ONLINE AUCTIONS:
THEORETICAL AND EMPIRICAL INVESTIGATIONS

A Dissertation

by

YU ZHANG

Submitted to the Office of Graduate Studies of
Texas A&M University
in partial fulfillment of the requirements for the degree of

DOCTOR OF PHILOSOPHY

August 2010

Major Subject: Economics

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Approved by:

Co-Chairs of Committee,	Brit Grosskopf
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ABSTRACT

Online Auctions:

Theoretical and Empirical Investigations. (August 2010)

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This dissertation, which consists of three essays, studies online auctions both theoretically and empirically.

The first essay studies a special online auction format used by eBay, "Buy-It-Now" (BIN) auctions, in which bidders are allowed to buy the item at a fixed BIN price set by the seller and end the auction immediately. I construct a two-stage model in which the BIN price is only available to one group of bidders. I find that bidders cutoff is lower in this model, which means, bidders are more likely to accept the BIN option, compared with the models assuming all bidders are offered the BIN. The results explain the high frequency of bidders accepting BIN price, and may also help explain the popularity of temporary BIN auctions in online auction sites, such as eBay, where BIN option is only offered to early bidders.

In the second essay, I study how bidders' risk attitude and time preference affect their behavior in Buy-It-Now auctions. I consider two cases, when both bidders enter the auction at the same time (homogenous bidders) thus BIN option is offered to both of them, and when two bidders enter the auction at two different stages (heterogenous bidders) thus the BIN option is only offered to the early bidder. Bidders' optimal strategies are derived explicitly in both cases. In particular, given bidders' risk atti-

tude and time preference, the cutoff valuation, such that a bidder will accept BIN if his valuation is higher than the cutoff valuation and reject it otherwise, is calculated. I find that the cutoff valuation in the case of heterogenous bidders is lower than that in the case of homogenous bidders.

The third essay focuses on the empirical modeling of the price processes of online auctions. I generalize the monotone series estimator to model the pooled price processes. Then I apply the model and the estimator to eBay auction data of a palm PDA. The results are shown to capture closely the overall pattern of observed price dynamics. In particular, early bidding, mid-auction draught, and sniping are well approximated by the estimated price curve.

To My parents, husband, and my son, Michael

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CHAPTER I

INTRODUCTION

Auction is a market mechanism of buying and selling goods by offering them up for bid, and then selling the item to the highest bidder. Auctions are used when the seller has uncertainty about the maximum amount each bidder is willing to pay, or the values that bidders attach to the item. Otherwise, the seller could just offer the item to the bidder with the highest willingness to pay (value) at or just below what this bidder is willing to pay. The uncertainty regarding values facing both sellers and buyers is an inherent feature of auctions.

Common auction institutions include English auction, Dutch auction, First-price sealed-bid auction and Second-price sealed-bid auction. In the English auction, which is also called open ascending price auction, the auctioneer begins by calling out a low price and raise it. The auction stops when there is only one interested bidder. This bidder wins the item and pays the price at which the second-last bidder dropped out. Dutch auction is named for its best known example, the Dutch flower auctions. The auctioneer begins by calling out a high price. The price is lowered until some bidder indicates his interest. The item is then sold to this bidder at the given price. In a first-price sealed-bid auction, bidders submit their bids privately. The highest bidder wins the auction and pays his own bidder. In a second-price sealed-bid auction, the highest bidder wins the auction and pays the second highest bid.

Auctions elicit information from potential buyers regarding their willingness to pay in the forms of bids. And the outcome of the auction, that is, who wins the auction and pays how much, is determined by the bids submitted by the buyers. For

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The journal model is *Econometrica*.

example, in an English auctions or a second-price sealed-bid auctions, the highest bidder wins and pays a price close to the second highest bidder valuation, and it is weakly dominant strategy for every bidder to bid his true valuation. In an auction, from the perspective of the seller, the revenue is most important. However, from the perspective of society, efficiency, that the bidder with the highest valuation wins the auction, may be more important.

The growth of computers and internet technology has increased the popularity of auctions. The online auction sites, such as eBay, ubid, overstock, etc., provide a new platform where people can buy and sell any products or services overcoming geographic barrier and fully participating in the benefits of the global economy. Trading on the internet allows a business to reach a large number of buyers and a large number of sellers with lower cost and around-the-clock availability. Increasing popularity of online auctions generate a great interest in the economics of auction. Besides the common auctions forms described above, there are some new formats online auctions that deserve in-depth investigation. In The large amount of real-time or historical data available to researchers also makes empirical auction studies possible.

My dissertation studies online auctions both theoretically and empirically.

The first essay of my dissertation studies a special online auction format used by eBay, "Buy-It-Now" (BIN) auctions, in which bidders are allowed to buy the item at a fixed "Buy-It-Now" (BIN) price set by the seller and end the auction immediately.

Founded in 1995, eBay has become the world leader in the online auction business with more items and categories than any of its competitors. eBay offers several types of auctions, including regular auction, auction with a Buy-It-Now option, Buy-It-Now Only (fixed price format), and fixed price format with best offer. As in April of 2008, eBay's Buy-It-Now business made up 42% of all goods sold on eBay. It is the fasted growing eBay shopping business. One important feature of eBay Bin auction is that, once any bidder submits a bid higher than the reserve price, the BIN price disappears and bidders arrive after have no BIN options. However, most existing models studying BIN auctions (e. g. Reynolds and Wooders [**34**], Mathews and Katzman [**25**], Mathews [**24**]) assume all bidders are offered BIN options. Few models (Gallien and Gupta [**11**], Ivanova-Stenzel and Kroger [**19**])assume bidders enter the auction at different time and the BIN option is available only to early bidders. However, under this condition, the game is difficulty to solve and these models have to make many simplifications and assume special forms of value distributions or utility functions.

In the first essay of my dissertation, I constructs a theoretical model for BIN auctions with two stages in which one group of early bidders are offered a BIN option. If it is not taken, the second stage starts with an additional group of bidders as a standard second-price sealed-bid auction. In the model, I assume bidders have any general form of value distribution functions, and general concave utility forms, rather than assuming risk neutral or CARA risk averse.

The main findings are as follows. If the seller sets the BIN price less than the maximum certainty equivalent payment, there exists a unique equilibrium cutoff for the early bidders to accept the BIN price as long as his valuation is higher than the cutoff point, and reject it otherwise. Bidders are more likely to accept the BIN option in this two-stage model, compared with previous models which assume all bidders are offered the BIN. The seller revenue in this model is also higher than that in previous models. These results may explain the high frequency of bidders accepting BIN price, and may also help explain the popularity of temporary BIN auctions in online auction sites, such as eBay, where BIN option is only offered to early bidders.

Fixing the total number of bidders, the equilibrium cutoff decreases with the number of late bidders. This result may explain the current experimental and field findings that the frequency of the BIN option being taken is higher than theoretical predictions under the assumption that all bidders enter the auction simultaneously.

In the second essay, I studied how bidders' risk attitude and time preference affect their behavior in Buy-It-Now auctions. Experimental and field studies (Shahriar and Wooders [**37**], Ivanova-Stenzal and Kroger [**19**]) have found that buyers accept the BIN price too frequently, compared with the theoretical predictions when assuming bidders are risk averse. This suggests, in addition to, risk attitude, other factors such as time preference may also affect bidders' buy decision. Extended from the model in the first essay, a simple two-bidder model was constructed including both risk attitude and time preference. I consider the case when both bidders enter the auction at the same time (homogenous bidders) and the BIN option is offered to both of them, and the case when two bidders enter the auction at two different stages (heterogenous bidders) and the BIN option is only offered to the early bidder. Bidders' optimal strategies, in particular, the cutoff valuation, such that a bidders will accept BIN if his valuation is higher than the cutoff valuation and reject it otherwise, are derived explicitly in both cases. The cutoff valuation in the case of heterogenous bidders is lower than that in the case of homogenous bidders. The theoretical results generated in this model can be tested using laboratory experiments. At the end of this essay, I propose an experiment design, consisting of four components, eliciting risk attitudes, eliciting time preference, two-stage BIN auctions with two homogenous bidders, and two-stage BIN auctions with heterogenous bidders, which may be used to validate the theoretical results.

The third essay focuses on the empirical modeling of the price processes of online auctions. The bidding process of online auctions is typically marked by some common features such as, monotonically increasing bids, uneven temporal distribution, bid sniping, and final price bounded by the market price. Several studies, for example, Shumeli and Jank [**38**] and Wang et al. [**42**] have investigated the online auction price process using functional data analysis. However, those studies didn't provide much economic implications.I generalize the monotone series estimator by Ramsay [**32**] to model the pooled price process. Then I apply the model and the estimator to eBay auction data of a palm PDA. The data consist of an unbalanced panel of bids arrived at unevenly-spaced times during the auctions, including seller reputation, starting bid, all the bids and bidding time, and winning price. The price processes are modeled as monotone price growth processes (Zhang et al. [**43**]) plus auction-specific effects. I incorporate individual specific slopes for the relative price growth curve, which is modeled as a fixed effect, and also include an auction specific intercept, which is modeled as a random effect. Moreover, I model the coefficients for the auction attributes as time varying functions since the price processes of typical online auctions are often bounded by the market price. The results are shown to capture closely the overall pattern of observed price dynamics. In particular, early bidding, mid-auction draught, and sniping are well approximated by the estimated price curve.

The main contribution of the third essay is that the model is driven by economically interesting and important features of auction data, such as the distinct uneven arrival process, the common upper bound, and bid sniping. In addition, this innovative econometric model also tackled several technical difficulties, such as monotone constraints, time varying effects and multi-level individual effects. This able is able to capture the pattern of online auction closely and provide a flexible and practical approach for the modeling and real time prediction of online auctions.

CHAPTER II

TWO-STAGE BUY-IT-NOW AUCTIONS

2.1. Introduction

In recent years, an auction format called “Buy-It-Now” (BIN) auction has appeared in many auction sites such as eBay, Yahoo and Ubid. In a BIN auction, the bidders are allowed to buy the items at a fixed “Buy-It-Now” (BIN) price set by the seller and end the auction immediately. One important feature of eBay’s “Buy-It-Now” auction is that the BIN price is temporary and disappears as soon as a bid is made at or above the reserve price.

As of April 2008, eBay’s “Buy-It-Now” business made up 42% of all goods sold on eBay (Business Week, 2008). It is growing at an annual 22% pace, the fastest among eBay’s shopping business. The increasing popularity of the BIN auctions raises several interesting questions for economists. Under what conditions would a buyer purchase the item at the BIN price? Under what conditions would a seller set a BIN price in an auction? How would a seller set the reserve price and the BIN price in a BIN auction? There are several theoretical studies attempting to answer these questions (e.g. Hidvegi et al. [**15**], Reynolds and Wooders [**34**]). While most of the existing models of BIN auctions assume the BIN option is offered to all bidders in an auction, our model is motivated by the observation that in eBay auctions, the BIN price is observed only by a portion of bidders who arrive early. If some early bidder places a bid above the reserve price, the BIN price disappears and the bidders who come late do not have the BIN option.

We construct a two-stage temporary BIN auction model with two groups of bidders entering the auction in different stages. In the first stage, a group of bidders (*early* bidders) are offered a "Buy-It-Now" (BIN) option by the seller to purchase the item immediately at a listed price (BIN price). If no early bidder accepts the BIN price, the BIN option disappears and the second stage starts with an additional group of bidders (*late* bidders). Both early and later bidders then participate in a second-price sealed-bid auction with no BIN option. Hence the late bidders do not have any information about the BIN price.

While most theoretical models in BIN auctions assume that the bidder utility functions are in special forms such as linear functions and CARA utility functions, our theoretical results are derived with general concave utilty functions. We establish the existence and uniqueness of a cutoff equilibrium such that an early bidder will accept the BIN option if his valuation is higher than the cutoff valuation. We show that if the seller sets the BIN price low enough, an early bidder accepts the BIN price only if his valuation is higher than an equilibrium cutoff valuation. This equilibrium cutoff valuation exists as long as the bidders' utility function is monotonic increasing and is unique when bidders are risk averse or risk neutral. Other things being equal, the cutoff valuation decreases with bidders' degree of risk aversion and increases with the number of early bidders. Therefore, risk averse bidders in our model are more likely to accept the BIN options than in the model which assumes that BIN option is available to all bidders.

When bidders are risk averse, by setting an appropriate BIN price, the seller can obtain a higher expected revenue in a two-stage BIN auction than in a second-price sealed-bid auction with the same reserve price, and the expected revenue decreases with the number of early bidders. Therefore, compared to an auction with BIN option available to all bidders, our model results in a higher expected seller revenue. These results may help explain the popularity of temporary BIN auctions on online auction sites such as eBay and the observed high acceptance rates of BIN prices in experimental and field studies.

The rest of the paper is organized as follows. In section 2 we review the current literature related to BIN auctions and discuss the contributions of this paper relative to the existing literature. We present our model in section 3, in which we characterize the bidders' equilibrium strategies in section 3.1, investigate the factors which affect the equilibrium in section 3.2, illustrate the equilibrium analysis of the entire auction using a simple example in section 3.3, and investigate the seller's revenue and identify the conditions under which a BIN auction raises more seller revenue than a standard second-price sealed-bid auction in section 3.4. Section 4 contains the conclusion of this paper. All mathematical proofs are gathered in Appendix.

2.2. Literature Review

Several studies have investigated the BIN auctions theoretically. Budish and Takeyama [**8**] show that a seller facing two risk averse bidders may improve its expected profit by using an optimal permanent BIN price. Hidvegi et al. [**15**] extend the model with an arbitrary number of bidders and continuous valuation distribution and showed that the seller receives higher expected revenue in an auction with a permanent BIN price than in a standard second-price auction, provided that buyers are risk averse.

Reynolds and Wooders [**34**] characterize equilibrium bidding strategies for bidders with CARA utility function in both temporary and permanent BIN auctions. They show that for both auctions, when bidders are risk averse with CARA utility function, by setting an appropriate BIN price, a seller can raise higher revenue than in a second-price sealed-bid auction. Mathews and Katzman [**25**] considers temporary BIN auctions with risk neutral bidders and a risk averse seller. They find that when seller is risk averse, setting a BIN price may result in a Pareto improvement compared to a sealed bid second price auction. Mathews [**24**] studies the impact of time discounting on the BIN auctions. He finds that time discounting by either the seller or the bidders can lead to the seller choosing a BIN price which results in the option being exercised with positive probability.

All the papers mentioned above assume that all bidders enter the auction simultaneously and are all offered a BIN option. However, one important feature of a temporary BIN price auction is that, if anyone places a bid higher than the reserve price, the BIN option disappears. Therefore, all bidders entering afterwards will not be able to observe the BIN price and thus participate in a standard second price auction. This creates an information heterogeneity among the bidders such that early bidders have observed the BIN price, while late bidders have no information about the BIN price. Consequently, those early bidders who observe the BIN price will not bid above the BIN price hence their valuation follow a truncated distribution. On the other hand, the bid distribution for the late bidders still follow the original distribution. Therefore, it is important to consider the case when only a portion of bidders are offered the BIN option.

Gallien and Gupta [**11**] formulate a model featuring time-sensitive bidders with uniform valuations and Poisson arrivals. They solved seller's utility maximization problem by simulation and showed that the permanent BIN price auction leads to higher seller utility than the temporary BIN price auction does. However, they assumed that the seller maximizes his utility only by setting an optimal BIN price, without considering the reserve price. While the BIN price sets the upper-bound for the winning bid, the reserve price sets the its lower-bound. In a standard English auction, a revenue maximizing seller always set a reserve price that exceeds his value (Myerson [**27**]). Therefore, it is important to consider the reserve price in analyzing the auction equilibrium.

Ivanova-Stenzel and Kroger [**19**] consider an auction in which the seller offers the BIN price only to one bidder, say bidder one. If bidder one accepts the BIN price then the auction ends. Otherwise, a standard second-price sealed-bid auction with no reserve price starts with n bidders. They derive equilibrium strategy for bidder one but not for the seller. Furthermore, they assume that a zero reserve price when solving the equilibrium. Because online auctions are usually monitored by a large number of potential bidders and last for more than a week, it is rare that only one bidder has observed the BIN price and has the opportunity to purchase the item immediately. Therefore, to better understand online auctions with a temporary BIN option, it is important to consider a general model that allows an arbitrary number of bidders who are offered the BIN option.

There are also some empirical and experimental studies on BIN auctions. Shahriar and Wooders [**37**] suggest that a suitably chosen BIN price raises seller revenue with risk averse bidders. However they report that the frequency of BIN price being accepted is higher than theoretical prediction under the assumption that all bidders are offered the BIN option (Reynolds and Wooders [**34**]). Similar results are also found by Ivanova-Stenzel and Kroger [**19**].

In this study, we propose a model that captures the information heterogeneity among bidders in a temporary BIN auction by assuming that the BIN option is only available to one group of bidders. While most theoretical models in BIN auctions assume that the bidder utility functions are in special forms such as linear functions and CARA utility fuctions, we establish our theoretical results for general concave utilty functions $u(x)$ with $u(0) = 0$, $u'(x) > 0$ and $u''(x) \leq 0$.

There are two groups of bidders in the model: early bidders and late bidders. Early bidders enter the auction in stage one and observe the BIN price. They decide simultaneously whether to accept the BIN price and win the item immediately or to reject it. If no early bidder has accepted the BIN price, the BIN option disappears and the stage two starts and late bidders enter the auction. In stage two auction, both early bidders and late bidders participate in a second-price sealed-bid auction. We solve bidder's equilibrium strategy in a model allowing for an arbitrary number of risk neutral or risk averse bidders and continuous valuation distribution. When bidders are risk averse, we show that with some appropriately chosen BIN price, the seller expected revenue is higher in a BIN auction than in a second-price sealed-bid auction with the same reserve price. Furthermore, the seller revenue decreases with the number of early bidders. These results may help explain the popularity of temporary BIN auctions in online auction sites such as eBay. They also provide an explanation of some experimental and field findings that the frequency of bidders accepting BIN price is often higher than the theoretical predictions under the assumption that all bidders are offered the BIN option. The model is described below.

2.3. The Model

We assume that there is one indivisible good in the auction. The seller sets a reserve price r and a BIN price p. There are n bidders with each bidder's valuation drawn from a common continuous distribution $F(v)$ with support $[\underline{v}, \overline{v}]$ and density $f(v) = F'(v)$. Bidders are risk averse or risk neutral with utility function $u(x)$. Assume $u(0) = 0$, $u'(x) > 0$ and $u''(x) \leq 0$.

The auction is assumed to consist of the following two stages.

Stage One There are n_1 $(0 < n_1 \leq n)$ early bidders at the beginning of the auction. All early bidders observe the BIN price p and decide simultaneously whether to accept p or not. If an early bidder accepts p, he wins the item immediately and the auction ends without entering the second stage. If there are more than one early bidder accept the BIN price, a winner will be selected randomly among them. If no one accepts p (i.e., all early bidders reject p), the BIN option disappears and auction enters the second stage.

Stage Two There are n_2 late bidders ($0 \leq n_2 < n$ and $n_1 + n_2 = n$) entering the auction at stage two with no information about the BIN price. Both early bidders and late bidders participate in a standard second-price sealed-bid auction with a reserve price r. The highest bidder among them wins the auction. The winning price is the second highest bid.

2.3.1. Bidders' Equilibrium Strategies

We first derive bidders' equilibrium strategy in the two-stage auction presented above. Reynolds and Wooders [**34**] shows that under CARA, there is no loss of generality in restricting attention to a "cutoff" strategy in equilibria of stage one. A cutoff strategy for an early bidder i in stage one is characterized by a value $c \in [p, \overline{v}]$ such that he accepts the BIN price p if $v_i \geq c$ and reject the BIN price p if $v_i < c$.

