GEOMETRIC DESIGN OF SPHERICAL SERIAL CHAINS WITH CURVATURE CONSTRAINTS IN THE ENVIRONMENT

A Thesis

by

ANURAG BHARADWAJ TOLETY

Submitted to the Office of Graduate Studies of
Texas A&M University
in partial fulfillment of the requirements for the degree of

MASTER OF SCIENCE

August 2011

Major Subject: Electrical Engineering

GEOMETRIC DESIGN OF SPHERICAL SERIAL CHAINS WITH CURVATURE CONSTRAINTS IN THE ENVIRONMENT

A Thesis

by

ANURAG BHARADWAJ TOLETY

Submitted to the Office of Graduate Studies of
Texas A&M University
in partial fulfillment of the requirements for the degree of

MASTER OF SCIENCE

Approved by:

Co-Chairs of Committee,	Mehrdad Ehsani
	Nina P. Robson
Committee Members,	Won-Jong Kim
	Shankar Bhattacharya
	Karen L. Butler-Purry
Head of Department,	Costas Georghiades

August 2011

Major Subject: Electrical Engineering

ABSTRACT

Geometric Design of Spherical Serial Chains

with Curvature Constraints in the Environment. (August 2011)

Anurag Bharadwaj Tolety, B. Tech., Indian Institute of Technology, Kharagpur

Co–Chairs of Advisory Committee: Dr. Mehrdad Ehsani
Dr. Nina P. Robson

This research builds up on recent results in planar kinematic synthesis with contact direction and curvature constraints on the workpiece. The synthesis of spherical serial chains is considered to guide a rigid body, such that it does not violate normal direction and curvature constraints imposed by contact with objects in the environment. These constraints are transformed into conditions on the velocity and acceleration of points in the moving body to obtain synthesis equations which can be solved by algebraic elimination. Trajectory interpolation formulas yield the movement of the chain with the desired contact properties in each of the task positions. Example shows the application of the developed theory to the failure recovery of a robot manipulator.

To my Teachers, my Parents and my Friends

ACKNOWLEDGMENTS

I thank Dr. Nina Robson for giving me an opportunity to work under her. I owe the knowledge that I've gained to her motivation and guidance.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

FIGURE Page

CHAPTER I

INTRODUCTION

Recent results on planar kinematic synthesis with contact directions and curvature constraints on the work piece [1] are expanded upon in this paper. The design of spherical linkages based on the contact and curvature constraints imposed on a work piece is considered. The specifications on the end points of the end effector which are in contact with objects in the environment are transformed into specifications on the end effector origin and frame orientation. These specifications are then used to synthesize serial TS chains. A Robotic Arm fixed on a Surface Mobility Platform (SMP) is used to show the application of serial chain synthesis to failure recovery of the Robotic Arm.

A. Mechanical Design Principles

Engineers build systems. System can be defined as an assemblage of sub-systems, hardware and software components, and people, designed to perform a set of tasks to satisfy specified functional requirements and constraints. Each of these systems performs many functions. The design of effective systems is the goal of many fields as engineering, business and government. System design has lacked a formal theoretical framework and thus, has been done heuristically/empirically. Heuristic approaches emphasize qualitative guidelines. After systems are designed they are sometimes modeled and simulated. In many cases they have to be constructed and tested. All these processes are done to improve the design after heuristic design solutions are implemented in hardware and software. Such an approach to system design includes

---

The journal model is *IEEE Transactions on Automatic Control.*

both technical and business risks because of the uncertainties associated with the performance and quality of a system that is created by means of empirical decisions.

Other approaches to check or optimize the system that have been already designed are dimensional analysis, decision theory and others. There are three issues with these approaches. First, they do not provide tools for coming up with rational system design, beginning from the definition of the design goals. Secondly, some of these methods simply confirm the result, if systems are correctly designed. Thirdly, they are not general principles for system design since they can not be applied to non-phisical systems such as software.

Systems with many functional Requirements (FRs), physical components and lines of computer codes can be complex in the sense that the probability of satisfying the highest FRs decreases with increase in the number of FRs and design parameters (DPs). One of the goals of axiomatic approach [2] is to reduce this complexity by being able to make right design decisions at all levels. From axiomatic design point of view, systems , machines and software must satisfy functional requirements, constraints, the Independence Axiom and the Information Axiom.

The first step in designing a system is to determine the customer needs (CNs) in the customer domain that the system must satisfy. Then, the FRs and the constraints (Cs) of the system in the functional domain are determined to satisfy the customer needs. The FRs must be determined without thinking about the solution, so as to come up with creative ideas. The FRs are defined as the minimum set of independent requirements that the design must satisfy.

The next step in axiomatic design is to map these FRs of the functional domain into the physical domain, conceiving the design embodiment and identifying the DPs. The DPs must be chosen so that there is no conflict with the constraints.

During the mapping process, the design must satisfy the Independence Axiom, which requires that the functional independence be satisfied through the development of an uncoupled/decoupled design. In an ideal design the number of FRs and DPs is equal, a consequence of the Independence Axiom and Information Axiom. The Information Axiom states that the design that has the least information content is the best design. This axiom provides a guide in selecting DPs, in addition to providing the selection criteria for best design among those designs that satisfy the Independence Axiom. Through the use of both axioms, one can easily optimize a multi-FR design solution.

CHAPTER II

LITERATURE SURVEY: GEOMETRIC DESIGN OF MECHANICAL LINKAGES WITH TASK CONSTRAINTS

Research in the synthesis of serial chains to achieve acceleration requirements related to curvature is limited. It is primarily found in the synthesis theory for planar RR chains, and the work by Chen and Roth [3] for spatial chains.

A. Geometric Design of Mechanical Linkages

The term kinematics defines the branch of mechanics which deals with the motion without considering the forces associated to it. It includes positions, velocities, accelerations and higher derivatives. A mechanical system consists of a series of rigid links connected by joints. The joints allow relative movement of different type between elements. Although any type of joint mechanism can be used to connect the links of a robot, traditionally the joints are chosen from a set of six mechanisms called lower pairs. These special type of mechanisms are revolute, prismatic, helical, cylindrical, spherical and planar joints. Most modern manipulators consist of a set of rigid links connected together by a set of joints. Motors are attached to the joints so that the overall motion of the mechanism can be controlled to perform a given task. A tool, typically a gripper of some sort is attached to the end of the robot to interact with the environment.

In the geometric design problem, also known as Rigid Body Guidance Problem, the designer specifies a certain task to be performed by a mechanical system. Therefore, major part of the design of a mechanism is the choice of type of the mechanical system, capable of realizing the task.The design process involves finding the most appropriate solution to satisfy a set of functional requirements (FRs). Among the FRs are those, related to the movement of the manipulator, while others may be related to different factors such as repeatability, cost, etc. It is the designers goal to calculate the design parameters (DPs) and to create a mechanical system that fits the best to the given task and at the same time agrees with a set of FRs. This problem is also called in literature as Geometric Design Problem. The precision points are usually described by three parameters (two for position and one for orientation) in plane and six parameters - three for position and three for orientation for spatial problems.

Traditionally, the robots are designed in such a way that the desired task needs only to lie inside the higher dimensional workspace of the robot by adjusting its link dimensions. Then, path-planning and control algorithms are used to solve the problem of reaching the task. However, such a general scheme is not needed in many cases and the actual way of designing a robot is using a trial and error approach, i.e. the only FRs are the needed degrees of freedom and a sketch of the desired workspace. The design is analyzed and its movement is compared to the desired task. If it is not satisfactory, the designer modifies some of the characteristics of the robot and analyzes it again. It is obvious that improvements are needed to be done in the design of robotic systems.

The geometric design of a mechanical system includes the following four main steps: task specification, choice of kinematic structure or topology selection, dimensional synthesis of the chosen mechanism and detailed design. Essentially, the designer's goal is to find a mechanism whose workspace contains the desired task. Of course, there are requirements for the workspace to be with the smallest possible dimensions, simple control scheme and path planning.

1. Task Specification

The Task is specified in several ways - either as a set of distinct points, an approximate path or as a region in which the robot can move. An assessment of the relation between positions and final design is made by the designer.

2. Topology Selection

The selection of a mechanism depends on matching the degrees of freedom with the type. The mechanical system is considered at it's most basic level. Several solutions for structuring the problem have presented in the past. Fang and Tsai [4] developed a systematic approach for structural synthesis for a class of 4-DOF and 5-DOF over-constrained parallel manipulators, based on the theory of screws and reciprocal screws. Different types of parallel structures [5], [6], [7], [8], [9] were based on Fang's and Tsai's approach.

3. Dimensional Synthesis

This part consists of making modifications to the workspace of the kinematic structure to approximate the task specification. Two categories of solutions exist here: approximate and exact synthesis [10].

Optimization algorithms are used by approximate synthesis such that even though the rigid body does not pass through the desired poses [11], [12], [13], [14], [15], [16], [17], [18].

Many researchers studied the dimensional/geometric synthesis of planar mechanism. Prominent among them are Roth's analytic design of two revolute open chains [19] in 1986, Innocenti's solution of the spatial Burmester problem in 1994 [20], and McCarthy's work on mechanism theory and robot design [21], [22] in 2000.

Perez and Mccarthy made contributions to the dimensional synthesis of mechanisms based on dual qauternion methodology for the synthesis of constrained robotic systems [23], [24], [25], [26], [27], [28]. In [29], Perez and McCarthy showed that the relative kinematics equations of a serial chain appear in the matrix exponential formulation of the kinematics equations for a robot manipulator. Also, see related work of Walbrecht, Su, Perez and McCarthy [30], Su, McCarthy, Wampler [31], Soh, Perez and McCarthy [32] and Perez, McCarthy at al [33].

A new set of design equations for the geometric design problem involving three link manipulator was put forward by Lee and Mavroidis [34], [35], [36], [10], [37], [38]. They use compuational methods to solve the open problems in design. Su and McCarthy [39], [40], [41] presented the design of a number of spatial constraint serial chains and formulated their solutions. General cases of CS and RPS polynomial systems were presented for the first time. The design equations for spatial SS chains for finitely and infinitesimally separated positions is found in Chen and Roth [3]. The case of seven finitely specified task positions is solved in Innocenti [20] and further studied by Liao and McCarthy [42].

This research is inspired by the work of Rimon and Burdick [43] who, in the mid nineties, introduced a novel configuration space method for analyzing the relative mobility of a body in frictionless contact with rigid stationary bodies. The authors studied the 1-st order properties of body's free motions, and relate them to Screw Theory. They introduce a mobility index that measures the effective 1-st order mobility of body in an equilibrium grasp. This index is shown to be a function of the number and location of the contact points. Examples show that the 1-st order mobility theory can not adequately differentiate between different equilibrium grasps involving the same number of fingers. This motivates the development of a 2-nd order theory in the companion paper [44]. The analysis leads to a 2-nd order mobility index, that captures the inherent body mobility in an equilibrium grasp. The index differentiates between grasps, which are considered equivalent by the first order, or instantaneous theories, but are physically different. The authors show that 2-nd order effects can be used to lower the mobility of an equilibrium grasp and can be used to prove lower bound on the number of contacting bodies needed to immobilize an object.

Our goal is to consider how this viewpoint can be used to synthesize a linkage that guides body such that it satisfies obstacle constraints. In the next chapter, the planar case is explained. We then move on to the spherical case.

B. Summary of Literature Survey

There are two fundamental approaches to current research in geometric design of robotic systems to achieve workspace specifications: (i) approximate synthesis of platform systems which generally begins with a platform topology and optimizes its dimensions to obtain a desired distribution of manipulability, a velocity based metric; and (ii) exact synthesis which generally focusses on single serial chains and computes the dimensions of the chain to achieve a specific set of task positions. Explicit computation of the dimensions of a chain to satisfy the task requirements is involved in Exact Synthesis. This results in an increase in the complexity once the number of design parameters increase. On the the other side, approximate synthesis can handle large no. of design parameters at the cost of finding an approximate solution.

