# EXPERIMENTAL CHARACTERIZATION AND MOLECULAR STUDY OF NATURAL GAS MIXTURES 

A Dissertation<br>by<br>DIEGO EDISON CRISTANCHO BLANCO

Submitted to the Office of Graduate Studies of Texas A\&M University<br>in partial fulfillment of the requirements for the degree of DOCTOR OF PHILOSOPHY

May 2010

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Approved by:

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May 2010

Major Subject: Chemical Engineering

ABSTRACT<br>Experimental Characterization and Molecular Study of Natural Gas Mixtures<br>(May 2010)<br>Diego Edison Cristancho Blanco, B.S.; B.S.; M.S., Universidad Industrial de Santander, Colombia Chair of Advisory Committee: Dr. Kenneth R. Hall

Natural Gas (NG) plays an important role in the energy demand in the United States and throughout the world. Its characteristics as a clean, versatile and a sustainable source of energy makes it an important alternative within the spectra of energy resources. Addressing industrial and academic needs in the natural gas research area requires an integrated plan of research among experimentation, modeling and simulation. In this work, high accuracy $P \rho T$ data have been measured with a high pressure single sinker magnetic suspension densimeter. An entire uncertainty analysis of this apparatus reveals that the uncertainty of the density data is less that $0.05 \%$ across the entire ranges of temperature ( 200 to 500 ) K and pressure (up to 200 MPa ). These characteristics make the $P \rho T$ data measured in this study unique in the world. Additionally, both a low pressure (up to 35 MPa ) and a high pressure (up to 200 MPa ) isochoric apparatus have been developed during the execution of this project. These apparatuses, in conjunction with a recently improved isochoric technique, allow determination of the phase envelope for NG mixtures with an uncertainty of $0.45 \%$ in temperature, $0.05 \%$ in pressure and $0.12 \%$ in density. Additionally, an innovative technique, based upon Coherent Anti-Stokes Raman Scattering (CARS) and Gas Chromatography (GC), was proposed in this research to minimize the high uncertainty introduced by the composition analyses of NG mixtures. The collected set of $P \rho T$ and saturation data are fundamental for thermodynamic formulations of these mixtures. A study at the molecular level has provided molecular data for a selected set of main
constituents of natural gas. A 50-50\% methane-ethane mixture was studied by molecular dynamics simulations. The result of this study showed that simulation time higher than 2 ns was necessary to obtain reasonable deviations for the density determinations when compared to accurate standards. Finally, this work proposed a new mixing rule to incorporate isomeric effects into cubic equations of state.

## DEDICATION

To<br>Jennifer, Juan Diego<br>and<br>Our Families

## ACKNOWLEDGMENTS

I hereby wish to thank Dr. Kenneth R. Hall for giving me the opportunity to work on this project and for his advice, both professional and personal. I appreciate his patience and understanding during the difficult times with the project. I am grateful to Dr. Hall for financially supporting me for last 4 years and making it possible for me to obtain my degrees at Texas A\&M University. I would like to thank my advisory committee members, Dr. Maria A. Barrufet, Dr. Perla Balbuena, and Dr. Tahir Cagin, and Dr James Holste, for their time and advices.

I would like to gratefully acknowledge the contributions of Dr. Mark O. McLinden and Dr. Eric W. Lemmon of the National Institute of Standards and Technology (NIST) for their technical support in calibrating our apparatus parts and for providing us access to the Reference Properties (REFPROP) programs. I also thank them for their friendship and advice during my internship in the NIST at Boulder, Colorado.

I would like to extend my appreciation for the help I received from the various people in the department, directly or indirectly related to my research. I would like to express my sincere gratitude to Mr. Randy A. Marek of the machine shop who helped us with our mechanical works, and Mr. Jason W. Caswell of the physics electronic shop for spending countless hours with us advising on electrical and electronic problems and advancements of our experimental setup.

I am indebted to all my colleagues in the research group for the valuable time spent with them. I appreciate the help I received from Dr. Mert Atilhan and Dr. Saquib Ejaz during the early stages of my lab experience. I find myself extremely lucky to have worked with Mr. Ali Abedi, Mr. Ivan Mantilla, Mr. Diego Ortiz, Mr. Alejandro Coy, Mrs. Andrea Tibaduiza, Mr. Pedro Acosta, and Mr. Ullas Pathak, in the same research group. I would like to thank them here for their help, time and friendship.

I am very indebted and grateful to my parents Jose Rafael Cristancho and Audrey Blanco for their support and understanding. I thank them hereby for teaching me how to pursue real happiness. Also, I would like to extend my appreciation to my mother-in-
law, Edda Diaz, for her support and understanding in all respects. I send thanks to my brother Byron Cristancho, and sister Deissy Cristancho, for being role models for me throughout all stages of my life. I also thank my brother-in-law Mauricio Diaz, for his support.

Above all, I find myself fortunate to have such a wonderful wife Jennifer Carvajal, and son, Juan Diego Cristancho. They have been the driving force that makes me look for a better world every day. They create an environment of love, understanding, friendship, and motivation around me, leading my life always to a better state.

## NOMENCLATURE

| $a$ | Calibration constant in the deviation equations of ITS-90 for a PRT or coil radius (in) |
| :---: | :---: |
| $b$ | Calibration constant in the deviation equations of ITS-90 for a PRT |
| $B$ | Second virial coefficient ( $\mathrm{cm}^{3} / \mathrm{mole}$ ) or Constant in the reference function of ITS-90 for a PRT calibration |
| C | Third virial coefficient ( $\left.\mathrm{cm}^{3} / \mathrm{mole}\right)^{2}$ |
| $C_{p}$ | Isobaric heat capacity |
| $C_{v}$ | Isochoric heat capacity |
| I | Current through platinum resistance thermometer (mA) |
| $m$ | Mass of sinker (g) |
| M | Molar mass (kg/kmole) or magnetic moment |
| $n$ | Number of moles |
| $N$ | Number of components in a natural gas mixture or constant in a polynomial equation |
| P, p | Pressure (MPa) [psia] |
| $R$ | Resistance of platinum resistance thermometer (ohm) or universal gas constant ( $8.314 \mathrm{~J} / \mathrm{mole} \mathrm{K}$ ) |
| $T$ | Temperature (K) |
| $u$ | Uncertainty or speed of sound |
| $V$ | Volume of sinker ( $\mathrm{cm}^{3}$ ) or voltage drop, volt |
| W | Ratio of the resistance of a platinum resistance thermometer at a temperature to its resistance at the triple point of water or balance reading or weight |
| $x$ | Composition as mole fraction |

## Abbreviations

| AC | Alternating Current |
| :--- | :--- |
| AGA | American Gas Association |
| AGA8-DC92 | Detailed Characterization Method of the American Gas Association |
| BP | British Petroleum |
| CARS | Coherent Anti-Stokes Raman Scattering |
| CB | Cricondenbar |
| CP | Critical Point |
| CT | Cricondentherm |
| DC | Direct Current |
| DMM | Digital Multimeter |
| DPI | Differential Pressure Indicator |
| EoS | Equation of State |
| FT | Fischer-Tropsch |
| FTE | Force Transmission Error |
| FWM | Four Wave Mixing Signal |
| GERG | European Group for Gas Research |
| GERG-2004 | GERG-2004 EOS for gas mixtures |
| GTL | Gas to Liquids |
| GTE | Gas to Ethylene |
| HIP | High Pressure Equipment Company |
| HP | Hand Pump |
| IC | Isochore |
| IEA | International Energy Agency |
| IT | Isotherm |
| ITS-90 | International Temperature Scale of 1990 |
| LNG | Liquefied Natural Gas |
| LVDT | Linear Variable Differential Transformer |
| F |  |


| MP | Measurement Point |
| :--- | :--- |
| MSD | Magnetic Suspension Densimeter |
| NIST | National Institute of Standards and Technology |
| NR | Non-Resonant |
| PID | Proportional Integral Derivative |
| PPM | Parts Per Million |
| PRT | Platinum Resistance Thermometer |
| PT6K | 6,000 psia Range Pressure Transducer |
| PT30K | 30,000 psia Range Pressure Transducer |
| SSR | Solid State Relay |
| T | Tee Fitting |
| Ta | Tantalum |
| Ti | Titanium |
| V | Valve |
| ZP | Zero Point |

## Greek letters

$\beta \quad$ Temperature distortion coefficient $\left(\mathrm{K}^{-1}\right)$ or thermal coefficient of expansion $\left(\mathrm{K}^{-1}\right)$ or balance calibration factor
Pressure distortion coefficient $\left(\mathrm{MPa}^{-1}\right)$
Difference or deviation
Internal temperature period of pressure transducer quartz crystal ( $\mu \mathrm{s}$ )

Saturation density $\left(\mathrm{kg} / \mathrm{m}^{3}\right)$
Standard deviation or deviation at the $68 \%$ confidence level or normal stress

Coupling factor
Apparatus contribution to the force transmission error
$\varepsilon$
$\chi$
$\omega$
$\gamma$

## Subscripts

$0 \quad$ Index for constants $B$ and $D$ in reference functions of ITS-90 for PRT calibration

6+ Hexane and components heavier than hexane such as heptane, octane, etc

Electromagnet
Reference condition of $23{ }^{\circ} \mathrm{C}$ for cross sectional area of piston cylinder assembly of dead weight gauge or reference condition of $20^{\circ} \mathrm{C}$ and 1 bar pressure for sinker volume
p-mag Permanent magnet
ref
Reference state
Initial set-point
Sinker
Vacuum condition

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## INTRODUCTION

### 1.1 Economic Impact of Natural Gas In the United States and In the World

Currently, all segments of society have a strong dependence upon energy. Figure 1 reveals a remarkable difference between consumption and production of energy over the last fifty years in the US ${ }^{1}$. Energy import in 2008 was almost 30 quadrillion Btu and shows an increasing trend for the coming years. Therefore, the US economy has a huge dependence upon fuel exporting countries for price and supply stability ${ }^{2}$.


Figure 1. Monthly consumption, production, imports and exports in the US. ${ }^{3}$

Petroleum, coal and natural gas have been the major sources of energy in the US in the past (Figure 2). The high consumption and import of petroleum and the potential domestic reserves of coal have led the US Department of Energy (DOE) and the industrial sector to investigate new coal-based technologies to get clean energy more efficiently at lower capital cost ${ }^{4}$. Integrated Gasification Combined Cycle (IGCC) ${ }^{5}$, advanced gas turbine ${ }^{5}$ and coal-based zero emissions power plant ${ }^{6}$ are some of the technological advances recently reported. It is important to note that these technologies are adaptable to natural gas feedstock. However, until clean coal technologies are well developed, production of natural gas must be sustained and increased to avoid dependence upon imported petroleum ${ }^{2}$.

[^0]

Figure 2. Consumption of energy from different sources US ${ }^{3}$.

In addition to coal-based technologies, Gas to Liquids (GTL) technology has been developed. GTL technology involves converting natural gas into higher aliphatic hydrocarbons in the range of $\mathrm{C}_{4}-\mathrm{C}_{10}{ }^{2}$, and it is more economical using natural gas feedstock than coal ${ }^{7}$. Recently, GTL plants have been commissioned in America, the Middle East, and Asia ${ }^{8-11}$. This information leads us to recognize that more understanding of liquid-gas mixtures is necessary to support technological developments that focus upon energy generation.

Natural gas exhibits reliable features as a clean, versatile and sustainable fuel. Figure 3 shows the flexibility of Natural Gas as an energy source when used in different demand sectors in the United States. As of December 31, 2007, the estimated total U.S proved reserves of dry natural gas were $237,726 \mathrm{Bcf}^{3}$. Additionally; large volumes of natural gas classified as undiscovered recoverable resources exist ${ }^{3}$. Approximately $79 \%$ of deepwater reserves of oil and gas remain to be explored, as well as $20 \%$ of shallowwater reserves ${ }^{12}$.

The conditions of deepwater reserves are in the pressure range of (89.6-124.1 $\mathrm{MPa})$ and a temperature range of $(365.6-408.15 \mathrm{~K})^{2}$. At the production wellhead, the temperature range can be (283.49-293.8 K) and the pressure can be (10.3-20.7 MPa). These abrupt changes of pressure and temperature create operational problems such as
natural gas hydrate formation ${ }^{13}$ and retrograde condensation ${ }^{14}$. A good base of data and an accurate EoS, validated in this range of operation, would help predict and avoid these inconveniences ${ }^{2}$.


Figure 3. U.S. primary natural gas consumption by sector ${ }^{3}$.

Although many years of research have emphasized determination of empirical and theoretical models for the prediction of equilibrium properties of natural gas, high uncertainty still exists in their determination ${ }^{15}$. This information is vital for design and operation of natural gas facilities and processes based upon natural gas as a feedstock. Figure 4 exemplifies the deviations of some EoS extensively used in the industry to predict phase envelopes for natural gas.

When natural gas undergoes custody transfer, the selling and buying process of the natural gas is quantified by measurements of mass flow rate using specific mass flow-meters such as orifice meters ${ }^{2}$. Equation 1 represents the methodology used for determining the Natural Gas price.


Figure 4. PT, PTV, SW, GD, MMM EoS predictions for synthetic natural gas mixture.

$$
\begin{equation*}
\frac{\$}{d a y}=\left(\frac{m^{3}}{d a y}\right)\left(\frac{k g}{m^{3}}\right)\left(\frac{J}{k g}\right)\left(\frac{\$}{J}\right) \tag{1.1.1}
\end{equation*}
$$

The first factor on the right-hand side of the equation corresponds to the volumetric flow rate. Flow meters, such as orifice meters, determine this quantity. The second factor is the density. Table 1 demonstrates the effect of the accuracy of the density in the final profit or loss for either the selling or the buying companies during custody transfer. The third factor corresponds to the heating value of the gas, which results from either calorimetric measurements or composition analysis. The final term is the gas price determined by competition. Therefore, uncertainties in the mass flow measurements and equivalent energy values become relevant in the financial transactions between selling and buying companies. The use of an accurate and validated EoS in this process helps facilitate custody transfer and reduce errors in large volume trades.

Table 1. Economic Impact of NG Custody Transfer ${ }^{3}$

| Accuracy | Density |  |  | Economic impact of inaccurate density measurements |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | Lower Boundary | True Value | Upper Boundary |  |  |
| \% Deviation | $\left(\mathrm{kg} / \mathrm{m}^{3}\right)$ |  |  | (\$/year) |  |
| 0.03 | 0.9343 | 0.9346 | 0.9349 | 498,225 | 498,225 |
| 0.50 | 0.9299 |  | 0.9392 | 8,305,210 | 8,305,210 |
| 3.00 | 0.9065 |  | 0.9626 | 49,830,165 | 49,830,165 |
| Producer $\rightarrow$ |  |  |  | Loss | Profit |
|  |  |  | Buyer $\rightarrow$ | Profit | Loss |

### 1.2 Objectives of the Current Research

After the analysis of the economic importance of natural gas for society, evaluation of the actual situation of the natural gas industry, and lack of knowledge in academia about these systems, different challenges have driven in this research project. Two principal objectives of this project exist for contributing to the solution of the industrial and academic problems and improving understanding of natural gas mixtures:

1. To measure high-accuracy $P \rho T$ and equilibrium data over a wide range of pressures and temperatures for natural gas mixtures and its main constituents.
2. To contribute understanding of the molecular behavior for natural gas mixtures by using state of the art theoretical techniques.

These two principal objectives have secondary objectives:

1. To compensate the Force Transmission Error experienced during the operation of the High Pressure Single-Sinker Magnetic Suspension Densimeter (MSD) in order to minimize the uncertainty of density data collected with this apparatus.
2. To measure high-accuracy $P \rho T$ data for the main constituents of natural gas and natural gas mixtures up to very high pressure ( 200 MPa ).
3. To develop a methodology that allows determination of phase equilibrium properties and isomolar derivatives for natural gas mixtures based upon isochoric data.
4. To design and build a new, high-pressure isochoric apparatus that allows determination of bubble points and expands the temperature and pressure range of the isochoric and $P \rho T$ high accuracy data for natural gas mixtures collected by the Thermodynamics Research group at Texas A\&M University.
5. To propose an alternative methodology for determining natural gas composition that minimizes the uncertainty associated with long chain hydrocarbons present in the mixture.
6. To perform molecular mechanics and molecular dynamics calculations in order to develop a systematic and consistent data base that contributes to a better understanding of natural gas mixtures.
7. To incorporate the conformational effects into an analytical model that improves the prediction of mixtures with long chain hydrocarbons.

The enormous and complex task that involves the thermodynamics of natural gas mixtures makes conclusive results difficult in this research area. However, the new highaccuracy experimental data and scientific tools provide a path toward better understanding of these systems. The following sections present a detailed description of the experimental and theoretical activities and results obtained during the execution of this work.

## EQUATIONS OF STATE FOR NATURAL GAS APPLICATIONS

### 2.1 State of the Art for EoS

A vast literature background exists in the EOS area ${ }^{16}$. Research in EoS includes experimental and theoretical activities, and both fields interact dynamically to improve their results. The experimental research includes collecting accurate thermodynamic data such as pressure, temperature, density, and thermal properties ${ }^{17}$ and compositions for gas mixtures ${ }^{2}$.

Numerous EoS have appeared in the past ${ }^{16}$, and they have been able to attain good accuracy in the prediction of thermodynamic behavior of pure substances. Cubic, molecular-based and multiparametric EoS have performed well in the prediction of complex systems as well ${ }^{16}$. However, the problems of phase equilibrium and prediction of behavior at high pressures and temperatures for natural gas mixtures remain unresolved ${ }^{18}$. Heavy hydrocarbon effects in natural gas mixtures produce remarkable differences between the EoS predictions and experimental data ${ }^{2,18}$.

The cubic EoS present a balance between accuracy, reliability, simplicity and speed of computation ${ }^{16}$. However, they normally lack the necessary structure in some regions of phase space and produce inappropriate behavior for first and second derivatives. Still, industry uses the cubic EoS to predict phase equilibrium in multicomponent mixtures. The $\mathrm{RK}^{19}, \mathrm{SRK}^{20}, \mathrm{PR}^{21}$ and $\mathrm{PT}^{22}$ are examples of cubic EoS ${ }^{23}$. The quintic EoS ${ }^{24}$ is a new attempt to develop an EOS with features similar to those of cubic EoS but with improved predictive capacity and flexibility. The quintic EoS is still in development phase and has not been extended to mixtures. The normal methods used to extend cubic EoS to mixtures have been classical, quadratic mixing rules, composition-dependent combining rules, density-dependent mixing rules and combining EoS with excess-Gibbs energy models ${ }^{16}$.

In contrast to the cubic EoS, molecular-based EoS describe the thermophysical behavior of multicomponet systems. The major contribution of these EoS is to construct a real molecular understanding of thermodynamic behavior of fluids ${ }^{25}$. Beginning with Van der Waals theory and finishing with sophisticated statistics, molecular methods have produced a wide range of EoS. Perturbation theories, integral equation theories,
mechanical and dynamical molecular simulations and Monte Carlo methods have been used to build molecular-based EoS ${ }^{16}$.

Inspired by the Van der Waals premise, which separates repulsive and attractive contributions to pressure, numerous authors ${ }^{16}$ have developed theoretical approaches allowing generation of EoS that account for molecular interactions. Among these EoS, called generalized Van der Waals EoS, are: BACK (Boublic-Alder-Chen-Kresglewski,), PHCT (perturbed-hard-chain-theory), COR (chain of rotators), SAFT (statistical associated fluid theory) ${ }^{25}$ and others ${ }^{16}$. Although the equations based upon the generalized Van der Waals theory can represent saturated data and PVT data outside the saturated region with few parameters, the complex functional forms of their universal parameters make them less desirable computationally than cubic EoS.

Multiparameter EoS are also an important family. The multiparameter EoS are based upon accurate experimental thermodynamic data ${ }^{16}$. The mathematical fitting of these experimental thermodynamic data using a mathematically arbitrary EoS produces remarkable agreement with experimental data. However, these EoS have a large number of parameters that affects computational time remarkably. The BWR (Benedict-WebbRubin), the Martin-Hou, and the Schimidt and Wagner are examples of multiparameter EoS ${ }^{16}$. Recently Kunz et al. ${ }^{15}$ have developed a new reference equation of state for natural gas applications. This equation, GERG-2004, claims an accuracy of $0.1 \%$ in density determination of natural gas mixtures. Efforts at the National Institute of Standards and Technology (NIST) have developed a fluid property computer program named RefProp $8.0^{26}$ that includes a set of reference and technical EoS for a variety of fluids.

The virial EoS ${ }^{27}$ is a valuable EoS because of its basis in statistical-mechanical theory. MGERG ${ }^{28}$ and SGERG ${ }^{29}$ are based upon the virial EoS. These EoS have been developed for pipeline custody transfer of natural gas. Iglesias-Silva and Hall (1996) extended the applicability range of SGERG. AGA8-DC92 ${ }^{30}$, which is the current industry standard EoS for natural gas custody transfer, is an MBWR EoS because it involves virial terms as well as SGERG ${ }^{2}$.

Theoretically, different alternatives have been developed to attack the EoS problem. The Van der Waals EoS has spawned multiple EoS ${ }^{16}$. The normal route used to solve the multicomponent themodynamic behavior has been to find an EoS with good prediction capability for pure components and to extend its scope using composition dependent mixing rules ${ }^{31}$.

Recently, new developments in computational and statistical mechanics techniques have enabled development of a robust methodology to solve the complex molecular behavior of multicomponent systems ${ }^{32-33}$. This new window of knowledge allows one to continue theoretical research focusing upon the molecular based EoS and improving mixing rules in order to predict the thermodynamic behavior and to understand the driving forces that generate it.

In molecular dynamics and molecular mechanics simulations and the Monte Carlo technique, research has related to binary mixtures for hydrocarbons and associated compounds for natural gas components ${ }^{34-36}$. NERD, COMPASS, OPLS, AUA, UA have been the most used force fields. The predictability and reliability agree with experimental data. Nevauer et al. ${ }^{34}$ employ Monte Carlo simulations using the NPT ensemble to determine the molar volume and compressibility factor of naturally occurring hydrocarbon mixtures. They had problems with calculations because of their pseudo-components representation of the real composition in the mixtures, the use of basic mixing rules for the Lennard-Jones potential parameters and the potential itself.

On the other hand, Boublík ${ }^{37-38}$ has addressed the molecular-based BACK EOS improving the molecular representation of both the non-sphericity parameter ( $\alpha$ ) and the closed-packed volume $\left(V^{00}\right)$. Starting from the old Scaled Particle Theory (SPT) developed by Reiss, Frisch, and Lebowitz ${ }^{35}$ and its recent improvement ${ }^{36}$, and modifying the respective values of $\alpha$ and $V^{00}$ using enlarged, fused hard sphere models, they have obtained new numerical values for these parameters and some relative improvements in BACK EOS performance.

### 2.2 Multiparameter EoS

Recently, EoS based upon the Helmholtz energy functional have led to the most accurate reproductions of experimental data ${ }^{39}$. Despite the fact that the complex multiparameter functional used during their construction makes them computational expensive for process design and simulation, their unquestionable capability for reproducing high accuracy data makes them an indispensable tool for comparing standard quality data. This research project deals with high accuracy experimental data, therefore the understanding of these models is fundamental throughout this dissertation.

These EoS use the Helmholtz energy, a fundamental property function of density and temperature,

$$
\begin{equation*}
a(\rho, T)=a^{0}(\rho, T)+a^{r}(\rho, T) \tag{2.2.1}
\end{equation*}
$$

where $a(\rho, T)$ is the Helmholtz energy, $a^{0}(\rho, T)$ is the ideal gas contribution to the Helmholtz energy, and $a^{r}(\rho, T)$ is the residual Helmholtz energy. Usually this formulation is represented better as dimensionless Helmholtz energy functions and dimensionless density and temperature:

$$
\begin{equation*}
\frac{a(\rho, T)}{R T}=\alpha(\delta, \tau)=\alpha^{0}(\delta, \tau)+\alpha^{r}(\delta, \tau) \tag{2.2.2}
\end{equation*}
$$

where $\delta=\rho / \rho_{c}$ and $\tau=T_{c} / T$. A major advantages of this formulation is that any thermodynamic property can be determined from derivatives of the Helmholtz energy

$$
\begin{equation*}
Z(\delta, \tau, \bar{x})=1+\delta \alpha_{\delta}^{r} \tag{2.2.3}
\end{equation*}
$$

where $\alpha_{\delta}^{r}=\left(\partial \alpha^{r} / \partial \delta\right)_{\tau, \bar{x}}$ is the first derivative of the residual Helmholtz energy with respect to the residual density. The ideal gas Helmholtz energy in dimensionless form is

$$
\begin{equation*}
\alpha^{0}=\ln \delta-\ln \tau+\sum a_{k} \tau^{i_{k}}+\sum a_{k} \ln \left[1-\exp \left(-b_{k} \tau\right)\right] \tag{2.2.4}
\end{equation*}
$$

where $\delta_{0}=\rho_{0} / \rho_{c}$ and $\tau_{0}=T_{c} / T_{0}$.
The usual functional form for Helmholtz energy EoS is

$$
\begin{equation*}
\alpha^{r}(\delta, \tau)=\sum N_{k} \delta^{d_{k}} \tau^{t_{k}}+\sum N_{k} \delta^{d_{k}} \tau^{t_{k}} \exp \left(-\delta^{t_{k}}\right) \tag{2.2.5}
\end{equation*}
$$

but some equations of state ${ }^{39}$ use additional, Gaussian bell-shaped terms, first proposed by Setzmann and Wagner ${ }^{40}$, for the methane EoS

$$
\begin{align*}
\alpha^{r}(\delta, \tau) & =\sum N_{k} \delta^{d_{k}} \tau^{t_{k}}+\sum N_{k} \delta^{d_{k}} \tau^{t_{k}} \exp \left(-\delta^{l_{k}}\right) \\
& +\sum N_{k} \delta^{d_{k}} \tau^{t_{k}} \exp \left(-\eta\left(\delta-\varepsilon_{k}\right)^{2}-\beta\left(\tau-\gamma_{k}\right)^{2}\right) \tag{2.2.6}
\end{align*}
$$

Each summation typically has 4 to 20 terms.
Lemmon et al. have recently developed a robust methodology to construct multiparameter EoS incorporating the previous contributions of different authors ${ }^{30,41-42}$. This methodology does not focus specifically upon the quantitative statistical analysis of the fitting procedure, but rather upon the quality and the physical sense of the model by analyzing different thermodynamic scenarios. The combination of accurate reproduction of experimental data and fulfillment of all the available thermodynamic constrains leads to the best, high-quality EoS developed so far.

As an example, two comparisons have appeared between the fundamental EoS for propane ${ }^{39}$ and a new EoS for propylene glycol that is still under development by Cristancho et al. ${ }^{43}$. Figures 4 and 5 come from the RefProp 8.0 program. Figure 5 reveals that both equations represent the temperature-density diagram correctly. However a closer inspection of these plots reveals that the rectilinear diameter for the propylene glycol equation of state has a smooth curvature, in contrast to the perfectly rectilinear diameter displayed by the propane equation of state, which is the correct behavior.


Figure 5. Temperature-density diagram for the propylene glycol EoS and propane EoS.


Figure 6. Cvr-Temperature diagram for propylene glycol EoS and propane EoS.

Figure 6 indicates that the propylene glycol EoS does not predict correct behavior for the residual heat capacity at constant volume $C v r=(C v-C p) / R T$. The propane EoS represents correct behavior. In conclusion, many different scenarios should be analyzed to develop an outstanding multiparameter equation of state.

### 2.2.1 GERG-2004

GERG-2004 is a multiparameter EoS adopted by GERG (Groupe Europeen de Recherches Gazieres) in 2004. GERG-2004 is based upon a multi-fluid approximation explicit in the reduced Helmholtz energy

$$
\begin{equation*}
\alpha(\delta, \tau, \bar{x})=\alpha^{0}(\rho, T, \bar{x})+\alpha^{r}(\delta, \tau, \bar{x}) \tag{2.2.1.1}
\end{equation*}
$$

where $\alpha^{0}(\rho, T, \bar{x})$ accounts for the ideal Helmholtz energy part of the mixture

$$
\begin{equation*}
\alpha^{0}(\rho, T, \bar{x})=\sum_{i=1}^{N} x_{i}\left[\alpha_{0 i}^{0}(\rho, T)+\ln x_{i}\right] \tag{2.2.1.2}
\end{equation*}
$$

and $\alpha^{r}(\delta, \tau, \bar{x})$ is the residual Helmholtz energy of the mixture

$$
\begin{equation*}
\alpha^{r}(\delta, \tau, \bar{x})=\sum_{i=1}^{N} x_{i} \alpha_{0 i}^{r}(\delta, \tau)+\sum_{i=1}^{N-1} \sum_{j=i+1}^{N} x_{i} x_{j} F_{i j} \alpha_{i j}^{r}(\delta, \tau) \tag{2.2.1.3}
\end{equation*}
$$

$\delta=\rho / \rho_{r}(\bar{x})$ is the reduced density of the mixture and $\tau=T_{r}(\bar{x}) / T$ is the residual temperature of the mixture. A more detailed description of the mathematical structure of this equation appears in the GERG TM 152007 by Kunz et al. ${ }^{15}$. The optimized methodology utilized in the GERG-2004 EoS structure allows determining all thermodynamic functions just with direct derivatives of the reduced Helmholtz energy as follows for pressure

$$
\begin{equation*}
\frac{p(\delta, \tau, \bar{x})}{\rho R T}=1+\delta \alpha_{\delta}^{r} \tag{2.2.1.4}
\end{equation*}
$$

where $\alpha_{\delta}^{r}=\left(\partial \alpha^{r} / \partial \delta\right)_{\tau, \bar{x}}$ is the first derivative of the residual Helmholtz energy with respect to the residual density. This peculiarity avoids integral calculations that can be difficult for the numerical performance of this EoS in process simulators. The most remarkable characteristic of this EoS is its high-accuracy predictions for thermodynamic properties in the homogeneous gas, liquid and supercritical regions as well as vaporliquid equilibrium states. The data base used to develop GERG-2004 contains more than 100,000 experimental data for volumetric and thermal properties of binary mixtures, natural gas mixtures and other types of mixtures. This robust experimental support makes GERG-2004 a reference EoS for application in industrial process design to make more accurate calculations and to minimize the over- and under-design of industrial units.