Suppose one early bidder accepts the BIN price. The probability that there are another $k_1 - 1$ early bidders ($1 \leq k_1 \leq n_1$) who accept the BIN price is

$$\binom{n_1 - 1}{k_1 - 1} (1 - F(c))^{k_1 - 1} F^{n_1 - k_1}(c).$$

Define the expected payoff of an early bidder with valuation v_i who accepts the BIN price as $\pi_p^1(v_i)$,

$$\pi_p^1(v_i) = \sum_{k_1=1}^{n_1} \binom{n_1 - 1}{k_1 - 1} (1 - F(c))^{k_1 - 1} F^{n_1 - k_1}(c) \frac{u(v_i - p)}{k_1} = u(v_i - p) \frac{1 - F^{n_1}(c)}{n_1(1 - F(c))}.$$

When a bidder rejects the BIN price, he will not obtain the item if any other early bidders accept the BIN price. If no bidder in stage one accepts the BIN price, all early bidders participate in a standard second-price sealed-bid auction in stage two with the addition of n_2 late bidders.

The auction enters the stage two with a probability of $F^{n_1}(c)$ when no early bidders accepts the BIN price. Since an early bidder rejects the BIN price only if his valuation is lower than the cutoff point c, the valuations of early bidders in stage two follows a truncated distribution $F_e(v)$ with support $[\underline{v},\ c]$. Define the truncated distribution

$$F_e(v) = \begin{cases} F(v)/F(c), & \text{if } x \in [\underline{v}, c] \\ 1, & \text{otherwise} \end{cases}$$

with corresponding density function

$$f_e(v) = \begin{cases} f(v)/F(c), & \text{if } x \in [\underline{v}, c] \\ 0, & \text{otherwise} \end{cases}$$

In a second-price sealed-bid auction, it is a weakly dominant strategy for a bidder to bid his true valuation (Vickrey [41]), which implies the following result.

Lemma 1 *If the auction enters stage two where a second-price sealed-bid auction occurs, both early bidders and late bidders bid their true valuations.*

According to Lemma 1, an early bidder wins the auction in stage two if: (a) all $n_1 - 1$ other early bidder also reject the BIN option in stage one; (b) his valuation is higher than all $n_1 - 1$ other early bidders and all n_2 late bidders. Define the expected payoff in stage two for an early bidder with valuation v_i who rejects the BIN price in stage one as $\pi_b^1(v_i)$,

$$\pi_b^1(v_i) = F^{n_1-1}(c)[u(v_i - r)(F_e^{n_1-1}(r)F^{n_2}(r)) + \int_r^{v_i} u(v_i - x)dF_e^{n_1-1}(x)F^{n_2}(x)].$$

Having obtained the expected payoff for an early bidder of accepting a BIN price and rejecting a BIN price, we proceed to derive an equilibrium cutoff valuation. A cutoff c^* is a symmetric Bayes-Nash equilibrium if $\pi_p^1(v_i) \leq \pi_b^1(v_i)$ for all $v_i \in [\underline{v}, c^*]$ and $\pi_b^1(v_i) < \pi_p^1(v_i)$ for all $v_i \in (c^*, \overline{v}]$. By the continuity of the payoff function, for the conjectured strategy to characterize an equilibrium in stage one, an early bidder's expected payoff from accepting the BIN price should be equal to his expected gain in stage two when his valuation equals c. Therefore, an equilibrium cutoff c^* satisfies $\pi_p^1(c^*) = \pi_b^1(c^*)$. Since

$$\begin{aligned}
\pi_p^1(c) &= u(c-p)\frac{1-F^{n_1}(c)}{n_1(1-F(c))} \\
\pi_b^1(c) &= F^{n_1-1}(c)[u(c-r)F_e^{n_1-1}(r)F^{n_2}(r) + \int_r^c u(c-x)dF_e^{n_1-1}(x)F^{n_2}(x)] \\
&= F^{n_1-1}(c)[u(c-r)\frac{F^{n_1-1}(r)}{F^{n_1-1}(c)}F^{n_2}(r) + \int_r^c u(c-x)dF_e^{n_1-1}(x)F^{n_2}(x)] \\
&= u(c-r)F^{n-1}(r) + \int_r^c u(c-x)dF^{n-1}(x)
\end{aligned}$$

it follows that

$$u(c^*-p)\frac{1-F^{n_1}(c^*)}{n_1(1-F(c^*))} = u(c^*-r)F^{n-1}(r) + \int_r^{c^*} u(c^*-x)dF^{n-1}(x).$$

In Theorem 1 we will show the conditions for the above equation to define an unique equilibrium cutoff valuation implicitly.

Following Reynold and Wooders [**34**], we apply the concept of certainty equivalent payment in the proof of Theorem 1. We denote a certainty equivalent payment by $\delta(v)$ such that a risk averse bidder with valuation $v \in [\underline{v}, \overline{v}]$ which follows a distribution function $F(v)$ is indifferent between the following two outcomes: (a) winning a standard second-price auction with n bidders thus paying a random amount of $\max\{r, y\}$ where r is the reserve price and y is the second highest bid; (b) winning the auction and paying a certain amount $\delta(v)$. Using the mean value theorem, one can show that there exists a unique $\delta(v)$ such that

$$u(v-\delta(v))F^{n-1}(v) = u(v-r)F^{n-1}(r) + \int_r^v u(v-x)dF^{n-1}(x),$$

where $\delta(v)$ is increasing in v. In particular, we define the certainty equivalent payment of a risk neutral bidder as $\delta_0(v)$.

We establish the following result.

Theorem 1 *Suppose that all bidders are risk averse with a twice-differentiable utility function $u(x)$ such that $u(0) = 0$, $u'(x) > 0$ and $u''(x) \leq 0$. Consider a two-stage temporary Buy-It-Now auction with n_1 early bidders and n_2 late bidders whose valuation follow a distribution $F(v)$ with support $[\underline{v}, \overline{v}]$. Suppose that the seller sets the reserve price at r and sets a BIN price at p.*

1. *If $p \leq \delta(\overline{v})$, there exists a unique symmetric equilibrium cutoff $c^* \in [p, \overline{v})$ for early bidders such that an early bidder accepts the BIN price if his valuation is higher than or equal to c^* and rejects the BIN price otherwise.*

   The equilibrium cutoff c^* is implicitly defined by

$$u(c^*-p)\frac{1-F^{n_1}(c^*)}{n_1(1-F(c^*))} = u(c^*-r)F^{n-1}(r) + \int_r^{c^*} u(c^*-x)dF^{n-1}(x).$$

   This equilibrium cutoff exists when $u(0) = 0$, $u'(x) > 0$ and is unique when $u''(x) \leq 0$.

2. *If $p > \delta(\overline{v})$, there does not exist an equilibrium cutoff strategy. Hence the BIN price is never accepted by a bidder. Therefore a second-price sealed-bid auction with n $(n = n_1 + n_2)$ bidders always occur in stage two.*

Proof. See Appendix. ■

Theorem 1 establishes the existence and uniqueness of an equilibrium cutoff strategy for early bidders such that an early bidder accepts the BIN price if his valuation is higher than or equal to a cutoff valuation and reject it otherwise. Note that this cutoff equilibrium exists as long as $u(0) = 0$ and $u'(x) > 0$. Hence risk aversion of the bidders is a sufficient but not necessary condition for the existence of the cutoff equilibrium. The cutoff equilibrium also exists when bidders are risk averse. Therefore, as long as seller sets the BIN price low enough, a bidder, who can be either risk neutral, risk averse or risk loving, will accept the BIN price as long as his valuation is lower than the cutoff valuation. However, the uniqueness of the equilibrium is established under the assumption that $u''(x) \leq 0$, i.e. bidders are risk averse or risk neutral.

2.3.2. Factors Influencing Equilibrium Cutoff Valuation

In this section we investigate how this equilibrium cutoff valuation is affected by factors such as the proportion of early bidders, the BIN price, and bidders' degree of risk aversion.

2.3.2.1. Proportion of Early Bidders

We first consider the proportion of early bidders. Fixing the number of total bidders, when there are fewer early bidders in stage one, an early bidder with valuation higher than the cutoff valuation has a greater probability of winning the auction. Meanwhile, the number of total bidders in stage two is the same thus the winning probability for an early bidder in stage two does not change. Therefore, its is more likely for an early bidder to accept the BIN price with fewer early bidders. This result is established formally below.

Corollary 1 *Consider a temporary Buy-It-Now auction with n_1 early bidders and n_2 late bidders whose valuation follow a distribution $F(v)$ with support $[\underline{v}, \overline{v}]$. Assume* *bidders are risk averse. Suppose that the seller sets the reserve price at r and the BIN price at $p < \delta(\overline{v})$. Fixing the total number of bidders, the unique symmetric equilibrium cutoff c^* for the early bidders to accept the BIN price increases with the number of early bidders.*

Proof. As we have shown in Theorem 1, when $p < \delta(\overline{v})$, a unique $c^* \in [p, \overline{v}]$ exists such that

$$u(c^* - p)\frac{1 - F^{n_1}(c^*)}{n_1(1 - F(c^*))} = u(c^* - r)F^{n-1}(r) + \int_r^{c^*} u(c^* - x)dF^{n-1}(x).$$

Define $L(c) = u(c-p)\frac{1-F^{n_1}(c)}{n_1(1-F(c))}$ and $R(c) = u(c-r)F^{n-1}(r) + \int_r^c u(c-x)dF^{n-1}(x)$, then we have

$$\begin{aligned}
&L(p) = 0 \\
&R(p) = u(p-r)F^{n-1}(r) + \int_r^p u(p-x)dF^{n-1}(x) \geq L(p) \\
&L(\overline{v}) = u(c-p) \\
&R(\overline{v}) = u(\overline{v} - \delta(\overline{v})).
\end{aligned}$$

The total number of the bidders n does not change, therefore, as n_1 increases, $R(c)$ remains the same. Also, both $L(p)$ and $L\left(\overline{v}\right)$ are not affected by n_1. We have

$$L(c) = u(c-p)Q(F(c)),$$

where $Q(F(c)) = (1 - F(v)^{n_1})/(n_1(1 - F(v)))$, which decreases as n_1 increases for $c < \overline{v}$. Thus, for any $c \in (p, \overline{v})$, $L(c)$ decreases n_1 increases.

In all, when n_1 increases, $R(c)$ remains the same for $c \in [p, \overline{v}]$, $L(c)$ remains the same only when $c = p$ and $c = \overline{v}$, and $L(c)$ becomes smaller for $c \in (p, \overline{v})$. We have also shown that $R(p) \geq L(p)$, $R(\overline{v}) \geq L(\overline{v})$ and $\partial L(c)/\partial c > \partial R(c)/\partial c$. Therefore the intersection of $L(c)$ and $R(c)$ shifts rightwards as n_1 increases. This suggests that the unique equilibrium cutoff c^* satisfying $L(c) = R(c)$ increases as n_1 increases. ■

Suppose that $n_1 = n$ and $n_2 = 0$. This is equivalent to the case where all bidders are offered the BIN option. Corollary 1 implies that, since the cutoff valuation attain its maximum in this case, a bidder is less likely to accept a BIN price compared with the case where only a portion of bidders are offered the BIN option. This may explain the high frequency of BIN price being taken in the empirical studies of BIN auctions.

2.3.2.2. BIN Price

We now consider the impact of the BIN price on the equilibrium cutoff valuation. We establish the following result.

Corollary 2 *Consider a temporary Buy-It-Now auction with n_1 early bidders and n_2 late bidders whose valuation follow a distribution $F(v)$ with support $[\underline{v}, \overline{v}]$. Assume bidders are risk averse. When a seller sets the reserve price at r and sets a BIN price at p such that $p < \delta(\overline{v})$, the unique symmetric equilibrium cutoff $c^* \in [p, \overline{v})$ increases with the BIN price p.*

Proof. When $p < \delta(\overline{v})$, c^* satisfies $L(c^*) = R(c^*)$, where $L(c) = u(c-p)\ Q((F(c))$ and $R(c) = u(c-r)F^{n-1}(r) + \int_r^c u(c-x)dF^{n-1}(x)$.

In the proof of Theorem 1, we have shown that for $c \in [p, \overline{v}]$, $L^{'}(c) > 0$ and $R^{'}(c) > 0$, $L(p) < R(p)$ and $L(\overline{v}) > R(\overline{v})$ if $p < \delta(\overline{v})$. When p increases, $L(c)$ decreases thus shifts rightward and $R(c)$ remains the same. Therefore the intersection of $L(c)$ and $R(c)$ shifts rightward, which means c^* increases. ■

This result is intuitive in the sense that a buyer will prefer a lower BIN price than a higher BIN price.

2.3.2.3. Risk Attitude

A BIN option offers the bidder an opportunity to win the item with high certainty. Therefore a more risk averse bidder should be more likely to accept the BIN option compared with a less risk averse bidder.

Corollary 3 *Consider a two-stage temporary Buy-It-Now auction with n_1 early bidders and n_2 late bidders. When a seller sets the reserve price at r and sets a BIN price at p such that $p < \delta(\overline{v})$, the unique symmetric equilibrium cutoff $c^* \in [p, \overline{v})$ decreases as bidders become more risk averse, that is, as the certainty equivalent payment , $\delta(v)$, of all bidders increases.*

Proof. Define

$$L(c) = u(c-p)\frac{1-F^{n_1}(c)}{n_1(1-F(c))} = u(c-p)Q(F(c))$$
$$R(c) = u(c-\delta(c))F^{n-1}(c)$$

For utility function $u_k(v)$, we define the certainty equivalence payment $\delta_k(v)$ as

$$u_k(v-\delta_k(v))F^{n-1}(v) = u(v-r)F^{n-1}(r) + \int_r^v u(v-x)dF^{n-1}(x)$$
$$u_k(v-\delta_k(v)) = \frac{u(v-r)F^{n-1}(r) + \int_r^v u(v-x)dF^{n-1}(x)}{F^{n-1}(v)}$$

and let

$$L_k(c) = u_k(c-p)\frac{1-F^{n_1}(c)}{n_1(1-F(c))} = u_k(c-p)Q(F(c))$$
$$R_k(c) = u_k(c-\delta_k(c))F^{n_1-1}(c).$$

Denote c_k^* as the solution for $L_k(c_k^*) = R_k(c_k^*)$, which means,

$$u_k(c_k^*-p)Q(F(c_k^*)) = u_k(c_k^*-\delta_k(c_k^*))F^{n_1-1}(c_k^*).$$

For the same v, suppose $\delta_1(v) < \delta_2(v)$ for different utility function u_1 and u_2.To show that c_k^* decreases in δ_k, it is sufficient to show that $c_1^* > c_2^*$.

As shown in the proof of Theorem One, we have shown that for $L(c) < R(c)$ if $p < c < c^*$, $L(c) = R(c)$ if $c = c^*$ and $L(c) > R(c)$ if $c^* < c < \overline{v}$. Therefore, to show that $c_1^* > c_{2,}^*$ it is sufficient to show that $L_1(c_2^*) < R_1(c_2^*)$, or $L_2(c_1^*) > R_2(c_1^*)$.

Because $p < \delta_1(c_1^*) < \delta_2(c_1^*)$, we have $u_2(c_1^* - p) > u_2(c_1^* - \delta_2(c_1^*))$. And since $Q(F(c_1^*)) > F^{n_1-1}(c_1^*)$, we have

$$u_2(c_1^* - p)Q(F(c_1^*)) > u_2(c_1^* - \delta_2(c_1^*))F^{n_1-1}(c_1^*),$$

that is $L_2(c_1^*) > R_2(c_1^*)$.

Hence, we have established that $c_1^* > c_2^*$. Therefore, when $p < \delta(\overline{v})$,the unique symmetric equilibrium cutoff $c^* \in [p, \overline{v}]$ decreases as a bidder's certainty equivalent payment $\delta(v)$ increases, that is, the bidder's utility function becomes more risk averse.

■

So far we have identified the equilibrium strategies for both early bidders and late bidders given the reserve price r and the BIN price p. To characterize the equilibrium for the entire auction, the seller's choice of reserve price and the BIN price must also be analyzed. We first assume that the seller is risk neutral and attaches no value to the good. The seller maximizes his revenue by setting a reserve price r and a BIN price p assuming that the bidders follow the equilibrium strategies characterized in Lemma 1 and Theorem 1. Below we illustrate the equilibrium analysis of a temporary two-stage BIN auction two risk neutral bidders.

2.3.3. Equilibrium Analysis of a BIN Auction with One Early Bidder and One Late Bidder

Suppose there is one risk neutral early bidder with valuation v_1 and one risk neutral late bidder with valuation v_2, whose valuations follow a uniform distribution with support [0,b]. A seller sets the reserve price r and BIN price p at the beginning of the auction. Assume one early bidder enters the auction at stage one. He faces two choices: 1) to accept the BIN price p and win the item immediately, or 2) to reject a bid then participate in a second price sealed-bid auction to compete with one late bidder.

Suppose the early bidder uses a cutoff strategy c that he accepts the BIN price if $v_1 \geq c$ and rejects the BIN price otherwise.

Define the expected utility of an early bidder from accepting the BIN price as π_p^1,

$$\pi_p^1 = u(v_1 - p).$$

If the early bidder does not accept the BIN price p, the auction becomes a standard second price auction with two bidders. Define the early bidder's expected payoff in the auction as π_b^1,

$$\pi_b^1 = u(v_1 - r)F(r) + \int_r^{v_1} u(v_1 - x)dF(x).$$

If the early bidder's valuation exceeds his cutoff point, he will purchase the item at the BIN price in stage one and the auction ends. Therefore, if an early bidder shows up in stage two, his valuation must be lower than his cutoff point c. A later bidder obtains the item if his valuation is higher than the reserve price and the early bidder's valuation. Therefore, the late bidder's expected payoff, π^2, is

$$\pi^2 = \frac{1}{F(c)}(u(v_2 - r)F(r) + \int_r^{v_2} u(v_2 - x)dF(x)).$$

By the continuity of the payoff function, for the cutoff strategy to characterize an equilibrium in stage one, an early bidder's expected payoff by taking the BIN price in stage one should be equal to his expected gain in stage two when his valuation equals to the equilibrium cutoff c^*, that is,

$$u(c^* - p) = u(c^* - r)F(r) + \int_r^{c^*} u(c^* - x)dF(x) \tag{2.1}$$

Since $F(v) = \frac{v}{b}$, and bidders are risk neutral equation 2.1 becomes,

$$c^* - p = (c^* - r)(\frac{r}{b}) + \int_r^{c^*} (c^* - x)d\frac{x}{b}.$$

The above equation have two roots,

$$c_l^* = b - \sqrt{b^2 - 2bp + r^2}$$
$$c_h^* = b + \sqrt{b^2 - 2bp + r^2}.$$

The higher one exceeds the highest bidder's valuation b, therefore there exists one unique cutoff point $c^* = b - \sqrt{b^2 - 2bp + r^2} \in [0, b]$ when $p \leq (b^2 + r^2)/2b$.