The focus of this dissertation is on exact synthesis for workspace constraints that include acceleration, with the goal of obtaining all the solutions to a given task specification. Rimon and Burdick in [43], [44] show that acceleration properties of movement can be used to effectively constrain a rigid body for part-fixturing and grasping applications. Research in the synthesis of serial chains to achieve acceleration requirements related to curvature is limited. It is primarily found in the synthesis theory for planar RR chains, and the work by Chen and Roth [3] for spatial chains.

The research proposed here develops a methodology for the exact synthesis of serial chains to achieve a specified relative contact curvature requirement between the end-effector and a workspace obstacle. It is shown in the planar synthesis work that the curvature requirements yield geometric constraints on position, velocity and acceleration. These constraints yield design equations that can be solved to determine the dimensions of the serial chain. In extending this work to spatial serial chains, a new method based on sparse matrix resultants was developed, which solves exact synthesis problems with acceleration constraints.

C. Contributions

The theoretical contributions of this research are:

1. Contact relationships for spherical movement [45]. Here the normal direction and curvature constraints imposed by contact with three objects in the workspace is used to define velocity and acceleration constraints on the end-effector of a spherical TS chain.

2. Theoretical foundation for the transition from contact geometry to kinematic synthesis for spherical movement.

3. Development of a Rover using a Surface Mobility Platform, Robotic Arm for current and future experimental validation of the obtained results.

The applications of this research have focussed on the design of spatial TS chain to maintain a specified contact geometry with objects in the environment.

CHAPTER III

PLANAR VELOCITY AND ACCELERATION CONSTRAINTS DEFINED BY CONTACT AND CURVATURE CONSTRAINTS

This chapter briefly revises recently developed work by Robson and McCarthy in [1]. It covers the geometric design of planar RR chain that guides an end-effector such that it maintains contact with objects in the workspace. This contact imposes contact and curvature constraints on the movement of the end-effector that must be provided by the RR chain. The problem is solved by transforming the contact and curvature constraints into conditions on the velocity and acceleration of the moving body, and by obtaining a set of synthesis equations that allow the computation of the design parameters of the RR chain.

The ability to design RR chains that achieve specific contact curvature can be used to design four-, six- and eight-bar linkages that achieve specific curvature requirements for applications in fixturing and grasping.

A. Task Specification

Planar RR chains, shown in Figure 1 are fundamental component for the construction of a variety of mechanical linkages.

Let the movement of a rigid body be defined by the parameterized set of 3×3 homogeneous transforms $[T(t)] = [R(t), \mathbf{d}(t)]$ constructed from a rotation matrix, $A(t)$, and translation vector $\mathbf{d}(t)$. A point $\mathbf{p}$ fixed in the moving body traces a

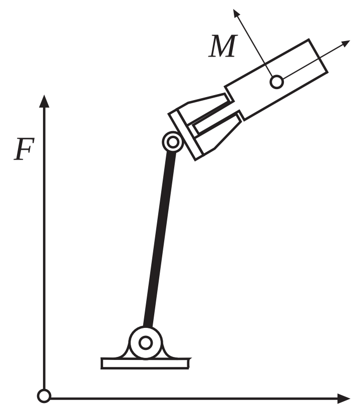

Fig. 1. A serial RR chain.

trajectory $\mathbf{P}(t)$ in a fixed coordinate frame F, given by

$$\begin{Bmatrix} P_x(t) \\ P_y(t) \\ 1 \end{Bmatrix} = \begin{bmatrix} \cos\phi(t) & -\sin\phi(t) & d_x(t) \\ \sin\phi(t) & \cos\phi(t) & d_y(t) \\ 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} p_x \\ p_y \\ 1 \end{Bmatrix}, \tag{3.1}$$

or

$$\mathbf{P}(t) = [K(t)]\mathbf{p}. \tag{3.2}$$

The goal is to determine the movement $[K(t)]$ that has the property that two points in the moving body have the trajectories $\mathbf{A}(t)$ and $\mathbf{B}(t)$ consistent with contact with two circular objects, as shown Figure 2.

By positioning the coordinate frame M such that its origin coincides with $\mathbf{A}(t)$, its x-axis is directed along the line $\mathbf{B}-\mathbf{A}$. This defines $[K(t)]$ with translation vector $\mathbf{d}(t)$ and rotation angle, $\phi(t)$ given by

$$\mathbf{d}(t) = \mathbf{A}(t), \quad \phi(t) = \arctan\frac{\vec{k}\times(\mathbf{B}-\mathbf{A})\cdot(\mathbf{B}-\mathbf{A})}{(\mathbf{B}-\mathbf{A})\cdot(\mathbf{B}-\mathbf{A})}, \tag{3.3}$$

where $\vec{k}$ is a vector perpendicular to the plane of movement. The vector $\vec{k}$ combines with the cross product operation to perform a 90° rotation in the plane, so for a

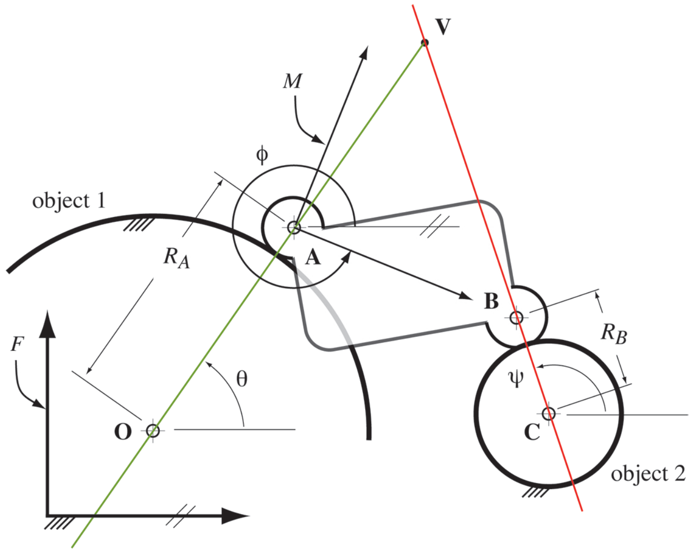

Fig. 2. The body M moves in contact with two objects such that the trajectories of **A** and **B** have the radii of curvature, R_A and R_B, respectively.

vector $\mathbf{y}$, it can be written that

$$\vec{k} \times \mathbf{y} = [J]\mathbf{y}, \quad \text{where} \quad [J] = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}. \tag{3.4}$$

In what follows, the position, velocity and acceleration of the body at an instant denoted $t = 0$ are determined, from properties of the trajectories $\mathbf{A}(t)$ and $\mathbf{B}(t)$ imposed by contact with two objects.

1. The Position Specification

A moving body M is assumed to be in contact with two fixed objects whose point trajectories $\mathbf{A}(t)$ and $\mathbf{B}(t)$ are constrained to follow circles in the vicinity of a reference position denoted by $t = 0$. Using Taylor's Series expansion, the movement of M in the vicinity of $t = 0$ can be expressed as,

$$[K(t)] = [K_0^j] + [K_1^j]t + \frac{1}{2}[K_2^j]t^2 + ..., \quad \text{where} \quad [K_i^j] = \frac{d^i[K^j]}{dt^i}. \tag{3.5}$$

Evaluating the derivatives of $[K(t)]$, we obtain

$$[K_0^j] = \begin{bmatrix} \cos\phi_0 & -\sin\phi_0 & d_{x,0} \\ \sin\phi_0 & \cos\phi_0 & d_{y,0} \\ 0 & 0 & 1 \end{bmatrix},$$

$$[K_1] = \begin{bmatrix} -\phi_1\sin\phi_0 & -\phi_1\cos\phi_0 & d_{x,1} \\ \phi_1\cos\phi_0 & -\phi_1\sin\phi_0 & d_{y,1} \\ 0 & 0 & 0 \end{bmatrix},$$

$$\text{and} \quad [K_2] = \begin{bmatrix} -\phi_2\sin\phi_0 - \phi_1^2\cos\phi_0 & -\phi_2\cos\phi_0 + \phi_1^2\sin\phi_0 & d_{x,2} \\ \phi_2\cos\phi_0 - \phi_1^2\sin\phi_0 & -\phi_2\sin\phi_0 - \phi_1^2\cos\phi_0 & d_{y,2} \\ 0 & 0 & 0 \end{bmatrix}. \tag{3.6}$$

Here, the notation $d^i\phi/dt^i(0) = \phi_i$ and $d^i\mathbf{d}/dt^i(0) = (d_{x,i}, d_{y,i})^T = \mathbf{d}_i$ was introduced.

2. The Velocity Specification

In order to satisfy the force constraints at the prescribed positions, the directions of the velocity vectors $\dot{\mathbf{A}}$ and $\dot{\mathbf{B}}$ are determined, which are perpendicular to the forces $\mathbf{F}_A = \mathbf{A} - \mathbf{O}$ and $\mathbf{F}_B = \mathbf{B} - \mathbf{C}$, respectively (see Figure 2). This is achieved by defining the point of intersection $\mathbf{V}$ of the lines of actions of these two forces, and ensuring that it is the velocity pole of the movement of M in this position.

Let us define the angular velocity of M as $\mathbf{w} = \dot{\phi}\vec{k}$, then $\mathbf{A}$ and $\mathbf{B}$ follow circles around $\mathbf{O}$ and $\mathbf{C}$, respectively. In this configuration, it is known that the point of intersection of the lines of action L_A^i and L_B^i of the constraint forces $\mathbf{F}_A^i$ and $\mathbf{F}_B^i$, is the velocity pole, $\mathbf{V}$ of the body and has zero velocity. The point $\mathbf{V}$ is obtained by solving the two linear equations

$$L_A^i: \quad \mathbf{x} - \mathbf{A}^i) \cdot [J]\mathbf{F}_A^i = 0,$$
$$L_B^i: \quad (\mathbf{x} - \mathbf{B}^i) \cdot [J]\mathbf{F}_B^i = 0, \quad i = 1, 2, \tag{3.7}$$

where $\mathbf{x} = (x, y)$ are variable point coordinates.

Thus, the velocities of $\mathbf{A}$ and $\mathbf{B}$ are determined as

$$(\dot{\mathbf{A}} = \mathbf{w} \times (\mathbf{A} - \mathbf{V}), \quad \text{and} \quad \dot{\mathbf{B}} = \mathbf{w} \times (\mathbf{B} - \mathbf{V}). \tag{3.8}$$

The angular velocities $\mathbf{w}_{OA} = \dot{\theta}\vec{k}$ and $\mathbf{w}_{CB} = \dot{\psi}\vec{k}$ can be obtained from the relations

$$\dot{\mathbf{A}} = \mathbf{w}_{OA} \times (\mathbf{A} - \mathbf{O}), \quad \text{and} \quad \dot{\mathbf{B}} = \mathbf{w}_{CB} \times (\mathbf{B} - \mathbf{C}). \tag{3.9}$$

Collecting these results into the velocity loop equations of the quadrilateral **OABC**, to compute the velocity $\dot{\mathbf{B}} = \dot{\mathbf{A}} + \mathbf{w} \times (\mathbf{B} - \mathbf{A})$ to be

$$\mathbf{w}_{OA} \times (\mathbf{A} - \mathbf{O}) + \mathbf{w} \times (\mathbf{B} - \mathbf{A}) = \mathbf{w}_{CB} \times (\mathbf{B} - \mathbf{C}). \tag{3.10}$$

The define The elements of the velocity matrix $[T_1]$ are defined by the angular velocity $\phi_1 = \dot{\phi}(0)$, and the velocity $\mathbf{d}_1 = \dot{\mathbf{A}}(0)$.