Table 2. Estimated Relative Experimental Uncertainties of the Most Accurate Binary and Multicomponent Mixture Data ${ }^{15}$

| Data Type | Property | Relative Uncertainty |
| :---: | :---: | :---: |
| $P \rho T$ data | $\Delta \rho / \rho$ | $\leq(0.05-0.1) \%$ |
| Isochoric heat capacity | $\Delta c_{v} / c_{v}$ | $\leq(1-2) \%$ |
| Speed of sound (gas phase) | $\Delta w / w$ | $\leq(0.05-0.1) \%$ |
| Isobaric heat capacity | $\Delta c_{p} / c_{p}$ | $\leq(1-2) \%$ |
| Enthalpy differences | $\Delta(\Delta h) / \Delta h$ | $\leq(0.2-0.5) \%$ |
| Saturated liquid density | $\Delta \rho^{\prime} / \rho^{\prime}$ | $\leq(0.1-0.2) \%$ |
| $V L E$ data | $\Delta p_{s} / p_{s}$ | $\leq(1-3) \%$ |

The estimated relative uncertainties for the most accurate binary and multicomponent mixtures appear in the Table $2{ }^{15}$. These values support the outstanding reliability of this equation to predict thermodynamic properties for natural gas mixtures.

However, this EoS requires more computational time when used in process design and optimization. Within this work, GERG-2004 provides is the reference values for all comparisons to experimental data.

### 2.3 The Quintic Equation of State

Since the work of Hall and Atilhan ${ }^{24}$ in which they propose a quintic EoS, additional improvements have been developed for this idea. The hypothesis behind this approach is to find a new quintic functional form that achieves accurate prediction like multiparameter equations but with a small number of parameters. This can lead to more efficient computing algorithms useful for process design and simulation. The generalized quintic form is:

$$
\begin{equation*}
Z=\frac{1+n_{1} \rho+n_{2} \rho^{2}+n_{3} \rho^{3}+n_{4} \rho^{4}}{1+d_{1} \rho+d_{2} \rho^{2}+d_{3} \rho^{3}} \tag{2.3.1}
\end{equation*}
$$

where $Z$ is the compressibility factor, $\rho$ is the density and the $n_{i}$ and $d_{i}$ are temperature dependent parameters. One of the principal challenges with this equation has been the unique determination of the set $\left\{n_{i}, d_{i}\right\}$. The non-linearity of the quintic equation leads to multiples solutions, most of them inconsistent with physical constraints. Therefore, Cristancho et al. ${ }^{44}$ start a systematic approach seeking the most reliable and consistent functional in order to include thermodynamic constraints in the quintic form using the minimum number of parameters.

To simplify inclusion of additional thermodynamic constraints such as the phase equilibrium constraints, the original form of the quintic equation is transformed into a Helmholtz energy function. Starting with factorizing the denominator

$$
\begin{equation*}
Z=\frac{1+n_{1} \rho+n_{2} \rho^{2}+n_{3} \rho^{3}+n_{4} \rho^{4}}{(1+b \rho)\left(1+D_{1} \rho+D_{2} \rho^{2}\right)} \tag{2.3.2}
\end{equation*}
$$

where
$D_{1}=d_{1}+b$
$D_{2}=d_{2}+b d_{1}+b^{2}=-d_{3} / b$
$d_{3}+b d_{2}+b^{2} d_{1}+b^{3}=0$
Then,

$$
\begin{equation*}
\frac{Z-1}{\rho}=\frac{N_{1}+N_{2} \rho+N_{3} \rho^{2}+N_{4} \rho^{3}}{(1+b \rho)\left(1+D_{1} \rho+D_{2} \rho^{2}\right)} \tag{2.3.3}
\end{equation*}
$$

where
$N_{1}=n_{1}-d_{1}$
$N_{2}=n_{2}-d_{2}$
$N_{3}=n_{3}-d_{3}$
$N_{4}=n_{4}$
The residual Helmholtz Free Energy comes from

$$
\begin{equation*}
\frac{A^{r}}{R T}=\int_{0}^{\rho} \frac{Z-1}{\rho} d \rho \tag{2.3.4}
\end{equation*}
$$

and

$$
\begin{equation*}
\frac{Z-1}{\rho}=\frac{F_{1}(\rho)}{1-b \rho}+\frac{F_{2}(\rho)}{1+D_{1} \rho+D_{2} \rho^{2}} \tag{2.3.5}
\end{equation*}
$$

where $F_{1}(\rho)$ and $F_{2}(\rho)$ can take different combinations as represented in Table 3. After performing all the multiple possibilities in the determination of the Helmholtz energy, different cases result.

Table 3. Mathematical Combinations for Functions $F_{1}(\rho)$ and $F_{2}(\rho)$

| $F_{1}(\rho)$ | $F_{2}(\rho)$ |
| :---: | :---: |
| $N_{1}$ | $a_{1} \rho+a_{2} \rho^{2}$ or $a_{2} \rho^{2}$ |
| $N_{1}+N_{2} \rho$ | $a_{1} \rho+a_{2} \rho^{2}$ or $a_{1} \rho$ or $a_{2} \rho^{2}$ |
| $N_{2} \rho$ | $N_{1}$ or $N_{1}+a_{1} \rho+a_{2} \rho^{2}$ or $N_{1}+a_{2} \rho^{2}$ |

## Case I.

(a) $F_{1}(\rho)=N_{1}$ and $F_{2}(\rho)=a_{1} \rho+a_{2} \rho^{2}$

$$
\begin{align*}
\frac{A^{r}}{R T}= & -\frac{N_{1}}{b} \ln (1-b \rho)+\frac{a_{2}}{D_{2}} \rho+\frac{\left(a_{1} D_{2}-a_{2} D_{1}\right)}{2 D_{2}^{2}} \ln \left(1+D_{1} \rho+D_{2} \rho^{2}\right)+\ldots \\
& \frac{1}{2}\left\{\frac{D_{1} a_{2} D_{2}+2 D_{2} a_{2}-D_{1}^{2} a_{2}}{D_{2}^{2} \sqrt{D_{1}^{2}-4 D_{2}}}\right\} \ln \left(\frac{1+\frac{2 D_{2} \rho}{\sqrt{D_{1}^{2}-4 D_{2}}+1}}{1-\frac{2 D_{2} \rho}{\sqrt{D_{1}^{2}-4 D_{2}}-1}}\right) \tag{2.3.6}
\end{align*}
$$

(b) $F_{1}(\rho)=N_{1}$ and $F_{2}(\rho)=a_{2} \rho^{2}$

$$
\begin{align*}
\frac{A^{r}}{R T}= & -\frac{N_{1}}{b} \ln (1-b \rho)+\frac{a_{2}}{D_{2}} \rho-\frac{a_{2} D_{1}}{2 D_{2}^{2}} \ln \left(1+D_{1} \rho+D_{2} \rho^{2}\right)+\ldots \\
& \frac{a_{2}}{2}\left\{\frac{2 D_{2}^{2}-D_{1}^{2}}{D_{2}^{2} \sqrt{D_{1}^{2}-4 D_{2}}}\right\} \ln \left(\frac{1+\frac{2 D_{2} \rho}{\sqrt{D_{1}^{2}-4 D_{2}}+1}}{1-\frac{2 D_{2} \rho}{\sqrt{D_{1}^{2}-4 D_{2}}-1}}\right) \tag{2.3.7}
\end{align*}
$$

Case II.
a) $F_{1}(\rho)=N_{1}+N_{2} \rho$ and $F_{2}(\rho)=a_{1} \rho+a_{2} \rho^{2}$

$$
\begin{align*}
\frac{A^{r}}{R T}= & -\left\{\frac{N_{1}}{b}+\frac{N_{2}}{b^{2}}\right\} \ln (1-b \rho)+\left\{\frac{a_{2}}{D_{2}}+\frac{N_{2}}{b}\right\} \rho+\frac{\left(a_{1} D_{2}-a_{2} D_{1}\right)}{2 D_{2}^{2}} \ln \left(1+D_{1} \rho+D_{2} \rho^{2}\right)+\ldots \\
& \frac{1}{2}\left\{\frac{D_{1}\left(a_{1} D_{2}-a_{2} D_{1}\right)+2 D_{2} a_{2}}{D_{2}^{2} \sqrt{D_{1}^{2}-4 D_{2}}}\right\} \ln \left(\frac{1+\frac{2 D_{2} \rho}{\sqrt{D_{1}^{2}-4 D_{2}}+1}}{1-\frac{2 D_{2} \rho}{\sqrt{D_{1}^{2}-4 D_{2}}-1}}\right) \tag{2.3.8}
\end{align*}
$$

b) $F_{1}(\rho)=N_{1}+N_{2} \rho$ and $F_{2}(\rho)=a_{1} \rho$

$$
\begin{align*}
\frac{A^{r}}{R T}= & -\left\{\frac{N_{1}}{b}+\frac{N_{2}}{b^{2}}\right\} \ln (1-b \rho)+\frac{a_{2}}{D_{2}} \rho+\frac{a_{1}}{2 D_{2}} \ln \left(1+D_{1} \rho+D_{2} \rho^{2}\right)+\ldots \\
& \frac{1}{2}\left\{\frac{D_{1} a_{1}}{D_{2}^{2} \sqrt{D_{1}^{2}-4 D_{2}}}\right\} \ln \left(\frac{1+\frac{2 D_{2} \rho}{\sqrt{D_{1}^{2}-4 D_{2}}+1}}{1-\frac{2 D_{2} \rho}{\sqrt{D_{1}^{2}-4 D_{2}}-1}}\right) \tag{2.3.9}
\end{align*}
$$

c) $F_{1}(\rho)=N_{1}+N_{2} \rho$ and $F_{2}(\rho)=a_{2} \rho^{2}$

$$
\begin{align*}
\frac{A^{r}}{R T}= & -\left\{\frac{N_{1}}{b}+\frac{N_{2}}{b^{2}}\right\} \ln (1-b \rho)+\frac{a_{2}}{D_{2}} \rho+\frac{a_{1}}{2 D_{2}} \ln \left(1+D_{1} \rho+D_{2} \rho^{2}\right)+\ldots \\
& \frac{1}{2}\left\{\frac{a_{2}\left(2 D_{2}-D_{1}^{2}\right)}{\left.D_{2}^{2} \sqrt{D_{1}^{2}-4 D_{2}}\right\} \ln \left(\frac{1+\frac{2 D_{2} \rho}{\sqrt{D_{1}^{2}-4 D_{2}}+1}}{1-\frac{2 D_{2} \rho}{\sqrt{D_{1}^{2}-4 D_{2}}-1}}\right\}}\right. \tag{2.3.10}
\end{align*}
$$

## Case III

a) $F_{1}(\rho)=N_{2} \rho$ and $F_{2}(\rho)=N_{1}$

$$
\begin{equation*}
\frac{A^{r}}{R T}=-\frac{N_{2}}{b^{2}} \ln (1-b \rho)+\frac{N_{2}}{b} \rho-\frac{N_{1}}{\sqrt{D_{1}^{2}-4 D_{2}}} \ln \left(\frac{1+\frac{2 D_{2} \rho}{\sqrt{D_{1}^{2}-4 D_{2}}+1}}{1-\frac{2 D_{2} \rho}{\sqrt{D_{1}^{2}-4 D_{2}}-1}}\right) \tag{2.3.11}
\end{equation*}
$$

b) $F_{1}(\rho)=N_{2} \rho$ and $F_{2}(\rho)=N_{1}+a_{1} \rho+a_{2} \rho^{2}$

$$
\begin{align*}
\frac{A^{r}}{R T}= & -\frac{N_{2}}{b^{2}} \ln (1-b \rho)+\left\{\frac{a_{2}}{D_{2}}+\frac{N_{2}}{b}\right\} \rho+\frac{\left(a_{1} D_{2}-a_{2} D_{1}\right)}{2 D_{2}^{2}} \ln \left(1+D_{1} \rho+D_{2} \rho^{2}\right)+\ldots \\
& \frac{1}{2}\left\{\frac{D_{1}\left(a_{1} D_{2}-a_{2} D_{1}\right)+2 D_{2} a_{2}-2 N_{1} D_{2}^{2}}{D_{2}^{2} \sqrt{D_{1}^{2}-4 D_{2}}}\right\} \ln \left(\frac{1+\frac{2 D_{2} \rho}{\sqrt{D_{1}^{2}-4 D_{2}}+1}}{1-\frac{2 D_{2} \rho}{\sqrt{D_{1}^{2}-4 D_{2}}-1}}\right) \tag{2.3.12}
\end{align*}
$$

c) $F_{1}(\rho)=N_{2} \rho$ and $F_{2}(\rho)=N_{1}+a_{2} \rho^{2}$

$$
\begin{align*}
\frac{A^{r}}{R T}= & -\frac{N_{2}}{b^{2}} \ln (1-b \rho)+\left\{\frac{a_{2}}{D_{2}}+\frac{N_{2}}{b}\right\} \rho-\frac{a_{2} D_{1}}{2 D_{2}^{2}} \ln \left(1+D_{1} \rho+D_{2} \rho^{2}\right)+\ldots \\
& \frac{1}{2}\left\{\frac{2 D_{2} a_{2}-a_{2} D_{1}^{2}-2 N_{1} D_{2}^{2}}{D_{2}^{2} \sqrt{D_{1}^{2}-4 D_{2}}}\right\} \ln \left(\frac{1+\frac{2 D_{2} \rho}{\sqrt{D_{1}^{2}-4 D_{2}}+1}}{1-\frac{2 D_{2} \rho}{\sqrt{D_{1}^{2}-4 D_{2}}-1}}\right) \tag{2.3.13}
\end{align*}
$$

Each of these cases represents a possible way to define the quintic equation as a Helmholtz energy function. Initial work has focused upon determining the correct number of parameters to include in the quintic form and their corresponding functionalities with temperature ${ }^{44}$.

An alternative procedure is to correlate this quintic functional based upon the multiparameter fitter developed by Lemmon et al. ${ }^{26}$. Basically a new functional form for the Helmholtz energy is defined based upon the different cases obtained earlier from the Helmholtz integral. Now, the Helmholtz energy function and the density are dimensionless:

$$
\begin{equation*}
\alpha^{r}(\delta, \tau)=\frac{A^{r}}{R T}=\int_{0}^{\rho} \frac{Z-1}{\delta} d \delta \tag{2.3.14}
\end{equation*}
$$

where $\alpha^{r}(\delta, \tau)$ is the dimensionless residual Helmholtz free energy, $\delta=\rho / \rho_{c}$ and $\tau=T_{c} / T$. When

$$
\begin{equation*}
\frac{Z-1}{\delta}=\frac{F_{1}(\delta)}{1-b \delta}+\frac{F_{2}(\delta)}{1+D_{1} \delta+D_{2} \delta^{2}} \tag{2.3.15}
\end{equation*}
$$

and taking $F_{1}(\delta)=S_{2} \delta$ and $F_{2}(\delta)=N_{1}$, and $S_{2}=N_{4} / D_{2}$

$$
\begin{equation*}
\alpha^{r}(\delta, \tau)=\frac{A^{r}}{R T}=-\frac{S_{2}}{b^{2}} \ln (1-b \delta)+\frac{S_{2}}{b} \delta-\frac{N_{1}}{\sqrt{D_{1}^{2}-4 D_{2}}} \ln \left(\frac{1+\frac{2 D_{2} \delta}{\sqrt{D_{1}^{2}-4 D_{2}}+1}}{1-\frac{2 D_{2} \delta}{\sqrt{D_{1}^{2}-4 D_{2}}-1}}\right) \tag{2.3.16}
\end{equation*}
$$

This functional form leads to a simpler functional:

$$
\begin{equation*}
\alpha^{r}(\delta, \tau)=\eta(\tau)\left[\delta-\frac{\ln (1-b(\tau) \delta)}{b(\tau)}\right]-\varepsilon(\tau) \ln \left(\frac{1+\gamma(\tau) \delta}{1-\beta(\tau) \delta}\right) \tag{2.3.17}
\end{equation*}
$$

where
$\eta=S_{2} / b$
$\varepsilon=N_{1} / \sqrt{D_{1}^{2}-4 D_{2}}$
$\gamma=\frac{2 D_{2}}{\sqrt{D_{1}^{2}-4 D_{2}}+1}$
$\beta=\frac{2 D_{2}}{\sqrt{D_{1}^{2}-4 D_{2}}-1}$

Now in order to take advantage of the multiparamer fitter developed by Lemmon et al. ${ }^{26}$ the first and second derivatives have to be calculated. The results are

$$
\begin{align*}
& \alpha_{\delta}^{r}(\delta, \tau)=\eta\left\{1+\frac{1}{(1-b \delta)}\right\}-\varepsilon\left\{\frac{\gamma}{1+\gamma \delta}+\frac{\beta}{1-\beta \delta}\right\}  \tag{2.3.18}\\
& \alpha_{\delta \delta}^{r}(\delta, \tau)=\frac{\eta b}{(1-b \delta)^{2}}+\varepsilon\left\{\frac{\gamma^{2}}{(1+\gamma \delta)^{2}}-\frac{\beta^{2}}{(1-\beta \delta)^{2}}\right\} \tag{2.3.19}
\end{align*}
$$

$$
\begin{gather*}
\alpha_{\delta \delta \delta}^{r}(\delta, \tau)=2 \frac{\eta b^{2}}{(1-b \delta)^{3}}-2 \varepsilon\left\{\frac{\gamma^{3}}{(1+\gamma \delta)^{3}}+\frac{\beta^{3}}{(1-\beta \delta)^{3}}\right\}  \tag{2.3.20}\\
\alpha_{\tau}^{r}=\eta_{\tau}\left\{\delta-\frac{\ln (1-b \delta)}{b}\right\}+\frac{\eta b_{\tau}}{b}\left\{\frac{\ln (1-b \delta)}{b}+\frac{\delta}{(1-b \delta)}\right\}+\ldots \\
-\varepsilon_{\tau} \ln \left(\frac{1+\gamma \delta}{1-\beta \delta}\right)-\varepsilon \delta\left\{\frac{\gamma_{\tau}}{1+\gamma \delta}+\frac{\beta_{\tau}}{1-\beta \delta}\right\}  \tag{2.3.21}\\
\alpha_{\tau \tau}^{r}= \\
\eta_{\tau \tau}\left\{\delta-\frac{\ln (1-b \delta)}{b}\right\}+\frac{1}{b}\left[2 \eta_{\tau} b_{\tau}+\eta b_{\tau \tau}-\frac{\eta b_{\tau}^{2}}{b}\right]\left\{\frac{\ln (1-b \delta)}{b}+\frac{\delta}{(1-b \delta)}\right\}+. .  \tag{2.3.22}\\
b^{2}\left\{\frac{\delta(2 b \delta-1)}{(1-b \delta)^{2}}-\frac{\ln (1-b \delta)}{b}\right\}-\varepsilon_{\tau \tau} \ln \left(\frac{1+\gamma \delta}{1-\beta \delta}\right)-2 \varepsilon_{\tau} \delta\left\{\frac{\gamma_{\tau}}{1+\gamma \delta}-\frac{\beta_{\tau}}{1-\beta \delta}\right\}+. . \\
-\varepsilon \delta\left\{\frac{\gamma_{\tau \tau}}{1+\gamma \delta}+\frac{\beta_{\tau \tau}}{1-\beta \delta}-\delta\left(\frac{\gamma_{\tau}^{2}}{(1+\gamma \delta)^{2}}-\frac{\beta_{\tau}^{2}}{(1-\beta \delta)^{2}}\right)\right\} \\
\alpha_{\delta \tau}^{r}(\delta, \tau)=\eta_{\tau}\left\{1+\frac{1}{(1-b \delta)}\right\}+\frac{\eta b_{\tau} \delta}{(1-b \delta)^{2}}-\varepsilon_{\tau}\left\{\frac{\gamma}{1+\gamma \delta}+\frac{\beta}{1-\beta \delta}\right\}+\ldots  \tag{2.3.23}\\
-\varepsilon\left\{\frac{\gamma_{\tau}}{1+\gamma \delta}+\frac{\beta_{\tau}}{1-\beta \delta}-\delta\left(\frac{\gamma \gamma_{\tau}}{(1+\gamma \delta)^{2}}-\frac{\beta \beta_{\tau}}{(1-\beta \delta)^{2}}\right)\right\}
\end{gather*}
$$

where $A_{x}=A(x, y)_{x}=(\partial A / \partial x)_{y}$. This equation is being initially fit to methane data and with simple temperature functions for $\eta(T), \varepsilon(T), \gamma(T)$ and $\beta(T)$.

The quintic EoS is an important approach that promises excellent performance. However, more work has to be done until a final conclusion can be drawn for this new functional.

## EXPERIMENTAL CHARACTERIZATION OF NATURAL GAS MIXTURES*

It is well known that any thermodynamics property of a fluid can be determined from a set of volumetric, composition and saturation data ${ }^{45}$. With the former information over a wide range of thermodynamics states, a thermodynamic formulation results for a fluid. Obviously the quantitative quality of the predictions based upon any type of thermodynamics formulation relies upon the accuracy of the data used during the fitting process.

One of the main challenges of this research project has been to establish accurately a robust set of thermodynamic volumetric, composition and saturation data for natural gas, its main constituents and associated mixtures. Achieving this goal has required a detailed uncertainty analysis of the recent techniques developed in the Thermodynamics Research Group at Texas A\&M ${ }^{2,17,46-47}$. Additional improvements for these techniques and a new high pressure isochoric apparatus have been developed. Finally a new technique for minimizing the uncertainty of the gas composition has been proposed and tested. The next sections contain descriptions and analysis of each data collection technique.

### 3.1 PoT Data

Magnetic suspension densimeters have provided high accuracy $P \rho T$ data for different substances over wide ranges of temperature and pressure for three decades ${ }^{48-49}$. Wagner and Kleinrahm ${ }^{50}$ discuss the advantages of this technique and contrast it to with

[^1]other techniques. The thermodynamics research group at Texas A\&M University has utilized a unique, high-accuracy, high-pressure, single-sinker magnetic suspension densimeter, Figure 7. The main characteristics of this apparatus appear in the ref 2 with recent modifications in refs 46 and 47. Additionally, this research project implemented several improvements to the magnetic suspension densimeter. First, speeding the heating cycle minimized the heating time during high temperature measurements; second, technical analysis of the isothermal shields indicated that the external isothermal shield was not necessary to achieve the desired temperature stability, and, finally, anchoring the high-precision electronic balance to the aluminum platform avoided displacement of the electromagnet and minimized uncertainty in the measurements caused by vertical and horizontal alignment.


Figure 7. High pressure single-sinker magnetic suspension densimeter.

The capability of going to high pressure ( 200 MPa ) in the single-sinker magnetic suspension densimeter allows determination of high-accuracy density data covering a wider range than any other MSD. Although preliminary analysis of the uncertainty for $P \rho T$ data measured using this apparatus have appeared ${ }^{46-47}$, none have accounted properly for one of the main source of uncertainty for this apparatus, the force transmission error (FTE). Therefore, an important task for this research project was the accurate determination of the FTE.

### 3.1.1 Force Transmission Error Analysis

An MSD utilizes the Archimedes principle, and consists of a pressurized cell with an internal sinker that senses a buoyancy force when the cell contains a fluid. A magnetic suspension coupling system transmits the change its apparent weight to a highprecision mass balance without direct connection between the sample cell and the balance. Knowledge of the apparent weight of the sinker and its properties allows determination of accurate densities for the fluid.

The FTE is a significant source of uncertainty in this technique. FTE is the error caused by the magnetic behavior of the cell, the suspension coupling and the measured fluid that leads to inaccuracies in the transmitted force measured at the high-precision balance. Different approaches exist to estimate and compensate for the FTE ${ }^{51-52}$. Kano et al. ${ }^{52}$ have proposed an analysis based upon a magnetostatic study of the MSD suspension. This analysis accounts for all the magnetic, gravitational and buoyancy forces during the measuring process. Unfortunately, this approach requires a detailed knowledge of both the magnetic and geometric properties of the MSD and the fluid, which are not always available. McLinden et al. ${ }^{51}$ have developed an empirical analysis applicable to both two- and single-sinker magnetic densimeters. In their analysis, they attribute the FTE to two different sources, one is the magnetic error introduce by the MSD, the "apparatus effect", which accounts for error caused by the magnetic characteristics of the densimeter cell and the suspension coupling. In principle, the apparatus effect is available from vacuum measurements of the sinker mass. The other source of error depends upon the magnetic properties of the fluid, the "fluid-specific effect".

McLinden et al. ${ }^{51}$ present a detailed explanation of each of the different sources of error and a mathematical model for its determination. The mathematical model results from a force balance during the different measurements steps in the MSD. For a singlesinker MSD, they suggest performing experiments using two different sinkers to determine unknowns in the mathematical model. They applied their technique to the single-sinker densimeter developed by Brachthäuser et al. ${ }^{53}$. The results show different values for the apparatus effect for each type of densimeter, and they conclude that "the
apparatus effect must be determined for each densimeter." An additional important result in their analysis is that "with careful measurements and the necessary calibrations, a single-sinker densimeter can yield $\mathrm{P} \rho \mathrm{T}$ data nearly as accurate as those from a twosinker densimeter" presuming the two-sinker densimeter data are correct.

Rubotherm Präzisionsmesstechnik GmbH manufactured our single-sinker MSD, specially designed to work at pressures up to 200 MPa over a temperature range of 193523 K . This apparatus has been used to measure densities of natural gas and associated mixtures as well as for low- and high-pressure densities for pure components ${ }^{2,46-47}$. The FTE analysis for this apparatus follows both the analysis and the results of McLinden et al. ${ }^{51}$. It was not possible to implement exactly the same procedure because of the peculiarities of our MSD (thicker cell wall and a higher apparatus contribution to the FTE) reflected in the scatter of the apparatus constant determined from the McLinden et al. ${ }^{51}$ approach, Figure 8. The apparatus constant results display an average value of 55.5 $\pm 211.5 \mathrm{ppm}$ which cannot be considered as a statistically significant value. This result dictates a slightly different approach to quantify and compensate for of this source of error.

### 3.1.1.1 Theoretical Model

Figure 9 presents the operation of the single-sinker MSD. In the (a) position, the balance is tared; in the (b) position, the permanent magnet ( pm ) is weighted; and in the (c) position, the permanent magnet and the sinker are weighted. In all positions, the electromagnet (em) is weighted.

The forces on the balance are

$$
\begin{align*}
& w_{1}=\alpha\left\{\phi\left[m_{\mathrm{pm}}-\rho_{\mathrm{f}} V_{\mathrm{pm}}\right]+m_{\mathrm{em}}+m_{\mathrm{c} 1}-\rho_{\mathrm{a}}\left(V_{\mathrm{em}}+V_{\mathrm{c} 1}\right)+w_{\mathrm{zero}}\right\}  \tag{3.1.1.1.1}\\
& w_{2}=\alpha\left\{\phi\left[m_{\mathrm{s}}+m_{\mathrm{pm}}-\rho_{\mathrm{f}}\left(V_{\mathrm{s}}+V_{\mathrm{pm}}\right)\right]+m_{\mathrm{em}}+m_{\mathrm{c} 2}-\rho_{\mathrm{a}}\left(V_{\mathrm{em}}+V_{\mathrm{c} 2}\right)+w_{\mathrm{zero}}\right\} \\
& w_{2}-w_{1}=\alpha\left\{\phi\left[m_{\mathrm{s}}-\rho_{\mathrm{f}} V_{\mathrm{s}}\right]+m_{\mathrm{c} 2}-m_{\mathrm{c} 1}-\rho_{\mathrm{a}}\left(V_{\mathrm{c} 1}-V_{\mathrm{c} 2}\right)\right\} \tag{3.1.1.1.3}
\end{align*}
$$

where $\alpha$ is the balance calibration factor, $\phi$ is the coupling factor, $\rho_{\mathrm{f}}$ is the fluid density, $\rho_{\mathrm{a}}$ is the density of the purge gas in the balance chamber (nitrogen), $V$ is the total volume, $m$ is the mass, $w$ is the balance reading, and $w_{\text {zero }}$ is the balance reading with nothing on the balance pan or weighing hook. The subscripts are: 1 denotes balance position 1, pm denotes the permanent magnet and includes the lifting fork, em denotes the electromagnet and includes linkage to the balance, c1 denotes compensation weight 1 (tantalum), and c 2 denotes compensation weight 2 (titanium).


Figure 8. Apparatus constant calculations based on experimental data measured in the high pressure single sinker MSD. Nitrogen • $265 \mathrm{~K}, \mathbf{\Delta} 298 \mathrm{~K}$,* 350 K ; Carbon dioxide च $310 \mathrm{~K}, 350 \mathrm{~K}$; Methane -305 K , i $340 \mathrm{~K}, \square 400 \mathrm{~K}$; Ethane $\times 298 \mathrm{~K}$, ○ 350 K , $400 \mathrm{~K}, \diamond 450 \mathrm{~K}$.


Figure 9. Operation of the single-sinker MSD: a) Suspension control (SC) 'off’, Ti and Ta both raised, (b) SC 'on' zero position (ZP), Ta lowered, Ti raised, (c) Measuring position, SC 'on', Ta raised, Ti lowered.

