% confidence interval, and both the minimum and maximum of the summary output values across the replications. These measures demonstrate that the outputs show minor variation and are meaningful for further comparisons to be made.

Various scenarios are now simulated similarly and compared to the base case so as to analyze the value of traceability. In this work, we are mainly concerned with the impact of supply chain structure, properties and the level of traceability on the performance metrics and in turn the value of traceability. A snapshot of the simulation model in ARENA is shown in Figure 22. The complete ARENA model is presented in the Appendix.
4.5. **Supply Chain Properties and Structure**

The supply chain properties that are primarily of interest are the demand fulfilled by $S_1$ and the lead time from $S_1$ as they determine the size and location of the contamination origin. The effect of a change in the supply chain structure on the value of traceability is also analyzed by changing the number of processing centers in the supply chain.

![ARENA simulation from supplier to processing center](image_url)

**Figure 22:** ARENA simulation from supplier to processing center

### 4.5.1. Demand

Initially, the demand fulfilled by $S_1$ centers is varied to study the impact of low and high values of demand on traceability. This can help in assessing the effect of the
size of \( S_1 \) on traceability. For example, the demand fulfilled by smaller tomato farms will be lower than that fulfilled by larger sized ones.

It was assumed that the supplier \( S_1 \) fulfills 25% of the total demand at the processing centers in the base case. Now, the demand fulfilled by \( S_1 \) to the processing centers is first decreased to 13% of the total demand for the low demand case and then increased to 52% of the total demand at the processing centers for the high demand case. As in the base case the simulation is terminated when 200 food borne illnesses occur and the length of the simulation will correspond to the time to detection of the contamination. The outputs of the simulation for 50 replications are summarized in Tables 12 and 13. The total demand at the retailer(s), processing centers, lead time and the other parameters of the model are the same as in the base case.

The bar chart in Figure 23 depicts the breakdown of contaminated stock at different stages of the supply chain for the low demand, high demand as well as the base case. For example from Table 10, for the base case the breakdown of the contaminated stock (raw material + finished products) at various stages of the supply chain will be as follows:

1. Raw material in transit from \( S_1 \) to \( P_1 \) or in queue at \( P_1 \), 57 lbs.
2. Raw material in transit from \( S_1 \) to \( P_2 \) or in queue at \( P_2 \), 58 lbs.
3. Finished products originating from \( P_1 \) to be recalled, 393 lbs.
4. Finished products originating from \( P_2 \), 392 lbs.
Table 10: Base case: Distribution of contaminated stock

<table>
<thead>
<tr>
<th>Location of contaminated stock from $S_1$</th>
<th>Amount of contaminated stock</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time to detection = 17.66 days</td>
<td></td>
</tr>
<tr>
<td>Total raw material shipped to $P_1$ and $P_2$ each</td>
<td>450 lbs</td>
</tr>
<tr>
<td>Raw material in transit to $P_1$ and $P_2$ each</td>
<td>50 lbs</td>
</tr>
<tr>
<td>Total raw material at $P_1$ and $P_2$ each</td>
<td>400 lbs</td>
</tr>
<tr>
<td>Raw material in queue at $P_1$ and $P_2$ each</td>
<td>7 lbs, 8 lbs</td>
</tr>
<tr>
<td>Finished products from $P_1$ and $P_2$ each</td>
<td>393 lbs, 392 lbs</td>
</tr>
</tbody>
</table>

Table 11: Statistical measures of the outputs for the base case

<table>
<thead>
<tr>
<th>Outputs</th>
<th>Sample Mean</th>
<th>95% Confidence Interval Half Width</th>
<th>Minimum Output</th>
<th>Maximum Output</th>
</tr>
</thead>
<tbody>
<tr>
<td>Raw material in transit to $P_1$</td>
<td>50 lbs</td>
<td>0.6432</td>
<td>48</td>
<td>54</td>
</tr>
<tr>
<td>Raw material in queue at $P_1$</td>
<td>7 lbs</td>
<td>0.0245</td>
<td>4</td>
<td>10</td>
</tr>
<tr>
<td>Finished products from $P_1$</td>
<td>393 lbs</td>
<td>2.5278</td>
<td>390</td>
<td>400</td>
</tr>
</tbody>
</table>
Therefore the total contaminated stock originating from \( S_1 \) processed by \( P_1 \) is 450 lbs of which 57 lbs is in the form of raw material, 393 lbs is in the form of finished products. Similarly, the total contaminated stock originating from \( S_1 \) processed by \( P_2 \) is 450 lbs of which 58 lbs is in the form of raw material, 392 lbs is in the form of finished products.

It can be seen that that a change in demand has a direct proportional impact on the total amount of contaminated stock. For example, in the high demand case when the demand fulfilled by \( S_1 \) increases by close to two times as compared to the base case (25 lbs to 52 lbs), the total contaminated stock too increases by nearly twice the amount (900 lbs to 1640 lbs). When the demand decreases by close to half (25 lbs to 13 lbs), the total contaminated stock also decreases almost proportionately (900 lbs 560 lbs). This is because as the supplier fulfills higher or lower demand its production of contaminated stock also increases or decreases respectively.

