DYNAMIC TRAFFIC ASSIGNMENT INCORPORATING
COMMUTERS’ TRIP CHAINING BEHAVIOR

A Thesis
by
WEN WANG

Submitted to the Office of Graduate Studies of
Texas A&M University
in partial fulfillment of the requirements for the degree of

MASTER OF SCIENCE

August 2011

Major Subject: Civil Engineering
Dynamic Traffic Assignment Incorporating Commuters’ Trip Chaining Behavior

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ABSTRACT

Dynamic Traffic Assignment Incorporating Commuters’ Trip Chaining Behavior.

(August 2011)

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Traffic assignment is the last step in the conventional four-step transportation planning model, following trip generation, trip distribution, and mode choice. It concerns selection of routes between origins and destinations on the traffic network. Traditional traffic assignment methods do not consider trip chaining behavior. Since commuters always make daily trips in the form of trip chains, meaning a traveler’s trips are sequentially made with spatial correlation, it makes sense to develop models to feature this trip chaining behavior. Network performance in congested areas depends not only on the total daily traffic volume but also on the trip distribution over the course of a day. Therefore, this research makes an effort to propose a network traffic assignment framework featuring commuters’ trip chaining behavior. Travelers make decisions on their departure time and route choices under a capacity-constrained network.

The modeling framework sequentially consists of an activity origin-destination (OD) choice model and a dynamic user equilibrium (DUE) traffic assignment model. A heuristic algorithm in an iterative process is proposed. A solution tells commuters’ daily travel patterns and departure distributions. Finally, a numerical test on a simple
transportation network with simulation data is provided. In the numerical test, sensitivity analysis is additionally conducted on modeling parameters.
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1. INTRODUCTION

Transportation systems play a critical role by supporting the development and the interactions of socio-economic systems. They allow for efficient and safe movement of people and goods, thus contributing to improved quality of life and benefits to the economy. At the same time, transportation systems affect the environment via their integration with land-use policies and the travel behavior they encourage. An understanding of these complex relationships is crucial to the solution of some transportation-related problems, such as traffic congestion, fuel consumption, greenhouse gas (GHG) emissions, and global climate change. In such a framework, activity-based approaches to travel behavior analysis explicitly recognize interactions among activities, trips, and individuals in time and space. Such an analysis can facilitate the identification, evaluation, and implementation of more effective and reliable land-use and transportation policies.

Travel demand is a derived product from travelers’ social activities. It is necessary to explore what drives people to travel in order to fully understand and predict their travel demand for the sake of planning. The activity-based approach compared with trip-based approach focuses on a better understanding of travel behavior. A better understanding will help develop a better capability to predict how travelers respond to their travel environment changes and how their responses are temporally and spatially correlated.

This thesis follows the style of Transportation Research Part B.
Trip chaining is a typical travel phenomenon but lacks sufficient investigation. This probably results from the difficulty in defining trip chains, in extracting related information from travel diary surveys, or establishing and analyzing all the possible trip chain patterns (Shiftan, Y., 1998; Primerano et al., 2008; Bernadin et al., 2009). In this research, trip chaining is defined as activity scheduling with a set of connected trips from when an individual leaves the origin to when he or she returns to the final destination within a day, linking secondary activities to primary activities through travel made. A simple trip chain is illustrated in Fig. 1, which depicts the daily travel pattern for commuters who depart from home early in the morning and come back home at the end of the day. There are some special characteristics for a typical trip chain. For instance, the destination in one trip is the origin of the next, and the duration at one destination will affect the departure time of the successive trip. However, these chained trips are simply treated as separate, independent ones in traditional trip-based traffic assignment models (Sheffi, 1985).

**Figure 1** A simple trip chain illustration
Traffic assignment addresses the selection of routes between origins and destinations on the transportation networks. Conventional trip assignment techniques based on static traffic assignment have been widely used for decades. The limitations of the static traffic assignment methods and the improvement of computational capacity have allowed this study area to move toward more behaviorally realistic dynamic traffic assignment (DTA) models. DTA techniques have a number of advantages over the static traffic assignment, such as representing time-dependent interactions of the travel demand and supply of the network and the capability to capture traffic congestion buildup and dissipation.

Network performance under congestion relies not only on the total traffic volume but also on the temporal distribution of trips (Boyce, D., 2007). Therefore, modeling trip departure time is an important topic to understand and predict how congestion arises from individual travel decisions. In particular, how individuals adjust their departure time in response to congestion occurring on the network and how departure time is affected by policies such as improved accessibility, pricing, flexible work hours, and improved traffic information are also worth exploring.
2. LITERATURE REVIEW

2.1 Dynamic Traffic Assignment

Some attempts have been made to address the DTA problems. The related model formulations in prior studies are classified as the mathematical programming method (Merchant and Nemhauser, 1978; Janson, 1991), the optimal control theory method (Friesz et al., 1989; Ran and Boyce, 1994; Lam and Huang, 1995), the variational inequality method (Friesz et al., 1993; Ran and Boyce, 1996; Lam and Yin, 2001), the graphical solution method (Munoz and Laval, 2005; Laval, 2009) and the simulation method (Mahmassani et al., 1992; Mahmassani, 2001; Brown et al., 2009). Those short-period (10-15min) dynamic models representing real-time traffic conditions can be integrated into some advanced traffic management systems and intelligent route guidance systems, but they have not been widely implemented in practice except in some simulation approach software packages due to burdensome computation for large-scale transportation networks. On the other hand, some hourly period time-dependent models (Bell et al., 1996; Lam and Zhang, 1999; Lam and Yin, 2001) have been presented to mainly investigate the daily travel distribution patterns by providing the traffic forecast in each time interval (1-2hr). They make some simplifications on the dynamics of traffic in transportation networks but still have the advantage of efficient computation and effective travel estimation for the purpose of long-term planning.
2.2 Utility of Activity