Because $V_{\mathrm{c} 2} \approx V_{\mathrm{cl}}$ and $\rho_{\mathrm{a}}$ are small

$$
\begin{equation*}
w_{2}-w_{1}=\alpha\left\{\phi\left[m_{\mathrm{s}}-\rho_{\mathrm{f}} V_{\mathrm{s}}\right]+\left(m_{\mathrm{c} 2}-m_{\mathrm{c} 1}\right)\right\} \tag{3.1.1.4}
\end{equation*}
$$

The coupling factor, $\phi$, represents the correction to the force balance in the MSD caused by the FTE. For a vacuum measurement in the cell, $\rho_{f}=0$ and

$$
\begin{equation*}
\left(w_{2}-w_{1}\right)_{0}=\alpha\left\{\phi_{0} m_{\mathrm{s}}+\left(m_{\mathrm{c} 2}-m_{\mathrm{c} 1}\right)\right\} \tag{3.1.1.1.5}
\end{equation*}
$$

Here, $\phi_{0}$ accounts for the apparatus effect of the force transmission error. Combining Eqs. 3.1.1.1.4 and 5,

$$
\begin{equation*}
\rho_{\mathrm{f}}=\rho_{\mathrm{s}}\left(1-\frac{\phi_{0}}{\phi}\right)+\frac{\left(w_{2}-w_{1}\right)_{0}-\left(w_{2}-w_{1}\right)_{\mathrm{f}}}{\phi \alpha V_{\mathrm{s}}} \tag{3.1.1.1.6}
\end{equation*}
$$

and from Eq. 3.1.1.1.4

$$
\begin{equation*}
\rho_{\mathrm{f}}=\rho_{\mathrm{s}}+\frac{\left(m_{\mathrm{c} 2}-m_{\mathrm{c} 1}\right)-\left(w_{2}-w_{1}\right)_{\mathrm{f}} / \alpha}{\phi V_{\mathrm{s}}} \tag{3.1.1.1.7}
\end{equation*}
$$

Eqs. 3.1.1.1.6 and 7 are equivalent. Now, assuming

$$
\begin{equation*}
\rho_{\mathrm{f}}=\rho_{\phi=1}+\Delta \rho_{\mathrm{FTE}} \tag{3.1.1.1.8}
\end{equation*}
$$

and postulating

$$
\begin{equation*}
\rho_{\mathrm{f}}=\rho_{\phi=1}+\Delta \rho_{\text {Apparatuseffect }}+\Delta \rho_{\text {FluidEffect }} \tag{3.1.1.1.9}
\end{equation*}
$$

and rewrite Eq. 3.1.1.1.8 as

$$
\begin{equation*}
\rho_{\mathrm{f}}=\rho_{\phi=\phi_{0}}+\Delta \rho_{\text {Fluideffect }} \tag{3.1.1.1.10}
\end{equation*}
$$

where

$$
\begin{equation*}
\rho_{\phi=\phi_{0}} \equiv \rho_{\phi=1}+\Delta \rho_{\text {ApparatusEffect }} \tag{3.1.1.1.11}
\end{equation*}
$$

which basically corrects the raw density data with the apparatus effect. One important detail is that although the true fluid density $\rho_{\mathrm{f}}$ appears as two terms, the right hand side of Eq. 3.1.11.8 does not change. Density measurements for different fluids performed with two different sinkers (copper and titanium) determine the coupling factor by using Eqs. 3.1.1.1.7 and 8. Now, assuming

$$
\begin{equation*}
\Delta \rho_{\text {Fluideffict }}\left(T_{1}, P_{1}\right) \cong \Delta \rho_{\text {Fluideffict }}\left(T_{2}, P_{2}\right) \tag{3.1.1.1.12}
\end{equation*}
$$

when $\left(T_{1}, P_{1}\right)$ and $\left(T_{2}, P_{2}\right)$ are nearly the same values for the same fluid. Now, combining Eqs. 3.1.1.1.7 and 11 for both titanium and copper sinkers

$$
\begin{align*}
& \rho_{\phi=\phi_{0}}^{\mathrm{s} 1}\left(T_{1}, P_{1}\right)+\Delta \rho_{\text {Fluideffect }}\left(T_{1}, P_{1}\right)=\rho_{\mathrm{s} 1}+\frac{\left(m_{\mathrm{c} 2}-m_{\mathrm{c} 1}\right)-\left(w_{2}-w_{1}\right)_{\mathrm{f}}^{\mathrm{s} 1} / \alpha}{\phi V_{\mathrm{s} 1}}  \tag{3.1.1.1.13}\\
& \rho_{\phi=\phi_{0}}^{\mathrm{s2}}\left(T_{2}, P_{2}\right)+\Delta \rho_{\text {Fluideffect }}\left(T_{2}, P_{2}\right)=\rho_{\mathrm{s} 2}+\frac{\left(m_{\mathrm{c} 2}-m_{\mathrm{c} 1}\right)-\left(w_{2}-w_{1}\right)_{\mathrm{f}}^{\mathrm{f}} / \alpha}{\phi V_{\mathrm{s} 2}} \tag{3.1.1.1.14}
\end{align*}
$$

where s1 and s2 stand for sinker 1 (titanium) and sinker 2 (copper). Finally, with Eqs. 3.1.1.1.13 and 14, and the assumption of Eq. 3.1.1.1.1:

$$
\begin{equation*}
\phi=\frac{\left(m_{\mathrm{c} 2}-m_{\mathrm{c} 1}\right)\left(\frac{1}{V_{\mathrm{s} 2}}-\frac{1}{V_{\mathrm{s} 1}}\right)-\frac{1}{\alpha}\left[\frac{\left(w_{2}-w_{1}\right)_{\mathrm{f}}^{\mathrm{s}}}{V_{\mathrm{s} 2}}-\frac{\left(w_{2}-w_{1}\right)_{\mathrm{f}}^{\mathrm{s} 1}}{V_{\mathrm{s} 1}}\right]}{\left[\rho_{\phi=\phi_{0}}^{\mathrm{s} 2}-\rho_{\phi=\phi_{0}}^{\mathrm{s} 1}\right]-\left[\rho_{\mathrm{s} 2}-\rho_{\mathrm{s} 1}\right]} \tag{3.1.1.1.15}
\end{equation*}
$$

Equation 3.1.1.1.15 is the expression to determine our FTE from the two sinkers experiments.

### 3.1.1.2 Experimental

To perform the two sinkers experiment, we collected data for four pure compounds (methane, ethane, carbon dioxide, and nitrogen) up to 180 MPa at temperatures ranging between 265 K and 400 K . Methane and nitrogen came from Scott Specialty Gases having a grade of Ultra High Purity (UHP) with mole fractions of 99.99 \% and 99.9995 \% respectively. Ethane and $\mathrm{CO}_{2}$ came from Matheson Tri Gas with mole fractions of $99.95 \%$ and $99.999 \%$ respectively. The titanium sinker mass and volume are 30.39159 g and $6.741043 \mathrm{~cm}^{3}$, and the copper sinker mass and volume are 30.398939 g and $3.403268 \mathrm{~cm}^{3}$ determined using procedure described by McLinden and Splett ${ }^{54}$. Patil et al. ${ }^{2,55}$ describe the single-sinker MSD, and additional modifications to expand the range of measured temperature appear in refrs 46 and 47. The PRT (Minco Products
model S1059PA5X6) has calibration at fixed temperature points defined by ITS-90 and by a calibrated PRT traceable to NIST. The temperature stability was $\pm 5 \mathrm{mK}$ and the uncertainty of the PRT was 2 mK with respect to the triple point of water ${ }^{46}$. Two Digiquartz® transducers ( 40 MPa and 200 MPa ) from Paroscientific Inc measure pressure. The uncertainty for these transducers is $0.01 \%$ of full scale.

### 3.1.1.3 Results and Analysis

Table 4 contains the temperatures of the two sinkers experiments performed at pressures up to 180 MPa . All the data were collected at similar pressure and temperature conditions for both sinkers to justify Eq. 3.1.1.1.12. Then, additional data were collected for all the fluids to validate the FTE results The deviations for the raw densities (densities without FTE compensation) compared to densities calculated from EoS ${ }^{40,42,}$ $56-57$ as implemented in the NIST REPROP $8.0^{26}$ appear in Figures 10 and 11. The data have considerable deviations in the low-pressure range. This reflects that the FTE affects the low-pressure densities to a greater extent as mentioned by McLinden et al. ${ }^{51}$

Table 4. Two Sinkers Experiment Temperatures

| Fluid | Copper - Titanium Sinkers |
| :---: | :---: |
| N 2 | $(265,298,350) \mathrm{K}$ |
| CO 2 | $(310,350) \mathrm{K}$ |
| CH 4 | $(305,340,400) \mathrm{K}$ |
| C2H6 | $(298,400,450) \mathrm{K}$ |



Figure 10. Titanium sinker raw densities ( $\phi=1$ ) deviations. Nitrogen • $265 \mathrm{~K}, \boldsymbol{\Delta} 298$ K,* 350 K ; Carbon dioxide $\nabla 310 \mathrm{~K}$, 350 K ; Methane -305 K , i 340 K , $\square 400 \mathrm{~K}$; Ethane $\times 298 \mathrm{~K}, \circ 350 \mathrm{~K}, \square 400 \mathrm{~K}, \diamond 450 \mathrm{~K}$.


Figure 11. Copper sinker raw densities ( $\phi=1$ ) deviations. Nitrogen • $265 \mathrm{~K}, \boldsymbol{\Delta} 298 \mathrm{~K}$,* 350 K ; Carbon dioxide $\nabla 310 \mathrm{~K}, ~ 350 \mathrm{~K}$; Methane -305 K , 1340 K , $\square 400 \mathrm{~K}$; Ethane $\times 298 \mathrm{~K}, \circ 350 \mathrm{~K}, \square 400 \mathrm{~K}, \diamond 450 \mathrm{~K}$.

Then Eqs. 3.1.1.1.11 and 15 determine both $\rho_{\phi=\phi_{0}}$ and the coupling factor, $\phi$, for the copper and titanium sinkers. The determined value for the apparatus effect is about $\left(\phi_{0}-1\right)=189 \pm 16 \mathrm{ppm}$ for all the experiments. This value corresponds to a correction of about 5.17 mg for our vacuum measurement. The apparatus contribution, $\phi_{0}$, from the FTE for our high-pressure, single-sinker MSD is higher than any reported by McLinden et al. ${ }^{51}$. This result occurs because our cell design accommodates higher pressures (thick walls), and it is diamagnetic (beryllium copper material). The large fluctuations in the apparatus contribution to the FTE result from the two sinkers experiment; every time we change the sinkers, we introduce a considerable uncertainty into the experiment. This is one of the most important difficulties when performing the two sinkers experiment, and it is worse for the high-pressure single-sinker MSD.

Figure 12 shows the value for $\left(\phi-\phi_{0}\right)$ as a function of pressure. This plot reveals two important characteristics of our FTE: our coupling factor does not display significant pressure dependence (i.e. density and the fluctuations for the value $\left(\phi-\phi_{0}\right)$ are clear indications that the fluid contribution is not statistically significant for our MSD. These fundamental facts lead us to conclude that the fluid contribution of the FTE is negligible, and we can assume its independence with temperature and pressure without introducing much uncertainty. This does not mean that a fluid specific effect does not exist, but the apparatus contributions mask that effect. Thus, we compensate our experiments using only the apparatus effect. On the other hand, the 16 ppm of fluctuation in $\phi_{0}$ is in total agreement with the fluctuations of the data when compared to the EoS as implemented in REFPROP 8.0 for the different fluids. This behavior is obvious in Figure 13.


Figure 12. $\left(\phi-\phi_{0}\right)$ Values for the two-sinkers experiment. Nitrogen - 265 K, $\mathbf{\Delta} 298$ K ; Carbon dioxide $\nabla 310 \mathrm{~K}, 350 \mathrm{~K}$; Methane -305 K , i 340 K , $\square 400 \mathrm{~K}$; Ethane $\times$ $298 \mathrm{~K}, \circ 350 \mathrm{~K}, \diamond 450 \mathrm{~K}$.


Figure 13. Titanium sinker corrected densities deviations. Nitrogen • $265 \mathrm{~K}, \boldsymbol{\Delta} 298 \mathrm{~K}$,* 350 K ; Carbon dioxide $\nabla 310 \mathrm{~K}, 350 \mathrm{~K}$; Methane -305 K , i $340 \mathrm{~K}, \square 400 \mathrm{~K}$; Ethane $\times 298 \mathrm{~K}, \circ 350 \mathrm{~K}, \square 400 \mathrm{~K}, \diamond 450 \mathrm{~K}$.

Figure 13 indicates that our experimental data for pure compounds lie within a $2 \sigma$ deviation band of $0.05 \%$. Higher deviations appear at pressures below 7 MPa caused by the intrinsic characteristics of the high-pressure, single-sinker MSD and its ancillary equipment. However, the low-pressure data fall within the experimental uncertainty.

An additional experimental observation is that the data measured with the copper sinker have higher deviations compared to RefProp 8.0 than the densities measured with the titanium sinker as apparent in Figure 14. This could be an effect caused by the differences between the sinker densities and the fluid densities. The density of copper is almost twice the density of titanium ( $8905.54 \mathrm{~kg} \cdot \mathrm{~m}^{-3}$ and $4508.44 \mathrm{~kg} \cdot \mathrm{~m}^{-3}$ respectively). Therefore, for more experimental accuracy, the density of the sinker should be as close as possible to the fluid density. McLinden et al. ${ }^{51}$ and Wagner et al. ${ }^{50}$ also reach this conclusion.


Figure 14. Copper sinker corrected densities deviations. Nitrogen • $265 \mathrm{~K}, \boldsymbol{\Delta} 298 \mathrm{~K}$,* 350 K ; Carbon dioxide $\nabla 310 \mathrm{~K}, ~ 350 \mathrm{~K}$; Methane -305 K , 1340 K , $\square 400 \mathrm{~K}$; Ethane $\times 298 \mathrm{~K}, \circ 350 \mathrm{~K}, \quad 400 \mathrm{~K}, \diamond 450 \mathrm{~K}$.

### 3.1.2 Pure Compounds P $\boldsymbol{P} \boldsymbol{T}$ Data

A first step to determine the performance of our MSD, after the FTE determination, is to measure and compare high purity pure compound $P \rho T$ data to the RefProp $8.0{ }^{26}$. Data for methane and ethane, the primary constituents of natural gas, have been measured. A detailed analysis for these data follows.

### 3.1.2.1 Accurate PpT Data for Methane From (300 To 450) K Up to 180 Mpa

Methane is the principal constituent of natural gas and an important raw material for many industrial processes. Accurate thermophysical property data for methane are necessary for design and evaluation of these processes. Setzmann and Wagner ${ }^{40}$ make an extensive analysis of the thermodynamic data for methane reported before 1991. On the basis of the uncertainty analysis of the data sources, they define three groups of data: group 1 has the most consistent sets of data and lower experimental uncertainties, and the other two groups do not follow their predefined quality standards. They have developed an EoS, using the group 1 data, based upon an explicit Helmholtz energy function with 40 coefficients. They claim a relative uncertainty in the density predictions of $0.03 \%$ up to 12 MPa and from $0.03 \%$ to $0.15 \%$ for higher pressures.

Setzmann and Wagner ${ }^{40}$ provide a detailed description of the data used for fitting their EoS. Four sets of data reside in group 1 for pressures greater than 35 MPa : Trappeniers et al., ${ }^{58}$ (2 to 260) MPa; Morris, ${ }^{59}$ (130 to 690) MPa; Mollerup, ${ }^{60}$ (0.2 to 72) MPa; and Kortbeek and Schouten, ${ }^{61}$ (150 to 1000) MPa. Mollerup reports an uncertainty in density of $10^{-3} \rho$, and Setzman and Wagner estimate the uncertainties for Kortbeek and Schouten at $10^{-3} \rho$ with Trappeniers et al. and Morris at $5 \cdot 10^{-4} \rho$.

Data for methane at $(298,305,338,400$ and 450$) \mathrm{K}$ up to 180 MPa were measured. The methane came from Scott Specialty Gases having a grade of Ultra High Purity (UHP) with a mole fraction of 99.99 \% methane. The characteristics of the MSD are the same as described in the FTE analysis. After compensation for the FTE in the raw data and based upon the assumption of uncorrelated errors for the different sources of error such as temperature and pressure, the uncertainty for our data is $3 \cdot 10^{-4} \rho$ for pressures greater than 7 MPa and up to $5 \cdot 10^{-4} \rho$ for pressures between 5 MPa and 7 MPa .

The two reported uncertainties exist because our MSD uses two different pressure transducers ( 40 MPa and 200 MPa ), and they do not produce a uniform uncertainty across the entire range of pressures.

### 3.1.2.1.1 Results and Analysis

The five sets of isothermal data appear in Table 5, along with the predicted densities obtained from RefProp $8.0^{26}$. The last column in the table contains the deviations with respect to the experimental data. Figure 15 shows a comparison between our experimental data and those of Trappeniers et al. ${ }^{58}$, Mollerup ${ }^{60}$, and Kortbeek and Schouten ${ }^{61}$ referenced to RefProp 8.0 predictions.

Table 5. Measured Density Values for Methane

| $\boldsymbol{T} / \mathbf{K}$ | $\boldsymbol{P} / \mathbf{M P a}$ | $\boldsymbol{\rho} / \mathbf{k g} \cdot \mathbf{m}^{-\mathbf{3}}$ | $\boldsymbol{\rho / \mathbf { k g } \cdot \mathbf { m } ^ { - \mathbf { 3 } }}$ <br> RefProp 8.0) | $\mathbf{1 0 0} \cdot\left(\boldsymbol{\rho}-\boldsymbol{\rho}_{\text {RefProp }}\right) / \boldsymbol{\rho}$ |
| :---: | :---: | :---: | :---: | :---: |
| 298.156 | 1.012 | $\mathrm{~T}=298.15 \mathrm{~K}$ |  |  |
| 298.189 | 5.009 | 35.316 | 35.329 | -0.065 |
| 298.149 | 10.010 | 76.080 | 76.078 | 0.001 |
| 298.249 | 14.994 | 118.506 | 118.512 | -0.005 |
| 298.145 | 20.012 | 157.172 | 157.183 | -0.007 |
| 298.138 | 29.958 | 212.508 | 212.485 | 0.011 |
| 298.144 | 35.056 | 232.422 | 232.377 | 0.019 |
| 298.139 | 49.959 | 273.189 | 273.169 | 0.007 |
| 298.157 | 66.961 | 303.605 | 303.563 | 0.014 |
| 298.150 | 79.980 | 321.098 | 321.030 | 0.021 |
| 298.144 | 99.874 | 342.269 | 342.183 | 0.025 |
| 298.138 | 124.934 | 363.139 | 363.026 | 0.031 |

Table 5. Continued

| T/K | $\boldsymbol{P} / \mathbf{M P a}$ | $\rho / \mathbf{k g} \cdot \mathbf{m}^{-3}$ | $\begin{gathered} \rho / \mathrm{kg} \cdot \mathbf{m}^{-3} \\ \text { RefProp 8.0) } \end{gathered}$ | $100 \cdot\left(\rho-\rho_{\text {RefProp }}\right) / \rho$ |
| :---: | :---: | :---: | :---: | :---: |
| 298.141 | 150.062 | 380.039 | 379.913 | 0.033 |
| 298.138 | 159.617 | 385.731 | 385.591 | 0.036 |
| 298.142 | 170.054 | 391.531 | 391.416 | 0.029 |
| 298.143 | 185.333 | 399.489 | 399.340 | 0.037 |
| 298.145 | 186.931 | 400.257 | 400.131 | 0.032 |
| 298.142 | 188.059 | 400.837 | 400.688 | 0.037 |
| $\mathrm{T}=305.24 \mathrm{~K}$ |  |  |  |  |
| 305.236 | 5.001 | 34.173 | 34.175 | -0.007 |
| 305.235 | 6.897 | 48.435 | 48.438 | -0.006 |
| 305.231 | 9.993 | 72.932 | 72.931 | 0.002 |
| 305.240 | 15.006 | 113.440 | 113.440 | 0.000 |
| 305.242 | 20.696 | 155.024 | 155.030 | -0.004 |
| 305.239 | 29.976 | 205.443 | 205.430 | 0.006 |
| 305.239 | 34.563 | 223.779 | 223.755 | 0.011 |
| 305.230 | 49.968 | 267.232 | 267.252 | -0.008 |
| 305.233 | 60.012 | 287.019 | 287.012 | 0.002 |
| 305.239 | 69.988 | 302.917 | 302.884 | 0.011 |
| 305.227 | 79.855 | 316.163 | 316.098 | 0.020 |
| 305.234 | 99.904 | 337.939 | 337.879 | 0.018 |
| 305.233 | 124.930 | 359.159 | 359.090 | 0.019 |
| 305.225 | 149.862 | 376.248 | 376.153 | 0.025 |
|  |  | $\mathrm{T}=338 \mathrm{~K}$ |  |  |
| 338.049 | 5.000 | 29.983 | 29.986 | -0.009 |
| 338.037 | 6.905 | 42.093 | 42.086 | 0.017 |

Table 5. Continued

| T/K | P/MPa | $\rho / \mathbf{k g} \cdot \mathrm{m}^{-3}$ | $\begin{gathered} \rho / \mathrm{kg} \cdot \mathbf{m}^{-3} \\ \text { RefProp 8.0) } \end{gathered}$ | $100 \cdot\left(\rho-\rho_{\text {RefProp }}\right) / \rho$ |
| :---: | :---: | :---: | :---: | :---: |
| 338.103 | 9.969 | 62.054 | 62.044 | 0.016 |
| 338.079 | 15.026 | 95.246 | 95.243 | 0.003 |
| 338.082 | 20.687 | 130.132 | 130.139 | -0.005 |
| 338.080 | 30.005 | 177.380 | 177.371 | 0.005 |
| 338.048 | 34.473 | 195.446 | 195.453 | -0.004 |
| 338.083 | 50.031 | 241.969 | 242.001 | -0.013 |
| 338.079 | 59.971 | 263.229 | 263.250 | -0.008 |
| 338.063 | 70.001 | 280.639 | 280.679 | -0.014 |
| 338.103 | 80.310 | 295.610 | 295.648 | -0.013 |
| 338.112 | 99.908 | 318.744 | 318.719 | 0.008 |
| 338.068 | 124.895 | 341.685 | 341.626 | 0.017 |
| 338.066 | 149.542 | 359.787 | 359.752 | 0.010 |
| 338.094 | 164.905 | 369.665 | 369.505 | 0.043 |
| 338.121 | 179.829 | 378.164 | 378.121 | 0.011 |
|  |  | $\mathrm{T}=400 \mathrm{~K}$ |  |  |
| 400.068 | 5.005 | 24.610 | 24.618 | -0.031 |
| 400.013 | 6.915 | 34.195 | 34.199 | -0.011 |
| 400.015 | 10.002 | 49.746 | 49.744 | 0.005 |
| 400.029 | 13.795 | 68.723 | 68.704 | 0.027 |
| 399.988 | 15.027 | 74.789 | 74.786 | 0.005 |
| 400.025 | 20.675 | 101.642 | 101.644 | -0.002 |
| 399.984 | 30.014 | 141.158 | 141.137 | 0.015 |
| 400.036 | 34.510 | 157.640 | 157.611 | 0.019 |
| 400.001 | 50.037 | 203.469 | 203.518 | -0.024 |

Table 5. Continued

| $\boldsymbol{T} / \mathbf{K}$ | $\boldsymbol{P} / \mathbf{M P a}$ | $\boldsymbol{\rho} / \mathbf{k g} \cdot \mathbf{m}^{-\mathbf{3}}$ | $\boldsymbol{\rho} / \mathbf{k g} \cdot \mathbf{m}^{-\mathbf{3}}$ <br> RefProp 8.0 $)$ | $\mathbf{1 0 0} \cdot\left(\boldsymbol{\rho}-\boldsymbol{\rho}_{\text {RefProp }}\right) / \boldsymbol{\rho}$ |
| :---: | :---: | :---: | :---: | :---: |
| 399.967 | 59.973 | 225.951 | 225.977 | -0.011 |
| 400.042 | 69.978 | 244.725 | 244.750 | -0.011 |
| 400.003 | 79.920 | 260.652 | 260.674 | -0.009 |
| 399.943 | 89.964 | 274.637 | 274.631 | 0.002 |
| 400.089 | 99.984 | 286.800 | 286.796 | 0.001 |
| 400.022 | 124.882 | 312.062 | 312.064 | -0.001 |
| 400.023 | 149.627 | 332.136 | 332.150 | -0.004 |
| 450.091 | 6.886 | $\mathrm{~T}=450 \mathrm{~K}$ |  | -0.022 |
| 450.010 | 20.697 | 87.478 | 87.464 | 0.016 |
| 450.048 | 30.002 | 121.791 | 121.792 | -0.001 |
| 450.115 | 34.492 | 136.647 | 136.635 | 0.009 |
| 450.057 | 50.036 | 179.955 | 179.987 | -0.018 |
| 450.027 | 59.975 | 202.137 | 202.170 | -0.017 |
| 450.064 | 69.966 | 221.073 | 221.139 | -0.030 |
| 450.083 | 80.008 | 237.571 | 237.606 | -0.015 |
| 450.025 | 89.981 | 251.923 | 251.957 | -0.013 |
| 450.011 | 99.932 | 264.608 | 264.645 | -0.014 |
| 450.018 | 119.918 | 286.376 | 286.412 | -0.013 |
| 450.034 | 139.476 | 304.187 | 304.188 | 0.000 |

Figure 15 demonstrates the deviations compared to our experimental data. It is clear that the calculations from the Setzmann and Wagner EoS are in excellent agreement with our experimental data, and that the predictions from the equation are better than expected for pressures greater than 12 MPa .


Figure 15. Percentage deviation of the experimental $P \rho T$ data from values calculated using Setzmann and Wagner ${ }^{40}$. This work 298 K , • 305 K , $\boldsymbol{\Delta} 338 \mathrm{~K}$, ■ 400 K , $\boldsymbol{\nabla}$ 450 K ; $\operatorname{ref} 58, \Delta(273.25$ to 423.25$) \mathrm{K}$; ref 60, o 310 K : ref 61, x 298.15 K.

Second and third virial coefficients determined from the $P \rho T$ data indicate that extrapolation of the data into the low pressure range is reliable. Figure 16 presents the methodology to evaluate the second and third virial coefficients at 298 K . The selected low-density data exhibit both a linear trend and have good correlation coefficients.


Figure 16. Procedure for the determination of the second virial coefficient using 298 K data. The intercept value, determined by mean square regression, is $-0.002681 \mathrm{~m}^{3} \mathrm{~kg}^{-1}$, the slope is $9.82 \times 10^{-6}\left(\mathrm{~m}^{3} \mathrm{~kg}^{-1}\right)^{2}$ and the correlation coefficient $\mathrm{R}^{2}$ is 0.999 .

Figures 17 and 18 present comparisons of literature data ${ }^{62-65}$ to the current data referenced to the Setzmann and Wagner equation. Most of these data lie in a band with an absolute deviation of $0.2 \mathrm{~cm}^{3} \cdot \mathrm{~mol}^{-1}$ for the second virial coefficients and of 150 $\left(\mathrm{cm}^{3} \cdot \mathrm{~mol}^{-1}\right)^{2}$ for the third virial coefficients. The estimated uncertainty for the second and the third virial coefficients are respectively $0.57 \mathrm{~cm}^{3} \cdot \mathrm{~mol}^{-1}$ and $125\left(\mathrm{~cm}^{3} \cdot \mathrm{~mol}^{-1}\right)^{2}$. The current values for the 450 K virial coefficients have a higher absolute deviation of $0.48 \mathrm{~cm}^{3} \cdot \mathrm{~mol}^{-1}$ and $301.2\left(\mathrm{~cm}^{3} \cdot \mathrm{~mol}^{-1}\right)^{2}$, which is a reflection of the paucity of lowdensity data taken for this isotherm. However, it appears that the apparatus is capable of determining second and third virial coefficients. The second and third virial coefficients values appear in Table 6.


Figure 17. Absolute deviations for second virial coefficient from values calculated using the Setzmann and Wagner ${ }^{40}$ equation of state $\Delta \mathrm{B}=\left(\mathrm{B}_{\text {exp }}-\mathrm{B}_{\text {calc }}\right)$. This work $■$; ref $62 \bullet$; ref $63 \star$; ref $64 \boldsymbol{\Delta}$; ref $65 \boldsymbol{\nabla}$.


Figure 18. Absolute deviations for third virial coefficient from values calculated using the Setzmann and Wagner ${ }^{40}$ equation of state $\Delta \mathrm{C}=\left(\mathrm{C}_{\text {exp }}-\mathrm{C}_{\text {calc }}\right)$. This work $\bullet$; ref 62 © ; ref 63 ■; ref 65 *.

Table 6. Second and Third Virial Coefficients for Methane

| $\mathbf{T} / \mathbf{K}$ | $\mathbf{B} /\left(\mathbf{c m}^{\mathbf{3}} \mathbf{m o l}^{-1}\right)$ | $\mathbf{C} /\left(\mathbf{c m}^{\mathbf{3}} \mathbf{m o l}^{-1}\right)^{\mathbf{2}}$ |
| :---: | :---: | :---: |
| 298.190 | -43.01 | 2527.8 |
| 305.235 | -40.40 | 2430.1 |
| 338.092 | -30.01 | 2220.5 |
| 400.017 | -15.72 | 1978.8 |
| 450.038 | -7.78 | 1973.2 |

### 3.1.2.2 Accurate PןT Data for Ethane From (298To 450) K up to 200 MPa

Ethane is the second most abundant constituent of natural gas and an important raw material for many industrial processes and scientific applications. Accurate thermophysical property data for ethane are necessary for design and evaluation of these processes. Bücker and Wagner ${ }^{42}$ make an extensive analysis of the thermodynamic data for ethane reported before of 2006. Based upon the uncertainty analysis of the data sources, they define three different groups of data: group 1 has the most consistent sets of data and lower experimental uncertainties, the other two groups do not follow their predefined quality standards. They have developed an EoS, using the group 1 data, based upon an explicit Helmholtz energy function with 44 coefficients. They claim a relative uncertainty in the density predictions of $0.02 \%-0.03 \%$ from the melting line up to temperatures of 520 K and pressures of 30 MPa .

Bücker and Wagner ${ }^{42}$ provide a detailed description of the data used for fitting their EoS. Two sets of data reside in group 1 for pressures greater than 30 MPa and below than 200 MPa , Pal et al. ${ }^{66}$ ( 0.52 to 73 ) MPa, Golovskii et al. ${ }^{67}$ ( 1.2 to 60) MPa. The estimated relative uncertainties by Bücker and Wagner for the Pal et al. and Golovskii et al. data are $0.40 \%$ and $0.25 \%$ respectively. Byun et al. ${ }^{68}$ published an additional set of high pressure data from ( 15 to 276 ) MPa ; however these data have high relative deviations, up to $7 \%$, compared to Bücker and Wagner EoS. Therefore, we do not include these data in our analysis. No additional, reliable data within the range temperatures and pressures of concern to this publication appear in the literature for comparison.

Density data for ethane at (298, 350, 400 and 450 ) K up to 200 MPa were measured. The ethane came from Matheson Tri Gas having a grade of Ultra High Purity (UHP) with mole fractions of $99.95 \%$ ethane. The data was corrected using the FTE analysis.

### 3.1.2.2.1 Results and Analysis

The four sets of isothermal data appear in Table 7, along with the predicted densities obtained from the Bücker and Wagner EoS as implemented in RefProp 8.0. ${ }^{26}$. The last column in the table contains the deviations with respect to the experimental data.