Further, as we analyze the proportion of raw material and finished products in the total contaminated stock for all cases. In the base case the raw material forms approximately 13% of the total contaminated stock and the finished products (originating at \( P_1 \) or \( P_2 \)) form the remaining 87% of the stock. In the case of high demand the proportions remain approximately the same.

However, in the low demand case the raw material forms just 8% of the total contaminated stock whereas the finished products form 92% of the stock. This is because the other model parameters such as processing times, lead times etc. are held the
same but since the demand and subsequent production is low, a greater percentage of the raw material will be processed into finished products.

It was noted that though for the present high demand case the proportions of raw material and finished products are same as in the base case, for a further higher demand the proportion of raw material in the contaminated stock increases and the amount of finished products decreases.

Table 12: Low demand: Distribution of contaminated stock

<table>
<thead>
<tr>
<th>Location of contaminated stock from $S_1$</th>
<th>Amount of contaminated stock</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time to detection = 21.338 days</td>
<td></td>
</tr>
<tr>
<td>Total raw material shipped to $P_1$ and $P_2$ each</td>
<td>280 lbs</td>
</tr>
<tr>
<td>Raw material in transit to $P_1$ and $P_2$ each</td>
<td>20 lbs</td>
</tr>
<tr>
<td>Total raw material at $P_1$ and $P_2$ each</td>
<td>260 lbs</td>
</tr>
<tr>
<td>Raw material in queue at $P_1$ and $P_2$ each</td>
<td>0 lbs</td>
</tr>
<tr>
<td>Finished products from $P_1$ and $P_2$ each</td>
<td>260 lbs</td>
</tr>
</tbody>
</table>

4.5.2 Lead Time

Now the lead time from $S_1$ to the processing centers is varied to study the impact of changing lead time on traceability. This can help in assessing the effect of the location of $S_1$ on traceability. For example, tomato farms located in Mexico supplying to a processing center in the U.S. will have a longer lead time as compared to farms in Texas.
Table 13: High demand: Distribution of contaminated stock

<table>
<thead>
<tr>
<th>Location of contaminated stock from $S_1$</th>
<th>Amount of contaminated stock</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time to detection = 15.76 days</td>
<td></td>
</tr>
<tr>
<td>Total raw material shipped to $P_1$ and $P_2$ each</td>
<td>820</td>
</tr>
<tr>
<td>Raw material in transit to $P_1$ and $P_2$ each</td>
<td>100 lbs</td>
</tr>
<tr>
<td>Total raw material at $P_1$ and $P_2$ each</td>
<td>720 lbs</td>
</tr>
<tr>
<td>Raw material in queue at $P_1$ and $P_2$ each</td>
<td>0 lbs</td>
</tr>
<tr>
<td>Finished products from $P_1$ and $P_2$ each</td>
<td>720 lbs</td>
</tr>
</tbody>
</table>

Figure 23: Distribution of contaminated stock (Changing demand)

The lead time from the supplier to the processing centers is lowered to 1 day and then increased to 4 days as compared to the base case. That is, the lead time is decreased to half of the lead time in the base case and then increased to twice that in the base case.
The total demand fulfilled by all the nodes in the supply chain and the other input parameters are the same as in the base case. The corresponding results of the simulation are recorded in Tables 14 and 15. The bar chart depicting the amount of contaminated stock at different stages of the supply chain for low and high lead times as well as the base case is shown in Figure 24.

Table 14: Low lead time: Distribution of contaminated stock

<table>
<thead>
<tr>
<th>Location of contaminated stock from S_1</th>
<th>Amount of contaminated stock</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Time to detection = 16.922 days</td>
</tr>
<tr>
<td>Total raw material shipped to P_1 and P_2 each</td>
<td>430 lbs</td>
</tr>
<tr>
<td>Raw material in transit to P_1 and P_2 each</td>
<td>20 lbs</td>
</tr>
<tr>
<td>Total raw material at P_1 and P_2 each</td>
<td>410 lbs</td>
</tr>
<tr>
<td>Raw material in queue at P_1 and P_2 each</td>
<td>4 lbs, 5 lbs</td>
</tr>
<tr>
<td>Finished products from P_1 and P_2 each</td>
<td>406 lbs, 405 lbs</td>
</tr>
</tbody>
</table>

For example from Table 14, for the case of low lead time the breakdown of the contaminated stock (raw material + finished products) at various stages of the supply chain as shown in the bar chart will be as follows:

1. Raw material in transit from S_1 to P_1 and in queue at P_1, 24 lbs.
2. Raw material in transit from S_1 to P_2 and in queue at P_2, 25 lbs.
3. Finished products originating from P_1, 406 lbs.
4. Finished products originating from P_2, 405 lbs.
It can be seen from Table 14 that for the case of low lead time the total contaminated stock originating from \( S_1 \) processed by \( P_1 \) is 430 lbs of which 24 lbs is in the form of raw material and 406 lbs as finished products. Similarly, the total contaminated stock originating from \( S_1 \) processed by \( P_2 \) is 430 lbs of which 25 lbs is in the form of raw material and 405 lbs as finished products.

