DESIGN OF A WIDE BANDWIDTH CONTINUOUS-TIME LOW-PASS
SIGMA-DELTA MODULATOR

A Thesis
by
CHENG-MING CHIEN

Submitted to the Office of Graduate Studies of
Texas A&M University
in partial fulfillment of the requirements for the degree of

MASTER OF SCIENCE

December 2011

Major Subject: Electrical Engineering
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Approved by:

Co-Chairs of Committee, Aydin I. Karsilayan
Jose Silva-Martinez

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Duncan M. Walker

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ABSTRACT

Design of a Wide Bandwidth Continuous-time Low-pass Sigma-delta Modulator.

(December 2011)

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M.S., The University of Texas at Arlington

Co-Chairs of Advisory Committee: Dr. Aydin I. Karsilayan
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The emergence of bandwidth-intensive services has created a need for high speed and high resolution data converters. Towards this end, system level design of a continuous-time sigma-delta modulator achieving 11 bits resolution over 100 MHz signal bandwidth by using a feed-forward topology is presented. The system is first built in the Simulink environment in MATLAB. The building blocks in the loop filter are modeled with non-idealities, and specifications for these blocks are obtained by simulations. An operational transconductor amplifier (OTA) with 100 mS transconductance, 70 dB linearity, and 34.2 mW power dissipation is designed to be used in the loop filter. Simulation results indicate that the 5th order loop filter implemented in the feed-forward architecture in transistor level shows lower power consumption, 105 mW, compared to the loop filter implemented by feedback architecture, 152 mW.
ACKNOWLEDGEMENTS

First and foremost, I would like to express my deep appreciation to my advisor Aydin I. Karsilayan and co-advisor Dr. Jose Silva-Martinez for their invaluable guidance, insightful thinking, kindness and support throughout my graduate study. Without his patient support and encouragement, this work would not have been completed. I would also like to thank all my committee members, Dr. Peng Li and Dr. Duncan M. Walker for their time, support and valuable suggestions.

I would like to thank my team members Vijayaramalingam Periasamy, Aravind Padyana and Seokmin Hwang in the CT LP ΣΔ data converter project. Special thanks go to Vijayaramalingam Periasamy who supported and helped me a lot not only in academic field, also in my personal life. I would like to thank all my friends, especially Yung-Chung Lo, Hsien-Pu Chen, Dr. Xi Chen, Shan Huang, Jiayi Jin, HongBo Chen, Yuan Luo, Jackie Zou, Yang Liu, Jun Yan, Cheng Li, Haoyu Qian, Xuan Zhao for all the valuable discussions, encouragements and friendship. I also want to thank Ella and Tammy for their assistance.

Finally, I would like to express my deepest gratitude to my parents, my wife, Shu-Man Lu, and my son, Hudson Chien, for their unconditional love, constant encouragement and support. I couldn’t go this far without their support.
# TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>ABSTRACT</td>
<td>iii</td>
</tr>
<tr>
<td>ACKNOWLEDGEMENTS</td>
<td>iv</td>
</tr>
<tr>
<td>TABLE OF CONTENTS</td>
<td>v</td>
</tr>
<tr>
<td>LIST OF FIGURES</td>
<td>vii</td>
</tr>
<tr>
<td>LIST OF TABLES</td>
<td>ix</td>
</tr>
<tr>
<td>1. INTRODUCTION</td>
<td>1</td>
</tr>
<tr>
<td>1.1 Nyquest ADC</td>
<td>2</td>
</tr>
<tr>
<td>1.2 Oversampling ADC</td>
<td>4</td>
</tr>
<tr>
<td>1.3 Sigma-delta ADC</td>
<td>5</td>
</tr>
<tr>
<td>1.4 Thesis organization</td>
<td>7</td>
</tr>
<tr>
<td>2. SYSTEM LEVEL DESIGN OF $\Sigma\Delta$ MODULATOR</td>
<td>8</td>
</tr>
<tr>
<td>2.1 Design considerations</td>
<td>9</td>
</tr>
<tr>
<td>2.2 Loop filter implementation</td>
<td>15</td>
</tr>
<tr>
<td>2.3 Simulation results</td>
<td>21</td>
</tr>
<tr>
<td>2.4 Non-ideal models of the transconductors</td>
<td>25</td>
</tr>
<tr>
<td>3. TRANSISTOR LEVEL DESIGN OF TRANSCONDUCTOR</td>
<td>28</td>
</tr>
<tr>
<td>3.1 Design considerations</td>
<td>28</td>
</tr>
<tr>
<td>3.2 Design procedures</td>
<td>30</td>
</tr>
<tr>
<td>3.3 Results</td>
<td>44</td>
</tr>
<tr>
<td>4. SIMULATION RESULTS</td>
<td>46</td>
</tr>
<tr>
<td>4.1 Filter simulation results</td>
<td>46</td>
</tr>
<tr>
<td>4.2 System simulation results</td>
<td>48</td>
</tr>
<tr>
<td>4.3 Comparison</td>
<td>49</td>
</tr>
<tr>
<td>5. CONCLUSIONS</td>
<td>51</td>
</tr>
<tr>
<td>5.1 Future work</td>
<td>51</td>
</tr>
</tbody>
</table>
REFERENCES .......................................................................................................... 52
VITA ......................................................................................................................... 55
LIST OF FIGURES

Page

Figure 1: Comparison of radio architectures ................................................................. 1
Figure 2: Signal processing chain .................................................................................. 2
Figure 3: Quantization noise probability density function .............................................. 3
Figure 4: Oversampled quantization noise power spectral density profile ..................... 4
Figure 5: First order sigma-delta modulator ................................................................. 5
Figure 6: First order modulator quantization noise profile .......................................... 6
Figure 7: Continuous-time $\Sigma\Delta$ modulator ............................................................. 8
Figure 8: Noise transfer function .................................................................................. 11
Figure 9: Zero and pole locations of NTF ................................................................. 11
Figure 10: Equivalence of continuous and discrete-time modulators ......................... 13
Figure 11: NRZ DAC pulses ......................................................................................... 14
Figure 12: Low-pass and band-pass nodes of biquad ................................................... 16
Figure 13: Proposed low-pass feed-forward LC loop filter .......................................... 17
Figure 14: MATLAB model of the system ................................................................. 20
Figure 15: Output spectrum from ideal behavioral model .......................................... 21
Figure 16: Loop filter .................................................................................................. 22
Figure 17: STF from $V_{in}$ to node A ................................................................. 22
Figure 18: STF from $V_{in}$ to node B ................................................................. 23
Figure 19: STF from $V_{in}$ to node C ................................................................. 23
Figure 20: STF from $V_{in}$ to node D ................................................................. 24
Figure 21: STF from $V_{\text{in}}$ to node E ................................................................. 24
Figure 22: Transconductor MATLAB model ....................................................... 26
Figure 23: Transconductor topology .................................................................... 29
Figure 24: Small-signal model of the OTA .......................................................... 30
Figure 25: Non-linearity model of the OTA ......................................................... 32
Figure 26: HD3 plot of the OTA ........................................................................... 34
Figure 27: Power consumption of the OTA V.S. degeneration resistor .............. 35
Figure 28: OTA with dominant noise sources ...................................................... 36
Figure 29: Shot noise of BJT presented in the OTA ............................................. 37
Figure 30: Thermal noise of BJT presented in the OTA ....................................... 39
Figure 31: Noise distribution .............................................................................. 41
Figure 32: The small signal model for frequency response. ............................... 42
Figure 33: CMFB circuit .................................................................................... 43
Figure 34: Two tone test of OTA-$G_{\text{m}1}$ (71 dB) ................................................. 45
Figure 35: Loop filter schematic ....................................................................... 46
Figure 36: The loop filter magnitude response ..................................................... 47
Figure 37: The loop filter phase response ............................................................. 47
Figure 38: System simulation with filter in transistor level .................................. 48
Figure 39: SNR calculation in MATLAB (SNR = 68 dB) ..................................... 49
# LIST OF TABLES