Table 7. Measured Density Values for Ethane

| $P / \mathbf{M P a}$ | $\rho / \mathrm{kg} \cdot \mathrm{m}^{-3}$ | $\begin{gathered} \rho / \mathrm{kg} \cdot \mathrm{~m}^{-3} \\ (\text { RefProp 8.0) } \end{gathered}$ | $100 \cdot\left(\rho-\rho_{\text {RefProp }}\right) / \rho$ |
| :---: | :---: | :---: | :---: |
|  |  | $\mathrm{T}=298.150 \mathrm{~K}$ |  |
| 2.000 | 29.226 | 29.228 | -0.004 |
| 5.987 | 352.999 | 353.033 | -0.010 |
| 7.909 | 371.593 | 371.571 | 0.006 |
| 10.071 | 385.867 | 385.799 | 0.018 |
| 14.959 | 407.911 | 407.811 | 0.024 |
| 20.004 | 423.934 | 423.827 | 0.025 |
| 24.927 | 436.235 | 436.121 | 0.026 |
| 29.893 | 446.599 | 446.474 | 0.028 |
| 35.020 | 455.820 | 455.683 | 0.030 |
| 39.905 | 463.582 | 463.434 | 0.032 |
| 49.977 | 477.362 | 477.162 | 0.042 |
| 75.091 | 503.592 | 503.383 | 0.041 |

Table 7. Continued

| $\boldsymbol{P} / \mathrm{MPa}$ | $\rho / \mathbf{k g} \cdot \mathrm{m}^{-3}$ | $\begin{gathered} \rho / \mathrm{kg} \cdot \mathrm{~m}^{-3} \\ (\text { RefProp 8.0) } \end{gathered}$ | $100 \cdot\left(\rho-\rho_{\text {RefProp }}\right) / \rho$ |
| :---: | :---: | :---: | :---: |
| 99.827 | 523.125 | 522.954 | 0.033 |
| 119.760 | 536.225 | 536.070 | 0.029 |
| 149.807 | 552.995 | 552.905 | 0.016 |
|  |  | $\mathrm{T}=350.000 \mathrm{~K}$ |  |
| 1.999 | 22.819 | 22.814 | 0.024 |
| 29.974 | 384.889 | 384.905 | -0.004 |
| 49.928 | 430.315 | 430.124 | 0.044 |
| 74.959 | 464.696 | 464.424 | 0.058 |
| 100.019 | 489.111 | 488.825 | 0.058 |
| 124.863 | 508.192 | 507.902 | 0.057 |
| 149.906 | 524.195 | 523.931 | 0.050 |
| 172.157 | 536.528 | 536.301 | 0.042 |
|  |  | $\mathrm{T}=400.000 \mathrm{~K}$ |  |
| 0.805 | 7.452 | 7.453 | -0.016 |
| 7.325 | 84.481 | 84.487 | -0.006 |
| 27.286 | 311.921 | 311.899 | 0.007 |
| 48.575 | 383.533 | 383.354 | 0.047 |
| 69.346 | 421.214 | 421.012 | 0.048 |
| 92.289 | 450.020 | 449.796 | 0.050 |
| 117.459 | 473.901 | 473.683 | 0.046 |
| 139.686 | 491.026 | 490.822 | 0.041 |
| 154.657 | 501.119 | 500.927 | 0.038 |
| 166.558 | 508.474 | 508.315 | 0.031 |
| 181.565 | 517.111 | 516.954 | 0.030 |

Table 7. Continued

| $P / \mathbf{M P a}$ | $\rho / \mathbf{k g} \cdot \mathbf{m}^{-3}$ | $\rho / \mathrm{kg} \cdot \mathrm{~m}^{-3}$ <br> (RefProp 8.0) | $100 \cdot\left(\rho-\rho_{\text {RefProp }}\right) / \rho$ |
| :---: | :---: | :---: | :---: |
| 196.062 | 524.823 | 524.691 | 0.025 |
|  |  | $\mathrm{T}=450.000 \mathrm{~K}$ |  |
| 1.998 | 16.685 | 16.685 | 0.002 |
| 5.021 | 44.467 | 44.464 | 0.007 |
| 10.001 | 96.599 | 96.581 | 0.019 |
| 20.005 | 202.282 | 202.249 | 0.016 |
| 30.018 | 272.417 | 272.340 | 0.028 |
| 49.974 | 346.238 | 346.020 | 0.063 |
| 69.967 | 388.105 | 387.843 | 0.068 |
| 79.928 | 403.693 | 403.405 | 0.072 |
| 99.962 | 429.113 | 428.804 | 0.072 |
| 119.860 | 449.290 | 448.977 | 0.070 |
| 139.478 | 465.963 | 465.650 | 0.067 |

Figure 19 is a comparison between our experimental data and those of Pal et al. ${ }^{66}$ and Golovskii et al. ${ }^{67}$ referenced to RefProp 8.0 predictions. It is clear that the calculations from the Bücker and Wagner EoS are in excellent agreement with our experimental data, and that the predictions from the equation are better than expected for pressures greater than 30 MPa .


Figure 19. Percentage deviation of the experimental $P \rho T$ data from values calculated using the Bücker and Wagner ${ }^{42}$ equation of state. This work $\bullet 298.150 \mathrm{~K}, \boldsymbol{\Delta} 350.000$ $\mathrm{K}, \pm 400 \mathrm{~K}, \star 450 \mathrm{~K}$; ref 66 , o ( 290 to 345 ) K ; ref673, $\Delta$ ( 255 to 270 ) K.

Second and third virial coefficients determined from the $P \rho T$ data (as described earlier) indicate that extrapolation of the data into the low pressure range is reliable. No virial coefficients were determined for the isotherm 298.150 K because only a single vapor datum was available for the extrapolation. Figures 20 and 21 present comparisons of experimental literature data ${ }^{62,69-70}$ along with the current data based upon the Bücker and Wagner equation.


Figure 20. Absolute deviations for second virial coefficient from values calculated using the Bücker and Wagner ${ }^{42}$ equation of state $\Delta \mathrm{B}=\left(\mathrm{B}_{\exp }-\mathrm{B}_{\text {calc }}\right)$. This work $\bullet$; ref $62 \boldsymbol{\Delta}$; ref 1569 ; ref $70 \star$.


Figure 21. Absolute deviations for third virial coefficient from values calculated using the Bücker and Wagner ${ }^{42}$ equation of state $\Delta \mathrm{C}=\left(\mathrm{C}_{\text {exp }}-\mathrm{C}_{\text {calc }}\right)$. This work $\bullet$; ref $62 \boldsymbol{\Delta}$; ref $69 \boldsymbol{\nabla} ; \operatorname{ref} 70 \star$.

Most of these data lie in a band with absolute deviation of $0.5 \mathrm{~cm}^{3} \cdot \mathrm{~mol}^{-1}$ for the second virial coefficients and of $500\left(\mathrm{~cm}^{3} \cdot \mathrm{~mol}^{-1}\right)^{2}$ for the third virial coefficients. The estimated uncertainty for the second and the third virial coefficients are respectively 0.57 $\mathrm{cm}^{3} \cdot \mathrm{~mol}^{-1}$ and $270\left(\mathrm{~cm}^{3} \cdot \mathrm{~mol}^{-1}\right)^{2}$. Therefore, it appears that our apparatus is capable of determining second and third virial coefficients. The second and third virial coefficients values for ethane appear in Table 8.

Table 8. Second and Third Virial Coefficients for Ethane

| $\mathbf{T} / \mathbf{K}$ | $\mathbf{B} /\left(\mathbf{c m}^{\mathbf{3}} \mathbf{m o l}^{-1}\right)$ | $\mathbf{C} /\left(\mathbf{c m}^{\mathbf{3}} \mathbf{m o l}^{-1}\right)^{\mathbf{2}}$ |
| :---: | :---: | :---: |
| 350.000 | -130.71 | 8084 |
| 400.000 | -96.43 | 7327 |
| 450.000 | -71.29 | 5912 |

### 3.2 Isochoric and Phase Equilibrium Data

Isochoric data are a valuable source of data for densities and vapor-liquid equilibrium of mixtures ${ }^{71}$. Since the publication of the Griffiths' proof of the collinearity constraints ${ }^{72}$ and some alternatives approaches using isochoric data as a source of equilibrium information for mixtures ${ }^{73}$, several theoretical and experimental investigations have taken advantage of the "change of slope" method ${ }^{74-76}$. The "change of slope" utilizes the break in the slope of the isochoric data when moving from the onephase region to the two-phase region. With this information it is possible to determine dew and bubble points of phase envelopes for mixtures.

The Thermodynamics research group at Texas A\&M University has built a low pressure isochoric apparatus to use the "change of slope" technique for the determination of natural gas mixtures phase loops ${ }^{17,77}$. Coupling isochoric data with the isothermal density data from the MSD provides complementary information for these techniques. The pressure limitations of the isochoric apparatus developed in the past restrict the determination of the bubble points for natural gas mixtures. Therefore we decided to extend the range of the isochoric data by building a high pressure apparatus. The high
pressure isochoric apparatus was designed and constructed to work up to 200 MPa over the range of ( 100 to 500 ) K . With the capabilities of this new apparatus, the low pressure isochoric apparatus and the MSD, we now possess the infrastructure to characterize natural gas mixtures over a wide thermodynamic space with high accuracy.

Finally, we determined the uncertainty for the "change of slope" method. A new methodology to determine the saturation density from the isochoric and the isothermal data was resulted as well as a technique to correct the isochoric slopes.

### 3.2.1 Low Pressure Isochoric Apparatus

Zhou ${ }^{17}$ developed the low pressure isochoric apparatus. A detailed description appears in ${ }^{17,77}$. Table 9 contains the principal features of this apparatus.

## Table 9. Low Pressure Isochoric Apparatus Features

|  | 100 K to 500 K <br> Temperature Range <br> $0.02 \%$ accuracy <br> $\pm 3 \mathrm{mK}$ stability <br> $\pm 2 \mathrm{mK} \Delta \mathrm{T}$ across the cell |
| :---: | :--- |
| Pressure Range | up to $35 \mathrm{MPa} 0.01 \%$ accuracy (full scale) |

However, during this project we made some improvements to this apparatus while conserving its main characteristics. A new Varian DS-202 mechanical pump improved the vacuum system. We reassembled the apparatus in a new portable frame, which allowed a more versatile positioning in the lab space, Figure 22. These new features also facilitated implementation of simultaneous experiments using both the MSD and the new high-pressure isochoric apparatus. Additional modifications were implemented to the automated control system in LabView 6.1. Changing the control loop strategy achieved a better stability and almost zero gradients over the entire range of temperatures. Now the electrical trim heater at the top of the isochoric cell possesses an independent PID control, with the temperature gradient between the bottom and the
top part of the cell used as a set point. This modification reduces gradients present at low temperature.


Figure 22. Low-pressure isochoric apparatus.

### 3.2.2 High Pressure Isochoric Apparatus

The new high pressure isochoric apparatus system includes several principal and ancillary instruments. It consists of an isochoric cell of beryllium cupper ( $\mathrm{Cu}-\mathrm{Be} 175$ ). The cell has been tested up to 340 MPa at room temperature and has a volume of 10.5 $\mathrm{cm}^{378}$. Figure 23 presents a view of the isochoric apparatus.


Figure 23. Cut view of new isochoric apparatus.

The ancillary instruments are: a cylinder storage hot box, feed charging and discharging manifolds, temperature control heat exchangers around the high pressure cell, two external isothermal shields, pressure and temperature measurement systems, a compressor, a vacuum system, a heating/cooling liquid constant-temperature circulation bath and a computer for data acquisition and control. Figure 24 is a picture of the high pressure isochoric apparatus.

The cylinder storage hot box and the feed charging manifold are those described by Atilhan ${ }^{79}$ and Ejaz ${ }^{46}$. Some modifications that allow the simultaneous use for both the MSD and the high pressure isochoric apparatus appear in Figure 25. A system of two three way/two stem connection valves (FV1 and FV2) allow use the vacuum and the feed charging manifold as desired. The valves came from HIP and are operable up to 60,000 psia.


Figure 24. High pressure isochoric apparatus.


Figure 25. Feeding section of both the MSD and the high-pressure isochoric apparatus.

A high pressure manifold allows feeding and discharging the high-pressure isochoric cell. It consists of four high-pressure valves ( $30,000 \mathrm{psia}$ ), a high-pressure hand pump model \#37-6-30 with a capacity of 11 cc , POLYPAK B-1372 (30,000 psia) for fine-tuning the pressure and the tubing lines (up to $60,000 \mathrm{psia}$ ) all coming from HIP. Finally it has a high-pressure gauge for monitoring the inlet pressure to the isochoric cell.


Figure 26. High pressure isochoric manifold.

The configuration of the high pressure isochoric cell and the pressure transducer appears in Figure 27. The tubing line between the isochoric cell and the pressure transducer has a volume less than $0.1 \%$ the volume of the cell as recommended by Matabe ${ }^{80}$. The pressure transducer is an oil-free, absolute pressure resonating crystal pressure transducer, model 430K-101 (Paroscientific, Inc). This instrument has automatic temperature compensation (the equation and parameters are in Appendix A). Location in an aluminum block allows temperature control and guarantees better stability. The aluminum block thermostating system includes a three-lead PRT, an auto-
tune PID temperature controller, a solid state relay (SSR) switch, a cartridge heater (all supplied by Omega Engineering) and a variable AC power supply. The PRT used in the thermostat is a three lead, ceramic encapsulated, $100 \Omega$ PRT (Omega model: RTD-2$1 \mathrm{PT} 100 \mathrm{KN} 2528-108-\mathrm{T})$. The temperature stability achieved by this system is of $\pm 0.1^{\circ} \mathrm{C}$. Additional temperature control of the feeding line uses a simple PID control scheme with Clayborn precision heat tape. The temperature was set to $60{ }^{\circ} \mathrm{C}$ to minimize the moles contained in the tubing line during operation.


Figure 27. Isochoric cell and pressure transducer configuration.

A vacuum system achieves high vacuum in the isochoric apparatus. It consists of a mechanical vacuum pump from Welch Duo-Seal®, model 1402, a diffusion pump model 0159 and a vacuum gauge model 801 from Varian Inc. A cold trap located between the diffusion pump and the vacuum line going to the isochoric apparatus
reduces the backflow of oil molecules. A schematic of the vacuum system appears in Figure 28.


Figure 28. Vacuum system.

### 3.2.2.1 Temperature Control

A robust physical and digital control scheme establishes an optimum temperature control for the high-pressure isochoric apparatus. Figure 29 is a detailed, cut view of the isochoric apparatus. The isochoric apparatus consists of an external aluminum chamber, insulated with a fiberglass layer and an additional layer of spiral-on thermal insulated tape made of a high quality cork and synthetic rubber (Parker Products, Inc). This insulation provides an excellent isolation for the isochoric system from the surroundings. Two internal and external shields are between the external chamber and the isochoric cell. These shields are sources or sinks of heat for temperature control. High vacuum applied to the interior of the external chamber makes radiation the predominant mechanism for heat transfer between the shields and the isochoric cell.

A four wire PRT (Platinum Resistance Thermometer) from Minco® measures the temperature at the bottom of the high-pressure isochoric cell. The measurement methodology is similar to that described by Zhou ${ }^{17}$, Atilhan ${ }^{47}$ and Ejaz ${ }^{46}$. Based upon this technique, the temperature measurements have a resolution of less than 0.1 mK . The calibration parameters for the PRT used in this apparatus are in Appendix B.

Four heaters are located in the isochoric system (labeled as H1, H2, H3 and H4). H1 and H4 control the temperature gradient across the cell. They have a separate PID control loop to keep the temperature gradient below 3 mK . At very low temperatures (around 120 K ), this task becomes more difficult because of heat conduction from the feed line and the aluminum platform to the isochoric cell. Therefore, the low temperature range has a gradient of 10 mK . However, a detailed analysis proves this gradient does not have a significant effect upon the measured pressure. Heaters H2 and H3 are responsible for the cell temperature control. Figure 30 represents the implemented methodology for temperature control. The control scheme was implemented totally in LabView® 8.0. The data acquisition and control occured through a connector block SCB-68 and a data acquisition (DAQ) card PCI-6704 both from National Instrument (NI) Company. The computer uses and sends TTL (transistor-transistor-logic) signals through a PCI-DAQ card to control the on/off action of solid state relays (SSR) that control the heater. Figure 31 shows the control box.


Figure 29. Isochoric apparatus cut view.


Figure 30. Temperature control methodology.


Figure 31. Control box set-up.

### 3.2.3 Phase Loop Determination Technique

A phase envelope is the pressure-temperature boundary for a mixture that separates its single-phase region from its two-phase region. Accurate phase envelope conditions have practical and theoretical uses. For example, the dew point of a natural gas mixture is important because liquid dropout can adversely affect flow measurement. It is also important to avoid heterogeneous flow in gas systems because liquids can damage compressors. A common practice is to operate the gas-gathering operation in the dense phase region above the cricondenbar, Melvin ${ }^{81}$. Accurate measurements are important to check the reliability of EoS. Hydrocarbon dew points in natural gas mixtures are important quality parameters stipulated in contracts and enforced throughout the supply chain.

Experimentally, it is possible to obtain dew or bubble points visually by observing the first appearance of a liquid drop or gas bubble in a vapor-liquid equilibrium cell. This procedure has large uncertainties near the critical point because the liquid and gas have identical properties. Also the procedure depends upon the visual skills of the experimenter. Industrially, a chilled mirror apparatus is popular because of speed and ease of use. However, sensitivity and repeatability depend upon the rate of mirror cooling and the flow rate of the fluid, Warner et al. ${ }^{82}$. Another method for obtaining the pressure-temperature conditions of the phase envelope is measuring pressure and temperature at constant density. Biswas and Ten Seldam ${ }^{83}$ suggested this for pure fluids. Fluid isochores for mixtures at constant overall composition change slope on passing across the boundary. They are collinear only at the cricondentherm. Griffiths proved this behavior theoretically by as shown by Doiron et al. ${ }^{84}$, and Rowlinson et al. ${ }^{85}$ demonstrated the effect theoretically and experimentally.

Duarte-Garza et al. ${ }^{76}$ determined the phase boundary by first fitting a linear or second order temperature function to the single-phase isochoric data and then fit another polynomial to the two-phase isochoric data. The intersection of these two functions determined the phase boundary. Later, Di Nicola et al. ${ }^{86}$ used the same procedure, but employed quadratic polynomials in the single-phase region and an Antoine-type equation for the two-phase region. Zhou et al. ${ }^{77}$ developed a procedure based upon the
residuals obtained from the fit of the temperature function to the single-phase isochoric data.

This work uses the changing slope at the phase envelope to obtain the temperature and pressure at the dew and bubble point. The method developed by Zhou et al. ${ }^{77}$ is modified to improve the selection of the boundary point and to reduce the uncertainty in its determination. Different samples with fixed overall composition were tested and an uncertainty that depends upon the number of components was established.

### 3.2.3.1 Technique

The current technique fits a regression model to the experimental data. It assumes that the errors are uncorrelated random variables with mean zero and constant variance and that the errors have a normal distribution as suggested by Montgomery and Runger ${ }^{87}$. Because it is important that these assumptions be valid, we perform an analysis of the residuals to determine if the model requires additional terms. The following procedure provides the phase boundary pressure and temperature:

1. The isochoric experiment produces a set of data in the single-phase region and another in the two-phase region. As the data approach the phase boundary, it becomes difficult to determine if a point is in the single- or the two-phase region. To avoid omitting any point from the single-phase region, we fit a linear function to the experimental points that we are confident lie in the homogeneous region. If a systematic error appears in the residuals, we use a second-order function. Then, we add points, one-by-one, to the fitting procedure and analyze the residuals. After adding the first point of the heterogeneous region, the residuals exhibit a discontinuous jump. We exclude this point from the fit and retain the isochoric equation for the homogeneous region. The outliers are removed from the single phase by calculating standardized residuals with respect the fit. Nearly $95 \%$ of them should be in the interval $(-2,+2)$ according to the assumption that the errors have a normal distribution (Montgomery and Runger ${ }^{87}$ ). The highest errors
occur near the cricondentherm because the discontinuous change in slope of an isochore is difficult to detect.
2. The pressure values at the experimental temperature of the single- and twophase regions result from using the equation from step 1 and calculating the pressure residuals. These differences increase when crossing into the heterogeneous region as shown in Figure 32.
3. The residuals from the two-phase region are fit with a linear or second order polynomial. Rarely, a third order polynomial is necessary.
4. The temperature intercept is where the pressure residual is null. Figure 32 also shows the residuals and the polynomial passing through the residuals in the twophase region. In the Zhou et al. ${ }^{77}$ procedure, the intersection of the two equations determines the temperature. Their procedure can lead to higher errors in the temperature if the polynomial does not cross the zero line of the residual.
5. Finally, the pressure is estimated using the $P-T$ polynomial obtained in step 1 . This intercept belongs to phase loop, $T_{\mathrm{E}}$, in Figure 33.
6. Because this is an isochoric experiment, the composition of the mixture is known from gravimetric construction.


Figure 32. Pressure residuals in the single and two-phase region: Solid circles are single phase data and hollow circles are two-phase data.

Sometimes, outliers exist in the heterogeneous region (Figure 33 shows an example). The point is an outlier because lies outside the trend line.

Our procedure does not consider the correlation coefficient $\left(\mathrm{R}^{2}\right)$ as a condition of checking a change in slope. In general, a large value of $\mathrm{R}^{2}$ does not imply a steep slope. In addition, it always increases if a variable is added to the model, but this does not necessarily mean that the new model is better than the old one.


Figure 33. Interval errors for the pressure and temperature.

### 3.2.3.2 Uncertainties in Temperature and Pressure

The confidence interval for pressure and temperature to determine the error involved in the procedure is calculable. To obtain the standard deviation of the isochoric equation, we use the number of data points in the single-phase and the Student $t$, the uncertainty for pressure using a confidence level of $95 \%$. The uncertainty for the pressure appears in Figure 33. The uncertainty bounds intercept the polynomial of the heterogeneous region residuals. Hence, two values for the temperature $T_{\mathrm{B}}$ and $T_{\mathrm{C}}$ are
obtained. The phase envelope temperature lies between them, and their difference is the temperature confidence interval. We use the largest temperature and pressure confidences for a given sample. The final value of the uncertainty for our procedure is the square-root of the sum of the quadratic pressure and temperature confidences, Figure 34.


Figure 34. Phase envelope for a 7-component natural gas mixture near the criconderbar.

### 3.2.3.3 Results and Analysis

Mixtures with different numbers of components comprise the samples to calculate the dew and bubble temperatures and pressures. The results for some mixtures that appear in the literature are presented. The first example is an equimolar mixture of $\mathrm{CO}_{2}+\mathrm{N}_{2}$. Duarte-Garza et al. ${ }^{76}$ report the isochoric measurements for this mixture. The data consist of 19 isochores, but the last four isochores have only two points in the twophase region. Consequently, it is not possible to apply the procedure for these data. Table 8 contains the calculated values for the phase boundary with the uncertainty at each point. The uncertainty is established at each point with this procedure along with an overall percentage deviation.

Table 10. Comparison Between Determined Boundary Pressures and Temperatures and Literature Values

| $P$, MPa <br> This work | $\sigma_{P}$ | $T, \mathrm{~K}$ <br> This work | $\sigma_{T}$ | $P$, MPa <br> Ref. ${ }^{76}$ | $T, \mathrm{~K}$ <br> Ref. |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |  |
| $\mathrm{CO}_{2}+\mathrm{N}_{2}$ |  |  |  |  |  |
|  |  |  |  |  |  |
| 21.436 | 0.064 | 209.54 | 0.105 | 21.416 | 208.929 |
| 19.620 | 0.053 | 216.67 | 0.120 | 19.417 | 215.579 |
| 18.189 | 0.087 | 224.78 | 0.213 | 18.153 | 224.230 |
| 17.279 | 0.056 | 232.29 | 0.157 | 17.220 | 231.416 |
| 15.875 | 0.050 | 242.75 | 0.183 | 15.842 | 242.167 |
| 15.313 | 0.044 | 247.02 | 0.191 | 15.282 | 246.524 |
| 14.809 | 0.021 | 251.04 | 0.104 | 14.578 | 250.677 |
| 14.421 | 0.034 | 254.14 | 0.204 | 14.376 | 253.563 |
| 13.950 | 0.016 | 257.02 | 0.109 | 13.916 | 256.581 |
| 13.505 | 0.005 | 259.41 | 0.042 | 13.492 | 259.245 |
| 13.126 | 0.004 | 261.61 | 0.040 | 13.121 | 261.559 |
| 12.364 | 0.005 | 264.68 | 0.078 | 12.387 | 264.922 |
| 11.576 | 0.004 | 266.57 | 0.107 | 11.614 | 266.953 |
| 10.695 | 0.005 | 266.99 | 0.290 | 10.798 | 268.039 |
|  |  |  |  |  |  |
| $\mathrm{CO}_{2}+\mathrm{N}_{2} \mathrm{O}$ |  |  |  |  |  |
|  |  |  |  | Ref. ${ }^{86}$ | Ref. ${ }^{86}$ |
|  |  |  |  |  |  |
|  |  |  |  |  |  |
| 3.0805 | 0.0015 | 272.368 | 0.029 | 3.076 | 272 |
| 2.0752 | 0.0024 | 257.523 | 0.054 | 2.072 | 257 |
| 2.1891 | 0.0008 | 258.785 | 0.018 | 2.187 | 259 |
| 2.8008 | 0.0013 | 267.194 | 0.025 | 2.797 | 267 |
| 2.4121 | 0.0010 | 261.381 | 0.020 | 2.413 | 261 |
| 0.9787 | 0.0004 | 233.168 | 0.017 | 0.981 | 233 |
| 3.8045 | 0.0012 | 278.092 | 0.017 | 3.788 | 278 |
| 3.3226 | 0.0015 | 272.452 | 0.027 | 3.319 | 272 |
| 1.3004 | 0.0008 | 240.645 | 0.023 | 1.301 | 241 |
| 2.8903 | 0.0013 | 267.060 | 0.024 | 2.885 | 267 |
| 4.1892 | 0.0013 | 280.992 | 0.020 | 4.185 | 281 |
| 2.6398 | 0.0017 | 263.310 | 0.020 | 2.635 | 263 |

The second example uses the data for $\mathrm{CO}_{2}+\mathrm{N}_{2} \mathrm{O}$ from Di Nicola et al. ${ }^{86}$. After checking the standardized residuals, it appears that three points of the single-phase region could be outliers or belong to the two-phase region, specifically for the overall compositions $0.6099,0.6953$, and 0.9071 . Including those values in the two-phase region does not result in deterioration of the curve-fit. Linear and quadratic functions represent attempts to fit the isochoric data in the single- and two-phase regions, but in all cases a quadratic polynomial is superior. Table 10 contains the results together with the values given by Di Nicola et al. ${ }^{86}$. They estimate uncertainties of $\pm 1 \mathrm{kPa}$ and $\pm 0.3 \mathrm{~K}$ while our calculations indicate a maximum value of 2.4 kPa and 0.1 K .

The next example is a 7-component synthetic natural gas mixture measured in our laboratory. The accuracy of the temperature and pressure measurements is 0.01 K and 0.002 MPa . These small errors in pressure and temperature measurements are negligible in phase boundary determinations. In this mixture, straight lines represent the single-phase data. The region near the cricondenbar requires a second-order polynomial. In the two-phase region, selection of the order of the polynomial requires a different procedure. The order of the polynomial is increased until the difference between two consecutive, calculated phase boundary temperatures are within the uncertainty and no significant jump in the value occurs. This procedure requires care to avoid over-fitting the data. This particular mixture requires second and third order polynomials. The average deviation in the calculations of the phase boundary temperatures is $2 \sigma=1.3 \mathrm{~K}$ and the average percentage deviation of the pressure is $0.027 \%$. This is the most complicated example examined because it is located in a narrow interval close to the cricondentherm $T>0.93 T_{\text {criconderthem }}$. In this work, the percentage deviation is:
$\%$ deviation $=\frac{\Delta Y}{Y} \times 100$
where $\Delta Y$ is the uncertainty interval in the temperature or pressure and $Y$ is the temperature or pressure at the phase boundary.

Figure 35 presents the phase envelope of a 22 -component mixture ${ }^{88}$. The average deviation in the temperature is $2 \sigma=0.82 \mathrm{~K}$. As seen in Figure 35, the closer the point is to the cricondentherm, the higher the uncertainty of the calculated temperature. Without the points near the cricondentherm, the value of $2 \sigma$ drops to 0.3 K . The average percentage deviation in pressure is $0.022 \%$. Here, second order polynomials and straight lines in the single- and two- phase regions represent the data.


Figure 35. Phase envelope for a 22-component natural gas mixture.

Finally, Figures 36 and 37 present the percentage deviation of the temperature and pressure of all the multicomponent mixtures analyzed in this work. The pressure uncertainty increases near the cricondenbar $(d P / d T=0)$ and the temperature uncertainty increases near the cricondenthem $(\mathrm{d} T / \mathrm{d} P=0)$. The average percentage deviations for the temperature and pressure are $0.45 \%$ and $0.04 \%$.


Figure 36. Percentage deviation of the phase boundary temperature. Natural gas mixtures. $\bigcirc$, 9 -component; $\square$, 7-component; $\triangle$, 9-component; $\nabla$, 9-component; ${ }^{\circ}$, 14component; $\boldsymbol{\Delta}, 15$-component; $\boldsymbol{\nabla}$, 22-component; $\boldsymbol{\bullet}, \mathrm{CH}_{4}+\mathrm{C}_{2} \mathrm{H}_{6}$.


Figure 37. Percentage deviation in the calculation of the phase boundary pressure. Symbols description is the same as in Figure 36.

### 3.2.4 Saturated Density Determination and Isochoric Derivative Corrections

These two effects require compensation to determine the saturation density using isochoric data (actually, the "isochoric" data are isomolar) and phase boundaries. First, a volume change with temperature and pressure exists in the sample cell and in the transmission line between the isochoric cell and the pressure transducer. This effect can be correlated using the thermomechanical properties of both the gas cell and the transmission line materials:

$$
\begin{equation*}
\frac{V(T, P)}{V_{r e f}\left(T_{r e f}, P_{r e f}\right)}=1+\beta\left(T-T_{r e f}\right)+\kappa\left(P-P_{r e f}\right) \tag{3.2.4.1}
\end{equation*}
$$

where $\boldsymbol{\beta}$ is the thermal expansion coefficient, $\boldsymbol{\kappa}$ is the pressure distortion parameter, and $T_{\text {ref }}$ and $P_{\text {ref }}$ are the reference temperature and pressure.