The breakdown of the contaminated stock at different stages of the supply chain as shown in the bar chart in Figure 24 for high lead time can also be computed and accounted for as above.

It can be seen that as the lead time from \( S_1 \) to the processing centers increases (2 days to 4 days) or decreases (2 days to 1 day) as compared to the base case the total contaminated stock produced also increases (900 lbs to 960 lbs) or decreases (900 lbs to 860 lbs) respectively.

This is because as the lead time from the origin of contamination, \( S_1 \) to the processing centers increases the time taken for the contaminated products to reach the customer and a food borne illness outbreak to be reported also increases.

Hence the time to detection increases as can be seen from Tables 10, 14 and 15; however, as the demand rate fulfilled by the supplier during this period remains the same as in the base case, greater amount of contaminated stock will be produced when the lead time is longer. Conversely, lesser amount of contaminated stock will be produced for a decrease in lead time. It can also be noted that the increase and decrease in contaminated stock for changing lead times is not as significant as that for changing demand.
Table 15: High lead time: Distribution of contaminated stock

<table>
<thead>
<tr>
<th>Location of contaminated stock from $S_1$</th>
<th>Amount of contaminated stock</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total raw material shipped to $P_1$ and $P_2$ each</td>
<td>480 lbs</td>
</tr>
<tr>
<td>Raw material in transit to $P_1$ and $P_2$ each</td>
<td>100 lbs</td>
</tr>
<tr>
<td>Total raw material at $P_1$ and $P_2$ each</td>
<td>380 lbs</td>
</tr>
<tr>
<td>Raw material in queue at $P_1$ and $P_2$ each</td>
<td>0 lbs</td>
</tr>
<tr>
<td>Finished products from $P_1$ and $P_2$ each</td>
<td>380 lbs</td>
</tr>
</tbody>
</table>

Further as we analyze the proportion of raw material and finished products in the total contaminated stock in the low lead time case the raw material forms just 6% of the total contaminated stock whereas the finished products form 94% of the stock. This is because as lead time decreases lesser amount of raw material will be in transit to the processing center as compared to the high lead time case and a greater amount will be processed into finished products. For the case of high lead time the raw material will form 21% of the total contaminated stock while the finished products form 79% of the stock.

Therefore it can be seen that though the total amount of contaminated stocks are not significantly different for changing lead times, the distribution of the stock as raw material upstream or finished products downstream differs significantly. Hence, for case of smaller lead times from the supplier to the processing center a traceability system downstream is more important in order to track the contaminated stock present in the
form of finished products at the processing center FGI, in transit to the retailer, on the retailer shelf or with the customer. Whereas for longer lead times from the supplier to the processing center a traceability system upstream to track the raw material at the supplier, in transit to the processing center or in queue at the processing center is important.

4.5.3. Number of Processing Centers

Now the impact of a change in the supply chain structure on the value of traceability is analyzed. Initially, the number of processing centers is decreased to 1 processing center $P_1$ which fulfills the total demand at the retailer(s). Then the number of processing centers is increased to 4 by incorporating two additional processing centers.
$P_3$ and $P_4$ to the supply chain. The processing centers $P_3$ and $P_4$ are modeled similar to the processing centers $P_1$ and $P_2$, with each processing center fulfilling a demand of 50 lbs/day. All the other parameters are as in the base case.

The results of the simulation are summarized in Tables 16 and 17. The bar chart in Figure 25 depicts the amount of contaminated stock at different stages of the supply chain for low and high number of processing centers as well as the base case.

For example from Table 17, for the case of high number of processing centers the breakdown of the contaminated stock (raw material + finished products) at various stages of the supply chain as shown in the bar chart will be as follows:

1. Raw material in transit from $S_1$ to $P_1$, $P_2$, $P_3$, $P_4$, 30 lbs.
2. Finished products originating from $P_1$, $P_2$, $P_3$, $P_4$, 200 lbs.

Therefore for the case of high number of processing centers the total contaminated stock originating from $S_1$ processed by $P_1$ is 230 lbs of which 30 lbs is in the form of raw material and 200 lbs in the form of finished products. The distribution of the total contaminated stock originating from $S_1$ and processed by $P_2$, $P_3$, $P_4$ is also similar.

The breakdown of the contaminated stock at different stages of the supply chain as shown in the bar chart in Figure 25 for low number of processing centers can also be evaluated and accounted for as above from Table 16. It can be seen that as the number of processing centers increase (2 to 4) or decrease (2 to 1) the amount of contaminated stock also increases (900 lbs to 920 lbs) or decreases (900 lbs to 890 lbs) marginally.
Table 16: Low number of processing centers: Distribution of contaminated stock

<table>
<thead>
<tr>
<th>Location of contaminated stock from $S_1$</th>
<th>Amount of contaminated stock</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time to detection = 17.62 days</td>
<td></td>
</tr>
<tr>
<td>Total raw material shipped to $P_1$</td>
<td>890 lbs</td>
</tr>
<tr>
<td>Raw material in transit to $P_1$</td>
<td>100 lbs</td>
</tr>
<tr>
<td>Total raw material at $P_1$</td>
<td>790 lbs</td>
</tr>
<tr>
<td>Raw material in queue at $P_1$</td>
<td>54 lbs</td>
</tr>
<tr>
<td>Finished products from $P_1$</td>
<td>736 lbs</td>
</tr>
</tbody>
</table>

