APPLICATIONS OF TIME SERIES IN FINANCE AND MACROECONOMICS

A Dissertation

by

RAUL IBARRA RAMIREZ

Submitted to the Office of Graduate Studies of
Texas A&M University
in partial fulfillment of the requirements for the degree of

DOCTOR OF PHILOSOPHY

May 2010

Major Subject: Economics

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Approved by:

Chair of Committee,	Dennis W. Jansen
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ABSTRACT

Applications of Time Series in Finance and Macroeconomics. (May 2010)

Raul Ibarra Ramirez, B.A., Universidad de Monterrey

Chair of Advisory Committee: Dr. Dennis W. Jansen

This dissertation contains three applications of time series in finance and macroeconomics. The first essay compares the cumulative returns for stocks and bonds at investment horizons from one to ten years by using a test for spatial dominance. Spatial dominance is a variation of stochastic dominance for nonstationary variables. The results suggest that for investment horizons of one year, bonds spatially dominate stocks. In contrast, for investment horizons longer than five years, stocks spatially dominate bonds. This result is consistent with the advice given by practitioners to long term investors of allocating a higher proportion of stocks in their portfolio decisions.

The second essay presents a method that allows testing of whether or not an asset stochastically dominates the other when the time horizon is uncertain. In this setup, the expected utility depends on the distribution of the value of the asset as well as the distribution of the time horizon, which together form the weighted spatial distribution. The testing procedure is based on the Kolmogorov Smirnov distance between the empirical weighted spatial distributions. An empirical application is presented assuming that the event of exit time follows an independent Poisson process with constant intensity.

The last essay applies a dynamic factor model to generate out-of-sample forecasts for the inflation rate in Mexico. Factor models are useful to summarize the information contained in large datasets. We evaluate the role of using a wide range of macroeconomic variables to forecast inflation, with particular interest on the importance of using the consumer price index disaggregated data. The data set contains 54 macroeconomic series and 243 consumer price subcomponents from 1988 to 2008. The results indicate that factor models outperform the benchmark autoregressive model at horizons of one, two, four and six quarters. It is also found that using disaggregated price data improves forecasting performance.

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CHAPTER I

INTRODUCTION

This dissertation contains three applications of time series in the broad fields of finance and macroeconomics. In the next chapter, I will use a novel methodology called spatial dominance to compare the distributions of cumulative returns for stocks and bonds at different investment horizons. Should stock be preferred to bonds in the long run? Financial advisers typically recommend allocating a greater proportion of stocks for long-term investors than for short-term investors. However, this conclusion is not supported, in general, by theoretical models. Merton and Samuelson [40] have found that investor's optimal portfolio should be independent of the planning holding period.

The results in Chapter II from using the spatial dominance test are consistent with the advice given by practitioners of allocating a greater proportion of stocks in their portfolios. The spatial dominance method employed was introduced by Park [40] and it is useful to compare the performance of two assets over a given period of time when the utility of the investor is based on the level of wealth at each point in time. An important advantage of using this approach is that we impose minimal assumptions about the utility function such as monotonicity and risk aversion. This study will present the results of the test for spatial dominance for investment horizons from 1 to 10 years using data for stock and bond returns from the US from 1965 to 2008.

Chapter III of this dissertation introduces a generalization of stochastic dominance to situations that involve an uncertain time horizon. Existing methods to compare the performance of two different assets are based on the assumption that an

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The journal model is *Journal of Economic Theory*.

investor knows with certainty the time of exit at the moment of making an investment decision. In practice, however, the investment horizon is never known with certainty. For example, an investor might plan to invest for 1 year. After 6 months, due to an emergency or sudden need for money, the portfolio may have to be liquidated. There are many factors that can drive the exit of an investor, such as of purchasing or selling a house, loss of a job, early retirement, disability, bequest, among others. This chapter presents a method that allows to test whether an asset stochastically dominates the other when the time horizon is uncertain.

When two series are compared using the standard method of stochastic dominance, we need to estimate the empirical distribution function. We say that one series dominates the other if the distribution function is located to the right of the other distribution for all points of the support. In our setup we cannot usual the typical distribution as in the standard stochastic dominance approach since the value of the asset is likely to be nonstationary and the time horizon is uncertain. One of the contributions of this paper is to introduce the weighted spatial distribution, which combines the distribution function of a nonstationary series with the distribution of the time horizon. The weighted spatial distribution is a spatial distribution weighted by the density of the uncertain time horizon. Chapter III will present the test statistic and critical values which are based on stochastic dominance literature. An empirical application will be presented assuming that that can drive the exit of the investor occur at some constant rate. The data employed are for the S&P 500 and the 3 month Treasury Bill. This application will examine how the investment decision depends on the frequency that the investor has to liquidate the portfolio.

Chapter IV of this dissertation applies a factor model to generate out of sample forecasts for the inflation rate in Mexico and to evaluate the role of using the Consumer Price Index (CPI) disaggregated data to forecast inflation. Factor models are useful to summarize the information content in large datasets. This is an appealing feature for forecasting purposes since it allows us to concentrate on a few common factors instead of a large number of explanatory variables. Factor models have been used to forecast inflation in industrialized countries such as the US and Euro Area and to evaluate the role of using macroeconomic variables such as interest rates and monetary aggregates. In this chapter we will also look at the role of using CPI disaggregated data to forecast inflation. This is the first application of factor models for Mexico. We will use a large data set containing 54 macroeconomic series and 243 CPI subcomponents from 1988 to 2008. The results from the factor model will be compared with those of a benchmark autoregressive model using an out of sample simulation exercise. We will show that factor models outperform the conventional autoregressive model at horizons from one to six quarters ahead and using the CPI disaggregated data contribute to substantial improvements in the forecasting performance of the factor model. We present a summary of this dissertation in Chapter V.

CHAPTER II

STOCKS, BONDS AND THE INVESTMENT HORIZON: A SPATIAL DOMINANCE APPROACH

A. Introduction

Financial advisers typically recommend to allocate a greater proportion of stocks for long-term investors than for short-term investors.[1] The advice given by practitioners suggests that optimal investment strategies are horizon dependent and it is motivated by the idea that the risk of stocks decreases in the long run, which is called time diversification.[2] However, this conclusion is not supported, in general, by theoretical models. Merton and Samuelson [39] conclude that that lengthening the investment horizon should not reduce risk, which implies investor's optimal portfolio should be independent of the planning holding period. On the other hand, Chung et al. [13] find that the investment horizon might affect the investment decisions in the presence of human wealth, guaranteed consumption or mean reverting returns.

There are several approaches to examine empirically the question of whether stocks should be preferred over bonds in the long run. One approach consists of directly calculate the terminal wealth distributions for various portfolios with different asset allocations, and to evaluate the expected utility for each portfolio. The drawback of this approach is that it requires one to assume a specific utility function,

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[1]For example, the popular book on investment advice by Siegel [44] recommends buying and holding stocks for long periods since the risk of stocks decreases with the investment horizon. In addition, Malkiel [34] states that "The longer an individual's investment horizon, the more likely is that stocks will outperform bonds".

[2]Chung et al. [13] make a distinction between time series diversification, which means that investors should reduce the holding of risky assets as they become older, and cross sectional diversification, which means that an older person should hold a smaller percentage of his wealth in risky assets than a younger person. This chapter is related with cross sectional diversification.

hence no general conclusions can be reached. Another possible approach is to employ the Markowitz [36] mean variance analysis.[3] For example, Levy and Spector [28] and Hansson and Persson [22] concluded that the optimal allocation for stocks is significantly larger for long investment horizons than a one-year horizon. The problem of using a mean variance approach approach is that it assumes that the investor preferences depend only on the mean and variance of portfolio returns over a single period. A more general approach is to employ a test for stochastic dominance. Stochastic dominance tests have been proposed by Mc Fadden [37] and extended by Linton et al. [29]. This approach has the advantage of being non parametric and hence it provides criteria for entire preference classes. Furthermore, this approach can be applied whether the returns distributions are normal or not.

One conclusion from previous research that employs dominance criteria is that stochastic dominance does not provide evidence that stocks dominate bonds as the investment horizon lengthens (Hodges and Yoder [23], Strong and Taylor [50]). This conclusion is based on the assumption that stock and bond returns are iid. However, empirical evidence suggests that the assumption of iid stocks returns is not supported by the data. In particular, Campbell [9] and Fama and French [16] show that there is strong evidence on the predictability of stock returns, which in turn implies that the optimal investment strategies are horizon dependent.

In this chapter, we follow a nonparametric approach by using a test for spatial dominance introduced by Park [40] to compare the distributions of stock and bond returns for horizons from 1 to 10 years. Spatial dominance is a generalization of the concept of stochastic dominance to compare the performance of two assets over a given period of time. In other words, while the concept of stochastic dominance

[3]For an empirical application of the expected utility and the mean variance approaches, see Thorley [52].

is static and it is only useful to compare two distributions at a fixed time, spatial dominance is useful to compare two distributions over a period of time. In contrast to the standard stochastic dominance approach, this test is valid for the nonstationary difussion processes commonly employed in continuous time finance.

Roughly speaking, we say that one distribution spatially dominates another distribution when it gives a higher level of utility over a given period of time. Spatial dominance is based on buy and hold strategies. That is, an investor with an investment horizon of T years chooses an allocation at the beginning of the first year and does not touch his portfolio again until the T years are over. The investor is not allowed to rebalance his portfolio. One possible justification for this assumption is the existence of transaction costs (Liu and Loewenstein, [32]). Our analysis assumes that the investor holds only one type of asset in order to focus on the effect the holding period has on the investor's preferences for stocks versus bonds. Diversification across asset categories is not considered.[4]

This chapter makes several contributions to the time diversification literature. First, it is based on a new methodology to evaluate the performance between alternative investments. This methodology allows us to compare the entire return distributions of two portfolios instead of just the mean or median portfolio returns used in most conventional studies. Second, the approach followed in this paper relaxes the parametric assumptions about preferences that are considered in other papers. Only a few restrictions on the form of utility function (i.e., nonsatiation, risk aversion and time separable preferences) are imposed. Third, the approach is valid for the nonstationary diffusion processes commonly used in finance. Finally, the test employs

[4]Recently, Post [42] and Linton et al. [30] have extended the standard pairwise stochastic dominance to compare a given portfolio with all possible portfolios constructed from a set of financial assets. This concept might be useful in our analysis, but we do not pursue this direction in this paper.

information from the entire path of the asset price instead of using only the the asset values at two fixed points in time.

The data for this study are U.S. stock and bond returns obtained from Datastream. The study period is from 1965 to 2008. The variable stock price refers to the S&P 500 including dividends. Bond returns are based on the three month U.S. Treasury Bill. The empirical results suggest that for investment horizons of one year or less, bonds second order spatially dominate stocks, which means that risk averse investors obtain higher levels of utility by investing in bonds. In contrast, for investment horizons between five and eight years, stocks second order spatially dominates bonds. For horizons of nine years or more, any investor whose preferences are characterized by nonsatiation will obtain a higher expected utility by investing in stocks. These results are consistent with the common advice that investors with long-term horizons should allocate more heavily to stocks.

This chapter is organized as follows. The next section presents the econometric methodology. Section C discusses the test for spatial dominance. Section D analyzes the empirical results. Concluding remarks are presented in Section E.

B. Econometric Methodology

Spatial analysis is based on the study of the distribution function of nonstationary time series. It was introduced by Park [40]. The spatial analysis consists of the study of a time series along the spatial axis rather than the time axis. This methodology is designed for nonstationary time series, but the theory is also valid for stationary time series.

The advantages of using this methodology are: i) Adopt a nonparametric approach which relaxes the parametric assumptions about preferences. ii) The analysis is appropriate for both stationary and nonstationary time series. iii) Since this methodology is derived in continuous time, the analysis is appropriate for high frequency data.

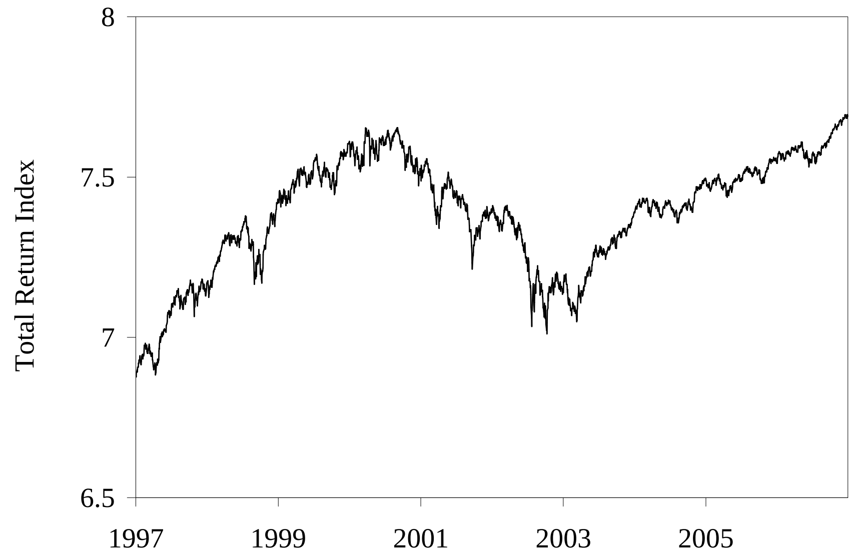
Figure 1 is useful to explain the intuition behind spatial analysis. Usually we plot the data on the xy plane where x represents the time axis and y represents the space (figure 1(a)). However, this representation is meaningful only under the assumption of stationarity, as we can interpret these readings as repeated realizations from a common distribution. In contrast, for nonstationary data this representation is no appropriate since the distribution changes over time. Clearly, the data for stock prices are nonstationary. For this case, it is useful to read the data along the spatial axis. This is in particular useful for series that take repeated values over a certain range. The idea of spatial analysis is to calculate the frequency for each point on the spatial axis (figure 1(b)), which can be interpreted as a distribution function. The statistical properties of this distribution function are the main object of study in spatial analysis.

1. Preliminaries on Spatial Analysis

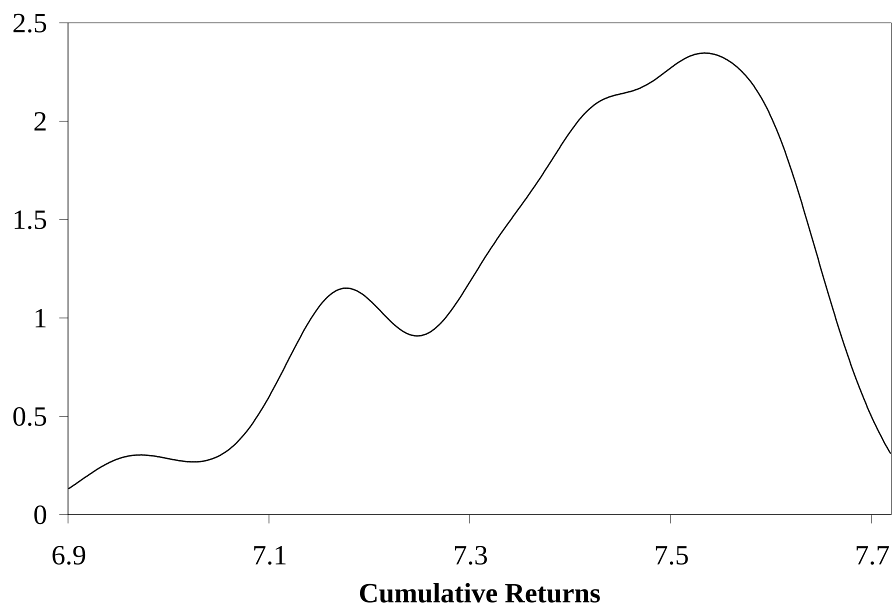
In order to explain the test for spatial dominance, it is necessary to introduce some important definitions. Let

$$X = (X)_t, t \in [0, T] \tag{2.1}$$

be a stochastic process. The local time, represented as $\ell(T, x)$, is defined as the frequency at which the process visits the spatial point x up to time T. Notice that the local time itself is a stochastic process. It has two parameters, the time parameter T and the spatial parameter x. If the local time of a process is continuous, then we



(a) S&P 500 Total Return Index



(b) Local Time

Fig. 1. Spatial Analysis

may deduce that,

$$\ell(T,x) = \lim_{\varepsilon \to 0} \frac{1}{2\varepsilon} \int_0^T 1\{|X_t - x| < \varepsilon\} dt. \tag{2.2}$$

Therefore, we may interpret the local time of a process as a density function. [5]

The corresponding distribution function called integrated local time is defined as:

$$L(T,x) = \int_{-\infty}^{x} \ell(T,y)dy = \int_0^T 1\{X_t \le x\} dt. \tag{2.3}$$

The local time is known to be well defined for a broad class of stochastic processes. Notice that the local time itself is a stochastic process and random. Taking the expectation of this random variable, we can define the spatial density function as:

$$\lambda(T,x) = E\ell(T,x) = \lim_{\varepsilon \to 0} \frac{1}{2\varepsilon} \int_0^T P\{|X_t - x| < \varepsilon\} dt. \tag{2.4}$$

The corresponding spatial distribution function is defined as:

$$\Lambda(T,x) = EL(T,x) = \int_0^T P\{X_t \le x\} dt. \tag{2.5}$$

Consider a continuous utility function u that depends on the value of the stochastic process X. By occupation times formula, we may deduce that:

$$E \int_0^T u(X_t)dt = \int_{-\infty}^{\infty} u(x)\lambda(T,x)dx. \tag{2.6}$$

The equation above implies that, for any given utility function, the sum of expected future utilities generated by a stochastic process over a period of time is determined by and only by its spatial distribution. Therefore, the spatial distribution is useful to

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[5] To understand this definition, recall that

$$f(x) = \frac{dF(x)}{dx} = \frac{dP(X \le x)}{dx} = \lim_{\varepsilon \to 0} \frac{1}{2\varepsilon} P\left\{|X_t - x| < \varepsilon\right\}.$$

analyze dynamic decision problems that involve utility maximization.

Since we are interested in the sum of expected future utilities, we might consider a discount rate r for the level of utility. In this case, the discounted local time would be defined as:

$$\ell^r(T,x) = \int_0^T e^{-rt}\ell(dt,x).$$

The equation above implies that the discounted integrated local time can be defined as:

$$L^r(T,x) = \int_{-\infty}^x e^{-rt}\ell(T,x) = \int_0^T e^{-rt}1\{X_t \le x\}dt.$$

Similarly, the discounted spatial density can be defined as:

$$\lambda^r(T,x) = E\ell^r(T,x) = \int_0^T e^{-rt}\lambda(dt,x).$$

The discounted spatial distribution is given by:

$$\Lambda^r(T,x) = EL^r(T,x) = \int_0^T e^{-rt}P\{X_t \le x\}dt.$$

2. Spatial Dominance

The usual approach to compare two distribution functions is to employ the concept of stochastic dominance. More specifically, if we have two stationary stochastic processes, X and Y with cumulative distribution functions Π^X and Π^Y, then we say that X stochastically dominates Y if,

$$\Pi^X(x) \le \Pi^Y(x) \tag{2.7}$$

for all $x \epsilon \mathbb{R}$ with strict inequality for some x. This is equivalent to say:

$$Eu(X_t) \ge Eu(Y_t) \tag{2.8}$$

for every utility function u such that $u'(x) > 0$. In other words, the process X stochastic dominates the process Y if and only if it yields a higher level of utility for any non decreasing utility function. Therefore, the notion of stochastic dominance is static an it is restricted to the study of stationary time series.

In this paper, the concept of stochastic dominance is generalized for dynamic settings, by introducing the notion of spatial dominance. Spatial dominance can be applied to compare the distribution function of two stochastic processes over a period of time. Suppose we have two nonstationary stochastic processes, X and Y defined over the same time interval. Then, we say that the stochastic process X spatially dominates the stochastic process Y if and only if

$$\Lambda^{r,X}(T,x) \leq \Lambda^{r,Y}(T,x) \tag{2.9}$$

for all $x \epsilon \mathbb{R}$ with strict inequality for some x.

