

**LEAKAGE PREDICTION OF LABRYNTH SEALS HAVING
ADVANCED CAVITY SHAPES**

A Thesis

by

SUNIL MURLIDHER PANICKER

Submitted to the Office of Graduate Studies of
Texas A&M University
in partial fulfillment of the requirements for the degree of

MASTER OF SCIENCE

December 2010

Major Subject: Mechanical Engineering

Leakage Prediction of Labyrinth Seals Having

Advanced Cavity Shapes

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Approved by:

Chair of Committee,	Gerald L. Morrison
Committee Members,	Robert E. Randall
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ABSTRACT

Leakage Prediction of Labyrinth Seals Having Advanced Cavity Shapes.

(December 2010)

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Chair of Advisory Committee: Dr. Gerald L. Morrison

Labyrinth seals are widely used in various turbo machines including turbines, compressors and pumps. Their purpose is to prevent the backflow of the working fluid. This backflow is due to the leakage of the seal. This loss affects the efficiency of the turbo machine, so it becomes critically important to assess the leakage of the seals under the given operating conditions. The accuracy of prediction of leakage is also important for performing rotodynamic analysis. The geometric shape of the seal plays an important role in influencing the fluid flowing through the seals and the leakage rate. Many empirical seal leakage prediction models, useful from a design/analysis point of view, have been developed.

Saikishan Suryanarayanan and Gerald. L .Morrison studied the influence of various geometric and flow parameters on the leakage of labyrinth seals with rectangular cavities. They proposed a leakage equation based on their Computational Fluid Dynamics (CFD) simulations using software FLUENT.

However, many real world labyrinth seals do not have simple rectangular cavities. In particular, this thesis focuses on seals with Isosceles triangle shaped teeth, right triangle shaped teeth, and a NASA seal. In the present work, CFD simulations of labyrinth seals with advanced cavity shapes are performed and the results are compared with the predictions of the rectangular seal model. The results show that the advanced cavities like, Isosceles shaped seal were more efficient as compared to rectangular seals. The pressure drop, which was taken as one of the key parameters to adjudge the efficiency of seals showed negative behavior in some of the advanced cavity shaped seal. The advanced cavity shaped seals are used in various turbo machinery equipments like steam and gas turbines. This study shows that Isosceles cavity shaped seals are the most efficient among all the advanced cavity shapes used in the present study.

DEDICATION

Dedicated to Lord Hanuman who inspired me to follow the path of truth, who has given me the strength and determination to pursue knowledge in right the direction.

ACKNOWLEDGEMENTS

I would like to thank my advisor, Dr. Gerald L. Morrison, for his consistent support and guidance throughout the course of this research. He has been a very good academic advisor, has wished the best for me and has inspired me to do well.

I am also thankful to my committee members, Dr. Robert E. Randall and Dr. Devesh Ranjan, for providing valuable input which has enabled me to pursue research in my chosen field.

I would also like to thank the Turbomachinery research consortium for providing me the facilities without which my research would not have been possible. Also, thanks to Texas A&M University who gave me the opportunity to study in this prestigious institution. It has been a great learning experience.

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NOMENCLATURE

A - Clearance area, $\pi D c$

c - Radial clearance, m

C_d - Discharge coefficient for a given tooth of a multi tooth labyrinth seal

D - Shaft diameter, m

\dot{m} - Mass flow rate of leakage flow (kg/s)

P_i - Seal inlet pressure, Pa

P_e - Seal exit pressure, Pa

P_i - Tooth inlet pressure, Pa

P_e - Tooth exit pressure, Pa

Pr - Pressure ratio, $\frac{P_e}{P_i}$

Re - Reynolds number based on clearance, $\frac{\dot{m}}{\pi D \mu}$

s - Tooth pitch, m

w - Tooth width, m

x - Axial distance along seal, m

β - Divergence angle of jet, radians

γ - Kinetic energy carry over coefficient

ε - Dissipation of turbulent kinetic energy

κ - Turbulent kinetic energy

μ - Dynamic viscosity, Pa/s

ρ_i - Fluid density at seal inlet, kg/m³

ρ_{in} - Fluid density at tooth inlet, kg/m³

χ - Percentage of kinetic energy carried over

ψ - Expansion factor

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CHAPTER I

INTRODUCTION



Figure 1 A Labyrinth Seal

A seal is an important component in all kinds of turbo machinery such as turbines, compressors, pumps etc. A seal is the device which reduces the leakage of the fluid from the high pressure regions of assembly and casing of the component to the low pressure regions and avoids backflow. It influences the stability of the stator –rotor core and is a highly reliable device. The Figure 1 illustrated above shows an assembly of labyrinth seals located inside steam turbine. The labyrinth seal is one of the most common type of seal used today, the design of the cavity can be from simple rectangular to a highly complex design, with its efficiency depending on the seal design. Labyrinth seals are also very feasible designs in sense they easily fit into the assembly.

This thesis follows the style of Journal of Turbo Machinery.

Labyrinth seal reduces the internal fluid leakage by enhancement of friction inside the seal and dissipation of kinetic energy. The mode of achieving the friction is different in different seals. In labyrinth seals the friction to the leakage flow is enhanced by the following mechanism. The fluid passes through a series of restrictions and cavities, where a portion of the kinetic energy is dissipated by the effects of turbulence. The flow of the fluid can be analyzed through Computational Fluid Dynamics with appropriate turbulence models.

The cavity shape geometry is one of the most important parameters affecting the performance of the seals as it is generally found that changing the shape of the cavity can result in a change in flow pattern and thereby seal leakage to a large extent. So it becomes extremely important to analyze the effects of cavity shape on seal performance. It has been found that the rectangular cavity shapes have high kinetic energy dissipation.

A seal can be described by two parts, one which restricts the flow and the other part where the kinetic energy created by the restriction is dissipated. The effectiveness of the turbulent dissipation of kinetic energy in the cavity depends on the spread rate of the jet issuing out from under the tooth (Hodkinson [5]).

The carry over coefficient is related to the percentage of kinetic energy carried over to the next cavity, χ , by

$$\gamma^2 = 1 - \chi \text{-----(1.1)}$$

The carry over coefficient has a direct impact on the performance of the seal. It may be different for different cavity shapes and thus seal performance may vary with cavity shape. The use of modern techniques, like computational fluid dynamics tools, has

become very important in predicting the nature of different flows and thermodynamic process. However one should always keep in mind the accuracy and the stability of the CFD model which is used to predict the nature of the problem. This study will use a CFD tool to simulate the problem and analyze the nature of the turbulent flow through the seal.

The CFD tool used is to calculate the coefficient of discharge C_d (for the first tooth) .The carry over coefficient for a single cavity seal is used to predict the kinetic energy approaching the second tooth. The kinetic energy tends to increase as the value of carry over coefficient increases for a rectangular cavity seal, but for other geometries it may vary.

This study aims to obtain an accurate empirical prediction model of the seal leakage in the labyrinth seals with non rectangular shaped cavities.

CHAPTER II

PRIOR RESEARCH WORK

The research on seals began as early as 1908 by Martin [4] when he presented a leakage model for labyrinth seals. He derived a simple leakage rate equation based on the work done to achieve the flow rate. This analytical approached equation was used for the incompressible flow and was not verified experimentally. Also, in his model, he neglected the effect of kinetic energy carry over and assumed $\gamma = 1$. This model is not very accurate but it is taken as the basic model for many advanced models.

$$\dot{m} = A P_i / \sqrt{R T_i} \sqrt{(1 - (P_e/P_i)^2) / (n - \ln (P_e/P_i))} \text{-----} (2.1)$$

Compressible flow was analyzed by Stodola [8] and he gave the equations for subsonic and choked flows. He also mentioned the fact that mass flow rate across the seal is inversely proportional to the number of teeth. His model was similar to Martin's [4] in a sense that he did not consider the effect of kinetic energy carry over. Gercke [3] considered the variable areas used in a labyrinth seal but he also did not consider the carry over coefficient.

Egli [6] performed extensive research in labyrinth seals. His model (shown in Eqn. 2.2) was more accurate in a sense that, it considered the carry over coefficient and the kinetic energy dissipation. He assumed that "the steam flowing through the labyrinth seal experienced a pressure drop across each throttling. After each throttling, a fraction of kinetic energy was converted as pressure energy, while some amount was transformed into heat and got destroyed. The left over kinetic energy was available for subsequent

throttling". He was able to modify the basic Martins equation by introducing an empirical flow constant which accounted for the carry over coefficient across the seal.

$$\dot{m} = \gamma_{\text{empirical}} A P_i / \sqrt{R T_i} \sqrt{(1 - (P_e/P_i)^2) / (n - \ln(P_e/P_i))} \text{-----(2.2)}$$

Hodkinson [5] developed a model, which is considered as a step further from Egli's model. In his model, he considered a gas jet geometry through which the working fluid flowing expanded from tip of the tooth (upstream), he described the expansion to be conical. This expansion is divided into two parts - the first part of the jet impinges on the downstream tooth and is forced to re-circulate within the cavity, which leads to the kinetic energy dissipation, while the other part of the jet flows under the cavity to the next cavity, carrying the kinetic energy, to the next cavity along with it.

$$\dot{m} = A \sqrt{(P_i \rho_i)} \psi \gamma \alpha \text{-----(2.3a)}$$

where, ψ is the expansion coefficient accounting for compressible effects, expressed as:-

$$\psi = \sqrt{(1 - (P_e/P_i)^2) / (n - \ln(P_e/P_i))} \text{-----(2.3b)}$$

One more important thing he considered was the relation between the seal geometry and the carry over coefficient, as he thought that seal geometry variation can vary the carry over coefficient. The carry over coefficient which was taken as the function of seal geometry was expressed by

$$\gamma = \sqrt{1 - (n-1/n)(c/s/(c/s+0.02))} \text{-----(2.4)}$$

Experiments on seal leakage were performed by Ahmed Gamal [2]. He considered various seals and provided a detailed discussion on the behavior pertinent to labyrinth seals through a set of experiments. Gamal's experimental results were

compared with various models of seals. A comparative study of these models was presented. He was able to find that certain models produced accurate results at high Reynolds number while others were very good at low Reynolds number. He tried some hybrid models and checked out how they perform under different conditions.

Morrison and Al Ghasem [7] studied effect of leakage in a special type of seals known as wind back seals. They concluded that the Hodkinson model [5] of the carry over coefficient was not effective in finding seal leakage of the wind back seals. They obtained results by analyzing the flow pattern inside the seal cavity. The result was that γ differed with changes in the pressure ratio.

This observation was taken as the basis of studies by Saikishan Suryanarayanan [1], who analyzed the dependence of γ on flow parameters for labyrinth seals. Suryanarayanan performed a number of CFD simulations using FLUENT for both incompressible and compressible flow through rectangular cavity labyrinth seals. He studied the effect of different geometry and flow parameters on carry over coefficient, and discharge coefficient. A leakage model for labyrinth seals that accounts for the Reynolds number dependence of the carry over coefficient was developed. The details of the model are as follows:

$$C_d = C_d^{1\text{tooth}} \quad \text{for first constriction}$$

$$C_d = C_d^{1\text{tooth}} (0.925\gamma^{0.861}) \quad \text{for subsequent teeth} \text{-----} (2.5a)$$

$$C_d^{1\text{tooth}} = (0.7757 - 0.002051(w/c)) / [1 + \{44.86(w/c)/R \text{-----} (2.5b)$$

$$\gamma = [1 - 6.5(c/s) - 8.638(c/s)(w/s)] [R_e + R_o]^{(2.454(c/s) + 2.268(c/s)(w/s)^{1.673})} \text{-----} (2.5c)$$

$$R_o = (1 - 6.5(c/s) - 8.638(c/s)(w/s))^{(-1/2.454(c/s) + 2.268(c/s)(w/s)^{1.673})} \text{-----} (2.5d)$$

$$Re = m / \pi \mu D \text{-----} (2.6)$$

where Re is the Reynolds Number

$$\psi_v = 0.558 P_{n+1} / P_n + 0.442 \text{-----} (2.7)$$

It has to be noted that most of the earlier work has been done for labyrinth seals with rectangular cavities. Labyrinth seal cavities come with a variety of shapes and the flow field changes based on the cavity shape. So it becomes very important to study seals with non-rectangular cavity shapes and to understand the effects on the carry over coefficient and discharge coefficient of the seal. By knowing this, the exact shape of the seal which will minimize the seal leakage can be predicted. Steam and gas turbine companies use a variety of cavity shapes for labyrinth seals including Isosceles triangle shaped and right triangle shaped designs. Analysis of seals with such advanced cavity shapes would enable development of a generalized pattern according to the services and usage.

CHAPTER III

OBJECTIVES

The purpose of this study is to evaluate the effect of different cavity shapes upon labyrinth seal performance. This will enable researchers and industrial people to accurately predict how the different cavity shapes and clearances affect the seal leakage. The different seals are simulated using computational fluid dynamics (CFD) tools. The specific objectives of this study are as follows:

1. To perform simulations of leakage flow through labyrinth seals with right triangular, Isosceles triangular and NASA cavity shapes with different clearances for a range of Reynolds numbers using FLUENT. This requires grid generation using GAMBIT.
2. Analyze the flow field and study the effect of cavity shape and clearance on carry over coefficient.
3. Determine how varying the shape of the labyrinth seal cavity affects the discharge coefficient.
4. Determine the effects of cavity shape and clearance on expansion factor.
5. Compare the above results with earlier models for seals with rectangular cavities.

CHAPTER IV

COMPUTATIONAL METHOD

Computational fluid dynamic software results have been used to calculate the various parameters such as coefficient of discharge, carry over coefficient, and compressibility effects.

The seals have been modeled in Gambit and then analyzed using Fluent. Based upon previous studies Al Ghasem and Morrison [7] convergence was assumed when the residual were less than 10^{-4} . This assures the pressure distribution is converged.

The CFD software employed uses the finite volume technique for solving the problem. This method is unconditionally stable, the over relaxation factors have been set in between ranges of 0 to 1. The finite volume method divides a continuous domain into various discretized parts. These parts are known as grid and are analyzed with the required parameter determined. The basic fact behind the finite volume scheme is that a generalized solution, which is a cubic polynomial, is used as a default solution and is taken up initially then based on this solution and problem specific conditions more accurate solutions are obtained.

This study uses a k- ϵ turbulence model. This model is based upon turbulent kinetic energy dissipation and diffusion techniques. Further details of this method can be found in the Fluent manual. This method uses the Navier's Stokes equations to solve the problem. This method gives an accurate simulation of the working condition of the seal by providing basic details of parameters such as pressure, temperature, diffusion kinematic viscosity etc. Three different seal designs were considered the first two

consisted of triangular shapes as shown in Figures 2 and 3. Figure 2 of Isosceles cavity shaped seal is given below :

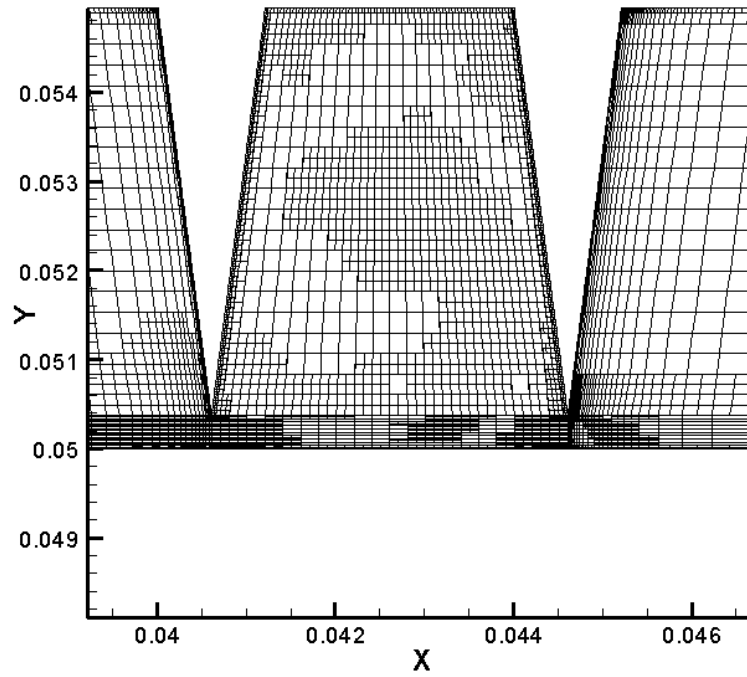


Figure 2 Grid Isosceles Triangle Shaped

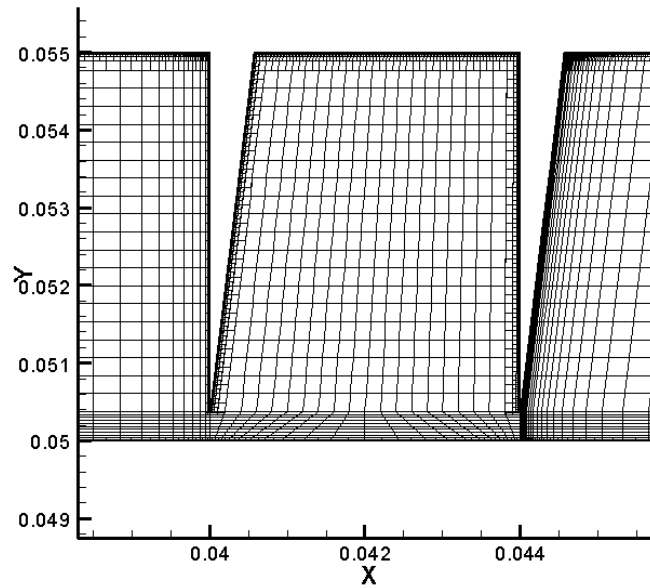


Figure 3 Grid Right Triangle Seal

Figure 3 shows the grid distribution of right triangle shaped seal cavity. Besides these seals another type of seals used by NASA is also studied. These seals are typically used in steam turbines. The third seal is a design that has been used in the space shuttle main engine turbo pumps This seal possesses a more complex geometry as shown in Figure 4.

