# PRESSURE DROP IN A PEBBLE BED REACTOR 

A Thesis<br>by<br>CHANGWOO KANG

Submitted to the Office of Graduate Studies of Texas A\&M University in partial fulfillment of the requirements for the degree of MASTER OF SCIENCE

August 2010

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Approved by:

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August 2010

Major Subject: Nuclear Engineering

ABSTRACT<br>Pressure Drop in a Pebble Bed Reactor. (August 2010)<br>Changwoo Kang, B.S., Korea Military Academy<br>Chair of Advisory Committee: Dr. Yassin A. Hassan

Pressure drops over a packed bed of pebble bed reactor type are investigated. Measurement of porosity and pressure drop over the bed were carried out in a cylindrical packed bed facility. Air and water were used for working fluids.

There are several parameters of the pressure drop in packed beds. One of the most important factors is wall effect. The inhomogeneous porosity distribution in the bed and the additional wetted surface introduced by the wall cause the variation of pressure drop. The importance of the wall effects and porosity can be explained by using different bed-to-particle diameter ratios. Four different bed-to-particle ratios were used in these experiments $\left(\mathrm{D} / \mathrm{d}_{\mathrm{p}}=19,9.5,6.33\right.$ and 3.65$)$.

A comparison is made between the predictions by a number of empirical correlations including the Ergun equation (1952) and KTA (by the Nuclear Safety Commission of Germany) (1981) in the literature. Analysis of the data indicated the importance of the bed-to-particle size ratios on the pressure drop. The comparison between the present and the existing correlations showed that the pressure drop of large bed-to-particle diameter ratios $\left(\mathrm{D} / \mathrm{d}_{\mathrm{p}}=19,9.5\right.$ and 6.33$)$ matched very well with the original KTA correlation. However the published correlations cannot be expected to predict accurate pressure drop
for certain conditions, especially for pebble bed with $D / d_{p}$ (bed-to-particle diameter ratio) $\leq 5$. An improved correlation was obtained for a small bed-to-particle diameter ratio by fitting the coefficients of that equation to experimental database.

## DEDICATION

This work is dedicated to the following:

I want to thank my parents and sisters for their understanding and encouragement. In addition, I dedicate it to the Korea Army that supported me for two years.

## ACKNOWLEDGEMENTS

I would like to thank my committee chair, Dr. Yassin A. Hassan, and my committee members, Dr. William H. Marlow and Dr. Annamalai Kalyan, for their guidance and support throughout the course of this research.

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## CHAPTER I

## INTRODUCTION

## The importance of research

Pebble Bed Modular Reactor (PBMR), developed in South Africa, is type of packed bed. Mechanisms of heat and mass transfer, and the flow and pressure drop of the fluid through the bed of beads are considered for design of PBMR. Among these factors, pressure drop in a pebble bed reactor is important for design of PBMR and related to the pumping power and cost. The right description of the pressure drop explains the energy requirements of the pumps and compressors. Therefore an accurate correlation of pressure drop is required for a wide range of Reynolds number in packed bed. However, fluid velocity and pressure profile cannot be obtained easily for such packed column, particularly if the flow is turbulent. For such systems, experimental data can be used to build correlations of dimensionless variables that can give pressure profile in packed column. In addition, the porosity of the bed is an important factor for these mechanisms. Because the porosity gives affection to the velocity of the wall flow. The pressure loss due to friction in packed beds is part of the total pressure loss. Therefore, in this work, it is chosen to show pressure drop correlation in packed beds.

This thesis follows the style of International Journal for Numerical Methods in Fluids.

## Review of literature

There have been two main approaches for developing friction factor expressions for packed columns. In one method the packed column is visualized as a bundle of tangled tubes of weird cross section; the theory is then developed by applying the previous results for single straight tubes to the collection of crooked tubes. In the second method the packed column is regarded as a collection of submerged objects, and the pressure drop is obtained by summing up the resistances of the submerged particles. The tube bundle theories have been somewhat more successful.

A variety of materials may be used for the packing in column: spheres, cylinders, Berl saddles, and so on. It is assumed throughout the following discussion that the packing is statistically uniform, so that there is no "channeling" (in actual practice, channeling frequently occurs, and then the development given here does not apply). It is further assumed that the diameter of the packing is contained, and that the column diameter is uniform.

The friction factor for the packed column is

$$
\begin{equation*}
f=\frac{1}{4}\left(\frac{d_{p}}{L}\right)\left(\frac{\Delta P}{\frac{1}{2} \rho v^{2}}\right) \tag{1}
\end{equation*}
$$

in which $L$ is the length of the packed column, $\mathrm{d}_{\mathrm{p}}$ is the effective particle diameter, and v is the superficial velocity. This is the volume flow rate divided by the empty column cross section.

$$
\begin{equation*}
\mathrm{v}=\frac{m}{\rho A} \tag{2}
\end{equation*}
$$

The pressure drop through a representative tube in the tube bundle model is written as

$$
\begin{equation*}
\Delta P=\frac{1}{2} \rho\langle v\rangle^{2}\left(\frac{L}{R_{h}}\right) f_{\text {tube }} \tag{3}
\end{equation*}
$$

in which the friction factor for a single tube, $f_{\text {tube }}$, is a function of the Reynolds number.

$$
\begin{equation*}
\operatorname{Re}_{h}=4 R_{h}<v>\frac{\rho}{\mu} \tag{4}
\end{equation*}
$$

When this pressure difference is substituted, then the following equation is derived.

$$
\begin{equation*}
f=\frac{1}{4} \frac{d_{p}}{R_{h}} \frac{\left\langle v>^{2}\right.}{v^{2}} f_{\text {tube }}=\frac{1}{4 \varepsilon^{2}} \frac{d_{p}}{R_{h}} f_{\text {tube }} \tag{5}
\end{equation*}
$$

In the second expression, we have introduced the void fraction, $\varepsilon$, the fraction of space in the column not occupied by the packing. Then $v=\langle v\rangle \varepsilon$, which results from the definition of the superficial velocity.

The hydraulic radius, $\mathrm{R}_{\mathrm{h}}$, can be expressed in terms of the void fraction, $\varepsilon$, and the wetted surface per unit volume of bed as follows:

$$
\begin{equation*}
R_{h}=\frac{\text { cross section available for flow }}{\text { wetted perimeter }}=\frac{\text { volume available for flow }}{\text { total wetted surface }}=\frac{\left(\frac{\text { volume of void }}{\text { volume of bed }}\right)}{\left(\frac{\text { wetted surface }}{\text { volume of bed }}\right)}=\frac{\varepsilon}{d} \tag{6}
\end{equation*}
$$

The quantity a is related to the "specific surface", $\mathrm{a}_{\mathrm{v}}$ (total particle surface per volume of particles) by

$$
\begin{equation*}
a_{v}=\frac{a}{1-\varepsilon} \tag{7}
\end{equation*}
$$

The quantity, $a_{v}$, is in turn used to define the mean particle diameter $d_{p}$ as follows:

$$
\begin{equation*}
d_{p}=\frac{6}{a_{v}} \tag{8}
\end{equation*}
$$

This definition is chosen because, for spheres of uniform diameter, $d_{p}$, is exactly the diameter of a sphere. From the last three expressions we find that the hydraulic radius is

$$
\begin{equation*}
R_{h}=\frac{d_{p} \varepsilon}{6(1-\varepsilon)} \tag{9}
\end{equation*}
$$

Then, the friction factor is written as

$$
\begin{equation*}
f=\frac{3}{2}\left(\frac{1-\varepsilon}{\varepsilon^{3}}\right) f_{\text {tube }} \tag{10}
\end{equation*}
$$

We now adapt this result to laminar and turbulent flows by inserting appropriate expressions for $f_{\text {tube }}$.
(a) For laminar flow, $\mathrm{f}_{\text {tube }}=16 / \mathrm{Re}_{\mathrm{h}}$. This is exact for circular tubes only. To account for the non-cylindrical surfaces and tortuous fluid paths encountered in typical packedcolumn operations, it has been found that replacing 16 by 100/3 allows the tube bundle model to describe the packed-column data. When this method expression is used for the tube friction facto, then the friction factor becomes

$$
\begin{equation*}
f=\frac{3}{2}\left(\frac{(1-\varepsilon)^{2}}{\varepsilon^{3}}\right) \frac{75}{d_{p} \rho v / \mu} \tag{11}
\end{equation*}
$$

This f is used to get pressure difference, then

$$
\begin{equation*}
\frac{\Delta p}{L}=150\left(\frac{\mu v}{d_{p}^{2}}\right)\left(\frac{(1-\varepsilon)^{2}}{\varepsilon^{3}}\right) \tag{12}
\end{equation*}
$$

Which is the Blake-Kozeny equation.
It is good for $\operatorname{Re}_{\mathrm{m}}=\frac{\rho v \mathrm{~d}_{\mathrm{p}}}{\mu(1-\varepsilon)}<10$ and $\varepsilon<0.5$
(b) For highly turbulent flow, a treatment similar to the above can be given. We begin again with the expression for the friction factor definition for flow in a circular tube. This time, however, it is noted that, for highly turbulent flow in tubes with any appreciable roughness, the friction factor is a function of the roughness only, and is independent of Reynolds number. If it is assumed that the tubes in all packed columns have similar roughness characteristics, then the value of $\mathrm{f}_{\text {tube }}$ may be taken to be the same constant for all systems. Taking $\mathrm{f}_{\text {tube }}=7 / 12$ proves to be an acceptable choice. When this is inserted into eq.(10), then

$$
\begin{equation*}
f=\frac{7}{8}\left(\frac{1-\varepsilon}{\varepsilon^{3}}\right) \tag{13}
\end{equation*}
$$

When this is substituted into eq.(1), then

$$
\begin{equation*}
\frac{\Delta p}{L}=\frac{7}{4}\left(\frac{\rho v^{2}}{d_{p}}\right)\left(\frac{1-\varepsilon}{\varepsilon^{3}}\right) \tag{14}
\end{equation*}
$$

which is the Burke-Plummer equation, valid for $R e_{m}=\frac{\rho v d_{p}}{\mu(1-s)}>1000$.
(c) For the transition region, after superposition of the pressure drop expressions for (a) and (b) above to get

$$
\begin{equation*}
\frac{\Delta p}{L}=180\left(\frac{\mu v}{d_{p}^{2}}\right)\left(\frac{(1-\varepsilon)^{2}}{\varepsilon^{3}}\right)+1.8\left(\frac{\rho v^{2}}{d_{p}}\right)\left(\frac{1-\varepsilon}{\varepsilon^{3}}\right) \tag{15}
\end{equation*}
$$

For very small v , this simplifies to the Blake-Kozeny equation, and for very large v , to the Burke-Plummer equation. Such empirical super-positions of asymptotes often lead to satisfactory results. Again, it is rearranged to form dimensionless groups:

$$
\begin{equation*}
\left(\frac{\Delta p \rho}{G^{2}}\right)\left(\frac{d_{p}}{L}\right)\left(\frac{\varepsilon^{3}}{1-\varepsilon}\right)=150\left(\frac{1-\varepsilon}{d_{p} G / \mu}\right)+\frac{7}{4}=\frac{150}{\operatorname{Re}_{m}}+\frac{7}{4} \tag{16}
\end{equation*}
$$

The most widely used correlation is the Ergun equation(1952) [1].

$$
\begin{equation*}
\frac{\Delta p}{L}=150\left(\frac{\mu v}{d_{p}^{2}}\right)\left(\frac{(1-\varepsilon)^{2}}{\varepsilon^{3}}\right)+1.75\left(\frac{\rho v^{2}}{d_{p}}\right)\left(\frac{1-\varepsilon}{\varepsilon^{3}}\right) \tag{17}
\end{equation*}
$$

This equation is comprised of the pressure drop as the sum of the pressure losses coming from the viscous energy loss and the inertial energy loss. Therefore it is valid for laminar, turbulent as well as transitional region. It is also very simple and convenient to use and gives good results for predicting the pressure drop. However, the coefficients (150 and 1.75) in the Ergun equation [1] are not constants and don't have physical meanings but depend on many factors such as the Reynolds number, the porosity, and particle shape. Moreover, the obtained pressure drop results from the Ergun equation [1] are mostly less than other's data in the low Reynolds number regimes $\left(\frac{\mathrm{Re}}{(1-\mathrm{s})} \leq 10\right)$. Otherwise, the Ergun's predictions are larger than some experimental data by other researchers. Plus, one of their limitation is that their equation is mainly applicable for spherical particles in the porosity range of $0.35 \sim 0.55$. Therefore, researchers are in
agreement with the fact that the values of the Ergun constants should be determined empirically for each bed and many have tried to make proper correlations.

One of the most used correlations for predicting pressure drop through the pebble bed type reactor is the $\mathrm{KTA}(1981)$ correlation [2]. They performed an extensive investigation to give an empirical correlation for the pressure loss coefficient due to friction. KTA correlation [2] is valid for wide range of Modified Reynolds number $\left(10^{\circ}<\right.$ $\left.\operatorname{Re}_{\mathrm{m}}<10^{5}\right)$. However their valid range of porosity $(\varepsilon)$ is from 0.36 to 0.42 .

There are several influencing parameters of the pressure drop in packed beds. One of the most important factors is wall effect. The inhomogeneous porosity distribution in the bed and the additional wetted surface introduced by the wall cause the variation of pressure drop. The opinions about the resultant wall effect are contradictory. Some researchers found increase of the pressure drop due to the wall effect. But others said they have obtained a reduction due to the wall effect. Many researchers have concluded the following: The pressure drop can be increased by wall friction or decreased by an increase in porosity near the wall based on the type of flow regime. In the lower Reynolds number regime, the wall friction is highly affected. In the high Reynolds number regime, the porosity effect is dominant [3]. Some other published paper on the influence of the tube to particle diameter ratio shows that the increasing pressure drop due to the wall effect are based on experiments under streamline flow conditions or in the transitional range[4],[5],[6],[7]. Otherwise, the decreasing pressure drop due to the wall effect is measured at high Reynolds numbers[4],[8],[9],[10]. The general conclusion is that the Ergun equation[1] (with average values of porosity and superficial
velocity) is applicable down to tube-to-particle-diameter ratios of $\mathrm{D} / \mathrm{d}_{\mathrm{p}}>10$. He tested with the tube-to-particle-diameter ratio higher than 10 , so the experimentally determined Ergun constants should not be affected by the reactor column wall.

Mehta and Hawley(1969)[7] redefined the equivalent diameter in Ergun equation[1] and introduced the modified Ergun equation to use for the finite beds with wall effects. The hydraulic radius is shown as a characteristic length of the packed bed and it should depend on the wall for small bed-to-particle diameter ratios. And they used the wall correction factor that explains the effect of the bed-to-particle diameter ratios on the hydraulic diameter. This factor is used for modifying the hydraulic diameter. However, their result is only valid for the limited Reynolds number regimes ( $\operatorname{Re}_{\mathrm{m}}<10$ ).
R.E Hicks(1970)[11] also said that two coefficients are not constants but they are the functions of Reynolds number. Also, he found a friction factor. It is not intended as a general equation for packing beds but emphasized that the Ergun equation [1] is not valid for values of Reynolds number less than 500 .

Reichelt(1972)[12] modified Mehta and Hawley's correlation[1] and redefined a wall modified hydraulic radius and corresponding wall modified parameters.

Macdonald et al.(1979)[13] also changed two coefficients of the Ergun's equation[1]. Moreover, using $\varepsilon^{\text {a.s. }}$, instead of $\varepsilon^{\text {a }}$ is considered to make better fit to the data point. He divided the various published model into three categories: phenomenological model, model based on the conduit flow (a. geometrical model, b. statistical model, c. model utilizing the complete Navier-Stokes equation), and models based on flow around
submerged objects. Their equation is also valid for only limited Reynolds number regime.
R.M. Fand and R. Thinakaran(1990)[14] expressed their correlation with the respect to the porosity and the flow parameters as functions of the bed-to-particle diameter ratios. But their experiments were limited within circular cylinders.

Comiti and Renaud(1989)[15] generated the correlation of a capillary type model. Their correlation for the pressure drop has two terms like many correlations. The first term of the model explains the wall friction in the pore as well as at the bed wall, while the second one accounts for the energy loss and wall effects. Each term of the equation of the model is shown as a function of three structural parameters that are the porosity, the dynamic specific surface area, and the tortuosity.

Foumeny et al.(1993)[8] provided a correlation for mean porosity in packed beds of spherical particles. They have mentioned that one of common source of error is the assumption the mean porosity of packed beds with spherical particles is nearly equal to 0.4. And they tried to solve the problem of existing pressure drop correlations that didn't account for the strong wall effect of the low bed-to-particle diameter ratio. In addition, their porosity equation follow a general rule, decreasing the particle size reduces the mean porosity of the bed and, therefore, increases the pressure drop. Their approach has followed from this paper.

Shijie Liu et al.(1994)[16] showed the fluid has different chances for mixing and less curvature effects is considered to the flow near the wall. A near the wall, the particle has less possibilities of fluid mixing due to less faces of incoming flow. They considered and
assumed that the mixing and the curvature effects are equally affected by the wall. The limitation of their equation is validation for limited Reynolds number.
R.E. Hayes(1995)[17] reported the Darcy law is available for low Reynolds number less than one. His new correlation for the permeability of a packed bed has been presented. Capillary and the cell models are applied for modeling pressure drop in porous media. The proposed porous micro structure of a square channel is not affordable for a physical model of a porous medium is filled with uniform spherical particles.

Eisfeld and Schnitzlein(2001)[3] made an improved correlation that accounts for the wall effect. For the inertial pressure loss term, they manipulated the coefficients of the wall correction factor. They mentioned the boundary layer theory that indicates the wall friction. The wall friction factor is restricted to a small boundary layer at high Reynolds numbers and it reaches further into the reactor at low Reynolds numbers. They concluded that the pressure drop can be increased by wall friction or decreased by an increase in porosity near the wall. In the low Reynolds number regime, the wall friction effects in more important and it causes the pressure drop to decrease. Otherwise, the porosity is more influential that the wall friction factor in the high Reynolds number regime and the pressure drop increases. They also explained that the predominance of one effect depends on fluid velocity. According to Foumeny et al.[8], their wall correction factor for the inertial pressure loss term doesn't come from physical reasoning and it is based on curve-fitting model. Moreover their equation makes a larger inertial pressure loss term that that of the Ergun equation[1] for the bed of $D / d_{p}$.

Niven(2002)[18] discusses a model of pore conduits consisting of alternating expanding and contracting sections can be used for analyzing of Ergun equation[1]. They obtained a model for the pressure drop in packed beds even though it has too many parameters to be determined.

Di Felice and Gibilaro(2004)[19] also suggested a model which explains the wall effect in packed beds. They used a corrected average superficial fluid velocity to predict the pressure drop. The parallel flow of fluid through two zones, the bulk zone and the wall zone, is indicated in their results. By using their simple model, the unusual trend of pressure loss - increasing with increasing of wall effects in the viscous flow regime and decreasing with increasing wall effects in the inertial flow regime - is explained. The limitation of their results is poor predictions in the high Reynolds number regime.

Agnes Montillet(2004)[20] mentioned the experimental pressure drop is lower than that predicted by classical models and it seems difficult to give a physical explanation to this phenomenon. The influence of the wall effect decreases with the bed-to-particle diameter ratio, but this influence is hard to estimate at large Reynolds numbers for the bed-to-particle diameter ratios more than 10. Their work indicated the pressure drop in a finite bed is not a power 2 terms in velocity for the turbulent flow regime. A correlating equation for $\mathrm{f}(\varepsilon)$ of Rose correlation[21], which account for the effect of the bed-toparticle diameter ratio, is proposed in their results.

Nemec and Levec(2005)[22] studied the effect of particle shapes and sizes and bed packing techniques on the single phase pressure drop in packed bed.
Y.S. Choi et al.(2008)[23] developed a semi-empirical pressure drop equation for the packed beds of spherical particles with small bed-to-particle diameter ratios. They used capillary-orifice model which treats a packed bed as a bundle of capillary tubes with orifice plates to explain a wall correction factor for the inertial pressure loss term.

Jinsui Wu. et al.(2008)[24] evaluated that the effect of the bed height on the pressure drop with constant ball diameter. It is found that the pressure drop increases with increasing of the bed height and the fluid velocity. The average hydraulic radius model and the contracting-expanding channel model are also used for their model.

The previously discussed correlations are obtained by limited empirical experiments. Table 1 Shows the pressure drop correlations, porosity ranges, bed-to-particle diameter ratios and Reynolds number ranges found in the literature. These correlations are limited in the sense of a narrow range of Reynolds number and limited porosity range used. This causes a problem when the use of a wide range of Reynolds number and porosities is needed.

The purposes of this paper are to verify the KTA correlation [2] that is used for Gas Cooled Pebble Bed Reactor and to formulate an accurate correlation for pressure drop that includes wall effect. This work also presents data for CFD validation. For these purpose, we made experimental set up of cylindrical packed bed and annular packed bed. The real pebble bed reactor geometry was changed from a cylindrical bed $\left(\mathrm{D} / \mathrm{d}_{\mathrm{p}}=61.7\right)$ to annular bed $\left.\left(D_{o}-D_{i}\right) / 2 d_{p}=14.17\right)$. These present experiments that were considering of real packed geometries would give good directions for predicting of pressure drop.