This definition implies that, for any non decreasing utility function u,

$$E\int_0^T e^{-rt}u(X_t)dt \geq E\int_0^T e^{-rt}u(Y_t)dt, \tag{2.10}$$

or, equivalently,

$$\int_{-\infty}^{\infty} u(x)^X \lambda^r(T,x)dx \geq \int_{-\infty}^{\infty} u(x)^Y \lambda^r(T,x)dx, \tag{2.11}$$

which means that the stochastic process X provides at least the same level of expected utility than the stochastic process Y over a given period of time. This result is showed in Park [40].

Several orders of spatial dominance can be defined, according to certain restrictions on the shape of the utility function. For the first four orders of spatial dominance, these restrictions consist of non satiation, risk aversion, preference for positive skewness and aversion to kurtosis, respectively (Levy, [27]).

The integrated local time of order at order $s \geq 2$ can be defined as:

$$L^{r,X,s}(T,x) = \int_{-\infty}^{x} L^{r,X,s-1}(T,x)dz. \tag{2.12}$$

A stochastic process X spatially dominates Y at order $s \geq 2$ if

$$\Lambda^{r,X,s}(T,x) \leq \Lambda^{r,Y,s}(T,x), \tag{2.13}$$

where,

$$\Lambda^{r,X,s}(T,x) = \int_{-\infty}^{x} \Lambda^{r,X,s-1}(T,x)dz. \tag{2.14}$$

It can be shown that the definition of spatial dominance implies that implies that the stochastic process X provides a higher level of expected utility than the stochastic process Y.

3. Motivation for Spatial Dominance

The concept of spatial dominance consists of comparing the sum of expected utilities $E\int_0^T e^{-rt}u(X_t)dt$ over a given period of time, where X_t is the cumulative return at time t. It is assumed that the investor follows a buy and hold strategy. One possible justification of this strategy is found in Liu and Lowenstein [32]. In that paper, it is shown that the presence of transaction costs together with a finite horizon imply a largely buy and hold and horizon dependent investment strategy.[6]

The spatial dominance employs information from the entire path of the value of the asset X_t. This is an appealing feature compared to the standard stochastic dominance which only depends on the value of the asset at two points in time, X_0 and X_T.

[6] For example, Liu and Lowenstein [32] find that for investors of three years or less, the expected time to sale after a purchase is roughly equal to the investment horizon in the presence of transaction costs.

In our setup, utility is a function of the cumulative return at each point in time. We can think of this function as an indirect utility function, where the investor consumes a constant fraction of the price of the asset at each point in time. Another way to justify this setup is a model in which the investor maximizes the expected utility of terminal wealth when the investment horizon is uncertain and follows an independent Poisson process with constant intensity (Merton, [38]).

Another advantage of using the method of spatial dominance is that it is valid to compare the nonstationary processes commonly used to model asset prices. Since the asset price X_t is not stationary, the distribution function of X_t for $t \epsilon [0, T]$ does not converge to the distribution function of a stationary random variable. For that reason, we cannot employ the standard stochastic dominance concept designed for stationary variables. Instead, this distribution converges to the local time distribution function. The spatial distribution employed in our paper will be estimated as an average of N observations of the local time distribution function.

4. Estimation Method

The estimation methods and asymptotic theory are derived in Park [40]. The theory presented before is built for continuous time processes. In practice, we need a estimation method for data in discrete time. Suppose that we have discrete observations $(X_{i\Delta})$ from a continuous stochastic process X on a time interval $[0, T]$ where $i = 1, 2, \ldots, n$ and Δ denotes the observation interval. The number of observations is given by $n = T/\Delta$. All the asymptotic theory assumes that $n \longrightarrow \infty$ via $\Delta \to 0$ for a fixed T. Notice that, in contrast with the conventional approach, the theory is based on the infill asymptotics instead of the long span asymptotics.

Under certain assumptions of continuity for the stochastic process, the integrated local time can be estimated as the frequency estimator of the spatial distribution:

$$\hat{L}(T,x) = \Delta \sum_{i=1}^{n} e^{-ri\Delta} 1\{X_{i\Delta} \leq x\}. \tag{2.15}$$

Park [40] shows that the estimator above is consistent. For orders $s > 1$ we have that,

$$\hat{L}^{X,r,s}(T,x) = \frac{\Delta}{(s-1)!} \sum_{i=1}^{n} e^{-ri\Delta}(x - X_{i\Delta})^{s-1} 1\{X_{i\Delta} \leq x\}. \tag{2.16}$$

To estimate the spatial distribution, we need to introduce a new process based on the original stochastic process. More precisely, a process with stationary increments is defined as:

$$X_t^k = X_{T(k-1)+t} - X_{T(k-1)} \tag{2.17}$$

for $k = 1, 2 \ldots, N$. Roughly speaking, this stochastic process is defined in terms of the increment with respect to the first observation for each interval. The assumptions on these models are satisfied for all diffusion models used in practice to model interest rates. The estimators for the spatial density and spatial distribution can be computed by taking the average of each of the N intervals:

$$\hat{\Lambda}_N^{r,s}(T,x) = \frac{1}{N} \sum_{i=1}^{N} \hat{L}^{r,s}(T,x). \tag{2.18}$$

C. Testing for Spatial Dominance

The test for the null hypothesis given in equation 3.14 that X first order spatially dominates Y can be rewritten as:

$$H_0 : \delta(T) = \sup_{x \in \mathbb{R}} (\Lambda^{r,X}(T,x) - \Lambda^{r,Y}(T,x)) \leq 0 \tag{2.19}$$

against the alternative:

$$H_1 : \delta(T) > 0. \tag{2.20}$$

As proposed in the stochastic dominance literature (Mc Fadden, [37]), the Kolmogorov Smirnov statistics are used to test for spatial dominance. The Kolmogorov Smirnov statistic can be written as:

$$D_N(T) = \sqrt{N} \sup_{x \in R} (\hat{\Lambda}_N^{r,X}(T,x) - \hat{\Lambda}_N^{r,Y}(T,x)). \tag{2.21}$$

Park [40] shows that assuming continuity and controlling for dependencies, under the null hypothesis,

$$D_N(T) \rightarrow_d \sup_{x \in R} (U^X(T,x) - U^Y(T,x)), \tag{2.22}$$

where $(U^X(T,x), U^Y(T,x))'$ is a mean zero vector Gaussian process.

If we are interested in testing for spatial dominance of order $s > 1$, then we need replace $\hat{\Lambda}_N^{r,X}(T,x)$ in equation 2.21 by $\hat{\Lambda}_N^{r,X,s}(T,x)$ given in equation 2.18.

Notice that the distribution of D_N depends upon the unknown probability law of the unknown stochastic processes X, Y. Thus, the asymptotic critical values cannot be tabulated. There are three alternatives to obtain the critical values: simulation methods, bootstraping methods and subsampling methods. The results presented here are based on subsampling methods to obtain the critical values. In the stochastic dominance literature, subsampling methods have been proposed by Linton et al. [29]. The general theory for subsampling methods is explained in Politis et al. [41].

Let N_s denote the subsample size. Then, we will have $N - N_s + 1$ overlapping subsamples. For each of these subsamples i, we calculate the test statistic for the spatial dominance test $D_{N_s,i}$, where $i = 1, \ldots, N - N_s + 1$. Then, we approximate the sampling distribution of D_N using the distribution of the values of $D_{N_s,i}$. Therefore, the critical value can be approximated as

$$g_{N_s,\alpha} = \inf_w \left(\frac{1}{N - N_s - 1} \sum_{i=1}^{N-N_s-1} 1\left\{D_{N_s,i} \leq w\right\} \geq 1 - \alpha \right). \tag{2.23}$$

Table I. Descriptive Statistics for Stock and Bond Returns: 1965-2008

Variable	S&P 500	Treasury Bill
Mean	0.0325	0.0222
Std. Dev.	1.0081	0.0108
Skewness	-1.1556	0.9870
Kurtosis	34.9269	4.8405
Median	0.0215	0.0203
Maximum	10.9679	0.0666
Minimum	-22.8218	0.0000
Observations	11484	11484

Note: Daily Stock Returns are measured as log changes in the total return index from 1/7/1965 to 1/6/2009.

Thus, we reject the null hypothesis at the significance level α if $D_N > g_{N_s,\alpha}$.

D. Empirical Results

This section applies the test of spatial dominance to a dataset of daily returns on the S&P 500 index and the 3 month Treasury Bill from 1965 to 2008. The descriptive statistics are reported in Table I. The means of these series are 0.033 and 0.022 respectively, while the standard deviations are 1.01 and 0.01. Following the standard macroeconomics literature (Kydland and Prescott, [26]) the annual discount rate r is set to 4%.

Figure 2 plots the estimated discounted spatial distribution $\hat{\Lambda}^r(T,x)$ and integrated discounted spatial distribution $\hat{\Lambda}^{r,2}(T,x)$ of the two series for an investment horizon of one year, that is, T=1.[7] As can be seen, the distributions cross in both cases, suggesting no evidence of spatial dominance over this time period. Figure 3 presents the case of a ten year horizon. The estimated spatial distribution for a ten

[7]The support of the estimated distributions is based on the range of data of cumulative returns with 500 intermediate points. Sensitivity analysis using different intermediate points for the estimation of the spatial distribution yield similar results.

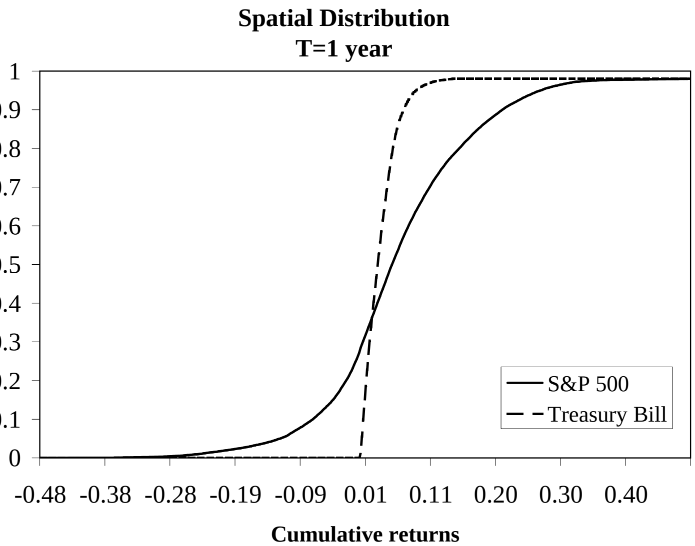
Table II. First Order Spatial Dominance Test

Horizon	KS	M	CV	PV
a) H_0: S&P 500 FOSD Treasury Bill				
1	6.89	54	2.56	0.00
2	5.45	43	2.42	0.00
3	4.59	56	2.34	0.00
4	4.02	31	3.10	0.00
5	3.67	57	2.62	0.00
6	3.41	34	3.03	0.00
7	3.04	49	2.53	0.00
8	2.55	30	2.31	0.02
9	1.97	38	2.28	0.14
10	1.53	63	2.85	0.37
b) H_0: Treasury Bill FOSD S&P 500				
1	7.42	50	3.04	0.00
2	7.14	74	2.72	0.00
3	6.91	69	2.85	0.00
4	6.43	61	2.96	0.00
5	6.03	64	3.10	0.00
6	5.73	58	3.08	0.00
7	5.38	59	3.19	0.00
8	5.24	60	3.31	0.00
9	5.18	58	3.35	0.00
10	5.27	59	3.45	0.00

Note: The number of subsamples M is based on the minimum volatility method. The p values are based on critical values at the 5% level.

year investment horizon suggests that the S&P 500 first order spatially dominates (FOSD) the Treasury Bill.

The first order spatial dominance test is reported in Table II. For the FOSD test, the null hypothesis is that $H_0 : \Lambda^{r,X}(T,x) \leq \Lambda^{r,Y}(T,x)$ for all x. The first column shows the investment horizon (in years), while the test statistic is showed in the second column. The next column reports the number of subsamples which is based on the minimum volatility method. The last two columns report the critical value and the p value respectively.



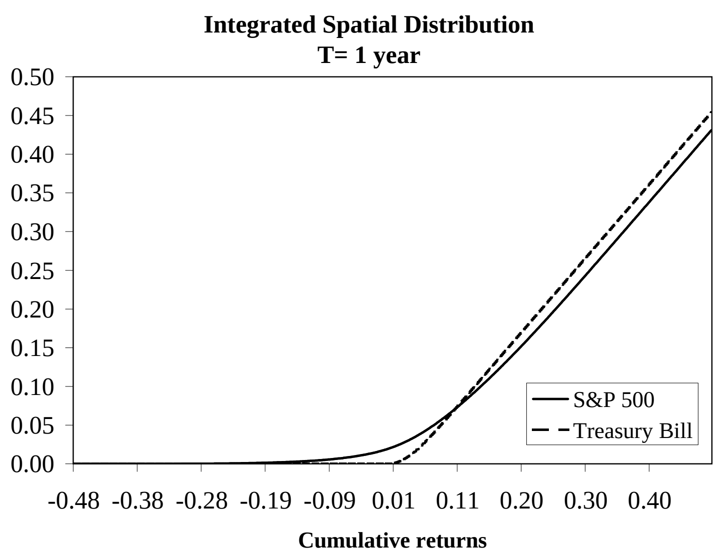


Fig. 2. Estimated Spatial Distribution and Integrated Spatial Distribution for a 1 year Investment Horizon

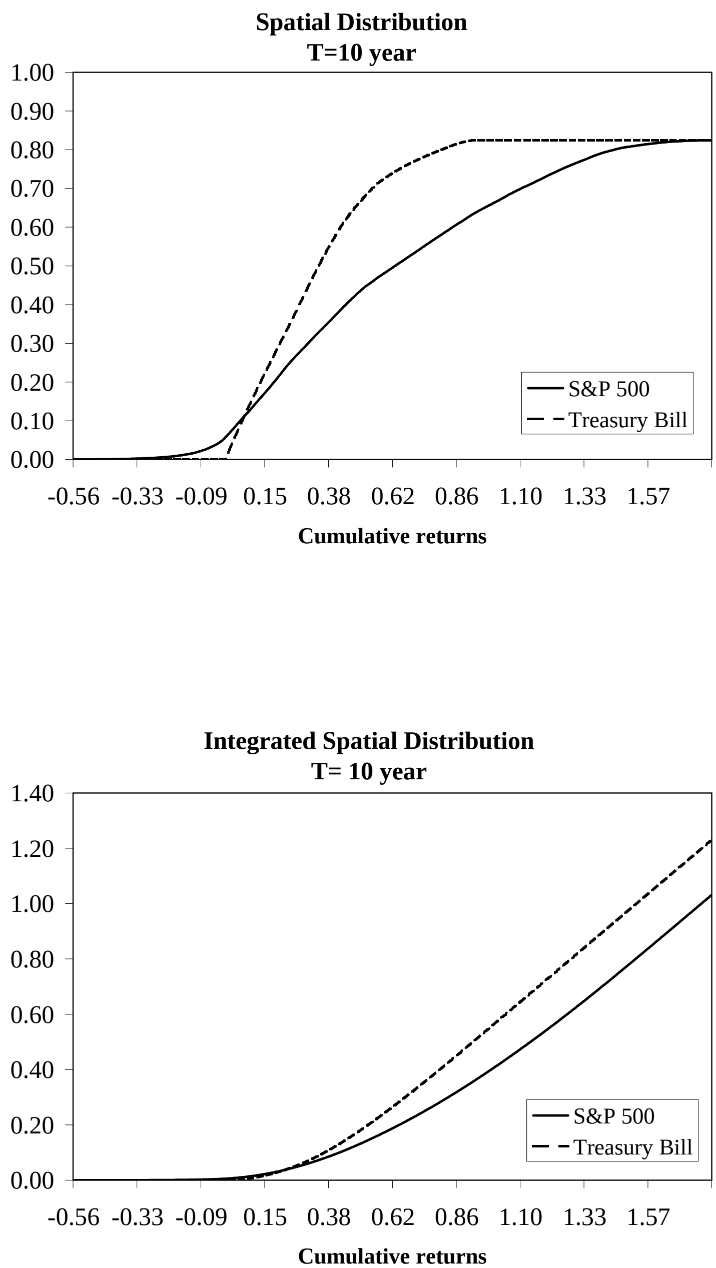


Fig. 3. Estimated Spatial Distribution and Integrated Spatial Distribution for a 10 year Investment Horizon

For choosing the optimal subsample size, the minimum volatility method is employed, as suggested by Politis et al. [41]. This method consists of calculating the local standard deviation of the critical value and then selecting the subsample size that minimizes this volatility measure. The local standard deviation is based on the critical values in the range $[N_s - b, N_s - b + 1, \ldots, N_s + b]$.[8] This method ensures that the critical values are relatively stable around the optimal subsample size. The sampling distribution of the test statistic is based on subsampling methods with overlapping subsamples.

The results suggests that, for investment horizons of one year or less, we reject the null hypothesis of first order spatial dominance of stocks over bonds at the conventional significance levels. However, we cannot reject the null hypothesis of first order spatial dominance of bonds over stocks. This result implies that any investor with monotonic preferences will obtain a higher level of expected utility by investing in bonds.[9]

The second order spatial dominance (SOSD) test is reported in Table III. For the SOSD test, the null hypothesis is that $H_0 : \Lambda^{r,X,2}(T, x) \leq \Lambda^{r,Y,2}(T, x)$ for all x. For investment horizons between two and four years, we reject the null hypothesis of FOSD and SOSD. For investment horizons between five and eight years we reject the null hypothesis of FOSD, but we cannot reject the null hypothesis of SOSD of stocks over bonds. This result implies that any investor with preferences characterized by nonsatiation and risk aversion will obtain a higher expected utility by investing in S&P 500 instead of Treasury Bills.

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[8] The results presented here are for b=5. Sensitivity analysis for different values of b yield similar results.

[9] Liu and Loewenstein [32] find that in a model with transaction costs, a short term investor might optimally hold only bonds even when there is a positive risk premium.

Table III. Second Order Spatial Dominance Test

Horizon	KS	M	CV	PV
a) H_0: S&P 500 SOSD Treasury Bill				
1	0.55	67	0.31	0.00
2	0.63	69	0.63	0.05
3	0.64	73	0.90	0.20
4	0.63	34	1.39	0.19
5	0.60	38	1.52	0.31
6	0.56	38	1.62	0.33
7	0.52	48	1.61	0.32
8	0.43	39	1.59	0.37
9	0.29	38	1.74	0.44
10	0.20	38	1.97	0.46
b) H_0: Treasury Bill SOSD S&P 500				
1	0.40	48	0.46	0.07
2	0.86	63	0.89	0.06
3	1.28	57	1.29	0.05
4	1.66	59	1.65	0.05
5	2.03	62	1.91	0.02
6	2.37	57	2.12	0.00
7	2.69	67	2.38	0.00
8	3.14	65	2.44	0.00
9	3.69	65	2.39	0.00
10	4.25	59	2.24	0.00

Note: The number of subsamples M is based on the minimum volatility method. The p values are based on critical values at the 5% level.

For investment horizons between nine and ten years, we cannot reject the null hypothesis that $H_0 : \Lambda^{r,X}(T,x) \leq \Lambda^{r,Y}(T,x)$ for all x. This result implies that any investor with preferences characterized by nonsatiation will attain a higher expected utility by investing in S&P 500 rather than Treasury Bills.[10]

These results are robust across different subsample sizes (N_s). Figures 4, 6 and 6 plot the p-value for the null hypothesis of spatial dominance, for investment horizons of six months, six years and ten years against subsample size (N_s).

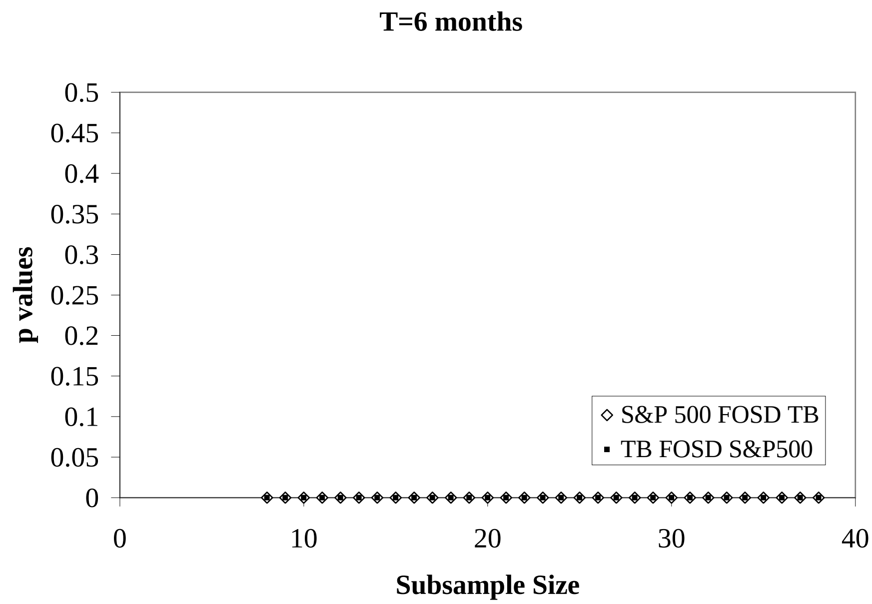
The p values support the results suggested by the estimated spatial distributions. For a six month investment horizon bonds second order spatially dominate stocks. For a six year investment horizon, the S&P 500 index SOSD the Treasury Bill, while for a ten year investment horizon, the S&P 500 index FOSD the Treasury Bill.