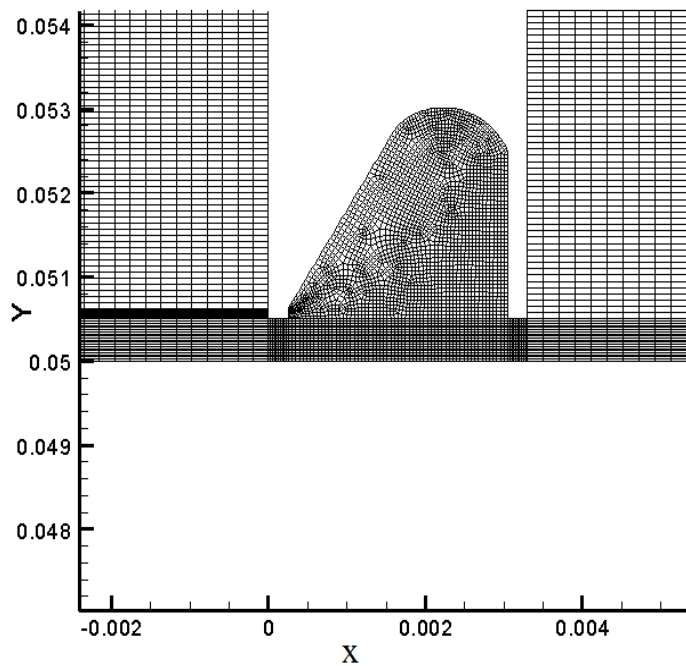


Figure 4 NASA Seal

The purpose of this study is to illustrate how varying labyrinth seal tooth and cavity shape change the leakage rate as represented by the carry over coefficient and individual tooth discharge coefficient. The results will be compared to the rectangular teeth and cavity labyrinth seals that were investigated by Suryanarayanan [1].

Al Ghasem and Morrison [7] performed a study on the effect of different wall functions and how they affect the accuracy of seal predictions. His study showed that Fluent enhanced wall functions performed the best as long as the node closest to the wall had a value of Y^+ less than 5.

GRID

To solve the flow simulations using computational fluid dynamics tools, it was very important to consider a grid which gave accurate results for the given specified problems. The problem was categorized as a two dimensional, axis symmetric case. The grid was made of Isosceles triangle shaped cavity, right angled shaped cavity, and NASA seal shaped cavity. The inlet boundary condition was taken as mass flow inlet and the outlet boundary condition was pressure outlet set as atmospheric pressure of 1atm. The wall Y^+ was considered as fewer than 5 for turbulent flow considerations. The grid was adapted as the Reynolds number was increased. It was noted that maximum number of nodes increased in the near wall region where viscous effects were dominant.

BOUNDARY CONDITIONS

The boundary conditions have to be assigned very carefully. In order to get an accurate prediction of the results it is suggested that, the inlet boundary condition should be taken as mass flow inlet which is denoted as \dot{m} . As the Reynolds number increases, the mass flow inlet increases. The outlet boundary condition is specified as pressure outlet which is kept at atmospheric pressure of 1 atm. The shaft and the top are fixed as stationary walls and the data changes have been recorded. Suryanarayanan [1] showed that for the tooth on the stator seals the leakage rate does not vary more than 1% with the shaft speed. Therefore the shaft is considered stationary in the study.

GRID INDEPENDENCE

A grid independent study has been conducted in all cases so that a close to real solution can be attained. In this technique the grid is refined using the adaptation capability in Fluent to reach a stable solution, i.e. Y^+ under 5 and then the main grid is further refined to obtain grid independence. This means the solution which has been obtained should not undergo any change when the grid is further refined.

Morrison and Al Ghasem [7] had shown in their studies that a precise calculation was obtained using the k- ϵ model with enhanced wall function for the flow pattern inside the seal.

CHAPTER V

COEFFICIENT OF DISCHARGE AND CARRY OVER

COEFFICIENT FOR DIFFERENT CAVITY SHAPES

DISCHARGE COEFFICIENT

Discharge coefficient is the measure of total losses incurred as the fluid flows through the cavity and under the tooth. In the ancient Roman Empire, Frontin who was the water supply in charge under the King Augustus used short pipes of certain fixed sizes to measure the amount of water supplied to the users. As it turned out, this kind of measurement was purely empirical as parameters to define amount of water supplied like head, flow, orifice sizes were not defined until 1643. It was Torricelli who showed that velocity of flux was $V_i = \sqrt{2gh}$. The relation between the discharge coefficient and velocity of flux were developed in later years.

The discharge coefficient is an important parameter in studying labyrinth seals, as it describes the tooth efficiency, i.e. the total losses due to the geometry of particular tooth. The discharge coefficient is also related to the geometry of cavity shapes as it turns out that different cavity shapes have different effects on the discharge coefficient.

The discharge coefficient mathematically is expressed as

$$C_d = \frac{m}{A \sqrt{2\rho(P_1 - P_e)}} \text{-----(5.1)}$$

where ρ is the density of the fluid upstream and of the tooth and, P_1 and P_e is the difference of pressures across the tooth.

The CFD results provide the information about the pressure distribution across the tooth. Typically, the discharge coefficient should be in the range of 0.5 to 1.

The goal of this research is to accurately predict the relation between the discharge coefficient and geometry of the seal and to examine the dependence of the discharge coefficient upon Reynolds number.

CARRY OVER COEFFICIENT

The carry over coefficient is one of the most important parameters to be found in this work. It develops a relationship for the amount of the kinetic energy carried over from the upstream tooth to the downstream tooth. Figure 5 below shows the stream function distribution inside the seal cavity.

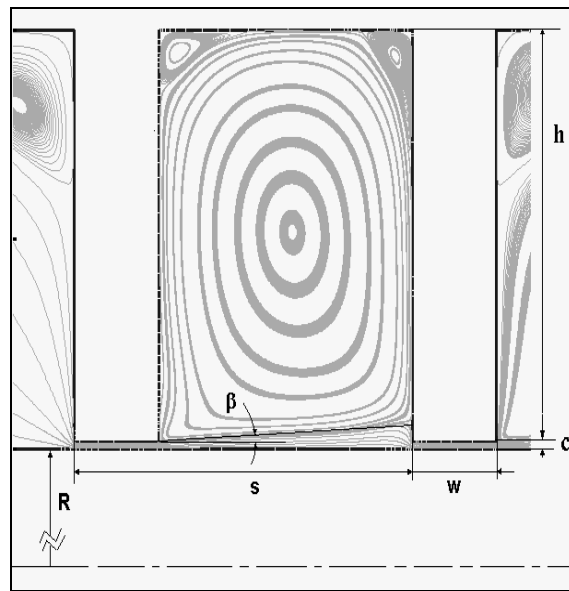


Figure 5 Carry Over Coefficient, Tan Beta Value

Hodkinson [5] presented a theory relating the spread rate of the jet issuing from the upstream tooth to the carry over coefficient. The most important aspect in calculating the carry over coefficient is the angle β which was calculated from the streamline which divides where the part of the fluid which passes under the downstream tooth and that of which is circulated back in the seal cavity. The relationship between β and the carry over coefficient was given by Hodkinson whose pivotal role in research of the seals has been described earlier.

$$\tan \beta = c(1-\chi)/\chi s \text{-----} (5.2)$$

where β was defined as the divergence angle.

β is measured as the angle which was made between the line connecting the tip of the tooth which is upstream to the point to where the jet velocity is impinged onto the downstream tooth is zero, and the line which can be drawn exactly parallel to the rotor surface.

This angle β was found using the Tecplot software which allowed measurement of the flow inside the cavity.

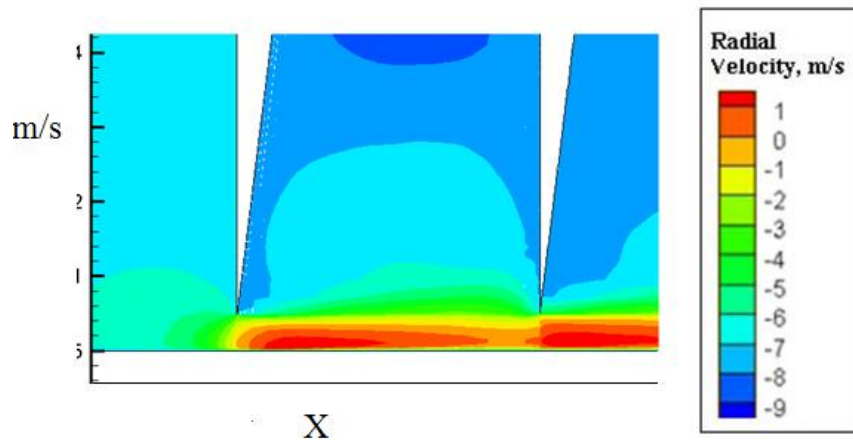


Figure 6 An Example of CFD Flow Simulation Case

The seal geometry is taken for a single tooth Isosceles cavity which is used extensively for various applications in steam turbines and gas turbines. The carry over coefficient and discharge coefficient calculated allows studying the behavior of these cavity shapes for compressible and incompressible fluids. The compressible fluid taken is air and the incompressible fluid which has been taken is water. The coefficient of discharge and the carry over coefficient for Isosceles cavity shape, right triangular cavity shape and NASA seal has been calculated. These results have been compared with rectangular cavity shapes results which were performed by Saikishan Suryanarayanan [1]. Figure 6 shows a typical flow simulation case. It shows the pattern of fluid flowing inside the cavity of the seal.

SINGLE TOOTH COMPRESSIBLE AIR

Saikishan Suryanarayanan [1] showed that for a rectangular shaped seal, the discharge coefficient for a single tooth provided the basis for developing empirical models for subsequent teeth in a seal.

The CFD simulation used to calculate the C_d for a single tooth is a 2-D axis symmetric seal with different geometry shapes. The shapes taken are the Isosceles triangle shaped geometry, the right angled shaped geometry and the complex cavity shape known as NASA seal which is straight at one side of the tooth and has an angular cavity on the other side of the tooth. Various tooth/rotor clearances were considered.

ISOSCELES SHAPED TRIANGLE SEAL

This shape of tooth is generally used by companies like GE , Elliot and Dresser Rand in steam turbines as a labyrinth seal component. The basic purpose of a labyrinth seal is to avoid the backflow of the fluid which can decrease machine efficiency and can cause vibrations. This study presents information on the coefficient of discharge as a function of discharge and the Reynolds number. To know the effect of the clearance, 4 different clearances are considered, which are 0.05 mm, 0.375mm, 0.5mm and 1mm. The Reynolds numbers have been chosen from the range of 1,000 to 100,000 which provides the information of the behavior of the seal under different types of flow namely from laminar to turbulent. The pressure distribution has also been studied for different Reynolds numbers and with different clearances.

Most of the earlier studies show that the discharge coefficient is the product of carry over coefficient and flow coefficient. For a single tooth the flow coefficient term is one. The energy loss under a single tooth can be measured experimentally or obtained from a given set of empirical formulas.

However, the present study uses a different approach altogether to predict the modeling of the discharge coefficient. It models the discharge coefficient in terms of net kinetic energy loss as a function of mass inflow rate and density, where mass flow rate is in turn related to the Reynolds number.

For a better understanding of the kinetic energy; the carry over coefficient and the coefficient of discharge of single tooth have been studied.

This work describes the effect on various parameters with change in the nature of the flow for a single tooth. The coefficient of discharge, Reynolds number, clearance, compressibility and the pressure drop are taken into consideration.

EFFECT OF REYNOLDS NUMBER

The effect of Reynolds number upon the single tooth is described in this part. It can be implied that with a change in the Reynolds number, the pressure drop varies and the coefficient of discharge also varies. The effect of Reynolds number for different clearances is also studied. For the larger clearance of 1mm, it can be predicted that, with increase in the Reynolds number the pressure drop seems to increase. The increase in pressure drop invariably decreases the coefficient of discharge; this trend is seen in very high Reynolds number from 10000 to 50000 for the case of the larger clearance, like 1mm. The second clearance value of 0.5mm seems to validate the prediction that with

increase in Reynolds number, the pressure drop will increase which in turn decreases the coefficient of discharge.

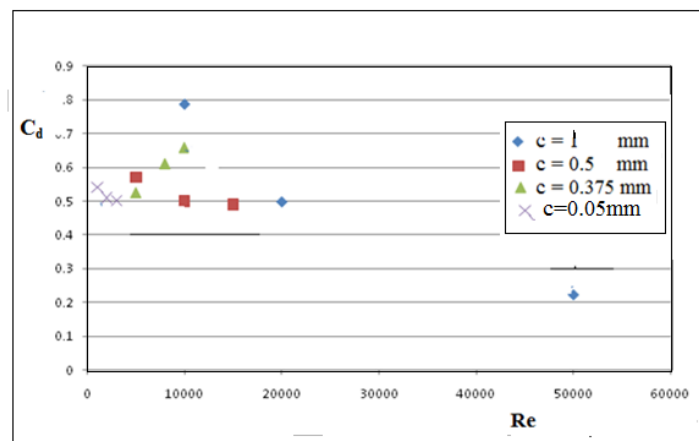
The lower clearance of 0.375 mm shows a unique behavior, which is, the C_d in this case does not seem to drop with increase in the Reynolds number. Rather it increases with Reynolds number, this prediction has been partly due to the development of flow of the fluid in the clearance of the tooth. Mathematically speaking, this is the clearance point at which, the fractional increase in mass flow rate with respect to clearance is more than the increase in the pressure drop across the seal, which results in the coefficient of discharge increasing. Moreover, a detailed analysis to the type of flow will be given in later chapters, to exactly know the behavior of the seal under the tooth.

It has to be noted that the flow is choked in the lower clearance (0.05mm) case for all the Reynolds numbers considered in this study. When the pressure ratio across the tooth reaches a value of 0.528, the mass flow rate cannot be increased by further reducing the exit pressure and the flow is said to be 'choked'.

For the lowest clearance of 0.05mm the coefficient of discharge decreases and it shows the similar trend as in case 1mm and 0.5mm, the flow behavior with respect to pressure drop and Reynolds number is explained elaborately in the earlier part of this section, and similar pattern seems to emerge in this case as well, as the pressure drop increases with Reynolds number which result in decrease in the coefficient of discharge.

Figure 7 on the next page shows how the coefficient of discharge varies with change in Reynolds number for the case of air.

The slope which can be defined as the change in coefficient of discharge to the Reynolds number can be used to predict how the flow develops across the different seals. The slope is negative for the most of the part in case of 0.5mm, 1mm and 0.05mm which shows how C_d decreases with increase in Reynolds number, but in case of 0.375 mm the slope becomes positive and C_d increases with increase in the Reynolds number.



**Figure 7 Coefficient of Discharge vs. Reynolds Number (Single Tooth Isosceles) Air
Case Cd 1st Tooth**

SINGLE TOOTH WATER SEAL

In the case of single tooth Isosceles seals which use incompressible media like water, the effect of Reynolds number is much more significant, in a sense that, the pressure drop varies rather quickly, within a limited range. Therefore, the coefficient of

discharge shows a trend where it first decreases, then after a certain range it starts to increase, but the slope of increase is not as much the slope of decrease.

The higher clearance of 1 mm show that the pressure drop, increases with increase in the Reynolds number. This trend also follows with the pressure ratio, as it increases with increase in the Reynolds number for a larger clearance. The sudden change in the nature of the slope can be attributed the change in the streamlines of the incompressible fluid flowing, the details of which will be explained in the later chapters.

The clearance 0.5 mm shows a similar trend, but in this case, the change in the nature of the slope occurs at a slightly lower Reynolds numbers which confirms the earlier hypothesis given that with decrease in the clearance, the coefficient of discharge increases. This is caused by the area of cross section decreasing resulting in the incompressible fluid facing more restriction while moving, which in turn results in higher turbulence.

The study also reveals how the nature of incompressible fluid changes with the lower clearance values. It can be clearly seen that with the same set of Reynolds numbers like (2000,5000,10000), the lower clearance like 0.375mm, 0.05mm shows greater pressure in the upstream tooth. Since the pressure upstream is greater; it results in greater pressure drop, and thereby increasing the coefficient of discharge.

The lower coefficient of discharge is due to the larger pressure drop across the cavity. The density in case of incompressible fluid like water is more than that of the air, so generally the pressure drop in water is much higher than that of the air. Figure 8 on

the next page shows how the coefficient of discharge behaves with increase in Reynolds number for the case of incompressible fluid like water.