Table 1. Pressure drop correlations, porosity, diameter ratios and Reynolds number found in the literature.

| Author | Pressure drop equation | $\varepsilon$ | $\mathrm{d}_{0}(\mathrm{~cm})$ | D/4, | Re or $\mathrm{Re}_{\mathrm{m}}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Ergun(1952) | $\frac{\Delta p}{L}=150\left(\frac{\mu v}{d_{p}^{2}}\right)\left(\frac{(1-\varepsilon)^{2}}{\varepsilon^{3}}\right)+\frac{7}{4}\left(\frac{\rho v^{2}}{d_{p}}\right)\left(\frac{1-\varepsilon}{\varepsilon^{3}}\right)$ |  |  |  | $1<\mathrm{Re}<10^{3}$ |
| Handley/Heggs(1968) | $\frac{\Delta p}{L}=368\left(\frac{(1-\varepsilon)^{2}}{\varepsilon^{3}}\right)\left(\frac{\mu v}{d_{p}^{2}}\right)+1.24\left(\frac{\rho v^{2}}{d_{p}}\right)\left(\frac{1-\varepsilon}{\varepsilon^{3}}\right)$ | 0.390 | 3.17 9.52 | 8 24 | 399<Re<3985 |
| Reichelt(1972) | $\begin{aligned} & \frac{\Delta p}{L}=\left(\frac{154 A_{w}^{2} \mu(1-\varepsilon)}{\rho v d_{p}}+\frac{A_{w}}{B_{w}}\right)\left(\frac{\mu v}{d_{p}^{2}}\right)\left(\frac{(1-\varepsilon)^{2}}{\varepsilon^{3}}\right) \\ & A_{w}=1+\left(\frac{2}{3 \frac{D}{d_{p}}(1-\varepsilon)}+1\right)^{2} \\ & B_{w}=\left(1.15\left(\frac{d_{p}}{D}\right)^{2}+0.87\right) \end{aligned}$ | $\underset{485}{0.360 .0}$ | 9.71~24.05 | ${ }_{2}^{3.32-14.3}$ | 0.01 -Re<17635 |
| Foumeny(1993) | $\frac{\Delta p}{L}=130\left(\frac{(1-\varepsilon)^{2}}{\varepsilon^{3}}\right)\left(\frac{\mu \nu}{d_{p}^{2}}\right)+\left(\frac{D / d_{p}}{\left(0.335\left(D / d_{p}\right)+2.28\right)}\left(\frac{\rho v^{2}}{d_{p}}\right)\left(\frac{1-\varepsilon}{\varepsilon^{3}}\right)\right)$ | $\begin{aligned} & 0.386-0 . \\ & 456 \end{aligned}$ | $2.1 \sim 15.48$ | 3.23-23.8 | $5<\mathrm{Re}_{\mathrm{m}}<8500$ |
| Yu(2002) | $\frac{\Delta p}{L}=203\left(\frac{\mu \nu}{d_{p}^{2}}\right)\left(\frac{(1-\varepsilon)^{2}}{\varepsilon^{3}}\right)+1.95\left(\frac{\rho \nu^{2}}{d_{p}}\right)\left(\frac{1-\varepsilon}{\varepsilon^{3}}\right)$ | $0.364-0$. 379 | 12-20 | 30 | 797<Re<2449 |
| Montillet(2004) | $\frac{\Delta p}{L}=\left(\frac{1410}{\operatorname{Re}}+16+\frac{45}{\operatorname{Re}^{0.45}}\right)\left(\frac{\rho v^{2}}{d_{p}}\right)$ | 0.367 | 4.92 | 12.2 | 30<Re<1500 |
| Y.S.Choi(2008) | $\frac{\Delta p}{L}=150\left(\frac{\mu \mathrm{M}^{2} v}{d_{p}^{2}}\right)\left(\frac{(1-\varepsilon)^{2}}{\varepsilon^{3}}\right)+1.75\left(\frac{\rho \mathrm{MC}_{\mathrm{w}} v^{2}}{d_{p}}\right)\left(\frac{1-\varepsilon}{\varepsilon^{3}}\right)$ |  |  | 3.2-91 | $0.01<$ Re $<10^{3}$ |
| J.Wu(2008) | $\begin{aligned} & \frac{\Delta p}{L}=72 \tau\left(\frac{\mu v}{d_{p}^{2}}\right)\left(\frac{(1-\varepsilon)^{2}}{\varepsilon^{3}}\right)+\frac{3}{4} \tau\left(\frac{\rho v^{2}}{d_{p}}\right)\left(\frac{1-\varepsilon}{\varepsilon^{3}}\right)\left(\frac{3}{2}+\frac{1}{\beta^{4}}-\frac{5}{2 \beta^{2}}\right) \\ & \beta=\frac{1}{1-\sqrt{1-\varepsilon}} \end{aligned}$ | 0.42 | 10 |  | $0<\mathrm{Re}_{\mathrm{m}}<4000$ |
| Leva(1951) | $\frac{\Delta p}{L}=200\left(\frac{\mu v}{d_{p}^{2}}\right)\left(\frac{(1-\varepsilon)^{2}}{\varepsilon^{3}}\right)+\frac{7}{4}\left(\frac{\rho v^{2}}{d_{p}}\right)\left(\frac{1-\varepsilon}{\varepsilon^{3}}\right)$ | $\begin{gathered} 0.354-0 . \\ 651 \end{gathered}$ |  | ${ }_{\sim}^{1.624} \sim 13.466$ | $1<$ Re<17635 |

Table 1. continued

| Author | Pressure drop equation | $\varepsilon$ | $\mathrm{d}_{8}(\mathrm{~cm})$ | D/4 ${ }_{6}$ | Re or $\mathrm{Re}_{\mathrm{m}}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Wentz and Thodos(1963) | $\frac{\Delta p}{L}=\left(\frac{\mu \nu}{d_{p}^{2}}\right)\left(\frac{(1-\varepsilon)^{2}}{\varepsilon^{3}}\right)\left(\frac{0.396 \mathrm{Re}_{m}}{\mathrm{Re}_{m}^{0.05}-1.20}\right)$ | ${ }_{882}^{0.3440 .}$ | 3.1242 | 11.382 | ${ }^{1460-R e 7661}$ |
| Tallmadge(1970) | $\frac{\Delta p}{L}=\left(\frac{\mu \nu}{d_{p}^{2}}\right)\left(\frac{(1-\varepsilon)^{2}}{\varepsilon^{3}}\right)\left(150+4.2 \mathrm{Re}^{0.833}\right)$ |  |  |  | $0.1<$ Re $10^{5}$ |
| Hicks(1970) | $\frac{\Delta p}{L}=6.8\left(\frac{(1-\varepsilon)^{1.2}}{\varepsilon^{3}}\right)\left(\frac{\rho^{1.2} v^{1.8} \mu^{0.2}}{d_{p}^{1.2}}\right)$ |  |  |  | $300<\mathrm{Re}_{\mathrm{m}} 660000$ |
| R.E. Hayes(1994) | $\frac{\Delta p}{L}=\left(\frac{\mu \nu}{d d_{p}^{2}}\right)\left(\frac{(1-\varepsilon)^{2}}{\varepsilon^{3}}\right)\left\{\frac{1}{\tau}\left[456+\frac{17.8(3 \tau-1)}{\tau(1-\varepsilon)(1-\tau)} \mathrm{Re}^{\text {Re }}{ }^{0.5} \frac{1}{\varepsilon}+1.3\left(\frac{\tau}{3 \tau-1}\right) \mathrm{Re}_{m}\right\}\right.$ | $\begin{gathered} \hline 0.402, \\ 0.408,0 . \\ 427, \\ 0.385 \\ \hline \end{gathered}$ | $\begin{gathered} 2.97,4.82, \\ 6.01,2.5 \end{gathered}$ |  | 3<Re<1000 |
| KTA(1981) | $\frac{\Delta p}{L}=\left(\frac{320}{\operatorname{Re} /(1-\varepsilon)}+\frac{6}{(\operatorname{Re} /(1-\varepsilon))^{0.1}}\right)\left(\frac{1-\varepsilon}{\varepsilon^{3}}\right)\left(\frac{\rho v^{2}}{d_{p}}\right)\left(\frac{1}{2 \rho}\right)$ | $\begin{gathered} 0.36 \\ -0.42 \end{gathered}$ |  |  | $10<\mathrm{Re}_{\mathrm{m}} \leq 10^{5}$ |
| Brauer(1960 | $\frac{\Delta p}{L}=\left(160+3.1 \mathrm{Re}_{m}^{0.9}\right)\left(\frac{\mu \nu}{d_{p}^{2}}\right)\left(\frac{(1-\varepsilon)^{2}}{\varepsilon^{3}}\right)$ |  |  |  | $2<\mathrm{Re}_{\mathrm{m}}<20000$ |
| Foscolo(1982) | $\frac{\Delta p}{L}=1.73\left(\frac{1-\varepsilon}{\varepsilon^{4.8}}\right)\left(\frac{\mu v}{d_{p}^{2}}\right)+0.336\left(\frac{1-\varepsilon}{\varepsilon^{4.8}}\right)\left(\frac{\rho v^{2}}{d_{p}}\right)$ |  |  |  | $0.2<$ Re 500 |
| Macdonald(1979) | $\frac{\Delta p}{L}=150\left(\frac{\mu \nu}{d_{p}^{2}}\right)\left(\frac{(1-\varepsilon)^{2}}{\varepsilon^{3}}\right)+\frac{7}{4}\left(\frac{\rho \nu^{2}}{d_{p}}\right)\left(\frac{1-\varepsilon}{\varepsilon^{3}}\right)$ |  |  |  |  |
| Shijie Liu(1994) | $\begin{aligned} & \frac{\Delta p}{L}=\left(\frac{\mu v}{d_{d}^{2}}\right)\left(\frac{(1-\varepsilon)^{2}}{\varepsilon^{1 / 3}}\right) \\ & \left\{85.2\left(1+\frac{\pi\left(d_{p} / D\right)}{6(1-\varepsilon)}\right)^{2}+0.09\left(1-\frac{\pi^{2}\left(d_{2} / D\right)}{24}\left(1-0.5\left(d_{p} / D\right)\right)\right)_{\mathrm{Re}_{m}} \frac{\mathrm{Re}_{m^{2}}{ }^{2} 6^{2}+\mathrm{Re}_{m}{ }^{2}}{}\right\} \end{aligned}$ | 0.6007 | 3.184 | 1.4039 | $\begin{gathered} 1328<\mathrm{Re}_{\mathrm{m}}< \\ 4081 \end{gathered}$ |
| Carman(1970) | $\frac{\Delta p}{L}=\left(180+2.87 \operatorname{Re}_{m}^{0.9}\right)\left(\frac{\mu \nu}{d_{p}^{2}}\right)\left(\frac{(1-\varepsilon)^{2}}{\varepsilon^{3}}\right)$ |  |  |  |  |
| Rose(1949) | $\frac{\Delta p}{L}=f(\varepsilon)\left(\frac{1000}{\operatorname{Re}}+\frac{60}{\operatorname{Re}^{1 / 2}}+12\right)\left(\frac{\rho v^{2}}{d_{p}}\right)$ <br> $\mathrm{f}(\varepsilon)$ is 1 for $\varepsilon=0.4$ | 0.373 0.480 | $\begin{aligned} & 1.12 \\ & 3.10 \end{aligned}$ | $\underset{\substack{10.25 \\ 2.7}}{ }$ | 1000 -2e6000 |
| Morcom(1946) | $\frac{\Delta p}{L}=\left(\frac{800}{\operatorname{Re}}+14\right)\left(\frac{\rho v^{2}}{d_{p}}\right)$ | $\begin{gathered} 0.425 \\ \sim 0.450 \end{gathered}$ | $0.56 \sim 1.01$ |  | 100<Re<500 |

## Theoretical consideration

There are several influencing parameters of pressure drop in packed beds. Wall effect and porosity are especially considered in this work.

## Wall effect

The wall effect exists for packed beds. The inhomogeneous porosity distribution in the bed and the additional wetted surface introduced by the wall cause the variation of pressure drop. It is important to predict the wall effect.

The opinions about the resultant wall effect are contradictory. Some researchers found increase of the pressure drop due to the wall effect. But others said they have got a reduction due to the wall effect.

Many researchers have concluded as follows: The pressure drop can be increased by wall friction or decreased by an increase in porosity near the wall. The flow regimes affect the predominance of one effect over the other. The wall friction effect is more important than the increased porosity effect in the low Reynolds number regime. On the other hand, the porosity effect is dominant in the high Reynolds number regime [3]. Some other published paper on the influence of the tube to particle diameter ratio shows that the increasing pressure drop due to the wall effect are based on experiments under streamline flow conditions or in the transitional range [4],[5],[6],[7]. Otherwise, the decreasing pressure drop due to the wall effect is measured at high Reynolds numbers[4],[8],[9],[10]. There are some efforts to account for the wall effect. The first
attempt to address the wall effect was made by Crman(1937)[5]. He considered that the wall effect on the inertial term is negligible and only the viscous term(Darcy's flow) needs to be corrected. Recent experimental studies showed that Carman's treatment is inadequate.

One of the main researchers is Metha and Hawley(1969)[7]. They defined a hydraulic radius,

$$
\begin{equation*}
R_{H}=\frac{\varepsilon}{6(1-\varepsilon) M} \tag{18}
\end{equation*}
$$

Where, $M=1+\frac{2}{3}\left[\frac{d_{p}}{D(1-\varepsilon)}\right]$.
Their conclusion is that wall effects are not significant if the diameter ratio is greater than 50.

Fand et al.(1990)[14] said that experimental data obtained by Metha and Hawley(1969)[7] indicates that this last conclusion is somewhat overly conservative. Finally they concluded that wall effects are not significant if the diameter ratio is greater than 40.

Riechelt(1972)[12] modified Metha and Hawley's correlation[7], and he defined a wall modified hydraulic radius,

$$
\begin{equation*}
R_{h w}=\frac{R_{H}}{M} \tag{19}
\end{equation*}
$$

He also yielded corresponding "wall-modified" parameters:

$$
\begin{align*}
f_{w} & =\frac{f^{\prime}}{M}  \tag{20}\\
\operatorname{Re}_{w} & =\frac{\mathrm{Re}^{\prime}}{M} \tag{21}
\end{align*}
$$

At last, he obtained the following modification equation,

$$
\begin{equation*}
f_{w}=A_{w} / \operatorname{Re}_{w}+B_{w} ; \quad A_{w}=150, \quad \frac{1}{\sqrt{B_{w}}}=\frac{1.5}{\left(D / d_{p}\right)^{2}}+0.88 \tag{22}
\end{equation*}
$$

Fand et al.(1990)[14] reported that, for cylindrical ducts packed with spheres, the "wall effect" becomes significant for $\mathrm{D} / \mathrm{d}_{\mathrm{p}}<40$, and consequently the flow parameters become functionally dependent upon $\mathrm{D} / \mathrm{d}_{\mathrm{p}}$ for $\mathrm{D} / \mathrm{d}_{\mathrm{p}}<40$.

Foumeny(1993)[8] also concluded that the wall effect is important when the diameter ratio, $D / d_{\mathrm{p}}$, is less than 50 , and it is pronounced at values less than 12 .

The general conclusion of all above works is that the Ergun equation[1](with average values of porosity and superficial velocity) from a practical point of view is applicable down to quite low tube-to-particle-diameter $\operatorname{ratios}\left(\mathrm{D} / \mathrm{d}_{\mathrm{p}} \geq 10\right)$. They tested with the tube-to-particle-diameter ratio higher than 10, so the experimentally determined Ergun constants should not be affected by the reactor column wall.

## Porosity

The pressure drop is extremely sensitive to changes in the mean void fraction, $\varepsilon$, This influence is described either empirically, using dimensional analysis [21], or theoretically, most often employing the hydraulic radius concept [1],[5]. The porosity, $\varepsilon$,
defined as the fraction of the total volume of a porous medium represented by the voids in its matrix, is a primary controlling geometry of the matrix of the medium. For the case of spherical particles contained in a circular cylinder, the porosity tends toward unity upon approach to the cylinder wall [14].

The constants of Ergun equation [1], A and B, can vary from macroscopic bed to bed even if repacked with the same batch of particles. If the repacking of the bed changes the values of the Ergun constants this could mean that the porosity is not adequately taken into account by the capillary model [22].

Rumph and Gupte(1971) [25] have analyzed the effect of various distributions of spherical particles over a relatively wide range of porosities $(0.35<\varepsilon<0.70)$ and proposed a different dependence upon porosity. For the region of packed bed reactor relevance $(0.35<\varepsilon<0.55)$ does not differ very much from that of the capillary model, considering an average difference of only about $10 \%$. Other porosity functions like the one determined by Liu et al.(1994)[26] in general yield values between those of the capillary model and the empirical model proposed by Rumph and Gupte(1971) [25]. Furthermore, it has to be pointed out that the results of Rumph and Gupte(1971) [25] have been obtained from media created with higher porosities than normally encountered in beds composed of spherical particles and could therefore lead to non-uniformly packed beds giving us the wrong impression. Thus, it was deemed necessary to recheck the porosity effect on pressure drop with more natural particle distributions.

Some additional differences between the porosities of beds, despite the same packing procedures, were due to wall effect. One can conclude that the porosity dependence
seems to be well described by the capillary model, reflected by the fact that all the data lay on a single curve for all packed beds. This is in agreement with a number of works for the viscous regime reviewed by Carman(1937)[5] as well as a more recent one of Endo et al.(2002)[27]. With regards to the porosity dependence within the inertial regime, Hill et al.(2001)[28] reported, on the basis of theoretical simulations of flow through random arrays of spheres, that the porosity function is also well taken into account as long as the porosity is around 0.4 as is indeed the case for packed bed reactors when made up of spheres. Ergun(1952)[1] also made an interesting point that if a transformation of his equation is made employing the fundamental expressions for the shear stress, hydraulic radius and interstitial velocity, this leads to complete elimination of porosity, in the field of aerodynamics. Therefore, the porosity function of the capillary model can be assumed as an accurate one within the region of interest $(0.35<\varepsilon<0.55)$ as the arguments for overweight those against [22].

## CHAPTER II

## EXPERIMENTAL METHODOLOGY

This chapter explains the channel flow facility, experimental techniques and detailed methods used for this investigation.

A pressure drop experimental setup had been designed for studying single-phase flow studies. The basic components of the test rig were the test section(column), different kinds of pump, reservoir water tank, hot film manometer to measure velocity of working fluid, several flow meters(electrical flow meter, Dwyer rate master flow meter and Hivolume air flow rate calibrator), different kinds of pressure measurer (Pressure transducers, Magnehelic differential pressure gages, Inclined-Vertical manometer and Digital manometer) and electrical thermometer. A cylindrical packed bed and an annular type packed bed were used for these experiments. Also, four different sizes of spherical particles were used for air and water test.

## Properties of working fluids

The working fluids are air and water. Tables 2 and 3 show the properties of working fluids, air and water.

Table 2. Air properties

| Temperature $\left({ }^{\circ} \mathrm{C}\right)$ | Density $(\mathrm{kg} / \mathrm{m} 3)$ | Dynamic Viscosity $(\mathrm{kg} / \mathrm{m} \cdot \mathrm{s})$ |
| :---: | :---: | :---: |
| 28 | 1.204 | 0.0000182 |

Table 3. Water properties

| Temperature $\left({ }^{\circ} \mathrm{C}\right)$ | Density $(\mathrm{kg} / \mathrm{m} 3)$ | Dynamic Viscosity $(\mathrm{kg} / \mathrm{m} \cdot \mathrm{s})$ |
| :---: | :---: | :---: |
| 25 | 997.13 | 0.000891 |
| 26 | 996.86 | 0.000871 |
| 27 | 996.59 | 0.000852 |
| 28 | 996.31 | 0.000833 |
| 29 | 996302 | 0.000815 |
| 30 | 995.71 | 0.000798 |
| 31 | 995.41 | 0.000781 |
| 32 | 995.09 | 0.000765 |
| 33 | 994.76 | 0.000749 |
| 34 | 994.43 | 0.000734 |
| 35 | 994.08 | 0.000720 |
| 36 | 993.73 | 0.000705 |
| 37 | 993.37 | 0.000692 |
| 38 | 993.00 | 0.000678 |
| 40 | 992.63 | 0.000666 |
| 39 | 992.25 | 0.000653 |

## Instruments for cylindrical bed experiments

## A test rig

The column is a cylindrical bed. The diameter of column is 4.75 " $(0.12065 \mathrm{~m})$. The length is 60 " $(1.534 \mathrm{~m})$. It has five tabs to measure pressure. Thin aluminum grids were placed at the column inlet and outlet to serve as a filter.

## Beads

The column was filled with spherical balls randomly. Four different sizes of sphere beads were used for this experiment. These sizes were $0.635 \mathrm{~cm}, 1.27 \mathrm{~cm}, 1.905 \mathrm{~cm}$ and 3.302 cm . By using these particles, pressure drops over the bed were measured. And each different bed-to-particle ratios gave different porosities. It affected the velocity on the bed wall. As the size of particles increases, a high Reynolds number was presented at same flow rate because of diameter of the particle as well as porosity affections to the Reynolds number.

## Pipes, valves, and unions

The diameter of cylindrical pipes that were used for these experiments were 1.5 inches ( 3.81 cm ). Fluid flows were controlled by using valves. Unions were also used for connection of pipes.

## A reservoir tank

A Reservoir tank was used for water experiments. The water should be enough to make a loop. The shape of tank was rectangular parallelepiped.

## Pumps (power source)

For water experiments: A 3 H.P. pump was used with Lancer GPD 502 that was pump controller. The RPM was 3450 . The maximum frequency was 60 Hz . The fluid was pumped from a reservoir tank through the packed bed using this centrifugal type pump. For air experiments: Two different power sources were used for these experiments.

- Air compressor
- Blower(Hi-Q) (it sucked the air from the bottom of the column.)


## Pressure instruments

4 different pressure instruments were used for these experiments in order to check wall pressures with high accuracy. It would be also good error estimator by comparing each result.

- Magnehelic differential pressure gages (6 different scale pressure gages)
- Inclined - Vertical Manometer (scale: 0 to 10 inches water)
- Digital Manometer (for 0 to 40 psi )
- Pressure transducer ( 5 sensors of the pressure transducer (for 0 to 30 psi ))
- Model : NI-SCXI model (1600/1200/1000)


## Flow meter

For water experiments, a magnetic electrical flow meters with a digital display was placed at the inlet of the channel to measure the bulk flow rate. The flow rate was checked by using a propeller type flow meter

For air experiments, 3 different flow meters were used.

- Dwyer rate master flow meter(scale)
- Hi-volume air flow rate calibrator: The HFC-XXC series units utilize a precision machined venturi tube coupled with a pressure differential gauge giving a direct reading in the volumetric units of our choice
- Hot-film manometer: It measures velocity of the pipe. By multiplying area to velocity, flow rate can be calculated.


## Experimental techniques

Pressure transducer is a device capable of measure the pressure that is present at certain point with a cross sectional area, A. The basic definition of pressure is the ratio of the force applied to a body divided by the cross sectional area where the force is being applied. The differential pressure transducer measures the difference of pressure at two different locations or in two different directions. The differential pressure transducer used in this study is the validyne model DP103. This device utilizes a central diaphragm as a sensor element and is of the variable reluctance type.

A variable reluctance pressure transducer is perhaps best described as an inductive halfbridge, and consists of a pressure sensing diaphragm and two coils. The coils are wired in series and are mounted so their axes are normal to the plane of the diaphragm. Clamped tightly between the coil housings, the diaphragm is free to move in response to differential pressure.

The coils are supplied with an AC excitation, typically 5 Vrms at 3 or 5 KHz . The coils are matched so that their impedances are approximately equal. When a differential pressure is applied to the sensor, the diaphragm deflects away from one coil and towards the opposite. The diaphragm material is magnetically permeable, and its presence nearer the one coil increases the magnetic flux density around the coil. The stronger magnetic field of the coil, in turn, causes its inductance to increase, which increases the impedance of one coil. At the same time, the opposite coil is decreasing its impedance. The change
in coil impedances brings the half-bridge out of balance, and small AC signal appears on the signal line.

The change in coil impedance is directly proportional to the position of the diaphragm, so the amplitude of the signal is directly proportional to the applied pressure. The phase of the signal with respect to the excitation is determined by the direction of movement of the diaphragm."

## Sensitivity to low pressures

Because the diaphragm need move only one or two thousandths of an inch to produce a full scale output, the thickness and area of the diaphragm determines the full-scale pressure range. A large diaphragm made of thin foil will respond to extremely low pressures. Conversely, a thick diaphragm with a small area responds to very high pressures.

## Frequency response

The ability of a pressure sensor to respond accurately to rapid pressure changes is a function of three variables: the mechanical response of the sensor itself, the frequency response of the sensor electronics, and the natural frequency of the plumbing that brings the pressure waveform to the transducer. The mechanical response of the sensor depends on the construction of the sensing element. The electronics connected to a pressure transducer will mostly likely include damping, or a low pass filter on the output stage that may even be the most limiting factor in system response. The tubing that leads
up to the transducer from the pressure source will also have a resonant frequency that will limit the usable response of the pressure measuring system. Each of these factors must be considered in order to arrive at a good estimate of the accurate response of the pressure measurement system.

## Sensor mechanical response

Almost all pressure-sensing technologies rely on a pressure-sensing diaphragm to transmit the dynamic pressure waveform to the electro-mechanical element of the pressure sensor. For sensing technologies other than variable reluctance, the sensing diaphragm is connected via linkages or other mechanical means to a strain gage, piezoelectric, capacitive, or some other electrical sensing element. The stiffness of the sensing diaphragm and the associated linkages create a mechanical spring-mass system whose natural frequency is usually specified by the manufacturer. If the sensor is underdamped, amplification and also dynamic error, of the incoming waveform occurs. If the sensor is over-damped, the incoming pressure waveform is attenuated. In either case, pressure measurement at or near the natural frequency of the sensor will result in undesirable distortion of the dynamic signal.

For variable reluctance pressure sensors, the only mechanical part that moves in response to pressure is the sensing diaphragm, and the total displacement over a full scale pressure excursion is less than 2 thousandths of an inch. There are no mechanical linkages or hydraulics inside the sensor to slow down the sensing element. The position of the diaphragm is measured inductively, and this is how the sensed pressure is
converted to an electrical signal. The natural frequency of variable reluctance transducer is a function of range. Because the sensing diaphragm is in contact with the flow being measured, the sensor is typically over-damped at its natural frequency.

