FRACTURE MODELING AND FLOW BEHAVIOR IN SHALE GAS RESERVOIRS USING DISCRETE FRACTURE NETWORKS

A Thesis

by

JOACHIM NWABUNWANNE OGBECHIE

Submitted to the Office of Graduate Studies of Texas A&M University in partial fulfillment of the requirements for the degree of

MASTER OF SCIENCE

December 2011

Major Subject: Petroleum Engineering

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Approved by:

Chair of Committee, David S. Schechter Committee Members, Jerome J. Schubert Yuefeng Sun Head of Department, Stephen A. Holditch

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ABSTRACT

Fracture Modeling and Flow Behavior in Shale Gas Reservoirs Using Discrete Fracture Networks.

(December 2011)

Joachim Nwabunwanne Ogbechie, B.Eng., University of Benin Chair of Advisory Committee: Dr. David S. Schechter

Fluid flow process in fractured reservoirs is controlled primarily by the connectivity of fractures. The presence of fractures in these reservoirs significantly affects the mechanism of fluid flow. They have led to problems in the reservoir which results in early water breakthroughs, reduced tertiary recovery efficiency due to channeling of injected gas or fluids, dynamic calculations of recoverable hydrocarbons that are much less than static mass balance ones due to reservoir compartmentalization, and dramatic production changes due to changes in reservoir pressure as fractures close down as conduits. These often lead to reduced ultimate recoveries or higher production costs.

Generally, modeling flow behavior and mass transport in fractured porous media is done using the dual-continuum concept in which fracture and matrix are modeled as two separate kinds of continua occupying the same control volume (element) in space. This type of numerical model cannot reproduce many commonly observed types of fractured reservoir behavior since they do not explicitly model the geometry of discrete fractures, solution features, and bedding that control flow pathway geometry. This inaccurate model of discrete feature connectivity results in inaccurate flow predictions in areas of the reservoir where there is not good well control.

Discrete Fracture Networks (DFN) model has been developed to aid is solving some of these problems experienced by using the dual continuum models. The Discrete Fracture Networks (DFN) approach involves analysis and modeling which explicitly incorporates the geometry and properties of discrete features as a central component controlling flow and transport. DFN are stochastic models of fracture architecture that incorporate statistical scaling rules derived from analysis of fracture length, height, spacing, orientation, and aperture.

This study is focused on developing a methodology for application of DFN to a shale gas reservoir and the practical application of DFN simulator (FracGen and NFflow) for fracture modeling of a shale gas reservoir and also studies the interaction of the different fracture properties on reservoir response. The most important results of the study are that a uniform fracture network distribution and fracture aperture produces the highest cumulative gas production for the different fracture networks and fracture/well properties considered.

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NOMENCLATURE

DFN	Discrete fracture network
DP/DK	Dual porosity/dual permeability
DP	Dual porosity
t	Time
km	Matrix permeability
kf	Fracture permeability
md	Millidarcy
nd	Nanodarcy
SD	Standard deviation
Frac	Fracture
Cv	Coefficient of variation
CI	Connectivity index
HF	Hydraulic fracture
cum	Cumulative
FA	Fracture aperture
FD	Fracture density
FL	Fracture length
HFA	Hydraulic fracture aperture
HFL	Hydraulic fracture length
NFS	Number of hydraulic fracture stages

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CHAPTER I

INTRODUCTION

As the world's conventional oil and gas fields are been depleted, the need to develop marginal and the more heterogeneous fields (fractured reservoirs) will become increasingly important.

Since the movement of hydrocarbons and other fluids in fractured reservoirs or in conventional reservoirs with significant fracture permeability often is not as expected or predicted. This behavior is due to the presence of fractures in these reservoirs which significantly affects the mechanism of fluid flow. It usually results in early water breakthroughs; reduced tertiary recovery efficiency due to channeling of injected gas or fluids; dynamic calculations of recoverable hydrocarbons that are much less than static mass balance ones due to reservoir compartmentalization; and dramatic production changes due to changes in reservoir pressure as fractures close down as conduits. These problems often lead to reduced ultimate recoveries or higher production costs.

Production experience with fractured reservoirs has repeatedly shown how understanding and exploiting the fracture connectivity at the reservoir scale is an important factor for optimizing reservoir performance (Dershowitz, 1998).

Generally, modeling flow behavior and mass transport in fractured porous media is done using the dual-continuum concept in which fracture and matrix are modeled as

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two separate kinds of continua occupying the same control volume (element) in space (Warren and Root 1963). This type of numerical model cannot reproduce many commonly observed types of fractured reservoir behavior since they do not explicitly model the geometry of discrete fractures, solution features, and bedding that control flow pathway geometry. This inaccurate model of discrete feature connectivity results in inaccurate flow predictions in areas of the reservoir where there is not good well control (Dershowitz et al., 2005).

The complexity of fracture networks means that a large quantity of data is required to characterize fracture systems adequately. Another modeling approach, the Discrete Fracture Network (DFN) model has been developed to aid is solving some of these problems experienced by using the dual continuum models. Discrete fracture network (DFN) models portray fractures and fracture connectivity very differently from other methods and it can leads to a more realistic representation of the fracture network in fractured reservoirs. The Discrete Fracture Networks (DFN) approach involves *"analysis and modeling which explicitly incorporates the geometry and properties of discrete features as a central component controlling flow and transport."* (Dershowitz et al., 2005). DFN are stochastic models of fracture architecture that incorporate statistical scaling rules derived from analysis of fracture length, height, spacing, orientation, and aperture (Guohai, 2008).

1.1 Motivation

The presence of fractures in the fractured reservoir significantly affects the mechanism of fluid flow (Gilman, 2003). The impact of fractures in these reservoirs has led to exploration and development problems; this has been a major problem for both reservoir engineers developing oil and gas fields and researchers studying nuclear repository sites. With regards to the oil and gas sector the approach is to simplify the reservoir modeling process in fractured reservoir using the dual-continuum concept in which fracture and matrix are modeled as two separate kinds of continua occupying the same control volume (element) in space (Warren and Root 1963), the shortcomings of these models is that they do not explicitly model the geometry and the discrete features of the fractures. This inaccurate model of discrete feature connectivity results in inaccurate flow predictions in the reservoir which leads to lots of reservoir development issues.

Another approach uses the discrete fracture network (DFN) models for fracture modeling of the reservoir. DFN are stochastic models of fracture architecture that incorporate statistical scaling rules derived from analysis of fracture length, height, spacing, orientation, and aperture (Guohai, 2008). DFN incorporates the geometry and properties of discrete features as a central component controlling flow and transport. This leads to a better representation of the reservoir and thus produces reservoir models that give more realistic flow predictions in the reservoir.

1.2 Objective

The aim of this project is to improve the understanding of the flow behavior in shale gas reservoirs by using a Discrete Fracture Network (DFN) simulator (FracGen and NFflow) for fracture modeling of a shale gas reservoir. It also studies the interaction of the different fracture properties on reservoir response. Finally based on the understanding from this study and also learning from literature, it will aim to improve understanding on the flow behavior in shale gas reservoirs and the fracture properties and their impact on reservoir response.

1.3 Methodology

The first stage of the study is a literature review to understand the current and the past applications of discrete fracture networks in fracture modeling for both shale gas reservoirs and other types of reservoirs and also study the flow behavior in shale gas reservoir.

The next stage will be creating fracture models with the DFN simulator and running flow simulations to see the response that would be observed from the input provided. Subsequently, the results obtained from the stage above will be interpreted and sensitivity analysis done to ensure that the right parameters where fed into the models. The final stage is to make conclusions based on the results obtained and learning from other published results in the literature.

CHAPTER II

FRACTURE LITERATURE REVIEW

2.1 Fractured Reservoir

Fractured reservoirs are primarily related to their tectonic history. Fractures form due to the loading and unloading forces in the history of the rocks.

Natural fractures are believed to represent the local stress at the time of fracturing, while large scale fractures and folds can be related to differential stress over time. Both the maximum and minimum horizontal stress components increase with increasing burial. Open fractures which are of interest in fracture modeling requires stress relief in at least one direction (Nelson, 2001).

2.2 Fracture

A fracture can be defined as any discontinuity within a rock mass that developed as a response to stress. There are two forms of fracturing modes and the classification depends on the stress system that resulted in the formation of the fracture system.

In mode I fracturing, fractures are in tensile or opening mode in which displacements are normal to the discontinuity walls. This form of fracture system forms *joints* and *veins*.

In mode II fractures which results from an in - plane shear mode, in which the displacements are in the plane of the discontinuity, this fracturing system results in faults (Golf-Racht, 1982).

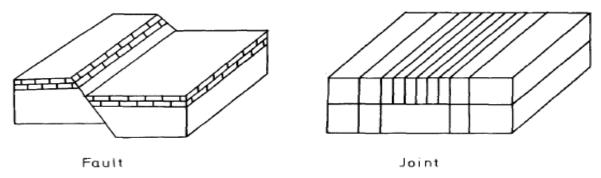


Figure 2.1: Schematic representation of a fault and a joint (Golf-Racht, 1982)

In general, a fracture in which relative *displacement has occurred* can be defined as a *fault*, while a fracture in which *no noticeable displacement* has occurred can be defined as a joint. See **Fig. 2.1**.

Fractures exist on a wide range of scales from microns to hundreds of kilometers, and it is known that throughout this scale range they have a significant effect on processes in the Earth's crust including fluid flow and rock strength (Bonnet et al., 2001).

2.3 Geometrical Description of Fractures

A fracture can be defined in a geometrical three-dimensional space with the following basic geometrical parameters (**Fig. 2.2**): dip angle, dip direction, persistence (dimension and shape) and aperture (the gap between two opposite surfaces of the discontinuity). Although there are other properties that can define a fracture these above are the basic properties that can be used to define a fracture (Jing and Stephansson, 2007).

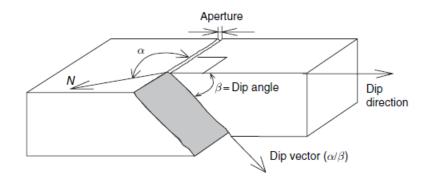


Figure 2.2: Geometric parameters associated with a discontinuity, the gap between the two surfaces is exaggerated to illustrate the aperture (Jing and Stephansson, 2007)

2.4 Fracture Mechanisms or Origins (Nelson, 2001)

2.4.1 Tectonic

- Tectonic or structure related fractures result of tectonic events like faulting and folding.
- Regional fractures normally covers large areas and have a pretty constant orientation. It can be difficult to unveil the exact cause of these fractures.

2.4.2 Non-Tectonic

- Contractional fractures cause of reduction in bulk volume. Due to thermal contraction and diagenesis.
- Surface-related fractures relate to weathering and unloading of stored stress.

2.5 Naturally Fractured Reservoir (NFR)

Natural fractures are normally generated through diagenesis or tectonic deformation and it affects most reservoirs in some way or another. This could be in a positive way - as extra fluid conduits – or in a negative way as barriers to flow or cross-flow shortcircuiting natural flow paths.

In carbonates, natural fractures typically create secondary porosity and permeability in low porosity matrix. However fractures are the fluid pathways but can also lead to early water breakthrough and inhibit secondary recovery.

In silisiclastics, they can add some permeability to existing matrix-dominated production.

Other reservoir types like basement rocks, volcanic rocks and coal-bed methane are also affected by fractures and have in later years opened the oil-companies eyes as new potential reservoirs.

2.6 Classification of Naturally Fractured Reservoirs (Nelson, 2001)

This classification is based on contribution of fractures to the total reservoir porosity and permeability (**Fig. 2.3**).

NFR's are reservoirs where naturally occuring fractures have/ are predicted to have significant effect on reservoir fluid flow and may change througout the production history:

- Increased/decreased permeability and/ porosity
- Increased permeability anisotropy

Type 1: Fractures provide primary porosity and permeability. It requires large drainage area, typically granite or quartize reservoirs with large fractures. Few wells needed for development, but a potential for high initial production and rapid decline (early water breakthrough). example: fractured granites.

Type 2: Fairly low matrix porosity and permeability, with good initial production given by fractures which provide essential permeability. example: carbonates.

Type 3: These reservoirs already have a high matrix porosity, with a fairly good permeability, but the fractures add an extra permeability component to productivity. example: sandstones.

Type 4: In compressional settings, folding may occur with differential strain, leading partly to extensional fractures, partly to compressional ones. In carbonates stylolites may form and in high porosity sandstones deformation bands may occur – all acting as barriers to flow, and some creating compartmentalization. Also existing open fractures may be mineralized or filled with clay.

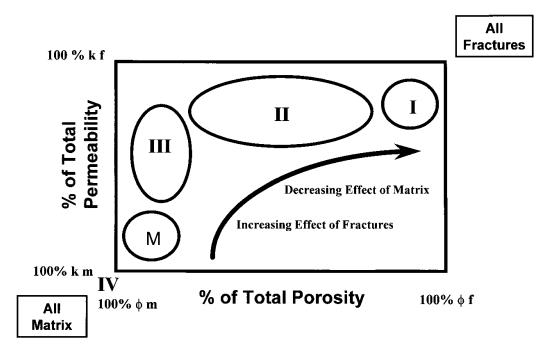


Figure 2.3: Schematic cross plot of percent reservoir porosity versus percent reservoir permeability (percent due to matrix versus percent due to fractures) for the fractured reservoir classification (Nelson, 2001)

2.7 Fracture Properties Definitions

2.7.1 Fracture Density

Fracture density is the number of fractures per unit area or volume (Davy et al., 1990). Fracture density expresses the extent of rock fracturing.

2.7.2 Fracture Length

Fracture length, i.e., the length of the fracture, Fracture length distribution is generally taken to be a lognormal.

2.7.3 Fracture Aperture

Fracture aperture also called the fracture width (e) is the distance between the fracture walls (Bonnet et al., 2001).

2.7.4 Fracture Systems

It is defined as a set of parallel fractures.

2.7.5 Fracture Cluster

A fracture cluster is a group of linked fractures. It is a term derived from percolation theory. A cluster that links opposite sides of the study is termed a "percolating cluster."

2.7.6 Fracture Intensity

Fracture intensity of a fracture set is measured either by the number of fractures per unit area or the summed lengths of fractures per unit area (Ghosh, 2009).

2.7.7 Fracture Network

A fracture network is generally defined as a set of individual fractures which may or may not intersect (Adler et al., 2009). It can also be defined as formed by two or several associated fracture sets.

2.7.8 Fracture Orientation

Fracture orientation gives the direction and tilt of the fracture. When characterizing the fracture orientation distribution, it is generally found that the fractures can be divided into a number of distinct fracture sets. These sets of fractures comprise fractures that can be characterized by common distributions of parameters, and which have a common origin and history. These fracture sets are often defined in terms of their orientation distributions which tend to be clustered around preferred orientations on a lower hemisphere projection of the poles to the fracture planes. This definition of the characteristic orientation is best achieved by using conventional statistical methods to identify distinct clusters. The fractures can then be separated into their distinct sets and further parameters inferred for each set independently. it is used for separation of different sets of fractures using stereographic projection techniques (Jing and Stephansson, 2007).

Fracture orientations measured for the outcrops are displayed on rose diagrams and equal-angle stereographic net projections for each of the measured locations (Ghosh, 2009). The best statistical analysis method for orientation of fractures is Fisher Distribution (Dershowitz et al., 2005).

2.7.9 Fracture Spacing

Fracture spacing (D) is the average distance between parallel regularly spaced fractures. It is the distance between two adjacent fractures of the same set following the same distribution function for their orientations. It is used for determination of the volume density of the fracture population (Jing and Stephansson, 2007).

2.7.10 Fracture Porosity

Fracture porosity is a percentage of void space in fractures compared to the total volume of the system. Fracture porosity is estimated using the following expression:

$$\phi_f = \left(\frac{e}{D+e}\right) * 100 \tag{2.1}$$

From the expression, it shows that fracture porosity is very scale-dependent. The value of ϕ_f can be 100% in a particular location of reservoir, but the value for the whole reservoir is generally less than 1%. According to (Nelson, 2001), fracture porosity is always less than 2%; in most reservoirs is less than 1% with a general value of less than 0.5%. An exception to these rules-of thumb is vuggy fractures where porosity can vary from 0 to a very large value.

The importance of fracture porosity in reservoir performance depends on the type of fractured reservoir. If the fracture system provides an essential porosity and permeability to the reservoir, then fracture porosity is a critical parameter to be determined in early stages of development. As contribution of matrix porosity to the whole system increases, the relevance of fracture porosity decreases. Fracture porosity is one of the fracture properties that are difficult to determine. The common sources of fracture porosity estimation are:

1) Core analysis

- 2) Porosity-permeability relationships
- 3) Field/lab determinations

4) Logs

5) Multiple-well tests

2.7.11 Fracture Permeability

Permeability defines the ability of porous medium to transmit fluids. The presence of open fractures has a great impact in reservoir flow capacity. Therefore, fracture permeability is an important factor that determines reservoir quality and productivity. Darcy's equation that is used to model fluid flow through porous media cannot be used to represent flow through fractures. Thus, parallel plate theory was developed to model fluid flow in fractures. The parallel plate model is based on fracture width and spacing concepts (Parsons, 1966), who combined the model for fracture and matrix fluid flow and obtained the following equation for fracture permeability:

$$k_f = \frac{e^2}{12} x \frac{\rho g}{\mu}$$
(2.2)

This equation assumes laminar flow between smooth, non-moving, parallel plates and homogeneous fractures with respect to orientation, width and spacing. Parson's relationship is simple but is applicable to fluid flow through fractured reservoirs.. In some cases, partially or total filled fractures can act as flow barriers. The effect of fractures on permeability depends on several factors such as morphology, orientation, and others. Fracture width and permeability are difficult to determine from direct sources such as core data or laboratory test. Well test analysis is the most common source of fracture permeability information.

2.8 Statistical Analysis in Fracture Characterization

Fracture characterization using Discrete Fracture Network (DFN) utilizes a number of statistical tools, the complexity of fracture networks means that a large quantity of data is required to characterize fracture systems adequately. These tools aid in ensuring that

the limited information provided by the various sources are applied to describe the entire well and characterize the reservoir properly.

2.8.1 Histograms

Histograms are used for the evaluation of the most frequent range of the variations of a given parameter. The data are generally collected in relation to a given criterion, such as lithology, pay interval, number of cores, types of fractures, etc. Histograms are applied to almost all parameters which define single fractures or multi-fractures characterization. From the frequency curve and cumulative frequency curve, the range of average values of a given parameter is obtained by a conventional procedure.

2.8.2 Geometrical Models

Geometrical models (especially in the case of matrix block units), using a stereographic projection approach for magnitude and shape. Polar stereogram and various other schematic representations are particularly useful in the identification of the preferential trends of certain parameters, which often help in the description of the properties for large groups of fractures.

2.8.3 Log – Normal Distribution (Davis, 1986)

The log normal distribution is closely related to the normal distribution. If the logarithm of a variable is normally distributed, then the variable itself is log normally distributed. The log-normal distribution is skewed with a long tail on the right hand side. However, after transforming the data by taking the log of the variable, the distribution becomes symmetric and normal. If we consider X to be a log normally distributed variable, then we can define $Y= \ln X$, where Y is the value of the natural logarithm of the random

variable X. If the mean of the variable Y is μ and the variance is σ^2 , we can write the probability density function for the variable X as,

$$f(x) = \frac{1}{x\beta\sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{\ln x - \alpha}{\beta}\right)^2\right]$$
(2.3)

The mean and variance of the random variable X, is related to the mean and variance of the transformed variable Y through,

$$\alpha = \ln \mu - \frac{\beta^2}{2} \tag{2.4}$$

$$\beta^2 = \ln\left[1 + \frac{\sigma^2}{\mu^2}\right] \tag{2.5}$$

where μ is the mean of the variable X, and σ^2 is the variance of the variable X. So given a mean and variance of a variable, we can generate the log-normal values by first deriving the mean and variance from **eqn. 2.4 and 2.5**, and then standardizing this using,

$$z = \frac{\ln x - \alpha}{\beta} \tag{2.6}$$

This is similar to the normal distribution. So by choosing a range of values for *z*, we can generate a range of values for x.

2.8.4 Exponential Law (Bonnet et al., 2001)

This law has been used to describe the size of discontinuities in continental rocks (Cruden, 1977; Hudson and Priest, 1979; Nur, 1982; Priest and Hudson, 1981) and in the vicinity of mid-oceanic ridges (Carbotte and McDonald, 1994; Cowie et al., 1993). In these cases, fracture growth results from a uniform stress distribution (Dershowitz and Einstein, 1988), and propagation of fractures can be compared to a Poisson process (Cruden, 1977) resulting in an exponential distribution given by:

$$n(w) = A_2 \exp(-w/w_0)$$
 (2.7)

where A_2 is a constant, The exponential law incorporates a characteristic scale w_0 eqn. 2.7 that reflects either a physical length in the system, such as the thickness of a sedimentary layer or the brittle crust (Cowie, 1998), or a spontaneous feedback processes during fracture growth (Renshaw, 1999). Numerical simulations performed by (Cowie, 1995), and experimental results of (Bonnet, 1997) have shown that exponential distributions of fracture length are also associated with the early stages of deformation, when fracture nucleation dominates over growth and coalescence processes.

2.8.5 Gamma Law (Bonnet et al., 2001)

The gamma distribution is a power law with an exponential tail (**Fig. 2.4**) and is in common use in fault or earthquake statistics and seismic hazard assessment (Davy, 1993; Kagan, 1997; Main, 1996; Sornette and Sornette, 1999). Any population that obeys this kind of distribution is characterized by a power law exponent *a*, a characteristic scale w_0 (**eqn. 2.8**).

$$n(w) = A_3 w^{-a} \exp(-w/w_o)$$
(2.8)

The characteristic scale w_0 may be related to the correlation length in the spatial pattern, where it implies an upper bound for fractal behavior (Stauffer and Aharony, 1994),or may depend on deformation rate (Main and Burton, 1984). When w_o is greater than the size of the system wmax, the gamma law reduces to a power law, and, conversely.

2.8.6 Power Law (Bonnet et al., 2001)

The power-law distribution is a straight line on a log-log plot. Numerous studies at various scales and in different tectonic settings have shown that the distribution of many fracture properties (i.e., length, displacement) often follows a power law.

$$n(w) = A_4 w^{-a} (2.9)$$

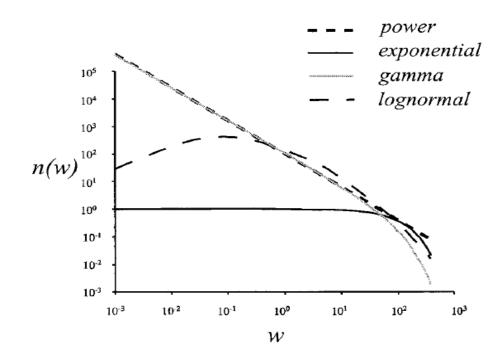


Figure 2.4: Plot illustrating the four different functions (power, lognormal, exponential and gamma law) most often used to fit data sets. Data over more than 1 order of magnitude are needed before these different distributions can be easily distinguished (Bonnet et al., 2001)

Power law distributions have the important consequence that they contain no characteristic length scale (**equation 2.9**). In nature the power laws have to be limited by physical length scales that form the upper and lower limits to the scale range over which they are valid. It is now generally recognized that resolution and finite size effects on a power law population can also result in distributions that appear to be exponential or lognormal.

But with the rise of scaling concepts in earth sciences, power law distributions have been favored over lognormal distributions because of their greater physical significance (Barton and Zoback, 1992). However, all power laws in nature must have upper and lower cutoffs.

2.9 Statistical Methods for Measuring Fracture Size Distributions

There are three different types of distribution that are commonly used to characterize fracture properties; these are:

- (a) Frequency
- (b) Frequency Density
- (c) Cumulative Frequency Distributions.

2.9.1 Frequency

The Frequency represents the occurrence of the fracture property.

2.9.2 Frequency Density

It represents the density of the occurrence of the fracture property.

2.9.3 Cumulative Frequency Distributions

The cumulative frequency distribution represents the number of fractures whose length is greater than a given length l and corresponds to the integral of the density distribution n(l).

$$C(l) = \int_{l}^{l_{max}} n(l)dl \tag{2.10}$$

where *l*max is the greatest length encountered in the network. Hence if n(w) is a power law characterized by an exponent equal to *a* (eqn. 2.9), the cumulative distribution will be a power law for $w \ll w$ max with an exponent equal to *a* - 1, the cumulative

distribution has been widely used because it is easily computed. In practice, it is constructed by summing incremental frequency data, and hence tends to give a smoother trend than the frequency or density distributions, increasing artificially the regression coefficient (Bonnet et al., 2001).

CHAPTER III

FRACTURE CHARACTERIZATION AND MODELLING

3.1 Fracture Modeling

Fractures are commonly found in most outcrops – and they affect fluid flow in most oil and gas reservoirs in the world. Natural fractures can introduce a high heterogeneity or a strong anisotropy in reservoirs, and affects fluid flow or mechanical stability. An accurate description and characterization of the fracture network is therefore of very high importance for all stages of reservoir management: drilling, well placement, stimulation, completion and production profile design.

There is an interrelationship between fractures, fluid-flow and effective stress state in the reservoir. Decisions must be made early to:

- 1. Identify and characterize fractures (at the exploration stage) and evaluate anisotropy etc. at the development stage for better infill drilling campaigns.
- 2. How does fractures (fracture network) affect fluid flow?
- 3. What is the effect of change in stress with production?

The purpose of fracture network modeling is to integrate geophysical, geological, petrophysical and reservoir engineering findings on fracture characterization to create a reliable fracture network model, with the aim to predict the distribution of fracture sets properties with the reservoir and build a fracture network model which will be used for flow computation and simulation.

This process aids in creating simulation properties for matrix and fractures and determines the interaction between the fracture and the matrix to be able to predict reservoir behavior. By modeling the fractures explicitly, the spatial relationships between properties in adjacent cells can be observed.

