

QUANTIFYING THE UNCERTAINTY IN ESTIMATES OF WORLD
CONVENTIONAL OIL RESOURCES

A Thesis

by

Chih-Ming Tien

Submitted to the Office of Graduate Studies of
Texas A&M University
in partial fulfillment of the requirements for the degree of
MASTER OF SCIENCE

December 2009

Major Subject: Petroleum Engineering

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Chair of Committee,	Duane A. McVay
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ABSTRACT

Quantifying the Uncertainty in Estimates of World Conventional Oil Resources.

(December 2009)

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Since Hubbert proposed the “peak oil” concept to forecast ultimate recovery of crude oil for the U.S. and the world, there have been countless debates over the timing of peak world conventional oil production rate and ultimate recovery. From review of the literature, forecasts were grouped into those that are like Hubbert’s with an imminent peak, and those that do not predict an imminent peak. Both groups have bases for their positions. Viewpoints from the two groups are polarized and the rhetoric is pointed and sometimes personal. A big reason for the large divide between the two groups is the failure of both to acknowledge the significant uncertainty in their estimates. Although some authors attempt to quantify uncertainty, most use deterministic methods and present single values, with no ranges. This research proposes that those that do attempt to quantify uncertainty underestimate it significantly.

The objective of this thesis is to rigorously quantify the uncertainty in estimates of ultimate world conventional oil production and time to peak rate. Two different methodologies are used. The first is a regression technique based on historical

production data using Hubbert's model and the other methodology uses mathematical models. However, I conduct the analysis probabilistically, considering errors in both the data and the model, which results in likelihood probability distributions for world conventional oil production and time to peak rate. In the second method, I use a multiple-experts analysis to combine estimates from the multitude of papers presented in the literature, yielding an overall distribution of estimated world conventional oil production.

Giving due consideration to uncertainty, Hubbert-type mathematical modeling results in large uncertainty ranges that encompass both groups of forecasts (imminent peak and no imminent peak). These ranges are consistent with those from the multiple-experts analysis. In short, the industry does not have enough information at this time to say with any reliability what the ultimate world conventional oil production will be. It could peak soon, somewhere in the distant future, or somewhere in between. It would be wise to consider all of these possible outcomes in planning and making decisions regarding capital investment and formulation of energy policy.

DEDICATION

This thesis is dedicated to my wife and parents who fully support me to pursue the knowledge.

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1. INTRODUCTION

Fossil fuel is still the main energy source used by humankind. However, with oil price exceeding \$100/bbl in recent years, questions regarding when we will run out of oil and how much oil we will produce drive us to reexamine this old discussion. In 1956, Hubbert (1956) presented the “peak oil” concept to explain that production of finite, nonrenewable resources should begin at zero, rise to a peak, then decline continuously. The viewpoint of “peak oil” is primarily concerned about when the production rate can no longer increase and will plateau or begin to decline, rather than about running out of oil.

There have been numerous debates during the past half-century (e.g., Al-Jarri and Startzman 1997; Campbell 2003; Lynch 2003; Ryan 1966) over the timing of “peak oil” production rate. Because our civilization still depends heavily on oil, and global energy demand is anticipated to increase in the next few decades, it is increasingly important to determine how long the oil supply can meet the world’s energy demand.

Bentley et al. (2008) summarized the methodologies and predictions for global oil production by many authors since 1956. The predictions include a wide range of opinions ranging from imminent peaks in oil production rate to no imminent peaks. The causes that resulted in significant prediction inconsistency between these forecasts are due to large uncertainties. The uncertainty comes from different aspects, e.g., measurement or reporting errors within the production data, predictive errors of forecast

models, different language definitions of crude oil, and so on. Besides these uncertainties, other factors that make it hard to determine the “peak oil” are (1) cost and investment risk, (2) technological, political and environmental challenges, and (3) future demand for world oil.

It is important to quantify uncertainty in peak oil in order to have a greater recognition of the range of possible outcomes. Unfortunately, this uncertainty has not been thoroughly studied and quantified, which makes it difficult to make decisions, such as planning capital spending and formulating energy policy.

2. BACKGROUND

Literature Review

In the middle of the twentieth century, Hubbert (1956) proposed the “peak oil” concept attempting to forecast the ultimate recovery of crude oil for the U.S. and the world. The concept of peak oil recognized that production of finite, nonrenewable resources should begin at zero, rise to a peak, then decline continuously. Furthermore, he emphasized that there is no requirement that such a production curve be symmetrical or that it have only a single maximum. The peak time in his publication is inferred by a hand-drawn production curve given an assessment of a region’s ultimate recoverable oil.

Later, Hubbert (1962) combined this idea with a mathematical model (logistic curve) to estimate the ultimate crude oil production of the U.S. from historical data including past production, proved reserves, and proved discoveries. He predicted that crude oil production would peak in the early 1970s for the US lower 48. This prediction turned out to be remarkably accurate.

Hubbert’s Model

Hubbert (1956) presented and illustrated the peak oil concept for finite or nonrenewable resources. Assumptions for the peak oil concept are:

1. In the production of any resource of fixed magnitude, the production rate must begin at zero and must decline again to zero after passing through one or more maxima.
2. The area under the production curve plotted against time equals ultimate oil recovery

as time approaches infinity.

Hubbert later used one complete production cycle and developed the mathematical model based on the logistic curve by assuming the production rate curve ($q(t)$ or dQ/dt) could be expressed as a parabolic function of the cumulative production, Q , (Hubbert 1962, 1982),

$$q(t) = \frac{dQ}{dt} = aQ + bQ^2 \dots\dots\dots (1)$$

When cumulative oil production is equal to ultimate oil recovery, Q_∞ , the production rate q will be zero. At this point, from Eq. (1),

$$b = -\frac{a}{Q_\infty} \dots\dots\dots (2)$$

Combining Eq. (2) and Eq. (3),

$$\frac{dQ}{\left(Q - \frac{Q}{Q_\infty}\right)} = a dt \dots\dots\dots (3)$$

Integrating Eq. (3) from $t = t_0$ to ∞ , the logistic cumulative oil production curve is

$$Q = \frac{Q_\infty}{\left[1 + N_0 e^{-a(t-t_0)}\right]} \dots\dots\dots (4)$$

$$\text{where } N_0 = \frac{Q_\infty - Q_0}{Q_\infty} \dots\dots\dots (5)$$

Substituting Eq. (4) into Eq. (1), the production rate curve as a function of time is

$$q(t) = \frac{dQ}{dt} = Q_{\infty} \frac{aN_0 e^{-a(t-t_0)}}{\left[1 + N_0 e^{-a(t-t_0)}\right]^2} \dots\dots\dots (6)$$

The maximum cumulative production rate is obtained by differentiating Eq. (1) and setting it to zero,

$$Q_M = \frac{Q_{\infty}}{2} \dots\dots\dots (7)$$

The maximum production rate can be obtained by substituting Eq. (7) into Eq. (1),

$$q_M = \frac{aQ_{\infty}}{4} \dots\dots\dots (8)$$

To find the time at the maximum production rate or maximum cumulative production rate, Eq. (7) can be equated with Eq. (4),

$$t_M = t_0 + \frac{\ln(N_0)}{a} \dots\dots\dots (9)$$

There are four parameters for Eq. (4) and Eq. (6). In order to analyze historical production data with these equations conveniently, I can reduce the number of parameters and the complexity of the equations. The cumulative production can be expressed in terms of ultimate oil recovery and peak time by substituting t_0 from Eq. (9) into Eq. (4),

$$Q = \frac{Q_{\infty}}{\left[1 + e^{-a(t-t_M)}\right]} \dots\dots\dots (10)$$

Differentiating Eq. (10),

$$q(t) = \frac{dQ}{dt} = \frac{Q_{\infty} a e^{-a(t-t_M)}}{\left[1 + e^{-a(t-t_M)}\right]^2} \dots\dots\dots(11)$$

Eq. (10) is the type of logistic curve found by Verhulst in 1845 with population studies, and its derivative, Eq. (11), is the so-called Hubbert curve. The curve in Eq. (11) can be simplified using the hyperbolic secant function,

$$q(t) = \frac{Q_{\infty} \sec h^2 \left[\frac{(t-t_M)}{2S_L} \right]}{4S_L} \dots\dots\dots(12)$$

$$\text{where } S_L = \frac{1}{a} \dots\dots\dots(13)$$

Hubbert's model has historical oil production data versus time following a symmetric bell-shaped curve. Hence, another mathematical model often used in fitting historical production data is the normal distribution model because of its similar curve shape compared to Hubbert's logistic model. Bartlett (2000) applied the normal distribution model to predict crude oil production for the U.S and the world.

Normal Distribution Model

The probability density function for a normal distribution is given by a formula

$$f(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}}, \quad -\infty < x < \infty \dots\dots\dots(14)$$

where μ = the mean (the peak of the bell curve), and

σ = the standard deviation (a measure of the width of the bell curve).

The area under the probability density function in Eq. (14) is equal to one. Thus, I can transfer the original mathematical form of the normal distribution into the production rate curve with the three parameters Q_{∞} , t_M , and S_N ,

$$q(t) = \frac{Q_{\infty}}{\sqrt{2\pi}S_N} e^{-\frac{(t-t_M)^2}{2S_N^2}} \dots\dots\dots (15)$$

where Q_{∞} = ultimate oil recovery,

t_M = the time at the maximum production rate, or the timing of peak oil, and

S_N = the width parameter.

Between Eq. (12) and Eq. (15), there exists only one difference between parameters of these two equations, i.e., the width of the curves. In addition, two important parameters, Q_{∞} and t_M , are both in Eq. (12) and Eq. (15), and it will be convenient to compare the difference between these two models for uncertainty analyses.

Debates over Peak Oil

After Hubbert, other authors (Bartlett 2000; Brandt 2007; Laherrere 1997) also developed or utilized different mathematical or statistical models to forecast oil production trends and predict ultimate recoverable resources. Laherrere (1997, 2000) pointed out that Hubbert's model applied for forecasting production only works well in nature's domain, unaffected by political or significant economic interference, and for areas having a large number of fields and unfettered activity. In addition, he also found

that historical production curves from different countries with a smaller number of basins and fields have more than one peak, but almost all of the peaks are individually symmetrical in their upper parts. Because of these constraints and weaknesses in predicting crude oil production with Hubbert's model, Laherrere (1997) modified the Hubbert model and applied multiple-cycle concepts to describe the production trends by fitting historical production data into two or more peaks instead of a complete production cycle. He called this approach "multiple-Hubbert modeling."

A number of authors in Group 1 of Bentley's paper (2008) evaluate conventional world oil resources by adding cumulative production, the 2P value of discovery data (P50 oil reserves), and estimates of yet-to-find. However, the 2P value of discovery data cannot be determined effectively because potential prices and technology development can drive it to increase significantly. Although the different kinds of models suggested by these papers rely heavily on analyzing historical production data or projecting the 2P value of discovery data to predict ultimate recoverable resources of crude oil, they generally neglect uncertainty within the models, the production data and discovery data.

Only a few publications (Brandt 2007; Laherrere 2007) acknowledge or attempt to quantify the considerable uncertainty associated with forecasts of world crude oil production. These papers indicate that uncertainty from inaccurate production data is primarily caused by the motive, language, and reporting data. Inaccurate data may result from political motivations to publish inexact amount of reserves, different reserves definitions, and ambiguous words in reporting data like oil, crude oil, and liquids. Most authors probably realize that uncertainty affects their forecasts, but they do not put much

emphasis on quantifying the uncertainty associated with forecasting crude oil production.

Since Hubbert presented the concept of peak oil, there has existed a long-standing debate over the optimism (e.g., Lynch 2003) and pessimism (e.g., Campbell 2003) associated with ultimate oil recovery and the timing of peak oil. Campbell (2003) relies primarily on Hubbert-type analyses and argues that there will be an imminent peak and decline in production rate from resource constraints. The position of Lynch (2003) is that there is no imminent peak. He argues that the main flaw in Hubbert-type models is that they consider ultimate recoverable resources as a fixed amount rather than a dynamic number that can be raised from application of new technology, knowledge, capability, and other factors. In addition, these conditions can be combined together to increase recovery by developing new producing areas and lowering economic limitations. Thus, he critiqued problems with Hubbert-type models by pointing out that they ignore the potential impact of price, technology, politics, and other factors.

Edwards (1997) and Bentley et al. (2008) both reviewed publications forecasting global oil production dating from 1956. From Tables 1 of both papers, I find that most previous publications focused on finding reasonable mathematical and statistical models to describe the past history production trend and project the future oil production rate, like Hubbert's, by fitting past production or discovery data in order to forecast oil production trends. Most authors attempted to illustrate how uncertainty played a key role in the forecasts. For example, Laherrere (2007) ever demonstrated that forecasts was affected significantly by uncertainty of data from reserves definitions, reporting errors

within production data, and language definition of conventional oil, but they still do not quantify the uncertainty well because their assessments usually depend on deterministic methods and yield only single values with no ranges. Bentley et al. (2008) classified the forecasts from individual, organizations and institutions into three different groups: Group 1- quantitative analyses which predict that global oil production will come to a resource-limited peak in the near term and certainly before 2020; Group 2 - forecasts that use quantitative methods, but which appear to have no production peak within the forecasted time period (2020 or 2030); and Group 3- non-quantitative analyses that do not predict a resource-limited oil peak within the foreseeable future. The predictions from these groups covered a wide range, from imminent peaks to no imminent peaks.

Based on review of the literature above, I find it useful to divide between those papers that are like Hubbert's (imminent peak) and those that do not predict an imminent peak. Both groups have bases for their position, but both fail to convincingly quantify the uncertainty in their forecasts. Because of this, viewpoints are polarized and the rhetoric is pointed, and sometimes personal.

The wide range of forecasts made by different groups and summarized by Bentley et al. (2008) easily make us realize that large uncertainty exists in estimates of world crude oil production and peak time. However, most only give a single value rather than a possible range. There is a clear need to develop and apply probabilistic methods in order to quantify the true uncertainty in estimates of world conventional oil resources and time to peak oil production rate.

3. OBJECTIVE

The overall objective of this study is to rigorously quantify the uncertainty in estimates of world conventional oil resources and time to peak world oil production rate. A secondary objective is to quantify the uncertainty in conventional oil production of the lower U.S. 48 as a contrast to the uncertainty in the estimates of world recoverable oil resources.

4. METHODOLOGY FOR QUANTIFYING THE UNCERTAINTY

Overview

The methodologies applied to quantify the uncertainty in my work are mathematical modeling and multiple experts system. The reason for choosing the mathematical modeling for developing the uncertainty quantification is to examine how the forecasts generated by the models can be significantly affected by the uncertainty existed in production data and model predictive errors. Besides, due consideration to wider range of forecasts for world conventional oil production, it is helpful to conduct multiple experts system for analyzing the uncertainty among these forecasts.

Finally, I will integrate the two methodologies into an overall assessment of the uncertainty in estimates of world conventional resources.

Mathematical Modeling of Uncertainty

In the petroleum industry, we often apply statistical and probabilistic methods to evaluate original hydrocarbons in place (OHIP) in a reservoir because of the large uncertainty due to scarcity and inaccuracy of data. For the same reason, I will use a regression method based on past production using Hubbert's model and other mathematical models to quantify the uncertainty. However, I conduct the analysis probabilistically, considering errors in both the data and the model, which results in likelihood probability distributions for world conventional oil production and time to peak rate.

In my analysis, definition of the likelihood function depends on the specification of a model for the uncertainty associated with the observed production data every year. The likelihood function is obtained by combining the production data, d^{obs} , with a forward model, $g(x)$, that predicts production data every year implicitly as a function of Q_∞ , t_M , and S_N for the normal distribution model, and also Q_∞ , t_M , and S_L for Hubbert's model. The vector $g(x)$ consists of the predicted production data using the normal distribution or Hubbert's model. The production data misfits (observed data minus forward model) is assumed to follow a normal distribution, which is represented as

$$f(d^{obs} | x) = \frac{1}{(2\pi)^{n_d/2} [\det(C_D)]^{1/2}} \cdot \exp \left\{ -\frac{1}{2} \left[(g(x) - d^{obs})^T C_D^{-1} (g(x) - d^{obs}) \right] \right\} \quad \dots (16)$$

where n_d = the number of measured (observed) data points.

In Eq. (16), d^{obs} is the vector of observed production data for every year and can be expressed as

$$d^{obs} = \begin{bmatrix} q_1^{obs} & q_2^{obs} & \cdot & \cdot & \cdot & q_n^{obs} \end{bmatrix}^T \dots (17)$$

In Eq. (16), C_D is the data covariance. I assume the errors in the observed production data are uncorrelated and the standard deviations of all observed production data are different. I assume that the variances of historical production data are a specific ratio (R) of the production value of every year, and I can get the covariance matrix:

$$C_D = \begin{bmatrix} \sigma_{q_1}^2 & 0 & \cdot & \cdot & 0 \\ 0 & \sigma_{q_2}^2 & 0 & \cdot & \cdot \\ \cdot & 0 & \cdot & 0 & \cdot \\ \cdot & \cdot & 0 & \cdot & 0 \\ 0 & \cdot & \cdot & 0 & \sigma_{q_{n_d}}^2 \end{bmatrix}_{n_d \times n_d} = R \begin{bmatrix} q_1^{obs} & 0 & \cdot & \cdot & 0 \\ 0 & q_2^{obs} & 0 & \cdot & \cdot \\ \cdot & 0 & \cdot & 0 & \cdot \\ \cdot & \cdot & 0 & \cdot & 0 \\ 0 & \cdot & \cdot & 0 & q_{n_d}^{obs} \end{bmatrix}_{n_d \times n_d} \dots (18)$$

In Eq. (16), $g(x)$ is the vector of production data of every year predicted by the forward model and can be written as

$$g(x) = \begin{bmatrix} q_1^{calc} & q_2^{calc} & \cdot & \cdot & \cdot & q_n^{calc} \end{bmatrix}^T \dots\dots\dots (19)$$

The variances in Eq. (18) include both the uncertainty in the model and errors within the data. I can effectively analyze the uncertainty existing in the model and data by choosing a reasonable variance and comparing the goodness of fit between the model prediction and data using either Hubbert's model or the normal distribution model. I use the sum-of-square norm to indicate the mismatch between the production data and model prediction. The sum-of-square norm can be defined as

$$S_o S = \frac{1}{n} \sum_{i=1}^n \left[\frac{(q_i^{obs} - q_i^{cal})}{\sigma_i} \right]^2 \dots\dots\dots (20)$$

The standard deviation in Eq. (20) is expressed as a ratio of the production data

$$\sigma_i^2 = R q_i^{obs} \dots\dots\dots (21)$$

Combining Eq. (20) and Eq. (21),

$$S_o S = \frac{1}{n} \sum_{i=1}^n \left[\frac{(q_i^{obs} - q_i^{cal})}{\sqrt{R q_i^{obs}}} \right]^2 = \frac{1}{nR} \sum_{i=1}^n \frac{(q_i^{obs} - q_i^{cal})^2}{q_i^{obs}} \dots\dots\dots (22)$$

Multiple Experts System for Analyzing Uncertainty

Over a long period of time, expert judgments have been used to provide useful information in forecasting, decision-making, and assessing risks, and its application

fields are quite diverse, including aerospace, medicine, the nuclear industry, veterinary science, agriculture, meteorology, economics, and psychology.

Experts play an important role in risk and decision analysis because their judgments can provide valuable information, especially considering the limited availability of hard data related to a great number of important uncertainties in risk analysis. Simply speaking, the purpose of using multiple experts is to obtain additional information that can generate more accurate forecasts or estimates. The decision-maker must choose and decide which expert(s) to consult to obtain information such as estimates or probabilities. Forecast accuracy can be improved by combining individual forecasts from multiple experts.

The objective of a multiple expert analysis is usually to obtain a single distribution that covers the beliefs of a number of experts. A specific distribution is specified by each expert individually. These distributions are usually considered to be independent of one another. Then, these distributions are aggregated into a single distribution mathematically.

A simple mathematical technique used widely to combine each the distributions of multiple experts is linear opinion pooling. The individual distributions $\{f_1(\varphi), \dots, f_n(\varphi)\}$, from each expert can be combined to obtain a consensus distribution, $f(\varphi)$, by the following function,

$$f(\varphi) = \sum_{i=1}^n \alpha_i f_i(\varphi) \dots\dots\dots (23)$$

Eq. (23) is a weighted average of the individual distributions from each expert, and

the weighting parameter α_i sums to 1. Here, I may choose larger weighting parameters for some experts than others. When the decision maker considers all expert's views reasonable and his/her beliefs encompass the full range of the opinions from all experts, the linear opinion pooling is more appropriate. This method puts more emphasis on the area where most values of φ overlap, but it does not exclude the low or high values of φ suggested by only one expert (Clemen 1989; O'Hagan et al. 2006).

In my study, I use a multiple-experts analysis to combine estimates from the multitude of papers presented in the literature to build an opinion pool, and then produce a probabilistic distribution of assessments in world conventional oil resources.

5. PRODUCTION DATA ANALYSIS WITH MATHEMATICAL MODEL

Conventional oil production data from the U.S. and the world are analyzed in many publications because of abundant historical production data and accessible data sources. I obtained production data from the Energy Information Administration (EIA 1996, 2009) for my analysis.

I select the Hubbert model and normal distribution model as my forward model, and then I attempt to find the ranges for the three parameters that yields the 3D parameter space that bounds all significant probability. I can obtain the overall possible predicted production data by sampling the three parameters within their ranges. Substituting the predicting production data into Eq. (16), I get the likelihood function value for every set of three parameters. In quantifying the uncertainty, all possible outcomes of uncertain events could be considered and designated probabilities to give a reliable probability density function of the result of interest. Hence, from the overall parameter space, I will find the marginal probability function and cumulative density function for each parameter, and classify the cumulative probability function into P90, P50, P10 and mean value.

From the standpoint of the probabilistic method, it will be helpful to realize the uncertainty in the estimates of ultimate oil recovery and maximum peak year. In addition, I am also interested to examine how the uncertainty from the scarcity of production data can significantly affect the forecasts by choosing past production data only before 1956.

6. ANALYSIS OF CONVENTIONAL OIL RESOURCES OF THE U.S. 48

I obtained conventional oil production data of the U.S. lower 48 from Energy Information Administration of the U.S. (EIA, 1996; 2009). **Fig. 1** is the production plot from the year 1860 to 2008. There is a clear peak in the production rate and a clearly established decline.