The utility of activity at a certain time is defined as a function of satisfaction for performing the activity itself and intensity with which the activity is performed. Both the satisfaction and the intensity are time dependent. Supernak (1992) proposed the concept of time-dependent utility considering the utility of an activity determined by its type and duration. Lam and Yin (2001), Huang et al. (2002) and Adnan et al. (2009) applied the time-dependent utility profile to the activity choice problem combined with the dynamic route choice. The utility of a given activity depends on when the traveler starts the activity and how long he or she performs the activity. Here, the time-varying utility profiles by activity type are used for assessing utilities from activity participation.

Assume the marginal utility function derived from a temporal utility profile for activity \( i \) is \( f_i(t) \), which represents the obtained utility from a time unit of performing activity \( i \) at time \( t \). The total activity utility with starting time \( t_1 \) and ending time \( t_2 \) is computed as:

\[
U_i(t_1, t_2) = \int_{t_1}^{t_2} f_i(t) dt
\]

It should be noted that the temporal utility profiles would be different between activity types and traveler groups. Some efforts have been made to measure the utility of activities with real data (Kawakami and Isobe 1986; Kitamura and Supernak 1997). In this research, the temporal utility profile of each potential activity is predetermined for the studied commuters.
2.3 Activity-based Demand Modeling

Since activity-based approaches to modeling travel demand are conceptually more appealing than the traditional trip-based methods, a number of activity-based travel demand forecast models have already been presented in prior studies. Ben-Akiva et al. (1996) proposed the activity schedule model system, and the system was implemented by using data from Boston (Bowman and Ben-Akiva, 2000). Oppenheim (1995) used a discrete choice model for activity locations together with static assignment on routes between the locations to combine activity location and travel choice. Lam and Yin (2001) incorporated the temporal utility profiles of activities into a DTA modeling framework to model travelers’ activity and route choice jointly. They developed a variational inequality-based formulation to assign traffic dynamically and brought consistency between choices and travel times. However, their framework does not consider network congestion and ignores the sequential selection process of trip chaining (i.e. linkages between consecutive activity-travel decisions). Abdelghany and Mahmassani (2003) explored a stochastic dynamic user equilibrium (SDUE) framework in which drivers simultaneously determine departure time, sequence of their activities, and path to the final destination at the origin in order to minimize their perceived travel disutility. However, their model only considered the fixed intermediate stops for individual traveler at the origin without dealing with the linkages between consecutive travel decisions, and they also treated the duration at intermediate stops exogenously. To overcome these deficiencies, Kim et al. (2006) presented a mathematical model for individual traveler’s activity chaining. The activity with the biggest utility among
activity alternatives was sequentially selected in the model, and then the starting time and duration were simultaneously determined based on the perceived time-dependent travel time. But they treated travel times as opportunity costs with a time constraint approach instead of converting to disutility in the model. Lin et al. (2008) developed a conceptual framework for integrating activity-based approaches and DTA techniques. Technical, computational, and practical issues involved in this integration were explored by using CEMDAP for activity-based modeling and VISTA for DTA modeling. However, their studies focused merely on realization of the module integration without proposing any theoretically sound model formulation. Zhang et al. (2005) analyzed the influence of bottleneck congestion on commuters by investigating departure time choice for the home–work tour as a trade-off between travel cost and the time-dependent activity utility. They established an equilibrium condition between the schedule choice pattern and network congestion through a fixed-point problem. However, they treated the travel time on links ideally as constants without considering the dynamic traffic conditions. Heydecker and Polak (2006) developed a model of tour scheduling with equilibrium analysis on congested network with peak-period tolling. Their model indicated how travelers could achieve identical utility by making travel choices within the network equilibrium. But their analysis on equilibrium behavior is difficult to address multi-stop tour situations due to the lack of consideration on the balance between sequential positions.

This research attempts to gain insights into the effect of commuters’ scheduling and dynamic traffic conditions on their daily trip chaining behavior and the network
performance, especially aim at addressing the sequential activity choice problem for trip chaining. The proposed modeling framework is expected to be used as an effective activity-based travel demand analysis tool for long-term transportation planning. The remainder of this paper is divided into the following sections. After reviewing the literature in Section 2, we specify the studied problem in Section 3 and propose the methodology in Section 4. Section 5 illustrates the algorithm. In Section 6, we describe the experimental results. Section 7 provides the conclusions.
3. PROBLEM STATEMENT

This research proposes a modeling framework for dynamic traffic assignment concerning commuters’ trip chaining behavior in order to grasp their activity-based travel feature and estimate daily travel distribution. A capacity-constrained network is designed such that network congestion would be accounted for.