3. The Acceleration Specification

When the rigid body M moves in contact with two objects, points $\mathbf{A}$ and $\mathbf{B}$ are guided along trajectories with radii of curvature R_A and R_B. This is the same as specifying points $\mathbf{O}$ and $\mathbf{C}$ which are the centers of curvature of the trajectories $\mathbf{A}(t)$ and $\mathbf{B}(t)$ at the instant $t = 0$.

By the usage of the acceleration loop equations of the quadrilateral **OABC**, the angular accelerations $\mathbf{a}_{OA} = \ddot{\theta}\vec{k}$ and $\mathbf{a}_{CB} = \ddot{\psi}\vec{k}$ for a given value of the angular acceleration $\mathbf{a} = \ddot{\phi}\vec{k}$ are determined. This in turn allows us to determine $\ddot{\mathbf{d}} = \ddot{\mathbf{A}}$.

The acceleration loop equations are obtained by computing the time derivative of the velocity loop equations (3.10) to yield

$$\begin{aligned}\mathbf{a}_{OA} \times (\mathbf{A} - \mathbf{O}) + &\mathbf{w}_{OA} \times (\mathbf{w}_{OA} \times (\mathbf{A} - \mathbf{O}))\\ &+\mathbf{a}\times(\mathbf{B} - \mathbf{A}) + \mathbf{w} \times (\mathbf{w} \times (\mathbf{B} - \mathbf{A}))\\ &\qquad = \mathbf{a}_{CB} \times (\mathbf{B} - \mathbf{C}) + \mathbf{w}_{CB} \times (\mathbf{w}_{CB} \times (\mathbf{B} - \mathbf{C})).\end{aligned} \tag{3.11}$$

The angular acceleration of the moving body M is specified to be $\ddot{\phi} = 0$, so $\mathbf{a} = 0$. The acceleration loop equations are obtained in the form

$$\ddot{\theta}\vec{k} \times (\mathbf{A} - \mathbf{O}) - \dot{\theta}^2(\mathbf{A} - \mathbf{O}) - \dot{\phi}^2(\mathbf{B} - \mathbf{A})) = \ddot{\psi}\vec{k} \times (\mathbf{B} - \mathbf{C}) - \dot{\psi}^2(\mathbf{B} - \mathbf{C})), \tag{3.12}$$

which have been simplified using the identity $\vec{k} \times (\vec{k} \times \mathbf{y}) = -\mathbf{y}$.

The angular acceleration $\ddot{\theta}$ is obtained by taking the dot product of (3.12) with $\mathbf{B} - \mathbf{C}$ to cancel the term containing $\ddot{\psi}$, that is

$$\ddot{\theta} = \frac{\big(\dot{\theta}^2(\mathbf{A} - \mathbf{O}) + \dot{\phi}^2(\mathbf{B} - \mathbf{A}) - \dot{\psi}^2(\mathbf{B} - \mathbf{C})\big) \cdot (\mathbf{B} - \mathbf{C})}{\vec{k} \times (\mathbf{A} - \mathbf{O}) \cdot (\mathbf{B} - \mathbf{C})} \tag{3.13}$$

In a similar way, we obtain $\ddot{\psi}$ by computing the dot product of (3.12) with $\mathbf{A} - \mathbf{O}$,

$$\ddot{\psi} = \frac{-\big(\dot{\theta}^2(\mathbf{A} - \mathbf{O}) + \dot{\phi}^2(\mathbf{B} - \mathbf{A}) - \dot{\psi}^2(\mathbf{B} - \mathbf{C})\big) \cdot (\mathbf{A} - \mathbf{O})}{\vec{k} \times (\mathbf{B} - \mathbf{C}) \cdot (\mathbf{A} - \mathbf{O})} \tag{3.14}$$

The result is a specification of the acceleration $\ddot{\mathbf{A}}$, given by

$$\ddot{\mathbf{A}} = \ddot{\theta}\vec{k} \times (\mathbf{A} - \mathbf{O}) - \dot{\theta}^2(\mathbf{A} - \mathbf{O}). \tag{3.15}$$

Thus, the values $\phi_2 = 0$ and $\mathbf{d}_2 = \ddot{\mathbf{A}}(0)$ determine the elements of the acceleration matrix $[T_2]$.

4. The Synthesis Equations

In this section, for a task that includes contact with the environment, the design equations of a planar RR chain are formulated. The chain has five design parameters, the coordinates of the fixed pivot $\mathbf{G} = (u, v, 1)$ in the fixed frame F, the coordinates of the moving pivot $\mathbf{w}$ in the moving frame M, and the length of the crank R.

The geometry of the RR chain satisfies the constraint equation

$$(\mathbf{W} - \mathbf{G}) \cdot (\mathbf{W} - \mathbf{G}) = R^2, \tag{3.16}$$

where $\mathbf{W}$ defines the fixed frame coordinates of the moving pivot $\mathbf{w}$. Two derivatives of this equation yield the additional constraint equations

$$\begin{aligned} \dot{\mathbf{W}} \cdot (\mathbf{W} - \mathbf{G}) &= 0, \\ \ddot{\mathbf{W}} \cdot (\mathbf{W} - \mathbf{G}) + \dot{\mathbf{W}} \cdot \dot{\mathbf{W}} &= 0. \end{aligned} \tag{3.17}$$

To use these equations for designing the RR chain, the movement of M is specified, such that $\mathbf{W}(t) = [K(t)]\mathbf{w}$. The resulting equations can be solved to determine the design parameters (see Robson and McCarthy [46]).

5. The Design Equations

Five design equations for the RR chain are obtained by determining the movement of M such that its position, velocity and acceleration are known in one location, and its position and velocity are known in a second location. From our previous results, this implies that contact curvatures are specified in the first location and constraint forces in the second location. Thus, the matrix functions

$$[K^1(t)] = [K_0^1] + [K_1^1]t + \frac{1}{2}[K_2^1]t^2, \quad [K^2(t)] = [K_0^2] + [K_1^2]t, \tag{3.18}$$

where $[K_0^j]$, $[K_1^j]$, $j=1,2$ and $[K_2^1]$, are known.

In what follows, it is convenient to use the coordinate of the moving pivot in the first location $\mathbf{W}^1=(x,y,1)$ as design parameters for the RR chain, rather than the moving pivot coordinates $\mathbf{w}$ in M, where $\mathbf{W}^1=[K_0^1]\mathbf{w}$. This allows us to define the trajectories $\mathbf{W}^1(t)$ and $\mathbf{W}^2(t)$ using the relative displacement matrices defined in (4.11), so we have

$$\begin{aligned}\mathbf{W}^1(t)=[D^1(t)]\mathbf{W}^1 =&[I+\Omega^1 t+\frac{1}{2}\Lambda^1 t^2]\mathbf{W}^1,\\ \mathbf{W}^2(t)=[D^2(t)]\mathbf{W}^2 =&[I+\Omega^2 t][D_{12}]\mathbf{W}^1,\end{aligned} \tag{3.19}$$

where $[D_{12}]=[K_0^2][K_0^1]^{-1}$ yields $\mathbf{W}^2=[D_{12}]\mathbf{W}^1$.

Substituting the trajectories (3.19) into the constraint equations (3.16) and (3.17) to obtain,

$$\begin{aligned}\mathcal{P}_1: &\quad 0=(\mathbf{W}^1-\mathbf{G})\cdot(\mathbf{W}^1-\mathbf{G})-R^2,\\ \mathcal{V}_1: &\quad 0=[\Omega^1]\mathbf{W}^1\cdot(\mathbf{W}^1-\mathbf{G}),\\ \mathcal{A}_1: &\quad 0=[\Lambda^1]\mathbf{W}^1\cdot(\mathbf{W}^1-\mathbf{G})+[\Omega^1]\mathbf{W}^1\cdot[\Omega^1]\mathbf{W}^1,\\ \mathcal{P}_2: &\quad 0=([D_{12}]\mathbf{W}^1-\mathbf{G})\cdot([D_{12}]\mathbf{W}^1-\mathbf{G})-R^2,\\ \mathcal{V}_2: &\quad 0=[\Omega^2][D_{12}]\mathbf{W}^1\cdot([D_{12}]\mathbf{W}^1-\mathbf{G}).\end{aligned} \tag{3.20}$$

These are the design equations for the RR chain.

6. Trajectory Planning

The inverse kinematics of the RR chain yields the joint parameter vector $\mathbf{q}=(\theta_1,\theta_2)$ at each of the task positions, $i=1,2$. In order to obtain the joint velocity vector $\dot{\mathbf{q}}$

at the ith position, we solve the equation

$$V_i = [J_i]\dot{\mathbf{q}}_i, \quad i = 1, 2, \tag{3.21}$$

where $V_i = (\omega, \mathbf{v})$ is the velocity prescribed at position i, and J_i is the Jacobian of the RR chain. Notice that because the Jacobian is a 3×2 matrix, the solution is obtained by pre-multiplying by the inverse of the Jacobian

$$\dot{\mathbf{q}}_{ii} = [J_i^T J_i]^{-1}[J_i^T]V_i, \quad i = 1, 2. \tag{3.22}$$

This is the well-known pseudo-inverse which provides an exact solution because the linkage was designed to satisfy this velocity requirement.

Now to determine the joint acceleration vector $\ddot{\mathbf{q}}$, the following equation is solved

$$A_i = \dot{J}_i\dot{\mathbf{q}}_i + J_i\ddot{\mathbf{q}}_i, \quad i = 1, \tag{3.23}$$

where $A_i = (\alpha, \mathbf{a})$ is the acceleration prescribed at the first position and $\dot{J}_i$ is the time derivative of the 3×2 Jacobian matrix. The vector $\dot{J}_i\dot{\mathbf{q}}_i$ is known, thus the solution is again obtained using the pseudo-inverse,

$$\ddot{\mathbf{q}}_i = [J_i^T J_i]^{-1}[J_i^T](A_i - \dot{J}_i\dot{\mathbf{q}}_i). \tag{3.24}$$

The trajectory between the joint parameters $(\theta_0, \dot{\theta}_0, \ddot{\theta}_0)$ and $(\theta_f, \dot{\theta}_f, \ddot{\theta}_f)$ over the range $0 \leq t \leq t_f$, shown in Figure 3, is generated by the fifth degree polynomial

$$\theta(t) = a_0 + a_1 t + a_2 t^2 + a_3 t^3 + a_4 t^4 + a_5 t^5, \tag{3.25}$$

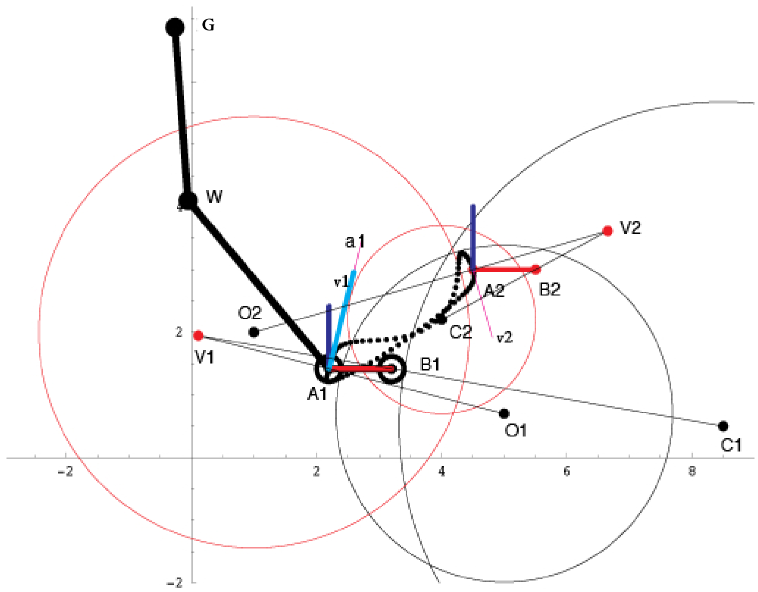

Fig. 3. The two task positions, velocities (v1, v2) and acceleration (a1), as well as the velocity poles location (V1, V2) for each position. The RR chain holding a work-piece maintains contact with two bodies in the first position.

where

$$
\begin{aligned}
a_0 =& \theta_0, \quad a_1 = \dot{\theta}_0, \quad a_2 = \frac{\ddot{\theta}_0}{2}, \\
a_3 =& \frac{20\theta_f - 20\theta_0 - (8\dot{\theta}_f + 12\dot{\theta}_0)t_f - (3\ddot{\theta}_0 - \ddot{\theta}_f)t_f^2}{2t_f^3}, \\
a_4 =& \frac{30\theta_0 - 30\theta_f + (14\dot{\theta}_f + 16\dot{\theta}_0)t_f + (3\ddot{\theta}_0 - 2\ddot{\theta}_f)t_f^2}{2t_f^4}, \\
a_5 =& \frac{12\theta_f - 12\theta_0 - (6\dot{\theta}_f + 6\dot{\theta}_0)t_f - (\ddot{\theta}_0 - \ddot{\theta}_f)t_f^2}{2t_f^5}.
\end{aligned} \tag{3.26}
$$

Equation (3.25) is obtained by solving the equations defining the joint position, velocity and acceleration evaluated at $t = 0$ and $t = t_f$ to compute the coefficients $a_i, i = 0, \ldots, 5$, see [47].