## Pressure transducer

Two PCI 6024 data acquisition boards from National Instruments acquire the analogsignal produced by the differential pressure transducer. This board counts with an A/C converter to convert the analog signal into a digital one. The internal clock board gives the maximum conversion rate for the $\mathrm{A} / \mathrm{D}$ converter. A code program developed in Labview was the responsible for the data acquisition start, storage and process. Basically, this program waits for the trigger signal generated by the frame grabber PCI 1424 and then starts the acquisition of data. The program has the capability to choose the acquisition rate.

## Porosity measurements

One of the most important parameters affecting the behavior of a packed bed is the accurate quantification of its porosity. However, its measurement is a challenging topic when talking about randomly packing beds due to the nature of the packing. Three independent methods were implemented in order to characterize the porosity or voids fraction in the column: water displacement, weighting and particle counting.

## Water displacement method

This method consists in pouring water in the empty column and measuring the total water volume poured. The second step is to randomly pack the spheres into the column. Water is poured into the randomly packed bed until the whole column was completely filled. The column was shaken during the water pouring in order to let the trapped air between the pores to escape. The water is then collected into an accurate scaled container. The voids fraction is obtained by subtracting the amount of water measured in the container when the column is packed from the one obtained from the empty column. The procedure is repeated several times and the average porosity is quantified.

## Weighting method

In this method the total number of spheres packed into the column is needed in addition to the volume of the empty column. The empty column volume is calculated using the cylinder dimensions. The total number of beads is obtained by weighting the randomly packing the beads in the column. The total bead's weight is calculated by subtracting the
empty column weight. In an independent procedure, the weight of groups of 5, 10, 15, 20 and 100 beads is measured in order to obtain an average weight per bead. The total number of beads in the column is calculated using the ratio of the total beads weight to an individual bead weight. Once the total number of spheres is known, the total volume occupied by the packing material is calculated using the volume of an individual sphere. The porosity value is calculated by subtracting the volume of the empty column from the volume occupied by the spheres. This method assumed that the beads are perfect spheres with tight tolerances in its diameter.

## Particle counting method

This method consists in manually counting the total number of beads inside the column. The total volume occupied by the spheres is calculated by multiplying the number of spheres times the volume of an ideal sphere. The porosity is obtained by subtracting the total volume occupied by the spheres from the volume of the empty column. The method is repeated several times to obtain a good statistical value of the averaged porosity.

The last one was most accurate even though it was time consuming. The porosity was also compared with several porosity correlations to confirm the porosity and to get the regime of porosity error.

## Comparison with existing porosity correlations

To compare present work for porosity measurements with existing porosity correlations, 6 correlations were chosen.

Fand and Thinakaran (1990) [14]

$$
\begin{equation*}
\varepsilon=\frac{0.151}{\left(\frac{D}{d_{p}}-1\right)}+0.360, \text { for } \mathrm{D} / \mathrm{d}_{\mathrm{p}} \geq 2.033 \tag{23}
\end{equation*}
$$

Beaver et al. (1973) [29]
Beaver et al.[29] said that the porosity increases with decreasing $D / d_{p}$ for small beds. When $D / d_{p}$ is greater than 15 , the size of the bed appears to have no effect on the porosity. The average value of the porosity for all beds having $\mathrm{D} / \mathrm{d}_{\mathrm{p}}$ is greater than 15 was found to be 0.368 , with a maximum deviation of less than 2 percent. They also reported that the trend of variation of $\varepsilon$ with $D / d_{p}$ can be predicted by employing a simple model first proposed by Rose. If it is assumed that the outer layer of spheres, to a depth of $\mathrm{d} / 2$ from the walls, has void fraction $\varepsilon_{\mathrm{w}}$ and that the inner core is randomly packed and has a void fraction $\varepsilon_{\infty}$, then the porosity of the whole bed is given by

$$
\begin{equation*}
w h \varepsilon=\left[w h-\left(w-d_{p}\right)\left(h-d_{p}\right)\right] \varepsilon_{w}+\left[\left(w-d_{p}\right)\left(h-d_{p}\right)\right] \varepsilon_{\infty} \tag{24}
\end{equation*}
$$

$$
\begin{equation*}
D_{e}=2 w h /(w+h) \tag{25}
\end{equation*}
$$

where, whe is the porosity of the whole bed.
$D_{e}$ is the equivalent diameter of the bed.
$h$ and $w$ are, respectively, the height and width of the bed.
The term involving $d_{p}$ is always very small and can be neglected. Then after rearrangement and introduction of $D_{e}$, there is obtained

$$
\begin{equation*}
\varepsilon=\varepsilon_{\infty}\left[1+2 \frac{d_{p}}{D_{e}}\left(\frac{\varepsilon_{w}}{\varepsilon_{\infty}}-1\right)\right] \tag{26}
\end{equation*}
$$

where, it has been assumed that $\varepsilon_{\infty}=0.368$, and that layer of spheres next to the walls is close-packed so that $\varepsilon_{\mathrm{w}}=0.476$ [29].

Foumeny et al. (1993) [8]
A common source of error is the assumption that mean porosity of packed beds of spherical particles is approximately equal to 0.4 . While this may be acceptable for beds with relatively large tube to particle diameter ratios, it is certainly not realistic tube to particle diameter ratios, it is certainly not realistic for low diameter ratios, $\mathrm{D} / \mathrm{d}_{\mathrm{p}}<10.0$. [8]
$\varepsilon=1-\frac{2}{3}\left(\frac{1}{\frac{D}{d_{p}}}\right)^{3} \frac{1}{\sqrt{\left(\frac{2}{d_{p}}-1\right)}}$, for $1 \leq \mathrm{D} / \mathrm{d}_{\mathrm{p}} \leq \sqrt{3} / 2$

$$
\begin{equation*}
\varepsilon=0.383+0.25\left(\frac{D}{d_{p}}\right)^{-0.923} \frac{1}{\sqrt{\left(0.723 \frac{D}{d_{p}}-1\right)}} \text {, for } 1+\frac{\sqrt{3}}{2} \geq \frac{D}{d_{p}} \tag{28}
\end{equation*}
$$

## Sato (1973) [30]

Sato[30] made 3 different porosity correlations for different packing ways.

- Gently dumped (without external impact).

$$
\begin{equation*}
\varepsilon=0.3517+0.4657\left(\frac{d_{p}}{D}\right), \quad \text { for } \quad \frac{\mathrm{d}_{\mathrm{p}}}{\mathrm{D}}<0.4 \tag{29}
\end{equation*}
$$

- Dumped with simultaneous vibration.

$$
\begin{equation*}
\varepsilon=0.3472+0.4417\left(\frac{d_{p}}{D}\right), \quad \text { for } \quad \frac{\mathrm{d}_{\mathrm{p}}}{\mathrm{D}}<0.4 \tag{30}
\end{equation*}
$$

- One minute vibration after dumped.

$$
\begin{equation*}
\varepsilon=0.3494+0.4381\left(\frac{d_{p}}{D}\right), \quad \text { for } \quad \frac{\mathrm{d}_{\mathrm{p}}}{\mathrm{D}}<0.4 \tag{31}
\end{equation*}
$$

## Zou and $\mathrm{Yu}(1995)$ [31]

- for the loose random packing

$$
\begin{gather*}
\varepsilon=0.400+0.010\left(e^{10.686\left(\frac{d_{p}}{D}\right)}-1\right), \quad \text { for } \frac{\mathrm{d}_{\mathrm{p}}}{\mathrm{D}}<0.256  \tag{32}\\
\varepsilon=0.8460-1.898\left(\frac{d_{p}}{D}\right)+2.725\left(\frac{d_{p}}{D}\right)^{2}, \quad \text { for } 0.256<\frac{\mathrm{d}_{\mathrm{p}}}{\mathrm{D}}<0.538  \tag{33}\\
\varepsilon=1-\frac{2}{3} \times 1 \frac{\left(\frac{d_{p}}{D}\right)^{3}}{\sqrt{2\left(\frac{d_{p}}{D}\right)-1}}, \text { for } 0.538<\frac{\mathrm{d}_{\mathrm{p}}}{\mathrm{D}} \tag{34}
\end{gather*}
$$

- for the dense packing

$$
\begin{gather*}
\varepsilon=0.372+0.002\left(e^{15.306\left(\frac{d_{p}}{D}\right)}-1\right), \text { for } \frac{\mathrm{d}_{\mathrm{p}}}{\mathrm{D}}<0.253  \tag{35}\\
\varepsilon=0.681-1.363\left(\frac{d_{p}}{D}\right)+2.241\left(\frac{d_{p}}{D}\right)^{2}, \text { for } 0.253<\frac{\mathrm{d}_{\mathrm{p}}}{\mathrm{D}}<0.530  \tag{36}\\
\varepsilon=1-\frac{2}{3} \times 1 \frac{\left(\frac{d_{p}}{D}\right)^{3}}{\sqrt{2\left(\frac{d_{p}}{D}\right)-1}}, \text { for } 0.530<\frac{\mathrm{d}_{\mathrm{p}}}{\mathrm{D}} \tag{37}
\end{gather*}
$$

## Chu(1989) [32]

Their actual porosities data were used in order to compare with present porosities.
Table 4. Actual porosities data for $\mathrm{Chu}(1989)$

| $\mathrm{D}(\mathrm{cm})$ | $\mathrm{d}_{\mathrm{p}}(\mathrm{cm})$ | $\mathrm{D} / \mathrm{d}_{\mathrm{p}}$ | $\varepsilon$ |
| :---: | :---: | :---: | :---: |
| 0.273 | 0.095 | 2.873684 | 0.462 |
| 0.491 | 0.057 | 8.614035 | 0.407 |
| 0.957 | 0.095 | 10.07368 | 0.387 |
| 0.802 | 0.034 | 23.58824 | 0.386 |

## Temperature measurements

A Density and viscosity of air and water are affected by temperature. Especially the properties of the water were sensitive to the temperature. Therefore, an exact temperature measurement at each experiment should be taken. As pump power increased, the temperature was also increased. On the previous experiments, average temperature was used for calculation of Reynolds number. However, properties of water were changed with temperature variation outstandingly. Instead of using of average temperature, the right temperatures at each flow rate measurement point were used to determine properties of water. Because an accurate temperature affects an exact Reynolds number at each point. Even small difference of viscosity's value gave different Reynolds number. It would be one of the error mechanisms.

## Methods of experiment for the pressure drop

Air experiments with a cylindrical packed bed
For experiments with air, pumps and air compressor are used to give power.

## With pump blower

The column is filled by slow feeding of the spherical particles of diameter is 0.635 cm . While the particles were poured into the bed, the bed was shaken to make sure the beads were firm and filled very well. It was connected to pump on the bottom of the bed. Two blowers of the pump sucked air from bottom of the Bed. Then air entered the column from the top of the bed in a downward direction. On the top of the bed, flow meter and open pipe were connected. The bed has two knit-mesh at the end of both sides to prevent beads from leaving the bed as well as the uniform distribution of the air. By using flow meter on the inlet of the bed, the flow rate of air was measured. The flow rate was controlled by the main valve located at the side of the blower. Pressure gauges and inclined vertical manometer checked the pressure at each tab. Same procedures were used for different size particles.

With air compressor
The bed was connected to air compressor. The air compressor forced air into the column. Also rate mate flow meter checked flow rate. The flow rate was controlled by the main valve located at the top of the air compressor. Pressure gages and inclined vertical manometer checked the pressure at each tab. On the bottom of the bed, the air went out to the room. It was not the loop but open system. Also we did same experiments for different size particles with same way.

## Water experiments with a cylindrical packed bed

The column was connected to flow meter, pump, and reservoir water tank. The water from the reservoir water tank went through the pump. Pump gave out the water to the column. The water flow rate was measured using flow meter. When the flow rate was constant, pressure transducer measured each pressure on the tabs. The Flow rate was controlled by Pump controller button. Also, an electronic thermometer was submerged into the reservoir tank to measure temperature of water. We did this experiment with different particle sizes and different bed set up(vertical and horizontal set up). For the vertical column set up, we did experiments for both flow directions (up-flow and downflow). All these experiments were performed with a data rate acquisition system. Data was acquired different flow rate for single-phase flow. These experiments were carried out under various Reynolds numbers.

Fig. 1. and 2. show the particles that were used in these experiments. Fig. 3. indicates a diagram of facility of air experiment. Also, Fig. 4. show a picture of facility of air experiment. Otherwise, Fig. 5. indicates a diagram of facility of water experiment. Fig. 6. show a picture of facility of water experiment.


Fig. 1. Different size sphere particles $\left(d_{p}=0.635 \mathrm{~cm}, 1.27 \mathrm{~cm}\right.$ and 1.905 cm$)$.


Fig. 2. Different size sphere particles $\left(d_{p}=0.635 \mathrm{~cm}, 1.27 \mathrm{~cm}, 1.905 \mathrm{~cm}\right.$ and 3.302 cm$)$.


Fig. 3. A diagram of facility of air experiment.


Fig. 4. A picture of facility of air experiment.


Fig. 5. A diagram of water experiment facility.


Fig. 6. A picture of water experimental facility.

## CHAPTER III

## DATA ANALYSIS AND RESULTS

## The average porosity

Average porosities were measured from three different experiments. Table 5 indicates the average porosity at each bed-to-particle diameter ratios. The average porosity for $D / d_{p}=19$ came from method 2 , and the average porosity for $D / d_{p}=9.5,6.33$ and 3.66 came from method 3. The results from method 2 and 3 did not have big differences. In case of $D / d_{p}=19$, the number of particles is a lot, so it was hard to count all the particles.

Table 5. Bed-to-particle diameter ratios and each porosity.

| $D / d_{p}$ | 19 | 9.5 | 6.33 | 3.66 |
| :---: | :---: | :---: | :---: | :---: |
| $\varepsilon$ | 0.385 | 0.397 | 0.416 | 0.465 |

The average porosities from present experiments were compared with correlations found in the literature.

Again, the porosity measurements were like below.

- Water displacement method
- Weighting method
- Particle counting method


## Water displacement method results

Table 6 shows water displacement method results for $D / d_{p}=19$.

Table 6. Water displacement method results for $\mathrm{D} / \mathrm{d}_{\mathrm{p}}=19$.

|  | Measured volume <br> of column with <br> water (liter) | Volume of water <br> with beads are <br> filled (liter) | Porosity | Average <br> Porosity |
| :---: | :---: | :---: | :---: | :---: |
| 1 | 17.371 | 6.55 | 0.377 |  |
| 2 | 17.37 | 6.53 | 0.376 |  |
| 2 | 17.36 | 6.57 | 0.378 |  |
| 4 | 17.41 | 6.6 | 0.379 |  |
| 5 | 17.4 | 6.9 | 0.396 |  |

## Weighting method results

Table 7 shows weighting method results for $D / d_{p}=19$.

Table 7. Weighting method results for $\mathrm{D} / \mathrm{d}_{\mathrm{p}}=19$.

| The number of <br> beads | Weight(g) | 1 bead <br> weight(g) | Estimated total <br> number of beads | Porosity |
| :---: | :---: | :---: | :---: | :---: |
| Total beads | 12655 g |  |  |  |
| 10 | 1.59 | 0.159 | 79591 | 0.387 |
| 25 | 3.96 | 0.1584 | 79892 | 0.385 |
| 100 | 15.85 | 0.1585 | 79842 | 0.386 |
| 110 | 17.44 | 0.15855 | 79819 | 0.386 |
| 210 | 33.3 | 0.15857 | 79806 | 0.386 |
| 245 | 54.59 | 0.15853 | 79826 | 0.386 |
| 345 | 0.15823 | 79977 | 0.385 |  |

## Weighting method results and particle counting method results

Table 8 shows Weighting method results and Particle counting method results for $\mathrm{D} / \mathrm{d}_{\mathrm{p}}=9.5$.

Table 8. Weighting method results and Particle counting method results for $\mathrm{D} / \mathrm{d}_{\mathrm{p}}=9.5$.

| The <br> number <br> of beads | Weight <br> $(\mathrm{g})$ | 1 bead <br> weight <br> $(\mathrm{g})$ | Estimated, <br> the number <br> of beads | Counted, <br> the number <br> of beads | Porosity | Average <br> Porosity |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Total | 7546 |  |  | 5990 | 0.397 | 0.397 |
| 10 | 12.64 | 1.264 | 5970 |  | 0.399 |  |
| 50 | 62 | 1.24 | 6085 |  | 0.387 |  |

Table 9 shows Weighting method results and Particle counting method results for $\mathrm{D} / \mathrm{d}_{\mathrm{p}}=6.33$.

Table 9. Weighting method results and Particle counting method results for $\mathrm{D} / \mathrm{d}_{\mathrm{p}}=6.33$.

| The <br> number <br> of beads | Weight (g) | 1 bead <br> weight (g) | Estimated, <br> the number <br> of beads | Counted, <br> the number <br> of beads | Porosity | Average <br> Porosity |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Total | 8550 |  |  | 1990 | 0.416 | 0.416 |
| 10 | 43.05 | 4.305 | 1986.9 |  | 0.417 |  |
| 50 | 212 | 4.24 | 2016.5 |  | 0.408 |  |

Table 10 shows Weighting method results and Particle counting method results for $\mathrm{D} / \mathrm{d}_{\mathrm{p}}=3.65$.

Table 10. Weighting method results and Particle counting method results for $\mathrm{D} / \mathrm{d}_{\mathrm{p}}=3.65$.

| The number of beads | Weight (g) | 1 bead weight (g) | Estimated, the number of beads | Counted, the number of beads | Porosity |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Total | 10520 |  |  | 489 | 0.465 |
| 5 | 108 | 21.6 | 487.03 |  | 0.471 |
| 10 | 215 | 21.5 |  |  |  |
| 20 | 430 | 21.5 |  |  |  |
| 30 | 645 | 21.5 |  |  |  |
| 40 | 860 | 21.5 |  |  |  |

## Porosities from existing correlations

Tables 11 and 12 show porosities from existing correlations.

Table 11. Porosities from existing correlations.

| $\begin{gathered} \mathrm{d}_{\mathrm{p}} \\ (\mathrm{~cm}) \end{gathered}$ | $\begin{gathered} \mathrm{D} \\ (\mathrm{~cm}) \end{gathered}$ | Foumeny | Beaver | R.M. <br> Fand | Sato | $\begin{aligned} & \text { Zou } \\ & \text { /Yu } \end{aligned}$ | Chu | Average experimenta 1 porosity |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0.635 | 12.065 | 0.387 | 0.395 | 0.368 | $\begin{aligned} & 0.376 \\ & 0.370 \\ & 0.372 \end{aligned}$ | $\begin{array}{\|c\|} \hline 0.407 \\ \text { (Loose) } \\ 0.374 \\ \text { (Dense) } \\ \hline \end{array}$ | . | 0.385 |
| 1.27 | 12.065 | 0.396 | 0.405 | 0.377 | $\begin{aligned} & 0.401 \\ & 0.394 \\ & 0.396 \end{aligned}$ | 0.42 (Loose) 0.38 (Dense) | . | 0.397 |
| 1.905 | 12.065 | 0.407 | 0.414 | 0.388 | $\begin{aligned} & 0.425 \\ & 0.417 \\ & 0.419 \end{aligned}$ | 0.444 (Loose) 0.392 (Dense) | . | 0.416 |
| 3.302 | 12.065 | 0.443 | 0.435 | 0.417 | $\begin{aligned} & 0.479 \\ & 0.468 \\ & 0.469 \end{aligned}$ | $\begin{array}{\|c\|} \hline 0.53 \\ \text { (Loose) } \\ 0.47 \\ \text { (Dense) } \end{array}$ | - | 0.465 |

Table 12. Porosities from present work (Summary for porosities from present work).

| $\mathrm{d}_{\mathrm{p}}(\mathrm{cm})$ | $\mathrm{D}(\mathrm{cm})$ | $\mathrm{D} / \mathrm{d}_{\mathrm{p}}$ | $\mathrm{d}_{\mathrm{p}} / \mathrm{D}$ | $\varepsilon(1)$ | $\varepsilon(2)$ | $\varepsilon(3)$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0.635 | 12.065 | 19 | 0.053 | 0.381 | 0.385 |  |
| 1.27 | 12.065 | 9.5 | 0.105 |  | 0.393 | 0.397 |
| 1.905 | 12.065 | 6.33 | 0.158 |  | 0.414 | 0.416 |
| 3.302 | 12.065 | 3.65 | 0.274 |  | 0.471 | 0.465 |

## Pressure drop analysis and results

## Dependence of bed porosity on $D / d_{p}$



Fig. 7. Comparison of present work with existing correlations (porosity as a function of bed-to-particle diameter ratios).

Fig. 7. shows porosities as a function of tube to particle diameter ratios. Present work is similar with Chu (1989)[32] for $\mathrm{D} / \mathrm{d}_{\mathrm{p}}=19$ and other porosities(for $\mathrm{D} / \mathrm{d}_{\mathrm{p}}=9.5,6.33$ and 3.66) are similar with Sato(1973)[30].

Small differences in the values of mean porosity can give rise to big differences in the constants A and B of the Ergun equation [1]. It is therefore important that the mean porosity of the bed is accurately known so that reliable pressure drop correlations can be
formulated. In order to obtain such information, the mean porosity values of the beds considered here have been determined experimentally using three different methods.

Fig. 7. shows present work for porosity with existing porosity correlations. For decreasing bed-to-particle diameter ratios, porosity also increases as seen in our calculations. The average porosity of $\mathrm{D} / \mathrm{d}_{\mathrm{p}}=19$ is 0.385 and it is most similar with the result of Foumeny(1993)[8], Chu(1989)[32]. And the average porosity of D/d $\mathrm{d}_{\mathrm{p}}=9.5$ is 0.397. This porosity is same value with the porosity from $\operatorname{Sato(1973)}$ [30]. The average porosity of $\mathrm{D} / \mathrm{d}_{\mathrm{p}}=6.33$ is 0.416 and it is also most similar with the result of $\operatorname{Sato}(1973)$ [30]. The average porosity of $\mathrm{D} / \mathrm{d}_{\mathrm{p}}=3.65$ is 0.465 and it is most similar with the result of Sato(1973) [30].

Table 13. The modified Reynolds number regimes for present work.

|  | $\mathrm{D} / \mathrm{d}_{\mathrm{p}}$ | 19 | 9.5 | 6.33 | 3.66 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Air | $\mathrm{Re}_{\mathrm{m}}$ | $528 \sim 1,197$ | $646 \sim 3,797$ | $1,118 \sim 8,010$ |  |
| Water | $\mathrm{Re}_{\mathrm{m}}$ | $547 \sim 3,114$ | $2,648 \sim 9,102$ | $4,877 \sim 13,313$ | $20,046 \sim 29,936$ |

Fig. 8. indicates pressure drops per unit length as a function of the modified Reynolds numbers for air working fluid experiments. The pressure drop for large bed-to-particle diameter ratios $\left(\mathrm{D} / \mathrm{d}_{\mathrm{p}}=19\right)$ has more pressure drop at a constant modified Reynolds number. And the pressure drop was decreased as the bed-to-particle diameter ratios was decreased. Fig. 9. depicts the modified friction factor as a function of the modified

Reynolds number for air working fluid experiments. The results are linear even $D / d_{p}$ is different.

Where, $\quad f_{m}($ the modified friction factor $)=\frac{\Delta p d_{p}^{2} \varepsilon^{3}}{L \mu v(1-\varepsilon)^{2}}$
$\operatorname{Re}_{m}$ (the modified Reynolds number) $=\frac{\rho v d_{p}}{\mu(1-\varepsilon)}$

Fig. 10 indicates pressure drops per unit length as a function of the modified Reynolds numbers for water working fluid experiments. Like Fig. 8., the pressure drops were increased as $D / d_{p}$ were increased at a constant modified Reynolds number. Fig. 11. shows that the modified friction factors are linear with $\mathrm{D} / \mathrm{d}_{\mathrm{p}}$ is larger than 6.33 . However, the slope of modified friction factor for $D / d_{p}=3.65$ is less than the slope of modified friction factor for other $D / d_{p}$. From these results and Foumeny (1993)[8], in case of $D / d_{p}$ is less than 5 , the wall effect became an important factor.