The studied network is denoted by \((N, A)\), where \(N\) is the set of nodes representing various activity zones such as residential zones, work zones, shopping zones, and \(A\) is the set of arcs connecting these zones. A set of commuters always make daily travel decisions on what activity to take next and which route to choose for that activity destination. For example, a commuter may depart from residential zone (home) to work zone in the morning, go to shopping zone after work and then come back to home, or directly return home without visiting any other leisure zones, which forms different daily trip-chain patterns.

In this research, given the number of potential travelers in each origin zone at initial time of study period and their temporal activity utility profiles, we discretely model commuters’ sequential activity choices and simultaneous route choices. The objective has two levels: to reach the user equilibrium condition for dynamic traffic assignment at each time interval and to achieve the stable daily time-dependent travel distributions.

The travel costs under dynamic traffic condition are taken into account based on some flow conservation and flow propagation constraints. Considering the interdependence for activity choice and dynamic traffic assignment as well as the
complexity of network congestion, existence of equilibrium solutions for the proposed modeling framework is explored. In addition, a numerical example is designed to validate the proposed model.
4. METHODOLOGY

We consider the following three basic assumptions in formulating the problem: 1) all the commuters on the studied network are considered to be behaviorally homogenous; 2) the possible interaction with other networks is neglected; and 3) the link travel time would be linearly increased with queues. The following presents the notations for the problem formulation:

Sets of Nodes

- \( N \) = all nodes representing various activity zones
- \( S \) = activity destination choice set, \( S \subseteq N \)

Sets of Arcs

- \( A \) = all arcs
- \( B(r) \) = the set of links with tail node \( r \)
- \( H(r) \) = the set of links with head node \( r \)

Index

- \( i, j, k \) = time slice index
- \( r, s, l, m \) = activity zone index
- \( a, b \) = link index
- \( p \) = path index

Parameters

- \( T \) = a fixed study period
- \( \sigma \) = the duration of each time interval
• $\alpha$ = the utility value of unit travel time

• $\beta$ = the influence factor of successive activity choices on current activity choice

• $Q_r$ = the number of potential travelers within zone $r$ at initial time of study period

• $t_a^0$ = the free-flow travel time to traverse the link $a$

• $s_a$ = the maximum exit flow rate of link $a$

**Variables**

• $U_{rs}(k)$ = the total utility of choosing activity destination $s$ after $r$ at interval $k$

• $V_s(k)$ = the activity utility at position $s$ during interval $k$

• $t_{rs}(k)$ = the estimated travel time from $r$ to $s$ departing at interval $k$

• $t_{rs}^r(k)$ = integer part of $(t_{rs}(k)/\sigma + 0.5)$

• $p_{rs}(k)$ = the probability of visiting location $s$ after $r$ at interval $k$

• $N_r(k)$ = the number of potential travelers from zone $r$ at interval $k$

• $d_{rs}(k)$ = the aggregate departure flow at interval $k$ from $r$ to $s$

• $f_{ps}(k)$ = the flow rate on path $p$ with OD pair $rs$ entering network at interval $k$

• $t_{ps}^r(k)$ = the travel time along path $p$ with OD pair $rs$ entering network at interval $k$

• $t_a(k)$ = the travel time on link $a$ for commuters entering this link at interval $k$

• $t_{rs}^{min}(k,f^*)$ = the minimum path travel time with OD pair $rs$

• $u_{ap}^{rs}(k)$ = the inflow rate on link $a$ at interval $k$ departing from $r$ to $s$ via path $p$

• $v_{ap}^{rs}(k)$ = the departure rate from link $a$ at interval $k$ departing from $r$ to $s$ via path $p$

• $u_a(k)$ = the total inflow rate on link $a$ at interval $k$

• $v_a(k)$ = the total departure rate from link $a$ at interval $k$

• $U_a(k)$ = the cumulative arrivals at link $a$ by interval $k$
• $V_a(k) = $ the cumulative departures from link $a$ by interval $k$
• $q_a(k) = $ the queue experienced by traveler entering link $a$ at interval $k$
• $\delta_{api}^{rs}(k) = $ 0-1 integer variable and it is equal to 1 when the flow departing from $r$ to $s$ during interval $i$ via path $p$ will arrive at link $a$ at interval $k$

### 4.1 OD Demand Formulation for Trip Chaining

To formulate the trip chaining process along with departure time choices, the study period $T$ is discretized into a number of equal time slices. A commuter located in zone $r$ at time interval $k-1$ will choose to perform a certain activity in the next time interval $k$ from activity destination set $S$ (zone $r$ is also included in set $S$, which means the commuter can also choose to keep his stay at zone $r$ for next time interval). Denote $U_{rs}(k)$ as the utility value of choosing destination $s$ after $r$ at time interval $k$.

Considering the systematic and random components of utility formulation, we have:

$$U_{rs}(k) = V_s(k) - \alpha t_{rs}(k) + \beta \sum_{l \in S} p_{sl} U_{sl}(k + t'_{rs}(k) + 1) + \epsilon_s$$  \hspace{1cm} (1)

where $V_s(k)$ is the activity utility at location $s$ during interval $k$, $t_{rs}(k)$ is the estimated travel time from $r$ to $s$ departing at interval $k$, $\alpha$ is the utility value of unit travel time, $t'_{rs}(k)$ is the equivalent number of time intervals for the travel time $t_{rs}(k)$, calculated as $\text{INT}(t_{rs}(k)/\sigma + 0.5)$ with the duration of each time interval $\sigma$, $U_{sl}(k + t'_{rs}(k) + 1)$ is the utility of commuter’s choice with activity destination $l$ at time interval $k + t'_{rs}(k) + 1$ after he arrives at location $s$, $p_{sl}$ is the probability of visiting location $l$ after $s$, $\beta$ measures the influence of potential successive activity
choices on the current activity choice, and $\varepsilon_s$ is the random utility component which reflects the unobservable or immeasurable factors of utility or the errors in factor measurements.