E. Concluding Remarks

This chapter employs a spatial dominance test to compare the distributions of stocks and bonds for different investment horizons. There are several advantages of using the concept of spatial dominance. First, we are able to rank investments without assuming any restriction on the form of the utility function. Second, we compare the entire distribution of returns rather than only the mean or the median return as used in the traditional studies. Third, this methodology is valid for either stationary or non stationary time series. The test employs subsampling methods since the limiting distribution of the test statistics depends on unknown data generating process.

Using a daily data set from 1965-2008, it is found that the spatial dominance

[10]Levy and Spector [28] find results that are consistent with ours in a model where borrowing and lending are not allowed or when borrowing takes place at a higher rate than lending. Using data for annual returns from 1926 to 1990, the authors find that investors having a log utility function and facing a long term horizon should invest all wealth in stocks.



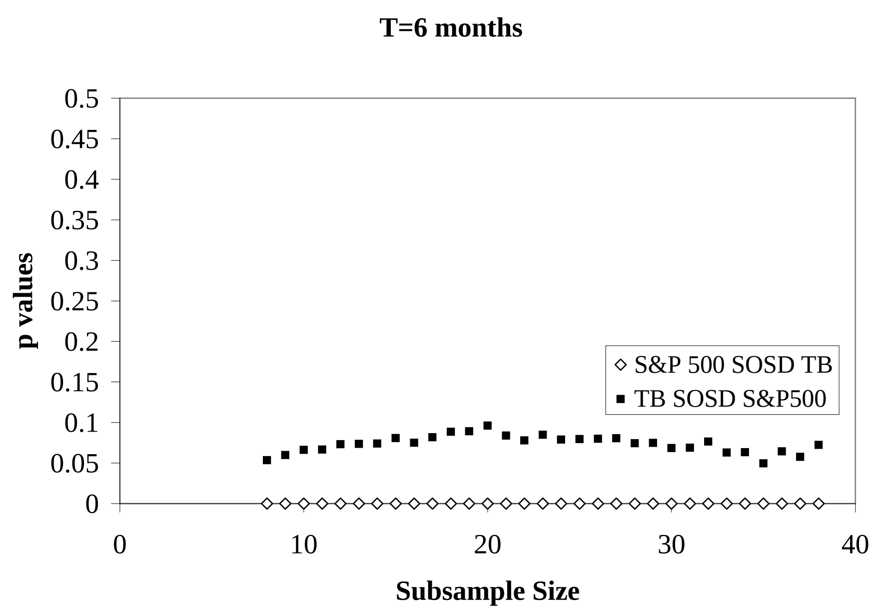
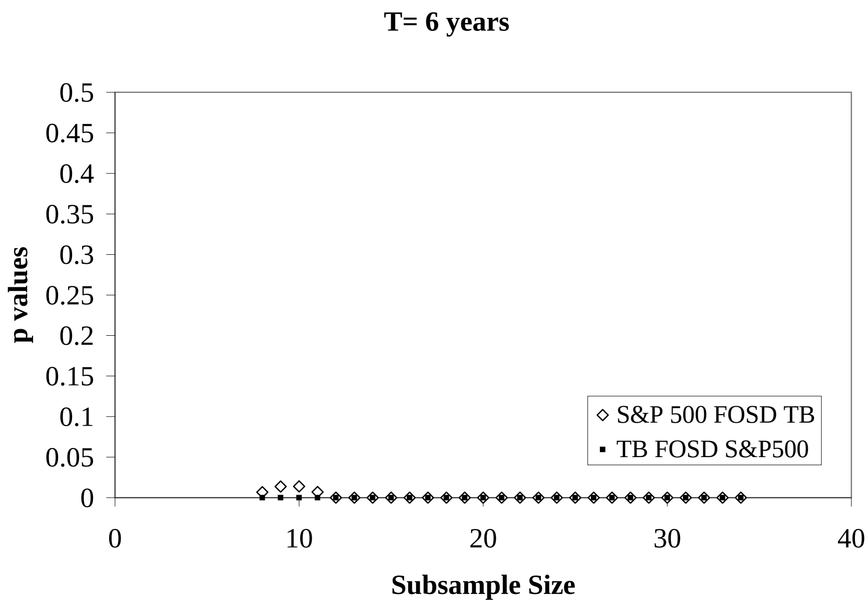


Fig. 4. P values for Spatial Dominance Test for a 6 Month Investment Horizon



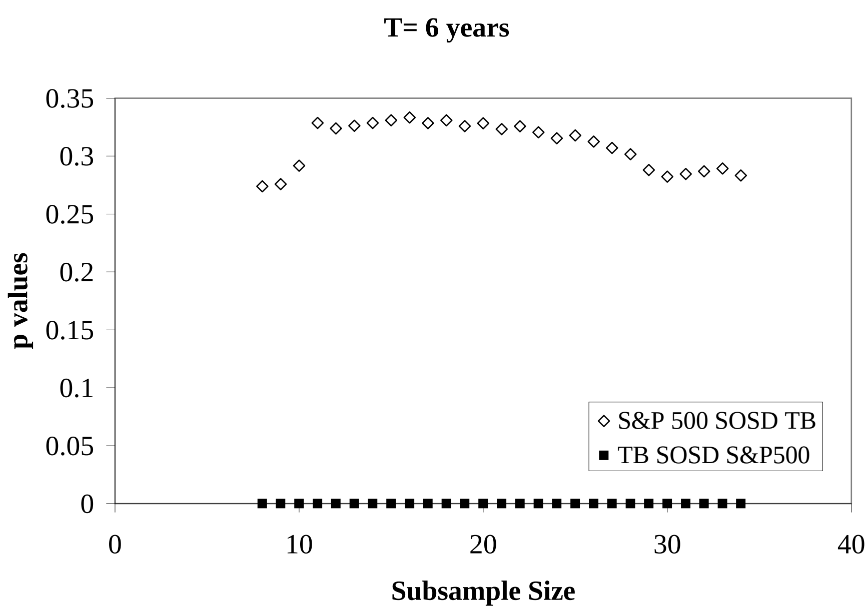
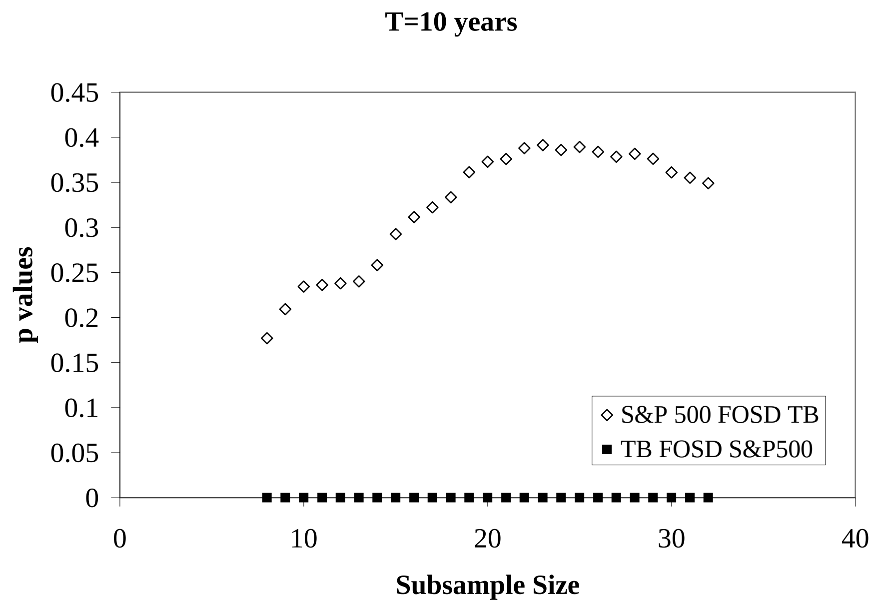


Fig. 5. P values for Spatial Dominance Test for a 6 Year Investment Horizon



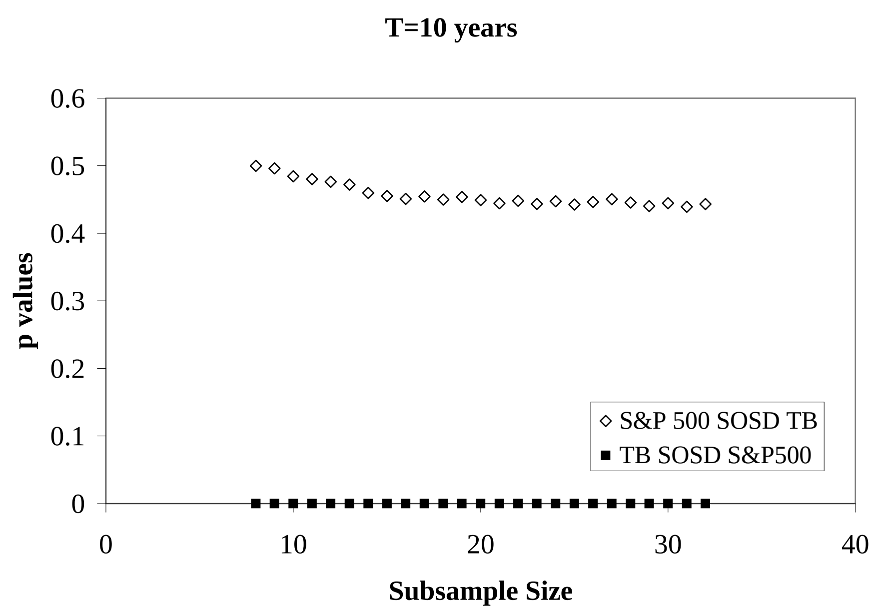


Fig. 6. P values for Spatial Dominance Test for a 10 year Investment Horizon

relations between S&P 500 and the Treasury Bill depend on the investment horizon. First, for investment horizons of one year or less, bonds second order spatially dominate stocks, which means that risk averse investors obtain higher levels of utility by investing in bonds. In contrast, for investment horizons between five and eight years, stocks second order spatially dominate bonds. For horizons of nine years or more, any investor whose preferences are characterized by nonsatiation will obtain a higher expected utility by investing in stocks.[11]

It is important to remember that, stochastic dominance, even where it exists, does not determine optimal asset weights within a portfolio choice setting. However the results of the study can be related to the advice given by practitioners to investors of allocating a higher proportion of stocks in their portfolio decisions, specially for investors following a buy and hold strategy. Our results suggest that an appropriate model to study investor optimal allocations should consider the investment horizon.

[11] Samuelson [44] examines the riskiness of stock at longer horizons, which might justify our empirical results. He finds that if returns are mean reverting, stocks will become less risky the longer the investment horizon is. Returns are negatively correlated so that volatility is reduced, because a positive or negative price movement tends to be followed by a price movement in the negative direction. Notice that Samuelson proves this result for an investor who optimally rebalances his portfolio at regular intervals, rather than the buy and hold investor that we consider here. Barberis [4] finds that, assuming a buy and hold investment horizon with utility defined over terminal wealth, predictability in stock returns implies that long term investors allocate more to equities than short term investors.

CHAPTER III

TESTING FOR STOCHASTIC DOMINANCE WITH UNCERTAIN TIME HORIZON

A. Introduction

The problem of portfolio evaluation has received considerable attention in the field of financial economics. Markowitz [36] introduced a popular approach for portfolio selection in his well known mean variance analysis. According to this approach, an investor will choose the portfolio that has the highest mean return for any given standard deviation. Although the mean-variance analysis has become popular among practitioners, it relies on restrictive assumptions on preferences and the distribution of returns.

The concept of stochastic dominance is considered a less restrictive approach than the mean variance analysis, since it is based on general assumptions on preferences such as nonsatiation and risk aversion. Stochastic dominance tests have been proposed by Mc Fadden [37] and extended by Linton et al. [29]. These methods are useful to compare the distribution of returns of two assets at a fixed period of time.

Park [40] introduces the notion of spatial dominance, which is a generalization of the concept of stochastic dominance for dynamic settings when the value of assets follow a nonstationary stochastic process. In other words, while the concept of stochastic dominance is static and it is only useful to compare two distributions at a fixed time, spatial dominance is useful to compare two distributions over a period of time.

All the approaches mentioned above are based on the assumption that an investor knows with certainty the time of exit at the moment of making an investment decision.

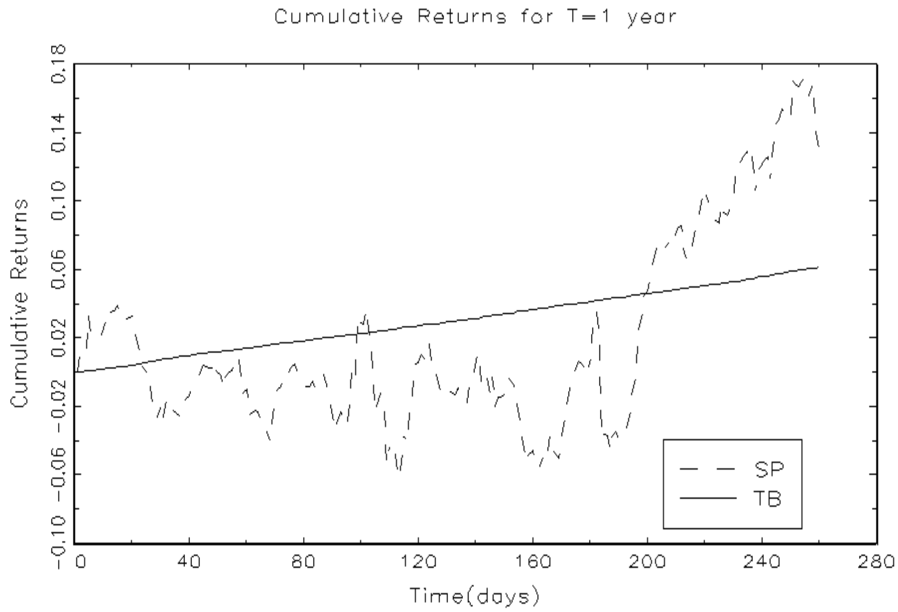


Fig. 7. Simulated Cumulative Returns

In practice, however, the investment horizon is never known with certainty at the time that the initial investment decisions are made. There are many factors that can drive the exit of an investor, such as of purchasing or selling a house, loss of a job, early retirement, disability, bequest, among others.

Figure 7 illustrates the comparison of two stochastic processes representing the values of cumulative returns for two different assets. The figure shows a simulation of the cumulative returns for the S&P 500 (dashed line) and the 3 month Treasury Bill (straight line) for an investment horizon of one year. Daily returns were obtained by random sampling with replacement from the original dataset for the period 1967-2006. According to existing studies, the investment decision will be determined only by the cumulative return at the end of the investment horizon. If the realization shown in the figure above occurs, the S&P 500 would be chosen since it has a higher value at the end of the period. This is true if the holding period is known with certainty. In reality, however, investment horizon is never certain. An investor might plan to invest for 1 year. After 6 months, due to an emergency or sudden need for money, the portfolio may have to be liquidated. This suggests that the investor should care about the distribution of the cumulative returns during the entire period of time rather than the final period alone.

Several authors have investigated the portfolio choice problem of an investor with an uncertain investment horizon. Yaari [54] examines the problem of optimal consumption for an individual with an uncertain date of death in discrete time. Hakansson [20, 21] analyzes a similar problem under the presence of uncertainty and a risky investment horizon. Merton [38] examines a dynamic optimal portfolio selection problem for an investor retiring at an uncertain date, defined as the date of a Poisson process with constant intensity. Richard [43] generalized these results to the presence of life insurance. Blanchet-Scaillet et al. [6] introduced a bequest motive and randomly time varying probabilities of exiting the market. Liu and Loewenstein [32] show that the solution of a portfolio choice problem of an investor with uncertain time horizon and transaction costs converges to the solution with deterministic finite horizon. The literature mentioned above examines the optimal portfolio problem by imposing restrictive assumptions on the form of the utility function or the distribution of returns.

The contribution of this chapter to the literature is to introduce the notion of stochastic dominance with uncertain time horizon and propose a statistical test based on spatial analysis. In this setup, utility is a function of wealth at a terminal period, which is a random variable. We assume that the investor knows the distribution of the investment horizon, which is independent of the distribution for the asset values.

Since the value of the assets follows a nonstationary stochastic process, the properties of their distribution function can be analyzed using the method of spatial analysis (Park, [40]). Under the assumptions mentioned above, the expected utility depends not only on the distribution for the nonstationary process (that is, the spatial distribution), but also on the distribution for the stochastic time horizon. If both distributions are combined, we obtain what we call weighted spatial distribution, which is a spatial distribution weighted by the density of the uncertain time horizon.

The distribution of the time horizon for the exit time employed in previous studies, such as Merton [38], Richard [43], Liu and Loewenstein [32] is exponential distribution, which is related to the first jump time of a Poisson process. Huang et al. [25] employ the truncated exponential distribution. In the empirical application, we also employ the exponential distribution since in practice we observe finite maximum investment horizons.

Following Mc Fadden [37], the test statistic for the stochastic dominance test is based in the Kolmogorov Smirnov distance between the two weighted spatial distributions. Following the spatial analysis method, the weighted version of the distribution function of the nonstationary process is estimated by using its sampling analog estimator. The distribution of the test statistic depends on the unknown probability law of the stochastic processes. Therefore the critical values are estimated by subsampling, as suggested by Linton et al. [29].

An empirical application is presented assuming that the time horizon is exponentially distributed with constant intensity. The data employed are for the S&P 500 and the 3 month Treasury Bill. The results suggest that, when the average number of arrivals, that is, the average number of times that the investor has to liquidate the portfolio, is lower than one per year, the S&P 500 second order stochastically dominates the Treasury Bill. In contrast, when the average number of arrivals is more than 20 per year, the Treasury Bill dominates the S&P 500.

This chapter is organized as follows. The next section explains the econometric method used to analyze the weighted spatial distribution. Section C presents the testing procedure. An empirical application is included in section D. The last section presents concluding remarks.

B. Methodology

1. Preliminaries on Spatial Analysis

As mentioned in the introduction, we use the theory of spatial analysis, developed by Park [40] to analyze the distribution of the value of the assets which follow a nonstationary process. The spatial analysis consists of the study of a time series along the spatial axis rather than the time axis. This methodology is developed for nonstationary time series, but the theory is also valid for stationary time series. By following this method, we adopt a nonparametric approach which relaxes the parametric assumptions about preferences used in other studies to compare alternative investments.

In order to explain the test for stochastic dominance with uncertain time horizon, it is necessary to introduce some important definitions. Let

$$X = (X)_t, t \in [0, T] \tag{3.1}$$

be a stochastic process. As it is well know, the local time, represented as $\ell(T, x)$, is defined as the frequency at which the process visits the spatial point x up to time T. Notice that the local time itself is a stochastic process. It has two parameters, the time parameter T and the spatial parameter x. If the local time of a process is continuous, then we may deduce that,

$$\ell(T,x) = \lim_{\varepsilon \to 0} \frac{1}{2\varepsilon} \int_0^T 1\{|X_t - x| < \varepsilon\} dt. \tag{3.2}$$

Therefore, we may interpret the local time of a process as a density function. The local time is known to be well defined for a broad class of stochastic processes. Notice that the local time itself is a stochastic process and random. Taking the expectation of this random variable, we can define the spatial density function as:

$$\lambda(T,x) = E\ell(T,x) = \lim_{\varepsilon \to 0} \frac{1}{2\varepsilon} \int_0^T P\{|X_t - x| < \varepsilon\} dt. \tag{3.3}$$

Consider a continuous utility function u that depends on the value of the stochastic process X. By occupation times formula, we may deduce that:

$$E \int_0^T u(X_t) dt = \int_{-\infty}^{\infty} u(x) \lambda(T,x) dx. \tag{3.4}$$

The equation above implies that, for any given utility function, the sum of expected future utilities generated by a stochastic process over a period of time is determined by and only by its spatial distribution. Therefore, the spatial analysis is useful to analyze dynamic decision problems that involve utility maximization.