Given the characterized bidder's equilibrium cutoff strategy, a seller maximize his expected revenue by choosing the reserve price r and the BIN price p. A seller's expected revenue from a stage one bidder is $\int_r^{c_1} \left\{ \int_0^r rdF(y) + \int_r^x ydF(y) \right\} dF(x) + \int_{c_1}^b pdF(x)$, and the expected revenue from a stage two bidder is

$$\int_r^b (\frac{1}{F(c)} \int_r^y xdF(x)dF(y)).$$

Therefore, a seller's expected revenue is,

$$\Pi = \int_r^{c_1} \left\{ \int_0^r r dF(y) + \int_r^x y dF(y) \right\} dF(x) + \int_{c_1}^b p dF\,(x) + \int_r^b (\frac{1}{F(c)} \int_r^y x dF(x) dF(y))$$
$$= \int_r^c \left\{ \int_0^r r d(\frac{y}{b}) + \int_r^x y d\frac{y}{b} \right\} d\frac{x}{b} + \int_{c_1}^b p d\frac{x}{b} + \int_r^b (\frac{1}{F(c)}) \int_r^y x d\frac{x}{b} d\frac{y-a}{b}$$
$$= \frac{1}{b^2}(\frac{c^3 - r^3}{3} + \frac{c^2 - r^2}{2}(b-c)) + \frac{(b-c)}{b}p.$$

Assume the early bidder is using the cutoff point $c^* = b - \sqrt{b^2 - 2bp + r^2}$ by plugging c^* into Π, then we have,

$$\Pi = \frac{1}{b^2}(\frac{(b - \sqrt{b^2 - 2bp + r^2})^3 - r^3}{3} + \frac{(b - \sqrt{b^2 - 2bp + r^2})^2 - r^2}{2}\sqrt{b^2 - 2bp + r^2})$$
$$+ \frac{\sqrt{b^2 - 2bp + r^2}}{b}p$$

The optimal reserve price r^* and the optimal BIN price p^* can be obtained by solving $\partial_1 \Pi(r,p) = 0$ and $\partial_2 \Pi(r,p) = 0$. Solve for $\partial_1 \Pi(r,p) = 0$, we have

$$r^* = 0$$

and $SOC = -\frac{1}{b} < 0$.

Solve for $\partial \Pi_c(r,p)/\partial p = 0$, we have

$$p^* = \frac{r^2 + b^2}{2b},$$

$SOC < 0$ when $p < b/2$ and SOC goes to infinity as $p \geq b/2$. Therefore

$$p^* \geq b/2.$$

Perviously, we found that in order for the cutoff point to exist, we need $p \leq (b^2 + r^2)/(2b)$. Hence the cutoff equilibrium exists when $p^* = b/2$ and does not exist when $p^* > b/2$. Therefore the optimal seller strategy given the bidder uses a cutoff strategy is $p^* = b/2$ and $r^* = 0$. Then, we have

$$c^* = b - \sqrt{b^2 - 2bp^* + (r^*)^2} = b,$$

which means bidder one will only accept the BIN price when his valuation is b.

When $r^* = 0$, $p^* = b/2$ and $c^* = b$, the seller's expected revenue if the seller sets the BIN price at $b/2$, Π_1^*, is,

$$\Pi_1^* = \frac{1}{b^2}(\frac{c^{*3} - r^3}{3} + \frac{c^{*2} - r^2}{2}(b - c_{1l}^*)) + \frac{(b - c^*)}{b}p = \frac{b}{3}$$

If $p^* > b/2$, then the cutoff equilibrium does not exist, which suggests that the early bidder will never accept the BIN price. In this case, a standard second-price sealed-bid auction with two bidders always occur. Thus a seller's expected revenue becomes

$$\begin{aligned}\Pi &= 2\int_r^b [rF(r) + \int_r^{v_i} xdF(x)]dF(v_i) \\ &= 2r[1 - F(r)]F(r) + \int_r^b 2x[1 - F(x)]f(x)dx. \qquad (2.2)\end{aligned}$$

It follows that

$$\begin{aligned}&\partial\Pi/\partial r = 2F(r)(1 - F(r) - rf(r)) \\ &\partial\Pi/\partial r = 0 \text{ when } r^* = \frac{b}{2}, \text{ and } SOC < 0.\end{aligned}$$

Plug $r^* = b/2$ into equation 2.2, we obtain the expected seller revenue, Π_2^*, when he sets a BIN price higher than $b/2$,

$$\Pi_2^* = \frac{5}{12}b > \Pi_1^*.$$

Therefore, in a temporary Buy-It-Now auction with one risk neutral early bidder and one risk neutral late bidder, a risk neutral seller will choose a reserve price

$r = b/2$ and a BIN price $p > b/2$ such that bidder one will never exercise the buy-it-now option and a second-price sealed-bid auction always occur. The maximum seller revenue in this case is $5b/12$, which is the same as the the maximum seller revenue in a second-price sealed-bid auction with two risk neutral bidders.

2.3.4. Seller Revenue

Now we investigate the seller revenue assuming arbitrary numbers of early bidders and late bidders. Define a seller's expected payment from an early bidder with valuation v_i if the bidder accepts the buy price as $\Pi_{1,1}(r,p)$,

$$\Pi_{1,1}(r,p) = \sum_{k_1=1}^{n_1} \binom{n_1-1}{k_1-1} (1-F(c))^{k_1-1} F^{n_1-k_1}(c)\frac{p}{k_1} = p\frac{1-F^{n_1}(c)}{n_1(1-F(c))}.$$

Define the seller's expected payment from an early bidder if the bidder rejects the BIN price and no other early bidders take the BIN option as $\Pi_{1,2}(r,p)$,

$$\Pi_{1,2}(r,p) = F^{n_1-1}(c)[r(F_e^{n_1-1}(r)F^{n_2}(r)) + \int_r^{v_i} xdF_e^{n_1-1}(x)F^{n_2}(x)].$$

On the other hand, a seller's expected revenue from a late bidder is

$$\Pi_2(r,p) = F^{n_1-1}(c)[r(F_e^{n_1}(r)F^{n_2-1}(r)) + \int_r^{v_i} xdF_e^{n_1}(x)F^{n_2-1}(x)].$$

Therefore, a seller's expected revenue is

$$\begin{aligned}
\Pi(r,p) &= n_1\int_c^{\overline{v}} \Pi_{1,1}(r,p)dF\left(v_i\right) + n_1\int_r^{c} \Pi_{1,2}(r,p)dF\left(v_i\right) + n_2\int_r^{\overline{v}} \Pi_2(r,p)dF(v_i) \\
&= n_1\int_c^{\overline{v}} p\frac{1-F^{n_1}(c)}{n_1(1-F(c))}dF\left(v_i\right) \\
&+ n_1\int_r^{c} \{F^{n_1-1}(c)[r(F_e^{n_1-1}(r)F^{n_2}(r)) + \int_r^{v_i} xdF_e^{n_1-1}(x)F^{n_2}(x)]\}dF\left(v_i\right) \\
&+ n_2\int_r^{\overline{v}} \{F^{n_1-1}(c)[r(F_e^{n_1}(r)F^{n_2-1}(r)) + \int_r^{v_i} xdF_e^{n_1}(x)F^{n_2-1}(x)]\}dF\left(v_i\right).
\end{aligned}$$

We now first derive seller's optimal strategy when assuming bidders are risk neutral.

Theorem 2 *Consider a two-stage Buy-It-Now auction with* n_1 *early bidders and* n_2 *late bidders. Assume bidders are risk neutral and their valuation follow a distribution* $F(v)$ *under the support of* $[\underline{v}, \overline{v}]$*. It is a weakly dominant strategy for a risk neutral seller to set a BIN price* $p > \delta_0(\overline{v})$ *such that the early bidders never accepts the BIN price, and a reserve* $r = (1 - F(r))/f(r)$*, which is the same optimal reserve price in a second-price sealed-bid auction. Thus the maximum revenue a risk neutral seller gains in a Buy-It-Now auction facing* n_1 *risk neutral bidders and* n_2 *risk neutral bidders is the same as that in a second-price sealed-bid auction with* n *bidders where* $n = n_1 + n_2$*.*

Proof. When bidders are risk neutral, a second-price sealed-bid auction with the optimal reserve price is revenue maximizing (Myerson [**27**]). Therefore, it is a weakly dominant strategy for the seller set the BIN price $p > \delta(\overline{v})$ such that the early bidders will never accept the BIN price thus a second-price sealed-bid auction with n bidders will always occur.

In a second-price auction with n bidders, the expected seller revenue is

$$\begin{aligned}\Pi &= n\int_r^{\overline{v}} [rF^{n-1}(r) + \int_r^{v_i} x dF^{n-1}(x)] dF(v_i) \\ &= rn[1 - F(r)]F(r)^{n-1} + \int_r^{\overline{v}} xn(n-1)F(x)^{n-2}[1 - F(x)]f(x)dx.\end{aligned}$$

Differentiating Π with respect to r, we have

$$\begin{aligned}\partial\Pi/\partial r &= nF(r)^{n-1}(1 - F(r) - rf(r)) \\ \partial\Pi/\partial r &= 0 \text{ when } r^* = \frac{1 - F(r^*)}{f(r^*)}.\end{aligned}$$

Therefore, the seller's expected revenue is maximized by a unique r^* with $r^* = (1 - F(r^*))/f(r^*)$. ■

Theorem 2 implies that when bidders are risk neutral, to maximize his revenue, a seller should not set a BIN price, or should set a BIN price high enough, such that no bidder will exercise the "Buy-It-Now" option. This is because if the BIN price $p < \delta_0(\overline{v})$, there is a positive possibility that the BIN price will be accepted by some bidder thus the revenue for the seller is p. However, in a second-price auction, the bidder with highest valuation wins and pays $\delta_0(\overline{v})$, which is higher than p.

Up to now we have investigated the seller's choice in a two-stage Buy-It-Now auction when bidders are risk neutral. We find that when bidders are risk neutral, the seller will set a BIN price high enough so that no bidders will accept the BIN price. Therefore a two-stage Buy-It-Now auction does not yield a higher seller revenue than a standard second-price sealed-bid auction. However, bidders are often risk averse. Bidders' risk attitudes do not affect seller revenue in a standard second-price sealed-bid auction (Milgrom and Weber [**26**]). Meanwhile, as we have shown in Corollary 2, the more risk averse a bidder is, the more likely he is to accept a BIN price. The result below establishes conditions under which a two-staged buy-it-now auction raises higher expected seller revenue than a standard second-price sealed-bid auction, when a seller faces risk averse bidders.

Corollary 4 *Consider a temporary Buy-It-Now auction with n_1 early bidders and n_2 late bidders whose valuation follow a distribution $F(v)$ with support $[\underline{v}, \overline{v}]$. If a seller sets the BIN price p such that $\delta_0(\overline{\nu}) < p < \delta(\overline{\nu})$, the two-stage BIN auction with n_1 early bidders and n_2 late bidders raises higher seller revenue compared with a second-price sealed-bid auction with the same reserve price r and n bidders ($n = n_1 + n_2$) when bidders are risk averse.*

Proof. As we have shown in Theorem 1, when $p < \delta(\overline{\nu})$, an early bidder with valuation $v_i > c^*$ will accept the BIN price, where c^* is implicitly given in theorem 1 and $\delta(v)$ is the certainty equivalent payment.

Without loss of generality, consider bidder one with valuation v_1, which is the highest among all n_1 early bidders and bidder two with valuation v_2,which is the highest among all n_2 late bidders.

(1) If $v_1 < c^*$, since bidder one has the highest valuation among all early bidders, then the BIN price is never accepted by any bidders. Hence a second-price sealed-bid auction starts in stage two with $n = n_1 + n_2$ bidders. In this case, the maximum seller revenue is the same as a second-price sealed-bid auction with n bidders.Therefore, if bidder one is the winner in strage two, then the seller's expected revenue is $\delta(\nu_1)$. If bidder one is not the winner in stage two, then bidder two must be the winner, therefore the seller's expected revenue is $\delta(\nu_2)$.

(2) If $v_1 > c^*$, then bidder one will accept the BIN price p if $p < \delta(\overline{\nu})$. Define $\delta_0(\overline{\nu})$ as the certainty equivalent payment for a risk neutral bidder with valuation $\overline{\nu}$. By the property of the certainty equivalent payment, we have $\delta_0(v_1) < \delta_0(\overline{\nu})$, $\delta_0(v_2) < \delta_0(\overline{\nu})$ and $\delta_0(\overline{\nu}) < \delta(\overline{\nu})$. If a seller sets the BIN price p such that $max\{\delta_0(v_1), \delta_0(\nu_2)\} < \delta_0(\overline{\nu}) < p < \delta(\overline{\nu})$, then bidder one will accept the BIN price. Hence the seller will by paid by p. Therefore the expected seller revenue when $v_1 > c^*$ is always higher than that when $v_1 < c^*$.

So we have shown that, when bidders are risk averse, compared with a second-price sealed-bid auction with reserve price r, if the seller sets the BIN price at $\delta_0(\overline{\nu}) < p < \delta(\overline{\nu})$ and the reserve price at r, the seller's expected revenue in a two-stage Buy-It-Now auction is (i) the same if $v_1 < c^*$; (ii) and higher if $v_1 > c^*$. Since $c^* \in [p, \overline{v})$, there is a positive possibility that $v_1 > c^*$. Therefore, the seller's *ex-ante* expected revenue is higher in the two-stage Buy-It-Now auction. ■

Note that as discussed in the proof the the Theorem 1, when bidders are risk loving, there is still a probability that the BIN price is accepted by a bidder as long as it is low enough. However, the certainty equivalent payment of a risk loving bidder is lower than a risk neutral bidder with the same valuation. Therefore, the expected seller revenue from a BIN auction is lower than that from a second-price sealed-bid auction when bidders are risk loving.

Next we show how seller's expected revenue is affected by bidders' risk attitude.

Corollary 5 *Suppose bidders are risk averse and their valuations follow a distribution function* $F(v)$ *with support* $[\underline{v}, \overline{v}]$*. If a seller sets the BIN price* p *such that* $\delta_0(\overline{v}) < p < \delta(\overline{v})$*, then the seller revenue in a two-stage BIN auction with* n_1 *early bidders and* n_2 *late bidders increases as bidders become more risk averse, that is, as the certainty equivalent payments of all bidders increase.*

Proof. Assume for the same v, $\delta_1(v) < \delta_2(v)$, for utility function u_1 and u_2. Hence bidders with utility function u_1 are more risk averse than bidders with utility function u_2. As shown in Corollary 2, the unique equilibrium cutoff c^* decreases as bidders become more risk averse, therefore $c_1^* > c_2^*$.

Without loss of generality, consider bidder one with valuation v_1, which is the highest among all n_1 early bidders and bidder two with valuation v_2,which is the highest among all n_2 late bidders.

When bidders' utility function is u_2, if $v_1 > c_1^*$, the BIN price will be accepted and the seller's expected revenue is p. If $v_1 < c_2^*$, the auction become second-price sealed-bid auction with $n_1 + n_2$ bidders, hence the seller's expected revenue is $\max\{\delta_0(v_1), \delta_0(v_2)\}$. If $c_2^* < v_1 < c_1^*$, again no bidder will accept the BIN price and a second-price sealed-bid auction starts.thus the seller's expected revenue is $\max\{\delta_0(v_1), \delta_0(v_2)\}$.

When bidders' utility function is u_1, if $v_1 > c_1^*$,the BIN price will be accepted and the seller's expected revenue is p. If $v_1 < c_2^*$, the seller's expected revenue is

$\max\{\delta_0(v_1), \delta_0(v_2)\}$. However, if $c_2^* < v_1 < c_1^*$ bidder one will accept the BIN price thus seller's expected revenue is $p > \delta_0(\overline{v}) > \max\{\delta_0(v_1), \delta_0(v_2)\}$. Hence, when bidders' utility function is u_1, the seller's ex-ante expected revenue is higher than the case when bidders' utility function is u_2.

Therefore, we show that the seller revenue increases as bidders become more risk averse. ■

Below we establish that a BIN auction with less early bidders will generate higher seller revenue.

Corollary 6 *Suppose bidders are risk averse and their valuations follow a distribution function $F(v)$ with support $[\underline{v}, \overline{v}]$. Given a fixed total number of bidders n $(n = n_1 + n_2)$, if a seller sets the BIN price p such that $\delta_0(\overline{v}) < p < \delta(\overline{v})$, then the seller revenue in a two-stage BIN auction with n_1 early bidders and n_2 late bidders increases as the proportion of the early bidders decreases.*

Proof. Fix the total number of bidders n. Let $0 < n_1 < n_1' < n$. Define the cutoff valuation as c^* and c'^* when the number of early bidders is n_1 and n_1' respectively. Proposition One shows that fixing the total number of bidders, the unique symmetric equilibrium cutoff c^* for the early bidders to accept the BIN price increases as n_1 increases.Therefore, $c^* < c'^*$.

Without loss of generality, consider bidder one with valuation v_1, which is the highest among all n_1 early bidders and bidder two with valuation v_2,which is the highest among all n_2 late bidders.

When there are n_1' early bidders, if $v_1 > c'^*$, bidder one will accept the BIN price thus the seller's expected revenue is p. If $v_1 < c^* < c'^*$, then no bidder will accept the BIN price and the seller's expected revenue is $\max\{\delta_0(v_1), \delta_0(v_2)\}$. If $c^* < v_1 < c'^*$, then again no bidder will accept the BIN price and a second-price sealed-bid auction starts.thus the seller's expected revenue is $\max\{\delta_0(v_1), \delta_0(v_2)\}$.

When there are n_1 bidders, if $v_1 > c^{'*} > c^*$, then bidder one will accept the BIN price, then the seller's expected revenue is p. If $v_1 < c^*$, then no bidder will accept the BIN price and the seller's expected revenue is $\max\{\delta_0(v_1), \delta_0(v_2)\}$. If $c^* < v_1 < c^{'*}$, then bidder one will accept the BIN price, therefore the seller's expected revenue is $p > \max\{\delta_0(v_1), \delta_0(v_2)\}$.

Hence, for a fixed total number of bidders n, the seller's ex-ante expected revenue is higher when there are n_1 bidders compared with when there are $n_1' > n_1$ bidders. Therefore, the seller's revenue increases as the number of the early bidders n_1 decreases. ■

Intuitively seller's expected revenue in a BIN auction increases with bidders' risk attitude: the more risk averse bidders are, the more likely they are to take the BIN option. On the other hand, the more early bidders in a two-stage BIN auction, the less likely is a bidder with valuation higher than the cutoff valuation to win the auction by accepting the BIN price. Thus the less likely he is to exercise the BIN option.

2.4. Conclusion

In this essay, we study auctions with a temporary Buy-it-now price using a two-stage model. We establish that when bidders are risk averse, if the seller sets the BIN price low enough, an early bidders will accept the BIN price only if his valuation is higher than a unique equilibrium cutoff valuation. Other things being equal, the cutoff valuation decreases with bidders' degree of risk aversion and increases with the proportion of early bidders. When bidders are risk neutral, we characterize the equilibrium for both seller and bidders. Facing risk neutral bidders, a seller will set the BIN high enough, such that no bidder will accept the BIN price thus starting a second-price sealed-bid auction. Therefore the seller revenue with risk neutral bidders in a BIN auction is the same as in a second-price sealed-bid auction. This result is consistent with previous theoretical results under the assumption that the BIN price is available to all bidders (e.g. Hidvegi et al. [**15**] and Reynolds and Wooders [**34**]).

We further show that when bidders are risk averse, by setting an appropriate BIN price, the seller obtains a higher expected revenue in a two-stage BIN auction than in a standard second-price sealed-bid auction, and the expected revenue decreases with the number of early bidders. Our result may help explain why empirical studies often report that the frequency of bidders accepting the BIN price is higher than theoretical predictions under the assumption that all bidders are offered the BIN option. Furthermore, seller revenue increases as the number of early bidders decreases. Therefore, by accounting for the fact that the BIN option is available only to one group of bidders, our model provides an explanation for the popularity of temporary BIN auctions in online auction sites such as eBay, where bidders enter the auction after the BIN option disappears do not have information about the BIN price.