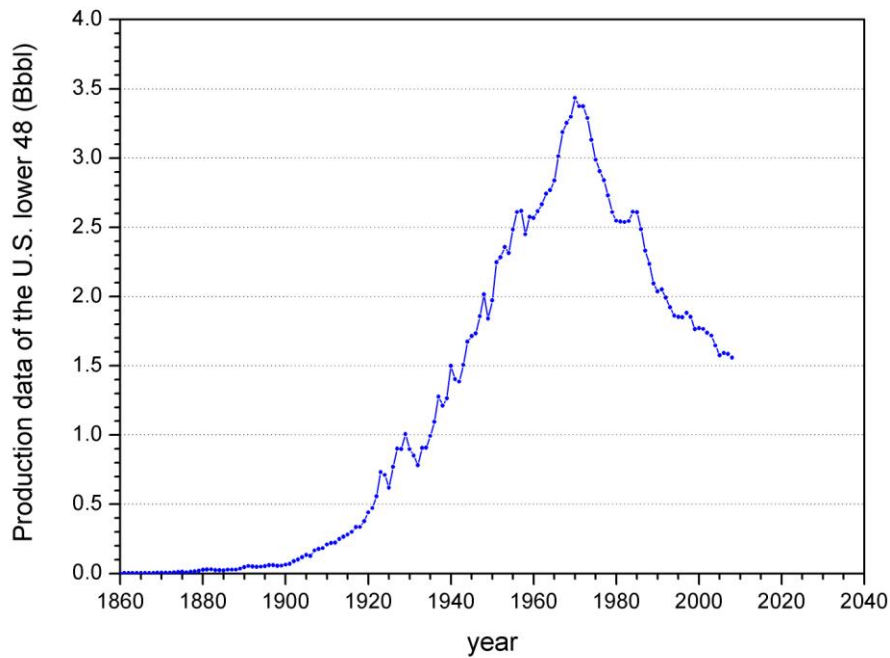


Fig. 1 - Production curve of the U.S. lower 48

The variance ratio (R) in Eq. (18) determines the model plus data errors for the uncertainty analyses in the likelihood function. Because I do not know what the actual variance ratio is, I choose different variance ratios to determine the extent of uncertainty

that can results from various data and model predictive errors. I developed and compared the uncertainty quantification in these scenarios for both Hubbert's model and the normal distribution model.

To do the analysis for any scenario, I have to determine the specific data range for each parameter. After obtaining the value of likelihood function of all parameter sets, I may find that some parameter sets have extremely small (or zero) probability or my parameter ranges do not include all possible values with significant probability. Thus, I need to examine and modify my data intervals until I cover all the possible parameter values with significant probability.

From the likelihood function values within the parameter space, marginal probability density functions (PDF) and cumulative density distributions (CDF) for each parameter can be easily obtained. In addition, I find the parameter set with maximum likelihood function value in the parameter space. This is the parameter set that would be obtained as the best fitting parameters estimated by using a least square method, which is independent of the variance ratio. I get the P90, P50, P10, and mean values for each parameter from the CDF plots, and find the other parameters with its corresponding maximum likelihood function value from the parameter space.

For the uncertainty analyses of conventional oil resources of the lower U.S. 48, I first take all production data up to 2008 to analyze with both Hubbert's model and Normal distribution model. Second, I only use the data before the year 1956, and this was done primarily to gain insights into the degree of uncertainty associated with forecasts made prior to peak rate (as was the case for U.S. lower 48 production in 1956), which should

help in the analysis of world oil production, which has apparently not yet reached a peak rate. This was also done as a historical curiosity, in order to quantify the uncertainty in U.S. production at the time Hubbert made his famous prediction. Finally, I compare and interpret the results of uncertainty analyses for these four schemes in the summary section.

Hubbert Model with Production Data up to 2008

Table 1 lists the data intervals and number of data points that define the discretized parameter space for different variance ratios.

Table 1 - Different variance ratio for Hubbert model with production data up to 2008 of the lower U.S. 48

Variance Ratio	R = 0.05		R=0.20		R=0.50	
	Data Interval	points	Data Interval	points	Data Interval	Points
Q_{∞} (Bbbl)	101~400	300	101~400	300	101~400	300
t_M (yr)	1961~1980.8	200	1961~1980.8	100	1961~1990.7	100
S_L (1/yr)	11~20.95	200	11~20.9	100	6~25.8	100

Fig. 2, **Fig. 4**, and **Fig. 6** are the plots of normalized PDF for each parameter Q_{∞} , t_M , and S_L . **Fig. 3**, **Fig. 5**, and **Fig. 7** are the plots of CDF for these three parameters, respectively. I obtained the P90, P50, P10, and mean values from the CDF of each parameter, which are shown in **Table 2**. Parameter sets found with maximum likelihood function value by taking the probabilistic value of each parameter are listed in **Table 3**, **Table 4**, and **Table 5**, and the sum-of-square norm (S_oS) values are also calculated. I choose one scenario ($R=0.5$) to plot the probabilistic forecasts for each parameter (**Fig. 8**, **Fig. 9**, and **Fig. 10**).

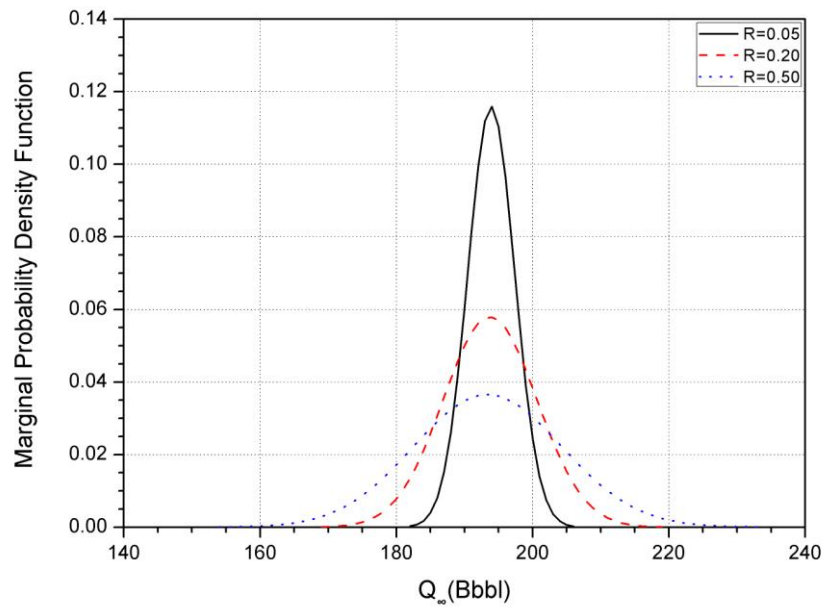


Fig. 2 - Marginal PDF of Q_{∞} for Hubbert model with production data up to 2008 of the lower U.S. 48

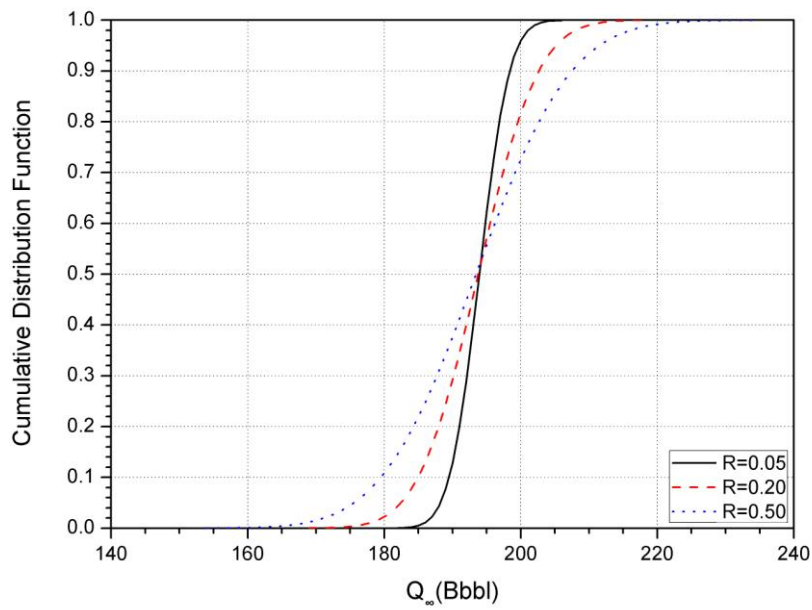


Fig. 3 - CDF of Q_{∞} for Hubbert model with production data up to 2008 of the lower U.S. 48

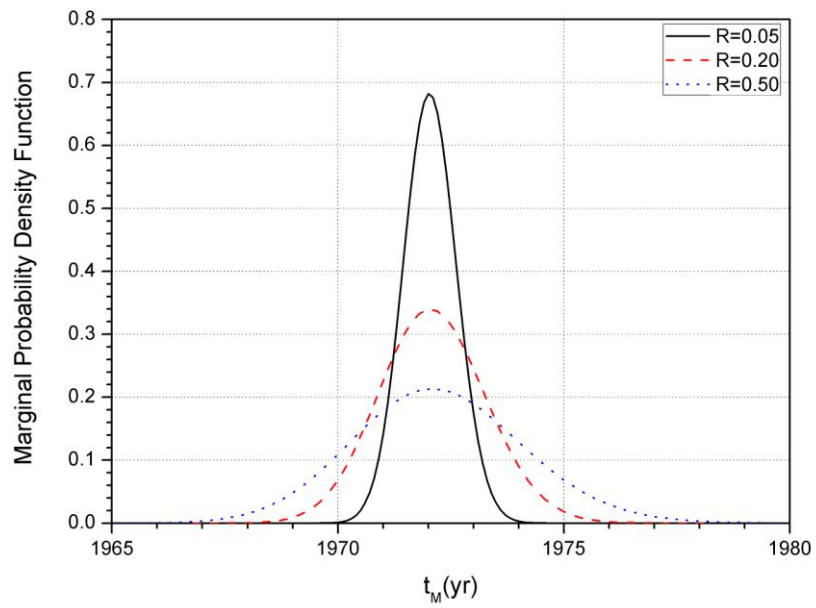


Fig. 4 - Marginal PDF of t_M for Hubbert model with production data up to 2008 of the lower U.S. 48

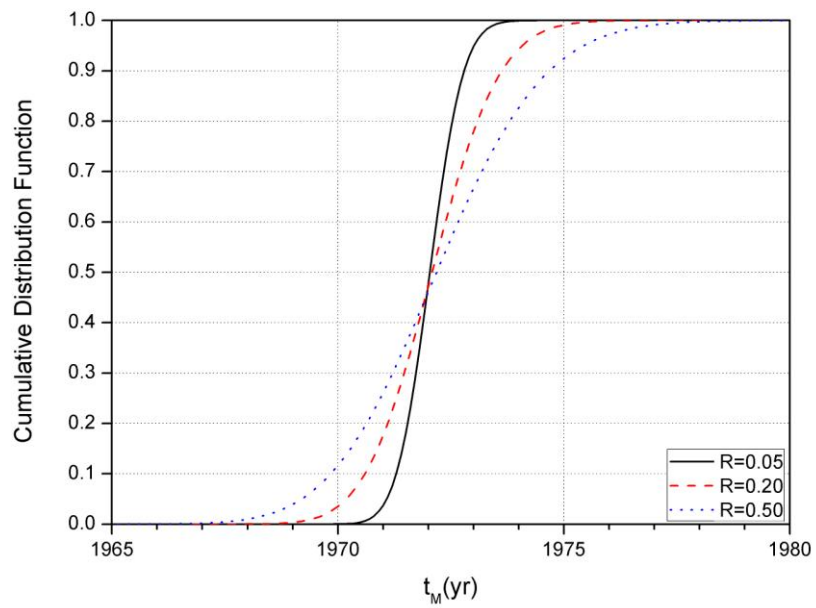


Fig. 5 - CDF of t_M for Hubbert model with production data up to 2008 of the lower U.S. 48

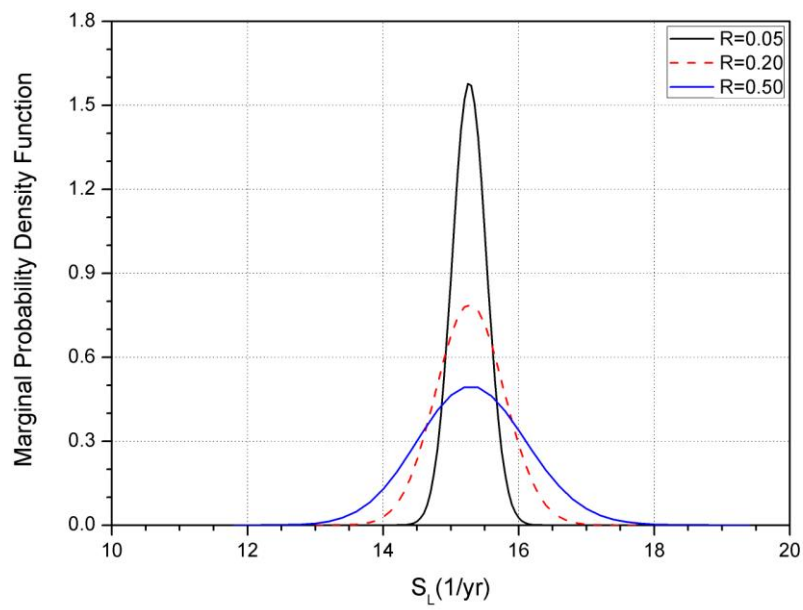


Fig. 6 - Marginal PDF of S_L for Hubbert model with production data up to 2008 of the lower U.S. 48

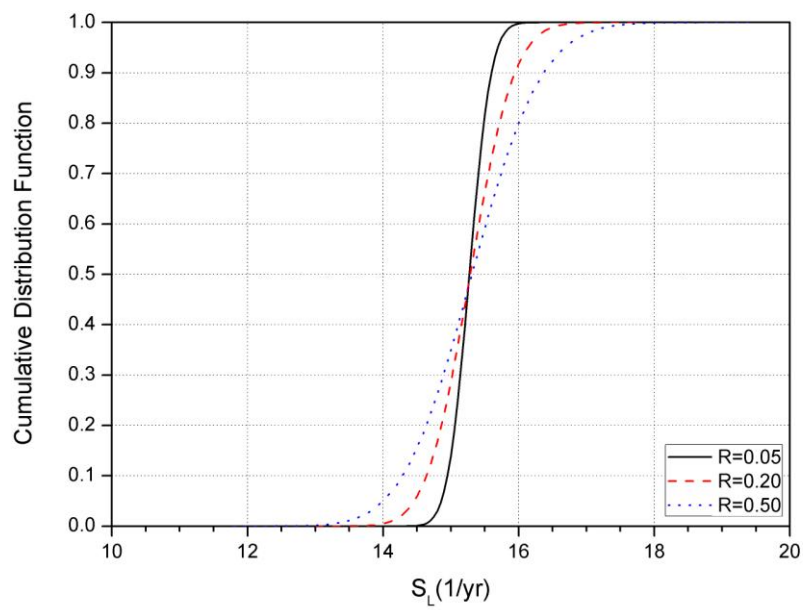


Fig. 7 - CDF of S_L for Hubbert model with production data up to 2008 of the lower U.S. 48

Table 2 - Probabilistic values of each parameter for Hubbert model with production data up to 2008 of the lower U.S. 48

Variance Ratio	R=0.05			R=0.20			R=0.50		
	Q_{∞} (Bbbl)	t_M (yr)	S_L (1/yr)	Q_{∞} (Bbbl)	t_M (yr)	S_L (1/yr)	Q_{∞} (Bbbl)	t_M (yr)	S_L (1/yr)
P90	189.497	1971.287	14.950	184.946	1970.600	14.641	179.497	1969.846	14.291
P50	193.925	1972.031	15.272	193.760	1972.080	15.285	193.435	1972.182	15.311
P10	198.365	1972.790	15.598	202.619	1973.622	15.942	207.487	1974.676	16.366
Mean	193.928	1972.035	15.273	193.774	1972.099	15.289	193.470	1972.231	15.322
Best-fit	194	1972.0	15.25	194	1972.0	15.30	194	1971.8	15.20

Table 3 - Parameter sets for Hubbert model with production data up to 2008 of the lower U.S. 48 (R=0.05)

R=0.05	Q_{∞}				t_M				S_L			
	Q_{∞} (Bbbl)	t_M (yr)	S_L (1/yr)	S_oS	Q_{∞} (Bbbl)	t_M (yr)	S_L (1/yr)	S_oS	Q_{∞} (Bbbl)	t_M (yr)	S_L (1/yr)	S_oS
P90	189	1971.8	15.15	0.4731	193	1971.3	15.10	0.4692	193	1971.7	14.95	0.4698
P50	194	1972.0	15.25	0.4589	194	1972.0	15.25	0.4589	194	1972.0	15.25	0.4589
P10	198	1972.2	15.35	0.4681	195	1972.8	15.45	0.4708	195	1972.4	15.60	0.4705
Mean	194	1972.0	15.25	0.4589	194	1972.0	15.25	0.4589	194	1972.0	15.25	0.4589

Table 4 - Parameter sets for Hubbert model with production data up to 2008 of the lower U.S. 48 (R=0.2)

R=0.2	Q_{∞}				t_M				S_L			
	Q_{∞} (Bbbl)	t_M (yr)	S_L (1/yr)	S_oS	Q_{∞} (Bbbl)	t_M (yr)	S_L (1/yr)	S_oS	Q_{∞} (Bbbl)	t_M (yr)	S_L (1/yr)	S_oS
P90	185	1971.6	15.1	0.1262	192	1970.6	15.0	0.1250	191	1971.4	14.6	0.1268
P50	194	1972.0	15.3	0.1148	194	1972.0	15.3	0.1148	194	1972.0	15.3	0.1148
P10	203	1972.4	15.4	0.1263	196	1973.6	15.6	0.1266	196	1972.8	15.9	0.1251
Mean	194	1972.0	15.3	0.1148	194	1972.0	15.3	0.1148	194	1972.0	15.3	0.1148

Table 5 - Parameter sets for Hubbert model with production data up to 2008 of the lower U.S. 48 (R=0.5)

R=0.5	Q_{∞}				t_M				S_L			
	Q_{∞} (Bbbl)	t_M (yr)	S_L (1/yr)	S_oS	Q_{∞} (Bbbl)	t_M (yr)	S_L (1/yr)	S_oS	Q_{∞} (Bbbl)	t_M (yr)	S_L (1/yr)	S_oS
P90	179	1971.5	15.0	0.0588	190	1969.7	14.8	0.0570	189	1970.9	14.2	0.0583
P50	193	1971.8	15.2	0.0460	194	1972.1	15.2	0.0460	194	1972.1	15.4	0.0461
P10	207	1972.7	15.6	0.0555	198	1974.8	16.0	0.0600	198	1973.6	16.4	0.0589
Mean	193	1971.8	15.2	0.0460	194	1972.1	15.2	0.0460	194	1972.1	15.4	0.0461

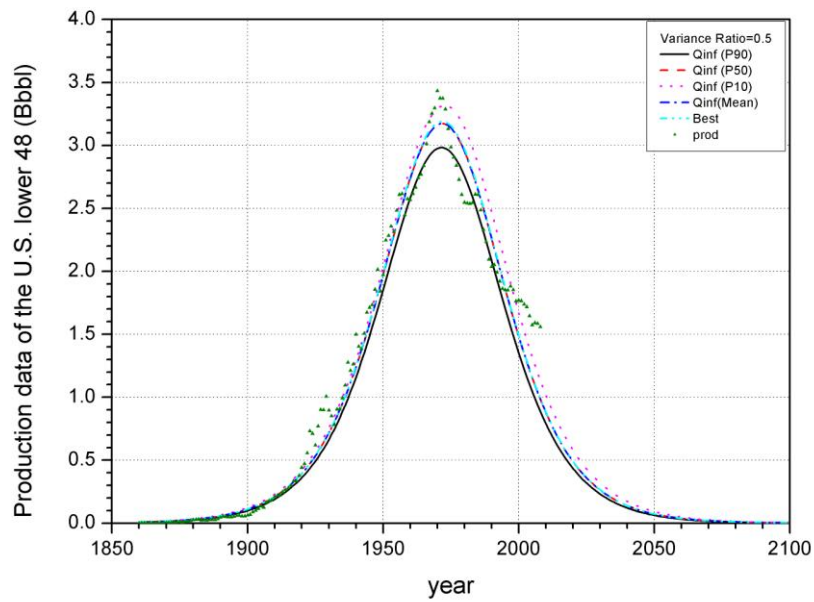


Fig. 8 - Probabilistic forecasts of Q_{∞} for Hubbert model with production data up to 2008 of the lower U.S. 48 ($R=0.5$)

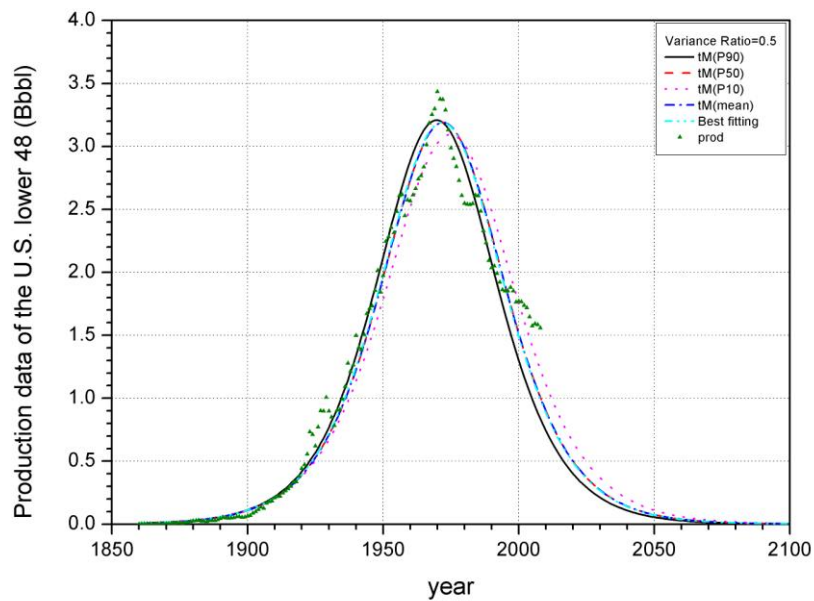


Fig. 9 - Probabilistic forecasts of t_M for Hubbert model with production data up to 2008 of the lower U.S. 48 ($R=0.5$)

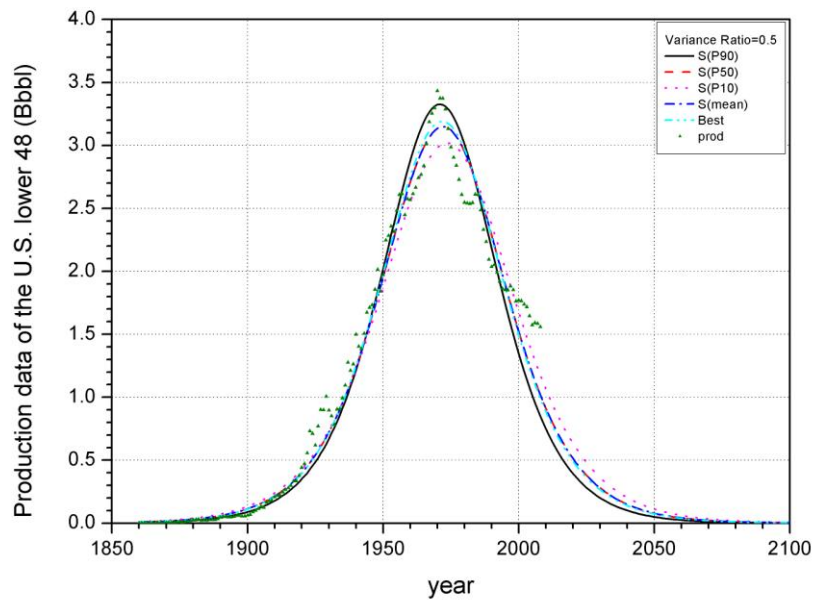


Fig. 10 - Probabilistic forecasts of S_L for Hubbert model with production data up to 2008 of the lower U.S. 48 ($R=0.5$)

Normal Distribution Model with Production Data up to 2008

Table 6 lists the data intervals and number of data points that define the discretized parameter space for different variance ratios.