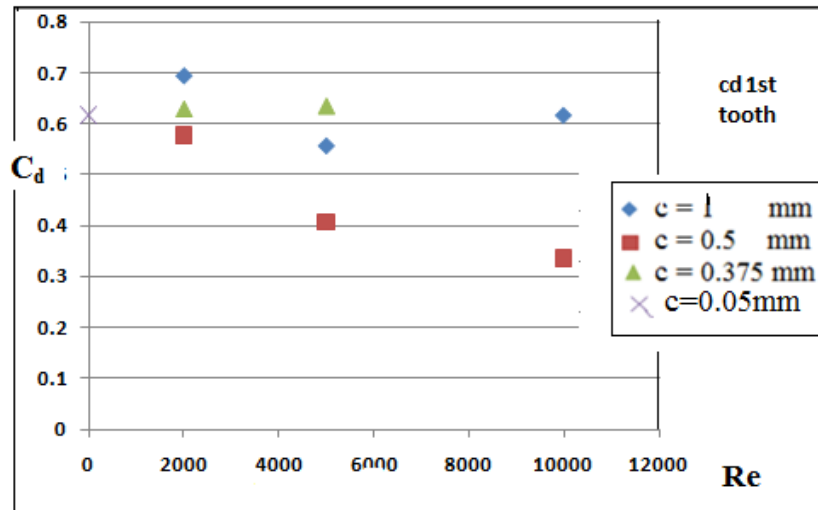


Figure 8 Coefficient of Discharge vs. Reynolds Number (Single Tooth Isosceles)

Water Case C_d 1st Tooth

DOUBLE TOOTH ISOSCELES COMPRESSIBLE AIR CASE COEFFICIENT OF DISCHARGE

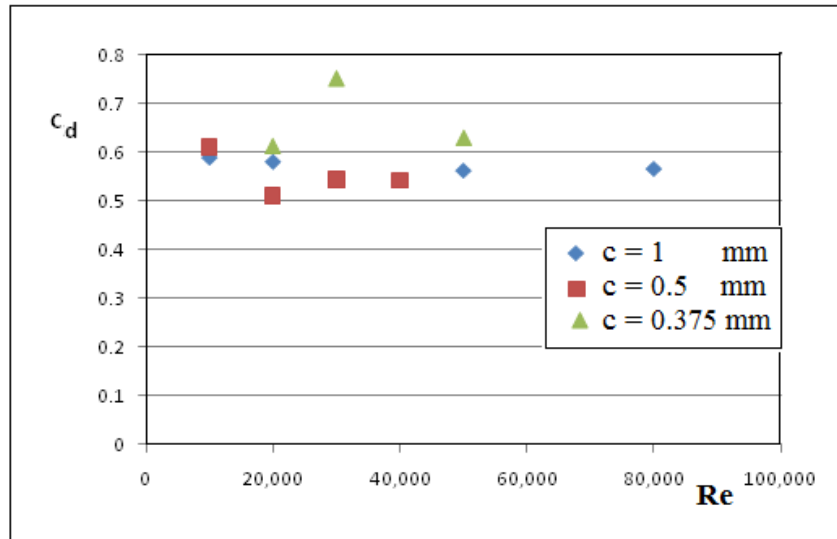
The behavior of the double tooth Isosceles triangle shaped seal shown in the Figure 9 will be adjudged with two parameters, which are the coefficient of discharge and the carry over coefficient. These two parameters can broadly define the criteria as to how the certain shape of the seal i.e. the geometry of seal, effects coefficient of discharge and carry over coefficient.

The variation in parameters in the first tooth is analyzed with clearances of 1mm, 0.5mm, 0.375mm etc. In the case of the larger clearance, 1mm, it is observed that pressure drop change is similar to the single tooth Isosceles seal type, which also shows that the coefficient of discharge is also similar in both cases. This trend is also seen the almost all of the clearances (1mm,0.5mm, 0.375mm), which leads to the conclusion that irrespective of the number of teeth in the seal, the coefficient of discharge values are similar for the first tooth for the same type of geometry.

In higher clearances, the C_d first decreases and then increases. This is attributed to the increase in the pressure drop with increase in the Reynolds number. The compressibility effect plays an important role, as it is clearly tangible that with increase in the Reynolds number, the drop in density increases. As a result, the change in the coefficient of discharge with respect to Reynolds number is more in air than in incompressible fluid like water. This behavior can be implied to the larger pressure drop in case of air.

One of the most important things to be observed in the case of the Isosceles triangle case is the Reynolds number effect on the pressure drop of the seal. As in previous cases, the higher clearance values of 1mm and 0.5mm show a similar trend, which is, with increase in the Reynolds number the pressure drop also increases which decreases the C_d values as one goes from lower Reynolds number to higher Reynolds number for the same clearance. In lower clearance values such as 0.375mm the same trend is seen again. With this observation, it can be concluded that similar geometry of seal effects in the same manner irrespective of the clearance changes or changes in cross

sectional area. Figure 9 shows the distribution of coefficient of discharge with Reynolds number.



**Figure 9 Coefficient of Discharge vs. Reynolds Number (Double Tooth Isosceles) Air
Case C_d 1st Tooth**

CARRY OVER COEFFICIENT FOR DOUBLE TOOTH ISOSCELES

AIR CASE

The carry over coefficient is an important parameter to be studied since it characterizes the effectiveness of the seal at dissipating the kinetic energy generated by the upstream tooth. To calculate the carry over coefficient, the relationship between the divergence angle of the jet and γ , given by Hodkinson [5], will be used in this thesis.

The carry over coefficient is one of the more important parameters that affect the amount of fluid flowing through the one tooth to another.

For the calculation of carry over coefficient, it is noted that the ΔP i.e., the difference between the upstream and downstream pressure is varied up to 10 atm. The effect of compressibility, the change in density and pressure with respect to Reynolds number has also been taken into account.

For the large tooth clearance of 1 mm, the carry over coefficient values are the largest and start from a value of 4 for a very low Reynolds number. With increase in the Reynolds number it is seen that the carry over coefficient values tend to decrease to minimum value close to 1.5, then they increase up to a value of 2.5, with increase in the Reynolds number.

The case of 0.5 mm shows a similar trend to the higher clearance of 1mm. At Reynolds number 10000, the flow separation does not occur completely, so it has not been considered in the study. Due to this effect, the carry over coefficient also tends to be low. However, at 20000, the flow separation occurs completely and gives a clearer value of 3 which continues to decrease as the Reynolds number increase for the same clearance of 1mm up to 30000. At this point, the reversal of trend occurs and the carry over coefficient increases with increase in Reynolds number.

The low clearance value of 0.375mm shows a similar trend; The carry over coefficient is taken for higher Reynolds number from 20000 to 50000. It is seen that the γ decreases first from 20000 to 30000 and then increases while moving from 30000 to

50000. This shows the beta increases from 20000 to 30000 and then decreases from 30000 to 50000.

In the case of rectangular cavities with small clearances, the carry over coefficient increases with increase in Reynolds number (Suryanarayanan [1]). This can be explained by the following physical argument. At higher Reynolds numbers the inertia of jet is high in comparison to the viscous forces. This results in a smaller divergence of the jet and higher kinetic energy carry over. However, as seen in Figure 10, the double tooth Isosceles triangle shaped geometry showed that with increase in Reynolds number the carry over coefficient first decreased and then increased. This trend in variation of carry over coefficient was noticed in all the clearances considered namely, 1 mm, 0.5 mm and 0.375 mm. This behavior may be attributed to the tendency of Isosceles cavity shape geometry to cause an increase in the divergence of the jet for certain range of Reynolds numbers (between 20000 to 30000). Above this range of Reynolds numbers, the carry over coefficient increases with increase in Reynolds number as in case of rectangular cavity and can be explained by the similar argument. Figure 10 on the next page shows the variation in carry over coefficient with Reynolds number for Isosceles shaped cavity. The fluid taken is air.

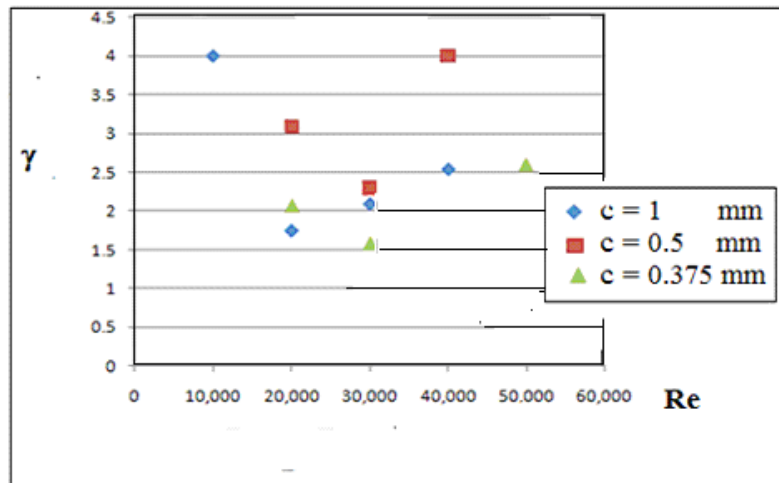


Figure 10 Carry Over Coefficient vs. Reynolds Number (Double Tooth Isosceles)

Air Case Cd 1st Tooth

DOUBLE TOOTH ISOSCELES SEAL WITH INCOMPRESSIBLE FLUID

WATER

The effect of incompressibility has to be taken into consideration as to check the working of the Isosceles cavity shaped seals. The clearance taken is 1mm, 0.5 mm, 0.375mm. The behavior of seals changes completely in case of a fluid like water, as the range of Reynolds number change with change in the value of coefficient of discharge and carry over coefficient.

DOUBLE TOOTH ISOSCELES WATER COEFFICIENT OF DISCHARGE

Saikishan Suryanarayanan [1] predicted that with an increase in the Reynolds number, the coefficient of discharge increases for an incompressible flow, for a

rectangular cavity shape seals. This thesis aims to show how the coefficients of discharge vary for different seals with different set of clearances. This study takes a large clearance of 1mm to lower clearance of 0.375mm which is generally the clearance values used in the companies for steam turbines and gas turbines.

On considering a bigger clearance seal of 1mm, it is clearly seen that at low Reynolds number, the coefficient of discharge is very low, around 0.3 to 0.4. These results in slightly higher pressure drop compared for low mass flow rate. However increasing Reynolds number, one can clearly observe the value tends to increase i.e. from 0.3 to 0.8. This means that with increase in the mass flow rate and the Reynolds number, the coefficient of discharge increases. The increase in pressure drop is lesser compared with increase in the mass flow rate which ultimately results in the greater value of C_d of the 1st tooth.

On looking at a medium range clearance of 0.5mm, some interesting facts have been observed. The coefficient of discharge again increases with an increase in the Reynolds number. The value coefficient of discharge is comparatively more than for the higher clearance of 1mm. At a low Reynolds number of 2000, the C_d value is around 0.45 which is comparatively larger than the 1mm case. The C_d for the 0.5mm clearance continues to increase as the Reynolds number increases, and ultimately to a point where C_d is close to 1. The increase in C_d value is due to an increase in the pressure drop, this is more for case of the higher Reynolds number for the same clearance. Isosceles shaped cavity enables lesser pressure drop which can be due to low β . Figure 11 below shows

how the coefficient of discharge changes with increase in the Reynolds number for the incompressible fluid.

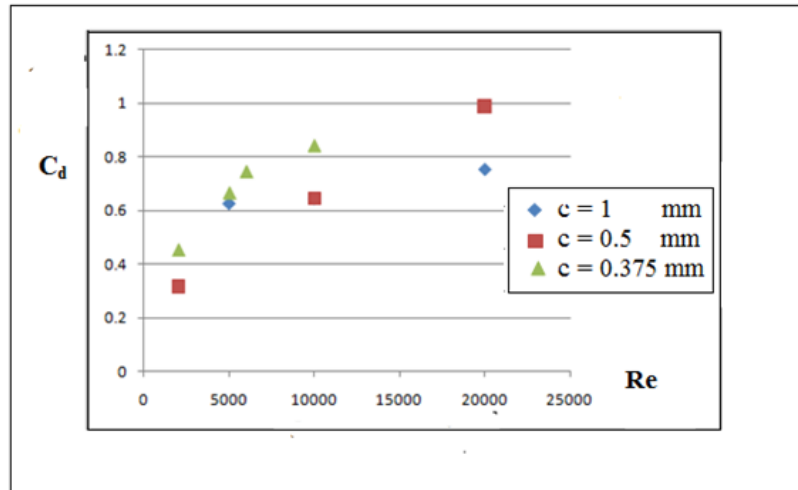


Figure 11 Coefficient of Discharge vs. Reynolds Number (Double Tooth Isosceles)

Water Case C_d 1st Tooth

The lowest clearance of 0.375mm reiterates the same fact, which had been found earlier, i.e. with lowering of the clearance, the pressure drop increases and ultimately results in increase in the coefficient of discharge value. The range of Reynolds number (2000 to 10000) in which coefficient of discharge in this case is found tends to be lower than the range of values (2000 to 20000) for all other clearance cases. This confirms the fact for a low clearance; one gets a larger pressure drop at lower Reynolds number.

Figure 11 shows that coefficient of discharge varies with Reynolds number for water case.

The behavior of seals with respect to the parameters like turbulent intensity, stream functions and compressibility effects will be explained in the later chapters to come.

DOUBLE TOOTH ISOSCELES WATER CARRY OVER COEFFICIENT

The carry over coefficient for the case of the Isosceles triangle is studied for the incompressible fluid like water. In the later chapter this carry over coefficient is calculated with Suryanarayanan equation model [1] and it is checked whether, this equation model is able to predict a range of carry over coefficient values with respect to Reynolds number for a given set of clearance.

The largest seal of 1mm has the lowest carry over coefficient. This means the beta value is high which can be attributed to the ratio of Y velocity component to the X velocity component. The lower carry over coefficient means that the seal is more efficient in kinetic energy dissipation. The Reynolds number is the ratio of inertia of the jet to the viscous forces. The higher clearance in the Isosceles based cavity results in more divergence of the wall jet resulting in higher values of β . The higher β value shows the lower carry over coefficient. The percentage kinetic carry over will be lower in this case.

The 0.5 mm clearance shows that the carry over coefficient is initially much higher than 1mm case. This is due to the small divergence in the wall jet resulting in much smaller $\tan\beta$. As discussed earlier, the smaller $\tan\beta$ results in higher carry over

coefficient. However as the Reynolds number increases, γ becomes smaller compared to the 1mm case.

The case of 0.375mm shows that carry over coefficient becomes much higher than 1mm and 0.5 mm case. This means the ratio of inertia of the jet is more than the viscose forces associated with the fluid. This results in smaller divergence of the jet resulting in lower $\tan \beta$. Since the $\tan \beta$ is inversely proportional to the carry over coefficient, a very high carry over coefficient is obtained.

The geometry of the seal affects the carry over coefficient a great deal. The details of the effect of cavity shapes will be illustrated in the coming chapters.

Carryover coefficient values increase with decrease in clearance for higher Reynolds number for all the clearances. This implies that the seal is much more efficient at dissipating the kinetic energy in the fluid exiting under the upstream tooth at lower clearances. Figure 12 on the next page shows how the carry over coefficient changes with change in the Reynolds number.

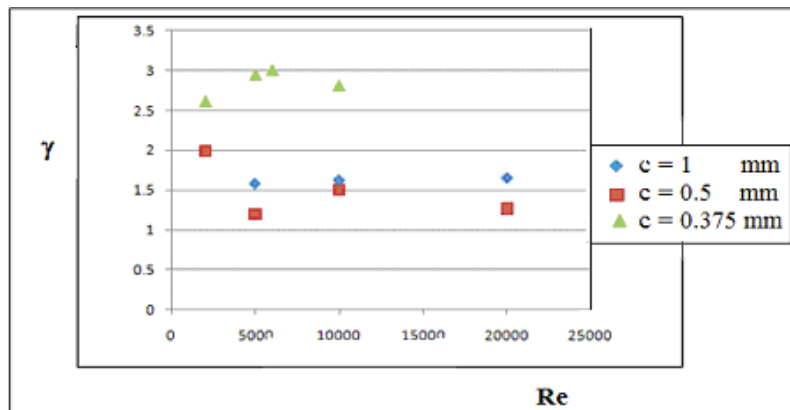
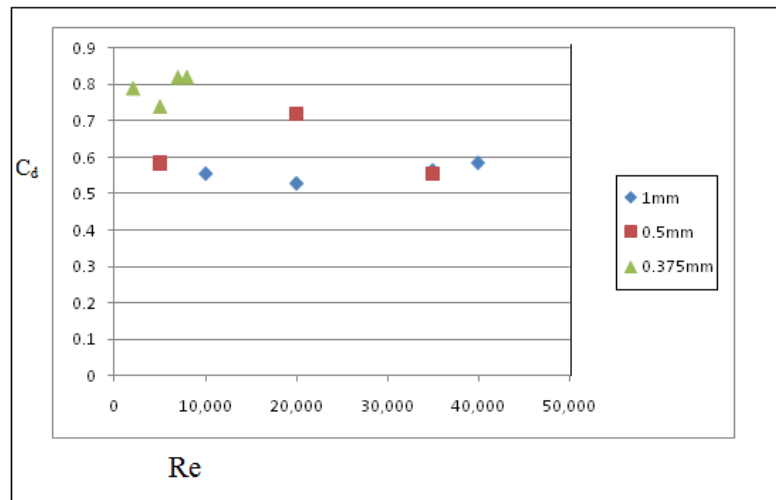


Figure 12 Carry Over Coefficient vs. Reynolds Number (Double Tooth Isosceles) Water Case

Similar trends in the variation of carry over coefficient as in the case of simulation with air as the working fluid were observed in simulations with water. The carry over coefficient initially decreases with increase in Reynolds number, but later increases with further increase in Re . The range over which the opposite trend is observed is different from the compressible flow results and is not observed for all clearances. The deviation in the jet is less and it decreases with increase in the Reynolds number. It has to be further noted that the carry over coefficient is lower for incompressible fluid compared to respective compressible simulations. It can also be stated that the clearances plays an equally important role as the shape of the cavity in deciding the percentage kinetic carry over.