CHAPTER III

RISK AVERSE OR IMPATIENT? MODELING BUY-IT-NOW AUCTIONS WITH RISK ATTITUDE AND TIME PREFERENCE

3.1. Introduction

Auction has always fascinated economists. There exists a large literature on auction theory. For example, see Milgrom and Weber [**26**], Wang et al. [**42**], Budish and Takeyama [**8**], and reference therein. Paarsch and Hong [**30**] discusses empirical modeling and estimation of auction data. Recently, the increasing popularity of online auctions has generated a renew interest in the economics of auction. The large amount of real-time or historical data available to researchers makes empirical auction studies possible. In addition, there are some new formats of online auctions that deserve in-depth investigation.

While a typical online auction takes the format of open bid, second-price English auction, in a "Buy-it-now" (BIN) auction, the bidders are allowed to buy the items at a fixed "buy-it-now" price set by the seller at any time during the auction. In Yahoo's "buy now" auction, a permanent buy price remains valid during the entire course of the auction. However, in eBay's "buy-it-now" auction, the buy price is temporary and disappears as soon as any bid is made at or above the reserve price.

Theoretical studies on BIN auctions suggested that when either buyers or sellers are risk averse, a properly set BIN price increases the expected utility of each agent (e.g. Harrison et al. [**13**], Hidvegi et al. [**15**], Reynolds and Wooders [**34**], Mathews and Katzman [**25**]). A permanent BIN price raises more revenue than a temporary BIN price when bidders have either constant or decreasing absolute risk aversion. (e.g. Reynolds and Wooders [**34**]). Besides risk attitude, time discounting can also account for the use of BIN auctions. Mathews [**24**] suggested that impatience on either side (or both sides) of the transaction can motivate the seller to choose a buy price resulting in the option being exercised.

Several studies have examined BIN auctions experimentally. Shahriar and Wooders [**37**] found that a suitably chosen BIN price raises seller revenue with risk averse bidders. However the BIN price is accepted too frequently compared with the prediction of theory when bidders are risk averse. Ivanova-Stenzel and Kroger [**19**] studied BIN auction in a sequential selling mechanism. In their setup, bidders enter the auction at different times. They estimate the risk attitudes of buyers from the auction data. Using existing population estimates of risk preferences they provide quantitative predictions for the distribution of sellers' BIN prices. They find that including risk preferences can only partly account for agents' behavior: it improves the fit for buyers, but is not sufficient to explain sellers' deviations from equilibrium BIN prices. In a follow-up study, Grebe, Ivanova-Stenzel, and Kroger [**12**] invited students who have eBay experience to participate in auctions set up by the experimenters on the eBay website. They elicited subjects' risk preferences using a lottery choice experiment conducted after a follow-up lottery experiment. They found that buyer behavior is still slightly biased towards overly frequent acceptance of the BIN price. Their conclusion is that risk aversion can only partly help in explaining buyer behavior but can not predict price setting behavior. These results suggested that besides risk aversion, other factors such as time preference may account for the use of BIN price in BIN auctions (see Mathews [**24**]). As in the webpage, eBay itself suggests to sellers and buyers when to use the "Buy-it-now" option. It is stated that the presence of a BIN price allow sellers to sell the item and get the payment faster, and allow buyers to get the item instantly.

Durham et al. [**10**] conducted a series of laboratory experiments that compare auctions without a BIN price to those with a temporary (eBay) or permanent (Yahoo!) BIN price. Their data indicate that seller revenue is highest with a permanent BIN price, that bidders utilize a permanent BIN price more frequently than a temporary BIN price, and that BIN prices can improve auction efficiency, which is most likely due to the mitigation of sniping behavior. However, the data generated by this experiment do not allow a direct test of any existing theoretical models since subjects' risk preferences were not elicited.

Experimental and field studies have found that buyers accept the BIN price too frequently, compared with the theoretical predictions when assuming bidders are risk averse. This suggests, in addition to, risk attitude, other factors such as time preference may also affect bidders' buy decision.

This essay explores the effects of bidders' risk attitudes and time preferences on BIN auctions using a simple two-bidder two-stage model based on the model in the first essay. Two cases are considered, when both bidders enter the auction at the same time (homogenous bidders) thus BIN option is offered to both of them, and when two bidders enter the auction at two different stages (heterogenous bidders) thus the BIN option is only offered to the early bidder. Bidders' optimal strategies are derived explicitly in both cases. In particular, given bidders' risk attitude and time preference, the cutoff valuation, such that a bidders will accept BIN if his valuation is higher than the cutoff valuation and reject it otherwise, can be calculated. The cutoff valuation in the case of heterogenous bidders is lower than that in the case of homogenous bidders. The theoretical results generated in this model can be tested using laboratory experiments. At the end of this essay, I propose an experiment design, consisting of four components, eliciting risk attitudes, eliciting time preference, two-stage BIN auctions with two homogenous bidders, and two-stage BIN auctions with heterogenous bidders, which may be used to validate the theoretical results.

3.2. Two-Stage BIN Auctions with Risk Attitude and Time Preference

The seller sets a buy-price p before the auction starts. There are 2 bidders, each with valuation v_i, independently drawn from a common continuous distribution $F(v)$ with the support of $[0, \overline{v}]$. Suppose a bidder, who enters an auction at t_0 and wins an item at time t., exhibits constant absolute risk aversion decided by the utility function $u(x,t) = \delta^{t-t_0}(1 - e^{-\alpha x})/\alpha$. Here α is the bidder's level of risk aversion and $\delta \in (0, 1]$ is the time discounting factor. By L'Hospital's rule, $\lim_{\alpha\to 0} u(x,t) = \delta^t x$, that is, bidders are risk neutral when $a = 0$, risk averse when $\alpha > 0$ and risk loving when $\alpha < 0$.

3.2.1. BIN Auctions with Heterogenous Bidders

We use a two-stage model with heterogenous bidders (Sarin and Zhang [**36**]).

Suppose a seller uses a temporary buy-price auction to sell one indivisible item. The seller sets the reserve price at 0 and the BIN price at p.

Stage one At $T_0 = 0$, one early bidder enter the auction. She observes the BIN price p and decides at t_1 whether to accept the BIN price or not. If she accepts the BIN price, she obtains the item and pays the BIN price. In this case, the auction ends without entering the second stage. If the early bidder decides not to accept the BIN price, then the auction enters the next stage.

Stage two Stage two auction starts at time $T_1 = t_1$ and ends at time T, which is fixed and set by the experimenter. The late bidder enters the auction at stage two She will not be able to observe the BIN price since it disappears at the end of stage one auction. Each bidder submits a sealed-bid in stage two. The bidder who places the higher bid wins the auction and pays the second highest bid, which is the lower bid.

Because a bidder discounts the utility at a rate of δ, in stage one it is weakly dominant for her to makes the decision at $t_1 = T_0$. Therefore, the pure strategies that a bidder who enters the auction in stage one can follow are: 1) place a bid and 2) take the BIN price.

Under CRRA, there is no loss of generality in restricting attention to equilibria in "cutoff" strategy (Reynolds and Wooders [**34**]). Suppose in stage one, a bidder uses a cutoff strategy that he places a bid if $v_1 \le c_1$ and takes the BIN price if $v_1 \ge c_1$.

There are two possible cases in stage one. In the first case, bidder one purchases the item at the BIN price in stage one. It is a weakly dominant strategy for him to make the decision at $t = 0$. His expected utility is

$$u_{11} = u(v_1 - p, 0) = (1 - e^{-\alpha(v_1 - p)})/\alpha.$$

In this case, bidder two has no chance to participate in the auction thus getting utility $u_2 = 0$.

In the second case, bidder one does not accept the BIN price. The auction becomes a standard second price auction with two bidders. Bidder one's expected utility is

$$u_{12} = \int_0^{v_1} u(v_1 - x, T)dF(x).$$

In this case, bidder two's expected utility is

$$u_2 = \int_0^{v_2} u(v_2 - x, T)dF(x).$$

By the continuity of the payoff function, for the cutoff strategy to characterize an equilibrium in stage one, a stage one bidder's expected payoff by taking the BIN price in stage one should be equal to his expected gain in stage two when his valuation equals c_1, that is,

$$\begin{aligned} u_{11} &= u_{12} \\ u(c_1 - p, 0) &= \int_0^{c_1} u(c_1 - x, T)dF(x) \qquad (3.1) \\ (1 - e^{-\alpha(c_1 - p)})/\alpha &= \int_0^{c_1} \delta^T (1 - e^{-\alpha(c_1 - x)})dF(x) \qquad (3.2) \end{aligned}$$

3.2.1.1. Risk Neutral Bidders

Suppose bidders are risk neutral and $\overline{v} = 1$, then $F(v) = v$, then equation 2 becomes,

$$\begin{aligned} c_1 - p &= \delta^T \int_0^{c_1} (c_1 - x)dx \\ c_{11} &= \frac{1 - \sqrt{1 - 2\delta^T p}}{\delta^T} \text{ and } c_{12} = \frac{1 + \sqrt{1 - 2\delta^T p}}{\delta^T} \qquad (3.3) \end{aligned}$$

A cutoff strategy exists when $c_1 \in (-\infty, 1]$, that is

$$p \leq \frac{1}{2\delta^T} \text{ and } p \ \leq 1 - \frac{\delta^T}{2}.$$

Since$1/(2\delta^T) \geq 1 - \delta^T/2$, a cutoff strategy exists when

$$p \ \leq 1 - \frac{\delta^T}{2}. \qquad (3.4)$$

We can ignore c_{12} since $c_{12} = (1 + \sqrt{1 - 2\delta^T p})/\delta^T > 1$, which exceeds the upper bound of any bidder's valuation. Thus the cutoff strategy is

$$c_1 = \frac{1 - \sqrt{1 - 2\delta^T p}}{\delta^T}.$$

Define $P_s = 1 - \delta^T/2$ as the threshold BIN price, which decreases as δ increases. If a seller sets a BIN price lower than P_s, bidder one will choose the BIN price as long as his valuation is higher than $c_1 = (1 - \sqrt{1 - 2\delta^T p})/\delta^T$, which decreases as p decreases or δ increases. This result suggests that an impatient buyer is more likely to purchase the item at the BIN price.

If a bidder does not discount the future ($\delta^T = 1$), then $P_s = 1/(2\delta^T) = 0.5$ and $c_1 = 1$, which means that if the BIN price is higher than 0.5, a bidder will never take BIN price, instead, he will place a bid and the second price auction starts. However, if a bidder discounts the future, for example, at a rate of 0.8 at time T ($\delta^T = 0.8$), then $P_s = 0.6$. That means that even if a seller sets a BIN price as high as 0.55, a bidder will purchase the item at BIN price as long as his valuation is higher than $c_1 = 0.82$. This explains previous finding in experimental data that the BIN price is accepted too frequently (Grebe, Ivanova-Stenzel, and Kroger [**12**], Ivanova-Stenzel and Kroger [**19**], Shahriar and Wooders [**37**]).

3.2.1.2. Risk Averse Bidders

Now assume bidders are risk averse with $u(x,t) = \delta^t(1-e^{-\alpha x})/\alpha$, then equation 2 becomes,

$$\begin{aligned}(1-e^{-\alpha(c_1-p)})/\alpha &= \delta^T \int_0^{c_1} (1-e^{-\alpha(c_1-x)})/\alpha dx \\ 1-e^{-\alpha(c_1-p)} &= \delta^T \int_0^{c_1} (1-e^{-\alpha(c_1-x)})dx \qquad (3.5)\end{aligned}$$

Let $Fy(c_1) = 1-e^{-\alpha(c_1-p)}$ and $Fb(c_1) = \delta^T \int_0^{c_1}(1-e^{-\alpha(c_1-x)})dx$. Fy and Fb are both monotonically increasing functions. $Fy(0) = 1-e^{\alpha p}$; $Fb(0) = 0$; $Fy(1) = 1-e^{-\alpha(1-p)}$; $Fb(1) = \delta^T(1-(1-e^{-\alpha})/\alpha)$. $Fy(0) \leq Fb(0)$.implies that a unique cutoff c_1 point exists when $Fy(1) \geq Fb(1)$, that is $p \leq Ps$, where Ps satisfies $1-e^{-\alpha(1-p)} = \delta^T(1-(1-e^{-\alpha})/\alpha)$.

As p increases, Fy decreases, therefore the cutoff point c_1 increases. As δ^T increases, Fb increases, therefore the cutoff point c_1 decreases. Both the risk neutral case and the risk averse case suggest that if a buyer is time impatient, she is more likely to accept a BIN price. This may explain that in experimental data, the BIN price was accepted too frequently even when theoretically it should be rejected when only risk aversion is considered (Grebe, Ivanova-Stenzel, and Kroger [**12**], Ivanova-Stenzel and Kroger [**19**], Shahriar and Wooders [**37**]). This result suggests that it is important to consider time preference in BIN auctions.

3.2.2. BIN Auctions with Homogenous Bidders

In a BIN auction with homogenous bidders, we assume all bidders enter the auction at the same time.

Now we consider the simple case when there are only two bidders with valuation v_1 and v_2 Suppose both bidders' valuations are drawn from the uniform distribution $F(v) = v/\overline{v}$ with support $(0, \overline{v}]$. Bidders have constant absolute risk aversion with $u(x,t) = \delta^t(1 - e^{-\alpha x})/\alpha$.

A bidder faces two choices: to place a bid or to purchase the item immediately at the buy-price p. If any bidder purchases the item at the BIN price, then the auction ends. If both bidders bake the BIN price, then the winner is chosen from them randomly. If no bidder takes the BIN price, then two bidders compete in a standard second price auction with no buy-price.

Suppose bidder one uses a cutoff strategy c_1 that he places a bid if $v \leq c_1$ and takes the buy-price if $v \geq c_1$

If a bidder purchases the item at the BIN price in stage one. It is a weakly dominant strategy for him to make the decision at $t = 0$. His expected utility is

$$\pi_p(v_i) = u(v_i - p, 0)\frac{1 - F^2(c)}{2(1 - F(c))} = (\frac{(1 - e^{-\alpha x})}{\alpha})\frac{1 - F^2(c)}{2(1 - F(c))}$$

If a bidder places a bid, his expected utility is

$$\pi_b(v_i) = \int_0^{\min\{v_i,c\}} u(v_i - x)dF(x)$$

By the continuity of the payoff function, for the cutoff strategy to characterize an equilibrium in stage one, a bidder's expected payoff by taking the BIN price in stage one should be equal to his expected gain in stage two when his valuation equals to c_1, that is,

$$\pi_p(v_i) = \pi_b(v_i) \tag{3.6}$$

$$\frac{1-F^2(c)}{2(1-F(c))}(1-e^{-\alpha(c_1-p)})/\alpha = \int_0^c \delta^T(1-e^{-\alpha(c-x)})/\alpha/\alpha dF(x) \tag{3.7}$$

3.2.2.1. Risk Neutral Bidders

Suppose bidders are risk neutral and $\overline{v} = 1$, then $F(v) = v$, then a bidder's utility from taking BIN price is $u_p = (1-c^2)(c-p)/(2(1-c))$ whereas his utility from placing a bid is $u_b = \delta^T \int_0^c (c-x)dx = \delta^T c^2/2$.

If a bidder does not discount the future, $\delta^T = 1$, then $u_p = (1-c^2)(c-p)/(2(1-c))$ and $u_b = c^2/2$. Solve $u_p = u_b$, we have

$$c = \frac{p}{1-p}.$$

We need $c = p/(1-p) \leq 1$, therefore for the cutoff point to exist, the BIN price $p \leq 1/2$.

If a bidder discounts the future such that $0 \leq \delta^T < 1$

$$\frac{1-c^2}{2(1-c)}(c-p) = \delta^T \int_0^c (c-x)dx$$

$$c_1 = \frac{(1-p) - \sqrt{1+2p-4\delta^T p + p^2}}{2(\delta^T - 1)} \tag{3.8}$$

$$c_2 = \frac{(1-p) + \sqrt{1+2p-4\delta^T p + p^2}}{2(\delta^T - 1)} \tag{3.9}$$

Cutoff strategy exists when $c \in (-\infty, 1]$, that is

$$p \leq 1 - \frac{\delta^T}{2}.$$

We can ignore c_2 since $c_2 < 0$, which is smaller than the lower bound of bidders' valuation. Thus the cutoff strategy is

$$c = \frac{(1-p) - \sqrt{1+2p-4\delta^T p + p^2}}{2(\delta^T - 1)}.$$

Define $P_s = 1 - \delta^T/2$ as threshold BIN price, which decreases as δ increases. If a seller sets a BIN price lower than P_s, bidder one will choose the BIN price as long as his valuation is higher than $c_1 = ((1-p) - \sqrt{1+2p-4\delta^T p + p^2})/(2(\delta^T - 1)),.$ which decreases as p decreases or δ increases This result suggests that an impatient buyer is more likely to purchase the item at a BIN price.

If a bidder does not discount future ($\delta^T = 1$), then $P_s = 1/(2\delta^T) = 0.5$ and $c_1 = 1$, which means if the BIN price is higher than 0.5, a bidder will never take BIN price, instead, he will place a bid then second price auction starts. However, if a bidder discounts the future, for example, at a rate of 0.8 at time T ($\delta^T = 0.8$), then $P_s = 0.6$. Even if a seller sets a BIN price as high as 0.55, a bidder will purchase the item at BIN price as long as his valuation is higher than $c_1 = 0.8789$. This again explains previous finding in experimental data that the BIN price is accepted frequently (Grebe, Ivanova-Stenzel, and Kroger [**12**], Ivanova-Stenzel and Kroger [**19**], Shahriar and Wooders [**37**]).

3.2.2.2. Risk Averse Bidders

Now assume bidders are risk averse with $u(x,t) = \delta^t(1 - e^{-\alpha x})/\alpha$. At cutoff point, a bidder gains the same utility from taking BIN price and placing a bid,

$$\frac{1-c^2}{2(1-c)}(1 - e^{-\alpha(c-p)})/\alpha = \int_0^c \delta^T(1 - e^{-\alpha(c-x)})/\alpha dx. \tag{3.10}$$

Let $F'y(c_1) = (1 - c^2)(1-e^{-\alpha(c-p)})/(2(1 - c))$ and $F'b(c_1) = \delta^T \int_0^{c_1} (1-e^{-\alpha(c_1-x)})dx$. $F'y(c_1)$ and $F'b(c_1)$ are both monotonically increasing functions. $F'y(0) = 0.5(1 - e^{\alpha p})$; $F'b(0) = 0$; $Fb(1) = \delta^T(1-(1 - e^{-\alpha})/\alpha)$. $F'y(0) \le F'b(0)$ implies that a unique cutoff c point exists when $F'y(1) \ge F'b(1)$, that is $p \le P's$, where $P's$ satisfies $0.5(1 - e^{-\alpha(c-p)}) = \delta^T(1 - (1 - e^{-\alpha})/\alpha)$. As p increases, $F'y$ decreases, therefore the cutoff point c increases. As δ^T increases, $F'b$ increases, therefore the cutoff point c decreases.Therefore, consistent with the BIN auctions with heterogenous bidders, both risk neutral case and risk averse case in BIN auctions with homogenous bidders suggest that if a buyer is time impatient, he is more likely to accept a BIN price.