Table 6 - Different variance ratio for Normal distribution model with production data up to 2008 of the lower U.S. 48

Variance Ratio	R = 0.05		R=0.20		R=0.50	
	Data Interval	points	Data Interval	points	Data Interval	points
Q_{∞} (Bbbl)	101~400	300	101~400	300	101~400	300
t_M (yr)	1961~1980.9	200	1961~1980.8	100	1961~1990.7	100
S_N (1/yr)	21~40.9	200	21~40.8	100	16~45.7	100

Fig. 11, **Fig. 13**, and **Fig. 15** are the plots of normalized PDF for each parameter Q_{∞} , t_M , and S_N . **Fig. 12**, **Fig. 14**, and **Fig. 16** are the plots of CDF for these three parameters, respectively. I obtained the P90, P50, P10, and mean value from the CDF of each parameter, which are shown in **Table 7**. Parameter sets found with maximum likelihood function value by taking the probabilistic value of each parameter are listed in **Table 8**, **Table 9**, and **Table 10**, and the sum-of-square norm (S_oS) values are also calculated. I chose one scenario ($R=0.5$) to plot the probabilistic forecasts for each parameter (**Fig. 17**, **Fig. 18** and **Fig. 19**).

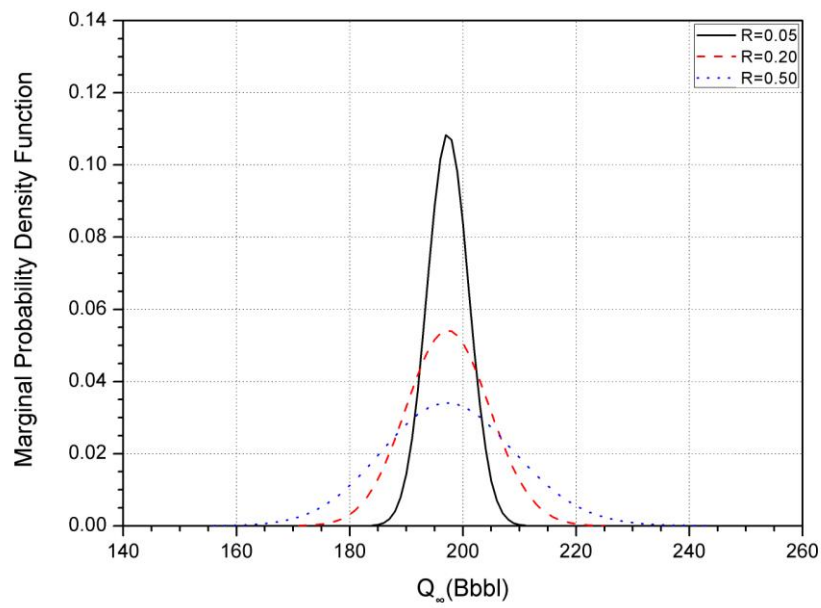


Fig. 11 - Marginal PDF of Q_{∞} for Normal distribution model with production data up to 2008 of the lower U.S. 48

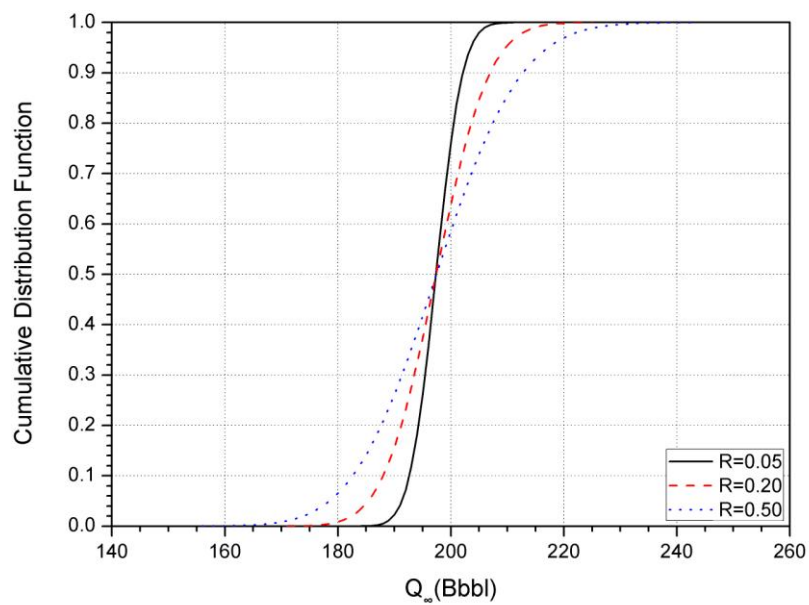


Fig. 12 - CDF of Q_{∞} for Normal distribution model with production data up to 2008 of the lower U.S. 48

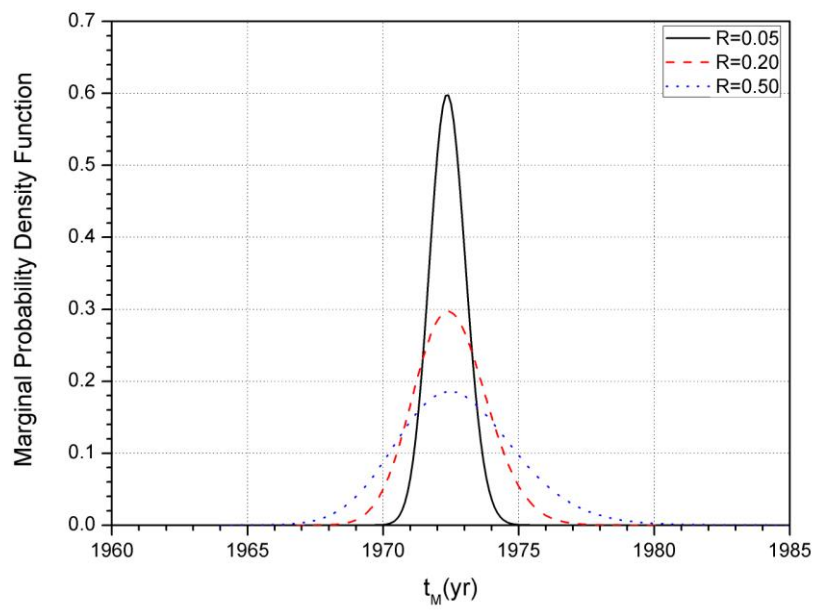


Fig. 13 - Marginal PDF of t_M for Normal distribution model with production data up to 2008 of the lower U.S. 48

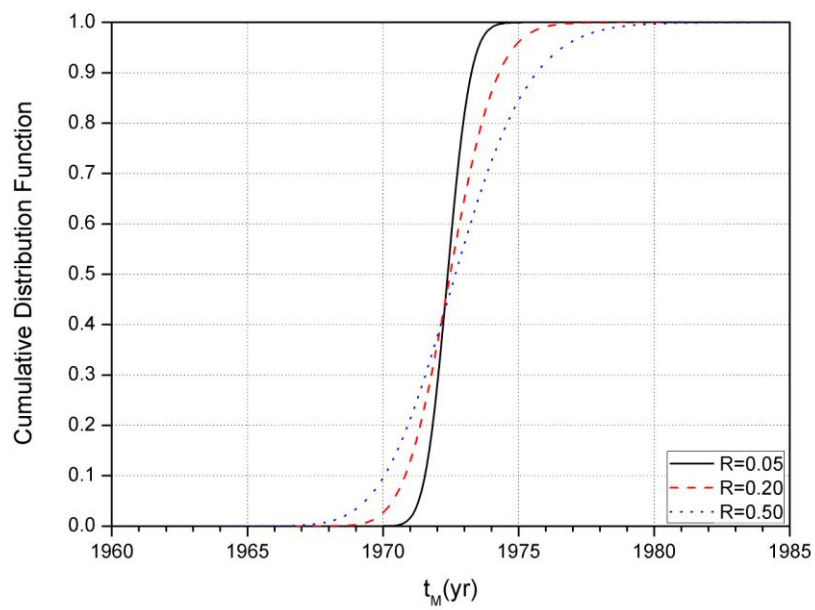


Fig. 14 - CDF of t_M for Normal distribution model with production data up to 2008 of the lower U.S. 48

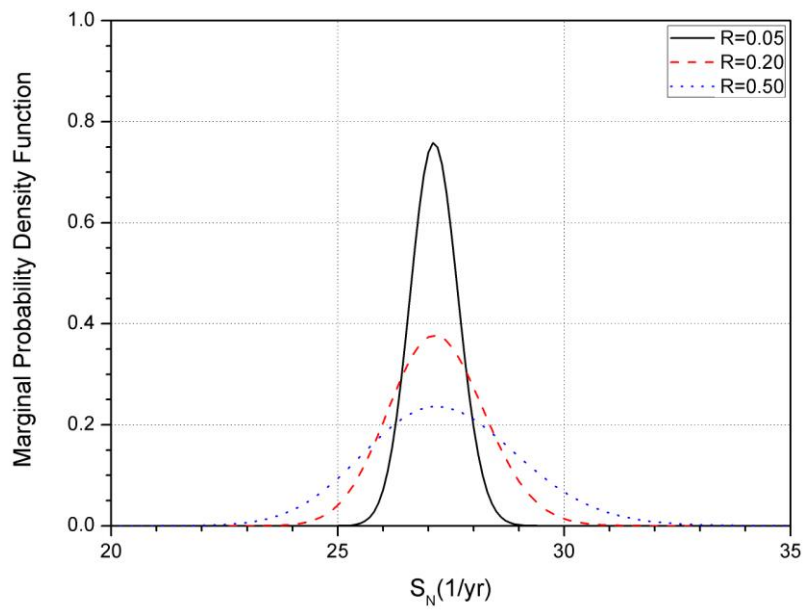


Fig. 15 - Marginal PDF of S_N for Normal distribution model with production data up to 2008 of the lower U.S. 48

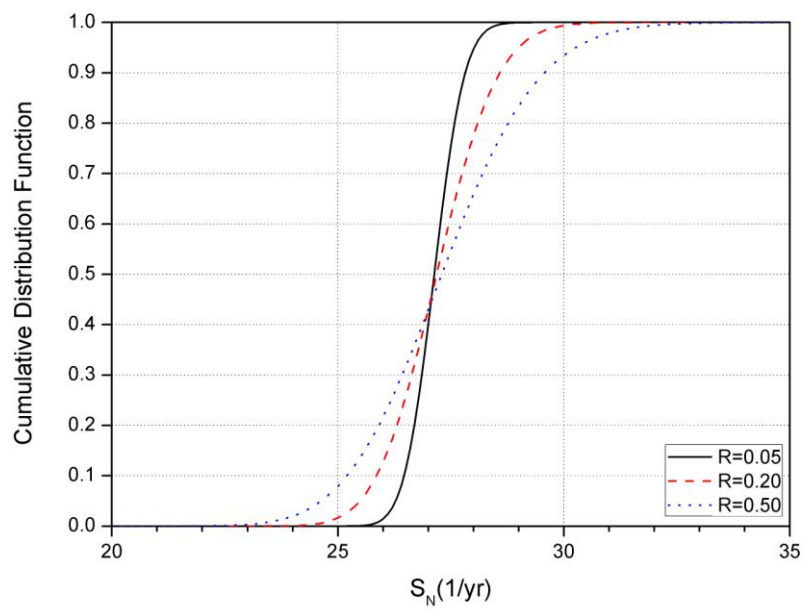


Fig. 16 - CDF of S_N for Normal distribution model with production data up to 2008 of the lower U.S. 48

Table 7 - Probabilistic values of each parameter for Normal distribution model with production data up to 2008 of the lower U.S. 48

Variance Ratio	R = 0.05			R=0.20			R=0.50		
	Q_{∞} (Bbbl)	t_M (yr)	S_N (1/yr)	Q_{∞} (Bbbl)	t_M (yr)	S_N (1/yr)	Q_{∞} (Bbbl)	t_M (yr)	S_N (1/yr)
P90	192.650	1971.538	26.461	188.012	1970.808	25.856	182.616	1970.046	25.205
P50	197.360	1972.378	27.129	197.379	1972.471	27.184	197.440	1972.664	27.297
P10	202.110	1973.251	27.814	206.914	1974.273	28.578	212.716	1975.652	29.565
Mean	197.372	1972.388	27.134	197.430	1972.514	27.204	197.579	1972.778	27.351
Best-fit	197	1972.3	27.1	198	1972.4	27.2	197	1972.4	27.1

Table 8 - Parameter sets for Normal distribution model with production data up to 2008 of the lower U.S. 48 (R=0.05)

R=0.05	Q_{∞}				t_M				S_N			
	Q_{∞} (Bbbl)	t_M (yr)	S_N (1/yr)	S_oS	Q_{∞} (Bbbl)	t_M (yr)	S_N (1/yr)	S_oS	Q_{∞} (Bbbl)	t_M (yr)	S_N (1/yr)	S_oS
P90	193	1972.1	26.9	0.2288	195	1971.5	26.7	0.2307	196	1971.9	26.5	0.2286
P50	197	1972.3	27.1	0.2193	197	1972.4	27.1	0.2194	197	1972.3	27.1	0.2193
P10	202	1971.7	27.4	0.2299	199	1973.2	27.5	0.2299	199	1972.9	27.8	0.2305
Mean	197	1972.3	27.1	0.2193	197	1972.4	27.1	0.2194	197	1972.3	27.1	0.2193

Table 9 - Parameter sets for Normal distribution model with production data up to 2008 of the lower U.S. 48 (R=0.20)

R=0.20	Q_{∞}				t_M				S_N			
	Q_{∞} (Bbbl)	t_M (yr)	S_N (1/yr)	S_oS	Q_{∞} (Bbbl)	t_M (yr)	S_N (1/yr)	S_oS	Q_{∞} (Bbbl)	t_M (yr)	S_N (1/yr)	S_oS
P90	188	1971.8	26.6	0.0660	194	1970.8	26.4	0.0645	194	1971.4	25.8	0.0658
P50	197	1972.2	27	0.0549	198	1972.4	27.2	0.0549	198	1972.4	27.2	0.0549
P10	207	1973	27.6	0.0663	201	1974.2	28	0.0668	201	1973.6	28.6	0.0676
Mean	197	1972.2	27	0.0549	198	1972.6	27.2	0.0551	198	1972.4	27.2	0.0549

Table 10 - Parameter sets for Normal distribution model with production data up to 2008 of the lower U.S. 48 (R=0.50)

R=0.50	Q_{∞}				t_M				S_N			
	Q_{∞} (Bbbl)	t_M (yr)	S_N (1/yr)	S_oS	Q_{∞} (Bbbl)	t_M (yr)	S_N (1/yr)	S_oS	Q_{∞} (Bbbl)	t_M (yr)	S_N (1/yr)	S_oS
P90	183	1971.5	26.5	0.0325	192	1970.0	26.2	0.0312	192	1971.2	25.3	0.0304
P50	197	1972.4	27.1	0.0219	198	1972.7	27.4	0.0222	198	1972.7	27.4	0.0222
P10	213	1973.6	28	0.0338	205	1975.7	28.9	0.0364	204	1974.5	29.5	0.0346
Mean	198	1972.4	27.1	0.0220	198	1972.7	27.4	0.0222	198	1972.7	27.4	0.0222

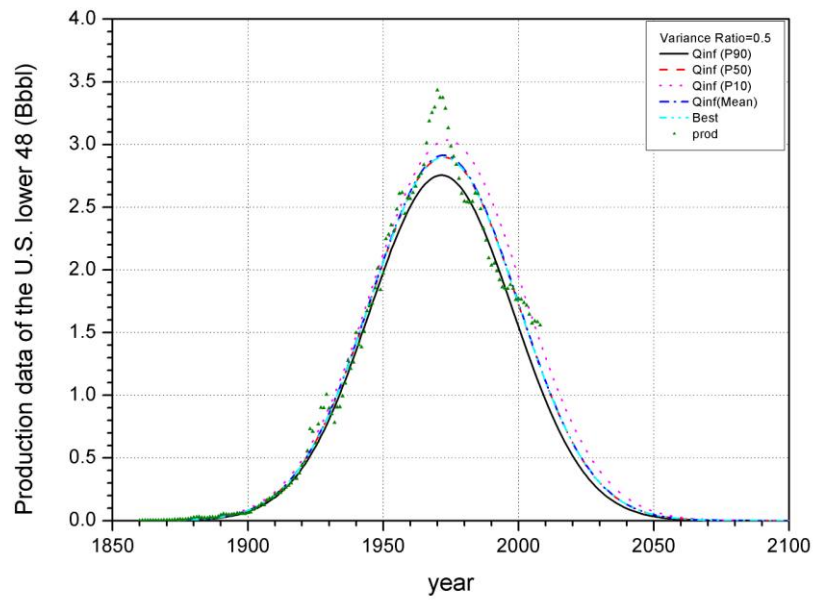


Fig. 17 - Probabilistic forecasts of Q_{∞} for Normal Distribution model with production data up to 2008 of the lower U.S. 48 ($R=0.5$)

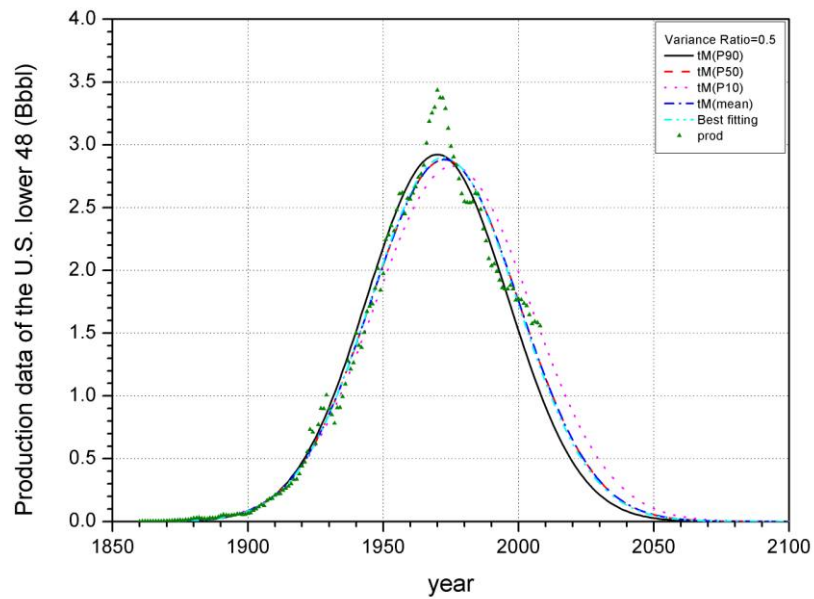


Fig. 18 - Probabilistic forecasts of t_M for Normal Distribution model with production data up to 2008 of the lower U.S. 48 ($R=0.5$)

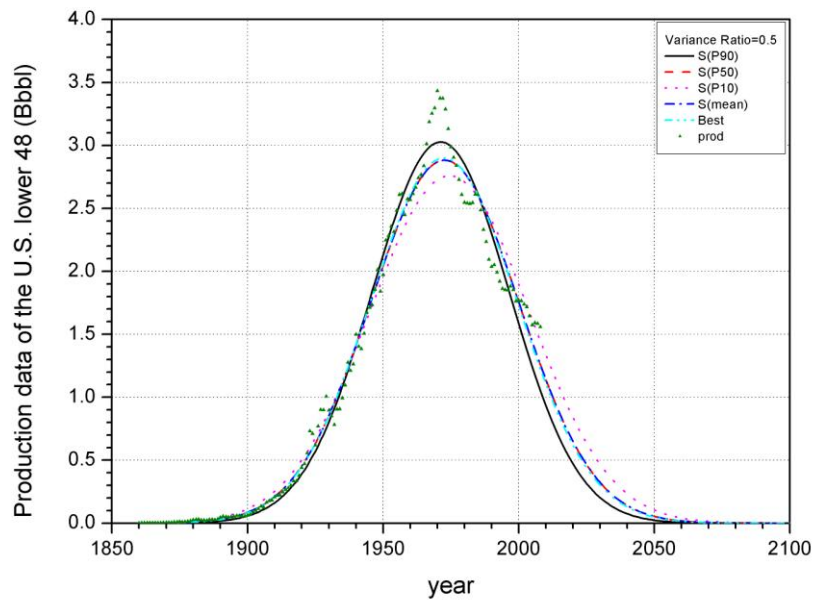


Fig. 19 - Probabilistic forecasts of S_N for Normal Distribution model with production data up to 2008 of the lower U.S. 48 ($R=0.5$)

Hubbert Model with Production Data up to 1956

Table 11 lists the data intervals and number of data points that define the discretized parameter space for different variance ratios.