($C = 1\text{mm}, 0.5\text{mm}, 0.375\text{mm}$ $S = 4\text{mm}$, $h = 50\text{mm}$)

Figure 13 Coefficient of Discharge vs Reynolds Number (Single Tooth Right Triangle)

Cavity Air Case Cd 1st Tooth

SINGLE TOOTH RIGHT TRIANGLE AIR

The right angled tooth is another cavity shape discussed in the present study. Values of the discharge coefficient are shown in Figure 13. Three different clearance of 1mm, 0.5mm, 0.375mm are analyzed for the right angled triangle shape. The results show that at the higher clearance of 1mm, the coefficient of discharge first decreases and then increase with increase in Reynolds number. A reversal of trend is observed for the case of 0.5mm. It shows that coefficient of discharge increases with increase in Reynolds number (from 10000 to 20000) then decreases for higher Reynolds number. For

0.375mm, the coefficient of discharge decreases and then increases, this trend was also observed in higher clearance of 1mm.

Saikishan Suryanarayanan [1] had shown that the Reynolds number based on the clearance was the only parameter governing the coefficient of discharge for incompressible fluid case for a fixed tooth clearance/pitch ratio. This trend was also seen for right triangle shaped seal as well with compressible air case.

The C_d for the right triangle shaped single tooth (Fig 14) shows the same trend as with C_d 1st tooth for double tooth of the same cavity shape. The double tooth right angled cavity shaped case will be discussed in the later part of this section. However, it can be concluded from this discussion that the coefficient of discharge for a single tooth right angled shaped and the C_d 1st tooth for the double tooth right angled shaped seal have similar values and show similar trends. This is due to the fact that coefficient of discharge in both the cases are influenced by the upstream condition of the fluid flowing through the 1st tooth, irrespective of number of teeth attached after it. The same argument also holds true for incompressible fluid as well which was already illustrated by Suryanarayanan [1] in his thesis.

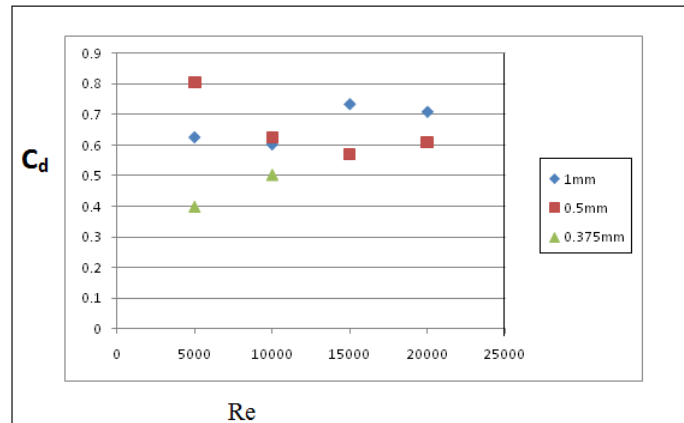


Figure 14 Coefficient of Discharge vs Reynolds Number (Single Tooth Right Triangle)

Cavity Water Case C_d 1st Tooth

SINGLE TOOTH RIGHT TRIANGLE WATER

The single tooth right triangle for incompressible fluid is shown in the above Figure 14. It shows that for higher clearance of 1mm the coefficient of discharge first decreases slightly (5000 to 10000) and then increases with increase in Reynolds number (10000 to 20000). The clearance value of 0.5mm shows that with increase in Reynolds number, the coefficient of discharge decreases (5000 to 15000) and then increases (15000 to 20000). The change in coefficient of discharge can be attributed to the variation in the pressure drop. The relation between coefficient of discharge and pressure drop has been broadly described in the earlier sections. The lower clearance of 0.375mm shows a constant increase of C_d with increase in the Reynolds number. The change in the behavior of different clearances with change in the Reynolds number is due to change in flow pattern as depicted by stream lines. This will be discussed in the later sections.

The C_d for single tooth right triangle and C_d 1st tooth for double tooth right triangle show a similar trend.

It can be clearly seen that the Reynolds number is the only governing factor for single tooth shaped right triangle cavity as cited by Suryanarayanan [1] for incompressible fluid provided tooth clearance/pith ratio is fixed.

DOUBLE TOOTH RIGHT TRIANGLE AIR

DOUBLE TOOTH RIGHT TRIANGLE COEFFICIENT OF DISCHARGE

The coefficient of discharge for the right triangle differs from the coefficient of discharge of the Isosceles triangle case, due to the geometry of the seal. In this case, if observed closely, it can be clearly noted that the higher clearance value, tends to have the carry over coefficient/Reynolds number negative and then it shows a positive value. However this cannot be said about a low clearance value. The research findings are discussed in a great detail below.

The clearance value of 1mm which is being taken has the highest value of the right angled seal, shows that with low Reynolds number the value seems to give a negative slope for the coefficient of discharge. Here it should be noted that the range of Reynolds number are taken from 10000 to 40000 and again the ΔP is maintained at around 10 atm absolute. With increase in Reynolds number the ΔP value is increased, this increase is inversely proportional to the coefficient of discharge.

The next clearance value taken is 0.5mm, which shows slightly more deviation than higher seal clearance of 1mm. The results obtained clearly show a positive increase

for the carry over coefficient/Reynolds number for Reynolds numbers from 10000 to 20000. This is due to the increase in the value of mass flow inlet and the reduction of ΔP , it may be noted that the mass flow inlet is directly proportional to the coefficient of discharge and inversely proportional to the ΔP . As the Reynolds number seems to increase from the 20000 to 30000, the slope shows a negative value, which means that the coefficient of discharge decreases with increase in the Reynolds number. It is seen that similar trend continues in case of the higher Reynolds number.

The clearance value of 0.375mm has been taken as the lowest clearance in this case primarily due to the fact that for lower clearance the coefficient of discharge becomes low. For the case of 0.375mm, the dependence of Reynolds number and pressure drop is seen. The lowering of clearance, gives that the coefficient of discharge to be taken in between 0.75 to 0.90 is seen between the Reynolds numbers 2000 to 10000, this implies that, the pressure drop of ΔP of 10 atm considered, is achieved in much lower range of Reynolds number. The results show one more tendency of analyzing the slope that it first decreases, and then increases, ie slope is negative between 2000 to 5000 Reynolds number values which is due to the increase in the pressure drop with Reynolds number. However this trend sets to reverse when the Reynolds number are taken from 5000 to 10000 as the slope carry over coefficient/Reynolds number becomes positive from 5000 to 10000, which is due to the lowering in ΔP with increase in Reynolds number. It has to be noted down that ΔP is inversely proportional to the coefficient of discharge which has been already stated

earlier. Figure 15 on the next page shows how the coefficient of discharge varies with change in Reynolds number for the right triangle shaped seal.

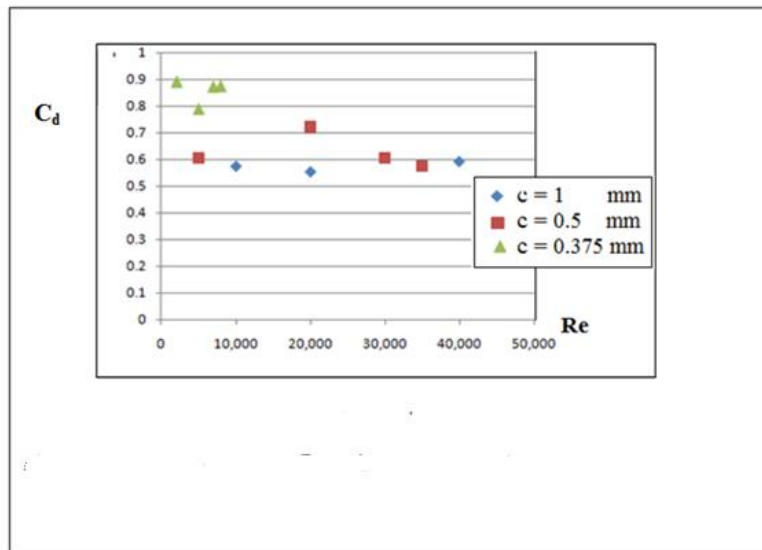


Figure 15 Coefficient of Discharge vs. Reynolds Number (Double Tooth Right Triangle)

Air Case C_d 1st Tooth

DOUBLE TOOTH RIGHT TRIANGLE CARRY OVER COEFFICIENT

The carry over coefficient for a compressible seal gives the information about the parametrical changes occurring inside the cavity. Here the effect of the seal geometry plays a very important role. It can be clearly seen that the carry over coefficient values for the case of the right triangle shape are invariably different than in the case of the Isosceles shaped seal.

The largest seal clearance taken in the research of 1mm shows the carry over coefficient lies between 2 to 2.5. This value does not seem to change very much even going to a higher Reynolds number, which can be due to the effect of the seal geometry. The slope of the carry over coefficient/ Reynolds number does not change very much with respect to the change in Reynolds number. The X-velocity component and Y-velocity component seems to be almost constant in case of these shapes which result in the beta value to be very low in value and constant in nature. This can be due to the fact that the streamlines in the second tooth tend to flattened after the first tooth. The deviation tends to be less and the results seem to be constant. The slight variation noted shows the slight change in beta value, however it reiterates the fact that change in Reynolds number does not change the carry over coefficient values drastically. Figure 16 on the next page shows how the carry over coefficient varies with change in Reynolds number for right triangle shaped seal.

The slightly lower clearance of 0.5mm also shows the same trend for the carry over coefficient as with lower Reynolds number the carry over coefficient value stays around 1.5 to 2.5. The x-velocity component and y-velocity component does not change

very much in case of this clearance. This shows the fact that how seal geometry tends to effect carry over coefficient. The beta value which has been defined as the ratio of change in Y-velocity component to the X-velocity component gives almost constant value which is considered to be due the streamlines and turbulent kinetic energy. This shows, that in all the clearances the value do not differ very much as all the cases, they generally lie in around a value of 2.

The lowest clearance is taken as 0.375mm shows the variation is near about same from 1.8 to 2.2 , the slope is negative as the carry over coefficient is decreasing, but the decrease is very slight, which seems to be prevalent in larger Reynolds number. However, considering all the cases, the carry over coefficient lie in between 1.8 to 2.2.

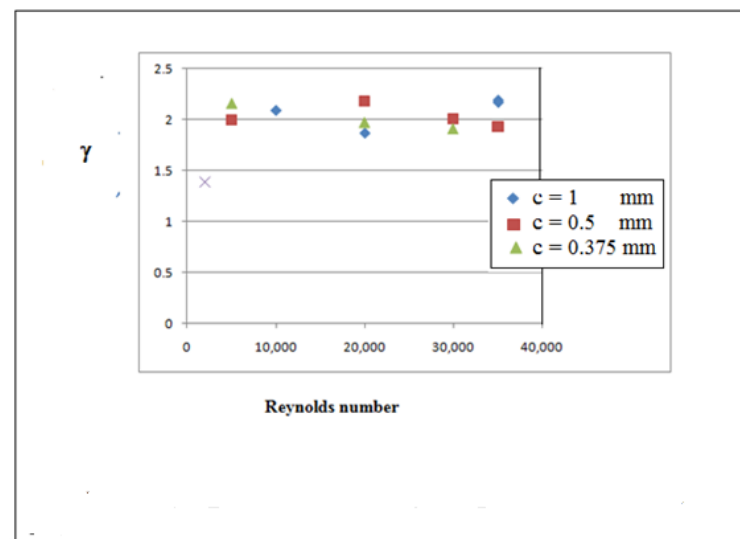


Figure 16 Carry Over Coefficient vs. Reynolds Number (Double Tooth Right Triangle) Air Case

The carry over coefficient is low for all the clearances. The value of the carry over coefficient lie around 1.8 to 2.2 for all the clearances. The β value is more in the case of the right triangle cavity shape and the carry over coefficient decreases with increases in the Reynolds number. The Reynolds number is the ratio of inertia of the jet to the viscous forces. The right angular cavity shape geometry results in more divergence of the wall jet resulting in higher values of β . This results in higher kinetic energy dissipation and lower carry over coefficient. This means the Isosceles cavity shape is more efficient for compressible fluids as compared to the right triangular shaped seal for compressible fluids. The amount of kinetic energy dissipation does not change much with clearance resulting in similar carry over coefficient for all the clearances.

DOUBLE TOOTH RIGHT TRIANGLE WATER

Incompressible flow is studied for the right triangle tooth; the density has been kept constant over the entire Reynolds number range.

Figure 17 on the next page shows how the coefficient of discharge varies with Reynolds number for the right triangle shaped seal for water.

DOUBLE TOOTH RIGHT TRIANGLE WATER COEFFICIENT OF DISCHARGE

The geometry of the shape of the seal greatly affects the discharge coefficient for incompressible flow. The seal taken into consideration has clearances of 1mm, 0.5mm, and 0.375mm. The clearance of 1mm shows that the coefficient of discharge

increases with an increase in Reynolds number from 5000 to 10000. The pressure drop across the seal is low at lower Reynolds number, hence the coefficient of discharge increases at lower Reynolds number. The highest clearance of 1mm shows that with an increase in Reynolds number from 15000 to 20000, the slope which can be taken as ratio of coefficient of discharge and Reynolds number is negative, the negative slope means that there is decline in the C_d values as one goes from 10000 to 20000. The value of C_d tends to decrease with the increase in Reynolds number as it seen that the ΔP or the pressure tends to increase with increase in Reynolds number. This is due to the turbulence and viscosity effects of the water. It might be noted that in the case of an incompressible fluid like water, the pressure drop is more than in case of the compressible fluid like air. This results in the increase in coefficient of discharge for air.

The seal 0.5 mm shows a different phenomenon altogether, in this case the pressure downstream becomes negative i.e. below the atmosphere, this happens in the Reynolds number from 15000 to 20000, the reasons will be explained in the later part of thesis. The effect of this negative pressure results in an increase in the overall pressure drop; this might in turn decrease the value of the coefficient of discharge. The stream lines and turbulent kinetic energy play a very important role as they are one of the most influential factors for a negative pressure. The slope of the coefficient of discharge and the Reynolds number becomes negative as the C_d value decreases from 5000 to 10000. However the slope becomes positive from 10000 to 20000 and a slight increase in C_d values occurs with respect to Reynolds number. It can be seen that incompressibility effects of the fluid clearly influence the pressure drop thereby affecting the discharge

coefficient. The negative pressure drop affects the coefficient of discharge for the 2nd tooth, making the coefficient of discharge of 2nd tooth undefined since the pressure actually rises across the last tooth.

As the clearance value for a given geometrical shape decreases, the area of cross section decreases which results in high turbulent flow in a small area of the seal. This in turn affects the carry over coefficient and discharge coefficient, as the discharge coefficient increases even in very small turbulent flow. This has been validated for other seal geometries.

The 0.375mm case of right triangle shaped seals gives the same range of pressure drop i.e. ΔP around 10 atm in a much lower value Reynolds number of 2000 to 10000. For the first case from 2000 to 5000 the pressure drop increases with increase in Reynolds number. The slope tends to positive. For 5000 to 10000 Reynolds number the slope increases more rapidly as pressure drop increases drastically around 10000 Reynolds number.

However, on analyzing all the clearance with respect to Reynolds number it can be clearly concluded, the slope may be positive or negative for a low Reynolds number, for all the cases, except that, the value of coefficient of discharge does not change as much when going to higher Reynolds number.

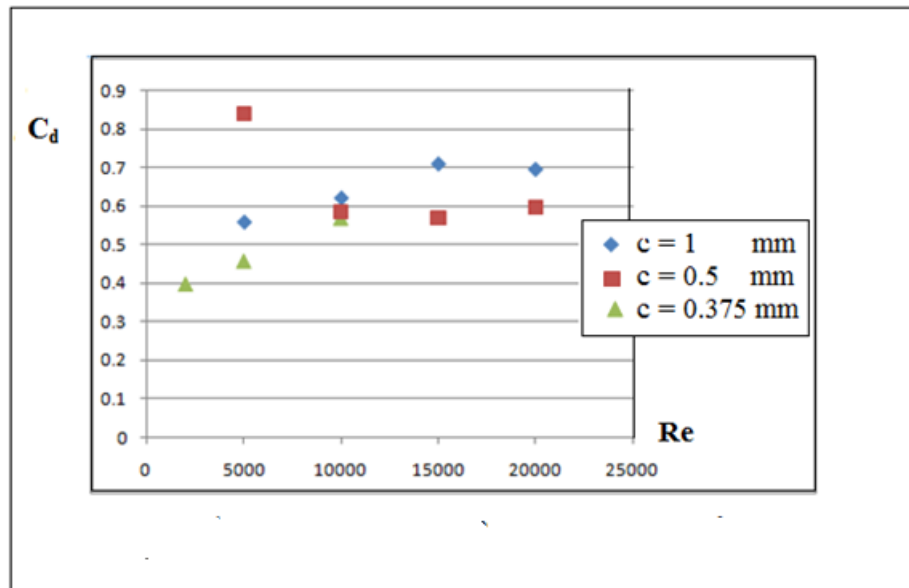


Figure 17 Coefficient of Discharge vs. Reynolds Number (Double Tooth Right Triangle) Water Case Cd 1st Tooth

PRESSURE RECOVERY ACROSS 2ND TOOTH

Pressure increase between the cavity and downstream of the 2nd tooth is caused by the flow undershooting the 2nd tooth which it never touches and slowing down past the 2nd tooth, resulting in the pressure downstream of the tooth being larger than the pressure upstream which results in a negative pressure rise across the tooth.

DOUBLE TOOTH RIGHT TRIANGLE WATER CARRY OVER COEFFICIENT NEGATIVE PRESSURE DROP

Figure 18 shows how the carry over coefficient varies with change in Reynolds number for the right triangle cavity shaped seal for water.

To analyze the incompressibility effects in the 2nd tooth and subsequent tooth, the carry over coefficient consideration should be taken into account. This specifies how the fluid works in different conditions, under different Reynolds number.