Fracture modeling is the most important aspect of the process to decipher the role of natural fractures in reservoir performance, and, eventually, it is the primary method for optimizing hydrocarbon recovery from fractured reservoir. It is a multi-step process involving several disciplines within reservoir characterization and simulation. The main idea is to build on geological concepts and gathered data such as interpretation of beds, faults and fractures from image log data, use field outcrop studies as analogs for conceptual models, seismic attributes used as fracture drivers etc. The next step is to transfer these data into a description of fracture intensity which can be populated into a geological framework model. Depending on the analysis of the fracture data, multiple sets of fractures can be identified; these can be the result of different tectonic events such as over-trusts and extensional faults, conjugate fractures related to bending or flexure of geological layers or simply related to difference in lithology.

Once data have been identified, analyzed and categorized, the fracture model itself can be built. From our initial intensity description we need to populate the fracture intensity in the reservoir model stochastically or deterministically. The deterministic method needs to have a very good idea of where and how the fractures behave in the reservoir; if no such data exist, a stochastic method should be used. The ultimate goal is to find reservoir grid properties which describe permeability and porosity for fractures as well as the standard permeability and porosity for the matrix they are either naturally fractured or consists of e.g. carbonates which are vugular or heavily fractured due to tectonic processes. Some of these reservoir rocks are originally dense and have little flow or storage capacity in the matrix, but once fractured, certain areas will become high flow zones. To try resolving this problem a Discrete Fracture Network (DFN) model based on intensity can be built. Up scaling properties based on a DFN model will generate a seconds set of properties of permeability, porosity and a sigma factor – describing the connectivity. This sigma/connectivity is essential in connecting 'duplicate' cells in a simulator describing the matrix and fracture porosities and permeabilities.

Fractures can increase effective porosity and permeability and introduce permeability anisotropy, particularly in rocks with low matrix permeability (Nelson, 2001). Faults can also function as fluid migration pathways, barriers, or a combination of both. For modeling and production purposes it is important to document directions of preferred fracture and fault orientations within primary hydrocarbon traps, such as anticlines. By understanding controls on fracture and fault orientation and distribution in a given reservoir the accuracy of flow modeling can be improved, thereby increasing primary and secondary hydrocarbon recovery.

The relationships between bed thickness, structural curvature and fracture porosity and permeability can be effective in evaluating geologic structures as hydrocarbon reservoirs. Rocks, in general, exhibit increased fracture density with increased deformation (Nelson, 2001).

Improving the recovery from fractured reservoirs is an increasingly important focus for many oil companies. The recovery from reservoirs where fractures dominate permeability is often a fraction of the resource recovered from conventional reservoirs in which matrix permeability dominates. The lower recovery and higher risks relate to the difficulty in forecasting how various completion placements, gel treatments, surfactant injection, and tertiary recovery processes will actually perform. A reduction in risk and an improvement in understanding of reservoir behavior will lead to enhanced profitability from under-exploited fractured fields.

Fractures do more than simply increase reservoir permeability. Fractures fundamentally alter reservoir connectivity and heterogeneity. Fractured reservoirs could be modeled with the same level of confidence and success as matrix-dominated reservoirs.

More successful exploitation of fractured reservoirs has been hindered by the lack of reservoir management tools that incorporate the unique flow behavior of reservoir fracture systems.

3.2 Fracture Modeling Approach

There are two broad methods for fracture modeling that is commonly used to model field-scale fluid flow in naturally fractured petroleum reservoirs.

- (a) Continuum Model
- (b) Discrete-Fracture Network (DFN) Models

3.2.1 Continuum Model

The Continuum model (**Fig. 3.1**) is one in which fracture and matrix are modeled as two separate kinds of continua occupying the same control volume (element) in space (Warren and Root 1963).

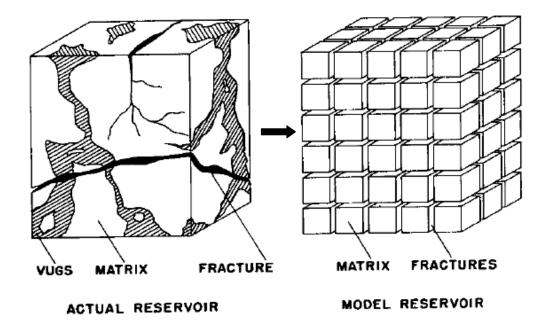


Figure 3.1: Dual-porosity model (Warren and Root 1963)

The fracture and matrix each has its own set of properties. Fluid transfer between the porous media is controlled by the pressure difference between components, fluid viscosity, matrix permeability, and a geometric factor known as shape factor. The shape factor is an important parameter in understanding the transfer function or fluid mechanism between the matrix system and fracture system vice versa. The shape factor (σ) introduced by (Kazemi et al., 1976) is given by:

$$\sigma = 4 * \left(\frac{1}{L_x^2} + \frac{1}{L_y^y} + \frac{1}{L_z^2}\right)$$
(3.1)

The shape factor represents the geometry of the matrix elements and controls flow between two porous media (matrix and fracture).

The continuum model is subdivided into two methods based on the nature of the fracture- matrix fluid interaction fractured reservoir (**Fig. 3.2**).

a) Dual Porosity

b) Dual Porosity/Dual Permeability

3.2.1.1 Dual Porosity

The first approach is a dual-porosity (DP) idealization of the reservoir, where a typical representative elementary volume (REV) of reservoir rock is assumed to contain a large number of equal size matrix blocks separated by interconnected fracture planes. In this method, flow to the well is only through the fracture system. Thus the fracture system acts as the primary flow path while the matrix system is the storage system and the fluid flow is from the matrix to the fracture. There is no matrix-to-matrix flow.

3.2.1.2 Dual Porosity/Dual Permeability

The second approach is the dual-porosity/dual-permeability (DP/DK) idealization of the reservoir, where, contrary to the dual-porosity case, the matrix blocks also communicate with each other; therefore, there is matrix-to-matrix flow in addition to matrix-to-fracture flow.

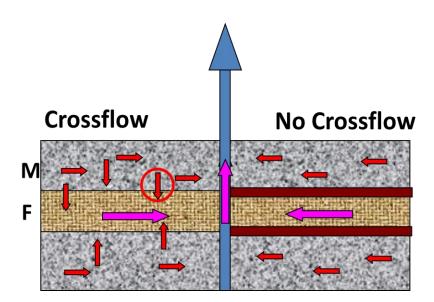


Figure 3.2: Illustration of dual porosity/single permeability and dual porosity/dual permeability models (Schlumberger, 2010)

3.2.1.3 Dual Porosity Model Types

Dual porosity idealization is a simplification of the real reservoir which is done with dual porosity model, fluid flow and transport exist in both the connected fractures and matrix blocks, there are two overlapping continua, where both are treated as porous media.

There are 3 dual porosity models that are widely used for simplification of the

reservoir (Fig 3.3).

- Slab model (sheet of parallel fracture sets)
- Matchsticks model (2 orthogonal fracture sets)
- Sugar cube model (3 orthogonal fracture sets)

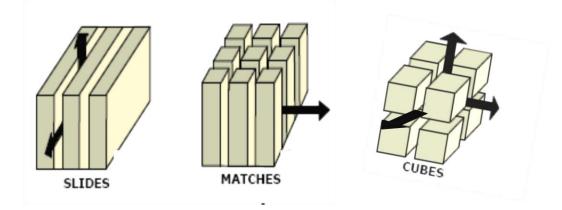


Figure 3.3: Illustration of the 3 types of dual porosity models (Reiss, 1980)

3.2.2 Discrete-Fracture Network (DFN) Models

The second fracture modeling approach is the Discrete Fracture Network (DFN) flow modeling; it is a more recent method, which relies on spatial mapping of fracture to construct an interconnected network of fracture surfaces.

The DFN model was developed to aid is solving some of these problems experienced by using the dual continuum models. The Discrete Fracture Network (DFN) approach involves "analysis and modeling which explicitly incorporates the geometry and properties of discrete features as a central component controlling flow and transport." (Dershowitz et al., 2005). DFN are stochastic models of fracture architecture that incorporate statistical scaling rules derived from analysis of fracture length, height, spacing, orientation, and aperture (Guohai, 2008).

There are two methods of definition of fracture network geometry description using the DFN models.

(a) Deterministic Modeling Method

(b) Stochastic Modeling Method

3.2.2.1 Deterministic Modeling Method

In the deterministic method the fracture network system is described explicitly, with the location and orientation of all the fractures incorporated into the model. This is clearly not possible for almost all networks of interest in the reservoir scale, since the details of the fracture network in the rock away from exposures or boreholes cannot be known. In practice it is not possible to simulate using this method, where a continuum model is used this may be appropriate, and a best estimate made of the effective properties that should apply to the region. The uncertainty due to the random location of many small features is no longer considered since only larger scale average results are being predicted (Herbert, 1996).

3.2.2.2 Stochastic Modeling Method

Stochastic modeling method is based on a statistical description of the fracture system to be represented. The statistical properties of the fracture network system are measured and fracture networks are generated that exhibit the same statistics. This means that our models are not exact representations of the real physical fracture network, and one would not expect any individual model to give an accurate prediction of the detailed flow in the real network. However, if one simulates many different realizations of the fracture network flow system, each having the same statistical properties as the real network, then the range of model results should bound the behavior of the real network (if a good statistical description of the fracture network has been used). For this to be the case, it is important that sufficiently many realizations of the fracture network have been generated and simulated. If only a few realizations are used then the distribution of possible behavior will not be accurately predicted and in particular the likelihood of more extreme behavior will not be known. Ideally, several hundred realizations may be necessary to determine this probability distribution of equally likely results and to predict, say 95% confidence limits. In practice, it is not always possible to simulate efficiently many realizations and often more qualitative bounds are estimated from a smaller sample of model results (Herbert, 1996).

CHAPTER IV

ANALYTICAL APPROACH

4.1 Study Approach

This study utilizes a set of programs that was developed by the national energy technology laboratory (NETL), to simulate gas reservoirs that consist of irregular, discontinuous or clustered strata-bound fracture networks within a tight matrix. It generates fracture networks, to simulate reservoir drainage/recharge, and to plot the fracture networks and reservoir pressures. These programs are suitable for reservoir modeling of reservoirs that produces relatively dry gas, with little interference from water or oil. The reservoir rock (matrix) has less than 1 md permeability. Variations in fracture apertures, density and connectivity are the dominant causes of heterogeneity in gas flow. Flow conductors are oriented nearly vertical and are strata-bound extending from bottom to top of beds that can be modeled individually (Boyle et al., 2010).

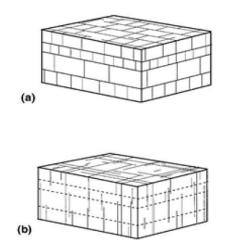


Figure 4.1: Diagram showing the major features of (a) strata-bound (b) non-strata- bound joint systems (Odling et al., 1999)

Fig. 4.1 shows a strata-bound system and a non-strata-bound joint system. In strata-bound systems, joints are largely confined to individual beds, their size is limited to a narrow range, and spacing is regular. In non-strata-bound cases, joint sizes cover a wider range, fractures cross-cut bedding and spacing tends to be clustered (Odling et al., 1999).

The five main components to the software package are:

(1) Fracture network generator (FracGen)

(2) Flow simulator (NFflow),

(3) Graphical output software (Fracout),

(4) PC-based User Interface (Fracflow)

(5) Ancillary programs for input data analysis or manipulation.

4.2 DFN Simulator (FracGen and NFflow)

The two main components of the DFN simulator program are FracGen and NFflow.

4.2.1 FracGen: Fracture Network Generation (Boyle et al., 2010)

FracGen is a computer program that uses an input file containing parameters and statistics for fracture attributes to generate a stochastic network of lines that represent some of the patterns of fractures commonly found within thin strata-bound petroleum reservoirs and aquifers. The stochastic network is generated by in an increasing complexity through a Monte Carlo process that samples fitted statistical distributions for various network attributes of each fracture set. FracGen fracture patterns are inherently two-dimensional and single layered, although the realizations of fractured layers can be

stacked to represent multi-layer reservoirs. Within stacked single layers or multi-layer realizations, individual fractures can penetrate two or more layers.

FracGen generates fracture patterns that range across the spectrum from regular to random, to clustered. It assumes that all fractures are strata-bound and vertical also the bed thickness is constant. If the host rock unit is massive or highly variable in thickness, or if many fractures fail to extend across definable beds (i.e., average fracture height is much less than bed thickness), other models might be more appropriate. Likewise, if many fractures are not perpendicular to bedding or if low-angle faults penetrate the reservoir; other network generators should be used.

Fractures are stochastically generated using one of three models, referred to as Model 1, Model 2, and Model 3. Model 0 supports Models 1 and 2 in the generation of fractures from borehole and sample trace information. (Boyle et al., 2010)

Model 1 generates randomly located fractures, although the connectivity controls can be used to produce various degrees of clustering, including unintended clustering.

Model 2 generates fracture swarms (elongated clusters), whereby the swarms are randomly located and can overlap.

Model 3 generates fractures in a spectrum of patterns that range from regular to uniform to random to clustered.

4.2.1.1 Connectivity Control in FracGen (Hatzignatiou and McKoy, 2000)

There are three methods for controlling connectivity:

(1) Fracture end-point shifting,

(2) T-termination frequency control,

(3) Intersection frequency control.

The first method involves moving each fracture end-point toward or away from the center point to the first point of intersection found with another fracture. The user can specify whether this point of intersection is with pre-existing fractures, or with subsequently generated fractures, or both. Termination and intersection frequency control is implicit in the selection of the percentage of fracture length over which end point shifting is allowed.

T-termination frequency control involves moving fractures to new locations or, preferentially, swapping fracture orientation until the end-point shifting function improves the match with the user-specified percentages of fractures having zero, one or two T-terminations. The computer program controls the percentage of fracture length over which end-point shifting occurs.

Intersection frequency control is used when it is important to explicitly control the frequencies of intersections, as when modeling cross-fractures and late formed fractures, it starts with a proposed fracture of maximum allowable length and counts all fracture intersections with the proposed fracture. If an acceptable number of intersections are found, it truncates the proposed fracture at the optimal number of intersections needed to improve the match with the user-specified distribution of intersection frequencies. If too few intersections are found, it swaps the proposed fracture orientation or location and repeats the process until the desired number of intersections is found. The user-specified frequency of T-terminations is matched also, but this is of lower priority. To control the resulting fracture length distribution, maximum and minimum acceptable fracture lengths are established and varied by the computer program to produce a distribution with lognormal statistics that match those specified by the user.

Another useful connectivity control mechanism is the synthetic annealing process which can be used in all three models to move fractures to locations where userspecified terminations or intersections are achieved. Both the second and third methods of connectivity control use a type of synthetic annealing in which the user specifies the frequency for swaps of location, the total number of swaps allowed per fracture, and the percentage of fractures generated in a set before synthetic annealing begins. These controls limit unintended parent-daughter clustering.

The user has the option to specify the locations of individual fractures and individual clusters. Therefore, it is possible to include within a generated network, known fault/fracture locations, seismically interpreted faults and fractures, and interpreted regional or local variations in fracture density or orientation.

4.2.2 NFflow Natural Gas Flow Modeling

NFflow is a numerical model for simulating naturally fractured tight gas tight (< 1 md matrix permeability) fractured reservoirs based on stochastically derived fracture patterns that mimic a complex system of interconnected natural fractures in the reservoir (Sams, July 1995.). NFflow is for single-layer and multi-layer reservoirs with the possibility of tilt and curvature. This approach permits a more accurate and realistic representation of fractured porous media fluid flow compared to traditional deterministic naturally fractured reservoir formulations.

The NFflow simulator is a single-phase (dry-gas), two-dimensional numerical model that solves fluid flow equations in both matrix and fracture domains sequentially for strata bound naturally fractured reservoirs. The mathematical model decouples fluid flow in fractures and matrix, and solves a one-dimensional un-steady state flow problem in the matrix domain to compute the volumetric flow rates from matrix into fractures and wellbores. Subsequently, the model uses the computed recharge fluid rates entering the fractures at the middle point of each fracture segment to compute the pressure and flow rate distributions at each node of the system by solving a two-dimensional fluid flow problem in the fracture domain (Sams, 1995). Flow through the rock is modeled with Darcy's Law, while flow along fractures is modeled as a linear (cubic law) function of the pressure difference between the recharge points and the fracture intersections. It assumes that there is no flow from matrix block to matrix block.

NFflow can operate in either rate-controlled mode or pressure-controlled mode. NFflow is designed to work with the fracture network files from FracGen but can be used with any fracture network descriptions having the proper format regardless of the source (Hatzignatiou and McKoy, 2000).

The flow simulator requires two input files, a run control file (*.RES) and fracture network description file (*.FLO). The *.RES file contains all data except the flow network description, which is contained in *.FLO. The flow network description consists of the fracture network plus the user-specified well descriptions (Boyle et al., 2010).

4.3 Ancillary Program

There are a number of ancillary programs but the one that was utilized most for this study is the fracture aperture reduction model (FARM).

4.3.1 Fracture Aperture Reduction Model (FARM)

It aids the user to reduce the hydraulic apertures within a specified distance around the well bore by running the ancillary program FARM.EXE. This program may be used in a trial-and-error process until the best possible match with production data or well test data is obtained for each of one or more networks (Boyle et al., 2010). Thus, the user estimates the amount and distance of fracture obstruction by drilling fluids, stimulation fluids, condensates, precipitates, or bio-fouling. It aids in reducing the flow from one or more fracture networks, this is done by shrink the apertures of the fractures nearby the well to account for the drilling damage. This adjustment is done on the fracture apertures in *.FLO file.

4.4 Summary of Sample Files for FracGen and NFflow (Boyle et al., 2010)

The list below is a summary of different FracGen and NFflow files.

- a) DAT File is the Input data file for FracGen; it is used to create fracture network realizations.
- b) FLO file is an output file from FracGen.
- c) DIA file is the output file from FracGen. It contains the diagnostics for the fracture network in the FLO file. It contains information on how many fractures are in the network, the connectivity of the fractures, and other

statistics. It aids the user to see if the network generated is what was desired and what inputs should be changed to generate better networks.

- RES file is the Input file for NFflow. It contains the reservoir parameters, PVT data, recurrent data for the flow simulation and the well performance schedule.
- e) BHP file contains the field data file for the performance of well. The file is can be produced by the ancillary program BHP.EXE, which calculates bottom-hole pressures using the Cullender and Smith method. This file contains a number of lines of information related to the calculation of bottom-hole pressures. At the bottom of the file is a table of well head flowing pressures and calculated bottom-hole flowing pressures for various times indicated. This file's specification must be listed in the *.RES file if the historical data is to be shown on any pressure versus time plots made by NFflow.
- f) LDF file is the file that gives the layer depths across the reservoir. This is the input needed to describe the upper and lower bounds of each layer and properties that are allowed to vary across the layer. The layer boundary definition is designed to allow the user to add continuous, smooth features to a reservoir (i.e. tilt, curvature); describing discontinuous features (i.e. faults) may produce unexpected results.

4.5 Mechanism for Fracture Generation in FracGen

Fractures are generated by randomly selecting fracture center-points and then assigning a length, aperture, and orientation to each center-point to define each fracture. Fracture attributes are defined by fixed variables and statistical distributions.

We begin to derive our fracture network models by assuming that an investigator can treat fractures as straight line segments (or rectangles in three-dimensional space) that are defined by a center point, a length, and an azimuth (all fractures are perpendicular to bedding).

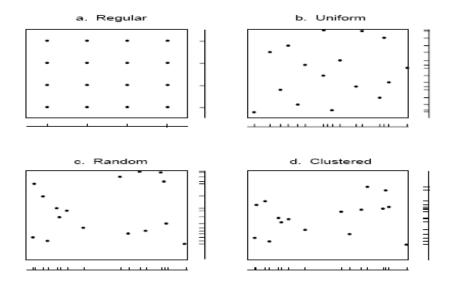


Figure 4.2: Fundamental spatial patterns of points (1-D & 2-D) (McKoy and Sams, 1997)

Fig. 4.2 illustrates the fundamental two-dimensional patterns of points in space. These patterns and are associated with different probability distributions. On the bottom and right sides of each two-dimensional illustration, **Fig. 4.2** shows the equivalent one dimensional pattern of point locations. The one-dimensional pattern is similar to the patterns that would arise if the points in the two-dimensional illustrations were converted into lines (fractures) oriented perpendicular to the one-dimensional illustrations.

Fig. 4.3 represents relative frequency histograms for the fundamental distributions of spacing, as observed along a sample line. These distributions of spacing arise from the fundamental patterns of point location. The pattern of points on a map is "random" (**Fig. 4.2c**) if each quadrant has the same probability of containing a center point as all other quadrants of equal size and if all points are placed without regard to the placement of other points. The distribution of spacing between random points along a sample line is exponential (**Fig. 4.2c**), with more short spacing's than long spacing's. The pattern of points on a map is "clustered" (**Fig. 4.2d**) if equal-size quadrants have different probabilities of containing a center point and if any points are placed in relation to the placement of other points.

In nature, things tend to be related, and fractures develop more closely together in some areas because of stress variations around primary or structural features in the rock or because of local stress variations originating in the underlying rock. In other areas, fractures are more uniformly spaced because each fracture relieved the ambient stress as it formed. Consequently, truly random fracture locations in rock are probably rare. Locations that appear random probably result from a combination of a uniform distribution and a clustered distribution.

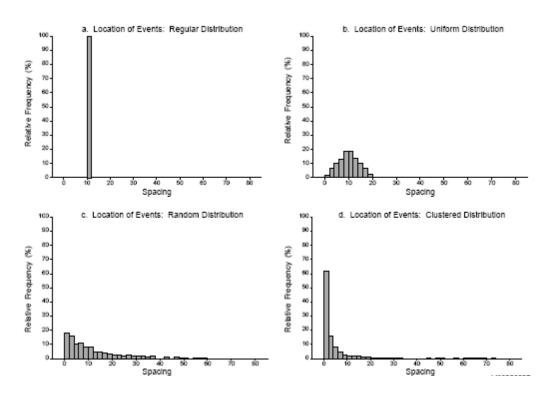


Figure 4.3: Fundamental distributions of spacing that arise from the fundamental spatial patterns of points (McKoy and Sams, 1997)

4.6 Drainage of Complex Fracture Network and Matrix

The mathematical model described below was developed to study reservoir performance as a function of fracture pattern, fracture network connectivity, and variations in matrix block size populations and locations. To this end, the model characterizes the reservoir in terms of the fluid carrying capacity of fractures, fluid flow paths, and the effective volumes drained by fracture segments (McKoy and Sams, 1997).

4.7 Approach: Mathematical/Numerical Model

The material balance and flow equations that constitute the mathematical model for the gas reservoir simulator is developed below.

The cubic law which is a linear function of the pressure difference between the recharge points and the fracture intersections shown in **eqn. 4.1**.

$$Q_{\nu} = -\frac{hw^3}{12\mu} \frac{dp}{ds} \tag{4.1}$$

where *h* is the formation thickness, *w* is the fracture aperture, μ is the gas viscosity, and $\frac{dp}{ds}$ is the gas pressure gradient. The molar gas density is given by the real gas equation of state.

$$\rho = \frac{1}{RT} \frac{p}{z} \tag{4.2}$$

where R is the gas constant, T is the temperature, and Z is the Z factor which is a function of T and p. The reservoir is assumed to be isothermal so the temperature dependence of Z is suppressed in the derivation.

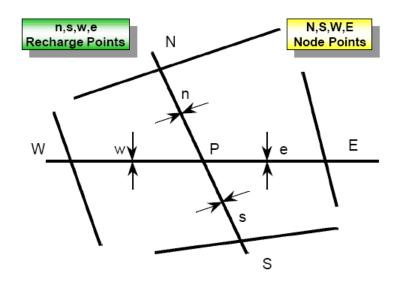


Figure 4.4: Schematic of fracture network and flow nodes modeling (Hatzignatiou, 1999)

Writing the material balance at P (Figure 4.4) yields,

$$-\left(\frac{hw^{3}}{12\mu}\right)_{w}\left(\frac{1-p\ dp}{RT\ Z\ ds}\right)_{P-w} + \left(\frac{hw^{3}}{12\mu}\right)_{e}\left(\frac{1-p\ dp}{RT\ Z\ ds}\right)_{P-e} - \left(\frac{hw^{3}}{12\mu}\right)_{s}\left(\frac{1-p\ dp}{RT\ Z\ ds}\right)_{P-s} + \left(\frac{hw^{3}}{12\mu}\right)_{n}\left(\frac{1-p\ dp}{RT\ Z\ ds}\right)_{P-n} = 0$$

$$(4.3)$$

By defining the real-gas pseudopotential (potential for short) as

$$\Phi = \int_0^p \frac{p' \, dp'}{\mu(p') Z(p')} \tag{4.4}$$

The primary nodes of the finite difference numerical system are placed in active fractures (fractures intersecting other fractures or wells), whereas nodes are not assigned in the matrix volume of the reservoir. **Fig. 4.4** illustrates two main fractures with north-south and west-east directions intersected at point P, and four additional fractures intersecting with the two main ones defining the intersection points W, E, N, and S. In this system there are five nodes (P, W, E, N, S) and four fracture segments WP, EP, NP, and SP. Fluid is allowed to flow from a matrix element into a fracture segment in both directions and at recharge points w (west), e (east), n (north) and s (south) defined at the middle of the four fracture segments. Note that any fracture which does not intersect another fracture or a well is considered as inactive and not a part of the fluid transportation network (Hatzignatiou, 1999).

The recharge fluid volumes from matrix into active fractures are computed by discretizing the matrix volume drained by each fracture segment and solving a onedimensional, linear, unsteady-state, porous media flow problem. The grid blocks in the matrix volumes, generated internally by the simulator, are non-uniform (grids located next to the fracture or wellbore are small and become progressively larger as a function of distance). The matrix block volumes are essentially "effective pore volumes" computed by NFflow to preserve fluid in place. The matrix of the system is considered homogeneous and isotropic and gravitational forces are neglected. The compressibility of both fracture and matrix is considered to be negligible, i.e., the matrix and fracture porosities are pressure independent (Hatzignatiou, 1999).

