ANALYSIS AND CONTROL OF BATCH ORDER PICKING PROCESSES CONSIDERING PICKER BLOCKING

A Dissertation

by

SOONDO HONG

Submitted to the Office of Graduate Studies of
Texas A&M University
in partial fulfillment of the requirements for the degree of

DOCTOR OF PHILOSOPHY

August 2010

Major Subject: Industrial Engineering

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Considering Picker Blocking

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Approved by:

Co-Chairs of Committee,	Andrew A. Johnson
	Brett A. Peters
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	Vivek Sarin
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ABSTRACT

Analysis and Control of Batch Order Picking Processes Considering

Picker Blocking. (August 2010)

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Co-Chairs of Advisory Committee: Dr. Andrew A. Johnson
Dr. Brett A. Peters

Order picking operations play a critical role in the order fulfillment process of distribution centers (DCs). Picking a batch of orders is often favored when customers' demands create a large number of small orders, since the traditional single-order picking process results in low utilization of order pickers and significant operational costs. Specifically, batch picking improves order picking performance by consolidating multiple orders in a "batch" to reduce the number of trips and total travel distance required to retrieve the items. As more pickers are added to meet increased demand, order picking performance is likely to decline due to significant picker blocking. However, in batch picking, the process of assigning orders to particular batches allows additional flexibility to reduce picker blocking.

This dissertation aims to identify, analyze, and control, or mitigate, picker blocking while batch picking in picker-to-part systems. We first develop a large-scale proximity-batching procedure that can enhance the solution quality of traditional batching models to near-optimality as measured by travel distance. Through simulation studies, picker blocking is quantified. The results illustrate: a) a complex relationship

between picker blocking and batch formation; and b) a significant productivity loss due to picker blocking.

Based on our analysis, we develop additional analytical and simulation models to investigate the effects of picker blocking in batch picking and to identify the picking, batching, and sorting strategies that reduce congestion. A new batching model (called Indexed order Batching Model (IBM)) is proposed to consider both order proximity and picker blocking to optimize the total order picking time. We also apply the proposed approach to bucket brigade picking systems where hand-off delay as well as picker blocking must be considered.

The research offers new insights about picker blocking in batch picking operations, develops batch picking models, and provides complete control procedures for large-scale, dynamic batch picking situations. The twin goals of added flexibility and reduced costs are highlighted throughout the analysis.

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CHAPTER I

INTRODUCTION

Distribution centers (DC) are a fundamental part of the supply chain, which links manufacturers to customers. Within the supply chain, DCs consolidate and store products, fulfill stocked products as requests arrive, and provide various value-added functions in response to product requirements. DCs are also of economic importance; according to the annual "State of Logistics Report" (Wilson, 2008), warehousing costs in the United States are approximately 8% of the total logistic cost, or 0.8% of total gross domestic product (GDP).

Online retailers' DCs are often termed "order fulfillment facilities." Their functions include distributing customer orders and sustaining the online retail business. Clearly, order picking operations represent significant cost and service drivers for these retailers. According to Tompkins *et al.* (2003), order picking typically comprises almost 50% of the total operating costs of a typical DC. For example, in 2003, Amazon.com's fulfillment expense was $477 million, which accounts for 48% of total operation expenses (Amazon.com, 2004). Amazon's order picking operations contribute between 10-15% of its fulfillment-related expenditure, including fulfillment and customer service centers (Lieu, 2005).

Despite the recent enhancements in order picking technology, 75 to 80% of all DCs still rely on manual order picking (De Koster, 2004; Napolitano, 2008).

This dissertation follows the style of *IIE Transactions*.

Manual order picking is cost effective because the initial setup cost is relatively low. Moreover, human pickers are flexible relative to mechanical systems (Ruben and Jacobs, 1999) and can more easily handle irregular shapes and sizes and employ diverse sets of picking vehicles as needed.

Since customer demands in online retailers' order fulfillment facilities are characterized by diverse, small-sized orders (De Koster, 2003), manual order picking faces a critical operational issue to ensure good performance. The problem involves determining the set of orders, i.e., the batch, to be picked by a worker, and the worker's route through the facility to retrieve the items in the batch. The traditional single-order picking mode of operation can result in many costly trips, particularly if the orders are small. In contrast, a batch order picking strategy groups orders to reduce the number of trips required, and consequently, reduces operational costs. Additionally, the latter strategy provides some robustness to the variation and operational difficulties caused by small order sizes. Therefore, an efficient order batching algorithm can have a significant impact on costs in an order picking environment with small order sizes.

In general, the number of items picked per unit of time is an important criterion for evaluating warehouse performance (De Koster and Balk, 2008). When a shorter fulfillment period is required, manual order picking systems tend to add more pickers to shorten the response time. However, using a batch picking strategy with multiple pickers introduces a new issue relevant to picker utilization, namely, that multiple pickers will create congestion and delays that "waste" productive work time. This increase in nonproductive time is known as *picker blocking*. The impact of the number of pickers on

order picking throughput and picker utilization indicates that warehouse managers should focus on picker blocking when assigning a large number of pickers to a particular retrieval process. We note, however, that traditional batching algorithms do not consider picker blocking or its impact on order picking productivity.

This dissertation is interested in order batching procedures in large-scale picking situations with *k*-pickers, where picker blocking can become a significant issue. We begin by considering a narrow-aisle picking environment, which is very attractive in terms of storage capability. However, since one-way passage in an aisle may be inevitable in this configuration (Gue *et al.*, 2006), the order fulfillment time can lengthen and the operational cost can increase, because the one-way travel characteristic leads to longer trips and the narrow-aisle configuration produces heavy congestion Bartholdi and Eisenstein.

Thus, we first examine the significance of picker blocking in the traditional proximity-based batching approach. This sub-study presents a new large-scale, near-optimal *distance-based* batch order picking procedure with traversal routing methods. The operational policy identified by a gap error comparison is near-optimal based on a travel distance criterion, but also reduces picker blocking relative to other order batching methods. However, management is still required to reduce productivity loss due to blocking.

Second, since the prior simulation study identifies picker blocking which is not fully modeled by the available literature, we focus on developing an analytical model that is suitable for batch picking and examining situations of varying levels of picker

activity.

Because batch picking with *k*-pickers appears to produce a significant level of picker blocking when *k* increases to fulfill high demand levels, we propose a combined batching and sequencing model, referred to as the indexed batching model (IBM), to simultaneously control both the trip distance and the time blocked.

We also analyze bucket brigade picking, a popular order picking situation, where picker blocking is still an issue but the routing issue is replaced with hand-off delay issue. We identify analytical throughput models, build an integrated control framework to reduce both picker blocking and hand-off delay, and derive control algorithms for each delay case.

This dissertation is organized as follows. Chapter II describes general knowledge and background on order picking in distribution centers. Chapter III reviews the literature and identifies new opportunities. Chapter IV explores managing a large-size order batching situation more efficiently and describes the effects of picker blocking. Chapter V examines picker blocking in batch picking using analytical models and simulation study. A new batching model that considers both proximity and congestion is developed in Chapter VI. In Chapter VII, we discuss an application of the proposed approach to bucket brigade systems. Chapter VIII summarizes the contributions and highlights future research opportunities.

CHAPTER II

BACKGROUND

1. ORDER PICKING SYSTEMS

Order picking operation involves retrieving customer orders from storage in an order picking system (OPS) in a DC. Commonly, a DC is composed of multiple OPSs classified by the relevant storage and retrieval mechanism. Specifically, in part-to-picker systems, an automated device transfers items requested to a stationary order picker (Figure 1(a)). In picker-to-part systems, pickers travel to item storage locations and collect the items (Figure 1 (b)). In the latter, pickers must traverse multiple aisles and areas to fulfill orders. The travel mode can include walking with a cart or riding on a retrieval vehicle. The skill and flexibility of the human pickers are critical, as pickers visit multiple locations on each tour and handle diverse items.

(a) (b)

Figure 1. Examples of order picking systems: (a) part-to-picker system (Warehouse-rx.com); (b) picker-to-part system (Amazon.com).

Figure 2 shows a typical, and popular, picker-to-part picking system, i.e., a bin-shelving picking system with a parallel-aisle configuration and two cross aisles located

in the front and back of the layout that connect the parallel aisles. A loading/unloading (L/U) station is located in the front of the leftmost aisle. Bin-shelving storage on each side of the aisles allows order pickers to easily retrieve items. One pick face includes multiple pick locations. To collect a batch, the picker starts from the L/U station, circumnavigates the aisles of pick area via the cross aisles, and returns to the L/U station; this operation forms a *trip*.

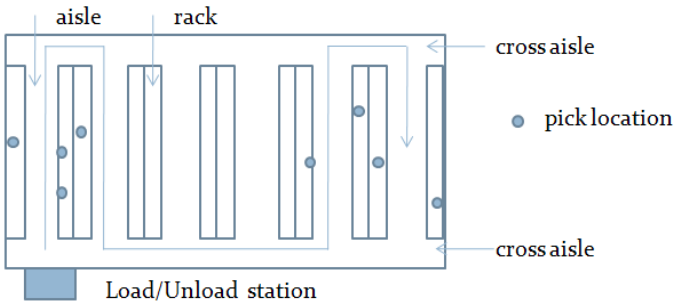


Figure 2. A typical picker-to-part system: parallel-aisle OPS layout (Gademann *et al.*, 2001).

A specified routing method (based on pickers' experience or management-determined) plays an important role in improving order picking performance, because it determines the travel distance, which is a fundamental throughput measure. Heuristics are often preferred because they produce more straightforward and natural routes for pickers than an optimal strategy (Petersen and Schmenner, 1999). The heuristics include the traversal method, the return method, the mid-point method, the largest gap method, and the combined method (Petersen, 1997). The traversal routing method in Figure 3 is most frequently cited in the literature because of its simplicity and popularity in industry. When this method is used in parallel-aisle OPS, any aisle containing at least one pick is

traversed entirely.

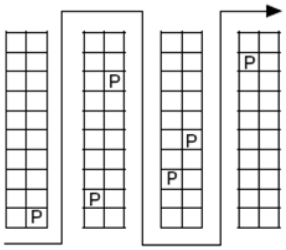


Figure 3. Traversal route method (Petersen, 1997).

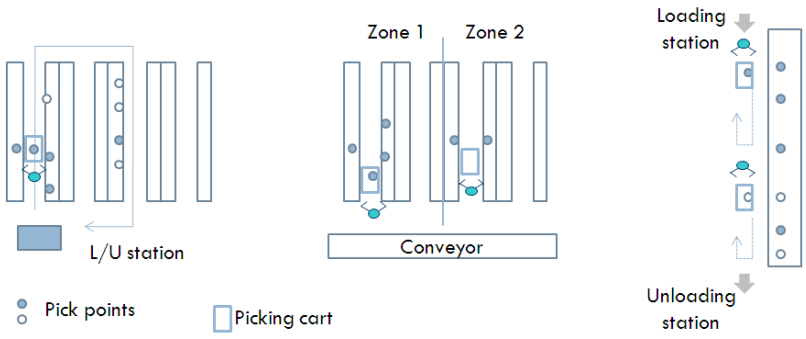
2. ORDER PICKING POLICY

From an operational view, organizing and batching orders for pickers to reduce travel and blocking time is as important as designing optimal routing strategies. Single-order picking allocates one order to one picker. Alternatively, to increase efficiency, several orders can be consolidated in a batch. Figure 4 (a) illustrates batch picking by a single picker. Since one picker picks multiple orders in the same trip, the total retrieval time is reduced. When multiple orders are collected in a trip, their disassembly into orders is termed a sorting operation. There are two efficient strategies relevant to the sorting operation while batch picking. In the *sort-while-pick strategy*, pickers sort products while traveling between picking locations. A cart carries bins for orders. The picked items are identified as belonging to a particular order and deposited in the correct bins. The *pick-then-sort strategy* separates the two operations into a sorting operation executed by manual workers or by sortation equipment to separate the items into orders after completing a trip to retrieve the items in a batch.

Picking a large-size batch (or order) may be assigned to multiple pickers and is

called zone picking (Figure 4 (b)). Order pickers travel only in their specialized zone. There are two protocols to assign a batch to each zone. In *synchronized zone picking*, each zone collects one batch simultaneously. Retrieval time for a batch can be shorter than a full retrieval time by a single picker, because several pickers process partitioned portions of a batch. In *progressive zone picking*, a batch is processed in individual zones sequentially. A batch is passed between zones, and items are collected in various zones to complete the orders in the batch. In general, a buffer of work-in-process batches is formed between two zones to insure pickers in downstream zones are not idle.

Bucket brigade picking is similar to progressive zone picking, but employs a variable zone boundary policy where zone size is not predetermined and is resized automatically and dynamically (Figure 4 (c)). No buffer between pickers is necessary (see, for example, Bartholdi and Eisenstein (1996a)). A batch must pass all pick faces and collect items at related pick faces in sequence to be completed. Pickers are ordered from upstream to downstream in a row, and the order is maintained across the zones. A picker picks an item and places it in the tote assigned to the particular batch. The picker then moves to the next pick face to continue processing the batch if there is no picker at the next pick face. The upstream picker hands off the current batch when the upstream picker meets a downstream picker who has no assigned batch.

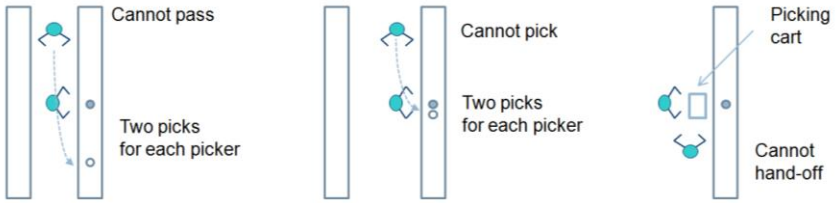


(a) (b) (c)

Figure 4. Order picking policies: (a) batch picking; (b) zone picking; and (c) bucket brigade picking.

3. PICKER BLOCKING

In a typical picker-to-part system, adding pickers is expected to enhance the system's order picking throughput. However, the benefits to throughput are increasingly offset by picker blocking (Ruben and Jacobs, 1999). Picker blocking occurs when multiple pickers traverse a pick area while maintaining a no passing restriction, or two or more pickers attempt to occupy the same space or the same resource simultaneously. When a picker prevents another picker from passing, *in-the-aisle blocking* arises as depicted in Figure 5(a), and when pickers attempt to pick from the same storage location, *pick-face blocking* occurs as depicted in Figure 5 (b). In this dissertation, picker blocking refers to in-the-aisle blocking unless otherwise stated.



(a) (b) (c)

Figure 5. Types of picker blocking: (a) in-the-aisle picker blocking; (b) pick-face blocking (Parikh and Meller, 2009); and (c) hand-off delay.

Bucket brigade picking also encounters picker blocking situations, because, as mentioned, this protocol sets a zone boundary between pickers in a variant manner. While an upstream picker moves in a forward direction, the next pick face may be occupied by a busy downstream picker (Figure 5(a)). Hence, the upstream picker cannot "hand off" the current batch to the downstream picker since the downstream picker is currently allocated to a retrieval task. The upstream picker also cannot pass over the downstream picker because the zone restriction disallows passing. Further, when the downstream picker is idle, he/she moves in a backward direction to take a hand-off from an upstream picker. If the upstream picker is picking when the downstream picker encounters the upstream picker, the downstream picker must wait for the completion, which is termed *hand-off delay* as shown in Figure 5(c).

CHAPTER III

LITERATURE REVIEW

1. BATCH PICKING WITH *K*-PICKERS

Depending upon pickers' organization, batch picking with *k*-pickers can be classified by

1) (single-zone) batch picking
2) (multiple-) zone batch picking
3) bucket brigade batch picking.

Batch picking is most commonly single-zone, *multiple-picker batch picking*. Since multiple pickers work in a zone, an interaction among *k*-pickers arises, which leads to picker blocking. In studying the relationship between picker blocking and batching algorithms, Ruben and Jacobs (1999) find that congestion impacts the selection of batching procedures and storage policies. Their simulation studies show that a turnover-based storage policy[1] causes more congestion than family-based[2] or random storage[3] strategies. Gue *et al.* (2006) and Parikh and Meller (2009; 2010) investigate effects of picker blocking using analytical and simulation studies. The authors introduce analytical models related to picker blocking in specified-order picking environments, both picker blocking in narrow-aisle (Gue *et al.*, 2006; Parikh and Meller, 2010) and pick-face blocking in wide-aisle (Parikh and Meller, 2009). Gue *et al.* (2006) explain

[1] A turnover-based storage policy determines storage locations of products according to the demand popularity of products. Popular products are stored in locations to reduce the retrieval time.

[2] The demand affinity between products is used to determine storage locations of products. Thus, it can reduce the time to reach the next item in an order.

[3] A random strategy randomly determines storage locations of products.

that the batch picking strategy in narrow-aisle OPSs can experience less picker blocking when the pick density is either very low or high. Parikh and Meller (2010) find that even though the pick density is high, picker blocking can be significant when the variation of the pick density is high. Parikh and Meller (2009) do not consider batching, but distinguish the effects of congestion in the wide-aisle picking situation of a single-pick model versus a multiple-pick model. The single-pick model assumes that at most a single pick occurs at a pick face, which is often true in single-order picking, whereas the multiple-pick model considers repeated picks at a pick face, which is more likely in batch picking. Parikh and Meller (2009) suggest wide-aisle OPSs may experience significant blocking when multiple-picks are required at each pick face. They also find that the variation of pick density plays a vital role in the significance of pick-face blocking.

From the standpoint of picker blocking, zone picking is a preferred alternative for heavy picker blocking environments. However, restricting pickers movement creates additional idleness from workload imbalances and increases work in process (WIP). There is some research on how to achieve equal balance among zones (Jane, 2000; Jane and Laih, 2005) by examining historical customer orders and the items assigned to storage zones. Le-Duc (2005) presents a procedure to find the optimal number of picking zones by using mixed integer programming. Jane and Laih (2005) propose an assignment algorithm in a synchronized zone picking system where all zone pickers fulfill the same order simultaneously. A similarity coefficient of any two items is presented for measuring the co-appearance of both items in the same order. To minimize

the idle time of the synchronized zone picking system, the items most frequently requested (i.e., with high similarity coefficient) are assigned to different zones.

As Bartholdi and Eisenstein (1996a) indicate, the balanced workload model in zone picking exhibits three major problems in practice. First, available approaches tend to depend on historical data; even though workloads are balanced for historical data, current and future demand patterns experience imbalances. Second, non-demand based uncertainties exist, e.g., equipment breakdown, absenteeism, etc., leading to workload imbalances. Third, picker capability is not identical and varies with pickers' learning.

To solve these problems, an order picking system with bucket brigades is an alternative to zone picking (Bartholdi and Eisenstein, 1996a). The bucket brigade picking system is a promising strategy that can solve load balance issues, a significant concern within multiple pickers OPSs. The bucket brigade method provides a self-balancing characteristic using minimal WIP (Bartholdi and Eisenstein, 1996a; Bartholdi and Eisenstein, 1996b). Yet, this strategy faces two operational delays: hand-off delay and picker blocking delay (Koo, 2009). The literature notes that it encounters less picker blocking when pickers are arranged in ascending capability order (Bartholdi and Eisenstein, 1996a; Bartholdi and Eisenstein, 1996b; Koo, 2009). However, the only available research on picker blocking in bucket brigade order picking has been conducted by Koo (2009), who proposes a model combining a zone picking policy and the bucket brigade order picking policy. Under his modified strategy, pickers' downstream travel is allowed to a predefined point at which pickers leave their current tote and move upstream. Since a downstream range is limited, picker blocking lessens,

and the number of direct hand-offs also drops since WIP is allowed. However, this method can significantly increase WIP and may disrupt the load-balancing characteristics.

2. ORDER BATCHING ALGORITHMS

The first component of our research focuses on the proximity batching relevant to parallel-aisle picking systems, where nearby orders are grouped based on travel distance. The proximity batching algorithms for parallel-aisle picking systems can be categorized as 1) optimal approaches; 2) meta-heuristics; 3) seed heuristics; and 4) saving heuristics.

An optimal approach is to solve the batching and routing problem exactly through a mixed integer programming model using branch-and-bound to minimize the maximum route length (Gademann and van de Velde, 2005; Gademann *et al.*, 2001). Despite enhanced branch-and-price methods, exact methods based on branch-and-bound face a limitation in scalability of the number of orders and batches (we verify this with our computational experiments in Section IV).

Hsu *et al.* (2005) propose a meta-heuristic approach, a genetic order batching algorithm, to minimize the total travel distance. The problem complexity of the genetic algorithm is strongly dependent on the number of batches, the number of orders, and the number of aisles. Similarly, it is not clear whether the proposed genetic algorithm can solve large-scale problems, because the algorithm appears to be inefficient over medium-size problems with low routing complexity.

De Koster *et al.* (1999) conduct a comparison study of seed and saving

algorithms. Our independent analysis in Chapter IV confirms that only seed and saving algorithms are able to analyze large-sized problems. However, the solution quality of these methods is uncertain in medium- and large-size problems, because the exact value of the optimal solution cannot be identified and lower bound estimates are not available in the literature.

3. RESEARCH ISSUES

Reviewing the available methods we identify three critical issues:

1) *The impacts on picker blocking of batch picking in a narrow-aisle system are not fully understood.* Within the proximity batching literature, Ruben and Jacobs (1999) discuss the limitation of the available batching methods on picker blocking control. Two studies (Gue *et al.*, 2006; Parikh and Meller, 2010) observe the impacts by the size and the variation of pick density throughout analytical and simulation models. However, the relationship between batch picking situations (i.e., batching algorithms, sorting strategies, and storage policies) and the results of analytical studies has not been fully examined despite its significance upon warehouse design and operations. For example, the literature is silent on whether batch picking always produces heavy picker blocking. If it does not, what conditions should be satisfied for higher order picking throughput?

2) *Proximity-based batching algorithms can handle only distance-related performance.* The literature on batching algorithms does not address the trade-off between travel distance and time blocked. Namely, to manage heavy picker blocking situations, a new order batching model and relevant solution procedure is

needed. The new batching algorithm requires quantifying picker blocking as well as travel distance.

3) *Bucket brigade picking systems also face significant congestion issues.* Picker blocking and the hand-off models in bucket brigade picking systems are not well understood with respect to analytical models and direct control. Only a simulation-based approach (Koo, 2009) has been used to quantify picker blocking, and a direct mitigation of picker blocking has yet to be addressed. Hand-off issues are frequently neglected in the available literature despite the possibility of productivity loss. Moreover, Koo's hand-off model fails to deliver an exact model; thus, we introduce such a model in Chapter VII. We conclude that understanding picker blocking and hand-off delays is very restricted and partially incorrect, and we provide a mechanism to improve operations in a bucket brigade system by explicitly addressing both issues in determining the operational plans.

CHAPTER IV

LARGE-SCALE ORDER BATCHING WITH TRAVERSAL ROUTING METHODS

This chapter investigates the effects by picker blocking when an order picking situation employs traditional batching models to reduce the pickers' total travel distance. In practice, some order picking systems retrieve 500~2000 orders per hour and include ten or more aisles. Available proximity batching methods are not suitable for the study proposed, because all large-scale approaches are implemented to obtain a heuristic solution, and those heuristic algorithms only demonstrate their improvement relative to a random batching strategy or prior batching algorithms. Thus, we employ a new, near-optimal proximity-batching procedure, a solution validation procedure, and relevant picker blocking experiments. The quality of the solutions is demonstrated by comparing with a lower bound developed as a linear programming relaxation of the batching formulation described in this chapter. A simulation study indicates that the proposed heuristic is relatively robust to picker blocking.

1. INTRODUCTION

From a computational view, the route selection problem is typically easy, but difficulty arises mainly due to the combinatorial number of potential batches. The routing problem in rectangular parallel-aisle systems can be optimally solved with polynomial complexity (Ratliff and Rosenthal, 1983). Furthermore, pickers often prefer heuristic routing methods (De Koster *et al.*, 1999; Gademann and van de Velde, 2005),

which can be computationally simpler than the optimal routing method. In contrast, the computational burden associated with the partitioning decision is a primary source of complexity for the batching problem. For example, when the number of orders is 100 and the capacity of the order picker is 10 orders per trip, the number of possible combinations for batching the orders is $6.5*10^{85}$. Hence, only heuristic batching algorithms can solve large-size problems in a timely manner. We note, too, that the complexity of the batching problem affects the assessment of solution quality. The performance of the various proposed methods for batching have not been demonstrated quantitatively in any practical size problem because lower bound estimates were not previously available.

We, therefore, examine picking systems that process 500-2000 orders in a one-hour time window. This picking environment has one-way narrow aisles, and we assume pickers use traversal routes through the DC.[4] We consider both sort-while-pick and pick-then-sort strategies, and both random and class-based storage policies. Ideally, we want to exploit the advantage of the traversal routing method in developing a computationally efficient procedure to solve large-size problems and determine a tight lower bound to evaluate performance.

We approach the batching problem using a selection-based routing method, not the more common construction-based routing method, and derive a new batching procedure by first assigning orders to routes and then constructing batches within route

[4] Throughout most of this dissertation we assume one-way narrow aisles since this is a typical setting for the batch picking problem where congestion is a concern; however, these methods can be extended to multi-directional travel with some increase in computational burden, as discussed in Section 6.3.

sets. Even though the routing mechanism occupies a small portion of the computational time, it influences solution approaches for order batching algorithms. The traditional order batching algorithms build a route for a given batch and calculate the route length. This route construction concept then guides the search procedure narrowing order-to-batch assignments to identify batches with potentially shorter routes. Initially, we identify a set of potential routes and match orders to potential routes. As the routes and their lengths are predetermined, it is possible to match orders to routes without identifying batches. The direct assignment of orders to routes can improve the solution quality, reduce the computational time, and obtain a lower bound. Accordingly, we build an efficient heuristic procedure to pack batches from orders within routes.

This chapter makes three important contributions to the extant literature. First, a large-scale, near-optimal order batching procedure for parallel-aisle picking systems is demonstrated for the first time; the environments cover both narrow-aisle and wide-aisle systems and are extendible to other layouts using traversal routing methods. Second, it introduces a new order batching formulation and relevant relaxation models utilizing a bin-packing problem. The bin-packing problem can be solved more efficiently on large-size problems compared to a batching problem even though both require complex analysis. Third, the proposed algorithm is compared with available heuristic algorithms in terms of both total travel distance and total travel time, since the shortest routing distance does not guarantee the shortest retrieval time in environments with picker blocking. A simulation study is used to evaluate the performance of the proposed algorithm considering picker blocking.

The remainder of the chapter is organized as follows. In Section 2, we review related studies regarding order batching algorithms in parallel-aisle picking systems. The details of the new formulation and the relaxed models are discussed in Sections 3 and 4, respectively. Section 5 describes a heuristic batching procedure based on the relaxation model. Section 6 discusses the computational experiments and comparison results. We conclude with directions for future research and the model's extension.

2. RELATED LITERATURE

The literature review in this chapter expands on the relevant portions from the general literature review presented in Chapter III. This chapter focuses on the proximity batching relevant to parallel-aisle picking systems, where nearby orders are grouped based on travel distance. The prior work in proximity batching algorithms for parallel-aisle picking systems can be categorized into 1) seed heuristics; 2) saving heuristics; 3) meta-heuristics; and 4) optimal approaches.

In conducting a comparison study of seed and saving algorithms, De Koster *et al.* (1999) conclude that the best seed algorithms combine three control factors: select the seed order as the order that must visit the largest number of aisles, choose the next order to minimize the number of additional aisles, and cumulatively update the seed information based on orders in the seed. Alternatively, the same paper develops the savings algorithm (which is a modified Clarke and Wright method (1964)) in which a savings list is updated until there are no remaining savings pairs. The authors find the savings algorithm is preferable to the seed algorithm. Our independent analysis also confirms that only seed and saving algorithms are able to analyze large-size problems.

However, the solution quality of these methods is uncertain in medium- to large-size problems, because the exact value of the optimal solution cannot be identified and lower bound estimates are not available in the literature.

Hsu *et al.* (2005) propose a meta-heuristic approach, a genetic order batching algorithm, to minimize the total travel distance. The problem complexity of the genetic algorithm is strongly dependent on the number of batches, the number of orders, and the number of aisles. Their tests are conducted on ~300 orders to generate ~40 batches in a five-aisle warehouse; this size problem required ~2500 seconds to execute the heuristic. It is not clear whether the proposed genetic algorithm can solve large-scale problems, because the algorithm appears to be computationally inefficient over medium-size problems with low routing complexity.

An optimal approach is to solve the batching and routing problem exactly through a mixed integer programming model (Gademann and van de Velde, 2005; Gademann *et al.*, 2001). Gademann *et al.* (2001) present a branch-and-bound solution for a wave picking environment, where a large number of orders are partitioned into multiple batches to minimize the maximum route length. Gademann and Van de Velde (2005) develop a branch-and-price formulation for the sort-while-pick order picking strategy. The authors present two important findings: 1) the number of aisles and the number of batches significantly impact the computational time; and 2) the average time to identify an optimal solution is very short compared to the time necessary to verify its optimality. Despite enhanced branch-and-price methods, Gademann and Van de Velde (2005) are only able to solve problems sizes of ~30 orders and ~8 batches. We infer and

confirm with our own experiments that exact methods based on branch-and-bound face a limitation in scalability of the number of orders and batches.

Summarizing the available methods, we identify two critical issues. First, all approaches are implemented to obtain a solution with a partitioning first, routing second method. The route construction procedure is necessary and follows a partitioning decision because the route length varies according to pick locations in a batch. However, the partitioning problem is complex, requiring the construction of all combinations of orders to batch assignments. Second, within the batching literature there is no research on lower bound algorithms for a large-scale problem. Heuristic algorithms only demonstrate their improvement relative to random batching strategy or prior batching algorithms. Without a lower bound, one cannot quantify the performance of the heuristics in absolute terms.

3. ROUTE-SELECTING ORDER BATCHING MODEL (RSB)

3.1 Problem definition

We consider an order picking environment similar to those described in Petersen II (2000) and Gong and De Koster (2008). The order profile assumes an average order size is two line items per order and 1080 orders arrive per hour. Figure 6 shows a ten-aisle bin-shelving OPS with a narrow parallel-aisle configuration and two cross-aisles located in the front and back of the layout, which connect the parallel aisles. An L/U station is located in front of the leftmost aisle. There are forty pick faces per aisle in which order pickers retrieve items. The height of the shelves does not impact the travel length. To collect a batch, a picker starts from the L/U station, circumnavigates aisles of

pick locations via the cross-aisles, and returns to the L/U station. While retrieving items, pickers take a one-way traversal route and do not make U-turns within an aisle. In other words, if they enter an aisle, pickers pass completely through it. However, they need not traverse every aisle. Further, each aisle is traversed in a fixed direction to prevent pickers from being blocked in an aisle by pickers approaching from the opposite direction, i.e., one-way traversal routing (Gue *et al.*, 2006) is used. One order picker can carry ten bins on a cart allowing him/her to simultaneously pick up to ten different orders. We assume a constant walking speed and pick time per item. In determining batches, blocking delays are ignored and total retrieval distance is minimized. The issue of blocking is revisited in more detail in Section 6.2.3. In addition, some parameters (e.g., sorting strategy, storage policy, capacity, and number of aisles) are varied to investigate robustness in the quality of solutions across differing environments.

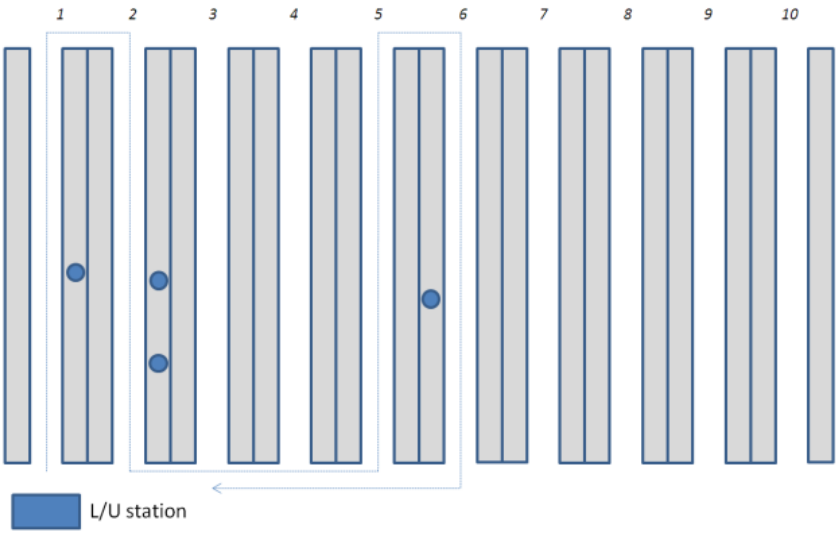


Figure 6. A ten-aisle order picking system

3.2 Formulation

A new order batching model is formulated that takes advantage of the traversal routing method. When traversal routing methods are used, all possible routes can be constructed from the warehouse layout. Thus, given a batch, a best fit route can be selected as a matching problem, referred to as the route-selecting order batching model (RSB).

The formulation is flexible and can handle both sort-while-pick and pick-then-sort operational strategies. The capacity of the cart is represented by CAPA. Q_o denotes the portion of CAPA that order o consumes. In the case of sort-while-pick strategy, CAPA is measured in units of orders, thus Q_o is 1. In the case of pick-then-sort strategy, CAPA is measured in units of items, thus Q_o becomes the number of items in order o. OA_{oa} is set to 1 if aisle a must be visited to gather the items in order o. Route information and length are initially constructed for all routes r in the route set R. Route information is expressed with the aisle visiting vector (RA_{ra}) and the route length is LT_r. Given pickers' one-way traversal routing, for pick areas of size $|A| = 2, 4, 6, 8, 10$, and 12, where A is the number of aisles, the sizes of route set $|R|$ are 1, 4, 12, 33, 88, and 232, respectively. Though the size of $|R|$ increases exponentially, for reasonable-size problems, for example 10 aisles, there are only 88 potential routes. We define a set of batches, B, initially $|B|=|O|$, allowing each order a separate batch. If batch b in B is set to include an order, batch b is active. RSB is formulated to determine if batch b is active, which is indicated by BV_b, if order o is assigned to batch b, which is indicated by X_{ob}, and the route of batch b, which is indicated by Y_{br}.

Indices and parameters

B, b = the set of batches, and its index $b \in B$

O, o = the set of orders, and its index $o \in O$

A, a = the set of aisles, and its index $a \in A = \{1, \cdots, |A|\}$

R, r = the set of routes, and its index $r \in R$

Q_o = the number of line items in order o

OA_{oa} = 1 if order o passes through aisle a (=order o has at least one pick in aisle a)
0 otherwise

LT_r = the length of route r

RA_{ra} = 1 if route r passes through aisle a
0 otherwise

$CAPA$ = the capacity of a cart

Decision variables

X_{ob} = 1 if order o is assigned to batch b
0 otherwise

Y_{br} = 1 if batch b takes route r
0 otherwise

BV_b = 1 if batch b is valid
0 otherwise

Formulation

(RSB) $$Min \quad \sum_{b \in B} \sum_{r \in R} LT_r Y_{br} \tag{4.1}$$

s.t. $$\sum_{b \in B} X_{ob} = 1, \qquad \forall o \in O, \tag{4.2}$$

$$\sum_{o \in O} Q_o \cdot X_{ob} \le CAPA, \qquad \forall b \in B, \tag{4.3}$$

$$X_{ob} \le BV_b \qquad \forall o \in O, \forall b \in B, \tag{4.4}$$

$$\sum_{r \in R} Y_{br} = 1, \qquad \forall\, b \in B' = \{b \mid BV_b = 1, b \in B\}, \tag{4.5}$$

$$X_{ob} \cdot OA_{oa} \le \sum_{r \in R} RA_{ra} Y_{br}, \qquad \forall\, a \in A, \forall\, o \in O, \ \forall\, b \in B' = \{b \mid BV_b = 1, b \in B\}, \tag{4.6}$$

$$X_{ob} = \{0,1\} \qquad \forall\, o \in O, \forall\, b \in B,$$

$$Y_{br} = \{0,1\} \qquad \forall\, b \in B, \forall\, r \in R,$$

The goal is to minimize the total travel distance (4.1). The basic function of the given algorithm is to partition orders into batches. An order cannot be separated into multiple batches and all orders should be assigned to batches (4.2); a batch should not exceed the capacity constraint of the cart (4.3). The maximum number of batches is limited to the number of orders. BV_b is active if at least one order is assigned to batch b (4.4). A batch must have one route (4.5). The aisle visiting incidence vector of route b should contain the aisle visiting incidence vector of orders in batch b (4.6).

3.3 Validation

To validate our model, we derive general requirements of the formulation as in Gademann and Van de Velde (2005).

- Requirement 1 (No splitting of an order and all orders are fulfilled). Every order is included in exactly one batch.
- Requirement 2 (Capacity). The number of items in a batch is less than or equal to the maximum batch size.
- Requirement 3 (Complete route). A route starts at the L/U station and returns to the L/U station.
- Requirement 4 (One-way directionality). Each aisle has its own moving direction.

Similar to Gademann and Van de Velde, we require 1, 2, 3, and 4. The requirements are modeled by (4.2) for requirement 1 and (4.3) for requirement 2. Requirements 3 and 4 are enforced while generating the candidate routes in set *R*.

4. ROUTE-BIN PACKING PROBLEM (RPP) AND ITS LP RELAXATION (RPP-LP)

This section develops two relaxation models for the route-selecting order batching formulation (RSB) model, both of which can serve as lower bounds for the RSB model. The RSB model stated above simplifies the batching problem; however, it still contains partitioning constraints (4.2), which have been proven to be NP-complete (Gademann *et al.*, 2001; Ruben and Jacobs, 1999). However, the partitioning stage can be postponed and a route-bin packing problem (RPP) is developed by assigning orders directly to routes. This allows a lower bound to be constructed, but additional reformulations using a linear programming relaxation are needed to solve large-size problems.

4.1 Route-bin packing problem (RPP)

RSB can be simplified by removing the batching variables to develop a new partitioning problem. When the partitioning stage is skipped, the batching problem is relaxed to obtain the number of routes required to retrieve orders. Then, within route types, batches can be identified similar to a generic bin-packing problem; this formulation is referred to as a route-bin packing problem (RPP). To further describe the details, we reuse two decision variables, X_{ob} and Y_{br}, introduced in the prior section.

Using the following two equations, $x_{or} = \sum_{b \in B} X_{ob} \cdot Y_{br}$, $y_r = \sum_{b \in B} Y_{br}$, we further define x_{or} indicating order *o* is assigned to route *r* $_r$ and y_r is the count of batches taking route *r*.

Based on these two new variables, we derive three new constraints (4.8), (4.9), and (4.10) using Gaussian elimination processes and Lagrangian relaxations. A constraint in (4.2) specified by order *o* is matched to a constraint in (4.8) having the same order *o*. The inequalities (4.9) and (4.10) also are valid after aggregating the constraints related to route *r*. Basically, we aggregate constraints in (4.3) for batches *b* using route *r*. We can replace batching index *b* with route index *r* by aggregating the constraints having the same route *r*; thus, (4.9) has no batch index. We repeat the same process for (4.6) to obtain (4.10). Finally, we relax constraints (4.4) and (4.5), and RPP without batching variables results. The proof appears in Appendix A.1.

Decision variables

x_{or} = 1 if order *o* is assigned to route *r*
0 otherwise

y_r = the number of batches assigned to route *r*

(Basic RPP) $Min \quad \sum_{r \in R} LT_r \cdot y_r$ (4.7)

s.t. $\sum_{r \in R} x_{or} = 1, \qquad \forall o \in O,$ (4.8)

$\sum_{o \in O} Q_o x_{or} \le CAPA \cdot y_r, \qquad \forall r \in R,$ (4.9)

$x_{or} \cdot OA_{oa} \le RA_{ra} y_r, \qquad \forall o \in O, \forall a \in A, \forall r \in R,$ (4.10)

$x_{or} = \{0,1\} \qquad \forall r \in R, \forall o \in O,$

$$y_r = \{0,1,2,..\} \qquad \forall\, r \in R,$$

The objective is to minimize the sum of the length of assigned routes (4.7). All orders are assigned to exactly one route (4.8). The capacity of the assigned routes *r* should be greater than or equal to the total quantity of items to be picked (4.9). The aisle visiting incidence vector of route *r* should contain the aisle visiting incidence vector of each order *o* that has been assigned to route *r* (4.10).