It is noted that $\sum_{t \in S} p_{st} U_{st}(k + t'_{rs}(k) + 1)$ can represent the interdependencies of consecutive activity choices in trip chaining, but it would lead to the recursiveness and burdensome computation for large transportation networks. This item was always ignored for simplicity as some previous activity-based models did (Fellendorf et al., 1997; Lam and Yin, 2001). In this research, since we mainly focus on a small or medium sized network with commuters delimited to a local region, the different levels on the possible connection between activity locations can be enumerated depending on commuters’ potential choices. As illustrated in Fig. 2, suppose commuter is now located at Point 1 as the first level, then Points 2, 3, 4 are the potential location choices after 1 as the second level, and there are also different location choices following Points 2, 3, 4 respectively as the third level. Considering the influence of successive activity choices on the current choice will become weaker at higher level, this significant item will be limited to the third level for trip-chain modeling here. Thus the complicated recursiveness can be avoided.
The random component $\varepsilon_s$ is assumed to be independent and identically Gumbel distributed, then the probability of selecting position $s$ at time interval $k$ can be estimated by the multinomial Logit model:

$$p_{rs}(k) = \frac{e^{V_s(k) - \alpha t_{rs}(k) + \beta \sum_{d \in S} p_{ds} u_s \left( k + t'_{rs}(k) + 1 \right)}}{\sum_{m \in S} e^{V_m(k) - \alpha t_{rm}(k) + \beta \sum_{l \in S} p_{ml} u_m \left( k + t'_{rm}(k) + 1 \right)}}$$ (2)
Within each zone \( r \) at each time interval \( k \), there will be a number of potential travelers \( N_r(k) \), and only the number of potential travelers in each origin zone at initial time of study period is predetermined:

\[
N_r(0) = Q_r, \quad r \in R
\]  

Then, the aggregate departure flow at interval \( k \) from \( r \) to \( s \) can be formulated as:

\[
d_{rs}(k) = p_{rs}(k) \cdot N_r(k - 1)
\]

By calculating the aggregate departure flow at each time interval from each zone with Eq. (4), the computed travel demand distribution is elastic to the estimated utility value which depends on the dynamic travel time and temporal activity utility. Therefore, this proposed model can be used as OD demand analysis tool for trip chaining process.

**4.2 DTA on Capacity-constrained Network**

The activity choice (i.e. consecutive OD choice) behavior has been formulated in the last section, and now we consider modeling the combined activity and route choices in this section. The ideal dynamic user equilibrium (ideal DUE) condition is adopted here, as defined by Ran and Boyce (1996), which means “if for each OD pair at each time period, the actual travel times experienced by travelers departing at the same time are equal and minimal”. That is, the commuters who depart at the same time with the same destination choice will reach their destination simultaneously under the ideal DUE condition. The ideal DUE is one level of objective function aimed at each time interval.
within a study day. The other level is to achieve the stable daily time-dependent travel distributions, which will be elaborated in Section 5.

The ideal DUE formulation is equivalent to finding the optimal path flow vector \( f^* \) such that the following conditions hold:

\[
\begin{align*}
    t_p^{rs}(k, f^*) & = t_{min}^{rs}(k, f^*) \quad \text{if } f_p^{rs}(k) > 0 \quad \forall p, r, s, k \\
    & \geq t_{min}^{rs}(k, f^*) \quad \text{if } f_p^{rs}(k) = 0 \quad \forall p, r, s, k \\
    \sum_p \sigma f_p^{rs}(k) & = d_{rs}(k) \quad \forall r, s, k \\
    f_p^{rs}(k) & \geq 0 \quad \forall p, r, s, k \\
\end{align*}
\]  

where \( t_{min}^{rs}(k, f^*) \) is the minimum path travel time with OD pair \( rs \), \( \sigma \) is the duration of each time interval, and \( f_p^{rs}(k) \) is the path flow rate with OD pair \( rs \) entering the network at interval \( k \). Eq. (5) represents that at equilibrium, for each OD pair, only those paths and departure times that have minimum travel time would be used, while the paths and departure times that are not used would have the travel time higher than or equal to the minimum travel time. Eq. (6) represents the flow conservation and Eq. (7) guarantees the non-negativity conditions.

Besides, other constraints for flow conservation are:

\[
\begin{align*}
    N_r(k) & = N_r(k-1) + \sigma \sum_{a \in H(r)} v_a(k-1) - \sigma \sum_{a \in B(r)} u_a(k-1) \quad \forall r, k \\
    \sum_s d_{rs}(k) & = N_r(k-1) \quad \forall r, k
\end{align*}
\]
where $H(r)$ is the set of links with head node $r$, $B(r)$ is the set of links with tail node $r$, $u_a(k-1)$ is the inflow rate on link $a$ during interval $k-1$, and $v_a(k-1)$ is the departure rate from link $a$ during interval $k-1$.