4.7.1 Fracture Flow

The flow in the fracture network is described by Poiseulle's law. The numerical model is developed based on the gas pseudo pressure function which for convenience is defined as the half of the well-known gas pseudo pressure function. The finite difference approximation of the material balance equation at node P (see **Fig. 4.4**) (Sams, July 1995.):

$$(R_p^f)^l + 2\sum_j TX_j (\delta \phi_j^f - \delta \phi_j^f) = 0 \quad ; \ j = w, e, n, s$$
(4.5)

where R_p^f denotes the residual function,

$$\delta \phi_j^f = (\phi_j^f)^{n+1} - (\phi_j^f)^k \quad ; \ j = w, e, n, s \tag{4.6}$$

The gas pseudo pressure function difference, and

$$TX_j = \left(\frac{hw^3}{12\Delta x}\right)_j; \ j = w, e, n, s$$
(4.7)

Equation 4.7 defines the transmissibility in the fracture at fracture segment midpoints w, e, n and s. A material balance in a control volume in the fracture segment network expressed in a finite difference form yields relationships that relate pseudo pressure function values at fracture segment mid-points and node points (P, W, E, N and S):

$$\delta\phi_j^f = \frac{r_j(R_j^m)^l}{2TX_j} + r_j \left(\delta\phi_j^f - \delta\phi_P^f\right) \; ; \; j, i = w, W, e, E; N, n, S, s \tag{4.8}$$

With the residual function at recharge points in fracture segments expressed as

$$(R_j^m)^l + 2TX_j \left(\delta \phi_j^f - \delta \phi_p^f\right) - 4TX_j \delta \phi_j^f + T_j^m \left[\left(\frac{\partial \phi^m}{\partial \phi^f}\right)_j - 1 \right] + \delta \phi_j^f - \frac{V_f}{\Delta t} \left[\frac{d}{d\phi^f} \left(\frac{p}{z}\right) \right]_j \delta \phi_j^f = 0 \ ; \ j = w, e, n, s$$

$$(4.9)$$

In eqn. 4.8, r_i at recharge point j is equal to,

$$r_j = 2\left(\frac{TX}{4TX+T^c}\right)_j; \quad j = w, e, n, s$$
(4.10)

With the matrix transmissibility at *j* defined as

$$T_j^m = 2\left(\frac{A_m k_m}{\Delta x_m}\right)_j; \quad j = w, e, n, s$$
(4.11)

and

$$T_j^c = T_j^m \left(1 - \frac{\partial \phi^m}{\partial \phi^f}\right)_j + \frac{v_f}{\Delta t} \left[\frac{\partial}{\partial \phi} \left(\frac{p}{z}\right)\right]_j \quad ; \quad j = w, e, n, s$$
(4.12)

The expressions for fracture segments WP, EP, NP and SP from **eqn. 4.8** can be substituted into the nodal material balance expression, **eqn. 4.17**, to obtain the following finite difference equation:

$$(R_{p}^{f})^{l} + \sum_{j} r_{j} (R_{j}^{m})^{l} + 2 \sum_{j,i} T X_{j} r_{j} \delta \phi_{j}^{f} + 2 \delta \phi_{p}^{f} \sum_{j} T X_{j} (r_{j} - 1) = 0; \qquad (4.13)$$
$$j, i = w, We, E; N, n, S, s$$

This expression applied to the fracture and wellbore flow nodes yields a system of equations with unknowns the flow nodes gas pseudo pressure function difference. The solution of this system is obtained via the Gauss-Seidel over-relaxation method (Aziz and Settari, 1985; Sams, July 1995.)

4.7.2 Matrix Flow

The flow in matrix is governed by Darcy's equation provided that turbulence flow effects are not present; the fluid flow problem in the matrix is solved numerically as a onedimensional problem with the grid block system drained into a well or a fracture. Following standard techniques, a fully-implicit, finite difference formulation of the material balance equation for dry-gas flow in porous media and in residual form yields:

$$R_{m}^{k} = T_{m,i+1/2} \Delta \phi_{m,i+1} - \left(\left(\frac{PV_{m} \mu c}{\Delta t} \right)_{i}^{k} + T_{m,i+1/2} + T_{m,i-1/2} \right) + \Delta \phi_{m,i} + T_{m,i-1/2} \Delta \phi_{m,i-1}$$
(4.14)

where

$$R_{m}^{k} = T_{m,i+1/2} (\phi_{i}^{k} - \phi_{i+1}^{k}) - T_{m,i-1/2} (\phi_{i-1}^{k} - \phi_{i}^{k}) + \left(\frac{PV_{m}\mu c}{\Delta t}\right)_{i}^{k} (\phi_{i}^{k} - \phi_{i}^{n})$$

$$(4.15)$$

 A_m denotes the matrix block area, Note that the matrix transmissibility for describing the fluid flow among matrix grid blocks is defined as

$$T_{m,i\pm 1/2} = \frac{1}{2}T_i \tag{4.16}$$

The solution of **eqn. 4.15** can be obtained iteratively by solving a tri-diagonal matrix, using Thomas algorithm (Aziz and Settari, 1985), to obtain the values of in the reservoir matrix which are then used to compute the pseudo pressure function at each time step.

4.7.3 Initial & Boundary Conditions

The conditions applied to obtain the numerical solution are presented in this subsection. Inner boundary condition: The two types of inner boundary conditions considered are the constant surface gas production rate and constant bottomhole production pressure. For the constant surface gas production rate the model allows gas to flow into the wellbore from both matrix blocks and fractures. Therefore, the gas production rate at the well is the summation from both these contributions, i.e.

$$q_{gsc} = \sum_{i=1}^{NS} J_i [\phi_{mi} - \phi(P_{wf})] + 2 \sum_{j=1}^{NF} T X_j [\phi_{fj} - \phi(P_{wf})]$$
(4.17)

4.8 Flow Behavior Fractured Networks

The two important factors that affect flow behavior in fractured reservoirs are:

- 1) Fracture connectivity.
- 2) Fracture scaling.

4.8.1 Fracture Connectivity

Fracture connectivity refers to how individual fractures link to form coherent networks. Another definition of connectivity defines in terms of fracture clusters defines it quantitatively as the proportion of the total trace length that belongs to the largest cluster. It is sensitive to network geometry and fracture characteristics such as length, size distributions, orientation, density and aperture of individual fractures. Connectivity also depends on the spatial distribution and interaction of different fracture sets to form a continuous network (Odling et al., 1999). Clustering of fractures into swarms is a key factor affecting the fracture interconnectivity and it is used as a measure for assessing flow transport in rocks (Xu et al., 2006).

In general, connectivity increases as:

1) An increasing number of fractures of the same set are added to the system resulting in an increase in fracture density.

- 2) The length of the fractures increases.
- 3) The orientation of fractures in a set exhibits a higher degree of dispersion.
- 4) Fractures of multiple sets are added to the system.

4.8.2 Fracture Scalability

Fracture scalability refers to how small features are related to large (Odling et al., 1999). The scalability of fracture system is an important parameter that affects the properties of the fractures and ultimately the flow behavior of a reservoir. Considering that access to the reservoir is usually limited to just the area where the wellbore is exposed, this is usually a small area compared to the overall area of the reservoir so different schemes will be employed to be able to estimate what the fracture network will be in the entire reservoir scale from the information that is obtained from the wellbore region.

4.9 Parameters for Quantifying Connectivity in Fracture Networks

Three parameters from literature that can be used to quantify the connectivity of a fracture system are:

- 1) Percolation Threshold
- 2) Connectivity Index
- 3) Coefficient of Variation (Cv)

Percolation theory is a general mathematical theory of connectivity and transport in geometrically complex systems (King et al., 2002). It describes the effect of the connectivity of the small-scale, or microscopic, parts of a disordered system on its large-scale, or macroscopic, properties, (Hunt, 2005). From percolation theory which is applied to fracture systems. When fracture density is low, clusters of connected fractures

are small and no cluster spans the entire area with increasing fracture density, clusters grow and at the '*percolation threshold*' the largest cluster spans the sample area (Odling et al., 1999).

Thus the '*percolation threshold*' is a value once reached the fracture network changes from fracture clusters to fracture networks, it is an important parameter, since it gives us information about the state of the fracture network.

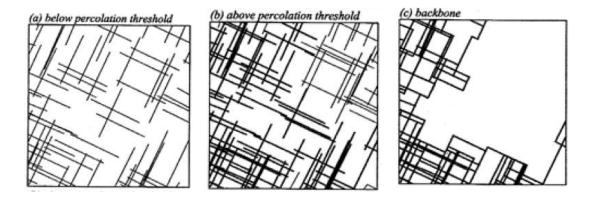


Figure 4.5 Diagram showing fracture networks below (a) and above (b) the percolation threshold and also the backbone network (c) (Odling et al., 1999)

In **Fig. 4.5a** isolated fractures with few interconnections can be observed below the percolation threshold while in **Fig. 4.5b** a fracture network spanning the entire area can be seen above the percolation threshold. **Fig. 4.5c** shows the 'backbone'; defined as all direct routes through the fracture network across the sample area. This is the largest cluster with all 'dead-ends' removed. The backbone is the part of the spanning cluster which can conduct fluid. The connectivity index (CI) quantifies the connectivity between any two points in space Monte Carlo simulation is used to evaluate the connectivity index for stationary cases and relationships between the connectivity index and the parameters of the discrete fracture model are analyzed.

The CI is the probability that two arbitrary points within the region are connected. The CI is, therefore, local and independent of the scale of the region. The CI can, however, be used to evaluate the percolation thresholds when combined with percolation criteria and the scale of the system. Monte Carlo simulation is used to evaluate the connectivity index and to assess the influence on connectivity of different fracture network models (Xu et al., 2006).

A more practical approach at understanding fracture connectivity in the reservoir is using the coefficient of variation, Cv which is a measure of the spatial distribution. It is defined as the standard deviation divided by the mean spacing (Cox and Lewis, 1966). If the fractures are located randomly, the intersections with line sampling define a Poisson point process and spacing's have a negative exponential distribution where Cv =1. If the traces are regularly spaced the standard deviation of spacing is small and Cv < 1. If however, fractures are clustered, the standard deviation is large and Cv > 1 (Xu et al., 2006).

CHAPTER V

FRACTURE MODELLING OF A SHALE GAS RESERVOIR USING FRACGEN AND NFFLOW

5.1 Modeling Approach

From the previous chapter the principle of operation of FracGen and NFflow has been discussed at length, in this chapter the application of FracGen and NFflow for fracture modeling of a shale gas reservoir in the Eagle Ford shale will be highlighted.

From the well information and also from basin information for an Eagle Ford shale gas reservoir an input data for FracGen was created and fed into it to create the fracture map that represents the reservoir. This study will utilize data from an Eagle Ford shale gas reservoir data obtained from SPE 138425.

The focus is to study the interaction of different fracture/well properties on reservoir response, four cases will be considered for this analysis and they represent the following fracture sets.

- 1) Uniformly distributed fracture pattern of one fracture set.
- 2) Clustered fracture pattern of one fracture set.
- 3) Uniformly distributed fracture pattern of two fracture sets.
- 4) A uniform distributed fracture pattern of one fracture set and a clustered fracture set.

The fracture/well properties are subdivided into two broad categories.

- 1) Controllable
- 2) Non-Controllable

Fracture/Well Properties	
Controllable Well Properties	Non-Controllable Geological
Hydraulic Fracture Aperture	Natural Fracture Aperture
Number of Stages Hydraulic Fracture	Natural Fracture Length
Hydraulic Fracture Length	Natural Fracture Density

Table 5.1: Fracture/well properties classification

This classification of the fracture/well properties (**Table 5.1**) is based on their nature; natural fracture and hydraulic fracture. Natural fractures are fractures that are created by forces of nature, while hydraulic fracture are fractures created by human influences on the reservoir either by hydraulic fracturing or other well activities, for example drilling. For this study the focus of the hydraulic fractures are fractures created by hydraulic fracturing process in shale gas reservoir.

5.2 Fracture Modeling Procedure

The steps below are the procedure taken for fracture modeling of the shale gas reservoir using FracGen and NFflow.

- A FracGen input file was created from the well/reservoir parameters and fed into FracGen.
- (2) An output FLO file is created from FracGen and then well parameters are added before it is fed into the NFflow for flow simulation.
- (3) Also from FracGen the layer description file (LDF) is also created which specifies the depths, porosity & permeability of the layers.

- (4) After that the BHP files is created from the rate versus time data as well as the pressure versus time data. The BHP file contains data that is used for history matching while running the NFflow.
- (5) Then the RES file is created to aid in running the flow simulation in NFflow. The RES file contains the PVT properties and also the file directory paths of both the LDF and the BHP file.

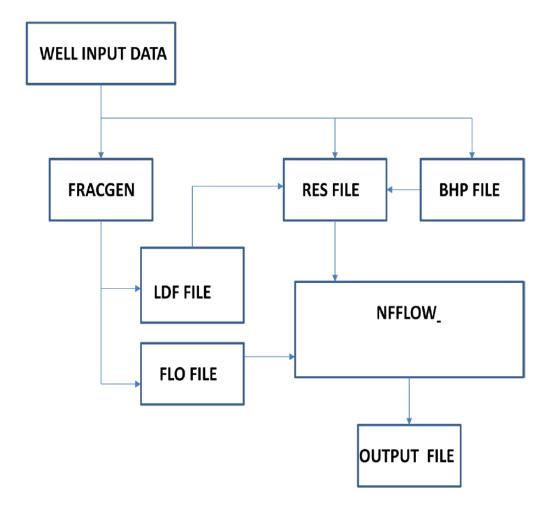


Figure 5.1: FracGen and NFflow simulation process

5.3 Case 1: Uniformly Distributed Fracture Pattern of One Fracture Set

Reservoir and Fracture Properties for Well A	
Parameter	Value
Wellbore Radius (ft)	0.33
Wellbore Lateral Length (ft)	3710
Number of Fracture Stages	10
Depth (ft)	10875
Pay Zones Thickness (ft)	283
Reservoir Pressure	7000
Specific Gravity	0.621
Temperature (F)	285
BHFP (psi)	3600
Drainage Area (Acres)	80
Reservoir Size (ft)	(933.38, 3733.52)
Reservoir Permeability (nd)	55
Reservoir Porosity (%)	4

Table 5.2: Reservoir parameters for an Eagle Ford shale gas well for case 1

From the FracGen and NFflow procedure shown in **Fig. 5.1**, a FracGen input **A1.1** (**Appendix 1**) was created from the fracture properties and other reservoir parameters for the Eagle Ford Shale gas (**Table 5.2**) and fed into FracGen to generate a fracture map **Fig. 5.2**, the well control data is shown in **Table 5.3**.

Fracture Property	Value	Source
Fracture Aperture (ft)	2.E-04	Assumed
Mean Fracture Length (ft)	280	Assumed
Fracture Orientation (degree)	N 45° E	Assumed
Fracture Density (ft/ft ²)	0.056	Assumed
Hydraulic Fracture Aperture (ft)	2.E-04	Assumed
Hydraulic Fracture Length (ft)	400	Well Data
Hydraulic Fracture Orientation	E-W	Assumed
Number of Hydraulic Fracture Stages	10	Well Data

Table 5.3: (Control data	ı fed into	FracGen	for case 1
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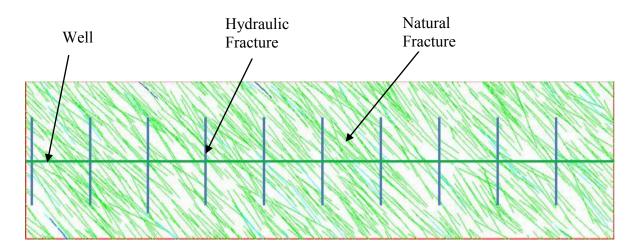


Figure 5.2: FracGen fracture map showing the well profile of the reservoir for case 1

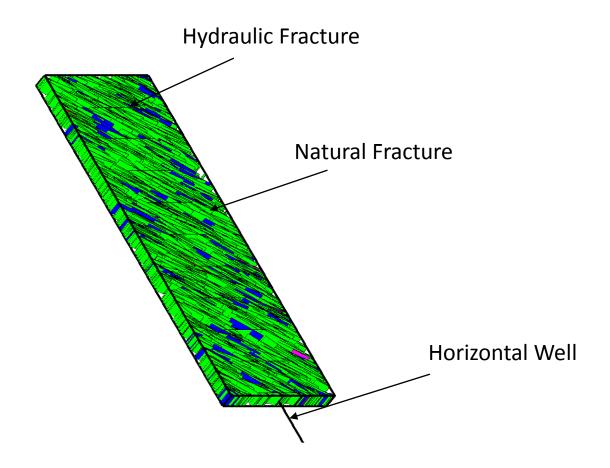


Figure 5.3: 3d representation of the well and fracture distribution

The FLO file **A1.2** and LDF file **A1.3** are used to run the flow simulation. The next step is to add well parameters to the FLO file; this basically involves the well location, well radius and some key words that identify it as a well. Then a BHP file **A1.4** and RES file **A1.5** is created. The file path to the LDF and BHP files are added to the RES file to enable NFflow access them during the flow simulation. Another important process carried out before flow simulation is to apply a FARM factor if necessary to the FLO file which represents damage around the well due to drilling activities, it can also

represent a skin factor. Then the RES and FLO file is fed into NFflow and the flow simulation is run and the reservoir parameter is adjusted to obtain a match for the reservoir data.

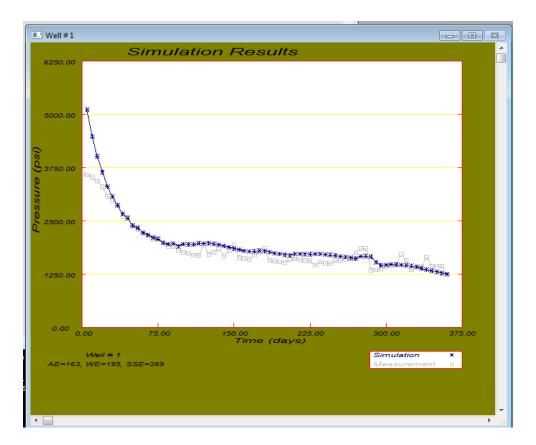


Figure 5.4: NFflow output showing the simulation result for case 1

Fig. 5.3 is a 3d representation of the well and fracture distribution while **Fig. 5.4** shows the simulation result for the case run above and is taken to be the control case as we have a match with the observed data. The next step is to vary the fracture/well control properties stated in **Table 5.3** and observe the changes to the reservoir response. The values in **Table 5.4** are used as input parameters for the flow simulation.

Fracture/well Prop.	-50%	-25%	Control	+25%	+50%
Fracture Aperture (ft)	0.00001	0.00015	0.00002	0.000025	0.00003
Fracture Density (points/ft ²)	0.0001	0.00015	0.0002	0.00025	0.0003
Mean Fracture Length (ft)	140	210	280	350	420
Hydraulic Fracture Aperture (ft)	0.00001	0.000015	0.00002	0.000025	0.00003
Hydraulic Fracture Length(ft)	200	300	400	500	600

 Table 5.4: Summary of fracture/well parameters for case 1

Some preliminary calculations were carried out shown in **Appendix 2**, this is to ensure that the 2d- fracture density is either kept constant or varied when altering fracture properties by using the relationship relating 2d-fracture density with the other properties.

It is important to note that for all cases in this project, 2 simulation runs were carried out for each input parameter, one is for the pressure response using a rate controlled RES & BHP files and the other uses a pressure controlled file for cumulative gas produced.

5.3.1 Fracture Aperture

This represents the width of the fracture opening; the value in the control file is 0.00002 ft, this value is varied in steps and the values obtained are used for running simulations **Figs. 5.5-5.6** shows summaries of the responses observed.

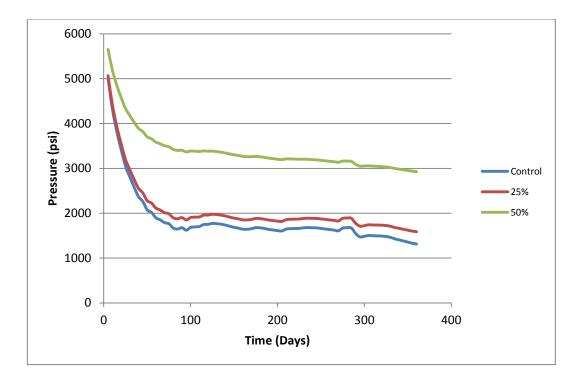


Figure 5.5: Change in pressure response due to increase in fracture aperture for case 1

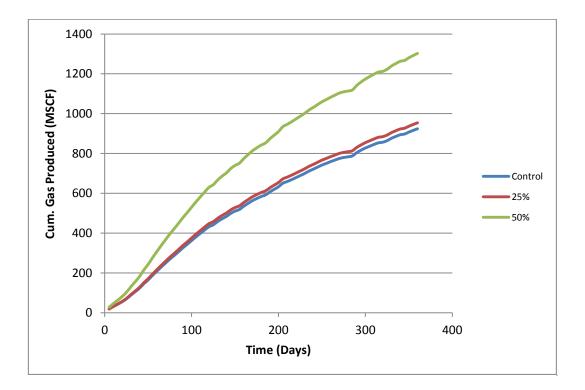


Figure 5.6: Change in cum. gas produced due to increase in fracture aperture for case 1

5.3.2 Fracture Density

The fracture density represents the number of fractures per unit area/volume, the value in the control file is $0.0002 \text{ points/ft}^2$, this value is varied in steps from the control and the values obtained were used to run simulations and results are shown in **Fig. 5.7 - 5.10**.

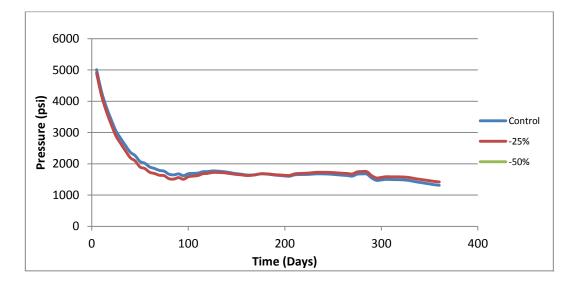


Figure 5.7: Change in pressure response due to decrease in fracture density for case 1

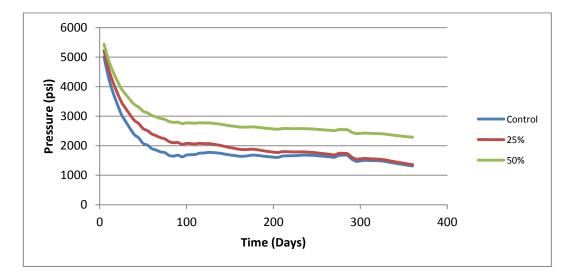


Figure 5.8: Change in pressure response due to increase in fracture density for case 1

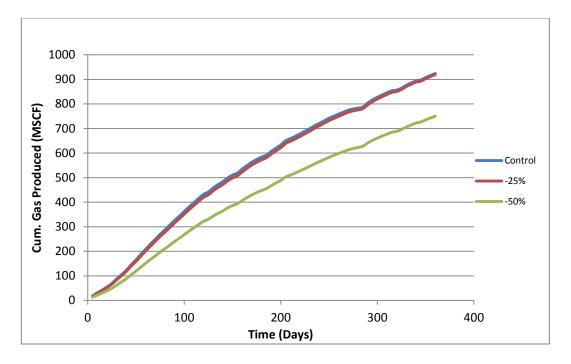


Figure 5.9: Change in cum. gas produced due to decrease in fracture density for case1

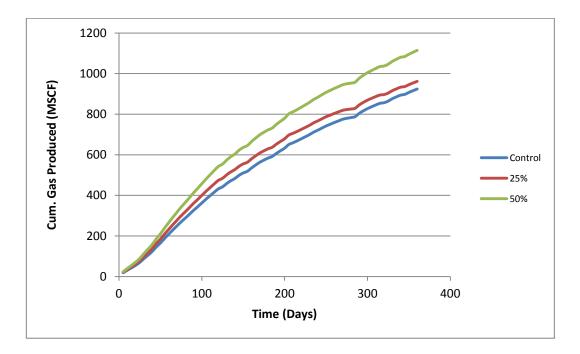


Figure 5.10: Change in cum. gas produced due to increase in fracture density for case1

5.3.3 Fracture Length

This represents the mean length of the fracture with a fracture set, the value in the control file is mean of 280 ft, this value is varied in steps and values obtained were used to run simulations and results obtained are shown in **Fig. 5.11 - 5.14**.

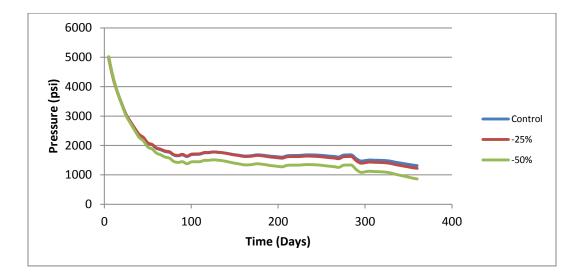


Figure 5.11: Change in pressure response due to decrease in fracture length for case 1

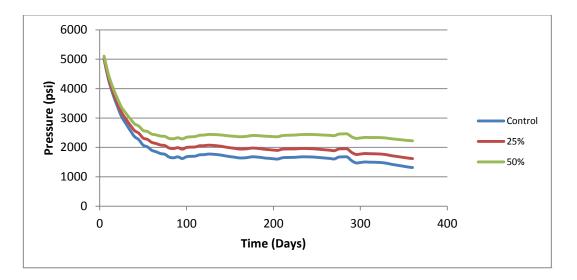


Figure 5.12: Change in pressure response due to increase in fracture length for case 1

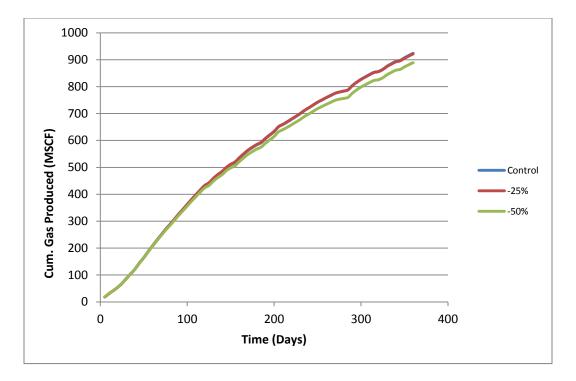


Figure 5.13: Change in cum. gas produced due to decrease in fracture length for case 1

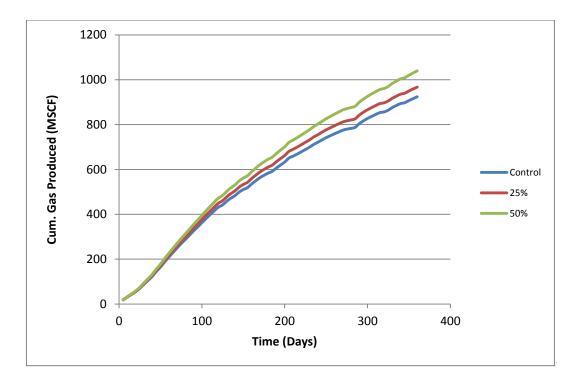


Figure 5.14: Change in cum. gas produced due to increase in fracture length for case 1

The summary plots (**Fig.5.15 &5.16**) for the non-controllable fracture properties for the change in cumulative gas production.