Since we are considering an uncertain time horizon, we need to introduce a density function $f_H(t)$ with $t \in [0,T]$, which will give a weight for each point in time of the stochastic process. Then, the weighted local time or *w-local time* will be defined as:

$$\ell^H(T,x) = \int_0^T f_H(t) \ell(dt, x). \tag{3.5}$$

The weighted local time can be interpreted as a density function. Figure 8 illustrates the concept of weighted local time. The upper panel shows a sample path for Brownian motion with drift, $dX_t = \mu dt + \sigma dW_t$, where the drift parameter $\mu = 1$, the variance parameter $\sigma^2 = 1$, and W is a standard Brownian motion. The lower panel shows the estimated local time (straight line) and the estimated weighted local time (dashed line), with an exponential density for the investment horizon, that is, $f_H(t) = \frac{\lambda e^{-\lambda t}}{1-e^{-\lambda T}}$. The exponential distribution gives a higher weight at the initial points in time, when the process takes lower values. Therefore, the weighted local time is above the usual local time for lower values of the process and viceversa.

The *w-integrated local time* can be defined accordingly as:

$$L^H(T,x) = \int_{-\infty}^{x} \ell^H(T,y)dy = \int_0^T f_H(t)1\{X_t \leq x\}dt. \tag{3.6}$$

Similarly, the *weighted spatial density* can be defined as:

$$\lambda^H(T,x) = \mathbb{E}\ell^H(T,x) = \int_0^T f_H(t)\lambda(dt,x). \tag{3.7}$$

The corresponding distribution function, called *weighted spatial distribution*, is given by:

$$\Lambda^H(T,x) = \mathbb{E}L^H(T,x) = \int_0^T f_H(t)P\{X_t \leq x\}dt. \tag{3.8}$$

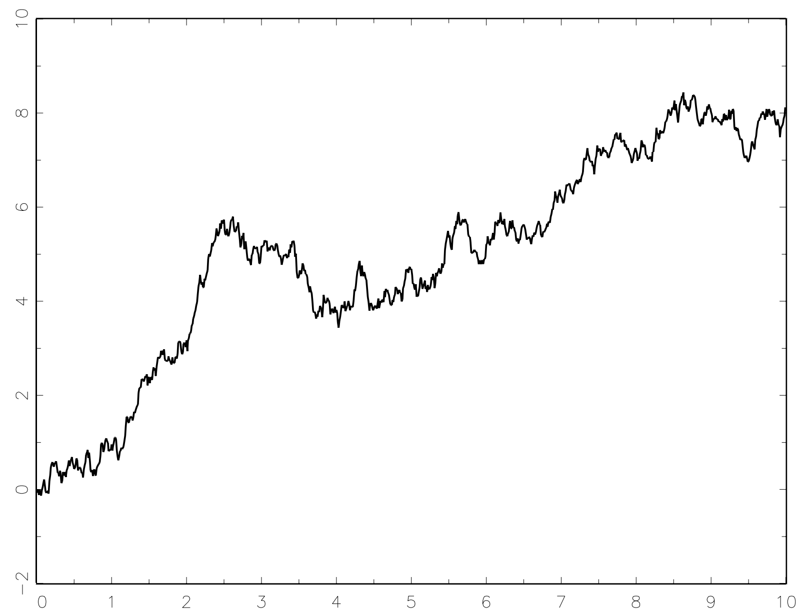
2. Stochastic Dominance with Uncertain Time Horizon

The usual approach to compare two distribution functions is to employ the concept of stochastic dominance. More specifically, if we have two stationary stochastic processes, X and Y with cumulative distribution functions Π^X and Π^Y, then we say that X stochastically dominates Y if,

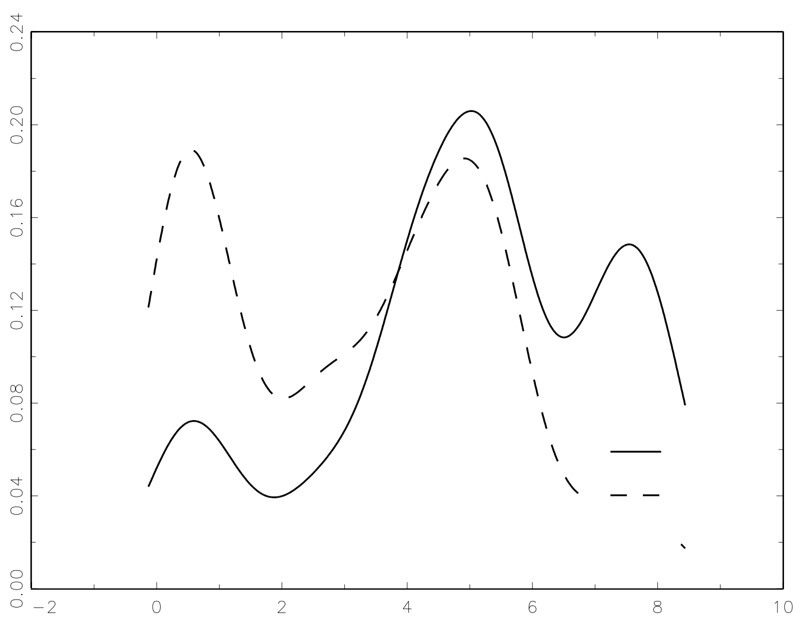
$$\Pi^X(x) \leq \Pi^Y(x) \tag{3.9}$$

for all $x \epsilon \mathbb{R}$ with strict inequality for some x. This definition is equivalent to:

$$Eu(X_t) \geq Eu(Y_t) \tag{3.10}$$



(a)



(b)

Fig. 8. Illustration of Weighted Local Time

for every utility function u such that $u'(x) > 0$. In other words, the process X stochastic dominates the process Y if and only if it yields a higher level of utility for any non decreasing utility function. Therefore, the notion of stochastic dominance is static an it is restricted to the study of stationary time series. Moreover, this concept is based on the assumption that the time of exit is known with certainty by the time that the investment decision is made.

In this chapter, the concept of stochastic dominance is extended by considering the case in which the investment horizon is uncertain. Assume that the investment horizon H is a random variable with support $[0, T]$ and density $f_H(t)$. Let X_t and Y_t represent two nonstationary stochastic processes representing the cumulative return at time t for two different assets. To make the investment decision, the investor will consider the utility at the end of the liquidation period, $\mathbb{E}u(X_H)$, or equivalently, $\mathbb{E}\int_0^T f_H(t)u(X_t)dt$. Notice that T is defined as a maximum investment horizon. Considering that X_t is a nonstationary stochastic process, the extended version of the occupation times formula implies that

$$\mathbb{E}u(X_H) = \int_{-\infty}^{\infty} u(x)\lambda^{H,X}(T, x)dx. \tag{3.11}$$

In other words, the expected utility can be written as a function of the weighted spatial density.

Definition 1. *X first order stochastically dominates Y with uncertain time horizon H if and only if*

$$\Lambda^{H,X}(T, x) \leq \Lambda^{H,Y}(T, x) \tag{3.12}$$

for all $x \epsilon \mathbb{R}$ with strict inequality for some x, or, equivalently,

$$\mathbb{E}u(X_H) \geq \mathbb{E}u(Y_H)$$

for any non decreasing utility function u.

Notice that the definition of stochastic dominance with uncertain time horizon includes the definitions of traditional stochastic dominance and spatial dominance as special cases. That is, if $f_H(t)$ is degenerate and X_t, Y_t are stationary, this definition is equivalent to the traditional stochastic dominance. If $f_H(t)$ is uniform, this definition is equivalent to the spatial dominance.

The definition above implies that, for any non decreasing utility function u,

$$\int_{-\infty}^{\infty} u(x)^X \lambda^H(T,x)dx \geq \int_{-\infty}^{\infty} u(x)^Y \lambda^H(T,x)dx, \tag{3.13}$$

which means that the stochastic process X provides at least the same level of expected utility than the stochastic process Y over a given period of time.

Several orders of stochastic dominance can be defined, according to certain restrictions on the shape of the utility function. For the first four orders of stochastic dominance, these restrictions consist of non satiation, risk aversion, preference for positive skewness and aversion to kurtosis, respectively (Levy, [27]).

In order to introduce the test for second order stochastic dominance, we need to define the weighted integrated local time and the weighted spatial distribution. The weighted integrated local time of order at order 2, $L^{H,X,2}(T,x)$ can be defined as:

$$L^{H,X,2}(T,x) = \int_{-\infty}^{x} L^{H,X}(T,x)dz. \tag{3.14}$$

Definition 2. *A stochastic process X second order stochastically dominates Y with uncertain time horizon H if,*

$$\Lambda^{H,X,2}(T,x) \leq \Lambda^{H,Y,2}(T,x) \tag{3.15}$$

for all $x \epsilon \mathbb{R}$ with strict inequality for some x, where,

$$\Lambda^{H,X,2}(T,x) = \int_{-\infty}^{x} \Lambda^{H,X}(T,x)dz. \tag{3.16}$$

It can be shown that the definition of stochastic dominance at an order $s \geq 2$ implies that implies that the stochastic process X provides a higher level of expected utility than the stochastic process Y.

3. Estimation Method

The theory presented before is built for continuous time processes. In practice, we need a estimation method for data in discrete time. Suppose that we have discrete observations $(X_{i\Delta})$ from a continuous stochastic process X on a time interval $[0,T]$ where $i = 1, 2, \ldots, n$ and Δ denotes the observation interval. The number of observations is given by $n = T/\Delta$. All the asymptotic theory assumes that $n \longrightarrow \infty$ via $\Delta \to 0$ for a fixed T. Notice that, in contrast with the conventional approach, the theory is based on the infill asymptotics instead of the long span asymptotics.

Under certain assumptions of continuity for the stochastic process, the weighted integrated local time can be estimated as,

$$\hat{L}(T,x) = \Delta \sum_{i=1}^{n} f_H(i\Delta) 1\{X_{i\Delta} \leq x\}. \tag{3.17}$$

The second order weighted integrated local time can be estimated as,

$$\hat{L}^{H,2}(T,x) = \Delta \sum_{i=1}^{n} f_H(i\Delta)(x - X_{i\Delta}) 1\{X_{i\Delta} \leq x\}. \tag{3.18}$$

Following Park [40], we will assume that the modulus of continuity for the stochastic process X will be given by

$$\omega(\Delta) = \max_{|t-s| \leq \Delta} |X_t - X_s|$$

for all $s, t \geq 0$. We assume that $\omega(\Delta) = o(N^{-1/2})$. The appendix shows the uniform consistency of the estimator given in equation 3.17.

To estimate the weighted spatial distribution, we need to introduce a new process based on the original stochastic process. More precisely, a process with stationary increments is defined as:

$$X_t^k = X_{T(k-1)+t} - X_{T(k-1)} \tag{3.19}$$

for $k = 1, 2 \ldots, N$. Roughly speaking, this stochastic process is defined in terms of the increment with respect to the first observation for each interval. The assumptions on these models are satisfied for all diffusion models used in practice to model interest rates. The estimators for the weighted spatial density and weighted spatial distribution can be computed by taking the average of each of the N intervals:

$$\hat{\Lambda}_N^{H,s}(T, x) = \frac{1}{N} \sum_{k=1}^{N} \hat{L}_k^{H,s}(T, x). \tag{3.20}$$

C. Testing for Stochastic Dominance with Uncertain Time Horizon

The test for the null hypothesis given in equation 3.12 that X first order stochastically dominates Y with uncertain time horizon H can be rewritten as:

$$H_0 : \delta(T) = \sup_{x \in \mathbb{R}} (\Lambda^{H,X}(T, x) - \Lambda^{H,Y}(T, x)) \leq 0, \tag{3.21}$$

against the alternative:

$$H_1 : \delta(T) > 0. \tag{3.22}$$

As proposed in the stochastic dominance literature (Mc Fadden, [37]), the Kolmogorov Smirnov statistic is used to test for stochastic dominance. The Kolmogorov Smirnov statistic can be written as:

$$D_N(T) = \sqrt{N} \sup_{x \in R} (\hat{\Lambda}_N^{H,X}(T, x) - \hat{\Lambda}_N^{H,Y}(T, x)). \tag{3.23}$$

The appendix shows that assuming continuity and controlling for dependencies, then, under the null hypothesis,

$$D_N(T) \to_d \sup_{x \in R} (U^X(T,x) - U^Y(T,x)), \tag{3.24}$$

where $(U^X(T,x), U^Y(T,x))'$ is a mean zero vector Gaussian process.

If we are interested in testing for second order stochastic domminance, then we need replace $\hat{\Lambda}_N^{H,X}(T,x)$ in equation 3.23 by $\hat{\Lambda}_N^{H,X,2}(T,x)$ given in equation 3.20.

Notice that the distribution of D_N depends upon the unknown probability law of the unknown stochastic processes X, Y. Thus, the asymptotic critical values cannot be tabulated. There are three alternatives to obtain the critical values: simulation methods, bootstraping methods and subsampling methods. The results presented here are based on subsampling methods to obtain the critical values. In the stochastic dominance literature, subsampling methods have been proposed by Linton et al. [29]. The general theory for subsampling methods is explained in Politis et al. [41].

Let N_s denote the subsample size. Then, we will have $N - N_s + 1$ overlapping subsamples. For each of these subsamples i, we calculate the test statistic for the spatial dominance test, $D_{N_s,i}$, where $i = 1, \ldots, N - N_s + 1$. Then, we approximate the sampling distribution of D_N using the distribution of the values of $D_{N_s,i}$. Therefore, the critical value can be approximated as

$$g_{N_s,\alpha} = \inf_w \left(\frac{1}{N - N_s - 1} \sum_{i=1}^{N-N_s-1} 1\left\{ D_{N_s,i} \le w \right\} \ge 1 - \alpha \right). \tag{3.25}$$

Thus, we reject the null hypothesis at the significance level α if $D_N > g_{N_s,\alpha}$.

D. Empirical Results

1. Distribution Function of Stochastic Time Horizon

The test presented above can be implemented for any distribution of the investment horizon $f_H(t)$. Following Huang et al. [25], we use the truncated exponential distribution $f_H(t) = \frac{\lambda e^{-\lambda t}}{1-e^{-\lambda t}}$. We also employ the exponential distribution since in practice we observe finite maximum investment horizons. Moreover, the exponential distribution is related to the jump time of a Poisson process, which has some properties that are relevant in our framework. More specifically, an uncertain sudden exit can be modelled as the jump of a Poisson process, and the amount of time until the first jump occurs follows an exponential distribution.

Suppose there are n different factors which can drive the exit of an investor (i.e., loss of a job, disability). For each of these possible events, let h_i for $i = 1, \ldots, n$ be independent exponentially distributed random variables representing the amount of time until the first arrival occurs. Then λ_i represents the average number of arrivals (i.e., sudden exits from the market) that occur per unit of time. Let $H = \min(h_1, h_2, \ldots h_n)$. Then H is exponentially distributed with parameter $\lambda = \sum_{i=1}^{n} \lambda_i$. Therefore, the parameter λ in our results can be interpreted as the sum of the intensities of several independent Poisson processes. We will find that the results of stochastic dominance between two assets might depend on the parameter λ, which represents the average number of sudden exits from the market.

2. Empirical Results

This section applies the test of stochastic dominance with uncertain time horizon to a dataset of daily returns on the S&P 500 index and the 3 month Treasury Bill from 1967 to 2006. The descriptive statistics are reported in Table IV. The means of these

Table IV. Descriptive Statistics for Stock and Bond Returns: 1967-2006

Variable	S&P 500	Treasury Bill
Mean	0.0400	0.0230
Std. Dev.	0.9545	0.0109
Skewness	-1.3653	0.9683
Kurtosis	38.9815	4.7143
Median	0.0215	0.0212
Maximum	8.7222	0.0666
Minimum	-22.8218	0.0031
Observations	10434	10434

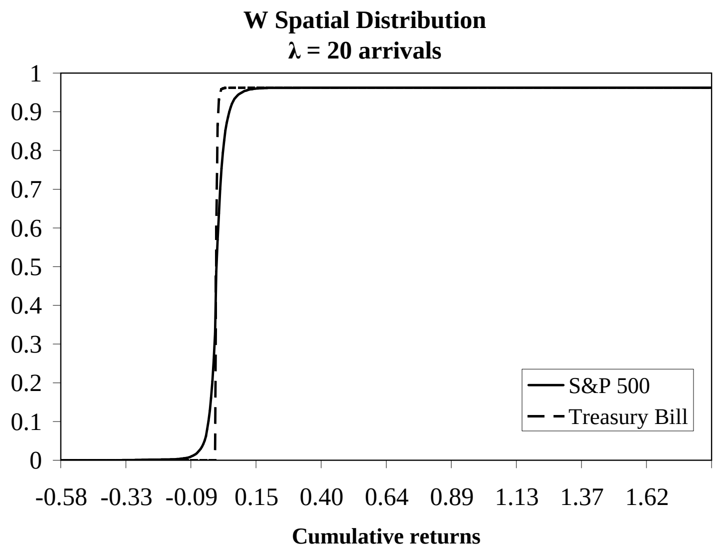
series are 0.04 and 0.023 respectively, while the standard deviations are 0.95 and 0.01. For this example, the maximum time horizon T is 10 years.

Figure 9 plots the estimated weighted spatial distribution $\hat{\Lambda}^{H}(T,x)$ and the weighted integrated spatial distribution $\hat{\Lambda}^{H,2}(T,x)$ of the two series for an average number of arrivals λ of 20 per year.[1] As can be seen, the weighted spatial distributions cross, which suggests no evidence of first order stochastic dominance. However, this is less clear for the integrated weighted spatial distribution.

Figure 10 presents the weighted spatial distribution and weighted integrated spatial distribution when the average number of arrivals is .5 arrivals per year. The estimated weighted spatial distribution suggests that the S&P 500 second order stochastically dominates (SOSD) the Treasury Bill.

The stochastic dominance tests are reported in Table V. For the SOSD test, the null hypothesis is that $H_0 : \Lambda^{H,X,2}(T,x) \leq \Lambda^{H,Y,2}(T,x)$ for all x. The first column shows the average number of arrivals per year λ, while the test statistic is showed in the second column. The next column reports the number of subsamples which is

[1]The support of the estimated distributions is based on the range of data of cumulative returns with 500 intermediate points. Sensitivity analysis using different intermediate points for the estimation of the spatial distribution yield similar results.