Table 11 - Different variance ratio for Hubbert model with production data up to 1956 of the lower U.S. 48

Variance Ratio	R = 0.01		R=0.02		R=0.155	
	Data Interval	points	Data Interval	points	Data Interval	points
Q_{∞} (Bbbl)	101~200.5	200	81~180	100	51~1049	500
t_M (yr)	1951~1970.9	200	1951~1970.8	100	1941~2040	100
S_L (1/yr)	6~15.95	200	6~15.95	200	6~25.8	100

Fig. 20, **Fig. 22**, and **Fig. 24** are the plots of normalized PDF for each parameter Q_{∞} , t_M , and S_L . **Fig. 21**, **Fig. 23**, and **Fig. 25** are the plots of CDF for these three parameters, respectively. I obtained the P90, P50, P10, and mean value from the CDF of each parameter – **Table 12**. Parameter sets found with maximum likelihood function value by taking the probabilistic value of each parameter are listed in **Table 13**, **Table 14**, and **Table 15**, and the sum-of-square norm (S_oS) values are also calculated. I choose one scenario ($R=0.155$) to plot the probabilistic forecasts for each parameter (**Fig. 26**, **Fig. 27**, and **Fig. 28**).

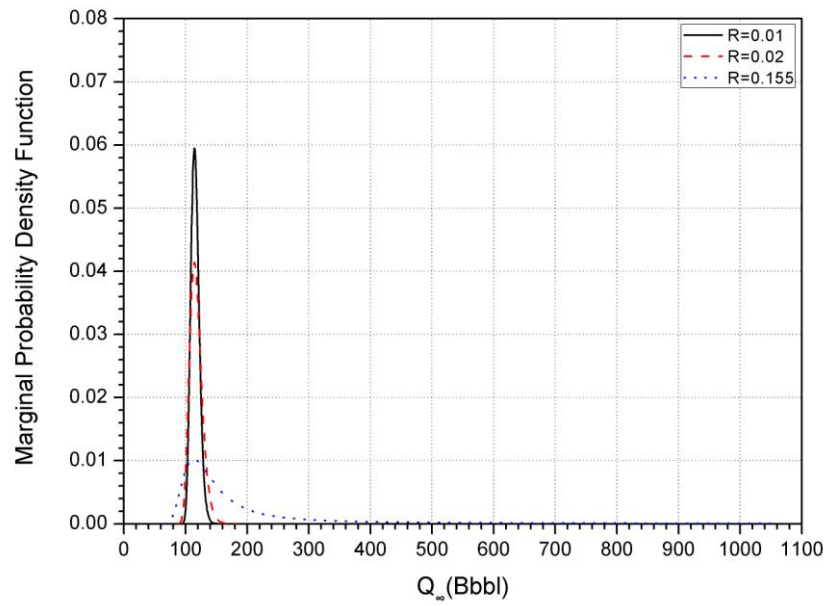


Fig. 20 - Marginal PDF of Q_{∞} for Hubbert model with production data up to 1956 of the lower U.S. 48

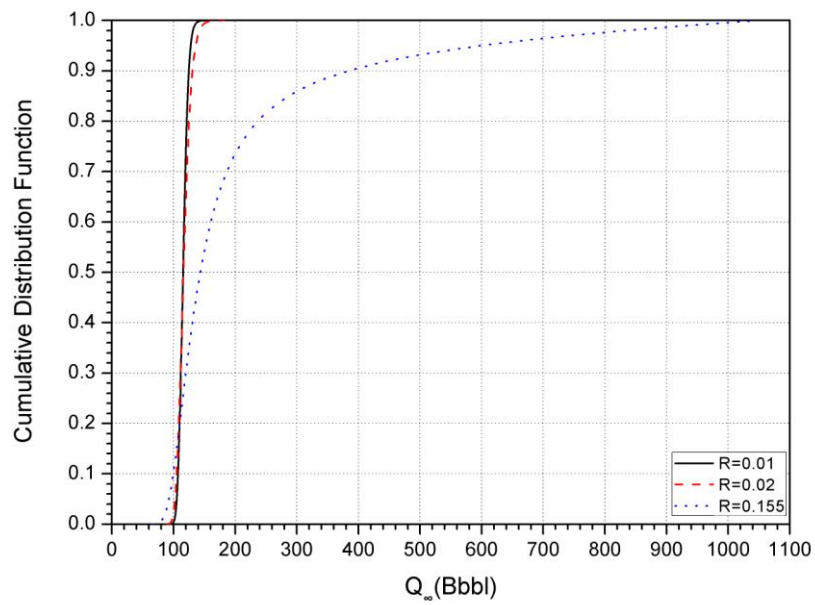


Fig. 21 - CDF of Q_{∞} for Hubbert model with production data up to 1956 of the lower U.S. 48

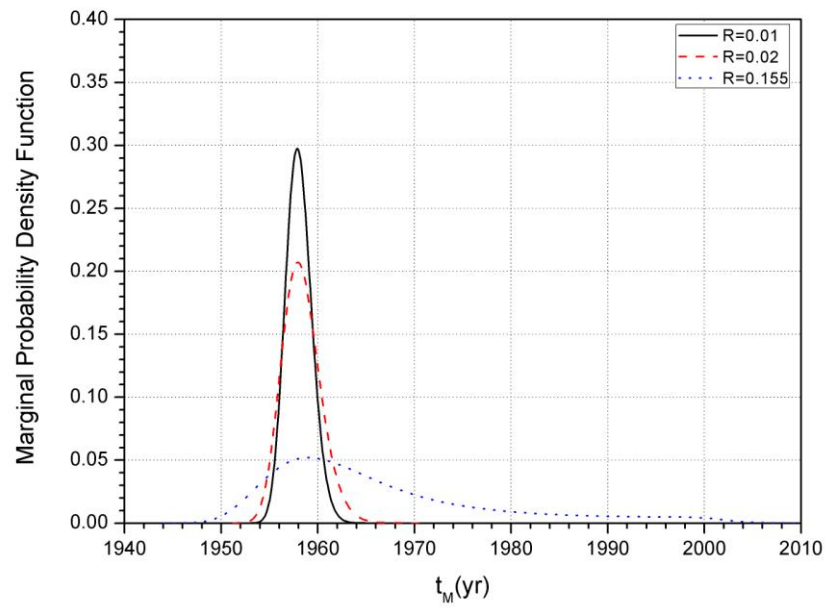


Fig. 22 - Marginal PDF of t_M for Hubbert model with production data up to 1956 of the lower U.S. 48

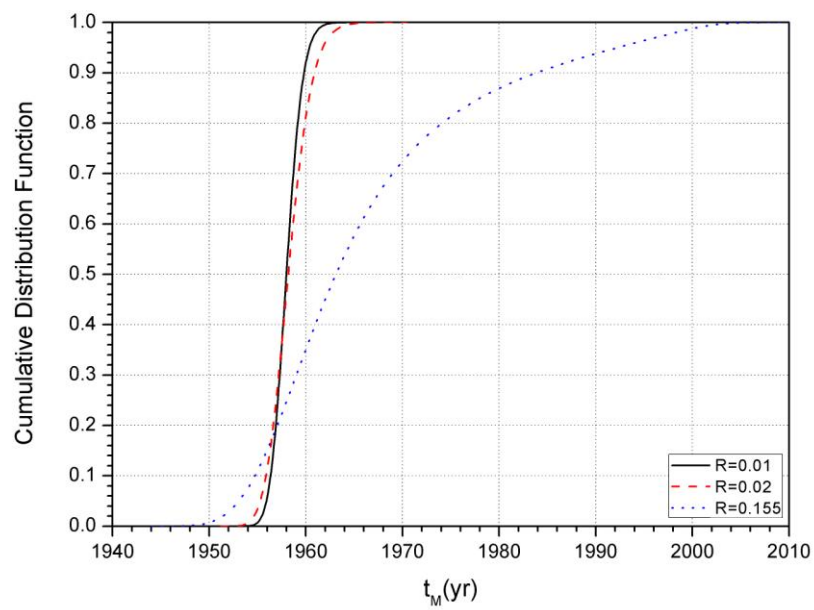


Fig. 23 - CDF of t_M for Hubbert model with production data up to 1956 of the lower U.S. 48

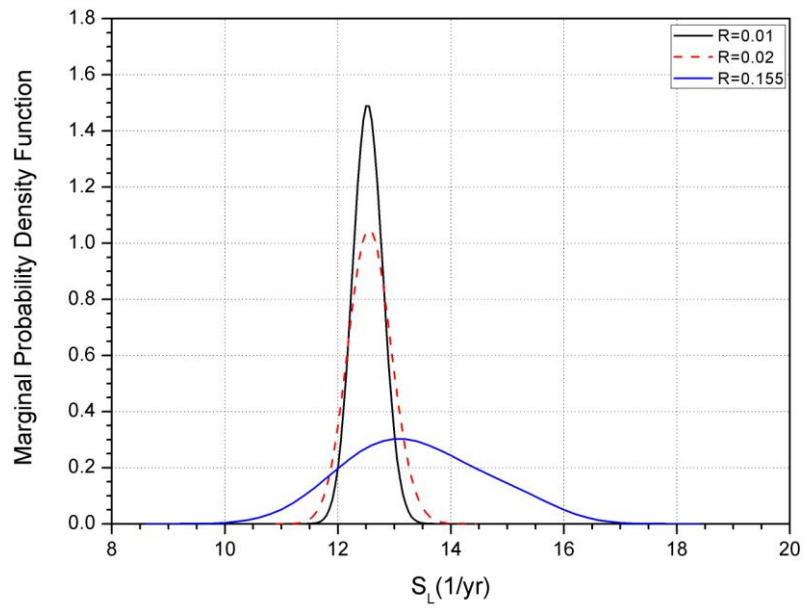


Fig. 24 - Marginal PDF of S_L for Hubbert model with production data up to 1956 of the lower U.S. 48

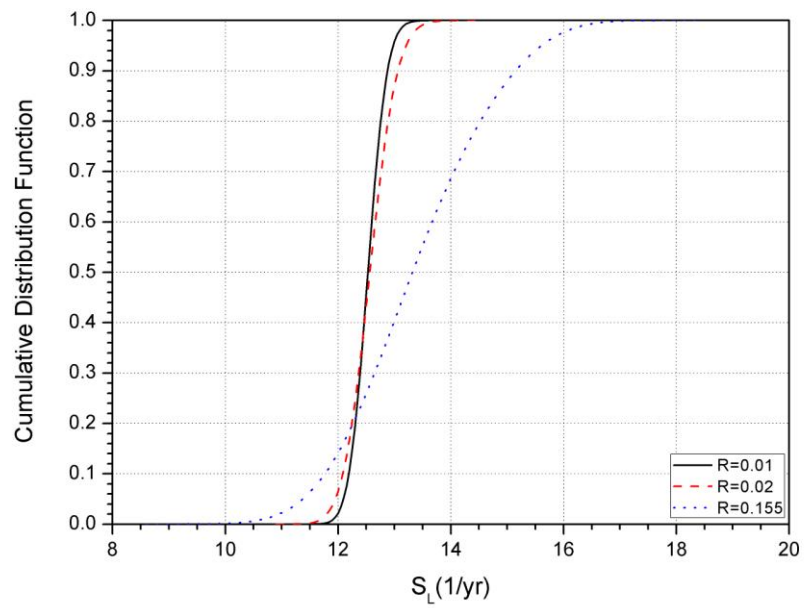


Fig. 25 - CDF of S_L for Hubbert model with production data up to 1956 of the lower U.S. 48

Table 12 - Probabilistic values of each parameter for Hubbert model with production data up to 1956 of the lower U.S. 48

Variance Ratio	R = 0.01			R=0.02			R=0.155		
	Q_{∞} (Bbbl)	t_M (yr)	S_L (1/yr)	Q_{∞} (Bbbl)	t_M (yr)	S_L (1/yr)	Q_{∞} (Bbbl)	t_M (yr)	S_L (1/yr)
P90	107.259	1956.345	12.190	105.020	1955.871	12.083	99.426	1954.733	11.769
P50	115.221	1957.980	12.529	116.163	1958.175	12.562	143.350	1963.093	13.317
P10	124.790	1959.807	12.876	130.739	1960.884	13.059	384.210	1983.830	15.144
Mean	115.714	1958.039	12.531	117.228	1958.299	12.568	203.82	1966.247	13.388
Best-fit	115	1957.9	12.5	114	1957.8	12.5	115	1958	12.6

Table 13 - Parameter sets for Hubbert model with production data up to 1956 of the lower U.S. 48 (R=0.01)

R=0.01	Q_{∞}				t_M				S_N			
	Q_{∞} (Bbbl)	t_M (yr)	S_L (1/yr)	S_oS	Q_{∞} (Bbbl)	t_M (yr)	S_L (1/yr)	S_oS	Q_{∞} (Bbbl)	t_M (yr)	S_L (1/yr)	S_oS
P90	107	1956.3	12.25	0.7179	107	1956.3	12.25	0.7179	109	1956.6	12.20	0.7162
P50	115	1957.9	12.50	0.7028	115	1958.0	12.55	0.7034	116	1958.1	12.55	0.7032
P10	125	1959.8	12.80	0.7247	125	1959.8	12.80	0.7247	123	1959.6	12.90	0.7263
Mean	116	1958.1	12.55	0.7032	115	1958.0	12.55	0.7034	116	1958.1	12.55	0.7032

Table 14 - Parameter sets for Hubbert model with production data up to 1956 of the lower U.S. 48 (R=0.02)

R=0.02	Q_{∞}				t_M				S_N			
	Q_{∞} (Bbbl)	t_M (yr)	S_L (1/yr)	S_oS	Q_{∞} (Bbbl)	t_M (yr)	S_L (1/yr)	S_oS	Q_{∞} (Bbbl)	t_M (yr)	S_L (1/yr)	S_oS
P90	105	1956.0	12.20	0.3645	105	1955.8	12.15	0.3648	107	1956.2	12.10	0.3633
P50	116	1958.2	12.55	0.3520	116	1958.2	12.55	0.3520	115	1958.0	12.55	0.3517
P10	131	1960.8	12.95	0.3753	130	1960.8	13.00	0.3737	126	1960.2	13.05	0.3735
Mean	117	1958.4	12.60	0.3525	116	1958.2	12.55	0.3520	115	1958.0	12.55	0.3517

Table 15 - Parameter sets for Hubbert model with production data up to 1956 of the lower U.S. 48 (R=0.155)

R=0.155	Q_{∞}				t_M				S_N			
	Q_{∞} (Bbbl)	t_M (yr)	S_L (1/yr)	S_oS	Q_{∞} (Bbbl)	t_M (yr)	S_L (1/yr)	S_oS	Q_{∞} (Bbbl)	t_M (yr)	S_L (1/yr)	S_oS
P90	99	1955	12.0	0.0513	101	1955	12.0	0.0491	101	1955	11.8	0.0503
P50	143	1963	13.2	0.0533	143	1963	13.2	0.0533	137	1962	13.4	0.0528
P10	385	1984	15.2	0.1142	383	1984	15.2	0.1141	249	1976	15.2	0.1028
Mean	203	1971	14.2	0.0774	163	1966	13.6	0.0615	137	1962	13.4	0.0528

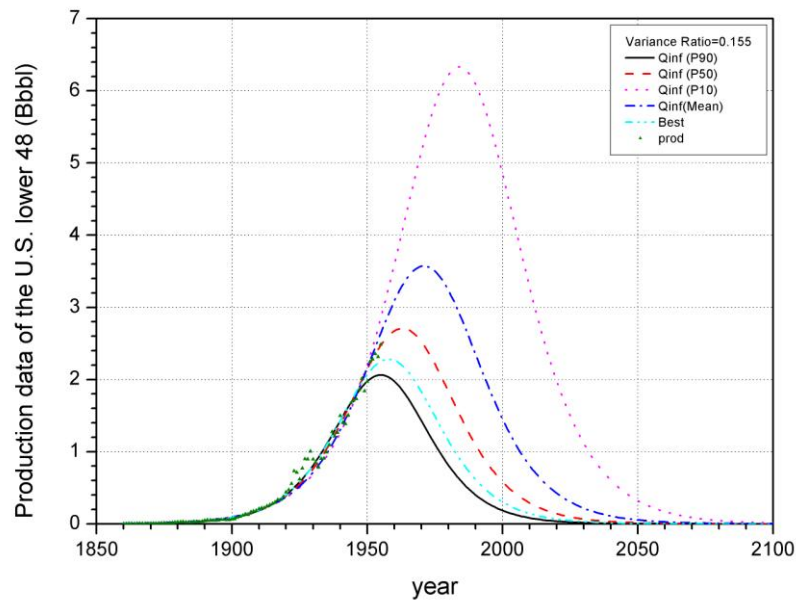


Fig. 26 - Probabilistic forecasts of Q_{∞} for Hubbert model with production data up to 1956 of the lower U.S. 48 ($R=0.155$)

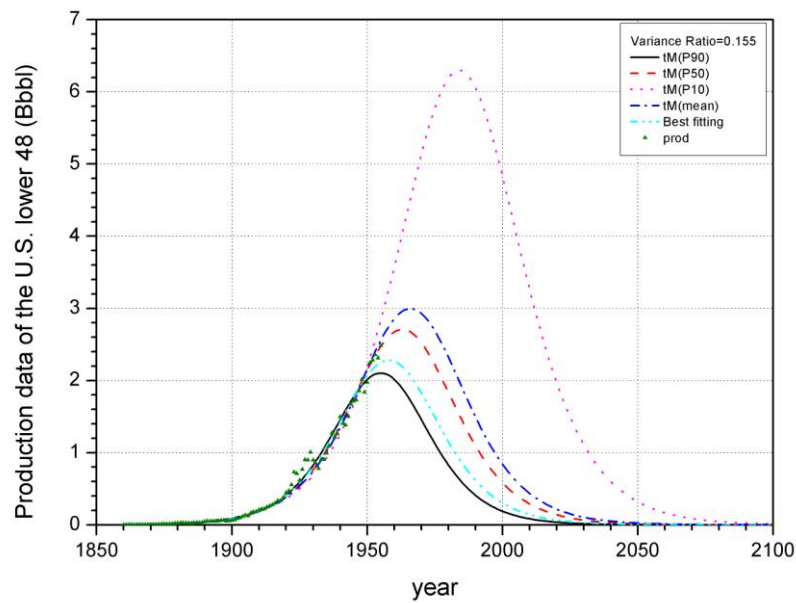


Fig. 27 - Probabilistic forecasts of t_M for Hubbert model with production data up to 1956 of the lower U.S. 48 ($R=0.155$)

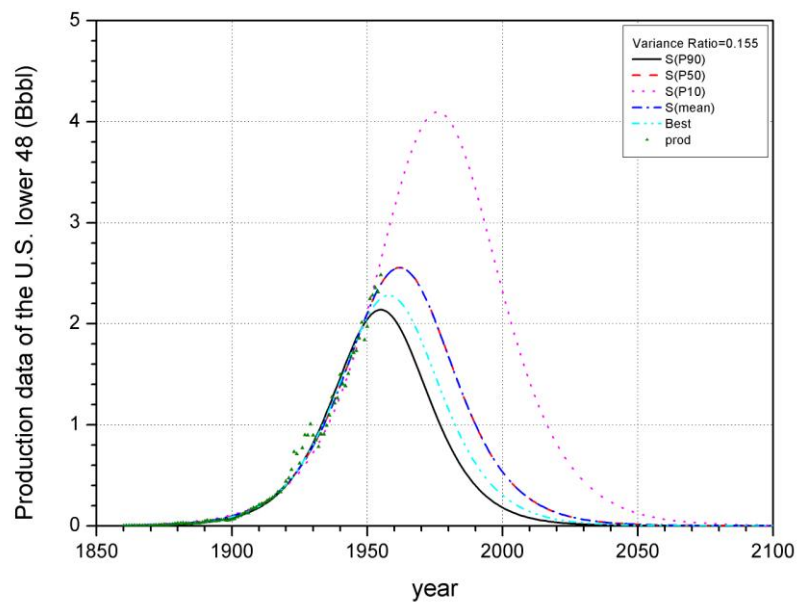


Fig. 28 - Probabilistic forecasts of S_L for Hubbert model with production data up to 1956 of the lower U.S. 48 ($R=0.155$)

Normal Distribution Model with Production Data up to 1956

Table 16 lists the data intervals and number of data points that define the discretized parameter space for different variance ratios.

Table 16 - Different variance ratio for Normal distribution model with production data up to 1956 of the lower U.S. 48

Variance Ratio	R = 0.01		R=0.02	
	Data Interval	points	Data Interval	points
Q_{∞} (Bbbl)	101~1099	500	101~1598	500
t_M (yr)	1961~2020.4	100	1961~2040.2	100
S_N (1/yr)	21~50.7	100	21~50.7	100

Fig. 29, **Fig. 31**, and **Fig. 33** are the plots of normalized PDF for each parameter Q_{∞} , t_M , and S_N . **Fig. 30**, **Fig. 32**, and **Fig. 34** are the plots of CDF for these three parameters, respectively. I obtained the P90, P50, P10, and mean value from the CDF of each parameter, in which are shown in **Table 17**. Parameter sets found with maximum likelihood function value by taking the probabilistic value of each parameter are listed in **Table 18**, and **Table 19**, and the sum-of-square norm (S_oS) values are also calculated. I choose one scenario ($R=0.01$) to plot the probabilistic forecasts for each parameter (**Fig. 35**, **Fig. 36**, and **Fig. 37**).