The highest clearance value of 1mm shows that carries over coefficient increases for a low Reynolds number range, from 5000 to 10000. The Y-velocity component is greater with respect to X-velocity component which results in greater beta value. This is one of the flow cases where the pressure rises across the 2nd tooth. The fact that beta is negative shows that jet of the fluid exiting from under the upstream tooth does not spread out and hit the downstream tooth. The Vena Contracta affect from the first tooth is substantial enough to cause the jet to pass under the tooth undisturbed. This results in the 2nd tooth having minimal influence upon the seal leakage. It appears that this is a range of clearance to tooth pitch ratios where to pitch is too short allowing the jet to pass under the 2nd tooth relatively undisturbed. The slope becomes negative when moved from 10000 to 15000, which shows clearly the effect that the Y- velocity component decreases with increase in X-velocity component. It can be clearly seen the carry over coefficient-Reynolds number in case right triangle based cavity shaped seal does not change for as much for high Reynolds number compared with low Reynolds number.

The clearance value of 0.5mm produces carry over coefficients which do not change very much from lower Reynolds number to higher Reynolds number. This seal geometry plays a very important role as the right angled tooth has almost flattened stream line which is making the beta values almost constant. This gives slight increase in

higher Reynolds number but does not change very much, all the values varies around 1 to 2. For this clearance the jet does not entirely pass under the second tooth undisturbed.

The 0.375 mm clearance values show a slightly different nature, in case of the lower value of Reynolds number, the value of carry over coefficient drops from 6 to 1; as the jet spreads more rapidly as the Reynolds number decreases. It can be seen that the governing factor is the Reynolds number again as small Reynolds number gives a large carry over coefficient for a low clearance value. The effect of clearance value is also seen as with the respect to the fixed pitch. The carry over coefficient is low for low clearance and low Reynolds number. The change in carry over coefficient is more at this range. The beta value again plays an important role in case of the lower Reynolds number, as value of beta tends to be very low for very low Reynolds number indicating a slow spread rate of the jet.

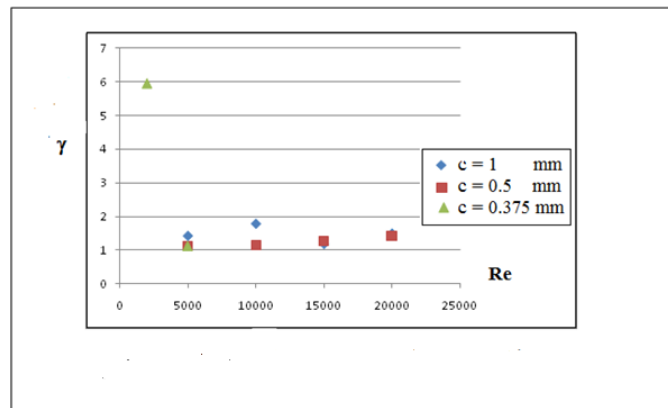


Figure 18 Carry Over Coefficient vs. Reynolds Number (Double Tooth Right Triangle) Water Case

The next values of carry over coefficient stay almost constant even with increase in the Reynolds number. This is again due to the invariant value of beta indicating a constant spread rate of the jet. The next values of carry over coefficient stay almost constant even with increase in the Reynolds number. This is again due to the invariant value of beta indicating a constant spread rate of the jet. The carry over coefficient of labyrinth seals with a right triangular geometry was found to be much lower than the corresponding cases with either Isosceles or NASA designs for all clearance values. This implies that the right triangle cavity shape allows more kinetic dissipation for the fluid exiting under the upstream tooth by causing a large deviation in the jet issuing from the upstream tooth. Hence it can be concluded that this cavity shape is very efficient in dissipating kinetic energy of the working fluid and possibly reducing seal leakage. Figure 18 shows the carry over coefficient varies with Reynolds number for right triangle seal.

PRESSURE RECOVERY ACROSS 2ND TOOTH AND RESULTING C_D

The pressure drop is considered as one of the most influential factors to decide the coefficient of discharge. This thesis involves different cavity shapes and pressure drop across the teeth. Under some conditions, the pressure in the seal cavity upstream of the last tooth can be less than the pressure at the seal exit. Therefore, the pressure difference across this last seal tooth is negative as define for the calculation of the discharge coefficient of this tooth. In this study the seal exit pressure was assumed to be one 1 atmospheric absolute. Thus, if the last seal cavity pressure is less than one

atmosphere the pressure actually raises across the last tooth. For these conditions, the discharge coefficient for the last tooth is undefined since $dp = P_{up\ stream} - P_{down\ stream}$ is negative and C_d becomes an imaginary number. For incompressible flow simulations, Fluent can predict a pressure less than a perfect vacuum. The 1 atm exit pressure is just a reference value added to the simulation values. For compressible flows where the ideal gas law is used to calculate density, this situation does not occur. This phenomenon was found for the case of incompressible fluid, which is water in this case; with both Isosceles cavity shape and right triangle shaped tooth.

Cd 2nd TOOTH

This is the most important finding of this research. As noticed previously, the pressure drop across the cavity is negative. This negative pressure drop results in the coefficient of discharge which becomes undefined for the 2nd tooth, due to this pressure drop. The details of the cavity shapes and pressure drop are given below in the table.

s.no	Cavity shape	clearance	Reynolds number	Pressure 1 st Tooth(upstream)	Pressure 1 st tooth (downstream)	Cd 1 st tooth	Cd 2 nd tooth
1	isosceles	1mm	20000	4.26	0.74	0.75	undefined
2	isosceles	1mm	15000	2.02	0.81	0.64	undefined
3	issocelae	0.5mm	10000	4.91	0.1	0.65	undefined
4	Right triangle	1mm	20000	4.26	0.1	0.69	undefined
5	Right triangle	0.5mm	20000	21	-1.49	0.595	undefined
6	Right triangle	0.5mm	15000	13.01	-0.9	0.57	undefined
7	Right triangle	0.5mm	10000	6.26	0.385	0.583	undefined

Table 5.1 showing the cases of the negative pressure drop

NEGATIVE PRESSURE DROP

The flow conditions where the pressure difference across the last tooth increases have a common flow characteristic. The jet of the fluid issuing from under the tooth upstream of the last tooth does not diverge (spread) radially sufficiently to be wider than the tooth clearance. This results in β being negative and the carry over coefficient being undefined as well as the discharge coefficient. The jet does spread radially after the last tooth resulting in a pressure recovery. This flow pattern can be observed by evaluating contour plots of stream function. Figure 19(a) shows the distribution of negative pressure across the labyrinth seal cavity.

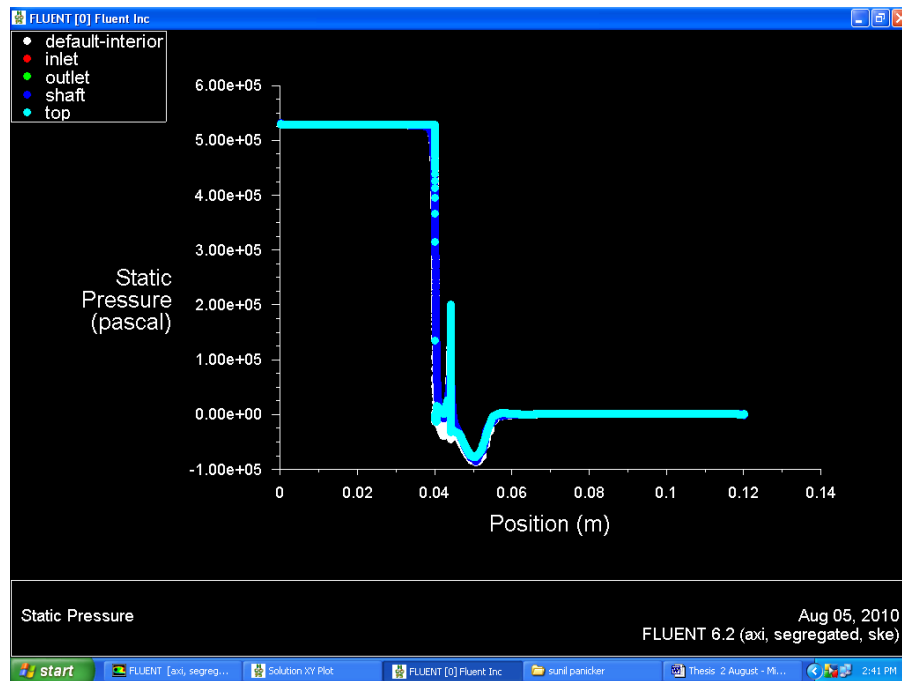


Figure 19 (a) A Case of Negative Pressure Drop, $Clr=0.5mm$, $Re=10000$

THE STREAM FUNCTION AND ITS IMPORTANCE

Seal performance is most often represented by its coefficient of discharge, which is related to the pressure drop across the seal. This thesis involves the study of how different cavity shapes influence the pressure drop. In the case of labyrinth seals with rectangular cavities and low values of c/s , the pressure drops from higher values to lower values across each tooth as shown in earlier studies [1].

However in the current simulations with advanced cavity shapes and larger clearances, pressure sometimes increases between the upstream cavities and downstream

of the 2nd tooth resulting in the pressure downstream of the tooth being larger than the pressure upstream which is represented negative pressure drop. This phenomenon was found for cases of incompressible fluid, (water); with both Isosceles cavity shape and right triangle shaped tooth.

ANALYSIS BASED ON STREAM FUNCTION

The observation of the flow pattern within the cavity and under the tooth might provide reasons for the strange behavior of negative pressure drop observed in the current simulations involving Isosceles and right triangle shaped geometries that were considered. Stream function, Ψ is defined in the following manner for 2D incompressible flows.

$$u = \delta\psi/\delta y$$

$$v = -\delta\psi/\delta x$$

where u and v are the x and y -components of the velocity. $\Psi = \text{constant}$ represents a streamline. By definition, velocity vectors are tangents to streamlines. Fluid particles flow along a streamline (no fluid particles can cross a streamline). Hence computation of the streamfunction and display of streamlines provides a visualisation of the flow pattern. It was seen in the earlier section as to how the streamlines are related to the carry over coefficient.

STREAM FUNCTION IN ISOSCELES TRIANGLE

The Isosceles triangle is one of the most commonly used types of seals in steam turbines. These seals show different characteristics for certain Reynolds number and set

of clearances than for rectangular shaped teeth. The comparison with a normal behavior seal and this negative pressure drop behavior will be discussed in this section.

The case of 1mm is explained here. The stream function alignment shows (Figure 19) that they indicate large β for low Reynolds number (Re No less than 10000). However around Reynolds number 10000 to 15000 the value of β becomes less resulting in an abrupt decrease in pressure drop. The orientation of the stream function becomes unidirectional and linear under the cavity. The reason is explained as follows. On considering the pressure drop for the 1st tooth, the pressure before and after 1st tooth is similar to the normal pressure drop. However, for the 2nd tooth of the seal, the pressure after the exit of the 2nd tooth is more than the pressure before the inlet of the 2nd tooth. Therefore, the coefficient of discharge becomes undefined in this case. This phenomenon is observed more in Isosceles and right triangle shaped cavity shapes. The measurement of the stream function is taken under the cavity in the near wall region where viscous effects are prevalent. The jet of fluid issuing through the upstream tooth does not spread radially very much and does not become wider than the tooth clearance. Hence the pressure recovery does not take place. However at the last tooth (after the 2nd tooth) tooth the jet spreads radially more than the tooth clearance, hence pressure recovery is more at the last tooth.

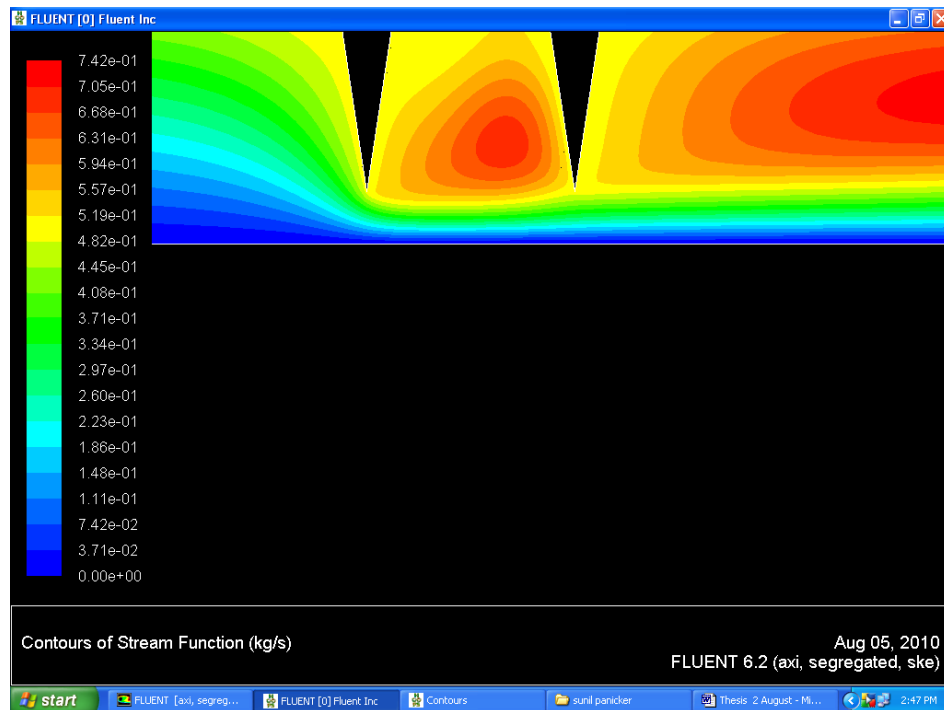


Figure 19(b) Stream Function Distribution for Isosceles Seal, Clr= 1mm, Re= 10000

The contours of stream functions are given above. As discussed previously, the distribution of stream functions is depicted by the change in color. It can be deduced that the Isosceles cavity shape greatly effects the direction of stream lines. It has to be noted that pressure drop is reflected by these streamlines. Figure 19(b) shows the distribution of stream function across the labyrinth seal (in case of Isosceles triangle shaped seal).

Figure 20 below shows the distribution of the stream functions across the Isosceles seal with Reynolds number of 20000 and the clearance of 1mm.

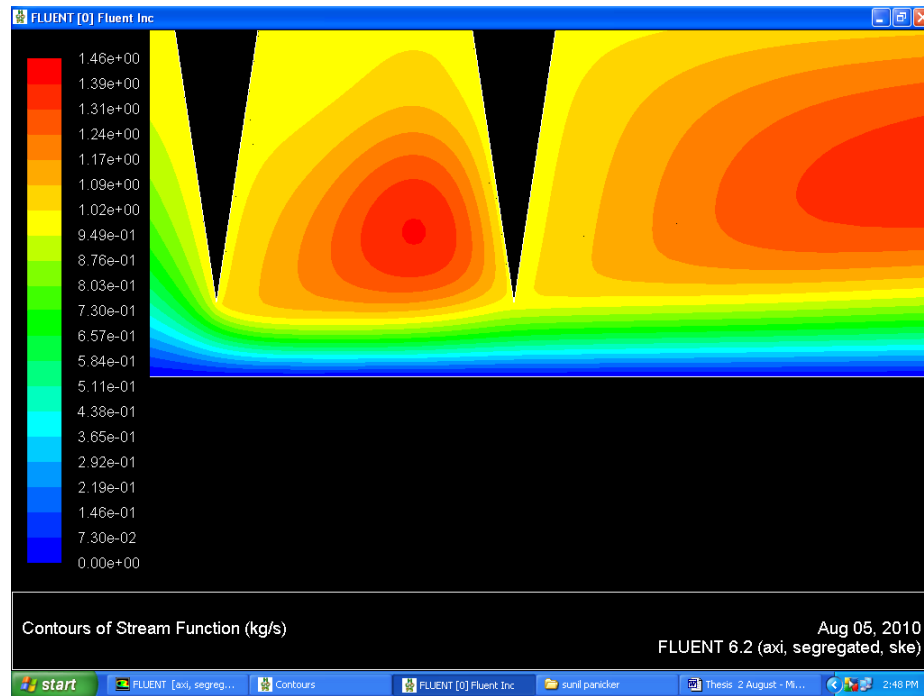


Figure 20 Stream Function Isoceles Shaped Seal C=1 mm Re=20000

The distribution of stream lines for the smaller clearance (Figure 21) of the same cavity shaped seal tend to be different than for the larger clearance seal of same cavity shape. It is also deduced that not only the clearance, but the Reynolds number also affects the direction of the stream lines throughout the seal. The seal of 0.5mm with Reynolds number of 10000 is seen broadly below. Figure 21 shows the distribution of stream function for a small clearance across the seal. Figure 20 and 21 invariably show the effect of Reynolds number on the different seal clearance. They also substantiate the fact that Reynolds number is a significant parameter while analyzing the seal performance.