Table 17: High number of processing centers: Distribution of contaminated stock

<table>
<thead>
<tr>
<th>Location of contaminated stock from $S_1$</th>
<th>Amount of contaminated stock</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time to detection = 17.76 days</td>
<td></td>
</tr>
<tr>
<td>Total raw material shipped to $P_1$, $P_2$, $P_3$, $P_4$ each</td>
<td>230 lbs</td>
</tr>
<tr>
<td>Raw material in transit to $P_1$, $P_2$, $P_3$, $P_4$ each</td>
<td>30 lbs</td>
</tr>
<tr>
<td>Total raw material at $P_1$, $P_2$, $P_3$, $P_4$ each</td>
<td>200 lbs</td>
</tr>
<tr>
<td>Raw material in queue at $P_1$, $P_2$, $P_3$, $P_4$ each</td>
<td>0 lbs</td>
</tr>
<tr>
<td>Finished products from $P_1$, $P_2$, $P_3$, $P_4$ each</td>
<td>200 lbs</td>
</tr>
</tbody>
</table>

However, of greater significance is the fact that the dispersion of the contaminated stock in the supply chain varies as the number of processing centers change. Since, the contaminated stock will transit through greater or lesser number of processing centers as the number of processing centers to which $S_1$ supplies varies. The
dispersion of the product is an important factor in determining the value of traceability as it has a high impact on the scale of track and trace operations to be conducted in the event of a recall (Buhr 2003).

Table 18: Dispersion factor

<table>
<thead>
<tr>
<th>Scenario</th>
<th>Number of forms of the contaminated stock (f)</th>
<th>Number of processing centers (p)</th>
<th>Dispersion factor ((\rho))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Base case</td>
<td>2</td>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>Decreasing processing centers</td>
<td>2</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>Increasing processing centers</td>
<td>2</td>
<td>4</td>
<td>8</td>
</tr>
</tbody>
</table>

To estimate the extent of dispersion in our model due to the change in the number of processing centers we define a factor of dispersion \(\rho\). The factor of dispersion \(\rho\) is defined as:

\[
\rho = p \times f
\]

(4.1)

where \(p\) is the number of processing centers in the supply chain where the contaminated stock is processed and \(f\) is the number of different forms of contaminated stock (raw material, finished products) that has to be tracked and traced at the processing centers. This is because we need to track and trace contaminated stock at all the processing
centers in the form of raw material as well as finished products, i.e.; the raw material arriving from $S_1$ to the processing centers and the finished products departing from all processing centers. The value of $\rho$ for different scenarios is summarized in Table 18.

As the value of the dispersion factor increases the stock is more widely dispersed. The dispersion factor represents the location as well as the type of contaminated stock in which has to be tracked and traced in the event of a recall. The corresponding dispersion of stock in the supply chain can also be seen visually in the bar chart in Figure 25.

4.6. Levels of Traceability

The traceability system in totality is the combination of information on tags on the product and a real time sensor or software system which tracks and traces the product. The level of traceability refers to how far back and the extent to which the traceability system is capable of tracking and tracing products in the supply chain. The level of traceability will be partly determined by the information available from the product regarding its origin and the nodes it has transited through in the supply chain. Therefore partial information will translate into a partial level of traceability and complete information can lead to complete traceability.
Figure 25: Distribution of contaminated stock (Changing number of processing centers)

Analytically for our supply chain model, this information can be considered to be in the form of an information set \(\{S_i, P_j, R_k\}\) attached to each finished product where the elements of the set are defined as:

- \(S_i\): Supplier(s) from where the raw material in the product originated, \(i = 1, \cdots, 4\).
- \(P_j\): Processing center(s) where the product was processed, \(j = 1,2\).
- \(R_k\): Retailer where the product is sold, \(k = 1\).

Therefore if the information set attached to the product is of the form \(\{S_1, P_2, R_1\}\) it indicates that the raw material in the product originated from supplier \(S_1\), the product was processed at processing centers \(P_2\) and sold at retailer \(R_1\). Here, we approximate the cluster of retailers as a single retailer where all products are sold as we are interested in analyzing the impact of information about the suppliers which includes the node of origin of contamination on the value of traceability.
The impact of the level of traceability on the amount of contaminated stock is now analyzed by considering cases where there is partial and no information to track and trace the supply chain in the event of a product contamination and recall.

4.6.1. Complete Supply Chain Information

We assume that for the base case complete information is available on each product about the supplier, processing center and the retailer that the product has transited through.

This implies that for the base case the information set attached to each contaminated product will be of the form \{S_1, P_1, R_1\} or \{S_1, P_2, R_1\} indicating that the raw material in the product originated from supplier \(S_1\), the product was processed at processing centers \(P_1\) and sold at retailer \(R_1\) or the raw material in the product originated from supplier \(S_1\), the product was processed at processing centers \(P_2\) and sold at retailer \(R_1\). This information set provides complete information about the origin and the nodes that the contaminated product has transited through in the supply chain. This information can then be utilized to trace and track the contaminated stock in the supply chain.