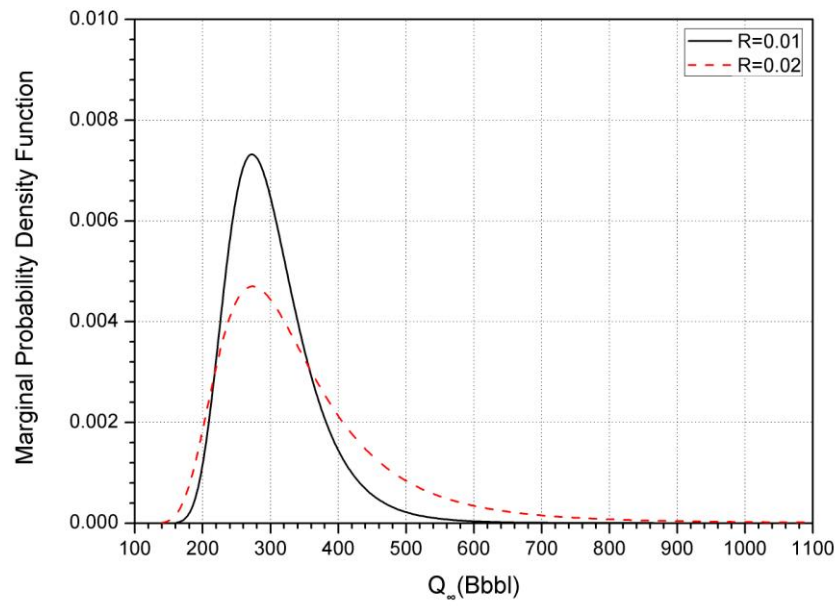


Fig. 29 - Marginal PDF of Q_{∞} for Normal distribution model with production data up to 1956 of the lower U.S. 48

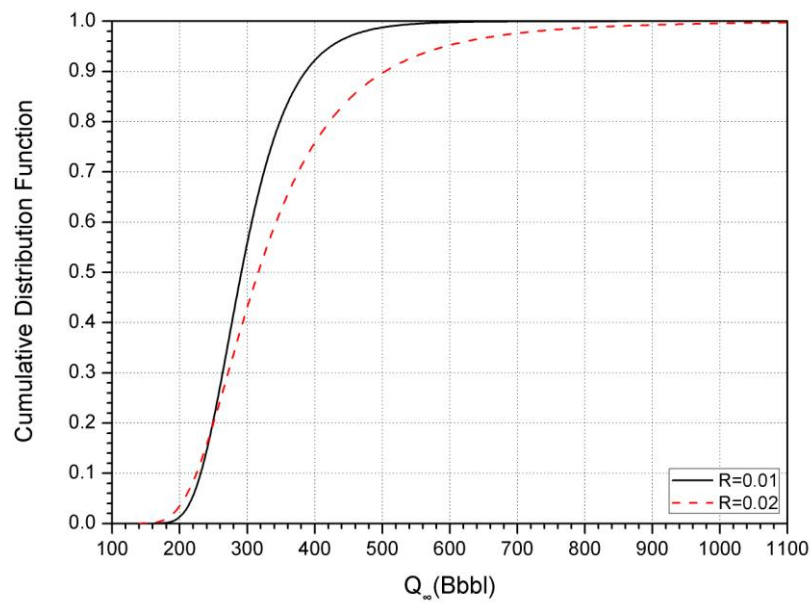


Fig. 30 - CDF of Q_{∞} for Normal distribution model with production data up to 1956 of the lower U.S. 48

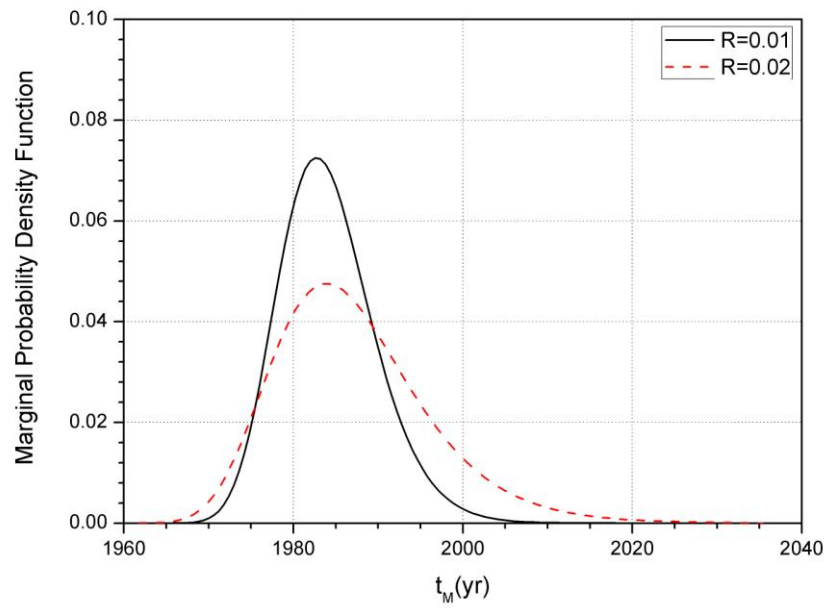


Fig. 31 - Marginal PDF of t_M for Normal distribution model with production data up to 1956 of the lower U.S. 48

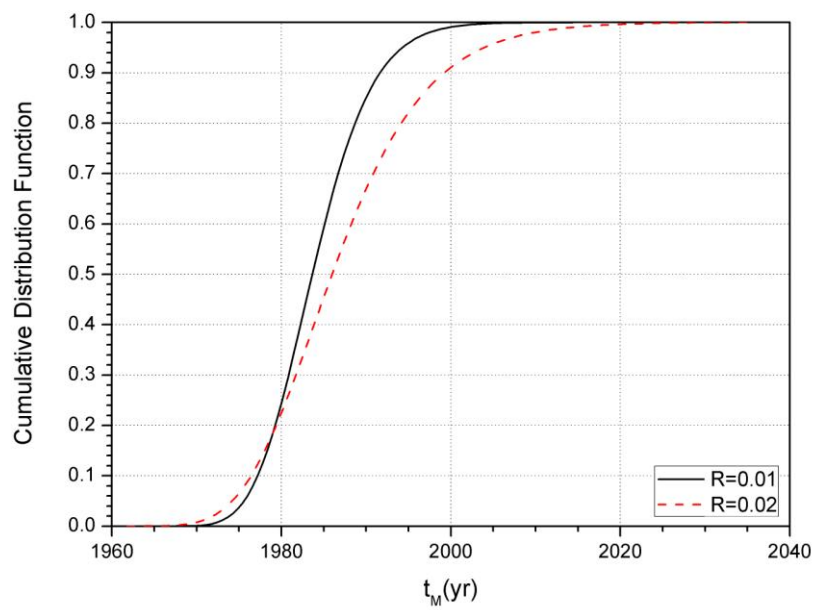


Fig. 32 - CDF of t_M for Normal distribution model with production data up to 1956 of the lower U.S. 48

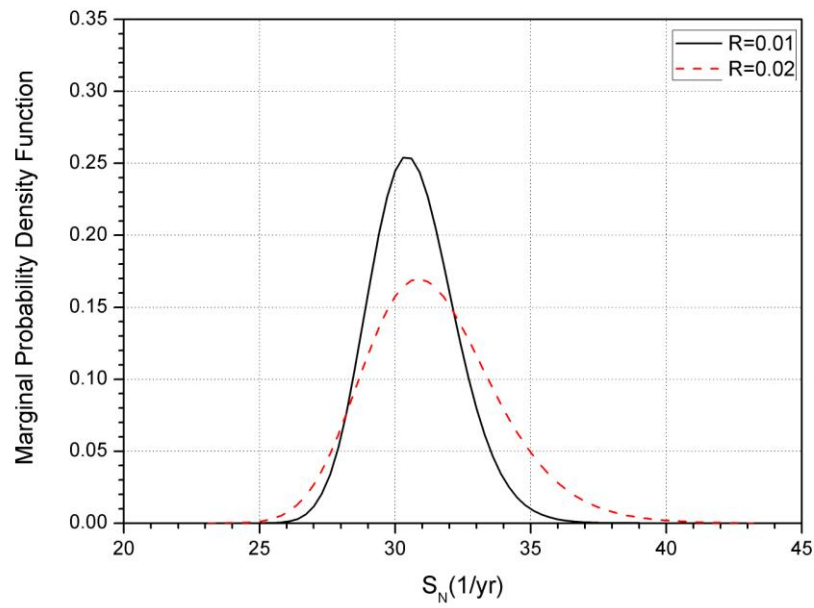


Fig. 33 - Marginal PDF of S_N for Normal distribution model with production data up to 1956 of the lower U.S. 48

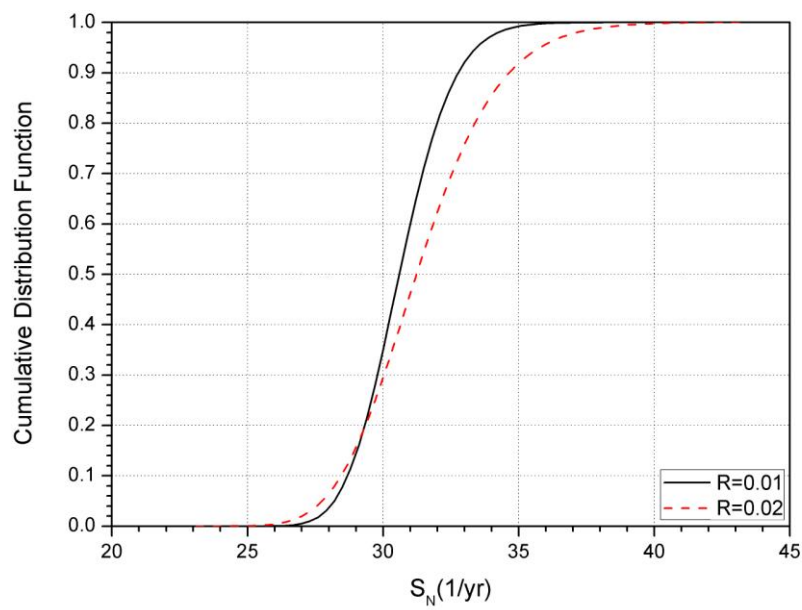


Fig. 34 - CDF of S_N for Normal distribution model with production data up to 1956 of the lower U.S. 48

Table 17 - Probabilistic values of each parameter for Normal distribution model with production data up to 1956 of the lower U.S. 48

Variance Ratio	R = 0.01			R=0.02		
	Q_{∞} (Bbbl)	t_M (yr)	S_N (1/yr)	Q_{∞} (Bbbl)	t_M (yr)	S_N (1/yr)
P90	231.00	1977.189	28.695	224.67	1976.43	28.455
P50	291.19	1983.645	30.594	316.22	1985.970	31.231
P10	386.41	1991.638	32.762	503.47	1999.193	34.644
Mean	302.22	1984.117	30.676	347.73	1987.100	31.425
Best-fit	281	1982.6	30.3	281	1982.6	30.3

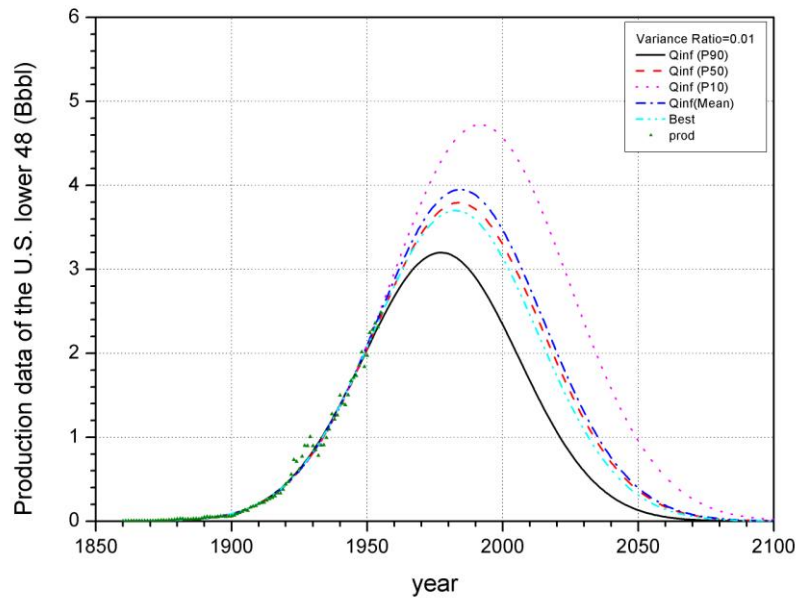


Fig. 35 - Probabilistic forecasts of Q_{∞} for Normal Distribution model with production data up to 1956 of the lower U.S. 48 (R=0.01)

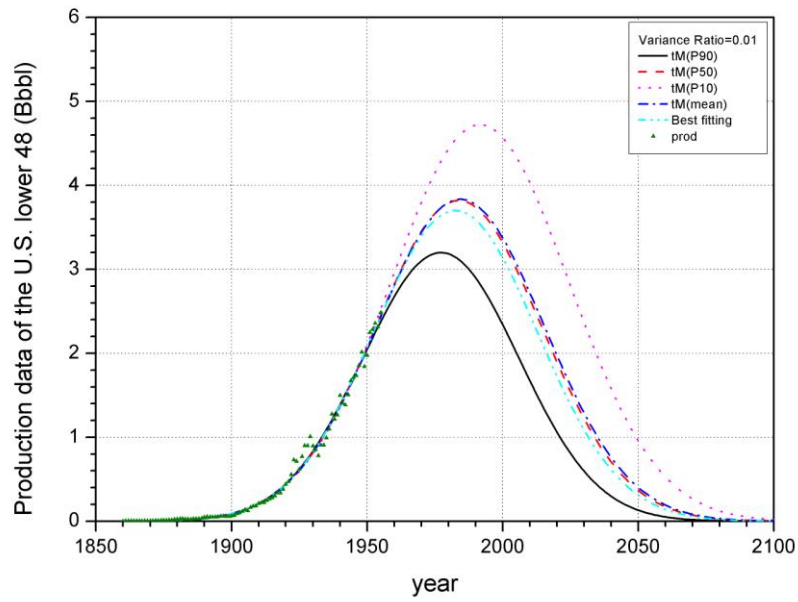


Fig. 36 - Probabilistic forecasts of t_M for Normal Distribution model with production data up to 1956 of the lower U.S. 48 ($R=0.01$)

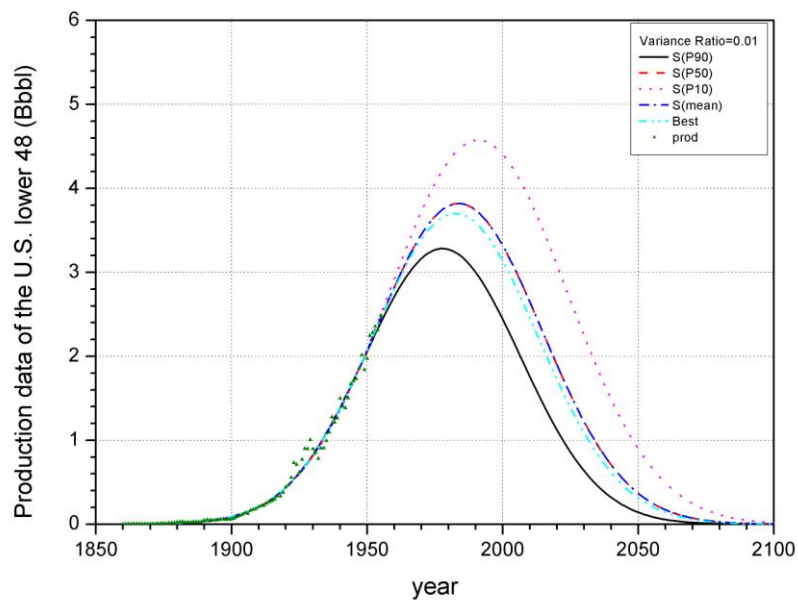


Fig. 37 - Probabilistic forecasts of S_N for Normal Distribution model with production data up to 1956 of the lower U.S. 48 ($R=0.01$)

Table 18 - Parameter sets for Normal distribution model with production data up to 1956 of the lower U.S. 48 (R=0.01)

R=0.01	Q_{∞}				t_M				S_N			
	Q_{∞} (Bbbl)	t_M (yr)	S_N (1/yr)	S_oS	Q_{∞} (Bbbl)	t_M (yr)	S_N (1/yr)	S_oS	Q_{∞} (Bbbl)	t_M (yr)	S_N (1/yr)	S_oS
P90	231	1977.2	28.8	0.5346	231	1977.2	28.8	0.5346	237	1977.8	28.8	0.5336
P50	291	1983.8	30.6	0.5300	293	1983.8	30.6	0.5267	293	1983.8	30.6	0.5267
P10	387	1991.6	32.7	0.5544	387	1991.6	32.7	0.5544	375	1991	32.7	0.5535
Mean	303	1984.4	30.6	0.5346	297	1984.4	30.9	0.5292	293	1983.8	30.6	0.5267

Table 19 - Parameter sets for Normal distribution model with production data up to 1956 of the lower U.S. 48 (R=0.02)

R=0.02	Q_{∞}				t_M				S_N			
	Q_{∞} (Bbbl)	t_M (yr)	S_N (1/yr)	S_oS	Q_{∞} (Bbbl)	t_M (yr)	S_N (1/yr)	S_oS	Q_{∞} (Bbbl)	t_M (yr)	S_N (1/yr)	S_oS
P90	224	1976.2	28.5	0.2709	224	1976.2	28.5	0.2709	230	1977.0	28.5	0.2703
P50	317	1985.8	31.2	0.2676	314	1985.8	31.2	0.2653	314	1985.8	31.2	0.2653
P10	503	1999.4	34.8	0.3023	503	1999.4	34.8	0.3023	491	1998.6	34.5	0.2983
Mean	347	1988.2	31.8	0.2737	335	1987.4	31.5	0.2690	323	1986.6	31.5	0.2675

Summary of Uncertainty Analyses for the Lower U.S. 48 by Mathematical Modeling

1. Because the conventional oil production curve of the lower U.S. 48 is declining significantly (**Fig. 1**), there are no large differences between the probabilistic forecasts of the Hubbert and Normal distribution models even if I apply a larger variance ratio (**Table 2, Table 7**).
2. The results by applying the scarcity of production data to develop the uncertainty quantification demonstrate the significant influence and discrepancy for the probabilistic forecasts of both two models, and a tiny variance ratio ($R=0.01$) results in wider range of probabilistic forecasts for normal distribution model than Hubbert model (**Table 12, Table 17**). Furthermore, due to the scarcity of production data or unclear declining production curve, it will make us easily recognize much uncertainty within the forecasts by applying the mathematical models to predict.
3. For the Hubbert's model, there are existed the noticeable differences in the results of uncertainty analyses between the pre-1956 (pre peak) and overall (post peak). For example, the range of probabilistic values of Q_{∞} and t_M with $R=0.155$ in pre-peak data analyses (**Table 12**) is wider than the scenario $R=0.2$ in post-peak (**Table 2**). Besides, it also can be observed the similar results for Normal distribution model. Because of this, it will provide insights into the world analysis (pre peak).
4. From **Table 12** and **Table 17**, I find that the P10 value of Q_{∞} ($R=0.155$) in Hubbert model is nearly equal to the corresponding value of the scenario ($R=0.01$) in Normal distribution model, and it means that there is much uncertainty for applying normal

distribution model to forecast the production trend.

5. Although the Hubbert model is derived based on the concept of peak oil and logistic curve, I compare the sum-of-square norm (S_oS) values of both models, and it shows that normal distribution model fits the production data well than Hubbert model.
6. The tiny difference between best fitting values for different variance ratio in Hubbert model and Normal distribution model is due to numerical errors.

7. ANALYSIS FOR CONVENTIONAL OIL RESOURCES OF THE WORLD

I obtained conventional oil production data of the world from Energy Information Administration of the U.S. (EIA, 1996; 2009).

Fig. 38 is the production trend from the year 1860 to 2008. While there is a local peak in the 1970's, there is no overall peak in world oil production rate.

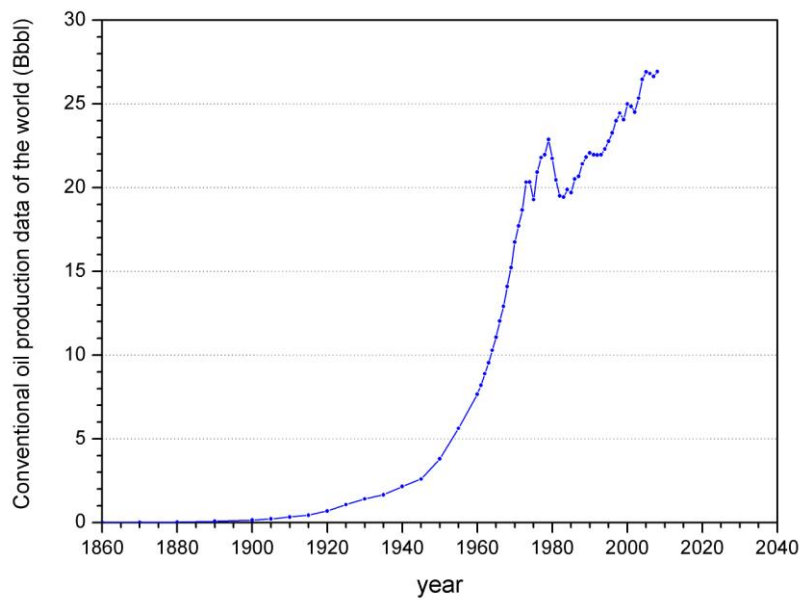


Fig. 38 - Production curve of world conventional oil

Here, I only take all production data up to 2008 to analyze with both Hubbert's model and Normal distribution model, and compare and interpret the results of uncertainty analyses for these two schemes in the summary section.

Hubbert Model with Production Data up to 2008

Table 20 lists the data intervals and number of data points that define the discretized parameter space for different variance ratios.

Table 20 - Different variance ratio for Hubbert model with production data up to 2008 of the world

Variance Ratio	R=2		R=4		R=8	
	Data Interval	points	Data Interval	points	Data Interval	points
Q_{∞} (Bbbl)	1001~3098	1000	101~6095	1000	101~10090	1000
t_M (yr)	1951~2050	100	1951~2099.5	100	1951~2099.5	100
S_L (1/yr)	5~24.8	100	5~54.5	100	5~54.5	100

Fig. 39, **Fig. 41**, and **Fig. 43** are the plots of normalized PDF for each parameter Q_{∞} , t_M , and S_L . **Fig. 40**, **Fig. 42**, and **Fig. 44** are the plots of CDF for these three parameters, respectively. I obtained the P90, P50, P10, and mean value from the CDF of each parameter, which are shown in **Table 21**. Parameter sets found with maximum likelihood function value by taking the probabilistic value of each parameter are listed in **Table 22**, **Table 23**, and **Table 24**, and the sum-of-square norm (S_oS) values are also calculated. I choose one scenario ($R=8$) to plot the probabilistic forecasts for each parameter (**Fig. 45**, **Fig. 46**, and **Fig. 47**).