3.2.3. Comparison between BIN Auctions with Heterogenous Bidders and BIN Auctions with Homogenous Bidders

For both types of auctions, a risk neutral bidder's cutoff strategy exists when $p \le 1 - \delta^T/2$. If a bidder does not discount the future, her cutoff point in a BIN auction with heterogenous bidders is $c_{tHEBIN} = 1-\sqrt{1-2p}$, whereas his cutoff point in a BIN auction with homogeneous bidders is $c_{tHOBIN} = p/(1-p)$. $c_{tHEBIN} < c_{tHOBIN}$ when $p \le 0.5$. Therefore, a bidder's cutoff point is lower in a BIN auction with heterogenous bidders than in a BIN auction with homogenous bidders. Similar results hold for risk averse and time impatient bidders. This is because a bidder gets higher utility in a

BIN auction with heterogenous bidders from taking the BIN price because she has less opponents if she enters the auction before the BIN price disappear. Therefore, a model of BIN auctions with heterogenous bidders may explain that bidders accept the BIN price with a higher frequency than the theoretical prediction using models with homogenous bidders.

3.3. A Proposed Experiment Design for Testing the Model

Given risk attitude and time discounting rate, the cutoff valuation can be calculated in the above model. Therefore, this model generate results that can be tested in the laboratory environment.

We can conduct both BIN auctions with heterogenous bidders(HEBIN), BIN auctions with homogeneous bidders(HOBIN), and also standard second-price sealed-bid auction (SP) as the benchmark. In a standard second-price sealed-bid auction, each bidder places a sealed-bid and the person who submits the higher bid becomes the winner. The winner pays the equivalence of the second highest bid, which is the bid submitted by the loser. In a BIN auction with heterogenous bidders, there are two types of bidders: one *early* bidder and *one* late bidder. The early bidder will have an option to purchase the good at a listed price, the BIN price, set by the experimenter. If the early bidder accepts the BIN price, he/she can obtain the good immediately. If the early bidder chooses not to accept the BIN price, the BIN option disappears and a standard second-price sealed bid auction starts. A late bidder enters the auction at this stage. Both bidders are then asked to submit a sealed-bid. The person who submits the higher bid in the auction becomes the winner. In a BIN auction with homogeneous bidders, two bidders enter the auction simultaneously and both have the option of purchase the item immediately at the BIN price. If one of the bidder accepts the BIN price, he/she wins the auction immediately by paying the BIN price. If both bidders accept the BIN price, the winner will be selected randomly. If none of them accept the BIN price, the BIN option disappears and a standard second-price sealed bid auction starts. Each bidder will then place a sealed-bid. The person who submits the higher bid in the auction becomes the winner and pays the equivalence of the second highest bid.

To explore the effect of bidders' time preferences, three treatments can be conducted. The outcome of an auction is either revealed to the bidders immediately (Immediate Revelation) or one week after the auction ends (Delayed Revelation). If the auction outcome is revealed to the bidders, we further vary whether participants receive the payment the next day after the experiment (Early Payment) or 8 days after (Delayed Payment).

For each treatment, we can recruit 48 subjects, with 144 subjects recruited in total. Each treatment started off with 10 rounds of standard second price auctions. In order to eliminate potential order effects, we conduct two orders: SP followed by HEBIN followed HOBIN and SP followed by HOBIN followed by HEBIN. Bidders are matched from one session into 4 groups of 6 so that we can obtain 4 observations in each session. Every auction is repeated for 10 rounds. In each round, a bidder is randomly paired with a different bidder within a group[1]. A bidder's valuation is randomly drawn from the $U[0, 100]$ distribution in every round. A bidder begins with an initial balance of 500 in the first period and will be declared bankrupt if his balance falls below 0[2]. Once a subject goes bankrupt, he will not be allowed to participate in

[1]We do not inform bidders that they are matched within groups so that they do not guess their opponents' valuations.

[2]In the treatments when the auction results will not be revealed immediately, subjects are not able to track their balances. Once a subject's balance reaches 0, he will be announced bankrupt by the computer

any more auctions and will be regarded as placing 0 bids in all the following periods. Net earning of the winner in each period is his valuation minus the price he pays and the loser earns 0. At the end of the experiment, the computer will randomly select 12 rounds, 4 from each auction type, for payment. A bidder's final earnings will be the sum of the profits he/she earns in these six rounds. If the sum falls below 0, he/she will be paid 0.

After the auctions end, the participants are first given one payoff tables for elicitation of their risk attitudes (Holt and Laury [**16**], Andersen et al [**1**]). Each participant chooses lottery A or B in each row, and one row is later selected at random (10% chance) for payout. Only risk-loving subjects would take lottery B in the first row. And only very risk averse subjects would take lottery A in the second-to-last row. The last row is a test that the subject understood the instructions.

Then the participants are given another payoff table for eliciting their time preference (Andersen et al. [**1**], Coller and Williams [**9**]). They are asked to choose between Option A (payment in 1 day) and Option B (payment in 8 days) for each of the 10 payoff alternatives (Andersen et al. [**1**]). One decision row will be selected at random (10% chance) to be paid out at the chosen date. One out of 24 subjects will be drawn randomly out to receive the real payment.

Individual risk attitude and time preference can be estimated by the two surveys illustrated above. Then, given these two parameters, we can calculate each bidder's cutoff valuation using the method discussed in previous section. Therefore, we will be able to predict each bidder's decision and the seller revenue in each auction.

3.4. Conclusion

In this essay, I constructed a simple two-bidder model including both risk attitude and time preference. Two cases were considered, when BIN option is offered to both bidders(homogenous bidders) and when BIN option is offered to only one of them (heterogenous bidders). I showed that a bidder's cutoff point is lower in a BIN auction with heterogenous bidders than in a BIN auction with homogenous bidders. Similar results hold for risk averse and time impatient bidders. This is because a bidder gets higher utility in a BIN auction with heterogenous bidders from taking the BIN price because she has less opponents if she enters the auction before the BIN price disappear. Therefore, a model of BIN auctions with heterogenous bidders may explain that bidders accept the BIN price with a higher frequency than the theoretical prediction using models with homogenous bidders.

Given individual risk attitude and time preference, in this model, the bidders cutoff valuation can be explicitly calculated thus bidders behavior and seller revenue in the auctions can be predicted thus testable using laboratory experiment. In the last section of this essay, I propose an experiment procedure that can be used to test this model and may reconcile experimental findings with theoretical predictions.

CHAPTER IV

A MODEL OF ONLINE AUCTION PRICE PROCESSES: MONOTONICITY, TIME-VARYING COEFFICIENTS AND MULTI-LEVEL INDIVIDUAL EFFECTS

4.1. Introduction

Auctions have always intrigued economists. The early work on auction focuses on its theoretical properties. Applying game theory to the study of auction, Vickey [**41**] in his seminal work established the optimal bidding strategies in the English ascending-bid auction, the (first-price) sealed-bid auction, and the Dutch declining-price auction. He also demonstrated the strategic equivalent of the last two auctions. More importantly, he then proposed a novel auction format that he showed to be strategically equivalent to the English ascending-bid auction – the second price auction, which is also widely known as the Vickrey auction.

The second price auction, and its various extensions, has since been studied intensively by economists, see for example, Milgrom and Weber [**26**] and Krishna [**21**]. One can show through simple deduction that the optimal strategy in a second price auction is to bid one's true valuation. In this sense, the second price auction is characterized as incentive compatible or preference revealing. Despite its popularity among theoretical economists, the second price auction remained largely a theoretical curiosity for a long time since its debut.[1]

The increasing popularity of online auctions has created much public interest, both commercially and academically, on this new form of e-commerce. One of the

[1] See Lucking-Reily et al. [**23**] for some early adapters of the second price auction, and a fascinating account of the history of the second price auction.

main drivers of this interest is the online auction site eBay. Established in 1995, eBay has since grown rapidly and evolved into one of the world's largest online market places, where anyone can sell practically anything at any time. With a presence in 39 markets, including the U.S., and approximately 84 million active users worldwide, eBay has changed the face of internet commerce. In 2007, the total value of sold items on eBay's trading platforms was nearly $60 billion. The benefit to consumers is clear: eBay provides an open trading platform where the market determines the value of items that are sold. Over the years, the site has become a cultural barometer of sorts, providing a view into what objects consumers want most at any time. At the same time, since eBay archives detailed auction information and makes it publicly available, it provides a unique opportunity for researchers to investigate the economics of auctions and agents' behavior in this market environment.

Online auctions differ from their offline counterparts in their duration, anonymity of participants, low barrier of entry, global reach and around-the-clock availability. Therefore, online auctions have become a serious competitor to offline auctions. Moreover, they also create new phenomena that depart from and cannot be explained by classical auction theory. According to classical auction theory, the final price of an auction is determined by *a priori* information about the number of bidders, their valuation, and the auction format (Milgrom and Weber [**26**], Krishna [**21**]).[2]

Investigation of conventional auctions is often constrained by limited amount of auction information and non-availability of data. In contrast, online auctions provide rich and detailed information that is publicly available. Consequently, there is a rapidly-growing literature on online auctions. For research in economics, marketing and information system, see among others, Ba and Pavlou [**2**], Bajari and Hortaçsu

[2]Paarsch and Hong [30] provides a review of classical auction theories and an in-depth coverage on econometric modeling and estimation of auction data.

[**3**,**4**], Bapna et al. [**5**], Lucking-Reiley, et al. [**23**], and Ockenfels and Roth [**28**,**29**]. A second strand of the literature use the method of functional data analysis to analyze online auction data. See, for example, Hyde et al. [**17,18**], Jank and Shmueli [**20**], Shmueli et al. [**39,40**], Shmueli and Jank [**38**], Wang et al. [**42**], Zhang et al. [**43**] and references therein. This line of research considers the bidding histories of auctions functional data, and thus uses the functional data analysis approach to extract common features of the data on identical or similar auctions. One of the key findings of this literature is that what happens during the auction also matters in online auctions. The dynamics of bidding process, especially the evolution of bid arrival process and bidding process, is shown to influence the final price of auctions.

In this study, we focus on the econometric modeling of the price process of online auctions. The price dynamics of online auctions has important implications. For example, knowledge of what drives price dynamics can help the seller in designing better auctions and can help the auctioneer make adjustment that changes the auction experience (e.g., controlling bid increment policies to alleviate the commonly experienced "bidding drought" in the middle of the auction). In addition, Wang et al. [**42**] showed that price dynamics, when incorporated into a forecasting model, can lead to real-time predictions and result in higher accuracy compared with classical forecasting models.

One feature of the price process of online auctions is the sparse and unevenly-spaced (in terms of arrival time) bid information. The functional data analysis approach treats the price history of every auction as one unit of functional data, and uses methods such as the principal component analysis to extract the commonality among auctions. This analysis typically entails smoothing the bid history of individual auctions as the first step. However, the quality of the smoothing can be questionable when the number of bids of the auctions is small. Moreover, individual auctions are treated as independent processes in this stage. Instead, assuming that similar auctions share a common underlying process, we use panel model method to pool similar auctions in our model, resulting in an unbalanced panel because the number of bids and their arrival times differ across auctions.

Since the price history of online auctions is a monotone process, we generalize the monotone series estimator by Ramsay [**32**] to model the pooled price processes. However, the pooled bid history is not necessarily monotone because bids in multiple auctions arrived at different points of time, and a bid submitted earlier in one auction is not necessarily lower than a bid submitted at a later time in a different auction. Thus a naive application of the monotone estimator to the pooled data imposes unduly restrictive constraint on the estimation. Instead, we present a nonparametric panel estimator with auction-specific slope coefficients, which ensures the monotonicity of bids of individual auctions and allows a common (relative) price growth curve among all auctions.

We apply the proposed method to price histories of eBay auctions of a popular model of Palm handhelds. It is shown that we are able to capture the price evolution of online auctions closely with a relatively simple yet flexible model. In particular, the typical patterns of online auction bid histories, including the early bidding, mid-auction bidding draught, and end-auction bid sniping, are well represented by the estimated price growth curve.

The rest of the paper is organized as follows. Section 2 introduces the online auctions and discusses some features of the price process of online auctions. Section 3 presents a panel model that pools sparse and unevenly-spaced bid histories and assumes a common price evolution among similar auctions. Section 4 proposes a monotone series estimator with auction-specific slopes for a common relative price growth curve that accommodates the unique nature of pooled bid histories. Section

5 presents the data of eBay online auctions used in this study. Section 6 discusses the estimation results and some noteworthy features of the estimated price growth curve. The last section concludes the paper.

4.2. Online Auctions and Its Price Process

The online auction business model is one in which participants bid for products and services over the Internet. Several types of online auctions are possible. In an English auction the initial price starts low and is bid up by successive bidders. In a Dutch auction, multiple identical items are offered in one auction, with all winning bidders paying the same price – the highest price at which all items will be sold (treasury bills, for example, are auctioned this way). Almost all online auctions use the English auction method.

Compared with conventional onsite auctions, online auctions offer some unique advantages as follows.

Around-the-clock availability Bids can be placed at any time (24/7). Items are listed for a number of days (usually between 1 and 10, at the discretion of the seller), giving purchasers time to search, decide, and bid. This convenience increases the number of bidders.

Global reach and low barrier of entry Sellers and bidders can participate from anywhere that has internet access. This makes them more accessible and reduces the cost of "attending" an auction. This increases the number of listed items (i.e., number of sellers) and the number of bids for each item (i.e., number of bidders). The items do not need to be shipped to a central location, reducing costs, and reducing the seller's minimum acceptable price.

Anonymity of participants Sellers and buyers do not reveal their identities in online auctions. This anonymity protects participants' privacy and thus encourages participation.

Large number of bidders Because of the potential for a relatively low price, the broad scope of products and services available, the ease of access, and the social benefits of the auction process, there are a large numbers of bidders.

Large number of sellers Because of the large number of bidders, the potential for a relatively high price, reduced selling costs, and ease of access, there are a large number of sellers.

Network economies The large number of bidders will encourage more sellers, which, in turn, will encourage more bidders, which will encourage more sellers, etc., in a virtuous circle. The more the circle operates, the larger the system becomes, and the more valuable the business model becomes for all participants.

Online auctions also distinguish itself from onsite auctions in some other aspects. For example, in conventional onsite auctions, the number of bidders is observed by the bidders and oftentimes fixed. In contrast, bidders in online auctions can enter the auction at any time during the auction, giving rise to an additional source of uncertainty in online auctions. The arrival of bidders and bids are often modeled as a Poisson process, or more generally, a non-homogeneous Poisson process (see, for example, Shmueli et al. [**39**]). On the other hand, unlike the sealed-bid auction (English auction), the entire bidding process is observed by all potential bidders. Consequently, bidders can adjust their bidding strategies and timing decision based on real-time evolution of the ongoing bidding process. Sellers can also learn from completed auctions of similar items to improve their auction designs. Examination of bid histories of previous auctions can provide insight into questions such as what is the impact of minimum bid/secret reserve price on the final price, and how to determine an optimal minimum bid/secret reserve price, and so on. Therefore the price process of online auctions is of prominent importance to both bidders and sellers.

The high quality and publicly available online auction data provide a unique opportunity to investigate the price process and its dynamics. In this study, we focus on one particular kind of auctions: auctions with a *hard* deadline. The hard-closed auction is the dominant format used by eBay, which usually lasts from three to ten days.[3] A hard-closed auction features an pre-announced deadline. Upon the end of the auction, the bidder with the highest bid is announced the winner, while the winning price is the second highest bid (plus a bid increment) in the auction. In other words, eBay uses the Vickrey auction or the second price auction. The entire bidding history is observable to the public, except for the highest bid.

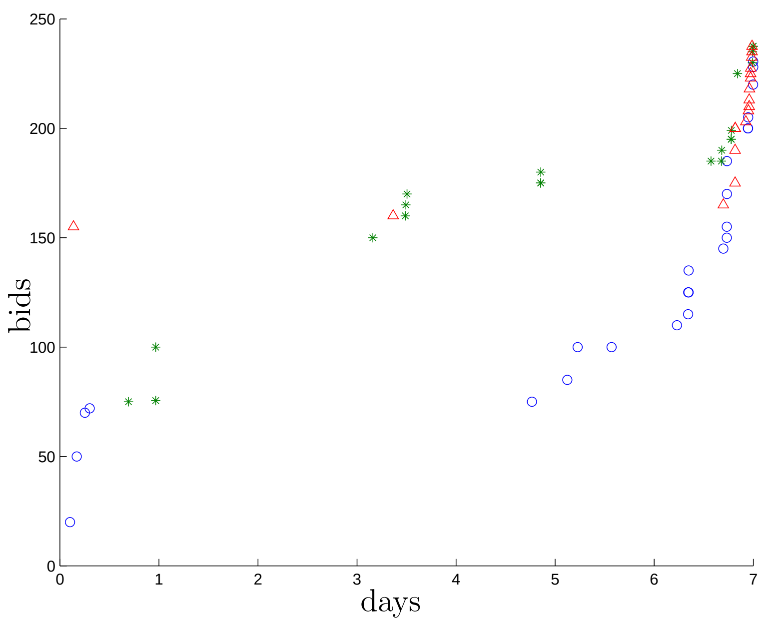


Fig. 1. Bid History of an Online Auction

Figure 1 depicts bidding histories of three eBay auctions that lasted seven days

[3]eBay also uses some variations of the hard-closed auction. For example, some auctions offer a "Buy It Now" price. If during the auction, someone opts to purchase the item in question at the posted "Buy It Now" price, the auction ends immediately and that particular bidder becomes the winner of the auction.

(see Section 5 for details on the sample used in this study).

The bidding process of online auctions is typically marked by some of the following empirical regularities.

Montononicity The price process is monotonically increasing. Bidders are not allowed to submit a bid lower than the currently posted highest bid (plus a minimum increment).

Uneven temporal distribution Bids of an online auction arrive at unevenly-spaced discrete time points chosen by bidders.[4] There is usually some bidding at the auction start, followed by a period of very little activity, and cumulating in a surge of bidding at the end of the auction. Many online auctions observe early biding in the first one or two days. Early bidding often is used as a strategy to establish a bidder's sincere interest and to deter other bidders. Another reason for early bidding is to establish time priority (when the two highest bids are tied, the earliest bidder is the winner). The mid-auction "bidding draught" is possibly because bidders avoid revealing their willingness to pay too early, to avoid early price increases. Finally, during the last hours of the auction, bidding picks up again and peaks dramatically during the last minutes of the auction.

Bid sniping The last-moment bidding, or "bid sniping", is particularly common in auctions with a hard close time. This phenonmon has been studied intensively. Ockenfels and Roth [**29**] reports that 14% of all bidders submit their final bids in the last five minutes, 9% in the last minute, and more than 2% in the last ten seconds before the closing of the auction. There are various explanations as to why bidders engage in bid sniping. Ockenfels and Roth [**29**] and Bajari and Hortaçsu [**4**] identify

[4] To recover the underlying common continuous price process, researchers often rely on a large number of similar auctions with usually small number of bids, which results in an unbalanced panel of bids. The consequences of this particular nature of pooled bid histories on model construction and estimation are discussed below.

several reasons for bidding as late as possible in the auction. Bid sniping avoids bidding wars with incremental bidders, who, rather than bidding their secret valuation for an item, only place bids that are marginally above the current high-bid in order to gain the auction lead. Late-bidding also avoids revealing one's true valuation for an item to uninformed bidders who look to others' bids in order to determine their value for the item. Thus, placing the final bid as late as possible is a valuable solution, although it carries the risk of not being recorded by the auction mechanism due to high server and network traffic (Roth and Ockenfels [**35**]). Ockenfels and Roth [**29**] offer a second explanation that the presence of naive bidders on eBay who do not understand the proxy-bidding mechanism, and hence bid incrementally in response to competitors' bids. They demonstrate that last minute bidding is a best-response by rational bidders against such naive bidders. For a survey on bid sniping, see Bajari and Hortaçsu [**3**].