The number of constraints in the basic RPP formulation for constraint set (4.10) is $|O||A||R|$. This can be simplified as follows:

1) For each *r* in *R*, we evaluate whether order *o* is covered by route *r* and, if so, include order *o* in set O_r.
2) Then for *o* in $O \backslash O_r$, x_{or} is 0, because route *r* does not cover order *o*.

Thus, constraint set (4.11) is constructed, which has no more than $|O||R|$ constraints. Relaxing constraint (4.10) to (4.11) reduces the complexity of the formulation with only a minimal expansion of the solution space.

(RPP) $Min \quad \sum_{r \in R} LT_r \cdot y_r$

s.t. (4.8), (4.9), and

$$x_{or} = 0, \qquad \forall\, o \in O \backslash O_r, \forall\, r \in R, \qquad (4.11)$$

Rather than (4.11), there is another way to reduce the number of constraints. We can penalize Q_{or} = INFINITY instead of each constraint in (4.11). Then, x_{or} is forced to be 0, because Q_{or} is larger than CAPA. The resulting formulation has a smaller number

of constraints. However, using a general MIP solver, the computational performance of this strategy to reduce the number of constraints in (4.11) is poor. Thus, we use (4.11) for computational purposes. The RPP without constraints (4.11) is equivalent to a generalized bin-packing problem (Lewis and Parker, 1982).

4.2 Linear programming relaxation on RPP (RPP-LP)

We derive a lower bound algorithm by relaxing the integer restrictions within RPP. This LP relaxation of RPP provides a weak lower bound. To strengthen the lower bound, we add valid inequalities based on the original constraint (4.10). This is implemented by enforcing y_r to be equal to maximal x_{or} for route r as shown in (4.12).

(RPP-LP) $Min \quad \sum_{r \in R} LT_r \cdot y_r$

s.t. (4.8), (4.9), (4.11), and

$$x_{or} \le y_r, \qquad \forall\, o \in O_r, \forall\, r \in R, \tag{4.12}$$

$$0 \le y_r \qquad \forall\, r \in R,$$

Constraints (4.12) ensure that if any order o is assigned to route r, then there is at least one batch within route r.

4.3 Relationship and optimality

A simple lower bound can be constructed by assuming that each order uses an optimal route (LT_o) and each cart is fully loaded during each trip. We define the travel distance under this construction to be the ideal batching (IB) bound represented by

Obj(IB).

$$\text{Obj}(IB) = \sum_{o \in O} 1/CAPA \cdot LT_o = \sum_{o \in O} LT_o / CAPA$$

Obj(IB) is equal to or less than Obj(RPP-LP), because RPP-LP without constraints (4.11) and (4.12) is the formulation to find the travel distance under ideal batching.

For Obj(RPP-LP), Obj(RPP), and Obj(RSB), the following inequalities hold as a definition of relaxation:

$$\text{Obj(IB)} \leq \text{Obj(RPP-LP)} \leq \text{Obj(RPP)} \leq \text{Obj(RSB)}$$

The solution to RPP is optimal if Obj (RPP) = Obj (restored batches from RPP solution), because the upper bound is the same as the lower bound. The solution by RPP-LP is also optimal if the solution by RPP-LP is integral and Obj (RPP-LP) is equal to Obj (restored batches from RPP-LP solution).

5. A HEURISTIC ROUTE-PACKING BASED ORDER BATCHING PROCEDURE (RBP)

This section describes a heuristic solution procedure to solve the batching problem based on the RPP formulation. The RPP model is preferred, because batches can easily be constructed from the solution to RPP. However, RPP is still computationally difficult, so two further computational improvements are considered: 1) a partial route set; and 2) a truncated branch-and-bound approach. The proposed heuristic procedure is composed of three steps:

Step 1: identify and construct potential route sets.

Step 2: assign orders to routes using RPP

Step 3: restore a feasible solution from the infeasible solution obtained from the relaxed model.

These steps are described below.

Step 1. : Identify and construct potential route sets

We have already shown in section 3.2 that $|R|$ increases exponentially as $|A|$ increases. Consequently, variables and constraints in the RPP formulation, including the route index, increase exponentially. The set of routes is constructed in two steps: first, an elementary route set (R_e) is selected to guarantee each order can be picked using one of the routes in the route set. This is done by completely enumerating all routes and sequencing them in ascending order by route length. For order o, we select a first fit from the set, and update R_e U $\{r\}$ ties are broken randomly. The elementary route set is only part of the reduced route set (R_r) used in RPP. Second, we consider combined route set (R_c), because these routes will be useful when the number of orders assigned to a route do not divide evenly into the batch size.

To generate the combined route set, we employ the Clark and Wright II algorithm (CW II) (Clarke and Wright, 1964; De Koster *et al.*, 1999). The modified CW II algorithm constructs routes with relatively short travel distances. As part of the CW II algorithm, a composite level, indicating the maximum number of routes covered by a combined route, must be specified. A detail of the route-set selection procedure follows.

Route-set selection procedure:

1. Initialize O = all orders, R_e ={}, R_c ={}.
2. Construct R_e
   For o = 1 to $|O|$
      If R_e does not include an optimal route for order o
         R = optimal route of o
         $R_e = R_e \cup \{r\}$
      End if
   End for
3. Construct R_c from R_e using a route composition algorithm
   Set the composite limit C
   Do
      Calculate the savings s_{ij} for all possible route pairs i,j in $R_e \cup R_c$
      Sort the savings in decreasing order.
      Do
         Select the pair with the non-selected highest savings. In the case of a tie, select a random pair.
         If the pair does not violate composite level C
            Combine both "routes" to form a new element r in R_c
      While (remaining pair in the savings or any composite candidate)
   While (all r's in R_e have not been included in R_c)
4. $R_r = R_e \cup R_c$

The route construct step can be illustrated by the example shown in Figure 7. Assume that the number of aisles is six and six orders are given. In this aisle configuration, 12 different routes are available. From the orders to be picked, the elementary route set is constructed as $\{e_1, e_2, e_3, e_4\}$. For four elementary routes, CW II creates c_1 when the composite level is four. R_r becomes $\{e_1, e_2, e_3, e_4\}$, because c_1 is already a route in R_e.

Incidence vectors of orders	*Incidence vectors of Elementary route set*	*Incidence vectors of Combined route set*
o_1: {1,0,0,0,0,0}	e_1: {1,1,0,0,0,0}	c_1: {1,1,1,1,0,0}
o_2: {1,1,0,0,0,0}	e_2: {0,0,1,1,0,0}	
o_3: {0,0,1,1,0,0}	e_3: {1,0,0,1,0,0}	
o_4: {1,0,0,1,0,0}	e_4: {1,1,1,1,0,0}	
o_5: {1,1,0,0,0,0}		
o_6: {1,0,1,1,0,0}		

Figure 7. An example of elementary route set and combined route set.

Step 2. Assign orders to routes using RPP

This step solves *RPP* using an IP solver with a time-truncated branch-and-bound method. Gademann and Van de Velde (2005) indicate that the branch-and-bound approach to solving the batching formulation converges to a near-optimal solution quickly and most of the computational time is spent validating the optimality of the solution. Because RPP considers a simpler set of potential routes the computational time will be faster, but we also truncate the search with a time-limitation. However, later we will construct a lower bound, thus we can estimate the impact on the solution quality caused by the time truncation.

Step 3. Build batches from orders within routes

Step three, BP_r , constructs batches with routes using the order-to-route assignment information. After constructing the batches, residual orders must be merged into additional batches. The solution of the BP_r sub-procedure depends on the sortation strategy.

i) Sort-while-pick strategy

In this case, since the size of a batch is based on number of orders, not items, BP_r

can be solved using a greedy algorithm. By assigning orders to batches on a first-come-first-serve basis, we can obtain an optimal solution. Figure 8 illustrates a procedure to cluster 10 orders into two 5-order batches, where y_r is 2. Then, orders are grouped into two batches, b_1 and b_2.

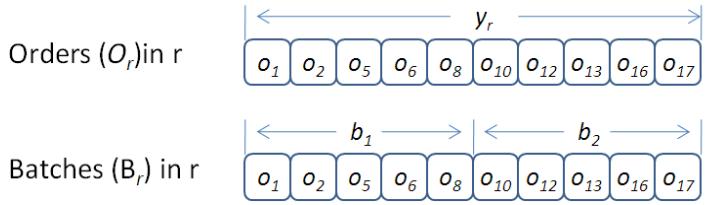


Figure 8. Batches b_1 and b_2 are constructed by grouping y_r orders assigned to route *r*.

Note that the routes from the combined route set can be used to handle residual orders from the elementary route sets. The remaining residual analysis is typically trivial under a sort-while-pick strategy.

ii) Pick-then-sort order picking strategy

Here, CAPA is defined in terms of items. Further, orders can have multiple items. Thus, assigning orders to batches using a greedy algorithm produces a poor solution. Instead, we solve IP formulation BR_r shown below to allocate orders to batches more efficiently while maintaining CAPA. When there are remaining orders (i.e., not fully packed batches), we merge them into new batches. When there are residual batches of less than half of CAPA, the CW II algorithm is applied to merge these remaining batches.

$$(BP_r) \quad Min \quad \sum_{b \in B_r} z_b \tag{4.13}$$

$$\text{s.t.} \quad \sum_{b \in B_r} x_{ob} = 1, \qquad \forall\, o \in O, \tag{4.14}$$

$$\sum_{o \in O} Q_o x_{ob} \le CAPA \cdot z_b, \qquad \forall\, b \in B_r, \tag{4.15}$$

$$x_{ob} = \{0,1\} \qquad \forall\, b \in B_r, \forall\, o \in O,$$

$$z_b = \{0,1.\} \qquad \forall\, b \in B_r,$$

6. IMPLEMENTATION AND COMPUTATIONAL RESULTS

We first test the performance of the proposed heuristic on different problem sizes assuming a one-way traversal routing method. We then extend the experiments to the two-way traversal routing method.

6.1 Implementation

The following analysis using the MIP formulations developed above are implemented using the ILOG CPLEX Callable Library C API 11.0.4. The data-set generator and comparison algorithms are developed using the C language. To test the computational performance, the executable files are run on a Windows NT-based server system with the Windows Vista operating system (Xeon 2.66 Ghz CPU, 12 GB memory). While compiling the CPLEX source, the stand-alone dynamic-linked library (DLL) is used. Both the branch-and-cut option and the heuristic search option are disabled to evaluate the exact computational time. While solving RPP and BP_r, we use the truncated branch-and-bound method with a time limit of 60 seconds. Instead of the optimal solutions, we evaluate solutions of the RBP by comparing with their LP lower bound generated with a full route set. Note that RPP-LP does not require the time limit and BP_r

is only applicable for the pick-then-sort strategy.

Each experiment is repeated for 20 random instances. The number of orders in an instance is fixed. The number of items in an order is determined by a simple density function where p(1) = 0.5/0.95, p(n)=(1/2*(n-1)-1/2*n)/(0.95) when n = 2,…,10, and p(n) = 0 otherwise. This order size distribution generates a result similar to that of Frazelle's (2002) small picking example. The average order-size is 2.02. Item locations are determined by the within-aisle class-based storage policy where A:B:C ratio is 0.7:0.2:0.1. Further, class A, B and C items are stored in aisles 1-2, 3-4, and 5-10, respectively. The time to travel the length of one pick-face is 1 time unit. The time to travel the length and the width of the aisle is 21 and 2 time units, respectively. The time to travel the length of the aisle includes the time from the center of cross aisles to the front end of a passage aisle, and the time aisles from a back end of an aisle to the center of cross, which are assumed to be half of a pick face. Thus, the time to travel the length of the aisle becomes 40/2+0.5+0.5= 21. The L/U station is located in front of the leftmost aisle. To combine routes in the route set reduction stage, the composite level is set to 3 routes.

In discussing the performance of the algorithms, we use the following notation throughout the remainder of this section.

FCFS: partition orders into batches based on a first-come first-serve policy

Seed: the seed algorithm in De Koster *et al.* (1999): 1) select a seed having the largest number of aisles, 2) choose the order minimizing the number of additional aisles, and 3) update the seed as an order is added it.

CW II: the Clarke and Wright algorithm (II) in De Koster *et al.* (1999). See

Appendix A.2 for more detail.

RBP: the heuristic route-selection-based batching algorithm

LB: the linear relaxation model of RPP (RPP-LP)

IB: the ideal batching model

Obj: the objective value of an algorithm

ObjL: the objective value of RPP, L stands for a lower bound

ObjU: the objective value of restored solution of RPP, U stands for an upper bound

CPU: computational time in seconds

LU gap: gap between an objective function value and the RPP-LP objective function value expressed as a percentage (= (an objective function value – LB)/(LB) %)

6.2 Experimental results

6.2.1 Computational time and the total travel distance

The performance of the proposed RBP method is compared to FCFS, seed, CWII, and the LB to understand the relative performance. These problems are computationally difficult so the total travel distance, the run time and the percentage deviation from the lower bound are calculated and reported in Table 1. The RBP produced near-optimal solutions within about 2 minutes and outperformed the seed and the CW II algorithms. Moreover, RBP improvement over alternative methods was larger for scenarios in which the number of orders was smaller.

Table 1. Computational results over different algorithms

Sort	#	FCFS		Seed			CW II			RBP				LB		IB
Strategy	orders	Obj	LU gap	Obj	CPU	LU gap	Obj	CPU	LU gap	ObjL	ObjU	CPU	LU gap	Obj	CPU	Obj
Sort-	360	5923.0	57.97%	3549.3	0.00	29.87%	2899.1	0.40	14.14%	2546.9	2546.9	11.47	2.26%	2489.3	0.77	2305.8
while-	720	11892.5	59.80%	6332.3	0.02	24.51%	5501.9	4.96	13.12%	4844.6	4844.6	40.33	1.33%	4780.3	1.83	4615.9
pick	1080	17915.3	60.48%	8970.1	0.05	21.06%	8033.3	16.20	11.86%	7177.2	7177.2	56.95	1.34%	7080.8	2.68	6938.6
	1440	23961.0	60.82%	11573.1	0.09	18.88%	10505.0	39.09	10.63%	9504.9	9504.9	60.26	1.23%	9388.3	3.63	9256.0
	1800	29989.7	60.95%	14122.7	0.14	17.08%	12942.6	75.68	9.52%	11849.0	11849.0	60.34	1.17%	11710.5	4.58	11587.2
	2160	36033.8	61.06%	16605.7	0.21	15.50%	15412.0	137.30	8.96%	14183.3	14183.3	60.40	1.07%	14031.8	5.69	13916.0
Pick-	360	4645.5	55.74%	3147.4	0.01	34.67%	2476.9	0.46	16.98%	2128.7	2128.7	17.54	3.40%	2056.2	4.93	1897.4
then-	720	9342.6	57.37%	5539.1	0.02	28.09%	4659.0	4.79	14.51%	4107.7	4107.7	67.11	3.04%	3983.0	11.98	3814.4
sort	1080	14126.7	57.85%	7967.5	0.05	25.26%	6868.9	14.70	13.31%	6136.5	6160.5	75.30	3.34%	5955.0	12.87	5783.4
	1440	18831.5	58.35%	10198.8	0.09	23.09%	8927.0	33.69	12.14%	8076.2	8145.3	96.46	3.70%	7843.7	18.14	7689.6
	1800	23522.5	58.55%	12476.8	0.14	21.85%	10979.5	62.21	11.20%	10024.7	10100.9	105.02	3.47%	9750.3	22.80	9614.6
	2160	28257.9	58.69%	14683.5	0.20	20.51%	13065.3	104.09	10.66%	12002.4	12108.5	140.54	3.60%	11672.5	27.71	11550.7

Specifically, in the sort-while-picking strategy, the seed algorithm requires a run time of 0.2 seconds. However, the LU gap is between 15 and 30%. CW II has a shorter total travel distance, but took a longer computational time (which was also noted by De Koster *et al.* (1999)). As the problem size increased, its computational time increased exponentially. When the number of orders was 2160, it took on average 137.30 seconds. RBP demonstrated a considerable improvement in travel distance. The LU gap ranged from 1.07 to 2.26% when the computational time was limited to 60 seconds, whereas the best approach identified in De Koster *et al.* (1999), CW II, showed a gap ranging from 9 to 14%.

The LU gap of RBP was larger under the sort-while-pick strategy. The increase in the gap is because RBP produced some batches that were not filled to capacity because of fixed non-uniform order sizes. Note that this has been partially improved by forming additional batches by merging these remaining batches using the CW II algorithm. To investigate additional possibility and improve the solution quality, we conducted a neighborhood search considering different combinations of batches. We observed a small performance improvement, i.e., less than 0.2% of the total retrieval

distance. The details and experimental results are summarized in Appendix A.3.

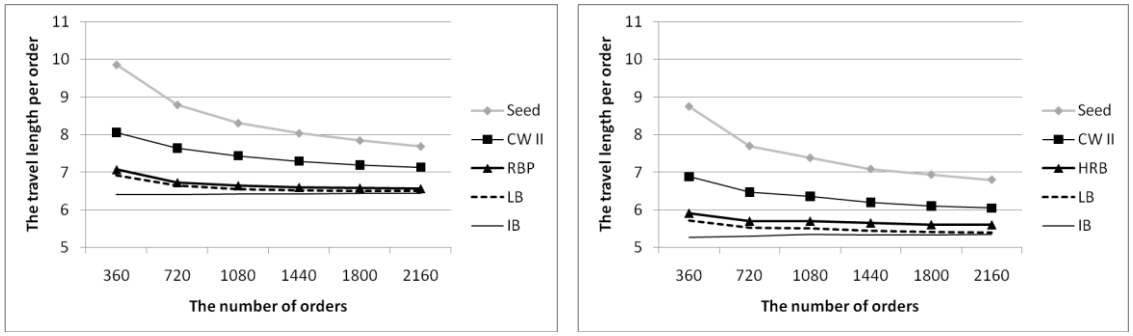
While the computational time of RBP and CW II was almost equal under the sort-while-pick strategy, the run-time of RBP increased under the pick-then-sort strategy, because the batch packing stage was computationally intensive using the IP bin-packing algorithm. However, run-times were still smaller than 150 seconds for all cases. While the IP-based batch packing process may take slightly longer, this is not a significant computational burden. Note that in both RPP and BP_r, the time limit for the branch-and-bound procedure is 60 seconds, and the solution procedure requires multiple iterations of BP_r.

The seed and CWII algorithms depend on having a large number of orders to improve performance. When the number of orders was 360 or 720, the algorithms experienced a large LU gap. Thus, the benefits of RBP are significant for large-size problems, but are even more prominent when the number of orders is small.

6.2.2 The average travel length per order

The average travel length per order is another metric that can evaluate the performance of various batching methods, assuming all orders construct similar numbers of batches. With this objective, a large-size batching problem is preferred since larger problems can produce more efficient batches, thus reducing trip distance. The previous methods developed for batching demonstrate a significant improvement in average travel length per order as shown in Figure 9. The improvement declined as the number of orders increased. When the number of orders increased from 1800 to 2160, there were minimal gains in throughput of the order picking system. In all cases, RBP dominated

other heuristics in solution quality with very small gaps to IB and LB.



(a) (b)

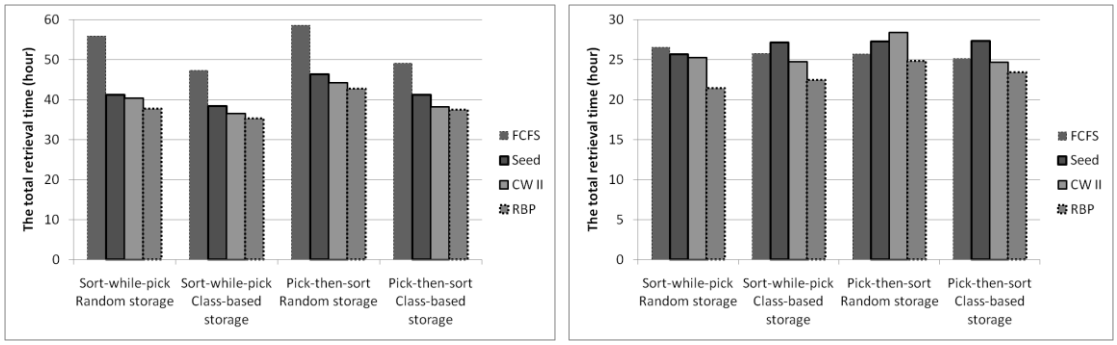
Figure 9. The average travel length per order with the one-way traversal routing method: (a) sort-while-pick strategy; and (b) pick-then-sort strategy.

6.2.3 Impacts on picker blocking in narrow-aisle configuration

In narrow-aisle picking systems, the shorter travel length does not guarantee a shorter retrieval time due to picker blocking (Gue *et al.*, 2006). Thus, we conduct a simulation study to quantify the effect on picker blocking on the various batching algorithms. Two situations are considered: a light congestion situation and a heavy congestion situation. A light congestion environment is defined as: the number of orders in a time window = 1080 orders, 4 time windows, pick:walk time ratio = 5:1, 5 pickers, setup time per batch = 120, and cart capacity = 10 orders or 20 items. A heavy congestion environment is defined as: pick:walk time ratio = 10:1, 15 pickers, and cart capacity = 25 orders or 50 items.

Figure 10 depicts the comparison of the total retrieval time. RBP was relatively robust to picker blocking situation, while seed and CW II produced very poor results under heavy congestion. These findings emphasize the importance of picker blocking and selecting a batching algorithm that not only reduces travel distance, but also does not

create excessive picker blocking.



(a) (b)

Figure 10. The total retrieval time comparison via a simulation study: (a) light congestion case; and (b) heavy congestion case.

Other experimental results are summarized in Appendix A.4. RBP demonstrated consistent performance over other order picking profiles, including variations in both OPS sizes and storage policies.

6.3 Application: wide-aisle picking systems

The previous framework considered pick areas characterized by one-way narrow-aisles. The proposed framework described in this study can be extended to operations with two-way wide-aisle pick areas. The wide-aisle picking system is used in industry to reduce picker blocking or to accommodate storage/retrieval vehicles.

6.3.1 Two-way traversal routing method

Here, pickers have greater flexibility in route selection. Consider constructing an extended route set R based on a two-way traversal routing method. The number of unique routes required grows quickly in the number of aisles. For example, for $|A| = 2, 4, 6, 8, 10, 12$, the corresponding number of routes is 1, 7, 31, 127, 511, 2047. The number of routes for any even value of A can be calculated using the following equation:

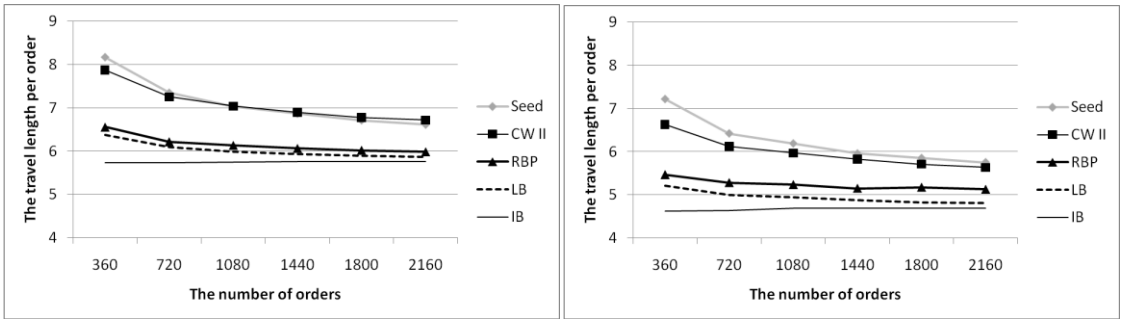
$L(A) = {}_{|A|}C_2 + {}_{|A|}C_4 + {}_{|A|}C_6 + .. + {}_{|A|}C_{|A|}$, where $|A| = 2,4,\ldots$ and ${}_{|A|}C_a = \binom{|A|}{a}$.

6.3.2 Computational result

In Table 2, the previous four methods for batching were used in a two-way traversal routing situation. Further, Figure 11 compares the average travel length per order in a ten-aisle picking system. The impact of optimally batching was more significant as the routing methods grew more complex. With the two-way traversal routing method, RBP continued to dominate CW II and the other methods and the improvement achieved by using RBP was larger for two-way traversal routing. The RBP route set included a smaller proportion of the total number of possible routes to attempt to balance performance with computation time. This is the primary source of the deterioration of the performance for both RBP and the lower bound estimates.

Table 2. Computational results with the two-way traversal routing method in the ten-aisle picking system

Sort	#	FCFS		Seed			CW II			RBP				LB		IB
Strategy	orders	Obj	LU gap	Obj	CPU	LU gap	Obj	CPU	LU gap	ObjL	ObjU	CPU	LU gap	Obj	CPU	Obj
Sort-	360	5385.1	57.42%	2938.6	0.01	21.97%	2833.4	0.43	19.08%	2359.6	2359.6	30.55	2.83%	2292.8	64.78	2063.2
while-	720	10808.0	59.43%	5287.2	0.03	17.06%	5219.4	4.17	15.98%	4476.7	4476.7	60.15	2.05%	4385.1	119.97	4128.7
pick	1080	16242.0	60.16%	7596.5	0.05	14.83%	7597.8	13.57	14.84%	6622.3	6622.3	60.28	2.30%	6470.2	185.28	6206.9
	1440	21716.9	60.66%	9883.7	0.09	13.55%	9922.5	31.43	13.89%	8729.0	8729.0	60.41	2.12%	8544.3	258.77	8286.1
	1800	27202.7	60.98%	12077.1	0.15	12.12%	12186.6	63.88	12.91%	10833.6	10833.6	60.61	2.03%	10613.7	422.94	10364.8
	2160	32725.9	61.25%	14273.7	0.21	11.17%	14506.4	111.40	12.59%	12924.7	12924.7	60.81	1.89%	12679.8	429.91	12443.9
Pick-	360	4243.8	55.79%	2598.4	0.01	27.79%	2385.9	0.49	21.36%	1968.7	1968.7	50.71	4.69%	1876.4	1267.52	1666.5
then-	720	8488.7	57.60%	4622.1	0.03	22.14%	4407.6	4.91	18.35%	3802.5	3802.5	60.76	5.35%	3598.9	6833.24	3343.0
sort	1080	12836.7	58.38%	6681.9	0.05	20.05%	6445.7	17.53	17.12%	5654.0	5654.0	64.55	5.51%	5342.2	13546.39	5070.2
	1440	17131.7	59.01%	8576.9	0.09	18.12%	8384.6	42.77	16.24%	7400.1	7416.8	79.92	5.31%	7022.8	19910.84	6752.1
	1800	21426.1	59.42%	10527.9	0.14	17.41%	10282.5	85.27	15.44%	9255.8	9314.1	98.83	6.65%	8694.6	16521.80	8436.3
	2160	25743.9	59.67%	12423.5	0.21	16.43%	12168.7	146.69	14.68%	11039.6	11073.2	127.08	6.24%	10382.3	24644.21	10137.0



(a) (b)

Figure 11. The average travel length per order with the two-way traversal routing method: (a) sort-while-pick strategy; and (b) pick-then-sort strategy.

7. CONCLUSIONS

This chapter introduced a route-selecting order batching formulation (RSB), its bound model (RPP-LP), and a heuristic solution procedure (RBP) to solve large-scale order batching problems. The special structure of RPP was exploited in developing the formulations and the solution. RBP produced near-optimal solutions in a narrow-aisle order picking system, where the number of aisles was ten and the number of orders was 2180. The computational time required was about 70 seconds on average, with a maximum of 140 seconds. The solution quality was demonstrated by comparing with a tight lower bound developed from the proposed model.

The procedure we have described is an important step toward efficient and effective DC design/operation, where both space utilization and operational throughput are major considerations. A narrow-aisle picking area in a DC is advantageous in terms of space utilization, but produces more picker blocking (Gue *et al.*, 2006; Napolitano and Gross&Associates, 2003). Solutions by RBP not only shortened the total travel distance to near-optimal solutions, but were robust to picker blocking.

A variety of direct extensions of RBP are possible. We showed the RBP framework was extendible to wide-aisle picking systems with a two-way traversal route. Some order picking systems, such as a multiple cross-aisle system (Roodbergen and de Koster, 2001) and a 2-block warehouse (Le-Duc and de Koster, 2007), can also be modeled using the RBP batching procedure. In those systems, it is possible to enumerate available or preferred routes (R) and to define matching relationships between routes and orders (O_r) for general situations. As long as the warehouse manager can construct a preferred route set (R), the proposed algorithm can solve the problem with only slight modifications.

Extending this research to consider other routing methods and to explicitly account for picker blocking will be useful. First, the proposed procedure can be a key enabler when developing an efficient batching algorithm with different routing methods as discussed in Section 6.3. Second, picker blocking should be scrutinized and managed in order picking operations. Our experimental results indicate that using the RBP method for batching can have significant benefits in terms of reduced picker blocking. However, productivity loss by picker blocking still remains an issue. These observations provide motivation for the research described in the next chapters.

CHAPTER V

ANALYSIS OF PICKER BLOCKING IN NARROW-AISLE BATCH PICKING

This chapter identifies sources of picker blocking in batch picking in a narrow-aisle situation and determines satisfactory operational situations, e.g., batching algorithm, sorting strategy, for reducing picker blocking. We present new multiple-pick analytical models to more accurately evaluate picker blocking in a closed-form expression of pick density and the number of pick faces. We compare the results developed from a conventional single-pick order picking models to our multiple-pick models to quantify and identify sources of picker blocking. Note a single pick model assumes there can be at most one picker per pick face, whereas a multiple-pick model allows multiple picks at a pick face. Finally, a simulation study over a variety of batching situations is presented. We highlight three findings for narrow-aisle batch picking processes: 1) variation in pick density across aisles affects picker blocking as much as the magnitude of pick density; 2) a near-optimal distance-based batching algorithm can reduce picker blocking when an appropriate sorting strategy is employed, because it reduces both the number of aisles visited and the variation in the number of picks per aisle; and 3) the sorting strategy (i.e., a pick-then-sort strategy or a sort-while-pick strategy) causes varying amounts of congestion, depending on possible routing options used.

1. INTRODUCTION

We consider a narrow-aisle picking environment, which is very attractive for its

storage capability. However, the narrow-aisle configuration can produce picker blocking, even though one-way traversal routing is used to mitigate congestion (Gue *et al.*, 2006). Accordingly, the order fulfillment time can lengthen and operational costs increase. In practice, the effects of batch formation on picker blocking vary according to the batching algorithm, sorting strategy, and storage policy.

A principle of batch picking is to have pickers gather items that are closely located within the storage space when feasible. Basically, a batch has a higher pick density compared to a single order, which leads to higher picker utilizations. Two studies (Gue *et al.*, 2006; Skufca, 2005) consider a model under a single-pick assumption defined as a situation in which only a single product type is picked at a particular pick face. However, in batch picking, the probability of needing to pick more than one product type at a particular pick face increases. Thus, multiple-pick models that consider repeated picks at a particular pick face can be useful. Parikh and Meller's (2010) recent analytical models of picker blocking considering multiple-picks in narrow-aisle configurations begin to develop an understanding of the impact of non-deterministic pick times as well as multiple-picks at a stop on order picking performance.

However, to date, researchers do not fully understand the relationship between picker blocking and batch formation. Gue *et al.* (2006) and Parikh and Meller (2010) have identified two sources of picker blocking as the size and variation of pick density. However, the impact of batch formation on picker blocking has not been characterized. It is evident that practical picking situations (i.e., batching algorithm, sorting strategy, and storage policy) influence batch formation and thus can have differing effects on two

sources of picker blocking.

In general, an analytical model characterizing picker blocking with a closed-form expression in terms of the number of pickers, *k*, is desirable. The *k*-picker model can help researchers analyze the impacts of increasing the number of pickers. The closed-form expression can suggest diverse numerical analysis over different operations without the use of simulations. Available analytical studies (Gue *et al.*, 2006; Parikh and Meller, 2009; Parikh and Meller, 2010; Skufca, 2005) develop models of two extreme cases: pickers' walk speed is infinite or slow. Neither model exists in practice, but they can bound actual situations and provide an excellent understanding of picker blocking. Parikh and Meller's (2010) two-pickers multiple-pick analytical models for narrow-aisle configurations raise two issues : 1) the analytical model for the slow walk speed case is developed based on four combinations of pick and walk tasks of two pickers; consideration of picking and walking states restricts the extension of the models as well as increases the computational complexity; and 2) a closed-form expression for the infinite walk speed case has not been developed; thus their experimental study does not provide analytical measures of picker blocking for varying pick density. In other words, similar to a simulation, the experimental study requires a computational calculation. Note that our study has been conducted independently from Parikh and Meller's recent study, but both studies produce similar analytical models and address an identical opinion which is relevant for the impacts of multiple-picks on picker blocking. Although Parikh and Meller's study was published first, we show the differences between two results. From the standpoint of the analytical models, the differences described above

have been identified. In terms of the research aim, however, we focus on both developing analytical models over multiple-pick situations, and also scrutinizing order batch picking situations which can give throughput benefits in a narrow-aisle configuration by satisfying the analytical results (Parikh and Meller cover only the impacts by multiple-picks on picker blocking).

This chapter develops new analytical models of picker blocking considering multiple-picks in narrow-aisle configurations, which are simpler compared to Parikh and Meller (2010) and can facilitate the derivation of two closed-form equations for the probability of being blocked. Further relevant convergence characteristics are addressed from the two closed-form expressions. More importantly, we conduct simulation studies over different batch picking situations to relate characteristics of the picking environment and picker blocking to determine appropriate batching strategies for high order picking throughput.

This chapter is organized as follows. Section 2 details the relevant order picking literature and identifies new research opportunities. Section 3 defines a circular blocking model. In Section 4, we derive new blocking models under the assumption of two-pickers and multiple-picks per location. We apply the models to two extreme cases. Relevant insights about the differences between the multiple-pick models and a single-pick model and the impacts of the size of variation in batch size are discussed. Section 5 examines the relationship between analytical models and batching situations. Section 6 summarizes the findings and offers suggestions for future research.

2. LITERATURE SURVEY

Picker blocking analysis in parallel-aisle picking systems can be distinguished by the aisle width, which defines the physical form of the system. A narrow-aisle system is typically characterized by no-passing in an aisle. The picker blocking created by the no-passing condition is termed *in-the-aisle blocking*. Skufca (2005) presents a *k*-picker congestion model of a circular no-passing system in the case of infinite walk speed. Gue *et al.* (2006) address two-picker congestion models of a parallel-aisle pick area approximated by a circular no-passing system considering infinite and unit walk speeds. In the unit walk speed, the unit walk time to pass a pick face is identical to the unit pick time. They also conduct additional simulations to investigate picker behavior under more practical walk speed assumptions. The authors focus on identifying the effects of "pick density" on picker blocking under the single-pick assumption. Their results indicate that a batch picking strategy in narrow-aisle OPSs is advantageous when the pick density is either very low or very high. Parikh and Meller (2010) find that picker blocking can also be significant when the variation of the pick density is high. They develop two-picker congestion models under extreme walk speed assumptions and investigate other scenarios via a simulation study. A closed-form expression was only derived for the unit walk speed scenario. Their unit speed Markov chain model is relatively complex compared to our model. In addition, their analytical model over the infinite walk speed scenario experiences a gap compared to our result which is independently conducted and more clearly satisfies a common characteristic of picker blocking models. Both issues will be discussed in Section 4.

A wide-aisle system experiences a different type of picker blocking, referred to as *pick face blocking*. Parikh and Meller (2009) investigate analytical models under both the single-pick and multiple-pick assumption. The multiple-pick model, which allows a picker to repeatedly pick at a pick face, can reflect a more realistic situation. The authors indicate that the variation of pick density plays a vital role in increasing picker blocking and find that the wide-aisle picking systems can encounter significant pick face blocking when multiple picks occur at a pick face. Their comparison of the two models points to the equal importance of the variation of pick time as well as the variation in pick density.

Several comparison studies to select a best-performing batching algorithm (De Koster *et al.*, 1999; Ho and Tseng, 2006; Pan and Liu, 1995; Ruben and Jacobs, 1999) have been conducted during the last two decades. However, most studies (De Koster *et al.*, 1999; Ho and Tseng, 2006; Pan and Liu, 1995) evaluate performance in terms of travel distance; only Ruben and Jacobs (1999) study the relationship between picker blocking and batching algorithms. The latter authors indicate that the level of congestion is affected by the selection of batching procedures and storage policies, although they don't provide a clear rationale for the congestion. Through simulation studies, they find that a turnover-based storage policy, where popular products with large demand are stored based on shortest-possible travel retrieval, generates more congestion than family-based, where higher-demand products are stored closer together, or random storage policies. Their blocking model approximates congestion by splitting an aisle in two and disallowing other pickers to access an occupied area. This type of unique control policy leads to different levels of congestion compared to recent studies (Gue *et al.*, 2006;

Parikh and Meller, 2009; Skufca, 2005).

Reviewing the available literature, we identify two critical issues with respect to the expression and analysis of picker blocking. First, the multiple-pick picker blocking models by Parikh and Meller (2010) are complex and inaccurate. To establish a Markov property, their analytical model for the slow walk speed case requires four sub states of the pick and walk tasks of two pickers; consideration of pick and walk states restricts the extension of the models as well as increases the computational complexity. In addition their model lacks a closed-form expression of infinite walk speed despite the fact that this type of expression can facilitate additional analysis of picker blocking.

Second, no analytical studies have fully investigated the relationship with the batching algorithm even though, in practice, the batching algorithm may change both the pick density level and its variation. Ruben and Jacobs's (1999) result fails to explain picker blocking in connection with batching algorithms, and there is no clear theoretical rationale for the congestion. Gue *et al.*'s (2006) notion, i.e., less picker blocking when pick density is very low or very high, also requires additional investigation as Parikh and Meller (2010) finds a higher picker blocking situation. Furthermore, since both Gue *et al.* and Parikh and Meller do not conduct their studies on batch picking environments, their results do not explain practical situations.

3. PROBLEM DEFINITION

3.1 Batch picking in narrow-aisle picking systems

In narrow-aisle picking systems, pickers circumnavigate one-way aisles to retrieve items from shelves and place them in a cart as shown in Figure 12. When an

aisle includes no items assigned to the picker, the aisle can be skipped to shorten the travel distance, but the unidirectional characteristic of the aisles must still be maintained. In practice, the order size is relatively small compared to the cart capacity; thus, orders may be batched to reduce total retrieval time by allowing pickers to collect multiple orders in the same trip. Orders cannot be split between multiple batches, and batch size is determined by the cart's carrying capacity.

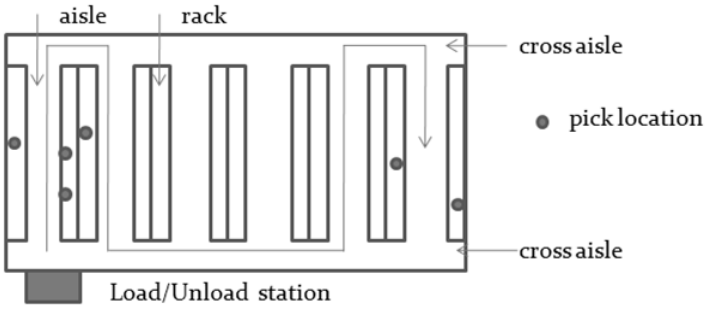


Figure 12. A narrow-aisle system and a routing example (modified from Gademann and Van de Velde (2005)).

In a narrow-aisle picking system, picker blocking can occur when multiple pickers traverse a pick area while maintaining a no-passing restriction. An upstream picker cannot pass a downstream picker as shown in Figure 13.

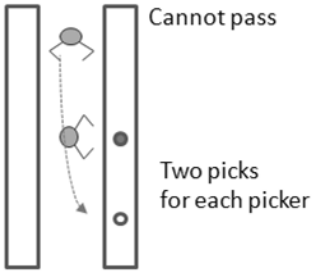


Figure 13. Picker blocking (Parikh and Meller, 2009).

3.2 Throughput model

Order picking systems are often characterized by the *ratio of time spent to pick* an item(s) to *time spent at a stop.* This ratio will be strictly less than one when picker blocking occurs. Gue *et al.* (2006) introduce a throughput model for an order picking system with k pickers in a single-pick situation. To reflect a multiple-pick situation, we generalize their model as Equation (5.1). When each picker is blocked $b(k)$ fraction of the time, $0 \le b(k) \le 1$, the throughput is

$$\lambda(k) = k \cdot \left[\frac{E[pt]}{E[pt]t_p + t_w} \right] (1 - b(k)), \tag{5.1}$$

where $E[pt]$ stands for the expected number of picks at a stop. The time to pick (t_p) represents the average time the picker is stopped and includes the time spent picking items. The time to walk (t_w) indicates the average time to walk past a pick face (location). In a single-pick model, $E[pt]$ is equal to p (Gue *et al.*, 2006), but a multiple-pick model is affected by the number of expected picks at a particular pick face as described in Parikh and Meller (2009).