The following scenarios of partial and no information are now compared to the base case. The other input parameters remain the same as in the base case.

4.6.2. Partial Supplier Information

It is assumed that the traceability system is capable of imparting information that states that the raw material in the product has originated from suppliers \(S_1\) and \(S_2\) but is
not capable of giving further information about which specific supplier or farm the raw material in the contaminated product originates from. This can correspond to a scenario where there might be information about tomatoes originating from farms in California but there is no information about which specific farm in California they belong to.

The information set attached to the contaminated product will be of the form \(\{s_1, s_2, p_1, r_1\}\) or \(\{s_1, s_2, p_2, r_1\}\) indicating that the raw material in the product originated from supplier \(s_1\) and \(s_2\), was processed at processing centers \(p_1\) and sold at retailer \(r_1\) or that the raw material in the product originated from supplier \(s_1\) and \(s_2\), was processed at processing centers \(p_2\) and sold at retailer \(r_1\).

Due to the availability of only partial information in this case the search space for all recall operations will include potentially contaminated stock from suppliers \(s_1\) as well as \(s_2\) thus widening the search space. This implies that on detection of the contamination as the information set attached to the contaminated product suggests that the raw material in the product belongs to \(s_1\) and \(s_2\) we will be forced to consider stock originating from suppliers \(s_1\) as well as \(s_2\) as contaminated and this total amount is referred to as the “potentially contaminated stock”.

The “potentially contaminated stock” represents an error in evaluating the contaminated stock due to the lack of complete information about the suppliers. The contaminated stock in this case will actually originate only from supplier \(s_1\) as in the base case and the “actual contaminated stock” is the same as the amount of contaminated stock in the base case. It can be assumed that the “potentially contaminated stock” will be used as an indicator for initiating initial recall operations. The simulation in this case
evaluates the “potentially contaminated stock” by considering stock originating from both $S_1$ and $S_2$ as contaminated.

Table 19: Partial supplier information: Distribution of contaminated stock

<table>
<thead>
<tr>
<th>Location of stock from $S_1$ and $S_2$</th>
<th>Potentially contaminated stock</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Time to detection = 17.69 days</td>
</tr>
<tr>
<td>Total raw material shipped to $P_1$ and $P_2$ each</td>
<td>900 lbs</td>
</tr>
<tr>
<td>Raw material in transit to $P_1$ and $P_2$ each</td>
<td>100 lbs</td>
</tr>
<tr>
<td>Total raw material at $P_1$ and $P_2$ each</td>
<td>800 lbs</td>
</tr>
<tr>
<td>Raw material in queue at $P_1$ and $P_2$ each</td>
<td>13 lbs, 15 lbs</td>
</tr>
<tr>
<td>Finished products from $P_1$ and $P_2$ each</td>
<td>787 lbs, 785 lbs</td>
</tr>
</tbody>
</table>

The results from the simulation of this scenario are summarized in Table 19. The breakdown of the “potentially contaminated stock” (raw material + finished products) at various stages of the supply chain as shown in the bar chart in Figure 26 will be as follows:

1. Raw material in transit from $S_1$ or $S_2$ to $P_1$ and in queue at $P_1$, 113 lbs.
2. Raw material in transit from $S_1$ or $S_2$ to $P_2$ and in queue at $P_2$, 115 lbs.
3. Finished products originating from $P_1$, 787 lbs.
4. Finished products originating from $P_2$, 785 lbs.

The total “potential amount of contaminated stock” from $S_1$ and $S_2$ to $P_1$ is 900 lbs, of which 113 lbs is in the form of raw material and 787 lbs is in the form of finished
products from P1. Similarly, the total “potential amount of contaminated stock” from S1 and S2 to P2 is 900 lbs, of which 115 lbs is in the form of raw material and 785 lbs is in the form of finished products from P2.

It can be noted that the total “potentially contaminated stock” (1800 lbs) in for the partial supplier information case is twice the amount of contaminated stock in the base case (900 lbs). This is because stock originating from both S1 and S2 is considered potentially contaminated” instead of the “actual contaminated stock” from just S1.

4.6.3. No Supplier Information

In this case it is assumed that the traceability system is capable of imparting information about the processing centers that the product has been processed at and the retailer it has been sold at but there is no further information regarding the supplier. Such a scenario is possible when suppliers are located overseas and it may be difficult to implement traceability systems at those locations, for example: tomatoes grown on farms in Mexico.

Table 20: No supplier information: Distribution of contaminated stock

<table>
<thead>
<tr>
<th>Location of products from processing centers P1 and P2</th>
<th>Potentially contaminated stock</th>
</tr>
</thead>
<tbody>
<tr>
<td>Finished products from P1 and P2 each</td>
<td>1572 lbs</td>
</tr>
<tr>
<td>Time to detection = 17.69 days</td>
<td></td>
</tr>
</tbody>
</table>
The information set attached to the contaminated product will be of the form \( \{P_1, R_1\} \) or \( \{P_2, R_1\} \). Due to the availability of no information about suppliers in this case the search space for recall operations will include all products processed at the processing centers \( P_1 \) and \( P_2 \) irrespective of the supplier they have originated from, thus widening the search space further.