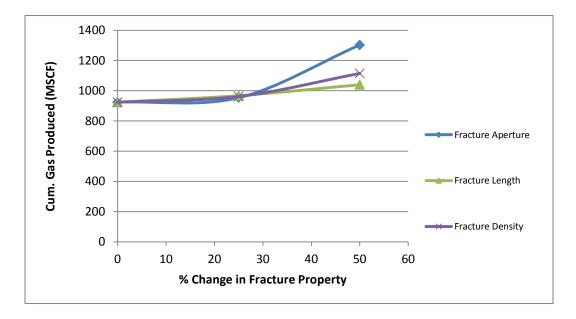


Figure 5.15: Change in cum. gas produced due to increase fracture property for case 1

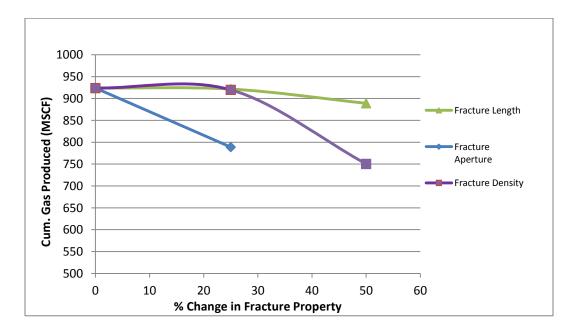


Figure 5.16: Change in cum. gas produced due to decrease in fracture property for case 1

5.3.4 Number of Hydraulic Fracture Stages

This represents the number of fracture stages; it is 10 in the control file. This number is varied and was used to run flow simulation and results are shown in **Fig. 5.17 & 5.18**.

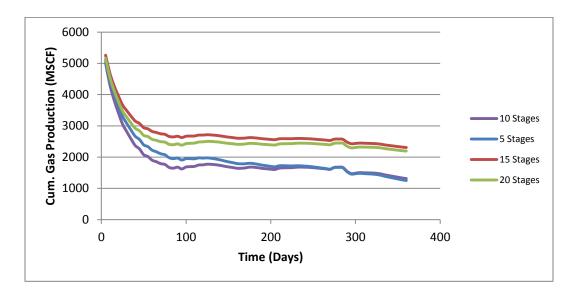


Figure 5.17: Change in pressure for different hydraulic fracture stages for case 1

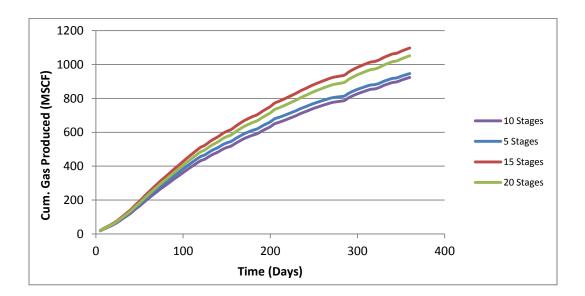


Figure 5.18: Change in cum. gas produced for different numbers of hydraulic fracture stages for case 1

5.3.5 Hydraulic Fracture Aperture

This represents the aperture of the hydraulic fracture, the value in the control file is 0.00002 ft, this value is varied in steps from the control and were used to run flow simulation and results obtained is shown in **Fig. 5.19 - Fig. 5.22**.

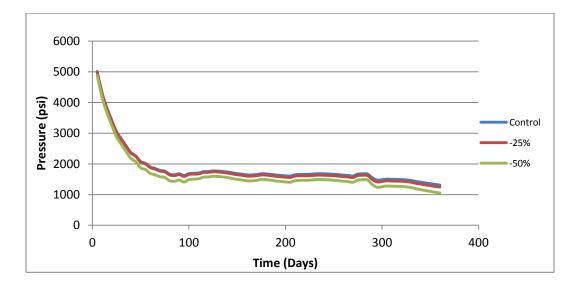


Figure 5.19: Change in pressure due to decrease in hydraulic fracture aperture for case 1

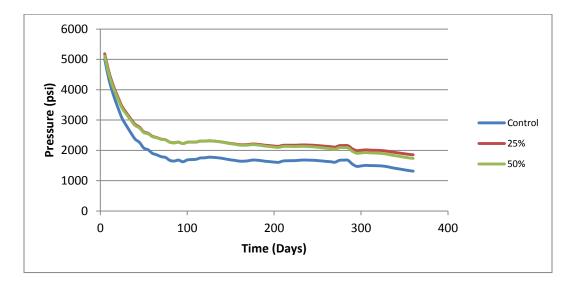


Figure 5.20: Change in pressure due to increase in hydraulic fracture aperture for case 1

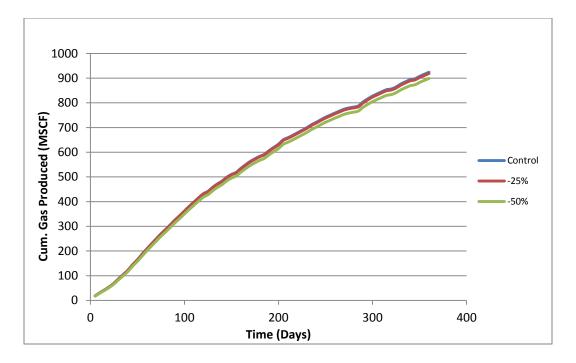


Figure 5.21: Change in cum. gas produced due to decrease in hydraulic fracture aperture for case 1

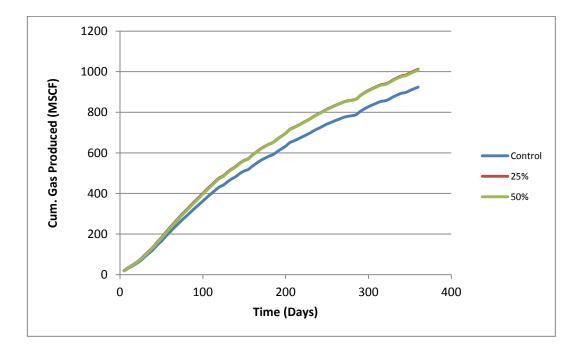


Figure 5.22: Change in cum. gas produced due to increase in hydraulic fracture aperture for case 1

5.3.6 Hydraulic Fracture Length

This represents the length of the hydraulic fracture; the value is 400 ft. in the control file. This value is varied in steps from the control and the values obtained were used to run flow simulation and results are shown in **Fig. 5.23 - 5.26**.

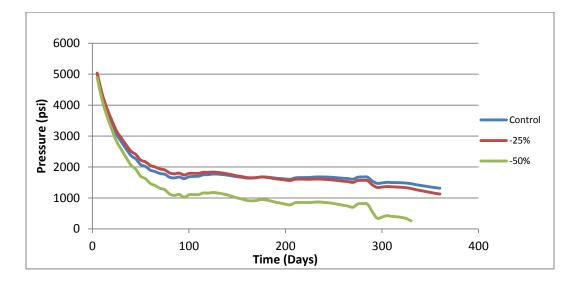


Figure 5.23: Change in pressure due to decrease in hydraulic fracture length for case 1

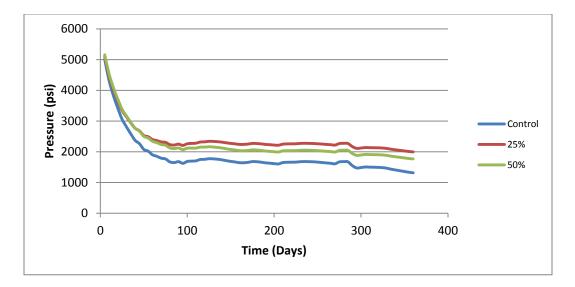


Figure 5.24: Change in pressure due to increase in hydraulic fracture length for case 1

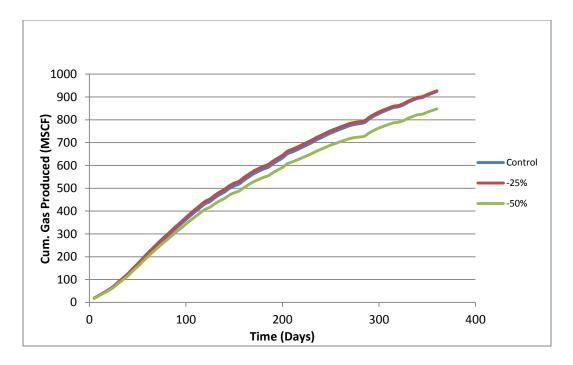


Figure 5.25: Change in cumulative gas produced due to decrease in hydraulic fracture length for case 1

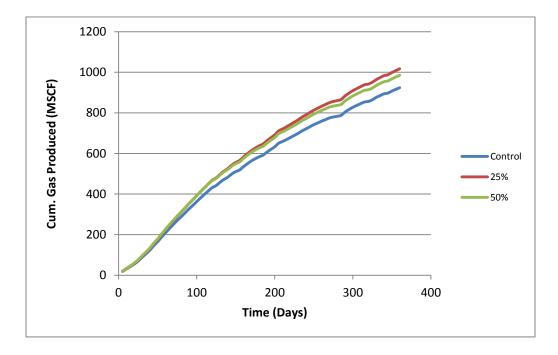


Figure 5.26: Change in cumulative gas produced due to increase in hydraulic fracture length for case 1

The chart (**Table 5.5**) is a summary of the different effects on reservoir response observed.

Fracture Property	Increasing Property	Decreasing Property
Fracture Length Fracture Aperture	The bottomhole pressure increases and the cum. gas produced increases The bottomhole	The bottomhole pressure decreases. The Cum. Gas produced decreases steadily.
	pressure increases. The Cum. Gas produced remains increases.	
Fracture Density	The bottomhole pressure increases and the cum. gas produced increases	The bottomhole pressure decreases. The Cum. Gas produced decreases steadily.
Hydraulic Fracture Stages	The bottomhole pressure increases and cum. gas produced increases till it reaches an optimum value	The bottomhole pressure decreases and cum. gas produced decreases.
HF Fracture Aperture	The bottomhole pressure remain largely constant at lower values but increases with larges values in hydraulic fracture Aperture and cum. gas produced follows the same trend	The bottomhole pressure and cum. gas produced remain constant
HF Fracture Length	The bottomhole pressure increases steadily and cum. gas produced increases till it reaches an optimum value	The bottomhole pressure decreases steadily and cumulative gas produced decreases.

 Table 5.5: Summary of effect of fracture property on reservoir response for case 1

5.4 Case 2: Clustered Fracture Pattern of One Fracture Set

For the analysis due to the highly stochastic nature of the clustered fracture network, we considered more increments and also for each case considered a number of simulation runs and the output were averaged to improve the accuracy of the solution and reduce the effect of the stochastic distribution of parameters.

Some preliminary calculations were carried out shown in **Appendix 2**, this is to ensure that the 2d- fracture density is either kept constant or varied when altering fracture properties by using the relationship relating 2d-fracture density with the other properties. **Table 5.6** shows the reservoir parameters for the Eagle Ford shale gas.

Reservoir and Fracture Properties for Well A				
Parameter	Value			
Wellbore Radius (ft)	0.33			
Wellbore Lateral Length (ft)	3710			
Number of Fracture Stages	10			
Depth (ft)	10875			
Pay Zones Thickness (ft)	283			
Reservoir Pressure	7000			
Specific Gravity	0.621			
Temperature (F)	285			
BHFP (psi)	3600			
Drainage Area (Acres)	80			
Reservoir Size (ft)	(933.38, 3733.52)			
Reservoir Permeability (nd)	55			
Reservoir Porosity (%)	4			

Table 5.6: Reservoir parameters for an Eagle Ford shale gas well for case 2

The list outlines properties used to describe a clustered fracture network in FracGen,

listed in Table 5.7 and used as input to produce the fracture map Fig. 5.27.

- 1) Cluster length
- 2) Fracture length
- 3) Intra-cluster fracture density/spacing
- 4) Density of cluster center-points
- 5) Fracture aperture
- 6) Hydraulic fracture aperture
- 7) Hydraulic fracture length
- 8) Hydraulic fracture stages

Fracture Property	Value	Source
Fracture Aperture (ft)	0.00002	Assumed
Mean Fracture Length (ft)	280	Assumed
Fracture Orientation (deg.) (Mean, SD.)	N 45° W, 8.0	Assumed
2d Fracture Density (ft/ft ²)	0.056	From Case
		1
Hydraulic Fracture Aperture (ft)	0.00002	Assumed
Hydraulic Fracture Length (ft)	400	Well Data
Hydraulic Fracture Orientation	E-W	Assumed
Number of Hydraulic Fracture Stages	10	Well Data
Cluster Orientation (deg.) (Mean, STD)	N 40° W, 10.0	Assumed
Mean cluster Length (ft)	1500	Assumed
Mean Intra-cluster Fracture spacing (ft)	40	Assumed
Mean Intra-cluster Fracture density (ft)	0.00006	Assumed
Density of cluster center-point (pts/ft ²)	0.0000057	Assumed

Table 5.7: Control data fed into FracGen for case 2

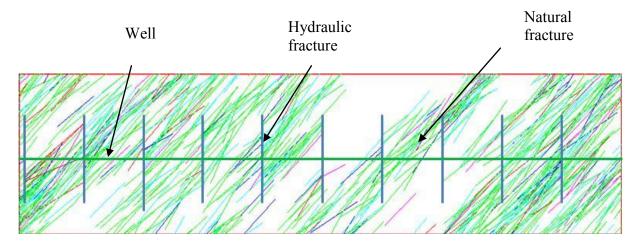


Figure 5.27: FracGen fracture map showing the well profile for case 2

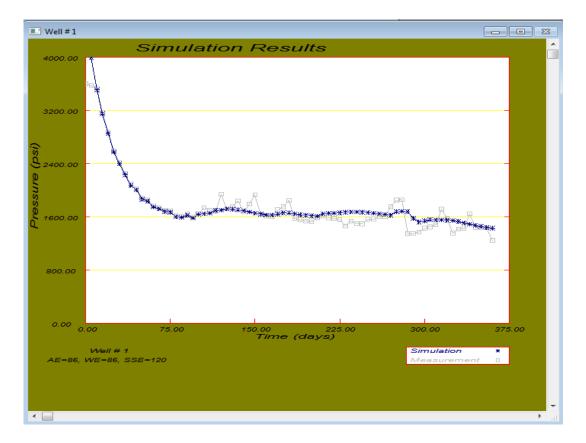


Figure 5.28: NFflow output showing the simulation result for case 2

Using the same procedure described in case 1 the simulation output is obtained and

shown in **Fig. 5.28**.

Fig. 5.28 shows the simulation result for the case run above and is taken to be the control data. The next step is to vary the control fracture/well properties stated in Table5.8 and observe the changes to the reservoir response.

Fracture/well	-50%	-25%	Control	25%	50%	100%
Prop.						
Fracture Aperture (ft)	0.000010	0.00001 5	0.000020	0.00002 5	0.000030	-
Fracture Density (ft/ft ²)	0.028	0.042	0.056	0.070	0.084	0.112
Fracture Length (ft)	140	210	280	350	420	560
Hydraulic Fracture Aperture (ft)	0.00001	0.00001 5	0.00002	0.00002 5	0.00003	0.0000 4
Hydraulic Fracture Length(ft)	200	300	400	500	600	800
Density of cluster center point (pts/ft ²)	1.030E- 05	7.241E- 06	5.742E- 06	4.874E- 06	4.324E-06	3.726E- 06

 Table 5.8: Summary of fracture/well parameters plotted for case 2

5.4.1 Fracture Length

This represents the mean length of the fracture within a fracture set, the value in the control file is 280 ft, this value is varied in steps from the control and values obtained were used to run simulations and results obtained are shown in **Fig. 5.29 & 5.30**.

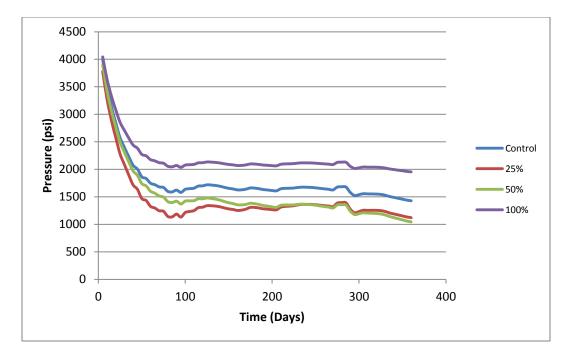


Figure 5.29: Change in pressure response due to increase in fracture length for case 2

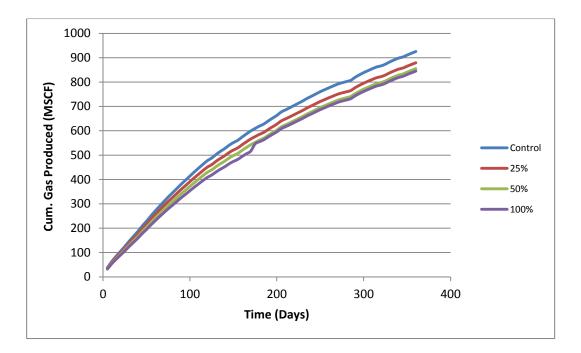


Figure 5.30: Change in cum. gas produced due to increase in fracture length for case 2

5.4.2 Fracture Density

This represents the density of fractures within the clusters, the control value is 0.0000057 pts/ft and it is varied in steps. The simulation results shown **Fig. 5.31 - 5.32**.

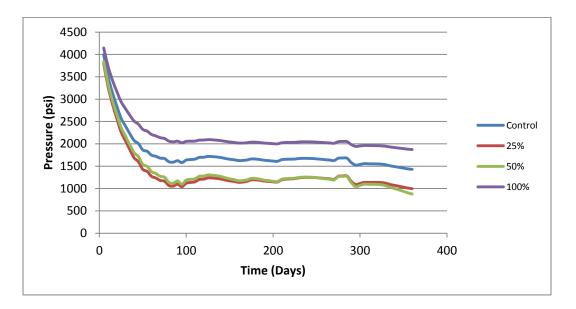


Figure 5.31: Change in pressure response due to increase in fracture density for case 2

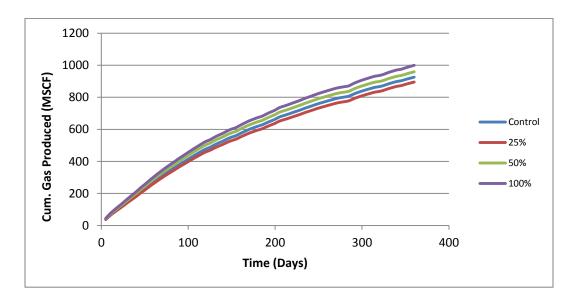


Figure 5.32: Change in cum. gas produced due to increase in fracture density for case 2

5.4.3 Fracture Aperture

This represents the aperture of the fractures, the value in the control file is 0.00002 ft, and simulations results obtained are shown in Fig. 5.33 - 5.34.

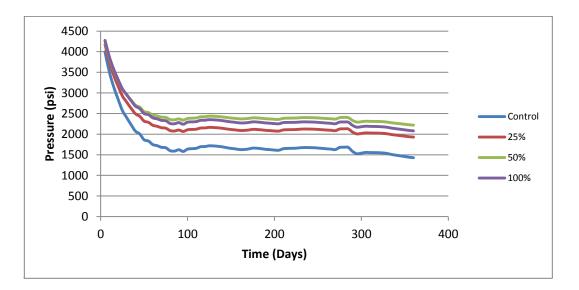


Figure 5.33: Change in pressure due to increase in fracture aperture for case 2

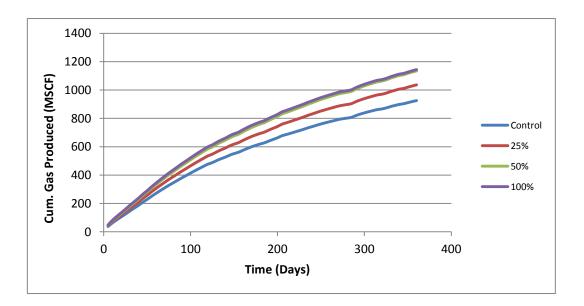


Figure 5.34: Change in cum. gas produced due to increase in fracture aperture for case 2

5.4.4 Hydraulic Fracture Aperture

This represents the aperture of the hydraulic fractures, the value in the control file is 0.00002 ft and simulations results obtained are shown in Fig. 5.35 - 5.36.

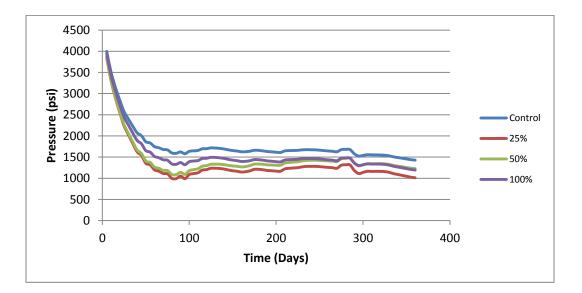


Figure 5.35: Change in pressure due to increase in hydraulic fracture aperture for case 2

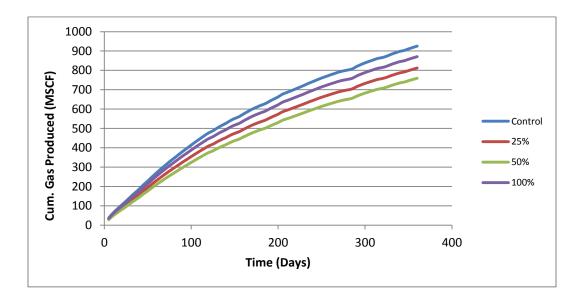


Figure 5.36: Change in cum. gas produced due to increase in hydraulic fracture aperture for case 2

5.4.5 Hydraulic Fracture Length

This represents the length of the hydraulic fractures, the value in the control file is 400 ft, and simulations results obtained are shown in **Fig. 5.37 - 5.40**.

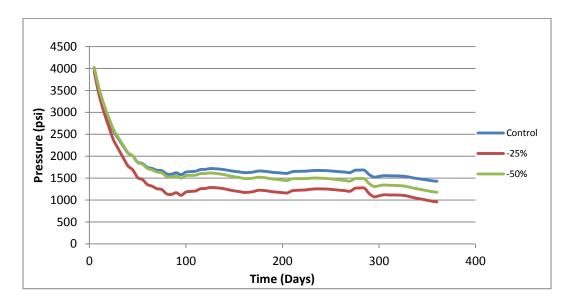


Figure 5.37: Change in pressure due to decrease in hydraulic fracture length for case 2

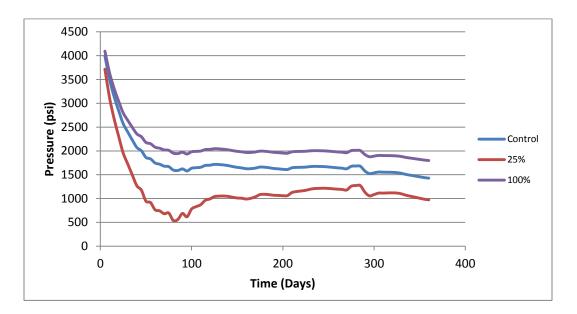


Figure 5.38: Change in pressure due to increase in hydraulic fracture length for case 2

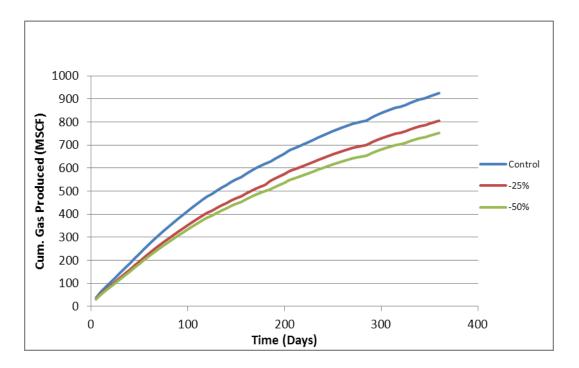


Figure 5.39: Change in cum. gas produced due to decrease in hydraulic fracture length for case 2

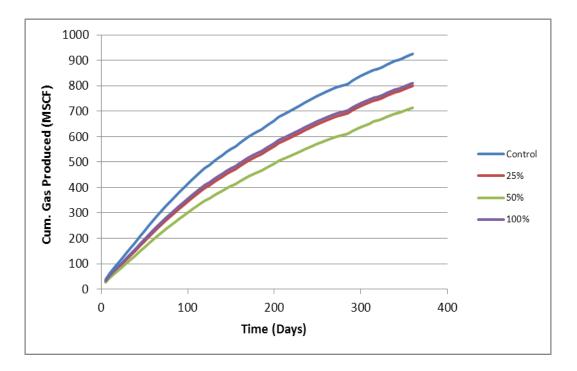


Figure 5.40: Change in cum. gas produced due to increase in hydraulic fracture length for case 2

5.4.6 Number of Hydraulic Fracture Stages

This represents the number of hydraulic fracture stages, the value in the control file is 10 stages, it is varied in steps and the simulation results are shown in, **Fig. 5.41 & 5.42**.