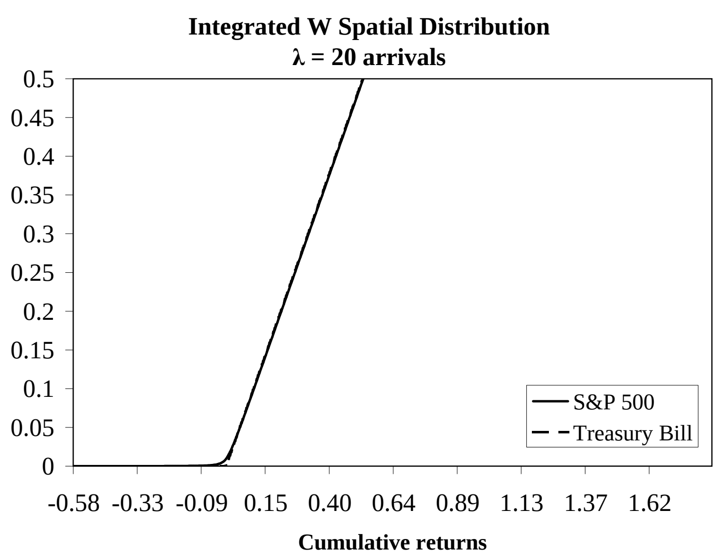


Fig. 9. Estimated Spatial Distribution and Integrated Spatial Distribution for an Average Number of Arrivals λ of 20 per Year

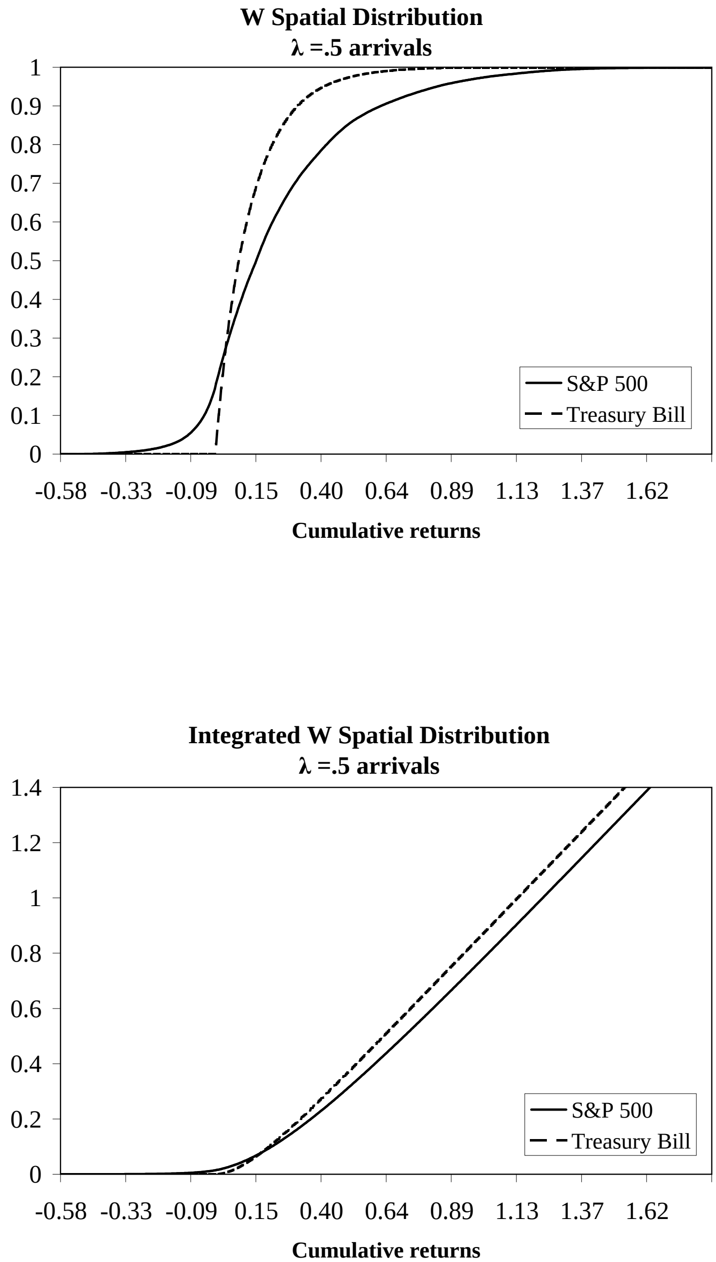


Fig. 10. Estimated Spatial Distribution and Integrated Spatial Distribution for an Average Number of Arrivals λ of .5 per Year

based on the minimum volatility method. The last two columns report the p value and the critical value respectively.

The sampling distribution of the test statistic is based on subsampling methods with overlapping subsamples. For choosing the optimal subsample size, the minimum volatility method is employed, as suggested by Politis et al. [41]. This method consists of calculating the local standard deviation of the critical value and then selecting the subsample size that minimizes this volatility measure. The local standard deviation is based on the critical values in the range $[N_s - b, N_s - b + 1, \ldots, N_s + b]$.[2] This method ensures that the critical values are relatively stable around the optimal subsample size.

The results suggest that when the average number of arrivals is lower than 1 per year, the S&P 500 second order stochastically dominates the Treasury Bill. In contrast, if the average number of arrivals is higher than 20 per year, the Treasury Bill dominates the S&P 500.

The results are robust across different subsample sizes. Figures 11 and 12 plot the p values for the null hypothesis of SOSD for λ of .5 and 20 arrivals per year against subsample size. The p values support the results given by the weighted spatial distribution. For λ=20, the Treasury Bill dominates the S&P 500, while for λ=5, the S&P 500 dominates the Treasury Bill.

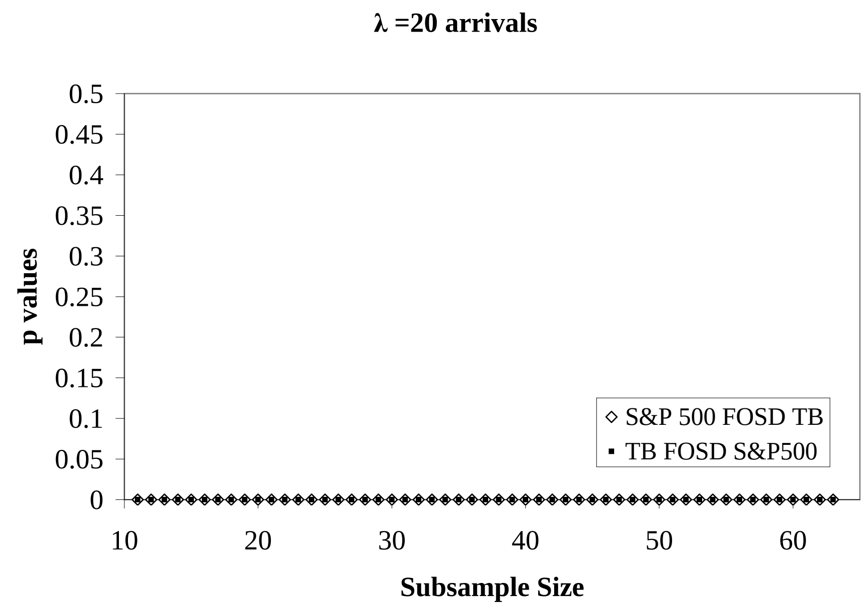
E. Concluding Remarks

This chapter introduces the concept of stochastic dominance with uncertain time horizon and proposes a statistical test based on spatial analysis. In this setup, utility is a function of wealth at a terminal period which is a random variable. The expected

[2] The results presented here are for b=5. Sensitivity analysis for different values of b yield similar results

Table V. Stochastic Dominance Test

λ	KS	M	PV	CV
a) H_0: S&P 500 SOSD Treasury Bill				
0.2	0.2462	35	0.4613	1.6223
0.4	0.3105	33	0.3785	1.2090
0.6	0.3488	32	0.2577	0.9434
0.8	0.3677	51	0.2638	0.7121
1	0.3752	35	0.1300	0.6525
2	0.3548	50	0.0000	0.3000
3	0.3227	32	0.0184	0.2533
4	0.2969	50	0.0000	0.1497
5	0.2761	50	0.0000	0.1207
10	0.2150	52	0.0000	0.0583
15	0.1825	54	0.0000	0.0398
20	0.1595	52	0.0000	0.0334
25	0.1417	53	0.0000	0.0288
b) H_0: Treasury Bill SOSD S&P 500				
0.2	3.2505	51	0.0000	1.6489
0.4	2.1569	51	0.0000	1.1844
0.6	1.5009	51	0.0000	0.8348
0.8	1.1164	37	0.0000	0.6119
1	0.8784	54	0.0000	0.5008
2	0.4145	22	0.0000	0.3067
3	0.2685	55	0.0000	0.1683
4	0.1974	38	0.0000	0.1150
5	0.1555	38	0.0000	0.0959
10	0.0735	23	0.0507	0.0732
15	0.0467	40	0.0314	0.0443
20	0.0335	42	0.1013	0.0362
25	0.0257	44	0.1465	0.0324



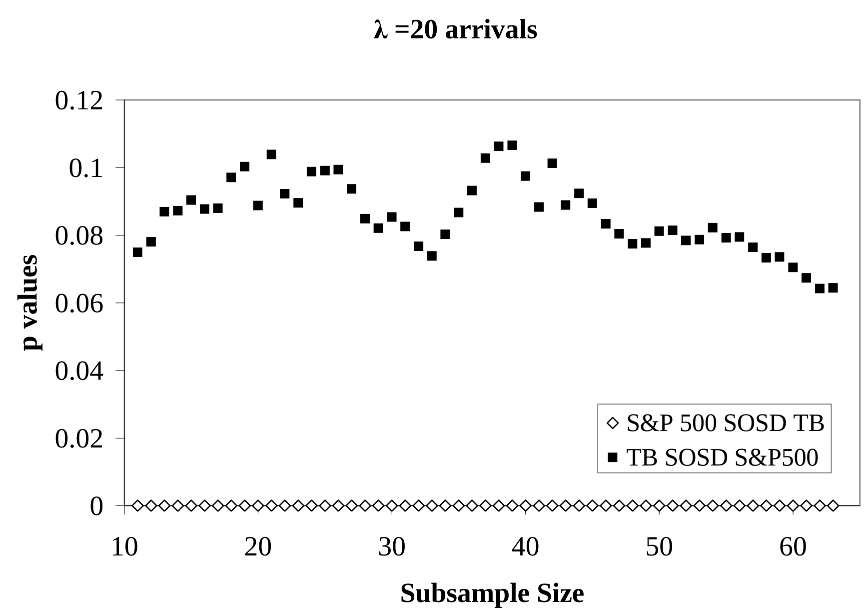
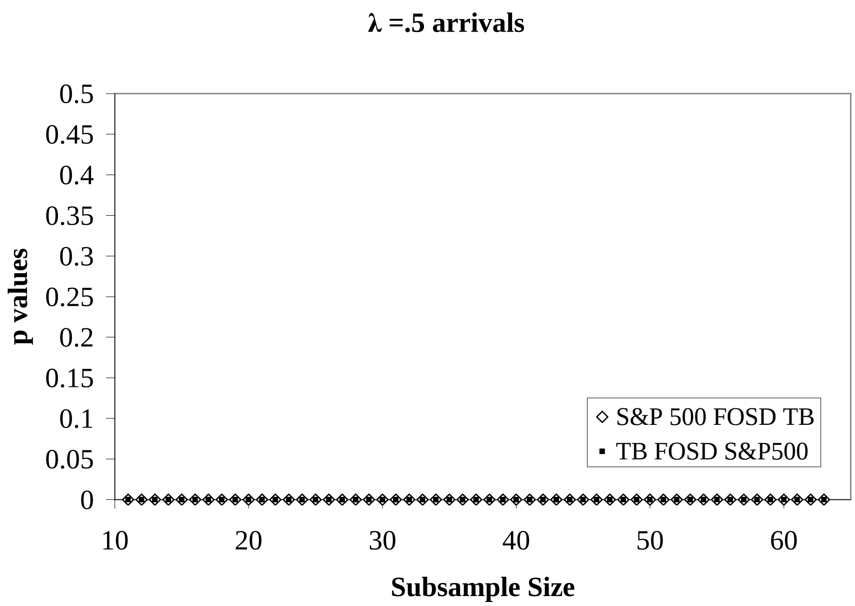


Fig. 11. P values for Stochastic Dominance Test with Uncertain Time Horizon for λ=20



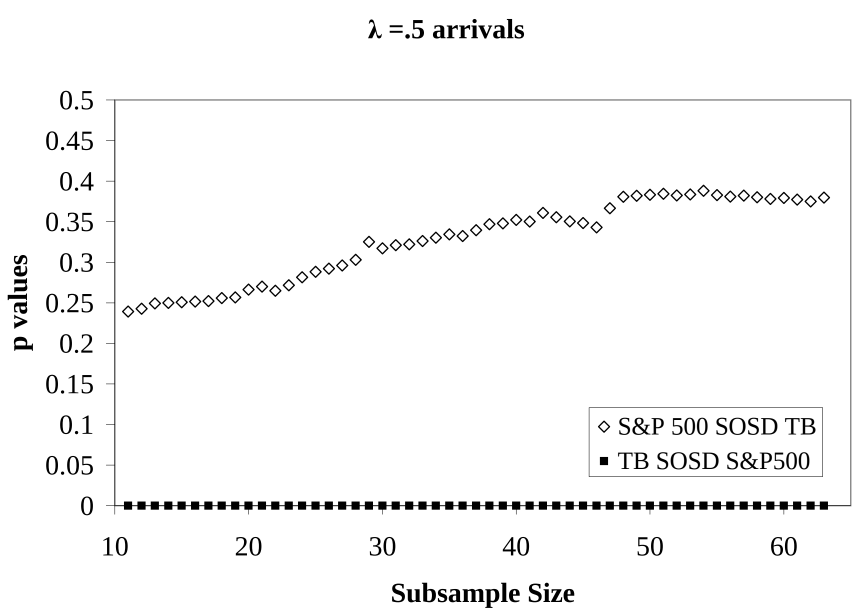


Fig. 12. P values for Stochastic Dominance Test with Uncertain Time Horizon for λ=.5

utility depends on the distribution for the nonstationary process (that is, the spatial distribution), as well as the distribution for the stochastic time horizon, which together form the weighted spatial distribution.

The distribution of the test statistic depends on the unknown probability law of the stochastic processes. Therefore the critical values are estimated by subsampling methods.

An empirical application is presented assuming that the time horizon is exponentially distributed with constant intensity. The data employed are for the S&P 500 and the 3 month Treasury Bill. The results suggest that, when the intensity parameter (which represents the number of arrivals per unit of time) is lower than one per year, the S&P 500 second order stochastically dominates the Treasury Bill. In contrast, when the average number of arrivals is more than 20 per year, the Treasury Bill dominates the S&P 500.

CHAPTER IV

FORECASTING INFLATION IN MEXICO USING FACTOR MODELS: DO DISAGGREGATED CPI DATA IMPROVE FORECAST ACCURACY?

A. Introduction

Inflation forecasts play an important role to effectively implement an inflation targeting regime (Svensson, [51]). Moreover, many economic decisions, whether made by policymakers, firms, investors, or consumers, are often based on inflation forecasts. The accuracy of these forecasts can thus have important repercussions in the economy.

This chapter focuses on forecasting inflation in Mexico. The forecasting framework is based on the factor model proposed by Stock and Watson [47]. Factor models incorporate the information content of a wide range of macroeconomic series. Recent advances in data collection have increased the amount of information available for economic analysis. As it is discussed in Bernanke and Boivin [5], economists have literally thousands of macroeconomic series available from different sources, including data at different frequencies and levels of aggregation, with and without seasonal and other adjustments. This opens the possibility of using a large number of time series to forecast important macroeconomic variables such as inflation in a more accurate and informative way. In spite of this, most empirical studies exploit only a limited amount of information. For example, vector autoregressions typically contain fewer than 10 variables because of the computation burden involved with large models.

The method used in this chapter summarizes the information contained in a large number of macroeconomic series into a few predictors of the inflation rate. The underlying assumption in our framework is that a small number of unobservable factors is the driving force behind the series under consideration. This is an appealing feature for forecasting purposes since it allows us to concentrate on a few common factors instead of a large number of explanatory variables. Recent empirical applications on factor models to forecast U.S. and Euro area inflation include Stock and Watson [45], [47], Marcellino et al. [35], Forni et al. [18], Angelini et al. [2], among others. To our knowledge, this is the first application of factor models for Mexico.

Previous applications of factor models including Stock and Watson [47] have only considered macroeconomic variables such as output, monetary aggregates and financial variables to forecast the inflation rate. In addition to those macroeconomic variables, our paper exploits the information contained in the subcomponents of the CPI at the highest degree of disaggregation. We investigate whether by pooling this information to construct common factors we can obtain better predictors of the inflation rate. Our dataset contains 243 CPI subcomponents from 1988 to 2008. We also include 54 macroeconomic series including real output, prices, monetary aggregates, financial variables and several components of the balance of payments, providing a complete description of the Mexican economy. Using this information, we estimate the common factors and use those factors to forecast the headline, core and non core inflation rate at the one, two, four and six quarters ahead horizons. Forecasting performances are evaluated through an out-of-sample simulation exercise. The factor forecasts are then compared with the alternative benchmark autoregressive model.

An important determinant of forecasting performance in factor models is the trade off between the information content from adding more data and the estimation uncertainty that is introduced. Boivin and Ng [8] find that more data to estimate the factors is not necessarily better for forecasting. This suggests the need to evaluate the role of adding the CPI components on forecasting performance. For this purpose, we estimate the model using datasets containing different blocks of variables, and evaluate changes in the forecasting performance when the CPI components are excluded.

We find that factor models have a higher predictive accuracy for headline, core and non-core inflation, in most cases producing out-of-sample root mean square forecast errors that are one-third less than those of the benchmark model. Our results also suggest that the estimated factors are related to relevant subsets of key macroeconomic variables, such as output and price inflation, which justifies their interpretation as major sources of the Mexican economy. Finally, we provide evidence that using CPI disaggregated data to extract the factors results in more accurate forecasts of the inflation rate.

The reminder of this chapter is organized as follows. Section B briefly discusses factor models. A description of the data is discussed in Section C. The forecasting framework is described in Section D. Section E presents the forecasting results. Section F concludes the paper.

B. The Factor Model

Suppose we are given time series data on a large number of predictors. Let y_t be the variable to forecast and X_t be the N predictor variables observed for $t = 1, \ldots, T$. We can think of the comovement in these economic time series as arising form a relatively few economic factors. One way of representing this notion is by using a dynamic factor model,

$$X_{it} = \lambda_i(L) f_t + e_{it}, \tag{4.1}$$

where f_t is a $\overline{r} \times 1$ vector of common factors, $\lambda_i(L)$ are lag polynomials in nonnegative powers of L, representing the factor loadings, and e_{it} is an idiosyncratic disturbance with limited cross sectional and temporal dependence. The factors can be considered as the driving forces of the economy and will therefore be useful for forecasting. If the lag polynomials $\lambda_i(L)$ are modelled as having finite orders of at most q, the factor model can be written as:

$$X_t = \Lambda F_t + e_t, \tag{4.2}$$

where $F_t = (f_t', \ldots, f_{t-q}')'$ is $r \times 1$, where $r \leq (q+1)\bar{r}$, the ith row of Λ is $\lambda_i = (\lambda_{i0}, \ldots, \lambda_{iq})$ and $e_t = (e_{1t}, \ldots, e_{Nt})'$.

Stock and Watson [46] show that, if the number of predictors N and time series T grow large, the factors can be estimated by the principal components of the $T \times T$ covariance matrix of X_t. The method of principal components minimizes the residual sum of squares,

$$V(F, \Lambda) = \min_{\Lambda, F} \frac{1}{NT} \sum_{i=1}^{N} \sum_{t=1}^{T} (X_{it} - \lambda_i F_t)^2, \tag{4.3}$$

subject to the normalization that $\frac{F'F}{T} = I_r$, where I_r is a $r \times r$ identity matrix. Concentrating out Λ, the problem is identical to maximizing $tr[F'(XX')F]$. The estimated factor matrix, denoted by $\hat{F}$, is $\sqrt{T}$ times the eigenvectors corresponding to the r largest eigenvalues of the $T \times T$ matrix XX'. The corresponding loading matrix is $\hat{\Lambda}' = (\hat{F}'\hat{F})^{-1}\hat{F}'X = \frac{\hat{F}'X}{T}$. See Stock and Watson [46] for more details.

Recent empirical applications for the US and Euro Area including Stock and Watson [47] and Marcellino et al. [35] have found important gains from using the factor forecasts based on the method of principal components. An alternative approach to estimate the factors proposed by Forni et al. [17] is to extract the principal components from the frequency domain using spectral methods. However, Boivin and Ng [7] conclude that the method proposed by Stock and Watson has smaller forecast errors in the empirical analysis. By imposing fewer constraints, and having to estimate a smaller number of auxiliary parameters, this method appears to be less vulnerable to misidentification, leading to better forecasts than the method of Forni et al. [17].

We will consider h step ahead forecasts for which the predictive relationship between X_t and y_{t+h} is represented as:

$$y^h_{t+h} = \alpha_h + \beta_h(L)F_t + \gamma_h(L)y_t + \varepsilon_{t+h}, \tag{4.4}$$

where $\gamma_h(L)$ and $\beta_h(L)$ are lag polynomial in non negative powers of L and ε_{t+h} are the forecast errors.