Standard joint trajectory interpolation formulas yield the movement of an RR chain with the desired contact properties in each task position, Figure 3.

B. Summary of Planar Research for Contact Specifications

The synthesis of planar RR chains that guide a rigid body, such that it does not violate normal direction and curvature constraints imposed by contact with two objects in the environment, developed in [1], was revised in this chapter. The contact direction and curvature constraints were transformed into conditions on the velocity and acceleration of certain points in the moving body. Joint trajectory interpolation formulas yield the movement of an RR chain with the desired contact properties in each task position.

CHAPTER IV

SPHERICAL VELOCITY AND ACCELERATION CONSTRAINTS DEFINED BY CONTACT AND CURVATURE CONSTRAINTS

A. Task Specification

Let the the movement of a rigid body be defined by the parameterized set of 4×4 homogeneous transforms $[K(t)] = [R(t), \mathbf{d(t)}]$ constructed from a rotation matrix, $R(t)$, and translation vector $\mathbf{d(t)}$. A point $\mathbf{p}$ fixed in the moving body traces a a trajectory $\mathbf{P(t)}$ in a fixed coordinate frame F, given by

$$\mathbf{P(t)} = \begin{bmatrix} c_{\phi(t)}c_{\theta(t)} & -s_{\phi(t)}c_{\psi(t)} + c_{\phi(t)}s_{\theta(t)}s_{\psi(t)} & s_{\phi(t)}s_{\psi(t)} + c_{\phi(t)}s_{\theta(t)}c_{\psi(t)} & d_x \\ s_{\phi(t)}c_{\theta(t)} & c_{\phi(t)}c_{\psi(t)} + s_{\phi(t)}s_{\theta(t)}s_{\psi(t)} & -c_{\phi(t)}s_{\psi(t)} + s_{\phi(t)}s_{\theta(t)}s_{\psi(t)} & d_y \\ -s_{\theta(t)} & c_{\theta(t)}s_{\psi(t)} & c_{\theta(t)}c_{\psi(t)} & d_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} p_x \\ p_y \\ p_z \\ 1 \end{Bmatrix}, \tag{4.1}$$

or

$$\mathbf{P(t)} = [K(t)]\mathbf{p}. \tag{4.2}$$

Our goal is to determine the movement $[K(t)]$ that has the property that three points in the moving body have the trajectories $\mathbf{A(t)}$, $\mathbf{B(t)}$ and $\mathbf{C(t)}$ consistent with contact with three spherical objects, as shown in Figure 4.

Now position the coordinate frame M such that its origin coincides with $\mathbf{A(t)}$ and its x-axis is directed along the line $\mathbf{B} - \mathbf{A}$. The z-axis of M is defined to be perpendicular to the plane containing $\mathbf{B} - \mathbf{A}$ and $\mathbf{C} - \mathbf{A}$. The y-axis is defined according to the right hand rule. This defines $[K(t)]$ with translation vector $\mathbf{d(t)}$ and the roll, pitch and yaw angles $\phi(t), \theta(t), \psi(t)$ are given by

$$\mathbf{d(t)} = \mathbf{A(t)}, \quad \theta(t) = -\arcsin\frac{\vec{k}\cdot(\mathbf{B}-\mathbf{A})}{|\mathbf{B}-\mathbf{A}|},$$

$$\phi(t) = Atan2(\frac{\vec{j}\cdot(\mathbf{B}-\mathbf{A})}{|\mathbf{B}-\mathbf{A}|}, \frac{\vec{i}\cdot(\mathbf{B}-\mathbf{A})}{|\mathbf{B}-\mathbf{A}|}),$$

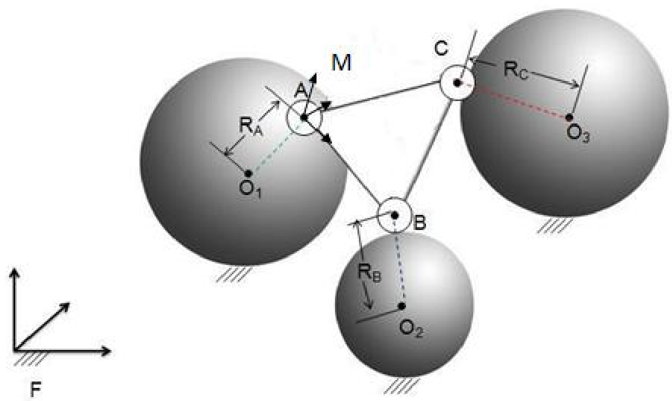

Fig. 4. Contact specifications in a specified position. The triangular shaped end effector is in contact with three objects. The objects are represented by spheres, whose radii are same as the radii of curvature R_A, R_B, R_C of the objects at A, B and C.

$$\psi(t) = Atan2(\frac{\vec{k}\cdot((\mathbf{B}-\mathbf{A})\times(\mathbf{C}-\mathbf{A}))\times(\mathbf{B}-\mathbf{A})}{|((\mathbf{B}-\mathbf{A})\times(\mathbf{C}-\mathbf{A}))\times(\mathbf{B}-\mathbf{A})|}, \frac{\vec{k}\cdot((\mathbf{B}-\mathbf{A})\times(\mathbf{C}-\mathbf{A}))}{|((\mathbf{B}-\mathbf{A})\times(\mathbf{C}-\mathbf{A}))|}) \tag{4.3}$$

where $\vec{i}$, $\vec{j}$ and $\vec{k}$ are vectors along the axis of the fixed frame.

In what follows, we determine on the position, velocity and acceleration of the body at an instant denoted $t = 0$, from properties of the trajectories $\mathbf{A(t)}$, $\mathbf{B(t)}$ and $\mathbf{C(t)}$ imposed by contact with two objects.

B. The Position Specification

We assume that the contact of our moving body M with three fixed objects constrain the point trajectories $\mathbf{A(t)}$, $\mathbf{B(t)}$ and $\mathbf{C(t)}$ to follow circles in the vicinity of a reference position denoted by $t = 0$. The movement of M in the vicinity of $t = 0$ can be expressed as the Taylor series expansion,

$$[K(t)] = [K_0] + [K_1]t + \frac{1}{2}[K_2]t^2 + ..., \quad \text{where} \quad [K_i] = \frac{d^i[K]}{dt^i}. \tag{4.4}$$

Evaluating the derivatives of [K(t)], we obtain

$$[K_0] = \begin{bmatrix} c_{\phi_0}c_{\theta_0} & -s_{\phi_0}c_{\psi_0} + c_{\phi_0}s_{\theta_0}s_{\psi_0} & s_{\phi_0}s_{\psi_0} + c_{\phi_0}s_{\theta_0}c_{\psi_0} & d_x \\ s_{\phi_0}c_{\theta_0} & c_{\phi_0}c_{\psi_0} + s_{\phi_0}s_{\theta_0}s_{\psi_0)} & -c_{\phi_0}s_{\psi_0} + s_{\phi_0}s_{\theta_0}s_{\psi_0} & d_y \\ -s_{\theta_0} & c_{\theta_0}s_{\psi_0} & c_{\theta_0}c_{\psi_0} & d_z \\ 0 & 0 & 0 & 1 \end{bmatrix}, \tag{4.5}$$

We have introduced the notation $d^i\phi/dt^i(0) = \phi_i$ and $d^i\mathbf{d}/\mathbf{dt^i}(\mathbf{0}) = (d_{x,i}, d_{y,i})^T = \mathbf{d_i}$.

The data provided for the position of the moving frame M identifies the coordinates of the contact points $\mathbf{A_0} = \mathbf{A}(\mathbf{0})$, $\mathbf{B_0} = \mathbf{B}(\mathbf{0})$ and $\mathbf{C_0} = \mathbf{C}(\mathbf{0})$. Using (4.3), we have ϕ_0, θ_0, ψ_0.

This defines the coordinate transformation $[K_0]$.

C. The Velocity Specification

In order to satisfy the force constraints at the prescribed positions, we determine directions of the velocity vectors $\dot{\mathbf{A}}$, $\dot{\mathbf{B}}$ and $\dot{\mathbf{C}}$ that are perpendicular to the forces $\mathbf{F}_A$, $\mathbf{F}_B$ and $\mathbf{F}_C$ that are along the direction of the radii of the spheres(Figure 4). The equations defining these constraints are as follows, where $\mathbf{w}$ defines the screw axis.

$$\dot{\mathbf{d}} = \dot{\mathbf{A}}.$$

$$\dot{\mathbf{A}} = \mathbf{w} \times (\mathbf{A} - \mathbf{d}) + \dot{\mathbf{d}} = \mathbf{w}_{O_1A} \times (\mathbf{A} - \mathbf{O_1}).$$

$$\dot{\mathbf{B}} = \mathbf{w} \times (\mathbf{B} - \mathbf{d}) + \dot{\mathbf{d}} = \mathbf{w}_{O_2B} \times (\mathbf{B} - \mathbf{O_2}).$$

$$\dot{\mathbf{C}} = \mathbf{w} \times (\mathbf{C} - \mathbf{d}) + \dot{\mathbf{d}} = \mathbf{w}_{O_3C} \times (\mathbf{C} - \mathbf{O_3}). \tag{4.6}$$

To find the values of $\dot{A}, \dot{\theta}, \dot{\phi}, \dot{\psi}$, we use (4.6) where $\mathbf{w} = R_0(R_1)^{-1}$ is a function of $\dot{\theta}, \dot{\phi}, \dot{\psi}$.

This completes the velocity specification $[K_1]$ from (4.4).

D. The Acceleration Specification

As the body M moves in contact with three objects, the points A, B and C are guided along trajectories with radii of curvature R_A, R_B and R_C. This is equivalent to the specification of points $\mathbf{O_1}$, $\mathbf{O_2}$ and $\mathbf{O_3}$ which are the centers of curvature of the trajectories $\mathbf{A(t)}$, $\mathbf{B(t)}$ and $\mathbf{C(t)}$ at the instant $t = 0$.