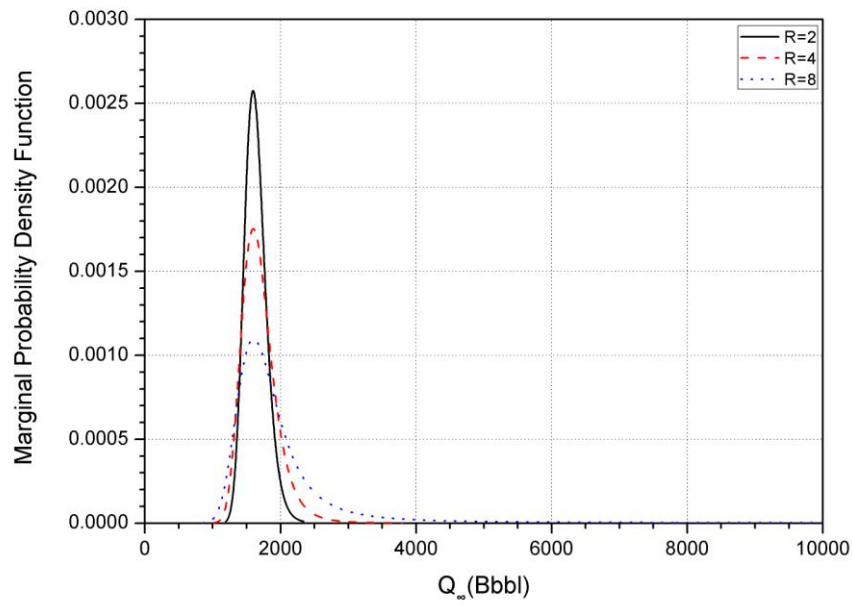


Fig. 39 - Marginal PDF of Q_{∞} for Hubbert model with production data up to 2008 of the world

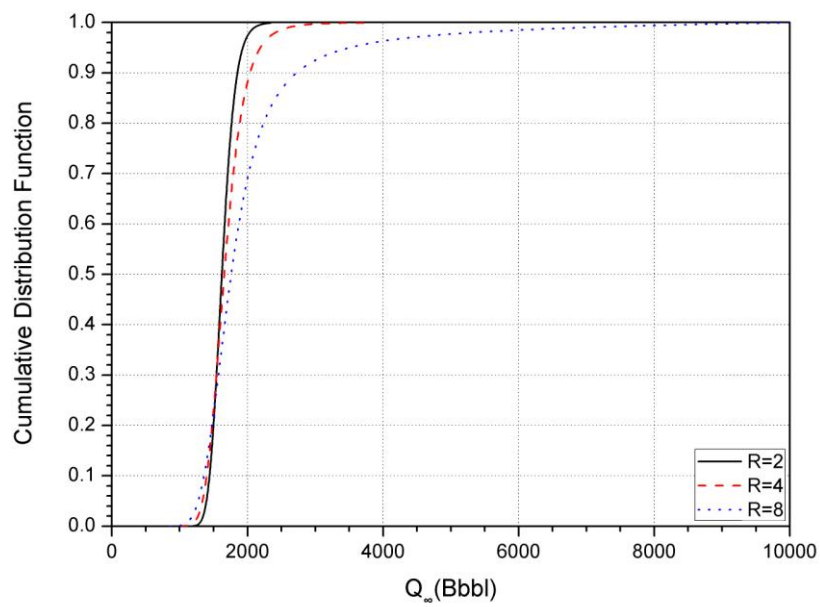


Fig. 40 - CDF of Q_{∞} for Hubbert model with production data up to 2008 of the world

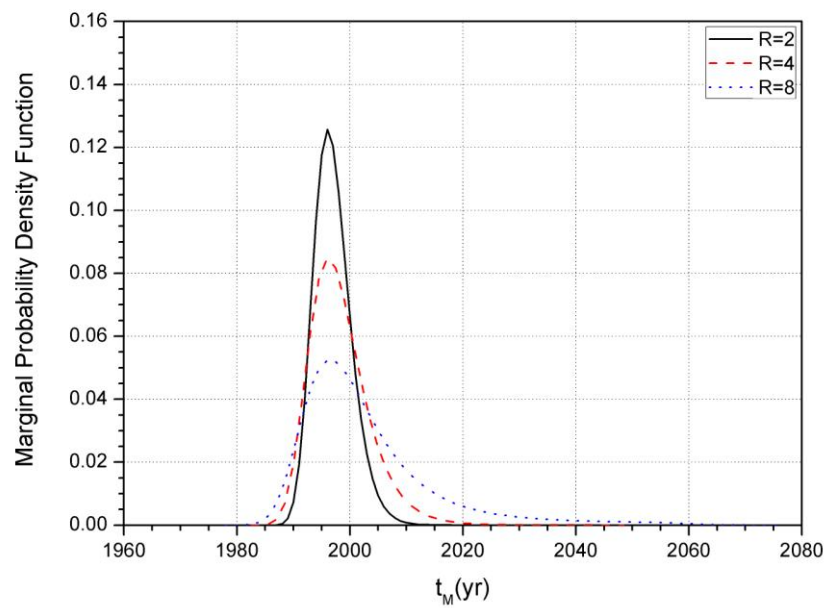


Fig. 41 - Marginal PDF of t_M for Hubbert model with production data up to 2008 of the world

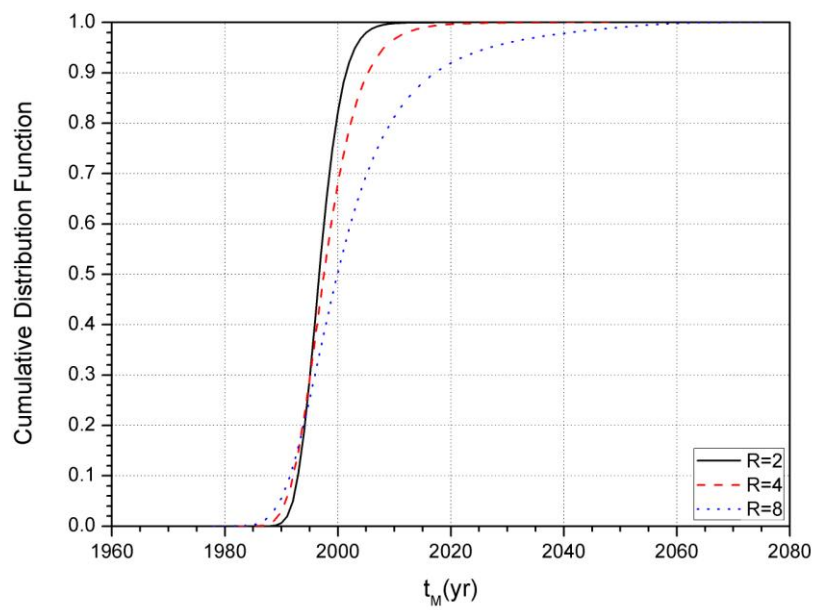


Fig. 42 - CDF of t_M for Hubbert model with production data up to 2008 of the world

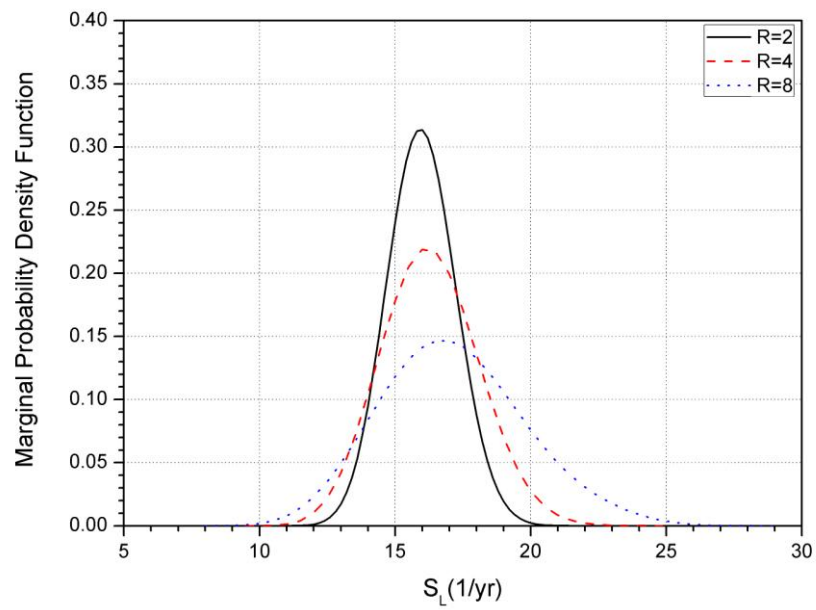


Fig. 43 - Marginal PDF of S_L for Hubbert model with production data up to 2008 of the world

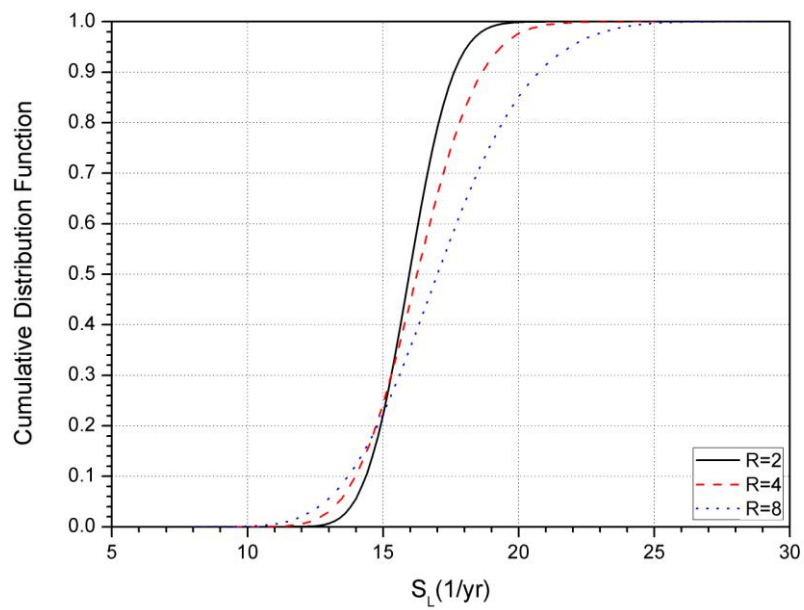


Fig. 44 - CDF of S_L for Hubbert model with production data up to 2008 of the world

Table 21 - Probabilistic values of each parameter for Hubbert model with production data up to 2008 of the world

Variance Ratio	R = 2			R=4			R=8		
	Q_{∞} (Bbbl)	t_M (yr)	S_L (1/yr)	Q_{∞} (Bbbl)	t_M (yr)	S_L (1/yr)	Q_{∞} (Bbbl)	t_M (yr)	S_L (1/yr)
P90	1439.8	1992.939	14.366	1396.6	1992.132	13.990	1358.6	1991.530	13.716
P50	1621.5	1996.680	15.961	1656.1	1997.509	16.251	1759.4	1999.894	17.002
P10	1850.2	2001.428	17.612	2034.8	2005.266	18.636	2722.2	2017.158	20.734
Mean	1636.0	1996.991	15.978	1695.8	1998.266	16.291	2005.3	2002.773	17.131
Best-Fit	1598	1996	15.8	1583	1996	15.5	1581	1996	15.5

Table 22 - Parameter sets for Hubbert model with production data up to 2008 of the world (R=2)

R=2	Q_{∞}				t_M				S_L			
	Q_{∞} (Bbbl)	t_M (yr)	S_L (1/yr)	S_oS	Q_{∞} (Bbbl)	t_M (yr)	S_L (1/yr)	S_oS	Q_{∞} (Bbbl)	t_M (yr)	S_L (1/yr)	S_oS
P90	1439	1994	14.6	7.4962	1478	1993	14.8	7.4662	1475	1994	14.4	7.5031
P50	1622	1996	15.8	7.4248	1634	1997	16.0	7.4251	1634	1997	16.0	7.4251
P10	1850	2001	17.2	7.3765	1811	2001	17.2	7.3825	1796	2000	17.6	7.3422
Mean	1637	1997	16.0	7.4246	1634	1997	16.0	7.4251	1634	1997	16.0	7.4251

Table 23 - Parameter sets for Hubbert model with production data up to 2008 of the world (R=4)

R=4	Q_{∞}				t_M				S_L			
	Q_{∞} (Bbbl)	t_M (yr)	S_L (1/yr)	S_oS	Q_{∞} (Bbbl)	t_M (yr)	S_L (1/yr)	S_oS	Q_{∞} (Bbbl)	t_M (yr)	S_L (1/yr)	S_oS
P90	1397	1993.0	14.5	3.7475	1433	1991.5	14.5	3.7352	1433	1993	14.0	3.7591
P50	1655	1997.5	16.0	3.7147	1649	1997.5	16.0	3.7152	1673	1997.5	16.5	3.6970
P10	2033	2003.5	18.0	3.6689	1991	2005.0	18.0	3.6841	1901	2002	18.5	3.6510
Mean	1697	1997.5	16.5	3.6950	1715	1999	16.5	3.7055	1673	1997.5	16.5	3.6970

Table 24 - Parameter sets for Hubbert model with production data up to 2008 of the world (R=8)

R=8	Q_{∞}				t_M				S_L			
	Q_{∞} (Bbbl)	t_M (yr)	S_L (1/yr)	S_oS	Q_{∞} (Bbbl)	t_M (yr)	S_L (1/yr)	S_oS	Q_{∞} (Bbbl)	t_M (yr)	S_L (1/yr)	S_oS
P90	1361	1993	14	1.8816	1431	1991.5	14.5	1.8677	1401	1993	13.5	1.8867
P50	1761	1999	17	1.8427	1781	2000.5	17	1.8480	1741	1999	17.0	1.8435
P10	2721	2015.5	20	1.8305	2691	2017.0	20.5	1.8306	2261	2009.5	20.5	1.8171
Mean	2001	2003.5	18	1.8356	1931	2003.5	18	1.8383	1741	1999	17.0	1.8435

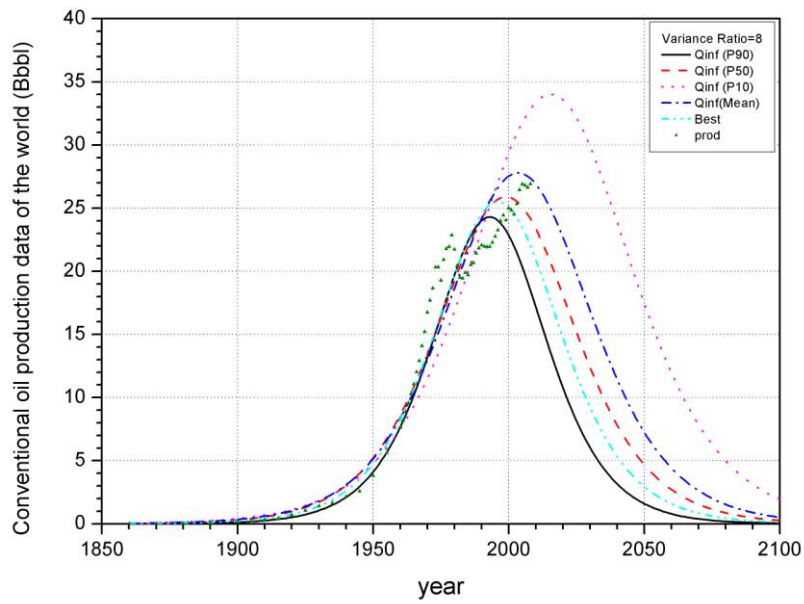


Fig. 45 - Probabilistic forecasts of Q_{∞} for Hubbert model with production data up to 2008 of the world ($R=8$)

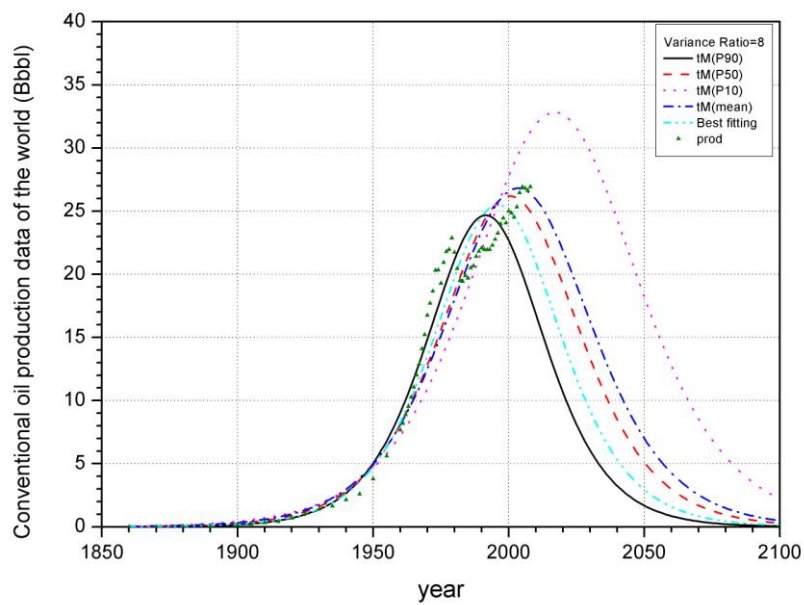


Fig. 46 - Probabilistic forecasts of t_M for Hubbert model with production data up to 2008 of the world ($R=8$)

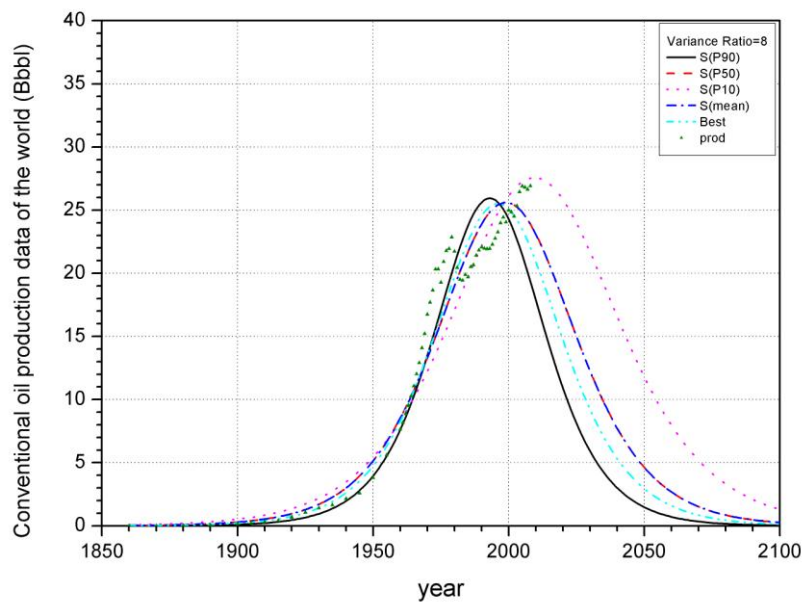


Fig. 47 - Probabilistic forecasts of S_L for Hubbert model with production data up to 2008 of the world ($R=8$)

Normal Distribution Model with Production Data up to 2008

Table 25 lists the data intervals and number of data points that define the discretized parameter space for different variance ratios.

Table 25 - Different variance ratio for normal distribution model with production data up to 2008 of the world

Variance Ratio	R = 0.2		R=2		R=4	
	Data Interval	points	Data Interval	points	Data Interval	points
Q_{∞} (Bbbl)	1001~2999	1000	901~5896	1000	900~10890	1000
t_M (yr)	1981~2030.5	100	1971~2070	100	1971~2119.5	100
S_N (1/yr)	21~40.8	100	11~60.5	100	1~100	100

Fig. 48, **Fig. 50**, and **Fig. 52** are the plots of normalized PDF for each parameter Q_{∞} , t_M , and S_L . **Fig. 49**, **Fig. 51**, and **Fig. 53** are the plots of CDF for these three parameters, respectively. I obtained the P90, P50, P10, and mean value from the CDF of each parameter, which are shown in **Table 26**. Parameter sets found with maximum likelihood function value by taking the probabilistic value of each parameter are listed in **Table 27**, **Table 28**, and **Table 29**, and the sum-of-square norm (S_oS) values are also calculated. I choose one scenario ($R=2$) to plot the probabilistic forecasts of each parameter (**Fig. 54**, **Fig. 55**, and **Fig. 56**).