Many correlations found in the literature considered wall effect. However, their consideration doesn't cover the small bed-to-particle diameter ratio $\left(\mathrm{D} / \mathrm{d}_{\mathrm{p}}\right)$ and their valid modified Reynolds number is low even though they considered it. This present work was compared with existing correlations. For this purpose, 20 correlations provided by Ergun(1952)[1], Foscolo(1983)[33], Handley and Heggs(1968)[34], Hicks(1970)[11], J.Wu(2008)[24], $\quad \operatorname{Leva}(1947)[9], \quad \operatorname{Macdonald}(1979)[13], \quad \operatorname{Montillet}(2004)[20]$, Reichelt(1972)[12], $\quad \mathrm{Yu}(2002)[35], \quad$ Shijie $\operatorname{Liu}(19994)[16], \quad$ Tallmadge(1970)[36], Brauer(1960)[37], Carman(1937)[5], Morcom(1946)[38], Foumeny(1993)[8], R.E.

Hayes(1995)[17], Rose(1949)[21], Wentz and Thodos(1963)[39] and KTA(1981)[2] were used. These correlations were plotted with their valid regime of the modified Reynolds number found in the literature. Fig. 12, 13 and 14 show comparison present work with correlations found in literatures at each bed-to-particle diameter ratios with air experiments. In these plots of Fig. 19 and 20, the Ergun[1] is matched well with present work for less than 1000 of $\mathrm{Re}_{\mathrm{m}}$. The Ergun[1] is only valid for modified Reynolds number less than 1000 . However, as the modified Reynolds number increases, the equation from Ergun does not match with present experiment results.

Fig. 15, 16, 17 and 18 indicate comparison present work of water working fluid with correlations in the literature at each bed-to-particle diameter ratios. KTA[2], Brauer[37], Carman[5] have similar trends with this present work for bed-to-particle diameter ratios of $19,9.5,6.33$. However, for $\mathrm{D} / \mathrm{d}_{\mathrm{p}}=3.66$, present work has less value than other correlations. In this bed-to-particle diameter ratios and in this modified Reynolds number region, present work is similar with Hicks[11] and Tallmadge[36].

Fig. 21, 22 explain this phenomenon by comparing with the KTA[2]. KTA[2] modified friction factor is similar with present work for $D / d_{p}=19,9.5$ and 6.33. However, the modified friction factor of KTA[2] has higher values than present experiment results. KTA[2] porosity is from 0.36 to 0.42 . It is limited and it is considered wall effect for relatively large bed-to-particle diameter ratios $\mathrm{D} / \mathrm{d}_{\mathrm{p}}=6.33$. However, as founded from Fig. 11, the wall effects become more important factors for pressure drop for $D / d_{p}$ is less than 5. From KTA[2] paper, it was presented that KTA[2] .is valid for only $0.36 \sim 0.42$ porosity regions. When $\mathrm{D} / \mathrm{d}_{\mathrm{p}}=3.65$, the porosity is 0.465 . It means that $\mathrm{KTA}[2]$ could
not consider high wall effects from their experiments. Therefore, it is concluded that the wall effect makes the difference between KTA[2] and present experiment results. In addition, existing correlations proposed by low bed-to-particle diameter ratios $\left(\mathrm{D} / \mathrm{d}_{\mathrm{p}}<5\right)$ has limited valid Reynolds number regimes. It signifies existing correlations couldn't apply widely. This present work is quite important, since the pressure drop of Gas Cooled Pebble Bed Reactor has to be predicted with high Reynolds numbers.

In case of $\mathrm{D} / \mathrm{d}_{\mathrm{p}}<5$, by fitting the KTA[2], in order to match with present work, a new correlation was derived.

$$
\begin{equation*}
f_{m}=\frac{\Delta p d_{p}^{2} \varepsilon^{3}}{L \mu v(1-\varepsilon)^{2}}=\frac{\frac{D}{d_{p}}}{0.2 \frac{D}{d_{p}}+3.6} \frac{\rho v d_{p}}{\mu(1-\varepsilon)}+160 \tag{39}
\end{equation*}
$$

This correlation is valid for $20,000<\operatorname{Re}_{\mathrm{m}}<29,936$ and $\mathrm{D} / \mathrm{d}_{\mathrm{p}}<5$.


Fig. 8. Pressure drops per unit length as a function of the modified Reynolds numbers for air working fluid experiments.

It is evident that increasing the Reynolds number increases the pressure drop across the bed. It can be seen that, under similar hydrodynamic conditions, decreasing the particle size or increasing the bed-to-particle diameter ratio results in a very large increase in the pressure drop. Since the mean porosity of the bed goes down significantly as the particle size decreases, thereby increasing the resistance to flow of the fluid. For a given bed diameter, the wall effect increases with increasing particle size. Thus, the fluid experiences more channeling in a bed of large-size particles than small ones and, therefore, provides a lower pressure drop. As a general rule, decreasing the particle size reduces the mean porosity of the bed and, thereby, increases the pressure drop across it.


Fig. 9. The modified friction factor as a function of the modified Reynolds number for air working fluid experiments.

When the column diameter is much greater than the particle diameter, the wall should have little effect on the characteristic length. In this air pressure drop experiments, $D / d_{p}$ $=19,9.5$ and 6.33. As we mensioned eariler, when $\mathrm{D} / \mathrm{d}_{\mathrm{p}}$ is less than 40 , wall effects have to be considered for pressure drop. Therefore, the wall effects may be important for these present experiments.


Fig. 10. Pressure drops per unit length as a function of the modified Reynolds numbers for water working fluid experiments

Like air results, in the case of water experiments, an increasing the Reynolds number increases the pressure drop across the bed. It can be also seen that, under similar hydrodynamic conditions, decreasing the particle size or increasing the bed-to-particle diameter ratio results in a very large increase in the pressure drop. Since the mean porosity of the bed goes down significantly as the particle size decreases, thereby increasing the resistance to flow of the fluid. In the case of $D / d_{p}=3.65$, the average porosity is 0.465 . This big increase of porosity causes small pressure drop as shown in Fig.10.


Fig. 11. The modified friction factor as a function of the modified Reynolds number for water working fluid experiments.

The pressure drop measurements for beds of spheres were plotted in the linear Ergun[1] manner, which in terms of dimensionless modified friction factor and the modified Reynolds number. The experimental pressure drop measurements should be affected by the presence of the wall, since the ratio of bed-to-particle diameter was small. In order to quantify the wall effect, the experiment was done at various bed-to-particle diameter ratios.

When the ratios of bed-to-particle diameter are $19,9.5$ and 6.33 , the modified fricton factor, $\mathrm{f}_{\mathrm{m}}$, has a similar value at a constant modified Reynolds number, $\mathrm{Re}_{\mathrm{m}}$. However, the modified friction factor is different in the case of $\mathrm{D} / \mathrm{d}_{\mathrm{p}}=3.65$ at a constant modified Reynolds number, $\mathrm{Re}_{\mathrm{m}}$. It is found from the slope of the linear line in Fig. 11.

Such a behavior was also observed by Foumeny et al.(1993)[8]. In the Foumeny et al.(1993) [8], the slope changed very shaply from $D / d_{p}=5.62$. Even though they also have differen slope between $D / d_{p}=19$ to $D / d_{p}=6.33$, the difference of that slope is not big. The cause of its difference reuslts between this work and Foumeny (1993) [8] is believed to be due to the experimental and fitting errors which tend to be more pronounced at extreme operating conditions. In addition, Foumeny[8] did experiments at a lower Reynolds number than these experiments. From this result and from Foumeny (1993) [8], it is concluded that the wall effect is important when the bed-to-particle diameter ratio is small, especially, when it is below around 5 . Therefore, the wall effect causes less pressure drop.

Indeed, a closer inspection of several other publications on the influence of the bed-toparticle diameter ratio reveals that statements of an increasing pressure drop due to the wall effect are generally based on experiments under stream flow conditions or in the transitional range. In contrast, an independent or decreasing pressure drop due to the wall effect is reported mainly for measurements at high Reynolds numbers. At least for streamline flow, the wall effect is important only for bed-to-particle diameter ratios below 10 , as stated frequently in the literature.


Fig. 12. The modified friction factor as a function of the modified Reynolds number for $\mathrm{D} / \mathrm{d}_{\mathrm{p}}=19$ with air working fluid.

It is desirable to compare the present work with existing correlations in the literature. For this purpose, 18 correlations provided by Ergun(1952) [1], Foscolo(1983) [33], Handley and Heggs(1968) [34], Hicks(1970) [11], J.Wu et al(2008) [24], Leva(2002) [9], Macdonald(1979) [13], Montillet(2004) [20], Reichelt(1972) [12], Yu et al(2002) [35], Shijie Liu(1994) [16], Tallmadge(1970) [36], Brauer(1960) [37], Carman(1970) [5], Morcom(1946) [38], Foumeny(1993) [8], R.E. Hayes(1995) [17] and KTA(1981) [2] were used.

Fig. 12 indicates the modified friction factor as a function of the modified Reynolds number, as defined by Ergun(1952) [1]. This figure shows the present experimental results as well as the published pressure drop correlations, with the following parameters of $\quad \mathrm{D}=12.065 \mathrm{~cm}, \mathrm{~d}_{\mathrm{p}}=0.635 \mathrm{~cm}$, and $\varepsilon=0.385$. Here, the bed-to-particle diameter ratio is 19 . These parameters are plugged into the above correlations. In addition, the air property like as density and dynamic viscosity at room temperature were considered. In this plot, the modified Reynolds number range goes from 353 to 1197 . The published correlations were plotted with their valid regime of modified Reynolds number found in the literature. As can be seen, all data points scatter around a general trend.

We observed that the current work (black line in the Figure X) agrees fairly well with the $\operatorname{Brauer}(1960)$ [37], $\operatorname{Hicks}(1970)$ [11], $\operatorname{KTA}(1981)$ [2], $\operatorname{Ergun}(1952)$ [1], Carman(1970) [5], Foumeny(1993) [8]. While R.E. Hayes(1995) [17], Handley and Heggs(1968) [34], Yu et al(2002) [35], J.Wu et al(2008) [24], Leva(2002) [9], Montillet(2004) [20] overpredict the present work in th entire range studied. This can be easily explained by the fact that the existing correlations were derived from experiments performed at high $D / d_{p}$ that has less wall effect. On the other hand, Morcom(1946) [38], Shijie Liu(1994) [16], Tallmadge(1970) [36] and Reichelt(1972) [12] underpredict the current work. This reason is because these correlations were derived from experiments at low $\mathrm{D} / \mathrm{d}_{\mathrm{p}}$ and at high $\mathrm{Re}_{\mathrm{m}}$. This clearly signifies the limited applicability of the existing correlations.


Fig. 13. The modified friction factor as a function of the modified Reynolds number for $\mathrm{D} / \mathrm{d}_{\mathrm{p}}=9.5$ with air working fluid.

In the case of $\mathrm{D} / \mathrm{d}_{\mathrm{p}}=9.5$, it was also compared with existing correlations. 19 correlations provided by Ergun(1952)[1], Foscolo(1983)[33], Handley and Heggs(1968)[34], Hicks(1970)[11], J.Wu(2008)[24], Leva(1947)[9], Macdonald(1979)[13], Montillet(2004)[20], $\quad \operatorname{Reichelt(1972)[12],~} \quad \mathrm{Yu}(2002)[35], \quad$ Shijie $\operatorname{Liu}(19994)[16]$, Tallmadge(1970)[36], Brauer(1960)[37], Carman(1937)[5], Foumeny(1993)[8], R.E. Hayes(1995)[17], Rose(1949)[21], Wentz and Thodos(1963)[39] and KTA(1981)[2] were Considered for this comparison.

Fig. 13 indicates the modified friction factor as a function of the modified Reynolds number, as defined by Ergun(1952) [1]. This figure shows the present experimental results as well as the published pressure drop correlations, with the following parameters of $\mathrm{D}=12.065 \mathrm{~cm}, \mathrm{~d}_{\mathrm{p}}=1.27 \mathrm{~cm}, \mathrm{D} / \mathrm{d}_{\mathrm{p}}=9.5$ and $\varepsilon=0.397$. In addition, the air property like as density and dynamic viscosity at room temperature were considered. These parameters are plugged into the above correlations. In this plot, the modified Reynolds number range goes from 647 to 4141 . The published correlations were plotted with their valid regime of modified Reynolds number found in the literature. As can be seen, all data points scatter around a general trend.

We observed that the current work (black line in the Fig. 13) agrees fairly well with the Brauer(1960)[37], Hicks(1970)[11], KTA(1981)[2], Foumeny(1993)[8], Carman(1970) [5], Foumeny(1993) [8], Wentz and Thodos(1963) [39]. While Ergun(1952) [1] doesn't match with the present work. Ergun overestimates present work when the Reynolds number is larger than 1000.


Fig. 14. The modified friction factor as a function of the modified Reynolds number for $\mathrm{D} / \mathrm{d}_{\mathrm{p}}=6.33$ with air working fluid.

In the case of $\mathrm{D} / \mathrm{d}_{\mathrm{p}}=6.33$, it was also compared with existing correlations. 14 correlations provided by Handley and Heggs(1968)[34], Hicks(1970)[11], J.Wu(2008)[24], $\operatorname{Leva(1947)[9],~} \quad \mathrm{Yu}(2002)[35], \quad \operatorname{Montillet(2004)[20],~}$ Reichelt(1972)[12], $\quad \operatorname{Rose}(1949)[21], \quad$ Tallmadge(1970)[36], $\quad \operatorname{Brauer}(1960)[37]$, Foumeny(1993)[8], R.E. Hayes(1995)[17], Wentz and Thodos(1963)[39] and KTA(1981)[2] were Considered for this comparison.

Fig. 14 indicates the modified friction factor as a function of the modified Reynolds number, as defined by $\operatorname{Ergun}(1952)$ [1]. This figure shows the present experimental
results as well as the published pressure drop correlations, with the following parameters of $\quad \mathrm{D}=12.065 \mathrm{~cm}, \mathrm{~d}_{\mathrm{p}}=1.905 \mathrm{~cm}, \mathrm{D} / \mathrm{d}_{\mathrm{p}}=9.5$ and $\varepsilon=0.416$. In addition, the air property like as density and dynamic viscosity at room temperature were considered. These parameters are plugged into the above correlations. In this plot, the modified Reynolds number range goes from 1118 to 7901 . The published correlations were plotted with their valid regime of modified Reynolds number found in the literature. As can be seen, all data points scatter around a general trend.

We observed that the current work (black line in the Fig. 14) agrees fairly well with the Handley and $\operatorname{Heggs(1968)~[34],~Brauer(1960)~[37],~KTA(1981)~[2].~}$


Fig. 15. The modified friction factor as a function of the modified Reynolds number for $\mathrm{D} / \mathrm{d}_{\mathrm{p}}=19$ with water working fluid.

It is desirable to compare the present work with existing correlations in the literature. For this purpose, 16 correlations provided by Ergun(1952)[1], Handley and Heggs(1968)[34], Hicks(1970)[11], J.Wu(2008)[24], Leva(1947)[9], Yu(2002)[35] Montillet(2004)[20], Reichelt(1972)[12], Shijie Liu(19994)[16], Tallmadge(1970)[36], Brauer(1960)[37], Carman(1937)[5], Foumeny(1993)[8], Rose(1949)[21], Wentz and Thodos(1963)[39] and KTA(1981)[2] were used.

Fig. 15 indicates the modified friction factor as a function of the modified Reynolds number, as defined by Ergun(1952) [1]. This figure shows the present experimental
results as well as the published pressure drop correlations, with the following parameters of $D=12.065 \mathrm{~cm}, \mathrm{~d}_{\mathrm{p}}=0.635 \mathrm{~cm}$, and $\varepsilon=0.385$ for water working fluid. Here, the bed-to-particle diameter ratio is 19 . These parameters are plugged into the above correlations. In addition, the water property like as density and dynamic viscosity at measured temperature were considered. In this plot, the modified Reynolds number range goes from 353 to 1197 . The published correlations were plotted with their valid regime of modified Reynolds number found in the literature. As can be seen, all data points scatter around a general trend.

We observed that the current work (black line in the Fig. 15) agrees fairly well with the Handley and Heggs(1968)[34], Hicks(1970)[11], Reichelt(1972)[12], Wentz and Thodos(1963)[39], $\quad \operatorname{Rose}(1949)[21], \quad \operatorname{Brauer}(1960)[37], \quad \operatorname{Carman}(1937)[5], \quad$ and KTA(1981)[2].

While $\operatorname{Ergun}(1952)[1], \mathrm{Yu}(2002)[35], \mathrm{J} . \mathrm{Wu}(2008)[24]$, Leva(1947)[9], Montillet(2004)[20] overpredict the present work in th entire range studied. This can be easily explained by the fact that the existing correlations were derived from experiments performed at high $\mathrm{D} / \mathrm{d}_{\mathrm{p}}$ that has less wall effect.

On the other hand, Shijie $\operatorname{Liu}(19994)[16]$, Tallmadge(1970)[36] underpredict the current work. This reason is because these correlations were derived from experiments at low $\mathrm{D} / \mathrm{d}_{\mathrm{p}}$ and at high $\mathrm{Re}_{\mathrm{m}}$. This clearly signifies the limited applicability of the existing correlations.


Fig. 16. The modified friction factor as a function of the modified Reynolds number for $\mathrm{D} / \mathrm{d}_{\mathrm{p}}=9.5$ with water working fluid.

13 correlations provided by Handley and Heggs(1968)[34], Hicks(1970)[11], J.Wu(2008)[24], Leva(1947)[9], Reichelt(1972)[12], Rose(1949)[21], Yu(2002)[35], Tallmadge(1970)[36], Brauer(1960)[37], Carman(1937)[5], Foumeny(1993)[8], Wentz and Thodos(1963)[39] and KTA(1981)[2] were used.

Fig. 16 indicates the modified friction factor as a function of the modified Reynolds number, as defined by Ergun(1952)[1]. This figure shows the present experimental results as well as the published pressure drop correlations, with the following parameters of $\mathrm{D}=12.065 \mathrm{~cm}, \mathrm{~d}_{\mathrm{p}}=1.27 \mathrm{~cm}$, and $\varepsilon=0.397$ for water working fluid. Here, the bed-
to-particle diameter ratio is 9.5 . These parameters are plugged into the above correlations. In addition, the water property like as density and dynamic viscosity at measured temperature were considered. In this plot, the modified Reynolds number range goes from 2524 to 9283 . The published correlations were plotted with their valid regime of modified Reynolds number found in the literature. As can be seen, all data points scatter around a general trend.

We observed that the current work (black line in the Fig. 16) agrees fairly well with the Handley and Heggs(1968)[34], Brauer(1960)[37], Carman(1937)[5], and KTA(1981)[2]. While Reichelt(1972)[12], Yu(2002)[35], Rose(1949)[21], J.Wu(2008)[24], Leva(2002), Foumeny(1993)[8] overpredict the present work in th entire range studied. This can be easily explained by the fact that the existing correlations were derived from experiments performed at high $\mathrm{D} / \mathrm{d}_{\mathrm{p}}$ that has less wall effect.

On the other hand, Hicks(1970)[11], Wentz and Thodos(1963)[39], Tallmadge(1970)[36] underpredict the current work. This reason is because these correlations were derived from experiments at low $D / d_{p}$ and at high $R e_{m}$. This clearly signifies the limited applicability of the existing correlations.

In the case of Rose(1949), it is matched very well by the modified Reynolds number is 6000. However, as it increases over $\operatorname{Re}_{\mathrm{m}}=6000$, it over-predicts the present work.


Fig. 17. The modified friction factor as a function of the modified Reynolds number for $\mathrm{D} / \mathrm{d}_{\mathrm{p}}=6.33$ with water working fluid.

For comparison of pressure drop in the case of $\mathrm{D} / \mathrm{d}_{\mathrm{p}}=6.33,11$ correlations provided by Handley and Heggs(1968)[34], Hicks(1970)[11], Leva(1947)[9], Reichelt(1972)[12], Tallmadge(1970)[36], Wentz and Thodos(1963)[39], Rose(1949)[21], Brauer(1960)[37], Carman(1937)[5], Foumeny(1993)[8] and KTA(1981)[2] were used.

Fig. 17 indicates the modified friction factor as a function of the modified Reynolds number, as defined by Ergun(1952)[1]. This figure shows the present experimental results as well as the published pressure drop correlations, with the following parameters of $\mathrm{D}=12.065 \mathrm{~cm}, \mathrm{~d}_{\mathrm{p}}=1.27 \mathrm{~cm}$, and $\varepsilon=0.397$ for water working fluid. Here, the bed-
to-particle diameter ratio is 9.5 . These parameters are plugged into the above correlations. In addition, the water property like as density and dynamic viscosity at measured temperature were considered. In this plot, the modified Reynolds number range goes from 4667 to 13920 . The published correlations were plotted with their valid regime of modified Reynolds number found in the literature. As can be seen, all data points scatter around a general trend.

We observed that the current work (black line in the Fig. 17) agrees fairly well with the Handley and Heggs(1968)[34], Brauer(1960)[37], and KTA(1981)[2].

While Reichelt(1972)[12], Rose(1949)[21], Leva(1947)[9], Foumeny(1993)[8] overpredict the present work in th entire range studied. This can be easily explained by the fact that the existing correlations were derived from experiments performed at high $D / d_{p}$ that has less wall effect.

On the other hand, Hicks(1970)[11], Wentz and Thodos(1963)[39], Carman(1937)[5], Tallmadge(1970)[36] underpredict the current work. This reason is because these correlations were derived from experiments at low $\mathrm{D} / \mathrm{d}_{\mathrm{p}}$ and at high $\mathrm{Re}_{\mathrm{m}}$. This clearly signifies the limited applicability of the existing correlations.


Fig. 18. The modified friction factor as a function of the modified Reynolds number for $\mathrm{D} / \mathrm{d}_{\mathrm{p}}=3.65$ with water working fluid.

For a bed-to-particle diameter ratio of 3.65 , a corresponding porosity is 0.465 was found in Fig. 18. In this plot, the modified Reynolds number range goes from 20,036 to 29,936 .

It is very difficult to do experiments with high Reynolds number. The reason is that there is not much of experiement results in the literature.

The pressure drop should be lower in this case, as a consequence of the existence of larger channels in the wall regions than those formed between spheres in the absense of a wall. The original Ergun equation[1] would thus overestimate the pressure drop. The

Ergun equation[1] wasn't plotted in Fig. 18. Because the modified Reynolds number is over their valid $\mathrm{Re}_{\mathrm{m}}$ regime.

By previouse cases, $\mathrm{D} / \mathrm{d}_{\mathrm{p}}=19,9.5$ and 6.33, Handley and Heggs(1968)[34], Brauer(1960)[37], and KTA(1981)[2] were matched very well with the present experiment work. However, $\operatorname{Brauer}(1960)$ [37] and KTA(1981)[2] didn't agree with the present work of $\mathrm{D} / \mathrm{d}_{\mathrm{p}}=3.65$ anymore because of large wall effects. Here, , Handley and Heggs(1968)[34] wasn't plottted in this figure due to their limitation Reynolds number. However, the current work (black line in the Fig. 18) agrees well with Hicks(1970)[11], and Wentz and Thodos (1963)[39]. Unfortunately, their correlations didn't match with current work in all of previous cases. Hicks(1970)[11] correlations is not the results of their experiments. They just made a best matched correlation by comparing many correlations like as Ergun(1952)[1], Handley and Heggs(1968)[34], Carman(1937)[5], Wentz and Thodos(1963)[39] and Morcom(1946)[38]. Otherwise, Wentz and Thodos(1963)[39] did experiments with high porosities $(\varepsilon=0.354,0.480,0.615$ and 0.728$)$.

Therefore, their experiments had large wall effects.


Fig. 19. Comparison of present work with Ergun equation (D/d $\mathrm{d}_{\mathrm{p}}=19$ with air).

As menstioned earlier, a couple of coefficients in Ergun's equation[1] have ever been disputed although this equation has been widely used in the engineering field.