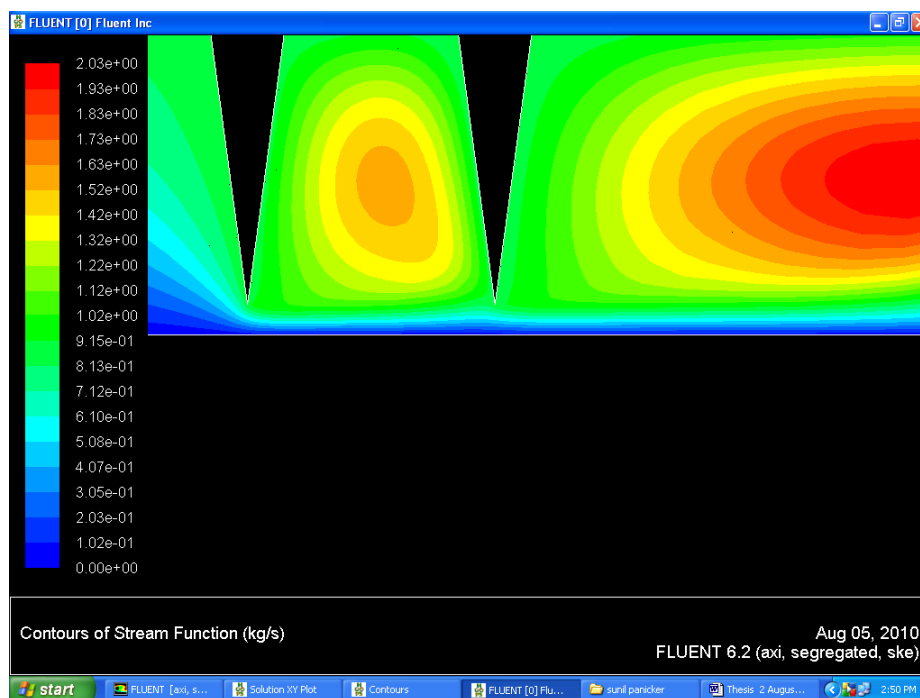


Figure 21 Stream Function Isosceles Shaped Seal Clr =0.5mm Re=10000

The fluid enters the seal and the stream lines are analyzed according to the flow of the seal. The direction of the stream lines shows the movement of the fluid particle is linear in the near wall region. The streamlines across the top appear to be similar to the bottom. The most important fact is the direction of the stream lines is straight under the tooth cavity which gives a minimum angle of $\tan \beta$. This is the reason for negative pressure drop in case of this seal.

RIGHT TRIANGLE SHAPED SEAL WITH NEGATIVE PRESSURE DROP

Another important type of seal studied is the right triangle cavity shaped seal which shows a negative pressure drop for specific Reynolds numbers and clearances. A

directory showing all the cases where all the negative pressure drop occurs has been already discussed previously.

The range of the Reynolds number and clearance values which possess a negative pressure drop for right triangle case are similar to the case of Isosceles shaped triangle. The change in the stream function with respect to the change in the cavity is described in this part of the section.

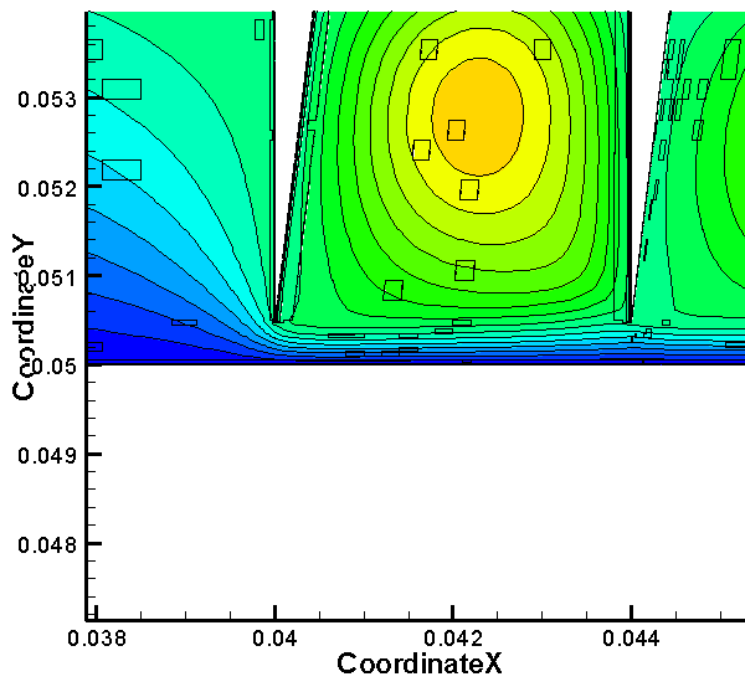


Figure 22 Stream Function Distribution for Right Triangle Shaped Seal , C=0.5mm,

Re=10000

The streamlines for the right triangle shape geometry (clearance 0.5mm) are shown in Figure 22 for a Reynolds number of 10000.

The observed flow pattern shows that:

- Most of the fluid flows right under the 2nd tooth, while the rest of the fluid re-circulates above the jet. Hence there is very little pressure recovery in the cavity since the jet did not spread.
- Due to the tooth edge being very narrow, there is no re-attachment of the flow.

The vena contracta effect is dominant just after 1st tooth due to large downward velocity of fluid entering in the seal. This results in the jet width being smaller than the tooth clearance. The jet of fluid then spreads, at a rate insufficient to hit the downstream tooth. Hence, there is very little pressure recovery in the cavity. The vena contracta effect after the 2nd tooth is much less than that of the 1st tooth. The jet spreads much more rapidly than in the seal cavity resulting in a net pressure rise between the cavity and downstream of the 2nd tooth.

The above factors lead to a lower pressure upstream of the second tooth (in comparison with seals with rectangular cavities) and cause a negative pressure drop.

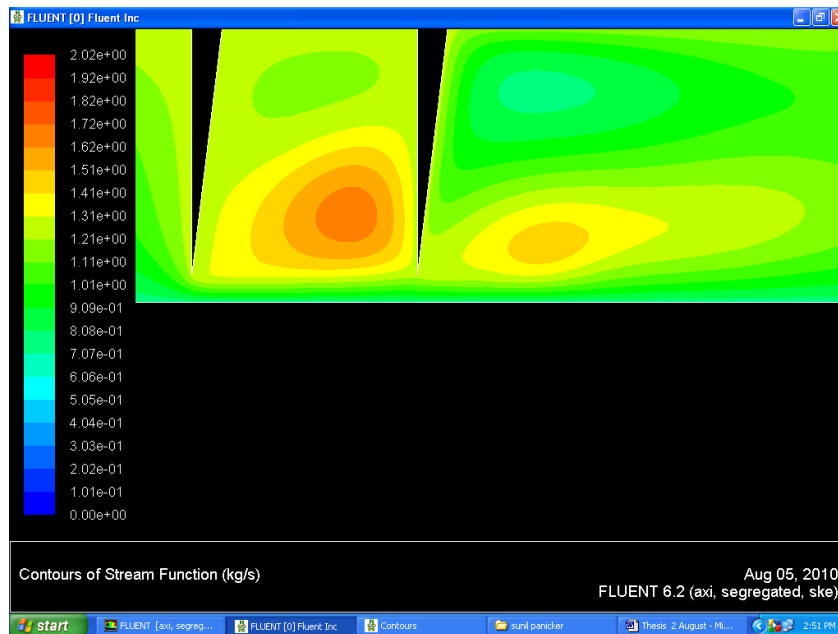


Figure 23 Right Triangle Shaped Stream Function, $C=0.5\text{mm}$, $Re=10000$

Similar distribution is witnessed in all the other cases of right triangular seals and Isosceles seals (stream functions shown in Figure 20 to 23), although the magnitudes of the stream functions might differ in other cases, the qualitative nature and pattern of the stream lines are similar in all cases. Hence the same physical reasoning holds in the other cases where negative pressure drop is observed. It appears that this phenomenon is caused by a combination of high clearances and cavity shapes that involve narrow tooth tips as well as large C/S ratios. Figure 23 show how the stream functions are aligned across the seal for different cavity shapes.

COMPARISON OF CFD PREDICTED CARRY OVER COEFFICIENT TO EXISTING MODEL FOR RECTANGULAR CAVITY

In this section, the carry over coefficient calculated from the CFD predicted flow field within the seal cavity for the Isosceles and right triangle seals are compared against earlier proposed models for carry over coefficient for Rectangular cavities.

The recent model proposed by Suryanarayanan [1] is valid only for clearance to pitch ratios less than 0.0375 which is at least three times smaller than the C/S values investigated in the present study. Therefore, Hodkinson's model [5] for Carry over coefficient is used for higher clearances studied in this work. It has to be noted that the Hodkinson's formula neglects Reynolds number dependence but it is used in this comparison to provide a rough understanding of the effect of cavity shapes.

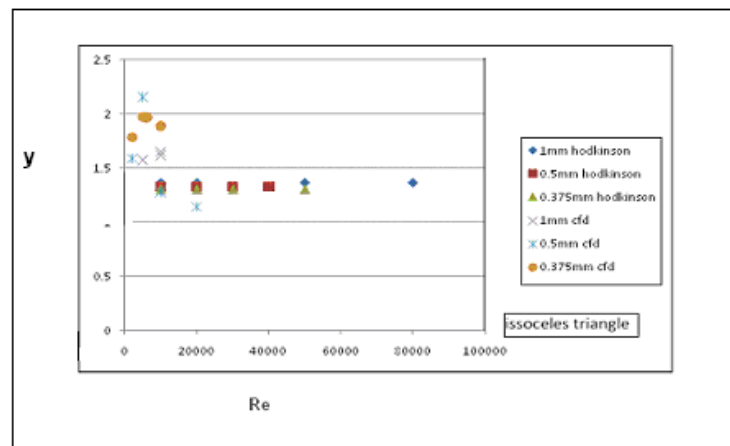


Figure 24 Comparison for Hodkinson Model with CFD Results for Isosceles Cavity Shape

It can be seen from the above Figure 24, while at lower Reynolds numbers the Isosceles seal has higher carry over coefficients than predicted by Hodkinson's formula, it is lower at higher Reynolds number. Though there are not sufficient data points at high Re to strongly support this conclusion, the trend indicates that Isosceles triangle seals could be efficient at high Reynolds numbers. Relatively, the carry over coefficient for right triangle design is much closer to Hodkinson's prediction as seen in Figure 25. Figures 24 and 25 illustrate the carry over coefficient are dependent upon the Reynolds number as well as the geometry. Hodkinson's formula only includes geometry dependence.

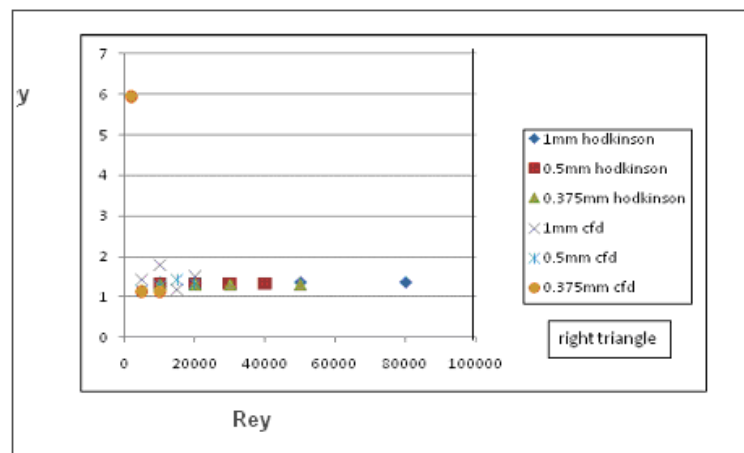


Figure25 Comparison for Hodkinson Model with CFD Results for Right Triangle Cavity

Shape

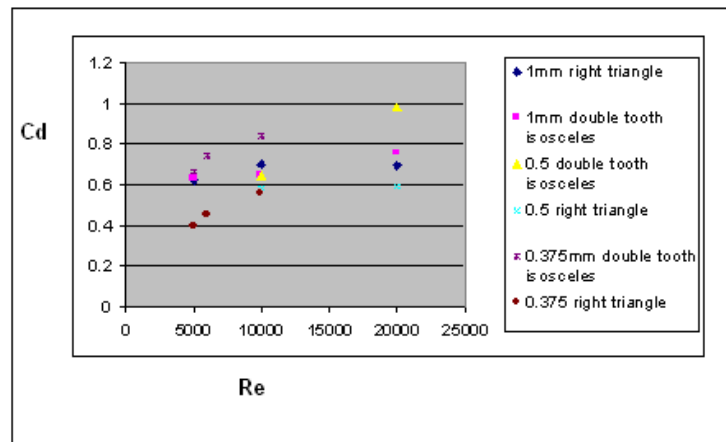


Figure 26 Comparison for Coefficient of Discharge for Isosceles Shaped Seal and Right Triangle Shaped Seal

Figure 26 shows the comparison between coefficients of discharge of Isosceles cavity shapes with respect to the right triangle cavity shape. The figure shows that at higher clearances of 1mm, the Cd of Isosceles triangle cavity shape have similar values to that of the right angled cavity shape. On moving to lower clearance of 0.5 mm, the Isosceles triangle shaped seal gives higher value of Cd than the right triangle. The difference in the Cd Isosceles, with respect to Cd right triangle, becomes even larger with increase in Reynolds number. As with increase in Reynolds number, the Cd Isosceles increases more as compared to Cd right triangle. This tendency is also seen in case of 0.375 mm, where the coefficient of discharge for the Isosceles triangle is much higher than that of the right triangle. The pressure drop across the cavity in the case of the Isosceles cavity shape is less than that of the right triangle cavity shape at lower clearances. This illustrates the fact that the pressure recovery around the 2nd tooth is

more for the Isosceles cavity as compared to the right triangle based cavity shape for the same range of clearance (clearance of 0.5mm and 0.375mm). This trend increases with increase in the Reynolds number. So it can be concluded that the Isosceles based cavity is more efficient than right triangle shaped cavity at lower clearances.

NASA SEAL

The NASA seal was used in the fuel pumps of the NASA space shuttle. The basic difference between this seal and other seals was its unique cavity shape (Figure A3). This seal has a cavity which is arc shaped between the first tooth and second tooth. The seal shaped showed a different nature for measuring parameters like coefficient of discharge and carry over coefficient.

NASA SEAL COEFFICIENT OF DISCHARGE

As already stated, the coefficient of discharge is an important parameter to gauge the performance of the seal. Therefore, it becomes evidently important to study the behavior of the coefficient of discharge with change in Reynolds number. For this reason two different seal clearances have been taken which are 1mm and 0.375mm respectively. The maximum pressure drop to be considered for measuring the coefficient of discharge values was taken as around 10 atm.

NASA SEAL WATER CASE

The NASA seal study was taken considered two clearances for 1mm and 0.375mm. These cases are discussed below. The larger clearance of 1mm shows that the range of coefficient of discharge is very low. The low coefficient of discharge signifies that pressure drop between the 1st tooth and 2nd tooth is very large. It is also seen that the

increase in Reynolds number the coefficient of discharge increases. It should be noted that these Reynolds numbers are very small and may not represent true operating conditions where C_d will be larger.

The circular shape at the top of the NASA seal plays a very important role in changing the coefficient of discharge. It has to be noted that this seal operates at very high Reynolds number in space, so the operating conditions differ with respect to earth. At low Reynolds number, the range of pressure drop observed is very high. However it can be seen from Figure 27, the coefficient of discharge increases for a smaller clearance (0.375 mm) with increase in Reynolds number; it means the pressure drop reduces with increase in Reynolds number. Thus, at very high Reynolds number, the pressure drop will be even lower. The pressure recovery across the 2nd tooth seems to increase with increase in high Reynolds number.

The higher clearance of 1mm shows the coefficient of discharge is low even at higher Reynolds number. This means that the pressure drop is high for the clearance of 1mm. This behavior is very different from the earlier cases, as it was seen that higher clearance implied lower coefficient of discharge for Isosceles and right triangle cavity shapes. The lower clearance of 0.375mm also shows a large pressure drop even at smaller Reynolds number. This trend continued to prevail at Reynolds number up to 5000. This concludes that the seal design was suitable at very high Reynolds number, and for very high pressure. It can be seen that the pressure recovery across the second tooth will be more even at higher Reynolds number. Figure 27 shows how the coefficient

of discharge changes with increase in the Reynolds number for incompressible fluid water.

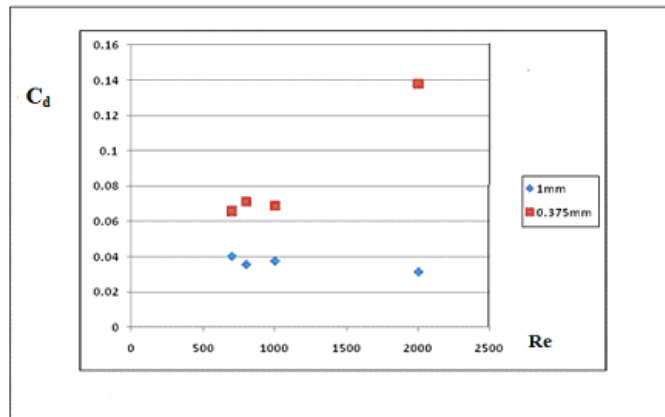


Figure 27 Effect of Coefficient of Discharge with Change in Reynolds Number

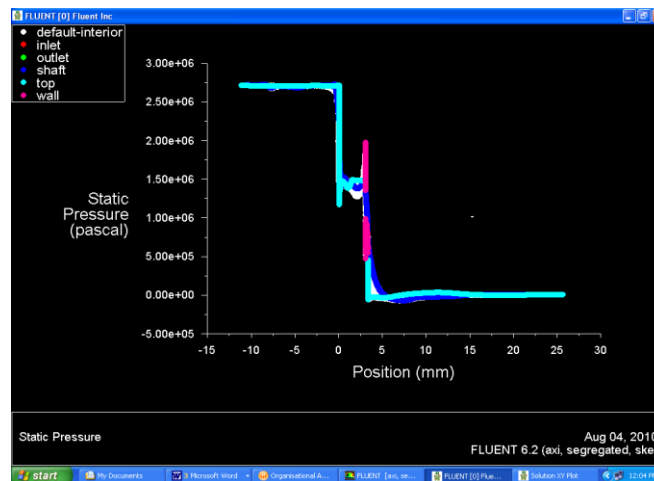


Figure 28 (a) NASA Seal Pressure Drop, Fluid= Water Clr= 0.375 mm, Re= 2000

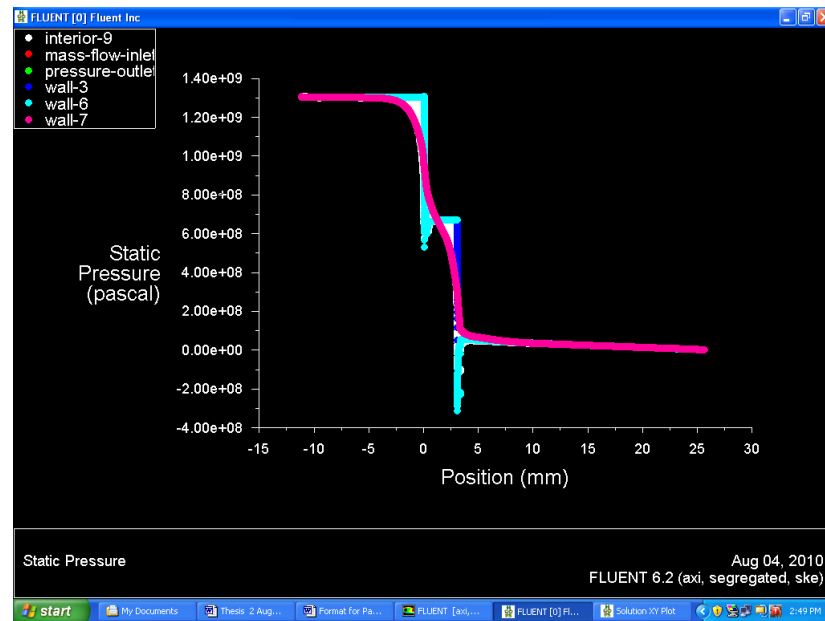


Figure 28(b) NASA Seal Pressure Drop, Fluid= Water Clr= 1mm, Re= 20000

Figures 28(a) and (b) shows how pressure drop varies for $clr = 1\text{mm}$, 0.375mm . The Reynolds number taken is 2000 and 20000 respectively in this case. It can be noted that the magnitude of the pressure increase with decrease in clearance.