Then, we formulate the link travel time and path travel time in a general capacity-constrained network. First assume the flow rates during each time interval are constant, and we have:

$$U_a(k) = U_a(k-1) + \sigma u_a(k) \quad \forall a, k$$

$$V_a(k + t_a(k)) = V_a(k - 1 + t_a(k-1)) + \sigma [k + t_a(k) - (k - 1 + t_a(k-1))] \cdot v_a(k + t_a(k)) \quad \forall a, k$$

where $U_a(k)$ is the cumulative arrivals at link $a$ by time interval $k$, $V_a(k)$ is the cumulative departures from link $a$ by time interval $k$. $t_a(k)$ is the travel time on link $a$ for commuters entering this link at interval $k$. Here the departure rate during $[k - 1 + t_a(k-1), k + t_a(k)]$ is supposed to be constant as $v_a(k + t_a(k))$.

Following the FIFO discipline, travelers would leave a link in the same order as that of their arrival at this link. Thus $U_a(k) = V_a(k + t_a(k))$ would always hold for any interval $k$, which leads to:

$$u_a(k) = [1 + t_a(k) - t_a(k-1)] \cdot v_a(k + t_a(k)) \quad \forall a, k$$

For a capacity-constrained network, we consider that there is a bottleneck at the end of each link with maximum flow rate $s_a$. For simplicity, the point queue concept is adopted here without considering the physical length of vehicles. Then, as Huang and Lam (2002) did, we formulate the travel time on link $a$ for commuters entering this link at interval $k$ as:
\[ t_a(k) = t_a^0 + \frac{q_a(k)}{\sigma s_a} \quad \forall a, k \tag{13} \]

where \( t_a^0 \) is the free-flow travel time to traverse the link \( a \), \( q_a(k) \) is the queue experienced by traveler entering link \( a \) at interval \( k \). By combining Eq. (12) and Eq. (13):

\[ u_a(k) = \left[ 1 + \frac{q_a(k) - q_a(k - 1)}{\sigma s_a} \right] \cdot v_a(k + t_a(k)) \quad \forall a, k \tag{14} \]

Considering deterministic queuing theory, the departure rate from link \( a \) is formulated regardless of any possible effect from the downstream traffic flow:

\[ v_a(k + t_a(k)) = \begin{cases} s_a & \text{if } t_a(k) > t_a^0 \text{ or } u_a(k) > s_a \\ u_a(k), & \text{otherwise} \end{cases} \tag{15} \]

By combining Eq. (14) and Eq. (15), the formulation of queue can be got:

\[ q_a(k) = \max\{q_a(k - 1) + \sigma (u_a(k) - s_a), 0\} \tag{16} \]

The link travel time can be calculated by Eq. (13) and Eq. (16) with the preservation of FIFO principle, as proved by Huang and Lam (2002). And it is obvious that in order to compute the link travel times, link inflow rates must be specified for each interval.

The link-based flow propagation constraints are as follows:

\[ u_{bp}^{rs}(k) = v_{ap}^{rs}(k) \cdot \mu_{ba}^{rsp} \quad \forall r, s, p, a, k \tag{17} \]

\[ \mu_{ba}^{rsp} = \begin{cases} 1 & \text{if } a \text{ is the preceding link of } b \text{ on path } p \\ 0 & \text{otherwise} \end{cases} \tag{18} \]

\[ v_{ap}^{rs}(k) = \begin{cases} u_{ap}^{rs}(k) - t_a^0 & \text{if there is no queue at interval } k \\ s_a \frac{u_{ap}^{rs}(j)}{u_a(j)} & \text{otherwise, where } j + t_a(j) = k \end{cases} \tag{19} \]

\[ u_a(k) = \sum_{rs} \sum_{p} u_{ap}^{rs}(k) \quad \forall a, k \tag{20} \]
where \( u_{bp}^{rs}(k) \) and \( \nu_{ap}^{rs}(k) \) are the path-specified link inflow rate and departure rate, respectively. So the link inflow rates can always be computed for each interval based on these path-specified flow propagation constraints.

Then, the path travel time can be formulated as the sum of all the link travel times along this path:

\[
t_p^{rs}(i) = \sum_{a \in p} \sum_{k \in i} t_a(k) \cdot \delta_{apl}^r(k) \quad \forall r, s, p, i
\]  

(21)

\[
\sum_k \delta_{apl}^r(k) = 1 \quad \forall r, s, p, a, i
\]  

(22)

\[
\delta_{apl}^r(k) = \{0, 1\} \quad \forall r, s, p, a, i, k
\]  

(23)

where \( \delta_{apl}^r(k) \) is equal to 1 when the flow departing from \( r \) to \( s \) during interval \( i \) via path \( p \) will arrive at link \( a \) at interval \( k \). It can be got that this path travel time formulation is non-linear and non-convex.

Now the modeling framework for commuter’s daily trip chaining behavior has already been proposed. Since the activity choices are interdependent with real-time traffic conditions and the DTA technique is adopted in a capacity-constrained network, it is crucial that whether there exist equilibrium solutions for this discrete-time trip chaining problem when applying some feasible rule to recursively updating commuters’ departure flows. The discussions on the existence of equilibrium solutions will be conducted later.
4.3 Analysis on Equilibrium Solutions

To analyze the existence of equilibrium solutions for the proposed modeling framework, we first consider the update mechanism for DTA process. Once the link travel times $t_a(k)$ for all links are estimated, the indicator variables $\delta_{apl}(k)$ can be determined accordingly and the associated path travel times can be computed. Then some route swapping rule can be employed to update the time-dependent path inflow rates, thus the link inflow rates, link queues and link travel times would be updated accordingly. Therefore, the path travel time formulation is significant for this iterative process.