The Ergun equation[1] predicts the experimental data well in the low modified Reynolds number as is shown in Fig. 19. and overpredicts the present work as the modified Reynolds is increased. And the discrepancy is increased at high modified Reynolds number as is shown in Fig. 20. The disagreement shown is probably due to the non consideration of wall effects in Ergun's equation[1]. A possible explanation is that, at high velocities, there is even more tendency for the flow to go through the larger channels close to the walls, where there is less friction.


Fig. 20. Comparison of present work with Ergun equation ( $D / d_{p}=9.5$ with air).

For turbulent flow, the kinetic effects on the flow friction (second term in the right-hand side of Ergun's equation[1]) are preponderant over the viscous effects (the first term in the right hand side of Ergun's equation[1]). For laminar flow (low Reynolds number), when the viscous effects are predominant, the friction area of the channels plays a major role.

Fig. 21, 22. indicate comparison of present work with KTA[2]. KTA[2] modified friction factor is similar with present work for $\mathrm{D} / \mathrm{d}_{\mathrm{p}}=19,9.5$ and 6.33 . KTA[2] porosity is from 0.36 to 0.42 . It is limited and it is considered wall effect for relatively large bed-to-particle diameter ratios $\mathrm{D} / \mathrm{d}_{\mathrm{p}}=6.33$.


Fig. 21. Comparison of present work with KTA
$\left(D / d_{p}=19,9.5\right.$ and 6.33 with air and water working fluids).


Fig. 22. Comparison of present work with KTA
$\left(D / d_{p}=3.65\right.$ with water working fluid $)$.

## Different friction factor

Many authors used modified friction factor to represent their result about pressure drop in packed beds. The modified friction factor came from the friction factor. The friction factor for packed bed is shown in equation 40.
f (friction factor for packed bed) $=\frac{\left(p_{\mathrm{O}}-\mathfrak{p}_{\mathrm{L}}\right) \rho}{\mathrm{G}^{2}}\left(\frac{\mathrm{~d}_{\mathrm{p}}}{\mathrm{L}}\right)\left(\frac{\varepsilon^{3}}{1-\varepsilon}\right)$
And the relationship between the modified friction factor and the friction factor is shown as:
$f_{m}$ (Modified friction factor for packed bed $)=f \times \operatorname{Re}_{m}$

Where, $\mathrm{f} \times \mathrm{Re}$ is essentially a dimensionless velocity gradient averaged over the surface. Therefore, the modified friction factor can be like this equation 42.

$$
\begin{align*}
\mathrm{f}_{\mathrm{m}}=\mathrm{f} \times \operatorname{Re}_{\mathrm{m}} & =\frac{\left(p_{\mathrm{o}}-p_{\mathrm{L}}\right) \rho}{(\rho \mathrm{v})^{2}}\left(\frac{\mathrm{~d}_{\mathrm{p}}}{\mathrm{~L}}\right)\left(\frac{\varepsilon^{3}}{1-\varepsilon}\right) \times \frac{\rho \mathrm{vd} \mathrm{p}^{2}}{\mu(1-\varepsilon)} \\
& =\frac{\left(p_{\mathrm{o}}-p_{\mathrm{L}}\right) \mathrm{d}_{\mathrm{p}}^{2}}{\mathrm{~L} \mu \mathrm{v}}\left(\frac{\varepsilon^{3}}{(1-\varepsilon)^{2}}\right) \tag{42}
\end{align*}
$$

The interesting thing is that $\mathrm{KTA}(1981)[2]$ used friction correlation, $\psi$ by modifying the friction factor in their paper. $\operatorname{KTA}(1981)[2]$ pressure drop correlation is shown as:

$$
\begin{equation*}
\frac{\Delta P}{L}=\left\{\frac{320}{\left(\frac{\operatorname{Re}}{1-\varepsilon}\right)}+\frac{6}{\left(\frac{\operatorname{Re}}{1-\varepsilon}\right)^{0.1}}\right\}\left\{\left(\frac{(1-\varepsilon)}{\varepsilon^{3}}\right)\left(\frac{1}{d_{p}}\right)\left(\frac{1}{2 \rho}\right)(\rho U)^{2}\right\} \tag{43}
\end{equation*}
$$

The friction correlation for $\operatorname{KTA}(1981)[2]$ is shown as:
$\Psi\left(\right.$ friction correlation for KTA) $=\left\{\frac{320}{\left(\frac{\operatorname{Re}}{1-\varepsilon}\right)}+\frac{6}{\left(\frac{\operatorname{Re}}{1-\varepsilon}\right)^{0.1}}\right\}$
The relationship between the friction factor and the friction correlation for KTA is represented like shown as:
$\mathrm{f}=\frac{\left(p_{\mathrm{O}}-p_{\mathrm{L}}\right) \rho}{\mathrm{G}^{2}}\left(\frac{\mathrm{~d}_{\mathrm{p}}}{\mathrm{L}}\right)\left(\frac{\varepsilon^{3}}{1-\varepsilon}\right)=\frac{1}{2}\left\{\frac{320}{\left(\frac{\operatorname{Re}}{1-\varepsilon}\right)}+\frac{6}{\left(\frac{\operatorname{Re}}{1-\varepsilon}\right)^{0.1}}\right\}=\frac{1}{2} \Psi$
Based on these relationship, the $\mathrm{KTA}(1981)$ correlation[2] can be represented with the modified friction factor.

$$
\begin{align*}
& \mathrm{f}_{\mathrm{m}}=\frac{\Delta \mathrm{Pd}_{\mathrm{p}}^{2} \varepsilon^{3}}{\mathrm{~L} \mu \mathrm{U}(1-\varepsilon)^{2}}=\mathrm{f} \times \mathrm{Re}_{\mathrm{m}}=\frac{1}{2} \Psi \times \operatorname{Re}_{\mathrm{m}} \\
& =\frac{1}{2}\left\{\frac{320}{\left(\frac{\operatorname{Re}}{1-\varepsilon}\right)}+\frac{6}{\left(\frac{\operatorname{Re}}{1-\varepsilon}\right)^{0.1}}\right\} \times \operatorname{Re}_{\mathrm{m}}=160+3 \operatorname{Re}_{\mathrm{m}}^{0.9} \tag{46}
\end{align*}
$$

Fig. 23 to 30 shows the comparison of present work with KTA by using the friction factor and the friction correlation that used in the $\operatorname{KTA(1981)[2]~paper.~}$


Fig. 23. The comparison of present work with KTA by using the friction factor $\left(D / d_{p}=19\right)$.


Fig. 24. The comparison of present work with KTA by using the friction correlation used in $\operatorname{KTA}\left(\mathrm{D} / \mathrm{d}_{\mathrm{p}}=19\right)$.


Fig. 25. The comparison of present work with KTA by using the friction factor $\left(\mathrm{D} / \mathrm{d}_{\mathrm{p}}=9.5\right)$.


Fig. 26. The comparison of present work with KTA by using the friction correlation used in KTA (D/d $\left.\mathrm{d}_{\mathrm{p}}=9.5\right)$.


Fig. 27. The comparison of present work with KTA by using the friction factor $\left(\mathrm{D} / \mathrm{d}_{\mathrm{p}}=6.33\right)$.


Fig. 28. The comparison of present work with KTA by using the friction correlation used in KTA ( $\mathrm{D} / \mathrm{d}_{\mathrm{p}}=6.33$ ).
$D / d_{p}=3.65$


Fig. 29. The comparison of present work with KTA by using the friction factor $\left(\mathrm{D} / \mathrm{d}_{\mathrm{p}}=3.65\right)$.


Fig. 30. The comparison of present work with KTA by using the friction correlation used in KTA ( $\mathrm{D} / \mathrm{d}_{\mathrm{p}}=3.65$ ).

## Error analysis

There are many factors to reduce error in present work. One of the most important factors is to keep the same porosity. When the bed is filled with beads, its porosity is changed even though same methods were used to fill particles into the bed. And the different porosity causes different pressure drop. It makes difficulties for us to compare air and water experiments. Therefore, the experiment of air and water were done continuously without repacking.

In addition, temperature is important factor. An exact checking of temperature gives accurate properties of air and water. The temperature of air didn't change very much. However, the water temperature was changed even while we were doing experiment. The temperature were checked when the pressure were measured. Also the differences of temperature were checked to estimate error regime of these experiments. A type of thermocouple was submerged into the reservoir tank to monitor the bulk liquid temperature. The flow rate was kept constant while the pressure transducer was reading pressures. Even while the pressure sensor was measuring the pressure, the temperature of the reservoir tank was increased about $1 \sim 2^{\circ} \mathrm{C}$. Total change of the temperature over one set experiment was about $6 \sim 10^{\circ} \mathrm{C}$.

Next effort for reducing error is to wait enough time for steady state. After pump works, it has to be waited for steady state of flow. When the flow meter gives same flow rate, the pressure transducer was started to measure pressures.

Another big issue is that it has to be no air between pressure sensor and bed water. It causes pressure measuring error. While the water flows, the sensor tab has to be connected loosely and water leaks through the pressure sensor and the water makes the left air out. After then tab should be locked totally. By doing these procedures, the air can be removed from the gap between the sensor and the water of bed.

Lastly, when the water flow the bed, any bubble has to be removed from the bed. The water of tank enters into the pump. And the water rotates the test loop. When it enters into the reservoir water tank again, some bubbles are made in the tank. The air might enter the water loop.

## Error analysis of air experiments

The uncertainty for the determination of the flow rate is $\pm 4 \%$. In addition, from the manufacturer, Dwyer Rate-master flow meter gives $\pm 2 \%$ uncertainty. The HiQ ventury digital instrument for measurement of flow rate has $\pm 4 \%$ uncertainty. The uncertainty for average bed porosity is $\pm 3 \%$.

Table 14 shows the uncertainty for porosity at each particle diameter case.

Table 14. The uncertainty for porosity measurement.

| $\mathrm{d}_{\mathrm{p}}$ | 0.635 cm | 1.27 cm | 1.905 cm | 3.3 cm |
| :---: | :---: | :---: | :---: | :---: |
| Porosity 1 | 0.381 | $\cdot$ | $\cdot$ | $\cdot$ |
| Porosity 2 | 0.385 | 0.393 | 0.414 | 0.471 |
| Porosity 3 | $\cdot$ | 0.397 | 0.416 | 0.465 |
| Porosity <br> (chosen) | 0385 | 0.397 | 0.416 | 0.465 |
| Regime from <br> other's <br> correlations | $0.368 \sim 0.395$ | $0.377 \sim 0.405$ | $0.388 \sim 0.419$ | $0.417 \sim 0.479$ |
| Uncertainty | 2.53 | 1.97 | 0.71 | 2.92 |

The uncertainty for pressure measurement is $\pm 1 \%$. Table 15 shows the uncertainty for pressure measurement.

Table 15. The uncertainty for pressure measurement.

| Instrument | Uncertainty $(\sigma)$ |
| :---: | :---: |
| Digital manometer (chosen) | $\pm 1 \%$ |
| Inclined manometer | $\pm 1 \%$ |
| Magnehelic differential pressure gauge | $\pm 2 \%$ |

The estimated uncertainty associated to the determination of the modified Reynolds number is $\pm 5 \%$. The uncertainty of the modified Reynolds number is consist of velocity (v) and porosity $(\varepsilon)$. The other factors like density, viscosity of air and particle diameter are assumed that they are constant. The estimated uncertainty associated to the determination of the modified friction factor is $\pm 5.1 \%$. This uncertainty is composed of differential pressure, porosity and velocity.

These uncertainties were determined by the Kline and McClintock [40] method.

## Error analysis for water experiments

The uncertainty for the determination of the flow rate is $\pm 1 \%$. From the manufacturer, G2 Industrial Grade flow meter has $\pm 1 \%$ error. The uncertainty for average bed porosity is $\pm 3 \%$. Table 14 shows the uncertainty for porosity at each particle diameter case.

The accuracy of the pressure transducer (sensor: PX309 series, 30 psi) from the manufacturer is $\pm 2 \%$. FSO (except $1 \mathrm{psi}= \pm 4.5 \%$ and $2 \mathrm{psi}= \pm 3 \%$ ). This error band includes linearity, hysteresis, repeatability, thermal hysteresis, and thermal errors.

The estimated uncertainty associated to the determination of the modified Reynolds number is $\pm 5.1 \%$. The uncertainty of the modified Reynolds number is consist of velocity(v), porosity( $\varepsilon$ ) and density, viscosity of water. Particle diameter is assumed that it is constant and could be ignored. The density and viscosity of water is changed with water temperature. The water is increased because of pump heat while we are measuring the flow rate. And the temperature of water is measured from the reservoir not the bed. The estimated uncertainty associated to the determination of the modified friction factor is $\pm 5.5 \%$. This uncertainty is composed of differential pressure, porosity, velocity and viscosity of water.

These uncertainties were determined by the Kline and McClintock[40] method.

Fig. 31 to 34 show the error estimation of present work of different particle size. In addition, Fig. 35 to 38 show the error comparison of present work with KTA. KTA has $\pm$ $15 \%$ error in their experiments. When the error is considered both KTA and present work they match very well each other except for the case of $D / d_{p}=3.65$.


Fig. 31. An error estimation for $D / d_{p}=19$.


Fig. 32. An error estimation for $\mathrm{D} / \mathrm{d}_{\mathrm{p}}=9.5$.


Fig. 33. An error estimation for $\mathrm{D} / \mathrm{d}_{\mathrm{p}}=6.33$.


Fig. 34. An error estimation for $D / d_{p}=3.65$

$$
\mathrm{D} / \mathrm{d}_{\mathrm{p}}=19
$$



Fig. 35. A comparison with KTA including error bar $\left(D / d_{p}=19\right)$.


Fig. 36. A comparison with KTA including error bar $\left(D / d_{p}=9.5\right)$.


Fig. 37. A comparison with KTA including error bar $\left(D / d_{p}=6.33\right)$.


Fig. 38. A comparison with KTA including error bar ( $D / d_{p}=3.65$ ).

## CHAPTER IV

## CONCLUSIONS

The average bed porosities based on experiments was compared with existing correlations. In addition, from data analysis of present work, it is demonstrated that correlations for pressure drop found in the literature doesn't predict pressure drop at high Reynolds numbers correctly. Even the system has wall effects, it is more difficult to predict pressure drop accurately from existing correlations. The wall effects became more prominent at $\mathrm{D} / \mathrm{d}_{\mathrm{p}}<5$. This thesis is based on a series of experiments from low Reynolds numbers to high Reynolds numbers ( $528<\mathrm{Re}_{\mathrm{m}}<29936$ ), with different bed-to-particle diameter ratios( 19, 9.56 .33 and 3.65). KTA [2] correlation was matched well with present work for bed-to-particle diameter ratios of 19, 9.5 and 6.33 . However, in the case of $D / d_{p}=3.65$, KTA [2] was over predicted because of wall effect of the system. Therefore, a new correlation for pressure drop at high Reynolds numbers ( $20000<\operatorname{Re}_{\mathrm{m}}$ $<29936$ ) and low bed-to-particle diameter ratios ( $\mathrm{D} / \mathrm{d}_{\mathrm{p}}<5$ ) was developed.

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## APPENDIX

## Data from the present experiments

## Water experiment data

Table 16. Water experiment data for $\mathrm{D} / \mathrm{d}_{\mathrm{p}}=19$ experiments of vertical bed set up with up-flow direction.

| HZ | $\mathrm{Q}(\mathrm{gpm})$ | $\mathrm{Q}(\mathrm{cfm})$ | $\mathrm{Q}\left(\mathrm{m}^{\wedge} 3 / \mathrm{s}\right)$ | T | $\mu(34 \mathrm{C})$ | $\rho(34 \mathrm{C})$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 17.5 | 11.17 | 1.493429 | 0.0007048 | 32.3 | 0.000715 | 993.9346 |
| 20 | 12.97 | 1.734089 | 0.0008184 | 23 | 0.0007038 | 993.6555 |
| 25 | 16.42 | 2.195354 | 0.0010361 | 26 | 0.000752 | 994.77355 |
| 30 | 20.03 | 2.678011 | 0.0012639 | 33 | 0.0007024 | 993.62026 |
| 35 | 23.56 | 3.149972 | 0.0014866 | 33.2 | 0.0007567 | 994.87078 |
| 40 | 27.12 | 3.625944 | 0.0017113 | 33.1 | 0.0007024 | 993.62026 |
| 45 | 30.75 | 4.111275 | 0.0019403 | 33.5 | 0.0007505 | 994.74097 |
| 50 | 34.33 | 4.589921 | 0.0021662 | 33.5 | 0.0007052 | 993.69067 |
| 55 | 38 | 5.0806 | 0.0023978 | 33.3 | 0.0007324 | 994.34364 |
| 60 | 41.58 | 5.559246 | 0.0026237 | 33 | 0.0007136 | 993.89999 |

(a)

| P1(PSI) | P2(PSI) | P3(PSI) | P4(PSI) | P5(PSI) |
| :---: | :---: | :---: | :---: | :---: |
| 1.7632461 | 1.06277 | 0.2046166 | -0.540239 | -1.371802 |
| 2.6002624 | 1.7808707 | 0.8054075 | -0.063736 | -1.01844 |
| 4.6352071 | 3.5388656 | 2.28219 | 1.10657 | -0.13904 |
| 7.0782088 | 5.6503061 | 4.0606152 | 2.5503655 | 0.9596776 |
| 9.9841893 | 8.159239 | 6.1557719 | 4.2323134 | 2.2293742 |
| 13.278702 | 11.019763 | 8.5919063 | 6.220436 | 3.7625058 |
| 17.034853 | 14.263273 | 11.325116 | 8.4029013 | 5.4116326 |
| 21.168358 | 17.877355 | 14.429607 | 10.949264 | 7.4191917 |
| 25.740563 | 21.832859 | 17.773202 | 13.631383 | 9.4582808 |
| 30.01416 | 26.25478 | 21.583846 | 16.776899 | 11.954107 |

(b)

Table 16. continued.

| $\mathrm{Re}_{\mathrm{m}}$ | $\mathrm{Q}\left(\mathrm{m}^{\wedge} 3 / \mathrm{s}\right)$ | $\mathrm{Q}(\mathrm{cfm})$ | $\mathrm{P}(\mathrm{Pa})$ | P <br> (inches water) | $\mathrm{f}_{\mathrm{m}}$ | $\mathrm{v}(\mathrm{m} / \mathrm{s})$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 899.47 | 0.0007 | 1.4928 | 5916.23 | 23.77 | 1794 | 0.0617 |
| 1060.77 | 0.0008 | 1.7334 | 7623.39 | 30.63 | 2022 | 0.0716 |
| 1258.20 | 0.0010 | 2.1944 | 11736.16 | 47.16 | 2301 | 0.0906 |
| 1641.35 | 0.0013 | 2.6769 | 16427.80 | 66.01 | 2827 | 0.1106 |
| 1794.39 | 0.0015 | 3.1487 | 22112.27 | 88.85 | 3003 | 0.1300 |
| 2222.33 | 0.0017 | 3.6244 | 28343.78 | 113.89 | 3603 | 0.1497 |
| 2361.01 | 0.0019 | 4.1096 | 35811.84 | 143.89 | 3758 | 0.1697 |
| 2802.30 | 0.0022 | 4.5880 | 43379.30 | 174.30 | 4339 | 0.1895 |
| 2988.53 | 0.0024 | 5.0785 | 52369.28 | 210.42 | 4556 | 0.2097 |
| 3354.77 | 0.0026 | 5.5569 | 61435.81 | 246.85 | 5014 | 0.2295 |

(c)

Table 17. Water experiment data for $\mathrm{D} / \mathrm{d}_{\mathrm{p}}=19$ experiments of horizontal bed set-up.

| HZ | $\mathrm{Q}(\mathrm{gpm})$ | $\mathrm{Q}(\mathrm{cfm})$ | $\mathrm{Q}\left(\mathrm{m}^{\wedge} 3 / \mathrm{s}\right)$ | T | $\mu(28 \mathrm{C})$ | $\rho(28 \mathrm{C})$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 12.5 | 6.83 | 0.913 | 0.0004 | 32.3 | 0.00076 | 994.9352 |
| 15 | 8.6 | 1.150 | 0.0005 | $23 ?$ | 0.000749 | 994.7083 |
| 17.5 | 10.42 | 1.393 | 0.0007 | $26 ?$ | 0.000705 | 993.6907 |
| 20 | 12.2 | 1.631 | 0.0008 | 33 | 0.000749 | 994.7083 |
| 25 | 15.42 | 2.062 | 0.0010 | 33.2 | 0.000746 | 994.6428 |
| 30 | 18.81 | 2.515 | 0.0012 | 33.1 | 0.000747 | 994.6756 |
| 35 | 22.11 | 2.956 | 0.0014 | 33.5 | 0.000741 | 994.5438 |
| 40 | 25.43 | 3.400 | 0.0016 | 33.5 | 0.000741 | 994.5438 |
| 45 | 28.75 | 3.844 | 0.0018 | 33.3 | 0.000744 | 994.6098 |
| 50 | 32.16 | 4.300 | 0.0020 | 33 | 0.000749 | 994.7083 |
| 55 | 35.47 | 4.742 | 0.0022 | 32 | 0.000765 | 995.0312 |
| 60 | 38.88 | 5.198 | 0.0025 | 30.4 | 0.000791 | 995.5301 |

(a)

| P1(PSI) | P2(PSI) | P3(PSI) | P4(PSI) | P5(PSI) |
| :---: | :---: | :---: | :---: | :---: |
| 0.46 | 0.33 | 0.05 | -0.08 | -0.31 |
| 1.10 | 0.90 | 0.53 | 0.32 | 0.01 |
| 1.86 | 1.55 | 1.08 | 0.78 | 0.39 |
| 2.71 | 2.30 | 1.73 | 1.33 | 0.83 |
| 4.78 | 4.10 | 3.27 | 2.63 | 1.89 |
| 7.27 | 6.29 | 5.13 | 4.21 | 3.20 |
| 10.20 | 8.85 | 7.34 | 6.09 | 4.76 |
| 13.56 | 11.82 | 9.90 | 8.27 | 6.57 |
| 17.37 | 15.17 | 12.80 | 10.74 | 8.62 |
| 21.62 | 18.91 | 16.04 | 13.50 | 10.91 |
| 26.27 | 23.01 | 19.61 | 16.53 | 13.43 |
| 30.01 | 27.54 | 23.54 | 19.88 | 16.23 |

(b)

Table 17. continued.

| $\mathrm{Re}_{\mathrm{m}}$ | $\mathrm{Q}\left(\mathrm{m}^{\wedge} 3 / \mathrm{s}\right)$ | $\mathrm{Q}(\mathrm{cfm})$ | $\Delta \mathrm{P}(3-5)(\mathrm{Pa})$ | $\Delta \mathrm{P}(3-5)$ <br> (inches <br> water) | $\mathrm{f}_{\mathrm{m}}$ | $\mathrm{v}(\mathrm{m} / \mathrm{s})$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 518.0825 | 0.0004 | 0.9128 | 2510.7169 | 10.0881 | 1171.5105 | 0.0377 |
| 661.6456 | 0.0005 | 1.1493 | 3547.2579 | 14.2529 | 1333.5559 | 0.0475 |
| 850.5672 | 0.0007 | 1.3926 | 4791.6428 | 19.2528 | 1579.0364 | 0.0575 |
| 938.6135 | 0.0008 | 1.6305 | 6166.0695 | 24.7753 | 1634.0506 | 0.0673 |
| 1191.1256 | 0.0010 | 2.0608 | 9474.5127 | 38.0686 | 1994.6389 | 0.0851 |
| 1450.0716 | 0.0012 | 2.5139 | 13298.1944 | 53.4321 | 2290.3866 | 0.1038 |
| 1718.1952 | 0.0014 | 2.9549 | 17793.4736 | 71.4942 | 2628.5581 | 0.1220 |
| 1976.1965 | 0.0016 | 3.3986 | 22988.9251 | 92.3695 | 2952.6902 | 0.1404 |
| 2225.2683 | 0.0018 | 3.8423 | 28856.6472 | 115.9460 | 3265.0175 | 0.1587 |
| 2474.2467 | 0.0020 | 4.2980 | 35383.1653 | 142.1696 | 3557.1122 | 0.1775 |
| 2674.1540 | 0.0022 | 4.7404 | 42559.7324 | 171.0050 | 3800.2483 | 0.1958 |
| 2836.0785 | 0.0025 | 5.1961 | 50416.0331 | 202.5716 | 3971.6019 | 0.2146 |