In addition, moles move between the sample cell and the transmission line during the experiment, and the phenomenon is a function of the pressure and the temperature in the sample cell and the transmission line. Figure 38 illustrates the apparatus. Compensation for this effect requires a model. The mole balance is

$$
\begin{equation*}
n_{T}=n_{0}+n_{\text {cell }}, \tag{3.2.4.2}
\end{equation*}
$$

where $n_{T}$ is the total number of moles, $n_{0}$ is the number of moles in the transmission line and $n_{\text {cell }}$ is the number of moles in the cell. Now, substituting the real gas equation into Eq. 3.2.4.2

$$
\begin{equation*}
n_{T}=\frac{P V_{0}}{R T_{0} Z_{0}}+\frac{P V_{\text {cell }}}{R T_{\text {cell }} Z_{\text {cell }}} \tag{3.2.4.3}
\end{equation*}
$$

and using Eq. 3.2.3-1 for the volumes while considering the temperature of the transmission line is constant during the experiment, Eq. 3.2.4.3 becomes

$$
\begin{equation*}
n_{T}=\frac{P}{R}\left\{\frac{V_{r e f}^{0}\left(1+\kappa_{0}\left(P-P_{r e f}\right)\right)}{T_{0} Z_{0}}+\frac{V_{r e f}^{\text {cell }}\left(1+\alpha_{\text {cell }}\left(T-T_{r e f}\right)+\kappa_{\text {cell }}\left(P-P_{r e f}\right)\right)}{T_{\text {cell }} Z_{\text {cell }}}\right\} \tag{3.2.4.4}
\end{equation*}
$$



Figure 38. Schematic of the mass interchange in the low-pressure isochoric apparatus.

The unknown parameters in Eq. 3.2.4.4 are $V_{r e f}^{0}, V_{r e f}^{\text {cell }}$ and $n_{T}$. The total number of moles $n_{T}$ is different for each set of isochoric data. The $Z$-factor is available from the MSD $P \rho T$ data or any reliable EoS such as GERG-2004 ${ }^{15}$. The unknown parameters in Eq. 3.2.4.4 come from fitting the isochoric data. Table 11 contains the low-pressure and the high-pressure isochoric apparatuses parameters. The error introduced during this step corresponds to approximately 30 ppm in density. Determination of saturation densities ( $\rho^{\prime}$ ) requires the number of moles in the cell as a function of temperature. Then extrapolation to the corresponding isochoric temperature provides the saturation value as shown in Figure 39.


Figure 39. Methodology to determine the number of moles for calculating saturation densities.

Table 11. Low and High Pressure Isochoric Apparatus Parameters

| Apparatus | Low <br> Pressure | High <br> Pressure |
| :---: | :---: | :---: |
| $V_{\text {ref }}^{0} / \mathrm{m}^{3}$ | $1.55 \mathrm{E}-07$ | $1.08 \mathrm{E}-05$ |
| $V_{\text {ref }}^{\text {cel }} / \mathrm{m}^{3}$ | $6.08 \mathrm{E}-05$ | $1.08 \mathrm{E}-08$ |
| $\kappa_{\text {cell }} / \mathrm{MPa}^{-1}$ | $4.86 \mathrm{E}-05$ | $1.60 \mathrm{E}-4$ |
| $\beta_{\text {cell }}, \beta_{0} / \mathrm{K}^{-1}$ | $2.53 \mathrm{E}-05$ | $1.6 \mathrm{E}-4$ <br> $12.53 \mathrm{E}-05$ |

Finally, with the number of moles in the cell corresponding to the phase boundary temperature and the volume of the cell, the saturation density is

$$
\begin{equation*}
\rho^{\prime}=\frac{n_{\text {cell }}^{\prime}}{V\left(T^{\prime}, P^{\prime}\right)} \tag{3.2.4.5}
\end{equation*}
$$

where $T^{\prime}$ and $P^{\prime}$ are the phase boundary temperature and pressure. The estimated relative uncertainty for the saturation densities is $0.12 \%$.

The isochoric derivative $(d P / d T)_{\rho}$ is necessary for thermal properties determination as illustrated by Eqs. 3.2.3-6 and 7

$$
\begin{align*}
& U\left(T, V_{2}\right)-U\left(T, V_{1}\right)=\int_{V_{1}}^{V_{2}}\left[T\left(\frac{\partial P}{\partial T}\right)_{V}-P\right] d V  \tag{3.2.4.6}\\
& S\left(T, V_{2}\right)-S\left(T, V_{1}\right)=\int_{V_{1}}^{V_{2}}\left(\frac{\partial P}{\partial T}\right)_{V} d V \tag{3.2.4.7}
\end{align*}
$$

Accurate determination of this derivative leads to accurate thermal data. To compensate for volume change and the mass interchange in the isochoric apparatus, the derivation for the derivative follows. The pressure in the sample cell is a function of temperature and density

$$
\begin{equation*}
P=P(T, \rho) \tag{3.2.4.8}
\end{equation*}
$$

The differential of this function is

$$
\begin{equation*}
d P=\left(\frac{\partial P}{\partial T}\right)_{\rho} d T+\left(\frac{\partial P}{\partial \rho}\right)_{T} d \rho \tag{3.2.4.9}
\end{equation*}
$$

Dividing both sides by $d T$ and imposing the condition of the experiment

$$
\begin{equation*}
\left.\frac{d P}{d T}\right|_{\exp }=\left(\frac{\partial P}{\partial T}\right)_{\rho}+\left.\left(\frac{\partial P}{\partial \rho}\right)_{T} \frac{d \rho}{d T}\right|_{\exp } \tag{3.2.4.10}
\end{equation*}
$$

where the left side of the equation is the derivative measured in the experiment. Then, the quantity required to evaluate the energy functions is

$$
\begin{equation*}
\left(\frac{\partial P}{\partial T}\right)_{\rho}=\left.\frac{d P}{d T}\right|_{\exp }-\left.\left(\frac{\partial P}{\partial \rho}\right)_{T} \frac{d \rho}{d T}\right|_{\exp } \tag{3.2.4.11}
\end{equation*}
$$

The second term in this expression contains the contributions from the non-isochoric nature of the experiment. The density is related to the total volume of the cell, $V_{\text {cell }}$, and the number of moles of fluid in the cell, $n$, by

$$
\begin{equation*}
\rho=\left.\frac{n}{V_{\text {cell }}} \Rightarrow \frac{d \rho}{d T}\right|_{\exp }=\left.\left(\frac{1}{V_{\text {cell }}}\right) \frac{d n}{d T}\right|_{\exp }-\left.\left(\frac{n}{V_{\text {cell }}}\right)\left(\frac{1}{V_{\text {cell }}}\right) \frac{d V_{\text {cell }}}{d T}\right|_{\text {exp }} \tag{3.2.4.12}
\end{equation*}
$$

The first term contains changes that occur because of a noxious volume. A noxious volume is a portion of the sample container that is not at the same temperature as the measuring cell. The second term describes the changes that arise when the volume of the measuring cell varies with temperature and internal pressure.

Cell Volume Changes. The cell volume varies with both temperature and pressure, therefore

$$
\begin{equation*}
d V_{\text {cell }}=\left(\frac{\partial V_{\text {cell }}}{\partial T}\right)_{P} d T+\left(\frac{\partial V_{\text {cell }}}{\partial P}\right)_{T} d P \tag{3.2.4.13}
\end{equation*}
$$

then

$$
\begin{equation*}
\left.\frac{d V_{\text {cell }}}{d T}\right|_{\text {exp }}=V_{\text {cell }}\left(\beta_{\text {cell }}+\left.\kappa_{\text {cell }} \frac{d P}{d T}\right|_{\text {exp }}\right) \tag{3.2.4.14}
\end{equation*}
$$

where the thermal expansion and pressure distortion of the cell are

$$
\begin{align*}
& \text { Thermal expansion: } \beta_{\text {cell }} \equiv \frac{1}{V_{\text {cell }}}\left(\frac{\partial V_{\text {cell }}}{\partial T}\right)_{P}  \tag{3.2.4.15}\\
& \text { Pressure distortion: } \kappa_{\text {cell }} \equiv \frac{1}{V_{\text {cell }}}\left(\frac{\partial V_{\text {cell }}}{\partial P}\right)_{T} \tag{3.2.4.16}
\end{align*}
$$

The numerical values of the thermal expansion and pressure distortion of the cell come from the materials of construction and the geometry of the cell design, and

$$
\begin{equation*}
\left.\frac{d \rho}{d T}\right|_{\exp }=\left.\left(\frac{1}{V_{\text {cell }}}\right) \frac{d n}{d T}\right|_{\exp }-\rho\left(\beta_{\text {cell }}+\left.\kappa_{\text {cell }} \frac{d P}{d T}\right|_{\text {exp }}\right) \tag{3.2.4.17}
\end{equation*}
$$

Noxious Volume Effects. If a portion of the volume containing the sample is at a fixed temperature (in a pressure transducer, for example), then

$$
\begin{equation*}
n+n_{0}=\text { constant } \Rightarrow d n=-d n_{0} \tag{3.2.4.18}
\end{equation*}
$$

where the subscript 0 denotes values for the noxious volume. Then

$$
\begin{equation*}
\left.\frac{d n}{d T}\right|_{\exp }=-\left.\frac{d n_{0}}{d T}\right|_{\exp } \quad \text { where } \quad n_{0}=\rho_{0} V_{0} \tag{3.2.4.19}
\end{equation*}
$$

Here, $\rho_{0}$ is the density in the noxious volume, and $V_{0}$ is the total volume of the noxious volume. Then

$$
\begin{equation*}
\left.\frac{d n_{0}}{d T}\right|_{\exp }=\left.\rho_{0} \frac{d V_{0}}{d T}\right|_{\exp }+\left.V_{0} \frac{d \rho_{0}}{d T}\right|_{\exp } \tag{3.2.4.20}
\end{equation*}
$$

The first term in this equation describes the contribution from changes in the noxious volume during the experiment, and the second term describes effects that arise from the $P-V-T$ behavior of the fluid in the noxious volume.

Noxious Volume Changes. For the noxious volume

$$
\begin{equation*}
d V_{0}=\left(\frac{\partial V_{0}}{\partial T_{0}}\right)_{P} d T_{0}+\left(\frac{\partial V_{0}}{\partial P}\right)_{T_{0}} d P_{0} \tag{3.2.4.21}
\end{equation*}
$$

The noxious volume and measuring volume have direct connection, therefore

$$
\begin{equation*}
P=P_{0} \quad \Rightarrow \quad d P=d P_{0} \tag{3.2.4.22}
\end{equation*}
$$

Dividing by $d T$ and imposing the experimental conditions

$$
\begin{equation*}
\left.\frac{d V_{0}}{d T}\right|_{\exp }=\left.\left(\frac{\partial V_{0}}{\partial T_{0}}\right)_{P} \frac{d T_{0}}{d T}\right|_{\exp }+\left.\left(\frac{\partial V_{0}}{\partial P}\right)_{T_{0}} \frac{d P}{d T}\right|_{\exp }=\left.\beta_{0} V_{0} \frac{d T_{0}}{d T}\right|_{\exp }+\left.\kappa_{0} V_{0} \frac{d P}{d T}\right|_{\exp } \tag{3.2.4.23}
\end{equation*}
$$

where the thermal expansion and pressure distortion for the noxious volume are analogous to those for the measuring cell

$$
\begin{equation*}
\text { Thermal expansion: } \beta_{0} \equiv \frac{1}{V_{0}}\left(\frac{\partial V_{0}}{\partial T}\right)_{P} \tag{3.2.4.24}
\end{equation*}
$$

$$
\begin{equation*}
\text { Pressure distortion: } \kappa_{0} \equiv \frac{1}{V_{0}}\left(\frac{\partial V_{0}}{\partial P}\right)_{T} \tag{3.2.4.25}
\end{equation*}
$$

If the noxious volume is constant, then

$$
\begin{equation*}
\left.\frac{d T_{0}}{d T}\right|_{\exp }=0 \tag{3.2.4.26}
\end{equation*}
$$

and

$$
\begin{equation*}
\left.\frac{d V_{0}}{d T}\right|_{\exp }=\left.\kappa_{0} V_{0} \frac{d P}{d T}\right|_{\exp } \tag{3.2.4.27}
\end{equation*}
$$

Fluid P-V-T Effects. Writing the fluid density as a function of temperature and pressure

$$
\begin{equation*}
d \rho_{0}=\left(\frac{\partial \rho_{0}}{\partial T_{0}}\right)_{P} d T_{0}+\left(\frac{\partial \rho_{0}}{\partial P}\right)_{T_{0}} d P \tag{3.2.4.28}
\end{equation*}
$$

and, as for the previous case,

$$
\begin{align*}
\left.\frac{d \rho_{0}}{d T}\right|_{\exp } & =\left.\left(\frac{\partial \rho_{0}}{\partial T_{0}}\right)_{P} \frac{d T_{0}}{d T}\right|_{\exp }+\left.\left(\frac{\partial \rho_{0}}{\partial P}\right)_{T_{0}} \frac{d P}{d T}\right|_{\exp }=0+\left.\left(\frac{\partial \rho_{0}}{\partial P}\right)_{T_{0}} \frac{d P}{d T}\right|_{\exp }  \tag{3.2.4.29}\\
& =\left.\left(\frac{\partial \rho_{0}}{\partial P}\right)_{T_{0}} \frac{d P}{d T}\right|_{\exp }
\end{align*}
$$

Combined Effects. Collecting the noxious volume results gives

$$
\begin{equation*}
\left.\frac{d n_{0}}{d T}\right|_{\exp }=\left.\rho_{0} V_{0} \kappa_{0} \frac{d P}{d T}\right|_{\exp }+\left.V_{0}\left(\frac{\partial \rho_{0}}{\partial P}\right)_{T_{0}} \frac{d P}{d T}\right|_{\exp }=\left.V_{0}\left(\rho_{0} \kappa_{0}+\left(\frac{\partial \rho_{0}}{\partial P}\right)_{T_{0}}\right) \frac{d P}{d T}\right|_{\exp } \tag{3.2.4.30}
\end{equation*}
$$

and

$$
\begin{align*}
\left.\frac{d \rho}{d T}\right|_{\text {exp }} & =-\left.\frac{V_{0}}{V_{\text {cell }}}\left(\rho_{0} \kappa_{0}+\left(\frac{\partial \rho_{0}}{\partial P}\right)_{T_{0}}\right) \frac{d P}{d T}\right|_{\text {exp }}-\rho\left(\beta_{\text {cell }}+\left.\kappa_{\text {cell }} \frac{d P}{d T}\right|_{\text {exp }}\right)  \tag{3.2.4.31}\\
& =-\rho \beta_{\text {cell }}-\left.\left[\rho \kappa_{\text {cell }}+\gamma\left(\rho_{0} \kappa_{0}+\left(\frac{\partial \rho_{0}}{\partial P}\right)_{T_{0}}\right)\right] \frac{d P}{d T}\right|_{\exp }
\end{align*}
$$

Combining the measuring cell and noxious volume results

$$
\begin{align*}
\left(\frac{\partial P}{\partial T}\right)_{\rho} & =\left.\frac{d P}{d T}\right|_{\text {exp }}-\left\{-\rho \beta_{\text {cell }}-\left.\left[\rho \kappa_{\text {cell }}+\gamma\left(\rho_{0} \kappa_{0}+\left(\frac{\partial \rho_{0}}{\partial P}\right)_{T_{0}}\right)\right] \frac{d P}{d T}\right|_{\exp }\right\}\left(\frac{\partial P}{\partial \rho}\right)_{T}  \tag{3.2.4.32}\\
& =\left.\frac{d P}{d T}\right|_{\text {exp }}+\left(\frac{\partial P}{\partial \rho}\right)_{T}\left\{\rho \beta_{\text {cell }}+\left.\left[\gamma\left(\frac{\partial \rho_{0}}{\partial P}\right)_{T_{0}}+\rho \kappa_{\text {cell }}+\gamma \rho_{0} \kappa_{0}\right] \frac{d P}{d T}\right|_{\text {exp }}\right\}
\end{align*}
$$

Rearranging Eq. 3.2.4.32:

$$
\begin{align*}
\left(\frac{\partial P}{\partial T}\right)_{\rho} & =\left.\frac{d P}{d T}\right|_{\exp }+P \frac{\rho}{P}\left(\frac{\partial P}{\partial \rho}\right)_{T}\left\{\beta_{\text {cell }}+\left.\left[\frac{\gamma}{\rho}\left(\frac{\partial \rho_{0}}{\partial P}\right)_{T_{0}}+\kappa_{\text {cell }}+\gamma \kappa_{0} \frac{\rho_{0}}{\rho}\right] \frac{d P}{d T}\right|_{\text {exp }}\right\} \\
& =\left.\frac{d P}{d T}\right|_{\exp }+P\left(\frac{\partial \ln P}{\partial \ln \rho}\right)_{T}\left\{\beta_{\text {cell }}+\left.\left[\frac{\gamma}{\rho}\left(\frac{\partial \rho_{0}}{\partial P}\right)_{T_{0}}+\kappa_{\text {cell }}+\gamma \kappa_{0} \frac{\rho_{0}}{\rho}\right] \frac{d P}{d T}\right|_{\exp }\right\} \tag{3.2.4.33}
\end{align*}
$$

Eq. 3.2.4.33 provides the correction to the experimental isochoric derivatives and allows the determination of caloric properties more accurately.

### 3.3 Composition Data

In order to establish the actual coordinates into the thermodynamics space for a specific state of a fluid requires the composition. This fact is the most important and difficult task when determining mixtures properties. Composition is the major source of error and uncertainty in the specification of natural gas properties. In fact, it contributes around $75 \%$ with of the total $2 \sigma$ uncertainty in the determination of natural gas densities, Figure 40.

Therefore special emphasis has been given to the determination of the natural gas mixtures composition, which is usually done by gas chromatography (GC). In its final report in 2003, "Preparation of Natural Gas Blends Used as Calibration Standards: Sources of Uncertainty and Best Preparation Practices", the Southwest Research Institute ${ }^{\circledR}$ provides an important analysis of the influence of natural gas composition in the determination of natural gas price during custody transfer ${ }^{89}$. Two hypothetic scenarios were evaluated to examine the effect of GC accuracy on the computed heating values:


Figure 40. Temperature, pressure and composition contribution to the total density uncertainty for a synthetic natural gas mixture at $1 \sigma$.

1. A simulated error in the analytical hexane content of $\pm 0.1 \%$ for a 1050 Btu/scf gas. The other components were renormalized proportionately to return the total $100 \mathrm{~mol} \%$.
2. The modification of the original composition to simulate errors of $\pm 0.1 \%$ into a lumped $\mathrm{C}_{6+}$ fraction.

The two evaluated examples lead to errors in the heating value of up to -4.27 Btu/scf, or $-0.4 \%$ that corresponds to an error of $\$ 584,000$ per year for a pipeline gas flowing at 100 MMSCFD. The report concludes that: "Large uncertainties in the heavier components introduce the potential for errors in GC calibration, specifically by biasing the computed response factor for those components" ${ }^{89}$. Additionally, the report notes that few certified gas standards were available that contained ppm levels of $\mathrm{C}_{9}$ and heavier components.

Based upon the relevance of the mixture composition in the determination of accurate thermodynamics properties, this project proposes an alternative methodology. The fundamental idea is to take advantage of the high accuracy GC analysis for key compounds in the gas mixtures and propagate it to the heavy hydrocarbons via Coherent Anti-Stoke Raman Scattering (CARS).

### 3.3.1 Coherent Anti-Stokes Raman Scattering (CARS)

Since 1928, when Raman found that a substance irradiated with light of a certain frequency scatters not only light with the same frequency of the incident beam but also lines with lower (Stokes) and higher (anti-Stokes) frequencies, the use of the Raman spectrum has become an important tool for characterizing molecular structure of materials ${ }^{90}$.

Recently, Pestov et al. ${ }^{91}$ have developed a hybrid technique to enhance the efficiency of the Raman scattering process caused by the broadband preparation with an order of $10^{5}$ compared to former techniques. Figure 41 illustrates the general principle of this technique.


Figure 41. CARS energy scheme.

The coherent anti-stokes Raman scattering technique is based upon the irradiation of a sample with two coherent laser beams (pump and stokes) that generate a resonant state between the ground state and the vibrational state of the molecules in the sample; then a third laser beam (probe beam) is used to scatter off an additional blueshift high frequency signal $\left(\omega_{\text {Pump }}-\omega_{\text {Stokes }}\right)+\omega_{\text {Probe }}$ (anti-stokes). Essentially, this new technique uses the shifting of the probe beam to minimize the non-resonant (NR) fourwave mixing (FWM) signal from other molecules. Therefore, using the ratios between the Raman lines in the normalized background-free CARS spectra can provide real-time monitoring of gas composition.

In order to evaluate the performance of this new approach determining natural gas mixture compositions using CARS, a high pressure gas chamber has been designed and built. The characteristics of the gas chamber appear in the Table 12 and in Figure
42. The gas cell needed to work at high-pressure to improve the signal intensity in the spectrogram.

## Table 12. CARS Gas Chamber Characteristics

| Material:Ultra-Violet quality synthetic fused silica. |
| :--- |
| Refractive index: $1.46 @ 500 \mathrm{~nm}$ |
| Surface flatness: $1 / 10$ |
| Parallelism: $\leq 5 \operatorname{arcsec}$ or 3 arcmin |
| Pressures up to 1000 psia |



Figure 42. CARS gas chamber.

This preliminary evaluation was performed in cooperation with the quantum optics group and the thermodynamics research group at Texas A\&M University. The schematic for the Raman technique is Figure 43.


Figure 43. Setup schematics. G1,2, gratings; DS1,2, delay stages; BS1,2, beam splitters, ND+SPF, a set of neutral density and shortpass filters; CCD, charge-coupled device ${ }^{92}$.

The test was performed with a gravimetrically prepared sample, at room temperature and pressure. The background-free CARS spectra were determined by tuning the probe signal. The CARS spectrogram appears in Figure 44. The normalized spectra and the gas composition are Figure 45. A remarkable base line results for the normalized spectra in Figure 45. This result leads to the hypothesis of using the "ratiomethod" ${ }^{13}$ coupled with GC analysis to minimize the uncertainty of composition determination for natural gas mixtures.

The proposed methodology is: first, characterize some key compound using GC analysis. Uncertainties of less than $0.04 \%$ are achievable for these key compounds (methane, carbon dioxide, etc). Then at the beginning of sampling, the compositions for these key compounds must be determined via GC analysis. Second, the CARS analysis must be performed at constant temperature and pressure. Here, an accurate control of these variables is necessary because of the strong interdependency of the signal intensity,
temperature and pressure. Finally, based upon the composition of the key compound and the spectra information, the composition of the unknown compounds can be determined using the "ratio-method" from the correlation between the gas compositions, the area of the bands $A_{x\left(v_{x}\right)}$ at frequencies $v_{x}$ and the relative normalized differential Raman scattering (RNDRS) cross sections (Eq 3.3.1.1). From Eq. 1, a direct propagation of the low uncertainty of the key compounds compositions from GC can be expected.

$$
\begin{equation*}
\frac{C_{x}}{C_{\text {methane }}}=\frac{A_{x\left(v_{x}\right)} \cdot \sum_{\text {methane }\left(v_{l}\right)}}{A_{\text {methane }\left(v_{l}\right)} \cdot \sum_{x(x x)}} \tag{3.3.1.1}
\end{equation*}
$$

Raman spectroscopy has been used in the past for the determination of natural gas compositions. However, the characteristics of the precedent studies ${ }^{93-95}$ were not as good as the new methodology for CARS developed by Pestov et al. ${ }^{92}$. Uncertainties of 0.002 in the mole fraction were obtained in the past, while the new methodology should achieve uncertainties less than 0.0005 in the mole fraction for the long chain and low composition compounds present in natural gas mixtures.


Figure 44. CARS spectrogram.


Figure 45. CARS spectra normalized on the reference FWM profiles.

## EXPERIMENTAL RESULTS FOR SYNTHETIC NATURAL GAS MIXTURES

After the development and improvement of all the necessary apparatus and methodologies for the isothermal densimeter and the isochoric apparatus, new measurements for natural gas mixtures were collected and old measurements were corrected and used to determine additional properties such us the saturation densities. The following analysis focuses upon the measurement of a new, ternary natural gas sample and the correction of density and isochoric data of four synthetic natural mixtures.

### 4.1 Ternary P $\boldsymbol{P} \boldsymbol{T}$ Data

Residual or pipeline natural gas is the principal product of a natural gas processing plant. Its importance as an energy source for industrial process, residential and commercial uses, transportation and generation of electric power is unquestionable ${ }^{3}$. Although its composition is variable, a ternary mixture of methane, ethane and propane is a suitable surrogate. Accurate characterization of such mixtures has been a fundamental research problem for many years ${ }^{15,96}$. Accurate $P \rho T$ data combined with experimental phase boundaries are necessary to develop and validate reference EoS. The formulation of highly accurate EoS for mixtures is an important research topic that depends upon the reliability of experimental data ${ }^{97}$.

During the past twenty years, new reference EoS for the main compounds of natural gas have appeared ${ }^{5-9}$. Recently, the Thermodynamics Laboratory at Texas A\&M University has produced new, highly-accurate data at high pressure for methane ${ }^{98}$, ethane ${ }^{99}$, nitrogen ${ }^{100}$ and carbon dioxide ${ }^{101}$ using a magnetic-suspension, single-sinker densimeter. These data have demonstrated that the fundamental EoS used as reference standards for these compounds ${ }^{26}$ behaves well at high pressure. This is apparent in the preceeding chapter. This work presents new $P \rho T$ data for a residual natural gas sample as part of a systematic study to validate and support natural gas standard EoS such as AGA8 DC92 ${ }^{102}$ and GERG-2004 ${ }^{15}$ at high pressure.

In addition, equilibrium data have been measured with the low-pressure isochoric apparatus. The results are compared to the Peng-Robinson ${ }^{21}$ EoS, which is commonly used in industry for phase equilibrium calculations using a phase behavior simulator developed in the Laboratory. Finally, the new methodology to predict saturation densities using isochoric data and to correct isochoric derivatives is applied.

### 4.1.1 Experimental

Isothermal density data for a ternary mixture at $(300,350$ and 400) K up to 200 MPa as well as isochoric and equilibrium data were measured. The ternary mixture came from DCG PARTNERSHIP Inc. having a molar composition of 95.039 \% methane, $3.961 \%$ ethane and $1.000 \%$ propane with $\pm 0.037 \%$ estimated gravimetric uncertainty (NIST traceable by weight). The characteristics of the MSD and the isochoric apparatuses appeared in the third chapter.

### 4.1.2 Results and Analysis

The density data for the sample and their comparisons to GERG-2004 and AGA8-DC2 predictions (implemented in RefProp 8.0, ${ }^{26}$ ) appear in Table 13, and the deviations are in Figure 46. This figure indicates that GERG-2004 has better predictive capability across the range of pressure than AGA8-DC2. GERG-2004 predicts density data with a relative deviation of approximately $0.02 \%$ up to 170 MPa . This result is consistent with those found previously for pure component density data at high pressure ${ }^{98-99}$. Therefore, it appears that the approach developed by different authors recently ${ }^{15,} 97$ to construct multiparameter EoS can predict high-pressure density data with excellent accuracy, at least up to 200 MPa . AGA8-DC2 has a relative deviation of $0.04 \%$ across the pressure range. Using the procedure described by Cristancho et al. ${ }^{98}$ provides the second and the third virial coefficients, which appear in Table 14. The estimated absolute uncertainty for the second and the third virial coefficients are $0.57 \mathrm{~cm}^{3} \cdot \mathrm{~mol}^{-1}$ and $125\left(\mathrm{~cm}^{3} \cdot \mathrm{~mol}^{-1}\right)^{2}$ respectively.

The procedure using isochoric data to determine the phase boundaries was described in Chapter 3 and represented in Figure 47. The predicted phase envelope that appears in Figure 48 is a calculation using the Peng-Robinson equation with binary interaction parameters determined from equilibrium data for natural gas mixtures in Table 15. The equation predictions follow the trend of the experimental data, but they have higher deviations approaching the cricondenbar.

Table 13. Density Values

| P/MPa | $\rho / \underset{3}{\mathrm{~kg}} \cdot \mathrm{~m}^{-}$ | $\rho / \mathrm{kg} \cdot \mathrm{~m}^{-3}$ <br> (GERG-2004) | $\rho / \mathrm{kg} \cdot \mathrm{~m}^{-3}$ <br> (AGA8-DC2) | 100 $\cdot(\rho-\rho G E R G) / \rho$ | $\begin{gathered} 100 \cdot(\rho-\rho A G A 8- \\ D C 2) / \rho \end{gathered}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{T}=300.000 \mathrm{~K}$ |  |  |  |  |  |
| 4.965 | 36.976 | 36.988 | 36.991 | -0.032 | -0.040 |
| 5.998 | 45.559 | 45.570 | 45.574 | -0.023 | -0.032 |
| 6.994 | 54.107 | 54.122 | 54.125 | -0.027 | -0.032 |
| 8.002 | 63.008 | 63.024 | 63.028 | -0.025 | -0.032 |
| 9.998 | 81.252 | 81.275 | 81.278 | -0.028 | -0.032 |
| 12.427 | 103.999 | 104.020 | 104.030 | -0.020 | -0.029 |
| 14.992 | 127.615 | 127.640 | 127.640 | -0.019 | -0.019 |
| 20.017 | 168.796 | 168.810 | 168.780 | -0.008 | 0.009 |
| 25.012 | 200.944 | 200.940 | 200.910 | 0.002 | 0.017 |
| 30.021 | 225.761 | 225.740 | 225.710 | 0.009 | 0.023 |
| 44.944 | 274.513 | 274.470 | 274.350 | 0.016 | 0.059 |
| 49.920 | 285.997 | 285.970 | 285.820 | 0.010 | 0.062 |
| 50.254 | 286.701 | 286.680 | 286.540 | 0.007 | 0.056 |
| $\mathrm{T}=350.000 \mathrm{~K}$ |  |  |  |  |  |
| 2.002 | 11.854 | 11.857 | 11.857 | -0.024 | -0.024 |
| 4.998 | 30.447 | 30.452 | 30.455 | -0.017 | -0.027 |
| 10.001 | 63.246 | 63.251 | 63.263 | -0.007 | -0.026 |

Table 13. Continued

| $P / \mathbf{M P a}$ | $\rho / \underset{3}{\mathbf{k g} \cdot \mathbf{m}^{-}}$ | $\rho / \mathrm{kg} \cdot \mathrm{~m}^{-3}$ <br> (GERG-2004) | $\rho / \mathbf{k g} \cdot \mathrm{m}^{-3}$ <br> (AGA8-DC2) | 100•( $\rho-\rho$ gerg $) / \rho$ | $\begin{gathered} 100 \cdot(\rho-\rho A G A 8- \\ D C 2) / \rho \end{gathered}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 19.987 | 128.080 | 128.070 | 128.090 | 0.008 | -0.008 |
| 30.019 | 180.470 | 180.460 | 180.410 | 0.006 | 0.033 |
| 39.996 | 218.303 | 218.300 | 218.280 | 0.001 | 0.011 |
| 49.953 | 246.399 | 246.350 | 246.320 | 0.020 | 0.032 |
| 69.915 | 286.052 | 286.050 | 285.960 | 0.001 | 0.032 |
| 79.946 | 301.044 | 301.060 | 300.940 | -0.005 | 0.035 |
| 99.921 | 325.172 | 325.190 | 325.060 | -0.006 | 0.034 |
| 119.976 | 344.390 | 344.400 | 344.270 | -0.003 | 0.035 |
| 139.909 | 360.259 | 360.240 | 360.150 | 0.005 | 0.030 |
| 149.927 | 367.348 | 367.310 | 367.240 | 0.010 | 0.030 |
| 169.848 | 380.063 | 379.990 | 379.970 | 0.019 | 0.025 |
| $\mathrm{T}=400.000 \mathrm{~K}$ |  |  |  |  |  |
| 4.999 | 25.994 | 25.993 | 25.996 | 0.003 | -0.008 |
| 10.004 | 52.804 | 52.802 | 52.814 | 0.003 | -0.019 |
| 19.992 | 104.944 | 104.920 | 104.960 | 0.023 | -0.015 |
| 30.035 | 150.199 | 150.170 | 150.170 | 0.019 | 0.019 |
| 40.009 | 186.143 | 186.130 | 186.090 | 0.007 | 0.028 |
| 49.929 | 214.526 | 214.470 | 214.450 | 0.026 | 0.035 |
| 59.959 | 237.631 | 237.590 | 237.600 | 0.017 | 0.013 |
| 69.948 | 256.657 | 256.640 | 256.640 | 0.006 | 0.006 |
| 79.919 | 272.755 | 272.750 | 272.740 | 0.002 | 0.005 |
| 89.963 | 286.787 | 286.790 | 286.760 | -0.001 | 0.009 |
| 99.971 | 299.071 | 299.080 | 299.030 | -0.003 | 0.014 |
| 119.622 | 319.587 | 319.580 | 319.510 | 0.002 | 0.024 |
| 139.783 | 337.056 | 337.020 | 336.940 | 0.011 | 0.035 |



Figure 46. Percentage deviations of the experimental $P \rho T$ data from values calculated using the GERG-2004 and AGA8-DC2 ${ }^{102}$. GERG-2004 • $300.000 \mathrm{~K}, ~ \triangle 350.000 \mathrm{~K}$, 400 ; AGA8-DC2 $\circ 400 \mathrm{~K}, \Delta 350.000 \mathrm{~K}, \diamond 450.000 \mathrm{~K}$.