Differentiating the velocity equations (4.6), we find the acceleration specifications.

$$\mathbf{a}_{\mathbf{O_1}A}\times(\mathbf{A}-\mathbf{O}_1)+\mathbf{w}_{O_1A}\times(\mathbf{w}_{\mathbf{O_1}A}\times(\mathbf{A}-\mathbf{O}_1)) = \mathbf{a}\times(\mathbf{A}-\mathbf{d})+\mathbf{w}\times(\mathbf{w}\times(\mathbf{A}-\mathbf{d})+\dot{\mathbf{d}}) \quad (4.7)$$

$$\mathbf{a}_{\mathbf{O_2}B}\times(\mathbf{B}-\mathbf{O}_2)+\mathbf{w}_{O_2B}\times(\mathbf{w}_{\mathbf{O_2}B}\times(\mathbf{B}-\mathbf{O}_2)) = \mathbf{a}\times(\mathbf{B}-\mathbf{d})+\mathbf{w}\times(\mathbf{w}\times(\mathbf{B}-\mathbf{d})+\dot{\mathbf{d}}) \quad (4.8)$$

$$\mathbf{a}_{\mathbf{O_3}C}\times(\mathbf{C}-\mathbf{O}_3)+\mathbf{w}_{O_3C}\times(\mathbf{w}_{\mathbf{O_3}C}\times(\mathbf{C}-\mathbf{O}_3)) = \mathbf{a}\times(\mathbf{C}-\mathbf{d})+\mathbf{w}\times(\mathbf{w}\times(\mathbf{C}-\mathbf{d})+\dot{\mathbf{d}}) \quad (4.9)$$

where $\mathbf{a}$ is the derivative of $\mathbf{w}$, and is a function of $\ddot{\theta}, \ddot{\phi}, \ddot{\psi}$ after substituting the values of $\theta, \phi, \psi, \dot{\theta}, \dot{\phi}, \dot{\psi}$. $\ddot{\mathbf{d}}$ is directly obtained from $\ddot{A}$. Similar equations are written for B and C, and solved to get $\ddot{\theta}, \ddot{\phi}, \ddot{\psi}$. This completes the acceleration matrix $[K_2]$ from (4.4).

E. Relative Movement

For convenience in what follows, we introduce the relative transformation $[D(t)] = [K(t)][K_0]^{-1}$ that operates on point coordinates measured in the fixed frame at the instant $t = 0$. Recall that a point $\mathbf{p}$ in M has the trajectory $\mathbf{P}(t)$ defined by the equation

$$\mathbf{P}(t) = [K(t)]\mathbf{p} = [K_0 + K_1 t + \frac{1}{2}K_2 t^2 + ...]\mathbf{p}. \tag{4.10}$$

Now let $\mathbf{P} = [K_0]\mathbf{p}$, then we have

$$\begin{aligned} \mathbf{P}(t) =& [K_0 + K_1 t + \frac{1}{2}K_2 t^2 + \cdots][K_0]^{-1}\mathbf{P}, \\ =& [I + \Omega t + \frac{1}{2}\Lambda t^2 + \cdots]\mathbf{P} = [D(t)]\mathbf{P}, \end{aligned} \tag{4.11}$$

where

$$\Omega = K_1 K_0^{-1}, \quad \Lambda = K_2 K_0^{-1} \tag{4.12}$$

In the following section, we use this formulation of the movement of M to describe the trajectory of a general point $\mathbf{P}(t)$.

F. Differential Kinematics

Given the relative movement of the moving frame M, we can examine the trajectories of all points in the workpiece. In particular, we ask if there is a point in the body that has zero curvature.

G. Spherical Motion

Let A define the rotation matrix of a moving frame M. If p is a point in the moving frame, the coordinates of p in the fixed frame F is given by α

$$\alpha(t, \vec{p} = A(t)\vec{p} \tag{4.13}$$

H. Taylor Series Expansion

The Taylor expansion of an orthogonal matrix is

$$A(t) = A_0 + A_1 t + \frac{1}{2!}A_2 t^2 + \frac{1}{3!}A_3 t^3 + ... \frac{1}{k!}A_k t^k ...$$
$$where \quad A_k = \frac{d^k A(t)}{dt^k}|_{t=0} \quad is \quad a \quad constant \quad matrix. \tag{4.14}$$

Simplify A_0 to I by aligning M with F. Recall $A(t)$ is orthogonal, hence for all values of t,

$$AA^T = I \quad or \quad (I + A_1 t + \frac{1}{2!}A_2 t^2 + \frac{1}{3!}A_3 t^3 + ...)(I + (A_1)^T t + \frac{1}{2!}A_2 t^2 + \frac{1}{3!}A_3 t^3 + ...) = I \tag{4.15}$$

The coefficients of t^k should vanish for i=1,2,...

$$t^1 : A_1 + (A_1)^T = 0 \tag{4.16}$$

implies A_1 is skew symmetric. Let $A_1 = B_1$ (skew symmetric) and let $(B_1)^2 = C_2$ which is symmetric

$$A_1(A_1)^T = -(A_1)^T(A_1)^T = -(B_1)^2 = -C_2 \tag{4.17}$$

Working along in a similar way, we can rewrite $A(t)$ as

$$A(t) = I + B_1 t + \frac{1}{2!}(C_2 + B_2)t^2 + \frac{1}{3!}(C_3 + B_3)t^3$$
$$where \quad C_2 = (B_1)^2, C_3 = \frac{3}{2}(B_2 B_1 + B_1 B_2) \tag{4.18}$$

I. Angular Velocity

In order to reduce the number of independent parameters, we will be parameterizing ϕ to normalize the equations so that $|\Omega| = 1$. To do this, we need to derive the Taylor expansion for the angular velocity matrix.

We know the rotational matrix is given by $\alpha = A(t)p$. It follows that the velocity vector of a point P of the moving space is $\dot{\alpha} = \dot{A}(t)p$. Elimination p we have $\dot{\alpha} =$

$\dot{A}A^{-1}\alpha$. Now if we denote $\dot{A}A^{-1}$ as Ω, we get the angular velocity matrix

$$\Omega(t) = \dot{A}(t)A^{-1}(t)$$
$$where \quad \dot{A} = B_1 + (C_2 + B_2)t + \frac{1}{2}(C_2 + B_2)t^2 + ...$$
$$A^{-1} = A^T = I - B_1 t + \frac{1}{2!}(C_2 - B_2)t^2 + ... \tag{4.19}$$

Expanding $\Omega(t)$ using Taylor expansion yields

$$\Omega(t) = \Omega_0 + \Omega_1 t + \frac{1}{2!}\Omega_2 t^2 + ... \tag{4.20}$$

Plugging [4.19],[4.20] into the above and equation coefficients upto the second order, we find

$$\Omega_0 = B_1$$
$$\Omega_1 = B_2$$
$$\Omega_2 = -(B_1)^3 + \frac{1}{2}(B_1B_2 - B_2B_1) + B_3$$

J. Canonical Reference Frame

We shall introduce a special coordinate system in order to simplify the equation. Let b_i be the vector associated with the skew symmetric matrix B_i and define

$$b_1 = \omega, \quad b_2 = \epsilon \quad b_3 = \gamma$$

or

$$B_1 = [\omega], \quad B_2 = [\epsilon] \quad B_3 = [\gamma] \tag{4.21}$$

Recall the fixed and moving frames coincide at the center of the sphere. Let us align the z-axis along ω and the x-axis along $-\omega \times \epsilon = \epsilon \times \omega$ (i.e put ϵ in the y-z plane). This reduces ω and ϵ to

$$\omega = \begin{Bmatrix} 0 \\ 0 \\ \omega_z \end{Bmatrix} \quad \epsilon = \begin{Bmatrix} 0 \\ \epsilon_y \\ \epsilon_z \end{Bmatrix} \tag{4.22}$$

To further simplify things, let us introduce a parameter ϕ such that $\Omega(t)$ is normalized so that $|\Omega| = 1$. This requires $\omega^2 = 1$ and all higher orders to be zero. It follows that

$$|\omega| = 1, \quad \omega \cdot \epsilon = 0, \quad \omega \cdot \gamma = -(1 + \epsilon^2) \tag{4.23}$$

which implies

$$\epsilon_z = 0, \quad \epsilon_y = \epsilon, \quad \gamma_z = -(1 + \epsilon^2) \tag{4.24}$$

hence

$$\omega = \begin{Bmatrix} 0 \\ 0 \\ 1 \end{Bmatrix} \quad \epsilon = \begin{Bmatrix} 0 \\ \epsilon \\ 0 \end{Bmatrix} \quad \gamma = \begin{Bmatrix} \gamma_x \\ \gamma_y \\ -(1+\epsilon^2) \end{Bmatrix} \tag{4.25}$$

Plugging [4.25] into [4.14], we have the series expansion for $A(t)$ in the canonical system for the unit angular velocity parameterization $t = g(\phi)$

$$A(\phi) = I + [\omega]\phi + \frac{1}{2!}([\omega]^2 + [\epsilon])\phi^2 + \frac{1}{3!}(\frac{3}{2}([\omega][\epsilon] + [\epsilon][\omega] + [\omega])\phi^3 + ... \tag{4.26}$$

where ω, ϵ, γ are given in [4.25]. Notice that $A(\phi)$ relies only on three instantaneous invariants $\epsilon, \gamma_x, \gamma_y$.

The trajectory of any point on a body that can move about a spherical motion can be written as

$$\alpha(\phi, p) = A(\phi)p$$

Expanding α, its first three derivatives at $\phi = 0$ are

$$\alpha_0 = \begin{Bmatrix} x \\ y \\ z \end{Bmatrix}, \quad \alpha_1 = \begin{Bmatrix} -y \\ x \\ 0 \end{Bmatrix}, \quad \alpha_2 = \begin{Bmatrix} -x + \epsilon z \\ -y \\ -\epsilon x \end{Bmatrix}$$

$$\alpha_3 = \begin{Bmatrix} (1+\epsilon^2)y + \gamma_y z \\ -(1+\epsilon^2)x + (\frac{3}{2}\epsilon - \gamma_x)z \\ -\gamma_y x + (\frac{3}{2}\epsilon + \gamma_x)y \end{Bmatrix} \tag{4.27}$$

We are interested in the geodesic curvature since it contains information of the curve's behaviour. A general expression for the curvature is

$$R = \frac{\alpha_0 \cdot (\alpha_1 \times \alpha_2)}{(\alpha_1 \cdot \alpha_1)^{\frac{3}{2}}} \tag{4.28}$$

Using [4.27], we get

$$R = \frac{z(x^2+y^2) - x\epsilon(x^2+y^2+z^2)}{(x^2+y^2)^{\frac{3}{2}}} \tag{4.29}$$

We are interested in the points whose trajectories have zero curvature i.e points that move along a great circle. The locus is defined by the equation

$$0 = z(x^2+y^2) - x\epsilon(x^2+y^2+z^2) \tag{4.30}$$

defines a cubic cone in the moving body. This is known as the inflection circle. For $\epsilon = 0.5$, the inflection cone looks like Figure 5.

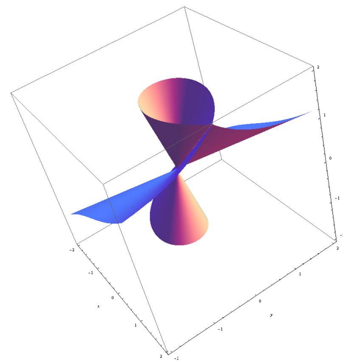

Fig. 5. Inflection cone.

K. Geometric Design of a TS Chain for Task Constraints Imposed by Objects in the Environment

Similar to the RR chain, which plays major role in the planar linkage synthesis, the TS chain is central to the structure of open and closed loop spatial robot manipulators. There are number of linkage systems that can be constructed from RR and TS kinematic chains, such as RRRR, RRTS and 5TS platform mechanisms. The TS chain is a special case of the five degree of freedom RRRRR, in which the first two revolute joints intersect and are perpendicular to each other and the last three revolute joints intersect in a point to define a spherical wrist.