3.3 A circular order picking aisle model

To simplify the analysis of the picker blocking phenomena in a narrow-aisle picking system, a parallel-aisle system is often modeled as a circular order picking aisle (Gue *et al.*, 2006) as shown in Figure 14. In developing the blocking models, we assume the following: 1) the circular order picking aisle consists of n pick faces; 2) two pickers perform the order picking; 3) they take a one-way traversal route, meaning that they travel through that aisle in only one direction (or in the circular model this implies that

they move only in a clockwise direction); 4) pick time is constant regardless of the pick face characteristics, such as shelf height; 5) at a pick face, pickers pick with a probability p; q denotes 1-p, the probability of walking past a pick-face; 6) a picker can only be picking, walking, or standing idle due to blocking; 7) the pick time and the walk time between two pick faces are deterministic, termed as t_p and t_w, respectively.

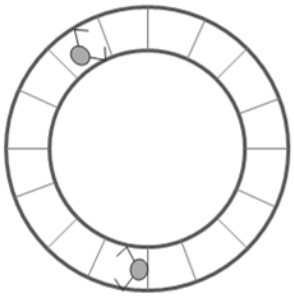


Figure 14. A circular order picking aisle (Gue *et al.*, 2006).

As a performance measurement, we obtain the percentage of time blocked, denoted as $b^m_{pt:wt}(k)$, where m stands for a multiple-pick situation and pt:wt represents the pick:walk time ratio. In the case of a single-pick situation (s), Skufca (2005) previously derived the analytical model for $b^s_{1:0}(k)$. Gue *et al.* (2006) studied single-pick models, i.e., $b^s_{1:1}(2)$ and $b^s_{1:0}(2)$, analytically, and generalized to other cases (e.g., $b^s_{1:0.5}(2)$, $b^s_{1:0.25}(2)$,…, $b^s_{1:1}(10)$) using simulation models. Parikh and Meller (2010) conducted another study for multiple-pick models ($b^m_{1:1}(2)$ and $b^m_{1:0}(2)$), where the analytical model for $b^m_{1:1}(2)$ is presented in a closed-form expression using a discrete Markov chain with $4(n-1)$ states, and $b^m_{1:0}(2)$ is built on $(n+1)$ state Markov chain, but does not have a closed-form expression.

3.4 Scope of study

We wish to develop new analytical models for $b^m_{1:1}(2)$ and $b^m_{1:0}(2)$ [5] and to investigate the more general case $b^m_{pt:wt}(k)$ over varying pick density variation, e.g., different pick density functions, using a simulation study in a circular order picking aisle. For a more complete understanding of picker blocking and batch picking and their relationships to other aspects of warehouse operations, we conduct an extended simulation study considering batching algorithms, sorting strategies, and storage policies in a parallel-aisle picking system.

4. ANALYSIS OF PICKER BLOCKING

We first build analytical models for two order pickers who conduct a retrieval operation in a parallel-aisle picking system using the circular aisle characterization to develop a general understanding, and then conduct a simulation study to reinforce the significance in more practical situations.

Our analytical study considers two extreme cases that do not exist in practice but provide bounds for realistic situations as well as help provide an excellent understanding of picker blocking: 1) walk speed is equal to unit pick time per pick face (pick:walk time = 1:1); and 2) walk speed is infinite (pick:walk time = 1:0). Our analytical model utilizes a Markov property in determining distances between two pickers, which is consistent with prior work, see also (Gue *et al.*, 2006; Parikh and Meller, 2009; Skufca, 2005).

[5] Our models replace and correct the Markov chains in Parikh and Meller (2010). First, we introduce a new Markov chain independent of picking or walking information in the infinite walk speed case. Second, we present an accurate Markov chain model to derive a closed-form expression of the unit walk speed case.

4.1 Pick:walk time = 1:1

Let D_t denote the distance between picker 1 and picker 2 at time t. Given the pick:walk time ratio is 1:1, the distance can be expressed as

(n + (picker 1 position) − (picker 2 position)) mod n (5.2)

and ranges from 1 to n-1. A Markov chain is introduced by defining state $S_t = D_t$, where $S_t = 0$ represents picker 1 blocking picker 2 and state $S_t = n$ represents picker 2 blocking picker 1. In other words, there are two blocking states and n-1 distance-related states. All states can be summarized by the vector [blocked, 1, 2, … , n-1, blocked].

These states allow us to distinguish four transition cases: 1) transition between unblocked states; 2) transition from an unblocked state to a blocked state; 3) transition from a blocked state to an unblocked state; and 4) transition between blocked states.

1) Transition probabilities between unblocked states

If both pickers pick (p*p) or walk (q*q), the current distance (D_t) does not change at t+1. However, when picker 1 picks while picker 2 walks (p*q), the distance decreases by 1. When picker 1 walks while picker 2 picks (q*p), the distance increases by 1.

2) Transition probabilities from an unblocked state to a blocked state

When the distance from picker 1 to picker 2 is 1, a blocked state can arise if picker 1 picks (with probability p) and picker 2 walks (with probability q). Vice versa, when the distance from picker 1 to picker 2 is n-1, the current state becomes a blocked state if picker 1 walks (with probability q) and picker 2 picks (with probability p).

3) Transition probabilities from a blocked state to an unblocked state

If picker 1 is blocked by picker 2, picker 1 must wait for picker 2 to walk (with probability q) to exit a blocked state. Vice versa, when picker 2 is blocked by picker 1, picker 2 must wait for picker 1 to walk (with probability q).

4) Transition probabilities between blocked states

When the current state is blocked, a pick can occur with probability p and the blocking status remains, i.e., a blocked state transitions to a blocked state with probability p.

In sum, when multiple picks are allowed, the transition probabilities can be described in a transition diagram as illustrated in Figure 15.

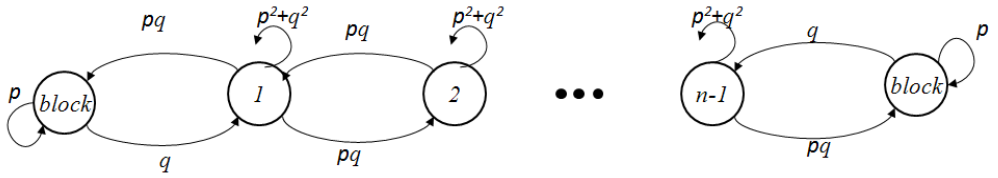


Figure 15. State space and transitions for the Markov chain model when picking time equals travel time.

The Markov chain model in Figure 15 does not include substates of picking or walking as the Gue *et al.* (2006) and Parikh and Meller (2010) models. Thus the transition matrix is more condensed. The resulting transition matrix, which has dimensions (n+1) x (n+1), is:

$$A=\begin{bmatrix} p & q & 0 & \cdots & 0 & 0 & 0 \\ pq & p^2+q^2 & pq & \cdots & 0 & 0 & 0 \\ 0 & pq & p^2+q^2 & \ddots & \ddots & 0 & 0 \\ \vdots & \vdots & \ddots & \ddots & \ddots & \vdots & \vdots \\ 0 & 0 & \ddots & \ddots & p^2+q^2 & pq & 0 \\ 0 & 0 & 0 & \cdots & pq & p^2+q^2 & pq \\ 0 & 0 & 0 & \cdots & 0 & q & p \end{bmatrix}$$

Stationary distribution

We obtain the following *v*, which satisfies *vA* = *v*.

$$v=\left[1,\frac{1}{p},\cdots,\frac{1}{p},1\right]$$

The stationary density using ||*v*|| is scaled to obtain a stationary probability. From *v* above, this implies:

$$\|v\|=2\cdot 1+(n-1)\frac{1}{p}=2+\frac{n-1}{p}$$

The blocking probability of one picker at one blocked state is

$$b_{1:1}^{m}(2)=\frac{v_{1^*}}{\|v\|}=\frac{1}{2+\frac{n-1}{p}}=\frac{p}{2p+n-1} \tag{5.3}$$

Equation (5.3) is identical to the results by Parikh and Meller (2010), whose transition matrix has dimensions 16*(*n*-1) x (*n*-1). Figure 16 plots percentage of time blocked over different number of aisles (*n*). The 1:1 picker blocking model estimates a smaller productivity loss when the picking area includes more pick faces as shown:

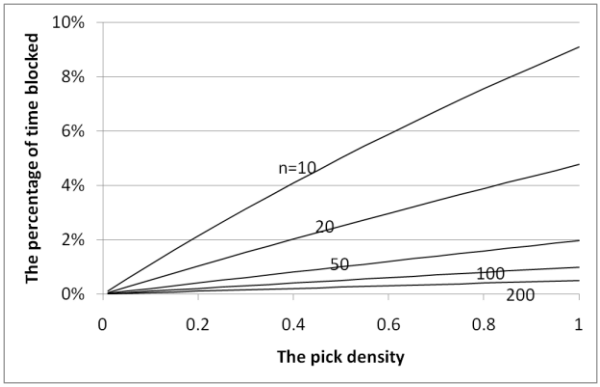


Figure 16. The percentage of time that pickers are blocked over different number of pick faces when two pickers work with pick:walk time = 1:1.

Productivity loss over pick density starts from 0, increases as pick density increases, and converges to 1/(*n*+1) as pick-density approaches 1. This result is summarized in the following theorem.

Theorem 1. When two pickers travel at unit speed, the percentage of time blocked is at least 0 and at most $\frac{1}{n+1}$.

Proof. (5.3) is a monotonic increasing function. Its limiting value is 0 when *p* goes to 0 and 1/(*n*+1) when *p* goes to 1 as follows: $\lim_{p\to 0}\frac{p}{n+2p-1}=0$,

$\lim_{p\to 1}\frac{p}{n+2p-1}=\frac{1}{n+1}$. The result is $0\le \text{timeblocked}\%\le\frac{1}{n+1}$. End of proof.

Figure 17 compares the relationship between a multiple-pick (*m*) model and a single-pick model (*s*) over two different numbers of pick faces (20 and 50 pick faces). Here, the x-axis is the average number of picks, not pick density. As Equation (5.1)

indicated, the throughput comparison over identical workloads (i.e., the number of picks) can express the impact of picker blocking. The multiple-pick results are monotonically increasing, while the single-pick results, developed by Gue *et al.* (2006), show a drop in picker blocking at high pick requirements.

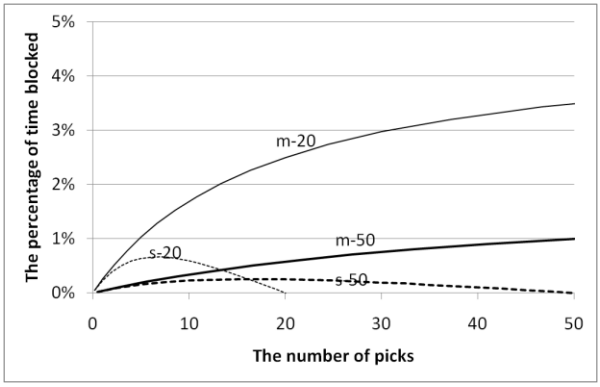


Figure 17. The comparison of single-pick and multiple-pick models when two pickers work with pick:walk time = 1:1.

Note that the proposed discrete-time Markov chain of picker blocking for multiple-picks with a pick:walk time = 1:1 differs from Parikh and Meller (2010) in that the distance is not conditioned on the operation modes of the pickers (i.e., walking or picking). As we addressed above, when multiple-picks are allowed, a Markov property of distance holds regardless of the previous walking or picking status. The conditional multiple-pick model is summarized in Appendix B.1, which is similar to the Parikh and Meller (2010) model. Moreover, the approach described in this chapter is applicable in wide-aisle systems discussed in Parikh and Meller (2009) (see Appendix B.2).

4.2 Pick:walk time = 1:0

The infinite speed assumption allows for transitions to multiple states in our

Markov chain model. Thus, the probability that a picker moves distance *x* is approximated, and then a probability function for the distance *y*, characterizing the change in the distance between the two pickers, is estimated.

Let random variables X^1_t and X^2_t represent the number of locations moved in time *t* by pickers 1 and 2, respectively. If a picker picks more than one pick at a pick face, the distribution of the location is defined over the infinite sample space with a random variable characterizing the number of locations between two pickers:

$$f(x) = q^x p \quad \text{for } x = 0,1,2,\ldots \tag{5.4}$$

$Y_t = X^1_t - X^2_t$ denote the change in distance between the two pickers when passing is not allowed. As described in Appendix B.3, the probability density function of Y_t ($g(y)$) becomes:

$$g(y) = \frac{pq^{|y|}}{1+q} \qquad \text{for } -\infty < y < \infty \tag{5.5}$$

Suppose the distance at the previous state is $D_{t-1} = r$. The actual change in distance is bounded by the physical blocking phenomenon and the amount of the change is limited by *r*. Like the previous 1:1 analysis, four transition cases are defined: 1) transition between unblocked states; 2) transition from an unblocked state to a blocked state; 3) transition from a blocked state to an unblocked state; and 4) transition between blocked states.

1) Transition probabilities between unblocked states

In this case, the distribution function (5.5) is used directly. Given *r*, the change is bounded between 1 and *n*-1 ruling out the possibility of the first picker catching up to the

second picker.

$$P(Y_t = y) = \frac{pq^{|y|}}{1+q} \qquad \text{for } 1-r < y < n-1-r, r = 1,\ldots,n-1$$

2) Transition probabilities from an unblocked state to a blocked state

The next step is calculating the probability of events with blocking. To obtain this probability, we need to accumulate all cases above the limits (0 or n). We note that there will be blocking at state 0 if $Y_t \leq -r$. $g(y)$ is symmetric and the probabilities for the bounding cases are calculated as:

$$P(Y_t \geq r) = \sum_{y=r}^{\infty} \frac{pq^{|y|}}{1+q} = \frac{p}{1+q} q^r \frac{1}{1-q} = \frac{q^r}{1+q}, \quad \text{for} 1 \leq r \leq n-1$$

$$P(Y_t \geq n-r) = \frac{q^{n-r}}{1+q}, \quad \text{for} 1 \leq r \leq n-1$$

3) Transition probabilities from a blocked state to an unblocked state

The distribution function (5.5) is again used directly. Note that r is 0 or n when a picker is blocked. Since the blocked picker walks first, initially, the distance between two pickers also becomes 0 or n.

$$P(Y_t = y) = \frac{pq^{|y|}}{1+q} \qquad \text{for } 1-r < y < n-1-r, r = 0 \text{ or } n$$

4) Transition probabilities between blocked states

Similar to 3), $r = 0$ or n express the blocked states. Without loss of generality, the probabilities in 2) are applicable. Thus, expressions for both the lower bound and the upper bound are as follows:

$$P(Y_t \geq r) = \frac{q^r}{1+q}, \quad \text{for } r = 0 \text{ or } n$$

$$P(Y_t \geq n-r) = \frac{q^{n-r}}{1+q}, \quad \text{for } r = 0 \text{ or } n$$

The probabilities that we derive are similar to Parikh and Meller (2010) with one exception. While managing the transition from *blocked* (0) to *blocked* (*n*) or *blocked* (*n*) to *blocked* (0), the equation above uses $q^n/(1+q)$ from the transition probability 4), which differs from $q^{n-2}/(1+q)$ in Parikh and Meller. Since they do not offer any comment on both values, the reason cannot be identified. Instead, we use a computational comparison, which will be discussed later.

The result forms the following transition matrix:

$$A = \begin{bmatrix} \frac{1}{1+q} & \frac{pq}{1+q} & \frac{pq^2}{1+q} & \cdots & \frac{pq^{n-2}}{1+q} & \frac{pq^{n-1}}{1+q} & \frac{q^n}{1+q} \\ \frac{q}{1+q} & \frac{p}{1+q} & \frac{pq}{1+q} & \cdots & \frac{pq^{n-3}}{1+q} & \frac{pq^{n-2}}{1+q} & \frac{q^{n-1}}{1+q} \\ \frac{q^2}{1+q} & \frac{pq}{1+q} & \ddots & \ddots & \ddots & \frac{pq^{n-3}}{1+q} & \frac{q^{n-2}}{1+q} \\ \vdots & \vdots & \ddots & \ddots & \ddots & \vdots & \vdots \\ \frac{q^{n-2}}{1+q} & \frac{pq^{n-3}}{1+\beta} & \ddots & \ddots & \ddots & \frac{pq}{1+q} & \frac{q^2}{1+q} \\ \frac{q^{n-1}}{1+q} & \frac{pq^{n-2}}{1+\beta} & \frac{pq^{n-3}}{1+\beta} & \cdots & \frac{pq}{1+q} & \frac{p}{1+q} & \frac{q}{1+q} \\ \frac{q^n}{1+q} & \frac{pq^{n-1}}{1+q} & \frac{pq^{n-2}}{1+q} & \cdots & \frac{pq^2}{1+q} & \frac{pq}{1+q} & \frac{1}{1+q} \end{bmatrix}$$

Stationary distribution

To identify a stationary distribution, a v which satisfies $vA = v$ is identified as:

$$v = [1, p, \ldots, p, 1]$$

We can scale the stationary density using $||v|| = 2+(n-1)p$. The blocking probability of a picker at one blocked state is:

$$b_{1:0}^{m}(2)=\frac{1}{2+(n-1)p} \tag{5.6}$$

Because of the differences in the expression for the transition probabilities from *blocked* (0) to *blocked* (*n*) or *blocked* (*n*) to *blocked* (0), the results given by the 1:0 analytical model we propose have a 0.032 to 0.170% error gap compared to the results from Parikh and Meller's (2010) model. According to Parikh and Meller (2009), when *p* = 1, both $b_{1:1}^{m}(2)$ and $b_{1:0}^{m}(2)$ should converge to the same value regardless of walk speed. A high pick density leads to the same congestion situation, which is observed in single-pick narrow-aisle models (Gue *et al.*, 2006) and wide aisle models (Parikh and Meller, 2010). When p=1, the equation in our 1:0 analytical model satisfies the general knowledge, but Parikh and Meller's model experiences a gap of 0.0083% when the number of pick faces = 20.

As the function is derived, the convergence characteristic of the 1:0 model can be investigated, and the following theorem is observed.

Theorem 2. When two pickers travel at infinite speed, the percentage of time blocked is at most 50% and at least $\frac{1}{n+1}$.

Proof. (5.6) is a monotonic decreasing function. There are two limiting characteristics. As *p* goes to 0, the upper limiting value is $\lim_{p\to 0}\frac{1}{2+(n-1)p}=\frac{1}{2}$. The lower limiting value is 1/(*n*+1) as follows: $\lim_{p\to 1}\frac{1}{2+(n-1)p}=\frac{1}{n+1}$.

The result is $\frac{1}{n+1} \le \text{timeblocked}\% \le \frac{1}{2}$. End of proof.

Figure 18 depicts the productivity loss over different numbers of pick faces. Picker blocking starts from picker utilization 50%, decreases as pick-density increases, and converges to 1/(*n*+1). As we observed in the 1:1 model, larger areas are less susceptible to picker blocking than smaller areas.

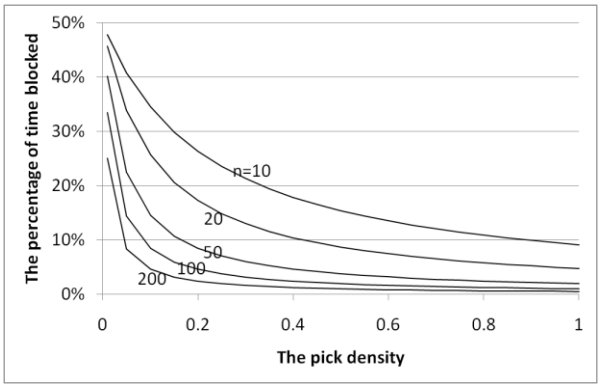


Figure 18. The percentage of time that pickers are blocked over different number of pick faces when two pickers work with pick:walk time = 1:0.

Figure 19 compares a multiple-pick (*m*) model and a single-pick model (*s*) over 20 pick faces and 50 pick faces. The percentage of time blocked for both the multiple-pick and single-pick models decreases monotonically as pick density increases. However, the multiple-pick results consistently experience a higher percentage of time blocked. Moreover, as Equation (5.6) indicated, the percentage of time blocked for the multiple-pick model goes to 1/(*n*+1), not to 0.

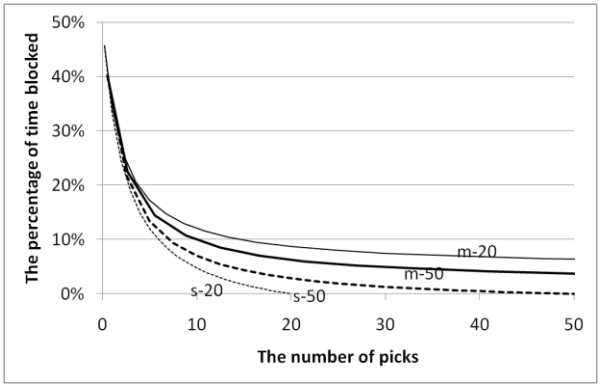


Figure 19. The comparison of single-pick and multiple-pick models when two pickers work with pick:walk time =1:0.

From theorems 1 and 2, a further important result can be derived.

Theorem 3. As pick density goes to 1, the percentage of time blocked converges to $\frac{1}{n+1}$ when there are two pickers.

Proof. This proof is a direct extension of the previous results. When the walk speed is equal to the pick time, we can use Equation (5.3) as follows: $\lim_{p\to 1}\frac{p}{n+2p-1}=\frac{1}{n+1}$. When pickers walk at infinite speed, Equation (5.6) experiences the same convergence: $\lim_{p\to 1}\frac{1}{2+(n-1)p}=\frac{1}{n+1}$. End of proof.

4.3 Simulation study

The two analytical models are based upon three assumptions: 1) extreme pick:walk time ratio; 2) a Markov property in distance between pickers; and 3) the

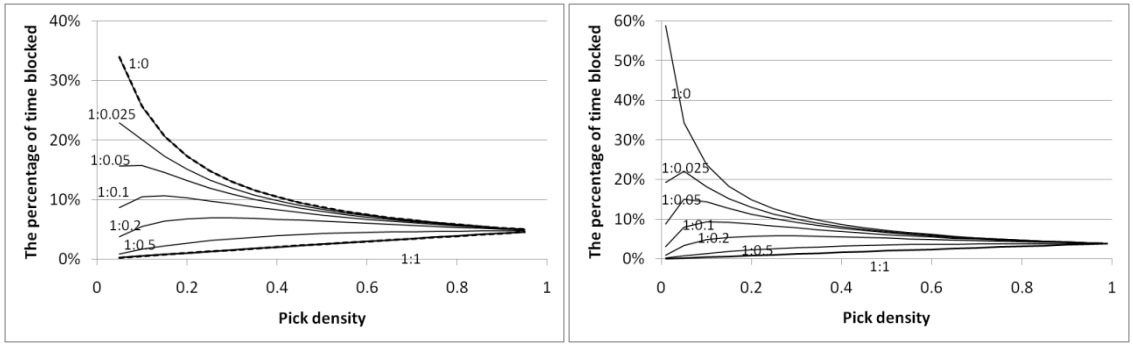
circular approximation to a parallel aisle order picking area. Below, assumptions 1 and 2 will be relaxed and investigated via a simulation study; assumption 3 will be maintained in Sections 4 and 5. Appendix B.4 discusses the validation of our analytical models and simulations by cross comparison among our analytical models, our simulation models, and Parikh and Meller's (2010) results.

4.3.1 Fractional walk speed

In practice, pickers are not extremely fast or slow. If the pick time is 1, most practical speeds for walking are on the range [0.05,1] (Gue *et al.*, 2006). For example, our literature review found a fast speed would have a pick to walk ratio of 1:0.1 (Petersen, 2000) and a slow speed would have a ratio of 1:0.2 (Yu and De Koster, 2009). We conduct a simulation study with pick:walk time = 1:0.025, 1:0.05, 1:0.1, 1:0.2, and 1:0.5. Figure 20 illustrates the simulations' results of a two-picker model (labeled a) and a five-picker model (labeled b). Solid lines are the results with pick:walk time = 1:0, 1:0.025, 1:0.05, 1:0.1, 1:0.2, 1:0.5, and 1:1 from top to bottom. The upper dotted line is an analytical result with pick:walk time = 1: 0. The lower dotted line is an analytical result with pick:walk time = 1:1.

As pick density increases, the percentage of time blocked converges to approximately the value derived in Theorem 3. For example, when $p = 0.95$, in Figure 20 (a) ranges [4.53, 5.00] of throughput loss by picker blocking in a 20-pick face circular picking system with two pickers. According to Theorem 3, the loss is 1/21 = 4.76 when two pickers are in the order picking system. Figure 20 (b), using five pickers, converges to [3.79, 3.86]. Our observation indicates that the multiple-pick characteristic of batch

picking increases picker blocking. In addition, picker blocking is an issue regardless of variation of pick density in a narrow-aisle order picking. This result supports the observations of Parikh and Meller (2010) in a narrow-aisle order picking and Parikh and Meller (2009) in a wide-aisle order picking.



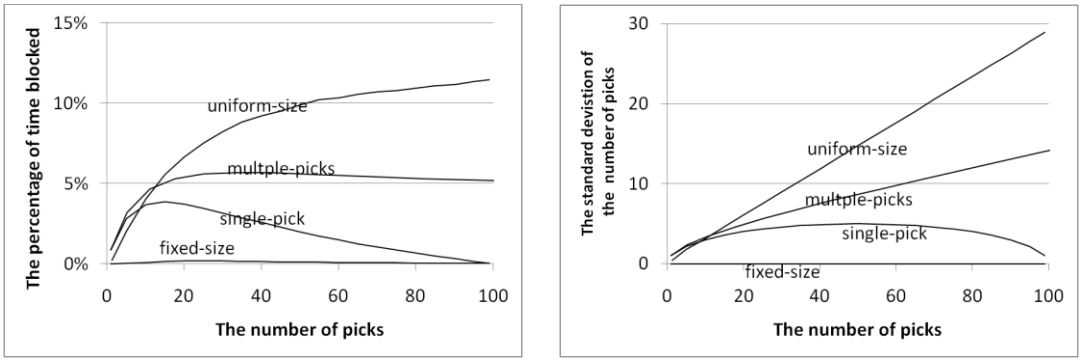
(a) (b)

Figure 20. The percentage of time blocked over different pick:walk time ratios: (a) two pickers in 20 pick faces; and (b) five pickers in 100 pick faces.

4.4.2 Non-Markov property in distance: Variation of the number of picks

In multiple-pick and single-pick analytical models, the number of picks in a trip (from the first pick face to the last pick face) is determined to maintain a Markov property of the distance between two pickers. That restriction is relaxed and investigated via simulation. A simulation model developed with the same pick probability restrictions as the single-pick analytical model (Gue *et al.* (2006)) is used. Several models are considered: a simulation model generated with the restrictions in the multiple-pick analytical model (described above), a fixed-size model (the number of picks in a trip is constant), and a uniform-size model (the number of picks in a trip follows a discrete uniform distribution [mean/2, mean *3/2]).

Figure 21(a) depicts the relationship between the percentage of time blocked and "the number of picks" for different assumptions regarding the distribution of items and Figure 21 (b) illustrates the relationship between "the number of picks" and the variation of "the number of picks" for different assumptions regarding the distribution of items. A high variation in the number of picks per trip results in more severe picker blocking, and conversely, even if the number of picks in a trip is large, i.e., pick density is high and multiple-picks are allowed, if the variation in the number of picks is low there is less picker blocking (i.e., fixed-size instance). Our observation extends Parikh and Meller's (2010) finding that variation of the number of picks in a trip is of similar importance as variation of pick time at a stop. In general, the order batching has additional flexibility to group orders into batches, thus, less variation of the number of picks in a unit distance can be constructed reducing picker blocking.



(a) (b)

Figure 21. Simulation results over different workload distributions (the number of pickers = 5, the number of pick faces = 100, and pick:walk time = 1:0.2) : (a) the percentage of time blocked; and (b) the standard deviation of the number of picks (workload).

5. COMPARISON STUDY IN PARALLEL-AISLE PICKING SYSTEMS

Another difficulty encountered when analyzing picker blocking in real picking situations arises due to the multiple-aisles characteristic and impacts by routing. In this section we describe an extended simulation study in a parallel-aisle order picking system. In particular, in a parallel-aisle order picking system with multiple aisles, decreasing the travel distance is a primary concern of management. Thus, a batching algorithm to efficiently reduce the travel distance is developed. In addition, a sorting strategy and a storage policy often are changed to maximize the retrieval performance (Frazelle, 2002; Tompkins *et al.*, 2003). The batching algorithm, sorting strategy, and storage policy can increase the expected number of picks at a stop, but they also impact picker blocking ($b(k)$). This section describes the effects of the batching algorithms, sorting strategies, and storage polices on picker blocking.

5.1 Simulation design

Various batching algorithms are available. Specifically, large-scale order batching situations will be considered, thus the comparison is limited to those that can handle large problems sizes. From the available literature, the following are considered:

- Seed: the seed algorithm developed in De Koster *et al.* (1999): 1) select a seed having the largest number of aisles; 2) choose the order minimizing the number of additional aisles; and 3) update the seed as an order is added to it.
- CW II: the Clarke and Wright algorithm (II) in De Koster *et al.* (1999). See Appendix B.5 for more detail.
- RBP: the heuristic route-selection-based batching algorithm. See Chapter IV and Appendix B.6 for more detail.

Seed and Clarke and Wright (CW) II are identified as the best algorithms in de Koster *et al.*'s comparison study (1999). The route-selection batching procedure (RBP) is a near-optimal batching algorithm discussed in Chapter IV.

A sorting strategy impacts the batching algorithms by affecting the units of measure determining the batch size. Basically, the pickers carry bins or boxes on the cart to store each order separately in a "sort-while-pick" operation. Thus, the batch size is determined by the number of bins, i.e., the number of orders. Another strategy, "pick-then-sort", does not carry bins (but it does require a sorting operation after the completion of the picking operation). In this case, the picker does not need to carry bins and separate orders, rather he/she can mix orders on the cart and orders can be batched to maximize capacity.

Products are typically stored in warehouses to minimize retrieval efforts. In general, a class-based storage policy stores the more frequently requested items closest to the loading station to reduce the trip distance in contrast to a random storage policy where items are stored in random locations in the warehouse.

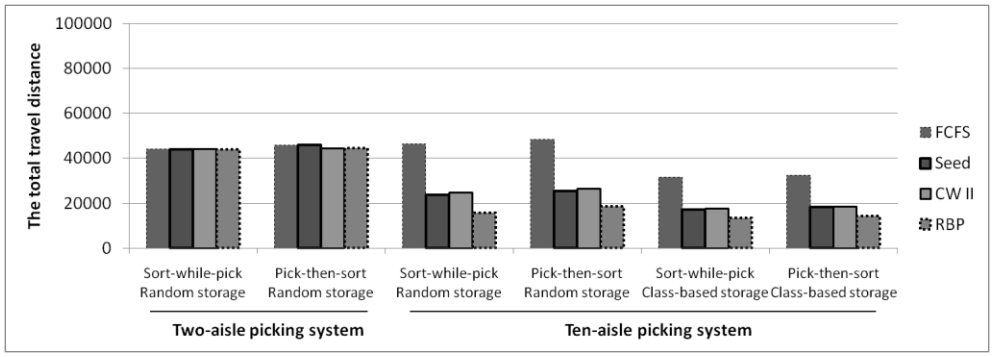
Consider a general order picking situation: the number of orders in a time window = 540 orders, eight time windows per shift, pick:walk time ratio = 5:1, setup time per batch = 0, average order size = two items (uniform [1,3]), five pickers, and cart capacity = 10 orders when sort-while-pick strategy and 20 items when pick-then-sort strategy. A two-aisle system and a ten-aisle system with identical total number of pick faces as 100 are considered to investigate the effects of pick density. While the two-aisle system is similar to a circular aisle model, the ten-aisle system captures the effects that

asiels can be skipped as long as the one-way travel within aisles is maintained. The number of simulation runs per instance (i.e., 20 runs per instance) following Ruben and Jacobs (1999). The percentage of time blocked and the standard deviation of the number of picks in an aisle (STD) are compared across scenarios.

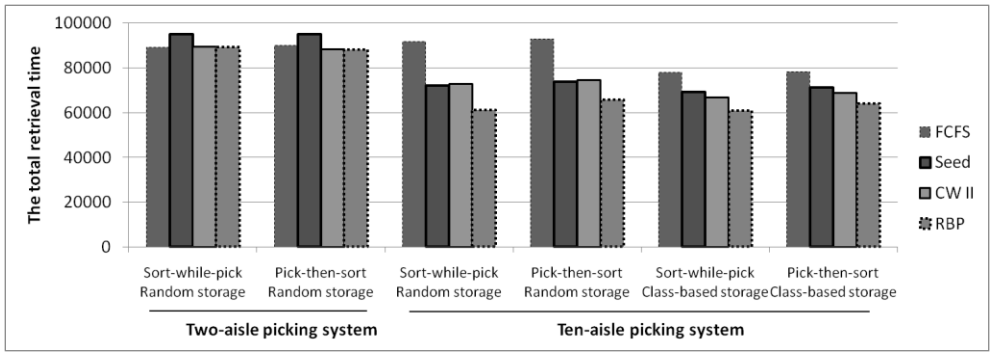
5.2 Experimental results

Figure 22 shows the total travel distance and the total retrieval times for eight different situations, while Figure 23 depicts the productivity loss for each batching algorithm. The two-aisle instances of FCFS in Figure 23 (a) is very similar to Gue *et al.* (2006). The productivity loss is approximately 1~3%. In the two-aisle models, other batching algorithms have similar or slightly better picker utilization, because there is a very small reduction of the total travel distance by decreasing the number of trips.

In the ten-aisle instances, the FCFS procedure in Figure 23 (a) shows a small percentage of time blocked, approximately 1.5~4.2%. However, with respect to overall performance, other batching algorithms achieve significantly larger reductions in the travel distance (Figure 22 (a)) and overall throughput improvement which is inversely related to the total retrieval time shown in Figure 22 (b). For batching algorithms other than FCFS, a productivity loss by picker blocking becomes an issue as noted by discussed in Section 4.3 and Parikh and Meller (2010).

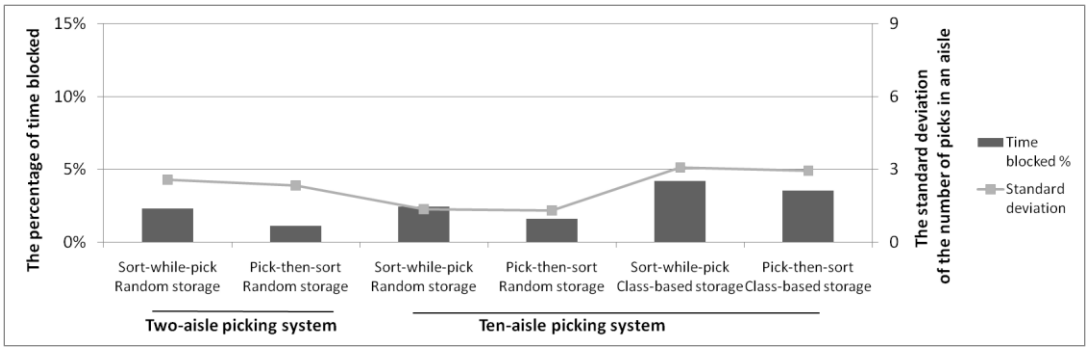


(a)

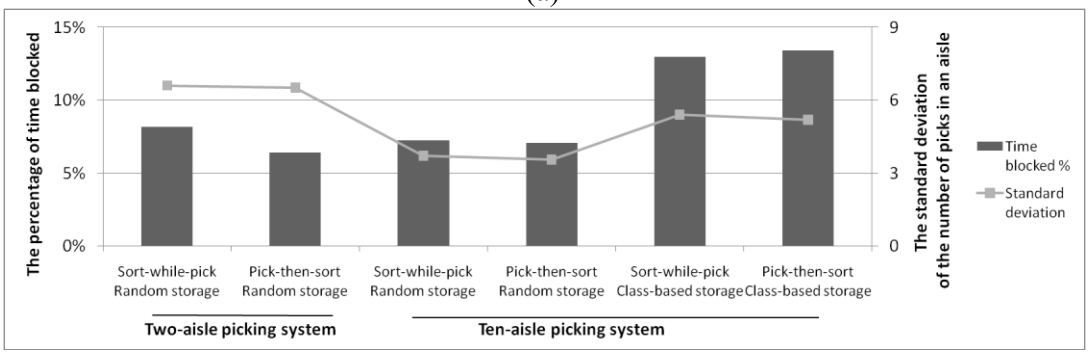


(b)

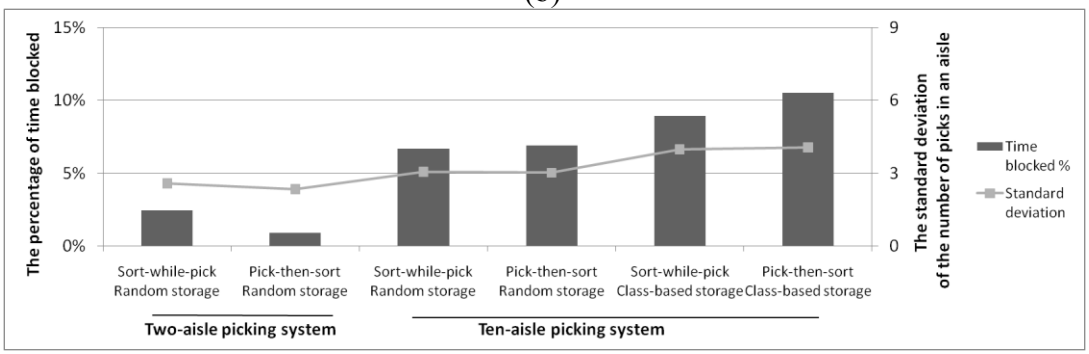
Figure 22. Comparison over different batching algorithms of: (a) total travel distance; and (b) total retrieval time.



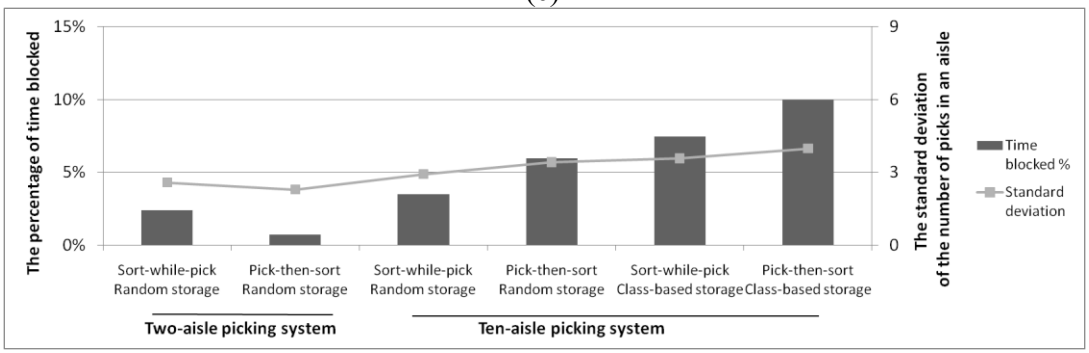
(a)



(b)



(c)



(d)

Figure 23. The percentage of time blocked and standard deviation of the number of picks per aisle over different batching algorithms: (a) FCFS; (b) seed; (c) CW II; and (d) RBP.

The results of the experiments provide insights regarding batching algorithms, sorting strategies, and storage policies as follows:

1) *Solution quality of batching algorithm impacts picker blocking when an appropriate sorting strategy is employed.*

The seed algorithm creates heavy congestion compared to FCFS, because the algorithm increases pick-density. CWII shows less picker blocking compared to the seed algorithm. However, the RBP solution exhibits less congestion due to reduced travel distance. Furthermore, the standard deviation of RBP is less than the standard deviation of the seed algorithm, and is less or a little more than the standard deviation of the CW II algorithm. Intuitively, an improved distance-based batching algorithm could encounter more congestion. However, RBP reduces congestion due to large reductions in the distance traveled, and relatively reasonable variation of picks per aisle as shown in Figure 23.