Table 14. Second and Third Virial Coefficients

| $\mathbf{T} / \mathbf{K}$ | $\mathbf{B} /\left(\mathbf{c m}^{\mathbf{3}} \mathbf{m o l}^{-\mathbf{1}}\right)$ | $\mathbf{C} /\left(\mathbf{c m}^{\mathbf{3}} \mathbf{m o l}^{\mathbf{- 1}}\right)^{\mathbf{2}}$ |
| :---: | :---: | :---: |
| 300.000 | -47.5407 | 2638.812 |
| 350.000 | -31.0707 | 2539.004 |
| 400.000 | -19.437 | 2338.606 |



Figure 47. Isochoric experimental design.


Figure 48. Experimental phase boundary. $\star$ Experimental data; ___ Peng-Robinson EoS.

## Table 15. Binary Coefficient Parameters for Peng -Robinson EoS

| Methane |  |  |  |
| :---: | :---: | :---: | :---: |
| Methane | - | Ethane |  |
| Ethane | -0.0021 | - | Propane |
|  |  | 0.008 | - |

The saturation densities and the correction for the isochoric derivatives used the procedures describes in the Chapter III. Figure 49 presents the experimental temperature and density diagram for the ternary sample. The numerical values are in Table 16.


Figure 49. Experimental temperature-density diagram.

Table 16. Phase Boundary Data

| $\mathbf{T} / \mathbf{K}$ | $\mathbf{P} / \mathbf{M P a}$ | $\boldsymbol{\rho}^{\boldsymbol{\prime}} / \mathbf{K g m}^{-\mathbf{3}}$ |
| :--- | :---: | :---: |
| 206.540 | 5.972 | 167.946 |
| 208.340 | 5.734 | 125.913 |
| 209.756 | 5.224 | 91.623 |
| 209.879 | 4.350 | 63.656 |
| 208.174 | 3.436 | 45.159 |
| 205.999 | 2.735 | 33.829 |
| 203.150 | 2.127 | 25.275 |
| 189.149 | 0.845 | 9.795 |

A comparison between the corrected experimental derivatives and GERG-2004 EoS appear in Figure 50. A deviation of $0.5 \%$ exists with GERG-2004 for points distant from the phase loop that becomes $1 \%$ for the points close to the phase loop.


Figure 50. Relative deviations of the corrected $(d P / d T)_{P}$ derivative compared to GERG-2004.

### 4.2 Synthetic Natural Gas

Density and equilibrium data were measured for four synthetic natural gas mixtures using the MSD and the low pressure isochoric apparatus. The final data have been corrected using the methodologies for the isothermal densities and the isochoric data as presented in Chapter III. The apparatus contribution selected for the FTE was $\phi_{0}=200 \mathrm{ppm}$. The compositions for the four synthetic samples were:

Table 17. Mixture Compositions (Mole Fraction)

|  | SAMPLE | SAMPLE | SAMPLE | SAMPLE |
| :--- | :---: | :---: | :---: | :---: |
|  | $\mathbf{1}$ | $\mathbf{2}$ | $\mathbf{3}$ | $\mathbf{4}$ |
| methane | 0.89990 | 0.89982 | 0.89975 | 0.90001 |
| ethane | 0.03150 | 0.03009 | 0.02855 | 0.04565 |
| propane | 0.01583 | 0.01506 | 0.01427 | 0.02243 |
| i-butane | 0.00781 | 0.00752 | 0.00709 | 0.01140 |
| n-butane | 0.00790 | 0.00753 | 0.00722 | 0.01151 |
| i-pentane | 0.00150 | 0.00300 | 0.00450 | 0.00450 |
| n-pentane | 0.00150 | 0.00300 | 0.00450 | 0.00450 |
| nitrogen | 0.01699 | 0.01697 | 0.01713 | - |
| carbon <br> dioxide | 0.01707 | 0.01701 | 0.01699 | - |

The $P \rho T$ data for samples 1 to 4 appear in Tables 18 to 21 along with comparisons to GERG-2004 ${ }^{15}$ and AGA8-DC2 ${ }^{30}$. Figures 51 to 54 represent the density deviations compared to GERG-2004 and AGA8-DC2 for the four synthetic samples. Highest deviations for all the samples occur at 250 K , therefore low temperature data are necessary to improve the predictability of these multiparameter equations of state. No significant difference occurs between the two EoS for the other temperatures.

Table 18. Po T Data for Sample 1

| T/K | P/MPa | $\rho / \mathrm{kgm}^{-3}$ | $\rho G E R G / \mathrm{kgm}^{-3}$ | $\begin{gathered} 100 \cdot(\rho- \\ \rho G E R G) / \rho \end{gathered}$ | $\begin{gathered} \rho A G A 8- \\ D C 2 / \mathrm{kgm}^{-3} \end{gathered}$ | $\begin{gathered} \hline 100 \cdot(\rho- \\ \rho A G A 8- \\ D C 2) / \rho \end{gathered}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 250.020 | 20.023 | 270.886 | 270.734 | -0.056 | 272.353 | 0.542 |
| 250.028 | 29.950 | 314.558 | 314.024 | -0.170 | 315.695 | 0.362 |
| 250.026 | 49.915 | 359.992 | 359.315 | -0.188 | 360.489 | 0.138 |
| 250.003 | 75.019 | 393.058 | 392.694 | -0.092 | 393.240 | 0.046 |
| 250.032 | 100.046 | 416.584 | 415.937 | -0.155 | 416.000 | -0.140 |
| 250.038 | 125.013 | 434.094 | 434.070 | -0.005 | 433.751 | -0.079 |
| 250.068 | 149.917 | 449.647 | 449.043 | -0.134 | 448.403 | -0.277 |
| 350.002 | 9.941 | 69.358 | 69.525 | 0.241 | 69.561 | 0.292 |
| 349.999 | 29.912 | 198.375 | 198.371 | -0.002 | 198.505 | 0.066 |
| 349.997 | 49.954 | 268.390 | 268.265 | -0.046 | 268.596 | 0.077 |
| 349.983 | 74.917 | 317.783 | 317.658 | -0.039 | 317.998 | 0.068 |
| 350.009 | 99.972 | 350.354 | 350.285 | -0.020 | 350.595 | 0.069 |
| 349.996 | 125.109 | 374.884 | 374.735 | -0.040 | 375.030 | 0.039 |
| 349.990 | 149.891 | 394.219 | 394.060 | -0.040 | 394.360 | 0.036 |
| 350.029 | 155.022 | 397.788 | 397.614 | -0.044 | 397.917 | 0.033 |
| 450.038 | 9.966 | 49.749 | 49.957 | 0.418 | 49.975 | 0.454 |
| 450.045 | 29.976 | 141.437 | 141.498 | 0.043 | 141.547 | 0.078 |
| 450.043 | 49.975 | 206.865 | 206.769 | -0.046 | 206.811 | -0.026 |
| 450.059 | 68.914 | 249.744 | 249.671 | -0.029 | 249.821 | 0.031 |
| 450.054 | 86.155 | 279.125 | 279.015 | -0.039 | 279.220 | 0.034 |
| 450.027 | 114.952 | 315.989 | 315.903 | -0.027 | 316.101 | 0.036 |
| 450.013 | 137.492 | 338.320 | 338.223 | -0.028 | 338.393 | 0.022 |

Table 19. PoT Data for Sample 2

| T/K | P/MPa | $\rho / \mathrm{kgm}^{-3}$ | $\rho G E R G / \mathrm{kgm}^{-3}$ | $\begin{gathered} 100 \cdot(\rho- \\ \rho G E R G) / \rho \end{gathered}$ | $\begin{gathered} \rho A G A 8- \\ D C 2 / \mathbf{k g m}^{-} \end{gathered}$ | $\begin{gathered} \hline 100 \cdot(\rho- \\ \rho A G A 8- \\ D C 2) / \rho \end{gathered}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 250.076 | 12.023 | 190.110 | 190.995 | 0.465 | 192.411 | 1.210 |
| 250.019 | 14.014 | 219.762 | 220.323 | 0.255 | 222.032 | 1.033 |
| 249.999 | 16.024 | 242.106 | 242.208 | 0.042 | 244.129 | 0.835 |
| 249.968 | 17.998 | 259.073 | 258.775 | -0.115 | 260.859 | 0.690 |
| 249.942 | 20.033 | 273.069 | 272.472 | -0.219 | 274.679 | 0.590 |
| 249.894 | 21.996 | 284.284 | 283.503 | -0.274 | 285.793 | 0.531 |
| 249.978 | 24.012 | 293.915 | 292.991 | -0.314 | 295.327 | 0.480 |
| 250.051 | 26.003 | 302.193 | 301.186 | -0.333 | 303.541 | 0.446 |
| 250.090 | 27.494 | 307.793 | 306.737 | -0.343 | 309.096 | 0.423 |
| 249.986 | 14.995 | 231.398 | 231.848 | 0.195 | 233.669 | 0.982 |
| 250.033 | 29.999 | 316.320 | 315.294 | -0.325 | 317.641 | 0.417 |
| 250.055 | 49.993 | 361.260 | 360.256 | -0.278 | 362.149 | 0.246 |
| 249.991 | 68.972 | 387.532 | 386.629 | -0.233 | 388.050 | 0.134 |
| 249.997 | 100.218 | 417.529 | 416.737 | -0.190 | 417.550 | 0.005 |
| 249.969 | 149.856 | 450.256 | 449.633 | -0.138 | 449.760 | -0.110 |
| 349.963 | 9.975 | 70.094 | 70.250 | 0.222 | 70.290 | 0.279 |
| 349.984 | 11.967 | 85.109 | 85.266 | 0.184 | 85.322 | 0.250 |
| 350.000 | 13.958 | 100.016 | 100.200 | 0.184 | 100.275 | 0.259 |
| 349.992 | 15.985 | 114.995 | 115.132 | 0.119 | 115.226 | 0.202 |
| 350.000 | 17.955 | 129.035 | 129.178 | 0.110 | 129.291 | 0.198 |
| 350.018 | 19.959 | 142.743 | 142.834 | 0.064 | 142.965 | 0.156 |
| 349.982 | 21.940 | 155.617 | 155.658 | 0.026 | 155.807 | 0.122 |
| 349.996 | 23.914 | 167.667 | 167.641 | -0.016 | 167.810 | 0.085 |

Table 19. Continued

| T/K | P/MPa | $\rho / \mathrm{kgm}^{-3}$ | $\rho G E R G / \mathrm{kgm}^{-3}$ | $\begin{gathered} 100 \cdot(\rho- \\ \rho G E R G) / \rho \end{gathered}$ | $\begin{gathered} \rho A G A 8- \\ D C 2 / \mathbf{k g m}^{-} \end{gathered}$ | $\begin{gathered} 100 \cdot(\rho- \\ \rho A G A B- \\ D C 2) / \rho \end{gathered}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 349.993 | 25.879 | 178.859 | 178.819 | -0.023 | 179.014 | 0.087 |
| 349.996 | 27.959 | 189.911 | 189.836 | -0.039 | 190.067 | 0.082 |
| 349.992 | 29.877 | 199.491 | 199.300 | -0.096 | 199.571 | 0.040 |
| 349.983 | 9.998 | 70.244 | 70.418 | 0.248 | 70.458 | 0.304 |
| 350.020 | 29.988 | 199.950 | 199.808 | -0.071 | 200.081 | 0.066 |
| 350.030 | 49.989 | 269.799 | 269.323 | -0.176 | 269.968 | 0.063 |
| 349.992 | 68.941 | 309.320 | 308.790 | -0.171 | 309.555 | 0.076 |
| 350.028 | 99.948 | 351.590 | 351.015 | -0.163 | 351.844 | 0.072 |
| 350.012 | 149.913 | 395.369 | 394.732 | -0.161 | 395.641 | 0.069 |
| 450.006 | 10.054 | 50.657 | 50.693 | 0.071 | 50.711 | 0.107 |
| 450.034 | 11.969 | 60.380 | 60.319 | -0.100 | 60.344 | -0.059 |
| 450.001 | 13.976 | 70.404 | 70.316 | -0.125 | 70.348 | -0.079 |
| 450.018 | 15.961 | 80.164 | 80.048 | -0.145 | 80.090 | -0.093 |
| 450.016 | 17.971 | 89.876 | 89.727 | -0.165 | 89.779 | -0.108 |
| 449.923 | 19.975 | 99.375 | 99.182 | -0.195 | 99.243 | -0.133 |
| 449.939 | 21.977 | 108.611 | 108.363 | -0.228 | 108.433 | -0.163 |
| 449.919 | 23.985 | 117.585 | 117.315 | -0.229 | 117.393 | -0.163 |
| 449.999 | 25.976 | 126.186 | 125.877 | -0.245 | 125.961 | -0.179 |
| 449.986 | 27.979 | 134.578 | 134.239 | -0.252 | 134.326 | -0.187 |
| 449.938 | 29.978 | 142.607 | 142.303 | -0.213 | 142.393 | -0.150 |
| 449.986 | 9.994 | 50.300 | 50.394 | 0.185 | 50.412 | 0.221 |
| 450.000 | 29.976 | 142.303 | 142.272 | -0.022 | 142.362 | 0.042 |
| 449.998 | 49.981 | 207.882 | 207.692 | -0.092 | 207.871 | -0.005 |
| 450.000 | 68.945 | 250.899 | 250.634 | -0.105 | 251.015 | 0.046 |

Table 19. Continued

| T/K | P/MPa | $\rho / \mathrm{kgm}^{-3}$ | $\rho G E R G / \mathrm{kgm}^{-3}$ | $\begin{gathered} 100 \cdot(\rho- \\ \rho G E R G) / \rho \end{gathered}$ | $\begin{gathered} \rho A G A 8- \\ D C 2 / \mathbf{k g m}^{-} \end{gathered}$ | $\begin{gathered} 100 \cdot(\rho- \\ \rho A G A 8- \\ D C 2) / \rho \end{gathered}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 449.989 | 99.967 | 299.379 | 299.025 | -0.118 | 299.581 | 0.068 |
| 450.000 | 149.899 | 350.038 | 349.589 | -0.128 | 350.212 | 0.050 |

Table 20. Po T Data for Sample 3

| T/K | P/MPa | $\rho / \mathrm{kgm}^{-3}$ | $\rho G E R G / \mathrm{kgm}^{-3}$ | $\begin{gathered} 100 \cdot(\rho- \\ \rho G E R G) / \rho \end{gathered}$ | $\begin{gathered} \rho A G A 8- \\ D C 2 / \mathbf{k g m}^{-} \end{gathered}$ | $\begin{gathered} 100 \cdot(\rho- \\ \rho A G A 8- \\ D C 2) / \rho \end{gathered}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 250.016 | 13.964 | 221.001 | 221.647 | 0.292 | 223.755 | 1.246 |
| 249.977 | 16.015 | 243.888 | 243.878 | -0.004 | 246.287 | 0.984 |
| 250.039 | 17.997 | 260.725 | 260.129 | -0.228 | 262.754 | 0.778 |
| 250.060 | 20.044 | 274.663 | 273.669 | -0.362 | 276.458 | 0.653 |
| 250.010 | 22.012 | 285.917 | 284.628 | -0.451 | 287.528 | 0.564 |
| 250.043 | 24.019 | 295.545 | 294.069 | -0.500 | 297.039 | 0.505 |
| 250.048 | 26.039 | 304.022 | 302.412 | -0.529 | 305.422 | 0.461 |
| 250.033 | 27.998 | 311.340 | 309.629 | -0.550 | 312.654 | 0.422 |
| 250.027 | 30.064 | 318.245 | 316.471 | -0.557 | 319.493 | 0.392 |
| 249.954 | 14.972 | 232.693 | 233.495 | 0.345 | 235.763 | 1.320 |
| 249.955 | 29.966 | 317.501 | 316.262 | -0.390 | 319.288 | 0.563 |
| 250.024 | 49.999 | 362.745 | 360.983 | -0.486 | 363.597 | 0.235 |
| 250.025 | 68.966 | 388.959 | 387.170 | -0.460 | 389.330 | 0.095 |
| 249.946 | 99.877 | 418.939 | 416.980 | -0.468 | 418.554 | -0.092 |
| 250.011 | 149.957 | 452.141 | 450.069 | -0.458 | 450.972 | -0.259 |
| 349.973 | 9.947 | 70.314 | 70.467 | 0.217 | 70.510 | 0.278 |

Table 20. Continued

| T/K | P/MPa | $\rho / \mathrm{kgm}^{-3}$ | $\rho G E R G / \mathrm{kgm}^{-3}$ | $\begin{gathered} 100 \cdot(\rho- \\ \rho G E R G) / \rho \end{gathered}$ | $\begin{gathered} \rho A G A 8- \\ D C 2 / \mathbf{k g m}^{-} \end{gathered}$ | $\begin{gathered} 100 \cdot(\rho- \\ \rho A G A 8- \\ D C 2) / \rho \end{gathered}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 349.982 | 11.980 | 85.763 | 85.901 | 0.162 | 85.965 | 0.235 |
| 349.977 | 13.975 | 100.887 | 100.977 | 0.089 | 101.064 | 0.176 |
| 349.984 | 15.980 | 115.755 | 115.838 | 0.072 | 115.953 | 0.172 |
| 349.988 | 17.987 | 130.202 | 130.221 | 0.015 | 130.366 | 0.126 |
| 349.970 | 19.973 | 143.863 | 143.834 | -0.020 | 144.010 | 0.102 |
| 349.976 | 21.985 | 156.940 | 156.870 | -0.044 | 157.080 | 0.090 |
| 349.970 | 23.988 | 169.138 | 169.055 | -0.050 | 169.304 | 0.098 |
| 349.981 | 25.975 | 180.228 | 180.334 | 0.059 | 180.630 | 0.223 |
| 349.976 | 27.967 | 191.240 | 190.886 | -0.185 | 191.238 | -0.001 |
| 349.986 | 29.925 | 200.884 | 200.529 | -0.177 | 200.943 | 0.029 |
| 349.950 | 9.988 | 70.602 | 70.786 | 0.261 | 70.829 | 0.322 |
| 349.980 | 29.997 | 201.665 | 200.878 | -0.390 | 201.295 | -0.183 |
| 350.011 | 49.989 | 272.162 | 270.210 | -0.717 | 271.175 | -0.363 |
| 349.977 | 68.881 | 310.868 | 309.454 | -0.455 | 310.642 | -0.073 |
| 349.977 | 99.889 | 353.595 | 351.613 | -0.560 | 352.967 | -0.177 |
| 350.015 | 149.888 | 397.800 | 395.238 | -0.644 | 396.761 | -0.261 |
| 450.004 | 9.978 | 50.549 | 50.579 | 0.057 | 50.596 | 0.092 |
| 450.011 | 11.975 | 60.650 | 60.677 | 0.044 | 60.702 | 0.086 |
| 449.999 | 13.975 | 70.681 | 70.690 | 0.012 | 70.725 | 0.062 |
| 450.006 | 15.980 | 80.599 | 80.575 | -0.030 | 80.621 | 0.027 |
| 450.011 | 15.981 | 80.598 | 80.576 | -0.027 | 80.622 | 0.030 |
| 450.007 | 17.996 | 90.377 | 90.324 | -0.058 | 90.383 | 0.007 |
| 449.972 | 19.972 | 99.746 | 99.681 | -0.065 | 99.754 | 0.008 |
| 449.972 | 19.971 | 99.747 | 99.674 | -0.074 | 99.746 | -0.001 |

Table 20. Continued

| T/K | P/MPa | $\rho / \mathrm{kgm}^{-3}$ | $\rho G E R G / \mathrm{kgm}^{-3}$ | $\begin{gathered} 100 \cdot(\rho- \\ \rho G E R G) / \rho \end{gathered}$ | $\begin{gathered} \rho A G A 8- \\ D C 2 / \mathrm{kgm}^{-} \end{gathered}$ | $\begin{gathered} 100 \cdot(\rho- \\ \rho A G A 8- \\ D C 2) / \rho \end{gathered}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 449.967 | 19.971 | 99.756 | 99.676 | -0.080 | 99.748 | -0.007 |
| 449.962 | 21.980 | 109.036 | 108.934 | -0.093 | 109.020 | -0.015 |
| 449.978 | 21.982 | 109.037 | 108.938 | -0.091 | 109.024 | -0.012 |
| 450.022 | 23.988 | 118.021 | 117.897 | -0.105 | 117.996 | -0.021 |
| 449.965 | 25.971 | 126.646 | 126.507 | -0.109 | 126.618 | -0.022 |
| 449.972 | 27.979 | 135.059 | 134.916 | -0.106 | 135.038 | -0.016 |
| 449.976 | 29.997 | 143.225 | 143.061 | -0.115 | 143.193 | -0.022 |
| 449.992 | 10.004 | 50.627 | 50.710 | 0.164 | 50.728 | 0.199 |
| 449.989 | 29.976 | 143.108 | 142.972 | -0.095 | 143.104 | -0.003 |
| 449.998 | 49.978 | 208.936 | 208.483 | -0.217 | 208.804 | -0.063 |
| 450.029 | 68.937 | 252.029 | 251.381 | -0.257 | 251.995 | -0.013 |
| 449.994 | 99.959 | 300.555 | 299.713 | -0.280 | 300.619 | 0.021 |
| 449.995 | 149.926 | 351.272 | 350.214 | -0.301 | 351.310 | 0.011 |

## Table 21. PoT Data for Sample 4

| T/K | P/MPa | $\rho / \mathrm{kgm}^{-3}$ | $\rho G E R G / \mathrm{kgm}^{-3}$ | $\begin{gathered} 100 \cdot(\rho- \\ \rho G E R G) / \rho \end{gathered}$ | $\underset{3}{\rho A G A 8-}$ | $\begin{gathered} 100 \cdot(\rho- \\ \rho A G A 8- \\ D C 2) / \rho \end{gathered}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 249.983 | 20.028 | 282.336 | 281.855 | -0.170 | 284.839 | 0.886 |
| 249.987 | 30.030 | 321.375 | 320.170 | -0.375 | 323.110 | 0.540 |
| 249.997 | 49.946 | 362.707 | 361.178 | -0.421 | 363.382 | 0.186 |
| 249.977 | 74.931 | 393.689 | 392.096 | -0.405 | 393.508 | -0.046 |
| 249.999 | 100.291 | 415.832 | 414.250 | -0.380 | 415.070 | -0.183 |

Table 21. Continued

| $\mathbf{T} / \mathrm{K}$ | $\mathbf{P} / \mathrm{MPa}$ | $\boldsymbol{\rho} / \mathrm{kgm}^{-3}$ | $\boldsymbol{\rho G E R G / \mathrm { kgm } ^ { - 3 }}$ | $\mathbf{1 0 0} \cdot(\boldsymbol{\rho}-$ <br> $\boldsymbol{\rho G E R G}) / \boldsymbol{\rho}$ | $\boldsymbol{\rho A G A 8} \mathbf{D C 2 / k g m}$ <br> $\mathbf{3}$ | $\mathbf{1 0 0} \cdot(\boldsymbol{\rho}-$ <br> $\boldsymbol{\rho A G A 8} \mathbf{-}$ <br> $\boldsymbol{D C 2}) / \boldsymbol{\rho}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 249.918 | 124.967 | 432.286 | 431.263 | -0.236 | 431.632 | -0.151 |
| 249.988 | 149.990 | 446.528 | 445.540 | -0.221 | 445.532 | -0.223 |
| 350.026 | 9.998 | 72.258 | 72.591 | 0.461 | 72.633 | 0.519 |
| 350.019 | 29.984 | 204.942 | 205.152 | 0.103 | 205.482 | 0.263 |
| 350.013 | 49.890 | 272.578 | 272.265 | -0.115 | 273.020 | 0.162 |
| 349.993 | 74.991 | 320.091 | 319.592 | -0.156 | 320.498 | 0.127 |
| 350.000 | 99.925 | 351.066 | 350.520 | -0.156 | 351.462 | 0.113 |
| 350.018 | 124.942 | 374.231 | 373.751 | -0.128 | 374.711 | 0.128 |
| 349.944 | 149.787 | 393.033 | 392.332 | -0.178 | 393.310 | 0.070 |
| 450.043 | 9.999 | 51.020 | 51.413 | 0.770 | 51.429 | 0.802 |
| 450.050 | 30.001 | 144.981 | 145.343 | 0.249 | 145.399 | 0.288 |
| 450.057 | 49.945 | 210.468 | 210.419 | -0.023 | 210.597 | 0.061 |
| 450.062 | 68.926 | 252.902 | 252.532 | -0.146 | 252.947 | 0.018 |
| 450.027 | 99.965 | 300.112 | 299.522 | -0.197 | 300.130 | 0.006 |
| 450.084 | 124.928 | 327.346 | 326.645 | -0.214 | 327.302 | -0.013 |
| 450.051 | 149.926 | 349.140 | 348.389 | -0.215 | 349.073 | -0.019 |



Figure 51. Sample 1 density deviations. GERG-2004, • $250 \mathrm{~K}, \Delta 350 \mathrm{~K}, ~ ■ 450 \mathrm{~K} ; \mathrm{AGA8}-\mathrm{DC} 2, \circ 250 \mathrm{~K}, \Delta 350 \mathrm{~K}, \square 450 \mathrm{~K}$.


Figure 52. Sample 2 density deviations. GERG-2004, • $250 \mathrm{~K}, ~ \triangle 350 \mathrm{~K}, ~ ■ 450 \mathrm{~K} ; \mathrm{AGA8}-\mathrm{DC} 2, \circ 250 \mathrm{~K}, \Delta 350 \mathrm{~K}, \square 450 \mathrm{~K}$.


Figure 53. Sample 3 density deviations. GERG-2004, • $250 \mathrm{~K}, ~ \triangle 350 \mathrm{~K}$, ■ 450 K ; AGA8-DC2, ○ $250 \mathrm{~K}, \Delta 350 \mathrm{~K}, \square 450 \mathrm{~K}$.


Figure 54. Sample 4 density deviations. GERG-2004, • $250 \mathrm{~K}, ~ \triangle 350 \mathrm{~K}$, ■ 450 K ; AGA8-DC2, o $250 \mathrm{~K}, \Delta 350 \mathrm{~K}$, $\square 450 \mathrm{~K}$.

Isochoric data determine the phase boundary data for the four synthetic samples. The raw isochoric data are in Appendix C. Tables 22 to 25 contains the phase boundary data for the synthetic samples. Figures 55 and 56 present the phase boundaries.