<table>
<thead>
<tr>
<th>Table</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Table 1: ΣΔ Modulator Target Specifications</td>
<td></td>
<td>9</td>
</tr>
<tr>
<td>Table 2: Noise transfer function parameters</td>
<td></td>
<td>10</td>
</tr>
<tr>
<td>Table 3: Modulator component values</td>
<td></td>
<td>19</td>
</tr>
<tr>
<td>Table 4: Transconductor non-linearity specifications</td>
<td></td>
<td>26</td>
</tr>
<tr>
<td>Table 5: Transconductor dominant pole specifications</td>
<td></td>
<td>27</td>
</tr>
<tr>
<td>Table 6: Transconductor specifications</td>
<td></td>
<td>28</td>
</tr>
<tr>
<td>Table 7: Comparisons between noise models and CADENCE simulations</td>
<td></td>
<td>41</td>
</tr>
<tr>
<td>Table 8: Performance summary of the OTAs $G_{m1}$ and $G_{m2}$</td>
<td></td>
<td>44</td>
</tr>
<tr>
<td>Table 9: Performance summary of the OTAs $F_3$ and $F_4$</td>
<td></td>
<td>44</td>
</tr>
<tr>
<td>Table 10: Performance summary of the OTA $F_5$</td>
<td></td>
<td>45</td>
</tr>
<tr>
<td>Table 11: Performance summary of the Filter</td>
<td></td>
<td>48</td>
</tr>
<tr>
<td>Table 12: Comparison between feedback and feed-forward topologies</td>
<td></td>
<td>49</td>
</tr>
<tr>
<td>Table 13: Brief summary of wide bandwidth ΣΔ ADC</td>
<td></td>
<td>50</td>
</tr>
</tbody>
</table>
1. INTRODUCTION

Receivers for wireless communications are becoming more and more digital, where analog functions are being realized by digital processors. As a result, analog-to-digital converters (ADC) are gradually moving towards the antenna. However, this places much higher requirements of bandwidth and dynamic range on the ADCs. In addition, the quest for higher data rates is leading to the rise of standards with larger signal bandwidths. This further increases the need for wide bandwidth ADCs. Fig. 1 shows the traditional super-heterodyne receiver architecture and software defined radio.

As can be seen in Fig. 1, in the traditional architecture, analog to digital conversion is preceded by significant analog processing. In the more digital-intensive software-defined radio architecture, the signal is digitized much earlier, enabling filtering and amplification operations in digital domain.

This thesis follows the style of *IEEE Journal of Solid State Circuits*. 

**Figure 1: Comparison of radio architectures**
1.1 Nyquist ADCs

Analog-to-digital converters are the interface between the real world which is continuous-time and continuous-amplitude in nature (e.g., voice, audio or video) and the digital world where only discrete-time and discrete-amplitude signals exist. Fig. 2 shows the major blocks involved in analog-to-digital conversion process.

Figure 2: Signal processing chain

Assume the input analog signal has useful content up to a frequency of $f_b$. The sample-and-hold block shown in Fig. 2 performs the operation of converting the continuous-time signal into a discrete-time signal. The sampling frequency ($f_s$) has to satisfy the Nyquist criterion shown in the following equation in order to avoid losing information when sampling:

$$f_s \geq 2 \cdot f_b$$  \hspace{1cm} (1.1)

The anti-aliasing filter (AAF) shown in Fig. 2 removes any signal with frequencies higher than $f_b$ to prevent aliasing. Therefore, the AAF is prior to the sample-and-hold block in the signal chain.

The quantizer shown in Fig. 2 performs the conversion from continuous to discrete amplitudes. Although the discretization process in time domain is lossless, the quantization process is inherently lossy. The error introduced by the quantization process is dependent on the number of bits of the quantizer.
For a quantizer with a full-scale value of $\pm V_{\text{ref}}$ and output of $N$ digital bits, the step size, $\Delta$, of the quantizer is given by,

$$\Delta = \frac{2V_{\text{ref}}}{2^N}$$  \hfill (1.2)

The quantization error, $\varepsilon$, introduced in the quantization operation at every sampling instant stays in the range $(-\Delta/2, \Delta/2)$. If the input signal of a quantizer is sufficiently ‘busy’, then the behavior of the quantization error can be considered a white noise process with the probability density function shown in Fig. 3 [1].

![Figure 3: Quantization noise probability density function](image-url)

The quantization noise introduced can be calculated as:

$$\sigma_{\varepsilon}^2 = \int_{-\Delta/2}^{\Delta/2} \varepsilon^2 \frac{1}{\Delta} \, d\varepsilon = \frac{\Delta^2}{12}$$  \hfill (1.3)

For a full-scale sine-wave as the input of the quantizer, the power of this sine wave is given by $(V_{\text{ref}}^2 / 2)$. Knowing the input signal and quantization noise power level, the signal to quantization noise can be obtained as:
For the more familiar expression relating SNR with an equivalent number of bits, equation (1.4) can be expressed in dB as:

\[ SQNR_{(dB)} = 6.02N + 1.76 \]  

(1.5)

ADCs with sampling rate equal to twice the signal bandwidth are called Nyquist-rate ADCs.

1.2 Oversampling ADC

The expression for quantization noise power was obtained in the previous section. Since this noise is considered as a white noise process, the power is distributed equally across all frequencies from DC to half the sampling rate. If the signal were to be sampled at a higher rate than the Nyquist rate, the same quantization noise power is spread over a higher frequency range. Thus, the in-band noise gets reduced, and the signal-to-quantization noise ratio (SQNR) is improved as shown in Fig. 4 [1].

![In-band quantization noise](image)

**Figure 4: Oversampled quantization noise power spectral density profile**
The improvement in SQNR due to oversampling is quantified by [2],

$$SQNR(dB) = 6.02N + 1.76 + 10\log_{10} OSR$$ \hfill (1.6)

where \( OSR = \frac{f_{\text{sample}}}{2f_b} \)

Based on equation (1.6), doubling OSR improves SQNR around 3 dB.

1.3 Sigma-delta (ΣΔ) ADC

Improving SQNR only by increasing OSR is an expensive and inefficient method. A more efficient way is by using some method that could shape the quantization noise from signal band to outside the band of interest. This is the principle of sigma-delta ADC. A simple block diagram of a first order sigma-delta converter is shown in Fig. 5.

![First order sigma-delta modulator](image)

**Figure 5: First order sigma-delta modulator**

Assume that quantization noise is an additive noise. Fig. 5 shows a linear system with two inputs and single output. The output can be expressed as:

$$v = STF \cdot u + NTF \cdot \varepsilon$$ \hfill (1.7)
where signal transfer function (STF) and noise transfer function (NTF) are defined as:

\[
STF = \frac{V(z)}{U(z)} = \frac{H(z)}{1 + H(z)} = z^{-1} \quad (1.8)
\]

\[
NTF = \frac{V(z)}{E(z)} = \frac{1}{1 + H(z)} = 1 - z^{-1} \quad (1.9)
\]

Based on equation (1.8), the signal appears unchanged at the output with just a delay. However, the noise is shaped as shown in Fig. 6.

**Figure 6: First order modulator quantization noise profile**

In the above example, the quantization noise is shaped out-of-band by a first order transfer function. In order to further improve SQNR, higher orders of the loop filter can be used. By using an \( L \) order loop filter, the NTF can be obtained as:

\[
NTF = (1 - z^{-1})^L \quad (1.10)
\]

The SQNR in such a case is calculated as [2],

\[
SQNR_{\text{max}} [dB] = 6.02N + 1.76 + (20L + 10)\log_{10} OSR - 10\log_{10} \frac{\pi^{2L}}{2L + 1} \quad (1.11)
\]
Therefore, the SQNR improves by increasing the order of the loop filter of sigma-delta ADC.

1.4 Thesis organization

The organization of this thesis is highlighted in this section.

Section 2 presents the system design of a continuous-time sigma-delta ADC, the loop filter implementation, and the simulated results using Simulink. The specifications of the loop filter components are derived based on the simulated results of the Simulink macro-model by adding non-idealities.

Section 3 presents the design of operational transconductor amplifier (OTA). The non-linear small signal model is built for optimizing power consumption. The noise and frequency response are analyzed by using small signal models as well. At last, the performances of OTAs implemented in transistor level are presented.

Section 4 presents simulated results of the loop filter and system simulated results with the loop filter in transistor level.