Price ceiling Many popular items in online auctions, for instance, small electronics such as cell phones and portable music players, are commonly available at conventional and online stores that offer a posted price. The price process of these kind of auctions is usually bounded above by this "market price". This is in contrast to conventional auctions of, say, art work where the final prices are usually hard to predict. Research on online auctions mostly focuses on auctions of these popular items due to their relative homogeneity and the abundance of data.

eBay requests that bidders submit their proxy bid: the highest amount that they are willing to pay for the item. eBay's automated bidding system records the proxy bid but displays only an increment above the second highest proxy bid during the ongoing auction. For example, suppose that bidder A is the first bidder to submit a proxy bid on an item with a minimum bid of \$10 and a minimum increment of \$1. Suppose that bidder A places a proxy bid of \$20. Then eBay's webpage automatically display

A as the highest bidder, with a bid of $10. Next, suppose that bidder B enters the auction with a proxy bid of $15. eBay still displays A as the highest bidder; however, it raises the displayed high bid to $16, one bid increment above the second-highest bid. If another bidder submits a proxy bid above $21, bidder A is no longer in the lead and will be notified by email and given a chance to update his bid. If bidder A wishes, he or she can submit a new higher proxy bid. The process continues until the auction ends. Under proxy bids, bidders only need to submit their maximum willingness to pay and eBay's bidding system will automatically 'bid' for them until their bids are overtaken by other bidders. From seller's point of view, the proxy bid also helps discourage bid sniping: Since the displayed high bid is in fact the second highest bid, bid sniping is only effective if a bid sniper outbids not only the current high bid, but the hidden highest bid.

eBay posts complete bid histories of closed auctions for at least 15 days on its web site. The final bid data that are publicly available on eBay's website are proxy bids, which are not equivalent to the prices displayed during the auction. Following Wang et al. [**42**], we construct 'live bids' based on the observed proxy bids. The series of live bids are essentially an ascending re-arrangement of the observed proxy bids, plus one bid increment. Although the proxy bid series generally is non-monotone, the live bids, which are the actual prices displayed by eBay during an ongoing auction, are strictly monotone. For the rest of the paper, price or bid refers to the live bids.

4.3. The Model

In this section, we lay out a model for the price process of online auctions over time. We stress that throughout this paper, the notion *time* refers to not the calendar clock time, but the 'auction clock' time or the current duration of an ongoing auction, which is between 0 and T, where T is the pre-specified length of the auction. Therefore, a bid submitted at time t indicates that the duration of the auction is t when this bid is submitted.

Consider for now the following model for a *single* hard-closed auction that lasts from time 0 through T (Zhang et al. [**43**]). For $0 \leq t \leq T$,

$$P(t) = \alpha + \beta m(t) + \boldsymbol{x'\gamma} + e_t,$$

where $m(t)$, the relative or baseline price growth curve, is a monotone function, and e_t is an unobserved idiosyncratic error with mean zero and finite variance. Without loss of generality, we assume that $m'(t) > 0$. Consequently, $P'(t)$ has the same sign as β, which is expected to be positive for the price process in question. Generally, the price process is affected by auction-specific attributes such as seller's reputation, opening bid set by the seller, and so on. Their effects are captured by $\boldsymbol{x'\gamma}$, where $\boldsymbol{x}$ is a $S \times 1$ vector of auction attributes, and $\boldsymbol{\gamma}$ is a comparable vector of unknown coefficients.

Although the price processes of online auctions are commonly marked by regularities such as monotonicity, early bidding and bid sniping, they also exhibit a considerable degree of heterogeneity, partly due to the relatively small number of bids in individual auctions. Thus investigation of a single online auction may not be sufficient to provide useful insight into the price process. Fortunately, there are usually a large number of auctions of identical or similar items in online auctions, which offer an opportunity to study their price processes systematically by pooling these auctions. There exists a small yet fast-growing literature on online auctions. A popular approach is to treat the price history of every single auction as a unit of functional data, and use functional data analysis method to explore the data. See, for example, Shmueli and Jank [**38**], Jank and Shmueli [**20**], Shmueli et al. [**39**], Wang et al. [**42**], and references therein. This method entails representing individual price history by a smooth curve via some smoothing methods and then using methods such as the principal component analysis to extract common patterns among the smooth curves. This approach, although powerful, has some limitations. The price processes are treated independently from each other in the first-stage smoothing, which may suffer a loss in statistical efficiency if the bid histories share some common structure. In addition, many auctions only register a small number of bids, rendering the quality of smoothed curves based on these "small" auctions questionable.

In this study, we adopt a different strategy. Suppose we observe bid histories of J similar auctions. We assume some common structure among them and estimate the common structure in question by pooling bid histories of these auctions. Our data now consist of an unbalanced panel of bids. To account for unobserved persistent auction-specific effects, we assume that $e_{j,t} = \eta_j + \varepsilon_{j,t}, j = 1, \ldots, J$, where η_j is the auction individual effect that is constant over time, and $\varepsilon_{j,t}$ is an error with mean zero and finite variance, which is iid across auctions and over time. For present, assume that all auctions share a common price growth process, $\alpha + \beta m(t)$. Let $P_j(t)$ be the price process of auction j. We can write the model as the following:

$$P_j(t) = \alpha + \beta m(t) + \boldsymbol{x_j'\gamma} + \eta_j + \varepsilon_{j,t}, \tag{4.1}$$

where $\boldsymbol{x}_j$ is a vector of attributes for auction j.

We next discuss some unique features of online auctions that command further modifications of model (4.1). First, model (4.1) imposes monotonicity via $\alpha + \beta m(t)$ for bids across all auctions. This is can be overly restrictive for the pooled bid histories, where monotonicity is only maintained within every individual bid history. To see this, note that when we pool bids from multiple auctions, the pooled bids may not be a monotone process: A bid submitted at time t_1 in one auction is not necessarily lower than a bid submitted at a later time t_2 in another auction, especially when t_1 and t_2 are close. In other words, monotonicity is not shared among the bids of multiple auctions, at least in a local sense. This non-monotonicity is evident from the three auctions depicted in Figure 1: the first bid of the auction represented by triangles is larger than a lot of bids of the other two auctions that occurred at later points of time.

Second, the assumption of a common price growth process $\alpha + \beta m(t)$ may not hold for a lot of online auctions. Suppose that the two auctions have different opening bids, but are otherwise identical *ex ante*. Under model (4.1), it follows that the *expected* final bid at time T for the auction with a higher opening bid is larger than that of another one. This expected discrepancy, however, contradicts an empirical regularity in a lot of popular online auctions, whose price processes oftentimes converge to the market price. For example, the three bid histories shown in Figure 1 clearly demonstrate the convergence of the winning bids in auctions with different opening bids.

To circumvent these problems posed by model (4.1), we need to allow for some flexibility for individual bid histories in our pooled estimation. In this study, we adopt a less restrictive assumption that the bid histories of similar auctions follow a common *relative* growth process $m(t)$. In particular, we allow the the magnitude (or phase) of the relative growth function, β, to differ across different auctions by replacing $\beta m(t)$ in model (4.1) with $\beta_j m(t)$, where β_j is an auction-specific "slope" for the relative growth function $m(t), j = 1, \ldots, J$. The introduction of this auction-specific slope helps accommodate heterogeneity that cannot be explained by observed auction-specific attributes and unobserved persistent individual effects.

The convergence of bids also commands a modification of how we model the effects of auction specific attributes. As discussed above, the price process is affected by auction-specific attributes such as seller's reputation, opening bid set by the seller and so on, whose effects are captured by $\boldsymbol{x'\gamma}$ in model (4.1). This linear structure assumes that $\boldsymbol{x}$ has a constant effect on the price process throughout the duration of the auction. This assumption, however, may not be consistent with the fact that the ending prices of a lot of items that are commonly available at other non-auction merchandisers tend to converge to their market price. Thus, factors such as opening bids may have a larger influence on the price process at the earlier stage of the auction, but play a progressively diminishing role toward the end of the auction. To allow for the possibility of time-varying effect of auction specifics, we propose to replace $\boldsymbol{x'\gamma}$ with a varying coefficient structure $\boldsymbol{x'\gamma}(t)$, where $\boldsymbol{\gamma}(t)$ is a smooth function of t. This approach allows us to capture the effects of auction-specific attributes that may evolve during the course of an auction by using a flexible construction of $\boldsymbol{\gamma}(t)$.

Gathering the above-mentioned modifications motivated by the features of online auction price process, we arrive at a more general model

$$P_j\left(t\right) = \alpha + \beta_j m\left(t\right) + \boldsymbol{x_j'\gamma}\left(t\right) + \eta_j + \varepsilon_{j,t}. \tag{4.2}$$

Next we shall construct an estimator for this model.

4.4. Estimation

Suppose we observe J similar auctions, each with n_j number of bids, $j = 1, \ldots, J$. Let D_j be a dummy variable for auction j. Denote by $p_{i,j}$ the bid placed at time $t_{i,j}$ in auction j, $i = 1, \ldots, n_j$. The panel version of model (4.2) can be written as

$$p_{i,j} = \alpha + \sum_{j=1}^{J} \beta_j D_j m(t_{i,j}) + \boldsymbol{x_j'\gamma}(t_{i,j}) + \eta_j + \varepsilon_{i,j}. \tag{4.3}$$

We shall first state some assumptions and the identification of model (4.3). We then present our estimation strategy of the monotone process $m(t)$, auction-specific coefficients η_j and β_j, and time-varying coefficients $\boldsymbol{\gamma}(t)$. A numerical algorithm for the proposed estimator is proposed. Because of the (semi)-nonparametric nature of the estimator, we conclude this section with a brief discussion on the selection of some smoothing parameters.

4.4.1. Monotone Process $m(t)$

There exist several approaches to model a monotone function. Earlier work includes the isotonic regression (Brunk [**7**]) and the Box-Cox transformation (Box and Cox [**6**]). Ramsay [**32**] presents a monotone series method. Racine et al. [**31**]) propose a kernel-based method for nonparametric estimation with general constraints.[5] Henderson and Parmeter [**14**] provide a brief survey of this literature.

In principle, any specification that warrants monotonicity suffices. In this study we adopt an approach based on Ramsay [**32**]. Suppose that the monotone relative/baseline price growth curve $m(t)$ in model (4.3) takes the form

$$m(t) = \int_0^t \exp(w(u))\, du, \tag{4.4}$$

where w is a real-valued function defined on $[0, T]$ with $w'(u) = d(w(u))/du$ being Lebesgue square integrable. It follows that $m'(t) = \exp(w(t)) > 0$, and thus $m(t)$ is a monotonic function.

The principal advantage brought by representation (4.4) is the transformation of the estimation problem from a problem of finding the constrained function m to a problem of computing the unconstrained function w. We can now model w using

[5]Henderson and Parmeter [**14**] use this approach to estimate a monotone bidding strategy of first-price auctions of Canadian lumbers.

some parametric specification or a flexible nonparametric smoother. Lacking any theoretical guidance, we opt to use a nonparametric series estimator. Popular choices of series basis functions include the polynomials, splines and trigonometric series (see, for example, Li and Racine [**22**] for a review of the series estimation). Let ϕ_k be a series of real-valued, linearly independent basis functions defined on $[0, T]$ that are Lebesgue square integrable. We set $w(t) = \sum_{k=1}^{K} c_k \phi_k(t)$, where $c_k, k = 1, \ldots, K$, are a vector of unknown parameters to estimate.[6] The special case of $w = c_0$ corresponds to the case of exponential $m(t)$ such that it has constant relative curvature, and $w = 0$ defines a linear function. Thus, small or zero values of $w(t)$ correspond to locally linear functions, whereas very large values correspond to regions of sharp curvature.

Compared with the isotonic regression which results in a step function, model (4.4) offers an additional advantage of differentiability. This is an appealing feature because a variable transformation problem requires the inversion of the transformation, perhaps to estimate argument values at points not corresponding to the original observations, i.e. the fitted function shall have a first derivative that is bounded away from 0. In the current study, it is desirable to also obtain the first and second derivatives functions of m, which represent the velocity (or speed) and acceleration (or rate of change of speed) of the price process respectively. One can show that $w'(t) = m''(t) / m'(t)$. Thus $w'(t)$ measures the relative curvature of the monotone function in the sense that it assesses the size of the curvature $m''(t)$ relative to the slope $m'(t)$. Alternatively, it can be viewed as the acceleration of $m(t)$ relative to its velocity.

Our model (4.4) is a variant of Ramsay [**32**]'s monotone series estimator. Ramsay

[6] Note that for $w(t)$, we do not have the usual constant term ϕ_0. This is because the coefficient for ϕ_0 and β cannot be identified separately.

proposed to use the following functional form

$$m^{*}(t)=\int_{0}^{t}\exp\left(\int_{0}^{u}w^{*}(v)\,dv\right)du. \tag{4.5}$$

to model a monotone function. It is seen that our $w'(t)$ in (4.4) plays a similar role as $w^{*}(t)$ does in (4.5). On the other hand, representation (4.4) has a simpler functional form and leads to a computationally more efficient estimation procedure.

4.4.2. Time-Varying Effects of $\boldsymbol{x}$

We model the time-varying effects of covariates using a varying coefficient model, which allows us to capture the differentiate effects of several factors during the life time of an auction. In particular, we use a series representation of the time-varying coefficients. Let $\boldsymbol{\gamma}(t)=\{\gamma_s(t)\}_{s=1}^{S}$ and $\gamma_s=\sum_{k=0}^{K_s}d_{s,k}\psi_{s,k}(t)$, where $\psi_{s,k}$ are a series of basis functions defined on $[0,T]$ with $\psi_{s,0}(t)=1$ for all s. It follows that

$$\boldsymbol{x}'\boldsymbol{\gamma}(t)=\sum_{s=1}^{S}\sum_{k=0}^{K_s}x_s d_{s,k}\psi_{s,k}(t). \tag{4.6}$$

Under this construction for $s=1,\ldots,S$, $d_{s,0}$ represents the baseline (time-invariant) component of γ_s, while the other terms capture how γ_s varies with time. The null hypothesis of $d_{s,k}=0$ for $k>0$ would suggest that the coefficient is unaffected by time and is therefore time-invariant. However, rejecting this null hypothesis would indicate that the coefficient is a function of time, and the values of $d_{s,k}$ would suggest how.

4.4.3. Auction Specific Effects

Model (4.3) has both an auction specific intercept and an auction specific slope (for the baseline price growth curve), and can be categorized as a nonlinear mixed effect model as is called in the statistics literature. In principle, we can use either the random effect or fixed effect model to estimate these auction specific effects. Since all auction attributes in this study are time-invariant, the fixed effect method for the auction specific intercept is not feasible. Therefore, we use random effect estimation to model the auction specific intercept.

On the other hand, we adopt the fixed effect approach for the auction specific slope for the following reasons. First, the estimated auction specific slope provides useful insight on the heterogeneity among procedurally similar auctions. Second, the estimated coefficients also serves as a diagnostic tool. By the positive monotonicity of price process, the individual slopes for the relative growth function are expected to be positive. Thus, negative estimated slopes signal mis-specification of our model. Third, the random effect method assumes independence between the individual slopes for $m(t)$ and other covariates in the model. Since the slope for $m(t)$ is likely correlated with the opening bid, as suggested by the auctions depicted in Figure 1, the independence assumption for random effect estimator is likely violated.

4.4.4. Estimation

Given the specifications of various components discussed above, our model can be written as

$$\begin{aligned} p_{i,j} &= \alpha + \sum_{j=1}^{J} \beta_j D_j m(t_{i,j}) + \boldsymbol{x_j'}\boldsymbol{\gamma}(t_{i,j}) + \eta_j + \varepsilon_{i,j} \\ &= \alpha + \sum_{j=1}^{J} \beta_j D_j \int_0^{t_{i,j}} \exp\left(\sum_{k=1}^{K} c_k \phi_k(u)\right) du + \sum_{s=1}^{S}\sum_{k=0}^{K_s} x_{j,s} d_{s,k} \psi_{s,k}\left(t_{i,j}\right) + \eta_j + \varepsilon_{i,j}, \end{aligned} \tag{4.7}$$

where $\boldsymbol{x}_s = \{x_{j,s}\}_{s=1}^{S}, j = 1, \ldots, J$.

Given the small T large N structure of the panel, we can safely assume stationarity of the data. Suppose that $\text{var}(\eta_i) = \sigma_\eta^2$ and $\text{var}(\varepsilon_{i,j}) = \sigma_\varepsilon^2$. The variance covariance of the composite error $e_{i,j} = \eta_j + \varepsilon_{i,j}$ for auction j takes the form

$$\boldsymbol{\Omega}_j = \begin{bmatrix} \sigma_\eta^2 + \sigma_\varepsilon^2 & \sigma_\eta^2 & \cdots & \sigma_\eta^2 \\ \sigma_\eta^2 & \sigma_\eta^2 + \sigma_\varepsilon^2 & & \vdots \\ \vdots & & \ddots & \sigma_\eta^2 \\ \sigma_\eta^2 & \cdots & \sigma_\eta^2 & \sigma_\eta^2 + \sigma_\varepsilon^2 \end{bmatrix},$$

which is $n_j \times n_j$. The variance covariance matrix for the entire sample is then given by

$$\boldsymbol{\Omega} = \begin{bmatrix} \boldsymbol{\Omega}_1 & 0 & \cdots & 0 \\ 0 & \boldsymbol{\Omega}_2 & & \vdots \\ \vdots & & \ddots & 0 \\ 0 & \cdots & 0 & \boldsymbol{\Omega}_J \end{bmatrix},$$

which is $n \times n$ with $n = \sum_{j=1}^{J} n_j$.

To balance the goodness of fit and the parsimony of the model, we propose to estimate model (4.7) using the penalized least squares estimator. Let $\boldsymbol{\beta} = \{\beta_j\}_{j=1}^{J}$, $\boldsymbol{c} = \{c_k\}_{k=1}^{K}$, and $\boldsymbol{d} = \{d_{s,k}\}_{s=1,k=1}^{S,K_s}$. Define $\boldsymbol{\theta} = \left\{\alpha, \boldsymbol{\beta}, \boldsymbol{c}, \boldsymbol{d}, \sigma_\eta^2, \sigma_\varepsilon^2\right\}$. Model (4.7), in matrix form, can be estimated by

$$\begin{aligned} \min_{\boldsymbol{\theta}} \ \frac{1}{n} & \left\{\boldsymbol{P} - \boldsymbol{\alpha} - \sum_{j=1}^{J} \beta_j \boldsymbol{D}_j \odot \boldsymbol{m} - \sum_{s=1}^{S} \boldsymbol{X}_s \odot \boldsymbol{\gamma}_s\right\}' \boldsymbol{\Omega}^{-1} \\ & \left\{\boldsymbol{P} - \boldsymbol{\alpha} - \sum_{j=1}^{J} \beta_j \odot \boldsymbol{D}_j \boldsymbol{m} - \sum_{s=1}^{S} \boldsymbol{X}_s \odot \boldsymbol{\gamma}_s\right\} + \lambda \int_0^T \left(\sum_{k=1}^{K} c_k \phi_k'(u)\right)^2 du, \end{aligned} \tag{4.8}$$

where $\odot$ indicates element-wise multiplication, $\boldsymbol{\alpha}$ is α times an $n \times 1$ vector of ones,

$\boldsymbol{D}_j$ is an $n \times 1$ vector that takes value one for auction j and zero otherwise, and

$$\boldsymbol{P} = [\boldsymbol{p}_1, \ldots, \boldsymbol{p}_J]', \boldsymbol{p}_j = \left[p_{1,j}, \ldots, p_{n_j,j}\right],$$

$$\boldsymbol{m} = [\boldsymbol{m}_1, \ldots, \boldsymbol{m}_J]', \boldsymbol{m}_j = \left[m(t_{1,j}), \ldots, m(t_{n_j,j})\right],$$

$$\boldsymbol{X}_s = [\boldsymbol{X}_{1,s}, \ldots, \boldsymbol{X}_{J,s}]', \boldsymbol{X}_{j,s} \text{ is an } n_j \times 1 \text{ vector of } x_{j,s} \text{ for auction } j,$$

$$\boldsymbol{\gamma}_s = \left[\boldsymbol{\gamma}_{1,s}, \ldots, \boldsymbol{\gamma}_{J,s}\right]', \boldsymbol{\gamma}_{j,s} = \left[\gamma_s(t_{1,j}), \ldots, \gamma_s(t_{n_j,j})\right],$$

for $j = 1, \ldots, J, s = 1, \ldots, S$. λ is a penalty parameter that determines the degree of penalty or regulation on $w'(t)$.