2) *Sorting strategy impacts picker blocking when combined with RBP*.

When the sorting operation is combined with an appropriate OPS size (i.e., the number of aisles) and as the solution quality of batching algorithms is close to optimality, e.g., RBP in most scenarios and CW II in a few particular cases, a distance-based batching model performs well in terms of picker blocking. In the two-aisle picking system with a single route, the pick-then-sort strategy experiences less picker blocking as shown in Figure 23 (d). Vice versa, in the ten-aisle pick system characterized by several routing lengths ((five cases of number of aisles visited: 2, 4, 6, 8, and 10), the sort-while-pick strategy is advantageous (see Figure 23 (d)).

In the two-aisle picking system, only single route is available under the traversal routing method. The pick-then-sort strategy determines the batch size by the number of picks. Then, the variation of picks across batches is 0 if batches are consolidated optimally. Accordingly, the variation of picks per aisle is 0, which is similar to a "fixed-size" case (see Section 4.4.2). Thus, RBP reduces picker blocking, whereas the sort-while-pick strategy packs each batch with a constant number of orders. Thus, the number of picks across batches can vary within range of the batch size * the order size. The sort-while-pick strategy results in greater picker blocking compared to the pick-then-sort strategy in the two-aisle picking situation.

The ten-aisle picking system faces a different situation as the number of aisles visited across batches becomes diverse. When the sorting operation is separated from the order picking operation (pick-then-sort strategy), there is more variation of the number of picks per aisle across batches. Intuitively, a batch should contain the same number of items, but the number of aisles visited is not identical. Thus, the variation of the number of picks per aisle among batches varies widely, as do the route lengths.

In the sort-while-pick strategy, less variation of picks per aisle can be achieved while obtaining a high quality solution. The sort-while-pick strategy constrains each batch to have the same number of orders, not number of items. A batch with a long route may include orders passing more aisles. To pass more aisles, each order may contain more items. Then, the batch with a long route may include more items because the batch size is determined by the number of orders, and vice versa. In conclusion, the expected number of picks of a batch will typically be proportional to the length of route, i.e., the

number of aisles visited, as batches are packed more optimally. Thus, compared to the pick-then-sort strategy, this characteristic can produce less variation of the number of picks per aisle, which reduces picker blocking.

3) *Similar to Ruben and Jacobs* (1999)*, class-based storage policies increase picker blocking.*

When a class-based storage policy is applied, picker blocking increases as Ruben and Jacobs observed. Even though the RBS algorithm implements a sort-while-pick strategy (Figure 23 (d)), the productivity loss due to congestion is 7.5%. In other words, the class-based storage policy offsets the gain of the travel distance with the losses related to picker blocking as shown in Figure 22. The previous observation (impacts by near-optimality and sorting strategy) is still valid since each aisle stores items evenly under the class-based storage policy.

6. CONCLUSION AND FURTHER STUDY

This chapter provided a new understanding of picker blocking in a narrow-aisle batching picking situation and scrutinized the relationship between picker blocking and order batching using both analytical models and simulation studies. New analytical models of two specific conditions in two-picker order picking situations (a slow walk speed and an infinite walk speed) are developed. Specifically, two closed-form expressions were derived and the relevant convergence characteristics addressed. Diverse simulations were conducted varying several warehouse policies including the batching algorithm, the sorting strategy, and the storage policy. Most importantly, simulation results showed that a near-optimal distance-based batch algorithm (RBP)

creates very little picker blocking. Furthermore, the sorting strategy affects the variation of the number of picks in an aisle, thus making specific sorting strategies (sort-while-pick) more effective in large facilities.

These experimental results reveal that batch strategies faces different levels of picker blocking and identify the conditions under which blocking can be reduced. First, we verified the importance of pick density on picker blocking (Parikh and Meller, 2010). Second, the distance-based batching (RBP) algorithm lessened picker blocking, because of a very significant reduction in the travel distance and a relatively uniform pick density. Third, a sort-while-pick strategy induces less picker blocking when a RBP was used in a large facility.

CHAPTER VI

BATCH PICKING IN NARROW-AISLE ORDER PICKING SYSTEMS WITH CONSIDERATION FOR PICKER BLOCKING

Reducing the time spent picking orders benefits warehouse operations by decreasing the resources required and by improving response time. The two primary components of the time spent picking orders are traveling time and blocking time. This chapter proposes a batching and sequencing procedure called the indexed batching model (IBM) with the objective of minimizing an aggregation of travel distance and congestion delay. The IBM differs from the traditional batching formulation in that it assigns orders to indexed batches, where a batch index represents the batch's release sequence. A mixed integer programming solution for exact control is developed and a simulated annealing procedure for a large-scale environment is demonstrated. Our results indicate that the integrated batching-and-sequencing approach achieves the throughput improvement not realized by the traditional approaches and allows for the development of batch picking strategies that are ideal for narrow-aisle order picking systems.

1. INTRODUCTION

DCs are constantly challenged to reduce the cost of their operations and to become more efficient. One common way to lower costs per unit shipped is to increase space utilization (Napolitano, 2009). According to the recent warehouse operations survey (Napolitano, 2008), the warehousing industry has three major cost sources:

inventory, investment, and order processing. For example, rising inventories often force warehouses to store more goods in less space (Gue *et al.*, 2006; Napolitano, 2009). Narrow-aisle picking systems are one alternative to increase space utilization with minimal investment costs. However, the narrow-aisle characteristic can add to order picking costs due to longer travel and more congestion (Gue *et al.*, 2006). Small order sizes exacerbate the problem, because they require more trips through the picking area. Implementing an efficient batch order picking strategy can help to reduce operational costs in a narrow-aisle order picking environment with small order sizes.

However, the combination of narrow-aisle OPS and a batch picking strategy can suffer from significant operational performance loss and control difficulties related to picker blocking (Gue *et al.*, 2006; Parikh and Meller, 2010). As more pickers travel in a picking area, well-designed control policies can reduce travel distances or improved design of the facility may elevate these congestion issues (Zhang *et al.*, 2009). Traditionally, an OPS can be designed with wide aisles to create less blocking, or can be operated using zone picking, where each zone contains a single picker. However, both of these approaches are not viable in many cases due to additional space (or cost) requirements.

Other approaches employ routing alternatives. Zhang *et al.* (2009) provide an alternative routing method where the path is dependent on the congestion amount. Gue *et al.* (2006) briefly introduce a routing strategy where a downstream (= blocking) picker exits an aisle and circulates back behind an upstream (blocked) picker using an empty aisle when there is significant congestion. However, the alternative paths or averted

routing approaches may lengthen a trip compared to the original route, and can be challenging to implement in practice.

Control policies to trade off travel distances and time blocked have not been addressed in the academic literature. Thus, the goals of this chapter are: 1) develop a control framework combining order batching and sequencing issues; 2) present a practical solution procedure to solve the integrated batching and sequencing problem; and 3) vary the order picking environments to investigate the performance of the proposed strategy. A new batching framework is developed including the sequencing problem. The proposed model is formulated as a mixed integer program (MIP). This formulation can only be solved optimally for small size problems. To overcome this limitation, we adapt a simulated annealing heuristic approach.

The chapter is organized as follows. Section 2 briefly reviews related studies. In section 3, a concise batching framework to handle blocking is developed. The framework considers a picking area with one-way aisles and uses insights from flow-shop scheduling problem to identify strategies to reduce picker blocking. Section 4 develops an indexed batching framework to address in-the-aisle picker blocking. Section 4 also addresses the sequencing of batches, how the multiple aisle impacts the framework, and how the proposed model can handle multiple trips. In Sections 5 and 6, we formulate a MIP and develop a simulated annealing heuristic solution approach, summarize the results, and discuss the importance of the findings.

2. LITERATURE SURVEY

When operational costs due to picker blocking are excessive, engineers prefer

alternative OPS configuration or order picking strategy to control blocking. Alternatives are available for a wide-aisle OPS (Parikh and Meller, 2009) or in the case of zone order picking (De Koster and Yu, 2008). However, for facilities in which space is a concern changing the layout and order picking operations to either of these alternatives may not be feasible. Further, to make the best use of their limited space some warehouses have narrow-aisles. However, if picker blocking is a concern in these settings the only solutions available in the literature are passing and rerouting strategies (Gue *et al.*, 2006; Zhang *et al.*, 2009). We review the previous studies of OPSs focusing on picker blocking. To structure our review of the related studies, each is classified based on their modeling methodology as: 1) analytical models of picker blocking; 2) routing methods with picker blocking; and 3) picker blocking while batching orders.

Gue *et al.*(2006) and Parikh and Meller (2009) introduce analytical models to quantify narrow-aisle and wide-aisle picker blocking, respectively. They determine the relationship between throughput and pick density demonstrating the significance of picker blocking. The results indicate that batch picking strategies in narrow-aisle OPS are advantageous when the pick density is either very low or very high (Gue *et al.*, 2006).

The problem of controlling or reducing picker blocking while routing has rarely been studied. Ratliff and Rosenthal (1983) present a polynomial timed dynamic model to optimally solve the order picking problem when the objective is to minimize travel distance. Hall (1993) surveys heuristics routing for practical purposes, and concludes that S-shape and largest-gap strategies are reasonable strategies for minimizing travel distance. These studies attempt to minimize travel distance, but when an order picking

area has significant traffic, picker blocking may result in additional distance traveled or time penalty; a structured analysis of additional travel distance or time delays is omitted in the literature. Gue *et al.*(2006) discuss practical methods to avoid picker blocking, such as allowing a trailing picker to pass while the leading picker unload collected items, or forcing a blocked picker to exit the current aisle and use an empty aisle to continue to traverse the pick area when significant blocking is expected.

Moreover, some literature indicates that batch picking tends to face less picker blocking. Gue *et al.* (2006) introduce an industry case with less picker blocking when pick density is very high. Ruben and Jacobs (1999) show the relationship between the batching algorithm and the storage policy and indicate this can increase congestion, picker blocking, and delays. The recent literature on batching algorithms ignores picker blocking or considers a single-order picker (Chen and Wu, 2005; De Koster *et al.*, 1999; Gademann and van de Velde, 2005; Gademann *et al.*, 2001; Ho and Tseng, 2006; Hsu *et al.*, 2005; Pan and Liu, 1995; Won and Olafsson, 2005). In Chapter V we discussed our finding that the near-optimal distance-based batching algorithm, RBP, experiences less picker blocking when a sort-while-picking strategy is applied.

3. PROBLEM DEFINITION

3.1 Narrow-aisle order picking system OPS and batch picking

We consider narrow-aisle OPS where pickers circumnavigate one-way aisles to retrieve items from shelves and place them on a cart. When an order picker has no items to retrieve in a particular aisle, the aisle can be skipped to shorten the travel distance if the unidirectional characteristic of aisles can still be maintained. In particular, the order

size is relatively small compared to the cart capacity; thus, consolidating many order retrievals into one trip ("batch picking") is considered to improve order picking throughput. The size of a batch is constrained by the number of orders that will fit on the cart. In other words, a picker carry bins on a cart and places each order in its own bin regardless of the order size. This sortation strategy is referred to as "sort-while-pick". Further, the number of items varies based on the order size, and orders cannot be split over multiple batches.

3.2 Multiple pickers and in-the-aisle picker blocking

In general, multiple pickers gather a set of orders prepared prior to the shift. Further, a picker who completes a trip through the picking area, to gather a particular batch, returns to the original starting position and begins picking a new batch without delay. When multiple pickers work in an OPS, they will encounter congestion while travelling and accessing pick faces. A narrow aisle layout has additional congestion created by the no passing policy (Gue *et al.*, 2006). In a narrow-aisle OPS, two types of picker blocking occur.

First, when two-way traversal of an aisle is possible, if a picker enters an aisle in which another picker is already present and moving towards the entering picker, deadlock arises. To avoid this, the approaching picker can be made to wait before entering. However, this forces the picker to stand idle. One-way traversal route is popular because this type of idleness or deadlock is avoided.

Second, congestion can occur even when pickers move in the same direction. If a trailing picker's next pick-location is occupied by a former picker, the trailing picker is

blocked until the former picker leaves. Gue *et al.*(2006) call this "in-the-aisle picker blocking". Whereas the deadlock in an aisle can be solved by the one-way traversal routing method, there is no simple rule to avoid the in-the-aisle blocking. When multiple aisles are visited, pickers can be re-sequenced at the end of aisles as Gue *et al.*(2006) point out; they observe less in-the-aisle blocking when another picker is allowed to pass in order to improve downstream blocking.

3.3 Performance criteria considering picker blocking

Two performance criteria can be considered for an OPS: total retrieval time and completion time. Total retrieval time maximizes pickers' throughput by reducing their work hours. Completion time is important especially when the completion time of the last order is important because of order commitment times. Either could be used depending on the firm's primary objective. In this chapter, the focus is to *minimize total retrieval time*.

The criterion, minimization of the total retrieval time, can be expressed by the sum of the cart loading (LT) and unloading time (UT), pick time (PT), walk time (WT), and delay time (DT) of all batches. Hence, the following objective is minimized:

Min LT+UT+PT+WT+DT

A trip requires a constant LT and picked-item UT. PT is approximated as the number of picks in a batch times the unit pick time. We ignore the effect of search time, height of shelves, and multiple picks in a pick face (i.e., to pick an item, a picker uses the same amount of time regardless of shelf height and consecutive picks at a same pick face). WT is the total travel distance times the unit walk time. We assume the

acceleration/deceleration time is negligible. DT is the gap between the planned leaving time at a pick face or an aisle entrance and the actual leaving time. When a downstream picker blocks the next pick face of an upstream picker, the upstream picker cannot leave the current location until the next pick face is available.

3.4 Batching models with in-the-aisle picker blocking

As the objective function is increased by the delay time caused by in-the-aisle picker blocking, the formulation of an order batching control model must also reflect the constraints regarding picker blocking. The scheduling literature provides several alternatives to estimate the time blocked. In particular, the in-the-aisle blocking is similar to the permutation flow shop scheduling problem with limited intermediate storage in the scheduling context, which is known to be a strongly NP-hard (nondeterministic polynomial-time hard) problem and is translated into a traveling salesman problem (TSP) (Pinedo, 1995).

The time lost by in-the-aisle picker blocking can be minimized for a given set of batches by optimal sequencing. To improve the benefits of batching, a batching sequencing problem is incorporated into the proposed model. Thus, the format of the new integrated problem *combines* the batch sequencing problem with the traditional batching problem, which we refer to as the batching and sequencing problem (BSP).

4. INDEXED ORDER BATCHING MODEL (IBM)

This section clarifies the BSP model and discusses sequencing issues, treatment of multiple aisles, and consideration of multiple trips for pickers.

4.1 Indexed batching (single aisle and infinite pickers)

To develop the intuition and basis for later models, consider an OPS that has a single aisle and an infinite number of pickers. Fundamentally, the sequencing problem determines a release sequence to obtain minimal delay given by batches. Thus, if the delay is measured and integrated into the objective function of the batching problem, the batching and sequencing problems can be solved simultaneously. We define this formulation as the IBM.

(Abstracted IBM with single aisle and infinite pickers) *Min* LUT+ WT+DT
Subject to
- Batching constraints
- One-way traversal routing constraints
- In-the-aisle picker blocking constraints

In the model, the one-way traversal routing constraints always hold since there is a single aisle. The IBM concept captures the sequencing decision in the in-the-aisle picker blocking constraints and the delay in the objective. The in-the-aisle blocking model developed in Gue *et al.* (2006) can be used here. Moreover, their model gives the same results as the permutation flow shops with finite intermediate storage in Pinedo (1995). Obviously, a permutation flow shop with identical machines and zero intermediate buffer storage is similar to an order picking situation in a narrow aisle. A job (batch) in the permutation flow shop with zero intermediate buffer storage stays at the current machine (pick face) if the next machine (pick face) is busy because of another job (batch). Pinedo calls the phenomenon *blocking*, which is the same as *picker blocking* in order picking.

Gue *et al.*'s model can express the in-the-aisle blocking given a set of batches, a release sequence, and the pickers' available start times. However, their model can only be applied directly for a single aisle with unlimited pickers. If these assumptions are relaxed, additional modeling is necessary to estimate picker blocking. These extensions are described below.

4.2 Aisle-entrance sequencing (multiple aisles and infinite pickers)

First, consider a multiple aisle setting. Thus, when seeking shorter travel distances, some batches skip some aisles (Figure 24) to avoid complete traversal of the facility. The routing alters the aisle entrance sequence, which can be enumerated in three different cases. First, at the first aisle, the release sequence is inherited from the indices of batches. Some batches may skip the first aisle, but it does not change the release sequence. The routing defines the subset of batches that traverse the first aisle. Second, we need to identify the batches entering the second aisle and update their entrance times. Some additional batches may skip the second aisle, but the entrance sequence at the second aisle remains the same as the initial index, because batches skipping the first aisle must also skip the second aisle because one-way traversal of aisles is enforced. After updating the aisle entrance time, picker blocking can be calculated. Third, for the third and higher aisles, the batches entering a particular aisle and their sequence must be identified, and the batches' entrance time calculated. The aisle-entrance sequence at the first aisle no longer holds since reentry occurs from batches that skipped previously aisles. Unlike the update of the aisle entrance time, the aisle-entrance sequence leads to additional constraints and decision variables in the programming problem because the

sequence is allowed to change.

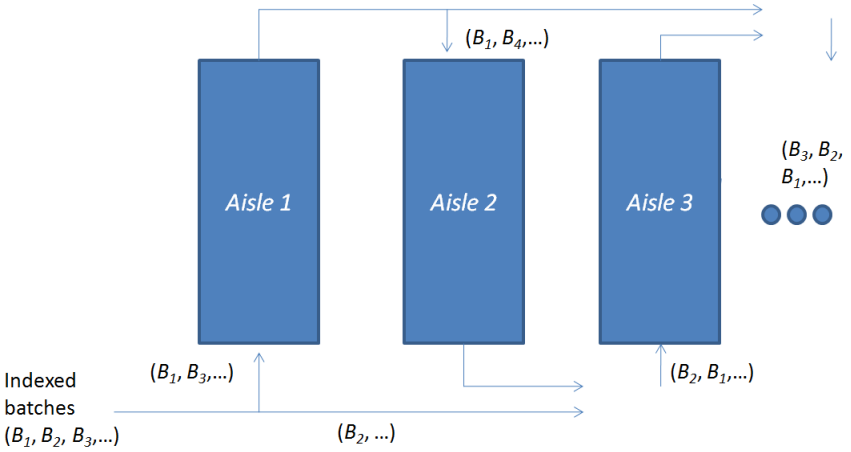


Figure 24. An example of different aisle-entrance orders due to batches skipping aisles (B_i=batch i).

Therefore, we need to re-index batches based on their arrival time at an aisle's entrance. We define this process as the "aisle-entrance sequencing problem." Interestingly, some batches may arrive simultaneously. When this happens, their entrance sequence should be determined by a simple tie-breaking strategy, such as random selection.

(Abstracted IBM with multiple aisles and infinite pickers) *Min* LUT+ WT+DT
Subject to
- Batching constraints
- One-way traversal routing constraints
- In-the-aisle picker blocking constraints
- Aisle-entrance sequencing constraints for 3,…,# aisles

4.3 Completion-time ordering (multiple aisles and finite pickers)

In practice, the number of batches is typically more than the number of pickers (*NP*). In this case, the starting time for the second trip of a picker should be updated

based on his/her previous completion time (Figure 25). The starting time of batch *i* is obtained by the completion time of the batch completed *NP* batches before *i* (*i-NP*). To facilitate this method, the completion-time is sorted in ascending order. We call the related constraints the completion-time ordering constraints.

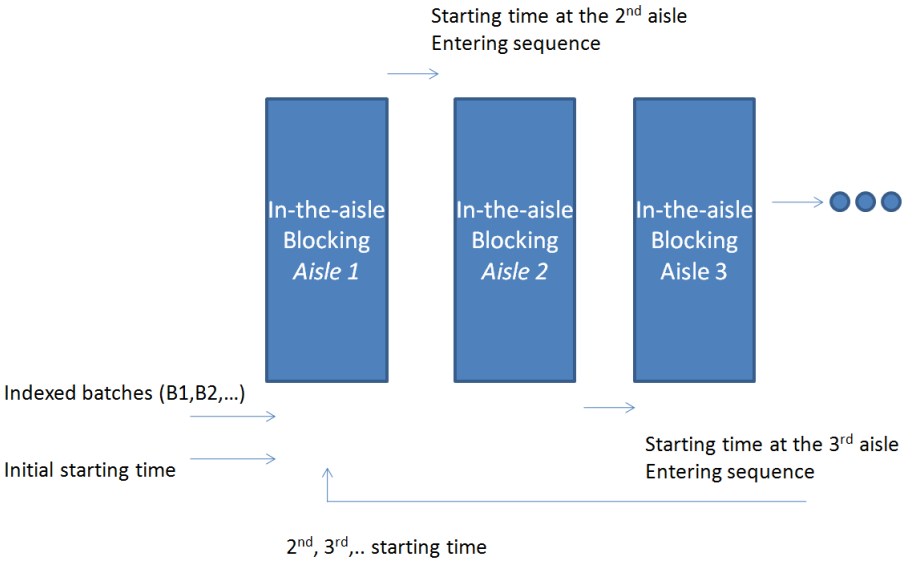


Figure 25. Order picker's retrieval trip starting time.

Returning to the optimization problem characterizing the batching and sequencing problem, the model now requires an additional constraint, the completion-time ordering constraints, due to the impact of multiple trips. Thus, the abstracted IBM becomes the following equation:

(Abstracted IBM with multiple aisles and finite pickers) *Min* LUT+ WT+DT
Subject to
- Batching constraints
- One-way traversal routing constraints
- In-the-aisle picker blocking constraints
- Aisle-entrance sequencing constraints for 3,…,# aisles
- Completion-time ordering constraints for all batches

Herein, the updates of the aisle-entrance sequencing, the in-the-aisle picker blocking, and the completion-time ordering are included in ascending order of time, i.e., a first event is processed first. Below, we give two solutions: an MIP formulation and a next-event advance approach.

5. AN EXACT MIXED-INTEGER PROGRAMMING (MIP) FORMULATION

In this section, we formulate the IBM as a mixed-integer program. We focus on a general formulation in this section, while the executable MIP formulation is shown in Appendix C.

5.1 Parameters and decision variables

We consider the general multiple aisle OPS layout as shown in Figure 26. The OPS has an even number of aisles to allow pickers to traverse the entire picking area without requiring a u-turn or back-tracking. The pick faces are numbered 0 to F^a+1 at every aisle. Pick faces 0 and F^a+1 represent the entrance and the exit of an aisle, respectively. In odd aisles, the entrance is located at the front cross aisle, and for even aisles entry is from the rear cross aisle. It takes time *AE* to travel from the entrance to the first pick face or from the last pick face to an exit. The travel time between neighboring pick faces is *PF*. The walk time from 0 to F^a+1 is equal to $PF*(|F^a|-1)+2*(AE+PF/2)$ =

AH when an aisle is passed through. The cross time between two parallel aisles is *AW*. The L/U station is located in the front of the leftmost aisle.

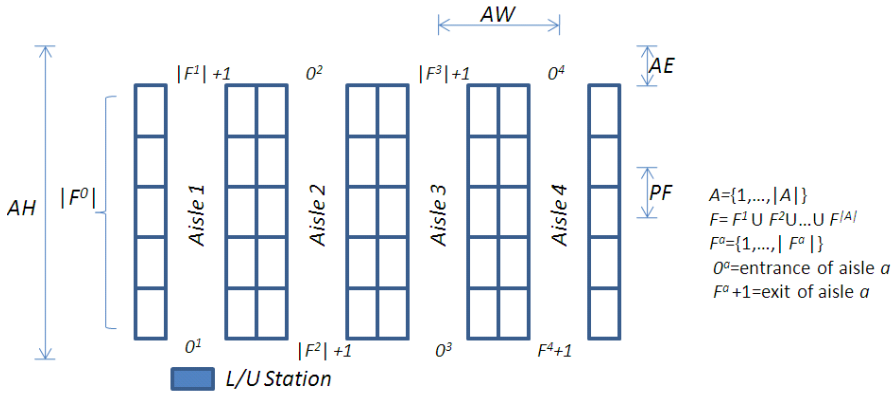


Figure 26. An OPS layout.

NP pickers work in the OPS, where *NP* is assumed to be smaller than the number of batches. The number of batches is not given, although the number of batches must be smaller than the number of orders. Two batch picking strategies—pick-then-sort and sort-while-pick—are considered; the choice of strategy impacts cart capacity. A picker who completes a trip is reassigned to the next available batch and all pickers are available initially.

Several decision variables associated with the IBM procedure must be defined: basically, orders are assigned to batches and to a release sequence through batching variables; each order includes multiple items; and each item is stored in only one pick face. X_{ob} is set to 1 when order *o* is assigned to batch *b*. The batch sequence at the third and later aisles is defined through variables (Y_{ij}^{a}). For all $a = 3,\ldots,|A|$, Y_{ij}^{a} is set to 1 when batch *j* is released in the i^{th} position of the sequence. The starting time for batches

picked on a picker's second or later trip (ST_i) can be captured using ordering variables (Z_{ij}) of the completion-time (CT_i). Similarly, Z_{ij} is set to 1 when batch j is completed in the i^{th} position of the sequence.

The routing is expressed by the aisle-visiting incident vector. Initially, the vector for order o at aisle a is given as OAV_{oa} (this vector can be obtained from items in an order). OAV_{oa} is set to 1 if any item in an order o is stored in aisle a. The route of a batch is determined by (BA_{ba}). If batch b has at least one pick in aisle a, BA_{ba} is set to 1. While evaluating picker blocking, AV_i^a expresses whether the i^{th} released batch enters in aisle a. Additional details follow.

Indices and parameters

F, f	=	the set of pick faces, its index, $f \in F = \{1, \cdots, \lvert F \rvert\}$, 0=L/U station or entrance
A, a, k	=	the set of aisles, and its indices $a, k \in A = \{1, \cdots, \lvert A \rvert\}$
F^a	=	the set of pick faces in aisle $a, F^a = \{1, \cdots, \lvert F^a \rvert\}, \lvert F \rvert = \lvert A \rvert \cdot \lvert F^a \rvert$
B, b, i, j	=	the set of for the batches, and its indices $b, i, j \in B, b$ stands for the initial batch number
O, o	=	the set of orders, and its index $o \in O$
OAV_{oa}	=	1 if order o passes through aisle a (order o has at least one pick in aisle a) 0 otherwise
OP_{of}	=	the number of picks of order o and pick face f
ST_b	=	the starting time of b^{th} batch
PF, PT	=	the walk time to pass one pick face, the pick time to pick an item
NP	=	the number of pickers
AH, AW, AE	=	the time to pass through an aisle, the width between two aisles, aisle enter/exit time

LT, UT = the loading time, the unloading time

τ = the time required for the transition between two batches in a pick face

$L^a_{-1,f}$ = the leaving time at the previous history

Decision variables

X_{ob} = 1 if order *o* is assigned to batch *b* (i.e. *b* is the release sequence at aisle 1)
0 otherwise

Y^a_{ij} = 1 if batch *j* enters aisle *a* at the i^{th} order, $a \in \{3,\ldots,|A|\}$
0 otherwise

Z_{ij} = 1 if batch *j* returns to the unloading station at the i^{th} order
0 otherwise

BV_b = 1 if batch *b* is valid
0 otherwise

BA_{ba} = 1 if batch *b* has at least one pick in aisle *a*
0 otherwise

AV^a_i = 1 if the i^{th} batch has at least one pick in aisle *a*
0 otherwise

NBV = the number of valid batches

BAC_{ba} = the completion time of batch *b* up to aisle *a*

NBA_b = the number of pairs of aisles visited to retrieve batch *b*

RBA_b = the right most aisle visited while retrieving batch *b*

BP_{bf} = the picking time of the i^{th} batch at pick face

P^a_{if}, CP^a_{if} = the pick time of the i^{th} batch at pick face *f* in aisle *a*, and its cumulative pick time

D^a_{if}, CD^a_{if} = the time delayed of the i^{th} batch at pick face *f* in aisle *a*, and its cumulative time delayed

L^a_{if} = the leaving time of the i^{th} batch at pick face *f* in aisle *a*

CW^a_{if} = the cumulative walk time of the i^{th} batch to pick face *f* in aisle *a*

CT_i = the completion time of the batch which has finished at the i^{th} order

$INT1_{ba}, INT2_{ba}$ = non-negative integer variables

5.2 Objective cost

The goal is to minimize loading/unloading time (*LUT*) + total walk time (*WT*) + total time delayed (*DT*) (6.1). *LUT* is proportional to the number of valid batches times the unit loading/unloading time. The travel time of a batch is the sum of the vertical travel times (= 2**NBA*$_b$**AH*) and the horizontal travel times (= 2**RBA*$_b$**AW*). *WT* is the sums of the travel times of all batches. *DT* is obtained by summing the cumulative delay at each aisle of all batches.

$$\text{Min} \quad (LT+UT)\cdot NBV + \sum_{b\in\{1,\cdots,NBV\}}\left(2\cdot NBA_b\cdot AH + 2\cdot RBA_b\cdot AW\right) + \sum_{a\in A}\sum_{b\in\{1,\cdots,NBV\}} CD^{a}_{i\left|F^a\right|} \tag{6.1}$$

5.3 Indexed batching constraints

The basic function of the given algorithm is to partition orders into batches. The actual decision includes the number-of-batches variable (*NBV*), batching variables (X_{ob}), and batch validity (BV_b). An order cannot be separated (6.2), and a batch should not exceed the capacity (6.3). When partitioning the orders, *NBV* should be determined simultaneously. The maximum number of batches is equal to the number of orders. We define a binary variable (BV_b) to represent the validity of a batch. BV_b is obtained from an OR operation among inclusion flags of orders in batch *b* (6.4). To avoid alternative identical solutions regarding batching, we set one additional comparison constraint such that lower-numbered batches are assigned first (6.5). Constraints (6.6) calculate the number of valid batches. From the batching information (X_{ob}), the pick time vector of batches is obtained (6.7).

$$\sum_{b\in B} X_{ob} = 1, \qquad \forall\, o\in O, \tag{6.2}$$

$$\sum_{o \in O} X_{ob} \le CAPA, \qquad \forall\, b \in B, \tag{6.3}$$

$$BV_b = \bigvee_{o \in O} X_{ob} \qquad \forall\, b \in B, \tag{6.4}$$

$$BV_b \ge BV_{b+1} \qquad \forall\ b \in B \setminus \{B\}, \tag{6.5}$$

$$NBV = \sum_{b \in B} BV_b \tag{6.6}$$

$$BP_{bf} = PT \cdot \sum_{o \in O} X_{ob} \cdot OP_{bf}, \qquad \forall\, b \in \{1, \cdots, NBV\}, \forall\, f \in F, \tag{6.7}$$

This set of constraints defines part of an integer programming problem and limits solutions to feasible batching and sequencing decisions.

5.4 One-way traversal routing constraints

The routing decision includes the routing incident variables (BA_{ba}), the number of aisles visited (NAV_b), and the rightmost aisle visited (RBA_b). Initially, OAV_{oa} is set to 1 if aisle a is visited to retrieve order o and 0 otherwise. If aisle a of any order in batch b is set to 1, aisle a should be set to 1 for batch b (BA_{ba}). In other words, BA_{ba} should be equal to or greater than the logical OR operation of OAV_{oa} (the aisle-incident vector) of orders in batch b (6.8). The formulation includes additional constraints to enforce unidirectional travel in aisles through constraints (6.9) and (6.10) for even-numbered aisles, respectively. The return to the front cross-aisle is guaranteed when the total number of visited aisles in a batch is even (6.11). RBA_b is used to calculate the travel distance and becomes the rightmost downstream aisle (6.12).

$$BA_{ba} \ge \bigvee_{o \in O} \left(OAV_{oa} \cdot X_{ob} \right) \qquad \forall\ b \in B, \forall a \in A, \tag{6.8}$$

$$2 \cdot INT1_{ba} + 1 = \sum_{k \in \{1,..,a\}} BA_{bk} \quad \text{if } BA_{ba} = 1 \qquad a = \{1,3,..,|A|-1\}, \forall\, b \in B \tag{6.9}$$

$$2 \cdot INT2_{ba} = \sum_{k \in \{1,..,a\}} BA_{bk} \quad \text{if } BA_{ba} = 1 \qquad a = \{2,4,..,|A|\}, \forall\, b \in B \tag{6.10}$$

$$2 \cdot NBA_b = \sum_{a \in A} BA_{ba}, \qquad \forall\, b \in B, \tag{6.11}$$

$$RBA_b = \underset{a \in A}{MAX}\left(a \cdot BA_{ba}\right), \qquad \forall\, b \in B, \tag{6.12}$$

5.5 In-the-aisle picker blocking constraints

In-the-aisle picker blocking constraints evaluate the blocking delay by the information composed of batches, the start-time of pickers (ST_i), the aisle-completion time of batch b (BAC_{ba}), and the trip-completion time of i^{th} batch (CT_i). The calculation requires the introduction of several intermediate variables: CP_{if}^{a}, CW_{if}^{a}, and CD_{if}^{a} stand for the cumulative pick time, the cumulative walk time, and the cumulative delay time before leaving pick face f in aisle a of batch b.

CW_{if}^{a} is the cumulative walk time when the picker picking batch i reaches pick face f in aisle a. The starting time is obtained from ST_i, CT_i and BAC_{ba}. Constraints (Eq. 5-13) update CW_{if}^{a} at aisle entrances and pick faces. At the loading station (aisle-entrance 1), CW_{if}^{a} is determined using the pickers' available time (ST_i) when the release sequence is smaller than the number of pickers, otherwise, using the completion time of the previous trip (CT_i). The starting time of batch NP+1 is the completion time of the first completed batch because the picker responsible for the first completed batch will be assigned to pick the NP+1st batch. At other aisle-entrances, CW_{if}^{a} is updated by the previous aisle completion time (BAC_{ba}) plus aisle crossing time (AW). Otherwise, CW_{if}^{a} is determined from the previous CW_{if-1}^{a} when i^{th} batch uses aisle a, i.e., $AV_i^a = 1$.

$$CW_{if}^{a}=\begin{cases} LT+ST_i & \text{if } i\le NP,\ f=0, \text{and } a=1 \\ LT+CT_{i-NP} & \text{if } i>NP,\ f=0, \text{and } a=1 \\ \sum_{j\in B} Y_{ij}^{a}\cdot BAC_{j,a-1}+AW & \text{if } f=0 \text{ and } a>1 \\ WT\cdot AV_i^{a}+CW_{i,f-1}^{a} & \text{otherwise} \end{cases} \quad \begin{array}{l} \forall\, i\in\{1,\cdots,NBV\}, \\ \forall\, f\in F^{a}\cup\{0\}, \forall a\in A, \end{array} \tag{6.13}$$

The delay time is expressed as the gap between the planned leaving time and the actual leaving time from a pick face or aisle entrance. An intermediate variable, leaving time (L_{if}^{a}), is introduced to simplify the calculation. This intermediate variable helps to establish the delay time as a function of the picker's leaving time and the pick face available time. Cases 1 through 3 below concern batch *i* passing through aisle *a*. In case 4, batch *i* skips aisle *a*.

Case-1) When pick face *f* is not the last pick face in an aisle. Figure 27 illustrates a timeline of a picker blocking situation in a pick face that is not the final pick face. A picker retrieving batch *i* leaves pick face *f* of aisle *a* at time $L_{if}^{a} = CP_{if}^{a} + CW_{if}^{a} + CD_{if}^{a}$. Herein, CW_{i0}^{a} stands for the arrival time at the aisle entrance. When the picker departs pick face *f*, pick face *f* is accessible by another picker after transition time (γ). When pick face *f* is already occupied, the picker must wait until pick face *f* is released. We describe blocking time as the gap between the pick face ready time ($L_{i-1,f}^{a}+\gamma$) and the planned-arrival time of the trailing picker ($CP_{if}^{a}+CW_{if}^{a}+CD_{if-1}^{a}+PF$). At pick face *f*, a trailing picker can depart pick face *f* at $CP_{if}^{a}+CW_{if}^{a}+CD_{if-1}^{a}$. If the next pick face *f*+1 is available without any picker blocking, the picker can arrive at $CP_{if}^{a}+CW_{if}^{a}+CD_{if-1}^{a}+PF$, where *PF* is the walk time between two neighboring pick faces. However, if the next

pick face f+1 is not available ($L^a_{i-1,f+1} + \gamma > CP^a_{if} + CW^a_{if} + CD^a_{if-1} + PF$), the picker should stay at the current pick face ($D^a_{if} = L^a_{i-1,f+1} + \gamma - (CP^a_{if} + CW^a_{if} + CD^a_{if-1} + PF)$), where $L^a_{i-1,f+1}$ is the departure time at the next pick face $f+1$ of the previous batch $i-1$. The leaving time, L^a_{if} , is updated to $CP^a_{if} + CW^a_{if} + CD^a_{if-1} + D^a_{if} = CP^a_{if} + CW^a_{if} + CD^a_{if}$, recursively.

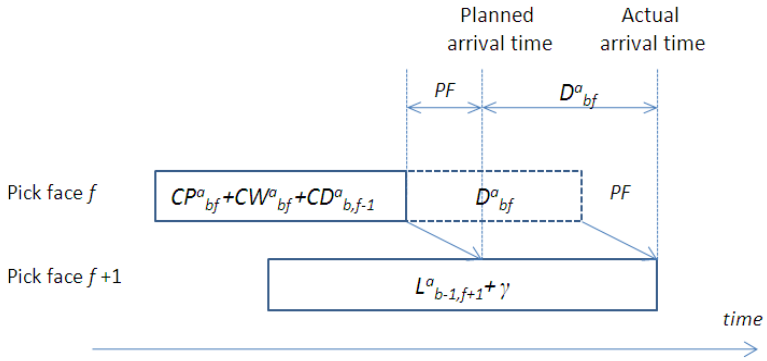


Figure 27. Delay time for batch b at pick face f when a picker is blocked.

Case-2) When pick face *f* is an aisle entrance. When multiple order pickers arrive together at an aisle entrance, or a picker intentionally waits at the aisle entrance (to improve downstream blocking), a waiting delay occurs. This delay is allowed at pick face 0^a. Since there is no picking time and no previous delay, if a delay occurs then D^a_{i0} becomes $L^a_{i-1,1} + \gamma - (CW^a_{i0} + AE)$.

Case-3) When pick face *f* is the last pick face, $|F^a|$. At the last pick face of an aisle, the calculation is unnecessary, because the picker exits an aisle. Thus, we do not consider picker blocking.

Case-4) When aisle *a* is skipped. While retrieving a batch, the picker passes through some aisles and skips others. When a batch skips an aisle, the batch does not

need to be used in calculating delay times in the skipped aisle. We update the leaving time of the batch skipping an aisle (L_{if}^{a}) using the leaving time of the previous batch ($L_{i-1,f}^{a}$). To detect if an aisle is being skipped, we use the routing information AV_i^a , which is a binary variable denoting the usage of aisle *a* by batch *b*. The detail is discussed in Section 5.6.