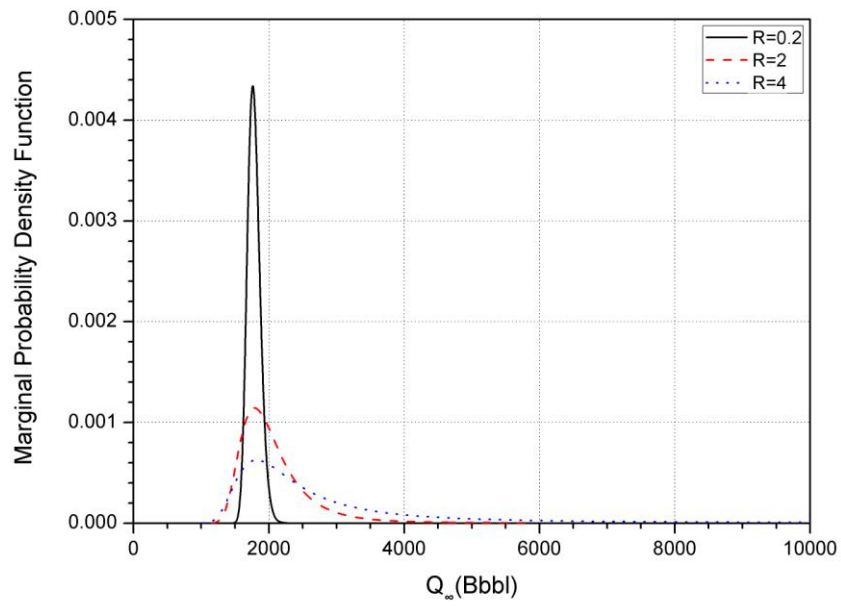


Fig. 48 - Marginal PDF of Q_{∞} for Normal distribution model with production data up to 2008 of the world

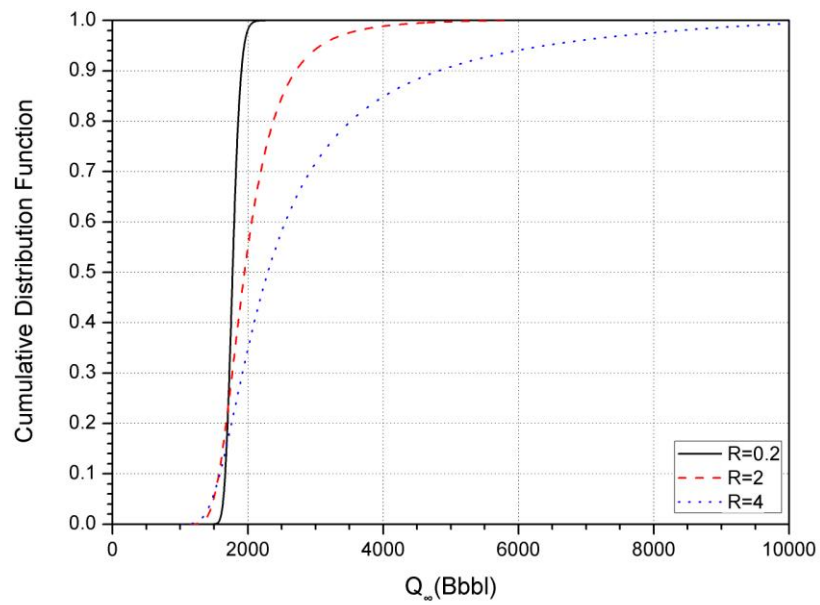


Fig. 49 - CDF of Q_{∞} for Normal distribution model with production data up to 2008 of the world

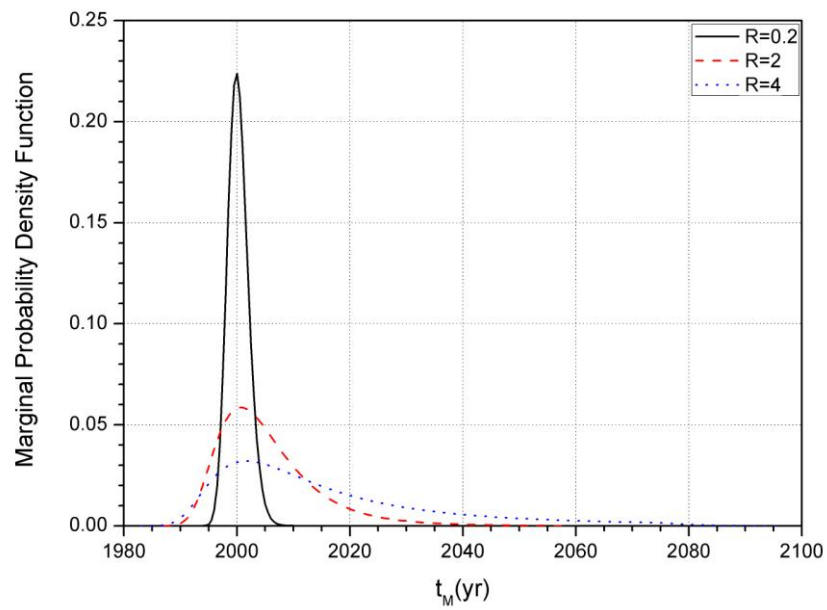


Fig. 50 - Marginal PDF of t_M for Normal distribution model with production data up to 2008 of the world

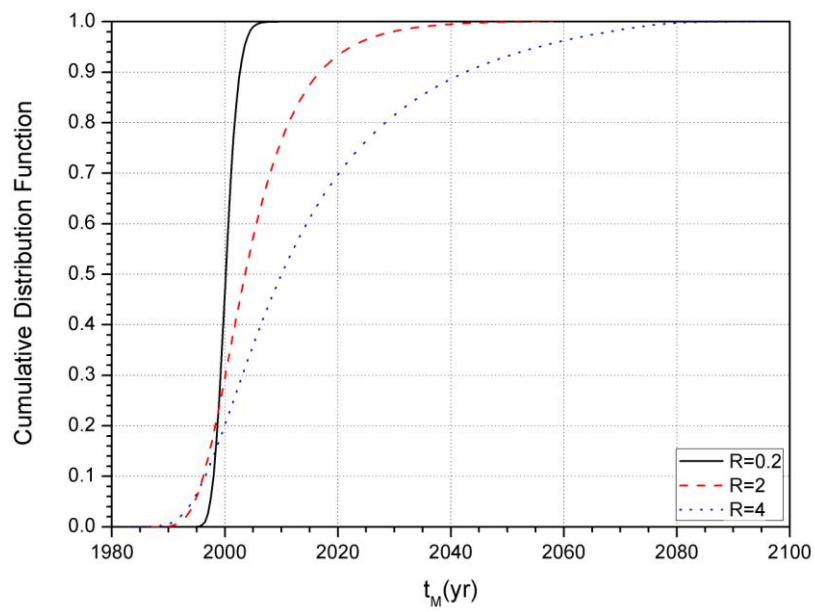


Fig. 51 - CDF of t_M for Normal distribution model with production data up to 2008 of the world

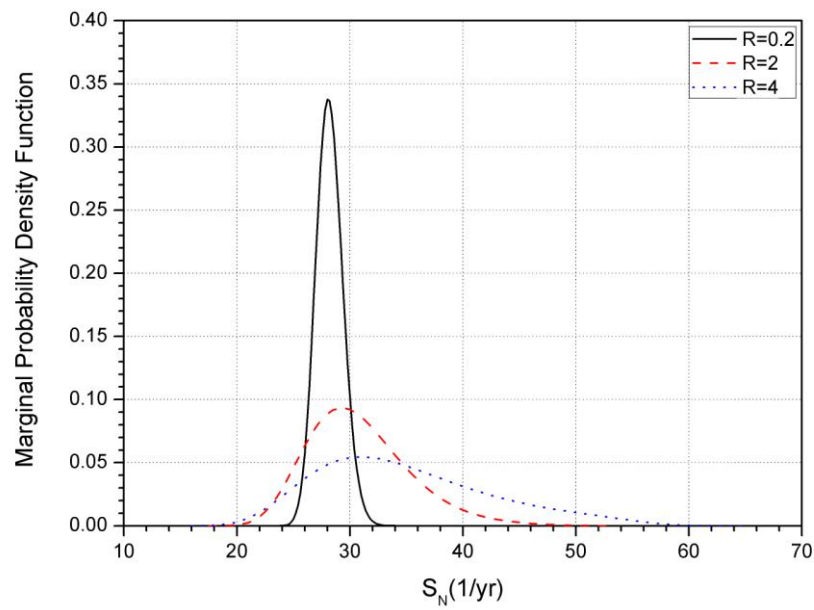


Fig. 52 - Marginal PDF of S_N for Normal distribution model with production data up to 2008 of the world

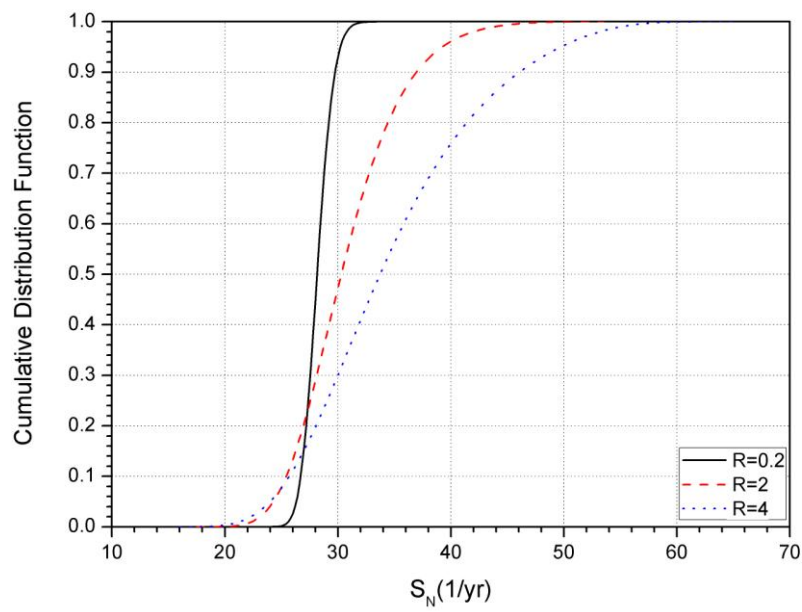


Fig. 53 - CDF of S_N for Normal distribution model with production data up to 2008 of the world

Table 26 - Probabilistic values of each parameter for Normal distribution model with production data up to 2008 of the world

Variance Ratio	R = 0.2			R=2			R=4		
	Q_{∞} (Bbbl)	t_M (yr)	S_N (1/yr)	Q_{∞} (Bbbl)	t_M (yr)	S_N (1/yr)	Q_{∞} (Bbbl)	t_M (yr)	S_N (1/yr)
P90	1664.96	1997.987	26.701	1570.6	1996.175	25.432	1585.2	1996.50	25.710
P50	1774.49	2000.131	28.155	1950.1	2003.574	30.299	2294.9	2010.01	33.730
P10	1904.51	2002.640	29.742	2707.5	2016.765	36.933	4817.3	2042.55	46.035
Mean	1780.76	2000.243	28.196	2073.7	2005.348	30.825	2838.6	2015.18	34.868
Best-fit	1765	2000	28.5	1766	2000	28.4	1750	1999.5	28

Table 27 - Parameter sets for Normal distribution model with production data up to 2008 of the world (R=0.2)

R=0.2	Q_{∞}				t_M				S_N			
	Q_{∞} (Bbbl)	t_M (yr)	S_N (1/yr)	S_oS	Q_{∞} (Bbbl)	t_M (yr)	S_N (1/yr)	S_oS	Q_{∞} (Bbbl)	t_M (yr)	S_N (1/yr)	S_oS
P90	1665	1998	26.8	76.009	1669	1998	26.8	76.007	1683	1998.5	26.8	76.044
P50	1775	2000	28.2	75.779	1765	2000	28	75.840	1773	2000	28.2	75.780
P10	1905	2002.5	29.6	75.563	1895	2002.5	29.6	75.569	1901	2002.5	29.8	75.501
Mean	1781	2000	28.2	75.776	1765	2000	28	75.840	1773	2000	28.2	75.780

Table 28 - Parameter sets for Normal distribution model with production data up to 2008 of the world (R=2)

R=2	Q_{∞}				t_M				S_N			
	Q_{∞} (Bbbl)	t_M (yr)	S_N (1/yr)	S_oS	Q_{∞} (Bbbl)	t_M (yr)	S_N (1/yr)	S_oS	Q_{∞} (Bbbl)	t_M (yr)	S_N (1/yr)	S_oS
P90	1571	1996	25.5	7.6178	1576	1996	25.5	7.6176	1601	1997	25.5	7.6237
P50	1951	2003	30	7.5465	1971	2004	30.5	7.5411	1971	2004	30.5	7.5411
P10	2706	2016	36.5	7.4289	2711	2017	36.5	7.4469	2661	2016	37.0	7.4089
Mean	2076	2006	31.5	7.5256	2026	2005	31	7.5333	1986	2004	31.0	7.5223

Table 29 - Parameter sets for Normal distribution model with production data up to 2008 of the world (R=4)

R=4	Q_{∞}				t_M				S_N			
	Q_{∞} (Bbbl)	t_M (yr)	S_N (1/yr)	S_oS	Q_{∞} (Bbbl)	t_M (yr)	S_N (1/yr)	S_oS	Q_{∞} (Bbbl)	t_M (yr)	S_N (1/yr)	S_oS
P90	1590	1996.5	26	3.8051	1600	1996.5	26	3.8049	1640	1998	26	3.8098
P50	2290	2010	33	3.7558	2280	2010.0	33	3.7562	2300	2010	34	3.7352
P10	4820	2041.5	45	3.6715	4860	2043.0	46	3.6589	4510	2040	46	3.6387
Mean	2840	2019	38	3.7010	2560	2014.5	36	3.7188	2390	2011.5	35	3.7221

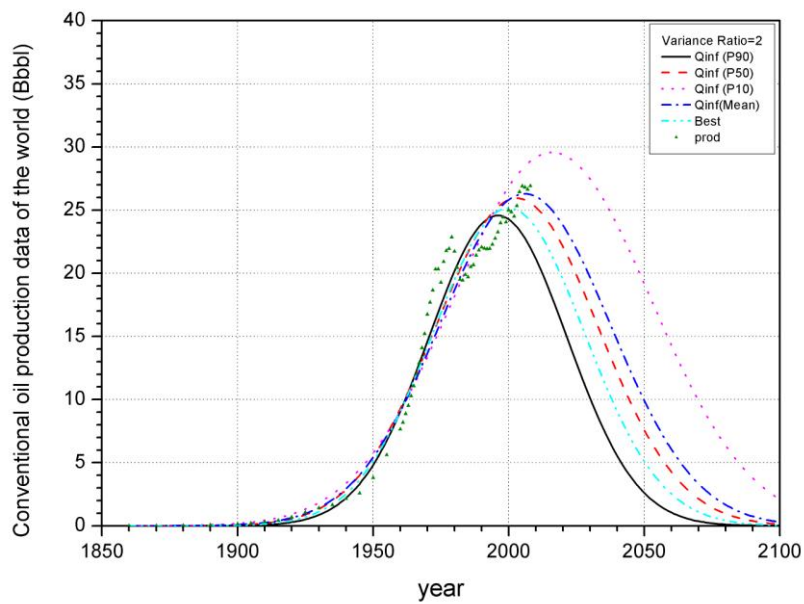


Fig. 54 - Probabilistic forecasts of Q_{∞} for Normal Distribution model with production data up to 2008 of the world ($R=2$)

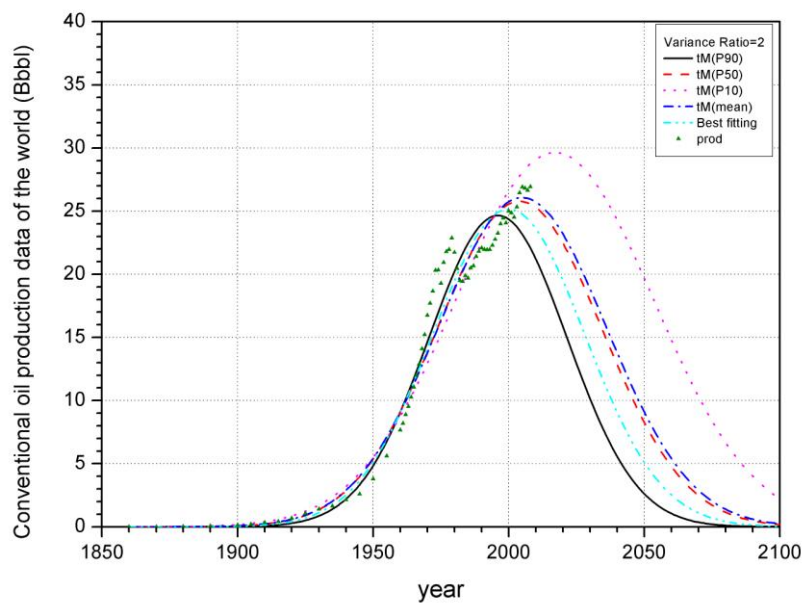


Fig. 55 - Probabilistic forecasts of t_M for Normal Distribution model with production data up to 2008 of the world ($R=2$)

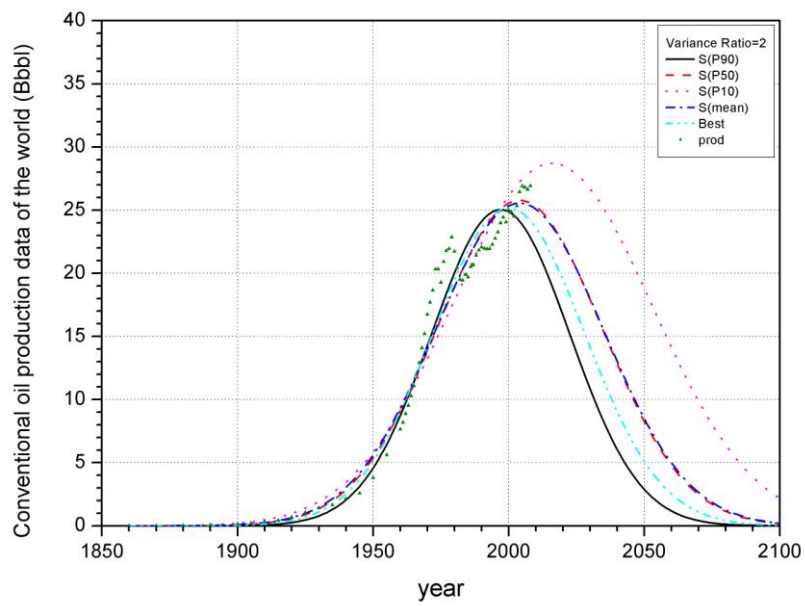


Fig. 56 - Probabilistic forecasts of S_N for Normal Distribution model with production data up to 2008 of the world ($R=2$)

Summary of Uncertainty Analysis for the World by Mathematical Modeling

1. By applying different variance ratio for mathematical modeling, it shows that uncertainty from mathematical modeling can be effectively quantified by estimating the probabilistic values - P90, P50, P10, and mean values. The larger variance ratio contributes much uncertainty for world oil production forecasts.
2. In the conventional oil production curve of the world (Fig. 38), there is no overall peak and period of declining production like for the production curve of the lower U.S. 48 (Fig. 1). Thus, in my analyses for both two models, there indeed existed significant differences in probabilistic forecasts for different variance ratios (Table 21, Table 26).
3. From **Table 21** and **Table 26**, it can be found that the range from P90 to P10 value of Q_{∞} and t_M ($R=8$) in the Hubbert model is nearly equal to the corresponding value of the scenario ($R=2$) in Normal distribution model, and it means that there existed much uncertainty for applying normal distribution model to forecast the world conventional oil production than Hubbert's model.
4. From the uncertainty analyses in the pre-peak section (production data up to 1956) of the lower U.S. 48, it is indicated that the variation of probabilistic forecasts is wider for global conventional oil production than the lower U.S. 48, because there is no peak and declining production curve for the world conventional oil production. It is implied there is lots of uncertainty in forecasting the world conventional oil production by using mathematical modeling.
5. I compare the sum-of-square norm (S_oS) of both two models, and it shows that the

Hubbert model is better for fitting the production data than the Normal distribution model.

6. Using the larger variance ratio for the Hubbert and Normal distribution models, the results from mathematical modeling comparable favorably with forecasts predicted by Group 2 and 3 in Bentley's (2008) paper. In particular, the uncertainty from mathematical modeling encompasses Group 2 and 3 estimates.

8. MULTIPLE EXPERTS ANALYSIS OF WORLD CONVENTIONAL OIL RESOURCES

Since Hubbert proposed the “peak oil” concept and attempted to predict the lower U.S 48 and world oil ultimate recoverable resources in 1956, there has been existed a great deal of publications related to forecasting global oil production. Bentley et al. (2008) reviewed the literatures relevant to the forecasts and methodologies of global oil production since 1956, and this paper classified the forecasts into three different groups: Group 1- quantitative analyses which predict that global oil production will come to a resource-limited peak in the near term and certainly before 2020; Group 2- forecasts that use quantitative methods, but which appear to have no production peak within the forecasted time period (2020 or 2030); Group 3- non-quantitative analyses that do not predict a resource-limited oil peak within the foreseeable future.

Before utilizing multiple experts analysis of world conventional oil resources, I need to find one distribution of each expert’s estimates. By Bentley’s (2008) paper, I regard the forecasts made by individuals, organizations and institutions as my experts. Unfortunately, these forecasts give only certain values for ultimate recoverable resources rather than a distribution. However, I can still combine these values to build a distribution for world conventional resources.

My objective is to analyze world conventional oil resources; hence, I take ultimate recoverable resources of world convention oil data from both Group 1 and Group 2 of Bentley’s (2008) paper – **Table 30**. Examining these results, I observed that

estimates of world conventional oil resources increase gradually with time (**Fig. 57**). Apparently, it means that there existed lots of uncertainty for these forecasts.

From the results of mathematical modeling in forecasting world conventional oil production, it can be found that large uncertainty ranges that encompass both groups of forecasts (imminent peak and no imminent peak). Because of this, I regard each forecast as the possible scenario, and classify those results into three groups such as in Bentley's paper (2008). Thus, if I apply multiple experts system to analyze world conventional oil resources, it is needed to encapsulate all possible forecasts (beliefs) made by the experts .

Group 3 did not predict a resource-limited oil peak within the predictable future (i.e., before 2030). Because there were no quantitative estimates from Group 3, I decided to use Hubbert mathematical modeling to generate a quantitative estimate that could be integrated with the estimates from Groups 1 and 2. My method was to assume, conservatively, year 2031 as the peak year for Group 3 estimates, and develop a probabilistic value for ultimate recoverable resources, Q_{∞} , by applying the mathematical modeling mentioned above.

Table 30 - World conventional oil ultimate recoverable resources from Group 1 & Group 2 of Bentley et al. (2008)

category	Date	Author	Ultimate (Bbbl)
Group 1	1956	Hubbert	1250
	1969	Hubbert	1350; 2100
	1972	ESSO	2100
	1972	Report: UN Conference	2500
	1974	SPRU, UK	1800-2480
	1977	Hubbert	2000
	1977	Ehrlich et al	1900
	1978	WEC/IFP	1803
	1981	WorldBank	1900
	1992	Meadows et al	1800-2500
	1995	Petroconsultants (xN)	1800
	1996	Ivanhoe	2000
	1997	Edwards	2836
	1998	IEA	2300
	1999	USGS	2000
	2000	Bartlett	2000 & 3000
	2002	BGR	2670
	2003	Energyfiles Ltd	2338
Group 2	2000	IEA (+N)	3345
	2001	US DoE/EIA	3303
	2002	Shell (+N)	3000
	2003	EU (WETO study)	4500

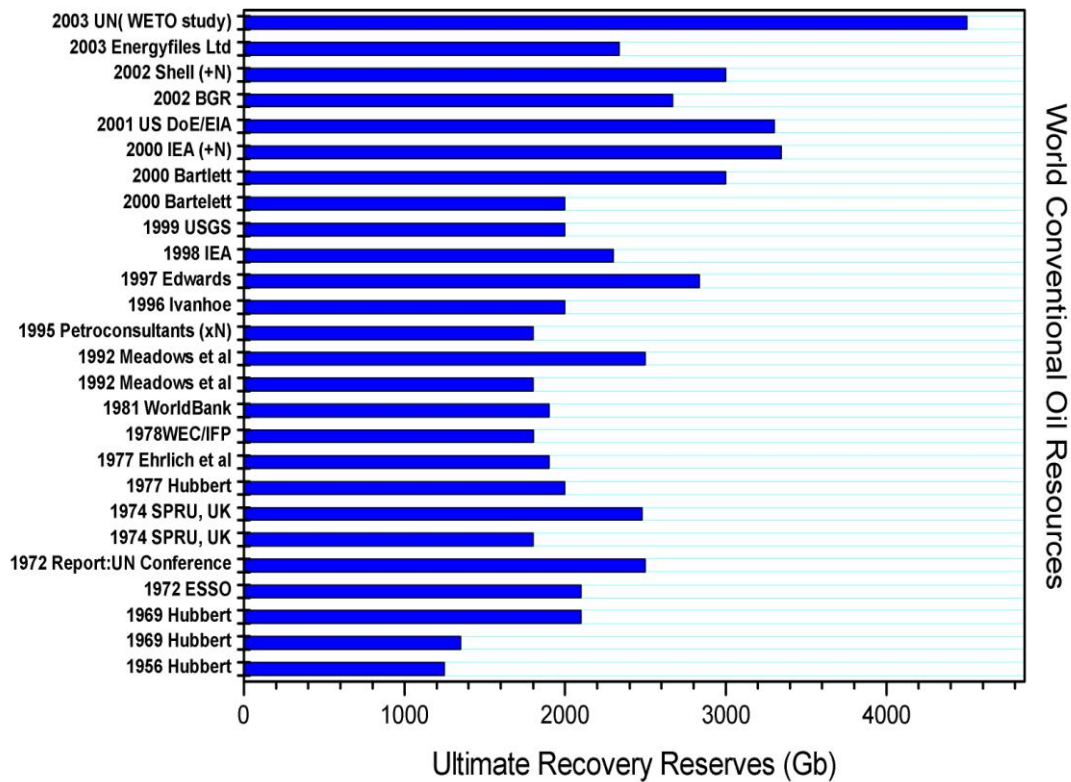


Fig. 57 - World conventional oil ultimate recoverable resources from Group 1 & Group 2 of Bentley et al. (2008)

Here, I use the variance ratio $R=8$ to do the analyses, and the probabilistic values of Q_{∞} and S_L at a peak year of 2031 are listed in **Table 31**. Parameter sets found with maximum likelihood function value by taking the probabilistic value of Q_{∞} and S_L are listed in **Table 32**. The plots of probabilistic forecasts of Q_{∞} and S_L are shown in **Fig. 58** and **Fig. 59**.