It is already proved that the proposed path travel time formulation is continuous with the path inflow rates (Huang and Lam, 2002). And it is known that the monotonicity of path travel time formulation can guarantee that some iteratively update process of DTA would converge to equilibrium solutions. Smith and Ghali (1990) proved that the path travel time is a monotonic function in terms of path inflow rate in a dynamic network with only single link. Smith and Wisten (1995) also proposed that the path travel time function is monotonic if no path contains more than one active bottleneck in network. However, for the network where more than one active bottleneck exists on each path, it is hard to deduce the monotonicity of path travel time based on the monotonicity of link travel times. Thus, we could not guarantee that an algorithm would surely converge to some equilibrium solutions for the proposed modeling framework with a capacity-constrained network, since it may lead to only a locally stable solution instead.
On the other hand, it should be noted that the utility formulation of choosing activity position \( s \) after \( r \) at time interval \( k \) (Eq. (1)) partly depends on the travel time estimation from \( r \) to \( s \), which would determine the time-dependent OD demand distribution for activity choices and then affect the OD travel time update process in return. In fact, the travel time estimation relies on the information provision for commuters. Considering that the feedback (i.e. dynamic OD travel times) from DTA model can update the input information for OD demand distribution, it is assumed that the OD travel times for commuters to make the next activity and route choices would always be estimated based on the prior available travel time information by use of time smoothing. The method of time smoothing is to create an approximating function that attempts to capture important trends in the data while leaving out noise. This real-time travel time estimation technique is employed in the proposed demand model.

By combining this sequential activity OD choice model and DUE traffic assignment model, some converged solutions can be obtained for commuters’ daily trip chaining behavior with reasonable design of algorithm.
5. ALGORITHM

In this section, an iterative algorithm is presented to solve the equilibrium solutions of the proposed modeling framework such that DUE conditions can be reached for DTA process along with converged daily travel distributions for commuters’ activity choices.

This iterative algorithm is developed on the basis of the day-to-day route and departure time swapping process (Smith and Wisten, 1995; Huang and Lam, 2002). Considering the interaction of sequential activity OD distributions with DTA process, the basic idea of this algorithm is specified as follows.

On a single day, to satisfy the DUE conditions for different time intervals, the time-dependent inflows for each OD pair on the non-cheapest paths are moved to the cheapest paths. The path inflow moved is proportional to the product of original path flows and travel time difference with cheapest paths, such that the commuters on path with large flow rate and with travel time far from the minimum travel time are more inclined to change route choices.

The travel time estimation to compute the activity OD distribution for one time interval is based on smoothing travel time data from previous intervals. Then after the activity OD demands are obtained, the DTA process will be conducted again for this interval until user equilibrium condition are met.
Figure 3 Operation mechanism
Meanwhile, the commuters will adjust their travel choices based on increasing daily travel experience, thus this iterative process will have different runs to addresses the updated daily travel distributions until the total time-dependent activity choices and associated travel times converge. Fig. 3 represents this heuristic iterative procedure for the proposed modeling framework.

The subscripts \( m, n \) indicates the travel day index, the iteration number for the DUE condition respectively. \( k \) represents the time interval index and \( K \) represents the total number of time intervals for daily study period. The algorithm is elaborated as following steps:

Step 0: System initialization.
Let \( m=1, k=0, n=0 \), for each activity choice OD pair, determine the initial travel time for all the enumerated paths under free-flow condition and find out the minimum OD travel time. Then go to Step 2.

Step 1: Check the travel day index \( m \) for daily travel initialization.
If \( m=2 \), estimate the initial minimum travel time for each activity choice OD pair based on the experience from the first travel day:
\[
t_{min}^{rs}(k)^m = t_{min}^{rs}(k)^{m-1}
\]
If \( m\geq3 \), estimate the initial minimum travel time for each activity choice OD pair by time smoothing:
\[
t_{min}^{rs}(k)^m = \theta t_{min}^{rs}(k)^{m-1} + (1-\theta) t_{min}^{rs}(k)^{m-2}
\]

Step 2:
For $k \in \{1, 2, \ldots, K\}$, compute the OD choice probability $p_{rs}(k)^m$ and demand distribution $d_{rs}(k)^m$ based on the minimum OD travel time $t_{min}^{rs}(k-1)^m$. Assign the initial inflow onto the shortest path for time interval $k$ by employing All-or-Nothing traffic assignment.

Step 3:

Compute the link inflow rates by Eq. (17-20), the link queues by Eq. (16) and the link travel time by Eq. (13).