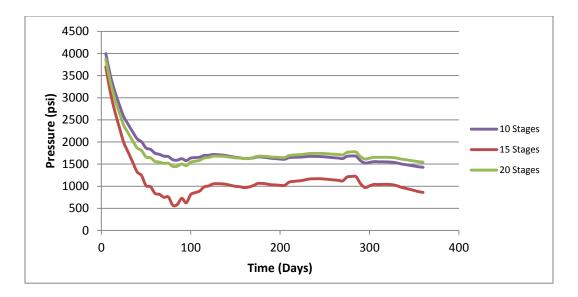


Figure 5.41: Change in pressure response due to change in hydraulic fracture stages for case 2

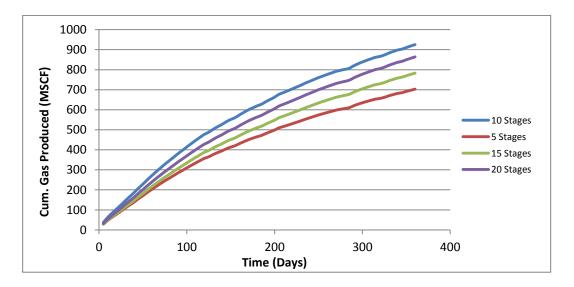


Figure 5.42: Change in cum. gas production due to change in hydraulic fracture stages for case 2

5.5 Case 3: Uniformly Distributed Fracture Pattern of Two Fracture Sets

We will consider the reservoir having 2 fracture sets which are uniformly distributed; one will be made the primary fracture set while the other is the secondary fracture set. Properties of the primary fracture set will be varied while the secondary is kept constant. **Table 5.9** shows the reservoir parameters for the Eagle Ford shale gas.

Reservoir and Fracture Properties for Well A			
Parameter	Value		
Wellbore Radius (ft)	0.33		
Wellbore Lateral Length (ft)	3710		
Number of Fracture Stages	10		
Depth (ft)	10875		
Pay Zones Thickness (ft)	283		
Reservoir Pressure	7000		
Specific Gravity	0.621		
Temperature (F)	285		
BHFP (psi)	3600		
Drainage Area (Acres)	80		
Reservoir Size (ft)	(933.38, 3733.52)		
Reservoir Permeability (nd)	55		
Reservoir Porosity (%)	4		

Table 5.9: Reservoir parameters for an Eagle Ford shale gas well for case 3

Table 5.10: Control data fed into FracGen for case 3

Fracture Property	Primary	Secondary	Source	
Fracture Aperture (ft)	2.E-04 2.E-04		Assumed	
Fracture Length (mean)	280	280	Assumed	
Fracture Orientation (degree)	N 45° E N 120° E		Assumed	
Fracture Density (ft/ft2)	0.042 0.014		From Case1	
Hydraulic Fracture Aperture (ft)	2.E-04		Assumed	
Hydraulic Fracture Length (ft)	40	00	Well Data	
Hydraulic Fracture Orientation	E-	W	Assumed	
Hydraulic Fracture Stages	10		Well Data	

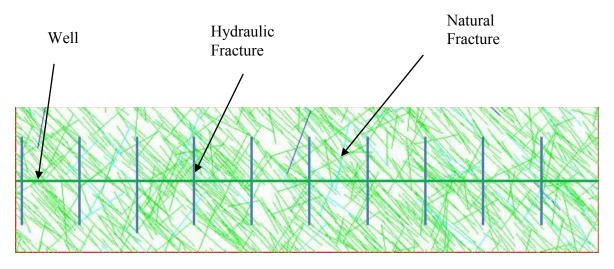


Figure 5.43: FracGen fracture map showing the well profile for case 3

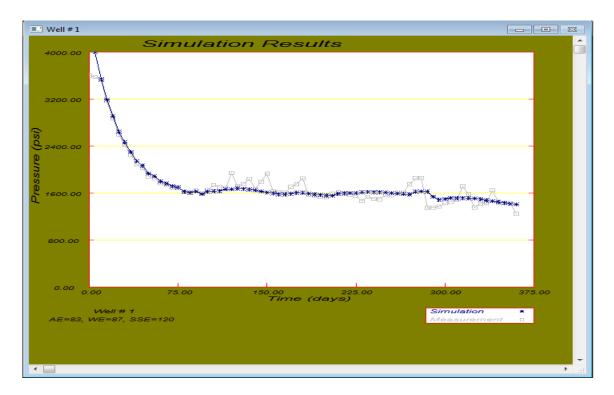


Figure 5.44: NFflow output showing the simulation result for case 3

from the input parameters that created the fracture map Fig. 5.43.

From the input parameters that created the fracture map **Fig. 5.43**, using the same procedure described in case 1 the simulation output is obtained, and shown in **Fig. 5.44**.

Fig. 5.44 shows the simulation result for the case run above and is taken to be the control case. From the control fracture/well properties stated in **Table 5.10**, the next step is to vary the parameters stated to obtain **Table 5.11** and then observe the changes in the reservoir response due to change in fracture/well properties, The values in **Table 5.11** are used as input parameters for FracGen.

Fracture/Well Prop.	-50%	-25%	Control	+25%	+50%
Fracture Aperture (ft)	0.00001	0.00015	0.00002	0.000025	0.00003
Fracture Density (pts/ft ²)	0.00005	0.0001	0.00015	0.0002	0.00025
Fracture Length (ft)	140	210	280	350	420
Hydraulic Fracture Aperture (ft)	0.00001	0.000015	0.00002	0.000025	0.00003
Hydraulic Fracture Length(ft)	200	300	400	500	600

 Table 5.11: Data variation fed into FracGen for the primary fracture set for case 3

Some preliminary calculations were carried out shown in **Appendix 2**, this is to ensure that the 2d fracture density is either kept constant or varied when altering fracture properties by using the relationship relating 2d fracture density with the other properties.

5.5.1 Fracture Aperture

This represents the width of the fracture opening; the value in the control file is 0.00002 ft. and standard deviation of 0.00002, this value is varied and the results obtained were used to run flow simulation and **Fig. 5.45 & 5.46** shows the response observed.

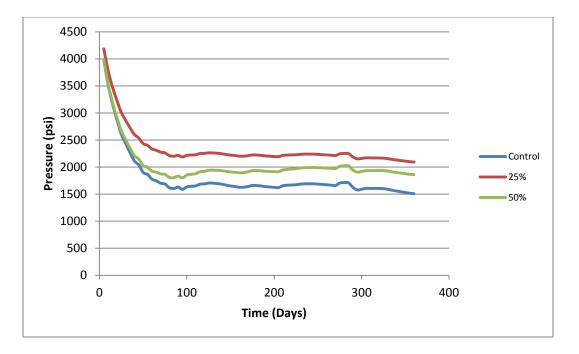


Figure 5.45: Change in pressure response due to increase in fracture aperture for case 3

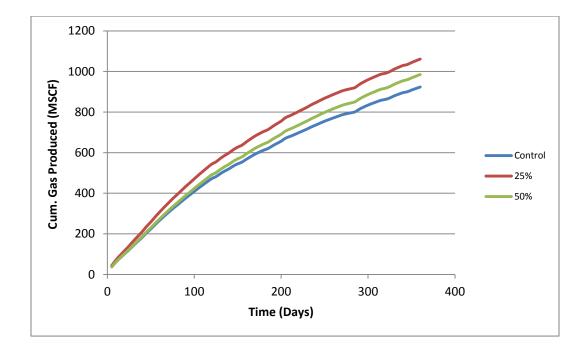


Figure 5.46: Change in cum. gas production due to increase in fracture aperture for case 3

5.5.2 Fracture Density

The fracture density represents the number of fractures per unit area/volume, the value in the control file is 0.00015 pts/ft and it is varied in steps from the control and the values obtained were used to run simulations and results are shown in **Fig. 5.47 & 5.48**.

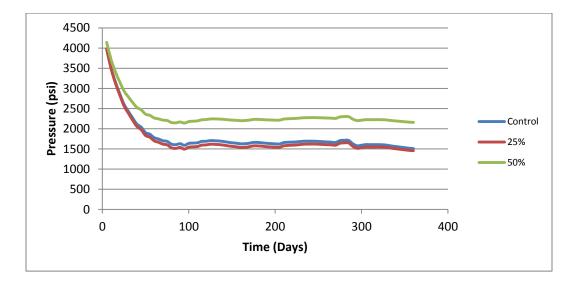


Figure 5.47: Change in pressure response due to increase in fracture density for case 3

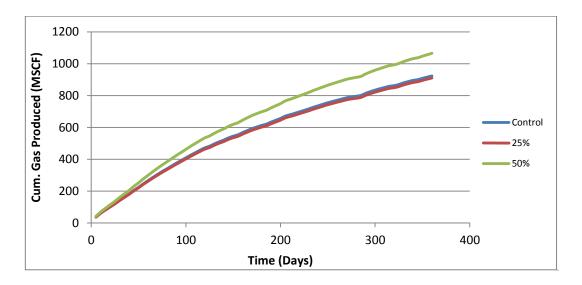


Figure 5.48: Change in cum. gas production due to increase in fracture density for case 3

5.5.3 Fracture Length

This represents the mean length of the fracture with a fracture set, the value in the control file is 280 ft, and the simulations results obtained are shown in **Fig. 5.49 & 5.50**.

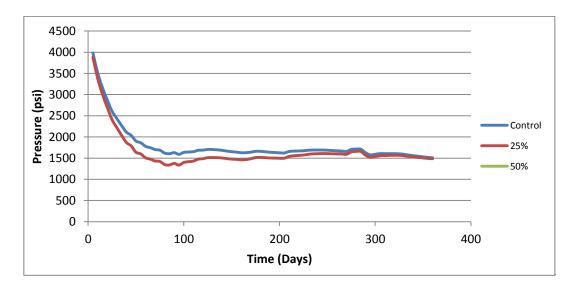


Figure 5.49: Change in pressure response due to increase in fracture length for case 3

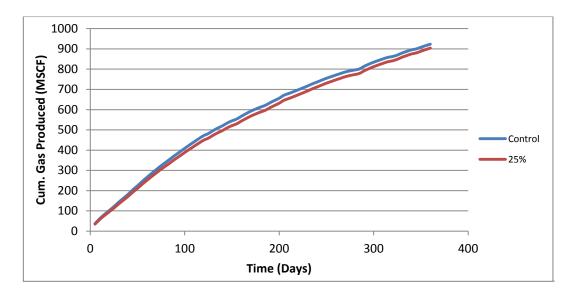


Figure 5.50: Change in cum. gas production due to increase in fracture length for case 3

5.5.4 Number of Hydraulic Fracture Stages

This represents the number of fracture stages; the number of stages is 10 in the control file. The simulation results are shown in **Fig. 5.51 & 5.52**.

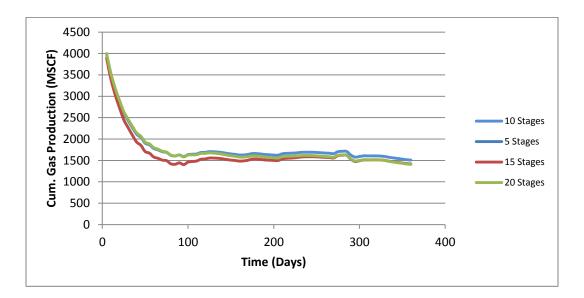


Figure 5.51: Change in pressure for different hydraulic fracture stages for case 3

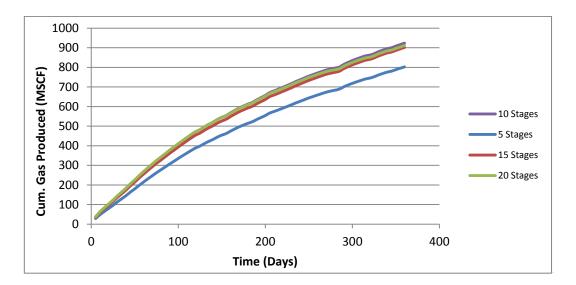


Figure 5.52: Change in cum. gas production for change in hydraulic fracture stages for case 3

5.5.5 Hydraulic Fracture Aperture

This represents the aperture of the hydraulic fractures, the value in the control file is 0.00002 ft and simulations results obtained are shown in **Fig. 5.53 – 5.56**.

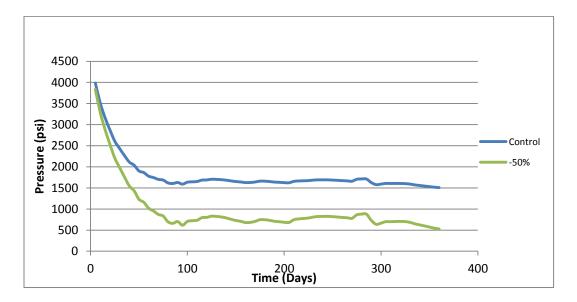


Figure 5.53: Change in pressure due to decrease in hydraulic fracture aperture for case 3

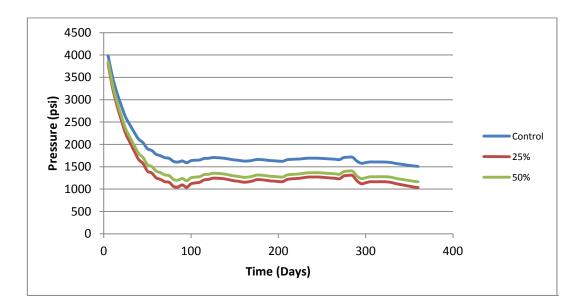


Figure 5.54: Change in pressure due to increase in hydraulic fracture aperture for case 3

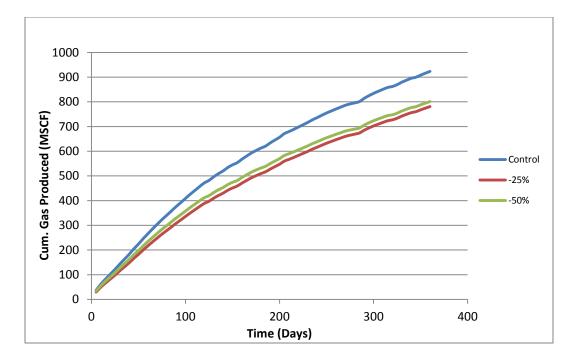


Figure 5.55: Change in cum. gas produced due to decrease in hydraulic fracture aperture for case 3

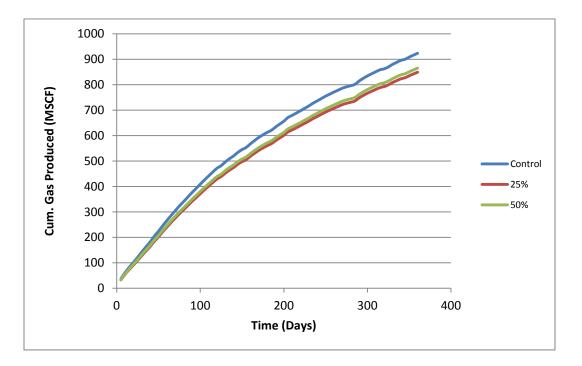


Figure 5.56: Change in cum. gas produced due to increase in hydraulic fracture aperture for case 3

5.5.6 Hydraulic Fracture Length

This represents the aperture of the hydraulic fractures, the value in the control file is 400 ft and simulations results obtained are shown in **Fig. 5.57 & 5.58**.

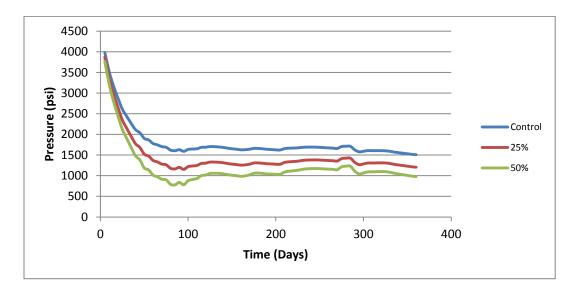


Figure 5.57: Change in pressure due to increase in hydraulic fracture length for case 3

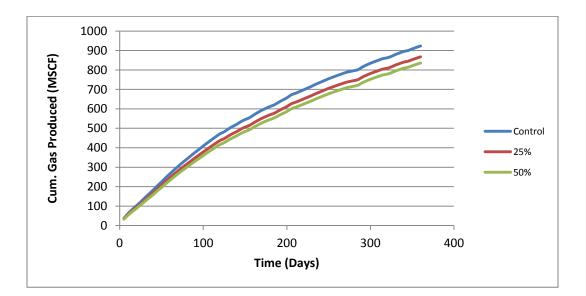


Figure 5.58: Change in cum. gas produced due to increase in hydraulic fracture length for case 3

5.6 Case 4: Two Fracture Sets with One Clustered and the Other Uniformly Distributed

We will consider the reservoir having 2 fracture sets where one is clustered and the other is uniformly distributed, the clustered fracture network is the primary fracture set while the uniformly distributed network is the secondary fracture set, the properties of the primary fracture set will be varied while that of the secondary is kept constant.

Table 5.12 shows the reservoir parameters for the Eagle Ford shale gas. To obtain some parameters in **Table 5.13** preliminary calculations were carried out shown in **Appendix 2**, this is to ensure that the 2d- fracture density is either kept constant or varied when altering fracture properties by using the relationship relating 2d-fracture density with the other properties.

Reservoir and Fracture Properties for Well A		
Parameter	Value	
Wellbore Radius (ft)	0.33	
Wellbore Lateral Length (ft)	3710	
Number of Fracture Stages	10	
Depth (ft)	10875	
Pay Zones Thickness (ft)	283	
Reservoir Pressure	7000	
Specific Gravity	0.621	
Temperature (F)	285	
BHFP (psi)	3600	
Drainage Area (Acres)	80	
Reservoir Size (ft)	(933.38, 3733.52)	
Reservoir Permeability (nd)	60	
Reservoir Porosity (%)	5.5	

Table 5.12: Reservoir parameters for an Eagle Ford shale gas well for case 4

Fracture Property	Primary	Secondary	Source
Fracture Aperture (ft)	0.0002	0.0002	Assumed
Mean Fracture Length (ft)	280	280	Assumed
Fracture Orientation (deg.) (Mean, St. dev.)	N 45° E, 8.0	N 120° E	Assumed
2D- Fracture density (ft/ft^2)	0.042	0.014	From Case 1
Hydraulic Fracture Aperture (ft)	2.E-04		Assumed
Hydraulic Fracture Length (ft)	400		Well Data
Hydraulic Fracture Orientation	E-S		Assumed
Number of Hydraulic Fracture Stages	10		Well Data
Cluster Orientation (deg.) (Mean, St. dev.)	N 40° E, 10.0	-	Assumed
Mean cluster Length (ft)	1500	-	Assumed
Mean Intra-cluster Fracture spacing (ft)	40	-	Assumed
Mean Intra-cluster Fracture density (ft)	0.00006	-	Assumed
Density of cluster center-point (pts/ft^2)	0.000004307		Assumed

Table 5.13: Control data fed into FracGen for case 4

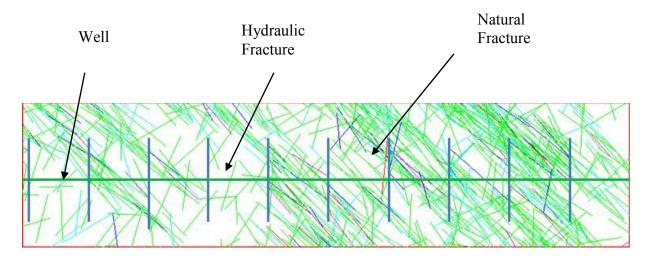


Figure 5.59: FracGen fracture map showing the well profile for case 4

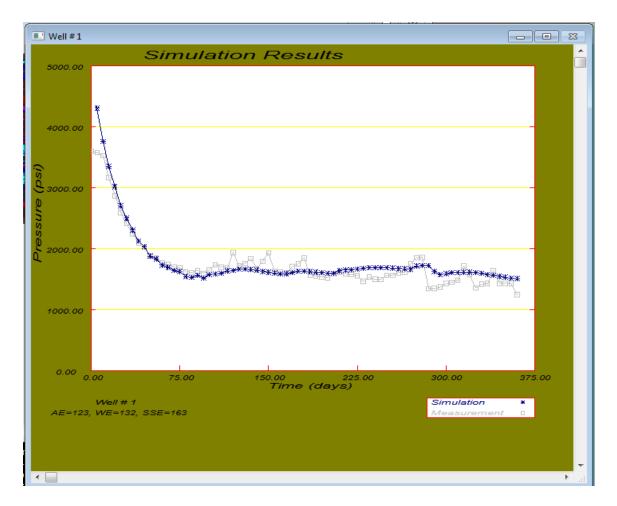


Figure 5.60: NFflow output showing the simulation result for case 4

Using the same procedure described in case 1, the fracture map **Fig. 5.59** and the simulation output is obtained shown in **Fig. 5.60**, from the input parameters that was used to create the fracture map.

Fig. 5.60 shows the simulation result for the case run above and is taken to be as the control case. The next step is to vary the fracture/well properties **Table 5.13** and observe the changes in the reservoir response. The values in **Table 5.14** are input parameters for FracGen.

Fracture/Well Prop.	-50%	-25%	Control	25%	50%	100%
Fracture Aperture (ft)	0.000010	0.000015	0.000020	0.000025	0.000030	-
Fracture Density (ft/ft ²)	0.021	0.032	0.042	0.053	0.063	0.084
Fracture Length (ft)	140	210	280	350	420	560
Hydraulic Fracture Aperture (ft)	0.00001	0.000015	0.00002	0.000025	0.00003	0.00004
Hydraulic Fracture Length(ft)	200	300	400	500	600	800
Density of cluster	7.727E-	5.431E-	4.307E-	3.655E-	3.243E-	2.795E-
center point (pts/ft ²)	06	06	06	06	06	06

Table 5.14: Data fed into FracGen for the different fracture properties for case 4

5.6.1 Fracture Aperture

This represents the width of the fracture opening; the value in the control file is 0.00002 ft, this value is varied in steps and the results obtained were used to run flow simulation and the response observed is shown in **Fig. 5.61 - 5.64**.

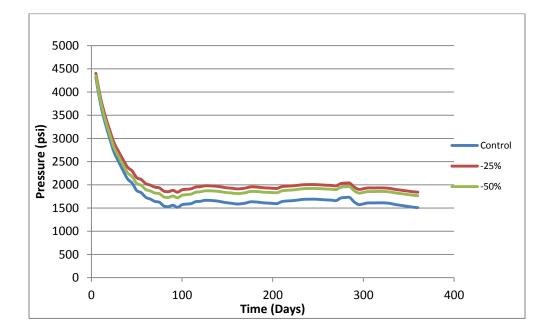


Figure 5.61: Change in pressure response due to decrease in fracture aperture for case 4

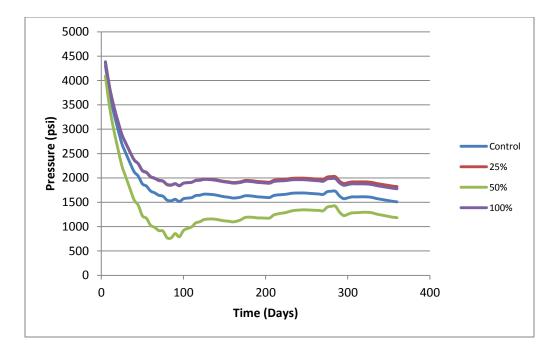


Figure 5.62: Change in pressure response due to increase in fracture aperture for case 4

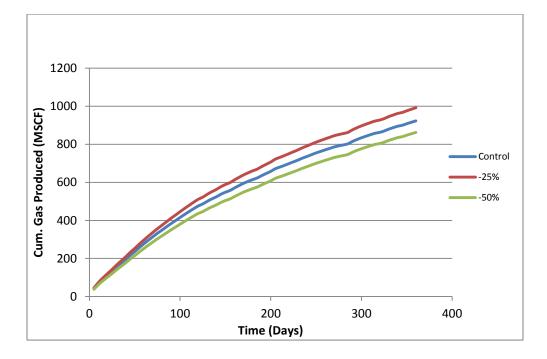


Figure 5.63: Change in cum. gas production due to decrease in fracture aperture for case 4

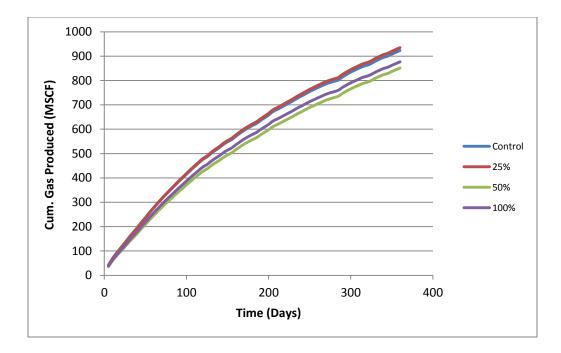


Figure 5.64: Change in cum. gas production due to increase in fracture aperture for case 4

5.6.2 Fracture Density

This represents the number of fractures per unit area/ volume, the value in the control file is 0.000004307 pts/ft. This value varied in steps from the control and the values obtained were used to run simulations and results are shown in **Fig. 5.65 - 5.68**.