The first term of (4.8) is the least squares fitting criterion. The second penalty or regularization term is similar to the norm of the second derivative used in cubic spline smoothing. In addition, since $w'(t) = m''(t)/m'(t)$, the penalty term keeps the fitted function away from the boundary condition $m'(t) = 0$.

We use an iterative updating procedure to find the solution for model (4.8). The rate of convergence is linear. Let $\boldsymbol{c}^{(v)}$ be the v^{th}-step value of $\boldsymbol{c}$. Beginning with an initial estimate $\boldsymbol{c}^{(0)}$, which may be a vector of 0s, we estimate $\alpha^{(0)}, \boldsymbol{\beta}^{(0)}, \boldsymbol{\gamma}^{(0)}, \sigma_\eta^{(0)}$ and $\sigma_\varepsilon^{(0)}$ by linear regression. Then, on any iteration $v > 1$ for which $\boldsymbol{\theta}^{(v-1)}$ are estimates on the previous iteration, we first optimize with respect to $\boldsymbol{c}$ by the Gauss-Jordan or scoring procedure for non-linear least squares problem to obtain $\boldsymbol{c}^{(v)}$, and then compute $\alpha^{(v)}, \boldsymbol{\beta}^{(v)}, \boldsymbol{\gamma}^{(v)}, \sigma_\eta^{(v)}$ and $\sigma_\varepsilon^{(v)}$ by linear regression. In particular, denote $\boldsymbol{\phi}(t) = [\phi_1(t), \ldots, \phi_K(t)]'$, $\tilde{\boldsymbol{\phi}}(t) = [\phi_1'(t), \ldots, \phi_K'(t)]'$, and $\boldsymbol{r}_j^{(v-1)} = [r_{1,j}^{(v-1)}, \ldots, r_{n_j,j}^{(v-1)}]'$. Define

$$M = \int_0^T \tilde{\boldsymbol{\phi}}(u)\, \tilde{\boldsymbol{\phi}}(u)' \, du,$$

$$m^{(v-1)}(t) = \int_0^t \exp\left(\boldsymbol{c}^{(v-1)\prime}\boldsymbol{\phi}(u)\right) du,$$

$$r_{i,j}^{(v-1)} = p_{i,j} - \alpha^{(v-1)} - \sum_{j=1}^{J} \beta_j^{(v-1)} D_j m^{(v-1)}(t_{i,j}) - \boldsymbol{x}_j' \boldsymbol{\gamma}^{(v-1)}(t_{i,j}),$$

and $n_j \times K$ matrix $\boldsymbol{Z}_j$, $j = 1, \ldots, J$, with rows

$$\boldsymbol{z}_j^{(v-1)}(t_{i,j}) = \frac{\partial m^{(v-1)}(t_{i,j})}{\partial \boldsymbol{c}} = \int_0^{t_{i,j}} \boldsymbol{\phi}^{(v-1)}(u) \exp\left(\boldsymbol{c}^{(v-1)\prime}\boldsymbol{\phi}(u)\right) du.$$

The Gauss-Jordan procedure requires that the update vector

$$\boldsymbol{\delta}^{(v)} = \boldsymbol{c}^{(v)} - \boldsymbol{c}^{(v-1)}$$

be the solution of the linear equation

$$\boldsymbol{R}^{(v-1)}\boldsymbol{\delta}^{(v-1)} = -\boldsymbol{s}^{(v-1)},$$

where

$$\boldsymbol{s}^{(v-1)} = -n^{-1} \sum_{j=1}^{J} \beta_j^{(v-1)} \left(\boldsymbol{Z}_j^{(v-1)}\right)' \left(\boldsymbol{\Omega}_j^{(v-1)}\right)^{-1} \boldsymbol{r}_j^{(v-1)} + \lambda M \boldsymbol{c}^{(v-1)},$$

$$\boldsymbol{R}^{(v-1)} = n^{-1} \sum_{j=1}^{J} \left(\beta_j^{(v-1)}\right)^2 \left(\boldsymbol{Z}_j^{(v-1)}\right)' \left(\boldsymbol{\Omega}_j^{(v-1)}\right)^{-1} \boldsymbol{Z}_j^{(v-1)} + \lambda M,$$

and $\left(\boldsymbol{\Omega}_j^{(v-1)}\right)^{-1}$ is the jth diagonal block of $\left(\boldsymbol{\Omega}^{(v-1)}\right)^{-1}$, which is block-diagonal. Note that $\boldsymbol{R}^{(v-1)}$ is not defined as $d\boldsymbol{s}^{(v-1)}/d\boldsymbol{c}^{(v-1)}$. Instead, the first term of $\boldsymbol{R}^{(v-1)}$ is

$$-n^{-1} \sum_{j=1}^{J} \beta_j^{(v-1)} \left(\boldsymbol{Z}_j^{(v-1)}\right)' \left(\boldsymbol{\Omega}_j^{(v-1)}\right)^{-1} \frac{d\boldsymbol{r}_j^{(v-1)}}{d\boldsymbol{c}^{(v-1)\prime}}.$$

This 'partial' directive of the first term of $\boldsymbol{s}^{(v-1)}$ with respect to $\boldsymbol{c}^{(v-1)}$is essentially an estimate of the outer product of the score function. This construction relieves the computational burden of calculating the full Hessian matrix, and ensures the string positive definiteness of $\boldsymbol{R}^{(v-1)}$.

Since σ_η^2 and σ_ε^2 are not observed, we replace them with their corresponding estimates. They can be estimated consistently by

$$(\sigma_\eta^{(v-1)})^2 = \frac{1}{J}\sum_{j=1}^{J}\frac{1}{n_j(n_j-1)/2}\sum_{i=1}^{n_j-1}\sum_{h=i+1}^{n_j} r_{i,j}^{(v-1)} r_{h,j}^{(v-1)},$$
$$(\sigma_\varepsilon^{(v-1)})^2 = \frac{1}{J}\sum_{j=1}^{J}\frac{1}{n_j}\sum_{i=1}^{n_j}(r_{i,j}^{(v-1)})^2 - (\sigma_\eta^{(v-1)})^2.$$

Lastly, we can select the smoothing parameter λ through some optimizing metrics such as the risk or the prediction risk, or through the method of cross validation or generalized cross validation.

4.5. Data

Auction data typically arrive in the form of a series of bids placed over a period between the start and end of an auction. In our empirical investigation of the price process of online auctions, we use a sample of eBay auctions of Palm M515 Personal Digital Assistant (PDA). At the time of data collection, these PDAs were popular items on eBay and had a market value of $249 (according to the popular online store Amazon.com). All auctions examined in this study are hard-closed with a duration of seven days, taking place between March 14th and May 25th of 2003. The same data are studied in Shmueli et al. [**39**], who employ a functional data analysis approach. In order to reduce external sources of variation, we include only auctions that are transacted in US dollars, of completely new items with no added-on features, having at least 10 bids, and where the seller did not set a secret reserve price. The number of auctions is 125 and the total number of bids is 3,351. For each auction, we observe the arrival time and magnitudes of bids, and auction-specific characteristics including seller's rating[7] and opening bids. Every auction in our data resulted in a sale.

Summary statistics of the auctions used in our analysis are given in Table I. These auctions vary considerably in many aspects. The number of bids ranges from 10 to 51 with an average around 27. The final price, which is essentially the second highest bid (plus one minimum increment), ranges from $177 to $250. It is known that many factors can influence the final price of online auctions, including the number of bidders, the opening bid, seller's reputation, shipping and handling, whether the item is under warranty, whether it is a brand-new or open-boxed item, whether the item comes with extra accessories, and so on. Despite that we only include auctions of brand-new Palm M515 with no add-on features in our sample, the final prices exhibit a substantial dispersion. At the same time, the final prices in online auctions are observed to converge to the market price ($249 in our case) from below. Not surprisingly, we observe that the bids top at $250 for all 125 auctions in our sample. The distribution of the final prices is centered around $230, $20 below the market price, and is approximately symmetric.

There is a considerable variation in the opening bids. The lowest opening bid equals the minimum increment ($0.01), and the highest ($180) is rather close to the market price. More than 50% of the opening bids are less than $1, and about 75% less than $10, indicating that most opening bids are likely speculative. It has been found that the opening bids have a direct and an indirect effect on price (Bapna et al. [**5**]). The opening bid is typically positively associated with final prices. Its indirect

[7] On eBay, a buyer can rate a seller by giving them a positive (+1), neutral (0), or negative (-1) score, along with a text comment. eBay records and displays all of these comments, including the ID of the person making the comment. eBay also displays some summary statistics of users' feedback.

influence has the oppositive direction: Lower opening bids attract more bidders, and the increased competition often results in a higher final price. This indirect effect on the number of bidders is evident from the three auctions depicted in Figure 1: the auction started with an opening bid around \$150 (represented by triangles in the plot) has a considerably smaller number of bids than the the other two do, especially in the early stage of the auction. As is discussed above, the effects of opening bids seem to diminish over time. The three auctions started with rather different opening bids then all converged to the market price. As for seller's ratings, they vary between 0 and 6,000. On eBay, a seller's rating is associated with trust: higher-rated sellers tend to extract price premiums due to their higher trust levels.

Table I. Summary Statistics

	Min.	1st Q.	Median	Mean	3rd Q.	Max.
number of bids	10	21	26	26.81	32	51
bid (unit: \$)	0.01	75	150	135.82	200	250
final price (unit: \$)	177	222	233	229.10	240	250
open bid (unit: \$)	0.01	0.01	1	12.40	10	180
seller rating	0	72	256	622	1,180	5,432

Next, Table II reports the frequency of bid arrivals during the course of seven-day auctions. Apparently, the time distribution of bid arrivals exhibits the typical patterns of online auctions: early bidding, mid-auction bidding draught, and bid sniping. In particular, bid sniping is rather evident: slightly more than 10% of bids arriving within an hour of auction end, and more than 5% within last 5 minutes.

Table II. Time Distribution of Bid Arrivals

	Day 1	Day 2	Day 3	Day 4	Day 5	Day 6	Day 7
Percentage (%)	13.28	6.81	6.81	5.21	9.08	13.58	45.23
	last hour		last 10 min		last 5 min		last 1 min
Percentage (%)	11.62		6.41		5.37		2.24

4.6. Estimation and Results

In this section, we first discuss the fitting of the proposed model to the eBay online auction data. We then present the estimation results, followed by some robustness check.

4.6.1. Estimation

Aiming to arrive at a parsimonious representation of the relative growth function $m(t)$, we use power series to model $w(t)$. In particular, we set $w(t) = \sum_{k=1}^{4} c_k t^k$. For simplicity, we use a power series of degree two to model the varying coefficients, as specified in (4.6), of open bids and seller's rating. Since the duration of hard-closed auctions is fixed, the domain of $w(t)$ is bounded. Thus the risk associated with power series due to potential outliers is greatly reduced. We use the GCV to select the regularization parameter λ. The auction-specific individual effects and slopes are modeled as random effects and fixed effects respectively. We use bootstrap for inferences. In particular, we use resampling of residuals to construct the bootstrap samples. This procedure is repeated for 300 times.

4.6.2. Results

The estimated results are reported below, where x_1 is the logarithm of seller's rating and x_2 is the opening bid. The bootstrap standard errors are reported in parenthesis below their corresponding coefficients.

$$\hat{P}(t) = \underset{(9.52)}{-25.71} + \sum_{j=1}^{J} \hat{\beta}_j D_j \hat{m}(t) + (\underset{(1.39)}{4.63} - \underset{(3.16)}{10.99}\, t + \underset{(0.26)}{1.38}\, t^2)x_1 + (\underset{(0.13)}{1.10} - \underset{(0.15)}{0.31}\, t + \underset{(0.01)}{0.03}\, t^2)x_2, \tag{4.9}$$

where

$$\hat{m}(t) = \int_0^t \exp(-\underset{(0.175)}{1.655}\, u + \underset{(0.159)}{0.784}\, u^2 - \underset{(0.039)}{0.193}\, u^3 + \underset{(0.003)}{0.016}\, u^4)du.$$

All the coefficients reported above are statistically significant at the 5% level. Because of the large number of auctions in our sample ($J = 125$), we choose not to report individual $\hat{\beta}_j$s but provide a summarization below. Consistent with the fact that the price process is a monotone increasing process, all estimated coefficients for β are positive. The coefficients $\hat{\beta}_j, j = 1, \ldots, J$, range from 114 to 258, with mean 183, median 187, and standard deviation 31. Regarding the error components, we obtain $\sigma_\varepsilon = 27.52$ and $\sigma_\eta = 2.26$ with bootstrap standard error 0.41 and 0.19 respectively. The adjusted R^2 is 0.85 for model (4.7), which is quite satisfactory given the parsimonious model with the small number of covariates (bid time, seller's rating, opening bid and auction dummies) and a high degree of freedom (2,860).[8]

Figure 2 plots the estimated price process (represented by solid line) evaluated at sample averages of auction attributes and the average slope for $\hat{m}(t)$ across all

[8]For comparison, we also estimate a simple model with a common slop β for all auctions and time-invariant coefficients for auction attributes. The corresponding adjusted R^2 is 0.66. Model (4.7) improves upon the simple model considerably by taking into account various features of the price processes.

auctions. In other words, the plotted price curve is constructed as

$$\hat{P}(t) = -25.71 + \bar{\hat{\beta}}\hat{m}(t) + (4.63 - 10.99t + 1.38t^2)\bar{x}_1 + (1.10 - 0.31t + 0.03t^2)\bar{x}_2,$$

where $\bar{\hat{\beta}} = 1/J \sum_{j=1}^{J} \hat{\beta}_j = 183$, and $\bar{x}_1$ and $\bar{x}_2$ are sample averages of seller's rating and opening bids respectively. The bootstrap 95% point-wise confidence band (represented by dash lines) is reported. Also reported is the scatter plot of the raw data. It is seen that the estimated curve is estimated precisely and closely captures the general pattern of price evolution.

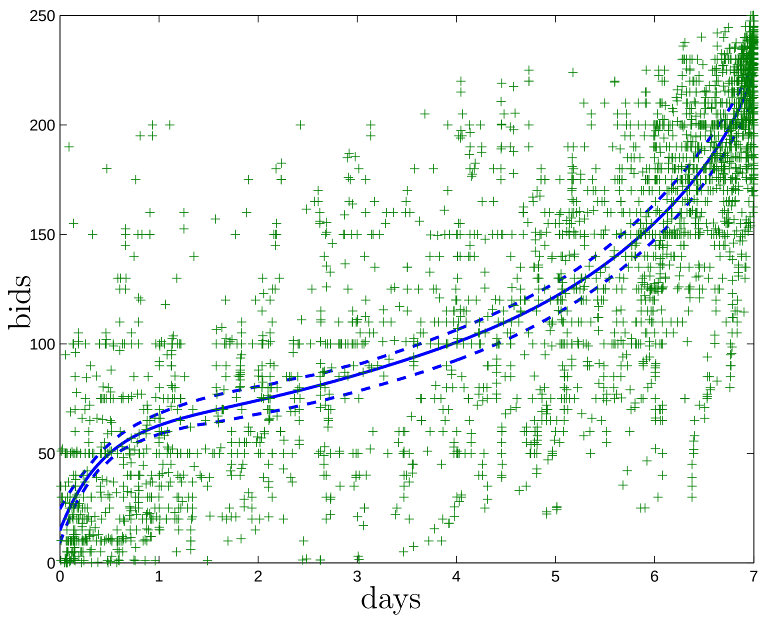


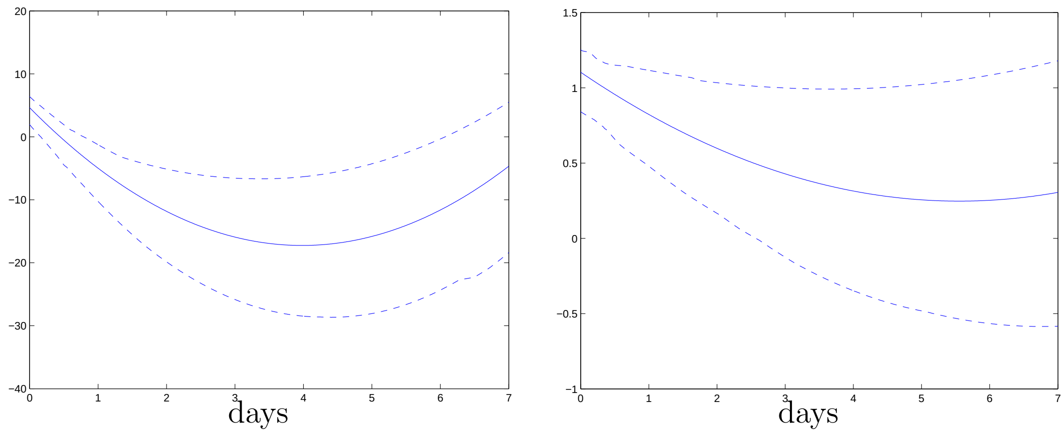
Fig. 2. Estimated Price Process

We next examine the estimated time-varying coefficients for the two auction-specific attributes. The results are plotted in Figure 3, in which the estimated curves are represented by solid lines and bootstrap 95% confidence band by dash lines. Both coefficients show distinct time-varying patterns. Furthermore, the estimates are relatively accurate at the early stage of the auctions, consistent with the fact that toward the end of the auctions, the bids converge to the market price, and hence the effects of various factors become increasingly small.[9] For seller's rating, the initial effect is positive. For most of the middle duration of the auctions, its effects are shown to be negative. Naturally, one would expect seller's rating to have a positive effect on the bids. Empirical evidence, however, provides mixed results. Resnick et al. [**33**] summarizes empirical studies on the effect of network reputation and find that the effects vary considerably across studies. Wang et al. [**42**] also find that seller's rating has a negative effect on the bids. One possible reason for the weakness of empirical evidence for seller's rating is that there are very few negative feedbacks.[10] Another confounding factor is that seller's rating is likely more a proxy seller's experience than seller's trustworthiness. This, in turn, might lead to lower transaction price since experienced sellers are on average more efficient. On the other hand, as expected, opening bids have a positive effects on the bids at the beginnings of the auctions. Its effects slowly decline during the course the auction, at the end of which it becomes not significantly different from zero.