Step 4:

Compute the path travel time by Eq. (21-23), and find out the minimum travel cost and the corresponding shortest paths:

$t_{min}^{rs}(k)_n^m = \min \{t_p^{rs}(k)_n^m, p \in P_{rs}\}$

$p_n^p = \{p: t_p^{rs}(k)_n^m = t_{min}^{rs}(k)_n^m, p \in P_{rs}\}$

Step 5:

Update the path inflow rates:

$f_p^{rs}(k)_{n+1}^m = f_p^{rs}(k)_n^m - \lambda_n f_p^{rs}(k)_n^m (t_p^{rs}(k)_n^m - t_{min}^{rs}(k)_n^m), p \in P_{rs}$

$T_n = \sum_{p \in P_{rs}} \lambda_n f_p^{rs}(k)_n^m (t_p^{rs}(k)_n^m - t_{min}^{rs}(k)_n^m)$

$f_p^{rs}(k)_{n+1}^m = f_p^{rs}(k)_n^m + \frac{T_n}{|p_n^p|}, p \in p_n^p$

Step 6: Check convergence of the inner loop for DUE.

If

$$\frac{\sum_{rs} \sum_{p \in P_{rs}} f_p^{rs}(k)_n^m (t_p^{rs}(k)_n^m - t_{min}^{rs}(k)_n^m)}{\sum_{rs} \sum_{p \in P_{rs}} f_p^{rs}(k)_n^m t_p^{rs}(k)_n^m} < \epsilon$$
set \( k = k + 1 \): if \( k < K \), go to Step 2, otherwise, go to Step 7;

Otherwise, set \( n = n + 1 \), go to step 3.

Step 7: Check convergence of the outer loop for daily OD demand distribution.

If

\[
\frac{\sum_{rs} \sum_{k} d_{rs} (k)^m | t_{rs}^{m} (k) - t_{rs}^{m-1} (k) |}{\sum_{rs} \sum_{k} d_{rs} (k)^m t_{rs}^{m} (k)} < \rho
\]

Stop. The current solution is the converged solution;

Otherwise, set \( m = m + 1 \), go to Step 1.
6. NUMERICAL EXAMPLE

6.1 Experimental Results

To validate the proposed modeling framework, we apply the solution algorithm to a simple transportation network as depicted in Fig. 4. It is delimited to a relatively small local region that consists of four activity zones: home zone, work zone, shopping zone and restaurant zone denoted as H, W, S and R respectively. The daily study time horizon is from 6am to 6pm. Initially there are totally 2000 behaviorally homogeneous commuters staying at home zone and they perceive the same temporal utility functions for these four activities as shown in Fig. 5. It is assumed that the vehicle occupancy is one person per vehicle here. For simplicity, single path with single link is set for different OD pairs in this simple test network such as from work zone to restaurant zone, except that two paths are set for the OD pair from home zone to work zone. We will test the DUE condition for these two alternative paths later.

Figure 4 A simple transportation network
Figure 5 Temporal utility profiles for four activities

As for the basic input of this test network, the parameters $\alpha$ and $\beta$ in Eq. (1) are set to be 1.0 and 0.1 respectively, $\theta$, $\lambda$ and $\epsilon$ for the convergence of proposed algorithm are set to be 0.6, $2e^{-3}$ and $1e^{-5}$ respectively. The free-flow travel time and maximum exit flow rate for each link are listed in Table 1. The heuristic algorithm presented in last section is coded in Microsoft C++ and run on a desktop computer with Core 2 CPU @3.00 GHz and 8GB RAM. The results are presented in Figs. 6 and 7 for commuters’ distribution at different locations and their departure rate by time of day respectively.
Table 1 The input values for travel time functions

<table>
<thead>
<tr>
<th>Link</th>
<th>Free-flow travel time (hr)</th>
<th>Maximum flow rate (vph)</th>
</tr>
</thead>
<tbody>
<tr>
<td>H to W</td>
<td></td>
<td></td>
</tr>
<tr>
<td>01</td>
<td>0.3</td>
<td>1000</td>
</tr>
<tr>
<td>02</td>
<td>0.4</td>
<td>900</td>
</tr>
<tr>
<td>H to S</td>
<td>0.5</td>
<td>800</td>
</tr>
<tr>
<td>W to H</td>
<td>0.3</td>
<td>800</td>
</tr>
<tr>
<td>W to S</td>
<td>0.3</td>
<td>800</td>
</tr>
<tr>
<td>W to R</td>
<td>0.2</td>
<td>1000</td>
</tr>
<tr>
<td>S to H</td>
<td>0.5</td>
<td>800</td>
</tr>
<tr>
<td>S to W</td>
<td>0.3</td>
<td>800</td>
</tr>
<tr>
<td>S to R</td>
<td>0.3</td>
<td>800</td>
</tr>
<tr>
<td>R to W</td>
<td>0.2</td>
<td>1000</td>
</tr>
<tr>
<td>R to S</td>
<td>0.3</td>
<td>800</td>
</tr>
</tbody>
</table>

Figure 6 Commuters’ distribution by time of day
From these two figures, we can get a clear picture of commuters’ daily travel patterns on the network both temporally and spatially. It is shown that due to the high work utility, a number of commuters depart from home to work during 6-7am period and the departure rate increases to the peak during 7-8am period. Most commuters tend to have lunch at noon and their departure rate from work to eating zone reaches the peak during 11am-12pm period. After lunch, they will go back to work such that the departure rate from eating zone to work increases to its peak during 1-2pm period. Similarly, the
departure rates from work to home and from work to shop reach the peak around 5pm and 6pm respectively, which indicates that a large number of commuters will go back to home or go to shopping when their daily working hours are over.