To obtain the forecasts, we use a three step forecast procedure. In the first step, we use the method of principal components to estimate the factors $\hat{F}_t$ from the predictors. In the second step, we use a linear regression to estimate the parameters given in model 4. Finally, the forecast is estimated as $\hat{y}^h_{t+h} = \hat{\alpha_h} + \hat{\beta}_h(L)\hat{F}_t + \hat{\gamma_h}(L)y_t$.

Stock and Watson [46] show that the principal components estimators and forecasts are robust to having temporal instability in the model, as long as the instability is relatively small and idiosyncratic (i.e., independent across series).[1]

C. The Data

The dataset consists of 54 quarterly macroeconomic series and 243 CPI subcomponents for the period 1988:I to 2008:IV. The frequency of the dataset is chosen considering that a larger range of macroeconomic variables are available on a quarterly basis than on a monthly basis.

The CPI subcomponents are obtained from Banco de Mexico. Since the CPI data are available on a monthly basis, we use the value for the last month of each quarter as the quarterly value. To form a balanced panel we have only considered the

[1] An empirical application about the stability of the method of principal components using data for the US is investigated in Stock and Watson [49]. The analysis shows that, in spite of the 1984 break for the inflation rate, the factors seem to be well estimated using the full sample period (i.e., 1959-2006).

series with available data for the entire period. Therefore, our dataset includes 243 out of the 315 CPI components.

The macroeconomic series are obtained from the OECD main economic indicators. This dataset has been used by Marcellino et al. [35] to construct forecasts for the Euro Area. The series include output variables (industrial production disaggregated by main sectors), employment, unemployment, prices (consumer and producer indexes), monetary aggregates, interest rates, stock prices, exchange rates and several components of the balance of payments. A complete list of the variables used in this paper is reported in the appendix. The macroeconomic series were selected from a longer list. We select those variables that have been employed in previous studies for the U.S. and Euro area, which are given in the appendix section of Stock and Watson [47] and Marcellino et al. [35]. If the series are available with and without seasonal adjustment, only the seasonally adjusted series are selected.

Following Marcellino et al. [35], the data are preprocessed in several steps before estimating the factors. First, we inspect each variable visually using a time series plot to detect inconsistencies in the series. We drop the series having discrepancies that could not be identified.

Second, the series are transformed to achieve stationarity as required by the factor model. Therefore, we take logs or first differences, as necessary. We apply the same transformation to all variables of the same type. In general, we transform output, prices, exchange rates, monetary aggregates and stock prices in growth rates.[2] Interest rates, unemployment rates and the components of the balance of payments are transformed to first differences. A summary of the transformations applied to the

[2]The inflation rate is modelled as being stationary. Chiquiar et al. [12] find that in 2000 the inflation rate in Mexico has switched from a nonstationary to a stationary process.

data is reported in the data appendix.[3]

Third, even though most of the series are reported as seasonally adjusted data, we pass all series through a seasonal adjustment procedure. The series are regressed against four seasonal variables and, if the HAC F-test for those coefficients is significant at the 10% level, the series are seasonally adjusted using the Wallis [53] linear approximation to X-11 ARIMA.

Fourth, the transformed seasonally adjusted series are screened for large outliers, that is, observations exceeding six times the interquartile range from the median. Since most outliers were identified with specific events, such as the 1995 economic crisis, we replace each outlying observation with the median of the series plus six times the interquartile range. Finally, the predictor series are normalized substracting their means and then dividing for their standard deviations.

The dataset described is used to forecast the inflation rate. In addition to forecasting the headline inflation, we will also present the results corresponding to the core and non-core inflation. The core index includes the least volatile components of the CPI. This index is thought to have a lagged response to macroeconomic variables, such as interest rates, exchange rates and wages. On the other hand, the non-core index contains the most volatile components, such as agricultural goods and those administered and concerted prices, such as gasoline, electricity, telephone and local transportation. This index mainly responds to external variables, such as international prices and other domestic non-market forces.

[3]Following previous studies in factor forecasting including Stock and Watson [47], we have not filtered the series using the method by Hodrick and Prescott [24]. This filtering method has been applied to construct business cycle indices based on common factors by Aiolfi, Catao and Timmermann [1]. However, Cogley and Nason [14] have shown that when the HP filter is applied to integrated processes, it can generate business cycle fluctuations even if they are not present in the original series, which would potentially misguide our forecasts.

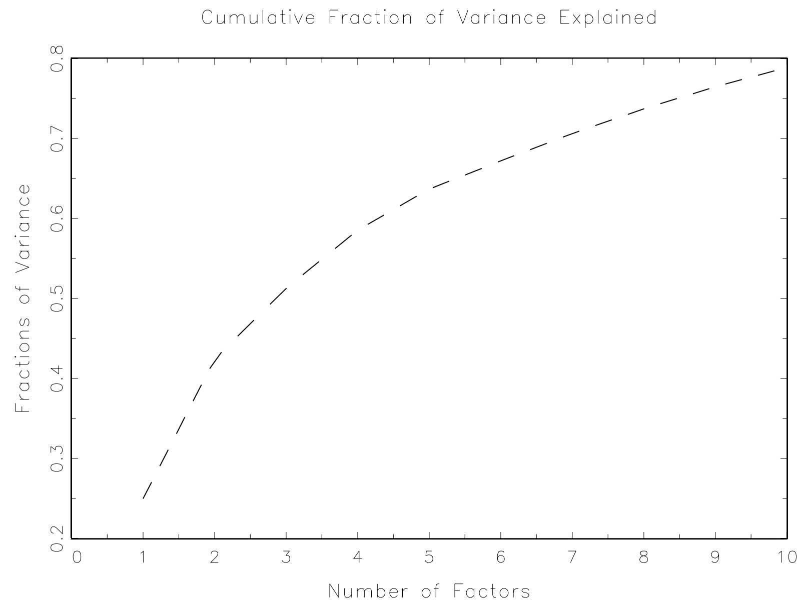


Fig. 13. Fractions of Variance

1. Estimation and Interpretation of Factors

Figure 13 shows the cumulative percentage of the total variation of the macroeconomic variables explained by the first 10 factors. As can be seen, with only 4 factors we are able to explain about 60% of the variation of the 54 series. One interpretation of this result is that there are only a few important sources of macroeconomic variability.

In order to characterize the first four estimated factors, we regress each variable in the dataset against each factor estimated over the full sample period. High values of R^2 in the resulting regressions suggest that the factor under analysis explains well that particular variable.

The results are shown in Figures 14 and 15. The horizontal axis indicates the code of the variables in the dataset as reported in the appendix, while the vertical axis gives the value of the R^2 of the factor corresponding to that particular variable. The vertical lines divide the variables into groups, as in the data appendix. The first factor appears to load primarily on output and employment, the second factor on price inflation, the third factor on trade and the fourth factor on exchange rates. Therefore, the extracted factors from our data are informative and interpretable from an economic point of view.

D. Forecasting Framework

1. Forecasting Design and Forecasting Models

Let π_t be the inflation at time t. We are interested in forecasting π_{t+h}, the annualized value of the inflation rate between t and $t+h$, defined as:

$$\pi^h_{t+h} = \frac{400}{h}[\ln(P_{t+h}/P_t)], \tag{4.5}$$

where P_t is the consumer price index at quarter t. Our factor model is specified as a linear projection of the h-step-ahead inflation rate π_{t+h} onto predictors observed at time t. The forecasting function can be written as:

$$\hat{\pi}^h_{t+h} = \hat{\alpha}_h + \sum_{j=1}^{m} \hat{\beta}'_{hj}\hat{F}_{t-j+1} + \sum_{j=1}^{p} \hat{\gamma}_{hj}\pi_{t-j+1} + \hat{\delta}' D_t, \tag{4.6}$$

where $\hat{F}$ are the estimated factors and the coefficients are defined as in equation 4.4. The number of factors k, the number of factor lags m and the number of autoregressive lags p are chosen by BIC with $k \leq 3$, $m \leq 4$, and $p \leq 5$.[4] We consider forecasting horizons of h =1, 2, 4 and 6 quarters ahead. The vector D_t contains seasonal dummy

[4]We have also constructed our forecasts including only contemporaneous values of the factors (i.e, m =1), although the results are not reported in this paper. The number of factors was estimated by the Bai and Ng [3] criterion and the number of autoregressive lags by BIC. Although the model yields similar conclusions, we find that including the lags of the factors results in more accurate forecasts.

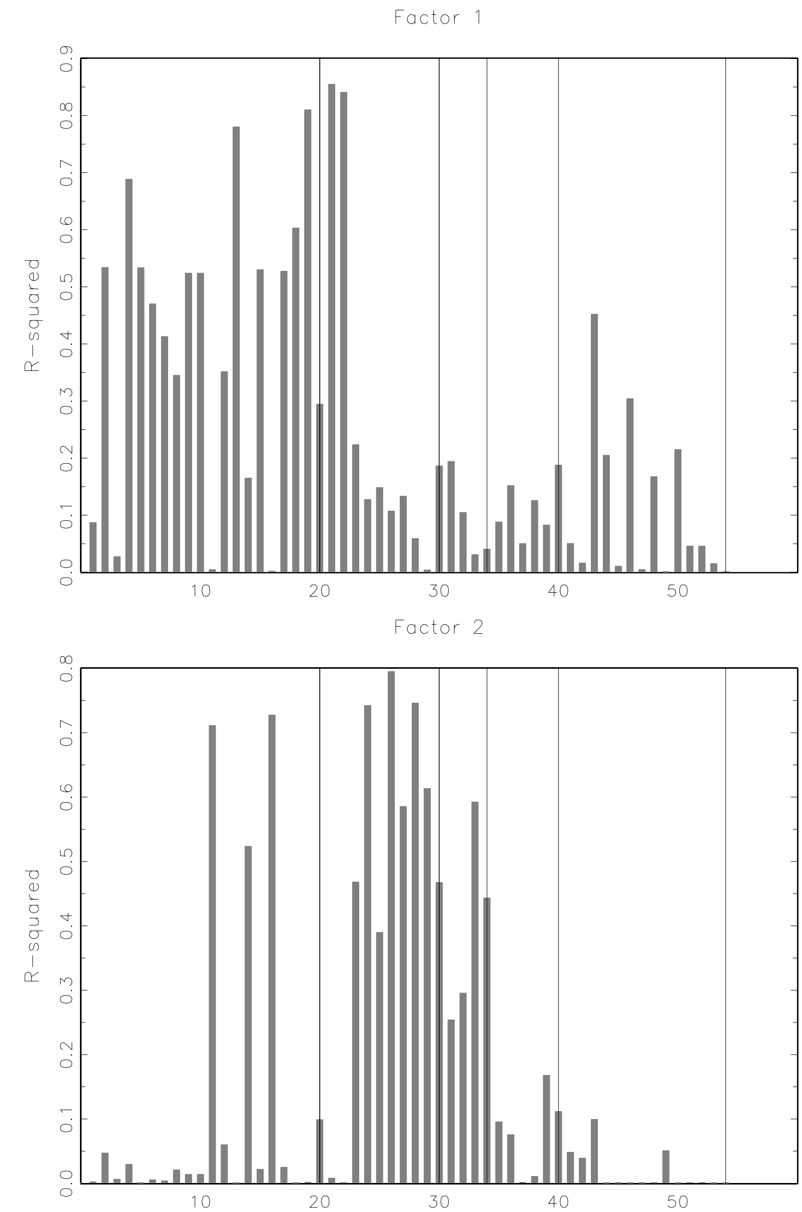


Fig. 14. Identification of Factors 1 and 2

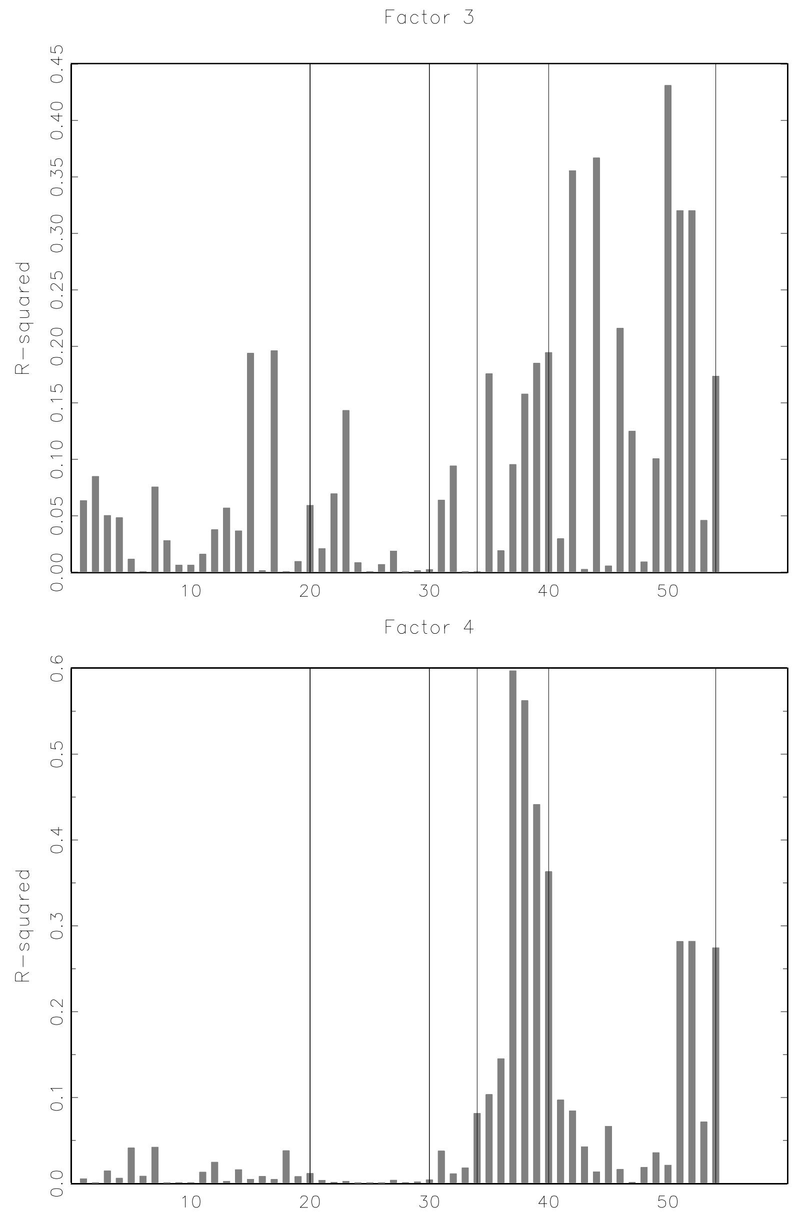


Fig. 15. Identification of Factors 3 and 4

variables.[5]

The direct approach used in this paper to construct the forecasts has some advantages over the standard iterative approach. First, it eliminates the need for additional equations to simultaneously forecast the regressors in equation 4.6. Second, it reduces the potential impact of specification error in the one-step ahead model by using the same horizon for estimation as for forecasting.

In addition to the macroeconomic variables considered by Stock and Watson [47], our approach to forecast inflation will extract the factors $\hat{F}_t$ from the the data set comprised of 243 CPI subcomponents. We compare our model with a benchmark univariate autoregressive forecast:

$$\hat{\pi}^h_{t+h} = \hat{\alpha}_h + \sum_{j=1}^{p} \hat{\gamma}_{hj} \pi_{t-j+1} + \hat{\delta}' D_t. \tag{4.7}$$

Capistran et al. [10] find that the autoregressive model with deterministic seasonality produces forecasts of equal performance compared to those taken from surveys of experts at the monthly frequency. The later in turn outperform other type of inflation forecasts in Mexico according to the evidence (Capistran and Lopez-Moctezuma, [11]).[6]

To analyze forecasting performance, we conduct an out-of-sample forecasting exercise. For each model, we estimate the factors and model parameters to obtain the forecasts of the inflation rate using a rolling scheme. According to Giacomini and

[5] Capistran et al. [10] provide empirical evidence that the seasonal components explain nearly 60% of the total variation of inflation rate during the period 2000-2005. For the core and non core inflation rate, the seasonal component explains above 60% and nearly 50% of their respective total variation.

[6] For the case of US inflation, Stock and Watson [48] find that since 1984 it has been difficult to outperform univariate models. Simple univariate models appear to generate relatively smooth and stable forecasts without suffering from large parameter estimation error.

White [19], the rolling windows scheme might be preferable if there are structural changes in the sample.

The out-of-sample forecasts are made for 2005:I to 2008:IV. The forecasting period is chosen considering the structural change in 2000, when the inflation rate switched from a non-stationary to a stationary process. Therefore, one part of the observations for the period when inflation is stationary is included in the estimation window and the remaining part is included in the forecasting period.

The length of the estimation window is 36 quarters. For instance, to construct the one step ahead forecast for 2005:I, we use data from 1996:I to 2004:IV to estimate the factors by the method of principal components. Then, we choose the number of factors, the number of factor lags and the number of autoregressive lags by BIC. Finally, we estimate the coefficients in equation 6 and use them to generate the out-of-sample forecast for 2005:I. Following the same forecasting procedure, we use data from 1996:II to 2005:I to make a one step ahead forecast for 2005:II. Notice that we drop the first observation and add a new observation at the end of the sample. This exercise is repeated until we obtain the forecast for 2008:IV using data from 1999:IV to 2008:III.[7]

To ensure that the length of the estimation windows and the number of out-of-sample forecasts is constant for the h steps ahead forecasts, we add $h-1$ observations at the beginning of the estimation period for $h = 2$, 4, and 6 quarters. For instance, to construct the $h = 2$ steps ahead forecast for 2005:I, we use data from 1995:IV to 2004:III. In moving forward the rolling procedure, the models are re-estimated each period. Therefore, the estimated factors as well as the number of factors, factor lags

[7]The results are robust for recursive forecasts and for different rolling window lengths. The forecasting results for windows lengths of 34, 38 and 40 quarters can be found in the appendix.

and autoregressive lags will be specific for each period and forecast horizon.

2. Forecast Comparison

To compare the forecast accuracy of the models, we calculate the root mean square error (RMSFE) of the factor models relative to the benchmark autoregressive model. To investigate whether the differences in the forecasting performance of the models are statistically significant, we use a test of equal predictive ability. Commonly used tests such as Diebold and Mariano [15] can only be applied to compare non-nested models. We apply the test by Giacomini and White [19] which is also useful to compare nested models.

The Giacomini and White (GW) test is a test of conditional forecasting ability. The test is constructed under the assumption that the forecasts are generated using a moving data window. Consider the loss differential $d_t = e_{1t}^2 - e_{2t}^2$, where e_{it} is the forecast error for forecast i.[8] The null hypothesis of equal forecasting accuracy can be written as:

$$H_0 : E[d_{t+\tau}|h_t] = 0, \tag{4.8}$$

where h_t is a $p \times 1$ vector of test functions or instruments and τ is the forecast horizon. If a constant is used as instrument, the test can be interpreted as an unconditional test of equal forecasting accuracy. The GW test statistic GW_T can be computed as the Wald statistic:

$$GW_T = T\left(T^{-1}\sum_{t=1}^{T-\tau} h_t d_{t+\tau}\right)' \hat{\Omega}_T^{-1} \left(T^{-1}\sum_{t=1}^{T-\tau} h_t d_{t+\tau}\right), \tag{4.9}$$

[8]The results reported in this paper are based on a MSE loss function which is the most common in the factor forecasting literature. We have also compared our results with those based on a Mean Absolute Error (MAE) loss, yielding similar conclusions. Those can be found in the appendix.

where $\hat{\Omega}_T$ is a consistent HAC estimator for the asymptotic variance of $h_t d_{t+\tau}$. Under the null hypothesis given in equation 4.8, the test statistic GW_T is asymptotically distributed as χ^2_p.

E. Forecasting Results

To estimate the model, we organize the data into six blocks: real output variables, price inflation, monetary aggregates, financial variables (interest rates, exchange rates and stock prices), balance of payments and CPI components. Then we follow Forni et al. [18] to analyze the marginal predictive content of these different groups of variables. That is, we estimate the factor model considering seven alternative datasets: The first group contains all variables except those in the real output block, the second group contains all variables except those in the price block, and so for the first six blocks. The seventh group contains all variables. In this way, we are able to evaluate the change in forecasting performance when each of the six groups of variables is excluded.

Table VI presents the RMSFE of the factor model estimated for each group of variables relative to the benchmark AR for the case of headline inflation. In general, the factors models outperform the benchmark AR model at all horizons, with an average gain in the range 30-40% with respect to the benchmark.