This section considers the algebraic synthesis of a spatial TS serial chain such that it guides a floating link through a set of positions with specified accelerations. The TS chain has seven design parameters, the coordinates of the center of the intersection of the perpendicular revolute joints (R joints) in the base frame, the coordinates of the intersection of the last three R joints in the moving frame and the length of the distance between these points. The synthesis procedure presented here, computes these design parameters by solving seven constraint equations that must be met for the chain to achieve a prescribed task.

A TS chain is formed when a body is connected to the ground by a gimbal joint and to a floating link by a spherical joint, Figure 6. Let the coordinates of the center of the fixed T joint be $\mathbf{B} = (B_x, B_y, B_z, 1)$ measured in the fixed frame F, and the point at the center of the moving spherical joint have the coordinates $\mathbf{p} = (p_x, p_y, p_z, 1)$ measured in the moving frame M. The movement of the floating link M is constrained by the TS chain such that the trajectory of $\mathbf{p}$ in F lies on a sphere about $\mathbf{B}$. This provides the conditions for formulating the design equations.

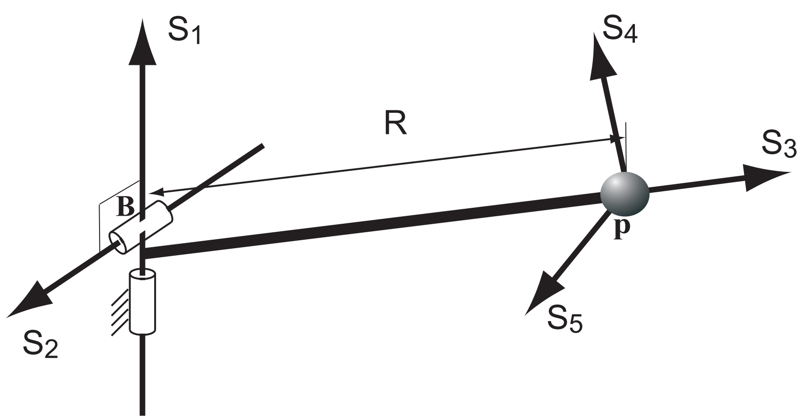

Fig. 6. Structure of the TS chain.

Let the orientation of the moving body M with the angles (θ, ϕ, ψ) is defined, which describe the longitude and latitude of the z-axis of the moving frame, and roll angles about that axis. This convention yields the rotation matrix

$$[A^j] = [Y(\theta)][X(-\phi)][Z(\psi)], \tag{4.31}$$

where $[X(\cdot)]$, $[Y(\cdot)]$, and $[Z(\cdot)]$ are rotations about the X, Y and Z axes respectively. The translation of the moving frame is defined by the vector $\mathbf{d} = (d_{x,j}, d_{y,j}, d_{z,j})$.

Thus, the matrix function $[T(t)] = [A^j(t), \mathbf{d}^j(t)]$ is given by

$$[K(t)] = \begin{Bmatrix} c_\phi c_\psi - s_\phi s_\theta s_\psi & -c_\psi s_\phi s_\theta - c_\phi s_\psi & c_\theta s_\phi & d_{x,j} \\ c_\psi s_\theta & c_\theta c_\psi & s_\theta & d_{y,j} \\ c_\psi s_\phi - c_\phi s_\theta s_\psi & -c_\phi c_\psi s_\theta + s_\phi s_\psi & c_\phi c_\theta & d_{z,j} \\ 0 & 0 & 0 & 1 \end{Bmatrix}, \tag{4.32}$$

where $sin(\cdot)$ and $cos(\cdot)$ are noted with $s(\cdot)$ and $c(\cdot)$, respectively.

The movement of the frame M relative to the fixed frame F in the vicinity of a reference position defined by $t = 0$ is defined by the Taylor series expansion,

$$[K^j(t)] = [K_0^j] + [K_1^j]t + \frac{1}{2}[K_2^j]t^2 + ..., \text{ where } [K_i^j] = \frac{d^i[K^j]}{dt^i}, \tag{4.33}$$

and $[K(0)] = [K_0]$ is the reference position. A point $\mathbf{p}$ in the moving frame has the trajectory $\mathbf{P}(t)$ in the fixed frame given by the equation

$$\mathbf{P}^j(t) = [K^j(t)]\mathbf{p} = [K_0^j + K_1^j t + \frac{1}{2}K_2^j t^2 + ...]\mathbf{p}. \tag{4.34}$$

It is convenient to substitute $\mathbf{p}^j = [K_0^j]^{-1}\mathbf{P}^j$ into (4.34) so that we obtain

$$\begin{aligned} \mathbf{P}^j(t) =& [K_0^j + K_1^j t + \frac{1}{2}K_2^j t^2 + \cdots][K_0^j]^{-1}\mathbf{P}, \\ =& [I + \Omega t + \frac{1}{2}\Lambda t^2 + \cdots]\mathbf{P} = [D_{ij}(t)]\mathbf{P}^j, \end{aligned} \tag{4.35}$$

The matrices $[K_0]$, $[K_1]$ and $[K_2]$ are defined by the position, velocity and acceleration task requirements. This data is transformed into the matrices $[\Omega]$ and $[\Lambda]$ and the relative displacements between the task positions in order to formulate the design equations. CraigCraigCraig

L. The Synthesis and Design Equations

We follow Suh and Radcliffe [48] and introduce relative transformations to obtain a standard set of design equations. Let $\mathbf{B} = (b_x, b_y, b_z, 1)$ be the homogeneous coordinates of the center of the T joint of the TS chain (Figure 7). And let $\mathbf{P^1} = (p_x, p_y, p_z, 1)$ be the homogeneous coordinates of the center of its spherical wrist measured in the fixed frame when the moving body is in the first task position, that is $\mathbf{P^1} = T_1\mathbf{p}$, where $\mathbf{p}$ is the vector measured in the frame of moving body. We can find the coordinates of $\mathbf{p}$ in the other ask positions using the relative transformation $[D_{1i}] = [T_i][T_1^{-1}]$, which yields $\mathbf{P^i} = [D_{1i}]\mathbf{P^1}$. We use the coordinate vectors $\mathbf{B}$ and $\mathbf{P^1}$ as our design parameters for the TS chain.

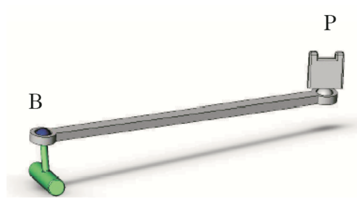

Fig. 7. A general TS chain. B and P define the coordinates of the T and S joints.

In order to lie on a sphere about the fixed pivot B, the moving pivot $\mathbf{P^i}$ must satisfy the constraint equations

$$(\mathbf{P^i} - \mathbf{B}) \cdot (\mathbf{P^i} - \mathbf{B}) = R^2 \tag{4.36}$$

in each of n specified positions, where R is the distance between the centers of T and S joints. We now substitute $\mathbf{P^i} = [D_{1i}]\mathbf{P^1}$ to obtain the position design equations.

$$([D_{1i}]\mathbf{P^1} - \mathbf{B}) \cdot ([D_{1i}]\mathbf{P^1} - \mathbf{B}) = R^2, i = 1, ..., n. \tag{4.37}$$

These are the position constraint equations.

The constraint equatioins on the velocities of the points in the moving body are obtained by computing the derivative of (4.37),

$$\frac{d}{dt}\mathbf{P^i} \cdot (\mathbf{P^i} - \mathbf{B}) = 0, i = 1, ...n \tag{4.38}$$

Recall that the velocity of a point in the ith task frame is defined by $\frac{d}{dt}\mathbf{P^i} = [W_i]\mathbf{P^i}$. Therefore, we have

$$\frac{d}{dt}\mathbf{P^i} = [W_i]D_{1i}\mathbf{P^1} \tag{4.39}$$

Notice that $[D_{11}]$ is the 4×4 identity matrix, so this equation also applies to velocities specified for position one. Substitute this equation into (4.38) and obtain the velocity design equations

$$([W_i]D_{1i}\mathbf{P^1}) \cdot (D_{1i}\mathbf{P^1} - \mathbf{B}) = 0, i = 1, ...n \tag{4.40}$$

Now, compute the second derivative of the position constraint (4.36) to determine the acceleration constraint equations.

$$\frac{d^2}{dt^2}\mathbf{P^i} \cdot (\mathbf{P^i} - \mathbf{B}) + (\frac{d}{dt}\mathbf{P^i}) \cdot (\frac{d}{dt}\mathbf{P^i}) = 0, i = 1, ...n \tag{4.41}$$

where i denotes the task position in which we define the acceleration. From our definition of the 4×4 acceleration matrix, we have $\frac{d^2}{dt^2}\mathbf{P^i} = [\Lambda_i]\mathbf{P^i}$, which yields

$$([\Lambda_i][D_{1i}]\mathbf{P^1}) \cdot ([D_{1i}]\mathbf{P^1} - \mathbf{B}) + ([W_i]D_{1i}\mathbf{P^1}) \cdot ([W_i]D_{1i}\mathbf{P^1}) = 0, i = 1, ...n. \tag{4.42}$$

These are the acceleration design equations.

M. Trajectory Planning

Standard trajectory planning techniques are adapted to the constrained movement of a spatial open chain with less than six degrees of freedom. The synthesis equations ensure that the end-effector can achieve the specified positions, velocities and accelerations, therefore the goal is to calculate the associated joint angles, joint velocities and joint accelerations. This is done using modifications of the fifth degree polynomial trajectory used to fit position, velocity and acceleration data that is described in Craig (1989)[47].

The inverse kinematics of a spatial open chain yields the joint parameter vector $\mathbf{q} = (\theta_1, \ldots, \theta_l)$ at each of the task positions, $j = 1, \ldots, n$. In order to obtain the joint velocity vector $\dot{\mathbf{q}}$ at the jth position, the following equation is solved:

$$V^j = [J_j]\dot{\mathbf{q}}_j, \quad j = 1, \ldots, n, \tag{4.43}$$

where $V^j = (W_j, \mathbf{v}_j)$ is the velocity prescribed at position j, and J_j is the Jacobian of the spatial chain. Notice, that because this Jacobian is an $m \times n$ matrix, a pre-multiplication of both sides by it's pseudo-inverse is needed to obtain

$$\dot{\mathbf{q}}_j = [J_j^T J_j]^{-1}[J_j^T]V_j, \quad i = 1, \ldots, n. \tag{4.44}$$

The pseudo-inverse of the Jacobian provides an exact solution, because the linkage is designed to satisfy this velocity requirement.

Now to determine the joint acceleration vector $\ddot{\mathbf{q}}_j$, the following equation is solved:

$$\dot{\Omega}^j = \dot{J}_j\dot{\mathbf{q}}_j + J_j\ddot{\mathbf{q}}_j, \tag{4.45}$$

where $\dot{\Omega}^j = (\alpha^j, \mathbf{a}^j)$ is the acceleration prescribed at the $j-th$ position and $\dot{J}_j$ is the time derivative of the $m \times n$ Jacobian matrix. The vector $\dot{J}_j \dot{\mathbf{q}}_j$ is known, thus the solution is again obtained using the pseudo-inverse of the Jacobian,

$$\ddot{\mathbf{q}}_j = [J_j^T J_j]^{-1}[J_j^T](A_j - \dot{J}_j \dot{\mathbf{q}}_j). \tag{4.46}$$

The $n-th$ degree interpolation of the joint parameters $\mathbf{q}_j, \dot{\mathbf{q}}_j, \ddot{\mathbf{q}}_j$ yields the spatial open chain trajectory.

N. Summary of Spherical Research for Contact Specifications

The synthesis of spatial TS chains that guide a rigid body, such that it does not violate normal direction and curvature constraints imposed by contact with two objects in the environment was presented for the first time in this chapter. The contact direction and curvature constraints were transformed into conditions on the velocity and acceleration of certain points in the moving body. Joint trajectory interpolation formulas yield the movement of the TS chain with the desired contact properties in each task position. There are two main contributions. The first one is that theoretical methods for the transition from contact geometry to kinematic synthesis for spherical movement has been developed. The second contribution lays in the development of contact relationships for spherical movement, imposed by contact with three objects in the workspace, is used to define velocity and acceleration constraints on the end-effector of the spherical TS chain.