Table 31 - Probabilistic values of Q_{∞} and S_L at a peak year of 2031 for Hubbert model with overall production data of the world

Variance Ratio	$R = 8, t_M = 2031 \text{ yr}$					
	Data Interval	points	P90	P50	P10	Mean Best-Fit
Q_{∞} (Bbbl)	1001~6000	5000	3437.4	3929	4426.7	3930.9
S_L (1/yr)	5~54.9	500	19.967	21.867	23.615	21.821

Table 32 - Parameter sets at the peak time 2031 for Hubbert model with overall production data of the world ($R=8$)

$R=8$	Q_{∞}				S_L			
	Q_{∞} (Bbbl)	t_M (yr)	S_L (1/yr)	S_oS	Q_{∞} (Bbbl)	t_M (yr)	S_L (1/yr)	S_oS
P90	3437	2031	22.5	1.8381	4063	2031	20.0	1.8545
P50	3929	2031	21.9	1.8346	3941	2031	21.9	1.8346
P10	4427	2031	21.5	1.8309	3781	2031	23.6	1.8321
Mean	3931	2031	22	1.8339	3949	2031	21.8	1.8353

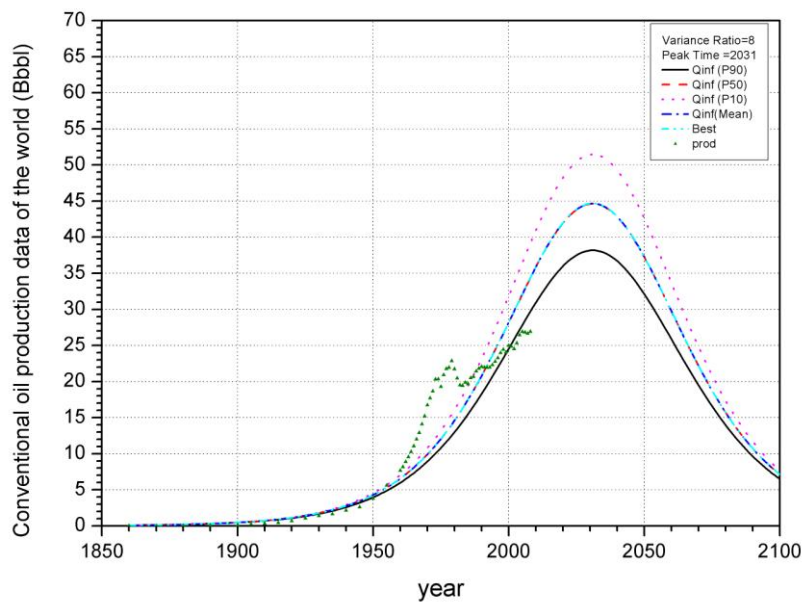


Fig. 58 - Probabilistic forecasts of Q_{∞} at a peak year of 2031 for Hubbert model with overall production data of the world ($R=8$)

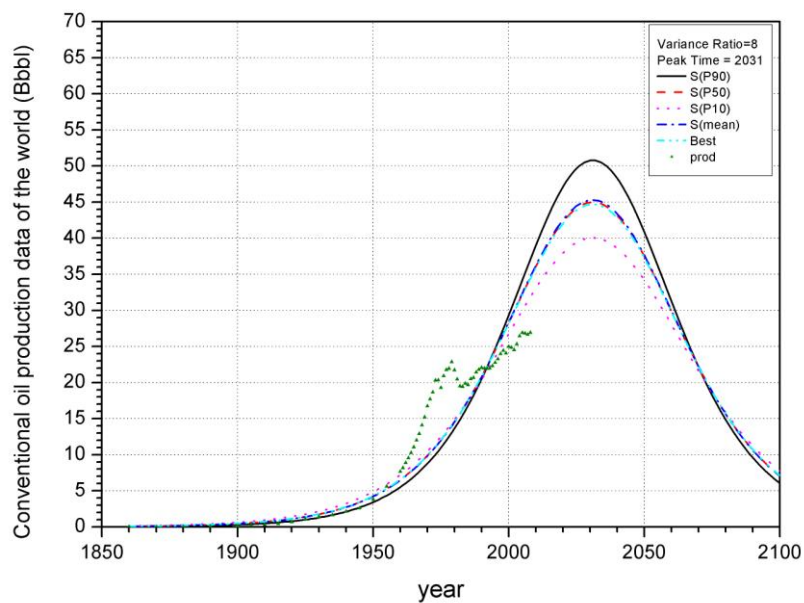


Fig. 59 - Probabilistic forecasts of S_L at a peak year of 2031 for Hubbert model with overall production data of the world ($R=8$)

Here, in order to make the linear pooling, I only take the P90, P50, P10, and mean values of ultimate recoverable resources Q_{∞} as the forecasts for Group 3. It means that I have four forecasts to represent Group 3 with single values (like the mean), and then pool with the single values from Groups 1 and 2 to obtain a probabilistic distribution.

Before applying the multiple experts method to get a probabilistic distribution for the world ultimate recoverable resources, it is needed to decide which mathematical technique is appropriate to combine the forecasts from these three groups. By comparing the results from mathematical modeling with the forecasts made by different groups in Bentley's paper (2008), I find that the forecasts from Groups 1 to 3 are all possible, and encompass the full range of opinions from all experts represented in the literature. Thus, the linear opinion pool is appropriate and each forecast from different groups should be given equal weighting.

Combining all forecasts from Group 1, Group 2, and Group 3, I get a histogram and cumulative distribution function for these all possible estimates (**Fig. 60**). Estimates in world conventional resources increase with the time (**Fig. 57**); thus, the question arises as to whether all forecasts should be integrated in the analysis or whether only more recent forecasts should be used. Next, I omit the forecasts before 1990 and generate another histogram and cumulative distribution function (**Fig. 61**). The probabilistic values of ultimate recoverable resources from both distributions generated by multiple experts system are listed in **Table 33**.

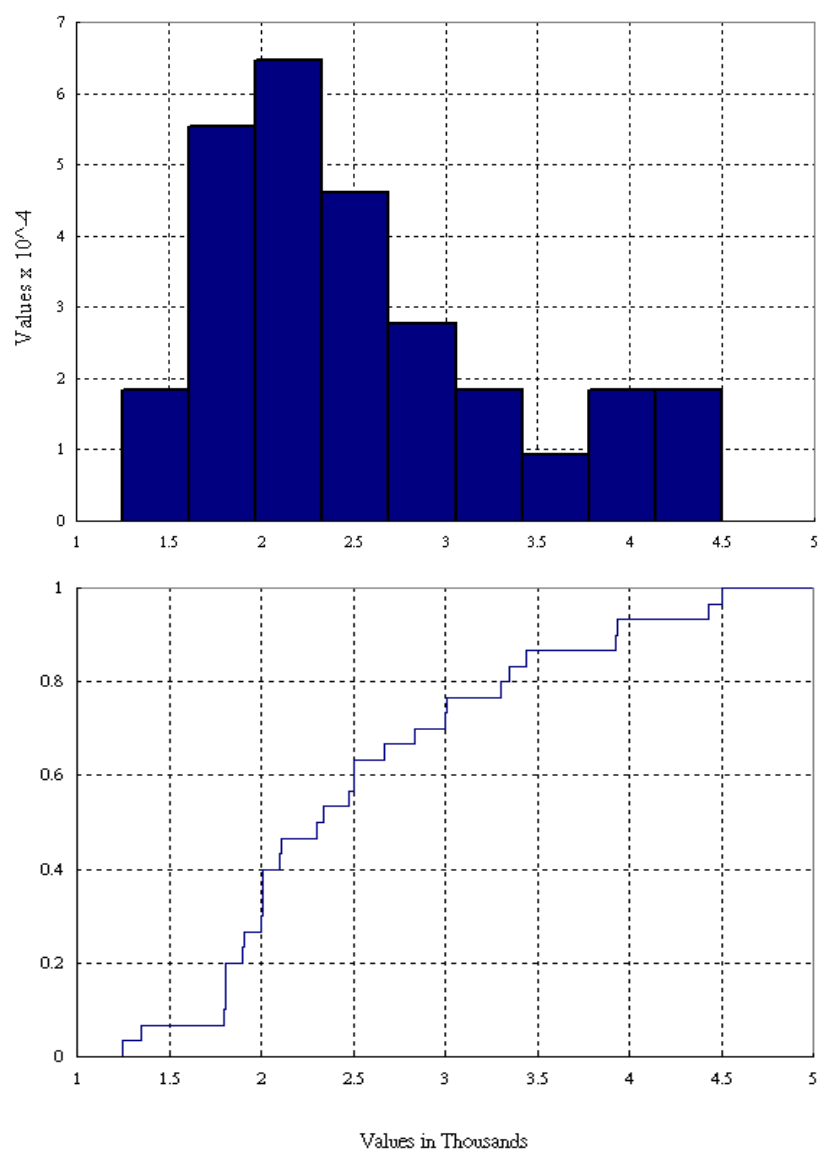


Fig. 60 - Histogram and CDF combining all forecasts of the three different groups

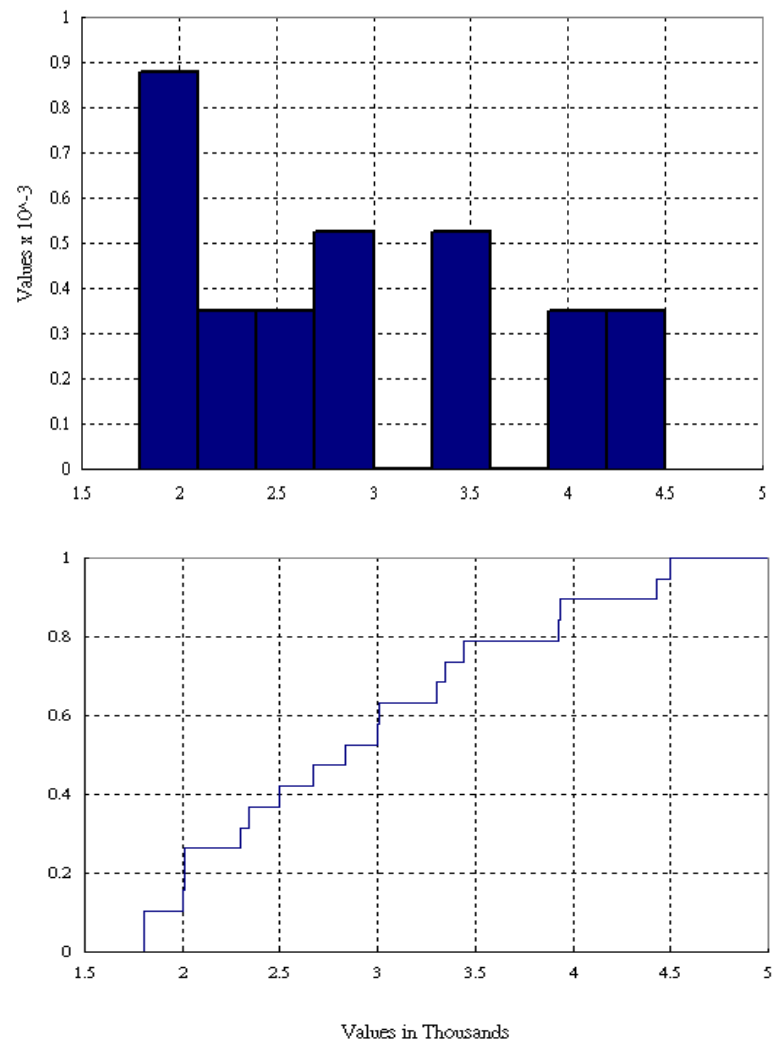


Fig. 61 - Histogram and CDF combining all forecasts after 1990 of the three different groups

Table 33 - Estimates of world conventional oil resources using multiple experts pooling

Q_{∞} (Bbbl)	All Forecasts	Forecasts after 1990
P90	1800.0	1800.0
P50	2319.0	2836.0
P10	3929.0	4427.0
Mean	2543.3	2900.8

From the mean values of ultimate recoverable resources in **Table 33**, we also can estimate the timing of peak oil for these two cases. Parameter sets found with maximum likelihood function value by taking the probabilistic values of t_M and S_L for both cases are listed in **Table 34** and **Table 35**. The plots of probabilistic forecasts of Q_∞ and S_L are shown in **Fig. 62** and **Fig. 63**.

Table 34 - Parameter sets at the $Q_\infty=2543.3$ Bbbl for Hubbert model with overall production data of the world ($R=8$)

R=8	t_M				S_L			
	Q_∞ (Bbbl)	t_M (yr)	S_L (1/yr)	S_oS	Q_∞ (Bbbl)	t_M (yr)	S_L (1/yr)	S_oS
P90	2543.3	2009.1	19.5	1.8174	2543.3	2012.5	18.1	1.8516
P50	2543.3	2013.5	19.9	1.8294	2543.3	2013.1	19.9	1.8280
P10	2543.3	2018.3	20.5	1.8387	2543.3	2014.9	21.7	1.8185
Mean	2543.3	2013.6	19.9	1.8298	2543.3	2013.1	19.9	1.8280

Table 35 - Parameter sets at the $Q_\infty=2900.8$ Bbbl for Hubbert model with overall production data of the world ($R=8$)

R=8	t_M				S_L			
	Q_∞ (Bbbl)	t_M (yr)	S_L (1/yr)	S_oS	Q_∞ (Bbbl)	t_M (yr)	S_L (1/yr)	S_oS
P90	2900.8	2014.6	20.2	1.8199	2900.8	2017.7	18.8	1.8505
P50	2900.8	2018.8	20.6	1.8303	2900.8	2018.4	20.6	1.8289
P10	2900.8	2023.4	21.4	1.8365	2900.8	2020.2	22.4	1.8227
Mean	2900.8	2018.9	20.6	1.8306	2900.8	2018.4	20.6	1.8289

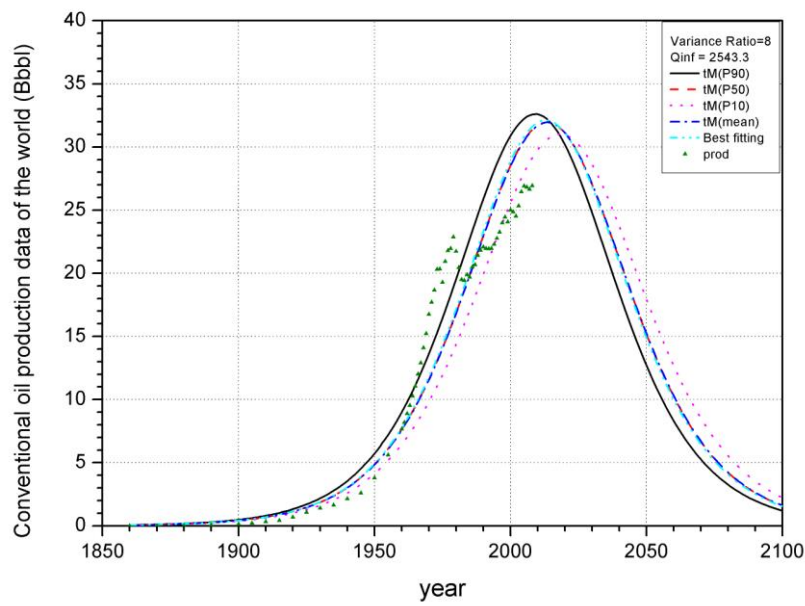


Fig. 62 - Probabilistic forecasts of t_M at the $Q_{\infty}=2543.3$ Bbbl for Hubbert model with overall production data of the world ($R=8$)

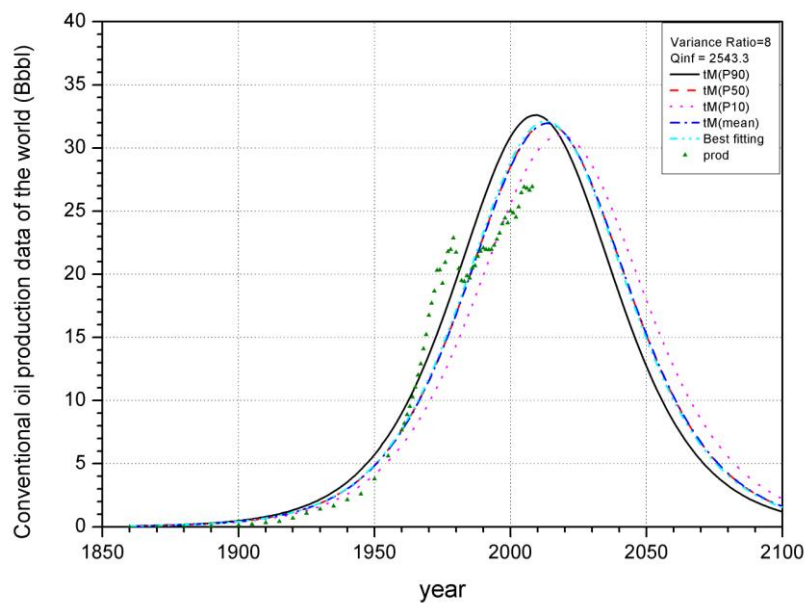


Fig. 63 - Probabilistic forecasts of t_M at the $Q_{\infty}=2900.8$ Bbbl for Hubbert model with overall production data of the world ($R=8$)

It is worth noting that I used conservative probabilistic forecasts for Group 3 for the multiple experts analysis. Group 3 authors did not quantify oil resources because they depend on economic and societal theory to explain that resources will grow from the improvement of technology, knowledge and economic incentives. Thus, the results obtained by the multiple experts analysis yields a conservative estimate for world conventional oil resources. Furthermore, because of the conservative estimation for Group 3, there is additional upside potential that is not represented in the composite distributions shown in **Fig. 60, Fig. 61, and Table 33**. Thus, these distributions also underestimate the uncertainty in ultimate world conventional oil resources. From **Fig. 57**, it indicated that the estimates for world conventional oil have significantly increased since 1990. Based on this reason, and because it results in larger uncertainty, it is better to represent the uncertainty in world conventional oil resources by taking the probabilistic distribution made with the forecasts after 1990.

9. CONCLUSIONS AND RECOMMENDATIONS

Conclusions

There have been countless debates over world remaining oil resources and the timing of peak oil. The very fact that there is wide diversity of opinion and heated debate indicates that there is a lot of uncertainty, which my studies confirm. I draw the following conclusions from the analyses described above:

1. The uncertainty in world conventional oil resources is best expressed by the results from the multiple experts analysis, taking the probabilistic distribution made with the forecasts after 1990. The results show that the mean value of ultimate recoverable resources is 2900.8 Bbbl. The P90 value is 1800 Bbbl, the P50 value is 2836 Bbbl, and the P10 value is 4427 Bbbl. The mean and standard deviation are both underestimated; i.e., there is actually more uncertainty than I have quantified, due to conservatively assuming the peak year of 2031 for the forecasts of Group3.
2. The results of uncertainty analyses from mathematical modeling show that we potentially have large uncertainty in conventional oil estimates of the world and the lower U.S. 48 when we only take data regression or model fitting as the forecasting tools. Furthermore, due to the scarcity of production data or indistinct declining production curve, it also makes us recognize much uncertainty within the forecasts by applying the mathematical models to predict.
3. Giving due consideration to uncertainty, Hubbert-type mathematical modeling results in large uncertainty ranges that encompass both groups of forecasts (imminent peak

and no imminent peak). In short, the industry does not have enough information at this time to say with any reliability what the ultimate world conventional oil production will be. It could peak soon, somewhere in the distant future, or somewhere in between. It would be wise to consider all of these possible outcomes in planning and making decisions regarding capital investment and formulation of energy policy.

4. Combining of expert assessments from the literatures, I can effectively build one distribution to encapsulate all beliefs regarding agreement or disagreement with “peak oil”. However, I attempt to conservatively estimate world conventional oil resources for the group that did not predict the peak oil. The resulting forecast distribution covers large uncertainty ranges similar to the results from the analyses of mathematical modeling.

Recommendation for Future Work

I expect that both methodologies can be applied to evaluate the uncertainty for conventional gas and unconventional hydrocarbon resources produced from tight gas reservoirs, coalbed methane, oil sands, gas shales, and gas hydrates.

NOMENCLATURE

a	The constant describing the shape of production curve (1/yr)
b :	The constant describing the shape of production curve (1/yr)
C_D	Covariance matrix
d^{obs}	The vector of observed production data of every year
n_d	The number of measured (observed) data points
N_0	Dimensionless cumulative production factor
$g(x)$	Forward model
q	Production rate (Bbbl/yr)
q_M	The maximum peak production rate (Bbbl/yr)
R	The variance ratio of production data
R_{MS}	Root mean square
S_N	The width parameter for normal distribution (yr)
S_L	The width parameter for Hubbert model (yr)
S_oS	The sum of square norms
t	The calendar time (yr)
t_M	The year at maximum production rate (yr)
t_0	Arbitrary time or reference point (yr)
Q	Cumulative production (Bbbl)
Q_0	Cumulative production at an arbitrary time or reference point (Bbbl)
Q_∞	Ultimate recovery resource (Bbbl)

Q_M	Maximum cumulative production (Bbbl)
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Subscript

M	Maximum
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N	Normal
---	--------

L	Logistic
---	----------

∞	Ultimate or infinity
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Superscript

cal	Calculated
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obs	Observed
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