Table 22. Phase Boundary Data for the Sample 1

| $\mathbf{T} / \mathbf{K}$ | $\mathbf{P} / \mathbf{M P a}$ | $\boldsymbol{\rho}^{\prime} / \mathbf{K g m}^{-3}$ |
| :---: | :---: | :---: |
| 225.6025 | 7.5882 | 158.7557 |
| 235.3435 | 7.9394 | 134.5477 |
| 241.0608 | 7.7931 | 116.7226 |
| 247.4203 | 7.1749 | 93.9198 |
| 252.5296 | 6.1611 | 71.8138 |
| 254.6000 | 4.8171 | 51.5162 |
| 252.0300 | 3.0468 | 30.5102 |
| 244.4553 | 1.5656 | 15.2759 |

Table 23. Phase Boundary Data for the Sample 2

| $\mathbf{T} / \mathbf{K}$ | $\mathbf{P} / \mathbf{M P a}$ | $\boldsymbol{\rho}^{\prime} / \mathbf{K g m}^{-\mathbf{3}}$ |
| :---: | :---: | :---: |
| 230.9600 | 8.7183 | 179.9815 |
| 240.1700 | 9.0836 | 157.2834 |
| 249.1000 | 9.1536 | 133.0381 |
| 255.6100 | 8.7152 | 114.3313 |
| 261.3700 | 7.7368 | 90.8631 |
| 263.7300 | 6.6819 | 73.2490 |
| 264.4200 | 5.3628 | 55.3067 |
| 263.6900 | 3.9726 | 38.9281 |
| 259.8900 | 2.3962 | 22.5349 |

Table 24. Phase Boundary Data for the Sample 3

| $\mathbf{T} / \mathbf{K}$ | $\mathbf{P / M P a}$ | $\boldsymbol{\rho}^{\boldsymbol{\prime}} / \mathbf{K g m}^{\mathbf{- 3}}$ |
| :---: | :---: | :---: |
| 239.3400 | 9.6843 | 174.5326 |
| 246.5200 | 9.9628 | 158.5842 |
| 255.7300 | 10.0323 | 139.7027 |
| 264.0200 | 9.1559 | 111.9532 |
| 268.9100 | 7.9602 | 88.5818 |
| 271.6600 | 6.5538 | 67.6517 |
| 272.0400 | 4.9297 | 47.8727 |
| 271.2800 | 3.3136 | 30.5582 |

Table 25. Phase Boundary Data for the Sample 4

| $\mathbf{T} / \mathbf{K}$ | $\mathbf{P} / \mathbf{M P a}$ | $\boldsymbol{\rho}^{\prime} / \mathbf{K g m}^{-\mathbf{3}}$ |
| :---: | :---: | :---: |
| 265.6700 | 11.7705 | 164.2590 |
| 270.3900 | 11.1496 | 144.7030 |
| 276.5500 | 9.8675 | 115.4050 |
| 280.5300 | 8.5594 | 92.4190 |
| 283.8200 | 6.9924 | 69.7680 |
| 284.1000 | 5.3831 | 50.8200 |
| 280.3100 | 3.4557 | 31.2580 |
| 273.6100 | 1.5884 | 13.9260 |



Figure 55. Phase boundaries for the samples $\star 1, \bullet 2, \boldsymbol{\nabla} 3, \star 4$.


Figure 56. Temperature-density boundaries for the samples $\downarrow 1, \bullet 2, \nabla 3, \star 4$.

Figures 55 and 56 indicate that as the long-chain hydrocarbons concentrations increase, so do the cricodentherms and cricodenbars, as expected.

### 4.3 High Pressure Isochoric Data for a Pipeline Sample

Isochoric data for a pipeline-type mixture has been measured. The composition for this mixture appears in Table 26. The estimated uncertainty for this composition is $\pm$ $0.04 \%$. The data cover the range of $(130-400) \mathrm{K}$ up to 200 M . Low temperature data at high pressure were measured in the region of $(130-250) \mathrm{K}$ and $(40-160) \mathrm{MPa}$. This region represents an important part of the thermodynamics space for natural gas mixtures unexplored in the past. Densities were determined by the intersection of densities measured at room temperature and using the methodology for the determination of saturation densities as explained in Chapter III.

## Table 26. Pipeline Composition

| Compound | mol \% |
| :---: | :---: |
| Methane | 95.014 |
| Ethane | 3.969 |
| Propane | 1.017 |

The experimental isochoric data and the determined densities appear in Tables 27 and 28. Additionally, bubble point determinations using the isochoric data establish the performance of the new isochoric apparatus for determining phase boundary data. Excellent consistency exists with the experimental data measured by Haynes et al. ${ }^{103}$ and the predictions from GERG-2004.

Table 27. Isochoric 1 Experimental Data

| $\mathbf{T} / \mathbf{K}$ | $\mathbf{P} / \mathbf{\rho P a} / \mathbf{k g} \cdot \mathbf{m}^{-3}$ | $\mathbf{1 0 0} \cdot(\boldsymbol{\rho}-\boldsymbol{\rho} \boldsymbol{G E R G}) / \boldsymbol{\rho}$ |  |
| :---: | :---: | :---: | :---: |
| 300.000 | 199.871 | 419.236 | 0.104 |
| 295.000 | 195.624 | 419.385 | 0.048 |
| 290.000 | 191.147 | 419.536 | 0.014 |
| 285.000 | 186.633 | 419.687 | -0.022 |
| 280.000 | 182.090 | 419.840 | -0.059 |
| 275.000 | 177.422 | 419.993 | -0.087 |
| 270.000 | 172.833 | 420.146 | -0.130 |
| 263.061 | 166.346 | 420.360 | -0.185 |
| 259.991 | 163.152 | 420.459 | -0.173 |
| 249.998 | 153.629 | 420.770 | -0.258 |
| 239.999 | 143.910 | 421.084 | -0.346 |
| 229.997 | 134.009 | 421.401 | -0.441 |

Table 27. Continued

| $\mathbf{T} / \mathbf{K}$ | $\mathbf{P} / \mathbf{M P a}$ | $\boldsymbol{\rho} / \mathbf{k g} \cdot \mathbf{m}^{-\mathbf{3}}$ | $\mathbf{1 0 0} \cdot(\boldsymbol{\rho}-\boldsymbol{\rho} \boldsymbol{G E R G}) / \boldsymbol{\rho}$ |
| :---: | :---: | :---: | :---: |
| 220.000 | 123.882 | 421.722 | -0.539 |
| 210.000 | 113.535 | 422.047 | -0.642 |
| 200.000 | 103.186 | 422.373 | -0.787 |
| 190.000 | 92.171 | 422.709 | -0.878 |
| 179.999 | 81.107 | 423.047 | -1.011 |
| 169.999 | 69.828 | 423.391 | -1.168 |
| 160.368 | 59.043 | 423.725 | -1.395 |
| 159.999 | 58.336 | 423.742 | -1.355 |
| 150.001 | 46.673 | 424.102 | -1.590 |
| 140.984 | 36.172 | 424.434 | -1.885 |
| 139.354 | 34.462 | 424.493 | -1.984 |
| 137.597 | 24.525 | 424.684 | -0.478 |

Table 28. Isochoric 2 Experimental Data

| $\mathbf{T} / \mathbf{K}$ | $\mathbf{P} / \mathbf{M P a}$ | $\boldsymbol{\rho} / \mathbf{k g} \cdot \mathbf{m}^{-\mathbf{3}}$ | $\mathbf{1 0 0} \cdot(\boldsymbol{\rho}-\boldsymbol{\rho} \boldsymbol{G E R G}) / \boldsymbol{\rho}$ |
| :---: | :---: | :---: | :---: |
| 390.000 | 200.548 | 380.181 | -0.09164 |
| 380.000 | 194.090 | 381.122 | 0.02012 |
| 375.000 | 190.858 | 381.594 | 0.07154 |
| 370.000 | 187.556 | 382.071 | 0.13057 |
| 360.000 | 180.895 | 383.032 | 0.24723 |
| 350.000 | 174.142 | 384.003 | 0.36404 |
| 340.000 | 167.295 | 384.985 | 0.48088 |

Table 28. Continued

| T/K | P/MPa | $\rho / \mathrm{kg} \cdot \mathrm{m}^{-3}$ | $100 \cdot(\rho-\rho G E R G) / \rho$ |
| :---: | :---: | :---: | :---: |
| 330.000 | 160.343 | 385.977 | 0.59870 |
| 325.000 | 156.729 | 386.484 | 0.67512 |
| 320.000 | 153.290 | 386.981 | 0.71626 |
| 310.000 | 146.138 | 387.996 | 0.83222 |
| 300.000 | 138.943 | 389.019 | 0.93515 |
| 290.000 | 131.439 | 390.066 | 1.07335 |
| 279.999 | 123.947 | 391.118 | 1.18617 |
| 275.000 | 120.005 | 391.658 | 1.27172 |
| 270.000 | 116.329 | 392.183 | 1.29814 |
| 260.000 | 108.608 | 393.261 | 1.40246 |
| 249.999 | 100.747 | 394.354 | 1.50477 |
| 239.999 | 92.751 | 395.461 | 1.60238 |
| 230.000 | 84.608 | 396.585 | 1.69537 |
| 219.991 | 76.332 | 397.724 | 1.77533 |
| 210.000 | 67.925 | 398.879 | 1.84362 |
| 199.994 | 59.366 | 400.053 | 1.89354 |
| 189.999 | 50.678 | 401.244 | 1.91874 |
| 180.000 | 41.880 | 402.454 | 1.90231 |
| 170.000 | 33.034 | 403.682 | 1.81474 |
| 160.000 | 24.266 | 404.923 | 1.59846 |
| 150.000 | 15.646 | 406.177 | 1.20011 |
| 140.000 | 6.959 | 407.450 | 0.65124 |
| 135.000 | 2.433 | 408.101 | 0.36151 |
| 131.999 | 0.521 | 408.435 | -0.11728 |
| 131.552 | 0.398 | 408.473 | -0.24556 |

Table 28. Continued

| T/K | $\mathbf{P} / \mathbf{M P a}$ | $\boldsymbol{\rho} / \mathbf{k g} \cdot \mathbf{m}^{-\mathbf{3}}$ | $\mathbf{1 0 0} \cdot(\boldsymbol{\rho} \boldsymbol{\rho} \boldsymbol{\rho}$ GERG $\boldsymbol{\prime}) / \boldsymbol{\rho}$ |
| :---: | :---: | :---: | :---: |
| 131.647 | 0.426 | 408.465 | -0.21877 |

Table 29 contains the experimental phase boundary data for isochoric 2. GERG2004 reproduces the phase boundary data within the expected deviation for the EoS ${ }^{15}$.

Table 29. Phase Boundary Data for the Isochoric 2

| Phase Boundary Data | Experimental Data | GERG-2004 EoS |
| :--- | :---: | :---: |
| Temperature $/ \mathrm{K}$ | 131.243 | 131.243 |
| Pressure $/ \mathrm{MPa}$ | 0.363 | 0.375 |
| Density $/ \mathrm{kg} \cdot \mathrm{m}^{-3}$ | 408.911 | 409.969 |

## MOLECULAR STUDY OF NATURAL GAS MIXTURES

A wide experimental characterization for natural gas, its main constituents and associated mixtures has appeared in the literature ${ }^{15}$. Thermal, volumetric and transport information have accumulated over almost a century of experimental research. However, because of variations in the compositions and the thermodynamic conditions in which natural gas occurs, important gaps remain for consideration ${ }^{104}$. Unfortunately, it is impossible to characterize all natural gas conditions experimentally, and in some cases experiments can be complex, unsafe and expensive.

Theoretical studies are always an important tool to overcome the limitations in experimental data. Starting from simple approximations such us the ideal gas equation, the theoretical understanding of natural gas mixtures has evolved to complex molecular simulations that involve state-of-the-art molecular theories and computational tools ${ }^{104}$. Despite all the results from these methods, the most useful and simple way to reproduce experimental data continues to be EOS. That is why in many cases theoretical effort focuses upon using modern molecular methods to generate molecular-based EOS ${ }^{105-106}$. These EOS use molecular understanding of the constituents to represent the possible interactions in the mixtures. However, gases, especially hydrocarbon gases, are "simple" systems compared to complex, highly polar and associated fluids. Still, molecular-based EOS for natural gas ${ }^{18,107-108}$ are not very accurate.

The main challenges for modeling natural gas from the molecular point of view are:

1. Introduce the conformational and shape effects of the long hydrocarbons $\left(\mathrm{C}_{4+}\right)$ in the calculations of pure and mixtures properties.
2. Develop realistic potential energy functions for the interactions of the constituents of natural gas that allow calculations of thermal, volumetric and transport properties. In some cases the so-called effective potentials improve considerably the predictions of the theoretical models ${ }^{109}$.
3. Generate molecular based models that avoid using experimental, binary data to fit the model. In this case the model is usually called a predictive model.
4. Achieve accurate predictions of thermophysical properties. Although the molecular models are very useful for determining qualitative behaviors of natural gas mixtures, its quantitative capabilities are still low compared to empirical models for industrial needs.

This research project contains an extensive literature review analyzing the different alternatives to approach the molecular understanding of natural gases. Concepts such as "shape factor" have been developed to build molecular equations such as BACK and its family of equations ${ }^{37-38,110-111}$. The Statistical Associated Fluid Theory (SAFT) has been used to develop molecular-based EOS ${ }^{105-106,}$, 112-115. Monte Carlo methods ${ }^{104,}$ ${ }^{116-118}$, molecular dynamics calculations ${ }^{119-124}$ and density functional approaches ${ }^{125-129}$ are some of the different alternatives that represent the wide spectrum of methodologies available to understand natural gas at the molecular level.

In order to establish a consistent set of molecular data based upon molecular mechanics calculations, this project used a detailed characterization using Gaussian03 ${ }^{130}$ for the main constituent of natural gas that appears in Table 30 as suggested by Kunz et al. ${ }^{15}$. However, Singh et al. ${ }^{106}$ and Leonhard et al. ${ }^{105}$ have performed an accurate description of the components. These results were tested and compared to experimental data; they suggested an excellent starting point for a systematic study of natural gases. Therefore, only calculations for molecules not considered in this reference were determined in this work. Here, the principal results are compiled for future developments, Table 31.

Singh et al. ${ }^{106}$ determined an excellent frame of molecular data that consists essentially of dipole and quadrupole moments, and isotropic polarizabilities, Table 31. For perturbation theory treatment, which is the fundamental methodology for development of molecular EOS such as SAFT, these properties are the fundamental parameters for calculating molecular interaction and proposing new model interactions ${ }^{105}$. The simplified representation of the different contribution to the Helmholtz energy model is:

$$
\begin{equation*}
A^{\text {res }}=A^{h s}+A^{\text {chain }}+A^{a s s o c}+A^{d i s p}+A^{Q Q}+A^{D D} \tag{5.1}
\end{equation*}
$$

where hs, chain, assoc, disp, QQ and DD refer to the hard sphere, chain, association, dispersion, quadrupole and dipole Helmholtz energy contributions respectively.

## Table 30. Natural Gas Main Constituents

Compound Compound

| Methane | Nonane |
| :--- | :--- |
| Ethane | Decane |
| Propane | Nitrogen |
| Butane | Carbon Dioxide |
| Iso-Butane | Carbon Monoxide |
| Pentane | Hydrogen Sulfide |
| Iso-Pentane | Hydrogen |
| Hexane | Water |
| Heptane | Oxygen |
| Octane |  |

Table 31. Dipole Moment, Quadrupole Moment and Isotropic Polarizability for the Main Constituents of Natural Gas

| Molecule | Method for $\boldsymbol{\mu}$ and $\boldsymbol{\theta}$ | $\mu_{x}(\mathrm{D})$ | $\mu_{y}(\mathrm{D})$ | $\mu_{z}(\mathrm{D})$ | $\theta_{x x}(\mathrm{~B})$ | $\theta_{y y}(\mathrm{~B})$ | $\theta_{z z}(\mathrm{~B})$ | $\begin{aligned} & \alpha_{1} \\ & \left(10^{-24} \mathrm{~cm}^{3}\right) \\ & \hline \end{aligned}$ | $\begin{aligned} & \alpha_{2} \\ & \left(10^{-24} \mathrm{~cm}^{3}\right) \end{aligned}$ | $\begin{aligned} & \alpha_{3} \\ & \left(10^{-24} \mathrm{~cm}^{3}\right) \end{aligned}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{CH}_{4}$ | B3LYP/6-31g(d) ${ }^{\text {a }}$ | 0 | 0 | 0 | 0 | 0 | 0 | $2.56{ }^{\text {b131 }}$ |  |  |
| $\mathrm{C}_{2} \mathrm{H}_{6}$ | MP2/aug-cc-pVDTZ//B3LYP/TZVP | 0 | 0 | 0 | 0.376 | 0.376 | -0.752 | 4.207 | 4.221 | - |
| $\mathrm{C}_{3} \mathrm{H}_{6}$ | MP2/aug-cc-pVDTZ//B3LYP/TZVP | 0 | 0.087 | 0 | -0.637 | -0.073 | 0.71 | 6.003 | 6.014 | - |
| $\mathrm{n}-\mathrm{C}_{4} \mathrm{H}_{6}$ | MP2/aug-cc-pVDTZ//B3LYP/TZVP | 0 | -0.016 | -0.033 | -0.903 | 0.151 | 0.752 | 7.780 | 7.791 | - |
| i- $\mathrm{C}_{4} \mathrm{H}_{6}$ | MP2/aug-cc-pVDTZ//B3LYP/TZVP | 0 | 0.000 | -0.133 | -0.343 | -0.346 | 0.689 | 7.814 | 7.827 | - |
| $\mathrm{n}-\mathrm{C}_{5} \mathrm{H}_{6}$ | MP2/aug-cc-pVDZ//B3LYP/TZVP | 0 | 0 | 0 | -1.036 | -0.404 | 1.44 | 9.368 | - | - |
| $\mathrm{i}-\mathrm{C}_{5} \mathrm{H}_{6}$ | B3LYP/6-31g(d) ${ }^{\text {a }}$ | 0.0843 | 0.0184 | 0 | -0.0969 | 0.0239 | 0.0730 | - | - | - |
| $\mathrm{n}-\mathrm{C}_{6} \mathrm{H}_{6}$ | MP2/aug-cc-pVDZ//B3LYP/TZVP | 0 | 0 | 0 | -1.286 | 0.0062 | 1.224 | 11.482 | - | - |
| $\mathrm{n}-\mathrm{C}_{7} \mathrm{H}_{6}$ | B3LYP/6-31g(d) ${ }^{\text {a }}$ | 0 | 0.0543 | 0 | -0.8542 | -0.1707 | 1.0249 | - | - | - |
| $\mathrm{n}-\mathrm{C}_{8} \mathrm{H}_{6}$ | MP2/aug-cc-pVDZ//B3LYP/TZVP | 0 | 0 | 0 | -2.400 | -0.121 | 2.521 | 15.195 | - | - |
| $\mathrm{n}-\mathrm{C}_{9} \mathrm{H}_{6}$ | B3LYP/6-31g(d) ${ }^{\text {a }}$ | 0 | 0.0543 | 0 | -1.1344 | -0.2411 | 1.3754 | - | - | - |
| $\mathrm{C}_{10} \mathrm{H}_{6}$ | MP2/aug-cc-pVDZ//B3LYP/TZVP | 0 | 0 | 0 | -0.362 | -0.367 | 0.729 | 10.485 | - | - |
| $\mathrm{N}_{2}$ | CCSD(T)/aug-cc-pVTZ | 0 | 0 | 0 | 0.744 | 0.744 | -1.488 | 1.679 | 1.707 | 1.753 |
| $\mathrm{CO}_{2}$ | CCSD(T)/aug-cc-pVTZ | 0 | 0 | 0 | 2.136 | 2.136 | -4.272 | 2.558 | 2.643 | 2.595 |
| CO | CCSD(T)/aug-cc-pVTZ | 0 | 0 | -0.101 | 0.985 | 0.985 | -1.970 | 1.923 | 1.968 | 1.951 |
| $\mathrm{H}_{2} \mathrm{~S}$ | CCSD(T)/aug-cc-pVTZ | 0 | 0 | 0.980 | -3.708 | 2.786 | 0.930 | 3.488 | 3.716 | 3.602 |
| $\mathrm{H}_{2}$ | B3LYP/6-31g(d) ${ }^{\text {a }}$ | 0 | 0 | 0 | -0.1415 | -0.1415 | 0.2830 | - | - | - |
| $\mathrm{H}_{2} \mathrm{O}$ | CCSD(T)/aug-cc-pVTZ | 0 | 0 | 1.841 | 2.573 | -2.473 | -0.125 | 1.372 | 1.450 | 1.391 |
| $\mathrm{O}_{2}$ | B3LYP/6-31g(d) ${ }^{\text {a }}$ | 0 | 0 | 0 | 0.1529 | 0.1529 | -0.3057 | - | - | - |

Multipole moments are reported in a quadropole principal axis. Methodology to calculate $\alpha_{i}$ appears in ${ }^{106}$.
${ }^{\mathrm{a}}$ Values determined in this research.
${ }^{\mathrm{b}}$ Parameter extracted from literature.

The implementation of the PCP-SAFT EOS ${ }^{105}$ using these parameters leads to a significant improvement in predictive capabilities when compared to similar molecular EOS such us PC-SAFT ${ }^{18}$. For instance, an average deviation of $0.6 \%$ exists for the vapor pressure predictions of pure components with PCP-SAFT; in the case of PC-SAFT, it is about $2 \%$. However, the experimental uncertainty for the vapor pressure of most of the pure compounds is around $0.02 \%$. The same result was found in the case of binary mixtures. Therefore, it appears that additional improvements are possible in the future.

### 5.1 Molecular Dynamics Simulations

Molecular dynamics calculations were performed during this research project to establish the accuracy of the method in the determination of volumetric properties such as density. A simple binary system of methane-ethane $0.5-.0 .5 \%$ was simulated using the DL_POLY 2.12 program. NPT simulations were performed using the NPT Hoover algorithm with 0.1 ps as the barostat and thermostat relaxation times. The number of molecules were 200 and the force field was OPLS-AA ${ }^{32}$. An equilibration time of 0.1 ns was used, and the average was taken with simulations of up to 10 ns to observe the effect of the simulation time on the accuracy of the density predictions compared to REFPROP $8.0^{26}$.

Highest deviations were found for pressures up to 4 bar with a simulation time of 1 ns at room temperature, Figure 57. Therefore, the effect of the simulation time was analyzed in respect of the quality of the predictions. The effect of the simulation time is in Figure 58. For a simulation time higher than 2 ns , the deviations of the density predictions fall within a band of $5 \%$. For higher pressures of 6,7 and 8 bar the deviation band is around 2\%, Figure 59. Therefore, increasing the simulation time and the pressure of the system leads to lower deviations compared to high accuracy standards. The highest deviations a low pressure and short simulation times can be attributed to high oscillations in the pressure during the simulations.


Figure 57. Density predictions for methane-ethane mixture. $298 \mathrm{~K}, 1 \mathrm{~ns}$ of simulation time $\mathbf{\triangle}$ REFPROP 8.0, • DL_POLY 2.12.


Figure 58. Density deviations for methane-ethane mixture at 5 bar and 298 K .

Finally, Figure 60 represents the radial distribution functions for this system at 298 K and 1 bar. This structural information can be used for further developments and comparisons of mixing rules using the energy equation.


Figure 59. Density deviations for methane-ethane mixture. $\bullet 6$ bar, $\boldsymbol{\Delta} 7$ bar, $\boldsymbol{\nabla} 8$ bar.

### 5.2 Conformational Analysis and New Mixing Rule

A new technique to account for isomeric interactions in gas mixtures follows. The analysis uses the fact that, for the same type of molecule in the mixtures, there are different rotational isomers. Therefore, there are interactions not just between different types of molecules but different types of isomers of the same molecule with isomers of different molecules. This assumption leads to:

$$
\begin{equation*}
a_{m i x}=\sum_{i j} x_{i} x_{j} a_{i j} \equiv \sum_{i j} \sum_{k l} x_{i}^{k} x_{j}^{l} a_{i j}^{k l}, \tag{5.2.1}
\end{equation*}
$$

where $x_{i}^{k}$ is the composition of the k -isomer of the i -molecule and $a_{i j}^{k l}$ is the binary interaction tensor that accounts for the interaction between the k -isomer of the i molecule with the 1 -isomer of the j-molecule.


Figure 60. Radial distribution functions for methane-ethane mixture at 1 bar and 298 K .

Now, the $x_{i}^{k}$ variable can be:

$$
\begin{equation*}
x_{i}^{k}=x_{i} P_{i}^{k} \tag{5.2.2}
\end{equation*}
$$

where $P_{i}^{k}(T)$ is the probability of having the k-isomer of the i-molecule at temperature $T$. Then,

$$
\begin{equation*}
a_{m i x}=\sum_{i j} \sum_{k l} x_{i} x_{j} P_{i}^{k} P_{j}^{l} a_{i j}^{k l} . \tag{5.2.3}
\end{equation*}
$$

Now, the correlation proposed by Bartell and Khol ${ }^{132}$ can determine the probability distribution of the i -isomer of the 1-molecule as a function of temperature

$$
\begin{equation*}
P_{l}^{i} / P_{l}^{j}=m^{i} / m^{j} \exp \left(-\left(n_{i}-n_{j}\right) \Delta G^{0} / R T\right) . \tag{5.2.4}
\end{equation*}
$$

where $m_{i}$ is the multiplicity of each isomer, $n_{i}$ is the number of gauche conformations in the isomer and $\Delta G^{0}$ is a fitting parameter. Then, the $a_{i}^{k}$ parameters can be found from the $a_{i}$ parameter for pure compounds determined from any EOS using equations 5.2.4 and 5.2.5

$$
\begin{equation*}
a_{i}(T)=\sum_{k} P_{i}^{k}(T) a_{i}^{k} . \tag{5.2.5}
\end{equation*}
$$

Finally the $a_{i j}^{k l}$ parameter can be calculated from the geometric average $\sqrt{a_{i}^{k} a_{j}^{l}}$.
In order to proof the former hypothesis, a binary system of methane-heptane 50$50 \mathrm{~mol} \%$ was analyzed. Volumetric data for methane and ethane came from REFPROP 8.0 to estimate the parameters of the model. The parameters for $n$-heptane are in Table 32. Methane does not have an isomer. The model to evaluate the hypothesis was the Redlich-Kwong EOS. Comparisons to the Peng-Robinson EOS and the Redlich-Kwong EOS used binary interaction parameters. Figures 61 to 63 indicate a considerable improvement for determining the compressibility factor, Z, compared to REFPROP 8.0, Peng-Robinson and Redlich-Kwong using binary interaction parameters. However,
these preliminary results are inconclusive without an extensive analysis of different systems and properties using this new formulation.

Table 32. Isomeric Data for Heptane Using Redlich-Kwong EOS

| Isomer | $\Delta \mathbf{G}$ | $\mathbf{2 3 9 9} \mathbf{~ J / m o l}$ | $\mathbf{m}_{\mathbf{i}}$ | $\mathbf{n}_{\mathbf{i}}$ |
| :---: | :---: | :---: | :---: | :---: |
| TTTT | a1 | 13135107 | 1 | 0 |
| TTTG | a2 | 24632138 | 4 | 1 |
| TTGT | a3 | 24632138 | 4 | 1 |
| TTGG | a4 | 2918096 | 4 | 2 |
| GTTG | a5 | 2918096 | 4 | 2 |
| TGTG | a6 | 11671933 | 8 | 2 |
| TGGT | a7 | 729498.7 | 2 | 2 |
| TGGG | a8 | 333030.7 | 4 | 3 |
| GTGG | a9 | 1385953 | 8 | 3 |
| GGGG | a10 | 8695.221 | 2 | 4 |



Figure 61. Compressibility factor deviations from new Redlich-Kwong EOS at 400 K .


Figure 62. Compressibility factor deviations for the new Redlich-Kwon EoS at 420 K .


Figure 63. Compressibility factor deviations for the new Redlich-Kwon EOS at 450 K .

## CONCLUSIONS AND RECOMMENDATIONS

Highly accurate experimental $P \rho T$ data for the main constituents of natural gas, methane and ethane, were measured up to 200 MPa . These are the first high-accuracy, high-pressure data reported in the literature for these compounds, they can be classified as reference data. After compensating for the force transmission error experienced by the high pressure MSD of the thermodynamics research group at Texas A\&M University, a relative uncertainty for the density measurements is $\leq 0.05 \%$. This result leads us to conclude that this apparatus is the most accurate, current, high-pressure densimeter.

Highly accurate experimental $P \rho T$ data were measured for synthetic natural gas mixtures up to 200 MPa . For two ternary mixtures of methane, ethane and propane, the GERG-2004 EOS displays better predictive capability than the AGA8-DC2 EOS. However, for four multicomponent synthetic natural gas mixtures (more than 8 components) the two EOS are comparable.

A new, high-accuracy high-pressure isochoric apparatus was developed to operate in the range of temperature from 100 to 500 K up to 200 MPa . This new apparatus allows determination of bubble points for natural gas mixtures and spans the range of high-pressure to low-temperature isochoric data; regions that were not available for previous experimental data. Additionally, a low-pressure isochoric apparatus was reassembled and calibrated to determine dew points. For these two apparatus, a new methodology for the determination of phase envelope data was developed. The estimated relative uncertainties for the pressure, temperature and saturation density are $0.04 \%$, $0.45 \%$ and $0.12 \%$ respectively. Finally, a methodology for correcting the isochoric experimental slope was developed; the estimated uncertainty for these data is $0.5 \%$ for temperatures distant from the phase loop of the mixture.

A new methodology for the determination of gas composition was proposed. This technique consists in coupling CARS-GC data to minimize the high uncertainty of the long-chain hydrocarbons in the mixture. A Raman gas chamber was built and initial tests performed. The well-defined line of the CARS spectrum leads to the conclusion that considerable improvements can be obtained using this new technique. The new
methodology can achieve uncertainties less than 0.0005 in the mole fraction for the longchain hydrocarbons and low composition compounds present in natural gas mixtures.

The main constituents of natural gas were characterized by quantum mechanics calculations from accurate data found in the literature and obtained in this research project. The implementation of these new data into a new, predictive, molecular-based EOS leads to considerable improvements for these types of equations. However, the deviations in the predictions of vapor pressures compared to experimental data for pure compounds shows that additional improvements are necessary to satisfy industrial needs.

Molecular dynamics calculations for a methane+ethane mixture were performed to establish the accuracy of the predictions of volumetric properties with this technique compared to high-accuracy EOS. The main conclusions drawn were that simulation times greater than 2 ns are necessary to improve the results of the simulation, and that increasing the pressure of the analyzed system minimizes the deviations of the density calculations.

A new scheme to introduce the isomeric effects in cubic EOS was proposed. This methodology accounts for the isomer-isomer interaction in the gas mixture. Preliminary results for a mixture of methane+octane suggest that the predictions of compressibility factor are improved without using binary interaction parameters for the mixing rule.

The following recommendations are suggested to extend this research:
The low-pressure capabilities of the high-pressure MSD should be improved by introducing an additional low-pressure transducer (1500 psia) to minimize the uncertainties of the low pressure measurements.

An analysis of the sinker mass effect should be performed. Preliminary results suggest that reducing the mass of the sinker decreases the apparatus effect of the force transmission error. If this hypothesis is true, it should be possible to compensate the fluid effect in the force transmission error of the MSD and improve the accuracy of the measurements.

The proposed models for the quitinc EOS should be fit to accurate thermodynamics data. Using the inversion problem methodology, the non-linear fit
technique developed by Lemmon et al. ${ }^{26}$ promises to be an excellent alternative for accurate predictions of thermodynamic properties using less computational time than a multiparameter EOS.

The CARS-GC methodology for determining gas compositions should be developed systematically; calibration curves should be determined for key components in the mixtures at constant temperature and pressure.

Real natural gas compositions for natural gas mixtures should be evaluated using molecular dynamics methods in order to establish the composition effect on the accuracy of the calculations. Additional analyses using Monte Carlo methods must be developed for equilibrium properties determinations.