Constraints (6.14) update the cumulative pick time. Constraints (6.15) calculate the cumulative delay time. Constraints (6.16) and (6.17) calculate the time delayed (D_{if}^{a}) and the leaving time (L_{if}^{a}) at pick face *f* with aisle-incidence vector (AV_i^a). Constraints (6.16) implement the delay calculation discussed in the four cases above. Constraints (6.17) update the leaving time. At an aisle entrance ($f = 0$), L_{if}^{a} is determined by $CW_{if}^{a} + CD_{if}^{a}$ since there is no pick operation. At a pick face ($f>0$), L_{if}^{a} is assigned with $CP_{if}^{a} + CW_{if}^{a} + CD_{if}^{a}$ if batch *i* passes through aisle *a*. When batch *i* skips aisle *a*, L_{if}^{a} is assigned to be equal to $L_{i-1,f}^{a}$.

$$CP_{if}^{a} = P_{if}^{a} + CP_{i,f-1}^{a}, \qquad \forall\, i \in \{1,\cdots,NBV\},\ \forall\, f \in F^{a}, \forall a \in A, \tag{6.14}$$

$$CD_{if}^{a} = D_{if}^{a} + CD_{i,f-1}^{a}, \quad \left(CD_{i0}^{a} = D_{i0}^{a}\right) \qquad \forall\, i \in \{1,\cdots,NBV\},\ f \in F^{a} \cup \{0\}, \forall a \in A, \tag{6.15}$$

$$D_{if}^{a} = \begin{cases} Max\left(L_{i-1,1}^{a} + \tau - CW_{if}^{a} - AE, 0\right) & \text{if } f = 0 \\ Max\left(\begin{matrix} L_{i-1,f+1}^{a} + \tau - CP_{if}^{a} - CW_{if}^{a} \\ - CD_{i,f-1}^{a} - PF, 0 \end{matrix}\right) & \text{if } AV_i^a = 1 \text{ and } f \in F^{a} \setminus \{0, \left|F^{a}\right|\} \\ 0 & \text{otherwise} \end{cases} \qquad \forall\, i \in \{1,\cdots,NBV\},\ \forall\, f \in F^{a} \cup \{0\}, \forall a \in A, \tag{6.16}$$

$$L_{if}^{a} = \begin{cases} CW_{if} + CD_{if} & \text{if } f = 0 \\ CP_{if} + CW_{if} + CD_{if} & \text{if } AV_{i,a(f)} = 1 \text{ and } f > 0 \\ L_{i-1,f}^{a} & \text{otherwise} \end{cases} \quad \forall\, i \in \{1,\cdots,NBV\},\ \forall\, f \in F^{a} \cup \{0\}, \forall a \in A, \tag{6.17}$$

5.6 Aisle-entrance sequencing constraints

We establish the release sequence at aisle *a* in $\{3,...,|A|\}$ as Y_{ij}^{a}. The index *i* defines a sequence and batch *j* is released as the i^{th} batch in a sequence when $Y_{ij}^{a} = 1$. Thus, only one batch can be assigned to each sequence position (6.18). Batch *j* is assigned to only one sequence position (6.19). Constraints (6.23) establish that the first completed batch at the previous aisle enters the current aisle first.

As the release sequence is determined, the related variables are assigned. The pick time vector of batch *i* at pick face *f* in aisle *a* is updated with batch *j*'s pick time (6.20). Additionally, the release sequence in each aisle updates the route information of i^{th} batch (AV_i^a) (6.21) and the batch completion time in aisle (*BAC*) using Y_{ij}^{a} (batch *j* is released at i^{th} time in aisle *a*) (6.22).

$$\sum_{j \in \{1..,NBV\}} Y_{ij}^{a} = 1 \quad \forall\, i \in \{1,\cdots,NBV\},\ \forall a \in \{3,\cdots,|A|\}, \tag{6.18}$$

$$\sum_{i \in \{1..,NBV\}} Y_{ij}^{a} = 1 \quad \forall\, i \in \{1,\cdots,NBV\},\ \forall a \in \{3,\cdots,|A|\}, \tag{6.19}$$

$$P_{if}^{a} = \begin{cases} BP_{if} & \text{if } a \in \{1,2\} \\ \sum_{j \in \{1..,NBV\}} Y_{ij}^{a} \cdot BP_{j,a|F^{a}|+f} & \text{if } a \notin \{1,2\} \end{cases} \quad \forall\, i \in \{1,\cdots,NBV\},\ \forall\, f \in F^{a}, \forall a \in A, \tag{6.20}$$

$$AV_i^a = \begin{cases} BA_{ia} & \text{if } a \in \{1,2\} \\ \sum_{j \in \{1..,NBV\}} Y_{ij}^{a} \cdot BA_{ja} & \text{if } a \notin \{1,2\} \end{cases} \quad \forall\, i \in \{1,\cdots,NBV\}, \forall a \in A, \tag{6.21}$$

$$BAC_{ba} = \begin{cases} CW_{b1} & \text{if } BA_{ba} = 0 \text{ and } a = 1 \\ L^{a}_{b\left|F^{a}\right|} + AE & \text{if } BA_{ba} = 1 \text{ and } a = 1 \\ BAC_{b,a-1} + AW & \text{if } BA_{ba} = 0 \text{ and } a \neq 1 \\ \sum_{i \in \{1..,NBV\}} Y^{a}_{ib} \cdot L^{a}_{i\left|F^{a}\right|} + AE & \text{if } BA_{ba} = 1 \text{ and } a \neq 1 \end{cases} \quad \forall\, i \in \{1,\cdots,NBV\}, \forall a \in A, \qquad (6.22)$$

$$\sum_{j} Y^{a}_{ij} \cdot BAC_{j,a-1} \leq \sum_{j} Y^{a}_{i+1,j} \cdot BAC_{j,a-1} \quad \forall\, i \in \{1,\cdots,NBV\}, \; \forall a \in \{3,\cdots,|A|\}, \qquad (6.23)$$

5.7 Completion-time ordering constraints

The completion time of the i^{th} batch (CT_i) is updated based on the completion time ordering variables of batches (Z_{ij}) and the completion time of the batch at the last aisle. Z_{ij} captures the completed batches such that batch j is the i^{th} batch completed (6.24), (6.25). CT_i shows the completion time of the i^{th} completed batch (the time when it returns to the unloading station) (6.26), where completion time = exit time at the last effective aisle + the return time to the L/U station + unloading time. Constraint (6.27) assures order completion times and the sequences are consistent.

$$\sum_{j \in \{1..,NBV\}} Z_{ij} = 1 \quad \forall\, i \in \{1..,NBV\}, \qquad (6.24)$$

$$\sum_{i \in \{1..,NBV\}} Z_{ij} = 1 \quad \forall\, j \in \{1..,NBV\}, \qquad (6.25)$$

$$CT_i = \sum_{j \in \{1..,NBV\}} \left(BAC_{j,RBA_j} + RBA_j \cdot AW\right) \cdot Z_{ij} + UT \quad \forall\, i \in \{1..,NBV\}, \qquad (6.26)$$

$$CT_i \leq CT_{i+1} \quad \forall\, i \in \{1..,NBV-1\}, \qquad (6.27)$$

The final executable MIP formulation is summarized in Appendix C.

5.8 Validation

From the traditional batching and scheduling models, the requirements necessary to define valid batches and sequences that are sufficiently flexible are defined through the following set of assumptions. Requirements 1 through 4 maintain the integrity of the

order, enforce the capacity of the cart, ensure that routes begin and end at the L/U station, and allow one-way travel only within aisles. While travelling in an aisle, two pickers cannot occupy a pick face at the same time and a picker cannot pass another picker in an aisle (outlined in requirements 5 and 6). When pickers move between aisles, they enter the next aisle they plan to traverse in a first-come, first-served (FCFS) priority ordering:

- Requirement 1 (No split of an order and all order fulfillments). Every order is included in exactly one batch.
- Requirement 2 (Capacity). The number of items in a batch is less than or equal to the maximum batch size.
- Requirement 3 (Complete route). A route must start at and return to the L/U station.
- Requirement 4 (One-way directionality). Each aisle has its own moving direction.
- Requirement 5 (A single picker at a pick face). Only one picker can occupy a pick face.
- Requirement 6 (No-passing in an aisle). Self-explanatory.
- Requirement 7 (FCFS at aisle entrance and LU station). The first picker to arrive starts first at every aisle and LU station.

Requirement 1 is enforced by (6.2) and requirement 2 is enforced by (6.3). Requirements 3 and 4 can be guaranteed when the constraints in (6.9), (6.10), and (6.11) are satisfied. (6.16) restricts a picker from entering a pick location occupied by the former picker; therefore, requirement 5 is enforced. Moreover, the delay time must be

greater than or equal to 0. Thus, a trailing picker cannot pass the former picker (Requirement 6). (6.13) enforces the FCFS sequencing at the LU station and at the beginning of each aisle (Requirement 7).

6. A SIMULATED ANNEALING (SA) ALGORITHM

Scalability is a major problem in order picking. The model above combines two NP-hard problems: the order batching problem and the sequencing problem. To handle large-scale instances, a simulated annealing heuristic procedure is used.

6.1 Simulated annealing procedure

Simulated annealing is widely used in sequencing problems and order batching problems. We employ an algorithm described in Pinedo (1995), which is illustrated in Figure 28. For a batching situation, an indexed batching solution is given as BS_1 and its total retrieval time as Obj(BS_1). The major characteristic is to accept a worse solution (BS) while progressively searching for a better candidate solution of solution BS_i with probability P(BS_i,BS) = e^((Obj(BS_i)-Obj(BS)/ β_i), where β_i is referred to as the cooling parameter or temperature. To update the cooling parameter (β_i), we use a simple function a^i where $0<a<1$, $a \in R$ (see Pinedo (1995) in detail). Thus, the probability to accept an incorrect solution gradually decreases as iteration i cumulatively updates the cooling parameter (β_i) using a, i.e., $\beta_i = a*\beta_{i-1}$ where $i>1$ and $0<a<1$. To generate an initial solution (BS_1), a large-scale order batching algorithm, RBP see Chapter V, is used which produces a near-optimal solution when the objective is to minimize the total retrieval distance. I_{max} is the maximum number of iterations. T is the updated temperature. Section 7.2.2 discusses how to develop a neighboring solution.

Step1. Set $i = 1$ and $T = a$.

Initialization

Obtain an initial feasible solution, BS_1

Set *Imax*

Set the best solution $BS^* = BS_1$

Step 2. Generate a new batch solution BS from BS_i, i.e. BS is the neighboring solution of BS_i.

If Obj(BS^*)<Obj(BS)<Obj(BS_i), set $BS_{i+1} = BS$;

Else If Obj(BS)<Obj(BS^*), set $BS^* = BS_{i+1} = BS$;

Else if Obj(BS)> Obj(BS_i), set $BS_{n+1} = BS$ with a probability of $e^{((\text{Obj}(BSi)-\text{Obj}(BS))/T)}$;

Otherwise, $BS_{i+1} = BS_i$

Step 3. Increase $i = i+ 1$.

Update the temperature $T = T * a$.

If $i = Imax$, then STOP; otherwise, go to Step 2.

Figure 28. A simulated annealing algorithm.

6.2 Picker blocking estimation (Obj(B))

Obj(B) quantifies the blocking time using the mechanism discussed in the previous section. Figure 29 describes the main procedure. In-the-aisle picker blocking follows the mechanism shown in Figure 27. We assume that there are *NP* pickers. *p* is index of pickers. $Status_p$ represents the current status of picker *p*, which includes a batch index when picker *p* has an assigned batch, *IDLE* when the picker is ready for picking or has no assigned batch, and *OFF-DUTY* when the last trip has been completed.

Step1. Set LUT_p, WT_p, DT_p, PT_p = 0.0, and $Status_p$ = IDLE for all pickers (p= 1,…,NP)
b= 1
Step 2. Select picker p of not OFF-DUTY and smallest LUT_p+WT_p+DT_p+PT_p. If tie, randomly choose
if no picker p, go to Step3
Switch ($Status_p$)
Case picker p has an assigned batch
If not the last visiting aisle // aisle-entrance ordering
In-the-aisle picker blocking on the assigned aisle
Update WT_p, DT_p, PT_p // picks, walks, delay at the aisle
$Status_p$ = Next aisle
Else the last visiting aisle // completion-time ordering
In-the-aisle picker blocking on the assigned aisle
Update LUT_p,WT_p, DT_p, PT_p // picks, walks, delay at the aisle
// walks to the L/U station, unloading
$Status_p$ = IDLE
Case picker p has no assigned batch
If $b \leq$ |B|
Assign the next batch // aisle-entrance ordering
Update LUT_p,WT_p // loading, walks to the first visiting aisle
$Status_{pk}$ = Batch b
$B = b$+1
else
$Status_p$ = OFF-DUTY
Step 3. Finish. Return sum of LUT_p,WT_p, DT_p, and PT_p

Figure 29. A picker blocking computation procedure.

7. IMPLEMENTATION AND COMPUTATIONAL RESULTS

This section summarizes the computational implementation and discusses insights from the results. The experiments analyze the impacts on walk time and delay time by the proposed integrated batch creation and sequencing framework compared to other order batching and release approaches. Different order picking strategies and

pick:walk time ratios are considered to explore the robustness of the proposed framework. Sensitivity tests are conducted over various order picking environments to observe the trends in throughput improvement and the computational performance of the proposed framework.

The MIP formulation is implemented using ILOG CPLEX Callable Library C API 11.0.4. The simulated annealing algorithm is programmed using C language as are the data-set generator and the simulation module. The executable files run on Windows Vista (Xeon 2.66 Ghz CPU, 24 GB memory, 32 bit implementation). For the MIP algorithm, we disable both the branch-and-cut option and the heuristic search option to evaluate the exact computational time. To validate the batching results, a discrete-event simulation method (Law and Kelton, 2000) is used, where the simulation clock is advanced in the "next-event time advance" approach. Three throughput performance measurements are reported: the average walk time plus delay time per order (WT+DT), the average retrieval time per order (RT), and the completion time (CT). The objective is the minimization of RT.

7.1 Exact approach

We implement the MIP solution described in Appendix C.1 directly and obtain the exact solution. The exact approach can manage only small problem sizes, which do not account for real-world problems. However, this approach allows us to test the impacts of the combined batching and sequencing problem and its computational improvement when an indexed batching model has been employed.

The profiles in Table 3 are used to generate data. For every parameter setting, we

run 20 instances. The item locations are generated according to the class-based storage policy with A:B:C ratio of 70:20:10 for first aisle: second aisle: remaining aisles, respectively. The term "interval" represents the inter-departure time between two pickers. Below, we also test the "pick-then-sort" method where CAPA determines a batch size. Then, (6.3) is replaced with $\sum_{o \in O} OS_o X_{ob} \leq CAPA$, where OS_o represents the order size.

Table 3. Default order picking and OPS profiles

Order picking operation profile				OPS profile			Order profile		Picker profile		
Strategy	Capacity	Pick time	L/U	#aisle	#pick faces	Width	#orders	Order size	Interval	#pickers	Speed
Sort-while-pick	**4 orders**	**5**	**10**	**4**	**10**	**2**	**16**	**2**	**1**	**4**	**1**
Pick-then-sort	**10 items**	**5**	**10**	**4**	**10**	**2**	**16**	**2**	**1**	**4**	**1**

We consider three different batching and release scenarios. B-then-R generates batches to minimize the total travel distance and releases batches randomly. B-then-S generates batches to minimize the total travel distance and sequences the batches to minimize the total delay time. BSP and IBM consider the release sequence while partitioning orders into batches. BSP does not use the indexed batching method, but rather combines the batching problem and the sequencing problem in a single model.

Table 4 illustrates the computational results. The table includes columns LT (loading and unload time), WT (walk time), DT (time blocked), PT (pick time), RT (retrieval time), CT (completion time), CPU (average run time in seconds), CPUmax (longest run time), and CPUmin (shortest run time). We note three important observations. First, the combined batching and sequencing approach dominates the other approaches. The BSP and IBM approaches show, on average, 13.2% retrieval time reduction compared to the B-then-R approach, whereas the B-then-S approach improves

the throughput on average about 8.0% with the same storage strategy. However, the CT lengthens (inevitable, since we use a small problem size). This issue will be revisited in the next section with a large problem size. Second, IBM dominates the BSP method when comparing the computational time. Third, despite the small problem size and an optimal IBM approach, we do not find a no-blocking result in an optimal model.

Table 4. Experimental results of the exact approach

Strategies	Scenarios	LT	WT	DT	PT	RT	CT	CPU	CPUmin	CPUmax
Sort-	B-then-R	20.0	33.9	16.1	37.9	107.9	130.1	0.29	0.23	0.47
while-	B-then-S	20.0	33.9	8.0	37.9	99.8	109.4	0.27	0.22	0.34
pick	BSP	20.0	34.1	1.6	37.9	93.6	122.2	1883.23	128.53	8507.01
	IBM	20.0	34.1	1.6	37.9	93.6	121.2	142.04	35.72	632.30
Pick-	B-then-R	20.0	35.1	16.6	44.3	115.9	141.6	0.26	0.19	0.41
then-	B-then-S	20.0	35.1	6.8	44.3	106.1	111.8	15.26	68.00	119.20
sort	BSP	20.0	35.1	1.2	44.3	100.5	125.1	512.36	63.10	2679.03
	IBM	20.0	35.1	1.2	44.3	100.5	125.8	63.38	13.82	180.72

7.2 Simulated annealing approach for large-size applications

7.2.1 A mail order company example

A mail order company warehouse operation is analyzed as an example of a large-scale order picking profile as described in Petersen (2000). The order picking environment, e.g., the number of aisles, the pick:walk time ratio, the number of pickers, the number of orders, etc., also derives from Petersen (2000); however, Petersen does not vary the pick to walk time ratios. To determine the ratios, Gue *et al.* (2006)'s recommendation of a ratio of 20:1 or smaller is used. Most academic studies have used 5:1~10:1 (Gong and De Koster, 2008; Gue *et al.*, 2006; Petersen, 2000). Thus, values on the range 2:1~20:1 were used in experimentation. Reported below are the two most commonly used ratios of 5:1 and 10:1. For every parameter setting, we test 20 instances.

The picking environment is summarized in Table 5.

Table 5. Configuration of an OPS (modified from Petersen example (Petersen, 2000))

Profiles	Values
Pick:walk time ratio	2:1, 5:1, 10:1 , 20:1
Number of aisles	10, 20, 30
Walk time	PF = 1 seconds/pick face, AE = 0.5 second, AW = 2 seconds
Number of pick faces / aisle	20 pick faces
Pick time	2, 5, 10, 20 seconds
Number of pickers	8, 16, 24 (starting interval = 1.0 seconds)
Cycle length	1 hour
Number of orders (per cycle)	360, 720, 1080, 1440
Loading / Unloading time	Each 60 seconds
Order size	2.02 (p(1) = 0.5/0.95, p(n)=(1/2*(n-1)-1/2*n)/(0.95) when n=2,…,10, and p(n) = 0 otherwise.), Unif(1,3), Uniform(3,9), Uniform(5,15)
SKU	1
P/D location	Center of the leftmost aisle
ABC class rule	Demand portion and aisle size 70%:20%:10% = 2:2:6, 50:30:20, Random
Capacity	10 orders, 30 items
Order picking strategy	Sort-while-pick, pick-then-sort

In discussing the performance of the algorithms, we use the following notation throughout the remainder of this section.

WT+DT: the average total walk time (WT) plus total time blocked (DT) per order

RT: the average retrieval time per order

CT: the completion time of the last completed batch

LB: the linear relaxation model of RBP

IBM_{sa}: the indexed batching procedure with simulation annealing, where this study uses a = 0.8 after a preliminary experiment

Obj: the objective value of an algorithm

Red : reduction ratio by an algorithm compared to the RBP + random release method expressed as a percentage (= (an objective function value of the RBP + random release method – the objective function value of an algorithm)/(an objective function value of the RBP + random release method) %)

LU gap: gap between an objective function value and the LB objective function

value expressed as a percentage (= (an objective function value – the LB objective function value)/(the LB objective function value) %)

CPU: the run-time in seconds

7.2.2 Neighborhood search

The method for defining a neighborhood in a simulated annealing procedure is critical to effective implementation (Pinedo, 1995). Four methods that can be used to define the neighborhood in which to search were investigated. In the first method, NB1, a general two-exchange method is employed where a pair of orders is exchanged. We randomly pick two batches (b_1, b_2) and two orders (o_1, o_2) from each batch. The new neighborhood becomes b_1 with o_2 and b_2 with o_1. Next, we develop three more neighborhood methods. The method NB2 switches b_1 with b_2, changing the sequence of batches to be picked. The orders in each batch do not change. NB3 and NB4 set an acceptance condition on NB1. In NB3, b_2 is selected among batches having the same route. In NB4, the new neighborhood must keep the current travel distance.

NB1. batches and orders change

NB2. batches change sequence

NB3. batches and orders change if two batches have the same route

NB4. batches and orders change if new batches have at most the same distance

Alternative neighborhood definitions are compared in Table 6 relative to different pick:walk time ratios. NB1 and NB4 are dominant. Specifically, when the congestion is light, NB4 is slightly better than NB1. Since the initial solution guarantees a near-optimal travel distance for the picker, the approach to search a neighbor in order to reduce the blocking time is effective. In contrast, when heavy congestion exists, NB1 identifies better solutions. NB1 searches a much larger solution space than NB4, because

NB1 considers both the distance reduction and the delay reduction.

Table 6. Comparison of neighborhood rules in simulated annealing approach

Rules	5:1		10:1	
	WT+DT	CPU	WT+DT	CPU
NB1	7.79	70.46	8.86	84.42
NB2	8.43	323.87	11.17	482.78
NB3	7.87	54.55	9.74	53.14
NB4	7.62	73.67	9.21	77.68

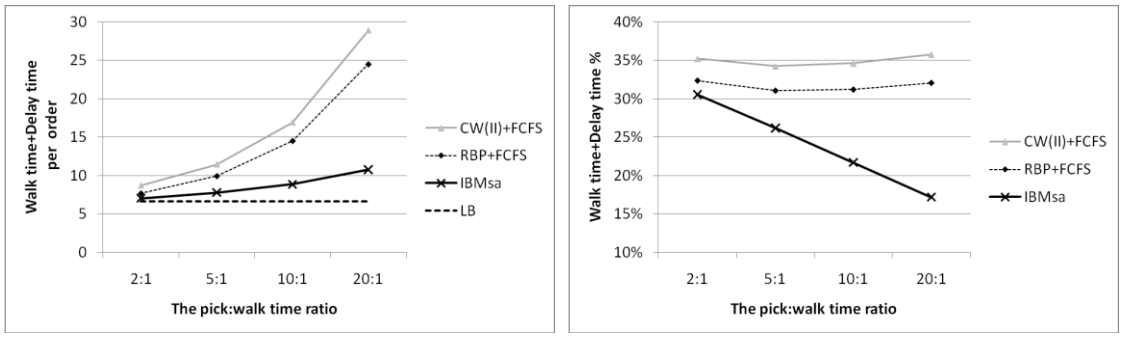
7.2.3 Comparison to available algorithms

Table 7 and Figure 30 compare the IBM_{sa} method to other available batching methods. The FCFS method groups orders into batches sequentially, and releases the batches as they arrive. The seed algorithm is one of the most common batching methods. The best seed algorithm in De Koster *et al.* (1999) is reported below. The CW II approach, a variation of the Clarke and Wright algorithm (1964) appearing in De Koster *et al.* (1999), is also considered. The table includes the performance of RBP. The LP relaxation described in Chapter IV is used to obtain a lower bound (LB). Seed, CW II, and RBP minimize only the travel distance and do not handle the release sequence. Thus, the grouped batches are released in a FCFS manner.

Table 7 and Figure 30 (a) show how IBM dominates the other methods based on the WT+DT criteria. The run time is less than 2 minutes. Specifically, the proposed IBM achieves a 2.5 to 18% reduction in the total retrieval time compared to the near-optimal distance-only approach, RBP, as depicted in Figure 30 (b).

Table 7. Comparison of WT+DT per order

	2:1		5:1		10:1		20:1	
	WT+DT	CPU	WT+DT	CPU	WT+DT	CPU	WT+DT	CPU
FCFS + Random release	17.69		20.25		25.39		36.84	
Seed algorithm + Random release	11.14	0.03	15.65	0.03	24.00	0.03	42.04	0.03
CW (II) + Random release	8.69	136.93	11.44	135.70	16.92	135.77	28.90	133.78
RBP + Random release	7.65	46.80	9.91	49.03	14.50	48.53	24.51	48.74
IBM_{sa}	7.04	67.34	7.79	81.92	8.86	90.76	10.78	101.32



(a) (b)

Figure 30. Algorithm comparison with different throughput measurements: (a) WT+DT per order; and (b) Walk time+delay time % in the total retrieval time.

7.2.4 Comparison across the number of orders

Table 8 summarizes the results across the number of orders over two batching strategies. Compared to the best distance-based algorithm (RBP), IBM_{sa} experiences approximately 5 % to 14.0% reduction of the total retrieval time. The solution from optimizing the retrieval time also results in 4% to 12% reduction of the completion time (CT). When the pick:walk time ratio is small, the percentage reduction in retrieval time decreases and the percentage gap to the lower bound is small. Both the sort-while-pick strategy and the pick-then-sort strategy show a stable improvement of the total retrieval time. When the problem size is small, IBM_{sa} performs better, because the search space is relatively smaller.

Table 8. Variation of the number of orders over two batching strategies

Pick: walk ratio	Stategy	# orders	RBP+Random release				LB	IBMsa -best								
									RT			CT				
			WT	WT+DT	RT	CT	RT	WT+DT	Obj	Red%	LU gap %	Obj	Red%	LU gap %	Rule	CPU
5:1	Sort	360	7.08	11.37	33.33	944.28	28.88	8.47	30.42	8.7%	5.3%	870.25	7.8%	32.4%	NB1	30.30
	while	720	6.72	9.91	31.89	1608.88	28.62	7.79	29.77	6.7%	4.0%	1504.50	6.5%	16.1%	NB1	69.00
	pick	1080	6.63	9.34	31.39	2305.15	28.61	7.40	29.45	6.2%	2.9%	2170.03	5.9%	11.9%	NB4	138.57
		1440	6.59	9.24	31.29	3008.38	28.57	7.33	29.38	6.1%	2.9%	2827.03	6.0%	9.6%	NB4	159.86
	Pick	360	5.92	11.34	29.41	929.15	23.63	8.36	26.44	10.1%	11.9%	875.33	5.8%	62.3%	NB1	111.21
	then	720	5.69	9.49	27.56	1493.90	23.50	7.25	25.32	8.1%	7.7%	1397.05	6.5%	31.2%	NB1	148.65
	sort	1080	5.70	8.62	26.78	2088.13	23.61	6.97	25.13	6.2%	6.4%	1957.08	6.3%	22.2%	NB1	225.22
		1440	5.61	8.25	26.42	2636.83	23.53	6.87	25.04	5.2%	6.4%	2521.55	4.4%	18.6%	NB4	298.70
10:1	Sort	360	7.08	16.83	48.74	1371.15	38.84	9.49	41.40	15.1%	6.6%	1220.65	11.0%	38.5%	NB1	37.12
	while	720	6.72	14.50	46.47	2351.93	38.60	8.86	40.82	12.1%	5.7%	2098.08	10.8%	20.3%	NB1	82.05
	pick	1080	6.63	13.62	45.72	3362.13	38.66	8.89	40.99	10.3%	6.0%	3027.43	10.0%	15.7%	NB1	154.65
		1440	6.59	13.51	45.60	4390.95	38.61	9.02	41.11	9.8%	6.5%	3973.35	9.5%	14.1%	NB1	180.00
	Pick	360	5.92	17.28	45.31	1369.80	33.59	10.25	38.27	15.5%	14.0%	1237.70	9.6%	62.2%	NB1	107.51
	then	720	5.69	13.86	41.91	2269.18	33.48	8.86	36.91	11.9%	10.2%	1995.40	12.1%	31.8%	NB1	148.53
	sort	1080	5.70	12.42	40.62	3085.15	33.66	8.53	36.73	9.6%	9.1%	2828.80	8.3%	24.1%	NB4	217.20
		1440	5.61	11.80	40.01	3958.68	33.58	8.29	36.50	8.8%	8.7%	3631.93	8.3%	19.9%	NB4	298.96

7.2.5 Other order picking profiles

Note that order picking environments can be more diverse. Table 9 summarizes the results of additional experiments varying the number of aisles, storage policy, number of pickers, and order size. The sort-while-pick strategy is evaluated for the pick:walk time ratio of 5:1.

Storage policy

The storage profile in Table 9 exhibits an interesting result. When the pick:walk time ratio = 10:1, the random storage policy performs best under no sequencing control. After applying the IBM, the class-based approaches perform better based on the total retrieval time criteria. This finding stresses a critical issue concerning the interdependence of the storage policy and the order-size pattern and number of pickers. If the class-based approach is used, the benefits of applying IBM are significant.

The number of pickers

More pickers cause more blocking. The proposed procedure shows an

improvement beyond the other methods investigated. RT reduction and CT reduction are 9.7% and 8.8%, respectively when the pick:walk time ratio = 5:1. With a higher pick:walk time ratio, more blocking occurs and the IBM algorithm shows greater benefits on a percentage basis.

The number of aisles in OPS

In larger OPS, pickers "spread out" in a picking area and there is less picker blocking. Thus, the benefit from IBM diminishes. "-" means that a lower bound solution could not be obtained because the problem size is too large.

Order size

We test three different order sizes. The IBM algorithm shows a robust benefit over all values.

Table 9. The experimental results over diverse order picking environments

Pick:			RBP + Random release				LB	IBMsa - Sort-while-picking policy - best								
walk	Profiles	Values							RT			CT				
ratio			WT	WT+DT	RT	CT	RT	WT+DT	Obj	Red %	LU gap%	Obj	Red %	LU gap%	Rule	CPU
5:1	Default		6.72	9.91	31.89	1608.88	28.62	7.79	29.77	0.07	0.04	1504.50	0.06	0.16	NB1	69.00
	Storage	5:3:2	7.69	9.64	31.62	1603.38	29.54	8.35	30.33	0.04	0.03	1532.53	0.04	0.15	NB4	153.76
		random	9.24	10.40	32.38	1624.33	31.04	9.68	31.66	0.02	0.02	1595.20	0.02	0.14	NB4	203.89
	#pickers	8	6.72	8.16	30.14	2832.10	28.62	6.93	28.91	0.04	0.01	2723.70	0.04	0.06	NB4	28.91
		24	6.72	11.87	33.85	1221.53	28.62	8.59	30.57	0.10	0.07	1114.35	0.09	0.28	NB1	87.37
	#aisles	20	10.14	11.77	33.77	1712.80	-	10.78	32.78	-	-	1664.03	-	-	NB4	444.29
		30	14.07	15.19	37.23	1850.35	-	14.41	36.46	-	-	1805.65	-	-	NB4	654.93
	order	U(1,3)	6.98	9.34	31.32	1590.30	28.87	7.56	29.53	0.06	0.02	1498.95	0.06	0.15	NB4	83.45
	size	U(3,9)	11.13	17.30	59.25	2942.38	52.98	12.88	54.83	0.07	0.03	2757.43	0.06	0.15	NB4	478.31
		U(5,15)	13.41	22.60	84.64	4194.13	75.34	16.22	78.26	0.08	0.04	3885.93	0.07	0.14	NB4	812.01
10:1	Default	0	6.72	14.50	46.47	2351.93	38.84	8.86	40.82	0.12	0.05	2098.08	0.11	1.38	NB1	82.05
	Storage	5:3:2	7.69	12.42	44.38	2269.83	39.52	9.26	41.23	0.07	0.04	2110.55	0.07	0.18	NB4	168.26
		random	9.24	12.09	44.05	2258.65	41.02	10.46	42.42	0.04	0.03	2166.98	0.04	0.17	NB4	213.22
	#pickers	8	6.72	10.35	42.31	3983.85	38.84	7.24	39.20	0.07	0.01	3703.53	0.07	0.06	NB4	64.74
		24	6.72	19.10	51.07	1850.28	38.84	10.61	42.57	0.17	0.10	1546.25	0.16	0.32	NB1	108.56
	#aisles	20	10.14	14.28	46.26	2367.10	-	11.79	43.77	-	-	2249.88	-	-	NB4	465.15
		30	14.07	17.08	49.11	2454.68	-	15.21	47.24	-	-	2360.50	-	-	NB4	677.16
	order	U(1,3)	6.98	12.90	44.86	2289.23	38.85	8.29	40.25	0.10	0.04	2042.75	0.11	0.16	NB4	120.33
	size	U(3,9)	11.13	24.82	96.72	4792.23	82.93	14.82	86.72	0.10	0.05	4360.45	0.09	0.17	NB4	1104.61
		U(5,15)	13.41	32.92	144.99	7165.48	125.37	19.26	131.34	0.09	0.05	6513.75	0.09	0.15	NB4	1772.90

7.2.6 Side effects in a large-scale application

The control of picker blocking minimizes both the RT per order and the

completion time. Table 10 includes additional analysis regarding the average and the standard deviation of inter-arrival times between pickers. We collect the inter-arrival time between pickers at the LU station and the 2nd aisle. With the proposed procedure, the inter-arrival time becomes smaller and less variable. The smaller variance may indicate that the pickers are more evenly spaced using the IBM method.

Table 10. Comparison of inter-completion time (the number of orders=2160, Imax=20000)

		5:1						10:1					
	Sequence			LU station		2nd aisle				LU station		2nd aisle	
#pickers	method	RT	CT	Avg	Std	Avg	Std	RT	CT	Avg	Std	Avg	Std
8 pickers	RBP+Rand	29.76	8165.88	36.87	33.71	47.43	47.54	41.86	11494.53	52.07	49.07	66.67	71.61
	IBMsa	28.69	7868.70	35.49	28.82	45.63	43.37	38.94	10696.53	48.39	40.63	61.83	64.65
16pickers	RBP+Rand	31.09	4376.40	19.24	19.09	24.36	28.43	45.26	6386.68	28.32	29.35	35.56	44.64
	IBMsa	29.18	4108.70	18.02	16.90	22.72	26.59	40.59	5688.20	25.09	23.42	31.17	32.20
24pickers	RBP+Rand	32.55	3148.18	13.53	14.60	16.87	21.08	48.80	4729.53	20.61	23.10	25.46	33.32
	IBMsa	29.95	2883.88	12.32	11.99	15.26	17.33	41.94	4010.88	17.22	16.79	21.21	23.01

8. CONCLUSION AND FURTHER STUDIES

This chapter presented: 1) the framework to optimize the order picking operation in a circumnavigational order picking system, where both travel distance and time blocked should be assessed; 2) the indexed order batching model (IBM) combining the order batching problem and the batch sequencing problem; and 3) a simulated annealing heuristic procedure to allow analysis of realistic problem sizes. The narrow-aisle structure was exploited in developing the framework, the algorithm, and the procedure. Experimental results showed that consideration for blocking in an integrated batching and sequencing approach can have substantial benefits on performance criteria such as total retrieval time or completion time.

This chapter has taken an initial step towards controlling congestion in a DC

facility. Specifically, we focused on the narrow-aisle order picking system, which is an attractive OPS layout due to its cost merit from the standpoint of the DC design. The proposed order picking operation procedure requires a reevaluation of some previous research findings. For example, Ruben and Jacobs (1999) recognize the possibility of productivity loss due to congestion under a class-based storage policy, which tends to increase pick-density to shorten the travel distance. However, our experimental results over the variation of the storage policy showed that if appropriate batching and sequencing procedures, such as IBM, are implemented the congestion in a class-based policy can be mitigated. IBM can also play a vital role in minimizing or preventing picker utilization from dropping as the number of pickers increases. According to Gue *et al.* (2006), the picker utilization drops as more pickers are staffed in an order picking system. Thus, it is clear that under IBM some order picking system design rules relevant to picker blocking should be reconsidered.

We suggest that our research be expanded to consider dynamic controls and to explicitly account for other idle factors. First, to handle real-world problems, more dynamic situations should be considered, for example, picking environments that encounter cart breakdowns, search failures, and order changes. IBM requires new planning when any of these difficulties are present. Second, while this study only considers picker blocking, some order picking strategies encounter different idle factors, such as hand-off delay in bucket brigade systems (Koo, 2009), which is a topic we will address in the next chapter.

CHAPTER VII

ANALYSIS AND CONTROL OF PICKER BLOCKING IN A BUCKET BRIGADE ORDER PICKING SYSTEM

Bucket brigades is an operation mode for order picking systems, which is characterized by its self-balancing nature and high pick rates (Bartholdi and Eisenstein, 1996a). However, due to variability and uncertainty of the pick locations within a particular order or batch, picker blocking can cause productivity losses. Furthermore, the hand-off operation, which involves transiting partially-picked orders or batches from upstream pickers to downstream pickers, can result in delays for the downstream pickers. This chapter examines the significance of picker blocking and hand-off delay in bucket brigade order picking and identifies the relevant analytical models, highlighting the issues of blocking and hand-off delay through simulation studies. Our analytical results identify several conditions for high order picking throughput, such as batch picking, stable picking performance, and intermediate hand-off. A complete control procedure for dynamic order picking is provided that mitigates both picker blocking and hand-off delay. The proposed framework experiences 7 to 12% improvement of utilization across diverse order picking situations when five pickers pick on average 20 items per tote.

1. INTRODUCTION

1.1 Bucket brigade order picking

A bucket brigade operational policy is attractive because the workload balancing

characteristic that allows dynamic reassignment of zones increases productivity with minimal managerial or planning requirements. In the warehousing industry, the order picking operation consists of retrieving customer orders from storage. To increase throughput, multiple orders are often grouped in a batch for more efficient picking operations.[6] The method by which batches are assigned to pickers can have a significant impact on picking performance. The bucket brigade concept used in general assembly-line operations can be applied to order picking to achieve valued properties, such as the self-balancing characteristic and minimum work-in-process (WIP) (Bartholdi and Eisenstein, 1996a; Bartholdi and Eisenstein, 1996b). In practice, a bucket brigade order picking strategy is often used with flow-rack shelving (Figure 31) in high throughput warehouse environments. In this study, the combination of flow-rack shelving and the bucket brigade strategy discussed in Bartholdi and Eisenstein (1996a) is referred to as a bucket brigade order picking system (OPS).

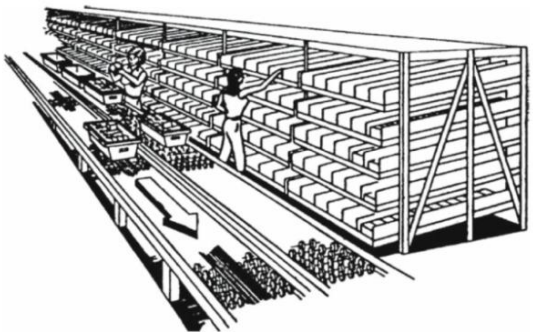


Figure 31. A flow-rack OPS (Bartholdi and Eisenstein, 1996a).

The bucket brigade OPS is characterized by limited WIP, high picking

[6] We discuss order pickers gathering batches. However, if batching is not used this would imply one order per batch.

throughput, high space utilization, and the self-balancing property (Bartholdi and Eisenstein, 1996a). However, as pick requirements are random over pick locations, pickers often encounter blocking when the downstream picker is busy. In addition, pickers may stand idle when the hand-off process is not synchronized.

In a bucket brigade OPS, multiple pickers help to pick a single batch. Pickers are sequenced from upstream to downstream, and the sequence is maintained throughout. Each batch is picked to a tote, and the tote is passed from one picker to the next traversing the aisle. Pickers collect items at related pick faces in sequence. A picker picks an item and places it in the tote assigned to a particular batch. The picker then moves to the next pick face to continue processing the batch if there is no picker at the next pick face. The upstream picker hands off the current tote upon meeting a downstream picker who has no assigned tote. The picker most upstream (the first picker) retrieves a new batch and tote from a loading station and begins picking at the first pick face. The last picker releases the completed batch to the unloading station. A work area for a picker is not predetermined and is dynamically resized through the pick-and-pass process. Thus, this strategy eliminates the need for work zone load balancing, which can be complicated and difficult (Bartholdi and Eisenstein, 1996a).

1.2 Performance under picker blocking and hand-off delay

A bucket brigade OPS does not allow pickers to pass due to the higher space utilization (Bartholdi and Eisenstein, 1996a). Not allowing pickers to pass one another can cause a delay in two ways. First, an upstream picker attempts to move forward to the next pick face that is occupied by a busy downstream picker as shown in Figure 32 (a).

In this situation the upstream picker cannot hand-off the current batch to the downstream picker because the downstream picker is currently executing a retrieval task. The upstream picker also cannot pass over the downstream picker, because passing is not allowed (*picker blocking*). Second, delay can occur when the downstream picker moves upstream to take a hand-off from an upstream picker. If the upstream picker is picking when the downstream picker encounters the upstream picker, the downstream picker must wait until the upstream picker completes the pick. This is termed *hand-off delay* as shown in Figure 32 (b).

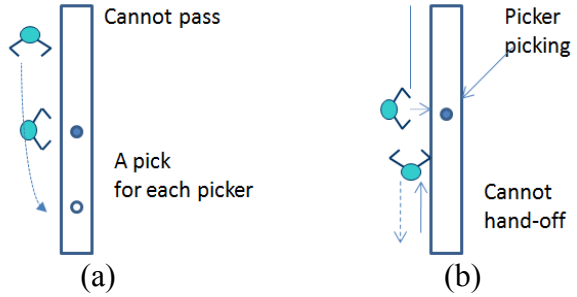


Figure 32. Delay situations in bucket brigade order picking: (a) picker blocking; and (b) hand-off delay.