This implies that on detection of the contamination we will be forced to consider products containing raw material originating from all the suppliers \( S_1, S_2, S_3 \) and \( S_4 \) as contaminated and this total amount is referred to as the “potentially contaminated stock”. The “potentially contaminated stock” represents an error in evaluation of the contaminated stock due to the lack of complete information.

The contaminated stock in this case too will actually originate only from supplier \( S_1 \) as in the base case and the “actual contaminated stock” is the same as the amount of contaminated stock in the base case. The results for this case are given in Table 20. The breakdown of the “potentially contaminated stock” in the form of finished products at various stages of the supply chain as shown in the bar chart will be as follows:

1. Finished products originating from \( P_1 \), 1572 lbs.
2. Finished products originating from \( P_2 \), 1572 lbs.

The “potential amount of contaminated products” from \( P_1 \) and \( P_2 \) each is 1572 lbs. It can be noted that the total “potentially contaminated products” (3144 lbs) in this case is approximately four times the number of finished contaminated products in the base case (3140 lbs). It can also be seen that due to the lack of complete information
regarding the suppliers, the amount of contaminated raw material cannot be estimated and the “potentially contaminated stock” just accounts for the finished products from the processing centers.

The bar chart in Figure 26 depicts the breakdown of the “potentially contaminated stock”. The proportion of the raw material and the finished products to the total contaminated stock is the same for the base case and the case of partial supplier information. This is because the operation of the supply chain and other parameters are the same in both cases and the only change is due to the “overestimation” of contaminated stock due to lack of complete information.

It can be seen that the “potentially contaminated stock” is greater in the case of no information about suppliers as compared to the case of partial information about suppliers. This is because as discussed earlier, in the partial information about suppliers case the contaminated stock is presumed to originate from both suppliers $S_1$ and $S_2$, whereas in the no information case stock from all the suppliers $S_1$, $S_2$, $S_3$ and $S_4$ is assumed to be contaminated.

4.7. Summary

In this chapter, the implementation of traceability systems to track and trace products in the event of a food product recall is analyzed. An ARENA based simulation model is built which incorporates traceability into the supply chain model presented earlier in this dissertation.
Various scenarios are modeled to analyze the value of traceability in the supply chain context. Supply chain properties, structure and levels of traceability are varied to evaluate the amount of contaminated stock in different forms at various locations in the supply chain. The next chapter provides a summary and discusses the conclusions and future research.

Figure 26: Distribution of contaminated stock (Changing information)
CHAPTER V

SUMMARY, CONCLUSION AND FUTURE RESEARCH

5.1. Summary

In this research the performance of a food supply chain model consisting of a supplier, processing center and retailer(s) in the event of a contamination leading to a food borne illness has been studied. Multi-stage supply chain models with a single retailer and multiple retailers are developed and compared. The impact of product contamination on the models is analyzed mathematically. The negative dependence of contamination on the origin and mode of detection of the contamination is quantified. A numerical study of the impact of a real-world contamination event in a tomato and lettuce supply chain is also conducted.

It is assumed that contamination can occur at the supplier or processing center. The mode of detection of contamination is through sampling at the supplier, processing center or the retailer(s). In addition, the contamination may also be discovered through the outbreak of a food borne illness.

In this model, the performance metrics of the model which are the expected amount of contaminated stock, expected time to detection of the contamination event and the subsequent costs of contamination for the different origins and modes of discovery of contamination have been quantified. The differences in individual food product attributes which can impact the cost of contamination are analyzed. The impact
of supply chain structure and properties and detection policies on the severity of potential contamination cases is also studied.

The cost effective sampling strategies which can be adopted by the players in the supply chain are derived. The payoff from the implementation of a quality control process which can eradicate contamination is evaluated.

Further, a traceability system capable of tracking and tracing back products in the event of a food product recall is incorporated in the supply chain model. The value of traceability for different supply chain scenarios is assessed through the implementation of ARENA based simulation model.

5.2. Conclusion

The research questions posed in Chapter I are now revisited and the findings of this research are discussed in the context of these questions.

The first research question was to determine the impact of contamination origin and discovery mode on the severity of the event.

The model in this research shows that the shows that regardless of how often one chooses to perform sampling the performance metrics are worse when sampling at the supplier (processing center) for the case of origin at the supplier (processing center). Our model shows that the closer one samples to the origin of contamination, the better the performance metrics will be. This implies that a sampling strategy implemented closest to the node with the highest probability of contamination is most effective. In fact, this reinforces a widely used principle which states that controls for high risk processes in
the food sector should be placed at the stage with a high failure probability (Bertolini et al. 2007). Hence, the location of sampling is more important than the sampling regimen adopted by companies or government agencies. Therefore an understanding of where and with what probability contamination originates, is of prime importance to the decision maker.