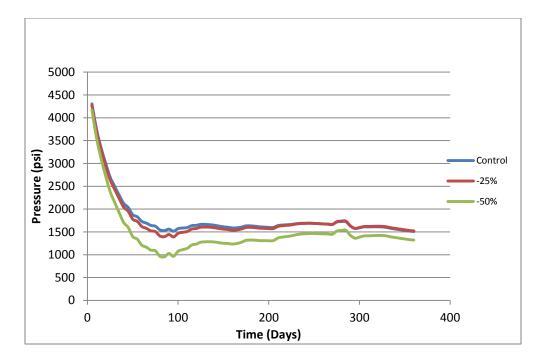


Figure 5.65: Change in pressure response due to decrease in fracture density for case 4

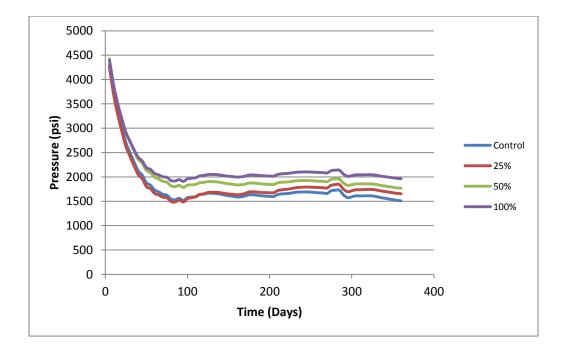


Figure 5.66: Change in pressure response due to increase in fracture density for case 4

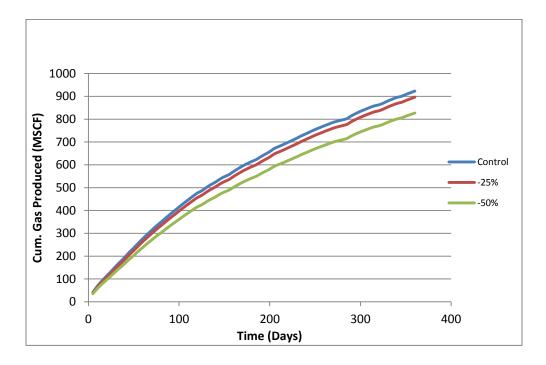


Figure 5.67: Change in cum. gas production due to decrease in fracture density for case 4

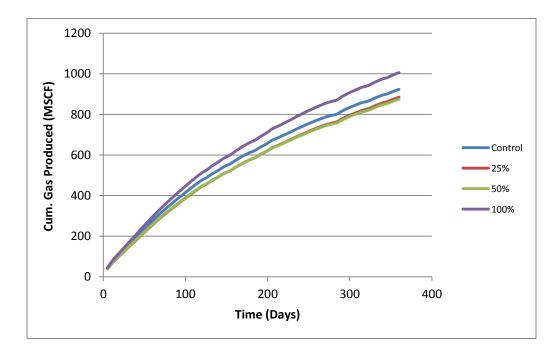


Figure 5.68: Change in cum. gas production due to increase in fracture density for case 4

5.6.3 Fracture Length

This represents the mean length of the fracture with a fracture set. The mean fracture length is 280 ft. The simulations results obtained are shown in **Fig. 5.69 - 5.72**.

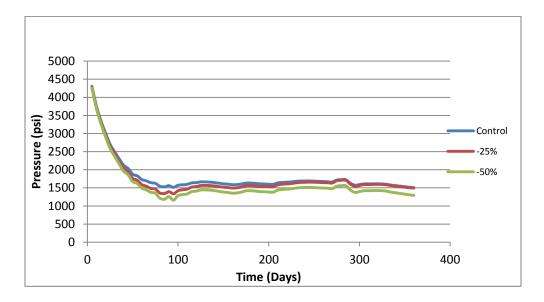


Figure 5.69: Change in pressure response due to decrease in fracture length for case 4

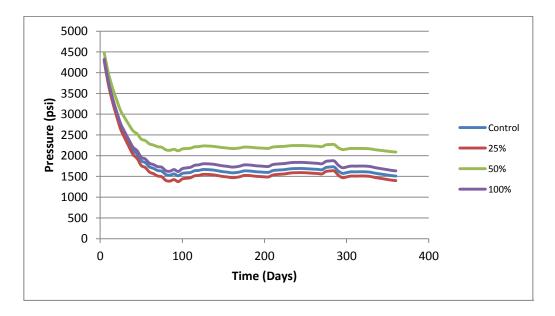


Figure 5.70: Change in pressure response due to increase in fracture length for case 4

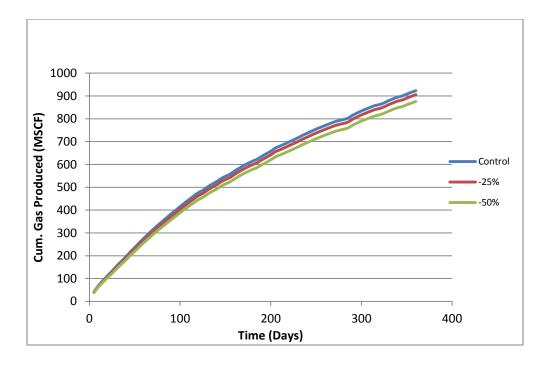


Figure 5.71: Change in cum. gas production due to decrease in fracture length for case 4

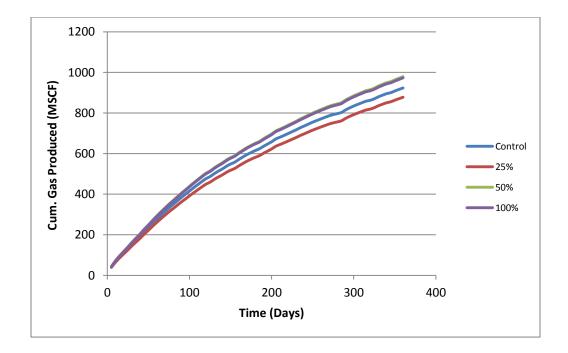


Figure 5.72: Change in cum. gas production due to increase in fracture length for case 4

5.6.4 Number of Hydraulic Fracture Stages

This represents the number of fracture stages; the number of stages is 10 in the control file. The simulation and results are shown in **Fig. 5.73 & 5.74**.

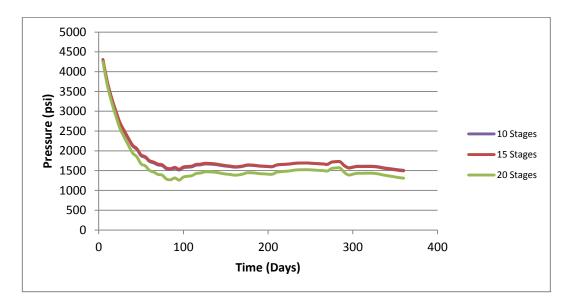


Figure 5.73: Change in pressure due to increase in hydraulic fracture stages for case 4

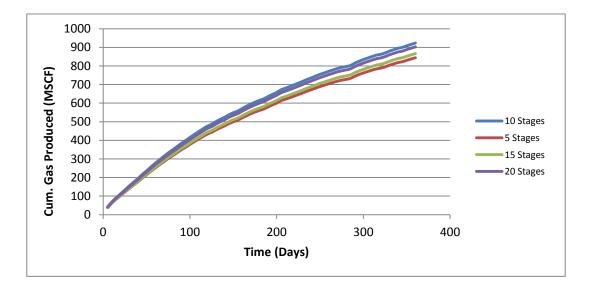


Figure 5.74: Change in cum. gas production due to change in hydraulic fracture stages for case 4

5.6.5 Hydraulic Fracture Aperture

This represents the aperture of the hydraulic fractures, the value in the control file is 0.00002 ft and simulations results obtained are shown in **Fig. 5.75 – 5.78**.

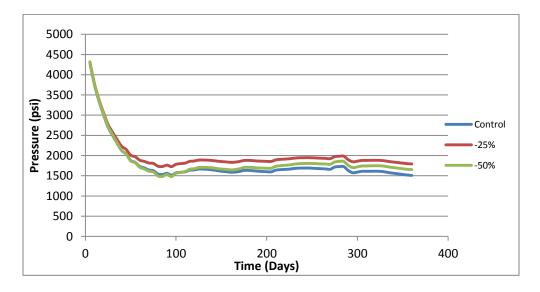


Figure 5.75: Change in pressure due to decrease in hydraulic fracture aperture for case 4

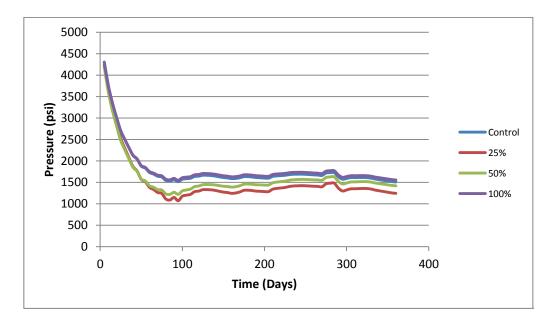


Figure 5.76: Change in pressure due to increase in hydraulic fracture aperture for case 4

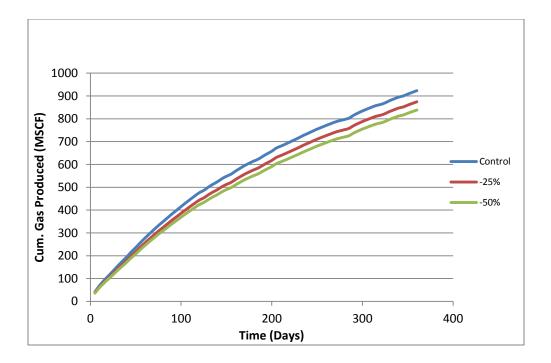


Figure 5.77: Change in cum. gas produced due to decrease in hydraulic fracture aperture for case 4

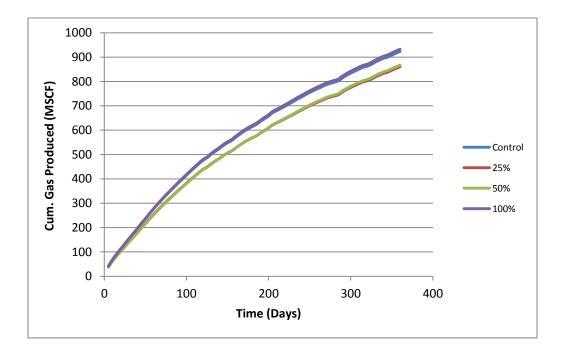


Figure 5.78: Change in cum. gas produced due to increase in hydraulic fracture aperture for case 4

5.6.6 Hydraulic Fracture Length

This represents the aperture of the hydraulic fractures, the value in the control file is 400 ft and simulations results obtained are shown in **Fig. 5.79 - 5.82**.

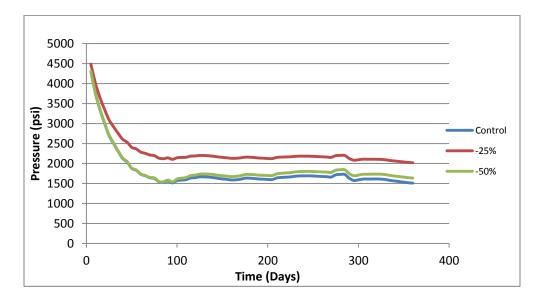


Figure 5.79: Change in pressure due to decrease in hydraulic fracture length for case 4

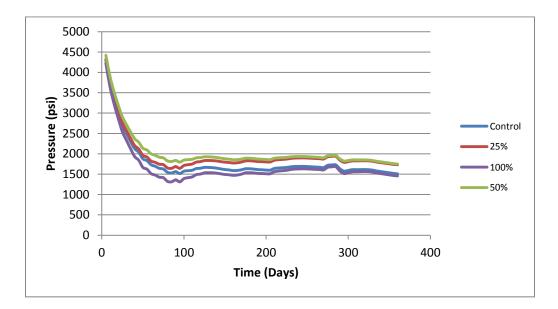


Figure 5.80: Change in pressure due to increase in hydraulic fracture length for case 4

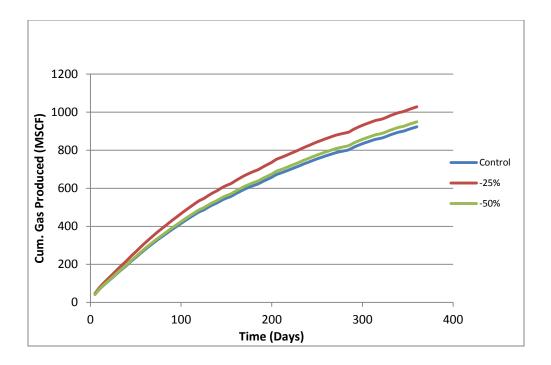


Figure 5.81: Change in cum. gas produced due to decrease in hydraulic fracture length for case 4

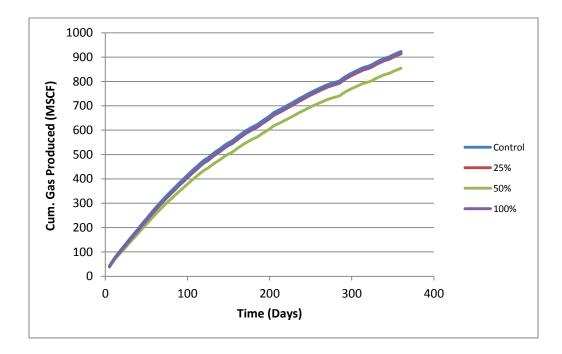


Figure 5.82: Change in cum. gas produced due to increase in hydraulic fracture length for case 4

5.7 Fracture/Well Properties Comparison Plots

5.7.1 Fracture Networks

For the fracture distribution comparison, **Fig. 5.83** shows that case 1 which represents the uniformly distributed fracture network has the highest cumulative gas production while case 3 which represents the uniformly distributed network of 2 fracture sets is next and the clustered fracture network with a passive uniform network is next also though very close to the single clustered network.

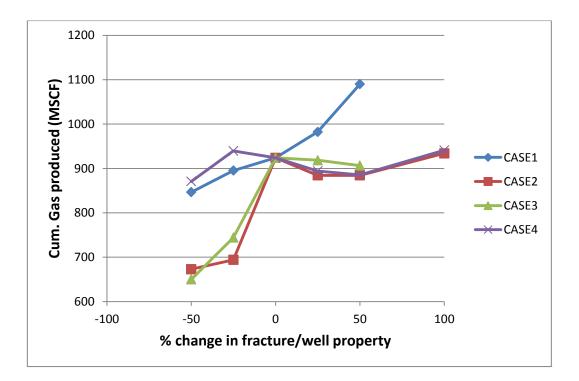


Figure 5.83: Change in cum. gas production for different fracture networks

5.7.2 Fracture/Well Property

For the Fracture/well properties comparison, **Fig. 5.84** shows the comparison plot for the different fracture and well properties studied, from the graph we can see that the fracture property that produced the highest change in cumulative gas production due to change in the fracture property is the fracture aperture, a similar trend is also seen when comparing the well properties, the hydraulic fracture aperture shows a slightly higher value to the hydraulic fracture length. Also the plot shows that the fracture/well properties usually reach a maximum value such that further increase will not results in increase in cumulative gas production.

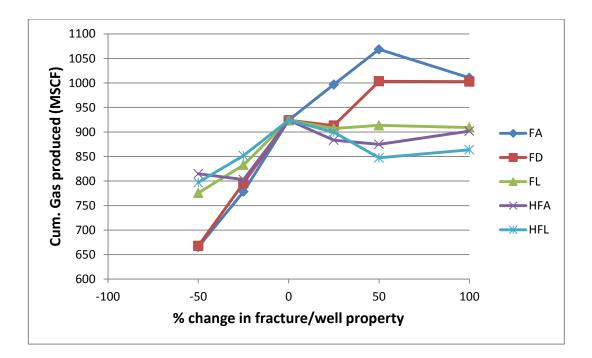


Figure 5.84: Change in cum. gas production for different fracture /well property

5.7.3 Hydraulic Fracture Stages

From the plot in **Fig. 5.85**, we can see that the maximum number of hydraulic fracture stages is 10 for most of the fracture networks except for case 1 where it is 15 and after the maximum value is reached the increase in the cumulative gas production is not appreciable to justify the investment.

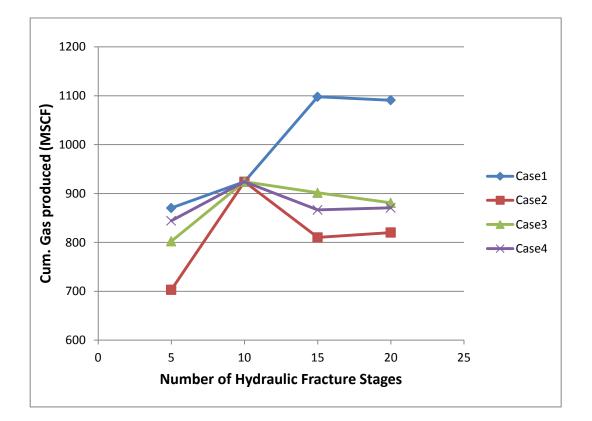


Figure 5.85: Change in cum. gas production for different hydraulic fracture stages

CHAPTER VI

CONCLUSIONS AND RECOMMENDATIONS

6.1 Conclusion

Based on the work done in this thesis, the following conclusions were drawn.

- 1) Uniform fracture network distribution produced the highest cumulative gas production of all the fracture networks studied.
- 2) 10 fracture stages is a suitable number of fracture stages as increasing the number of fracture stages does not necessarily increase productivity of the well significantly to justify the extra investment for the period studied except for case 1 were the optimal value is 15, and this may be due to the larger reservoir volume.
- 3) In order of their impact on cumulative production of the well, we have the following fracture/well properties arranged as follows: fracture aperture, fracture density, fracture length, hydraulic fracture aperture and hydraulic fracture length.
- For the same 2d- fracture density having more than one fracture set did not produce a higher cumulative gas production.

The study shows that for this reservoir having a fracture with wider aperture gives a better reservoir performance than having longer fractures.

An observation made in the course of this project which is important to note.

 Productivity in clustered fracture network depends solely on well placement, thus if the well placement is such that it intersects properly with the clustered fracture networks the productivity will be maximized but if not the productivity could be very poor thus fracture analysis of the reservoir could be an important factor that could affect productivity by influencing the well /hydraulic fracture placement.

6.2 Recommendations

The following are recommendations of the work that can be done to further test and confirm some observation obtained from this work.

- Other fractured shale gas reservoir should be modeled using FracGen & NFflow to confirm some of the observations made.
- A methodology to quantify the fracture connectivity using a parameter can also be investigated.

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APPENDIX 1

A1.1 FracGen Input File

FRACGEN 6th EDITION FILE IDENTIFICATION (<= 80 CHARACTERS) Well GU#3, Single-Layer Model X & Y DIMENSIONS OF FLOW REGION 933.38 3733.58 EFFECTIVE DEPTH OF MID-LAYER; EFFECTIVE THICKNESS OF FRACTURED LAYER 10875 283.0 NUMBER OF SETS (including LEVEL 0 sample trace set) 3 MODEL ----- SET 0 0 NUMBER OF SAMPLE TRACES (OR BOREHOLES) 0 SAMPLE TRACES: X-LEFT, Y-LEFT, X-RIGHT, Y-RIGHT, WIDTH, SHIFT(%) NAME OF DATA FILE FOR INTERSECTED FRACTURES MODEL ----- SET 1 1 SET IDENTIFICATION (<= 80 CHARACTERS) Seismically-resolved faults, interpretation of May 1999 MEAN AND SDEV OF FRACTURE ORIENTATION (360.0 = UNI) 0.0 0.0 MIN/MEAN AND MAX/DEV FRACTURE LENGTH, DIST. (0=UNI,1=EXP,2=LOG,3=INT) 0.0 0.0 0 MEAN AND SDEV OF FRACTURE APERTURE 0.0 0.0 DENSITY OF FRACTURE CENTER POINTS 0.0 CORRELATIONS (len=F(order), ori=F(len), wid=F(len)) 0.0 0.0 0.0 MAXIMUM PERCENT FRACTURE SHIFT: MODE I, II, III 0.0 0.0 0.0 SYNTHETIC ANNEALING CONTROLS (pstart, nswaps, swapl, ifreq) 100.0 0 0 0 **RELATIVE FREQUENCIES OF T-TERMINATIONS (T2,T1)** 0.0 0.0 FRACTURE INTERSECTION FREOUENCIES (%): ZERO TO 10+ INTERSECTIONS $0.0 \ 0.0 \ 0.0 \ 0.0 \ 0.0 \ 0.0 \ 0.0 \ 0.0 \ 0.0 \ 0.0 \ 0.0$ PERCENT FRACS PENETRATING OVERLYING LAYER; CORRELATION TO FRAC LENGTH 0.0 0.0 NUMBER OF USER-SUPPLIED FRACTURES 10 FRACTURES: X-LEFT, Y-LEFT, X-RIGHT, Y-RIGHT, WIDTH, SHIFT(%), PERCENT 266.0 370.0 666.0 370.0 0.000020 0.0 0.0 266.0 740.0 666.0 740.0 0.000020 0.0 0.0 266.0 1110.0 666.0 1110.0 0.000020 0.0 0.0 266.0 1480.0 666.0 1480.0 0.000020 0.0 0.0 266.0 1850.0 666.0 1850.0 0.000020 0.0 0.0

266.0 2220.0 666.0 2220.0 0.000020 0.0 0.0 266.0 2590.0 666.0 2590.0 0.000020 0.0 0.0 266.0 2960.0 666.0 2960.0 0.000020 0.0 0.0 266.0 3330.0 666.0 3330.0 0.000020 0.0 0.0 266.0 3700.0 666.0 3700.0 0.000020 0.0 0.0 MODEL ------ SET 3 1 SET IDENTIFICATION (<= 80 CHARACTERS) Set 1a, Regional Extension (Master) Fractures MEAN AND SDEV OF FRACTURE ORIENTATION (360.0=UNI) 45.0 8.0 MIN/MEAN AND MAX/DEV FRACTURE LENGTH, DIST. (0=UNI,1=EXP,2=LOG,3=INT) 260.00 300.00 0 MEAN AND SDEV OF FRACTURE APERTURE 0.00002 0.00002 DENSITY OF FRACTURE CENTER POINTS 0.0002 *CORRELATIONS* (*len=F(order*), *ori=F(len*), *wid=F(len*)) 1.0 0.8 0.7 MAXIMUM PERCENT FRACTURE SHIFT: MODE I, II, III 20.0 0.0 0.0 SYNTHETIC ANNEALING CONTROLS (pstart, nswaps, swapl, ifreq) 80.0 10 20 3 **RELATIVE FREQUENCIES OF T-TERMINATIONS (T2,T1)** 15.0 52.0 FRACTURE INTERSECTION FREQUENCIES (%): ZERO TO 10+ INTERSECTIONS $0.0 \ 0.0 \ 0.0 \ 0.0 \ 0.0 \ 0.0 \ 0.0 \ 0.0 \ 0.0 \ 0.0 \ 0.0$ PERCENT FRACS PENETRATING OVERLYING LAYER; CORRELATION TO FRAC LENGTH 0.0 0.0 NUMBER OF USER-SUPPLIED FRACTURES 0 FRACTURES: X-LEFT, Y-LEFT, X-RIGHT, Y-RIGHT, WIDTH, SHIFT(%), PERCENT

A1.2 FLO File

Well GU#3, Single-Layer Model		
933.380 3733.580 283.000 0.000		
X-left Y-left X-right Y-right Aperture Layer	Numb	er
266.000 370.000 666.000 370.000.200E-04	1	1
266.000 740.000 666.000 740.000 .200E-04	1	2
266.000 1110.000 666.000 1110.000 .200E-04	1	3
266.000 1480.000 666.000 1480.000 .200E-04	1	4
266.000 1850.000 666.000 1850.000 .200E-04	1	5
266.000 2220.000 666.000 2220.000 .200E-04	1	6
266.000 2590.000 666.000 2590.000 .200E-04	1	7
266.000 2960.000 666.000 2960.000 .200E-04	1	8
266.000 3330.000 666.000 3330.000 .200E-04	1	9
266.000 3700.000 666.000 3700.000 .200E-04	1	10
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314.160 3524.116 500.570 3700.000 .301E-04 142.893 2333.547 364.963 2581.487.153E-04 329.424 828.144 518.271 1039.381 .148E-04 0.000 313.969 56.391 396.626 .208E-04 383.764 1264.717 612.485 1480.000 .110E-04 877.570 0.000 878.241 1.058.124E-04 1 389 408.242 278.362 613.945 512.054 .118E-04 558.340 210.238 742.247 425.512 .410E-04 0.000 686.831 176.665 .188E-04 522.742 1 392 234.875 983.268 431.102 1181.403 .114E-04 222.006 1088.883 398.214 1254.075 .167E-04 206.200 2855.919 404.728 3057.591 .133E-04 392.287 2269.054 628.208 2425.278 .432E-04 66.019 2144.278 261.583 2348.728 .830E-05 0.000 1765.516 177.781 1944.675.272E-04 50.037 2310.058 199.309 2535.037 .254E-04 152.466 2558.551 345.706 2765.054 .915E-05 435.115 3700.000 464.377 3733.580.123E-04 0.000 2385.620 51.573 2428.954 .184E-04 711.734 753.807 909.369 955.936 .102E-04 9.913 3317.853 199.183 3492.351 .571E-05 697.954 3307.846 918.861 3532.930.178E-04 -1 0.000 2283.582 117.575 2411.848 .114E-04 436.954 2220.000 620.664 2411.421 .563E-05 593.998 327.612 809.556 510.209.264E-04 90.781 1191.094 298.993 1381.971 .347E-04 898.964 160.235 933.380 191.971 .283E-04 929.690 2692.957 933.380 2697.178.784E-05 851.501 2758.809 933.380 2831.598.183E-04 447.628 165.303 654.403 357.530.111E-04 0.000 471.293 8.119 480.488 .124E-04 333.798 2403.729 488.928 2590.000 .102E-04 0.000 1314.497 130.855 1474.956 .185E-04 669.382 1239.089 867.751 1439.783 .456E-04 14.075 772.620 197.756 986.791 .195E-04 178.549 2066.530 357.246 2220.000 .664E-05 0.000 234.065 81.401 315.181 .166E-04 0.000 3358.485 77.220 3421.334 .942E-05 540.531 687.401 751.254 874.499.403E-04 0.000 2165.669 183.001 2378.326 .146E-04 339.768 1480.000 607.709 1670.697 .807E-05 431.669 1030.265 637.618 1222.451 .222E-04 279.223 2322.557 471.785 2590.000 .174E-04 362.780 865.455 633.636 1110.000 .740E-05 586.813 1272.341 767.579 1535.252 .168E-04 0.000 3511.637 50.897 3557.559 .856E-05 526.293 2468.444 734.064 2658.319 .568E-05 318.496 1480.000 520.769 1689.952 .149E-04 52.268 3664.388 113.561 3733.580.138E-04 864.042 0.000 933.380 73.448.337E-04 1 433 809.633 2739.607 933.380 2899.995 .239E-04 0.000 2210.149 54.720 2263.922 .167E-04 1 435 892.447 2405.272 933.380 2449.488 .406E-04 1 436