Section 5 presents a summary of this research with some directions for future work.
2. SYSTEM LEVEL DESIGN OF ΣΔ MODULATOR

Continuous-time ΣΔ ADCs are becoming more popular recently than discrete-time ΣΔ ADCs primarily because of their reduced settling time requirements that results in better power efficiency shown in Fig. 7. In addition, they have some other advantages such as inherent anti-aliasing and reduced sample-and-hold requirements. However, they do have some disadvantages such as higher sensitivity to clock jitter and susceptibility to time-constant variations. In spite of these disadvantages, there has been a tremendous interest in continuous-time ΣΔ ADCs as seen by papers published in the recent literature [3], [4].

In the past decades, the design methodologies of discrete-time sigma-delta modulators have been thoroughly studied, and many good design tools are available, e.g. the ΔΣ MATLAB toolbox by Richard Schreier [5]. It is very common to obtain a discrete-time loop transfer function first, and then to use the impulse-invariant transformation to convert it to a continuous-time loop transfer function [6]. In this work, this approach is followed as well. However, it should be noted that this approach is not the only way to design a continuous-time ΣΔ modulator. The loop filter can be directly synthesized in the continuous-time domain [7], [2].

![Figure 7: Continuous-time ΣΔ modulator](image-url)
2.1 Design considerations

The target of this work is to realize a continuous-time $\Sigma\Delta$ modulator with the specifications given in Table 1 using Jazz 0.18 um technology.

<table>
<thead>
<tr>
<th>Performance parameter</th>
<th>Target specification</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bandwidth</td>
<td>100 MHz</td>
</tr>
<tr>
<td>Resolution</td>
<td>11 bits</td>
</tr>
<tr>
<td>Power consumption</td>
<td>$&lt; 500$ mW</td>
</tr>
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</table>

The first step of the design process is to obtain a noise transfer function (NTF) for the target specifications. In order to use the MATLAB toolbox by Richard Schreier to obtain a NTF, some variables for system-level design are required [8]. The required variables for system-level design are over-sampling ratio ($OSR$), number of levels in the internal quantizer ($N_L$), order of the loop filter ($L$) and out-of-band gain of the NTF ($NTF_{\text{max}}$).

The signal-to-quantization ratio (SQNR) of an $L^{th}$ order modulator with an N-bit internal quantizer and working at an oversampling ratio $OSR$ is given by.

$$SQNR_{\text{max}}[dB] = 6.02N + 1.76 + (20L + 10)\log_{10}OSR - 10\log_{10}\frac{\pi^2L}{2L+1} \quad (2.1)$$

Based on equation (2.1), increasing over-sampling ratio ($OSR$), number of bits in the internal quantizer ($N$), or order of the loop filter ($L$), improves the SQNR. However, each of the parameters has its limitation for realistic designs as discussed below.

First, increasing the OSR gives better SQNR. However, the maximum speed of operation is usually limited by technology to some finite value. Also, operation at higher speeds leads to higher power consumption.
Second, increasing number of bits of the internal quantizer obtaining smaller quantization noise improves the SQNR. However, increasing number of bits of the internal quantizer also increases the power and area of the implementation of the quantizer exponentially. Also, the number of bits of digital-to-analog converter (DAC) used in the feedback path increase simultaneously with increasing number of bits of the internal quantizer, which increases the area and complexity of the DAC design. In addition, parasitic capacitance of the internal quantizer increases when the number of bits of the quantizer increases and this may affect stability.

Third, increasing the order of the modulator leads to higher SQNR. However, higher order modulators are more difficult to stabilize, and usually the order of a practical system is limited to 5 [9].

Finally, increasing the maximum gain of the NTF outside the signal band (NTF_{\text{max}}) reduces the noise inside the signal band and improves SQNR. However, higher values of NTF_{\text{max}} degrade the performance of the modulator when high frequency noise such as clock jitter is present in the system.

Based on simulations in MATLAB using Richard Schreier ΣΔ toolbox, the above mentioned variables were picked for the targeted SQNR as shown in Table 2, The discrete-time NTF is given by:

\[
NTF(z) = \frac{z^5 - 4.433z^4 + 7.992z^3 - 7.320z^2 + 3.406z - 0.644}{z^5 - 2.680z^4 + 3.207z^3 - 2.035z^2 + 0.677z - 0.093}
\] (2.2)

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>OSR</td>
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</tr>
<tr>
<td>Quantizer bits (N)</td>
<td>3</td>
</tr>
<tr>
<td>Order (L)</td>
<td>5</td>
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<tr>
<td>NTF_{\text{max}}</td>
<td>3.4</td>
</tr>
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</table>

Table 2: Noise transfer function parameters
Figure 8 shows the magnitude plot of (2.2).

Figure 8: Noise transfer function

The zeros and poles of NTF located in Z-domain are shown in Fig. 9

Figure 9: Zero and pole locations of NTF
The second step is to obtain the discrete-time loop transfer function (LTF). Once the NTF is known, the LTF of the modulator can be obtained as:

\[
LTF = \frac{1}{NTF} - 1
\]  

(2.3)

The corresponding loop transfer function is

\[
LTF(z) = \frac{2.253z^4 - 5.539z^3 + 5.727z^2 - 2.801z + 0.539}{z^5 - 4.387z^4 + 7.797z^3 - 7.016z^2 + 3.196z - 0.589}
\]  

(2.4)

Usually, one clock cycle is needed for the operation of the quantizer and DAC. Therefore, \( z^{-1} \) (representing one cycle delay) is factored out from the LTF, which then becomes

\[
H(z) = \frac{2.253z^5 - 5.539z^4 + 5.727z^3 - 2.801z^2 + 0.539z}{z^5 - 4.387z^4 + 7.797z^3 - 7.016z^2 + 3.196z - 0.589}
\]  

(2.5)

The last step is to obtain the continuous-time loop filter transfer function, \( H(s) \), by converting \( H(z) \) using Impulse-Invariant transformation [9].

Impulse-Invariant Transformation: The equivalence of discrete-time and continuous-time \( \Sigma \Delta \) modulators is illustrated in Fig. 10. The main difference between two modulators is that signal is sampled at the input for a discrete-time \( \Sigma \Delta \) modulator, but, for a continuous-time \( \Sigma \Delta \) modulator, signal is sampled inside the loop before the quantizer. By breaking the loops at the digital-to-analog converter (DAC) inputs of discrete-time and continuous-time modulators, the corresponding open loop structures are obtained as shown in Fig. 10. For the same input, the outputs of the two open loops at the sampling instants should be the same if the two loops are identical:

\[
x(n)_d = x(t)\bigg|_{t=nT_s}
\]  

(2.6)
Figure 10: Equivalence of continuous and discrete-time modulators

If the impulse responses of the two loops are the same, then equation (2.6) can be rewritten as:

\[ Z^{-1}[H(z)] = L^{-1}[H_d(s) * H(s)] \] (2.7)

where \( Z^{-1} \) and \( L^{-1} \) are inverse Z and Laplace transforms, respectively. \( H_d(s) \) represents the frequency response of the DAC in the feedback loop.

In the time domain, equation (2.7) can be expressed as:

\[ h(n) = \{h_d(t) * h(t)\}|_{t=nT_s} \] (2.8)

where \( h(n) \), \( h_d(t) \) and \( h(t) \) represent the impulse responses of the discrete-time loop filter, the DAC and the continuous-time loop filter, respectively.
Based on equation (2.8), different DAC pulses result in different $H(s)$ for a given $H(z)$. Therefore, it is necessary to determine the shape of the feedback DAC waveform first.

![Figure 11: NRZ DAC pulses](image)

In this work, the non-return-to-zero (NRZ) DAC waveform shown in Fig. 11 is used for its better jitter tolerance. The return-to-zero (RZ) DAC waveforms have more number of transitions on average in each clock cycle. As a result, they are more sensitive to clock jitter than the NRZ DAC pulse shape.

For the case of the NRZ DAC, the impulse invariant transformation is available in MATLAB using the built-in function ‘d2c’. Then, the corresponding continuous-time loop filter transfer function is obtained as follows:

$$H(s) = \frac{2.253s^5 + 8.716 \times 10^9 s^4 + 1.719 \times 10^{10} s^3 + 2.212 \times 10^{28} s^2 + 1.811 \times 10^{37} s + 7.571 \times 10^{45}}{s^5 + 1.056 \times 10^9 s^4 + 9.471 \times 10^{17} s^3 + 4.115 \times 10^{26} s^2 + 1.418 \times 10^{35} s + 1.881 \times 10^{43}}$$

(2.9)
2.2 Loop filter implementation

In continuous-time ΣΔ modulators, two popular architectures are widely used: feedback and feed-forward architectures. Both of them have their own advantages and disadvantages. For feedback architectures, the advantages are a large adder is not required before the quantizer as compared to feed-forward architecture. However, for low-power design, the feed-forward architecture is preferred because the input swing of the first stage transconductor in the feed-forward loop filter is much smaller than that of the feedback counterpart. This significantly reduces the power requirement of the first stage transconductor, which is usually the major part of the total power consumption of the loop filter. However, the feed-forward architecture has its drawbacks as well. The out-of-band peaking in the signal transfer function of the feed-forward loop filter reduces the dynamic range of the modulator [10], [1].