(c)

Table 18. Water experiment data for $\mathrm{D} / \mathrm{d}_{\mathrm{p}}=9.5$ experiments of vertical bed set up with
up-flow direction.

| HZ | $\mathrm{Q}(\mathrm{gpm})$ | $\mathrm{Q}(\mathrm{cfm})$ | $\mathrm{Q}\left(\mathrm{m}^{\wedge} 3 / \mathrm{s}\right)$ | T | vis(34C) | $\operatorname{dens}(34 \mathrm{C})$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 22.5 | 20.1 | 2.6874 | 0.0013 | 37.5 | 0.00068 | 993.155 |
| 25 | 22.49 | 3.0069 | 0.0014 | 35.9 | 0.00071 | 993.726 |
| 27.5 | 24.69 | 3.3011 | 0.0016 | 37.5 | 0.00068 | 993.155 |
| 30 | 28.29 | 3.7824 | 0.0018 | 32.6 | 0.00076 | 994.838 |
| 32.5 | 29.35 | 3.9241 | 0.0019 | 37.7 | 0.00068 | 993.082 |
| 35 | 31.65 | 4.2316 | 0.0020 | 36 | 0.00071 | 993.691 |
| 37.5 | 33.91 | 4.5338 | 0.0021 | 37.6 | 0.00068 | 993.119 |
| 40 | 38.13 | 5.0980 | 0.0024 | 32.8 | 0.00075 | 994.774 |
| 42.5 | 38.53 | 5.1515 | 0.0024 | 37.6 | 0.00068 | 993.119 |
| 45 | 40.78 | 5.4523 | 0.0026 | 36.1 | 0.00070 | 993.656 |
| 47.5 | 43.15 | 5.7692 | 0.0027 | 37.3 | 0.00069 | 993.227 |
| 50 | 47.76 | 6.3855 | 0.0030 | 33.6 | 0.00074 | 994.511 |
| 52.5 | 47.57 | 6.3601 | 0.0030 | 36.8 | 0.00069 | 993.407 |
| 55 | 49.95 | 6.6783 | 0.0032 | 34.6 | 0.00073 | 994.175 |
| 57.5 | 51.89 | 6.9377 | 0.0033 | 36 | 0.00071 | 993.691 |

(a)

| P1(PSI) | P2(PSI) | P3(PSI) | P4(PSI) | P5(PSI) |
| :---: | :---: | :---: | :---: | :---: |
| 3.0973 | 2.6856 | 1.7417 | 0.9471 | 0.0332 |
| 4.0323 | 3.6242 | 2.5848 | 1.6745 | 0.6477 |
| 5.1268 | 4.6903 | 3.5425 | 2.5259 | 1.3832 |
| 5.9815 | 5.5241 | 4.1851 | 2.9693 | 1.6266 |
| 7.5756 | 7.0970 | 5.6961 | 4.4365 | 3.0234 |
| 8.9590 | 8.4614 | 6.9308 | 5.5291 | 3.9696 |
| 10.4490 | 9.9120 | 8.2462 | 6.6970 | 4.9808 |
| 11.3690 | 10.7927 | 8.8601 | 7.0144 | 4.9899 |
| 13.7131 | 13.1164 | 11.1607 | 9.2836 | 7.2285 |
| 15.5280 | 14.9077 | 12.8090 | 10.7554 | 8.5176 |
| 17.3824 | 16.7207 | 14.4591 | 12.2211 | 9.7971 |
| 18.2621 | 17.5414 | 14.9220 | 12.2705 | 9.4204 |
| 21.4524 | 20.7231 | 18.1219 | 15.4934 | 12.6661 |
| 23.4917 | 22.7220 | 19.9159 | 17.0423 | 13.9702 |
| 25.9376 | 25.1335 | 22.1779 | 19.1239 | 15.8659 |

(b)

Table 18. continued.

| $\mathrm{Re}_{\mathrm{m}}$ | $\mathrm{Q}\left(\mathrm{m}^{\wedge} 3 / \mathrm{s}\right)$ | $\mathrm{Q}(\mathrm{cfm})$ | $\mathrm{P}(\mathrm{Pa})$ | P (inches water) | $\mathrm{f}_{\mathrm{m}}$ | $\mathrm{v}(\mathrm{m} / \mathrm{s})$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 3165 | 0.0013 | 2.6863 | 6824 | 27 | 4579 | 0.1109 |
| 3542 | 0.0014 | 3.0057 | 8400 | 34 | 5037 | 0.1241 |
| 3888 | 0.0016 | 3.2997 | 9932 | 40 | 5425 | 0.1363 |
| 4455 | 0.0018 | 3.7808 | 12684 | 51 | 6047 | 0.1561 |
| 4622 | 0.0019 | 3.9225 | 13471 | 54 | 6190 | 0.1620 |
| 4984 | 0.0020 | 4.2298 | 15460 | 62 | 6588 | 0.1747 |
| 5340 | 0.0021 | 4.5319 | 17558 | 71 | 6983 | 0.1872 |
| 6005 | 0.0024 | 5.0959 | 21728 | 87 | 7685 | 0.2104 |
| 6068 | 0.0024 | 5.1493 | 22155 | 89 | 7755 | 0.2127 |
| 6422 | 0.0026 | 5.4500 | 24631 | 99 | 8146 | 0.2251 |
| 6796 | 0.0027 | 5.7668 | 27186 | 109 | 8497 | 0.2382 |
| 7522 | 0.0030 | 6.3829 | 32973 | 132 | 9311 | 0.2636 |
| 7492 | 0.0030 | 6.3575 | 32658 | 131 | 9259 | 0.2626 |
| 7866 | 0.0032 | 6.6755 | 36035 | 145 | 9730 | 0.2757 |
| 8172 | 0.0033 | 6.9348 | 38561 | 155 | 10022 | 0.2864 |

(c)

Table 19. Water experiment data for $\mathrm{D} / \mathrm{d}_{\mathrm{p}}=9.5$ experiments of horizontal bed set-up.

| HZ | $\mathrm{Q}(\mathrm{gpm})$ | $\mathrm{Q}(\mathrm{cfm})$ | $\mathrm{Q}\left(\mathrm{m}^{\wedge} 3 / \mathrm{s}\right)$ | T | vis(37C) | dens(37C) |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 17.5 | 15.87 | 2.121819 | 0.0010014 | 34.5 | 0.000727 | 994.20859 |
| 20 | 18.42 | 2.462754 | 0.0011623 | 34.8 | 0.000722 | 994.10644 |
| 20 | 18.31 | 2.448047 | 0.0011553 | 35 | 0.000719 | 994.03795 |
| 22.5 | 20.74 | 2.772938 | 0.0013087 | 36 | 0.000705 | 993.69067 |
| 25 | 23.18 | 3.099166 | 0.0014626 | 39.9 | 0.000654 | 992.26184 |
| 30 | 27.75 | 3.710175 | 0.001751 | 39.4 | 0.00066 | 992.4515 |
| 35 | 32.78 | 4.382686 | 0.0020684 | 39.3 | 0.00066 | 992.48921 |
| 40 | 37.14 | 4.965618 | 0.0023435 | 39.7 | 0.000657 | 992.33793 |
| 45 | 42.41 | 5.670217 | 0.002676 | 39.6 | 0.000658 | 992.37586 |
| 50 | 46.2 | 6.17694 | 0.0029152 | 40 | 0.000653 | 992.22368 |
| 55 | 51 | 6.8187 | 0.0032181 | 40 | 0.000653 | 992.22368 |
| 60 | 54.55 | 7.293335 | 0.0034421 | 38 | 0.000678 | 992.97252 |

(a)

| P1(PSI) | P2(PSI) | P3(PSI) | P4(PSI) | P5(PSI) |
| :---: | :---: | :---: | :---: | :---: |
| 1.636 | 1.555 | 1.250 | 0.944 | 0.627 |
| 2.354 | 2.330 | 1.836 | 1.467 | 0.988 |
| 2.382 | 2.291 | 1.895 | 1.497 | 1.087 |
| 3.217 | 3.115 | 2.618 | 2.118 | 1.602 |
| 4.124 | 4.061 | 3.371 | 2.790 | 2.093 |
| 6.330 | 6.220 | 5.300 | 4.490 | 3.538 |
| 8.818 | 8.651 | 7.454 | 6.342 | 5.070 |
| 11.930 | 11.705 | 10.265 | 8.853 | 7.272 |
| 15.072 | 14.771 | 12.978 | 11.164 | 9.167 |
| 19.215 | 18.856 | 16.815 | 14.682 | 12.354 |
| 23.038 | 22.593 | 20.140 | 17.544 | 14.732 |
| 28.102 | 27.587 | 24.863 | 21.894 | 18.706 |

(b)

Table 19. continued.

| $\mathrm{Re}_{\mathrm{m}}$ | $\mathrm{Q}\left(\mathrm{m}^{\wedge} 3 / \mathrm{s}\right)$ | $\mathrm{Q}(\mathrm{cfm})$ | $\mathrm{P}(\mathrm{Pa})$ | $\mathrm{P}($ inches water $)$ | $\mathrm{f}_{\mathrm{m}}$ | $\mathrm{v}(\mathrm{m} / \mathrm{s})$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 2524 | 0.00100 | 2.121 | 4298.56 | 17.27 | 3690.47 | 0.09 |
| 2947 | 0.00116 | 2.462 | 5849.37 | 23.50 | 4352.77 | 0.10 |
| 2941 | 0.00116 | 2.447 | 5568.56 | 22.37 | 4185.39 | 0.10 |
| 3397 | 0.00131 | 2.772 | 7007.37 | 28.16 | 4742.83 | 0.11 |
| 4087 | 0.00146 | 3.098 | 8812.29 | 35.41 | 5752.56 | 0.13 |
| 4848 | 0.00175 | 3.709 | 12145.13 | 48.80 | 6560.41 | 0.15 |
| 5727 | 0.00207 | 4.381 | 16437.04 | 66.04 | 7516.34 | 0.18 |
| 6524 | 0.00234 | 4.964 | 20633.43 | 82.91 | 8374.91 | 0.20 |
| 7436 | 0.00268 | 5.668 | 26275.04 | 105.57 | 9321.96 | 0.23 |
| 8161 | 0.00292 | 6.174 | 30752.66 | 123.56 | 10091.18 | 0.25 |
| 9008 | 0.00322 | 6.816 | 37283.94 | 149.81 | 11082.89 | 0.28 |
| 9283 | 0.00344 | 7.290 | 42442.47 | 170.53 | 11355.45 | 0.30 |

(c)

Table 20. Water experiment data for $\mathrm{D} / \mathrm{d}_{\mathrm{p}}=6.33$ experiments of vertical bed set up with up-flow direction.

| HZ | $\mathrm{Q}(\mathrm{gpm})$ | $\mathrm{Q}(\mathrm{cfm})$ | $\mathrm{Q}\left(\mathrm{m}^{\wedge} 3 / \mathrm{s}\right)$ | T | $\operatorname{vis}(30 \mathrm{C})$ | dens(30C) |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 27.5 | 28.15 | 3.763655 | 0.0017762 | 31.7 | 0.000769 | 995.126 |
| 30 | 30.78 | 4.115286 | 0.0019422 | 28.8 | 0.000818 | 996.007 |
| 35 | 36.2 | 4.83994 | 0.0022842 | 30 | 0.000797 | 995.651 |
| 40 | 41.73 | 5.579301 | 0.0026331 | 29.9 | 0.000799 | 995.682 |
| 45 | 47.09 | 6.295933 | 0.0029713 | 29.9 | 0.000797 | 995.651 |
| 50 | 52.3 | 6.99251 | 0.0033001 | 30.3 | 0.000792 | 995.561 |
| 55 | 57.46 | 7.682402 | 0.0036257 | 30.9 | 0.000782 | 995.377 |
| 60 | 62.38 | 8.340206 | 0.0039361 | 31.5 | 0.000773 | 995.189 |

(a)

| P1(PSI) | P2(PSI) | P3(PSI) | P4(PSI) | P5(PSI) |
| :---: | :---: | :---: | :---: | :---: |
| 3.753 | 3.001 | 2.050 | 1.205 | 0.284 |
| 4.588 | 3.761 | 2.723 | 1.780 | 0.768 |
| 6.594 | 5.601 | 4.354 | 3.212 | 1.994 |
| 8.910 | 7.731 | 6.254 | 4.878 | 3.419 |
| 11.535 | 10.138 | 8.414 | 6.767 | 5.045 |
| 14.429 | 12.810 | 10.815 | 8.865 | 6.852 |
| 17.639 | 15.772 | 13.484 | 11.200 | 8.863 |
| 21.321 | 19.207 | 16.616 | 14.002 | 11.346 |

(b)

| $\mathrm{Re}_{\mathrm{m}}$ | $\mathrm{Q}\left(\mathrm{m}^{\wedge} 3 / \mathrm{s}\right)$ | $\mathrm{Q}(\mathrm{cfm})$ | $\mathrm{P}(\mathrm{Pa})$ | $\mathrm{P}($ inches <br> water $)$ | $\mathrm{f}_{\mathrm{m}}$ | $\mathrm{v}(\mathrm{m} / \mathrm{s})$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 6324 | 0.0018 | 3.7621 | 7218.10 | 29.00 | 8779 | 0.155 |
| 6915 | 0.0019 | 4.1136 | 8518.38 | 34.23 | 9475 | 0.170 |
| 8132 | 0.0023 | 4.8379 | 11313.58 | 45.46 | 10700 | 0.200 |
| 9374 | 0.0026 | 5.5770 | 14580.83 | 58.59 | 11963 | 0.230 |
| 10578 | 0.0030 | 6.2933 | 18265.13 | 73.39 | 13280 | 0.260 |
| 11749 | 0.0033 | 6.9896 | 22362.14 | 89.85 | 14639 | 0.289 |
| 12908 | 0.0036 | 7.6792 | 26892.77 | 108.06 | 16024 | 0.317 |
| 14013 | 0.0039 | 8.3367 | 31374.06 | 126.06 | 17220 | 0.344 |

(c)

Table 21. Water experiment data for $\mathrm{D} / \mathrm{d}_{\mathrm{p}}=6.33$ experiments of horizontal bed set-up.

| HZ | $\mathrm{Q}(\mathrm{gpm})$ | $\mathrm{Q}(\mathrm{cfm})$ | $\mathrm{Q}\left(\mathrm{m}^{\wedge} 3 / \mathrm{s}\right)$ | T | vis(26C) | dens(26C) |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 22.5 | 23.67 | 3.1647 | 0.0015 | 24.1 | 0.00091 | 997.29881 |
| 25 | 26.4 | 3.5297 | 0.0017 | 24.4 | 0.00091 | 997.29881 |
| 27.5 | 29.18 | 3.9014 | 0.0018 | 24 | 0.00091 | 997.29881 |
| 30 | 31.99 | 4.2771 | 0.0020 | 26.3 | 0.00086 | 996.70648 |
| 35 | 37.56 | 5.0218 | 0.0024 | 26.5 | 0.00086 | 996.65262 |
| 40 | 43.27 | 5.7852 | 0.0027 | 26.5 | 0.00086 | 996.65262 |
| 45 | 48.71 | 6.5125 | 0.0031 | 26 | 0.00087 | 996.78656 |
| 50 | 54.09 | 7.2318 | 0.0034 | 26.9 | 0.00085 | 996.5438 |
| 55 | 59.22 | 7.9177 | 0.0037 | 25.3 | 0.00088 | 996.97016 |
| 60 | 64.61 | 8.6384 | 0.0041 | 27.8 | 0.00083 | 996.23701 |

(a)

| P1(PSI) | P2(PSI) | P3(PSI) | P4(PSI) | P5(PSI) |
| :---: | :---: | :---: | :---: | :---: |
| 2.413 | 1.684 | 2.015 | 0.946 | 1.311 |
| 3.167 | 2.279 | 2.681 | 1.367 | 1.818 |
| 3.999 | 2.935 | 3.424 | 1.833 | 2.375 |
| 4.924 | 4.274 | 3.560 | 2.936 | 2.244 |
| 6.976 | 6.100 | 5.171 | 4.335 | 3.420 |
| 9.301 | 8.161 | 6.991 | 5.901 | 4.747 |
| 11.956 | 10.545 | 9.116 | 7.734 | 6.300 |
| 14.895 | 13.154 | 11.441 | 9.729 | 7.994 |
| 18.076 | 16.007 | 13.996 | 11.939 | 9.872 |
| 22.069 | 19.671 | 17.319 | 14.898 | 12.483 |

(b)

| $\mathrm{Re}_{\mathrm{m}}$ | $\mathrm{Q}\left(\mathrm{m}^{\wedge} 3 / \mathrm{s}\right)$ | $\mathrm{Q}(\mathrm{cfm})$ | $\mathrm{P}(\mathrm{Pa})$ | $\mathrm{P}($ inches water $)$ | $\mathrm{f}_{\mathrm{m}}$ | $\mathrm{v}(\mathrm{m} / \mathrm{s})$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 4667 | 0.00149 | 3.16 | 4859.24 | 19.52 | 6159 | 0.13 |
| 5205 | 0.00167 | 3.53 | 5951.88 | 23.91 | 6763 | 0.15 |
| 5753 | 0.00184 | 3.90 | 7231.04 | 29.05 | 7434 | 0.16 |
| 6641 | 0.00202 | 4.28 | 8835.24 | 35.50 | 8730 | 0.18 |
| 7832 | 0.00237 | 5.02 | 11821.80 | 47.50 | 9993 | 0.21 |
| 9023 | 0.00273 | 5.78 | 15181.68 | 61.00 | 11140 | 0.24 |
| 10046 | 0.00307 | 6.51 | 19163.76 | 77.00 | 12352 | 0.27 |
| 11379 | 0.00341 | 7.23 | 23394.72 | 94.00 | 13855 | 0.30 |
| 12024 | 0.00374 | 7.91 | 28123.44 | 113.00 | 14677 | 0.33 |
| 13920 | 0.00408 | 8.63 | 32976.61 | 132.50 | 16751 | 0.36 |

(c)

Table 22. Water experiment data for $\mathrm{D} / \mathrm{d}_{\mathrm{p}}=3.65$ experiments of vertical bed set up with up-flow direction.

| HZ | $\mathrm{Q}(\mathrm{gpm})$ | $\mathrm{Q}(\mathrm{cfm})$ | $\mathrm{Q}\left(\mathrm{m}^{\wedge} 3 / \mathrm{s}\right)$ | T | vis(33C) | $\operatorname{dens}(33 \mathrm{C})$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 40 | 44.31 | 5.9242 | 0.0028 | 33 | 0.000749 | 994.7083 |
| 45 | 49.76 | 6.6529 | 0.0031 | 33.5 | 0.000741 | 994.5438 |
| 50 | 55.11 | 7.3682 | 0.0035 | 32 | 0.000765 | 995.0312 |
| 55 | 60.5 | 8.0889 | 0.0038 | 34 | 0.000734 | 994.3772 |
| 60 | 66.17 | 8.8469 | 0.0042 | 31 | 0.000781 | 995.3456 |

(a)

| P1(PSI) | P2(PSI) | P3(PSI) | P4(PSI) | P5(PSI) |
| :---: | :---: | :---: | :---: | :---: |
| 7.9423 | 7.1756 | 6.2522 | 5.6103 | 4.9277 |
| 10.2696 | 9.3829 | 8.3545 | 7.6202 | 6.8756 |
| 12.9339 | 11.9194 | 10.7720 | 9.9334 | 9.1161 |
| 15.7972 | 14.6463 | 13.3681 | 12.4261 | 11.5250 |
| 18.9943 | 17.6847 | 16.2683 | 15.2141 | 14.1998 |

(b)

| $\mathrm{Re}_{\mathrm{m}}$ | $\mathrm{Q}\left(\mathrm{m}^{\wedge} 3 / \mathrm{s}\right)$ | $\mathrm{Q}(\mathrm{cfm})$ | $\mathrm{P}(\mathrm{Pa})$ | $\mathrm{P}($ inches <br> water $)$ | $\mathrm{f}_{\mathrm{m}}$ | $\mathrm{v}(\mathrm{m} / \mathrm{s})$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 20047 | 0.00280 | 5.922 | 4174.95 | 16.77 | 17184 | 0.2446 |
| 22512 | 0.00314 | 6.650 | 5239.32 | 21.05 | 19203 | 0.2746 |
| 24933 | 0.00348 | 7.365 | 6459.98 | 25.96 | 21379 | 0.3042 |
| 27371 | 0.00382 | 8.085 | 7750.76 | 31.14 | 23365 | 0.3339 |
| 29936 | 0.00418 | 8.843 | 9304.54 | 37.39 | 25646 | 0.3652 |

(c)

Table 23. Water experiment data for $\mathrm{D} / \mathrm{d}_{\mathrm{p}}=3.65$ experiments of horizontal bed set-up.

| HZ | $\mathrm{Q}(\mathrm{gpm})$ | $\mathrm{Q}(\mathrm{cfm})$ | $\mathrm{Q}\left(\mathrm{m}^{\wedge} 3 / \mathrm{s}\right)$ | T | $\mathrm{vis}(29 \mathrm{C})$ | $\operatorname{dens}(29 \mathrm{C})$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 40 | 44.95 | 6.0098 | 0.0028 | 29.2 | 0.000815 | 995.948657 |
| 42.5 | 47.36 | 6.3320 | 0.0030 | 29.3 | 0.000815 | 995.948657 |
| 45 | 50.54 | 6.7572 | 0.0032 | 28.8 | 0.000815 | 995.948657 |
| 47.5 | 52.91 | 7.0741 | 0.0033 | 29.3 | 0.000815 | 995.948657 |
| 50 | 55.99 | 7.4859 | 0.0035 | 28.5 | 0.000815 | 995.948657 |
| 52.5 | 58.4 | 7.8081 | 0.0037 | 29.5 | 0.000815 | 995.948657 |
| 55 | 61.35 | 8.2025 | 0.0039 | 27.8 | 0.000832 | 996.237009 |
| 57.5 | 64.05 | 8.5635 | 0.0040 | 29.8 | 0.000797 | 995.651465 |
| 60 | 67.08 | 8.9686 | 0.0042 | 26.5 | 0.000860 | 996.652619 |

(a)

| P1(PSI) | P2(PSI) | P3(PSI) | P4(PSI) | P5(PSI) |
| :---: | :---: | :---: | :---: | :---: |
| 7.7068 | 7.2788 | 6.7101 | 6.3961 | 6.0704 |
| 8.6235 | 8.1389 | 7.5224 | 7.1766 | 6.8292 |
| 9.9445 | 9.3938 | 8.7089 | 8.3075 | 7.9096 |
| 10.9990 | 10.3866 | 9.6537 | 9.2150 | 8.7835 |
| 12.4340 | 11.7474 | 10.9411 | 10.4444 | 9.9701 |
| 13.6293 | 12.8794 | 12.0239 | 11.4803 | 10.9695 |
| 15.1823 | 14.3452 | 13.4119 | 12.7967 | 12.2396 |
| 16.5108 | 15.6043 | 14.6064 | 13.9451 | 13.3416 |
| 18.1659 | 17.1655 | 16.0875 | 15.3546 | 14.6998 |