Performance regarding picker blocking and hand-off delay in bucket brigade OPS is not well understood. In order to achieve the highest throughput, an individual order picker's region of operation within the aisle should stabilize so that the picker can become familiar with the set of items and their location within the region (Lim and Yang, 2009). In diverse bucket brigade situations researchers (Armbruster and Gel, 2006; Bartholdi and Eisenstein, 1996b; Bartholdi and Eisenstein, 2005; Bartholdi *et al.*, 2001) have identified operation rules or conditions that lead to stability. However, picker blocking and hand-off delay can impact picker utilization (Bartholdi and Eisenstein,

1996b) and this issue has received little attention in the literature to date. Only Koo (2009) investigates the productivity loss due to picker blocking and hand-off delays in a bucket brigade OPS using simulation under the assumption picker capability is identical. The throughput loss is 26.1% of the total working time, with 15.6% loss due to picker blocking and 10.5% loss due to hand-off delay. To our knowledge, there is no analytical model on picker blocking and hand-off delay in bucket brigade OPS which can help engineers develop more effective operational strategies.

1.3 Our scope and goals

Order picking throughput is often measured by the ratio of time spent to pick to time spent at a stop. Gue *et al.* (2006) introduced the throughput model for a narrow-aisle order picking system with k pickers. We generalize Gue *et al.*'s result for a bucket brigade OPS as described in Equation (7.1). When pickers are blocked with a fraction of the time, $b(k)$, where $0 \le b(k) \le 1$, and a hand-off takes $E[HO]$, where $0 \le E[HO] \le$ maximal pick time at a pick face, the throughput is:

$$\lambda(k) = k \cdot \left[\frac{E[pt]}{E[pt]t_p + t_w + (k-1)/n \cdot E[HO]} \right] (1 - b(k)) \tag{7.1}$$

where $E[pt]$ stands for the expected number of picks at a stop and n is the number of pick faces in bucket brigade OPS. The time to pick (t_p) represents the average time the picker is stopped and includes the time spent picking items. The time to walk (t_w) indicates the average time to walk past a pick face (location).

We assume that pickers perform identically, which is persuasive due to simplicity of order picking, the relatively easy learning curve in order picking, and the

use of technology. In our model, walk speed is not instantaneous for both forward and backward directions. Items in orders are randomly located in n pick faces and the number of pickers (k) is relatively small compared to n. The time to load and unload orders at the beginning and end of the aisle is negligible. Importantly, passing is not allowed in this high density bucket brigade operation.

We develop analytical models for picker blocking and hand-off delays in a bucket brigade OPS, where no correlation between two delays is assumed. We conduct a simulation study to clarify the source of delays in diverse situations. The analytical and simulation models allow for the size of delays to be quantified; however, a primary purpose of our examination is to assist operational decision-making. A control model and relevant algorithms are proposed to reduce the delays.

The chapter is organized as follows. Section 2 reviews the relevant order picking literature and identifies new opportunities. In Section 3, we introduce analytical models and control methods for picker blocking. Section 4 focuses on an analytical model of hand-off delay and details the proposed control policy for reducing hand-off delay. Section 5 describes a simulation study analyzing picker blocking and hand-off delay and summarizes the experimental results. Section 6 concludes this chapter.

2. LITERATURE REVIEW

Bucket brigade models are typically characterized by work content model (uniform or exponential), walk speed assumptions (finite or infinite speed in forward and backward walks), and pickers' velocity or capability (identical or non-identical). Bucket brigade was originally proposed for the manufacturing setting, thus descriptions of this

work have been adapted for an order picking setting.

2.1 Picker blocking and hand-off delay in bucket brigades

Bartholdi and Eisenstein (1996b) introduce the bucket brigade management method for manufacturing settings. Their three assumptions are: pickers travel with instantaneous walk speed (including backward walk speed), a picker's capability is distinct and not identical, and workloads are uniformly and randomly distributed. Their model considers non-identical capability and utilizes the capability difference to reduce blocking. The highest throughput is obtained when pickers are sequenced with the slowest picker in the location most upstream and the fastest picker in the location most downstream. Picker blocking can be minimized when there are large capability differences among pickers. The authors also suggest that hand-off delay can be reduced through practice.

Bartholdi and Eisenstein (1996a) present the bucket brigade for order picking and describe the productivity improvements through a physical implementation. In particular, the authors emphasize that bucket brigades can achieve both high space utilization and high picker utilization. However, since higher space utilization makes passing difficult, they recommend the bucket brigade for high-volume, limited space picking operations over the more traditional zone picking strategy. Further, the authors suggest another way to reduce picker blocking is cooperation between neighboring pickers, where a blocked picker aids a blocking picker with the help of pick-to-light technology. A blocked picker picks items of a blocking picker, which are identified by pick-to-light.

Bartholdi *et al*. (2001) develop a general performance model where the work

load is not uniform over the pick area. They show that bucket brigades is still advantageous and self-balancing despite the fact that pick locations are exponentially distributed. Their assumption states that when walk speed is instantaneous, pickers move rapidly. Thus, hand-offs of all pickers occur simultanesously and sychronously, and hand-off delays drop.

Bartholdi and Eisenstein (2005) analyze an assembly-line where the walk speed is not infinite and the return trip of a picker after handing off his/her workload requires significant time. Under these assumptions hand-off delay affects productivity. They find a considerable loss of productivity by walk-back time and hand-off delay; nonetheless, their practical application demonstrates a stable performance. Specifically, they assume constant hand-off time to identify the operational stability, but do not evaluate the productivity loss due to the hand-off operation. They do not observe the impacts of picker blocking.

Koo (2009) shows that picker blocking and hand-off delay reduce the productivity of the bucket brigade OPS when pickers have the same capability. The author assumes that work load is random, pick time is not deterministic, and walk time is infinite. He constrains each picker's picking area by defining a downstream boundary which he shows improves their productivity. Further, upstream pickers are allowed to leave totes at the boundary location if a downstream picker is not available to take over the tote. Under this set of assumptions Koo derives a closed form calculation for hand-off delay as $(k\text{-}1)*E[\text{pick time}]/2$.

2.2 Issues

Reviewing the available studies, we identify four critical issues:

1) *The impacts of picker blocking on a bucket brigade OPS when pickers have similar picking abilities are not quantified or well understood for realistic assumptions regarding pick and walk times.* Koo (2009) reports the productivity loss, but only considers a simulation study with an exponential pick time and infinite walk time. These two assumptions are not typical of realistic order picking operations.
2) *Available picker blocking mitigation methods are not appropriate for the general configuration described in this dissertation and do not maintain the standard bucket brigade protocol.* Cooperation between pickers (Bartholdi and Eisenstein, 1996a) is not clearly explained by the authors. Its realization would "break" a bucket brigade protocol because a blocked picker cannot assist a blocking picker under the standard bucket brigade protocol. The passing method proposed by Bartholdi and Eisenstein (2005) in the manufacturing setting also is not appropriate in the current order picking configuration because passing requires additional space for both pickers and totes. Moreover, it is not obvious that passing would improve performance in order picking because pickers may waste time passing over another picker. Koo (2009)'s approach violates the basic principle of bucket brigade by assigning WIPs at boundaries. In addition, stacking at boundaries increases WIPs and requires additional space.
3) *The impacts of hand-off delay on a bucket brigade order picking system have not been properly investigated.* The hand-off model by Koo (2009) is incorrect when a variation of pick time is not zero (see Section 5 below). Moreover, his study assumes instantaneous walk times. The impact of walk time on hand-off delay has not been discussed in the literature.
4) *Suggested methods to reduce hand-off delay lack operational details for implementation or are not practical for real settings.* Bartholdi and Eisenstein

(1996a) suggest a smooth hand-off operation; however, there is no description of the operational implementation. In addition, simultaneous and sychronous hand-off (Bartholdi *et al.*, 2001) does not apply when both pick time and walk time are finite.

3. ANALYSIS AND CONTROL OF PICKER BLOCKING

In this section, we develop analytical models of picker blocking and methods to mitigate picker blocking for bucket brigade OPS. Recognizing that both standard multiple-aisle rectangular order picking systems and bucket brigade order picking systems can be characterized using the circular-aisle OPS abstraction, we apply the blocking control model developed in Chapter V to a bucket brigade OPS under the assumption of no passing. Finally, we utilize the control model to demonstrate the reduction that can be achieved.

3.1 Picker blocking in a circular order picking aisle with two pickers

Gue *et al.* (2006) investigate the effects of picker blocking under a no-passing policy, considering only single-pick situations. The circular order picking aisle abstraction is used in developing both analytical models and a simulation study. Table 11, column 1, shows the closed-form expression for percentage of time blocked for two pick to walk time ratios developed in Gue *et al.* (2006). Column 2 presents our results in Chapter V. The analysis is undertaken for a two-picker OPS. Both approaches consider two extreme cases: 1) walk speed is equal to unit pick time per pick face (pick time:walk time = 1:1), and 2) walk speed is infinite (pick time:walk time = 1:0). The results in Table 11 are developed for a rectangular multiple aisle warehouse with cross aisles at the front and back of the picking area. Pickers take a one-way traversal route and passing is

not allowed. At a pick face, a batch includes an item with a probability *p*. Further, *q* denotes 1-*p*, the probability of no item at a pick face. The models of Gue *et al.* (2006) and Chapter V are distinguished by the number of picks per pick location, single vs. multiple. The multiple-pick model can repeat a pick at the same pick face with probability *p*.

Table 11. The percentage of time blocked when two pickers work (*p*=pick density, *n*=the number of pick faces)

Pick:walk time	Single-pick (Gue *et al.*, 2006)	Multiple-picks (see Chapter V)
1:0	$\frac{1-p}{2(1-p)+(n-1)p}$	$\frac{1}{2+(n-1)p}$
1:1	$\frac{pq}{(n-1)(p+1)^2-p^2}$	$\frac{p}{n+2p-1}$

Gue *et al.* (2006) explain that the batch picking strategy can experience less picker blocking when the pick density is either very low or very high. Chapter V and Parikh and Meller (2010) show that the variation in pick density can be as important as the level of pick density in determining the amount of blocking in a circular-aisle OPS. One important observation in Chapter V is that batch picking can reduce picker blocking.

3.2 Picker blocking in bucket brigade order picking

Bucket brigade order picking has a special release mechanism of a new batch and the mechanism impacts the picker blocking model. Thus, first, the release mechanism of a new batch is explained. Second, picker blocking will be discussed. Note that in this study we show the equivalence of the picker blocking models of the bucket brigade order picking and the circular-aisle abstraction under specific situations, instead of a

direct development of the picker blocking model of bucket brigade order picking.

Figure 33 describes a series of hand-offs after completion of a batch. k pickers are sequenced from the loading station to the unloading station in a decreasing sequence of k,k-1,…, 2,1. When a batch (denote this batch i^{th} batch) is finished by the picker most downstream (picker 1), a new batch must enter the system. Picker 1 becomes idle and moves backward to take over the batch of picker 2 who is moving forward with the i+1st batch. Obviously, the hand-off occurs when they meet. Picker 2 changes direction (backward towards the loading station) to take a new batch from a picker further upstream (i.e., picker 3), when he/she meets an upstream picker he/she takes over i+2nd batch, and then turns and continues picking in a forward direction. Finally, the picker most upstream (picker k) arrives at the loading station to take over i+k^{th} batch, and his/her arrival time at the loading station becomes the starting time of a new batch (i.e., i+k^{th} batch). The difference between the completion time of the i^{th} batch and the starting time of the i+k^{th} batch, which is a batch paired to the i^{th} released batch, equals the sum of backward walks and the hand-off delay occurring after completion of the i^{th} batch.

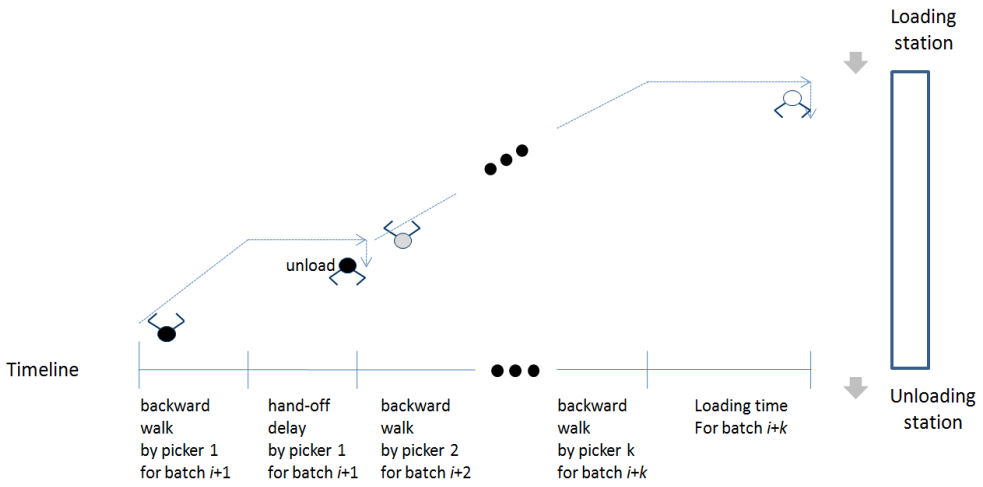


Figure 33. A description of chain reaction after completion of batch *i* to release a new batch *i*+*k*.

Assume that there is no hand-off delay and backward walk speed (empty travel walking speed) is instantaneous similar to Bartholdi and Eisenstein (1996a). In addition, *k* pickers have identical pick performance and walk speed as we assumed in Section 1. Interestingly, with infinite backward walk speed and no hand-off delays, the circular-aisle abstraction of the traversal routing rectangular picking system can be used to characterize a bucket brigade OPS in terms of picker blocking. Further, the same picker blocking model can be used for both analyses.

The equivalence can be easily shown by replacing “pickers” with “batches”. By definition, picker blocking occurs while pickers repeat picking, walking, and blocking, and the picking locations and durations are determined by batches. Thus, without loss of generality, the picker blocking mechanism can be derived from the batches. In bucket brigade order picking, picker blocking occurs when an upstream batch has no item to be picked, but a downstream batch has some picks at the next pick face and holds the next

pick face. Then, the upstream batch may stay at the current pick face, which causes a delay and becomes a picker blocking situation. A more rigorous proof follows.

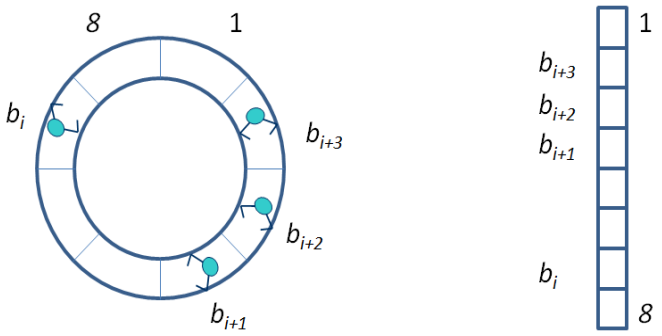
Theorem 4. When the backward walk time is instantaneous and the hand-off time is zero, the picker blocking model of bucket brigade order picking is equivalent to the picker blocking model of the circular-aisle abstraction.

Proof.

When the batch most downstream is completed, it disappears from the system, other batches in the system are handed off to the next pickers, and a new batch is released. The completion, backward walks, and hand-offs occur instantaneously and result in the release of a new batch. This proof shows that: order picking mechanisms of two models (bucket brigade order picking and circular-aisle abstraction) are equivalent until a batch is completed; the completion of a batch does not impact any locations and times of current batches; and the release of a new batch has the same locations, time, and batch.

1) Before completion of the batch most downstream

Without loss of generality, before completion of a batch, two models follow the same procedure. For example, consider batches i, i+1, i+2, and i+3 as depicted in Figure 34. Figure 34 (a) is a circular-aisle abstraction, and Figure 34 (b) is a bucket brigade order picking situation. The moving directions and batches are identical. Thus, until batch i (b_i) is completed, the two systems face the same situations of picker blocking.



(a) (b)

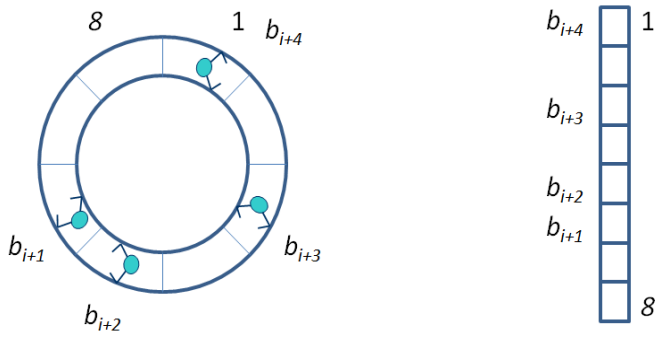
Figure 34. A normal situation example. In both models, four pickers process four batches. Two pickers (picker 3 and 4) may have a chance of blocking depending on items in batches *i*+2 and *i*+3 (the number of pick faces = 8, the number of pickers = 4): (a) a circular-aisle abstraction; and (b) a bucket brigade OPS.

2) Completion of the batch most downstream and occurrence of hand-off

Since batch *i* has been completed, the chain reaction discussed in Figure 33 arises. Due to the infinite backward walk speed and the zero hand-off delay, all batches will be handed off at the same time. Batch *i*+*k* enters the system (i.e., the first pick face) and its release time is identical to the completion time of batch *i*. The picker assignments of batches *i*+1, *i*+2, …, *i*+*k*-1 are changed from *2,3,..,k* to *1,…,k-1*. Picker *k* captures batch *i*+*k*. During this shift, there is no blocking. Then, recursively, case 1) repeats. In the circular-aisle abstraction, the release location of a new batch is the first pick face and the release time of a new batch is the completion time of k^{th} before. Thus, the two systems release a new batch into the same location at the same time when the backward walk speed is infinite and the hand-off delay is negligible (see (a)

(b)

Figure 35).



(a) (b)

Figure 35. A completion and release example. Both models release batch i+4 at the same time and it starts from pick face 1 (the number of pick faces = 8, the number of pickers = 4): (a) a circular-aisle abstraction; and (b) a bucket brigade OPS.

From Proofs 1 and 2, two systems are identical in steady state. Initialization and finalization stages are beyond the scope of the analysis of the steady state. However, technically, two models can start with the same procedure if they start together from the loading station. The finalization stage also can be the same if they do not allow any hand-off after the last batch enters the system. End of proof.

Having identified the equivalence of the picker blocking model in these two settings, we are now able to develop the following insights:

1) *Batch picking faces less picker blocking when the batch size is determined by the number of items, not the number of orders.* The batch size can be determined by quantity of items the tote can hold when using a pick-then-sort strategy, or number of orders (or the number of totes in a batch) in a sort-while-pick strategy. When a batch includes a fixed number of items, pick density is constant over batches. Thus, the variation of pick density decreases.
2) *With a finite backward walk time, picker blocking may become less than the infinite backward walk time case.* The release of the i+k^{th} batch requires a

duration after the completion of the i^{th} batch due to the backward walk times. The distance between $i+k^{th}$ and $i+k$-1^{st} lengthens compared to the infinite backward walk time case. Thus, picker blocking decreases.

3) *When walk speed is not infinite and is not unit walk speed, hand-off delay becomes more significant and picker blocking decreases.* As hand-off delay increases, the starting time of a new batch is delayed. Typically, picker blocking decreases as hand-off delay increases.

3.3 Indexed order batching model for control

Since the picker blocking mechanism of a circular-aisle OPS has been identified, and the equivalence of the bucket brigade OPS shown, the multiple-aisle IBM for picker blocking control described in Chapter VI can be employed.

We generalize the model by relaxing two assumptions: the IBM for bucket brigade order picking differs from the parallel-aisle IBM: 1) the starting time of the $i+k^{th}$ batch is determined by the cumulative sum of hand-off delay and backward walk time upon completion of the i^{th} batch; and 2) the IBM for bucket brigade order picking has no routing problem. Based on these two differences, the abstracted IBM becomes the following equation:

(Abstracted IBM with finite pickers) *Min* Walk time + Time delayed
Subject to
Indexed batching constraints
In-the-aisle picker blocking constraints
Release-time updating constraints

The indexed batching constraints associate the batching problem with the release sequence. In-the-aisle picker blocking constraints are required to calculate overall picker blocking. The IBM for bucket brigade OPS can update the release-time of batch $i+k$ using the following logic:

The starting time of batch $i+k$ at loading station
= the completion time of the i^{th} completed batch at unloading station
  + the expected backward travel time by picker 1 for batch $i+1$
  + the expected hand-off delay by picker 1 for batch $i+1$
  + the expected backward travel time by picker 2 for batch $i+2$
  + the expected hand-off delay by picker 2 for batch $i+2$
  …
  + the expected backward travel time by picker k for batch $i+k$
  + the expected loading time by picker k for batch $i+k$
= the completion time of the i^{th} completed batch at the unloading station
  + the expected backward travel time by picker 1,..,k linked by batch i's completion
  + the expected hand-off delay by picker 1,..,k-1
  + the expected loading time by picker k for batch $i+k$
= the completion time of the i^{th} completed batch at the unloading station
  + unit backward time*n
  + (k-1)E[HO]
  + the expected loading time by picker k for batch $i+k$

where k is the number of pickers and i is the index of a batch. For $E[HO]$ (the expected time delayed per hand-off occurrence), we introduce a weight factor α. Because an expected hand-off delay can vary depending on hand-off control (discussed below in Section 5), the weight factor α is necessary. Moreover, usually the loading time is 0 in a bucket brigade protocol. Thus, to obtain the starting time of the $i+k^{th}$ batch, we use the following equation:

The starting time of batch $i+k$ at the loading station
= the completion time of the i^{th} completed batch at the unloading station
  + unit backward time*n
  + α (k-1)E[HO]

For a detailed IBM formulation, see Appendix D.2.

4. ANALYSIS AND CONTROL OF HAND-OFF DELAY

In this section, we conduct an analytical study to quantify if hand-offs are a significant source of delay and thus a concern of management. We develop a renewal process model for the hand-off operation between two pickers when pick time is random and walk time is instantaneous. We also propose a method to control hand-off delay.

4.1 Renewal process model for hand-off operation

We assume that walk speed is infinite and the number of picks is large enough for analytical purposes. As in the previous blocking models, the first assumption (infinite walk speed) is common in the bucket brigade literature. Section 5 below provides further generalizations for cases with finite walk speed and fewer picks via a simulation study.

Consider that an upstream picker and a downstream picker are identical in terms of pick time and walk time. The upstream picker makes stops 1, 2,… for picks whenever a pick face contains at least one item to be picked. Note that each stop can process one or more picks and can come from different batches. $X_1, X_2, X_3,\ldots$ denotes the time spent for the upstream picker to pick all items in a pick face at a stop. In other words, $X_1, X_2, X_3,\ldots$ becomes an inter-arrival time between stops. The mean of the inter-arrival time of stops $[X_1, X_2, X_3,\ldots]$ is $E[X]$ and identical to average pick time per stop. The sequence, $A_1, A_2,\ldots$, represents the times at which the upstream picker completes the retrieval operation at 1st stop, the retrieval operation at 2nd stop, …. . The downstream picker's returning time is the sum of the walking time, picking time, blocking time (if blocked), and walk back for a particular batch. When the pick load is large enough, the returning time of a downstream picker is close to random arrival. Here, the sequence, $S_1, S_2, S_3,\ldots$

is the arrival time of the downstream picker to take over a tote from the upstream picker. The waiting time of a downstream picker is Y_1, Y_2, Y_3,... for each arrival. The waiting time for j^{th} hand-off (Y_j) becomes A_i-S_j where A_i stands for the completion time of pick(s) at i^{th} stop of the upstream picker.

The example in Figure 36 illustrates a hand-off delay of picker 1 when picker 2 processes the second item of batch 2 (B^2_2, where the superscript indicates the batch number and the subscript stands for items in a batch). Picker 1 has completed the last two picks of batch 1 (B^1_5 and B^1_6) and unloaded the collected batch, he/she is idle at the next pick face of picker 2. The idle time duration is Y_1 (=A_2-S_1) when picker 1 arrives at time S_1 and picker 2 finishes the second item of batch 2 (B^2_2) at time A_2.

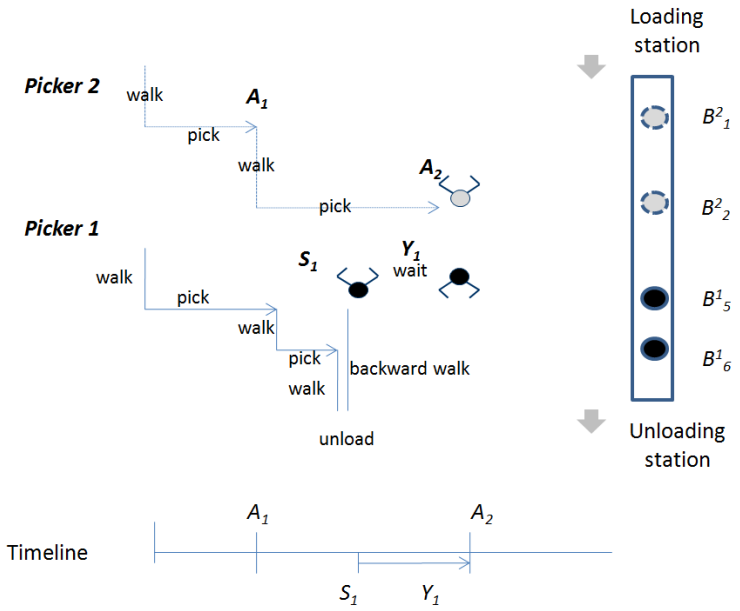


Figure 36. An example of hand-off and its appropriate renewal process.

From the situation we described above, the expected time delayed per hand-off occurrence and the expected time delayed per batch can be derived.

Theorem 5. The expected waiting time ($E[Y(t)]$) is $E[X^2]/2E[X]$ and the hand-off delay per batch is (k-1)* $E[X^2]/2E[X]$.

Proof.

We derive the renewal processes based on the definition in Ross (1996). By definition, $E[X^2]<\infty$, because X is the average pick time at a stop and X is finite as long as the pick is completed.

The expected waiting time ($E[Y(t)]$) can be expressed as:

$$E[Y(t)] = E\left[Y(t) \mid S_{N(t)} = 0\right] \cdot \overline{F}(t) + \int_0^t E\left[Y(t) \mid S_{N(t)} = y\right] \cdot \overline{F}(t-y) \cdot dm(y), \tag{7.2}$$

$$\text{where } \overline{F}(t) = 1 - F(t), m(x) = \sum_{n=1}^{\infty} F_n(x)$$

$$h(t) = E\left[X - t \mid X > t\right] \cdot \overline{F}(t)$$

Since $E[X^2]<\infty$, $h(t) = E\left[X - t \mid X > t\right] \cdot \overline{F}(t)$ is directly Riemann integrable. Thus, we can use the key renewal theorem:

$$E[Y(t)] = \int_0^{\infty} E\left[X - t \mid X > t\right] \cdot \overline{F}(t) \cdot dt / E[X] \tag{7.3}$$
$$= E\left[X^2\right] / 2E[X]$$

(See Appendix D.3 in detail.)

k-1 pickers are associated with hand-offs for a batch. Thus, the expected hand-off time per batch is (k-1)* E[X^2]/2E[X]. End of proof.

4.2 Intermediate hand-off rule and pick-first priority

The previous section estimated the expected wait time due to hand-off delay. Equation (7.3) indicates that multiple-picks can increase the hand-off delay as the

variation increases. Note that we assume that the pick time at a stop is dependent on the number of items, which is a multiple-pick situation at a pick face. Usually, the multiple-pick situation concerns multiple products. In practice, while retrieving multiple items from a pick face, an upstream picker may be able to yield remaining item(s) to a downstream picker after completion of an item, not all of the items. This yielding seems to be more practical and can prevent the stop-based model from overestimating the hand-off delay.

This study terms the yielding the *intermediate hand-off rule*. This practical rule and procedure can also reduce hand-off delay and simplify our other hand-off proofs. When an upstream picker sees a downstream picker while processing multiple items, he/she can yield the remaining picks to the downstream picker if he/she completes at least one pick. This method can prevent unnecessarily long hand-off delays when multiple-picks at a pick face are allowed. Then, under the intermediate hand-off rule, the mean of the inter-arrival time of picks $E[X]$ becomes the average pick time since a hand-off can occur at the completion of every pick, not stop.

However, when the downstream picker becomes idle simultaneously as the upstream picker starts the first pick, the upstream picker completes the first pick and yields the batch after the completion. This *exception* to the intermediate hand-off rule is called *pick-first priority.*

4.3 Control of hand-off delay: No-handshake hand-off policy

Initially, a hand-off policy to reduce delay is identified, and then an optimal control value is presented. The hand-off delay stems from poor synchronization between

two pickers. Typically in a bucket brigade system two pickers meet and the upstream picker hands the tote to the downstream picker. Pickers coming into direct contact is termed a handshake hand-off. Our new policy relaxes this restriction, which is termed a *no-handshake* hand-off policy. As depicted in Figure 37, an upstream picker decides to: 1) move forward to the next pick and retrieve the next pick; or 2) move backward, leaving a batch at the location of the next pick. In the latter case, the downstream picker will process the next pick upon taking over the batch.

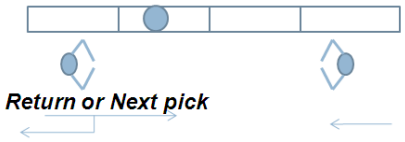


Figure 37. No-handshake hand-off policy.

Next, conditions which determine the upstream pickers' behavior are defined. Walk speed is infinite and the pickers are identical. It is assumed that pickers can accurately estimate expected hand-off delay. The assumption will be revisited when discussing a practical application in Section 5. Consider a hand-off between two pickers. Define τ as a threshold period of time. If the expected hand-off delay is longer than τ, the upstream picker does not perform the next pick. In Figure 38 (a), Y_2 is longer than τ, and the upstream picker does not start the second pick, but leaves the current batch and moves backward. The new hand-off time, zero, in Figure 38 (b) replaces Y_2 in Figure 38 (a). The remaining timeline of the no-handshake hand-off bucket brigade differs from the timeline of the regular bucket brigade because the upstream picker does not process a pick relevant to A_3 and instead the downstream picker retrieves the pick. Thus, the

remaining timeline uses A'_3,A'_4, A'_5, S'_2 and Y'_2. Note that at S_2, the downstream picker does not wait, but picks an item. The second hand-off occurs at S'_2.

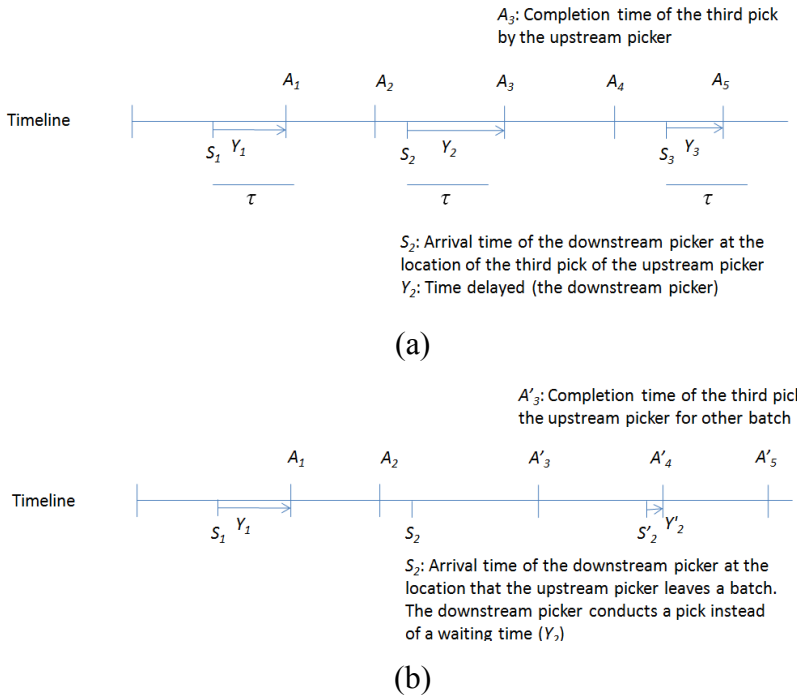


Figure 38. Comparing two bucket brigade methods: (a) regular bucket brigade; and (b) no-handshake hand-off bucket brigade.

Now we can derive an optimal policy. The waiting time by the downstream picker is conditioned on the expected wait time. The waiting time under the new policy is:

$$E[Y'(t)] = E\left[Y'(t) \mid S_{N(t)} = 0\right] \cdot \overline{F}(t) + \int_0^t E\left[Y'(t) \mid S_{N(t)} = y\right] \cdot \overline{F}(t-y) \cdot dm(y), \tag{7.4}$$

where $\overline{F}(t) = 1 - F(t), m(x) = \sum_{n=1}^{\infty} F_n(x)$

$$h(t) = E\left[X - t \mid X > t, X - t < \tau\right] \cdot \overline{F}(t)$$

Since $E[X^2] < \infty$, $h(t) = E\left[X - t \mid X > t, X - t < \tau\right] \cdot \overline{F}(t)$ is directly Riemann integrable.

Thus, we can use the key renewal theorem:

$$E[Y'(t)] = \int_0^\infty E[X - t \mid X > t, X - t < \tau] \cdot \overline{F}(t) \cdot dt / E[X] \qquad (7.5)$$
$$= \left[\int_0^\tau x^2 \cdot dF(x) + \tau^2 \int_\tau^\infty \cdot dF(x) \right] / 2E[X]$$

(See Appendix D.4 in detail.)

From Equation (7.5), we derive the following theorem.

Theorem 6. With a no-handshake hand-off policy, the minimum expected hand-off delay is zero.

Proof.

Equation (7.5) is always greater than and equal to 0 over τ. When $\tau = 0$, this value is always zero as shown below in Equation (7.6).

$$\int_0^\tau x^2 \cdot dF(x) + \tau^2 \int_\tau^\infty \cdot dF(x) = 0 \qquad (7.6)$$

End of proof.

5. SIMULATION AND EXPERIMENTAL RESULTS

In Sections 3 and 4, analytical and control models were presented to quantify and reduce picker blocking and hand-off delay. This section will verify the models using simulations. In addition, the simulations are extended into practical situations since several assumptions are inevitable in models: no hand-off delay in the picker blocking model, and a large number of picks and infinite walk speeds in the hand-off models. More importantly, the performance improvement will be evaluated in practical settings.

5.1 Simulation study on picker blocking

Figure 39 illustrates performance loss by picker blocking in 20-pick face bucket brigade systems and circular picking systems with two pickers whose speed is from 0 to infinite walk speed. Solid lines are the simulations' results when pick:walk time = 1:0, 1:0.025, 1:0.05, 1:0.1, 1:0.25, 1:0.5, and 1:1 from top to bottom. The upper dotted line is an analytical result with pick:walk time = 1: 0. The lower dotted line is a lower bound with pick:walk time = 1:1. Deterministic pick time and walk speed hold.

When pick:walk time = 1:1, the delay in the bucket brigade picking is almost identical to the pattern of the circular-aisle picking. When pick:walk time = 1:0, the bucket brigade order picking faces less blocking than the analytical model and the circular-aisle model. Our analysis indicates that the 1:0 model can include one hand-off situation at the first pick face. By chance, as an upstream picker arrives at the first pick face with a pick, he/she can face a downstream picker. According to the pick-first priority, the upstream picker picks and the downstream picker waits. Our observation indicates that when walk speed is not infinite or is unit walk speed, hand-off delay becomes more of a concern. As hand-off delay arises, the starting time of a new batch is delayed. Thus, picker blocking decreases.

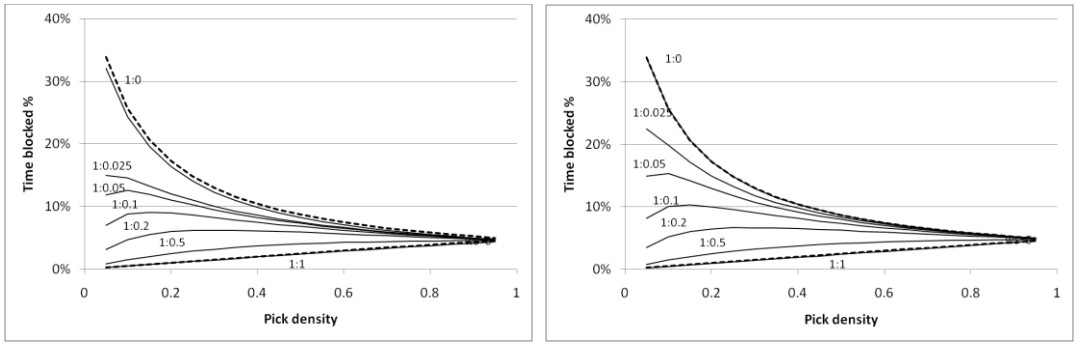


Figure 39. The percentage of time blocked (two-picker, 20 pick faces) with multiple-picks with infinite backward walk with allowance of intermediate hand-off: (a) bucket brigade system; and (b) circular-aisle system.

5.2 Simulation study on hand-off delay

A simulation study distinguishes when more and less hand-offs occur. The benefits of the hand-off control strategy are demonstrated.

5.2.1 Impacts on hand-off delay by practical situations

In practice, pickers are neither infinitely fast nor do they process an infinite number of picks. For a more realistic situation, we analyze the hand-off model using a discrete-event simulation under the intermediate hand-off rule and pick-first priority. More specifically, the walk time is classified by forward walk (i.e., loaded walk) and backward walk (i.e., empty walk) according to the moving direction or the carrying status of a tote. We consider five situations: 1) 100 pick faces and two pickers with 500 picks and infinite walk speed (notated 2NW-500); 2) 100 pick faces and five pickers with 50 picks and infinite walk speed (5NW-50); 3) 100 pick faces and five pickers with 20 picks and infinite walk speed (5NW-20); 4) 100 pick faces and five pickers with 20 picks and forward walk time = 0.1 time per pick face (5FW-20); and 5) 100 pick faces and five pickers with 20 picks, forward walk time = 0.1 time per pick face, and

backward walk time = 0.05 time per pick face (5BW-20). Additionally, we consider three pick time distributions: Uni = uniform [min,max] = [0.5, 1.5], Tri = triangular [min, mode, max] = [0.5, 1.0, 1.5], and Exp = exponential [mean] = [1.0], where the time unit represents a time spent to retrieve an item. There are 20 simulation runs with 2000 orders per run; the number of simulation runs is obtained from: 1) the comparison between our analytical models and simulation results; and 2) the experiment size proposed by the simulation environments in Ruben and Jacobs (1999).

The comparison results are summarized in Table 12. We are interested in the gap between analytical results and simulation values over order picking situations. 2NW-500 shows a very small gap compared to the analytical result. As the number of picks decreases and the number of pickers increases (5NW-50, 5NW-20) the hand-off delay decreases compared to the analytical value. Forward walk time and backward move also impact the delay (5FW-20, 5BW-20); less hand-off delay is observed. When walking takes positive time, pickers can confront each other while walking, not picking frequently. In this case, a hand-off operation can be conducted without delay. Thus, the average hand-off delay time is reduced.

Table 12. Average hand-off delay per occurrence over different order picking situations

Distribution	Uni		Tri		Exp	
	Time delayed	Gap	Time delayed	Gap	Time delayed	Gap
Analytical value	0.5466	-	0.5208	-	1.0000	-
2NW-500	0.5419	0.87%	0.5207	0.02%	1.0010	-0.10%
5NW-50	0.5233	4.27%	0.5101	2.06%	0.8663	13.37%
5NW-20	0.4730	13.46%	0.4628	11.14%	0.7046	29.54%
5FW-20	0.3333	39.02%	0.4039	22.46%	0.6253	37.47%
5BW-20	0.3286	39.87%	0.3223	38.13%	0.5225	47.75%

5.2.2 No-handshake hand-off policy

A simulation study is conducted to investigate the proposed control methods including the intermediate hand-off rule. The impact of picker blocking is minimized by fixing the batch size and only allowing single picks at a given pick face. Several picking environments are investigated by varying the pick time and the walk time distributions, the number of pick faces, and workloads.