In this work, the feed-forward architecture is chosen because of its low-power consumption property. Comparison of simulated results and discussion between feedback and feed-forward architectures will be shown in Section 4.

In continuous-time ΣΔ modulators, the loop filters are generally implemented as a cascade of integrators or resonators comprised of biquads. Consequently, when the bandwidth of interest in a modulator implementation increases, the power consumption in the amplifier within the integrators or biquads rises to have room for the higher frequency signal. Therefore, the use of passive filters comprised of LC sections is explored in this work.

In order to implement the loop transfer function (2.9) by cascading biquads, first, the denominator of the loop transfer function (2.9) can be rearranged as:

\[
(s + 2.11 \times 10^8)(s^2 + 2.35 \times 10^8 s + 3.36 \times 10^{17})(s^2 + 4.35 \times 10^8 s + 3.71 \times 10^{17})
\] (2.10)
From equation (2.10), we can see that there are two second order polynomials and one first order polynomial which can be implemented by cascading two biquads and one RC low-pass filter. A biquad to be used for feed-forward architecture must include both band-pass and low-pass output nodes for the same input node. The suitable biquad is shown in Fig. 12.

![Figure 12: Low-pass and band-pass nodes of biquad](image)

The corresponding transfer functions from $I_{in}$ to $V_{LP}$ and $V_{BP}$ can be found as:

\[
\frac{V_{LP}(s)}{I_{in}(s)} = \frac{R\omega_n^2}{s^2 + s\left(\frac{\omega_n}{Q}\right) + \omega_n^2}
\]

(2.11)

\[
\frac{V_{BP}(s)}{I_{in}(s)} = \frac{s\left(\frac{1}{C}\right) + R\omega_n^2}{s^2 + s\left(\frac{\omega_n}{Q}\right) + \omega_n^2}
\]

(2.12)

where $\omega_n = \frac{1}{\sqrt{LC}}$ and $Q = \frac{\omega_n L}{R}$

Next, the numerator of the loop transfer function (2.9) is implemented. Because the numerator has six coefficients from 5th order to constant, it requires six independent paths to implement it. However, with an implementation consisting only of two biquads,
which gives four paths and one fast-path directly from output of quantizer to the input of quantizer, the sum of the total number of paths is only five. Therefore, an additional feedback path is required for feed-forward LC loop filter implementation. The additional path is chosen from output of quantizer feeding back current to the first order RC section before the quantizer, which increases its denominator order by one compared to the fast-path. A single-ended representation of the proposed low-pass loop filter with feed-forward paths included is shown in Fig. 13.

Figure 13: Proposed low-pass feed-forward LC loop filter

The loop transfer function of the filter in Fig. 13 is given by,

$$\frac{V_o(s)}{V_{fb}(s)} = D_0 + \frac{1}{C_3 s + \omega_3} \frac{D_2}{C_3 s + \omega_3} \frac{D_1}{C_3 s + \omega_3} \frac{F_5}{C_3 s + \omega_3} \frac{s + R_1 \omega_1^2}{C_1 s + \omega_1^2} + \frac{D_4}{C_3 s + \omega_3} \frac{F_4}{C_3 s + \omega_3} \frac{R_1 \omega_1^2}{C_3 s + \omega_3} + \frac{D_1}{C_3 s + \omega_3} \frac{F_3}{C_3 s + \omega_3} \frac{R_1 \omega_1^2}{C_3 s + \omega_3}$$

$$+ \frac{D_1}{C_3 s + \omega_3} \frac{F_3}{C_3 s + \omega_3} \frac{R_1 \omega_1^2}{C_3 s + \omega_3} \frac{G_{m1} \left( \frac{s}{C_2} + R_2 \omega_2^2 \right)}{C_5 (s + \omega_3)} + \frac{G_{m2} R_2 \omega_2^2}{C_5 (s + \omega_3)} + \frac{G_{m3} R_3 \omega_3^2}{C_5 (s + \omega_3)} + \frac{R_1 \omega_1^2 D_1}{C_5 (s + \omega_3)}$$

(2.13)
where $\omega_1 = \frac{1}{\sqrt{L_1C_1}}$  $\omega_2 = \frac{1}{\sqrt{L_2C_2}}$  $\omega_3 = \frac{1}{R_3C_3}$  $Q_1 = \frac{\omega_1 L_1}{R_1}$  $Q_2 = \frac{\omega_2 L_2}{R_2}$

Now, it is time to calculate all the parameters of the loop filter. First, based on the equation (2.10) and the denominator of the equation (2.11), the biquad component values can be calculated. Since the inductor values ($L_1$ and $L_2$) are fixed to 100 nH, chosen to be the maximum value from the technology of Coilcraft, the capacitor values ($C_1$ and $C_2$) and resistor values ($R_1$ and $R_2$) are also fixed based on equation (2.10) and (2.11). The resistor value ($R_3$) is determined by DC gain of the loop transfer function (2.9). Then, the capacitor values ($C_3$) can be calculated from equation (2.10). $G_{m0}$ in Fig. 13 only performs unit voltage to current conversion, which value is 30 mS.

Next, in order to calculate the feedback and feed-forward coefficients in Fig. 13, by comparing the coefficients of the numerator of equation (2.9) to equation (1.13), each term with the same order in the numerator of equation (2.13) should be put together to form six polynomials from $5^{th}$ order to constant as follows:

- The $5^{th}$ order terms in the numerator of equation (2.13) is shown in the polynomial,

$$D_0 s^5 \quad (2.14)$$

- The $4^{th}$ order terms in the numerator of equation (2.13) is shown in the polynomial,

$$\left[ D_0 \frac{\omega_1}{Q_1} + D_0 \frac{\omega_2}{Q_2} + D_0 \omega_3 + \frac{D_2}{C_3} \right] s^4 \quad (2.15)$$

- The $3^{rd}$ order terms in the numerator of equation (2.13) is shown in the polynomial,

$$\left[ D_0 \omega_1^2 + D_0 \frac{\omega_2}{Q_1} \frac{\omega_1}{Q_2} + D_0 \omega_2^2 + D_0 \omega_3 \frac{\omega_1}{Q_1} + D_0 \omega_3 \frac{\omega_2}{Q_2} + \frac{D_2}{C_3} \frac{\omega_1}{Q_1} + \frac{D_2}{C_3} \frac{\omega_2}{Q_2} + \frac{D_1}{C_3} \frac{F_5}{C_1} \right] s^3 \quad (2.16)$$

- The $2^{nd}$ order terms in the numerator of equation (2.12) is shown in the polynomial,
\[
\left[ D_0 \omega_1^2 \frac{\omega_2}{Q_1} + D_6 \omega_2^2 \frac{\omega_1}{Q_1} + D_0 \omega_3 \omega_2^2 + D_0 \omega_3 \frac{\omega_2}{Q_1} Q_2 + D_0 \omega_4 \omega_2^2 + \omega_1^2 \frac{D_3}{C_1} + \frac{D_2}{C_3} \frac{\omega_1}{Q_1} \frac{\omega_2}{Q_2} + \omega_2^2 \frac{D_4}{C_3} + D_5 \omega_4 \omega_2^2 + D_4 R_1 \frac{\omega_1^2}{Q_1} + D_4 \frac{D_3}{C_3} \frac{\omega_1}{Q_1} \frac{\omega_2}{Q_2} + \omega_2^2 \frac{D_5}{C_3} \frac{\omega_1}{Q_1} \frac{\omega_2}{Q_2} \right] s^2
\]  

(2.17)