352.647 3222.713 498.239 3447.375 .625E-05 428.291 2686.329 593.439 2821.264 .430E-05 19.042 1951.083 207.421 2159.739.133E-04 0.000 3622.019 56.609 3678.927.213E-04 0.000 3096.116 4.140 3099.463 .114E-04 408.441 2261.123 618.644 2470.644 .159E-04 31.381 3592.527 164.237 3733.580.227E-04 0.000 2049.570 27.567 2079.948.184E-04 157.797 34.459 372.992 260.079.133E-04 804.237 848.413 933.380 942.313 .871E-05 436.361 2699.689 690.195 2941.865 .167E-04 538.107 816.191 737.506 993.086.185E-04 715.833 0.000 883.274 166.769.706E-05 913.943 1972.681 933.380 1989.355 .145E-04 734.724 2217.475 921.549 2383.174 .192E-04 68.553 3414.281 247.051 3649.802 .790E-05 0.000 3574.714 99.120 3684.797 .379E-05 0.000 3460.720 130.223 3555.326.142E-04 106.785 3711.001 123.507 3733.580.857E-05 0.000 1613.287 131.867 1738.720.118E-04 0.000 215.786 78.911 290.821.264E-04 719.845 610.007 933.380 799.327.179E-04 598.788 566.204 792.633 768.941 120E-04 881.578 3544.057 933.380 3589.625 .917E-05 710.289 3001.966 858.321 3203.711 .136E-04 760.390 3241.557 933.380 3418.212 .130E-04 260.582 1745.704 453.688 1948.999 .248E-04 260.067 3218.022 462.283 3408.145 .185E-04 821.256 1385.515 933.380 1483.912 .102E-04 118.759 1614.071 353.773 1850.000 .144E-04 0.000 1215.116 90.938 1353.418.221E-04 176.816 367.138 371.179 569.037 .405E-04 146.659 1906.248 344.327 2104.887 .902E-05 282.864 2260.982 468.605 2470.723 .115E-04 447.375 1782.474 639.493 1909.307 .281E-04 0.000 375.730 86.423 462.410.148E-03 621.722 3700.000 649.663 3733.580.117E-04 0.000 2174.389 29.396 2212.282 .251E-04 572.992 310.163 782.414 532.028.353E-04 51.360 1058.011 248.694 1256.586 .176E-04 108.940 2398.834 324.733 2590.000 .124E-04 886.225 3138.054 933.380 3178.911 .247E-04 323.084 1065.119 504.912 1240.493 .914E-05 191.050 2274.990 392.408 2474.105 .822E-05 221.858 1171.936 444.881 1389.170.173E-04 172.765 2553.037 380.935 2739.853 .106E-04 527.437 2220.000 705.068 2372.669.231E-04 178.813 2056.971 354.238 2220.000 .137E-04 640.284 3330.000 892.888 3547.311.209E-04 300.443 1850.000 479.596 2107.248 .829E-05 0.000 863.498 186.742 1022.077 .584E-05 532.959 562.691 696.641 789.263 .957E-05 146.958 1859.713 404.485 2065.622 .108E-04 427.421 1065.005 580.359 1252.121 .518E-05 286.068 2526.795 519.646 2760.971 .167E-04 547.377 2112.146 719.055 2261.900.161E-04 914.271 1585.594 933.380 1600.044 .125E-04 451.765 2497.967 622.475 2696.852 .137E-04 318.389 3013.791 513.589 3177.910.140E-04 0.000 2166.536 5.038 2171.524 .153E-04 442.913 3700.000 497.445 3733.580.279E-04 591.905 222.234 782.044 474.089.581E-05 290.919 3473.982 501.242 3700.000 .271E-04 125.593 2148.915 355.608 2306.575.228E-04 0.000 1801.545 190.054 1974.487 .395E-04 21.594 441.544 190.899 650.142.196E-04 364.583 211.595 510.182 449.347 .656E-05 746.697 2199.772 933.380 2404.218 .661E-05 236.639 455.063 417.586 740.000.353E-04 0.000 652.095 210.180 798.594.659E-05 0.000 3262.930 186.716 3435.096 .121E-04 734.082 1078.356 933.380 1264.166 .459E-04 217.610 0.000 343.402 169.052 .261E-04 422.193 1671.615 646.875 1850.000 .122E-04 657.286 1850.000 822.008 2075.407.114E-04 853.057 938.713 933.380 1017.987 .206E-04 151.774 3524.087 368.835 3651.954 .101E-04 203.996 2136.582 422.974 2308.421.719E-05 0.000 2631.675 41.872 2668.963 .703E-05 894.019 1957.846 933.380 1994.717 .747E-05 63.203 1456.721 270.172 1634.716 .260E-04 445.750 3228.247 630.677 3436.150.202E-04 142.249 2579.845 323.690 2790.774 .134E-04 688.602 1233.322 903.536 1409.918.134E-04 814.670 3335.714 933.380 3453.214 .931E-05 0.000 3509.240 187.434 3714.517 .131E-04 15.106 1260.483 184.400 1481.126 .188E-04 307.715 1318.717 471.545 1559.636 .283E-04 113.152 1918.096 299.338 2124.588 .235E-04 468.909 185.087 699.360 375.310.162E-04 203.181 1441.475 413.862 1622.826 .267E-04 253.144 770.344 452.809 986.575 .110E-04 470.771 2782.046 633.079 2960.000 .957E-05 270.674 2043.956 480.086 2220.000 .857E-05 769.474 1673.302 933.380 1836.053 .956E-05 871.566 3555.584 933.380 3611.802 .157E-04 342.008 167.726 528.247 370.000 .242E-04 99.807 1535.596 308.711 1737.296 .323E-04 0.000 36.245 193.807 170.038.184E-04 0.000 3406.773 36.728 3463.038 .483E-05 26.364 528.400 214.411 714.131 .557E-05 337.511 3252.024 508.714 3470.409.106E-04 647.817 1698.542 849.715 1888.832 .113E-04 636.286 1480.000 844.373 1747.673 .185E-04 82.025 2269.845 300.607 2402.300 .903E-05 874.601 617.864 933.380 677.790.133E-04

192.401 2996.540 400.389 3179.941 .133E-04 0.000 1357.469 91.333 1446.743 .566E-05 927.861 3611.458 933.380 3616.808.769E-05 0.000 3023.403 96.153 3129.032 .112E-04 190.498 2147.916 407.335 2315.879.170E-04 1 547 809.172 0.000 836.699 30.921.200E-04 35.840 582.443 226.037 842.133 .106E-04 185.107 3349.097 346.267 3530.725.733E-05 840.438 2079.414 933.380 2136.394 .336E-04 266.505 756.592 431.776 984.431 .686E-05 38.375 315.194 215.733 527.835 .122E-04 0.000 1751.878 174.343 1895.253 .131E-04 833.066 483.698 933.380 595.630.933E-05 165.687 585.466 390.512 795.804 .129E-04 0.000 3006.915 172.066 3159.355 .673E-05 45.890 1910.280 253.325 2093.460.178E-04 371.936 418.404 572.935 618.028.128E-04 93.536 2315.086 315.029 2581.403 .433E-04 696.963 1663.051 888.265 1862.803 .403E-04 92.915 1099.828 330.095 1326.390 .128E-04 695.639 1430.621 904.778 1629.104 .139E-04 238.333 2226.191 442.899 2429.214 .210E-04 0.000 1701.288 134.318 1800.731 .144E-04 829.941 652.499 933.380 748.171.122E-04 224.195 3450.869 437.806 3700.000 .358E-04 654.034 3168.225 838.949 3373.478 .957E-05 74.024 131.830 296.125 370.000.246E-04 759.913 2138.914 933.380 2257.033 .182E-04 0.000 3444.552 140.251 3570.545 .122E-04 0.000 1075.064 118.937 1216.906 .118E-04 178.112 2570.578 354.382 2782.892 .970E-05 1 573 427.882 0.000 532.017 102.565 .145E-04 1 574 583.695 1349.149 783.763 1516.962 .443E-04 74.763 2667.129 274.183 2857.637 .264E-04 415.692 2284.553 594.551 2480.769 .956E-05 903.317 3697.377 933.380 3731.966 .383E-04 436.013 440.702 677.587 630.854.256E-05 841.197 599.159 933.380 676.298.737E-05 126.585 2278.015 326.756 2467.452 .114E-04 515.478 3569.819 631.668 3733.580.959E-05 0.000 1306.604 147.548 1470.356 .999E-05 100.600 1906.952 288.596 2108.382 .295E-04 835.785 581.722 933.380 693.549.212E-04 244.403 1355.266 443.953 1554.150 .883E-05 18.626 2875.313 199.694 3038.963 .136E-04 319.952 2097.388 538.619 2220.000 .121E-04 511.938 1355.174 713.953 1542.181 .389E-05 807.628 1763.817 933.380 1945.727.259E-04 167.810 172.672 323.919 370.000.958E-05 92.465 174.120 271.591 303.856.247E-04 693.822 3600.827 790.765 3733.580.288E-04 691.482 3137.268 910.820 3395.174.109E-04 724.358 2624.768 917.320 2751.518.750E-04

0.000 1279.928 0.159 1280.055 .481E-04 0.000 1827.320 57.307 1870.663 .119E-04 453.855 555.881 601.378 740.000.107E-04 0.000 1526.313 83.413 1632.229 .271E-04 333.470 3578.314 537.100 3733.580.249E-04 0.000 1126.246 46.326 1168.945 .222E-04 614.820 3564.792 747.597 3733.580.175E-04 618.053 1297.684 782.623 1466.242 .134E-04 578.058 2752.391 761.386 2956.975 .107E-04 760.659 2751.058 920.780 2883.664 .226E-04 157.423 192.129 330.913 370.000 .134E-04 429.260 1179.053 670.610 1371.573 .558E-05 714.117 1405.055 906.211 1601.186 .618E-05 682.474 2861.205 821.510 3045.417 .255E-04 430.487 1678.200 648.673 1810.491 .137E-04 0.000 1267.553 95.151 1412.204 .838E-05 51.527 3356.218 199.080 3551.955 .122E-04 0.000 736.335 254.842 896.585 .558E-05 837.845 2663.003 933.380 2749.935 .646E-05 571.004 3330.000 719.062 3444.009.118E-04 163.387 3379.094 360.178 3570.087 .246E-04 506.712 2804.967 705.691 2993.652 .948E-05 708.537 2190.279 903.903 2381.934 .922E-05 645.736 0.000 834.677 145.190 .211E-04 513.770 51.631 698.989 253.509.168E-04 813.666 2528.265 933.380 2691.174 .130E-04 81.259 356.794 261.785 562.763 .111E-04 240.407 1308.988 427.258 1585.735 .165E-04 457.553 1480.000 639.328 1698.768.138E-04 714.039 0.000 767.851 53.972.175E-04 0.000 611.182 3.268 614.499.388E-05 159.188 2722.358 314.084 2906.887.152E-04 652.379 2355.548 891.066 2553.926.272E-04 890.702 203.565 933.380 241.436.137E-04 268.849 1294.911 451.663 1480.000 .125E-04 77.360 0.000 94.395 12.171.111E-04 - 1 462.982 1339.280 621.353 1534.596 .975E-05 359.347 2590.000 506.481 2750.214 .130E-04 452.023 1713.382 622.622 1926.593 .227E-04 867.092 3111.374 933.380 3172.616 .196E-04 167.170 2239.527 344.374 2447.189 .297E-04 832.371 877.187 933.380 999.508.337E-04 0.000 1110.527 18.554 1126.184 .183E-04 536.578 1742.526 733.772 1954.665 .126E-04 271.380 1179.519 445.300 1389.454 .275E-05 818.596 2889.083 933.380 3020.875 .954E-05 0.000 1870.320 31.849 1898.893 .128E-04 596.479 1716.927 811.268 1884.995 .208E-04 196.373 374.022 384.479 500.339 .866E-05 0.000 3499.593 16.699 3509.953 .116E-04 623.460 2003.071 780.873 2237.200.891E-05 64.078 1587.734 244.035 1772.697 .283E-04 0.000 2916.357 127.039 3071.791 .867E-05

0.000 3684.670 38.018 3733.580.341E-04 264.033 70.850 440.697 274.912 .149E-04 664.803 994.172 863.322 1180.386 .160E-04 560.676 2287.976 767.187 2485.450 .839E-05 0.000 3346.773 174.729 3494.468.311E-04 263.716 0.000 355.834 77.443 .848E-05 153.652 1039.081 335.300 1241.245 .113E-04 440.204 2416.983 600.112 2590.000 .451E-05 779.864 2813.256 933.380 3001.828.667E-05 690.132 384.746 891.751 566.792 .325E-04 81.193 1583.178 272.829 1775.604 .842E-05 670.458 3149.749 852.918 3365.211 .905E-05 772.423 80.457 933.380 274.763 .473E-04 799.597 2688.001 933.380 2824.260.730E-05 286.879 1480.000 517.370 1666.405 .888E-05 536.872 1593.237 717.340 1795.940 .357E-04 30.399 421.293 195.254 582.260.293E-04 0.000 437.600 125.889 570.044 .845E-05 606.366 2396.523 833.832 2555.707 .436E-04 594.174 3330.000 806.964 3538.633.782E-05 431.907 1611.953 618.155 1766.676 .224E-04 97.169 2286.408 261.168 2438.870 .136E-04 739.660 1164.220 933.380 1313.121 .126E-04 0.000 1658.456 32.001 1696.913 .367E-04 428.384 2517.302 625.002 2659.095 189E-04 864.304 2030.571 933.380 2101.465 .134E-04 13.810 2752.321 194.996 2977.814.296E-04 852.881 2201.490 933.380 2260.621 .387E-05 531.483 2129.304 722.345 2321.290.964E-05 510.425 413.001 712.618 652.003 .108E-04 0.000 1385.462 81.187 1458.123 .307E-05 545.333 2825.724 722.054 3018.001 .185E-04 517.149 0.000 585.873 67.969.136E-04 0.000 1539.191 37.958 1574.511 .150E-04 362.318 3330.000 588.046 3476.439.145E-04 205.268 161.574 427.741 370.000.213E-04 255.065 2793.657 406.904 3007.899.109E-04 0.000 2530.386 75.503 2606.591 .691E-05 132.897 2455.978 306.148 2672.738.108E-04 0.000 464.057 137.283 584.083 .119E-04 611.437 3700.000 635.817 3733.580.962E-05 888.536 250.544 933.380 315.257.175E-04 262.125 3296.693 474.806 3463.047 .853E-05 500.255 3557.825 644.174 3733.580.377E-04 843.790 47.388 933.380 152.064.138E-04 795.045 2605.660 933.380 2760.949 .760E-05 463.533 1893.075 680.215 2054.006 .119E-04 727.120 2246.108 877.565 2365.814 .127E-04 385.458 137.811 574.293 331.861.761E-05 240.308 3116.877 416.957 3330.000 .577E-05 426.592 205.595 654.002 435.475 .522E-05 0.000 2322.040 65.310 2386.289 .953E-05 0.000 1012.444 7.450 1018.512 .695E-05

0.000 2490.278 191.968 2639.829 .183E-04 1 702
588.970 370.000 750.174 523.683 .134E-04 1 703
18.763 1870.138 200.847 2068.802 .231E-04 1 704
890.062 2386.875 933.380 2426.271 .149E-04 1 705
665.998 3592.896 817.039 3733.580.130E-04 1 706
193.547 1550.488 378.867 1745.896 .220E-04 1 707
389.251 370.000 566.509 578.385.196E-04 1 708
475.156 3700.000 511.924 3733.580.489E-05 1 709
118.638 3128.107 308.033 3330.000 .329E-04 1 710
235.890 277.998 434.026 492.957 .163E-04 1 711
73.042 2103.748 279.336 2297.153 .117E-04 1 713
0.000 1772.746 113.071 1897.133.272E-04 1 714
472.793 841.530 651.280 1065.428 .864E-05 1 715
2.153 2611.220 173.296 2809.805 .173E-04 1 716
119.625 3715.202 136.231 3733.580.132E-04 1 717
851.749 3520.369 933.380 3592.203 .531E-05 1 718
145.277 2491.589 351.524 2663.969.209E-04 1 719
279.142 740.000 500.427 918.029.677E-05 1 720
510.094 3561.453 634.696 3733.580.133E-04 1 721
907.815 351.401 933.380 370.614.744E-05 1 722
248.067 3710.186 268.949 3733.580.838E-05 1 723
350.847 2440.214 575.361 2665.591 .290E-04 1 724
644.790 504.213 874.617 693.820.131E-04 1 725
<i>333.287 438.100 509.574 640.667 .990E-05 1 726</i>
726.999 2665.501 878.011 2848.244 .157E-04 1 727
200.176 537.164 444.341 740.000 .360E-04 1 728
733.729 332.073 930.107 514.919.118E-04 1 729
180.893 2288.383 333.131 2509.294 .686E-05 1 730
610.206 2431.280 774.535 2593.689.549E-05 1 731
704.645 2959.876 913.432 3126.849 .981E-05 1 732
870.489 955.918 933.380 1054.742 .164E-04 1 733
93.900 2966.818 266.208 3172.343 .108E-04 1 734
731.117 0.000 901.785 205.641 .363E-04 1 735
212.260 1944.460 417.163 2167.103 .154E-04 1 736
233.307 1690.227 428.575 1850.000 .949E-05 1 737
277.278 291.508 445.512 450.967 .148E-04 1 738
930.273 2686.945 933.380 2689.236.775E-05 1 739
286.943 3730.265 289.239 3733.580 793E-05 1 740
762.690 0.000 899.116 197.150.287E-04 1 741
482.057 1414.379 633.329 1577.303 .833E-05 1 742
514.328 1903.609 690.182 2130.690 .651E-05 1 743
214.381 2824.056 371.082 3050.938.210E-04 1 744
83.977 1716.263 336.815 1904.796.998E-05 1 745
0.000 457.701 157.965 575.710.238E-04 1 746
<i>309.891 285.505 468.729 536.960 .187E-04 1 747</i>
839.435 2638.383 933.380 2775.144.276E-04 1 748
737.507 3206.924 933.380 3362.593 .423E-04 1 749
587.364 209.453 742.419 350.001 .728E-05 1 750
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463.243 665.582 599.656 870.793 .111E-04 1 752
0.000 2834.700 71.541 2907.515 .805E-05 1 753
12.372 1120.968 167.052 1308.615 .133E-04 1 754

772.954 1977.793 933.380 2126.892 .122E-04 451.277 2220.000 598.176 2364.265 .136E-04 528.951 1480.000 750.710 1665.026 .138E-04 294.637 740.000 477.786 969.289.671E-05 818.744 3338.856 933.380 3507.077.197E-04 704.975 3316.799 924.634 3543.128.613E-05 770.758 2762.863 921.402 2884.471 .700E-05 0.000 378.975 175.776 530.860.206E-04 881.678 335.099 933.380 378.772 .104E-04 589.119 1657.467 758.031 1906.694 .141E-04 - 1 0.000 2869.320 43.067 2932.752 .903E-05 542.214 819.834 718.154 1079.405 .353E-04 811.281 0.000 933.380 82.745 .537E-05 1 767 642.097 2220.000 815.290 2424.150 .236E-04 238.639 1106.740 365.904 1294.814 .160E-04 0.000 159.690 136.327 361.326 .586E-05 726.427 1743.458 933.380 1883.348.764E-05 474.986 573.979 638.221 740.000.140E-04 0.000 2728.429 106.380 2885.972 .157E-04 660.586 3447.222 838.429 3630.836 .132E-04 1 774 29.937 490.231 171.047 694.133 .825E-05 1 775 769.763 1305.972 933.380 1435.726.889E-05 129.079 0.000 226.581 144.766 .499E-05 0.000 3164.682 168.229 3355.726 .868E-05 315.777 1688.784 549.349 1850.000 .907E-05 0.000 1599.249 6.757 1606.085 .278E-04 320.706 2960.000 450.028 3082.508.146E-04 298.087 3506.453 492.646 3636.635 .225E-04 383.359 401.811 626.624 640.952 .901E-05 395.135 740.000 541.829 960.024 .449E-05 550.614 676.643 624.873 740.000 .258E-05 0.000 810.060 161.675 917.695.138E-04 743.577 786.374 918.103 902.510 .622E-05 626.557 2590.000 863.139 2835.929.296E-04 - 1 588.031 740.000 830.813 993.598.897E-05 561.656 2704.712 837.927 2887.863 .184E-04 311.751 3520.534 582.591 3700.000 .204E-04 721.895 0.000 733.620 17.715.130E-04 *433.588 3627.297 481.682 3700.000 .689E-05* 0.000 323.085 26.078 362.543 .239E-04 449.431 370.000 635.682 652.614 .851E-05 505.689 816.954 720.325 1030.825 .133E-04 877.511 198.608 933.380 235.323 .809E-05 547.390 0.000 640.075 141.182 .933E-05 704.838 1568.182 928.268 1730.247 .791E-05 397.136 2022.651 565.796 2220.000 .632E-05 399.102 2590.000 630.750 2838.880 .645E-05 903.186 3494.808 933.380 3541.167 .663E-05 569.069 140.699 753.521 333.647.952E-05 461.139 3145.108 581.256 3330.000.250E-04 276.155 2235.904 412.009 2343.957 .801E-05 291.313 3589.772 461.287 3700.000 .127E-04 728.225 38.154 933.380 170.913 .553E-05 1

311.391 3330.000 626.013 3533.161 .710E-05 241.586 1563.116 426.611 1850.000 .153E-04 345.208 1615.099 648.350 1810.101 .143E-04 0.000 698.896 20.803 731.238.937E-05 1 811 592.428 2050.730 736.121 2274.364 .461E-05 674.265 2360.761 777.222 2426.733 .158E-04 871.354 1727.249 933.380 1766.971 .174E-04 0.000 553.751 99.124.194E-04 398.794 0.000 485.314 127.870.124E-04 343.196 70.904 1585.831 347.259 1850.000 .185E-04 312.249 3700.000 365.046 3733.580.181E-04 915.778 978.191 933.380 989.374 .832E-05 247.219 1906.217 348.292 2026.203 .118E-04 8.747 1236.790 165.470 1483.562 .654E-05 678.554 1269.730 857.131 1551.074 .677E-05 0.000 3015.410 33.059 3036.313 .359E-04 384.226 2590.000 581.659 2714.758 .206E-04 68.743 1382.896 333.335 1613.934 .150E-04 593.544 1237.836 922.324 1445.341 .841E-05 922.990 1688.692 933.380 1695.242 .466E-05 40.944 554.338 219.681 837.743.314E-04 85.493 1751.190 223.320 1837.860 .195E-04 702.314 663.117 933.380 808.329.644E-05 491.251 1245.235 619.784 1364.203 .110E-04 143.274 1084.588 300.001 1214.066 .773E-05 165.595 3120.418 500.762 3330.000 .770E-05 332.754 1850.000 453.945 2043.554 .187E-04 346.961 1616.227 521.698 1725.352 .649E-05 471.721 370.000 671.176 494.243 .116E-04 0.000 449.636 112.604 .113E-04 268.631 20.481 1072.689 217.323 1388.632 .133E-04 654.916 186.656 831.208 469.794 .206E-04 464.120 73.233 672.383 201.769.538E-05 451.973 3074.215 610.007 3330.000 .133E-04 744.345 2916.960 835.295 3064.360.818E-05 424.832 126.428 615.068 243.359.823E-05 305.014 2220.000 719.688 2474.709.845E-05 570.843 1850.000 670.520 2011.976.767E-05 286.977 638.086 420.235 740.000 .505E-05 0.000 203.527 54.081 .816E-05 170.314 886.572 2436.178 933.380 2512.817 .385E-05 265.728 0.000 563.805 180.620.124E-04 688.804 2129.537 889.447 2459.918 .134E-04 815.976 3114.092 933.380 3307.551 .107E-04 467.160 2803.594 587.782 2960.000 .638E-05 909.190 3291.430 933.380 3331.646 .156E-04 932.800 672.053 933.380 673.019.142E-04 498.115 2590.000 785.881 2761.986.103E-04 163.172 3056.077 431.780 3216.486 .795E-05 728.132 2382.965 933.380 2608.012 .941E-05 0.000 2109.064 23.700 2148.773 .217E-04 194.234 1201.785 396.568 1322.033 .887E-05 662.404 1110.000 781.291 1247.914 .553E-05 840.330 2497.241 895.490 2590.109.116E-04 788.823 1401.437 910.829 1607.014 .164E-04 593.187 827.466 878.907 1075.663 .260E-04 916.564 1771.493 933.380 1781.413 .622E-05 634.028 2341.498 892.395 2635.401 .601E-05 212.061 189.274 318.694 370.000 .174E-04 573.982 370.000 776.167 551.955 .470E-05 699.474 733.834 826.367 841.466.608E-04 0.000 3615.647 181.910 3721.743 .102E-04 739.796 712.023 933.380 824.723 .464E-05 739.757 2277.020 933.380 2608.951 .457E-05 - 1 82.158 3189.807 323.850 3330.000.789E-05 414.793 618.340 542.040 740.000 .303E-05 0.000 1878.367 203.199 1995.793 .943E-05 83.149 3013.813 170.687 3123.602 .187E-04 66.030 0.000 173.511 74.427.631E-05 1 876 545.482 2220.000 676.461 2315.915.300E-05 36.300 2828.018 171.576 3064.460.102E-04 200.903 1560.329 365.409 1850.000 .198E-04 603.045 740.000 817.901 909.206.185E-04 0.000 933.286 162.205 1024.680 .816E-05 538.551 2445.789 706.407 2626.248.138E-04 752.796 694.842 933.380 878.289.615E-05 38.271 993.195 189.510 1263.500 .558E-05 332.767 3700.000 351.533 3733.580.437E-05 346.396 0.000 413.417 55.008.569E-05 1 886 649.657 1480.000 860.712 1732.933 .968E-05 27.166 1839.796 85.360 1945.134 .410E-04 769.426 1162.194 901.185 1234.150.121E-04 455.789 3187.396 713.880 3328.155.653E-05 0.000 2641.159 74.381 2778.768 .485E-05 103.191 0.000 188.094 157.583 .861E-05 1 892 925.533 1143.920 933.380 1158.533 .513E-05 932.331 747.201 933.380 747.964.761E-05 764.591 603.049 933.380 774.671.218E-04 40.639 2490.516 174.944 2614.668 .609E-05 0.000 756.784 65.645 790.589.663E-05 639.722 2878.953 847.855 2985.405 .519E-05 16.083 1707.370 144.263 1772.107 .104E-04 13.433 2140.875 107.324 2187.460 .412E-05 6.751 640.349 215.339 743.499.798E-05 282.009 926.188 370.003 1110.000 .104E-04 121.868 38.667 262.395 195.145.104E-04 666.503 1816.146 699.096 1886.589 .125E-04 286.296 1850.000 455.117 2220.000 .100E-04 831.951 578.152 933.380 654.236 .410E-05 769.427 750.685 908.220 1065.251 .672E-05 0.000 3621.547 137.011 3733.580 .532E-05 865.415 1841.160 933.380 1869.870.740E-05 271.886 2231.520 611.452 2372.693 .534E-05 788.800 3662.241 816.185 3733.580.311E-05 25.560 2208.087 297.405 2312.438.714E-05 795.962 0.000 902.735 40.986.409E-05 1 913

```
724.999 1386.839 880.857 1575.299.763E-05 1 914
0.000 2909.649 386.771 3058.116.129E-04 1 915
460.371 236.884 678.054 320.444.564E-05 1 916
894.867 672.778 933.380 687.561.105E-04 1 917
WELLS
466.000 20.000 10875.000 0.330 1 10875.000
466.000 3730.000 0.330 1
END WELLS
```