As depicted in Figure 40, $\tau = 0$ achieves a minimum hand-off delay. While Figure 40 (a) shows almost zero hand-off delay, Figure 40 (b) shows a relatively significant hand-off delay in spite of $\tau = 0$. The expected hand-off delay of 5NW-50, 5NW-20, 5FW-20, and 5BW-20 situations in Figure 40 (b) increases as the variance and range of the pick time distribution increase. In particular, as the number of picks decreases (5NW-50, 5NW-20 situations in Figure 40 (b)), the values of time delayed increases when $\tau = 0$. Situations relevant to the pick-first priority for the exponential pick time cases occur more frequently because the number of picks is too small. When an upstream picker takes a long time to pick an item, a downstream picker reaches the hand-off location from the upstream picker before the upstream picker has completed the first pick. As walking speed impacts the downstream picker's performance (5FW-20 and 5FW-20 situations in Figure 40 (b)), the time to reach an upstream picker increases; thus the time delayed decreases when $\tau = 0$. In summary, while a significant portion of the hand-off delay can be reduced through the no-handshake hand-off rule, the portion of pick-first priority is exceptional, particularly for the exponential pick time cases in Figure 40 (b). When the variation of pick time is very high and the number of picks is

small, the no-handshake hand-off rule functions poorly. The stable retrieval performance plays an important role in employing the no-handshake hand-off rule appropriately.

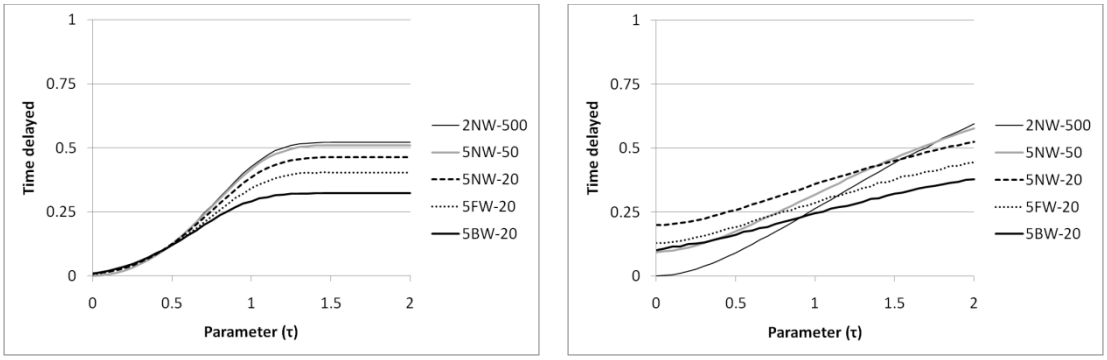


Figure 40. Impacts on hand-off delay of policy parameter over different picking environments: (a) triangular pick time; and (b) exponential pick time.

5.3 Integrated control of picker blocking and hand-off delay

This section summarizes the computational implementation and discusses insights from the analysis. IBM and no-handshake hand-off policy are implemented at different operational levels. IBM is proposed to determine the content of batches and the sequence of batches, while the no-handshake hand-off policy is an instruction given to the picker. Instead of integrating the two control strategies, a hierarchical structure is proposed. The details are as follows:

- Use IBM to reduce picker blocking
- Teach pickers the no-handshake hand-off policy to reduce hand-off delay

5.3.1 Experimental design

A modified order picking profile based on Koo (2009) is used to evaluate the proposed procedure. We consider 100 pick faces and five pickers. A picker performs

with pick:(forward) walk:backward walk ratio = 1.0:0.1:0.05. We employ a triangular distribution for pick time. Deterministic forward and backward walk times are assumed. We compare two control cases: FCFS = sequence orders into batches on a first-come-first-serve basis and release batches immediately after construction; and Cont = IBM + no-handshake hand-off operation.

We investigate four different scenarios listed in Table 13. First, a standard scenario uses the walk speed and picking capability configurations defined above. Second, a capability scenario differentiates picking capabilities across pickers. The unit time per pick for the five pickers in the simulation is differentiated into 1.5, 1.25, 1.0, 0.75, 0.5, where an average picker performs one pick per unit time. Third, the fast-walk scenario looks at the variations in the walk speed of pickers which frequently appear in the bucket brigade order picking literature (Bartholdi and Eisenstein, 1996b; Koo, 2009), where the authors assume pickers' travel with instantaneous walk speed. A fast-walk situation increases walk speed into pick:walk:back = 1:0.05:0.025; this value is a fast case in Gue *et al.* (2006). Fourth, in one small-OPS scenario, the walk speed is fast and the OPS is small in size. The OPS has 50 pick faces.

We evaluate single order picking and batch order picking. We consider five scenarios with varying average order sizes of 4, 6, 10, 20, and 50 items for the single order picking strategy, and two items per order in the batch picking strategy. The order size of each order is randomly selected based on a uniform distribution [min,max] = [mean/2, mean*3/2]. Pick time is drawn from a triangular distribution of [min, mode, max] = [0.5,1.0,1.5]. Note that according to our survey, a practical work load per picker

is 2~4 picks per batch (Koo, 2009) and four orders per batch (Bartholdi and Eisenstein, 1996a). Since an order size can vary, but is relatively small in a bucket brigade order picking, batch picking considers 20 items as a regular batch size (i.e., four picks per picker or two orders per picker) and 50 items for a heavy demand situation (i.e., 10 picks per picker or five orders per picker).

As a performance measure, we compare utilization (%), time blocked (%), and hand-off delay (%). Utilization is the percentage of time spent picking to overall operations. Time blocked represents a productivity loss. Hand-off delay includes the ratio of hand-off waiting time to the overall time. In addition, the column labeled Diff in the result tables (Table 14 and Table 15) shows the comparison between FCFS and Cont. Run time illustrates the computation time per cycle, where a cycle has k batches for k pickers.

The simulation is implemented using C language and the IBM formulations using ILOG CPLEX Callable Library C API 11.0.4. The executable files run on Windows Server 2008 (Xeon 2.66 Ghz CPU, 12 GB memory, 32 bit implementation). We disable both the branch-and-cut option and the heuristic search option to evaluate the exact computational time. One instance includes 2000 orders and 20 runs consistent with Ruben and Jacobs (1999). The picking environment is summarized in Table 13.

Table 13. Summary of experimental environments

Configuration	Values
Scenarios	Standard, Capability, Fast-walk, Small-OPS
Mean of order sizes	2, 4, 6, 10, 20, and 50
# items per order	Uniform distribution [min,max] = [mean/2, mean*3/2]
Pick time	Triangular distribution [min, mode, max] = [0.5, 1.0, 1.5]
Forward walk time	0.1 or 0.05
Backward walk time	0.05 or 0.025
$E[HO]$	0.5208
α	0.016
Performance measure	utilization (%), time blocked (%), and hand-off delay (%)
Runs per instance	20 runs with 2000 orders
The number of batches per one IBM	5 orders or batches per an IBM cycle.

5.3.2 Single order picking

Using FCFS, utilization is 19.95% to 67.16% (Table 14 (Standard)). The proposed approach (Cont) improves the utilization to 20.70~73.82%. In particular, when order sizes are medium or large, picker blocking is of increased concern and picker blocking control in the Cont approach is very effective. Compared to the batch picking, the single order picking produces more picker blocking since a higher variation of pick density is inevitable. IBM successfully manages the picker blocking. The reduction of picker blocking amounts to 58.20% compared to FCFS when the order size is 20 items per order. When the work load is higher and more pickers are used, blocking is more serious and the proposed methods exhibit robust and better performance over FCFS. Consistently, most hand-off delays are removed by the proposed control method. The runtimes for the IBM algorithm are 0.095~0.417 seconds per a cycle to determine the release sequence of five pickers. The FCFS in the capability scenario produces less picker blocking compared to the standard scenario. Thus, the Cont experiences small improvements. Fast-walk and small-OPS situations consistently show improvement in

terms of increased utilization.

Table 14. Experimental results on single order picking

Scenarios	Order	Utilization (%)			Time blocked (%)			Hand-off delay (%)			Run time
	Size	FCFS	Cont	Diff (%)	FCFS	Cont	Diff (%)	FCFS	Cont	Diff (%)	(seconds)
Standard	4	19.95	20.70	3.75	2.66	1.46	45.09	2.26	0.34	85.15	0.095
	6	26.67	28.05	5.17	3.85	1.91	50.41	2.68	0.41	84.67	0.125
	10	36.51	38.95	6.67	5.61	2.56	54.38	3.01	0.48	83.92	0.164
	20	50.82	55.21	8.64	8.09	3.38	58.20	2.92	0.53	81.97	0.229
	50	67.16	73.82	9.92	10.56	3.90	63.05	2.14	0.47	78.09	0.417
Capability	4	19.83	19.97	0.72	1.42	0.82	42.15	2.96	0.23	92.32	0.077
	6	26.33	27.06	2.75	1.98	1.03	47.94	3.49	0.26	92.42	0.106
	10	36.01	37.67	4.61	2.70	1.38	48.97	3.93	0.34	91.23	0.152
	20	50.84	53.90	6.02	3.55	1.74	50.99	3.90	0.46	88.30	0.215
	50	69.41	73.29	5.60	4.16	1.84	55.64	2.91	0.49	83.01	0.394
Fast-walk	4	30.58	32.85	7.41	6.81	4.73	30.61	5.13	1.30	74.69	0.148
	6	38.42	41.77	8.72	8.27	5.42	34.46	5.35	1.24	76.73	0.187
	10	48.49	53.33	9.97	10.01	6.17	38.41	5.12	1.12	78.06	0.230
	20	61.32	67.94	10.78	11.48	6.18	46.16	4.18	0.89	78.79	0.277
	50	73.92	82.07	11.03	12.44	5.26	57.74	2.55	0.61	76.23	0.463
Small-OPS	4	40.34	44.62	10.62	12.79	11.02	13.81	9.06	3.54	60.97	0.092
	6	47.99	53.52	11.51	13.46	11.19	16.85	8.63	2.94	65.90	0.109
	10	57.16	64.05	12.06	13.99	10.49	25.01	7.47	2.30	69.17	0.124
	20	67.98	76.10	11.94	13.79	8.61	37.54	5.53	1.57	71.64	0.166
	50	77.64	86.58	11.51	13.46	6.23	53.68	3.08	0.91	70.36	0.282

5.3.3 Batch order picking

Table 15 summarizes the results of varying the batch size in bucket brigade OPS. A 4.29~7.04% improvement of utilization in the standard picking situation is observed. As identified in Section 4, batch picking can reduce the variation of pick density and lead to less picker blocking. Thus, the results for batch picking are not as dramatic as observed in the single order picking scenario. Specifically, in the standard situation, the percentage of time blocked is 1.58~1.68%, whereas the FCFS situation is 2.60~2.89%. Hand-off control consistently shows improvement; the percentage difference between FCFS and Cont is 80.70~86.64%. The calculations related to the IBM average 0.387 seconds when the batch size is 20 and 1.254 seconds when the batch size is 50.

Interestingly, the proposed approach shows some improvement under capability instances, where the unit time per pick for the five pickers is not identical and pickers are optimally sequenced to maximize the picker performance. We note that the capability instance with batch picking is one of the best-performance order picking situations. Cont can still give a benefit. Capability instances slightly increase blocking delays, but achieve large reductions in hand-off delay, and thus lead to overall improvement in performance.

The fast-walk and small-OPS order picking scenario indicate higher utilization improvement by the proposed algorithm (3.87~9.45%). Computationally, fast-walk scenarios experience on average 0.524~1.048 seconds per five batches and small-OPS scenarios on average 0.338~0.737 seconds.

Table 15. Experimental results varying batch size

Scenarios	Batch	Utilization (%)			Time blocked (%)			Hand-off delay (%)			Run time
	Size	FCFS	Cont	Diff (%)	FCFS	Cont	Diff (%)	FCFS	Cont	Diff (%)	(seconds)
Standard	20	52.34	56.02	7.04	2.60	1.68	35.34	3.37	0.45	86.64	0.387
	50	72.56	75.67	4.29	2.89	1.58	45.28	2.37	0.46	80.70	1.254
Capability	20	51.10	54.29	6.25	1.05	1.06	-1.16	4.10	0.38	90.81	0.236
	50	71.66	74.18	3.52	0.78	0.79	-1.68	3.06	0.42	86.37	0.530
Fast-walk	20	64.91	70.39	8.45	4.25	2.81	33.96	5.01	0.71	85.88	0.524
	50	81.31	84.64	4.10	3.38	2.29	32.20	2.90	0.64	78.01	1.048
Small-OPS	20	73.03	79.94	9.45	5.94	4.19	29.46	6.51	1.40	78.43	0.338
	50	86.34	89.68	3.87	3.81	2.86	24.93	3.25	0.94	70.95	0.737

5.4 A distance-based heuristic approach for τ

Use of τ as a threshold is not practical in most circumstances since pickers probably cannot accurately estimate expected hand-off. However, this finding is easily transferrable to a distance-based heuristic approach. The difficulty of forecasting arises

because of the hand-off time of a downstream picker. We consider the situation that an upstream picker notices the downstream picker who completes a hand-off, and thus approaches in a backward direction. In this case, the upstream picker can decide to continue picking the current batch based on the expected arrival time of the immediately adjacent downstream picker. If the downstream picker is moving backward and the expected arrival time is less than the expected pick completion time, the upstream picker returns without picking. The expected arrival time can be measured by the distance from the downstream picker. A benefit of the distance-based heuristic approach is its ease of implementation, but the approach is also applicable when walk time is not so fast and stable.

The result shows a gap compared to the previous Cont results as depicted in Table 16, where $\alpha = 0.156$ is determined by a simulation study. The heuristic approach uses the distance = 20, which is derived from the average pick time divided by the backward walk time = 1.0/0.05. The heuristic approach (Hcont) experiences 5.66% improvement of utilization in a standard batch picking situation when the use of τ produces 7.04% improvement. The gap amounts to 0.72~0.85% of utilization because of increased hand-off delay. However, the results still outperform the FCFS with 3.12~5.66% improvement of utilization.

Table 16. Comparison of Cont and heuristic approach (Hcont)

Insta-	Batch	Utilization (%)			Time blocked (%)			Hand-off delay (%)			Run time
nce	Size	Cont	Hcont	Gap	Cont	Hcont	Gap	Cont	Hcont	Gap	(seconds)
Sta-	20	56.02	55.30	-0.72	1.68	1.81	-0.12	0.45	1.42	-0.97	0.740
ndard	50	75.67	74.82	-0.85	1.58	1.71	-0.13	0.46	1.22	-0.76	1.427

6. CONCLUSIONS

This chapter has made three important contributions to the analysis and understanding of bucket brigade OPS. First, analytical models of picker blocking and hand-off delay in bucket brigade OPS are developed. Second, based on analytical studies and additional simulation studies, the conditions are identified under which more efficient operations can be achieved. Third, control methodologies are developed to maximize order picking throughput.

Analytical models were developed to quantify the delays related to blocking and hand-offs by extending the analogy of a circular-aisle OPS to the bucket brigade OPS. The analytical results found: 1) batch picking can reduce picker blocking because of less variation of an average work load per batch; and 2) decreased variability in pick time reduces hand-off delay. Bartholdi and Eisenstein (1996b) emphasized the importance of a smooth hand-off operation, but did not clearly define the smooth operation and its rationale. Intermediate hand-off is one method which can reduce delays related to the hand-off operation. Moreover, the reduction stems from less variance of the expected pick time of an upstream picker.

Directly controlling picker blocking and hand-off delay also maximizes throughput. We found that IBM could mitigate picker blocking. Further, the analogy to a circular-aisle OPS facilitated the development of models to batch orders and assign batches to pickers to reduce blocking delays in bucket brigade systems. To reduce hand-off delay, the synchronization requirement in upstream-to-downstream hand-off was relaxed and strategies to coordinate the physical system were proposed. Both ideal

method and practical application were developed.

Based on our findings we suggest that future research should focus upon: 1) practical application; 2) generalization of the proposed approach for bucket brigades used in manufacturing operations; and 3) an integrated throughput model. The proposed methods such as intermedidate hand-off, no-handshake hand-off, and IBM may be difficult to implement in practice. In the case of the no-handshake hand-off, additional studies on realistic implementation approaches (e.g., a distance-based heuristic approach) could be undertaken. Since only order picking systems are considered, the next step is to identify possible applications in other manufacturing and service areas, for example, general manufacturing systems such as the assembly line described in Bartholdi and Eisenstein (2005). A more comprehensive solution that integrates the models could potentially contribute to a clearer understanding of bucket brigade operation.

CHAPTER VIII

CONTRIBUTIONS AND CONCLUSION

Order picking operations play a critical role in the order fulfilment process of warehouses and DCs. Picking a batch of orders is favored when customers' demands create a large number of small orders. Thus, constructing an appropriate order batching algorithm involves reducing the total retrieval workforce, and differs from a general batching in that scalability in the number of orders, simplicity in routing, and congestion must be addressed. This dissertation established four tasks:

- First, a large-scale and near-optimal order batching algorithm to minimize the travel distance is developed. The outcomes of this research highlighted critical observations of near-optimal, large-scale order batching: less congestion than expectation, but still significant under some situations.
- Second, since the available literature cannot explain the observations, an analysis and simulation study to identify the complex relationship between sources of picker blocking and the relevant situations of a real-world firm is undertaken.
- Third, a new order batching model and its large-scale solution to manage both distance and congestion simultaneously is developed.
- Fourth, we examined the significance of congestion and hand-off delays in bucket brigade order picking, followed by providing a structured control procedure for dynamic order picking which mitigates both picker blocking and hand-off delay directly.

This dissertation makes three major contributions. First, the proposed analytical studies give a clear understanding of picker blocking and hand-off delay in batch order picking. Second, it introduces for the first time in the literature exact batch picking

frameworks to handle picker blocking. Third, efficient solution methodologies are provided for two large-scale, practical order picking situations.

In particular, three new batching models are demonstrated:

1) A near-optimal, large-scale proximity-batching algorithm for traversal routing methods is developed. We express it as route-selecting batching formulation (RSB). To obtain an efficient and effective lower bound model for the batching problem, a route-bin packing problem (RPP) is derived from RSB.

2) A new order batching procedure with picker blocking in a narrow-aisle picking system is presented (IBM).

3) A new order batching procedure with picker blocking and hand-off delay is addressed for a bucket-brigade picking system.

REFERENCES

Amazon.com (2004) *2003 Amazon.com Annual Report*. Amazon.com, Inc., Seattle.

Armbruster, D., Gel, E.S. (2006) Bucket brigades revisted: Are they always effective? *European Journal of Operational Research*, **172**(1), 213-220.

Bartholdi, J.J., Eisenstein, D.D. (1996a) Bucket brigades: a self-organizing order-picking system for a warehouse. Working paper.

Bartholdi, J.J., Eisenstein, D.D. (1996b) A production line that balances itself. *Operations Research*, **44**(1), 21-34.

Bartholdi, J.J., Eisenstein, D.D. (2005) Using bucket brigades to migrate from craft manufacturing to assembly lines. *Manufacturing Service Operations Management*, **7**(2), 121-129.

Bartholdi, J.J., Eisenstein, D.D., Foley, R.D. (2001) Performance of bucket brigades when work is stochastic. *Operations Research*, **49**(5), 710-719.

Chen, M.-C., Wu, H.-P. (2005) An association-based clustering approach to order batching considering customer demand patterns. *Omega*, **33**(4), 333-343.

Clarke, G., Wright, J.W. (1964) Scheduling of vehicles from a central depot to a number of delivery points. *Operations Research*, **12**, 568-581.

De Koster, R. (2003) Distribution strategies for online retailers. *Engineering Management, IEEE Transactions on*, **50**(4), 448-457.

De Koster, R. (2004) *How To Assess a Warehouse Operation in a Single Tour*. RSM Erasmus University, Rotterdam, The Netherlands.

De Koster, R., Balk, B.M. (2008) Benchmarking and monitoring international warehouse operations in Europe. *Production and Operations Management*, **17**(2), 175-183.

De Koster, R., Van der Poort, E.S., Wolters, M. (1999) Efficient orderbatching methods in warehouses. *International Journal of Production Research*, **37**(7), 1479-1504.

De Koster, R., Yu, M. (2008) Minimizing makespan and throughput times at Aalsmeer flower auction. *The Journal of the Operational Research Society*, **59**(9), 9.

Frazelle, E. (2002) *World-class Warehousing and Material Handling*. McGraw-Hill,

New York.

Gademann, N., van de Velde, S. (2005) Order batching to minimize total travel time in a parallel-aisle warehouse. *IIE Transactions*, **37**(1), 63-75.

Gademann, N., Van den Berg, J., Van der Hoff, H. (2001) An order batching algorithm for wave picking in a parallel-aisle warehouse. *IIE Transactions*, **33**(5), 385.

Gong, Y., De Koster, R. (2008) A polling-based dynamic order picking system for online retailers. *IIE Transactions*, **40**, 1070-1082.

Gue, K.R., Meller, R.D., Skufca, J.D. (2006) The effects of pick density on order picking areas with narrow aisles. *IIE Transactions*, **38**(10), 859-868.

Hall, R.W. (1993) Distance approximations for routing manual pickers in a warehouse. *IIE Transactions*, **25**(4), 76.

Ho, Y.C., Tseng, Y.Y. (2006) A study on order-batching methods of order-picking in a distribution centre with two cross-aisles. *International Journal of Production Research*, **44**(17), 3391-3417.

Hsu, C.M., Chen, K.Y., Chen, M.C. (2005) Batching orders in warehouses by minimizing travel distance with genetic algorithms. *Computers in Industry*, **56**(2), 169-178.

Jane, C.C. (2000) Storage location assignment in a distribution center. *International Journal of Physical Distribution & Logistics Management*, **30**(1), 55.

Jane, C.C., Laih, Y.W. (2005) A clustering algorithm for item assignment in a synchronized zone order picking system. *European Journal of Operational Research*, **166**(2), 489-496.

Koo, P.-H. (2009) The use of bucket brigades in zone order picking systems. *OR Spectrum*, **31**(4).

Law, A.M., Kelton, W.D. (2000) *Simulation Modeling and Analysis*. McGraw-Hill, Boston.

Le-Duc, T. (2005) Design and control of efficient order picking processes. Ph.D. dissertation, Erasmus University, Rotterdam, The Netherlands.

Le-Duc, T., de Koster, R.M.B.M. (2007) Travel time estimation and order batching in a 2-block warehouse. *European Journal of Operational Research*, **176**(1), 374-388.

Lewis, R.T., Parker, R.G. (1982) On a generalized bin-packing problem. *Naval Research Logistics Quarterly*, **29**(1), 119-145.

Lieu, C.C.A. (2005) Impact of inventory storage and retrieval schemes on productivity. MBA. and MS. thesis, Massachusetts Institute of Technology, Cambridge, MA.

Lim, Y.F., Yang, K.K. (2009) Maximizing throughput of bucket brigades on discrete work stations. *Production and Operations Management*, **18**(1), 48-59.

Napolitano, M. (2008) Sitting tight - 2008 warehouse/DC operations survey results. *Logistics Management*, **47**(11), 47-50.

Napolitano, M. (2009) Real DC stories: low cost deep impact. *Logistics Management*, **48**(1), 46-49.

Napolitano, M., Gross&Associates (2003) *The Time, Space and Cost Guide to Better Warehouse Design*. The Distribution Group, Ogden, UT.

Pan, C.H., Liu, S.Y. (1995) A comparative study of order batching algorithms. *Omega*, **23**(6), 691-700.

Parikh, P.J., Meller, R.D. (2009) Estimating picker blocking in wide-aisle order picking systems. *IIE Transactions*, **41**, 232-246.

Parikh, P.J., Meller, R.D. (2010) A note on worker blocking in narrow-aisle order picking systems when pick time is non-deterministic. *IIE Transactions*, **42**(6), 392 - 404.

Petersen, C.G. (1997) An evaluation of order picking routeing policies. *International Journal of Operations & Production Management*, **17**(11), 1098-1111.

Petersen, C.G. (2000) An evaluation of order picking policies for mail order companies. *Production and Operations Management*, **9**(4), 319-335.

Petersen, C.G., Schmenner, R.W. (1999) An evaluation of routing and volume-based storage policies in an order picking operation. *Decision Sciences*, **30**(2), 481-501.

Pinedo, M. (1995) *Scheduling: Theory, Algorithms, and Systems*. Prentice Hall, Englewood Cliffs, NJ.

Ratliff, H.D., Rosenthal, A.S. (1983) Order-picking in a rectangular warehouse: a solvable case of the traveling salesman problem. *Operations Research*, **31**(3), 507-521.

Roodbergen, K.J., de Koster, R. (2001) Routing methods for warehouses with multiple cross aisles. *International Journal of Production Research*, **39**(9), 1865-1883.

Ross, S.M. (1996) *Stochastic Processes*. John Wiley & Sons, Inc., New York.

Ruben, R.A., Jacobs, F.R. (1999) Batch construction heuristics and storage assignment strategies for walk/ride and pick systems. *Management Science*, **45**(4), 575-596.

Skufca, J.D. (2005) k Workers in a circular warehouse: a random walk on a circle, without passing. *SIAM Review*, **47**(2), 301-314.

Tompkins, J.A., Bozer, Y.A., Tanchoco, J.M.A. (2003) *Facilities Planning*. J. Wiley, Hoboken, NJ.

Wilson, R. (2008) *19th Annual State of Logistics Report*. Council of Supply Chain Management Professionals, Washington DC.

Won, J., Olafsson, S. (2005) Joint order batching and order picking in warehouse operations. *International Journal of Production Research*, **43**(7), 1427-1442.

Yu, M., De Koster, R.B.M. (2009) The impact of order batching and picking area zoning on order picking system performance. *European Journal of Operational Research*, **198**(2), 480-490.

Zhang, M., Batta, R., Nagi, R. (2009) Modeling of workflow congestion and optimization of flow routing in a manufacturing/warehouse facility. *Management Science*, **55**(2), 267-280.

APPENDIX A

SUPPLEMENTARY FORMULATION, PROOF, ALGORITHM, AND RESULTS DISCUSSED IN CHAPTER IV

A.1 FORMULATION OF BASIC RPP FROM RSB

The basic RPP can be derived from RSB. In particular, each constraint in the basic RPP is derived from a constraint of RSB, or becomes a constraint aggregating relevant constraints in RSB.

1) Objective function

$$\sum_{b\in B}\sum_{r\in R} LT_r Y_{br} = \sum_{r\in R} LT_r \sum_{b\in B} Y_{br} = \sum_{r\in R} LT_r \cdot y_r$$

By definition, $\sum_{b\in B} Y_{br} = y_r$

2) Constraints (4.8)

From (4.2),

$$\sum_{b\in B} X_{ob} = 1 \qquad o\in O$$

$$X_{ob_1} + X_{ob_2} + \ldots + X_{ob_{|B|}} = 1 \qquad o\in O$$

$$X_{ob_1}\cdot\sum_{r\in R} Y_{b_1 r} + X_{ob_2}\cdot\sum_{r\in R} Y_{b_2 r} + \ldots + X_{ob_{|B|}}\cdot\sum_{r\in R} Y_{b_{|B|} r} = 1 \qquad o\in O$$

Since $\sum_{r\in R} Y_{br} = 1$

$$\sum_{r\in R} X_{ob_1}\cdot Y_{b_1 r} + \sum_{r\in R} X_{ob_2}\cdot Y_{b_2 r} + \ldots + \sum_{r\in R} X_{ob_{|B|}}\cdot Y_{b_{|B|} r} = 1 \qquad o\in O$$

$$\sum_{r\in R}(X_{ob_1}\cdot Y_{b_1 r} + X_{ob_2}\cdot Y_{b_2 r} + \ldots + X_{ob_{|B|}}\cdot Y_{b_{|B|} r}) = 1 \qquad o\in O$$

$$\sum_{r\in R}\sum_{b\in B}(X_{ob}\cdot Y_{br}) = 1 \qquad o\in O$$

By definition, $\sum_{b\in B} X_{ob}\cdot Y_{br} = x_{or}$

$$\sum_{r\in R} x_{or} = 1 \qquad o\in O$$

3) Constraints (4.9)

From (4.3),

$$\begin{cases} \sum_{o\in O} Q_o \cdot X_{ob} \le CAPA, & b \in B_1 = \{b \mid Y_{br_1} = 1, b \in B\} \\ \sum_{o\in O} Q_o \cdot X_{ob} \le CAPA, & b \in B_2 = \{b \mid Y_{br_2} = 1, b \in B\} \\ \quad\vdots & \\ \sum_{o\in O} Q_o \cdot X_{ob} \le CAPA, & b \in B_{|R|} = \{b \mid Y_{br_{|R|}} = 1, b \in B\} \end{cases}$$

Assume that all b's have at least one order

$$\begin{cases} \sum_{o\in O} Q_o \cdot X_{ob} \cdot Y_{br_1} \le CAPA \cdot Y_{br_1}, & b \in B_1 \\ \sum_{o\in O} Q_o \cdot X_{ob} \cdot Y_{br_2} \le CAPA \cdot Y_{br_2}, & b \in B_2 \\ \quad\vdots & \\ \sum_{o\in O} Q_o \cdot X_{ob} \cdot Y_{br_{|R|}} \le CAPA \cdot Y_{br_{|R|}}, & b \in B_{|R|} \end{cases}$$

By definition, $Y_{b_r r} = 1$

$$\begin{cases} \sum_{b\in B_1}\sum_{o\in O} Q_o \cdot X_{ob} \cdot Y_{br_1} \le \sum_{b\in B_1} CAPA \cdot Y_{br_1}, \\ \sum_{b\in B_2}\sum_{o\in O} Q_o \cdot X_{ob} \cdot Y_{br_2} \le \sum_{b\in B_2} CAPA \cdot Y_{br_2}, \\ \quad\vdots \\ \sum_{b\in B_{|R|}}\sum_{o\in O} Q_o \cdot X_{ob} \cdot Y_{br_{|R|}} \le \sum_{b\in B_{|R|}} CAPA \cdot Y_{br_{|R|}}, \end{cases}$$

Aggregate constraints indexed by r.
The new constraints become weaker, thus the new model becomes a relaxation of the original constraints

$$\sum_{o\in O} Q_o \cdot \sum_{b\in B_r} X_{ob} \cdot Y_{br} \le CAPA \cdot \sum_{b\in B_r} Y_{br} \qquad r \in R$$

$$\sum_{o\in O} Q_o \cdot x_{or} \le CAPA \cdot y_r, \qquad r \in R$$

By definition, $\sum_{b\in B} X_{ob} \cdot Y_{br} = x_{or}$ and $\sum_{b\in B} Y_{br} = y_r$

4) Constraints (4.10)

From (4.6),

$$\begin{cases} X_{ob} \cdot OA_{oa} \le \sum_{r \in R} RA_{ra} Y_{br}, & b \in B_1 = \{b \mid Y_{br_1} = 1, b \in B\}, a \in A \\ X_{ob} \cdot OA_{oa} \le \sum_{r \in R} RA_{ra} Y_{br}, & b \in B_2 = \{b \mid Y_{br_2} = 1, b \in B\}, a \in A \\ \quad \vdots & \\ X_{ob} \cdot OA_{oa} \le \sum_{r \in R} RA_{ra} Y_{br}, & b \in B_{|R|} = \{b \mid Y_{br_{|R|}} = 1, b \in B\}, a \in A \end{cases}$$

Assume that all b's have at least one order

$$\begin{cases} X_{ob} \cdot OA_{oa} \le RA_{r_1 a}, & b \in B_1, a \in A \\ X_{ob} \cdot OA_{oa} \le RA_{r_2 a}, & b \in B_2, a \in A \\ \quad \vdots & \\ X_{ob} \cdot OA_{oa} \le RA_{r_{|R|} a},, & b \in B_{|R|}, a \in A \end{cases}$$

By definition, $Y_{b_r r} = 1$

$$\begin{cases} X_{ob} \cdot OA_{oa} \cdot Y_{br_1} \le RA_{r_1 a} \cdot Y_{br_1}, & b \in B_1, a \in A \\ X_{ob} \cdot OA_{oa} \cdot Y_{br_2} \le RA_{r_2 a} \cdot Y_{br_2}, & b \in B_2, a \in A \\ \quad \vdots & \\ X_{ob} \cdot OA_{oa} \cdot Y_{br_{|R|}} \le RA_{r_{|R|} a} \cdot Y_{br_{|R|}}, & b \in B_{|R|}, a \in A \end{cases}$$

Since $Y_{br} \ge 0$, inequalities hold

$$\begin{cases} \sum_{b \in B_1} X_{ob} \cdot OA_{oa} \cdot Y_{br_1} \le \sum_{b \in B_1} RA_{r_1 a} \cdot Y_{br_1}, & a \in A \\ \sum_{b \in B_2} X_{ob} \cdot OA_{oa} \cdot Y_{br_2} \le \sum_{b \in B_2} RA_{r_2 a} \cdot Y_{br_2}, & a \in A \\ \quad \vdots & \\ \sum_{b \in B_{|R|}} X_{ob} \cdot OA_{oa} \cdot Y_{br_{|R|}} \le \sum_{b \in B_{|R|}} RA_{r_{|R|} a} \cdot Y_{br_{|R|}}, & a \in A \end{cases}$$

Aggregate constraints indexed by r. The new constraints become weaker, thus the new model becomes a relaxation of the original constraints

$$\begin{cases} OA_{oa} \cdot \sum_{b \in B_1} X_{ob} \cdot Y_{br_1} \le RA_{r_1 a} \cdot \sum_{b \in B_1} Y_{br_1}, & a \in A \\ OA_{oa} \cdot \sum_{b \in B_2} X_{ob} \cdot Y_{br_2} \le RA_{r_2 a} \cdot \sum_{b \in B_2} Y_{br_2}, & a \in A \\ \quad \vdots & \\ OA_{oa} \cdot \sum_{b \in B_{|R|}} X_{ob} \cdot Y_{br_{|R|}} \le RA_{r_{|R|} a} \cdot \sum_{b \in B_{|R|}} Y_{br_{|R|}}, & a \in A \end{cases}$$

$$OA_{oa} \cdot \sum_{b \in B_r} X_{ob} \cdot Y_{br} \le RA_{ra} \cdot \sum_{b \in B_r} Y_{br}, \qquad r \in R, a \in A$$

$$OA_{oa} \cdot y_{or} \le RA_{ra} \cdot y_r, \qquad r \in R, a \in A$$

By definition, $\sum_{b \in B} X_{ob} \cdot Y_{br} = x_{or}$ and $\sum_{b \in B} Y_{br} = y_r$

A.2 CLARKE AND WRIGHT II ALGORITHM (CLARKE AND WRIGHT, 1964; DE KOSTER *ET AL.*, 1999)

Step 1. Obtain the distance savings s_{ij} for all possible order pairs i,j when two orders are grouped, given the capacity of the pick device.

Step 2. Sort the savings in decreasing order.

Step 3. Select the pair with the highest savings. In the case of a tie, select a random pair.

Step 4. Combine both "orders" to form a new cluster, if allowed by the pickers' capacity. If not, choose the next combination on the list and repeat step 4.

Step 5. If not all order combinations have been included in a route, proceed with Step 1. In the calculation, all clusters are considered as orders. Otherwise, finish.

A.3 SIMULATED ANNEALING PROCEDURE FOR RBP

This section presents a simulated annealing algorithm for order batching to obtain an improved solution from RBP and summarizes the experimental results.

Simulated annealing procedure

Simulated annealing is widely used in sequencing problems and order batching problems. We employ an algorithm described in Pinedo (1995). For a batching situation, a batching solution is given as BS_1 and its total retrieval time as Obj(BS_1). The major characteristic is to accept a worse solution (BS) while progressively searching for a better candidate solution of solution BS_i with probability P(BS_i,BS) = e^((Obj(BS_i)-Obj(BS)/ β_i), where β_i is referred to as the cooling parameter or temperature. To update the cooling parameter (β_i), we use a simple function a^i where $0<a<1$, $a \in R$ (see Pinedo (1995) in detail). Thus, the probability to admit an solution with a worse objective value is gradually decreases as iteration i cumulatively updates the cooling parameter (β_i) using a, i.e., $\beta_i = a*\beta_{i-1}$ where $i>1$ and $0<a<1$. To generate an initial solution (BS_1), RBP is used, which produces a solution that nearly minimizes the total retrieval distance. I_{max} is the maximum number of iterations. T is the updated temperature.

Step1. Set $i = 1$ and $T = a$.

Initialization

Obtain an initial feasible solution, BS_1

Set *Imax*

Set the best solution $BS^* = BS_1$

Step 2. Generate a new batch solution BS from BS_i, i.e. BS is the neighboring solution of BS_i.

If Obj(BS^*)<Obj(BS)<Obj(BS_i), set $BS_{i+1} = BS$;

Else If Obj(BS)<Obj(BS^*), set $BS^* = BS_{i+1} = BS$;

Else if Obj(BS)> Obj(BS_i), set $BS_{n+1} = BS$ with a probability of $e^{((\text{Obj}(BSi)-\text{Obj}(BS))/T)}$;

Otherwise, $BS_{i+1} = BS_i$

Step 3. Increase $i = i+ 1$.

Update the temperature $T = T * a$.

If $i = Imax$, then STOP; otherwise, go to Step 2.

The method for defining a neighborhood in a simulated annealing procedure is critical to effective implementation (Pinedo, 1995). A general two-exchange method is employed where a pair of orders is exchanged. This method appears in Gademann and Van de Velde (2005).

Experimental results

Table A1 summarizes the experimental results over two capacity scenarios. The pick-then-sort strategy has been assumed, which produced a relative large LU gap. Two different capacity constraints are tested. We use Imax=10000 and the $a=0.8$.The SA+RBP columns include experimental results by the simulated annealing procedure. The Impv (%) column stands for the percentage of the objective value gap between RBP and SA+RBP divided by the objective value by RBP.

Our experimental results exhibit a very small improvement of the travel distance. Such small improvements stem from the solution quality by RBP and the limitation in

the neighborhood search approach. The solutions by RBP are very close to optimal relative to an objective function that minimizes travel distance. Thus, there are only minimal gains to be achieved in terms of travel distance.

Table A1 The experimental results over SA + RBP

Capa	# orders	RBP ObjU	CPU	LU gap	SA + RBP ObjU	CPU	LU gap	Impv (%)
20	360	3076.1	32.40	2.81%	3076.1	32.55	2.81%	0.00%
	720	6043.7	68.29	2.88%	6037.0	69.36	2.77%	0.11%
	1080	9073.8	103.32	2.95%	9060.5	104.03	2.81%	0.15%
	1440	12038.7	215.64	3.30%	12024.1	216.30	3.18%	0.12%
30	360	2132.3	19.29	3.57%	2132.3	20.10	3.57%	0.00%
	720	4116.0	64.21	3.23%	4116.0	65.29	3.23%	0.00%
	1080	6141.1	76.32	3.03%	6140.3	77.83	3.02%	0.01%
	1440	8095.1	122.26	3.11%	8092.8	123.96	3.08%	0.03%

A.4 COMPUTATIONAL PERFORMANCE OVER OTHER ORDER PICKING PROFILES

The number of aisles

Table A2 compares the CW II and RBP. The cardinality of the route set was strongly dependent on the number of aisles. RPP-LP can only solve ~14-aisle or smaller instances. Thus, Table A2 does not include LB results and LU gaps. Instead, we use the following comparison:

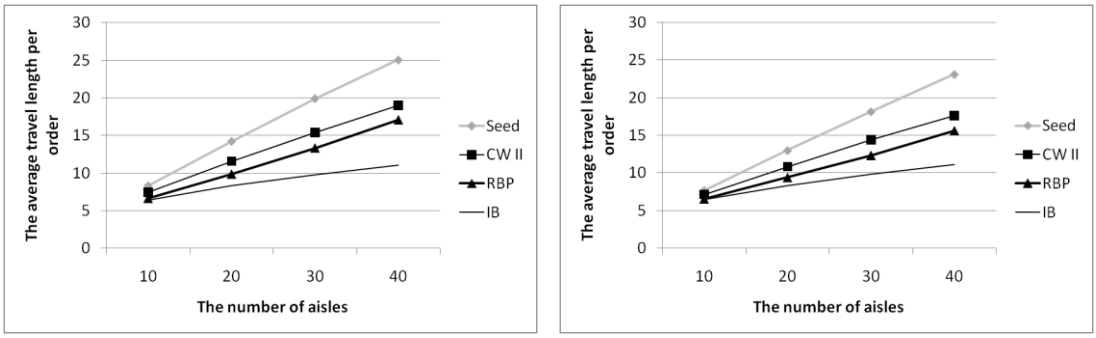
> RBP/CW: the ratio of ObjU to the objective function value of CW II. This measure is used where a lower bound is impossible.

RBP still dominated CW II in RBP/CW, but RBP required a long computational time as the number of aisles increased.