Results about the role of disaggregated CPI components are of particular interest. Excluding these variables results in a deterioration of forecasting performance at all horizons. The same is not true however, for the rest of the variables, since the forecasting performance sometimes improves when these variables are excluded. In other words, once the CPI components are considered, the real output variables, monetary aggregates, financial variables and the balance of payments components

Table VI. Forecasting Results: Headline Inflation

Excluded Block	h =1	h =2	h =4	h =6
Output	0.620 (0.018)	0.744 (0.001)	0.656 (0.007)	0.640 (0.006)
Prices	0.617 (0.025)	0.737 (0.001)	0.614 (0.009)	0.620 (0.004)
Mon. aggregates	0.615 (0.025)	0.723 (0.001)	0.616 (0.007)	0.622 (0.004)
Financial Var.	0.606 (0.025)	0.714 (0.000)	0.607 (0.006)	0.693 (0.001)
Bal. of Payments	0.619 (0.025)	0.721 (0.000)	0.664 (0.006)	0.620 (0.004)
CPI Components	0.849 (0.349)	0.848 (0.267)	0.957 (0.573)	0.807 (0.166)
None	0.611 (0.024)	0.719 (0.001)	0.611 (0.006)	0.618 (0.003)
RMSFE AR	2.522	2.188	1.612	1.598

Note: The table reports the RMSFE from using the factors for each dataset relative to the benchmark AR Model. The p-value for the Giacomini and White test of equal forecasting accuracy is presented in parenthesis. The RMSFE are calculated using out-of-sample forecasts from 2005:I-2008:IV with a rolling window of 36 quarters.

seem to have only a marginal effect on the forecasting ability of the model.

Table VI also reports the results of the predictive ability test for each model relative to the benchmark AR Model. More specifically, we present the p-values of the Giacomini and White [19] tests using a constant as an instrument. In general, we reject the null hypothesis of equal predictive ability for those models which include the CPI disaggregated data. However, when the factor model excludes the CPI components, the differences in forecasting performance with respect to the benchmark AR model are not statistically significant at any forecasting horizon.[9] In sum, the results show evidence of superior performance of the factor model over the benchmark AR model provided that the CPI components are included.

The forecasting results for core inflation and non-core inflation are reported in Tables VII and VIII respectively. The results suggest that the factor models consistently outperform the benchmark model at all horizons. The Giacomini and White

[9]This conclusion is consistent with the study by Giacomini and White [19] for the US. The authors find the null hypothesis of equal forecasting accuracy between the factor model that includes only the macroeconomic variables and the AR model cannot be rejected.

Table VII. Forecasting Results: Core Inflation

Excluded Block	h =1	h =2	h =4	h =6
Output	0.683 (0.141)	0.925 (0.299)	0.828 (0.048)	0.765 (0.000)
Prices	0.875 (0.492)	0.885 (0.250)	0.812 (0.038)	0.748 (0.000)
Mon. aggregates	0.915 (0.656)	0.879 (0.205)	0.840 (0.070)	0.804 (0.000)
Financial	0.765 (0.180)	0.865 (0.172)	0.830 (0.069)	0.819 (0.000)
Bal. of Payments	0.834 (0.189)	0.902 (0.335)	0.811 (0.042)	0.973 (0.830)
CPI Components	1.072 (0.042)	0.986 (0.880)	1.136 (0.149)	0.957 (0.615)
None	0.870 (0.471)	0.876 (0.203)	0.831 (0.063)	0.746 (0.000)
RMSFE AR	1.693	1.705	1.245	1.539

Note: The table reports the RMSFE from using the factors for each dataset relative to the benchmark AR Model. The p-value for the Giacomini and White test of equal forecasting accuracy is presented in parenthesis. The RMSFE are calculated using out-of-sample forecasts from 2005:I-2008:IV with a rolling window of 36 quarters.

test rejects the null hypothesis of equal predictive ability for horizons of $h = 4$ and 6 quarters ahead for the case of core inflation for those models including CPI disaggregated data. For the case of non-core inflation, we obtain the same conclusion for horizons of $h = 2$, 4 and 6 quarters ahead. According to this evidence, the CPI components are especially useful for medium horizon forecasts of $h = 4$ and 6 quarters of the core and non-core inflation. For horizons of $h = 1$ quarter ahead, the non-core index seems to be more difficult to predict since this index is subject to temporary shocks.[10]

In general, the relative performance of the factor models that include the CPI components improves as the forecast horizon increases. The factors capture the common component of the CPI disaggregated data, filtering out the idiosyncratic variations. This common component has a good predictive content especially for the long

[10]Notice that for horizons of h = 1 quarters ahead, the null hypothesis of equal predictive ability is rejected for headline inflation, but the same hypothesis is not rejected for core and non-core inflation. As it is shown by Lutkepohl [33], the forecasts from aggregated series might be superior to the forecasts from the disaggregated series when there the data generation process is unknown due to parameter uncertainty, which is commonly found in empirical applications.

Table VIII. Forecasting Results: Non-core Inflation

Excluded Block	h =1	h =2	h =4	h =6
Output	0.816 (0.111)	0.650 (0.042)	0.792 (0.006)	0.798 (0.000)
Prices	0.829 (0.128)	0.655 (0.044)	0.778 (0.001)	0.673 (0.000)
Mon. aggregates	0.828 (0.139)	0.652 (0.044)	0.765 (0.007)	0.680 (0.000)
Financial Var.	0.827 (0.133)	0.649 (0.042)	0.779 (0.002)	0.677 (0.000)
Bal. of Payments	0.830 (0.137)	0.652 (0.042)	0.772 (0.001)	0.621 (0.000)
CPI Components	0.855 (0.109)	0.629 (0.066)	1.069 (0.719)	0.883 (0.060)
None	0.828 (0.135)	0.651 (0.043)	0.778 (0.002)	0.678 (0.000)
RMSFE AR	6.814	4.630	2.615	2.117

Note: The table reports the RMSFE from using the factors for each dataset relative to the benchmark AR Model. The p-value for the Giacomini and White test of equal forecasting accuracy is presented in parenthesis. The RMSFE are calculated using out-of-sample forecasts from 2005:I-2008:IV with a rolling window of 36 quarters.

run component of inflation, resulting in higher improvements over the benchmark model as the horizon increases. In addition, the parameter uncertainty for the factor model is likely to be reduced at longer horizons, resulting in higher improvements.[11]

F. Conclusion

In this paper we use the dynamic factor model proposed by Stock and Watson [47] to forecast inflation in Mexico. This method exploits the information contained in a large number of economic series using a few common factors to construct the forecasts. We also investigate the role of using CPI disaggregated data to improve forecasting performance.

We use a large dataset consisting of 243 CPI components and 54 macroeconomic variables to extract the factors and simulate out-of-sample predictions of inflation. We estimate the model using datasets containing different blocks of variables to evaluate

[11]These results are in line with the simulations shown by Boivin and Ng [7] which suggest that the factor model significantly outperforms the autoregressive model at longer horizons. The results are also consistent with Stock and Watson [47].

the gains of including the CPI disaggregated data.

Our results indicate that factors model outperform the benchmark AR model at the one, two, four and six quarters ahead horizons, with gains of above 30% in terms of the RMSFE. Those gains are especially strong considering that Capistran et al. [10] have shown that the autoregressive model with deterministic seasonality performs as well as the surveys of experts. These results are in line with those from previous studies for the US and the Euro area. In addition, we provide evidence that using information from the CPI components contributes to substantial improvements in the accuracy of the inflation forecasts.

The results presented in this paper are promising enough to warrant further research. The Stock and Watson [47] methodology can be combined with more structural approaches to improve forecasting still further (Liu and Jansen, [31]). The method can also be applied to generate forecasts of inflation at the monthly frequency. The dynamic model proposed by Forni et al. [17] can also be applied to our dataset to compare the forecasting performance of the method used in this paper with an alternative factor model. Finally, we can also use the method of weighted principal components explained in Boivin and Ng [8] which considers the quality of the series to construct the factors.

CHAPTER V

CONCLUSION

Chapter II used a spatial dominance test to compare the distributions of stocks and bonds for investment horizons from one to ten years. The empirical approach used in this study has several advantages. We impose minimal assumptions about preferences, such as nonsatiation, risk aversion and time separable preferences. In addition, we use information from the entire path of the value of the asset instead of the ending points as in the standard stochastic dominance approach. Using a daily data set for the S&P 500 and the 3 month Treasury Bill from 1965-2008, it is found that the spatial dominance relations between these two assets depend on the investment horizon. For investment horizons of 1 year or less, bonds second order spatially dominate stocks, which means that risk averse investors obtain higher levels of utility by investing in bonds. In contrast, for investment horizons longer than 5 years, stocks second order spatially dominate bonds. This result is consistent with the advice given by practitioners to investors of allocating a higher proportion of stocks in their portfolio decisions. Our empirical results can be explained by examining the riskiness of stock at longer horizons. If returns are mean reverting, stocks will become less risky the longer the investment horizon is. Returns are negatively correlated so that volatility is reduced at longer horizons, because a positive or negative price movement tends to be followed by a price movement in the negative direction.

Chapter III presented a method that allows to test whether an asset stochastically dominates the other when the time horizon is uncertain. In this setup, the expected utility of the investor depends on the distribution function of the value of the asset asset as well as the distribution of the investment horizon. We have introduced the weighted spatial distribution, which combines the distribution of the value of the asset with the distribution of the time horizon. The weighted spatial distribution is a spatial distribution weighted by the density of the uncertain time horizon. We have followed the literature on stochastic dominance to estimate this distribution, the test statistic and the critical values. An empirical application has been presented assuming that the time horizon is exponentially distributed with constant intensity. The data employed are for the S&P 500 and the 3 month Treasury Bill. The results suggest that when the average number of arrivals, that is, the average number of times that the investor has to liquidate the portfolio is lower than one per year, the S&P 500 second order stochastically dominates the Treasury Bill. These results are consistent with the results found in the previous since a low expected number of arrivals implies a long investment horizon.

Chapter IV applied the dynamic factor model proposed by Stock and Watson [47] to forecast inflation in Mexico. We have been particularly interested in investigating the role of using CPI disaggregated data to improve forecasting performance. For this purpose, we have generated the forecasts using datasets containing different blocks of variables. The results indicate that factors model outperform the benchmark AR model at the one, two, four and six quarters ahead horizons, with gains of above 30% in terms of the RMSFE. Those gains are especially strong considering that Capistran et al. [10] have shown that the benchmark autoregressive model performs as well as the surveys of experts. We have also found that the factors used for forecasting have an economic interpretation as they are highly related with macroeconomic variables such as output and inflation. Finally, we provide evidence that using information from the CPI components contributes to substantial improvements in the accuracy of the inflation forecasts.

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APPENDIX A

PROOF OF CONVERGENCE FOR STOCHASTIC DOMINANCE TESTING

Let $\omega(\Delta) \to 0$ as $\Delta \to 0$. Then we have

$$\sup_{x \epsilon \mathbb{R}} \left| \hat{L}^{H,2}(T,x) - L^{H,2}(T,x) \right| \to_{a.s.} 0.$$

Proof

$$\begin{aligned}
&\sup_{x \epsilon \mathbb{R}} \left| \hat{L}^{H,2}(T,x) - L^{H,2}(T,x) \right| \\
=&\sup_{x \epsilon \mathbb{R}} \left| \sum_{i=1}^{\frac{T}{\Delta}} \int_{(i-1)\Delta}^{i\Delta} f_H(t) \left[(x - X_{i\Delta}^k) 1\{X_{i\Delta} \le x\} - (x - X_t^k) 1\{X_t^k \le x\} \right] \right| dt \\
\le&\frac{T}{\Delta} \int_{(i-1)\Delta}^{i\Delta} \left| f_H(t) \left((x - X_{i\Delta}^k) 1\{X_{i\Delta} \le x\} - (x - X_t^k) 1\{X_t^k \le x\} \right) \right| dt \\
\le&T w(\Delta) \\
=&o(N^{-\frac{1}{2}}).
\end{aligned}$$

where the fourth line comes from the fact that

$$\left| f_H(t) \left((x - X_{i\Delta}^k) 1\{X_{i\Delta} \le x\} - (x - X_t^k) 1\{X_t^k \le x\} \right) \right| \le w(\Delta).$$

This completes the proof.

For the convergence of the test statistic, consider X^k and Y^k be strictly stationary with a mixing coefficient $\alpha(k) = O(k^{-9-\delta})$ for some $\delta > 0$. Then we have that,

$$\begin{aligned}
D_N^{H,2} =& \sup_{x \in \mathbb{R}} \sqrt{N} \left(\widehat{\Lambda}_N^{X,2}(T,\cdot) - \Lambda_N^{Y,2}(T,\cdot) \right) \\
\to_d& \sup_{x \epsilon R} \left(U^{X,2}(T,x) - U^{Y,2}(T,x) \right)
\end{aligned}$$

where $U^{Z,2}(T,\cdot) = (U^{X,2}(T,x), U^{Y,2}(T,x))$ is a mean zero vector Gaussian process with covariance kernel,

$$
\begin{aligned}
&\mathbb{E}U^{Z,2}(T,x)\mathbb{E}U^{Z,2}(T,y)' \\
=& \lim_{N\to\infty}\frac{1}{N}\mathbb{E}\left(\Sigma_{k=1}^{N}\left[L_k^{Z,2}(T,x)-\Lambda^{Z,2}(T,x)\right]\right)\left(\Sigma_{k=1}^{N}\left[L_k^{Z,2}(T,y)-\Lambda^{Z,2}(T,y)\right]\right)' \\
=& \lim_{N\to\infty}\frac{1}{N}\sum_{i=1}^{N}\sum_{j=1}^{N}\int_0^T\int_0^T \mathbb{E}J_t^{Z,2}(i,x)J_t^{Z,2}(j,y)dtds,
\end{aligned}
$$

for which

$$
\begin{aligned}
J_t^{Z,2}(j,y) = &\left((w-X_t^k)1\{X_t^k\le w\}-\int_{-\infty}^{w}P\{X_t^k\le x\}dx\right) \\
&\left((w-Y_t^k)1\{Y_t^k\le w\}-\int_{-\infty}^{w}P\{Y_t^k\le x\}dx\right).
\end{aligned}
$$

Proof. We need to show that,

$$
\sqrt{N}\left(\widehat{\Lambda}_N^{X,2}(T,\cdot)-\Lambda^{X,2}(T,\cdot)\right)\to_d U^{X,s}(T,\cdot).
$$

We can write,

$$
\sqrt{N}\left(\widehat{\Lambda}_N^{X,2}(T,\cdot)-\Lambda_N^{X,2}(T,\cdot)\right)+\sqrt{N}\left(\Lambda_N^{X,2}(T,\cdot)-\Lambda^{X,2}(T,\cdot)\right).
$$

For the first term, we can easily deduce that

$$
\sqrt{N}\sup_{x\in\mathbb{R}}\left|\widehat{\Lambda}_N^{X,2}(T,\cdot)-\Lambda^{X,2}(T,\cdot)\right|\le\sqrt{N}\frac{1}{N}\sum_{k=1}^{N}Tw(\Delta)=o(1).
$$

For the second term of the equation, which we can write as $U_N(T,\cdot)$, we can use the same argument as Park [44]. We need to show the weak convergence od the finite dimensional distributions $U_N(T,\cdot)$ to those of $U(T,\cdot)$, which follows directly from the central limit theorem for the strong mixing sequences. In addition, we need to show that $U_N(T,\cdot)$ is stochastically equicontinuous. We introduce a pseudometric space ρ defined by

$$
\rho^2(x,y)=\mathbb{E}|L_k(T,x)-L_k(T,y)|^2.
$$

We first define:

$$x_i = \inf_{x\in\mathbb{R}} \left\{ \Lambda^{X,2}(T,x) \geq \frac{i\varepsilon^2}{T} \right\},$$

for $i = 1, \ldots, [T/\varepsilon^2]$. We also denote: $I_i = [x_{i-1}, x_i]$. Then,

$$\begin{aligned}
&\mathbb{E} \sup_{x,y\in I_i} \left| L_k^{X,s}(T,x) - L_k^{X,s}(T,y) \right| \\
=&\mathbb{E} \sup_{x,y\in I_i} \left(\int_0^T f_H(t) \left[(x - X_{i\Delta}^k) 1\{X_{i\Delta} \leq x\} - (x - X_t^k) 1\{X_t^k \leq x\} \right] dt \right)^2 \\
\leq& T^2 (x_i - x_{i-1})^2 = \varepsilon^2.
\end{aligned}$$

Notice that,

$$N(\varepsilon, \mathcal{F}) = [T/\varepsilon^2] + 1$$

for any $\varepsilon > 0$ given. This completes the proof for the case of one variable. The multivariate case follows directly from the univariate case.

APPENDIX B

DATA DESCRIPTION

This appendix lists the variables used to construct the estimated factors. The total number of series is 243 CPI components and 54 macroeconomic variables. The sample period is 1988:I to 2008:IV. The macroeconomic series are obtained from the OECD main economic indicators, and the CPI subcomponents are obtained from Banco de Mexico. The format is as follows: series number, transformation code, and series description. The transformations codes are 1= no transformation, 2=first difference, 5=first difference in logarithms.

Macroeconomic Variables

Real Output

1 . 5 Production in total mining sa - units: 2005=100

2 . 5 Production in total manufacturing sa - units: 2005=100

3 . 5 Production of total energy sa - units: 2005=100

4 . 5 Production of total industry including construction sa - units: 2005=100

5 . 5 Production of total construction sa - units: 2005=100

6 . 5 Total retail trade (Volume) sa - units: 2005=100

7 . 5 Total wholesale trade (Volume) sa - units: 2005=100

8 . 5 Insured workers - units: persons '000

9 . 2 Harmonized unemployment rate: all persons sa - units: %

10 . 2 Unemployment rate: survey-based (all persons) sa - units: %

11 . 5 Monthly earnings: manufacturing sa - units: 2005=100

12 . 5 Real monthly earnings: manufacturing - units: 2005=100

13 . 5 Benchmarked real output - Total - units: MXN mln

14 . 5 Benchmarked real output - Manufacturing - units: MXN mln

15 . 5 Benchmarked real output - Industry - units: MXN mln

16 . 5 Benchmarked real output - Construction - units: MXN mln

17 . 5 Benchmarked real output - Trade, transport and communication - units: MXN mln

18 . 5 Benchmarked real output - Financial and business services - units: MXN mln

19 . 5 Benchmarked real output - Market services - units: MXN mln

20 . 5 Benchmarked real output - Business sector - units: MXN mln

Prices

21 . 5 Benchmarked total labour costs - Manufacturing - units: MXN mln

22 . 5 Benchmarked unit labour costs - Manufacturing - units: 2005=100

23 . 5 Domestic PPI Finished goods - units: 2005=100

24 . 5 CPI All items Mexico - units: 2005=100

25 . 5 CPI Energy - units: 2005=100

26 . 5 CPI All items non-food non-energy - units: 2005=100

27 . 5 CPI Food excl. restaurants - units: 2005=100

28 . 5 CPI Services less housing - units: 2005=100

29 . 5 CPI Housing - units: 2005=100

30 . 5 Cost of construction: social housing - units: 2005=100

Monetary Aggregates

31 . 5 Narrow money (M1a) sa - units: 2005=100

32 . 5 Monetary aggregate M1 sa - units: MXN bln

33 . 5 Broad money (M3) sa - units: 2005=100

34 . 5 Monetary aggregate M4 sa - units: MXN mln

Financial Variables

35 . 2 Rate 91-day treasury certificates - units: % p.a.