STREAM FUNCTION OF NASA SEAL

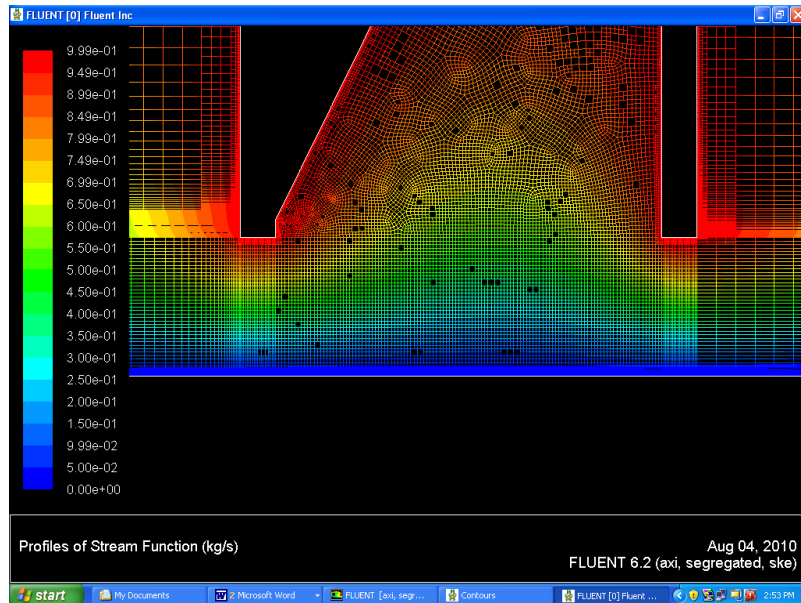


Figure 29(a) NASA Seal, Fluid= Water Clr= 1mm, Re= 20000 Stream Functions

The case of 1mm clearance is shown in Figure 29(a) in order to analyze the stream functions of the NASA seal. The stream functions near the 1st tooth deviate towards circular cavity of the seal. It is also seen here that stream functions form a curvature at the circular part of the seal. The rectangular tooth in front of the circular cavity restricts beta. The leakage rate is very high in $clr=1mm$ due to which fluid does not develop into a fully turbulent flow. This is due to a very high clearance of seal which offers minimum resistance to the fluid flowing through the cavity resulting in abnormal pressure drop. The pressure recovery across the second tooth is very less as most of fluid through the seal gets dissipated due to high clearance. The trend continues even at

higher Reynolds number of 20000. Therefore it is not advisable to use a seal with such a high clearance. Below Figure 29(b) shows the distribution of stream function across the NASA seal.

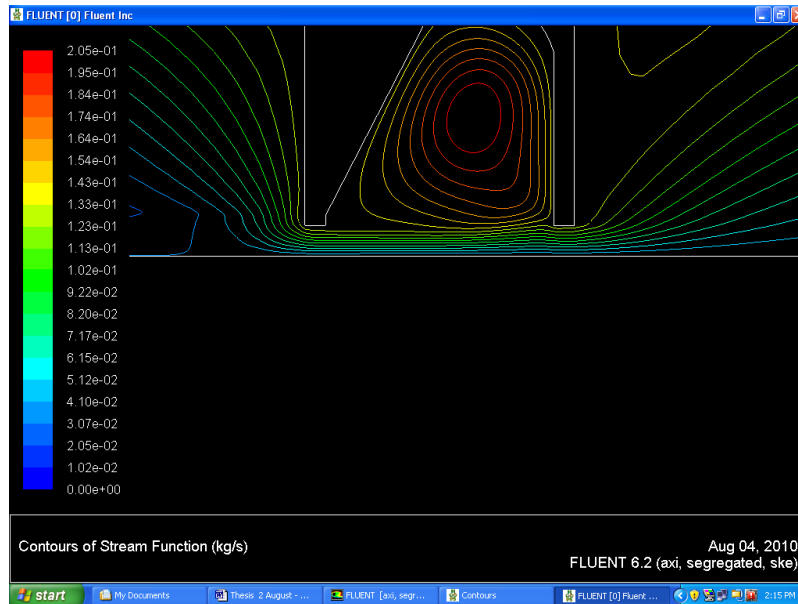


Figure 29(b) NASA Seal, Fluid= Water Clr= 0.375mm, Re= 2000

The smaller clearance of 0.375mm is analyzed here. It can be noted that at $Re=2000$ (shown here), the fluid flow becomes completely turbulent. The rectangular seal in front of the circular cavity restricts the flow of stream functions. The stream functions tend to direct towards the shaft assembly of the seal. Therefore beta becomes negative in this case. This due to the deviation of the stream functions towards the shaft (i.e. lower part of the seal). Here it can be seen from Figure 29(a) the pressure drop is high and abrupt (in order of 10^6 Pascals). However it is still lower than the pressure

drop for the 1mm case. This shows that fully developed turbulent flow gives lesser pressure drop and low leakage of the fluid.

NASA SEAL CARRY OVER COEFFICIENT

The carry over coefficient for the NASA seal has been investigated for two cases, one for the 1mm clearance and the other for the 0.375mm clearance. The carry over coefficient tends to be higher for the higher clearance and remains lower for the lower clearances. The elaborate discussion on case to case basis is given below.

The larger clearance of 1mm shows a large value of carry over coefficient i.e. around 10 to 14. The carry over coefficient tends to decrease from the Reynolds number 700 to 800 and increase from 800 to 1000. This can be attributed to the ratio of y velocity component to the ratio of x velocity component which is equal to $\tan \beta$ in this case. It has been already seen that $\tan \beta$ is inversely proportional to carry over coefficient. Hence, at 1 mm, the $\tan \beta$ is very low (sometimes even negative), which drastically increases the carry over coefficient. The negative beta angle is formed due to the restriction offered by the rectangular cavity to the stream functions. The stream lines near the 1st tooth tend to deviate downward towards the shaft assembly. This behavior is very different from other cavity shapes where β was positive and stream functions were inclined towards the upper part of the 2nd tooth. The change in the carry over coefficient with increase in the Reynolds number is depicted in the Figure 30.

The carry over coefficient increases with Reynolds number. This trend can be explained in a similar manner as done for Isosceles seals. Increase in the Reynolds number (ratio of the inertia of the stream jet issuing under the tooth to the viscous forces

under the tooth) results in smaller divergence of the jet. This results in the fluid passing under the tooth with low kinetic energy dissipation, resulting in a high carry over coefficient. This cavity shape shows a very high carry over coefficient at 1mm which is caused by most of the fluid passing underneath the tooth, whereas in a smaller clearance of 0.375mm most of the fluid dissipates kinetic energy due to turbulence viscosity interaction. It has to be noted that the carry over coefficient of this design is very sensitive to change in clearance.

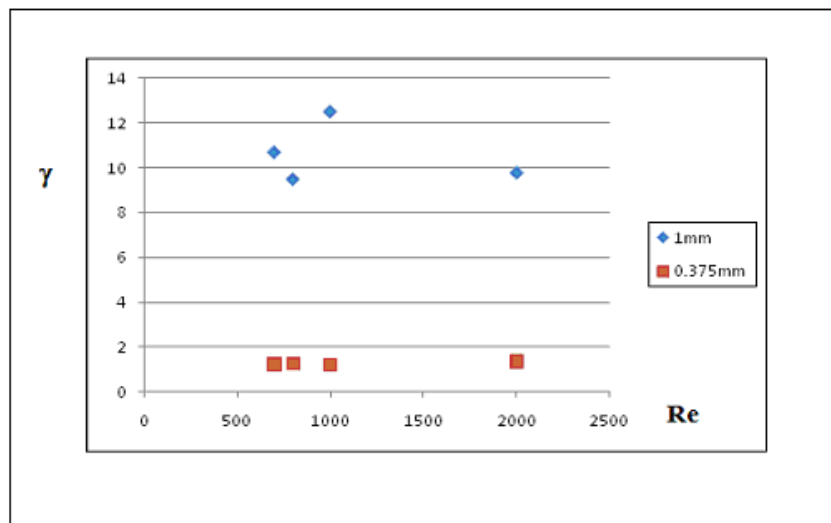


Figure 30 Carry Over Coefficient vs. Reynolds Number for NASA Seal, Water

The carry over coefficient for the lower clearance of 0.375mm, generally lie between 1 and 2. It is tangible that with increase in Reynolds number, the carry over

coefficient does not change very much and remains almost constant, this is due to a small change in the ratio of y-velocity component to x- velocity component which results in almost small change in tan beta.

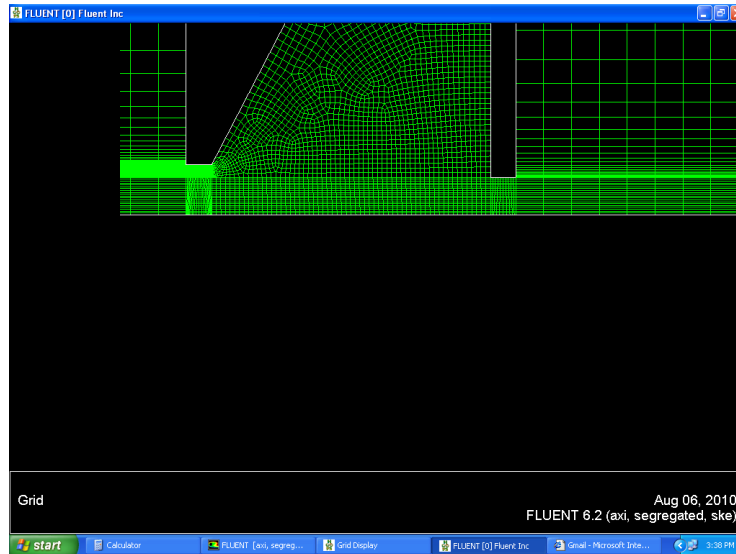


Figure 31(a) NASA Seal Modified Grid Shape (Trapezoidal 1st Tooth)

Figure 31(a) shows a proposed modification to the NASA seal. The new geometry shows that the line joining the inlet of the 1st tooth directly merges with the circular cavity of the seal. The geometry of the 1st tooth becomes trapezoidal, instead of rectangular. In the original seal, the shape of 1st tooth is rectangular.

The case of ‘Trapezoidal Nasa seal’ was treated as an axis symmetric, 2D problem. It was analyzed using FLUENT 6.3.2. The convergence criteria was set as 10^{-4} . The y^+ function was set as around 5. The case was analyzed for water (incompressible).

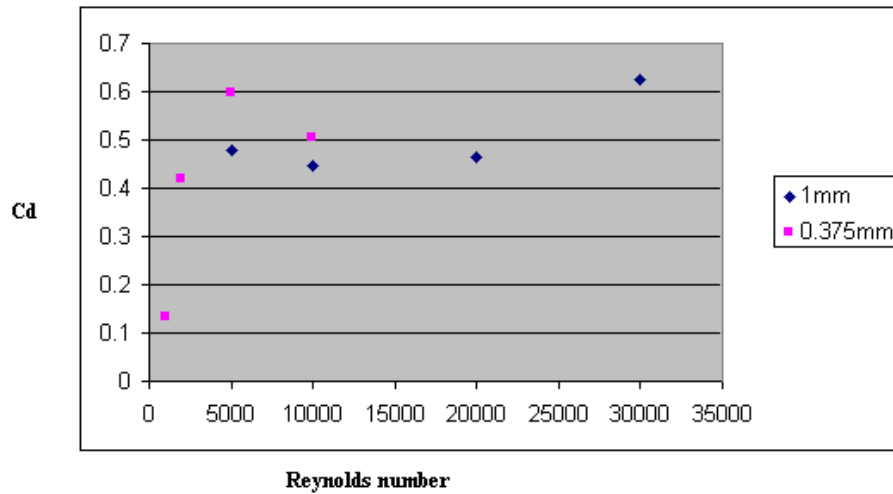


Figure 31(b) Coefficient of Discharge vs Reynolds Number for Modified 'Nasa Seal'

Figure 31 (b) shows the value of Cd for the 'Modified Nasa seal'. Again, two clearances of 1mm and 0.375mm have been taken in this case. For larger 1mm, the coefficient of discharge decreases from 0.48 to 0.45 for Reynolds number increase from 5000 to 10000. Cd then increases from 0.45 to 0.62 as the Reynolds number increases from 10000 to 30000. This is due to change in the pressure drop across the circular cavity of the seal. In case of 0.375mm, it was seen that coefficient of discharge increased from 0.013 to 0.60 as the Reynolds number increases from 1000 to 5000. Then Cd decreases from 0.6 to 0.5 as the Reynolds number increased from 5000 to 10000. The pressure recovery across the 2nd tooth was small. In some of the cases for both the clearances, the pressure was found negative across the 2nd tooth. This was due to negative Beta which will be discussed in detail in next section. The carry over coefficient needs to be checked for the modified NASA SEAL, to analyze the kinetic

energy carry over. Below Figure 32(a) shows the distribution of stream functions across NASA Seal.

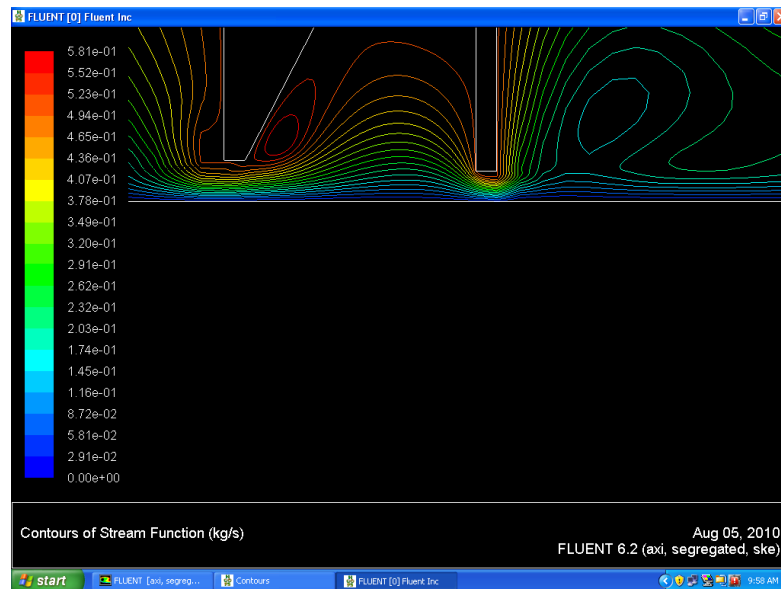


Figure 32(a) Nasa Seal (Modified1) Clr=0.375mm, Re=10000

The seal which was used by NASA in the turbojet pump gave a very high pressure drop, which was due to negative beta. However, a modified version of NASA seal is proposed in this thesis and its flow behavior is studied. Figure 32(a) shows the streamlines for this seal for a clearance of 0.375mm and a Reynolds number of 10000. In the modified NASA seal, the rectangular shaped profile present in front of the circular cavity was removed. The 1st tooth was directly connected to the circular cavity of the seal forming a trapezoidal shape around 1st tooth. The stream functions near the 1st tooth deviate towards the circular cavity making a positive 'beta angle. This behavior is very

different from the ‘original Nasa seal’ where the beta angle was negative due to the rectangular tooth profile situated at the beginning of the circular cavity (which resulted in abrupt pressure drop). The modified seal shows more uniform behavior and the pressure drop is also less (at standard conditions) in this seal.