Table A2 The experimental results with the variation of the number of aisles

# orders	# aisles	CW II Obj	CPU	RBP ObjL	ObjU	CPU	# routes	RBP/CW
1080	10	8033.3	15.6	7175.0	7175.0	56.7	40.4	0.89
	20	12492.8	17.0	10647.5	10647.5	121.0	147.2	0.85
	30	16614.3	17.2	14379.6	14379.6	242.9	254.4	0.87
	40	20517.8	18.7	18418.0	18418.0	366.8	342.4	0.90
2160	10	15412.0	141.5	14186.6	14186.6	60.5	47.8	0.92
	20	23365.4	129.5	20287.7	20287.7	123.1	214.1	0.87
	30	31102.9	147.8	26587.4	26587.4	253.5	393.4	0.85
	40	37971.8	142.0	33637.3	33637.3	394.3	552.9	0.89

The route reduction step is not effective in the 40-aisle instance. As the number of routes increased, we modulated the truncation time limit to produce good solutions; specifically, 120 seconds, 180 seconds, and 240 seconds were allowed for 20-aisle, 30-aisle, 40-aisle instances. However, despite this increase in the truncation time limit, RBP's performance suffered loss in the objective values. Figure A1 illustrates the variations of the average travel length over different algorithms with respect to the number of aisles. The performance gap between CW II and RBP did not widen as shown in Figure A1 when the number of aisles was 40.



(a) (b)

Figure A1 The average travel length per order over the variation of the number of aisles: (a) the number of orders = 1080, and (b) the number of orders = 2160.

Storage policy

Table A3 and Figure A2 include the test results with different storage policies. Picking systems can operate under different storage pattern or storage policies As orders were scattered more evenly, all algorithms had longer travel distance. In particular, the computational time of RBP lengthened. The storage policy has an impact on the route set of RBP. More uniformly-stored items produce more elementary routes. Thus, the elementary route set becomes larger, and the number of combined routes also increases. A larger route set results in longer computational time.

Table A3 The experimental results with the variation of storage policies

# Orders	# aisles	CW II Obj	CPU	RBP ObjL	ObjU	CPU	# routes	RBP/CW
ABC	10	18000.4	140.8	16181.4	16181.4	60.7	63.7	0.90
=0.5:0.3:0.2	20	28926.6	130.9	24043.8	24043.8	128.4	340.5	0.83
	30	38950.5	134.7	33104.8	33104.8	278.7	581.6	0.85
	40	47811.1	151.0	42441.0	42441.0	568.7	747.8	0.89
Random	10	22125.6	121.3	19310.4	19310.4	60.9	83.0	0.87
Storage	20	37872.4	126.9	34535.9	34535.9	150.0	554.5	0.91
	30	51343.2	138.2	46266.7	46266.7	347.9	796.4	0.90
	40	63794.8	155.1	57098.8	57098.8	699.6	901.9	0.90

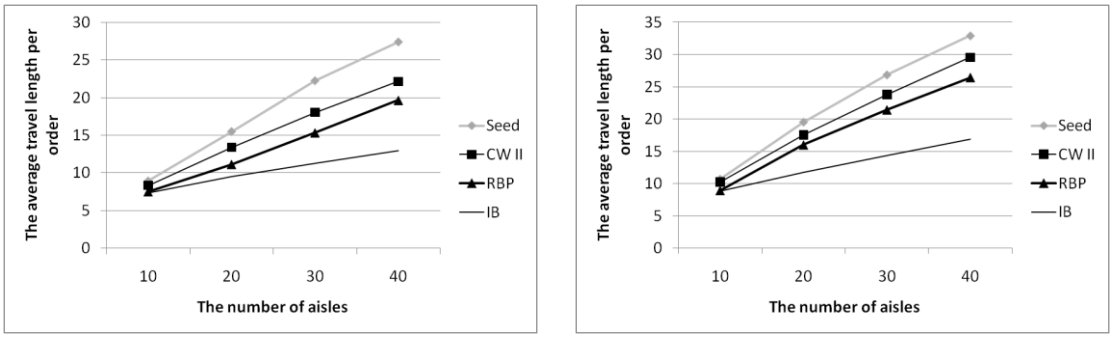


Figure A2 The average travel length per order over the variation of the storage policy (# orders = 1080): (a) ABC ratio = 0.5:0.3:0.2; and (b) random storage policy.

APPENDIX B

SUPPLEMENTARY EXAMPLES, PROOF, VALIDATION, ALGORITHM, AND RESULTS DISCUSSED IN CHAPTER V

B.1 PICKER BLOCKING MODEL OF PICK:WALK TIME = 1:1 IN A NARROW-AISLE USING PICK AND WALK TASKS

Let D_t denote the distance between picker 1 and picker 2 at time t. Given the pick:walk time ratio as 1:1, the distance d can be expressed as

(n+(picker 1 position)−(picker 2 position)) mod n

and ranges from 1 to n-1. To establish a Markov property, we can condition on the either pick or walk state of a previous distance and connect to the either pick or walk state of a posterior distance. Since there are two pickers and they can conduct either pick or walk, four sub states are available: d_{pp}, d_{wp}, d_{pw}, d_{ww} depending on the actions of pickers 1 and 2 and distance d, where p stands for a picking, w for a walking. In particular, two states, 1_{wp} and $n\text{-}1_{pw}$ are augmented into "blocked" because one picker attempts to walk toward one occupied pick face. Then all states can be described as the states [1_{pp}, 1_{pw}, blocked, 1_{ww}, 2_{pp}, 2_{pw}, 2_{wp}, 2_{ww}, . . . , $(n-1)_{pp}$, blocked, $(n-1)_{wp}$, $(n-1)_{ww}$]. When multiple-picks are allowed, their transition probability forms a new relationship. Figure A3 illustrates the transitions.

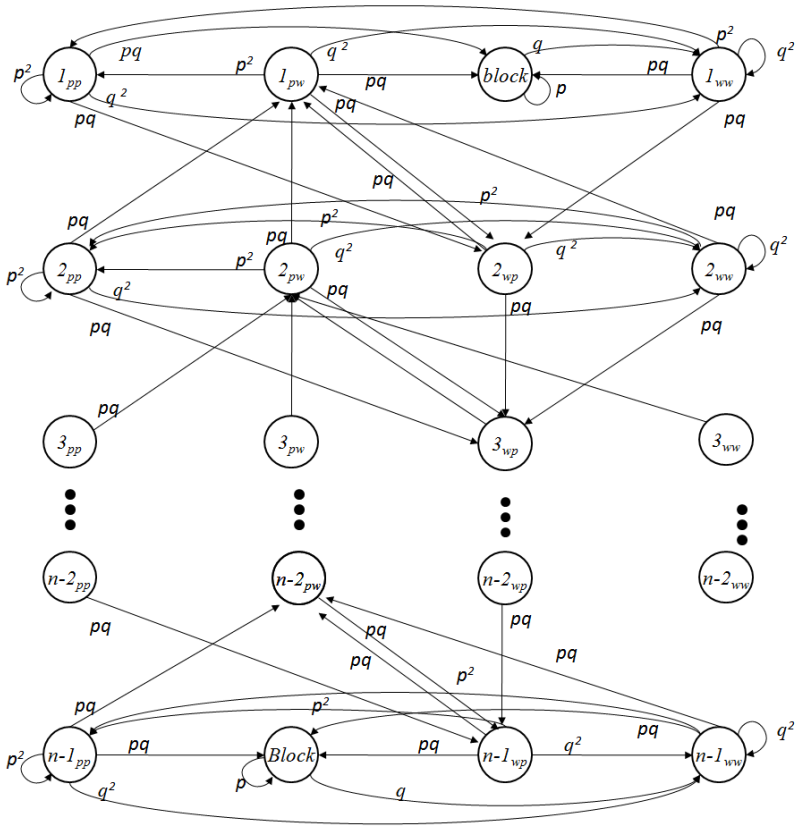


Figure A3. State space and transitions for the Markov chain model when the picking time equals travel time.

The resulting transition matrix is:

$$A = \begin{bmatrix} D_1 & U_1 & 0 & \cdots & 0 \\ L & D & U & \ddots & \vdots \\ 0 & \ddots & \ddots & \ddots & 0 \\ \vdots & \ddots & L & D & U \\ 0 & \cdots & 0 & L_{n-1} & D_{n-1} \end{bmatrix}$$

where

$$D_1 = \begin{bmatrix} p^2 & 0 & pq & q^2 \\ p^2 & 0 & pq & q^2 \\ 0 & 0 & p & q \\ p^2 & 0 & pq & q^2 \end{bmatrix}, D = \begin{bmatrix} p^2 & 0 & 0 & q^2 \\ p^2 & 0 & 0 & q^2 \\ p^2 & 0 & 0 & q^2 \\ p^2 & 0 & 0 & q^2 \end{bmatrix}, D_{n-1} = \begin{bmatrix} p^2 & pq & 0 & q^2 \\ 0 & p & 0 & q \\ p^2 & pq & 0 & q^2 \\ p^2 & pq & 0 & q^2 \end{bmatrix}$$

$$U_1 = \begin{bmatrix} 0 & 0 & pq & 0 \\ 0 & 0 & pq & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & pq & 0 \end{bmatrix}, U = \begin{bmatrix} 0 & 0 & pq & 0 \\ 0 & 0 & pq & 0 \\ 0 & 0 & pq & 0 \\ 0 & 0 & pq & 0 \end{bmatrix}, L = \begin{bmatrix} 0 & pq & 0 & 0 \\ 0 & pq & 0 & 0 \\ 0 & pq & 0 & 0 \\ 0 & pq & 0 & 0 \end{bmatrix}, L_{n-1} = \begin{bmatrix} 0 & pq & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & pq & 0 & 0 \\ 0 & pq & 0 & 0 \end{bmatrix}$$

Similar to Gue *et al.* (2006), we obtain the following v which satisfies $vA=v$.

$$v = \left[\overbrace{p^2, pq, p, q}^{d=1}, \overbrace{p^2, p, p, 1-2p-p^2}^{d=2}, \cdots, \overbrace{p^2, p, p, 1-2p-p^2}^{d=n-2}, \overbrace{p^2, p, pq, q}^{d=n-1} \right]$$

We can scale the stationary density using $\|v\|$.

$$\begin{aligned} \|v\| &= (n-3)\left(p^2 + p + p + 1 - 2p - p^2\right) + 2\left(p^2 + pq + p + q\right) \\ &= (n-3) + 2(p+1) \\ &= n + 2p - 1 \end{aligned}$$

The blocking probability of picker 2 is

$$b_{1:1}^{m}(2) = \frac{v_3}{\|v\|} = \frac{p}{n+2p-1}$$

B.2 PICKER BLOCKING WHEN PICK:WALK TIME = 1:1 IN A WIDE-AISLE

A Markov property is applied in pick:walk time = 1:1 for a wide-aisle situation. When multiple picks are allowed, their transition probability forms a transition diagram as illustrated in Figure A4.

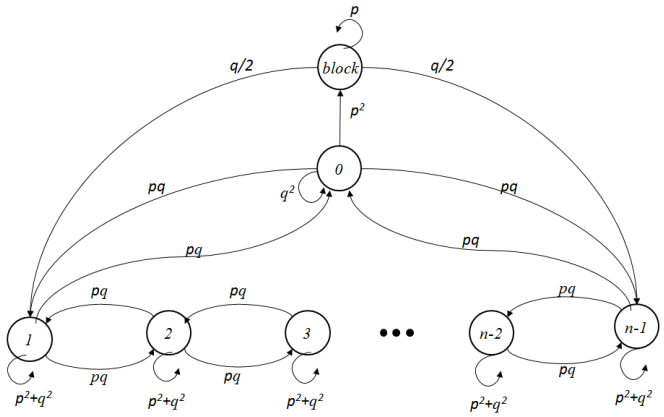


Figure A4. State space and transitions for the Markov chain model when picking time equals travel time in a wide-aisle situation with multiple-pick allowance.

The resulting transition matrix which has (n+1) x (n+1) is:

$$A = \begin{bmatrix} p & \frac{q}{2} & 0 & \cdots & 0 & 0 & \frac{q}{2} \\ p^2 & q^2 & pq & \cdots & 0 & 0 & pq \\ 0 & pq & p^2+q^2 & \ddots & \ddots & 0 & 0 \\ \vdots & \vdots & \ddots & \ddots & \ddots & \vdots & \vdots \\ 0 & 0 & \ddots & \ddots & p^2+q^2 & pq & 0 \\ 0 & 0 & 0 & \cdots & pq & p^2+q^2 & pq \\ 0 & pq & 0 & \cdots & 0 & qq & p^2+q^2 \end{bmatrix}$$

Stationary distribution

We obtain the following v which satisfies vA = v:

$$v = \left[1, \frac{q}{p^2}, \frac{1+q}{2p^2}, \cdots, \frac{1+q}{2p^2}\right]$$

We can scale the stationary density using $\|v\|$ to obtain a stationary probability.

From v above, we have:

$$\begin{aligned} \|v\| &= 1 + \frac{q}{p^2} + (n-1)\frac{1+q}{2p^2} = \frac{2p^2 + 2q + (n-1)(1+q)}{2p^2} \\ &= \frac{2p^2 + 2q + (n-1)(2-p)}{2p^2} \\ &= \frac{2p^2 + 2(1-p) + 2n - 2 - pn + p}{2p^2} \\ &= \frac{2p^2 - p + n(2-p)}{2p^2} \end{aligned}$$

The blocking probability of blocking state of a picker is

$$b(2) = \frac{v_{1^*}}{\|v\|} = \frac{\frac{1}{2}}{\frac{2p^2 - p + n(2-p)}{2p^2}} = \frac{p^2}{2p^2 - p + n(2-p)} \qquad \text{(A1)}$$

B.3 PROOF OF PROBABILITY WITHOUT PASSING

$$
\begin{aligned}
g(y) &= P(Y_t = y) \\
&= \sum_{x=0}^{\infty} P(Y_t = x+y)P(Y_t = x) = \sum_{x=0}^{\infty} f(x+y)f(x) \\
&= \sum_{x=0}^{\infty} \left(q^{x+y}p\right)\left(q^{x}p\right) = \sum_{x=0}^{\infty} q^{2x+y}p^2 \\
&= q^y p^2 \sum_{x=0}^{\infty} q^{2x} = q^y p^2 \left(\frac{1}{1-q^2}\right) = \frac{q^y p^2}{(1-q)(1+q)} \\
&= \frac{pq^y}{1+q}
\end{aligned}
$$

B.4 COMPARISON OF ANALYTICAL AND SIMULATION MODELS

Table A4 summarizes the results to validate the new analytical models and our simulation models. The 1:1 analytical model is already identical to the model by Parikh and Meller (2010). The results by the 1:0 analytical models also experienced 0.032~0.170% error gap compared to the results of Parikh and Meller (2010). The gap between the performances of the simulation model and the analytical model is 0.01~0.33% in terms of the percentage of the difference of the percentage of time blocked (i.e., Diff % = (the percentage of time blocked by the analytical model – the percentage of time blocked by the simulation model)/(the percentage of the time blocked by the analytical model) * 100) except one instance. When picker blocking occurs rarely, for example when p = 0.05 in pick:walk time = 1:1, the simulation model gives a relatively higher difference. For other cases, the difference percentage is smaller than 0.33%. These results show that the analytical model can well estimate a multiple-pick blocking situation.

Table A4. Comparison of analytical and simulation results of the percentage of time blocked in a circular aisle (20 pick faces)

Probability	Pick:walk time =1:1			Pick:walk time =1:0		
p	Analytical	Simulation	Diff %	Analytical	Simulation	Diff %
0.05	0.2618	0.2580	1.43	33.8983	33.8823	0.05
0.1	0.5208	0.5225	-0.33	25.6410	25.6283	0.05
0.2	1.0309	1.0313	-0.03	17.2414	17.2454	-0.02
0.3	1.5306	1.5256	0.33	12.9870	12.9916	-0.04
0.4	2.0202	2.0186	0.08	10.4167	10.4181	-0.01
0.5	2.5000	2.5005	-0.02	8.6957	8.6871	0.10
0.6	2.9703	2.9655	0.16	7.4627	7.4567	0.08
0.7	3.4314	3.4243	0.21	6.5359	6.5327	0.05
0.8	3.8835	3.8749	0.22	5.8140	5.8007	0.23
0.9	4.3269	4.3154	0.27	5.2356	5.2224	0.25
0.95	4.5455	4.5491	-0.08	4.9875	4.9917	-0.08

B.5 CLARKE AND WRIGHT II ALGORITHM (CLARKE AND WRIGHT, 1964; DE KOSTER *ET AL.*, 1999)

Step 1. Obtain the distance savings s_{ij} for all possible order pairs i,j when two orders are grouped, given the capacity of the pick device.

Step 2. Sort the savings in decreasing order.

Step 3. Select the pair with the highest savings. In the case of a tie, select a random pair.

Step 4. Combine both orders to form a new cluster, if allowed by the pickers' capacity. If not, choose the next combination on the list and repeat Step 4.

Step 5. If all order combinations have not been included in a route, proceed with Step 1. In the calculation, all clusters are considered as orders. Otherwise, finish.

B.6 A HEURISTIC ROUTE-PACKING BASED ORDER BATCHING PROCEDURE (RBP)

RBP takes advantage of the traversal routing method. When traversal routing

methods are used, all possible routes can be constructed from the warehouse layout. Thus, given a batch, a best fit route can be selected as a bin-packing problem (called the route-selecting order batching model (RSB)).

RBP is composed of three steps:

Step 1. Identifies potential route sets.

Step 2. Solves the RPP model heuristically. The RSB model stated above simplifies the batching problem, but still contains partitioning constraints. A route-bin packing problem (RPP) is developed by assigning orders to routes directly, which can skip the partitioning stage. However, RPP is still computationally difficult, and thus we consider two further computational improvements: a partial route set and a truncated branch-and-bound approach.

Step 3. Restores a feasible solution from the infeasible solution by the relaxed model.

APPENDIX C

EXECUTABLE MIP FORMULATION FOR INDEXED BATCH MODEL

Decision variables

$D_{if}^{a}, CD_{if}^{a}, DI_{if}^{a}$, = the time delay of the i^{th} batch at pick face *f* in aisle *a*, its cumulative time delay, and its intermediate variable

$L_{if}^{a}, LI_{if}^{a}, LF_{i}^{a}$, = the leaving time of the i^{th} batch at pick face *f* in aisle *a*

Formulation

$$(LT+UT)\cdot NBV + \sum_{b\in B}\left(2\cdot NBA_b\cdot AH\cdot WT + 2\cdot RBA_b\cdot AW\right) + \sum_{a\in A}\sum_{b\in B} CD_{i|F^a|}^{a}$$

$$\sum_{b\in B} X_{ob} = 1, \qquad \forall\ o\in O,$$

$$\sum_{o\in O} X_{ob} \le CAPA, \qquad \forall\ b\in B,$$

$$BV_b \ge X_{ob} \qquad \forall\ o\in O, \forall\ b\in B,$$

$$BV_b \le \sum_{o\in O} X_{ob} \qquad \forall\ b\in B,$$

$$BV_b \ge BV_{b+1} \qquad \forall\ b\in B\setminus\{|B|\},$$

$$NBV = \sum_{b\in B} BV_b$$

$$BP_{bf} = PT\cdot \sum_{o\in O} X_{ob}\cdot OP_{bf}, \qquad \forall\ b\in B, \forall\ f\in F,$$

$$BA_{ba} \ge OAV_{oa}\cdot X_{ob} \qquad \forall\ o\in O, \forall\ b\in B, \forall\ a\in A,$$

$$\begin{cases} 2\cdot INT_{ba} + 1 - M\cdot(1-BA_{ba}) \le \sum_{k\in\{0,\ldots,a\}} BA_{bk} + a\cdot BA_{ba} \\ 2\cdot INT_{ba} + 1 + M\cdot(1-BA_{ba}) \ge \sum_{k\in\{0\ldots a\}} BA_{bk} + a\cdot BA_{ba} \end{cases} \qquad \forall\ a\in A, \forall\ b\in B$$

$$2 \cdot NBA_b = \sum_{a \in A} BA_{ba}, \qquad \forall\ b \in B,$$

$$RBA_b \geq a \cdot BA_{ba}, \qquad \forall\ b \in B,$$

$$RBA_b \leq \sum_{k \in \{a,\ldots,|A|\}} k \cdot \left(BA_{bk} + M\left(1 - BA_{bk}\right)\right), \qquad \forall\ b \in B,$$

$$CW_{i0}^0 \leq LT + ST_i + \sum_{k \in \{i+1,\ldots,NP\}} M \cdot \left(1 - AP_k\right) \qquad \forall\ i \leq NP,\ b \in B, \forall\ f = 0, a = 0$$

$$CW_{i0}^0 \geq LT + ST_i - \sum_{k \in \{i+1,\ldots,NP\}} M \cdot \left(1 - AP_k\right) \qquad \forall\ i \leq NP,\ b \in B, \forall\ f = 0, a = 0$$

$$CW_{i0}^0 \leq LT + CT_{i-k} + M \cdot \left(1 - AP_k\right) \qquad \forall\ k \leq NP, \forall\ i \in B, \forall\ f = 0, a = 0$$

$$CW_{i0}^0 \geq LT + CT_{i-k} - M \cdot \left(1 - AP_k\right) \qquad \forall\ k \leq NP, \forall\ i \in B, \forall\ f = 0, a = 0$$

$$CW_{if}^a \leq BAC_{j,a-1} + AW + M \cdot \left(1 - Y_{ij}^a\right) \qquad \forall\ i, j \in B, \forall\ f = 0, \forall a \in A \setminus \{0\},$$

$$CW_{if}^a \geq BAC_{j,a-1} + AW - M \cdot \left(1 - Y_{ij}^a\right) \qquad \forall\ i, j \in B, \forall\ f = 0, \forall a \in A \setminus \{0\},$$

$$CW_{if}^a = WT \cdot AV_i^a + CW_{i,f-1}^a \qquad \forall\ i, j \in B, \forall\ f \in F^a, \forall a \in A \setminus \{0\},$$

$$CP_{if}^a = P_{if}^a + CP_{b,f-1}^a, \qquad \forall\ i \in B, \forall\ f \in F^a, \forall a \in A,$$

$$CD_{if}^a = D_{if}^a + CD_{i,f-1}^a, \quad \left(CD_{i0}^a = D_{i0}^a\right) \qquad \forall\ i \in M, \forall\ f \in F^a \cup \{0\}, \forall a \in A,$$

$$DX_{if}^a = \begin{cases} L_{i-1,f+1}^a + \tau - CW_{if}^a - AE & \text{if } f = 0 \\ L_{i-1,f+1}^a + \tau - CP_{if}^a - CW_{if}^a - CD_{i,f-1}^a - WT & \text{if } f > 0 \text{ and } f < F^a \\ 0 & \text{otherwise} \end{cases} \qquad \forall\ i \in B, \forall\ f \in F^a \cup \{0\}, \forall a \in A,$$

$$DX_{if}^a \leq M \cdot DF_{if}^a$$

$$DX_{if}^a \geq M \cdot \left(DF_{if}^a - 1\right)$$

$$DI_{if}^a \leq DX_{if}^a + M \cdot \left(1 - DF_{if}^a\right)$$

$$DI_{if}^a \geq DX_{if}^a - M \cdot \left(1 - DF_{if}^a\right)$$

$$DI_{if}^a \leq M \cdot DF_{if}^a$$

$$DI_{if}^a \geq -M \cdot DF_{if}^a$$

$$D_{if}^a \leq M \cdot AV_i^a \qquad \forall\ i \in B, \forall\ f \in F^a \cup \{0\}, \forall a \in A,$$

$$D_{if}^{a} \geq -M \cdot AV_{i}^{a} \qquad \forall\ i \in B, \forall\ f \in F^{a} \cup \{0\}, \forall a \in A,$$

$$D_{if}^{a} \leq DI_{if}^{a} + M \cdot \left(1 - AV_{i}^{a}\right) \qquad \forall\ i \in B, \forall\ f \in F^{a} \cup \{0\}, \forall a \in A,$$

$$D_{if}^{a} \geq DI_{if}^{a} - M \cdot \left(1 - AV_{i}^{a}\right) \qquad \forall\ i \in B, \forall\ f \in F^{a} \cup \{0\}, \forall a \in A,$$

$$LI_{if}^{a} = \begin{cases} CW_{if}^{a} + CD_{if}^{a} & \text{if } f = 0 \\ CP_{if}^{a} + CW_{if}^{a} + CD_{if}^{a} & \text{if } f > 0 \end{cases} \qquad \forall\ i \in B, \forall\ f \in F^{a} \cup \{0\}, \forall a \in A,$$

$$L_{if}^{a} \leq L_{i-1,f}^{a} + M \cdot AV_{i}^{a} \qquad \forall\ i \in B, \forall\ f \in F^{a} \cup \{0\}, \forall a \in A,$$

$$L_{if}^{a} \geq L_{i-1,f}^{a} - M \cdot AV_{i}^{a} \qquad \forall\ i \in B, \forall\ f \in F^{a} \cup \{0\}, \forall a \in A,$$

$$L_{if}^{a} \leq LI_{if}^{a} + M \cdot \left(1 - AV_{i}^{a}\right) \qquad \forall\ i \in B, \forall\ f \in F^{a} \cup \{0\}, \forall a \in A,$$

$$L_{if}^{a} \geq LI_{if}^{a} - M \cdot \left(1 - AV_{i}^{a}\right) \qquad \forall\ i \in B, \forall\ f \in F^{a} \cup \{0\}, \forall a \in A,$$

$$\sum_{j \in B} Y_{ij}^{a} = BV_{i} \qquad \forall\ i \in B, \forall\ a \in A \setminus \{1,2\},$$

$$\sum_{i \in B} Y_{ij}^{a} = BV_{j} \qquad \forall\ j \in B, \forall\ a \in A \setminus \{1,2\},$$

$$P_{if}^{a} = BP_{if} \qquad \forall\ i \in B, \forall\ f \in F^{a}, \forall a \in \{1,2\},$$

$$P_{if}^{a} \leq BP_{j,a\left|F^{a}\right|+f} + M \cdot \left(1 - Y_{ij}^{a}\right), \qquad \forall\ i, j \in B, \forall\ f \in F^{a}, \forall a \in A \setminus \{1,2\},$$

$$P_{if}^{a} \geq BP_{j,a\left|F^{a}\right|+f} - M \cdot \left(1 - Y_{ij}^{a}\right), \qquad \forall\ i, j \in B, \forall\ f \in F^{a}, \forall a \in A \setminus \{1,2\},$$

$$AV_{i}^{a} = BA_{ia}, \qquad \forall\ i \in B, \forall a \in \{1,2\},$$

$$AV_{i}^{a} \leq BA_{ja} + M \cdot \left(1 - Y_{ij}^{a}\right), \qquad \forall\ i, j \in B, \forall a \in A \setminus \{1,2\},$$

$$AV_{i}^{a} \geq BA_{ja} - M \cdot \left(1 - Y_{ij}^{a}\right), \qquad \forall\ i, j \in B, \forall a \in A \setminus \{1,2\},$$

$$BAC_{b0} \leq CW_{b0}^{0} + M \cdot BA_{b0}, \qquad \forall\ b \in B, \forall a \in A \setminus \{1\},$$

$$BAC_{b0} \geq CW_{b0}^{0} - M \cdot BA_{b0}, \qquad \forall\ b \in B, \forall a \in A \setminus \{1\},$$

$$BAC_{b0} \leq L_{b\left|F^{0}\right|}^{0} + AE + M \cdot (1 - BA_{b0}), \qquad \forall\ b \in B, \forall a \in A \setminus \{1\},$$

$$BAC_{b0} \geq L_{b\left|F^{0}\right|}^{0} + AE - M \cdot (1 - BA_{b0}), \qquad \forall\ b \in B, \forall a \in A \setminus \{1\},$$

$$BAC_{ba} \leq BAC_{b,a-1} + AW + M \cdot BA_{ba}, \qquad \forall\ b \in B, \forall a \in A \setminus \{1\},$$

$$BAC_{ba} \geq BAC_{b,a-1} + AW - M \cdot BA_{ba}, \qquad \forall\ b \in B, \forall a \in A \setminus \{1\},$$

$$BAC_{ba} \leq LF_{b}^{a} + WT + M \cdot (1 - BA_{ba}), \qquad \forall\ b \in B, \forall a \in A \setminus \{1\},$$

$$BAC_{ba} \geq LF_{b}^{a} + WT - M \cdot (1 - BA_{ba}), \qquad \forall\ b \in B, \forall a \in A \setminus \{1\},$$

$$LF_{b}^{a} \leq L_{i,\left|F^{a}\right|}^{a} + M \cdot (1 - Y_{ib}^{a}), \qquad \forall\ i, b \in B, \forall a \in A \setminus \{1\},$$

$$LF_{b}^{a} \geq L_{i,\left|F^{a}\right|}^{a} - M \cdot (1 - Y_{ib}^{a}), \qquad \forall\ i, b \in B, \forall a \in A \setminus \{1\},$$

$$BAC_{j,a-1} - M \cdot (1 - Y_{ij}^{a}) \qquad \forall\ i, j \in B, \forall a \in A \setminus \{1,2\},$$

$$\sum_{j \in B} Z_{ij} = BV_{i} \qquad \forall\ i \in B,$$

$$\sum_{i \in B} Z_{ij} = BV_{j} \qquad \forall\ i \in B,$$

$$CT_{i} \leq BAC_{j,|A|-1} - (|A| - 1 - RBA_{i}) \cdot AW + RBA_{j} \cdot AW + UT + M \cdot (1 - Z_{ij}) \qquad \forall\ i, j \in B,$$

$$CT_{i} \geq BAC_{j,|A|-1} - (|A| - 1 - RBA_{i}) \cdot AW + RBA_{j} \cdot AW + UT - M \cdot (1 - Z_{ij}) \qquad \forall\ i, j \in B,$$

$$CT_{i} \leq CT_{i+1} + M \cdot (1 - BV_{i+1}) \qquad \forall\ i \in B \setminus \{|B|\},$$

APPENDIX D

SUPPLEMENTARY FORMULATIONS AND PROOFS DISCUSSED IN CHAPTER VII

D.1 PACKING PROBLEM

The goal is to minimize the number of batches (A2). Y_b is 0 if batch b is selected and 0 otherwise. (A3) forces one order to be assigned once. (A4) is used to meet a capacity constraint if necessary.

$$\text{Min} \quad \sum_{b \in B} Y_b \tag{A2}$$

s.t.

$$\sum_{b \in B} X_{ob} = 1 \qquad \forall\, o \in O, \tag{A3}$$

$$\sum_{o \in O} OS_o \cdot X_{ob} \le CAPA \cdot Y_b \qquad \forall\, b \in B, \tag{A4}$$

D.2 INDEXED BATCHING MODEL (IBM) FOR BUCKET BRIGADE ORDER PICKING

Parameters and decision variables

An OPS has a linear aisle with $|F|$ pick faces. The pick faces are numbered 1 to F. L/U stations are numbered 0 and F+1, respectively. The forward travel time between neighboring pick faces is WT. The backward travel time between neighboring pick faces is BW. The walk time from 0 to F+1 is equal to WT*($|F|$+1) = AH. The L/U stations are

located in the front and rear of the aisle.

NP pickers work in the OPS, and the OPS is forced to assign all pickers. The number of batches is not given, although the number of batches must be smaller than the number of orders. Generally, the number of batches is greater than the number of pickers. Two batching picking policies — pick-then-sort policy and sort-while-pick policy — are considered; the policy impacts cart capacity. When a batch is completed, a new batch enters the system. Its entrance time is updated based on the backward walk time and the expected hand-off delay. All pickers are available initially.

Diverse decision variables are associated with the indexed order batching problem. Fundamentally, orders are assigned to batches and their release orders through index variables (X_{oi}). The starting time of batches in a picker's second or more trips (CW_i) is updated. The overall procedure includes more variables.

Indices and parameters

F, f	=	the set of pick faces, its index $f \in F$
O, o	=	the set of orders, and its index $o \in O$
B, i	=	the set of batches, and its index $i \in B$
OP_{of}	=	the number of picks of order o at pick face f
OS_o	=	the number of picks in order o
ST_i	=	the starting time of i^{th} batch
$CAPA$	=	the capacity of a cart (batch size)
PT	=	the pick time to pick an item
WT	=	the forward walk time between two pick faces
BW	=	the backward walk time between two pick faces

$E[HO]$	=	the expected hand-off delay per occurrence
NP	=	the number of pickers
α	=	the weight on hand-off delay
γ	=	the time required for the transition between two batches in a pick face

Decision variables

X_{oi}	=	1 if order *o* enters the i^{th} order; 0 otherwise
P_{if}, CP_{if}	=	the pick time of the i^{th} batch at pick face *f*, and its cumulative pick time
D_{if}, CD_{if}	=	the time delayed of the i^{th} batch at pick face *f*, and its cumulative time delayed
CW_{if}	=	the cumulative walk time of the i^{th} batch to pick face *f*
CT_i	=	the completion time of the order which has finished at the i^{th} batch

The goal is to minimize total walk time + total time delayed (A5). *Walk time* is the sums of the travel times of all batches. The travel time of the i^{th} batch is the sum of the forward travel times (= *AH*WT*), the backward travel times (= *AH*BW*) if *i* >*PK*, and the hand-off time if *i* >*PK*. *DT* is obtained by summing the cumulative delay at the last pick face of all batches.

$$\text{Min} \quad AH \cdot WT \cdot |B| + [AH \cdot BW + \alpha \cdot (NP-1)E[HO]] \cdot (|B| - PK) + \sum_{b \in \{1,\cdots,NBV\}} CD_{i|F|} \tag{A5}$$

s.t.

$$\sum_{i \in B} X_{oi} = 1 \qquad \forall\, o \in O, \tag{A6}$$

$$\sum_{o \in O} OS_o \cdot X_{oi} \le CAPA \qquad \forall\, i \in B, \tag{A7}$$

$$P_{if} = PT \cdot \sum_{o \in O} X_{oi} \cdot OP_{of}, \qquad \forall\ i \in B, \forall\ f \in F, \qquad \text{(A8)}$$

$$CW_{if} = \begin{cases} ST_i & \text{if } i \le NP,\ f = 0 \\ CP_{i-NP,|F|} + CW_{i-NP,|F|} + CD_{i-NP,|F|} & \\ \qquad + BW \cdot |F| + \alpha \cdot (NP-1) E[HO] & \text{if } i > NP,\ f = 0 \\ CW_{i,f-1} + WT & f > 0 \end{cases} \qquad \forall\ i \in B, \forall\ f \in F \cup \{0\}, \qquad \text{(A9)}$$

$$CP_{if} = P_{if} + CP_{i,f-1}, \qquad \forall\ i \in B, \forall\ f \in F, \qquad \text{(A10)}$$

$$CD_{if} = D_{if} + CD_{i,f-1}, \quad (CD_{i0} = D_{i0}) \qquad \forall\ i \in B, \forall\ f \in F \cup \{0\} \qquad \text{(A11)}$$

$$D_{if} = \begin{cases} Max\left(CW_{i-1,f+1} + CD_{i-1,f+1} + \gamma - CW_i, 0\right) & \text{if } f = 0 \\ Max\left(\begin{array}{l} CP_{i-1,f+1} + CW_{i-1,f+1} + CD_{i-1,f+1} + \gamma \\ \qquad - CP_{if} - CW_{if} - CD_{if} - WT, 0 \end{array}\right) & f \in F^a \setminus \{0, |F|\} \\ 0 & \text{otherwise} \end{cases} \qquad \forall\ i \in B, \forall\ f \in F \cup \{0\} \qquad \text{(A12)}$$

An order cannot be separated (A6) and a batch should keep the capacity constraint (A7). (A7) is set for the item-based capacity. When there is order-based capacity, constant 1 replaces OS_o. As the release sequence is determined, the related variables are assigned. The pick time vector of batch i at pick face f is updated with batch j's pick time (A8). Constraints (A9) update CW_{if} at the loading station and pick faces. At the loading station, CW_{if} is determined using the pickers' available time (ST_i) or the completion time of the NP^{th} previous trip ($CP_{i\text{-}NP,|F|} + CW_{i\text{-}NP,|F|} + CD_{i\text{-}NP,|F|}$) + the returning time to the entrance. The starting time of batch NP+1 can be derived from the completion time of the first completed batch, because the first responsible picker for the first batch will be assigned to pick the NP+1 batch. Backward travel time and the expected hand-off delay are added. Constraints (A10) and (A11) calculate the cumulative pick time and delay time. Constraint (A12) calculates the time delayed (D_{if}) using the leaving time at pick face f. At an $f = 0$, the leaving time of batch i is determined

by $CW_{if} + CD_{if}$ since there is no pick operation. At a pick face (f>0), the leaving time is assigned with $CP_{if} + CW_{if} + CD_{if}$.

D.3 HAND-OFF MODEL

$$E[Y(t)] = E\left[Y(t) \mid S_{N(t)} = 0\right] \cdot \overline{F}(t) + \int_0^t E\left[Y(t) \mid S_{N(t)} = y\right] \cdot \overline{F}(t-y) \cdot dm(y),$$

$$\text{where } \overline{F}(t) = 1 - F(t),\ m(x) = \sum_{n=1}^{\infty} F_n(x)$$

$$E\left[Y(t) \mid S_{N(t)} = 0\right] = E\left[X - t \mid X > t\right]$$

$$E\left[Y(t) \mid S_{N(t)} = y\right] = E\left[X - (t-y) \mid X > t - y\right]$$

$$E[Y(t)] = E\left[X - t \mid X > t\right] \cdot \overline{F}(t) + \int_0^t E\left[X - (t-y) \mid X > t - y\right] \cdot \overline{F}(t-y) \cdot dm(y)$$

$$then,\ h(t) = E\left[X - t \mid X > t\right] \cdot \overline{F}(t)$$

$$= \frac{1}{E[PT]} \int u dF(u)) = \frac{E[PT^2]}{2E[PT]}$$

$$E[Y(t)] = \int_0^{\infty} E\left[X - t \mid X > t\right] \cdot \overline{F}(t) \cdot dt / \mu$$

$$= \int_0^{\infty} \int_t^{\infty} (x - t) \cdot dF(x) \cdot dt / \mu$$

$$= \int_0^{\infty} \int_0^{x} (x - t) \cdot dt \cdot dF(x) / \mu$$

$$= \int_0^{\infty} x^2 dF(x) / 2\mu$$

$$= E\left[X^2\right] / 2\mu$$

D.4 HAND-OFF DELAY WITH NO-HANDSHAKE MODE

$$E[Y'(t)] = \int_0^{\infty} E\left[X - t \mid X > t, X - t < \tau\right] \cdot \overline{F}(t) \cdot dt / \mu$$

$$= \int_0^{\infty} \int_t^{t+\tau} (x - t) \cdot dF(x) \cdot dt / \mu$$

$$= \int_0^{\tau} \int_0^{x} (x - t) \cdot dt \cdot dF(x) / \mu + \int_{\tau}^{\infty} \int_{x-t}^{t} (x - t) \cdot dt \cdot dF(x) / \mu$$

$$= \int_0^{\tau} \frac{x^2}{2} \cdot dF(x) / \mu + \int_{\tau}^{\infty} \frac{\tau^2}{2} \cdot dF(x) / \mu$$

$$= \left[\int_0^{\tau} x^2 \cdot dF(x) + \tau^2 \int_{\tau}^{\infty} \cdot dF(x) \right] / 2\mu$$

$$E[HO]_{\exp} = \frac{\mu+\mu}{2\mu} - \frac{\tau\left[-xe^{-\mu x} - e^{-\mu x} + \tau e^{-\mu x}\right]_{\tau}^{\infty}}{\mu}$$

$$= 1 - \frac{\tau e^{-\mu\tau}}{\mu^2}$$

$$E[HO]_{unif} = \frac{\left[\frac{x^3}{3}\right]_{0.5}^{1.5}}{2\mu} - \frac{\tau\left[\frac{x^2}{2} - \tau x\right]_{\tau}^{1.5}}{\mu}$$

$$E[HO]_{tri} = \frac{\frac{0.5^2 + 1.5^2 + 1.0^2 - 0.5\cdot 1.5 - 0.5\cdot 1 - 0.5\cdot 1}{18} + \mu^2}{2\mu}$$

$$- \frac{\frac{2\tau}{0.5}\left[-\frac{x^3}{3} + \frac{(1.5+\tau)}{2}x^2 - 1.5\tau x\right]_{1}^{1.5} + \frac{2\tau}{0.5}\left[\frac{x^3}{3} - \frac{(0.5+\tau)}{2}x^2 + 0.5\tau x\right]_{\tau,0.5<\tau<1}^{1}}{2\mu}$$

VITA

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