CHAPTER V

DEVELOPMENT OF A ROVER PLATFORM AND ROBOTIC ARM SYSTEM FOR EXPERIMENTAL TESTING OF THE OBTAINED RESULTS

A. Control Architecture

In order to apply the described theoretical results to real world situations, a platform with a robotic arm mounted on it, similar to the NASA Mars Rover System was developed. A Surface Mobility Platform (SMP) with a TRT robotic arm mounted on it, is used to perform mission critical tasks (see Figure 8).

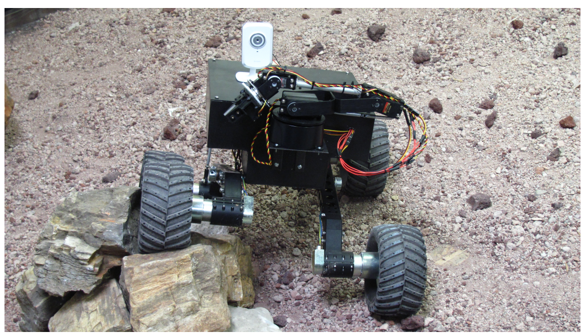

Fig. 8. Surface mobility platform (SMP) with the robotic arm.

The control architecture employed is a three level architecture. The top level consists of manual inputs or positions calculated using mathematica code. This level runs on a laptop. The inputs are transmitted by LabView, through a wireless connection to the controller on the Platform.

The controller is a National Instruments sbRIO 9632. This device controls the SMP motors directly. The Robotic Arm (Lynxmotion AL5D) is controlled by the sbRIO 9632 through an SSC-32 controller (Figure 9). Figure 10 shows the actual setup.

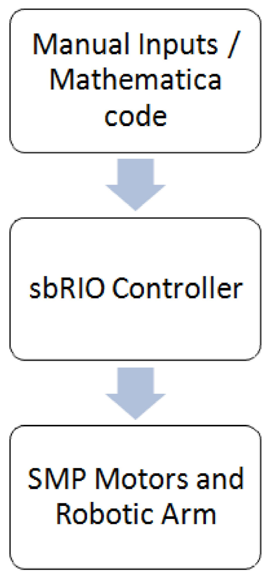

Fig. 9. Control architecture of the setup.

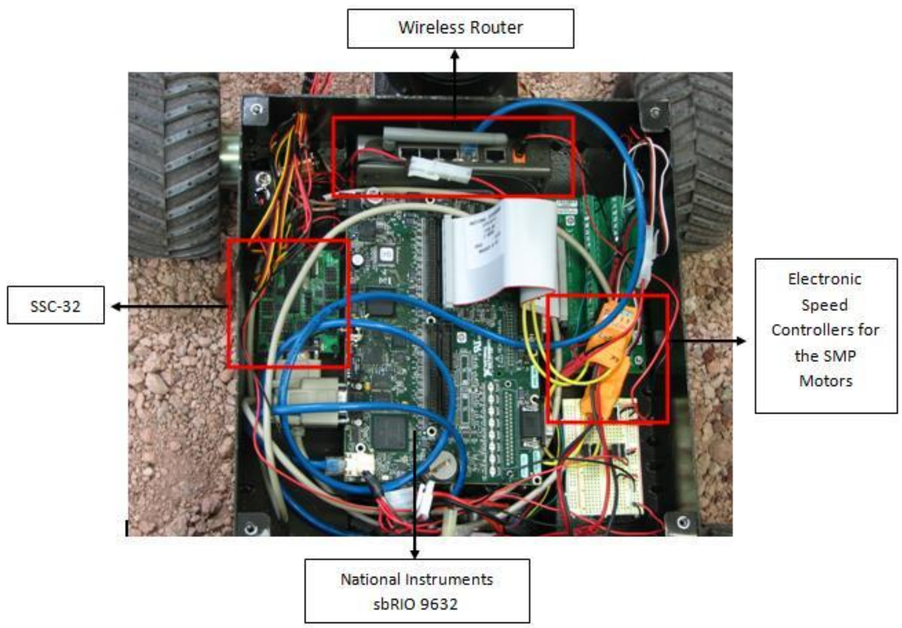

Fig. 10. Different parts of the control setup.

CHAPTER VI

APPLICATIONS

A. Failure Recovery

The spherical synthesis example is a part of our efforts to explore new efficient methods for the design of fault-tolerant robot manipulators. Particularly, we examine if a non-redundant general 5 DOF (TRT) robot manipulator mounted on a movable platform can go through the originally specified task after it's elbow joint fails and is locked in place (Figure 11).

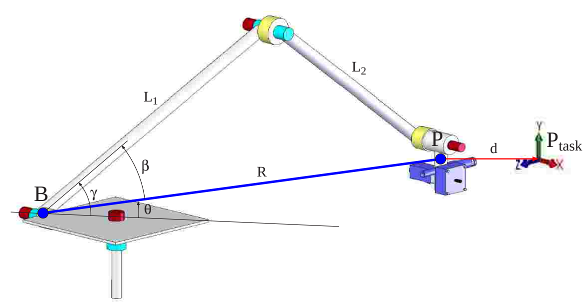

Fig. 11. The 5DOF serial TRT arm with a failed elbow joint defines a 4 DOF serial chain.

The recovery strategy is based on the ability to reposition the arm base, so that the base point $\mathbf{B} = (b_x, b_y, 14)$ of the Robotic Arm can be repositioned in a horizontal plane. Since the base of the Robotic Arm can not be moved in the z direction, the z component of $\mathbf{B}$ is taken as 14 cm. We assume that the TRT arm can grasp the tool frame where necessary so that the wrist center $\mathbf{P} = (p_x, p_y, p_z)$ can be located where needed in the tool frame.

The task consists of three positions, two velocities and one acceleration defined from contact and curvature specifications. The task data is given in Table I. The task specifications for the arm trajectory are obtained by placing markers on the tool and the robotic arm, and using a VICON Infrared Motion Capture System available (see Figure 12) at our Human Interactive Robotics Lab. The arm was to follow a trajectory specified on an object.

Fig. 12. VICON motion capture system consisting of infrared cameras.

Table I. Task Description.

Position Spec.(cm;rad)	$(d_x, d_y, d_z; \theta, \phi, \psi)$
Location 1	(25.31, 3.56, 15.23; -0.17, 0.897, 2.48)
Location 2	(12.53, 11.28, 5.63; -0.59, 0.55, 3.73)
Location 3	(18.11, 5.63, 10.2; 0.14, -0.44, 0.85)
Velocity Spec.(cm/s;rad/s)	$(\dot{d}_x, \dot{d}_y, \dot{d}_z; \dot{\theta}, \dot{\phi}, \dot{\psi})$
Location 1	(3.84, 0.65, 2.71; -0.03, 0.12, 0.16)
Location 2	(-8.59, -8.13, -5.69; 0.39, -0.5, 0.31)
Acceleration Spec.(cm/s^2;rad/s^2)	$(\ddot{d}_x, \ddot{d}_y, \ddot{d}_z, \ddot{\theta}, \ddot{\phi}, \ddot{\psi})$
Location 2	(0.3, -0.26, 2.36, -0.43, 0.49, 0.55)

The joint limits of the robotic arm were also measured by the VICON motion capture system. These are given in Table II.

If the elbow actuator of the third joint of the TRT arm fails, then we assume its brakes can be set so that θ_3(elbow joint angle) has a constant value. The remaining

Table II. Joint Limits of the Robotic Arm for Each Axis.

Joint	Min(degrees)	Max(degrees)
Base	-126.9	81.4
Shoulder	-18.8	177.4
Elbow	-193	13
Wrist 1	-20	182.4
Wrist 2	-90	93.6

joints of the arm form a TT chain that can position the wrist center **P** on a sphere with a radius R about the base point **B**(see Figure 4). The radius R is defined by the link lengths l_1 and l_2, the angle θ_3, and the cosine law to be

$$R^2 = {l_1}^2 + {l_2}^2 - 2l_1l_2cos(\theta_3) \tag{6.1}$$

The value of θ_3 can be obtained from the joint sensors of the actuator failure.

We seek the reconfiguration parameters $\mathbf{r} = (b_x, b_y, p_x, p_y, p_z)$ that allow the arm to perform the original task despite the failure. The polynomial system formed by the TS design equations consists of five quadratic equations in the unknowns $\mathbf{r}$ and has a total degree of $2^5 = 32$. The synthesis equations presented in the previous section are applied. The velocity and acceleration specifications given in the table above are used to derive the $[W], [D]$ and $[\Lambda]$ matrices. The solutions obtained are shown in Table III. The joint limits ensure that the solutions obtained above can be implemented.

Both the solutions can be chosen for finishing the task. Using Solution No.1, Figure 13 shows the healthy TRT arm going through the task, while Figure 14 shows the reconfigured platform system after the elbow failure.

Table III. Real Solutions Obtained after Solving the Synthesis Equations.

S.No	b_x(cm), b_y(cm)	p_x(cm), p_y(cm), p_z(cm)	R(cm)	θ_3(degree)
1	20.0579, 10.2074	34.8522, -2.83062, 3.631	22.27	85.68
2	9.62958, 7.81412	28.1166, 7.07669, 2.13764	21.97	84.2

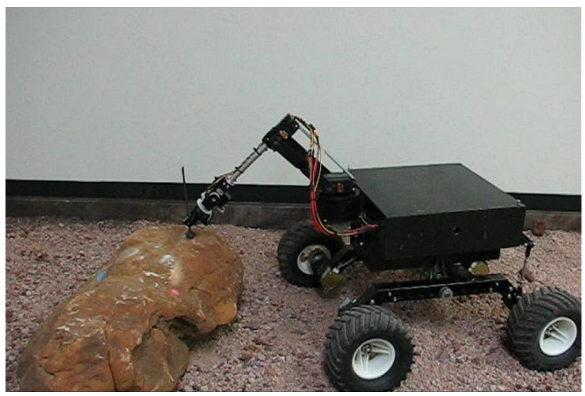

Fig. 13. SMP with healthy arm. The platform performs the task when the elbow joint is healthy.

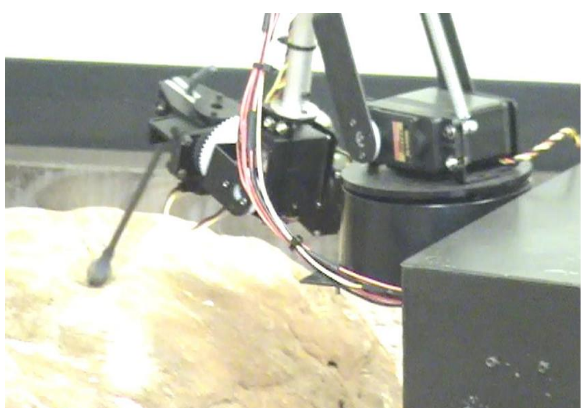

Fig. 14. SMP with crippled arm. The Rover performs the task after moving its base and the tool point to a new location.

CHAPTER VII

SUMMARY

This paper considers the synthesis of spherical TS chains that guide a rigid body such that it does not violate normal direction and curvature constraints imposed by contact with three objects in the environment. We transform the contact direction and curvature constraints into conditions on the velocity and acceleration of certain points in the moving body. These velocities and accelerations provide design degrees of freedom that allow for the arm to obtain the originally specified task, despite the joint failure. The synthesis equations are then solved by algebraic elimination. The application of this technique for joint failure recovery of robot manipulators was discussed.