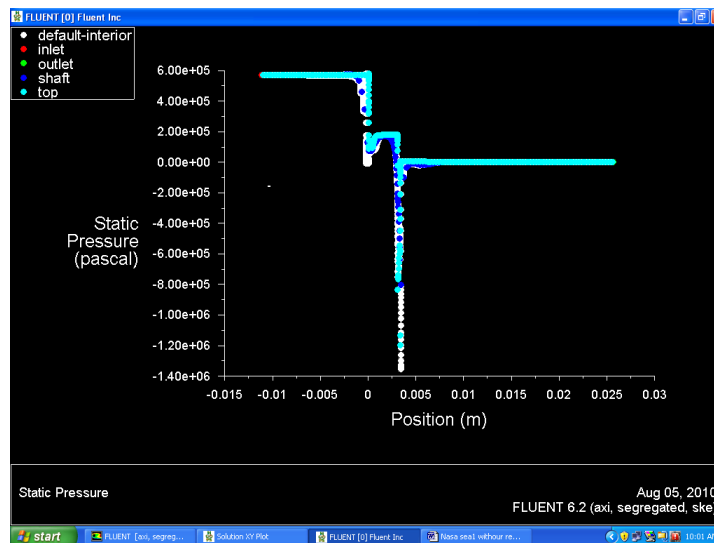


Figure 32(b) Nasa Seal (Modified2) Clr=0.375mm, Re=10000

However the stream functions continue to deviate towards the 2nd tooth from the circular cavity. This in turn results a negative pressure region around the 2nd tooth which can be seen in Figure 32(b). The trend of negative pressure drop continues to diminish as the stream functions become linear near the 2nd tooth. This behavior is attained at higher Reynolds number for the same clearance. Hence it is advisable to conclude that modified NASA seal behaves as a normal seal under standard conditions. However, it should be checked how it behaves in cryogenic fluids.

CHAPTER VI

COMPRESSIBILITY EFFECT

COMPRESSIBILITY EFFECT ON DIFFERENT TYPE OF SEALS

Many of the rotating equipment which use labyrinth seal operate on compressible fluids like air, CO₂ etc. So it becomes very important to study the behavior of the compressible fluid operating in a labyrinth seal. Saikishan Suryanaryanan [1] developed a model for air (compressible fluid model) which was able to predict the seal leakage accurately until the individual pressure ratio greater than 0.7. However, for lower air pressure ratios this model was not very accurate as the flow was choked. Hence, an expansion factor was introduced in the model by Saikishan Suryanaryanan [1] to compensate for the effects of compressibility.

Martin [4] and Egli [6] incorporated pressure ratio terms (given below) to compensate for the effect of compressibility.

$$\psi = \sqrt{(1 - (P_e/P_i)^2) / \sqrt{n - \ln(P_e/P_i)}} \quad \text{-----} \quad (6.1)$$

EXPANSION FACTOR

Saikishan Suryanarayanan [1] defined an expansion factor as the ratio of discharge coefficient of an individual tooth for a compressible flow to that of an incompressible flow through a seal having same geometry and at same Reynolds number. The leakage equation is modified by including this effect by the inclusion of ψ , where ψ is the ratio given in equation 6.1.

Suryanarayanan developed the following set of equations to predict the individual tooth discharge coefficient for incompressible flow. The range of pressure ratio is 0.1 to 1 .

where

$$C_d = C_d^{1\text{tooth}} \quad \text{for first constriction}$$

$$C_d = C_d^{1\text{tooth}}(0.925\gamma^{0.861}) \quad \text{for subsequent teeth}$$

$$C_d^{1\text{tooth}} = (0.7757 - 0.002051(w/c)) / [1 + \{44.86(w/c)/Re\}]$$

$$\gamma = [1 - 6.5(c/s) - 8.638(c/s)(w/s)] [R_e + R_o]^{(2.454(c/s) + 2.268(c/s)(w/s)^{1.673})}$$

$$R_o = (1 - 6.5(c/s) - 8.638(c/s)(w/s))^{(-1/2.454(c/s) + 2.268(c/s)(w/s)^{1.673})}$$

These equations are valid for rectangular cavity seals over the following ranges:-

For $0.0075 < c/s < 0.0375$, $0.0075 < w/s < 0.5$, $2.67 < w/c < 66.67$ and $0.75 < h/s < 4$, $250 < Re < 15000$

He also developed an expression for the expansion factor as a function of pressure ratio across each tooth.

The equation he developed is:

$$\psi = 0.558 Pr + 442 \text{-----} (6.2)$$

$$\text{for } 0.15 \ll Pr \ll 1$$

Figure 33(a) shows how the ψ (expansion factor) varies with change in pressure ratio for an Isosceles cavity shaped teeth. It is seen that the pressure ratio is able to depict the effects of compressibility. It is also assumed that regardless of the Reynolds number, exit pressure or other parameters, the ψ -Pr ratio establishes a linear

relationship. However, in case where negative pressure drop was seen at clearances like 0.5mm, the same does not hold true.

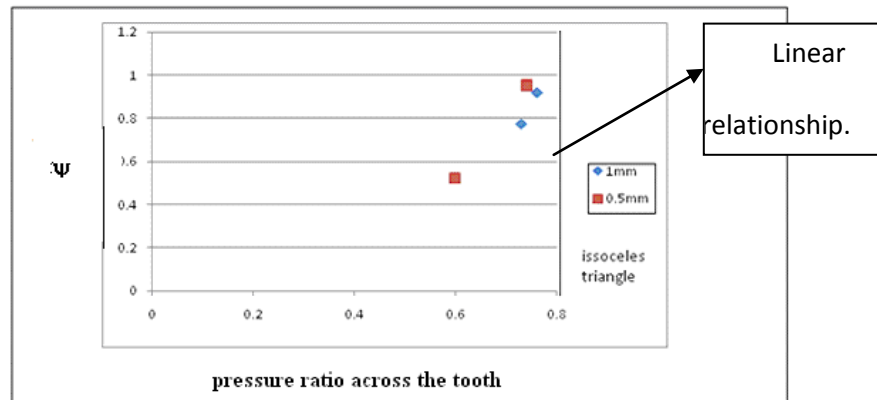


Figure 33(a) ψ vs. Pressure Ratio Across The Teeth for Isosceles Cavity Shaped Seal

Figure 33(b) shows the variation between the expansion factor ψ and the pressure ratio for right angular cavity shape. As discussed earlier, the pressure ratio of air is the governing factor to study the effect of the compressibility for various cavity shapes. The largest clearance of 1mm showed that the effects of compressibility were qualified by the pressure ratio. Hence a linear relationship of ψ -Pr was obtained in case of 1mm. However, in case of right triangle shaped cavity shape, it was noted that the negative pressure drop was observed in most of the especially at Clr=0.5mm. Therefore, in case of right triangle shaped cavity, the ψ -Pr does not show a linear relationship in case of 0.5mm.

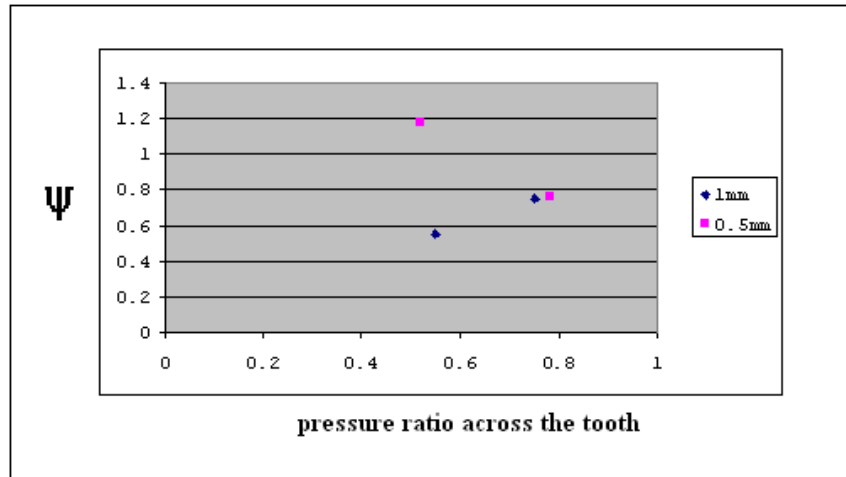


Figure 33(b) ψ vs. Pressure Ratio for Right Triangle Shaped Seal

CHAPTER VII

SUMMARY AND CONCLUSIONS

SUMMARY

The thesis narrates about how the seal behavior differs with change in the cavity shape. The seal which were considered were of 3 different cavity shapes. These were Isosceles triangle, right triangle cavity shaped and NASA seal. The effect of Reynolds number upon seal leakage was illustrated. The objective of the thesis was to investigate which cavity shape was best suitable for steam turbines and gas turbines. This thesis covered high Reynolds number values to observe, how the seals performed for very high Reynolds number. This thesis also addresses the different behavior of both compressible fluid like air and incompressible fluid like water.

The first case which was considered was a single tooth Isosceles tooth and was a simple design to start with. The analysis for this design included both water and air. The study showed that the coefficient of discharge decreased with increase in Reynolds number, in case of air, for clearance values 1mm, 0.5mm and 0.05mm, however a reversal of trend was observed in case of 0.375mm. It was estimated that at this clearance value, the fractional increase in the mass flow rate, with respect to the pressure drop, increased the coefficient of discharge with increase in the Reynolds number. The 2nd case which was considered was the study of a single Isosceles shaped tooth with water which was an incompressible fluid. This showed a more even behavior with change in the Reynolds number, the coefficient of discharge decreased for all the cases which were, 1mm, 0.5mm, 0.375mm and 0.05mm.

The next case was taken in order to measure the performance of the coefficient of discharge and carry over coefficient in presence of a 2nd tooth. In this case analysis was done for compressible air and incompressible water. For the case of air, four different clearances of 1mm, 0.5mm, 0.375mm and 0.05mm were considered. It was observed that the largest clearance produced a higher coefficient of discharge at low Reynolds number. As the Reynolds number increased it initially decreased and became constant with increase in Reynolds number. This behavior was predominantly seen in other clearances as well, with change in Reynolds number. The carry over coefficient also shows a change in value, as it decreases with increase in the Reynolds number up to 20000, and then starts increasing up to a Reynolds number of 50000. All other clearances considered performed in the same manner and possessed the same trend.

Incompressible water showed a different nature compared to compressible air. It was seen that the coefficient of discharge rose steeply with rise in the Reynolds number, similarly the Υ varied very little with change in Reynolds number.

Right triangular shaped cavity seals showed a decrease in coefficient of discharge with increase in Reynolds number for a low Reynolds number value, and then it started to increase for higher Reynolds number in case of air. This tendency was seen by all the clearance values. The carry over coefficient showed almost a constant value with increase in Reynolds number. All the clearances showed the same nature of curve, which was almost constant, but at the same Reynolds number for different clearances, it was seen the Υ appeared to be different.

The negative pressure drop across an individual seal tooth was one of the most important findings of this thesis. It was observed that for certain cavity shapes, a negative pressure drop is observed across the 2nd tooth of two teeth, resulting in an increase of coefficient of discharge and change in carry over coefficient. The negative pressure drop was mainly due to the straightening and flattening of stream lines at some specific Reynolds number, this phenomenon was observed in Isosceles cavity shape and right triangle cavity shaped seals. It was most prevalent for large clearance to pitch ratios.

The NASA seal showed a very low coefficient of discharge at low Reynolds number. This was due to the high pressure drop attained at very low Reynolds number. The NASA seal showed that circular geometrical shape was responsible for such high pressure drop in low Reynolds number. However, it appears C_d increases as the Reynolds number increases.

The ψ vs. pressure ratio (the effects of compressibility) of air showed a linear relationship in case of rectangular seals. The same relationship was observed for the case of the Isosceles cavity shaped seals. However, this relationship did not exist where the negative pressure drop was observed. It can be concluded that linear relationship is observed (in Isosceles cavity shapes) where there is positive or normal pressure drop. The same holds true for right triangle shaped cavity shapes as well.

The simulations showed that the Isosceles cavity shaped seals were more efficient than right triangle cavity shaped seals for both incompressible and compressible fluid for the same clearance. NASA seal was the most efficient seal for incompressible

fluid for higher clearances; however with decrease in clearance the efficiency of NASA seal became lower compared to Isosceles cavity shape, but was higher than right triangle cavity shapes.

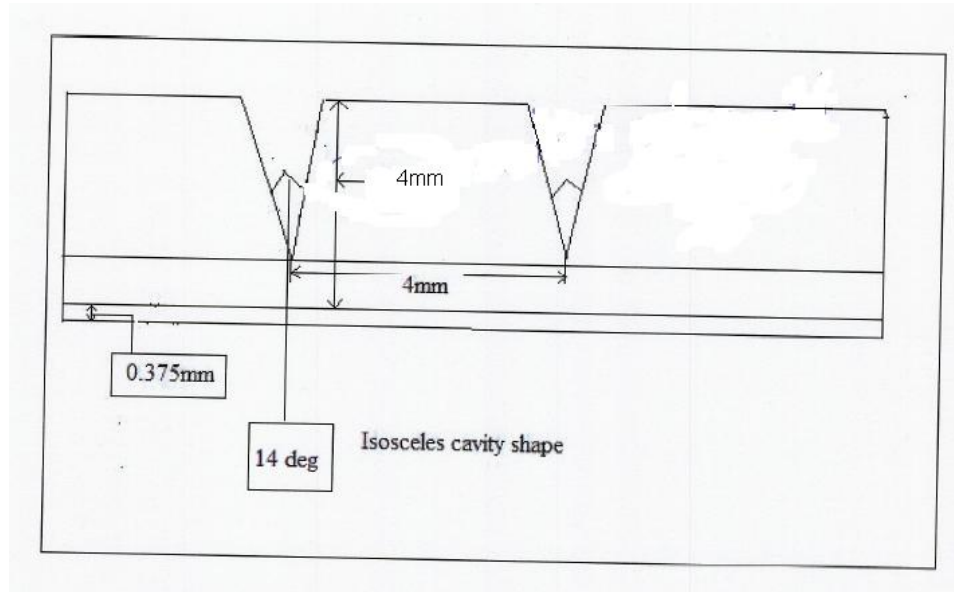
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APPENDIX A

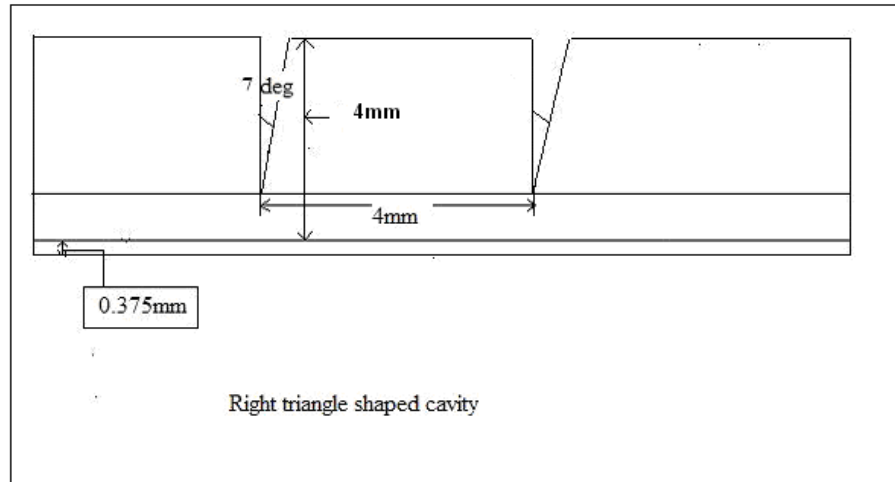
The drawings of mechanical seals are given below:

Isosceles shaped seal drawing



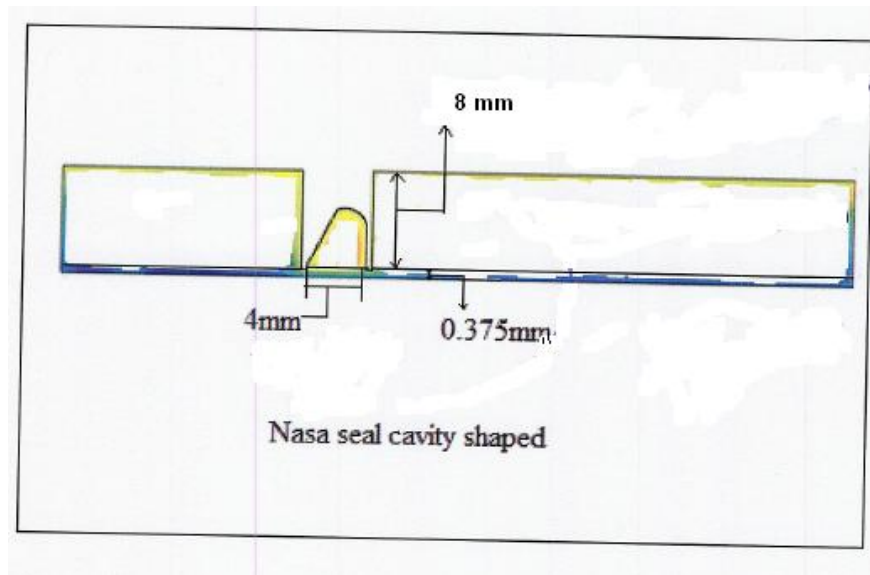
Figure# A1

Right triangle shaped cavity



Figure# A2

Nasa seal cavity shaped design



Figure# A3

VITA

Sunil Murlidher Panicker is the son of Mr. M.D. Panicker and Mrs. Sathi Devi Panicker. He was born in Trichur, Kerala. He completed his Bachelor of Engineering in mechanical engineering from Rajiv Gandhi Technical University, Madhya Pradesh with Honors. Prior to joining Texas A&M University, he worked at Reliance Industries Limited for 2 years.

He arrived in the United States of America to pursue his graduate studies in the field of mechanical engineering at Texas A&M University in fall 2008. He received his Master of Science in mechanical engineering in December 2010. He wishes to pursue a career in turbomachinery.

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