Next recall that the relative acceleration, $m''(t)/m'(t)$, is captured by $w'(t)$, which is plotted in Figure 4. The estimated curve is represented by a solid line, with bootstrap 95% confidence band in dash lines. It is seen that this relative acceleration curve is estimated with high precision. Since by construction $m'(t) > 0$ for the entire duration of auction, the sign of the relative acceleration only depends on $m''(t)$. The result provides a clear illustration of the price dynamics. The relative acceleration

[9] Note that the coefficients of the time-varying coefficients, as reported in (4.9), are estimated precisely. The increasingly widened confidence bands merely reflect the diminishing impacts of auction attributes toward the end of the auctions.

[10] Almost all of the feedback on eBay is positive. One interpretation can be that users are hesitant to leave negative feedback for fear of retaliation. Another factor limiting the potential usefulness of feedback, reported by Resnick et al. [**33**], is that feedback provision is an arguably costly activity that is completely voluntary, and that not all buyers (52.1 percent) actually provide reviews about their sellers.



(a) Time-varying coefficients for seller's rating

(b) Time-varying coefficients for opening bids

Fig. 3. Time-Varying Coefficients

increases gradually and persistently during the auction. The acceleration at the early stage of the auctions is negative, indicating that price movement has slowed downed since the onset of the auction. The second stage of the auction is characterized by a stable and slightly below-zero relative acceleration, corresponding to the middle segment of the price process, which is roughly linear and marked by relatively subdued bidding activities. The final stage of the price process exhibits a positive and increasing acceleration, accumulating in "bid sniping" type behavior near the end of the auction.

4.6.3. Specification Tests and Robustness Check

To investigate how sensitive the results are to various decisions or choices we have to make in our estimation, we conducted several robustness checks. The first two experiments are concerned with the basis functions used in the estimation: the number of terms and the specification of basis functions. The third experiment employs a transformation of the dependent variable.

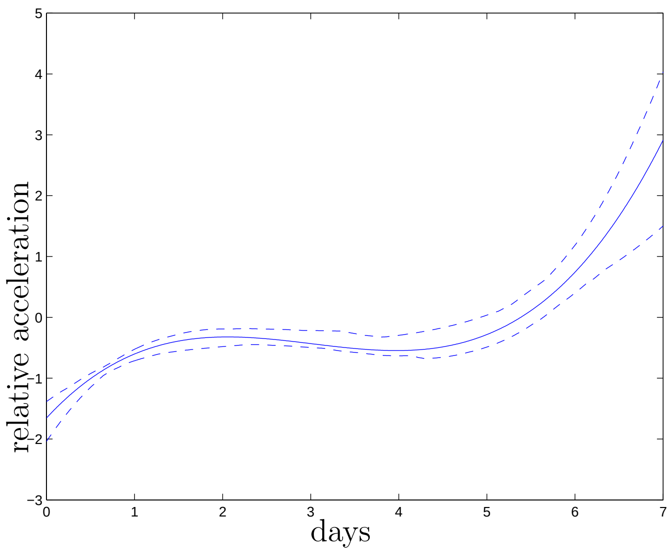


Fig. 4. Estimated Relative Acceleration of Price Process

Firstly, recall that we use a degree-four polynomial to model $m(t)$. Alternatively, we experiment with $K = 2, 6, 8$. The results are very similar to what is reported above with the penalty parameter selected automatically by the GCV method. Secondly, it is known that polynomials can be sensitive to outliers and is vulnerable to undesirable oscillations. Alternatively, we can use a spline basis to model $m(t)$. In our experiment, we use cubic spline with seven equally-spaced knots, and data-driven penalty parameter λ. Again, the results are similar to those obtained by using polynomials. The robustness of our results with respect to variations in the specification of basis function and number of terms are not surprising given that fact the the under price evolution appears to be a quite smooth function of time. For brevity, the results for the first two experiments are not reported. They are available from the author upon request.

Thirdly, to ensure that all predicted bids are positive, we can model logarithm of bids instead of the level of bids. Furthermore, it is observed all bids are below the market price. To ensure that the predicted bids are both positive and below the market price, we can use a logistic transformation $q = \log(p/(p^* - p))$, where $p^* = 250$ is the market price. To avoid numerical problems associated with y close to its lower bound (zero) and upper bound (250) in the logistic transformation, we modify the logistic transformation to $q = log((p + a)/(p^* + a - y))$ in our regression, where a is a small positive number. In our experiment, we set $a = 10$. We then estimate model (4.8) with p replaced by q. To compare with the results reported above, we evaluate $\tilde{p}(t) = ((p^* + a)\exp(\hat{q}(t)) - a) / (1+\exp(\hat{q}(t)))$, where $\hat{q}(t)$ is the estimate evaluated at sample averages. The estimated price processes based on $\hat{p}(t)$ (solid) and $\hat{q}(t)$ (dash) are reported in Figure 5 below. It is seen that our nonparametric models adapt to the transformation of the dependent variable and therefore the estimated price process is little affected by monotone transformation of the dependent variable.[11]

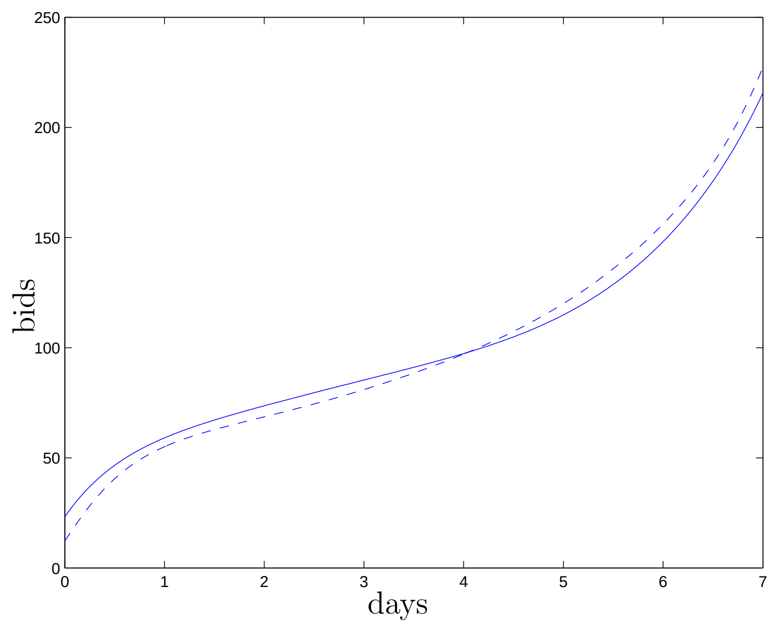


Fig. 5. Estimated Price Process (solid: p; dash: q)

[11]Modeling p directly may render fits to individual bid histories below zero or above the market price. Nonetheless, our experiment demonstrates that the estimated overall price process is rather robust to alternative transformation of p.

4.7. Conclusion

We observe some important and economically interesting empirical regularities in the bidding histories of online auctions. Based on our examination of online auction bidding process and guided by economic theories, we present a panel model of online auction price process that has the following features: (a) a common monotone price growth curve with a multiplicative auction-specific slope; (b) an additive auction-specific effect; (c) time varying effects for auction-specific attributes. A monotone nonparametric estimator is used for the common price growth process. The auction-specific slopes and intercepts are modeled as fixed effects and random effects respectively. A varying coefficient representation is used to model the time varying effects of auction attributes. Lastly, a penalized nonlinear least square estimator is proposed to estimate the coefficients and select the smoothing coefficients simultaneously. The results are shown to capture the overall pattern of online auction data closely.

The proposed model and estimator can be useful for researchers interested in modeling online auction price process and auction predictions. Future work that investigates the price process and bid arrival process jointly will be of interest.

CHAPTER V

SUMMARY

The classical economics argues the principle of one price. In real world, we know it is often the exception than the rule. There exist a lot of mechanisms or institutions that might lead to heterogeneous price. Auction is an important one. First, it offers a unique opportunity for us to observe the price determination process. The popularity of online auctions makes it even more relevant because of the availability of huge amount of data. Second, it is an economic subject where people can seamlessly combine economic theory and empirical investigation. This is exciting because for theorists, their models can be tested on real data. On the other hand, applied researchers can construct their model based on economic theories. Thirdly, auction as a mechanism or institution is rich enough, it offers so many variants that are both theoretical interesting and of practical importance. From the simple English auction or Vickrey auction to multi-unite auctions, or multi-subject auctions, where the strategic interactions among the bidders, and sellers and between these two parties pose so many interesting questions and challenges that remained to be answered.

In the first essay of this dissertation, I constructed a theoretical model to study BIN auctions. Different from a typical online auction which takes the format of a second-price open-bid English auction, in a BIN auction, bidders are allowed to purchase the item at a fixed price called Bin price set by the seller and end the auction immediately. One important feature of the eBay Bin auctions is that, once anyone submits a bid above the reserve price, the BIN option will disappear and a standard auction starts, so all the bidders who arrive after will not be offered the BIN option.

While most theoretical paper studying eBay BIN auctions assume that all bidders are offered the BIN option, for example, Reynolds and Wooders [**34**], Mathews and katzman [**25**], I constructed a two-stage model in which the BIN price is only available to one group of bidders. Although most theoretical models in BIN auctions assume that the bidder utility functions are in special forms, such as CARA or linear functions. I establish my theoretical results for generally concave utility functions with arbitrary number of bidders and any continuous valuation distributions.

I prove the existence and uniqueness of an equilibrium cutoff, such that bidders with valuations higher than that will accept the BIN, and reject it otherwise. I further find that bidders cutoff' is lower in this model, which means, bidders are more likely to accept the BIN option, compared with those models assuming all bidders are offered the BIN. This help explain the high acceptance rate of BIN options in the field data, which can not be explained by the previous other models. Moreover, I find that by setting an appropriate BIN price, when bidders are risk averse, the seller can obtain a higher revenue in BIN auction compared with second-price auction. Also, seller revenue in this model is higher than that in previous model when assuming all bidders are offered BIN option. So this model may better explain the popularity of BIN auctions.

In the second essay, I constructed a simple two-bidder model based on the model in the first essay. Besides risk attitude, I also include time discounting rate in this model. Then I predict equilibrium bidder strategy and seller revenues from the model. I consider two cases, when BIN option is offered to both bidders(homogenous bidders) and when BIN option is offered to only one of them (heterogenous bidders). I showed that a bidder's cutoff point is lower in a BIN auction with heterogenous bidders than in a BIN auction with homogenous bidders. Similar results hold for risk averse and time impatient bidders. Therefore, a model of BIN auctions with heterogenous bidders may explain that bidders accept the BIN price with a higher frequency than the theoretical prediction using models with homogenous bidders.

In the third essay, I focus on the econometrics modeling of the price processes of online auctions. The bidding process of online auctions is typically marked by some common features, such as, monotonically increasing bids, uneven temporal distribution, bid sniping, and final price bounded by the market price. We propose a monotone series estimator (Ramsay [**32**]) for panel data for a common relative price growth curve. And then we apply the model and the estimator to eBay auction data of a palm PDA. The results are shown to capture closely the overall pattern of observed price dynamics. In particular, early bidding, mid-auction draught, and snipping are well approximated by the estimated price curve.

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APPENDIX A

PROOF OF THEOREM 1

Proof. For a unique cutoff $c^* \in [p, \overline{v}]$ to exist such that an early bidder accepts the BIN price if his valuation $v_i > c$, rejects the BIN price if $v_i < c$ and indifferent between the two options if v_i=c, we need equation **??** to have one unique solution $c^* \in [p, \overline{v}]$.

Define $\gamma(x) = \frac{1-x^{n_1}}{n_1(1-x)} \leq 1$. It follows that $\gamma(1) = \lim_{x \to 1} \gamma(x) = 1$.

$$\gamma(x) = \frac{1 - x^{n_1}}{n_1(1-x)} > \frac{1 - x^n}{n(1-x)} > x^{n-1} \text{ for } x \in [0, 1) \text{ and } n > n_1.$$

Define $Q(v) = \gamma(F(v)) = \frac{1-F(v)^{n_1}}{n_1(1-F(v))}$ and $G(v) = F^n(v)$.

Then we have

$$Q(v) > G(v).$$

Define the left hand side of equation 1 as

$$L(c) = u(c - p)Q(c).$$

Apply the concept of the certainty equivalent payment, we define the right hand side as $R(c)$ and

$$R(c) = u(c - r)G(r) + \int_r^c u(c - x)dG(x) = u(c - \delta(c))G(c).$$

For a unique equilibrium cutoff $c^* \in [p, \overline{v}]$ to exist, $L(c)$ and $R(c)$ should have only one intersection when $p \leq c \leq \overline{v}$.

At equilibrium,

$$u(c - p)Q(c) = u(c - \delta(c))G(c)$$

Since $Q(c) > G(c)$ and $u(x)$ is concave, we have $u(c-p) < u(c-\delta(c))$ and $u'(c-p) > u'(c-\delta_\alpha(c))$.

Plugging $c = p$ and $\overline{v}$ into $L(c)$and $R(c)$, we have

$$\begin{aligned}
&L(p) = 0;\\
&R(p) = u(p-r)F^{n-1}(r) + \int_r^p u(p-x)dF^{n-1}(x) \geq L(p)\\
&L(\overline{v}) = u(c-p)\\
&R(\overline{v}) = u(\overline{v} - \delta(\overline{v}))
\end{aligned}$$

Lets consider the following cases.

Case 1: $p \leq \delta(\overline{v})$

We first consider the case when $p \leq \delta(\overline{v})$. Since $u'(x) > 0$, then $u(\overline{v}-p) > u(\overline{v} - \delta(\overline{v})$.Hence, we have

$$L(p) \leq R(p) \text{ and } L(\overline{v}) > R(\overline{v}).$$

Therefore, the solution always exists as long as $u'(x) > 0$.

If $\frac{\partial L(c)}{\partial c} > \frac{\partial R(c)}{\partial c}$ when $c \in [p, \overline{v}]$, then $L(c)$ and $R(c)$ have one unique intersection in this domain.

$$\begin{aligned}
\frac{\partial L(c)}{\partial c} &= u'(c-p)Q(c) + u(c-p)Q'(c)\\
\frac{\partial R(c)}{\partial c} &= u'(c-\delta(c))(1-\delta'(c))G(c)) + u(c-\delta(c))G'(c)\\
&= u'(c-\delta(c))G(c) + u(c-\delta(c))G'(c) - u'(c-\delta(c))\delta'(c)G'(c)
\end{aligned}$$

Since $u'(c-p) > u'(c-\delta(c))$ and $Q(c) > G(c)$, we have

$$u'(c-p)Q(c) > u'(c-\delta(c))G(c).$$

Hence $\frac{\partial L(c)}{\partial c} > \frac{\partial R(c)}{\partial c}$ if

$$\begin{aligned} u(c-\delta(c))G'(c) &\leq u'(c-\delta(c))\delta'(c)G(c) \\ \delta'(c) &\geq \frac{u(c-\delta(c))G'(c)}{u'(c-\delta(c))G(c)} \end{aligned}$$

By the defintion of the certainty equivalent payment,

$$u(c-\delta(c))G(c) = u(c-r)G(r) + \int_r^c u(c-x)dG(x).$$

Differentiate both sides, we obtain

$$u'(c-\delta(c))G(c) - u'(c-\delta(c))\delta'(c)G(c) + u(c-\delta(c))G'(c) = u'(c-r)G(r) + \int_r^c u'(c-x)dG(x).$$

Therefore,

$$\delta'(c) = \frac{u'(c-\delta(c))G(c) - u'(c-r)G(r) - \int_r^c u'(c-x)dG(x) + u(c-\delta(c))G'(c)}{u'(c-\delta(c))G(c)}.$$

Hence, $\delta'(c) \geq \frac{u(c-\delta(c))G'(c)}{u'(c-\delta(c))G(c)}$ if

$$u'(c-r)G(r) + \int_r^c u'(c-x)dG(x) \leq u'(c-\delta(c)G(c)$$

By mean value theorem, for any $x \in [r, \delta(c)]$, we can write

$$\begin{aligned} u'(c-x) &= u'(c-\delta(c)+\delta(c)-x) \\ &= u'(c-\delta(c)) + (\delta(c)-x)u''(y), \end{aligned}$$

where $y \in [c - \delta(c), c - x]$.Therefore,

$$\begin{aligned}
&u'(c-r)G(r) + \int_r^c u'(c-x)dG(x) \\
&= u'(c-r)G(r) + \int_r^c (u'(c-\delta(c)) + (\delta(c) - x)u''(y))dG(x) \\
&= u'(c-r)G(r) + u'(c-\delta(c)(G(c) - G(r)) + \int_r^c (\delta(c) - x)u''(y)dG(x) \\
&= G(r)(u'(c-r) - u'(c-\delta(c)) + u'(c-\delta(c)G(c) + \int_r^c (\delta(c) - x)u''(y)dG(x) \\
&\leq u'(c-\delta(c)G(c) + \int_r^c (\delta(c) - x)u''(y)dG(x) \text{ since } r \leq \delta(c).
\end{aligned}$$

Denote $M = \max u''(y)$. Since u is concave, $M < 0$. Therefore

$$\int_r^c (\delta(c) - x)u''(y)dG(x) \leq M \int_r^c (\delta(c) - x)dG(x) = M(\delta(c) - \int_r^c xdG(x)).$$

By the definition of $\delta(c)$ and the concavity of the utility function u, we have $\delta(c) \geq \int_r^c xdG(x)$ thus

$$\int_r^c (\delta(c) - x)u''(y)dG(x) \leq 0.$$

Hence, we obtain

$$u'(c-r)G(r) + \int_r^c u'(c-x)dG(x) \leq u'(c-\delta(c)G(c).$$

Therefore, we establish that

$$\frac{\partial L(c)}{\partial c} > \frac{\partial R(c)}{\partial c},$$

which, combining with $L(p) \leq R(p)$ and $L(\overline{v}) > R(\overline{v})$, implies the uniqueness and existence of the cutoff equilibrium equilibrium cutoff $c^* \in [p, \overline{v}]$

Case 2: $p > \delta(\overline{v})$

Since $p > \delta(\overline{v})$, we have

$$u(\overline{v} - p) \leq u(\overline{v} - \delta(\overline{v})),$$

which means

$$L(\overline{v}) \leq R(\overline{v}).$$

In the proof of case 1 we have shown that $\frac{\partial L(c)}{\partial c} > \frac{\partial R(c)}{\partial c}$. Therefore, for one equilibrium $c^* \in [p, \overline{v}]$ to exist, we need $L(p) \geq R(p)$. However, $L(p) = 0$ and $R(p) \geq 0$. So the only possible solution is $c^* = \overline{v}$ when $p = \delta(\overline{v})$, otherwise $L(c) \leq R(c)$.

In conclusion, when the BIN price $p > \delta(\overline{v})$, an early bidder never accepts the BIN price in the equilibrium; when $p = \delta(\overline{v})$, a bidder with valuation $\overline{v}$ is indifferent between accepting a BIN price or rejecting it and bidders with $v < \overline{v}$ will always reject the BIN price; when $p < \delta(\overline{v})$, there exists a unique symmetric equilibrium cutoff $c^* \in [p, \overline{v}]$ for early bidders such that an early bidder accepts the BIN price if his valuation is higher than c^* and rejects the BIN price otherwise. It follows that the equilibrium cutoff c^* is implicitly defined by

$$u(c^* - p)\frac{1 - F^{n_1}(c^*)}{n_1(1 - F(c^*))} = u(c^* - r)F^{n-1}(r) + \int_r^{c^*} u(c^* - x)dF^{n-1}(x),$$

which completes the proof of Theorem 1. ■

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