A1.3 LDF File

```
TITLE: Example Layer Description *** Imaginary Data to get program Running! ***
GRID Dimensions: Y X # of Layers
        2 2
               - 1
LAYER 1 TOP:
   10875.0 10875.0
   10875.0 10875.0
LAYER 1 BOTTOM:
   11016.5 11016.5
   11016.5 11016.5
LAYER 1 POROSITY:
  0.04
          0.04
   0.04
          0.04
LAYER 1 PERMEABILITY:
   0.000055 0.000055
   0.000055 0.000055
```

A1.4 BHP File

Calculation of Bottom-Hole Flowing Pressures by Method of Cullender and Smith

```
ID Header (<= 80 characters)
joachim
WELL Number (for NFflow)
1
True vertical DEPTH to middle of reservoir (feet)
10875
DISTANCE down hole to middle of producing zone (feet)
10875
Equivalent inside DIAMETER of tubing or casing (inches)
6.625
Pipe ROUGHNESS FACTOR (Fr; SPE Pet. Eng. Handbook Tables 33.10 & 33.11)
0.0007837
WELLHEAD & RESERVOIR TEMPERATURES (F)
 60.0 285.0
Gas SPECIFIC GRAVITY (air = 1.0)
0.621
PSEUDOCRITICAL TEMPERATURE (Rankine) & PRESSURE (psia)
353.8 672.9
```

RESULTS

day (d)	Qg (Mscf/d)	Ptf (psia) Pwf(psia)
0.00	5800.00	3600.00
5.00	5710.48	3576.50
10.00	5686.35	3527.37
15.00	5641.16	3435.33
20.00	5638.54	3294.15
25.00	5367.65	3074.58
30.00	5215.93	2982.88
35.00	5085.75	2870.71
40.00	4745.58	2609.69
45.00	4720.53	2580.59
50.00	4373.32	2370.39
55.00	4288.12	2298.38
60.00	4040.34	2215.51
65.00	3895.01	2157.38
70.00	3690.95	2079.06
75.00	3687.85	2078.32
80.00	3492.22	1994.44
85.00	3239.95	1906.87
90.00	3237.79	1906.24
95.00	2944.26	1813.32
100.00	2844.36	1758.23
105.00	2748.87	1738.35
110.00	2573.08	1697.67
115.00	2522.58	1688.12
120.00	2419.39	1939.23
125.00	2389.31	1719.66
130.00	2358.08	1757.94
135.00	2336.84	1837.43
140.00	2324.60	1672.77
145.00	2299.82	1793.37
150.00	2258.87	1931.80
155.00	2237.62	1633.82
160.00	2163.49	1616.04
165.00	2077.88	1602.80
170.00	1980.48	1704.92
175.00	1968.26	1750.68
180.00	1963.43	1846.17
185.00	1960.27	1571.69
190.00	1930.49	1556.28
195.00	1910.76	1534.93
200.00	1876.51	1525.17
205.00	1735.35	1584.51
210.00	1708.39	1611.94
215.00	1687.45	1588.53
220.00	1658.05	1575.39
225.00	1609.12	1557.57
230.00	1583.37	1463.57
235.00	1575.50	1534.20
240.00	1561.17	1498.76

245.00	1560.78	1495.06
245.00	1558.26	1565.69
255.00	1553.20	1565.79
260.00	1542.57	1605.20
265.00	1542.43	1606.85
270.00	1366.53	1750.85
275.00	1366.93	1852.76
280.00	1380.28	1856.58
285.00	1635.10	1346.89
290.00	1681.01	1350.34
295.00	1549.93	1370.56
300.00	1465.82	1429.35
305.00	1456.34	1449.10
310.00	1432.32	1480.53
315.00	1415.37	1715.87
320.00	1407.40	1578.67
325.00	1418.89	1355.15
330.00	1446.21	1417.98
335.00	1446.92	1428.72
340.00	1447.03	1645.48
345.00	1447.17	1433.77
350.00	1447.34	1433.84
355.00	1434.44	1433.85
360.00	1404.76	1245.66

A1.5 RES File (Rate controlled)

TITLE

Well GU#3, Abbreviated Simulation LDF FILE: C:\N\GoodResults\Uniform5\gu32.ldf PG_DATUM DATUM 7000.0 10875.0 TSC PSC TRES 60.0 14.7 285.0 PGZGUG300.0 .986 0.0138 600.0 .974 0.0142 900.0 .963 0.0146 1200.0 .956 0.0150 1500.0 .950 0.0154 .948 0.0159 1800.0 .948 2100.0 0.0163 2400.0.950 0.0168 .954 2700.0 0.0173 .961 3000.0 0.0178 .970 0.0183 3300.0 3600.0 .980 0.0189 3900.0 .992 0.0194 4200.01.0060.0200 4500.01.020 0.0206 4800.0 1.035 0.0212

```
5100.0
         1.052
                 0.0218
  5400.0
         1.069
                 0.0224
  5700.0
         1.086
                 0.0230
  6000.0
         1.105
                 0.0236
  6300.0
         1.123
                 0.0243
  6600.0
         1.143
                 0.0249
  6900.0
         1.162
                 0.0256
  7200.0
         1.182
                 0.0263
  7500.0
         1.202
                 0.0269
  7800.0
         1.222
                 0.0276
         1.243
  8100.0
                 0.0283
  8400.0
         1.263
                 0.0289
  8700.0
                 0.0296
         1.284
  9000.0
         1.305
                 0.0302
  9300.0
         1.326
                 0.0309
  9600.0
         1.347
                 0.0315
  9900.0
          1.369
                 0.0321
          1.390
 10200.0
                  0.0327
         1.411
 10500.0
                  0.0333
 10800.0
          1.432
                  0.0338
 11100.0
          1.454
                  0.0344
 11400.0
          1.475
                  0.0349
 11700.0
          1.497
                  0.0354
 12000.0
          1.518
                  0.0358
 12300.0
          1.539
                  0.0362
END PVT
NETWORK
NODES
DATA FOR WELL 1 IN FILE C:\N\GoodResults\Uniform5\Eagle2.bhp
TIME 0.0
OMEGA = 1.97
LOG = 6
DT = 15
OUTPUT WELL AT 5.00 EVERY 5.00 W/GRAPHICS
OUTPUT RES AT 30.00 EVERY 30.00 W/GRAPHICS NUMBERS TYPE
PRODUCE 1 AT 5800 MCF/D
TIME 5.00
PRODUCE 1 AT 5710 MCF/D
TIME 10.00
PRODUCE 1 AT 5686 MCF/D
TIME 15.00
PRODUCE 1 AT 5641 MCF/D
TIME 20.00
PRODUCE 1 AT 5638 MCF/D
TIME 25.00
PRODUCE 1 AT 5367 MCF/D
TIME 30.00
PRODUCE 1 AT 5215 MCF/D
TIME 35.00
PRODUCE 1 AT 5085 MCF/D
TIME 40.00
PRODUCE 1 AT 4745 MCF/D
TIME 45.00
```

PRODUCE 1 AT 4720 MCF/D TIME 50.0 PRODUCE 1 AT 4373 MCF/D *TIME 55.00* PRODUCE 1 AT 4288 MCF/D TIME 60.00 PRODUCE 1 AT 4040 MCF/D TIME 65.00 PRODUCE 1 AT 3895 MCF/D *TIME 70.00* PRODUCE 1 AT 3690 MCF/D *TIME* 75.00 PRODUCE 1 AT 3687 MCF/D *TIME 80.00* PRODUCE 1 AT 3492 MCF/D TIME 85.00 PRODUCE 1 AT 3239 MCF/D TIME 90.00 PRODUCE 1 AT 3237 MCF/D TIME 95.00 PRODUCE 1 AT 2944 MCF/D *TIME 100.00* PRODUCE 1 AT 2844 MCF/D TIME 105.00 PRODUCE 1 AT 2748 MCF/D TIME 110.00 PRODUCE 1 AT 2573 MCF/D TIME 115.00 PRODUCE 1 AT 2522 MCF/D TIME 120.00 PRODUCE 1 AT 2419 MCF/D TIME 125.00 PRODUCE 1 AT 2389 MCF/D *TIME 130.00* PRODUCE 1 AT 2358 MCF/D *TIME 135.00* PRODUCE 1 AT 2336 MCF/D TIME 140.00 PRODUCE 1 AT 2324 MCF/D TIME 145.00 PRODUCE 1 AT 2299 MCF/D *TIME 150.00* PRODUCE 1 AT 2258 MCF/D *TIME 155.00* PRODUCE 1 AT 2237 MCF/D *TIME 160.00* PRODUCE 1 AT 2163 MCF/D *TIME 165.00* PRODUCE 1 AT 2077 MCF/D *TIME 170.00* PRODUCE 1 AT 1980 MCF/D TIME 175.00 PRODUCE 1 AT 1968 MCF/D *TIME 180.00* PRODUCE 1 AT 1963 MCF/D *TIME 185.00* PRODUCE 1 AT 1960 MCF/D TIME 190.00 PRODUCE 1 AT 1930 MCF/D TIME 195.00 PRODUCE 1 AT 1910 MCF/D *TIME 200.00* PRODUCE 1 AT 1876 MCF/D *TIME 205.00* PRODUCE 1 AT 1735 MCF/D *TIME 210.00* PRODUCE 1 AT 1708 MCF/D *TIME 215.00* PRODUCE 1 AT 1687 MCF/D *TIME 220.00* PRODUCE 1 AT 1658 MCF/D *TIME 225.00* PRODUCE 1 AT 1609 MCF/D TIME 230.00 PRODUCE 1 AT 1583 MCF/D TIME 235.00 PRODUCE 1 AT 1575 MCF/D *TIME 240.00* PRODUCE 1 AT 1561 MCF/D TIME 245.00 PRODUCE 1 AT 1560 MCF/D TIME 250.00 PRODUCE 1 AT 1558 MCF/D *TIME 255.00* PRODUCE 1 AT 1553 MCF/D *TIME 260.00* PRODUCE 1 AT 1542 MCF/D TIME 265.00 PRODUCE 1 AT 1542 MCF/D *TIME 270.00* PRODUCE 1 AT 1366 MCF/D *TIME 275.00* PRODUCE 1 AT 1366 MCF/D TIME 280.00 PRODUCE 1 AT 1380 MCF/D TIME 285.00 PRODUCE 1 AT 1635 MCF/D *TIME 290.00* PRODUCE 1 AT 1681 MCF/D *TIME 295.00* PRODUCE 1 AT 1549 MCF/D *TIME 300.00* PRODUCE 1 AT 1465 MCF/D TIME 305.00 PRODUCE 1 AT 1456 MCF/D TIME 310.00

PRODUCE 1 AT 1432 MCF/D *TIME 315.00* PRODUCE 1 AT 1415 MCF/D *TIME 320.00* PRODUCE 1 AT 1407 MCF/D TIME 325.00 PRODUCE 1 AT 1418 MCF/D TIME 330.00 PRODUCE 1 AT 1446 MCF/D TIME 335.00 PRODUCE 1 AT 1446 MCF/D TIME 340.00 PRODUCE 1 AT 1447 MCF/D *TIME 345.00* PRODUCE 1 AT 1447 MCF/D TIME 350.00 PRODUCE 1 AT 1447 MCF/D TIME 355.00 PRODUCE 1 AT 1434 MCF/D *TIME 360.00* PRODUCE 1 AT 1404 MCF/D **STOP**

A1.6 **RES File** (Pressure controlled)

TITLE Well GU#3, Abbreviated Simulation LDF FILE: C:\N\GoodResults\Uniform5\Gu3.ldf PG_DATUM DATUM 4200.0 10875.0 TSC PSC TRES 60.0 14.7 285.0 PGZGUG300.0 .986 0.0138 600.0 .974 0.0142 900.0 .963 0.0146 1200.0 .956 0.0150 1500.0 .950 0.0154 1800.0 .948 0.0159 .948 2100.0 0.0163 .950 0.0168 2400.0 .954 2700.0 0.0173 0.0178 3000.0 .961 .970 3300.0 0.0183 3600.0 .980 0.0189 3900.0 .992 0.0194 4200.0 1.006 0.0200 4500.0 1.020 0.0206 4800.0 1.035 0.0212 5100.0 1.052 0.0218

0.0224

5400.0

1.069

5700.0 1.086 0.0230 6000.0 0.0236 1.105 6300.0 1.123 0.0243 6600.0 1.143 0.0249 6900.0 1.162 0.0256 7200.0 1.182 0.0263 7500.0 1.202 0.0269 7800.0 1.222 0.0276 8100.0 1.243 0.0283 8400.0 1.263 0.0289 8700.0 1.284 0.0296 9000.0 1.305 0.0302 9300.0 0.0309 1.326 9600.0 1.347 0.0315 9900.0 1.369 0.0321 10200.0 1.390 0.0327 10500.0 1.411 0.0333 10800.0 1.432 0.0338 1.454 11100.0 0.0344 11400.0 1.475 0.0349 11700.0 1.497 0.0354 1.518 12000.0 0.0358 12300.0 1.539 0.0362 END PVT **NETWORK NODES** DATA FOR WELL 1 IN FILE C:\N\GoodResults\Uniform5\Eagle2.bhp *TIME 0.0* OMEGA = 1.97LOG = 6DT = 15OUTPUT WELL AT 5.00 EVERY 5.00 W/GRAPHICS OUTPUT RES AT 30.00 EVERY 30.00 W/GRAPHICS NUMBERS TYPE PRODUCE 1 AT 3600 PSI TIME 5.00 PRODUCE 1 AT 3576 PSI TIME 10.00 PRODUCE 1 AT 3527 PSI TIME 15.00 PRODUCE 1 AT 3435 PSI TIME 20.00 PRODUCE 1 AT 3294 PSI TIME 25.00 PRODUCE 1 AT 3074 PSI *TIME 30.00* PRODUCE 1 AT 2982 PSI *TIME 35.00* PRODUCE 1 AT 2870 PSI *TIME 40.00* PRODUCE 1 AT 2609 PSI TIME 45.00 PRODUCE 1 AT 2580 PSI TIME 50.0

PRODUCE 1 AT 2370 PSI *TIME 55.00* PRODUCE 1 AT 2298 PSI *TIME 60.00* PRODUCE 1 AT 2215 PSI TIME 65.00 PRODUCE 1 AT 2157 PSI TIME 70.00 PRODUCE 1 AT 2079 PSI *TIME 75.00* PRODUCE 1 AT 2078 PSI *TIME 80.00* PRODUCE 1 AT 1994 PSI *TIME 85.00* PRODUCE 1 AT 1906 PSI TIME 90.00 PRODUCE 1 AT 1905 PSI TIME 95.00 PRODUCE 1 AT 1813 PSI *TIME 100.00* PRODUCE 1 AT 1758 PSI *TIME 105.00* PRODUCE 1 AT 1738 PSI TIME 110.00 PRODUCE 1 AT 1697 PSI *TIME 115.00* PRODUCE 1 AT 1688 PSI TIME 120.00 PRODUCE 1 AT 1938 PSI TIME 125.00 PRODUCE 1 AT 1719 PSI TIME 130.00 PRODUCE 1 AT 1757 PSI TIME 135.00 PRODUCE 1 AT 1837 PSI *TIME 140.00* PRODUCE 1 AT 1672 PSI TIME 145.00 PRODUCE 1 AT 1793 PSI TIME 150.00 PRODUCE 1 AT 1931 PSI *TIME 155.00* PRODUCE 1 AT 1633 PSI *TIME 160.00* PRODUCE 1 AT 1616 PSI *TIME 165.00* PRODUCE 1 AT 1602 PSI *TIME 170.00* PRODUCE 1 AT 1704 PSI *TIME 175.00* PRODUCE 1 AT 1750 PSI TIME 180.00 PRODUCE 1 AT 1846 PSI TIME 185.00 PRODUCE 1 AT 1571 PSI *TIME 190.00* PRODUCE 1 AT 1556 PSI TIME 195.00 PRODUCE 1 AT 1534 PSI TIME 200.00 PRODUCE 1 AT 1252 PSI TIME 205.00 PRODUCE 1 AT 1584 PSI *TIME 210.00* PRODUCE 1 AT 1611 PSI *TIME 215.00* PRODUCE 1 AT 1588 PSI *TIME 220.00* PRODUCE 1 AT 1575 PSI TIME 225.00 PRODUCE 1 AT 1557 PSI TIME 230.00 PRODUCE 1 AT 1463 PSI TIME 235.00 PRODUCE 1 AT 1534 PSI TIME 240.00 PRODUCE 1 AT 1498 PSI TIME 245.00 PRODUCE 1 AT 1495 PSI TIME 250.00 PRODUCE 1 AT 1565 PSI TIME 255.00 PRODUCE 1 AT 1566 PSI *TIME 260.00* PRODUCE 1 AT 1605 PSI *TIME 265.00* PRODUCE 1 AT 1606 PSI TIME 270.00 PRODUCE 1 AT 1750 PSI *TIME 275.00* PRODUCE 1 AT 1852 PSI TIME 280.00 PRODUCE 1 AT 1856 PSI TIME 285.00 PRODUCE 1 AT 1346 PSI TIME 290.00 PRODUCE 1 AT 1350 PSI *TIME 295.00* PRODUCE 1 AT 1370 PSI *TIME 300.00* PRODUCE 1 AT 1429 PSI *TIME 305.00* PRODUCE 1 AT 1449 PSI TIME 310.00 PRODUCE 1 AT 1480 PSI TIME 315.00

PRODUCE 1 AT 1715 PSI TIME 320.00 PRODUCE 1 AT 1578 PSI TIME 325.00 PRODUCE 1 AT 1355 PSI TIME 330.00 PRODUCE 1 AT 1417 PSI TIME 335.00 PRODUCE 1 AT 1428 PSI TIME 340.00 PRODUCE 1 AT 1645 PSI *TIME 345.00* PRODUCE 1 AT 1433 PSI *TIME 350.00* PRODUCE 1 AT 1434 PSI TIME 355.00 PRODUCE 1 AT 1435 PSI TIME 360.00 PRODUCE 1 AT 1245 PSI STOP

APPENDIX 2

A2.1 Case 1 Preliminary Calculation

A2.1.1 Fracture Length

When varying the fracture length to ensure that the 2d fracture density is kept constant by recalculating the values of the density of fracture center point such that only one parameter is varied at any point. The relationship between the density of fracture center points and the 2d-fracture density is given as

$$Di * Li = Dfi$$
 (A1)

Di	Li(ft)	Dfi (ft/ft ²)
0.0002	280	0.056
0.000267	210	0.056
0.0004	140	0.056
0.00016	350	0.056
0.000133	420	0.056

 Table A2.1: Case 1 fracture length FracGen input calculation

A2.1.2 Fracture Density

When varying the fracture density to ensure that the 2d fracture density is varied and not the density of fracture center point since it represents the fracture density. Using the same relationship stated above **eqn. A1**.

Di	Li(ft)	Dfi (ft/ft ²)
0.0002	280	0.056
0.00015	280	0.042
0.0001	280	0.028
0.00025	280	0.07
0.0003	280	0.084

 Table A2.2: Case 1 fracture density FracGen input calculation

A2.2 Case 2 Preliminary Calculation

For this analysis the 2d fracture density (0.056 ft/ft^2) obtained in case 1 was used as a known parameter subsequently other parameters needed was calculated from the 2d fracture density.

A2.2.1 Fracture Length

When varying the fracture length to ensure that the 2d fracture density is kept constant by recalculating the values of the density of fracture center point such that only one parameter is varied at any point.

The relationship between the two is shown in eqn. A2,

$$Dfi = 12 mi di (lci - li) li Dci$$
(A2)

where

mi = exponential mean offset (or "spacing") of fractures from their associated cluster axis (ft).

di = mean intra-cluster fracture center-point density (pts/ft2).

lci = mean length of clusters (ft).

li = mean fracture length (ft).

Dci = cluster center-point density (pts/ft2).

$$di = Dfi / (12 mi Dci (lci - li) li)$$
(A3)

From **eqn. A2** we can compute the different cluster center-point density as the mean fracture length changes.

	Dfi	mi	di	lci	li	Dci
Control	0.056	40	0.000059476	1500	280	0.000005742
-25%	0.056	40	0.000059476	1500	210	0.000007241
-50%	0.056	40	0.000059476	1500	140	0.000010302
25%	0.056	40	0.000059476	1500	350	0.000004874
50%	0.056	40	0.000059476	1500	420	0.000004324
100%	0.056	40	0.000059476	1500	560	0.000003726

 Table A2.3: Case 2 fracture length FracGen input calculation

A2.2.2 Fracture Density

When varying the fracture density some preliminary calculation was done to ensure that the 2d fracture density varied accurately by recalculating the values of the density of fracture center point, for this analysis the mean intra-cluster fracture center-point density was kept constant. The relationship between theses parameters is **eqn. A2**,

Also rearranging equation A2 we have

$$Dci = Dfi / (12 mi di (lci - li) li)$$
(A4)

From **eqn. A4** we can compute the density of fracture center point as the mean fracture length is kept constant.

	Dfi	mi	di	lci	li	Dci
Control	0.056	40	0.000059476	1500	280	0.000005742
-25%	0.042	40	0.000059476	1500	280	0.000004307
-50%	0.028	40	0.000059476	1500	280	0.000002871
25%	0.07	40	0.000059476	1500	280	0.000007178
50%	0.084	40	0.000059476	1500	280	0.000008614
100%	0.112	40	0.000059476	1500	280	0.000011485

 Table A2.4: Case 2 fracture density FracGen input calculation

A2.3 Case 3 Preliminary Calculation

A2.3.1 Fracture Length

In this case we have 2 uniformly distributed fracture sets, the primary and secondary fracture sets. The 2d-fracture density of the primary is made 75% of the value in case 1, which is 0.042 ft/ft². The 2d-fracture density of the secondary fracture set is then 0.014 ft/ft^2 and it is kept constant throughout the study. When varying the fracture length of the primary fracture set, to ensure that the 2d fracture density is kept constant the values of the density of fracture center point is recalculated such that only one parameter is varied at any point. The relationship between the density of fracture center points and the 2d-fracture density is given as **eqn. A1** above

Di	Li(ft)	Dfi (ft/ft ²)
0.00015	280	0.042
0.0002	210	0.042
0.0003	140	0.042
0.00012	350	0.042
0.0001	420	0.042

Table A2.5: Case 3 fracture length FracGen input calculation

A2.3.2 Fracture Density

When varying the fracture density of the primary fracture set to ensure that the 2d fracture density is varied and not the density of fracture center point since it represents the fracture density. Using the same relationship stated above we have that;

Di	Li(ft)	Dfi (ft/ft ²)
0.0002	280	0.042
0.00015	280	0.028
0.0001	280	0.014
0.00025	280	0.056
0.0003	280	0.07

Table A2.6: Case 3 fracture density FracGen input calculation

A2.4 Case 4 Preliminary Calculation

In this case we have 2 fracture sets, where one is clustered and the other is uniformly distributed, the primary and secondary fracture sets. The clustered fracture network is the primary while the uniform fracture network is the secondary. The 2d-fracture density

of the primary is made 75% of the value in case 1, which is 0.042 ft/ft². The 2d-fracture density of the secondary fracture set is then 0.014 ft/ft² and it is kept constant throughout the study. The 2d fracture density and other were used as known parameters subsequently other parameters needed was calculated from the relationship.

A2.4.1 Fracture Length

When varying the fracture length of the primary fracture set to ensure that the 2d fracture density is kept constant by recalculating the values of the density of fracture center point such that only one parameter is varied at any point.

The relationship between the two is eqn. A5.

$$di = Dfi / (12 mi Dci (lci - li) li)$$
(A5)

From **eqn. A5** we can compute the different cluster center-point density as the mean fracture length changes.

	Dfi	mi	di	lci	li	Dci
Control	0.042	40	0.000059476	1500	280	0.000004307
-25%	0.042	40	0.000059476	1500	210	0.000005431
-50%	0.042	40	0.000059476	1500	140	0.000007727
25%	0.042	40	0.000059476	1500	350	0.000003655
50%	0.042	40	0.000059476	1500	420	0.000003243
100%	0.042	40	0.000059476	1500	560	0.000002795

 Table A2.7: Case 4 fracture length FracGen input calculation

A2.4.2 Fracture Density

When varying the fracture density of the primary fracture set some preliminary calculation was done to ensure that the 2d fracture density varied accurately by recalculating the values of the density of fracture center point, for this analysis the mean intra-cluster fracture center-point density was kept constant. The relationship between these parameters is **eqn. A4**.

From **eqn. A4** we can compute the density of fracture center point as the mean fracture length is kept constant.

	Dfi	mi	di	lci	li	Dci
Control	0.042	40	0.000059476	1500	280	0.000004307
-25%	0.0315	40	0.000059476	1500	280	0.000003230
-50%	0.021	40	0.000059476	1500	280	0.000002153
25%	0.0525	40	0.000059476	1500	280	0.000005383
50%	0.063	40	0.000059476	1500	280	0.000006460
100%	0.084	40	0.000059476	1500	280	0.000008614

 Table A2.8: Case 4 fracture density FracGen input calculation

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