A systematic analysis should be completed for the new isomer-isomer interaction mixing rule for gases. Volumetric, equilibrium and thermal properties should be studied in order to evaluate the capabilities of this new approach.

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## APPENDIX A

## PRESSURE TRANSDUCER CALIBRATION COEFFICIENTS

Transducer model: 430K-101
Serial Number: 102124

## Temperature Coefficients

$\mathrm{X}=$ temperature period $(\mu \mathrm{s})$
$\mathrm{U}=\mathrm{X}-\mathrm{U}_{0}$
Temperature: $(\operatorname{deg} \mathrm{C})$
Temp $=Y_{1} U+Y_{2} U^{2}+Y_{3} U^{3}$

Table A.1. Temperature Coefficients

| $\mathrm{U}_{0}$ | $5.840168 \mu \mathrm{sec}$ |
| :--- | :--- |
| $\mathrm{Y}_{1}$ | $-3923.084 \mathrm{deg} \mathrm{C} / \mu \mathrm{sec}$ |
| $\mathrm{Y}_{2}$ | $-8226.233 \mathrm{deg} \mathrm{C} / \mu \mathrm{sec}^{2}$ |
| $\mathrm{Y}_{3}$ | 0 |

## Pressure Coefficients

$\mathrm{T}=$ pressure period ( $\mu \mathrm{sec}$ )
$\mathrm{C}=\mathrm{C}_{1}+\mathrm{C}_{2} \mathrm{U}+\mathrm{C}_{3} \mathrm{U}^{2}$
$\mathrm{D}=\mathrm{D}_{1}+\mathrm{D}_{2} \mathrm{U}$
$\mathrm{T}_{0}=\mathrm{T}_{1}+\mathrm{T}_{2} \mathrm{U}+\mathrm{T}_{3} \mathrm{U}^{2}+\mathrm{T}_{4} \mathrm{U}^{3}+\mathrm{T}_{5} \mathrm{U}^{4}$
Pressure: (psia)
$\mathrm{P}=\mathrm{C}\left(1-\mathrm{T}_{0}^{2} / \mathrm{T}^{2}\right)\left(1-\mathrm{D}\left(1-\mathrm{T}_{0}^{2} / \mathrm{T}^{2}\right)\right)$

## Table A. 2 Pressure Coefficients

| $\mathrm{C}_{1}$ | -182594.0 psia |
| :--- | :--- |
| $\mathrm{C}_{2}$ | $-13644.09 \mathrm{psia} / \mu \mathrm{sec}$ |
| $\mathrm{C}_{3}$ | $661604.7 \mathrm{psia} / \mu \mathrm{sec}^{2}$ |

## Table A. 3 Pressure Coefficients

| $\mathrm{D}_{1}$ | 0.015583 |
| :--- | :--- |
| $\mathrm{D}_{2}$ | 0 |

Table A. 4 Pressure Coefficients

| $\mathrm{T}_{1}$ | 29.94125 | $\mu \mathrm{sec}$ |
| :--- | :--- | :--- |
| $\mathrm{T}_{2}$ | 1.040748 | $\mu \mathrm{sec} / \mu \mathrm{sec}$ |
| $\mathrm{T}_{3}$ | 66.89682 | $\mu \mathrm{sec} / \mu \mathrm{sec}^{2}$ |
| $\mathrm{~T}_{4}$ | -44.90226 | $\mu \mathrm{sec} / \mu \mathrm{sec}^{3}$ |
| $\mathrm{~T}_{4}$ | 0 |  |

## APPENDIX B PRT CALIBRATION and ITS-90 COEFFICIENTS

The International Temperature Scale (ITS-90) is designed to characterize the absolute thermodynamic scale in the range of 0.65 K to 1358 K . ITS- 90 expresses the temperature in Kelvin in terms of the ratio of the measured resistance of the PRT at the temperature and its resistance at triple point of water; 273.16 K:

$$
\begin{equation*}
W(T)=\frac{R(T)}{R(273.16 \mathrm{~K})} \tag{B.1}
\end{equation*}
$$

The deviation equation given by equation D. 2 and reference function given by equation D. 3 are used to calculate the temperature below 273.16 K .

$$
\begin{align*}
& W_{\text {ref }}(T)=W(T)-a_{4}[W(T)-1]-b_{4}[W(T)-1] \ln W(T)  \tag{B.2}\\
& \frac{T}{273.16 K}=B_{0}+\sum_{1}^{15} B_{i}\left[\frac{W_{\text {ref }}(T)^{1 / 6}-0.65}{0.35}\right]^{i} \tag{B.3}
\end{align*}
$$

The deviation equation given by equation B. 4 and the reference function given by equation D. 5 is used to calculate the temperature above 273.16 K .

$$
\begin{align*}
& W_{\text {ref }}(T)=W(T)-a_{8}[W(T)-1]-b_{8}[W(T)-1]^{2}  \tag{B.4}\\
& T-273.15 K=D_{0}+\sum_{1}^{9} D_{i}\left[\frac{W_{\text {ref }}(T)-2.64}{1.64}\right]^{i} \tag{B.5}
\end{align*}
$$

The constants $a_{4}, b_{4}, a_{8}$ and $b_{8}$ were determined by Minco by calibrating the PRT at fixed temperature points defined by ITS-90. These constants and values of the constants $B_{\mathrm{i}}$ and $D_{\mathrm{i}}$ are given in Table B.1. The resistance of the PRT at the triple point of water measured during the original calibration done by Minco is $R(273.16 \mathrm{~K})=99.99731 \Omega$ and the ratio $R(373.15) / R(273.15)=1.3927$.

TABLE B. 1 Constants in Deviation Equations and Reference Functions of ITS-90

| a8 | $-1.3927129 \mathrm{D}-04$ | $\mathrm{~B}(0)$ | +0.183324722 |
| :--- | :--- | :--- | :--- |
| b8 | $-6.8843579 \mathrm{D}-04$ | $\mathrm{~B}(1)$ | +0.240975303 |
| a4 | $-7.0279407 \mathrm{D}-04$ | $\mathrm{~B}(2)$ | +0.209108771 |
| b4 | $-9.5605690 \mathrm{D}-06$ | $\mathrm{~B}(3)$ | +0.190439972 |
|  |  | $\mathrm{~B}(4)$ | +0.142648498 |
| $\mathrm{D}(0)$ | +439.932854 | $\mathrm{~B}(5)$ | +0.077993465 |
| $\mathrm{D}(1)$ | +472.418020 | $\mathrm{~B}(6)$ | +0.012475611 |
| $\mathrm{D}(2)$ | +37.684494 | $\mathrm{~B}(7)$ | -0.032267127 |
| $\mathrm{D}(3)$ | +7.472018 | $\mathrm{~B}(8)$ | -0.075291522 |
| $\mathrm{D}(4)$ | +2.920828 | $\mathrm{~B}(9)$ | -0.056470670 |
| $\mathrm{D}(5)$ | +0.005184 | $\mathrm{~B}(10)$ | +0.076201285 |
| $\mathrm{D}(6)$ | -0.963864 | $\mathrm{~B}(11)$ | +0.123893204 |
| $\mathrm{D}(7)$ | -0.188732 | $\mathrm{~B}(12)$ | -0.029201193 |
| $\mathrm{D}(8)$ | +0.191203 | $\mathrm{~B}(13)$ | -0.091173542 |
| $\mathrm{D}(9)$ | +0.049025 | $\mathrm{~B}(14)$ | +0.001317696 |
|  |  | $\mathrm{~B}(15)$ | +0.026025526 |

## APPENDIX C

## ISOCHORIC DATA FOR THE SYNTHETIC NARURAL GAS SAMPLES

## RESULTS FOR SAMPLE 1

Table C1. Isochoric Results for Sample 1. Black: Single Phase; Blue: Two Phase

| Isochore 1 |  | Isochore 2 |  | Isochore 3 |  | Isochore 4 |  | Isochore 5 |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\mathrm{P} /$ |  | $\mathrm{P} /$ |  | $\mathrm{P} /$ |  | $\mathrm{P} /$ |  | $\mathrm{P} /$ |
| $\mathrm{T} / \mathrm{K}$ | MPa | $\mathrm{T} / \mathrm{K}$ | MPa | $\mathrm{T} / \mathrm{K}$ | MPa | $\mathrm{T} / \mathrm{K}$ | MPa | $\mathrm{T} / \mathrm{K}$ | MPa |
| 343.15 | 21.236 | 343.15 | 17.924 | 343.15 | 15.561 | 343.15 | 12.663 | 343.15 | 9.876 |
| 333.15 | 20.088 | 333.15 | 17.010 | 333.15 | 14.820 | 333.15 | 12.097 | 333.15 | 9.471 |
| 323.15 | 18.937 | 323.15 | 16.088 | 323.15 | 14.056 | 323.15 | 11.529 | 323.15 | 9.064 |
| 313.15 | 17.781 | 313.15 | 15.165 | 313.15 | 13.296 | 313.15 | 10.956 | 313.15 | 8.654 |
| 303.15 | 16.617 | 303.15 | 14.241 | 303.15 | 12.533 | 303.15 | 10.378 | 303.15 | 8.241 |
| 293.15 | 15.455 | 293.15 | 13.313 | 293.15 | 11.763 | 293.15 | 9.800 | 293.15 | 7.827 |
| 283.15 | 14.280 | 283.15 | 12.387 | 283.15 | 10.994 | 283.15 | 9.224 | 283.15 | 7.414 |
| 273.15 | 13.112 | 273.15 | 11.453 | 273.15 | 10.229 | 273.15 | 8.648 | 273.15 | 7.000 |
| 263.15 | 12.017 | 263.15 | 10.528 | 263.15 | 9.469 | 263.15 | 8.075 | 263.15 | 6.602 |
| 258.15 | 11.383 | 258.15 | 10.076 | 258.15 | 9.096 | 258.15 | 7.792 | 258.15 | 6.392 |
| 253.15 | 10.816 | 253.15 | 9.601 | 253.15 | 8.726 | 253.15 | 7.507 | 253.15 | 6.188 |
| 248.15 | 10.222 | 248.15 | 9.137 | 248.15 | 8.334 | 248.15 | 7.221 | 248.15 | 5.991 |
| 243.15 | 9.630 | 243.15 | 8.665 | 243.15 | 7.955 | 243.15 | 6.947 | 243.15 | 5.795 |
| 238.15 | 9.066 | 238.15 | 8.203 | 238.15 | 7.585 | 238.25 | 6.689 | 238.25 | 5.606 |
| 233.15 | 8.463 | 233.15 | 7.762 | 233.15 | 7.227 | 233.50 | 6.433 |  |  |
| 228.15 | 7.882 | 228.15 | 7.341 | 228.15 | 6.871 | 228.15 | 6.150 |  |  |
| 223.15 | 7.353 | 223.15 | 6.927 | 223.15 | 6.514 |  |  |  |  |
| 218.15 | 6.867 | 218.15 | 6.521 |  |  |  |  |  |  |
| 213.25 | 6.398 |  |  |  |  |  |  |  |  |

Table C1. Continued

| Isochore 6 |  | Isochore 7 |  | Isochore 8 |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\mathrm{P} /$ |  | $\mathrm{P} /$ |  | $\mathrm{P} /$ |
| $\mathrm{T} / \mathrm{K}$ | MPa | $\mathrm{T} / \mathrm{K}$ | MPa | $\mathrm{T} / \mathrm{K}$ | MPa |
| 343.15 | 7.272 | 343.15 | 4.449 | 343.15 | 2.290 |
| 333.15 | 6.998 | 333.15 | 4.298 | 333.15 | 2.218 |
| 323.15 | 6.723 | 323.15 | 4.146 | 323.15 | 2.146 |
| 313.15 | 6.447 | 313.15 | 3.992 | 313.15 | 2.073 |
| 303.15 | 6.168 | 303.15 | 3.839 | 303.15 | 2.000 |
| 293.15 | 5.889 | 293.15 | 3.684 | 293.15 | 1.926 |
| 283.15 | 5.609 | 283.15 | 3.529 | 283.15 | 1.852 |
| 273.15 | 5.330 | 273.15 | 3.374 | 273.15 | 1.776 |
| 263.15 | 5.052 | 263.15 | 3.216 | 263.15 | 1.702 |
| 258.15 | 4.917 | 258.15 | 3.139 | 258.15 | 1.666 |
| 253.15 | 4.780 | 253.15 | 3.063 | 253.15 | 1.629 |
| 248.15 | 4.640 | 248.15 | 2.984 | 248.15 | 1.593 |
| 243.15 | 4.507 | 243.15 | 2.903 | 243.15 | 1.555 |
| 238.15 | 4.374 | 238.15 | 2.821 | 238.15 | 1.517 |
|  |  | 233.15 | 2.741 | 233.15 | 1.478 |
|  |  |  |  | 228.15 | 1.439 |
|  |  |  |  |  |  |
|  |  |  |  |  |  |

## RESULTS FOR SAMPLE 2

Table C2. Isochoric Results for Sample 2. Black: Single Phase; Blue: Two Phase

| Isochore 1 |  | Isochore 2 |  | Isochore 3 |  | Isochore 4 |  | Isochore 5 |  | Isochore 6 |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| T / K | $\begin{gathered} \hline \mathrm{P} / \\ \mathrm{MPa} \end{gathered}$ | T/ K | $\begin{gathered} \hline \mathrm{P} / \\ \mathrm{MPa} \\ \hline \end{gathered}$ | T/ K | $\begin{gathered} \hline \mathrm{P} / \\ \mathrm{MPa} \end{gathered}$ | T/K | $\begin{gathered} \hline \mathrm{P} / \\ \mathrm{MPa} \end{gathered}$ | T/K | $\begin{gathered} \hline \mathrm{P} / \\ \mathrm{MPa} \end{gathered}$ | T/K | $\begin{gathered} \hline \mathrm{P} / \\ \mathrm{MPa} \end{gathered}$ |
| 343.15 | 24.712 | 343.15 | 21.264 | 343.15 | 17.645 | 343.15 | 15.166 | 343.15 | 12.227 | 343.15 | 10.008 |
| 333.15 | 23.287 | 333.15 | 20.081 | 333.15 | 16.751 | 333.15 | 14.438 | 333.15 | 11.678 | 333.15 | 9.592 |
| 323.15 | 21.861 | 323.15 | 18.898 | 323.15 | 15.854 | 323.15 | 13.705 | 323.15 | 11.128 | 323.15 | 9.177 |
| 313.15 | 20.419 | 313.15 | 17.716 | 313.15 | 14.953 | 313.15 | 12.969 | 313.15 | 10.583 | 313.15 | 8.760 |
| 308.15 | 19.728 | 303.15 | 16.533 | 303.15 | 14.047 | 303.15 | 12.229 | 303.15 | 10.033 | 303.15 | 8.338 |
| 303.15 | 19.002 | 293.15 | 15.350 | 293.15 | 13.134 | 293.15 | 11.488 | 293.15 | 9.481 | 293.15 | 7.917 |
| 293.15 | 17.585 | 283.15 | 14.167 | 283.15 | 12.216 | 283.15 | 10.749 | 283.15 | 8.932 | 283.15 | 7.497 |
| 283.15 | 16.156 | 273.15 | 12.984 | 273.15 | 11.303 | 278.15 | 10.375 | 278.15 | 8.656 | 278.15 | 7.285 |
| 273.15 | 14.739 | 263.15 | 11.803 | 263.15 | 10.443 | 273.15 | 10.007 | 273.15 | 8.381 | 273.15 | 7.075 |
| 263.15 | 13.339 | 253.15 | 10.617 | 253.15 | 9.523 | 271.15 | 9.861 | 271.15 | 8.272 | 271.15 | 6.991 |
| 253.15 | 11.906 | 243.15 | 9.437 | 248.15 | 9.075 | 269.15 | 9.711 | 269.15 | 8.163 | 269.15 | 6.909 |
| 243.15 | 10.449 | 238.15 | 8.885 | 243.15 | 8.662 | 267.15 | 9.563 | 267.15 | 8.055 | 267.15 | 6.825 |
| 238.15 | 9.733 | 233.15 | 8.377 | 238.15 | 8.247 | 265.15 | 9.418 | 265.15 | 7.947 | 265.15 | 6.742 |
| 233.15 | 9.007 | 228.15 | 7.872 | 234.60 | 7.947 | 263.15 | 9.272 | 264.15 | 7.890 | 264.15 | 6.700 |
| 228.15 | 8.368 | 223.15 | 7.351 | 229.00 | 7.490 | 262.15 | 9.199 | 263.15 | 7.835 | 263.15 | 6.659 |
| 223.15 | 7.757 | 218.15 | 6.850 | 223.15 | 7.011 | 261.15 | 9.125 | 262.15 | 7.781 | 262.15 | 6.620 |
| 218.15 | 7.147 | 213.15 | 6.360 |  |  | 260.15 | 9.052 | 261.15 | 7.726 | 261.15 | 6.580 |
| 213.15 | 6.555 |  |  |  |  | 259.15 | 8.976 | 260.15 | 7.677 | 260.15 | 6.541 |
|  |  |  |  |  |  | 258.15 | 8.902 | 259.15 | 7.629 |  |  |
|  |  |  |  |  |  | 257.15 | 8.829 | 258.15 | 7.581 |  |  |
|  |  |  |  |  |  | 256.15 | 8.757 |  |  |  |  |
|  |  |  |  |  |  | 255.15 | 8.684 |  |  |  |  |
|  |  |  |  |  |  | 254.15 | 8.617 |  |  |  |  |
|  |  |  |  |  |  | 253.15 | 8.551 |  |  |  |  |

Table C2. Continued

| Isochore 7 |  | Isochore 8 |  | Isochore 9 |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| T/K | $\begin{gathered} \hline \mathrm{P} / \\ \mathrm{MPa} \end{gathered}$ | T/K | $\begin{gathered} \hline \mathrm{P} / \\ \mathrm{MPa} \\ \hline \end{gathered}$ | T / K | $\begin{gathered} \hline \mathrm{P} / \\ \mathrm{MPa} \\ \hline \end{gathered}$ |
| 343.15 | 7.735 | 343.15 | 5.582 | 343.15 | 3.329 |
| 333.15 | 7.433 | 333.15 | 5.380 | 333.15 | 3.217 |
| 323.15 | 7.131 | 323.15 | 5.177 | 323.15 | 3.105 |
| 313.15 | 6.831 | 313.15 | 4.973 | 313.15 | 2.992 |
| 303.15 | 6.530 | 303.15 | 4.771 | 303.15 | 2.881 |
| 293.15 | 6.229 | 293.15 | 4.571 | 293.15 | 2.769 |
| 283.15 | 5.929 | 283.15 | 4.368 | 283.15 | 2.658 |
| 278.15 | 5.776 | 278.15 | 4.266 | 278.15 | 2.601 |
| 273.15 | 5.625 | 273.15 | 4.164 | 273.15 | 2.546 |
| 271.15 | 5.565 | 271.15 | 4.123 | 271.15 | 2.522 |
| 269.15 | 5.505 | 269.15 | 4.083 | 269.15 | 2.500 |
| 267.15 | 5.445 | 267.15 | 4.042 | 267.15 | 2.477 |
| 265.15 | 5.385 | 265.15 | 4.002 | 265.15 | 2.455 |
| 264.15 | 5.356 | 264.15 | 3.982 | 264.15 | 2.444 |
| 263.15 | 5.326 | 263.15 | 3.961 | 263.15 | 2.433 |
| 262.15 | 5.297 | 262.15 | 3.939 | 262.15 | 2.421 |
| 261.15 | 5.267 | 261.15 | 3.918 | 261.15 | 2.410 |
| 260.15 | 5.239 | 260.15 | 3.896 | 260.15 | 2.399 |
|  |  |  |  | 259.15 | 2.387 |
|  |  |  |  | 258.15 | 2.375 |
|  |  |  |  | 257.15 | 2.362 |
|  |  |  |  |  |  |
|  |  |  |  |  |  |
|  |  |  |  |  |  |

## RESULTS FOR SAMPLE 3

Table C3. Isochoric Results for Sample 3. Black: Single Phase; Blue: Two Phase

| Isochore 1 |  | Isochore 2 |  | Isochore 3 |  | Isochore 4 |  | Isochore 5 |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| T/K | $\begin{gathered} \hline \mathrm{P} / \\ \mathrm{MPa} \\ \hline \end{gathered}$ | T/K | $\begin{gathered} \hline \mathrm{P} / \\ \mathrm{MPa} \\ \hline \end{gathered}$ | T/K | $\begin{gathered} \hline \mathrm{P} / \\ \mathrm{MPa} \\ \hline \end{gathered}$ | T/K | $\begin{gathered} \hline \mathrm{P} / \\ \mathrm{MPa} \\ \hline \end{gathered}$ | T/K | $\begin{gathered} \hline \mathrm{P} / \\ \mathrm{MPa} \\ \hline \end{gathered}$ |
| 323.15 | 20.829 | 343.15 | 21.114 | 343.15 | 18.470 | 343.15 | 14.803 | 343.15 | 11.873 |
| 313.15 | 19.506 | 333.15 | 19.976 | 333.15 | 17.525 | 333.15 | 14.096 | 333.15 | 11.350 |
| 303.15 | 18.172 | 323.15 | 18.828 | 323.15 | 16.569 | 323.15 | 13.387 | 323.15 | 10.825 |
| 293.15 | 16.836 | 313.15 | 17.675 | 313.15 | 15.605 | 313.15 | 12.675 | 313.15 | 10.298 |
| 283.15 | 15.495 | 303.15 | 16.516 | 303.15 | 14.636 | 303.15 | 11.958 | 303.15 | 9.769 |
| 273.15 | 14.151 | 293.15 | 15.348 | 293.15 | 13.660 | 293.15 | 11.236 | 293.15 | 9.238 |
| 263.15 | 12.826 | 283.15 | 14.177 | 283.15 | 12.685 | 283.15 | 10.522 | 283.15 | 8.709 |
| 258.15 | 12.222 | 273.15 | 13.006 | 273.15 | 11.708 | 278.15 | 10.162 | 278.15 | 8.445 |
| 253.15 | 11.542 | 263.15 | 11.914 | 263.15 | 10.742 | 273.15 | 9.804 | 273.15 | 8.183 |
| 243.15 | 10.182 | 253.15 | 10.731 | 253.15 | 9.808 | 271.15 | 9.662 | 271.15 | 8.079 |
| 238.15 | 9.555 | 243.15 | 9.621 | 243.15 | 8.883 | 269.15 | 9.521 | 269.15 | 7.975 |
| 233.15 | 8.943 | 238.15 | 9.078 | 238.15 | 8.429 | 267.15 | 9.382 | 267.15 | 7.872 |
| 228.15 | 8.358 | 233.15 | 8.537 | 233.24 | 7.979 | 265.15 | 9.241 | 265.15 | 7.771 |
| 223.21 | 7.773 | 228.15 | 8.015 | 228.40 | 7.541 | 263.15 | 9.102 | 263.15 | 7.670 |
| 218.37 | 7.210 | 223.15 | 7.496 | 223.55 | 7.123 | 262.15 | 9.038 | 262.15 | 7.621 |
| 213.50 | 6.634 | 218.15 | 6.981 |  |  | 261.15 | 8.977 |  |  |
|  |  | 213.29 | 6.456 |  |  |  |  |  |  |

Table C3. Continued

| Isochore 6 |  | Isochore 7 |  | Isochore 8 |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{T} / \mathrm{K}$ | $\mathrm{P} / \mathrm{MPa}$ | $\mathrm{T} / \mathrm{K}$ | $\mathrm{P} / \mathrm{MPa}$ | $\mathrm{T} / \mathrm{K}$ | $\mathrm{P} / \mathrm{MPa}$ |
| 343.15 | 9.262 | 343.15 | 6.724 | 343.15 | 4.414 |
| 333.15 | 8.884 | 333.15 | 6.474 | 333.15 | 4.263 |
| 323.15 | 8.507 | 323.15 | 6.224 | 323.15 | 4.111 |
| 313.15 | 8.128 | 313.15 | 5.972 | 313.15 | 3.958 |
| 303.15 | 7.748 | 303.15 | 5.717 | 303.15 | 3.805 |
| 293.15 | 7.367 | 293.15 | 5.463 | 293.15 | 3.650 |
| 283.15 | 6.989 | 283.15 | 5.210 | 283.15 | 3.496 |
| 278.15 | 6.800 | 278.15 | 5.083 | 278.15 | 3.418 |
| 273.15 | 6.610 | 273.15 | 4.957 | 273.15 | 3.341 |
| 271.15 | 6.536 | 271.15 | 4.906 | 271.15 | 3.311 |
| 269.15 | 6.461 | 269.15 | 4.854 | 269.15 | 3.279 |
| 267.15 | 6.386 | 267.15 | 4.803 | 267.15 | 3.247 |
| 265.15 | 6.312 | 266.15 | 4.777 | 265.15 | 3.214 |
| 263.15 | 6.241 | 265.15 | 4.750 | 264.15 | 3.198 |
|  |  | 264.15 | 4.725 | 263.15 | 3.182 |
|  |  | 263.15 | 4.698 |  |  |
|  |  |  |  |  |  |

## RESULTS FOR SAMPLE 4

Table C4. Isochoric Results for Sample 4. Black: Single Phase; Blue: Two Phase

| Isochore 1 |  | Isochore 2 |  | Isochore 3 |  | Isochore 4 |  | Isochore 5 |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\mathrm{P} /$ |  | $\mathrm{P} /$ |  | $\mathrm{P} /$ |  | $\mathrm{P} /$ |  | $\mathrm{P} /$ |
| $\mathrm{T} / \mathrm{K}$ | MPa | $\mathrm{T} / \mathrm{K}$ | MPa | $\mathrm{T} / \mathrm{K}$ | MPa | $\mathrm{T} / \mathrm{K}$ | MPa | $\mathrm{T} / \mathrm{K}$ | MPa |
| 343.15 | 21.395 | 343.15 | 18.614 | 343.15 | 14.846 | 343.15 | 12.059 | 343.15 | 9.331 |
| 333.15 | 20.150 | 333.15 | 17.584 | 333.15 | 14.101 | 333.15 | 11.505 | 333.15 | 8.933 |
| 323.15 | 18.912 | 323.15 | 16.564 | 323.15 | 13.357 | 323.15 | 10.947 | 323.15 | 8.541 |
| 313.15 | 17.671 | 313.15 | 15.541 | 313.15 | 12.611 | 313.15 | 10.389 | 313.15 | 8.148 |
| 303.15 | 16.435 | 303.15 | 14.515 | 303.15 | 11.862 | 303.15 | 9.823 | 303.15 | 7.754 |
| 293.15 | 15.183 | 293.15 | 13.488 | 293.15 | 11.105 | 293.15 | 9.263 | 293.15 | 7.360 |
| 283.15 | 13.939 | 283.15 | 12.455 | 283.15 | 10.359 | 283.15 | 8.706 | 283.15 | 6.967 |
| 278.15 | 13.321 | 278.15 | 11.946 | 278.15 | 9.987 | 278.15 | 8.434 | 278.15 | 6.778 |
| 273.15 | 12.702 | 273.15 | 11.430 | 273.15 | 9.629 | 273.15 | 8.171 | 273.15 | 6.590 |
| 268.15 | 12.075 | 268.15 | 10.937 | 270.15 | 9.413 | 271.15 | 8.063 | 271.15 | 6.513 |
| 264.15 | 11.604 | 264.15 | 10.542 | 268.15 | 9.277 | 269.15 | 7.960 | 269.15 | 6.440 |
| 263.15 | 11.494 | 263.15 | 10.440 | 266.15 | 9.131 | 267.15 | 7.849 | 267.15 | 6.364 |
| 262.15 | 11.385 | 262.15 | 10.345 | 264.15 | 8.988 | 265.15 | 7.742 | 265.15 | 6.289 |
| 261.15 | 11.275 | 261.15 | 10.249 | 262.15 | 8.845 |  |  |  |  |

Table C4. Continued

| Isochore 6 |  | Isochore 7 |  | Isochore 8 |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{T} / \mathrm{K}$ | $\mathrm{P} / \mathrm{MPa}$ | $\mathrm{T} / \mathrm{K}$ | $\mathrm{P} / \mathrm{MPa}$ | $\mathrm{T} / \mathrm{K}$ | $\mathrm{P} / \mathrm{MPa}$ |
| 343.15 | 6.981 | 343.15 | 4.448 | 343.15 | 2.054 |
| 333.15 | 6.713 | 333.15 | 4.292 | 333.15 | 1.989 |
| 323.15 | 6.443 | 323.15 | 4.134 | 323.15 | 1.922 |
| 313.15 | 6.172 | 313.15 | 3.976 | 313.15 | 1.854 |
| 303.15 | 5.900 | 303.15 | 3.818 | 303.15 | 1.788 |
| 293.15 | 5.626 | 293.15 | 3.659 | 293.15 | 1.720 |
| 283.15 | 5.352 | 283.15 | 3.499 | 283.15 | 1.652 |
| 278.15 | 5.217 | 278.15 | 3.419 | 278.15 | 1.618 |
| 273.15 | 5.081 | 273.15 | 3.336 | 273.15 | 1.584 |
| 271.15 | 5.026 | 271.15 | 3.303 | 271.15 | 1.567 |
| 269.15 | 4.971 | 269.15 | 3.270 | 269.15 | 1.549 |
| 267.15 | 4.917 | 267.15 | 3.237 | 267.15 | 1.533 |
| 265.15 | 4.863 | 265.15 | 3.204 | 265.15 | 1.517 |
| 263.15 | 4.808 | 263.15 | 3.170 | 263.15 | 1.500 |

## VITA

Diego Edison Cristancho Blanco is a son of Jose Rafael Cristancho, Audrey Blanco and younger brother of Deissy Cristancho and Byron Cristancho.

Diego Edison Cristancho Blanco has both a B.S. in chemical engineering and a B.S. in physics from the Industrial University of Santander, Colombia. He also has a M.S. in chemical engineering from the same university. He was a member of the Environmental Research Center of the Industrial University of Santander as a technical advisor for about three years. At the same time, he was part of the Gravitational and Cosmology research group in the Physics department. From 2002 to 2005, he taught courses in both the Chemical Engineering and the Physics departments. During his doctoral studies, he has had the opportunity to work for industry as an intern covering projects related to industrial software applications and supercritical extraction. Recently, he worked at the National Institute of Standards and Technology (NIST) in Boulder, Colorado developing a new fundamental equation of state for propylene glycol. He graduated with his Ph.D. in chemical engineering at Texas A\&M University in May 2010.

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