Ideally, a company will want to sample less frequently to keep sampling costs at a minimum. Proposition 2.7.4.2 shows the least frequent sampling strategy that a company can adopt to keep sampling costs at a minimum and preempt detection by customer reports of food borne illnesses.

In Proposition 2.7.4.3 it is demonstrated that for global supply chains and under realistic sampling strategies at the processing center or the retailer, the impact of contamination is most significant when the origin is at the supplier. These conditions are seen to hold for tomatoes in the numerical study.

In addition, the numerical study for tomato and lettuces also validate these analytical results. The numerical study shows that the choice of where to sample has a very significant impact on the overall cost of contamination. For example, in the case where the contamination origin is at the supplier, sampling at the processing center leads to an approximately ten times higher amount of contaminated stock, compared to sampling at the supplier.

It is also observed in the numerical study for all cases that sampling at the retailer detects contamination much later than detection through a food borne illness except when using an extremely stringent sampling strategy. This implies that sampling at the
retailer is redundant in a number of cases as detection through a food borne illness occurs much before a successful sampling event. Sampling at the processing center detects contamination only quicker than food borne illness reports when using a moderate sampling strategy.

Even though sampling at the supplier is the best option to detect contamination and minimize losses, in cases where the supplier/grower is located overseas (as in the tomato case study) it may be difficult to implement and monitor safety procedures. In such a scenario the company has to concentrate their efforts on sampling at the processing center (presumably located within the U.S.). For such a scenario, we derive the window of sampling frequencies that allow sampling to preempt discovery through food borne illness. A sampling strategy outside the specified window is not worth implementing as on average it will not yield any benefit.

If monitoring the supplier is possible, then there is more leeway and sampling can be done with lower frequency. We believe that these results provide useful decision support by aiding a company in making a decision about their safety efforts through sampling depending on their supply chain structure and the resources they have available.

The implementation of a quality control process which eradicates the contamination is also considered. In Proposition 2.7.5.1 we show that if a company cannot dedicate at least a certain budget to a quality control regimen, then it should not employ quality control at all. This is of particular managerial interest, because it shows that the benefits from quality control (in terms of avoiding a food borne illness scare) are
decidedly not linear in the effort and money spent on quality control, rather there is a
threshold of quality control effort below which there is no impact at all.

The second research question was to study the effect of product properties,
supply chain structure and attributes on the severity of a contamination event.

A supply chain with multiple retailers is modeled and compared to the single
retailer supply chain. It is found that the performance metrics are the same for mode of
detection through sampling at the supplier irrespective of the number of retailers in the
supply chain. However, the performance metrics worsen as the number of retailers
increase for all other modes of detection. Therefore, the time required to close the recall
case, which will depend on the total amount of contaminated stock as well as the
location and spread of the stock, will be smaller as the number of retailers in the supply
chain decreases. An identical sampling strategy implemented at the processing center or
retailer will result in contamination being detected faster as the number of retailers in the
supply chain decreases.

The results of a sensitivity analysis are presented in Proposition 2.12.1 to study
the impact of supply chain properties, product attributes and detection policies on the
overall cost of contamination. The analysis shows that a decrease in lead time(s) results
in better performance metrics. Therefore the impact of a food contamination event will
be higher in the case of products with a global supply chain with longer lead times.

It is also shown that an increase in the shelf life of the product results in a
worsening of the performance metrics. In addition, the performance metrics worsen with
an increase in the incubation time. Therefore, illnesses with longer incubation times such
as E. coli will result, ceteris paribus, in worse performance metrics as compared to an illness with lower incubation time such as bacillus.

It is also seen in the numerical study that a variation in the input parameters of customer demand rate, retailer service level, and the number of final products each item of raw material is processed into, have the highest impact on the outputs of the model.

Two products with different product attributes are contrasted and their performance in the case of a contamination event is analyzed. It is shown that for a given sampling strategy, the benefit from sampling vs. waiting for customer reports is higher for products that exhibit high values of expected incubation time of associated diseases, high shelf life and low retailer service levels. An example of this type of product is pork products such as ham and roasts. It is concluded that one can sample less frequently in the case of products like hams and roasts as compared to, e.g., tomatoes, which have lower shelf life, lower expected incubation times, and typically higher retail service levels. If a company wants a sampling strategy to be as successful for tomatoes as a given sampling strategy for pork products, then they need to sample more frequently, at higher cost.

The final research question was to assess the value of tracking and tracing products in the event of a food product contamination and recall.

To this end, traceability information is incorporated in the supply chain model followed by a simulation based study in ARENA in order to analyze the value of a traceability system for various scenarios. The impact of supply chain properties and the level of traceability implemented on the value of traceability is assessed.
Initially, the supply chain simulation is validated by comparing the numerical study results to the simulation results which coincide reasonably well. It is found that a change in demand has a direct proportional impact on the total amount of contaminated stock which is assumed to be an indicator of the value of traceability. This is because as the supplier fulfills higher or lower demand its production of contaminated stock also increases or decreases respectively.