- The 1st order terms in the numerator of equation (2.13) is shown in the polynomial,

\[
\left[ D_0 \omega_1^2 \omega_2^2 + D_6 \omega_3 \omega_1^2 \frac{\omega_2}{Q_1} + D_0 \omega_3 \omega_1^2 \frac{\omega_2}{Q_1} + \omega_1^2 \frac{D_2}{C_1} \frac{\omega_3}{Q_2} + \omega_2^2 \frac{D_3}{C_3} \frac{\omega_3}{Q_2} + \omega_2^2 \frac{D_4}{C_3} \frac{\omega_3}{Q_2} \right] s
\]  

(2.18)

- The constant terms in the numerator of equation (2.13) is shown in the polynomial,

\[
D_0 \omega_1^2 \omega_2^2 + \omega_2^2 \frac{D_2}{C_3} + F_3 R_1 \omega_1^2 \frac{D_1}{C_3} + F_4 R_1 \omega_1^2 \frac{D_4}{C_3} + F_5 R_1 \omega_1^2 G_{m1} \frac{D_1}{C_3} \frac{D_3}{C_2} \right] s
\]  

(2.19)

By comparing the coefficients of the numerator of equation (2.9) to polynomials (2.14) - (2.19), each equation only has one unknown which is either feedback or feed-forward coefficient. Thus, all unknown coefficients can be solved. The parameters of the modulator obtained after performing the comparison mentioned above shown in Table 3. The model of the system built in MATLAB Simulink is shown in Fig. 14.

<p>| | | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>(D_0)</td>
<td>0.45</td>
<td>(L_1)</td>
<td>100 nH</td>
<td>(L_2)</td>
<td>100 nH</td>
</tr>
<tr>
<td>(D_1)</td>
<td>6.9 mS</td>
<td>(C_1)</td>
<td>36 pF</td>
<td>(C_2)</td>
<td>31 pF</td>
</tr>
<tr>
<td>(D_2)</td>
<td>1.5 mS</td>
<td>(R_1)</td>
<td>33 Ω</td>
<td>(R_2)</td>
<td>51 Ω</td>
</tr>
<tr>
<td>(F_3)</td>
<td>1.7 mS</td>
<td>(G_{m1})</td>
<td>100 mS</td>
<td>(G_{m2})</td>
<td>1.3 mS</td>
</tr>
<tr>
<td>(F_4)</td>
<td>6.0 mS</td>
<td>(R_3)</td>
<td>4 kΩ</td>
<td>(C_3)</td>
<td>1.2 pF</td>
</tr>
<tr>
<td>(F_5)</td>
<td>2.4 mS</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 3: Modulator component values
Figure 14: MATLAB model of the system
2.3 Simulation results

A macro-model of the system was built in MATLAB using Simulink. The output obtained from a simulation of the system is shown in Fig. 15. The input is a -3 dBFS sine wave at 25 MHz and the SQNR is computed over a bandwidth of 100 MHz.

Figure 15: Output spectrum from ideal behavioral model

The closed loop transfer functions from the input to internal nodes of the loop filter were also obtained using Simulink. Fig. 16 shows the loop filter, and signal transfer function from $V_{in}$ to internal nodes are shown in Figs. 17 - 21.

Based on Figs. 17 - 20, the gain of STF to each node is less than unity. However, in Fig. 21, the response shows 4.5 dB magnitude peaking at 212 MHz. In wireless applications, the desired signal may be accompanied by a strong out-of-band interferer. Out-of-band peaking actually translates into a reduction of the dynamic range by limiting the maximum stable amplitude of the modulator.
Figure 16: Loop filter

Figure 17: STF from $V_{in}$ to node A

Peaking is 16 dB at 233 MHz
Figure 18: STF from $V_{in}$ to node B

Peaking is 3 dB at 212 MHz

Figure 19: STF from $V_{in}$ to node C

Peaking is 6 dB at 92 MHz
Figure 20: STF from $V_{in}$ to node D

Figure 21: STF from $V_{in}$ to node E

Peaking is 1 dB at 75 MHz

Peaking is 4.5 dB at 212 MHz
2.4 Non-ideal models of the transconductors

In the previous section, the design process assumed that all components used in the loop are ideal and behaved exactly as assumed in the model, and the simulation for the $\Sigma\Delta$ modulator was performed at the system level using Simulink. However, for real cases, this is not true. Many non-idealities are associated with different components in the loop. Therefore, in this section, the major non-idealities of transconductors in the loop filter will be discussed first, and then the specifications of transconductors in the loop filter will be given by simulations.

The first non-ideality of transconductors is non-linearity. Although the proposed architecture in this work uses passive components in signal paths to obtain the filtering action, which is inherently linear, a sufficient amount of gain is still required in the forward path requiring some active circuits. In this work, transconductors are the components providing gain in signal paths.

For a transconductor, the non-linearity can be expressed as follows:

$$i = c_1 \cdot v + c_3 \cdot v^3$$  \hspace{1cm} (2.20)

where $i$ is the output current of the transconductor, $v$ is the input voltage of the transconductor, $c_1$ is the linear transconductance coefficient, and $c_3$ is the third order non-linear transconductance coefficient of the transconductor.

Because of differential operation in this work, only odd-order non-linearities are considered here and even-order terms are neglected. From Fig. 16, the transconductors, $G_{m1}$, $F_4$ and $F_5$ do not have any gain components before them and hence any non-linearity introduced by those transconductors, when reflected back to the input, will appear without any attenuation. This non-linearity will then directly appear at the output due to the feedback action. Therefore, those transconductors require high linearity specification.
Conversely, the signal received by the transconductors, $G_{m2}$ and $F_3$ in Fig. 16, has already been amplified by transconductor, $G_{m1}$. Consequently, their non-linearities are not as critical. The non-linear model in MATLAB using equation (2.20) is shown in Fig. 22.

![Figure 22: Transconductor MATLAB model](image)

Using the non-linear model for all transconductors, system simulations were performed, and IM3 specifications were derived for each transconductor to obtain an overall system IM3 of 70 dB. The results are shown in Table 4.

**Table 4: Transconductor non-linearity specifications**

<table>
<thead>
<tr>
<th>Transconductor (Fig. 16)</th>
<th>Input swing (mV)</th>
<th>IM3 specification (dB)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$G_{m1}$</td>
<td>30</td>
<td>68</td>
</tr>
<tr>
<td>$G_{m2}$</td>
<td>60</td>
<td>55</td>
</tr>
<tr>
<td>$F_3$</td>
<td>90</td>
<td>55</td>
</tr>
<tr>
<td>$F_4$</td>
<td>30</td>
<td>68</td>
</tr>
<tr>
<td>$F_5$</td>
<td>60</td>
<td>68</td>
</tr>
</tbody>
</table>

The second non-ideality of transconductors is finite bandwidth. In the model of the transconductors considered so far, it is assumed that the transconductors have infinite bandwidth, which means the gain of the transconductors have the same value at all
frequencies. However, the gain of the transconductors starts to reduce at the frequency of
the internal pole. In this work, the transconductors are modeled as single-pole systems as:

\[ i = \frac{g_m}{1 + \frac{s}{\omega_p}} \cdot v \]  

(2.21)

where \( \omega_p \) represents the dominant pole of the transconductor.

Because the sigma-delta modulator is a closed loop system, excess delay due to
transconductor blocks can cause the loop to become unstable. The poles within the
transconductor block introduce additional delay in the signal path. Therefore, to make
sure that the delay introduced is not high, the minimum specifications for the dominant
poles of the transconductors were obtained as shown in Table 5.

<table>
<thead>
<tr>
<th>Transconductor (Fig. 16)</th>
<th>Dominant pole (GHz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( G_m1 )</td>
<td>1.8</td>
</tr>
<tr>
<td>( G_m2 )</td>
<td>0.9</td>
</tr>
<tr>
<td>( F_3 )</td>
<td>0.9</td>
</tr>
<tr>
<td>( F_4 )</td>
<td>1.8</td>
</tr>
<tr>
<td>( F_5 )</td>
<td>2.8</td>
</tr>
</tbody>
</table>
3. TRANSISTOR LEVEL DESIGN OF TRANSCONDUCTOR

For high frequency operation, operational transconductance amplifiers (OTA) have proven to be the best candidate for implementation of continuous-time filters compared to active-RC or switched capacitor topologies. This is because integrators based on OTA work in open loop instead of closed loop. The major disadvantage of OTAs is the input stage non-linearity at higher differential input voltages due to the non-linear characteristics of the input stage transistors. This problem causes inter-modulation distortion components at the output and degrades achievable dynamic range. Several techniques have been reported trying to improve the linearity of OTAs [11]. In this section, the design of OTAs used in wideband continuous-time sigma-delta modulator is presented in detail.