CHAPTER VIII

FUTURE WORK

The current thesis looks at the satisfying curvature constraints by synthesizing spherical chains. Future work consists of extending the theory to synthesizing spatial chains. Synthesizing spatial chains is even more complicated as it involves a larger set of equations to be solved. For example, in order to synthesize a TRS arm, there would be a total of 9 different variables which are the (x, y, z) co-ordinates of T, R and S joints. This requires a high degree of computational power.

REFERENCES

[1] N. Patarinsky-Robson and J.M. McCarthy, "Kinematic Synthesis with Contact Direction and Curvature Constraints on the Workpiece," *ASME Int. Design Eng. Conf.*, 2007.

[2] N. P. Suh, "Axiomatic Design Theory for Systems," *Research in Engineering Design*, vol. 10, pp. 189-209, 1998.

[3] P. Chen and B. Roth, "Design Equations for the Finitely and Infinitesimally Separated Position Synthesis of Binary Links and Combined Link Chains," *ASME Journal of Engineering for Industry*, vol. 91, pp. 209-219, 1969.

[4] Y. Fang and L. Tsai, "Structure Synthesis of a Class of 4-DOF and 5-DOF Parallel Manipulators with Identical Limb Structures," *The International Journal of Robotics Research*, vol. 21, no. 9, pp. 799-810, 2002.

[5] Z. Huang and Q. C. Li, "Type Synthesis of Symmetrical-lower Mobility Parallel Mechanisms Using the Constraint Synthesis Method," *The International Journal of Robotics Research*, vol. 22, no. 1, pp. 59-79, 2003.

[6] Z. Huang and Q. C. Li, "Type Synthesis of 4-DOF Parallel Manipulators," *Proc. of the IEEE International Conference on Robotics and Automation*, vol. 1, pp. 755-760, 2003.

[7] X. Kong and C. Gosselin, "Type Synthesis of 3T1R 4-DOF Parallel Manipulators Based on Screw Theory," *IEEE Transactions on Robotics and Automation*, vol. 20, no. 2, pp. 181-190, 2004.

[8] Q. Li, Z. Huang and J. M. Herve, "Type Synthesis of 3R2T 5-DOF Parallel Mechanisms Using the Lie Group of Displacements," *IEEE Transactions on Robotics and Automation*, vol. 20, no. 2, pp. 173-180, 2004.

[9] X. Kong and C. M. Gosselin, "Type Synthesis of 3-DOF Spherical Parallel Manipulators Based on Screw Theory," *Journal of Mechanical Design*, vol. 126, no. 1, pp. 101-108, 2004.

[10] E. Lee and C. Mavroidis, "Four Precision Points Geometric Design of Spatial 3R Manipulators," *Proc. of IFTOMMs 11th World Congress in Mechanism and Machine Science*, pp. 18-21, 2003.

[11] L. Saggere and S. Kota, "Synthesis of Planar Compliant Four-Bar Mechanisms for Compliant-Segment Motion Generation," *Journal of Mechanical Design*, vol. 123, pp. 535-541, 2001.

[12] H. Tokunaga, N. Matsuki, S. Imamura, F. Tanaka and T. Kishinami, "Kinematic Design of Mechanisms Using Motion Task Formulation and Constraint Reduction Based on Lie Algebra," *Journal of the Japan Society of Precision Engineering*, vol. 123, pp. 608-613, 2002.

[13] T. Sobh and D. Toundykov, "Kinematic Synthesis of Robotic Manipulators from Task Descriptions," *IEEE Conf. on Robotics and Automation*, pp. 1018-1023, 2003.

[14] T. Huang, Z. Li, M. Li, D. G. Chetwynd and C. M. Gosselin, "Conceptual Design and Dimensional Synthesis of a Novel 2-DOF Translational Parallel Robot for Pick-and-Place Operations," *Journal of Mechanical Design*, vol. 126, no. 3, pp. 449-455, 2004.

[15] E. Lee, C. Mavroidis, amd J. P. Merlet, "Five Precision Point Synthesis of Spatial

RRR manipulators Using Interval Analysis," *Transactions of the ASME*, vol. 26, pp. 842-849, 2004.

[16] I. Fernandez-Bustos, J. Aguirrebeitia, R. Aviles and C. Angulo, "Kinematical Synthesis of 1-dof Mechanisms Using Finite Elements and Generic Algorithms," *Finite Elements in Analysis and Design*, vol. 41, pp. 1441-1463, 2005.

[17] O. F. Jensen and J. M. Hansen, "Dimensional Synthesis of Spatial Mechanisms and the Problem of Non-Assembly," *Multibody System Dynamics*, vol. 15, no. 2, pp. 107-133, 2006.

[18] D. Mundo, J. Y. Liu, H. S. Yan, "Optimal Synthesis of Cam-Linkage Mechanisms for Precise Path Generation," *Journal of Mechanical Design*, vol. 128, pp. 1253-1259, 2006.

[19] B. Roth, "Analytical Design of Two Revolute Open Chains," *Proc. of the Sixth CISM-IFTOMM Symposium on the Theory and Practice of Robots and Manipulators*, pp. 207-214, 1986.

[20] C. Innocenti, "Polynomial Solution of the Spatial Burmester Problem," *Mechanism Synthesis and Analysis*, vol. 70, pp. 161-166, 1994.

[21] J. M. McCarthy, "Mechanism Synthesis Theory and the Design of Robots," *IEEE Int. Conf. on Robotics and Automation*, vol. 1, pp. 55-60, 2000.

[22] J. M. McCarthy, *Geometric Design of Linkages*, Springer-Verlag, New York, 2000.

[23] A. Perez and J. M. McCarthy, "Dimensional Synthesis of Spatial RR Robots," *Advances in Robot Kinematics*, Piran-Portoroz, Slovenia, pp. 26-30, 2000.

[24] A. Perez and J. M. McCarthy, “Dimensional Synthesis of CRR Serial Chains,” *ASME Design Engineering Technical Conferences*, Chicago, IL, pp. 1-8, 2003.

[25] A. Perez and J. M. McCarthy, “Dimensional Synthesis of RPC Serial Robots,” *ICAR: International Conference on Advanced Robotics*, Coimbra, 2003.

[26] A. Perez and J. M. McCarthy, “Dual Quaternion Synthesis of Constrained Robotic Systems,” *Journal of Mechanical Design*, vol. 126, no. 3, pp. 425-435, 2004.

[27] A. Perez and J. M. McCarthy, “Sizing a Serial Chain to Fit a Task Trajectory Using Clifford Algebra Exponentials,” *Proc. of the IEEE Int. Conf. on Robotics and Aut.*, pp. 4709-4715, 2005.

[28] A. Perez and J. M. McCarthy, “Geometric Design of RRP, RPR and PRR Serial Chains,” *Mechanism and Machine Theory*, vol. 40, no. 11, pp. 1294-1311, 2005.

[29] A. Perez and J. M. McCarthy, “Clifford Algebra Exponentials and Planar Linkage Synthesis Equations,” *ASME Journal of Mechanical Design*, vol. 127, no. 5, pp. 931-930, 2005.

[30] E. Wolbrecht, Su, A. Perez, J. M. McCarthy, “Geometric Design Of Symmetric 3-RRS Constrained Parallel Platforms,” *Proc. of the 2004 ASME International Mechanical Engineering Congress and Exposition*, pp. 13-19, 2004.

[31] H. Su, C. Wampler and J. M. McCarthy, “Geometric Design of Cylindric PRS Serial Chains,” *Journal of Mechanical Design*, vol. 126, no. 2, pp. 269-277, 2004.

[32] G. S. Soh, A. Perez Gracia and J. M. McCarthy, “The Kinematic Synthesis of Mechanically Constrained Planar 3R Chains,” *Proc. of the EuCoMeS, 1st European Conference on Mechanism Science*, 2006.

[33] M. C. Villa-Uriol, F. Kuester, N. Bagherzadeh, A. Perez and J. M. McCarthy, "Kinematic Synthesis of Avatar Skeletons from Visual Data," *9th International Symposium on advances in Robot Kinematics*, pp. 171-180, 2004.

[34] C. Mavroidis, Lee E. and M. Alam, "A New Polynomial Solution to the Geometric Design Problem of Spatial R-R Robot manipulators Using the Denavit and Hartenberg Parameters," *Transactions of the ASME*, vol. 123, pp. 58-67, 2001.

[35] E. Lee and C. Mavroidis, "Solving the Geometric Design Problem of Spatial 3R Robot Manipulators Using Polynomial Homotopy Continuation," *Transactions of the ASME*, vol. 124, pp. 652-661, 2002.

[36] E. Lee and C. Mavroidis, "Geometric Design of Spatial PRR Manipulators, Using Polynomial Elimination Techniques," *Proc of the ASME Design Tech. Conf.* , Paper Number DETC2002/MECH-34314, 2002.

[37] E. Lee and C. Mavroidis, "Geometric Design of 3R Robot Manipulators for Reaching Four End-Effector Poses," *International Journal of Robotics Research*, vol. 23, no. 3, pp. 247-254, 2004.

[38] E. Lee and C. Mavroidis, "An Elimination Procedure for Solving the Geometric Design of Spatial 3R Manipulators," *Transactions of the ASME*, vol. 128, pp. 142-145, 2006.

[39] H. Su and J. M. McCarthy, "Kinematic Synthesis of RPS Serial Chains," *Proceedings of ASME Design Eng. Tech. Conf.*, vol. 2B, pp. 1041-1047, 2003.

[40] H. Su and J. M. McCarthy, "The Synthesis of an RPS Serial Chain to Reach a Given Set of Task Positions," *Mechanism and Machine Theory*, vol. 40, no. 7, pp. 757-775, 2005.

[41] H. J. Su and J. M. McCarthy, "Synthesis of Bistable Compliant Four-bar Mechanisms using Polynomial Homotopy," *ASME Journal of Mechanical Design*, 2006.

[42] Q. Liao and J. M. McCarthy, "On the Seven Position synthesis of a 5-SS Platform Linkage," *ASME J. Mech. Des.*, vol. 123, no. 1, pp. 74-79, 2001.

[43] E. Rimon and J. Burdick, "A Configuration Space Analysis of Bodies in Contact - I. 1-st Order Mobility," *Mechanism and Machine Theory*, vol. 30, no. 6, pp. 897-912, 1995.

[44] E. Rimon and J. Burdick, "A Configuration Space Analysis of Bodies in Contact - II. 2-nd Order Mobility," *Mechanism and Machine Theory*, vol. 30, no. 6, pp. 913-928, 1995.

[45] N. P. Robson and A. Tolety, "Geometric Design of Spherical Serial Chains with Curvature Constraints in the Environment," *Submitted to the ASME 2011 International Design Engineering Technical Conferences Computers and Information in Engineering Conference*, Washington, 2011.

[46] N. Patarinsky-Robson and J. M. McCarthy, "The Synthesis of Planar 4R Linkages with Three Task Positions and Two Specigfed Velocities," *ASME Journal of Mech. Design*, 2005.

[47] J. Craig, *Introduction to Robotics, Mechanics and Control*, Addison - Wesley Publishing Company, Inc., New York, 1989.

[48] C. H. Suh and C. W. Radcliffe, *Kinematics and Mechanisms Design*, John Wiley and Sons, New York, 1978.

VITA

Anurag Bharadwaj Tolety

Dept. of Electrical Engineering

Texas A&M University

214 Zachry Engineering Center

TAMU 3128

College Station, Texas 77843-3128

tolety84@neo.tamu.edu

EDUCATION

Texas A&M University, College Station, Texas

Master of Science in Electrical Engineering with specialization in Control Systems

Graduation: August 2011

Indian Institute of Technology, Kharagpur, India

Bachelor of Technology in Electronics and Electrical Communication Engineering

Graduation: July 2006

The typist for this thesis was Anurag Bharadwaj Tolety.