36 . 5 Share prices: MSE IPC share price index - units: 2005=100

37 . 5 USD/MXN exchange rate end period - units: USD/MXN

38 . 5 MXN/USD exchange rate monthly average - units: MXN/USD

39 . 5 Real effective exchange rates - CPI Based - units: 2005=100

40 . 5 Real effective exchange rates - ULC Based - units: 2005=100

Balance of Payments

41 . 5 SDR Reserve assets - units: SDR bln

42 . 5 ITS Exports f.o.b. total sa - units: USD bln

43 . 5 ITS Imports f.o.b. total sa - units: USD bln

44 . 2 ITS Net trade (f.o.b. - f.o.b) sa - units: USD bln

45 . 1 Current account as a % of GDP - units: %

46 . 2 BOP Current balance USD sa - units: USD bln

47 . 2 BOP Balance on income sa - units: USD bln

48 . 2 BOP Balance on services sa - units: USD bln

49 . 2 BOP Balance on current transfers sa - units: USD mln

50 . 2 BOP Balance on goods sa - units: USD bln

51 . 2 BOP Cap. and fin. balance incl. reserves - units: USD bln

52 . 2 BOP Financial balance incl. reserves - units: USD mln

53 . 2 BOP Other investment, assets - units: USD mln

54 . 2 BOP Net errors and omissions - units: USD mln

CPI Components

1 . 5 Corn tortilla

2 . 5 Flour

3 . 5 Corn

4 . 5 Sweet bread

5 . 5 White bread

6 . 5 Loaf of Bread

7 . 5 Cakes and pastries

8 . 5 Pasta soup

9 . 5 Popular cookies

10 . 5 Other cookies

11 . 5 Wheat flour

12 . 5 Cereal flakes

13 . 5 Rice

14 . 5 Chicken pieces

15 . 5 Whole chicken

16 . 5 Pork meat

17 . 5 Chops and lard

18 . 5 Loin

19 . 5 Pork Leg

20 . 5 Steak

21 . 5 Ground beef

22 . 5 Pork shoulder

23 . 5 Special cuts of beef

24 . 5 Beef liver

25 . 5 Other beef offal

26 . 5 Ham

27 . 5 Sausages

28 . 5 Chorizo

29 . 5 Other meats

30 . 5 Dried meat

31 . 5 Bacon

32 . 5 Other fish

33 . 5 Shrimp

34 . 5 Mojarra

35 . 5 Other seafood

36 . 5 Sea bass and grouper

37 . 5 Red snapper

38 . 5 Canned tuna and sardines

39 . 5 Other canned fish and seafood

40 . 5 Pasteurized and fresh milk

41 . 5 Milk powder

42 . 5 Evaporated and condensed milk

43 . 5 Cheese

44 . 5 Yogurt

45 . 5 Cream

46 . 5 Manchego or Chihuahua cheese

47 . 5 Other cheeses

48 . 5 Ice cream

49 . 5 American cheese

50 . 5 Butter

51 . 5 Egg

52 . 5 Edible oils and fats

53 . 5 Apple

54 . 5 Bananas

55 . 5 Orange

56 . 5 Avocado

57 . 5 Mango

58 . 5 Papaya

59 . 5 Lime

60 . 5 Grape

61 . 5 Melon

62 . 5 Watermelon

63 . 5 Pear

64 . 5 Peach

65 . 5 Grapefruit

66 . 5 Pineapple

67 . 5 Guava

68 . 5 Tomato

69 . 5 Potato

70 . 5 Onion

71 . 5 Green tomato

72 . 5 Zucchini

73 . 5 Serrano pepper

74 . 5 Carrot

75 . 5 Poblano chile

76 . 5 Lettuce and cabbage

77 . 5 Pea

78 . 5 Chayote

79 . 5 Cucumber

80 . 5 Bean

81 . 5 Dried chile

82 . 5 Other pulses

83 . 5 Packaged juice or nectar

84 . 5 Processed peppers

85 . 5 Packaged vegetables

86 . 5 Mashed tomatoes and canned soups

87 . 5 Other canned fruit

88 . 5 Fruits and vegetables for babies

89 . 5 Sugar

90 . 5 Coffee

91 . 5 Roasted coffee

92 . 5 Soda

93 . 5 Mayonnaise and mustard

94 . 5 Chicken and salt concentrates

95 . 5 Potato chips and similar

96 . 5 Concentrates for soft drinks

97 . 5 Chocolate

98 . 5 Candies, honey and caramel topping

99 . 5 Jelly powder

100 . 5 Pieces of barbequed pork

101 . 5 Roasted chicken

102 . 5 Barbecue or birria

103 . 5 Beer

104 . 5 Tequila

105 . 5 Other liquors

106 . 5 Rum

107 . 5 Brandy

108 . 5 Wine

109 . 5 Cigarettes

110 . 5 Shirts

111 . 5 Men's underwear

112 . 5 Socks

113 . 5 Cotton trousers for men

114 . 5 Suits

115 . 5 Men's pants

116 . 5 Men's clothes

117 . 5 Blouses for women

118 . 5 Women's underwear

119 . 5 Stockings and panties

120 . 5 Cotton trousers for women

121 . 5 Pants for women

122 . 5 Sets and other clothing for women

123 . 5 Women's dresses

124 . 5 Women's skirts

125 . 5 Children cotton trousers

126 . 5 Pants for children

127 . 5 Shirts and t-shirts for kids

128 . 5 Girl dresses

129 . 5 Children's underwear

130 . 5 Underwear for girls

131 . 5 Baby costumes

132 . 5 Baby Shirts

133 . 5 Jackets and coats

134 . 5 Hats

135 . 5 Sweater for children

136 . 5 Uniforms for boy

137 . 5 Uniforms for girls

138 . 5 Tennis shoes

139 . 5 Women's shoes

140 . 5 Men's shoes

141 . 5 Children's Shoes

142 . 5 Other footwear expenses

143 . 5 Bags, suitcases and belts

144 . 5 Watches, jewelry and fashion jewelry

145 . 5 Rental housing

146 . 5 Electricity

147 . 5 Domestic gas

148 . 5 Domestic service

149 . 5 Kitchen furniture

150 . 5 Dining furniture

151 . 5 Stoves

152 . 5 Water heaters

153 . 5 Sofa sets

154 . 5 Dining furniture

155 . 5 Mattresses

156 . 5 Bed sets

157 . 5 Refrigerators

158 . 5 Laundry Machine

159 . 5 Irons

160 . 5 Blenders

161 . 5 Stereo equipments

162 . 5 Radios and tape recorders

163 . 5 Bulbs

164 . 5 Matches

165 . 5 Candles

166 . 5 Brooms

167 . 5 Glassware

168 . 5 Cooking batteries

169 . 5 Plastic utensils for the home

170 . 5 Bedspreads

171 . 5 Sheets

172 . 5 Blankets

173 . 5 Towels

174 . 5 Curtains

175 . 5 Detergents

176 . 5 Soap for washing

177 . 5 Deodorants

178 . 5 Antibiotics

179 . 5 Analgesics

180 . 5 Nutrition

181 . 5 Contraceptives

182 . 5 Gastrointestinal

183 . 5 Expectorants and decongestants

184 . 5 Flu medicine

185 . 5 Medical service

186 . 5 Surgery

187 . 5 Dental Care

188 . 5 Haircut

189 . 5 Beauty Salon

190 . 5 Hair products

191 . 5 Lotions and perfumes

192 . 5 Toilet soap

193 . 5 Toothpaste

194 . 5 Personal deodorants

195 . 5 Skin cream

196 . 5 Razors and shavers

197 . 5 Toilet paper

198 . 5 Diapers

199 . 5 Sanitary towels

200 . 5 Paper napkins

201 . 5 Bus

202 . 5 Taxi

203 . 5 Subway or electric transportation

204 . 5 Interstate bus

205 . 5 Air transportation

206 . 5 Cars

207 . 5 Bicycles

208 . 5 Lubrication

209 . 5 Tires

210 . 5 Other parts

211 . 5 Accumulators

212 . 5 Auto insurance

213 . 5 Road tax

214 . 5 Car maintenance

215 . 5 Parking

216 . 5 University

217 . 5 Primary school

218 . 5 High school

219 . 5 Secondary school

220 . 5 Community college

221 . 5 Kindergarten

222 . 5 Textbooks

223 . 5 Other books

224 . 5 Notebooks and folders

225 . 5 Pens, pencils and others

226 . 5 Hotels

227 . 5 Movies

228 . 5 Nightclub

229 . 5 Sports club

230 . 5 Sports shows

231 . 5 Newspapers

232 . 5 Journals

233 . 5 Toys

234 . 5 Discs and cassettes

235 . 5 Film Equipment

236 . 5 Musical instruments and other

237 . 5 Sporting goods

238 . 5 Snack bars

239 . 5 Restaurants

240 . 5 Bars

241 . 5 Cafeterias

242 . 5 Funerals

243 . 5 License fee and other documents

APPENDIX C

FORECASTING RESULTS USING ALTERNATIVE WINDOWS SIZES AND MAE LOSS FUNCTION

Table IX. Forecasting Results for Alternative Windows: Headline Inflation

Excluded Block	h =1	h =2	h =4	h =6
		Windows size=34 quarters		
Output	0.720 (0.020)	0.762 (0.036)	0.738 (0.034)	0.849 (0.150)
Prices	0.749 (0.020)	0.805 (0.060)	0.734 (0.032)	0.948 (0.542)
Mon. aggregates	0.730 (0.028)	0.797 (0.043)	0.744 (0.036)	0.913 (0.470)
Financial Var.	0.723 (0.025)	0.798 (0.055)	0.726 (0.028)	0.883 (0.333)
Bal. of Payments	0.714 (0.025)	0.785 (0.043)	0.727 (0.025)	0.865 (0.239)
CPI Components	0.968 (0.658)	1.031 (0.803)	1.123 (0.162)	1.284 (0.058)
None	0.723 (0.025)	0.797 (0.054)	0.730 (0.027)	0.882 (0.326)
RMSFE AR	2.454	2.126	1.392	1.030
		Windows size=38 quarters		
Output	0.628 (0.008)	0.831 (0.024)	0.751 (0.028)	0.788 (0.042)
Prices	0.642 (0.010)	0.787 (0.021)	0.716 (0.056)	0.797 (0.031)
Mon. aggregates	0.630 (0.010)	0.774 (0.016)	0.704 (0.040)	0.800 (0.034)
Financial Var.	0.624 (0.011)	0.778 (0.013)	0.674 (0.024)	0.795 (0.032)
Bal. of Payments	0.629 (0.011)	0.778 (0.013)	0.694 (0.029)	0.782 (0.036)
CPI Components	1.303 (0.458)	0.985 (0.858)	1.077 (0.309)	0.979 (0.444)
None	0.623 (0.010)	0.769 (0.012)	0.672 (0.024)	0.793 (0.031)
RMSFE AR	2.522	2.021	1.515	2.622
		Windows size=40 quarters		
Output	0.561 (0.038)	0.690 (0.062)	0.731 (0.097)	0.765 (0.044)
Prices	0.630 (0.066)	0.590 (0.148)	0.572 (0.125)	0.765 (0.037)
Mon. aggregates	0.593 (0.049)	0.739 (0.094)	0.732 (0.095)	0.768 (0.039)
Financial Var.	0.611 (0.060)	0.725 (0.086)	0.716 (0.087)	0.765 (0.038)
Bal. of Payments	0.607 (0.057)	0.742 (0.095)	0.717 (0.085)	0.757 (0.040)
CPI Components	1.560 (0.312)	0.980 (0.776)	1.469 (0.208)	1.176 (0.180)
None	0.616 (0.063)	0.733 (0.089)	0.716 (0.086)	0.765 (0.039)
RMSFE AR	2.248	2.401	1.939	2.982

Note: The table reports the RMSFE from using the factors for each dataset relative to the benchmark AR Model at different forecast horizons h. The p-value for the Giacomini and White test of equal forecasting accuracy is presented in parenthesis. The RMSFE are calculated using out-of-sample forecasts from 2005:I-2008:IV.

Table X. Forecasting Results for Alternative Windows: Core Inflation

Excluded Block	h =1	h =2	h =4	h =6
		Windows size=34 quarters		
Output	0.666 (0.122)	0.894 (0.455)	0.781 (0.006)	0.922 (0.322)
Prices	0.744 (0.162)	0.889 (0.280)	0.739 (0.000)	0.805 (0.043)
Mon. aggregates	0.801 (0.271)	0.906 (0.352)	0.759 (0.000)	0.793 (0.025)
Financial Var.	0.800 (0.241)	0.883 (0.244)	0.743 (0.000)	0.788 (0.036)
Bal. of Payments	0.798 (0.182)	0.880 (0.282)	0.731 (0.000)	0.740 (0.006)
CPI Components	0.984 (0.826)	1.006 (0.912)	1.186 (0.207)	1.090 (0.452)
None	0.723 (0.139)	0.884 (0.249)	0.743 (0.000)	0.778 (0.023)
RMSFE AR	1.753	1.691	1.333	1.291
		Windows size=38 quarters		
Output	1.117 (0.296)	0.929 (0.571)	0.896 (0.524)	0.841 (0.002)
Prices	0.885 (0.468)	0.858 (0.282)	0.805 (0.229)	0.875 (0.008)
Mon. aggregates	0.921 (0.588)	0.819 (0.164)	0.785 (0.178)	0.880 (0.013)
Financial Var.	0.935 (0.644)	0.769 (0.033)	0.897 (0.616)	0.834 (0.000)
Bal. of Payments	0.887 (0.413)	0.906 (0.506)	0.802 (0.223)	0.860 (0.009)
CPI Components	1.050 (0.120)	0.870 (0.255)	1.206 (0.075)	0.933 (0.413)
None	0.908 (0.539)	0.934 (0.645)	0.779 (0.175)	0.871 (0.006)
RMSFE AR	1.510	1.620	1.233	2.625
		Windows size=40 quarters		
Output	1.051 (0.792)	0.846 (0.038)	0.942 (0.605)	0.895 (0.001)
Prices	1.024 (0.908)	0.964 (0.754)	0.937 (0.596)	0.898 (0.000)
Mon. aggregates	0.872 (0.481)	0.955 (0.666)	0.945 (0.640)	0.901 (0.000)
Financial Var.	0.928 (0.737)	0.913 (0.274)	0.941 (0.625)	0.899 (0.000)
Bal. of Payments	0.815 (0.457)	0.921 (0.398)	0.913 (0.455)	0.892 (0.001)
CPI Components	1.091 (0.224)	0.878 (0.439)	0.793 (0.189)	1.012 (0.591)
None	0.953 (0.799)	0.935 (0.532)	0.937 (0.595)	0.898 (0.000)
RMSFE AR	1.499	1.670	1.476	2.890

Note: The table reports the RMSFE from using the factors for each dataset relative to the benchmark AR Model at different forecast horizons h. The p-value for the Giacomini and White test of equal forecasting accuracy is presented in parenthesis. The RMSFE are calculated using out-of-sample forecasts from 2005:I-2008:IV.

Table XI. Forecasting Results for Alternative Windows: Non-core Inflation

Excluded Block	h =1	h =2	h =4	h =6
		Windows size=34 quarters		
Output	0.834 (0.107)	0.712 (0.051)	0.691 (0.000)	0.752 (0.000)
Prices	0.821 (0.106)	0.725 (0.051)	0.696 (0.001)	0.779 (0.007)
Mon. aggregates	0.819 (0.108)	0.723 (0.052)	0.702 (0.000)	0.768 (0.001)
Financial Var.	0.819 (0.103)	0.720 (0.050)	0.687 (0.000)	0.757 (0.000)
Bal. of Payments	0.820 (0.105)	0.719 (0.051)	0.658 (0.000)	0.730 (0.000)
CPI Components	0.930 (0.168)	0.783 (0.101)	1.077 (0.385)	1.074 (0.297)
None	0.820 (0.105)	0.720 (0.050)	0.711 (0.001)	0.772 (0.005)
RMSFE AR	6.846	4.559	2.693	1.948
		Windows size=38 quarters		
Output	0.859 (0.221)	0.704 (0.020)	0.698 (0.068)	0.610 (0.041)
Prices	0.866 (0.220)	0.712 (0.021)	0.644 (0.032)	0.601 (0.045)
Mon. aggregates	0.812 (0.094)	0.710 (0.022)	0.662 (0.033)	0.617 (0.044)
Financial Var.	0.807 (0.083)	0.708 (0.020)	0.683 (0.056)	0.614 (0.044)
Bal. of Payments	0.862 (0.226)	0.710 (0.020)	0.682 (0.059)	0.607 (0.044)
CPI Components	0.956 (0.628)	0.840 (0.253)	0.974 (0.923)	0.703 (0.123)
None	0.863 (0.220)	0.708 (0.020)	0.651 (0.032)	0.615 (0.044)
RMSFE AR	6.559	4.344	2.766	3.269
		Windows size=40 quarters		
Output	0.851 (0.065)	0.642 (0.007)	0.535 (0.103)	0.589 (0.049)
Prices	0.844 (0.160)	0.642 (0.007)	0.553 (0.094)	0.565 (0.041)
Mon. aggregates	0.823 (0.134)	0.642 (0.007)	0.556 (0.095)	0.604 (0.049)
Financial Var.	0.766 (0.031)	0.638 (0.006)	0.514 (0.094)	0.655 (0.051)
Bal. of Payments	0.828 (0.149)	0.640 (0.007)	0.552 (0.092)	0.603 (0.052)
CPI Components	0.942 (0.561)	0.787 (0.103)	0.795 (0.285)	0.885 (0.289)
None	0.837 (0.160)	0.639 (0.007)	0.551 (0.094)	0.602 (0.049)
RMSFE AR	6.827	5.116	3.741	3.451

Note: The table reports the RMSFE from using the factors for each dataset relative to the benchmark AR Model at different forecast horizons h. The p-value for the Giacomini and White test of equal forecasting accuracy is presented in parenthesis. The RMSFE are calculated using out-of-sample forecasts from 2005:I-2008:IV.

Table XII. Forecasting Results Using a MAE Loss Function

Excluded Block	h =1	h =2	h =4	h =6
		Headline Inflation		
Output	0.605 (0.012)	0.716 (0.000)	0.631 (0.057)	0.567 (0.001)
Prices	0.606 (0.020)	0.690 (0.000)	0.634 (0.072)	0.617 (0.000)
Mon. aggregates	0.613 (0.019)	0.686 (0.000)	0.638 (0.056)	0.601 (0.000)
Financial Var.	0.584 (0.016)	0.674 (0.000)	0.623 (0.049)	0.703 (0.000)
Bal. of Payments	0.595 (0.018)	0.677 (0.000)	0.676 (0.068)	0.599 (0.000)
CPI Components	0.822 (0.320)	0.806 (0.146)	0.946 (0.592)	0.816 (0.154)
None	0.587 (0.016)	0.680 (0.000)	0.630 (0.052)	0.600 (0.000)
MAE AR	2.095	1.936	1.257	1.262
		Core Inflation		
Output	0.773 (0.220)	0.936 (0.579)	0.771 (0.022)	0.803 (0.027)
Prices	0.923 (0.702)	0.891 (0.355)	0.794 (0.044)	0.808 (0.070)
Mon. aggregates	0.932 (0.741)	0.875 (0.282)	0.841 (0.100)	0.859 (0.118)
Financial Var.	0.813 (0.290)	0.856 (0.218)	0.844 (0.144)	0.909 (0.340)
Bal. of Payments	0.924 (0.623)	0.892 (0.361)	0.796 (0.047)	1.039 (0.795)
CPI Components	1.181 (0.024)	0.899 (0.352)	1.086 (0.271)	0.970 (0.649)
None	0.899 (0.597)	0.884 (0.329)	0.838 (0.104)	0.816 (0.090)
MAE AR	1.240	1.460	1.040	1.164
		Non-core Inflation		
Output	0.767 (0.042)	0.663 (0.025)	0.691 (0.007)	0.666 (0.000)
Prices	0.806 (0.096)	0.661 (0.023)	0.617 (0.000)	0.578 (0.000)
Mon. aggregates	0.806 (0.113)	0.659 (0.026)	0.592 (0.000)	0.592 (0.000)
Financial Var.	0.806 (0.112)	0.655 (0.023)	0.609 (0.000)	0.585 (0.000)
Bal. of Payments	0.811 (0.117)	0.656 (0.022)	0.622 (0.000)	0.540 (0.000)
CPI Components	0.820 (0.100)	0.667 (0.115)	0.981 (0.919)	0.840 (0.032)
None	0.806 (0.112)	0.657 (0.024)	0.607 (0.000)	0.589 (0.000)
MAE AR	5.708	3.495	2.254	1.767

Note: The table reports the MAE from using the factors for each dataset relative to the benchmark AR Model at different forecast horizons h. The p-value for the Giacomini and White test of equal forecasting accuracy is presented in parenthesis. The MAE are calculated using out-of-sample forecasts from 2005:I-2008:IV.

VITA

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