3.1 Design considerations

Based on the simulated results from Section 2, the target specifications of OTAs are shown in Table 6. The implementation requires use of minimum power and area in Jazz 0.18 μm technology with supply voltage of 1.8 V.

Table 6: Transconductor specifications

<table>
<thead>
<tr>
<th>Transconductor (Fig. 16)</th>
<th>Transconductance (mS)</th>
<th>Linearity (dB)</th>
<th>Bandwidth (GHz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gm1</td>
<td>100</td>
<td>68</td>
<td>1.8</td>
</tr>
<tr>
<td>Gm2</td>
<td>1.3</td>
<td>55</td>
<td>0.9</td>
</tr>
<tr>
<td>F3</td>
<td>1.7</td>
<td>55</td>
<td>0.9</td>
</tr>
<tr>
<td>F4</td>
<td>6.0</td>
<td>68</td>
<td>1.8</td>
</tr>
<tr>
<td>F5</td>
<td>2.4</td>
<td>68</td>
<td>2.8</td>
</tr>
</tbody>
</table>
Based on Table 6, it is clear that the OTA $G_{m1}$ requires extremely high transconductance and high linearity.

An easy way to improve the linearity of an OTA is using source degeneration topology. For low-distortion operation, large bias currents are required that consume power and increase transistor thermal noise, and large degeneration resistors reduce OTA output swing and increase layout area. The OTA shown in Fig. 23 presented in [11] is chosen in this work. In this topology, the amplifier forces the input voltage across the degeneration resistor, creating a differential current that flows in the primary BJTs ($B_1$, $B_2$) and is replicated at the output by identical BJTs.

![Figure 23: Transconductor topology](image-url)
3.2 Design procedure

In order to find the relationship between input voltage and output current, a small-signal model of the OTA is created as shown in Fig. 24.

**Figure 24: Small-signal model of the OTA**

In Figure 24, $R_x$ is the output resistance at the drain node of PMOS transistor ($M_1$), $R$ is the degeneration resistor, $g_m$ is the transconductance of input PMOS transistor ($M_1$), and $g_{m(BJT)}$ is the tranconductance of BJT ($B_1$).

Ignoring the channel-length modulation of input PMOS and BJT, the tranconductance of the OTA is obtained as:

$$G_m = \frac{-g_m g_{m(BJT)} R_x N_m}{1 + g_m R + g_m g_{m(BJT)} RR_x} \quad (3.1)$$

where $N_m$ is the output mirroring factor.
For $g_m g_{m(BJT)} R R >> g_m R >> 1$, the transconductance of the OTA can be approximated as:

$$G_m \approx \frac{N_m}{R}$$  \hspace{1cm} (3.2)

Based on equation (3.2), in order to achieve high $G_m$ values, the degeneration resistor should be chosen as small as possible. However, when the size of the degeneration resistors reduces, the required collector biasing current of BJTs increases in order to keep the same linearity specification of the OTA. At the mean time, the increased collector biasing current is copied to the output by factor $N_m$, which significantly increases the total current consumed by the OTA, especially when $N_m$ is large for high $G_m$ case. Therefore, in order to find the optimum degeneration resistor size, which can achieve specifications with minimum power, the relationship between the degeneration resistor sizes and BJT collector biasing current required for the linearity specification needs to be found shown as followed.

**Non-linear model of the OTA**

Assume that the BJT ($B_1$) is the main source of non-linearity. Since the base node of the BJT ($B_1$) is a high impedance node as shown in Fig. 23, the base-emitter voltage increases significantly when the small-signal drain current of PMOS ($M_1$) increases, which makes most of the small signal current to flow through BJT ($B_1$). Since the drain current of PMOS ($M_1$) only has small variation, the assumption that the PMOS ($M_1$) is linear is reasonable. The third order harmonic source is added into BJT ($B_1$) in the small-signal model shown in Fig. 25.
In Fig. 25, \( g_{m3(BJT)} \) is the third order harmonic transconductance of BJT (B\(_1\)).

Assume the input signal of the OTA is a pure sine wave as:

\[
V_{in} = A \sin(\omega t)
\]  

(3.3)

Considering only the third order harmonic distortion, the output current of the OTA can be expressed as:

\[
I_o = a \sin(\omega t) + b \sin(3\omega t)
\]  

(3.4)

where \(a\) is the fundamental output signal amplitude and \(b\) is the third order harmonic tone amplitude.

Based on Fig. 25, the output current of the OTA can be expressed as:

\[
I_o = g_{m(BJT)} V_x + g_{m3(BJT)} V_x^3
\]  

(3.5)
The PMOS ($M_1$) node (drain and source) equations can be expressed as:

\[-(V_{in} - V_s) g_m R_x = V_x \quad (3.6)\]

\[(V_{in} - V_s) g_m - I_o R = V_x \quad (3.7)\]

From equations (3.6) and (3.7), the expression of $V_x$ can be obtained as:

\[V_x = \frac{-V_{in} g_m R_x - I_o R g_m R_x}{(1 + g_m R)} \quad (3.8)\]

Substituting equation (3.8) into equation (3.5),

\[I_o = g_{m(BJT)} \left[ \frac{-V_{in} g_m R_x - I_o R g_m R_x}{(1 + g_m R)} \right] + g_{m3(BJT)} \left[ \frac{-V_{in} g_m R_x - I_o R g_m R_x}{(1 + g_m R)} \right]^3 \quad (3.9)\]

Then, substituting equation (3.3) and (3.4) into equation (3.9) gives,

\[I_o = g_{m(BJT)} \left[ \frac{(-A g_m R_x - a R g_m R_x) \sin(\omega t) - b R g_m R_x \sin(3\omega t)}{1 + g_m R} \right] + g_{m3(BJT)} \left[ \frac{(-A g_m R_x - a R g_m R_x) \sin(\omega t) - b R g_m R_x \sin(3\omega t)}{1 + g_m R} \right]^3 \quad (3.10)\]

Combining all coefficients of $\sin(\omega t)$ term from equation (3.10), which should be equal to the coefficient $a$:

\[a = \frac{-g_m g_{m(BJT)} R_x A - a R g_m g_{m(BJT)} R_x}{1 + g_m R} \quad (3.11)\]

From equation (3.11), find $a$ expression shown as:

\[a = \frac{-g_m g_{m(BJT)} A R_x}{1 + g_m R + R g_m g_{m(BJT)} R_x} \quad (3.12)\]
Combining all coefficients of $\sin(3\omega t)$ term from equation (3.10), which should be equal to the coefficient $b$:

$$b = \frac{-b R g_{m(BJT)} g_m R_x}{1 + g_m R} - \frac{A^3}{4} g_{m3(BJT)} = \frac{3 A^2 b R g_{m} R_x g_{m3(BJT)}}{2 (1 + g_m R)} \quad (3.13)$$

From equation (3.13), find $b$ expression shown as:

$$b = -\frac{A^3}{4} g_{m3(BJT)} (1 + g_m R) \quad (3.14)$$

The third order harmonic distortion (HD3) of the OTA can be calculated as:

$$HD3 = 20 \log_{10} \left( \frac{b}{a} \right) \quad (3.15)$$

Based on the equations (3.12), (3.14) and (3.15), third order harmonic distortion, HD3, can be plotted as a function of collector current for different sizes of degeneration resistances, which is shown in Fig. 26.

Figure 26: HD3 plot of the OTA
It can be seen from Fig. 26 that, for the same linearity, the required collector current of BJT increases when the degeneration resistor size decreases.

Once the collector current of the BJT is decided, the total current consumed by OTA is approximately equal to $I_c N_m$. The total power consumed by OTA for differential amplifier case is

$$\text{Power} = 2I_c(1 + N_m)V_{DD}$$ \hspace{1cm} (3.16)$$

where $I_c$ is the collect current of the BJT ($B_1$), $N_m$ is the mirroring factor, and $V_{DD}$ is the supply voltage.

In order to optimize the total power consumed by the OTA, the mathematical model can be created by using MATLAB based on equation (3.12), (3.14), (3.15) and (3.16). Then, the relationship between the total power consumption of the OTA and the degeneration resistor size can be obtained by sweeping the degeneration resistor sizes as shown in Fig. 27.