Further, we analyze the proportion of raw material and finished products in the total contaminated stock for all cases. It is observed that finished products form a greater percentage of the total contaminated stock as compared to raw material for the low demand case. This is because the other model parameters such as processing times, lead times etc. are held the same but since the demand and subsequent production is low; a greater percentage of the raw material is processed into finished products.

In the case of high demand the proportions remain approximately the same as in the base case. It was noted that though for the present high demand case the proportions of raw material and finished products are same as in the base case, for a further higher demand the proportion of raw material in the contaminated stock increases and the amount of finished products decreases.

For the changing lead time scenario it was be seen that as the lead time increases or decreases as compared to the base case the total contaminated stock produced also increases or decreases respectively.

This is because as the lead time from the supplier to the processing centers increases the time taken for the contaminated products to reach the customer and a food
borne illness outbreak to be reported also increases. Hence the time to detection increases however, as the demand rate fulfilled by the supplier during this period remains the same as in the base case, greater amount of contaminated stock is produced when the lead time is longer. Conversely, lesser amount of contaminated stock is produced for a decrease in lead time.

It is also noted that the increase and decrease in contaminated stock for changing lead times is not as significant as that for changing demand. Further the proportion of raw material is lower as compared to the proportion of finished products in the total contaminated stock for the low lead time case. This is because as lead time decreases lesser amount of raw material is in transit to the processing center as compared to the high lead time case and a greater amount is processed into finished products. For the case of high lead time the raw material forms a greater proportion of the stock.

Therefore it is seen that though the total amount of contaminated stocks are not significantly different for changing lead times, the distribution of the stock as raw material upstream or finished products downstream differs significantly. Hence, for the case of smaller lead times from the supplier to the processing center a traceability system downstream is more important, whereas for longer lead times from the supplier to the processing center a traceability system upstream is important.

The impact of a change in the supply chain structure is studied by varying the number of processing centers in the supply chain. It is seen that as the number of processing centers increase or decrease the amount of contaminated stock also increases or decreases marginally.
However, of greater significance is the fact that the dispersion of the contaminated stock in the supply chain varies as the number of processing centers change. The dispersion of the product is an important factor in determining the value of traceability. A dispersion factor is formulated and quantified for varying number of processing centers. The dispersion factor increases as the number of processing centers increase in the supply chain.

The impact of different levels of traceability is also studied by considering scenarios with partial and no supplier information. It is found that the lack of complete information leads to a widening of the search space for recall. The potential amount of contaminated stock in the partial information case is twice that in the base case. Whereas the potential amount of contaminated stock in the no information case is four times that in the base case.

By providing a quantitative model of the impact of contamination in a supply chain, our research addresses a crucial gap in the existing literature. This research is expected to aid in mitigating risk due to product contamination in the global food supply chain. This research can also help in increasing food safety and quality control, which in turn can minimize the potential for bad publicity, liability and recalls for companies. The present analysis can assist companies in developing criterion for better selection of supply chain partners globally. We hope that our results will help companies better understand the risk profile of different stages of the food supply chain, and to be able to compare the relative merit of preventive quality control.
Overall, companies can benefit monetarily by using this information to improve supply-chain efficiency, build lower cost distribution networks and create product differentiation. The high risk of economic losses associated with food product contamination and recall may compel companies to delay product recall. Delaying product recall may however lead to higher costs especially in the case of large scale food contamination leading to a food borne illness. Quantifying the scope of the contamination event, the amount of contaminated stock and the time to detection for various scenarios as above can help companies make more informed decisions in the case of a contamination event. The assessment of the value of traceability for different scenarios can assist companies make more informed decisions when considering the implementation of traceability systems.

5.3. Future Research

For further research we can develop testing and trace back criteria based on our model using statistical hypothesis for more effective trace back and recall in the event of contamination. This will result in guidelines for better placement of food quality control checks for preventing high costs of contamination.

This research can be extended to study information sharing between supply chain partners for better inventory management in the event of a contamination to minimize the repercussions of a recall. Alternate routing and ordering policies in the case of a food contamination event can also be analyzed.
In real world food supply chains a 1:1 correspondence between the initial raw material and the final product may not exist. For example, spinach from several farms may be combined, sorted and then packaged into individual products at a processing center. Commingling complicates traceability and recall operations in the event of a contamination. The present model can be modified to study the effect of commingling on the impact of contamination and the value of traceability.

The prevalence of counterfeit drugs and sale of mispriced drugs by grey market sellers has been a growing cause for concern in the pharmaceutical supply chain. A study can also be undertaken to adopt the modeling approach in this research to model the use of RFID based technology in the pharmaceutical supply chain to track and trace counterfeit and mispriced drugs.
REFERENCES


World Health Organization. 2008. Melamine-contamination event: Expert meeting to review toxicological aspects of melamine and cyanuric acid. Held in


APPENDIX

SIMULATION OF THE SUPPLY CHAIN MODEL IN ARENA

Below is the supply chain model with traceability as simulated in ARENA depicted in different stages for a single supplier, processing center and retailer. The multiple nodes in the supply chain are modeled similarly.

Figure 27: ARENA model, supplier to processing center
Figure 28: ARENA model, processing center to retailer

Figure 29: ARENA model, retailer to customer
VITA

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