Table 2 displays the number of delayed commuters (i.e. hourly queue) at certain time periods. It is shown that there exist traffic congestions on Link 01 of Home-Work trip and also on Work-Restaurant and Restaurant-Work trips at their peak hours. Some transport policies on these congested areas such as expanding the road capacity can be implemented and evaluated on this network accordingly for the purpose of long-term strategic planning.

<table>
<thead>
<tr>
<th>Link</th>
<th>Num. of delayed commuters</th>
<th>Time period</th>
</tr>
</thead>
<tbody>
<tr>
<td>H to W 01</td>
<td>100</td>
<td>7am-8am</td>
</tr>
<tr>
<td>W to R</td>
<td>20</td>
<td>11am-12pm</td>
</tr>
<tr>
<td>R to W</td>
<td>90</td>
<td>1pm-2pm</td>
</tr>
</tbody>
</table>

In addition, to test if the dynamic user equilibrium has been achieved on this network, the dynamic travel times on two alternative paths for the Home-Work trip are recorded and displayed in Table 3. It is indicated that under the dynamic user equilibrium condition, the commuters always choose the path that has the minimum travel time for each time interval. In particular, it is noted that during 7-8am period, two alternative paths have the same travel time due to the congestion on the Link 01 such
that a small number of commuters choose to travel on Link 02 under the equilibrium condition.

Table 3 DUE conditions for OD pair: Home-Work

<table>
<thead>
<tr>
<th>Time period</th>
<th>Inflow rate (vph)</th>
<th>Real travel time (hr)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Link 01</td>
<td>Link 02</td>
</tr>
<tr>
<td>6am-7am</td>
<td>486</td>
<td>0</td>
</tr>
<tr>
<td>7am-8am</td>
<td>1100</td>
<td>59</td>
</tr>
<tr>
<td>8am-9am</td>
<td>365</td>
<td>0</td>
</tr>
<tr>
<td>9am-10am</td>
<td>85</td>
<td>0</td>
</tr>
<tr>
<td>10am-11am</td>
<td>43</td>
<td>0</td>
</tr>
<tr>
<td>11am-12pm</td>
<td>36</td>
<td>0</td>
</tr>
<tr>
<td>12pm-1pm</td>
<td>30</td>
<td>0</td>
</tr>
<tr>
<td>1pm-2pm</td>
<td>23</td>
<td>0</td>
</tr>
<tr>
<td>2pm-3pm</td>
<td>26</td>
<td>0</td>
</tr>
<tr>
<td>3pm-4pm</td>
<td>42</td>
<td>0</td>
</tr>
<tr>
<td>4pm-5pm</td>
<td>30</td>
<td>0</td>
</tr>
<tr>
<td>5pm-6pm</td>
<td>18</td>
<td>0</td>
</tr>
</tbody>
</table>

6.2 Sensitivity Test on Parameter $\alpha$

$\alpha$ represents the utility value of unit travel time, and commuters may have different travel patterns if $\alpha$ value varies. Here the sensitivity test on $\alpha$ is conducted and the test results are displayed in Figs. 8-11 and Table 4.
Figure 8 Num. of commuters at work by time of day (sensitivity test on alpha)

Figure 9 Home-Work departure flows by time of day
Figure 10 Work-Restaurant departure flows by time of day

Figure 11 Restaurant-Work departure flows by time of day
6.3 Sensitivity Test on Parameter $\beta$

$\beta$ represents the influence factor of successive activity choices on current activity choice, which features the linkages between consecutive activity-travel decisions. Here the sensitivity test on $\beta$ is conducted and the test results are displayed in Figs. 12 and 13. From these two figures, we can see that the commuters are not that forced to work
with larger beta value and a number of them tend to perform other activities instead such as shopping, which indicates the activity schedule is more flexible for commuters. With the increase of beta value, the commuters’ activity distribution becomes more spread out due to the interaction of different activity choice levels.

**Figure 12** Num. of commuters at work by time of day (sensitivity test on beta)
Figure 13 Commuters’ departure flows by time of day (sensitivity test on beta)
7. CONCLUSIONS

In this research, a dynamic traffic assignment modeling framework concerning commuters’ trip chaining behavior is proposed on a capacity-constrained transportation network. Commuters’ sequential activity choices and simultaneous route choices are discretely modeled with a comprehensive objective: to reach the user equilibrium condition for DTA at each time interval and to achieve the stable daily time-dependent demand distributions.

A heuristic solution method is proposed and applied to a simple transportation network. The numerical results illustrate the commuters’ daily travel patterns and network performance temporally and spatially. Through the numerical tests, the proposed formulations and algorithm are validated. Sensitivity analysis is also conducted on parameters $\alpha$, $\beta$. Commuters prefer to stay where they were to continue performing their present activities instead of switching to other activities when a higher disutility is associated with a unit travel time. The commuters’ spatial activity distribution becomes more spread out and their activity schedules are more flexible when their consecutive activity choices have more intense interaction.

We have provided a behaviorally realistic DTA model to feature trip chains. When this new model is employed as a new travel demand analysis tool for long-term transportation planning and transport policy evaluation, impact on the outcome from the traditional DTA models can be real. In addition, our proposed modeling framework can
assess policies such as employing time-varying tolls and staggered work hours in order to reduce network congestion.

Future work will include the development of more precise link travel time formulation and the application to the large-scale transportation network. It will be significant if travelers’ trip chaining travel data is available for model calibration.
REFERENCES


VITA

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