Figure 27: Power consumption of the OTA V.S. degeneration resistor
Based on Fig. 27, the degeneration resistor size is chosen to be 30 ohms for this work based on minimum power required to achieve the linearity specification compared to other degeneration resistor sizes. Since the degeneration resistor size and the collector current of BJT have been determined from power optimization, the next step of the OTA design is to decide transistor sizes. However, transistor sizes affect the OTA speed and noise. For large transistor sizes, parasitic capacitor will increase time-constant. For small BJTs size, the thermal noise of BJTs increase. Therefore, building a mathematical model for the noise and frequency performance of the OTA in terms of the circuit components is essential.

**Noise model of the OTA**

The noise sources of the OTA comes from the thermal noise of the current sources $M_3$, $M_4$, $M_5$, and $M_6$, the shot noise ($I_{\text{cn}}$) of the BJTs $B_1$ and $B_2$, the thermal noise ($V_{\text{bn}}$) of the BJTs $B_1$ and $B_2$, and the thermal noise of the degeneration resistors ($R$). Fig. 28 shows the OTA with dominant noise sources.

![OTA with dominant noise sources](image)
To calculate the total input referred noise of the OTA, assume that the noise sources are uncorrelated. Therefore, the superposition concept is applied when calculating the input referred noise of each component. Comparisons between the mathematical model of the input referred noise of each noise source and the simulated results will be presented later in this section.

(1) **Shot noise of the BJT (B1 and B2)**

The shot noise of the BJT is proportional to the collector current of BJT. The expression of the shot noise is given by

\[ I_{cn} = \sqrt{2qI_c} \]  

(3.17)

where \( I_c \) is the collector current of the BJT and \( q \) is the charge of an electron. The shot noise of the BJT presented in the small signal model of the OTA is shown in Fig. 29.

![Figure 29: Shot noise of BJT presented in the OTA](image)

**Figure 29: Shot noise of BJT presented in the OTA**
Based on Fig. 29, the input referred noise due to shot noise from BJT (B1) can be obtained as:

\[
\overline{V_n} = \frac{\sqrt{2}qI_c (1 + g_m R + g_m g_{m(BJT)} R R_s)}{-g_m g_{m(BJT)} R_s} 
\]  

(3.18)

If \( g_m g_{m(BJT)} R (R_s // r_x) >> g_m R \) and \( g_m R >> 1 \), the equation (3.19) can be simplified as:

\[
\overline{V_n} = -\sqrt{2}qI_c R 
\]  

(3.19)

Based on equation (3.19), in order to minimize the shot noise from BJT, the collector current and degeneration resistor should be chosen as small as possible. However, the collector current and the degeneration resistor have been fixed when optimizing the power consumption of the OTA.

(2) Thermal noise of the BJT (B1 and B2)

The thermal noise of the BJT is proportional to the internal resistance at the base node of the BJT. The expression of the thermal noise is:

\[
V_{bn} = \sqrt{4kTR_b} 
\]  

(3.20)

where \( R_b \) is the internal resistance at base node of the BJT, \( T \) is the absolute temperature, and \( k \) is Boltzmann constant.

The thermal noise of the BJT presented in the small signal model of the OTA is shown in Fig. 29.
Figure 30: Thermal noise of BJT presented in the OTA

Based on Fig. 30, the input referred noise of the thermal noise from BJT can be obtained as:

$$V_n = V_{bn}(R_x + g_m R R_x + g_m g_{m(BJT)} R r_i R_x)$$

$$= (r_i - R_x + r_i g_m R - g_m R R_x + g_m g_{m(BJT)} R r_i R_x)(g_m g_{m(BJT)} R_x)$$

where $r_i = r_i + R_h$. Based on equation (3.22), in order to minimize the thermal noise from BJTs, the internal resistance at its base node should be maximized. However, the BJT sizes are confined by technology. In addition, maximizing the base resistance increases the base capacitance, which decreases the OTA speed, and the OTA speed are given by the specifications obtained in Section 2.
(3) Thermal noise of the transistor (M₁ to M₆)

MOS transistors exhibit thermal noise. For long channel MOS devices operating in saturation, the channel noise is proportional to the transconductance of transistor and can be modeled by current source connected between the drain and source terminal. The expression of the thermal noise of MOS transistor is

\[ I_{cn} = \sqrt{4kTg_{m}\gamma} \]

where \( \gamma \) is 2/3 for long channel device.

The input referred noise equations are coded in MATLAB. Comparison between the calculations in MATLAB and the simulated results obtained from CADENCE is shown in Table 7.

(4) Thermal noise of the degeneration resistor (R)

The noise spectrum of a resistor is proportional to the absolute temperature and resistor value as:

\[ V_{bn} = \sqrt{4kTR} \]

Noise integrated from 1 MHz to 100 MHz are calculated in MATLAB and simulated in CADENCE. The comparison between the two is also shown in Table 7. Fig. 31 shows noise distribution.
Table 7: Comparisons between noise models and CADENCE simulations

<table>
<thead>
<tr>
<th>Noise Source</th>
<th>Noise models (μV)</th>
<th>Simulations (μV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shot noise of the BJT</td>
<td>2.9</td>
<td>3.0</td>
</tr>
<tr>
<td>Thermal noise of the BJT</td>
<td>5.2</td>
<td>5.1</td>
</tr>
<tr>
<td>Thermal noise of the degeneration resistor</td>
<td>1.7</td>
<td>1.5</td>
</tr>
<tr>
<td>Thermal noise of the input PMOS transistor</td>
<td>0.96</td>
<td>0.94</td>
</tr>
</tbody>
</table>

Figure 31: Noise distribution
Based on Table 7, for the topology of the OTA, over 50% of the noise comes from the BJTs. The thermal noise of the BJTs traded off with OTA speed, while the shot noise of the BJT traded off with OTA power. Therefore, when choosing the BJT sizes and biasing conditions, these trade-offs should be considered based on the specifications of each OTA.

**Frequency Response Model of the OTA**

In order to estimate the OTA speed limitation, knowledge of the OTA pole locations is essential. Since the chosen topology for the OTA is based on a closed loop, we can break the loop at the BJT base node, and calculate the pole at each node. The small signal model with parasitic capacitors is shown in Fig. 32.

![Figure 32: The small signal model for frequency response.](image)

In Fig.32, $C_{be}$ is the total parasitic capacitors from base to emitter terminal of BJT ($B_1$) and BJT array, $C_{bc}$ is the parasitic capacitor from base to collect terminal of BJT ($B_1$), $C_{cs}$ is the parasitic capacitor from collector to substrate terminal of BJT ($B_1$), $C_{gs}$ is the parasitic capacitor from gate to source terminal of PMOS ($M_1$), $C_{gd}$ is the parasitic capacitor from gate to drain terminal of PMOS ($M_1$), $C_{db}$ is the parasitic capacitor from drain to bulk terminal of PMOS ($M_1$), $Z_1$ is the resistance at the drain node of PMOS ($M_1$), and $Z_2$ is the resistance at the source node of PMOS ($M_1$). The values of the
parasitic capacitors are extracted from CADENCE. After the small signal model is created, the pole locations are obtained. The dominant pole is located at the base node of the BJT and the first non-dominant pole is located at source node of the input PMOS. Thus, in order to increase the OTA speed, the size of the BJTs and NMOS current mirrors are chosen as small as possible. However, for smaller BJT sizes, the thermal noise of BJT increases because the internal resistance increases at the same time. Therefore, there is a trade off between OTA speed and noise for choosing BJT sizes.

**CMFB (common mode feedback)**

In high gain fully differential amplifiers, the output common mode level is quite sensitive to mismatches and it cannot be stabilized by means of differential feedback. Thus, a common-mode feedback network must be added to sense the common mode level of the two outputs and accordingly adjust one of the bias currents in the amplifier. The tasks of the CMFB network is first sensing the output common mode level, comparison with a reference, and returning the error to the amplifier’s bias network. The CMFB used in this work is shown in Fig. 33.

![Figure 33: CMFB circuit](image-url)
3.3 Results

Based on power, noise and speed specifications, each OTA component sizes are determined. The performance summaries of each OTA are shown in Tables 8 - 10. Fig. 34 shows the two tone test of OTA $G_{m1}$.