(b)

| $\mathrm{Re}_{\mathrm{m}}$ | $\mathrm{Q}\left(\mathrm{m}^{\wedge} 3 / \mathrm{s}\right)$ | $\mathrm{Q}(\mathrm{cfm})$ | $\mathrm{P}(\mathrm{Pa})$ | $\mathrm{P}($ inches <br> water $)$ | $\mathrm{f}_{\mathrm{m}}$ | $\mathrm{v}(\mathrm{m} / \mathrm{s})$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 18721 | 0.0028 | 6.0098 | 4410.65 | 17.72 | 16440 | 0.2482 |
| 19725 | 0.0030 | 6.3320 | 4779.10 | 19.20 | 16907 | 0.2615 |
| 21049 | 0.0032 | 6.7572 | 5510.13 | 22.14 | 18266 | 0.2791 |
| 22036 | 0.0033 | 7.0741 | 5999.10 | 24.10 | 18996 | 0.2921 |
| 23319 | 0.0035 | 7.4859 | 6694.58 | 26.90 | 20033 | 0.3092 |
| 24322 | 0.0037 | 7.8081 | 7269.03 | 29.21 | 20854 | 0.3225 |
| 25551 | 0.0039 | 8.2025 | 8081.42 | 32.47 | 22070 | 0.3387 |
| 26676 | 0.0040 | 8.5635 | 8719.15 | 35.03 | 22808 | 0.3537 |
| 27938 | 0.0042 | 8.9686 | 9566.44 | 38.44 | 23894 | 0.3704 |

(c)

## Air experiment data

Table 24. The properties of air experiments.

| $\mathrm{D} / \mathrm{d}$ <br> p | $\varepsilon$ | $\mu(\mathrm{kg} / \mathrm{ms})$ | $\rho$ <br> $\left(\mathrm{kg} / \mathrm{m}^{\wedge} 3\right)$ | $\mathrm{d}_{\mathrm{p}}(\mathrm{m})$ | D | $\mathrm{A}\left(\mathrm{Bed}, \mathrm{m}^{2}\right)$ | L <br> $(\mathrm{m})$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 19 | 0.385 | 0.0000183538 | 1.1726 | 0.0063 <br> 5 | 0.12065 | 0.01143258 <br> 7 | 0.50 <br> 8 |
| 9.5 | 0.397 | 0.0000183538 | 1.1726 | 0.0127 | 0.12065 | 0.01143258 <br> 7 | 0.50 <br> 8 |
| 6.33 | 0.416 | 0.0000183538 | 1.1726 | 0.0190 <br> 5 | 0.12065 | 0.01143258 <br> 7 | 0.50 <br> 8 |

Table 25. Air experiment data for $\mathrm{D} / \mathrm{d}_{\mathrm{p}}=19$.

| $\mathrm{Re}_{\mathrm{m}}$ | $\mathrm{Q}\left(\mathrm{m}^{\wedge} 3 / \mathrm{s}\right)$ | $\mathrm{Q}(\mathrm{cfm})$ <br> $(\mathrm{ft} \wedge / \mathrm{min})$ | $\mathrm{P} 3-\mathrm{P} 5(\mathrm{~Pa})$ | $\mathrm{P} 3-\mathrm{P} 5$ <br> (inches water) | $\mathrm{f}_{\mathrm{m}}$ | $\mathrm{v}(\mathrm{m} / \mathrm{s})$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 263 | 0.00454 | 9.6056 | 373.32 | 1.50 | 617 | 0.39669 |
| 336 | 0.00579 | 12.2542 | 572.42 | 2.30 | 742 | 0.50607 |
| 409 | 0.00705 | 14.9381 | 796.42 | 3.20 | 847 | 0.61691 |
| 478 | 0.00824 | 17.4454 | 1045.30 | 4.20 | 952 | 0.72046 |
| 542 | 0.00934 | 19.7762 | 1343.95 | 5.40 | 1080 | 0.81672 |
| 603 | 0.01039 | 22.0010 | 1667.50 | 6.70 | 1204 | 0.90860 |
| 670 | 0.01155 | 24.4731 | 2015.93 | 8.10 | 1309 | 1.01069 |
| 732 | 0.01262 | 26.7332 | 2364.36 | 9.50 | 1405 | 1.10403 |
| 793 | 0.01366 | 28.9404 | 2737.68 | 11.00 | 1503 | 1.19518 |

Table 26. Air experiment data for $\mathrm{D} / \mathrm{d}_{\mathrm{p}}=9.5$.

| $\mathrm{Re}_{\mathrm{m}}$ | $\mathrm{Q}\left(\mathrm{m}^{\wedge} 3 / \mathrm{s}\right)$ | $\mathrm{Q}(\mathrm{cfm})$ <br> $\left(\mathrm{ft}^{\wedge} / \mathrm{min}\right)$ | $\mathrm{P} 3-\mathrm{P} 5(\mathrm{~Pa})$ | $\mathrm{P} 3-\mathrm{P} 5$ <br> (inches water) | $\mathrm{f}_{\mathrm{m}}$ | $\mathrm{v}(\mathrm{m} / \mathrm{s})$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 647 | 0.0055 | 11.58 | 177 | 0.71 | 1106 | 0.48 |
| 836 | 0.0071 | 14.97 | 296 | 1.19 | 1434 | 0.62 |
| 1000 | 0.0085 | 17.90 | 398 | 1.6 | 1613 | 0.74 |
| 1201 | 0.0102 | 21.50 | 548 | 2.2 | 1846 | 0.89 |
| 1404 | 0.0119 | 25.13 | 722 | 2.9 | 2081 | 1.04 |
| 1590 | 0.0134 | 28.45 | 916 | 3.68 | 2333 | 1.18 |
| 1848 | 0.0156 | 33.08 | 1202 | 4.83 | 2634 | 1.37 |
| 2071 | 0.0175 | 37.07 | 1493 | 6 | 2920 | 1.53 |
| 2268 | 0.0192 | 40.60 | 1755 | 7.05 | 3133 | 1.68 |
| 2418 | 0.0204 | 43.28 | 1954 | 7.85 | 3272 | 1.79 |
| 2562 | 0.0217 | 45.85 | 2153 | 8.65 | 3403 | 1.89 |
| 2683 | 0.0227 | 48.03 | 2297 | 9.23 | 3467 | 1.98 |
| 2921 | 0.0247 | 52.28 | 2663 | 10.7 | 3692 | 2.16 |
| 3148 | 0.0266 | 56.34 | 3061 | 12.3 | 3938 | 2.33 |
| 3344 | 0.0283 | 59.84 | 3492 | 14.03 | 4229 | 2.47 |
| 3551 | 0.0300 | 63.55 | 3820 | 15.35 | 4357 | 2.62 |
| 3645 | 0.0308 | 65.23 | 3982 | 16 | 4424 | 2.69 |
| 3798 | 0.0321 | 67.97 | 4338 | 17.43 | 4626 | 2.81 |
| 4142 | 0.0350 | 74.13 | 5002 | 20.1 | 4891 | 3.06 |

Table 27. Air experiment data for $\mathrm{D} / \mathrm{d}_{\mathrm{p}}=6.33$.

| $\mathrm{Re}_{\mathrm{m}}$ | $\mathrm{Q}\left(\mathrm{m}^{\wedge} 3 / \mathrm{s}\right)$ | $\mathrm{Q}(\mathrm{cfm})$ <br> $(\mathrm{ft} \wedge / \mathrm{min})$ | $\mathrm{P} 3-\mathrm{P} 5(\mathrm{~Pa})$ | $\mathrm{P} 3-\mathrm{P} 5$ <br> (inches water) | $\mathrm{f}_{\mathrm{m}}$ | $\mathrm{v}(\mathrm{m} / \mathrm{s})$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1118 | 0.0061 | 12.9198 | 116.97 | 0.4700 | 1811 | 0.53 |
| 1381 | 0.007533333 | 15.9556 | 169.24 | 0.6800 | 2122 | 0.66 |
| 1634 | 0.008916667 | 18.8855 | 233.95 | 0.9400 | 2478 | 0.78 |
| 2193 | 0.011966667 | 25.3454 | 435.54 | 1.7500 | 3437 | 1.05 |
| 2917 | 0.015916667 | 33.7115 | 696.86 | 2.8000 | 4135 | 1.39 |
| 3406 | 0.018583333 | 39.3595 | 933.30 | 3.7500 | 4743 | 1.63 |
| 3727 | 0.020333333 | 43.0660 | 1095.07 | 4.4000 | 5086 | 1.78 |
| 4296 | 0.023437406 | 49.6404 | 1566.88 | 6.2957 | 6314 | 2.05 |
| 4631 | 0.025267332 | 53.5162 | 1785.56 | 7.1744 | 6674 | 2.21 |
| 5055 | 0.027579476 | 58.4133 | 2077.27 | 8.3465 | 7113 | 2.41 |
| 5396 | 0.029440957 | 62.3559 | 2324.62 | 9.3403 | 7457 | 2.58 |
| 6005 | 0.032760549 | 69.3868 | 2793.41 | 11.2239 | 8053 | 2.87 |
| 6224 | 0.033956541 | 71.9200 | 2970.99 | 11.9374 | 8263 | 2.97 |
| 6633 | 0.03618687 | 76.6438 | 3314.45 | 13.3175 | 8650 | 3.17 |
| 6968 | 0.038018933 | 80.5241 | 3608.56 | 14.4992 | 8964 | 3.33 |
| 7462 | 0.040711047 | 86.2260 | 4060.33 | 16.3144 | 9419 | 3.56 |
| 7833 | 0.042734944 | 90.5126 | 4415.32 | 17.7408 | 9758 | 3.74 |
| 7902 | 0.043111963 | 91.3111 | 4482.91 | 18.0123 | 9821 | 3.77 |

## Experiment for the annular bed

## Methodology

Pressure drop experiment was also done by the annular packed bed. Fig. 39 shows the diagram of experiment apparatus. The outer diameter of this bed is 35 inches $(88.9 \mathrm{~cm})$ and inner diameter is 10 .inches $(26.67 \mathrm{~cm})$. The bed height is 86 inches $(218.44 \mathrm{~cm})$. The sphere particle diameter is 3.302 cm .

The bed has 10 tabs to measure wall pressures at each point. The distance of tab to tab was 6.5 inches ( 16.51 cm ). The working fluid was air. The flow rate of air was controlled by pump power controller. The only way to find flow rate was to measure velocity of the air in the tube. It was very difficult to measure velocities in the annular bed. Instead of measuring bed velocities, the pipe velocities were measured. At this point, the velocity measurement of the tube was also very difficult due to the size of the bed tube. The velocities were measured 16 points to get average velocity for this bed (Fig. 40). The velocities were fluctuated significantly. Therefore it was repeated 10 times. After then, the velocities in the annular bed were calculated. At last the flow rate was found by multiplying this average velocity to the area of the bed. And the flow rate of the inlet and the outlet flow rate were compared. The discrepancy was within $5 \%$. Table 29 shows the average velocities and area of the annular bed and pipe. The wall pressures (Table 30) were checked from the manometer at each tab. Table 31 shows final results of this experiment. Fig. 41 represents the modified friction factor of this packed bed as a function of the modified Reynolds number.


Fig. 39. A diagram and a picture of annular packed bed.


Fig. 40. A diagram of annular packed bed expeirment.

## Results and Analysis

The porosity of the annular bed was 0.404 . It was measured from particle counting method. The volume of the annular bed can be calculated from the bed dimensions. The only thing to know is the number of sphere. The number of sphere were filled were 31020. The measured porosity was 0.404 . It used to calculated pressure drop and applied to other correlations to get the pressure drop and to compare pressure drops. Table 28 shows the properties of these experiments.

Table 28. The properties of the experiments

| $\varepsilon$ | $\mu \_\mathrm{g}(\mathrm{kg} / \mathrm{ms})$ | $\rho \_\mathrm{g}\left(\mathrm{kg} / \mathrm{m}^{\wedge} 3\right)$ | $\mathrm{d} \_\mathrm{p}(\mathrm{m})$ | D | $\mathrm{A}\left(\right.$ Bed, $\left.\mathrm{m}^{\wedge} 2\right)$ | $\mathrm{A}\left(\right.$ pipe, $\left.\mathrm{m}^{\wedge} 2\right)$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0.404 | 0.00001803 | 1.18376 | 0.03302 | 0.6223 | 0.565 | 0.356 |

Table 29. The average velocities and areas
( v 1 : the average velocity of the pipe, A1 : the area of the pipe,
v2 : the average velocity of the annular bed and A2: the area of the annular bed).

| $\begin{gathered} \hline \mathrm{V} 1 \\ (\mathrm{~m} / \mathrm{s}) \end{gathered}$ | A1 (m ${ }^{2}$ ) | V2 (m/s) | $\begin{aligned} & \mathrm{A} 2 \\ & \left(\mathrm{~m}^{2}\right) \\ & \hline \end{aligned}$ |
| :---: | :---: | :---: | :---: |
| 0.2927 | 0.3563 | 0.1846 | 0.5649 |
| 0.5999 |  | 0.3784 |  |
| 0.8537 |  | 0.5385 |  |
| 1.1249 |  | 0.7096 |  |
| 1.5336 |  | 0.9673 |  |
| 1.7983 |  | 1.1343 |  |

Table 30. The measured pressures at each tab (unit : inches water).

| P1 | P2 | P3 | P4 | P5 | P6 | P7 | P8 | P9 | P10 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0.13 | 0.11 | 0.1 | 0.09 | 0.08 | 0.07 | 0.055 | 0.045 | 0.03 | 0.025 |
| 0.47 | 0.44 | 0.39 | 0.35 | 0.3 | 0.26 | 0.22 | 0.18 | 0.11 | 0.105 |
| 1.05 | 0.95 | 0.85 | 0.76 | 0.66 | 0.58 | 0.48 | 0.41 | 0.25 | 0.23 |
| 1.8 | 1.6 | 1.5 | 1.38 | 1.2 | 1.05 | 0.855 | 0.738 | 0.46 | 0.43 |
| 2.73 | 2.58 | 2.3 | 2.1 | 1.85 | 1.65 | 1.4 | 1.2 | 0.73 | 0.68 |

Table 31. The final summarized results (velocity, flow rate, the modified Reynolds number, pressure difference ( P 1 to P 8 ) and the modified friction factor).

| $\mathrm{V}(\mathrm{m} / \mathrm{s})$ | $\mathrm{Q}\left(\mathrm{m}^{3} / \mathrm{s}\right)$ | $\mathrm{Re}_{\mathrm{m}}$ | $\Delta \mathrm{P}(\mathrm{Pa})$ | $\Delta \mathrm{P}$ <br> (inches <br> water $)$ | $\mathrm{f}_{\mathrm{m}}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 0.185 | 0.104 | 671.625 | 21.155 | 0.085 | 1112.793 |
| 0.378 | 0.214 | 1376.467 | 72.175 | 0.290 | 1852.485 |
| 0.539 | 0.304 | 1958.756 | 159.283 | 0.640 | 2872.911 |
| 0.710 | 0.401 | 2580.905 | 264.311 | 1.062 | 3618.053 |
| 0.967 | 0.546 | 3518.441 | 415.630 | 1.670 | 4173.387 |
| 1.134 | 0.641 | 4125.904 | 547.536 | 2.200 | 4688.413 |



Fig. 41. The annular bed pressure drop results:
The modified friction factor as a function of the modified Reynolds number.


Fig. 42. The comparison of present work to KTA.

Fig. 42. shows the comparison of present work to KTA (1981)[2]. The modified friction factors of the present work were similar with KTA[2] by 3000 of the modified Reynolds number. However, as the modified Reynolds number increases, the difference of the modified friction factor between present work and KTA[2] increases. One of the possible reasons that made this difference is the velocity measurement error. As the velocity increases, there was more leaked air from the tube. The flow rate at the high velocity (=high pump power) was not accurate. In addition, the velocity measurement was not exact. The velocities from 16 points were not enough to get the exact average velocity of the pipe. The velocities of each point were fluctuated significantly. It caused the error of the pressure drop measurement. The new issue of this experiment is to get the flow rate exactly. The experiment set up has to be changed in order to get exact flow rate by reducing leaked air.

## Summary of the existing pressure drop correlations

Ergun(1952)

$$
\begin{aligned}
& \frac{\Delta P}{L}=150 \frac{(1-\varepsilon)^{2}}{\varepsilon^{3}} \frac{\mu}{d_{p}^{2}} U+1.75 \frac{(1-\varepsilon)}{\varepsilon^{3}} \frac{\rho}{d_{p}} U^{2} \\
& \frac{\Delta P}{L}=150 \frac{(1-\varepsilon)^{2}}{s^{s}} \frac{\mu_{f}}{d_{p}^{2}} U \text { (for laminar) } \\
& \frac{\Delta P}{L}=1.75 \frac{(1-s)}{\varepsilon^{s}} \frac{\rho}{d_{p}} U^{2} \text { (for turbulent) } \\
& f=\frac{\Delta P d_{p}^{z} s^{s}}{L \mu U(1-\varepsilon)^{2}}=150+1.75 \frac{p_{f} U d_{p}}{\mu(1-\varepsilon)} \\
& \operatorname{Re}_{m}<500
\end{aligned}
$$

Foscolo(1982)
$\frac{\Delta P}{L}=17.3 \frac{(1-\varepsilon)}{\varepsilon^{4.8}} \frac{\mu_{f}}{d_{p}^{2}} U+0.336 \frac{(1-\varepsilon)}{\varepsilon^{4.8}} \frac{\rho_{f}}{d_{p}} U^{2}$
$\frac{\Delta \mathrm{P}}{\mathrm{L}}=17.3 \frac{(1-\mathrm{s})}{\varepsilon^{4.8}} \frac{\mu_{\mathrm{f}}}{\mathrm{d}_{\mathrm{p}}^{2}}$ (for laminar)
$\frac{\Delta P}{L}=0.336 \frac{(1-s)}{s^{4 s}} \frac{P_{f}}{d_{p}} U^{2}$ (for turbulent)
$\mathrm{f}=\frac{\Delta \mathrm{Pd}_{p}^{\mathrm{z}} \mathrm{s}^{\mathrm{s}}}{\mathrm{L} \mu \mathrm{U}(1-\mathrm{s})^{2}}=17.3 \frac{1}{(1-\mathrm{s}) \mathrm{s}^{1 . \mathrm{s}}}+0.336 \frac{p_{\mathrm{f}} \mathrm{Ud}}{\mu(1-\mathrm{s}) \mathrm{s}^{1.8}}$
$0.2<\mathrm{Re}<500$

Foumeny et al. (1993, 1995)
$\frac{\Delta P}{L}=211 \frac{(1-s)^{2}}{s^{s}} \frac{\mu_{f}}{d_{p}^{2}} U+\left(3.81-\frac{5.265}{\frac{D}{d_{p e}}}-\frac{7.047}{\left(\frac{D}{d_{p e}}\right)^{2}} \frac{\left(1-s^{s}\right)}{s^{s}} \frac{\rho_{f}}{d_{p}} U^{2}\right.$
Where $\mathrm{d}_{\mathrm{pe}}=$ equivalent diameter, $6 \mathrm{~V}_{\mathrm{p}} / \mathrm{S}_{\mathrm{p}} \quad\left(\mathrm{S}_{\mathrm{p}}=\right.$ surface area of particle $)$
$1995 . \mathrm{f}=\frac{\Delta \mathrm{Pd}_{\mathrm{p}}^{\mathrm{p}} \mathrm{s}^{\mathrm{s}}}{\mathrm{L} \mu \mathrm{U}(1-\mathrm{z})^{2}}=211+\left(3.81-\frac{5.265}{\frac{D}{d_{p e}}}-\frac{7.047}{\left(\frac{D}{d_{p \mathrm{p}}}\right)^{2}}\right) \frac{p_{\mathrm{f}} \mathrm{Ud}}{\mu(1-\mathrm{p})}$
$5<\operatorname{Re}_{\mathrm{m}}<8500$
For cylindrical particle

$1993 . \mathrm{f}=\frac{\Delta \mathrm{Pd}_{\mathrm{p}}^{x} \varepsilon_{\mathrm{m}}^{\mathrm{s}}}{\mathrm{L} \mu \mathrm{u}\left(1-\varepsilon_{\mathrm{m}}\right)^{2}}=\frac{\frac{\mathrm{D}}{\mathrm{d}_{\mathrm{p}}}}{0.335 \frac{\mathrm{D}}{\mathrm{d}_{\mathrm{p}}}+2.28} \frac{\mathrm{pud}}{\mu\left(1-\varepsilon_{\mathrm{m}}\right)}+130$
$5<\operatorname{Re}_{\mathrm{m}}<8500$
$3<\frac{\mathrm{D}}{\mathrm{d}_{\mathrm{p}}}<24$

Handley and Heggs(1968)

$$
\begin{aligned}
& \frac{\Delta \mathrm{P}}{\mathrm{~L}}=368 \frac{(1-\varepsilon)^{z}}{s^{s}} \frac{\mu}{d_{p}^{z}} \mathrm{U}+1.24 \frac{(1-\varepsilon)}{s^{s}} \frac{\rho}{d_{p}} \mathrm{U}^{2} \\
& \mathrm{f}=\frac{\Delta \mathrm{Pd}_{p}^{z} s^{s}}{\mathrm{~L} \mu \mathrm{U}(1-s)^{2}}=368+1.24 \frac{p_{f} U d_{p}}{\mu(1-\varepsilon)} \\
& 1000<\operatorname{Re}_{\mathrm{m}}<5000
\end{aligned}
$$

Hicks(1970)
$\frac{\Delta \mathrm{P}}{\mathrm{L}}=6.8 \frac{(1-\mathrm{z})^{1.2} \mathrm{p}_{\mathrm{f}}{ }^{0.8} \mathrm{U}^{1, s} \mathrm{u}^{0 . \pi}}{\varepsilon^{\mathrm{d}} \mathrm{d}_{\mathrm{p}}^{1, z}}$
$\mathrm{f}=\frac{\Delta \mathrm{Pd}_{\mathrm{p}}^{\mathrm{z}} \mathrm{s}^{\mathrm{s}}}{\mathrm{L} \mu \mathrm{U}(1-\mathrm{s})^{2}}=6.8\left(\frac{\rho_{\mathrm{f}} \mathrm{U} \mathrm{d}}{\mu(1-\mathrm{p})}\right)^{0.8}$
$300<\operatorname{Re}_{\mathrm{m}}<60000$
J.Wu et al. (2008)
$\frac{\Delta P}{L}=72 \tau \frac{(1-\varepsilon)^{2}}{\varepsilon^{3}} \frac{\mu}{d_{p}^{2}} U+3 \tau \frac{(1-\varepsilon)}{4 \varepsilon^{3}} \frac{\rho}{d_{p}} U^{2}\left(\frac{3}{2}+\frac{1}{\beta^{4}}-\frac{5}{2 \beta^{2}}\right)$
Where, $\beta$ is the ratio of the pore diameter to the throat diameter
$\beta=\frac{1}{1-\sqrt{1-\varepsilon}}$
$\mathrm{f}=\frac{\Delta \mathrm{Pd}_{p}^{2} \mathrm{~s}^{\mathrm{s}}}{\mathrm{L} \mu \mathrm{U}(1-\mathrm{z})^{2}}=72 \tau+\frac{3 \tau}{4}\left(\frac{3}{2}+\frac{1}{\beta^{4}}-\frac{5}{2 \beta^{2}}\right) \frac{\mathrm{p}_{\mathrm{f}} \mathrm{Ud} \mathrm{d}_{\mathrm{p}}}{\mu(1-\mathrm{s})}$
$0<\operatorname{Re}_{\mathrm{m}}<4000$

Lakota (2002)

$$
\frac{\Delta P}{L}=160 \frac{(1-\varepsilon)^{2}}{\varepsilon^{3}} \frac{\mu}{d_{p}^{2}} U+1.6 \frac{(1-\varepsilon)}{\varepsilon^{3}} \frac{\rho}{d_{p}} U^{2}
$$

For nonporous spheres with $d_{p}=6 \mathrm{~mm}$.

$$
\begin{aligned}
& \mathrm{f}=\frac{\Delta \mathrm{Pd}_{\mathrm{g}}^{\mathrm{z}} \mathrm{~s}^{\mathrm{s}}}{\mathrm{~L} \mu \mathrm{U}(1-\mathrm{s})^{2}}=160+1.6 \frac{\mathrm{p}_{\mathrm{f}} \mathrm{U} \mathrm{~d}}{\mu(1-\varepsilon)} \\
& 18<\mathrm{Re}<110
\end{aligned}
$$