Table 8: Performance summary of the OTAs $G_{m1}$ and $G_{m2}$

<table>
<thead>
<tr>
<th>OTA</th>
<th>$G_{m1}$</th>
<th>$G_{m2}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Transconductance</td>
<td>101.7 mS</td>
<td>1.3 mS</td>
</tr>
<tr>
<td>$IM_3$</td>
<td>70.9 dB</td>
<td>58.1 dB</td>
</tr>
<tr>
<td>Power consumption</td>
<td>34.2 mW</td>
<td>14.1 mW</td>
</tr>
<tr>
<td>Input referred noise</td>
<td>5.2 uV</td>
<td>18 uV</td>
</tr>
<tr>
<td>Integration bandwidth</td>
<td>(1MHz~100MHz)</td>
<td>(1MHz~100MHz)</td>
</tr>
</tbody>
</table>

Table 9: Performance summary of the OTAs $F_3$ and $F_4$

<table>
<thead>
<tr>
<th>OTA</th>
<th>$F_3$</th>
<th>$F_4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Transconductance</td>
<td>1.7 mS</td>
<td>6.0 mS</td>
</tr>
<tr>
<td>$IM_3$</td>
<td>56.5 dB</td>
<td>70.2 dB</td>
</tr>
<tr>
<td>Power consumption</td>
<td>14.6 mW</td>
<td>16.4 mW</td>
</tr>
<tr>
<td>Input referred noise</td>
<td>18 uV</td>
<td>5.2 uV</td>
</tr>
<tr>
<td>Integration bandwidth</td>
<td>(1MHz~100MHz)</td>
<td>(1MHz~100MHz)</td>
</tr>
</tbody>
</table>
Table 10: Performance summary of the OTA F₅

<table>
<thead>
<tr>
<th>OTA parameters</th>
<th>F₅</th>
</tr>
</thead>
<tbody>
<tr>
<td>Transconductance</td>
<td>2.4 mS</td>
</tr>
<tr>
<td>IM₃</td>
<td>70.1 dB</td>
</tr>
<tr>
<td>Power consumption</td>
<td>25.4 mW</td>
</tr>
<tr>
<td>Input referred noise</td>
<td>15 uV</td>
</tr>
<tr>
<td>Integration bandwidth</td>
<td>(1MHz~100MHz)</td>
</tr>
</tbody>
</table>

Figure 34: Two tone test of OTA-Gm1 (71 dB)
4. SIMULATION RESULTS

This section mainly shows the measurements from the CADENCE simulations for both the loop filter in transistor level and the system with the loop filter in transistor level.

4.1 Filter simulation results

After all OTAs implemented in transistor level, the filter can be built following the loop filter design in Section 2. The schematic of the loop filter is shown in Fig. 35 and the magnitude and phase response of the loop filter is shown in Fig. 36 and Fig. 37. The performance summary of the loop filter is shown in Table 11.

Figure 35: Loop filter schematic
Figure 36: The loop filter magnitude response

Figure 37: The loop filter phase response
Table 11: Performance summary of the Filter

<table>
<thead>
<tr>
<th>Filter Parameters</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Order</td>
<td>5</td>
</tr>
<tr>
<td>DC gain</td>
<td>40 dB</td>
</tr>
<tr>
<td>Bandwidth</td>
<td>100 MHz</td>
</tr>
<tr>
<td>Power consumption</td>
<td>105 mW</td>
</tr>
<tr>
<td>Power supply</td>
<td>1.8 V</td>
</tr>
<tr>
<td>Technology</td>
<td>Jazz 0.18 um</td>
</tr>
</tbody>
</table>

4.2 System simulation results

Fig. 38 shows system simulation with the loop filter in transistor level. The simulated data in CADENCE are extracted, and then calculated in MATLAB shown in Fig. 39.

Figure 38: System simulation with filter in transistor level
4.3 Comparisons

The comparison between feedback and feed-forward topologies is shown in Table 12. Based on Table 12, the feed-forward topology has the advantage of lower power consumption. However, feed-forward topology also suffers from low blocker rejection. In addition, another drawback of feed-forward topology is out-of-band peaking discussed in Section 2. Table 13 shows the comparisons between this work and other wide bandwidth ΣΔ ADCs in specifications, power consumption and figure of merit (FOM).

Table 12: Comparison between feedback and feed-forward topologies

<table>
<thead>
<tr>
<th>Filter topology</th>
<th>Feed-forward</th>
<th>Feedback [9]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Power consumption</td>
<td>105 mW</td>
<td>152 mW</td>
</tr>
<tr>
<td>(filter only)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Blocker rejection</td>
<td>-4 dB @ 350 MHz</td>
<td>-26 dB @ 350 MHz</td>
</tr>
<tr>
<td>(attenuation)</td>
<td>-8 dB @ 450 MHz</td>
<td>-37 dB @ 450 MHz</td>
</tr>
</tbody>
</table>
Table 13: Brief summary of wide bandwidth $\Sigma\Delta$ ADC

<table>
<thead>
<tr>
<th>Year</th>
<th>Technology</th>
<th>Sampling Frequency</th>
<th>Bandwidth</th>
<th>SNDR (dB)</th>
<th>Power (mW)</th>
<th>FOM pJ/conv</th>
</tr>
</thead>
<tbody>
<tr>
<td>1998 [12]</td>
<td>InGaAs HEMT</td>
<td>5 GHz</td>
<td>100 MHz</td>
<td>39</td>
<td>400</td>
<td>31.2</td>
</tr>
<tr>
<td>2001 [13]</td>
<td>InGaAs HBT</td>
<td>18 GHz</td>
<td>500 MHz</td>
<td>42</td>
<td>1500</td>
<td>11.7</td>
</tr>
<tr>
<td>2003 [14]</td>
<td>InP HBT</td>
<td>8 GHz</td>
<td>250 MHz</td>
<td>40</td>
<td>1800</td>
<td>56.2</td>
</tr>
<tr>
<td>2006 [15]</td>
<td>SiGe HBT</td>
<td>20 GHz</td>
<td>313 MHz</td>
<td>-</td>
<td>490</td>
<td></td>
</tr>
<tr>
<td>2009 [3]</td>
<td>SiGe HBT</td>
<td>40 GHz</td>
<td>1 GHz</td>
<td>37</td>
<td>350</td>
<td>2.9</td>
</tr>
<tr>
<td>2011 [4]</td>
<td>45 nm CMOS</td>
<td>4 GHz</td>
<td>125 MHz</td>
<td>70</td>
<td>256</td>
<td>0.5</td>
</tr>
<tr>
<td>This Work</td>
<td>BiCMOS</td>
<td>2 GHz</td>
<td>100 MHz</td>
<td>67.6</td>
<td>300 (estimated)</td>
<td>0.7</td>
</tr>
</tbody>
</table>
5. CONCLUSIONS

In this thesis, a 100 MHz bandwidth 11-bit resolution \( \Sigma \Delta \) ADC employing LC filter sections in a feed-forward topology was presented. The system was designed in MATLAB using the \( \Sigma \Delta \) toolbox considering the various design variables and the trade-offs among them. The specifications of the individual building blocks were obtained taking into account the non-idealities. A high transconductance (100 mS) and high linearity (70 dB) OTA was designed for use in the loop filter with 34.2 mW power consumption in transistor level. The 5th order loop filter was implemented consuming less power (105 mW) compared to the loop filter implemented in the feedback topology (152 mW). The proposed topology shows 4 dB attenuation at 350 MHz and 8 dB attenuation at 450 MHz.

5.1 Future work

A couple of approaches to improve on the work presented in this thesis are noted below.

The out-of-band peaking inherent in feed-forward topology reduces the dynamic range of the ADC. Methods to reduce the peaking in the ADC need to be investigated.

The resolution of the ADC is limited primarily by the jitter of the clock signal in the feedback path. Approaches to mitigate the effect of jitter in the modulator need to be explored.
REFERENCES


VITA

Cheng-Ming Chien received the B.S. degree from National Cheng-Kung University Taiwan in 2000, and M.S. degree from The University of Texas at Arlington. He received the M.S. degree in electrical engineering from the Analog & Mixed Signal Center, Electrical & Computer Engineering Department, Texas A&M University, College Station in December 2011. His main research interests are in the field of data converters. He can be reached through the Department of Electrical Engineering, 3128 TAMU, College Station, TX 77843.