Leva(1947)
$\frac{\Delta \mathrm{P}}{\mathrm{L}}=\frac{3.50 \mathrm{G}^{1.9} \mathrm{p}^{0.1} \mathrm{Q}^{1.1}(1-\mathrm{s})}{\mathrm{d}_{\mathrm{p}}{ }^{1.4} \mathrm{P}_{\mathrm{f}} \mathrm{E}_{\mathrm{c}} \mathrm{s}^{5}}$
$\frac{\Delta \mathrm{P}}{\mathrm{L}}=200 \frac{(1-\mathrm{s})^{2}}{s^{\mathrm{s}}} \frac{\mu}{\mathrm{d}_{\mathrm{p}}^{2}} \mathrm{U}+1.75 \frac{(1-\mathrm{s})}{\mathrm{s}^{\mathrm{s}}} \frac{\rho}{\mathrm{d}_{\mathrm{p}}} \mathrm{U}^{2}$
$\mathrm{f}=\frac{\Delta \mathrm{Pd}_{\mathrm{p}}^{2} \mathrm{~s}^{\mathrm{s}}}{\mathrm{L} \mu \mathrm{U}(1-\mathrm{s})^{2}}=200+1.75 \frac{\mathrm{P}_{\mathrm{f}} \mathrm{Ud}}{\mu(1-\mathrm{s})}$
$1<\mathrm{Re}<17635$

Macdonald(1979)
$\frac{\Delta P}{L}=180 \frac{(1-s)^{2}}{s^{s}} \frac{\mu_{f}}{d_{p}^{2}} U+1.8 \frac{(1-s)}{s^{s}} \frac{\rho_{f}}{d_{p}} U^{2}$
$\mathrm{f}=\frac{\Delta \mathrm{Pd}_{\mathrm{p}}^{2} \mathrm{~s}^{\mathrm{s}}}{\mathrm{L} \mu \mathrm{U}(1-\mathrm{s})^{2}}=180+1.8 \frac{\mathrm{P}_{\mathrm{f}} \mathrm{U} \mathrm{d}}{\mu(1-\mathrm{p})}$
$\mathrm{Re}<500$

Montillet (2004)

$$
\begin{aligned}
& \frac{\Delta \mathrm{P}}{\mathrm{~L}}=\left\{\frac{1410}{\mathrm{Re}}+16+\frac{45}{\mathrm{Re}^{0.4 \mathrm{~s}}}\right\} \frac{\mathrm{P}_{\mathrm{f}} \mathrm{U}^{\mathrm{z}}}{\mathrm{~d}_{\mathrm{p}}} \\
& \mathrm{~F} \prime=\frac{\Delta \mathrm{Pd} \mathrm{p}_{\mathrm{p}}}{\mathrm{Lp}_{\mathrm{f}} \mathrm{U}^{2}=\frac{1410}{\mathrm{Re}}+16+\frac{45}{\mathrm{Re}^{0.45}}} \\
& \mathrm{f}=\frac{\Delta \mathrm{Pd}_{\mathrm{p}}^{\mathrm{s}} \mathrm{~s}^{\mathrm{s}}}{\mathrm{~L} \mu \mathrm{U}(1-\mathrm{s})^{2}}=\left\{\frac{1410}{\mathrm{Re}}+16+\frac{45}{\mathrm{Re}^{0.45}}\right\} \frac{\mathrm{d}_{\mathrm{p}} s^{\mathrm{s}} \mathrm{p}_{\mathrm{f}} \mathrm{U}}{\mu(1-\mathrm{s})^{2}}=\left\{\frac{1410}{\mathrm{Re}}+16+\frac{45}{\mathrm{Re}^{0.45}}\right\} \mathrm{Re}_{\mathrm{m}} \frac{s^{\mathrm{s}}}{(1-\mathrm{s})} \\
& 120<\operatorname{Re}<1540
\end{aligned}
$$

Rose(1949)
$\frac{\Delta P}{L}=f(\varepsilon)\left\{\frac{1000}{R e}+\frac{60}{R e^{1 / 2}}+12\right\} \frac{\rho_{f} U^{2}}{d_{p}}$
$\mathrm{f}=\frac{\Delta \mathrm{Pd}_{\mathrm{p}}^{\mathrm{z}} \mathrm{s}^{\mathrm{s}}}{\mathrm{L} \mu \mathrm{U}(1-s)^{2}}=\mathrm{f}(\varepsilon)\left\{\frac{1000}{\mathrm{Re}}+\frac{60}{\operatorname{Re}^{1 / 2}}+12\right\} \frac{\mathrm{d}_{\mathrm{p}} s^{\mathrm{s}} \mathrm{p}_{\mathrm{f}} \mathrm{U}}{\mu(1-\mathrm{s})^{2}}=\mathrm{f}(\varepsilon)\left\{\frac{\{000}{\mathrm{Re}}+\frac{60}{\operatorname{Re}^{1 / 2}}+12\right\} \operatorname{Re}_{\mathrm{m}} \frac{s^{\mathrm{s}}}{(1-s)}$
$\mathrm{f}(\varepsilon)$ is 1 for $\varepsilon=0.4$
$1000<\operatorname{Re}<6000$

Brauer(1960)
$\frac{\Delta P}{L}=\left\{160+3.1\left\{\operatorname{Re}_{\mathrm{m}}\right\}^{0.9}\right\} \frac{\mu \mathrm{U}(1-\mathrm{s})^{2}}{\mathrm{~d}_{\mathrm{p}}^{2} \mathrm{~s}^{\mathrm{s}}}$
$\mathrm{f}=\frac{\Delta \operatorname{Pd}_{p}^{\pi} \mathrm{s}^{\mathrm{s}}}{\mathrm{L} \mu \mathrm{U}(1-\mathrm{z})^{2}}=160+3.1\left\{\frac{\rho_{\mathrm{f}} \mathrm{U} \mathrm{d}}{\mu(1-\varepsilon)}\right\}^{0.9}$
$2<\operatorname{Re}_{\mathrm{m}}<20000$

Reichelt(1972)
$\frac{\Delta P}{L}=\left\{\frac{154 A_{w}^{2} \mu(1-z)}{P_{f} U d_{p}}+\frac{A_{W}}{B_{w}}\right\} \frac{\mu U(1-z)^{2}}{d_{p}^{2} z^{s}}$
$\mathrm{f}=\frac{\Delta \operatorname{Pd}_{\mathrm{p}}^{z} \varepsilon^{\mathrm{s}}}{\mathrm{L} \mu \mathrm{U}(1-\mathrm{s})^{2}}=\frac{154 \mathrm{~A}_{\mathrm{W}}^{z} \mu(1-\mathrm{s})}{\mathrm{P}_{\mathrm{f}} \mathrm{U} \mathrm{d}_{\mathrm{p}}}+\frac{\mathrm{A}_{\mathrm{W}}}{\mathrm{B}_{\mathrm{W}}}$
where, $A_{w}=1+\frac{2}{3 \frac{D}{d p}(1-\varepsilon)} \quad B_{w}=\left\{1.15\left(\frac{d_{p}}{D}\right)^{2}+0.87\right\}$ for Spheres beads
$0.01<\operatorname{Re}<17635$

Mehta and Hawley(1969)

$$
\begin{aligned}
& \frac{\Delta \mathrm{P}}{\mathrm{~L}}=\frac{150 \mathrm{G} \mu(1-\varepsilon)^{2}}{\mathrm{~d}_{\mathrm{p}}^{2} \rho \varepsilon^{3}}\left(\frac{4 \mathrm{~d}_{\mathrm{p}}}{6 \mathrm{D}(1-\varepsilon)}+1\right)^{2}+1.75 \frac{\mathrm{G}^{2}}{\rho \mathrm{~d}_{\mathrm{p}}} \frac{1-\varepsilon}{\varepsilon^{3}}\left(\frac{4 \mathrm{~d}_{\mathrm{p}}}{6 \mathrm{D}(1-\varepsilon)}+1\right) \\
& \mathrm{f}=150\left(\frac{4 \mathrm{~d}_{\mathrm{p}}}{6 \mathrm{D}(1-\varepsilon)}+1\right)^{2}+1.75 \operatorname{Re}_{\mathrm{m}}\left(\frac{4 \mathrm{~d}_{\mathrm{p}}}{6 \mathrm{D}(1-\varepsilon)}+1\right) \\
& 0.18<\operatorname{Re}<9.55
\end{aligned}
$$

Yu et al.(2002)
$\frac{\Delta P}{L}=203 \frac{(1-\varepsilon)^{2}}{\varepsilon^{3}} \frac{\mu_{f}}{d_{p}^{2}} U+1.95 \frac{(1-\varepsilon)}{\varepsilon^{3}} \frac{\rho_{f}}{d_{p}} U^{2}$
$\mathrm{f}=\frac{\Delta \mathrm{Pd}_{\mathrm{p}}^{\mathrm{z}} \mathrm{s}^{\mathrm{s}}}{\mathrm{L} \mu \mathrm{U}(1-\mathrm{s})^{2}}=203+1.95 \frac{\mathrm{p}_{\mathrm{f}} \mathrm{Ud}}{\mu(1-\mathrm{s})}$
$797<\mathrm{Re}<2449$

Shijie Liu(1994)

$$
\begin{aligned}
\frac{\Delta P}{L}= & \frac{\mu U(1-\varepsilon)^{2}}{d_{p}^{2} \varepsilon^{\frac{11}{3}}}\left\{85.2\left(1+\frac{\pi\left(d_{p} / D\right)}{6(1-\varepsilon)}\right)^{2}\right. \\
& \left.\quad+0.69\left[1-\frac{\pi^{2}\left(d_{p} / D\right)}{24}\left(1-0.5\left(d_{p} / D\right)\right)\right] \operatorname{Re}_{m} \frac{\operatorname{Re}_{m}^{2}}{16^{2}+\operatorname{Re}_{m}^{2}}\right\}
\end{aligned}
$$

$$
\mathrm{f}=\frac{1}{\varepsilon^{\frac{2}{3}}}\left\{85.2\left(1+\frac{\pi\left(\mathrm{d}_{\mathrm{p}} / \mathrm{D}\right)}{6(1-z)}\right)^{2}+0.69\left[1-\frac{\pi^{2}\left(\mathrm{~d}_{\mathrm{p}} / \mathrm{D}\right)}{24}\left(1-0.5\left(\mathrm{~d}_{\mathrm{p}} / \mathrm{D}\right)\right)\right] \mathrm{Re}_{\mathrm{m}} \frac{\mathrm{Re}_{\mathrm{m}}^{2}}{16^{2}+\mathrm{Re}_{\mathrm{m}}^{2}}\right\}
$$

$$
\operatorname{Re}_{\mathrm{m}}<1600
$$

KTA(1981)
$\frac{\Delta P}{L}=\left\{\frac{320}{\left(\frac{\operatorname{Re}}{1-\varepsilon}\right)}+\frac{6}{\left(\frac{\operatorname{Re}}{1-\varepsilon}\right)^{0.1}}\right\}\left\{\left(\frac{(1-\varepsilon)}{\varepsilon^{3}}\right)\left(\frac{1}{d_{p}}\right)\left(\frac{1}{2 \rho}\right)(\rho U)^{2}\right\}$
$\mathrm{f}=\frac{\Delta \mathrm{Pd}_{\mathrm{p}}^{\mathrm{s}} \mathrm{s}^{\mathrm{s}}}{\mathrm{L} \mu \mathrm{U}(1-\mathrm{z})^{2}}=160+3 \operatorname{Re}_{\mathrm{m}}{ }^{0.9}$
$10<\operatorname{Re}_{\mathrm{m}}<100000$
$0.36<\varepsilon<0.42$

## Chilton and Colburn(1931)

For low Reynolds number

$$
\begin{aligned}
& \frac{\Delta P}{L}=\frac{425 \mu U}{d_{p}^{2}} \\
& f=\frac{\Delta P_{p}^{z} s^{s}}{L \mu U(1-s)^{2}}=\frac{425 s^{s}}{(1-s)^{2}}
\end{aligned}
$$

For high Reynolds number

$$
\begin{aligned}
& \frac{\Delta P}{L}=\frac{19 \operatorname{Re}^{0.85} \mu \mathrm{U}}{\mathrm{~d}_{\mathrm{p}}^{2}} \\
& \mathrm{f}=\frac{\Delta \mathrm{Pd}_{\mathrm{p}}^{2} \varepsilon^{3}}{\mathrm{~L} \mu \mathrm{U}(1-\varepsilon)^{2}}=\frac{19 \mathrm{Re}^{0.85} \varepsilon^{3}}{(1-\varepsilon)^{2}}
\end{aligned}
$$

Wentz and Thodos(1963)

$$
\begin{aligned}
& d_{p}=3.12 \mathrm{~cm} \\
& \varepsilon=0.354,0.48,0.615,0.728 \\
& \frac{\Delta P}{L}=\frac{\mu U(1-s)^{2}}{d_{p}^{2} s^{s}} \frac{0.396 \operatorname{Re}_{m}}{R e_{m}^{0.05}-1.20}
\end{aligned}
$$

$\mathrm{f}=\frac{\Delta \mathrm{Pd}_{\mathrm{p}}^{\mathrm{p}} \mathrm{s}^{\mathrm{s}}}{\mathrm{L} \mu \mathrm{U}(1-\mathrm{s})^{2}}=\frac{0.396 \mathrm{Re}_{\mathrm{m}}}{\mathrm{Re}_{\mathrm{m}}{ }^{0.05}-1.20}$
$1460<\operatorname{Re}<7661$

Re. Hayes(1995)
$\frac{\Delta P}{L}=\frac{\mu \mathrm{U}(1-s)^{2}}{d_{p}^{2} s^{s}}\left\{\frac{1}{\tau}\left[456+\frac{17.8(3 \tau-1)}{\tau(1-s)(1-\tau)} \operatorname{Re}\right]^{0.5} \frac{1}{\varepsilon}+1.3\left(\frac{\tau}{3 \tau-1}\right) \operatorname{Re}_{\mathrm{m}}\right\}$
$\mathrm{f}=\frac{\Delta \mathrm{Pd}_{\mathrm{p}}^{\mathrm{p}} \varepsilon^{\mathrm{s}}}{\mathrm{L} \mathrm{\mu U}(1-z)^{2}}=\frac{1}{\tau}\left[456+\frac{17.8(3 \tau-1)}{\tau(1-z)(1-\tau)} \operatorname{Re}\right]^{0.5} \frac{1}{z}+1.3\left(\frac{\tau}{3 \tau-1}\right) \operatorname{Re}_{\mathrm{m}}$
$30<\operatorname{Re}<1000$

Tallmadge(1970)
$\frac{\Delta \mathrm{P}}{\mathrm{L}}=\frac{\mu \mathrm{U}(1-\varepsilon)^{2}}{\mathrm{~d}_{\mathrm{p}}^{2} \varepsilon^{3}}\left(150+4.2\left(\mathrm{Re}_{\mathrm{m}}\right)^{0.833}\right)$
$\mathrm{f}=\frac{\Delta \mathrm{Pd}_{\mathrm{p}}^{2} \mathrm{~s}^{\mathrm{s}}}{\mathrm{L} \mu \mathrm{U}(1-\mathrm{s})^{2}}=150+4.2\left(\operatorname{Re}_{\mathrm{m}}\right)^{0.833}$
$0.1<\operatorname{Re}<10^{5}$

Carman(1970)
$\frac{\Delta \mathrm{P}}{\mathrm{L}}=\left(180+2.87\left(\operatorname{Re}_{\mathrm{m}}\right)^{0.9}\right) \frac{\mu \mathrm{U}(1-\varepsilon)^{2}}{\mathrm{~d}_{\mathrm{p}}^{2} \varepsilon^{3}}$
$\mathrm{f}=\frac{\Delta \mathrm{Pd}_{\mathrm{p}}^{\mathrm{z}} \mathrm{s}^{\mathrm{s}}}{\mathrm{L} \mu \mathrm{U}(1-\mathrm{z})^{2}}=180+2.87\left(\operatorname{Re}_{\mathrm{m}}\right)^{0.9}$

Morcom(1946)
$\frac{\Delta P}{L}=\left(\frac{800}{\operatorname{Re}}+14\right) \frac{\rho U^{2}}{d_{p}}$
$\mathrm{f}=\frac{\Delta \operatorname{Pd}_{p}^{\mathrm{z}} \mathrm{s}^{\mathrm{s}}}{\mathrm{L} \mu \mathrm{U}(1-\mathrm{z})^{2}}=(800+14 \mathrm{Re}) \frac{\mathrm{s}^{\mathrm{s}}}{(1-\mathrm{s})^{2}}$
Y.S. Choi(2008)
$\frac{\Delta P}{L}=150 \frac{(1-\varepsilon)^{2}}{\varepsilon^{3}} \frac{\mu M^{2}}{d_{p}^{2}} U+1.75 \frac{(1-\varepsilon)}{\varepsilon^{3}} \frac{\rho \mathrm{MC}_{\mathrm{w}}}{\mathrm{d}_{\mathrm{p}}} \mathrm{U}^{2}$
$\mathrm{Re}_{\mathrm{m}}<1000$

Du plessis(1994)

$$
\begin{aligned}
& \frac{\Delta \mathrm{P}}{\mathrm{~L}}=207 \frac{(1-\varepsilon)^{2}}{\varepsilon^{3}} \frac{\mu_{\mathrm{f}}}{\mathrm{~d}_{\mathrm{p}}^{2}} \mathrm{U}+1.88 \frac{(1-\varepsilon)}{\varepsilon^{3}} \frac{\rho_{\mathrm{f}}}{\mathrm{~d}_{\mathrm{p}}} \mathrm{U}^{2} \\
& \mathrm{f}=\frac{\Delta \mathrm{Pd}_{\mathrm{p}}^{\mathrm{s}} s^{s}}{\mathrm{~L} \mu \mathrm{U}(1-\varepsilon)^{2}}=207+1.88 \frac{\mathrm{p}_{\mathrm{f}} \mathrm{Ud} \mathrm{p}_{\mathrm{p}}}{\mu(1-\varepsilon)}
\end{aligned}
$$

Table 32. Experimental parameters of referenced literature.

| Author | Bed type | Fuel type | $\varepsilon$ | $\mathrm{d}_{\mathrm{p}}$ | D/d ${ }_{\text {p }}$ | Re |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Oman/Waston (1944) | Cylindrical | Spherical | 0.3775~0.379 | 5.50 | 18.47 | 4.99~14.37 |
| Coulson(1949) | Cylindrical | Spherical | $0.393 \sim 0.420$ | 1.59~7.94 | $\begin{gathered} \hline 6.4 \sim \\ 31.95 \end{gathered}$ | 0.0095~1.125 |
| Andersson (1963) | Cylindrical | Spherical | $0.351 \sim 0.410$ | $\begin{gathered} 0.709 \sim \\ 5.49 \\ \hline \end{gathered}$ | $\begin{gathered} 6.53 \sim \\ 56.9 \end{gathered}$ | 1.5~1313 |
| Handley/ Heggs(1968) | Cylindrical | Spherical | 0.390 | $\begin{aligned} & 3.17 \\ & 9.52 \end{aligned}$ | $\begin{gathered} \hline 8 \\ 24 \\ \hline \end{gathered}$ | 1000~5000 |
| Mehta/Hawley (1969) | Cylindrical | Spherical | - | 0.14~1.65 | 7.7~91 | 0.18~9.55 |
| Reichelt(1972) | Cylindrical | Spherical | $0.366 \sim 0.485$ | $\begin{aligned} & \hline 9.71 \sim \\ & 24.05 \end{aligned}$ | $\begin{aligned} & \hline 3.32 \sim \\ & 14.32 \\ & \hline \end{aligned}$ | 0.01~17635 |
| Beavers(1973) | Cylindrical | Spherical | $0.364 \sim 0.376$ | 3 | $\begin{gathered} 19.02 \sim 4 \\ 4.33 \end{gathered}$ | 64~208 |
| Foumeny (1995) | Cylindrical | Spherical | $0.386 \sim 0.456$ | 2.1~15.48 | $\begin{gathered} 3.23 \sim \\ 23.8 \\ \hline \end{gathered}$ | 5~8500 |
| $\mathrm{Yu}(2002)$ | Cylindrical | Spherical | $0.364 \sim 0.379$ | 12~20 | 30 | 797~2449 |
| Lakota(2002) | Cylindrical | Spherical | 0.375 | 3 | 57.33 | 18~110 |
| $\begin{gathered} \text { Montillet } \\ (2004) \\ \hline \end{gathered}$ | Cylindrical | Spherical | 0.367 | 4.92 | 12.2 | 30~1500 |
| Nemec/Levec (2005) | Cylindrical | Spherical | 0.382~0.40 | 1.66~3.50 | $\begin{gathered} 11.7 \sim \\ 24.7 \end{gathered}$ | 3~147 |
| Y.S.Choi(2008) | Cylindrical | Spherical | - | - | 3.2~91 | 0.01~1000 |
| Wu(2008) | Cylindrical | Spherical | 0.42 | 10 |  | 0~4000 |
| Burke/Plummer (1928) | Cylindrical | Spherical | $0.363 \sim 0.421$ | . | $\begin{gathered} \hline 5.379 \sim 3 \\ 9.179 \\ \hline \end{gathered}$ | 0.8~1070 |
| $\begin{gathered} \text { Ergun/Orning } \\ (1949) \\ \hline \end{gathered}$ | Cylindrical | Spherical | $0.330 \sim 0.352$ | - | $\begin{gathered} 44.561 \sim \\ 51.107 \\ \hline \end{gathered}$ | 0.4~30 |
| Gupte(1970) | Cylindrical | Spherical | 0.366~0.640 | - | 50~250 | 0.01~184 |
| Leva(1951) | Cylindrical | Spherical | $0.354 \sim 0.651$ | - | $\begin{gathered} \hline 1.624 \sim 1 \\ 3.466 \\ \hline \end{gathered}$ | 1~17635 |
| Wentz/Thodos (1963) | Cylindrical | Spherical | $0.354 \sim 0.882$ | - | . | 2550~64900 |
| Fand(1989) | Cylindrical | Spherical | $0.3571 \sim 0.6168$ | $\begin{gathered} \hline 2.098 \sim \\ 4.029 \\ \hline \end{gathered}$ | $\begin{gathered} \hline 1.4 \sim \\ 41.28 \\ \hline \end{gathered}$ | 0.62~869 |
| Morcom $(1946)$ | Cylindrical | Spherical | $0.425 \sim 0.450$ | 0.56~1.01 | . | 100~500 |
| $\begin{gathered} \text { Wintersberg } \\ \text { /Tsotsas(2000) } \\ \hline \end{gathered}$ | Cylindrical | Spherical | 0.37 | ${ }^{\bullet}$ | 4~40 | $\begin{gathered} 1, \\ 1000 \\ \hline \end{gathered}$ |
| Shijie Liu(1994) | Cylindrical | Spherical | 0.6007 | 3.184 | 1.4039 | 0~6000 |
| $\begin{aligned} & \text { Eisfeld } \\ & (2001) \end{aligned}$ | Annular | . | $0.330 \sim 0.882$ | . | $\begin{gathered} 1.624 \sim 2 \\ 50 \end{gathered}$ | 0.01~17635 |
| Tallmadge (1970) | - | - | - | - | . | 0.1~100000 |
| Hicks(1970) | - | - | - | - | - | 300~60000 |
| $\begin{gathered} \text { R.E. } \\ \text { Hayes(1994) } \end{gathered}$ | Cylindrical | Spherical | $\begin{gathered} 0.402 \\ 0.408,0.427, \\ 0.385 \\ \hline \end{gathered}$ | $\begin{gathered} 2.97,4.82, \\ 6.01,2.5 \end{gathered}$ | - | 3~1000 |
| KTA(1981) | Annular, | Spherical | $0.36 \sim 0.42$ | - | - | 10~100000 |

## VITA

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